2

MOTION ALONG A STRAIGHT LINE

2.1. **IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

SET UP: We know the average velocity is 6.25 m/s.

EXECUTE: $\Delta x = v_{\text{av-}x}\Delta t = 25.0\text{ m}$

EVALUATE: In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

2.2. **IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

SET UP: $13.5\text{ days} = 1.166\times 10^6\text{ s}$. At the release point, $x = +5.150\times 10^6\text{ m}$.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{5.150\times 10^6\text{ m}}{1.166\times 10^6\text{ s}} = -4.42\text{ m/s}$

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq. (2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities:

$$\Delta x = v_{\text{av-}x}\Delta t = (105\text{ km/h})(1\text{ h}/60\text{ min})(140\text{ min}) = 245\text{ km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate Δt; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{245\text{ km}}{70\text{ km/h}} = 3.50\text{ h} = 3\text{ h and 30 min}.$$

The trip takes an additional 1 hour and 10 minutes.

EVALUATE: The time is inversely proportional to the average speed, so the time in traffic is $(105/70)(140\text{ min}) = 210\text{ min}$.

2.4. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200\text{ m} + 280\text{ m} = 480\text{ m}$.

EXECUTE: **(a)** The eastward run takes time $\dfrac{200\text{ m}}{5.0\text{ m/s}} = 40.0\text{ s}$ and the westward run takes $\dfrac{280\text{ m}}{4.0\text{ m/s}} = 70.0\text{ s}$. The average speed for the entire trip is $\dfrac{480\text{ m}}{110.0\text{ s}} = 4.4\text{ m/s}$.

(b) $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80\text{ m}}{110.0\text{ s}} = -0.73\text{ m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

2.5. **IDENTIFY:** Given two displacements, we want the average velocity and the average speed.

SET UP: The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ and the average speed is just the total distance walked divided by the total time to walk this distance.

EXECUTE: **(a)** Let +*x* be east. $\Delta x = 60.0\text{ m} - 40.0\text{ m} = 20.0\text{ m}$ and $\Delta t = 28.0\text{ s} + 36.0\text{ s} = 64.0\text{ s}$. So $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{20.0\text{ m}}{64.0\text{ s}} = 0.312\text{ m/s}.$

(b) $\text{average speed} = \dfrac{60.0\text{ m} + 40.0\text{ m}}{64.0\text{ s}} = 1.56\text{ m/s}$

EVALUATE: The average speed is much greater than the average velocity because the total distance walked is much greater than the magnitude of the displacement vector.

2.6. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find *x* for each *t*.

SET UP: $x(0) = 0$, $x(2.00\text{ s}) = 5.60\text{ m}$, and $x(4.00\text{ s}) = 20.8\text{ m}$

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{5.60\text{ m} - 0}{2.00\text{ s}} = +2.80\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 0}{4.00\text{ s}} = +5.20\text{ m/s}$

(c) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 5.60\text{ m}}{2.00\text{ s}} = +7.60\text{ m/s}$

EVALUATE: The average velocity depends on the time interval being considered.

2.7. **(a) IDENTIFY:** Calculate the average velocity using Eq. (2.2).

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0\text{ s}$: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}.$

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}.$

(b) IDENTIFY: Use Eq. (2.3) to calculate $v_x(t)$ and evaluate this expression at each specified *t*.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2.$

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}.$

(iii) $t = 10.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}.$

(c) IDENTIFY: Find the value of *t* when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0.$

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40\text{ m/s}^2)}{3(0.120\text{ m/s}^3)} = 13.3\text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.8. **IDENTIFY:** We know the position $x(t)$ of the bird as a function of time and want to find its instantaneous velocity at a particular time.

SET UP: The instantaneous velocity is $v_x(t)=\dfrac{dx}{dt}=\dfrac{d(28.0\text{ m}+(12.4\text{ m/s})t-(0.0450\text{ m/s}^3)t^3)}{dt}$.

EXECUTE: $v_x(t)=\dfrac{dx}{dt}=12.4\text{ m/s}-(0.135\text{ m/s}^3)t^2$. Evaluating this at $t=8.0\text{ s}$ gives $v_x=3.76\text{ m/s}$.

EVALUATE: The acceleration is not constant in this case.

2.9. **IDENTIFY:** The average velocity is given by $v_{\text{av-}x}=\dfrac{\Delta x}{\Delta t}$. We can find the displacement Δt for each constant velocity time interval. The average speed is the distance traveled divided by the time.

SET UP: For $t=0$ to $t=2.0\text{ s}$, $v_x=2.0\text{ m/s}$. For $t=2.0\text{ s}$ to $t=3.0\text{ s}$, $v_x=3.0\text{ m/s}$. In part (b), $v_x=2\ 3.0\text{ m/s}$ for $t=2.0\text{ s}$ to $t=3.0\text{ s}$. When the velocity is constant, $\Delta x=v_x\Delta t$.

EXECUTE: **(a)** For $t=0$ to $t=2.0\text{ s}$, $\Delta x=(2.0\text{ m/s})(2.0\text{ s})=4.0\text{ m}$. For $t=2.0\text{ s}$ to $t=3.0\text{ s}$, $\Delta x=(3.0\text{ m/s})(1.0\text{ s})=3.0\text{ m}$. For the first 3.0 s, $\Delta x=4.0\text{ m}+3.0\text{ m}=7.0\text{ m}$. The distance traveled is also 7.0 m. The average velocity is $v_{\text{av-}x}=\dfrac{\Delta x}{\Delta t}=\dfrac{7.0\text{ m}}{3.0\text{ s}}=2.33\text{ m/s}$. The average speed is also 2.33 m/s.

(b) For $t=2.0\text{ s to }3.0\text{ s}$, $\Delta x=(-3.0\text{ m/s})(1.0\text{ s})=-3.0\text{ m}$. For the first 3.0 s, $\Delta x=4.0\text{ m}+(-3.0\text{ m})=+1.0\text{ m}$. The dog runs 4.0 m in the +*x*-direction and then 3.0 m in the –*x*-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x}=\dfrac{\Delta x}{\Delta t}=\dfrac{1.0\text{ m}}{3.0\text{ s}}=0.33\text{ m/s}$. The average speed is $\dfrac{7.00\text{ m}}{3.00\text{ s}}=2.33\text{ m/s}$.

EVALUATE: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.10. **IDENTIFY** and **SET UP:** The instantaneous velocity is the slope of the tangent to the *x* versus *t* graph.

EXECUTE: **(a)** The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

EVALUATE: The sign of the velocity indicates its direction.

2.11. **IDENTIFY:** Find the instantaneous velocity of a car using a graph of its position as a function of time.

SET UP: The instantaneous velocity at any point is the slope of the *x* versus *t* graph at that point. Estimate the slope from the graph.

EXECUTE: *A*: $v_x=6.7\text{ m/s}$; *B*: $v_x=6.7\text{ m/s}$; *C*: $v_x=0$; *D*: $v_x=-40.0\text{ m/s}$; *E*: $v_x=-40.0\text{ m/s}$; *F*: $v_x=-40.0\text{ m/s}$; *G*: $v_x=0$.

EVALUATE: The sign of v_x shows the direction the car is moving. v_x is constant when *x* versus *t* is a straight line.

2.12. **IDENTIFY:** $a_{\text{av-}x}=\dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus *t* graph.

SET UP: $60\text{ km/h}=16.7\text{ m/s}$

EXECUTE: **(a)** (i) $a_{\text{av-}x}=\dfrac{16.7\text{ m/s}-0}{10\text{ s}}=1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x}=\dfrac{0-16.7\text{ m/s}}{10\text{ s}}=-1.7\text{ m/s}^2$.

(iii) $\Delta v_x=0$ and $a_{\text{av-}x}=0$. (iv) $\Delta v_x=0$ and $a_{\text{av-}x}=0$.

(b) At $t=20\text{ s}$, v_x is constant and $a_x=0$. At $t=35\text{ s}$, the graph of v_x versus *t* is a straight line and $a_x=a_{\text{av-}x}=-1.7\text{ m/s}^2$.

EVALUATE: When $a_{\text{av-}x}$ and v_x have the same sign the speed is increasing. When they have opposite sign the speed is decreasing.

2.13. **IDENTIFY:** The average acceleration for a time interval Δt is given by $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$.

SET UP: Assume the car is moving in the $+x$ direction. $1 \text{ mi/h} = 0.447 \text{ m/s}$, so $60 \text{ mi/h} = 26.82 \text{ m/s}$, $200 \text{ mi/h} = 89.40 \text{ m/s}$ and $253 \text{ mi/h} = 113.1 \text{ m/s}$.

EXECUTE: **(a)** The graph of v_x versus t is sketched in Figure 2.13. The graph is not a straight line, so the acceleration is not constant.

(b) (i) $a_{\text{av-}x} = \dfrac{26.82 \text{ m/s} - 0}{2.1 \text{ s}} = 12.8 \text{ m/s}^2$ (ii) $a_{\text{av-}x} = \dfrac{89.40 \text{ m/s} - 26.82 \text{ m/s}}{20.0 \text{ s} - 2.1 \text{ s}} = 3.50 \text{ m/s}^2$

(iii) $a_{\text{av-}x} = \dfrac{113.1 \text{ m/s} - 89.40 \text{ m/s}}{53 \text{ s} - 20.0 \text{ s}} = 0.718 \text{ m/s}^2$. The slope of the graph of v_x versus t decreases as t increases. This is consistent with an average acceleration that decreases in magnitude during each successive time interval.

EVALUATE: The average acceleration depends on the chosen time interval. For the interval between 0 and 53 s, $a_{\text{av-}x} = \dfrac{113.1 \text{ m/s} - 0}{53 \text{ s}} = 2.13 \text{ m/s}^2$.

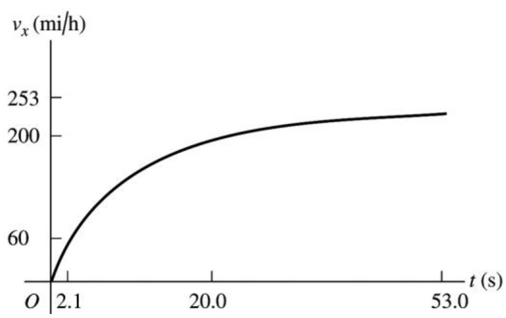


Figure 2.13

2.14. **IDENTIFY:** We know the velocity $v(t)$ of the car as a function of time and want to find its acceleration at the instant that its velocity is 16.0 m/s.

SET UP: $a_x(t) = \dfrac{dv_x}{dt} = \dfrac{d((0.860 \text{ m/s}^3)t^2)}{dt}$.

EXECUTE: $a_x(t) = \dfrac{dv_x}{dt} = (1.72 \text{ m/s}^3)t$. When $v_x = 16.0 \text{ m/s}$, $t = 4.313 \text{ s}$. At this time, $a_x = 7.42 \text{ m/s}^2$.

EVALUATE: The acceleration of this car is not constant.

2.15. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00 \text{ cm/s} - (0.125 \text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125 \text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0 \text{ cm}$, $v_x = 2.00 \text{ cm/s}$, $a_x = 2\ 0.125 \text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t: $t = 16.0 \text{ s}$.

(c) Set $x = 50.0 \text{ cm}$ and solve for t. This gives $t = 0$ and $t = 32.0 \text{ s}$. The turtle returns to the starting point after 32.0 s.

(d) The turtle is 10.0 cm from starting point when $x = 60.0 \text{ cm}$ or $x = 40.0 \text{ cm}$.
Set $x = 60.0 \text{ cm}$ and solve for t: $t = 6.20 \text{ s}$ and $t = 25.8 \text{ s}$.
At $t = 6.20 \text{ s}$, $v_x = +1.23 \text{ cm/s}$.
At $t = 25.8 \text{ s}$, $v_x = -1.23 \text{ cm/s}$.

Set $x = 40.0$ cm and solve for t: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4$ s, $v_x = 2$ 2.55 cm/s.

(e) The graphs are sketched in Figure 2.15.

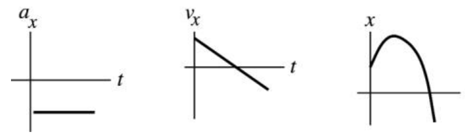


Figure 2.15

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.16. **IDENTIFY:** Use Eq. (2.4), with $\Delta t = 10$ s in all cases.

SET UP: v_x is negative if the motion is to the left.

EXECUTE: **(a)** $((5.0 \text{ m/s}) - (15.0 \text{ m/s}))/(10 \text{ s}) = -1.0 \text{ m/s}^2$

(b) $((-15.0 \text{ m/s}) - (-5.0 \text{ m/s}))/(10 \text{ s}) = -1.0 \text{ m/s}^2$

(c) $((-15.0 \text{ m/s}) - (+15.0 \text{ m/s}))/(10 \text{ s}) = -3.0 \text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each t. The instantaneous acceleration is $a_x = \dfrac{dv_x}{dt}$.

SET UP: $v_x(0) = 3.00$ m/s and $v_x(5.00 \text{ s}) = 5.50$ m/s.

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{5.50 \text{ m/s} - 3.00 \text{ m/s}}{5.00 \text{ s}} = 0.500 \text{ m/s}^2$

(b) $a_x = \dfrac{dv_x}{dt} = (0.100 \text{ m/s}^3)(2t) = (0.200 \text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00$ s, $a_x = 1.00 \text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.17.

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases as t increases. The average acceleration for $t = 0$ to $t = 5.00$ s equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50$ s, since $a_x(t)$ is a linear function of t.

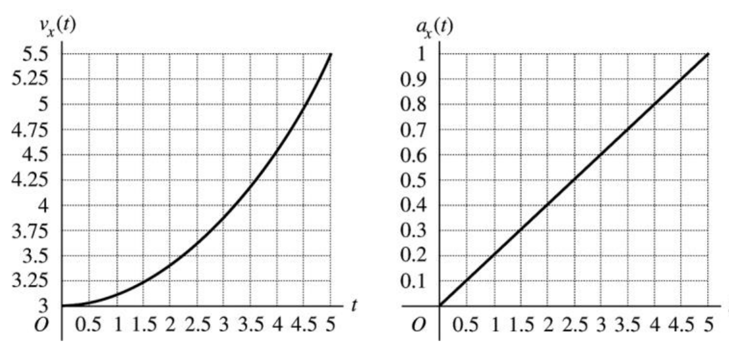


Figure 2.17

2.18. **IDENTIFY:** $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: **(a)** $v_x(t) = (9.60\ \text{m/s}^2)t - (0.600\ \text{m/s}^6)t^5$ and $a_x(t) = 9.60\ \text{m/s}^2 - (3.00\ \text{m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00$ s. At $t = 0$, $x = 2.17$ m and $a_x = 9.60\ \text{m/s}^2$. At $t = 2.00$ s, $x = 15.0$ m and $a_x = -38.4\ \text{m/s}^2$.

(b) The graphs are given in Figure 2.18.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00$ s, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

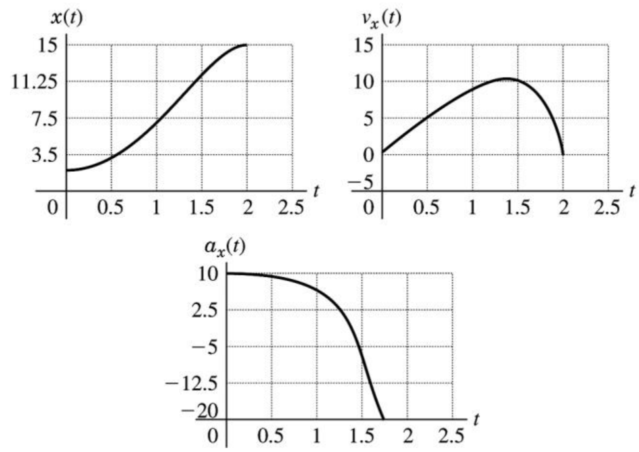
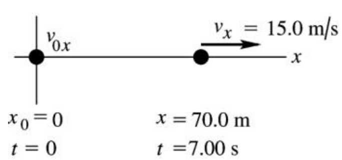


Figure 2.18

2.19. **IDENTIFY:** Use the constant acceleration equations to find v_{0x} and a_x.

(a) SET UP: The situation is sketched in Figure 2.19.



$x - x_0 = 70.0$ m
$t = 7.00$ s
$v_x = 15.0$ m/s
$v_{0x} = ?$

Figure 2.19

EXECUTE: Use $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \dfrac{2(x - x_0)}{t} - v_x = \dfrac{2(70.0\ \text{m})}{7.00\ \text{s}} - 15.0\ \text{m/s} = 5.0\ \text{m/s}$.

(b) Use $v_x = v_{0x} + a_x t$, so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{15.0\ \text{m/s} - 5.0\ \text{m/s}}{7.00\ \text{s}} = 1.43\ \text{m/s}^2$.

EVALUATE: The average velocity is $(70.0\ \text{m})/(7.00\ \text{s}) = 10.0$ m/s. The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0$.

2.20. **IDENTIFY:** In (a) find the time to reach the speed of sound with an acceleration of $5g$, and in (b) find his speed at the end of 5.0 s if he has an acceleration of $5g$.

SET UP: Let $+x$ be in his direction of motion and assume constant acceleration of $5g$ so the standard kinematics equations apply so $v_x = v_{0x} + a_x t$. **(a)** $v_x = 3(331\ \text{m/s}) = 993$ m/s, $v_{0x} = 0$, and $a_x = 5g = 49.0\ \text{m/s}^2$. **(b)** $t = 5.0$ s

EXECUTE: **(a)** $v_x = v_{0x} + a_x t$ and $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{993\text{ m/s} - 0}{49.0\text{ m/s}^2} = 20.3\text{ s}.$ Yes, the time required is larger than 5.0 s.

(b) $v_x = v_{0x} + a_x t = 0 + (49.0\text{ m/s}^2)(5.0\text{ s}) = 245\text{ m/s}.$

EVALUATE: In 5 s he can only reach about 2/3 the speed of sound without blacking out.

2.21. **IDENTIFY:** For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball starts from rest and moves in the $+x$-direction.

EXECUTE: **(a)** $x - x_0 = 1.50\text{ m},\ v_x = 45.0\text{ m/s}$ and $v_{0x} = 0.$ $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})} = 675\text{ m/s}^2.$$

(b) $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives $t = \dfrac{2(x - x_0)}{v_{0x} + v_x} = \dfrac{2(1.50\text{ m})}{45.0\text{ m/s}} = 0.0667\text{ s}$

EVALUATE: We could also use $v_x = v_{0x} + a_x t$ to find $t = \dfrac{v_x}{a_x} = \dfrac{45.0\text{ m/s}}{675\text{ m/s}^2} = 0.0667\text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

2.22. **IDENTIFY:** For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball moves in the $+x$ direction.

EXECUTE: **(a)** $v_x = 73.14\text{ m/s},\ v_{0x} = 0$ and $t = 30.0\text{ ms}.$ $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14\text{ m/s} - 0}{30.0 \times 10^{2\ 3}\text{ s}} = 2440\text{ m/s}^2.$$

(b) $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 73.14\text{ m/s}}{2}\right)(30.0 \times 10^{2\ 3}\text{ s}) = 1.10\text{ m}$

EVALUATE: We could also use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2}(2440\text{ m/s}^2)(30.0 \times 10^{2\ 3}\text{ s})^2 = 1.10\text{ m},$ which agrees with our previous result. The acceleration of the ball is very large.

2.23. **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = 2\ 250\text{ m/s}^2.$ $105\text{ km/h} = 29.17\text{ m/s}.$

EXECUTE: $v_{0x} = 1\ 29.17\text{ m/s}.$ $v_x = 0.$ $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (29.17\text{ m/s})^2}{2(-250\text{ m/s}^2)} = 1.70\text{ m}.$$

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.24. **IDENTIFY:** In (a) we want the time to reach Mach 4 with an acceleration of 4*g*, and in (b) we want to know how far he can travel if he maintains this acceleration during this time.

SET UP: Let $+x$ be the direction the jet travels and take $x_0 = 0.$ With constant acceleration, the equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply. $a_x = 4g = 39.2\text{ m/s}^2,$ $v_x = 4(331\text{ m/s}) = 1324\text{ m/s},$ and $v_{0x} = 0.$

EXECUTE: **(a)** Solving $v_x = v_{0x} + a_x t$ for t gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{1324\text{ m/s} - 0}{39.2\text{ m/s}^2} = 33.8\text{ s}.$

(b) $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(39.2\text{ m/s}^2)(33.8\text{ s})^2 = 2.24 \times 10^4\text{ m} = 22.4\text{ km}.$

EVALUATE: The answer in (a) is about ½ min, so if he wanted to reach Mach 4 any sooner than that, he would be in danger of blacking out.

2.25. **IDENTIFY:** If a person comes to a stop in 36 ms while slowing down with an acceleration of 60g, how far does he travel during this time?

SET UP: Let $+x$ be the direction the person travels. $v_x = 0$ (he stops), a_x is negative since it is opposite to the direction of the motion, and $t = 36\text{ ms} = 3.6 \times 10^{-2}\text{ s}$. The equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply since the acceleration is constant.

EXECUTE: Solving $v_x = v_{0x} + a_x t$ for v_{0x} gives $v_{0x} = -a_x t$. Then $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x = -\frac{1}{2}a_x t^2 = -\frac{1}{2}(-588\text{ m/s}^2)(3.6 \times 10^{-2}\text{ s})^2 = 38\text{ cm}.$

EVALUATE: Notice that we were not given the initial speed, but we could find it:

$$v_{0x} = -a_x t = -(-588\text{ m/s}^2)(36 \times 10^{2\ 3}\text{ s}) = 21\text{ m/s} = 47\text{ mph}.$$

2.26. **IDENTIFY:** In (a) the hip pad must reduce the person's speed from 2.0 m/s to 1.3 m/s over a distance of 2.0 cm, and we want the acceleration over this distance, assuming constant acceleration. In (b) we want to find out how the acceleration in (a) lasts.

SET UP: Let $+y$ be downward. $v_{0y} = 2.0\text{ m/s}$, $v_y = 1.3\text{ m/s}$, and $y - y_0 = 0.020\text{ m}$. The equations $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ apply for constant acceleration.

EXECUTE: **(a)** Solving $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for a_y gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(1.3\text{ m/s})^2 - (2.0\text{ m/s})^2}{2(0.020\text{ m})} = -58\text{ m/s}^2 = -5.9g.$$

(b) $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ gives $t = \frac{2(y - y_0)}{v_{0y} + v_y} = \frac{2(0.020\text{ m})}{2.0\text{ m/s} + 1.3\text{ m/s}} = 12\text{ ms}.$

EVALUATE: The acceleration is very large, but it only lasts for 12 ms so it produces a small velocity change.

2.27. **IDENTIFY:** We know the initial and final velocities of the object, and the distance over which the velocity change occurs. From this we want to find the magnitude and duration of the acceleration of the object.

SET UP: The constant-acceleration kinematics formulas apply. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, where $v_{0x} = 0$, $v_x = 5.0 \times 10^3\text{ m/s}$, and $x - x_0 = 4.0\text{ m}$.

EXECUTE: **(a)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(5.0 \times 10^3\text{ m/s})^2}{2(4.0\text{ m})} = 3.1 \times 10^6\text{ m/s}^2 = 3.2 \times 10^5\ g.$

(b) $v_x = v_{0x} + a_x t$ gives $t = \frac{v_x - v_{0x}}{a_x} = \frac{5.0 \times 10^3\text{ m/s}}{3.1 \times 10^6\text{ m/s}^2} = 1.6\text{ ms}.$

EVALUATE: **(c)** The calculated a is less than 450,000 g so the acceleration required doesn't rule out this hypothesis.

2.28. **IDENTIFY:** Apply constant acceleration equations to the motion of the car.

SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: **(a)** From Eq. (2.13), with $v_{0x} = 0$, $a_x = \frac{v_x^2}{2(x - x_0)} = \frac{(20\text{ m/s})^2}{2(120\text{ m})} = 1.67\text{ m/s}^2.$

(b) Using Eq. (2.14), $t = 2(x - x_0)/v_x = 2(120\text{ m})/(20\text{ m/s}) = 12\text{ s}.$

(c) $(12\text{ s})(20\text{ m/s}) = 240\text{ m}.$

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.29. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t}$. For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the shuttle travels in the $+x$ direction. $161\text{ km/h} = 44.72\text{ m/s}$ and $1610\text{ km/h} = 447.2\text{ m/s}.$ $1.00\text{ min} = 60.0\text{ s}$

EXECUTE: (a) (i) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{44.72\text{ m/s} - 0}{8.00\text{ s}} = 5.59\text{ m/s}^2$

(ii) $a_{\text{av-}x} = \dfrac{447.2\text{ m/s} - 44.72\text{ m/s}}{60.0\text{ s} - 8.00\text{ s}} = 7.74\text{ m/s}^2$

(b) (i) $t = 8.00$ s, $v_{0x} = 0$, and $v_x = 44.72$ m/s. $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t = \left(\dfrac{0+44.72\text{ m/s}}{2}\right)(8.00\text{ s}) = 179\text{ m}.$

(ii) $\Delta t = 60.0\text{ s} - 8.00\text{ s} = 52.0\text{ s}$, $v_{0x} = 44.72$ m/s, and $v_x = 447.2$ m/s.

$x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t = \left(\dfrac{44.72\text{ m/s}+447.2\text{ m/s}}{2}\right)(52.0\text{ s}) = 1.28\times 10^4\text{ m}.$

EVALUATE: When the acceleration is constant the instantaneous acceleration throughout the time interval equals the average acceleration for that time interval. We could have calculated the distance in part (a) as $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(5.59\text{ m/s}^2)(8.00\text{ s})^2 = 179\text{ m}$, which agrees with our previous calculation.

2.30. **IDENTIFY:** The acceleration a_x is the slope of the graph of v_x versus *t*.

SET UP: The signs of v_x and of a_x indicate their directions.

EXECUTE: (a) Reading from the graph, at $t = 4.0$ s, $v_x = 2.7$ cm/s, to the right and at $t = 7.0$ s, $v_x = 1.3$ cm/s, to the left.

(b) v_x versus *t* is a straight line with slope $-\dfrac{8.0\text{ cm/s}}{6.0\text{ s}} = -1.3\text{ cm/s}^2$. The acceleration is constant and equal to 1.3 cm/s^2, to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. For $t = 0$ to 4.5 s,

$x - x_0 = (8.0\text{ cm/s})(4.5\text{ s}) + \frac{1}{2}(-1.3\text{ cm/s}^2)(4.5\text{ s})^2 = 22.8\text{ cm}$. For $t = 0$ to 7.5 s,

$x - x_0 = (8.0\text{ cm/s})(7.5\text{ s}) + \frac{1}{2}(-1.3\text{ cm/s}^2)(7.5\text{ s})^2 = 23.4\text{ cm}$

(d) The graphs of a_x and *x* versus *t* are given in Figure 2.30.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t.$

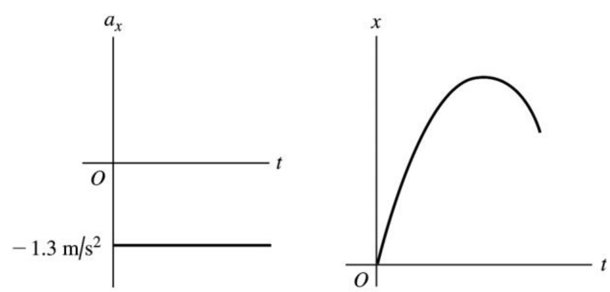


Figure 2.30

2.31. **(a) IDENTIFY** and **SET UP:** The acceleration a_x at time *t* is the slope of the tangent to the v_x versus *t* curve at time *t*.

EXECUTE: At $t = 3$ s, the v_x versus *t* curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus *t* curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11\text{ s}$ the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5\text{ s}$ the acceleration is constant and equal to zero. For the time interval $t = 5\text{ s}$ to $t = 9\text{ s}$ the acceleration is constant and equal to 6.25 m/s^2. For the interval $t = 9\text{ s}$ to $t = 13\text{ s}$ the acceleration is constant and equal to -11.2 m/s^2.

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}\quad a_x = 0\quad t = 5\text{ s}\quad x - x_0 = ?$

$x - x_0 = v_{0x}t$ ($a_x = 0$ so no $\frac{1}{2}a_x t^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5\text{ s}$ to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 4\text{ s}$. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9\text{ s}$ interval.)

$v_{0x} = 20\text{ m/s}\quad a_x = 6.25\text{ m/s}^2\quad t = 4\text{ s}\quad x_0 = 100\text{ m}\quad x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}.$

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}$.

At $t = 9\text{ s}$ the officer is at $x = 230\text{ m}$, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9\text{ s}$ to $t = 13\text{ s}$ the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but *not* for the whole $t = 0$ to $t = 13\text{ s}$ interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4\text{ s}$.

$v_{0x} = 45\text{ m/s}$ (at the start of this time interval)

$a_x = 2\ 11.2\text{ m/s}^2\quad t = 4\text{ s}\quad x_0 = 230\text{ m}\quad x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (45\text{ m/s})(4\text{ s}) + \frac{1}{2}(-11.2\text{ m/s}^2)(4\text{ s})^2 = 180\text{ m} - 89.6\text{ m} = 90.4\text{ m}.$

Thus $x = x_0 + 90.4\text{ m} = 230\text{ m} + 90.4\text{ m} = 320\text{ m}$.

At $t = 13\text{ s}$ the officer is at $x = 320\text{ m}$, so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-}x} = \Delta x/\Delta t$. For $t = 0$ to 5 s, $v_{\text{av-}x} = 20\text{ m/s}$. For $t = 0$ to 9 s, $v_{\text{av-}x} = 26\text{ m/s}$. For $t = 0$ to 13 s, $v_{\text{av-}x} = 25\text{ m/s}$. These results are consistent with Figure 2.37 in the textbook.

2.32. IDENTIFY: $v_x(t)$ is the slope of the x versus t graph. Car B moves with constant speed and zero acceleration. Car A moves with positive acceleration; assume the acceleration is constant.

SET UP: For car B, v_x is positive and $a_x = 0$. For car A, a_x is positive and v_x increases with t.

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.32a.

(b) The two cars have the same position at times when their x-t graphs cross. The figure in the problem shows this occurs at approximately $t = 1\text{ s}$ and $t = 3\text{ s}$.

(c) The graphs of v_x versus *t* for each car are sketched in Figure 2.32b.

(d) The cars have the same velocity when their *x*-*t* graphs have the same slope. This occurs at approximately $t = 2$ s.

(e) Car *A* passes car *B* when x_A moves above x_B in the *x*-*t* graph. This happens at $t = 3$ s.

(f) Car *B* passes car *A* when x_B moves above x_A in the *x*-*t* graph. This happens at $t = 1$ s.

EVALUATE: When $a_x = 0$, the graph of v_x versus *t* is a horizontal line. When a_x is positive, the graph of v_x versus *t* is a straight line with positive slope.

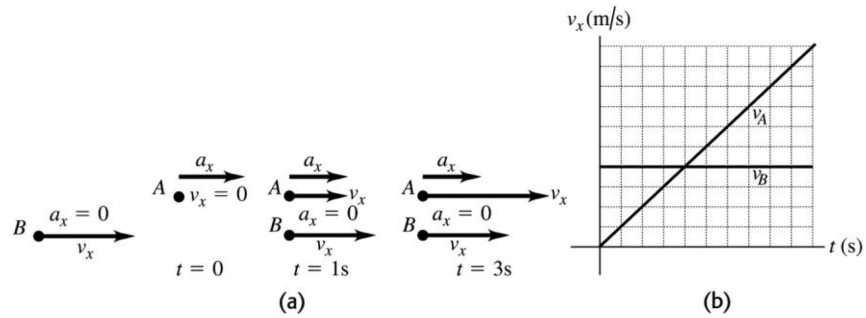


Figure 2.32a-b

2.33. **IDENTIFY:** For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Take $+y$ to be downward, so the motion is in the $+y$ direction. $19{,}300 \text{ km/h} = 5361 \text{ m/s}$, $1600 \text{ km/h} = 444.4 \text{ m/s}$, and $321 \text{ km/h} = 89.2 \text{ m/s}$. $4.0 \text{ min} = 240 \text{ s}$.

EXECUTE: **(a)** Stage *A*: $t = 240$ s, $v_{0y} = 5361$ m/s, $v_y = 444.4$ m/s. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{444.4 \text{ m/s} - 5361 \text{ m/s}}{240 \text{ s}} = -20.5 \text{ m/s}^2.$$

Stage *B*: $t = 94$ s, $v_{0y} = 444.4$ m/s, $v_y = 89.2$ m/s. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{89.2 \text{ m/s} - 444.4 \text{ m/s}}{94 \text{ s}} = -3.8 \text{ m/s}^2.$$

Stage *C*: $y - y_0 = 75$ m, $v_{0y} = 89.2$ m/s, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{0 - (89.2 \text{ m/s})^2}{2(75 \text{ m})} = -53.0 \text{ m/s}^2.$ In each case the negative sign means that the acceleration is upward.

(b) Stage *A*: $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t = \left(\dfrac{5361 \text{ m/s} + 444.4 \text{ m/s}}{2}\right)(240 \text{ s}) = 697 \text{ km}.$

Stage *B*: $y - y_0 = \left(\dfrac{444.4 \text{ m/s} + 89.2 \text{ m/s}}{2}\right)(94 \text{ s}) = 25 \text{ km}.$

Stage *C*: The problem states that $y - y_0 = 75 \text{ m} = 0.075 \text{ km}$.

The total distance traveled during all three stages is $697 \text{ km} + 25 \text{ km} + 0.075 \text{ km} = 722 \text{ km}$.

EVALUATE: The upward acceleration produced by friction in stage *A* is calculated to be greater than the upward acceleration due to the parachute in stage *B*. The effects of air resistance increase with increasing speed and in reality the acceleration was probably not constant during stages *A* and *B*.

2.34. **IDENTIFY:** Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same *x* at the same $t > 0$.

SET UP: The truck has $a_x = 0$. The car has $v_{0x} = 0$. Let $+x$ be in the direction of motion of the vehicles. Both vehicles start at $x_0 = 0$. The car has $a_C = 3.20 \text{ m/s}^2$. The truck has $v_x = 20.0$ m/s.

EXECUTE: (a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. Setting $x_T = x_C$ gives $t = 0$ and $v_{0T} = \frac{1}{2}a_C t$, so $t = \dfrac{2v_{0T}}{a_C} = \dfrac{2(20.0\ \text{m/s})}{3.20\ \text{m/s}^2} = 12.5\ \text{s}$. At this t, $x_T = (20.0\ \text{m/s})(12.5\ \text{s}) = 250\ \text{m}$ and $x = \frac{1}{2}(3.20\ \text{m/s}^2)(12.5\ \text{s})^2 = 250\ \text{m}$. The car and truck have each traveled 250 m.

(b) At $t = 12.5\ \text{s}$, the car has $v_x = v_{0x} + a_x t = (3.20\ \text{m/s}^2)(12.5\ \text{s}) = 40\ \text{m/s}$.

(c) $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. The x-t graph of the motion for each vehicle is sketched in Figure 2.34a.

(d) $v_T = v_{0T}$. $v_C = a_C t$. The v_x-t graph for each vehicle is sketched in Figure 2.34b.

EVALUATE: When the car overtakes the truck its speed is twice that of the truck.

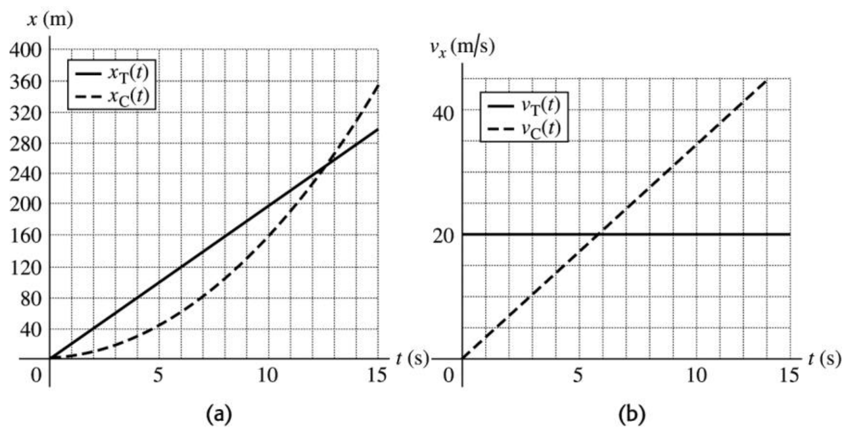


Figure 2.34a-b

2.35. IDENTIFY: Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g$, downward. Take the origin at the ground and the positive direction to be upward.

(a) SET UP: At the maximum height $v_y = 0$.

$v_y = 0 \quad y - y_0 = 0.440\ \text{m} \quad a_y = 2\ 9.80\ \text{m/s}^2 \quad v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.440\ \text{m})} = 2.94\ \text{m/s}$

(b) SET UP: When the flea has returned to the ground $y - y_0 = 0$.

$y - y_0 = 0 \quad v_{0y} = 1\ 2.94\ \text{m/s} \quad a_y = 2\ 9.80\ \text{m/s}^2 \quad t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94\ \text{m/s})}{-9.80\ \text{m/s}^2} = 0.600\ \text{s}$.

EVALUATE: We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94\ \text{m/s}$, $v_y = 0$ after 0.300 s.

2.36. IDENTIFY: The rock has a constant downward acceleration of $9.80\ \text{m/s}^2$. We know its initial velocity and position and its final position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: (a) $y - y_0 = -30\ \text{m}$, $v_{0y} = 18.0\ \text{m/s}$, $a_y = -9.8\ \text{m/s}^2$. The kinematics formulas give $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(18.0\ \text{m/s})^2 + 2(-9.8\ \text{m/s}^2)(-30\ \text{m})} = -30.2\ \text{m/s}$, so the speed is 30.2 m/s.

(b) $v_y = v_{0y} + a_y t$ and $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-30.3\text{ m/s} - 18.0\text{ m/s}}{-9.8\text{ m/s}^2} = 4.92\text{ s}.$

EVALUATE: The vertical velocity in part (a) is negative because the rock is moving downward, but the speed is always positive. The 4.92 s is the total time in the air.

2.37. **IDENTIFY:** The pin has a constant downward acceleration of 9.80 m/s^2 and returns to its initial position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: The kinematics formulas give $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. We know that $y - y_0 = 0$, so

$t = 2\dfrac{2v_{0y}}{a_y} = 2\dfrac{2(8.20\text{ m/s})}{-9.80\text{ m/s}^2} = +1.67\text{ s}.$

EVALUATE: It takes the pin half this time to reach its highest point and the remainder of the time to return.

2.38. **IDENTIFY:** The putty has a constant downward acceleration of 9.80 m/s^2. We know the initial velocity of the putty and the distance it travels.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: **(a)** v_{0y} = 9.50 m/s and $y - y_0$ = 3.60 m, which gives

$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(9.50\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(3.60\text{ m})} = 4.44\text{ m/s}$

(b) $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{4.44\text{ m/s} - 9.50\text{ m/s}}{-9.8\text{ m/s}^2} = 0.517\text{ s}$

EVALUATE: The putty is stopped by the ceiling, not by gravity.

2.39. **IDENTIFY:** A ball on Mars that is hit directly upward returns to the same level in 8.5 s with a constant downward acceleration of 0.379*g*. How high did it go and how fast was it initially traveling upward?

SET UP: Take $+y$ upward. $v_y = 0$ at the maximum height. $a_y = -0.379g = -3.71\text{ m/s}^2$. The constant-acceleration formulas $v_y = v_{0y} + a_y t$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ both apply.

EXECUTE: Consider the motion from the maximum height back to the initial level. For this motion $v_{0y} = 0$ and $t = 4.25$ s. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71\text{ m/s}^2)(4.25\text{ s})^2 = -33.5\text{ m}.$ The ball went 33.5 m above its original position.

(b) Consider the motion from just after it was hit to the maximum height. For this motion $v_y = 0$ and $t = 4.25$ s. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = -(-3.71\text{ m/s}^2)(4.25\text{ s}) = 15.8\text{ m/s}.$

(c) The graphs are sketched in Figure 2.39.

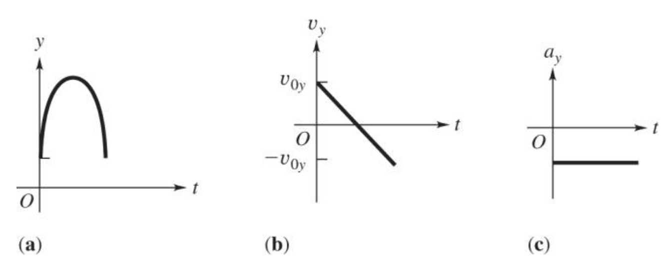


Figure 2.39

EVALUATE: The answers can be checked several ways. For example, $v_y = 0$, $v_{0y} = 15.8$ m/s, and $a_y = -3.7\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.8\text{ m/s})^2}{2(-3.71\text{ m/s}^2)} = 33.6\text{ m},$ which agrees with the height calculated in (a).

2.40. **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be positive. Since the lander is in free-fall, $a_y = +1.6\text{ m/s}^2$.

EXECUTE: $v_{0y} = 0.8$ m/s, $y - y_0 = 5.0$ m, $a_y = 1\ 1.6\ \text{m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8\ \text{m/s})^2 + 2(1.6\ \text{m/s}^2)(5.0\ \text{m})} = 4.1\ \text{m/s}.$

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

2.41. **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80\ \text{m/s}^2$. Let d be the distance the meterstick falls.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.90\ \text{m/s}^2)t^2$ and $t = \sqrt{\dfrac{d}{4.90\ \text{m/s}^2}}$.

(b) $t = \sqrt{\dfrac{0.176\ \text{m}}{4.90\ \text{m/s}^2}} = 0.190\ \text{s}$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

2.42. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80\ \text{m/s}^2$

EXECUTE: **(a)** $v_{0y} = 0, t = 2.50\ \text{s}, a_y = 9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(2.50\ \text{s})^2 = 30.6\ \text{m}.$
The building is 30.6 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\ \text{m/s}^2)(2.50\ \text{s}) = 24.5\ \text{m/s}$

(c) The graphs of a_y, v_y and y versus t are given in Figure 2.42. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

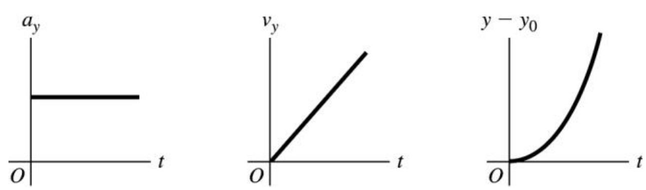


Figure 2.42

2.43. **IDENTIFY:** When the only force is gravity the acceleration is $9.80\ \text{m/s}^2$, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: **(a)** Find the velocity when the engines cut off. $y - y_0 = 525$ m, $a_y = 1\ 2.25\ \text{m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\ \text{m/s}^2)(525\ \text{m})} = 48.6\ \text{m/s}.$

Now consider the motion from engine cut-off to maximum height: $y_0 = 525$ m, $v_{0y} = +48.6$ m/s, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (48.6\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 121\ \text{m}$ and $y = 121\ \text{m} + 525\ \text{m} = 646\ \text{m}.$

(b) Consider the motion from engine failure until just before the rocket strikes the ground:
$y - y_0 = -525$ m, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +48.6$ m/s. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = 2\sqrt{(48.6\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-525\text{ m})} = -112\text{ m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-112\text{ m/s} - 48.6\text{ m/s}}{-9.80\text{ m/s}^2} = 16.4\text{ s}$.

(c) Find the time from blast-off until engine failure: $y - y_0 = 525\text{ m}$, $v_{0y} = 0$, $a_y = +2.25\text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(525\text{ m})}{2.25\text{ m/s}^2}} = 21.6\text{ s}$. The rocket strikes the launch pad $21.6\text{ s} + 16.4\text{ s} = 38.0\text{ s}$ after blast-off. The acceleration a_y is $+2.25\text{ m/s}^2$ from $t = 0$ to $t = 21.6\text{ s}$. It is -9.80 m/s^2 from $t = 21.6\text{ s}$ to 38.0 s. $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of v_y versus t is a straight line with positive slope of 2.25 m/s^2 during the blast-off phase and with negative slope of -9.80 m/s^2 after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of a_y determines the curvature of $y(t)$. At $t = 38.0\text{ s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.43.

EVALUATE: In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding v_y first allows us to avoid solving for t from a quadratic equation.

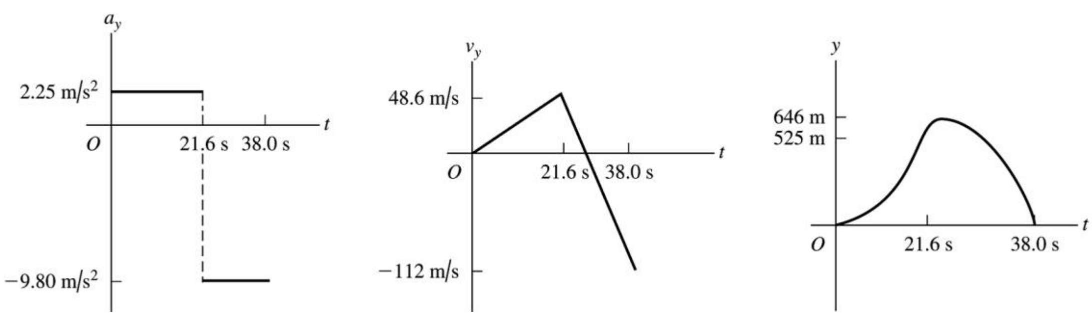


Figure 2.43

2.44. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80\text{ m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00\text{ m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0\text{ m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: **(a)** $t = 0.250\text{ s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00\text{ m/s})(0.250\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(0.250\text{ s})^2 = 0.94\text{ m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00\text{ m/s} + (-9.80\text{ m/s}^2)(0.250\text{ s}) = 2.55\text{ m/s}$.

$t = 1.00\text{ s}$: $y - y_0 = (5.00\text{ m/s})(1.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(1.00\text{ s})^2 = 0.10\text{ m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00\text{ m/s} + (-9.80\text{ m/s}^2)(1.00\text{ s}) = -4.80\text{ m/s}$.

(b) $y - y_0 = -40.0\text{ m}$, $v_{0y} = 5.00\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-40.0\text{ m} = (5.00\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. $(4.90\text{ m/s}^2)t^2 - (5.00\text{ m/s})t - 40.0\text{ m} = 0$ and

$t = \dfrac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right)\text{ s} = (0.51 \pm 2.90)\text{ s}$. t must be positive, so $t = 3.41\text{ s}$.

(c) $v_y = v_{0y} + a_y t = +5.00\text{ m/s} + (-9.80\text{ m/s}^2)(3.41\text{ s}) = -28.4\text{ m/s}$

(d) $v_{0y} = 5.00$ m/s, $a_y = -9.80$ m/s^2, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (5.00\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 1.28\text{ m.}$$ The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and y versus t are given in Figure 2.44. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

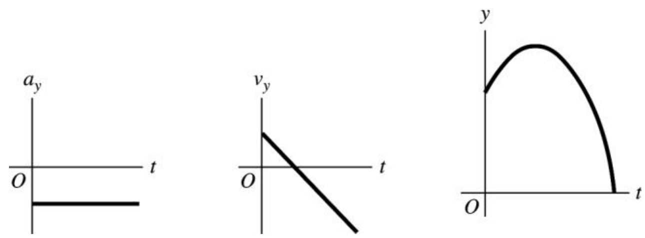


Figure 2.44

2.45. **IDENTIFY:** Use the constant acceleration equations to calculate a_x and $x - x_0$.

(a) SET UP: $v_x = 224$ m/s, $v_{0x} = 0$, $t = 0.900$ s, $a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $$a_x = \frac{v_x - v_{0x}}{t} = \frac{224\text{ m/s} - 0}{0.900\text{ s}} = 249\text{ m/s}^2$$

(b) $a_x/g = (249\text{ m/s}^2)/(9.80\text{ m/s}^2) = 25.4$

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(249\text{ m/s}^2)(0.900\text{ s})^2 = 101\text{ m}$

(d) SET UP: Calculate the acceleration, assuming it is constant:

$t = 1.40$ s, $v_{0x} = 283$ m/s, $v_x = 0$ (stops), $a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $$a_x = \frac{v_x - v_{0x}}{t} = \frac{0 - 283\text{ m/s}}{1.40\text{ s}} = -202\text{ m/s}^2$$

$a_x/g = (-202\text{ m/s}^2)/(9.80\text{ m/s}^2) = -20.6$; $a_x = 2\ 20.6g$

If the acceleration while the sled is stopping is constant then the magnitude of the acceleration is only 20.6*g*. But if the acceleration is not constant it is certainly possible that at some point the instantaneous acceleration could be as large as 40*g*.

EVALUATE: It is reasonable that for this motion the acceleration is much larger than *g*.

2.46. **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $y - y_0 = -30.0$ m, $t = 5.00$ s, $a_y = -9.80$ m/s^2. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \frac{1}{2}a_y t = \frac{-30.0\text{ m}}{5.00\text{ s}} - \frac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s}) = +18.5\text{ m/s.}$$

(b) $v_{0y} = +18.5$ m/s, $v_y = 0$ (at the maximum height), $a_y = -9.80$ m/s^2. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (18.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 17.5\text{ m.}$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to 9.80 m/s^2, downward, at all points in the motion, including at the maximum height.
(e) The graphs are sketched in Figure 2.46.

EVALUATE: The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-18.5 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.89 \text{ s}$. The egg has returned to the level of the cornice after 3.78 s and after 5.00 s it has traveled downward from the cornice for 1.22 s.

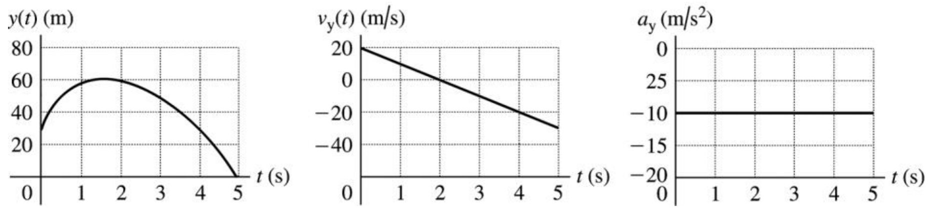


Figure 2.46

2.47. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall and acceleration due to gravity and setting up a ratio involving time of fall and acceleration due to gravity.
SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so $y - y_0 = h$. $v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $h = \tfrac{1}{2} g t_{\text{E}}^2$ and $h = \tfrac{1}{2} g_{\text{En}} t_{\text{En}}^2$. Combining these two equations gives

$$g t_{\text{E}}^2 = g_{\text{En}} t_{\text{En}}^2 \text{ and } g_{\text{En}} = g\left(\frac{t_{\text{E}}}{t_{\text{En}}}\right)^2 = (9.80 \text{ m/s}^2)\left(\frac{1.75 \text{ s}}{18.6 \text{ s}}\right)^2 = 0.0868 \text{ m/s}^2.$$

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.48. **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the boulder.
SET UP: Take $+y$ to be upward.

EXECUTE: **(a)** $v_{0y} = +40.0 \text{ m/s}$, $v_y = +20.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +2.04 \text{ s}.$$

(b) $v_y = -20.0 \text{ m/s}$. $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +6.12 \text{ s}$.

(c) $y - y_0 = 0$, $v_{0y} = +40.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = 0$ and

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(40.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = +8.16 \text{ s}.$$

(d) $v_y = 0$, $v_{0y} = +40.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 4.08 \text{ s}$.

(e) The acceleration is 9.80 m/s^2, downward, at all points in the motion.
(f) The graphs are sketched in Figure 2.48.
EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04 \text{ s} < 4.08 \text{ s} < 6.12 \text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

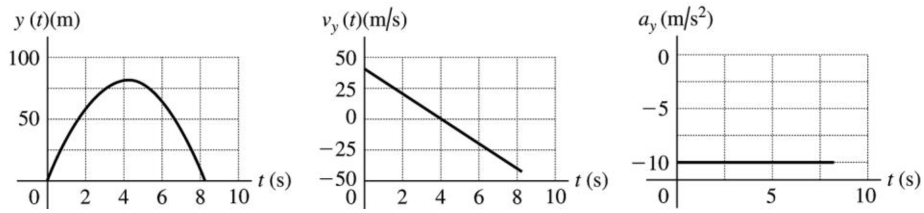


Figure 2.48

2.49. **IDENTIFY:** Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

SET UP: Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0\text{f}} = 3v_{0\text{s}}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

EXECUTE: **(a)** $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\dfrac{2v_{0y}}{t}$. Since both stones have the same a_y, $\dfrac{v_{0\text{f}}}{t_\text{f}} = \dfrac{v_{0\text{s}}}{t_\text{s}}$ and $t_\text{s} = t_\text{f}\left(\dfrac{v_{0\text{s}}}{v_{0\text{f}}}\right) = \left(\frac{1}{3}\right)(10\text{ s}) = 3.3\text{ s}.$

(b) Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\dfrac{v_{0y}^2}{2y}$. Since both have the same a_y, $\dfrac{v_{0\text{f}}^2}{y_\text{f}} = \dfrac{v_{0\text{s}}^2}{y_\text{s}}$ and $y_\text{f} = y_\text{s}\left(\dfrac{v_{0\text{f}}}{v_{0\text{s}}}\right)^2 = 9H.$

EVALUATE: The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

2.50. **IDENTIFY:** We start with the more general formulas and use them to derive the formulas for constant acceleration.

SET UP: The general formulas are $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt.$

EXECUTE: For constant acceleration, these formulas give $v_x = v_{0x} + \int_0^t a_x dt = v_{0x} + a_x \int_0^t dt = v_{0x} + a_x t$ and

$$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t (v_{0x} + a_x t)dt = x_0 + v_{0x}\int_0^t dt + a_x \int_0^t t\,dt = x_0 + v_{0x}t + \frac{1}{2}a_x t^2.$$

EVALUATE: The general formulas give the expected results for constant acceleration.

2.51. **IDENTIFY:** The acceleration is not constant, but we know how it varies with time. We can use the definitions of instantaneous velocity and position to find the rocket's position and speed.

SET UP: The basic definitions of velocity and position are $v_y(t) = \int_0^t a_y dt$ and $y - y_0 = \int_0^t v_y dt.$

EXECUTE: **(a)** $v_y(t) = \int_0^t a_y dt = \int_0^t (2.80\text{ m/s}^3)t\,dt = (1.40\text{ m/s}^3)t^2$

$y - y_0 = \int_0^t v_y dt = \int_0^t (1.40\text{ m/s}^3)t^2 dt = (0.4667\text{ m/s}^3)t^3.$ For $t = 10.0\text{ s}$, $y - y_0 = 467\text{ m}.$

(b) $y - y_0 = 325\text{ m}$ so $(0.4667\text{ m/s}^3)t^3 = 325\text{ m}$ and $t = 8.864\text{ s}.$ At this time $v_y = (1.40\text{ m/s}^3)(8.864\text{ s})^2 = 110\text{ m/s}.$

EVALUATE: The time in part (b) is less than 10.0 s, so the given formulas are valid.

2.52. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use Eqs. (2.17) and (2.18). Use the values of v_x and of x at $t = 1.0$ s to evaluate v_{0x} and x_0.

SET UP: $\int t^n dt = \frac{1}{n+1} t^{n+1}$, for $n \geq 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \frac{1}{2}\alpha t^2 = v_{0x} + (0.60 \text{ m/s}^3)t^2$. $v_x = 5.0$ m/s when $t = 1.0$ s gives $v_{0x} = 4.4$ m/s. Then, at $t = 2.0$ s, $v_x = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.8$ m/s.

(b) $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2}\alpha t^2) dt = x_0 + v_{0x}t + \frac{1}{6}\alpha t^3$. $x = 6.0$ m at $t = 1.0$ s gives $x_0 = 1.4$ m. Then, at $t = 2.0$ s, $x = 1.4 \text{ m} + (4.4 \text{ m/s})(2.0 \text{ s}) + \frac{1}{6}(1.2 \text{ m/s}^3)(2.0 \text{ s})^3 = 11.8$ m.

(c) $x(t) = 1.4 \text{ m} + (4.4 \text{ m/s})t + (0.20 \text{ m/s}^3)t^3$. $v_x(t) = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)t^2$. $a_x(t) = (1.20 \text{m/s}^3)t$. The graphs are sketched in Figure 2.52.

EVALUATE: We can verify that $a_x = \frac{dv_x}{dt}$ and $v_x = \frac{dx}{dt}$.

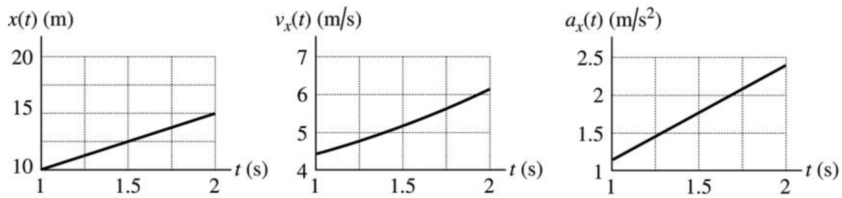


Figure 2.52

$a_x = At - Bt^2$ with $A = 1.50 \text{ m/s}^3$ and $B = 0.120 \text{ m/s}^4$

2.53. **(a) IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x \, dt$

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2)\, dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50 \text{ m/s}^3)t^2 - \frac{1}{3}(0.120 \text{ m/s}^4)t^3$

$v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$

SET UP: $x - x_0 + \int_0^t v_x \, dt$

EXECUTE: $x = x_0 + \int_0^t (\frac{1}{2}At^2 - \frac{1}{3}Bt^3)\, dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50 \text{ m/s}^3)t^3 - \frac{1}{12}(0.120 \text{ m/s}^4)t^4$

$x = (0.25 \text{ m/s}^3)t^3 - (0.010 \text{ m/s}^4)t^4$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}$.

(b) IDENTIFY and **SET UP:** At time t, when v_x is a maximum, $\frac{dv_x}{dt} = 0$. (Since $a_x = \frac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \frac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \frac{A}{B} = \frac{1.50 \text{ m/s}^3}{0.120 \text{ m/s}^4} = 12.5 \text{ s}$

At this time $v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$ gives

$v_x = (0.75 \text{ m/s}^3)(12.5 \text{ s})^2 - (0.040 \text{ m/s}^4)(12.5 \text{ s})^3 = 117.2 \text{ m/s} - 78.1 \text{ m/s} = 39.1 \text{ m/s}.$

EVALUATE: For $t < 12.5$ s, $a_x > 0$ and v_x is increasing. For $t > 12.5$ s, $a_x < 0$ and v_x is decreasing.

2.54. **IDENTIFY:** $a(t)$ is the slope of the *v* versus *t* graph and the distance traveled is the area under the *v* versus *t* graph.

SET UP: The *v* versus *t* graph can be approximated by the graph sketched in Figure 2.54.

EXECUTE: **(a)** Slope $= a = 0$ for $t \geq 1.3$ ms.

(b)

$h_{max} = \text{Area under } v\text{-}t \text{ graph} \approx A_{\text{Triangle}} + A_{\text{Rectangle}} \approx \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (2.5 \text{ ms} - 1.3 \text{ ms})(133 \text{ cm/s}) \approx 0.25 \text{ cm}$

(c) $a = \text{slope of } v\text{-}t$ graph. $a(0.5 \text{ ms}) \approx a(1.0 \text{ ms}) \approx \frac{133 \text{ cm/s}}{1.3 \text{ ms}} = 1.0 \times 10^5 \text{ cm/s}^2.$

$a(1.5 \text{ ms}) = 0$ because the slope is zero.

(d) $h = \text{area under } v\text{-}t$ graph. $h(0.5 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5 \text{ ms})(33 \text{ cm/s}) = 8.3 \times 10^{-3} \text{ cm}.$

$h(1.0 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0 \text{ ms})(100 \text{ cm/s}) = 5.0 \times 10^{-2} \text{ cm}.$

$h(1.5 \text{ ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (0.2 \text{ ms})1.33 \text{ cm/s} = 0.11 \text{ cm}$

EVALUATE: The acceleration is constant until $t = 1.3$ ms, and then it is zero. $g = 980 \text{ cm/s}^2$. The acceleration during the first 1.3 ms is much larger than this and gravity can be neglected for the portion of the jump that we are considering.

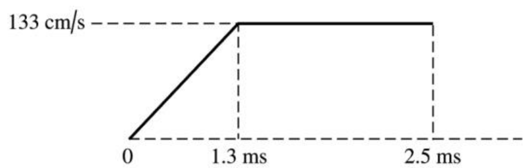


Figure 2.54

2.55. **IDENTIFY:** The sprinter's acceleration is constant for the first 2.0 s but zero after that, so it is not constant over the entire race. We need to break up the race into segments.

SET UP: When the acceleration is constant, the formula $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ applies. The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 10.0 \text{ m/s}}{2}\right)(2.0 \text{ s}) = 10.0 \text{ m}.$

(b) (i) 40.0 m at 10.0 m/s so time at constant speed is 4.0 s. The total time is 6.0 s, so

$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ m}}{6.0 \text{ s}} = 8.33 \text{ m/s}.$

(ii) He runs 90.0 m at 10.0 m/s so the time at constant speed is 9.0 s. The total time is 11.0 s, so

$v_{\text{av-}x} = \dfrac{100\text{ m}}{11.0\text{ s}} = 9.09\text{ m/s}.$

(iii) He runs 190 m at 10.0 m/s so time at constant speed is 19.0 s. His total time is 21.0 s, so

$v_{\text{av-}x} = \dfrac{200\text{ m}}{21.0\text{ s}} = 9.52\text{ m/s}.$

EVALUATE: His average velocity keeps increasing because he is running more and more of the race at his top speed.

2.56. **IDENTIFY:** The average speed is the total distance traveled divided by the total time. The elapsed time is the distance traveled divided by the average speed.

SET UP: The total distance traveled is 20 mi. With an average speed of 8 mi/h for 10 mi, the time for that first 10 miles is $\dfrac{10\text{ mi}}{8\text{ mi/h}} = 1.25\text{ h}.$

EXECUTE: **(a)** An average speed of 4 mi/h for 20 mi gives a total time of $\dfrac{20\text{ mi}}{4\text{ mi/h}} = 5.0\text{ h}.$ The second 10 mi must be covered in $5.0\text{ h} - 1.25\text{ h} = 3.75\text{ h}.$ This corresponds to an average speed of $\dfrac{10\text{ mi}}{3.75\text{ h}} = 2.7\text{ mi/h}.$

(b) An average speed of 12 mi/h for 20 mi gives a total time of $\dfrac{20\text{ mi}}{12\text{ mi/h}} = 1.67\text{ h}.$ The second 10 mi must be covered in $1.67\text{ h} - 1.25\text{ h} = 0.42\text{ h}.$ This corresponds to an average speed of $\dfrac{10\text{ mi}}{0.42\text{ h}} = 24\text{ mi/h}.$

(c) An average speed of 16 mi/h for 20 mi gives a total time of $\dfrac{20\text{ mi}}{16\text{ mi/h}} = 1.25\text{ h}.$ But 1.25 h was already spent during the first 10 miles and the second 10 miles would have to be covered in zero time. This is not possible and an average speed of 16 mi/h for the 20-mile ride is not possible.

EVALUATE: The average speed for the total trip is not the average of the average speeds for each 10-mile segment. The rider spends a different amount of time traveling at each of the two average speeds.

2.57. **IDENTIFY:** $v_x(t) = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}.$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1},$ for $n \ge 1.$

EXECUTE: **(a)** $v_x(t) = (9.00\text{ m/s}^3)t^2 - (20.0\text{ m/s}^2)t + 9.00\text{ m/s}.$ $a_x(t) = (18.0\text{ m/s}^3)t - 20.0\text{ m/s}^2.$ The graphs are sketched in Figure 2.57.

(b) The particle is instantaneously at rest when $v_x(t) = 0.$ $v_{0x} = 0$ and the quadratic formula gives

$t = \dfrac{1}{18.0}\left(20.0 \pm \sqrt{(20.0)^2 - 4(9.00)(9.00)}\right)\text{ s} = 1.11\text{ s} \pm 0.48\text{ s}.$ $t = 0.627\text{ s}$ and $t = 1.59\text{ s}.$ These results agree with the v_x-t graphs in part (a).

(c) For $t = 0.627\text{ s},$ $a_x = (18.0\text{ m/s}^3)(0.627\text{ s}) - 20.0\text{ m/s}^2 = -8.7\text{ m/s}^2.$ For $t = 1.59\text{ s},$ $a_x = +8.6\text{ m/s}^2.$ At $t = 0.627\text{ s}$ the slope of the v_x-t graph is negative and at $t = 1.59\text{ s}$ it is positive, so the same answer is deduced from the $v_x(t)$ graph as from the expression for $a_x(t).$

(d) $v_x(t)$ is instantaneously not changing when $a_x = 0.$ This occurs at $t = \dfrac{20.0\text{ m/s}^2}{18.0\text{ m/s}^3} = 1.11\text{ s}.$

(e) When the particle is at its greatest distance from the origin, $v_x = 0$ and $a_x < 0$ (so the particle is starting to move back toward the origin). This is the case for $t = 0.627\text{ s},$ which agrees with the *x*-*t* graph in part (a). At $t = 0.627\text{ s},$ $x = 2.45\text{ m}.$

(f) The particle's speed is changing at its greatest rate when a_x has its maximum magnitude. The a_x-t graph in part (a) shows this occurs at $t = 0$ and at $t = 2.00\text{ s}.$ Since v_x is always positive in this time interval, the particle is speeding up at its greatest rate when a_x is positive, and this is for $t = 2.00\text{ s}.$

The particle is slowing down at its greatest rate when a_x is negative and this is for $t = 0$.

EVALUATE: Since $a_x(t)$ is linear in t, $v_x(t)$ is a parabola and is symmetric around the point where $|v_x(t)|$ has its minimum value ($t = 1.11$ s). For this reason, the answer to part (d) is midway between the two times in part (c).

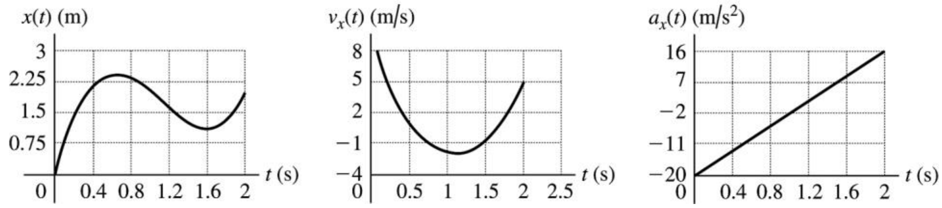


Figure 2.57

2.58. **IDENTIFY:** We know the vertical position of the lander as a function of time and want to use this to find its velocity initially and just before it hits the lunar surface.

SET UP: By definition, $v_y(t) = \dfrac{dy}{dt}$, so we can find v_y as a function of time and then evaluate it for the desired cases.

EXECUTE: **(a)** $v_y(t) = \dfrac{dy}{dt} = -c + 2dt$. At $t = 0$, $v_y(t) = -c = -60.0$ m/s. The initial velocity is 60.0 m/s downward.

(b) $y(t) = 0$ says $b - ct + dt^2 = 0$. The quadratic formula says $t = 28.57$ s ± 7.38 s. It reaches the surface at $t = 21.19$ s. At this time, $v_y = -60.0 \text{ m/s} + 2(1.05 \text{ m/s}^2)(21.19 \text{ s}) = -15.5$ m/s.

EVALUATE: The given formula for $y(t)$ is of the form $y = y_0 + v_{0y}t + \frac{1}{2}at^2$. For part (a), $v_{0y} = -c = -60$m/s.

2.59. **IDENTIFY:** In time t_S the S-waves travel a distance $d = v_S t_S$ and in time t_P the P-waves travel a distance $d = v_P t_P$.

SET UP: $t_S = t_P + 33$ s

EXECUTE: $\dfrac{d}{v_S} = \dfrac{d}{v_P} + 33 \text{ s}$. $d\left(\dfrac{1}{3.5 \text{ km/s}} - \dfrac{1}{6.5 \text{ km/s}}\right) = 33$ s and $d = 250$ km.

EVALUATE: The times of travel for each wave are $t_S = 71$ s and $t_P = 38$ s.

2.60. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. The average speed is the distance traveled divided by the elapsed time.

SET UP: Let $+x$ be in the direction of the first leg of the race. For the round trip, $\Delta x = 0$ and the total distance traveled is 50.0 m. For each leg of the race both the magnitude of the displacement and the distance traveled are 25.0 m.

EXECUTE: **(a)** $|v_{\text{av-}x}| = \left|\dfrac{\Delta x}{\Delta t}\right| = \dfrac{25.0 \text{ m}}{20.0 \text{ s}} = 1.25$ m/s. This is the same as the average speed for this leg of the race.

(b) $|v_{\text{av-}x}| = \left|\dfrac{\Delta x}{\Delta t}\right| = \dfrac{25.0 \text{ m}}{15.0 \text{ s}} = 1.67$ m/s. This is the same as the average speed for this leg of the race.

(c) $\Delta x = 0$ so $v_{\text{av-}x} = 0$.

(d) The average speed is $\dfrac{50.0 \text{ m}}{35.0 \text{ s}} = 1.43$ m/s.

EVALUATE: Note that the average speed for the round trip is not equal to the arithmetic average of the average speeds for each leg.

2.61. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}.$

SET UP: Let $+x$ be upward.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{\text{av-}x} = \dfrac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}.$ When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

2.62. **(a) IDENTIFY** and **SET UP:** The change in speed is the area under the a_x versus t curve between vertical lines at $t = 2.5\text{ s}$ and $t = 7.5\text{ s}$.

EXECUTE: This area is $\tfrac{1}{2}(4.00\text{ cm/s}^2 + 8.00\text{ cm/s}^2)(7.5\text{ s} - 2.5\text{ s}) = 30.0\text{ cm/s}$

This acceleration is positive so the change in velocity is positive.

(b) Slope of v_x versus t is positive and increasing with t. The graph is sketched in Figure 2.62.

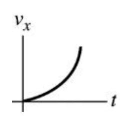


Figure 2.62

EVALUATE: The calculation in part (a) is equivalent to $\Delta v_x = (a_{\text{av-}x})\Delta t.$ Since a_x is linear in t, $a_{\text{av-}x} = (a_{0x} + a_x)/2.$ Thus $a_{\text{av-}x} = \tfrac{1}{2}(4.00\text{ cm/s}^2 + 8.00\text{ cm/s}^2)$ for the time interval $t = 2.5\text{ s}$ to $t = 7.5\text{ s}$.

2.63. **IDENTIFY:** Use information about displacement and time to calculate average speed and average velocity. Take the origin to be at Seward and the positive direction to be west.

(a) SET UP: average speed $= \dfrac{\text{distance traveled}}{\text{time}}$

EXECUTE: The distance traveled (different from the net displacement $(x - x_0)$) is $76\text{ km} + 34\text{ km} = 110\text{ km}.$

Find the total elapsed time by using $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{x - x_0}{t}$ to find t for each leg of the journey.

Seward to Auora: $t = \dfrac{x - x_0}{v_{\text{av-}x}} = \dfrac{76\text{ km}}{88\text{ km/h}} = 0.8636\text{ h}$

Auora to York: $t = \dfrac{x - x_0}{v_{\text{av-}x}} = \dfrac{-34\text{ km}}{-72\text{ km/h}} = 0.4722\text{ h}$

Total $t = 0.8636\text{ h} + 0.4722\text{ h} = 1.336\text{ h}.$

Then average speed $= \dfrac{110\text{ km}}{1.336\text{ h}} = 82\text{ km/h}.$

(b) SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t},$ where Δx is the displacement, not the total distance traveled.

For the whole trip he ends up $76\text{ km} - 34\text{ km} = 42\text{ km}$ west of his starting point. $v_{\text{av-}x} = \dfrac{42\text{ km}}{1.336\text{ h}} = 31\text{ km/h}.$

EVALUATE: The motion is not uniformly in the same direction so the displacement is less than the distance traveled and the magnitude of the average velocity is less than the average speed.

2.64. **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60 \text{ m/s}^2)(14.0 \text{ s})^2 = 157 \text{ m}$.

At $t = 14.0$ s, the speed is $v_x = v_{0x} + a_x t = (1.60 \text{ m/s}^2)(14.0 \text{ s}) = 22.4 \text{ m/s}$. In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4 \text{ m/s})(70.0 \text{ s}) = 1568 \text{ m}$.

For the interval during which the train is slowing down, $v_{0x} = 22.4$ m/s, $a_x = -3.50 \text{ m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4 \text{ m/s})^2}{2(-3.50 \text{ m/s}^2)} = 72 \text{ m}$.

The total distance traveled is $157 \text{ m} + 1568 \text{ m} + 72 \text{ m} = 1800 \text{ m}$.

EVALUATE: The acceleration is not constant for the entire motion but it does consist of constant acceleration segments and we can use constant acceleration equations for each segment.

2.65. **(a) IDENTIFY:** Calculate the average acceleration using $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_x - v_{0x}}{t}$. Use the information about the time and total distance to find his maximum speed.

SET UP: $v_{0x} = 0$ since the runner starts from rest.

$t = 4.0$ s, but we need to calculate v_x, the speed of the runner at the end of the acceleration period.

EXECUTE: For the last $9.1 \text{ s} - 4.0 \text{ s} = 5.1 \text{ s}$ the acceleration is zero and the runner travels a distance of $d_1 = (5.1 \text{ s})v_x$ (obtained using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$).

During the acceleration phase of 4.0 s, where the velocity goes from 0 to v_x, the runner travels a distance

$$d_2 = \left(\frac{v_{0x} + v_x}{2}\right)t = \frac{v_x}{2}(4.0 \text{ s}) = (2.0 \text{ s})v_x$$

The total distance traveled is 100 m, so $d_1 + d_2 = 100$ m. This gives $(5.1 \text{ s})v_x + (2.0 \text{ s})v_x = 100 \text{ m}$.

$v_x = \dfrac{100 \text{ m}}{7.1 \text{ s}} = 14.08 \text{ m/s}$.

Now we can calculate $a_{\text{av-}x}$: $a_{\text{av-}x} = \dfrac{v_x - v_{0x}}{t} = \dfrac{14.08 \text{ m/s} - 0}{4.0 \text{ s}} = 3.5 \text{ m/s}^2$.

(b) For this time interval the velocity is constant, so $a_{\text{av-}x} = 0$.

EVALUATE: Now that we have v_x we can calculate $d_1 = (5.1 \text{ s})(14.08 \text{ m/s}) = 71.8 \text{ m}$ and $d_2 = (2.0 \text{ s})(14.08 \text{ m/s}) = 28.2 \text{ m}$. So, $d_1 + d_2 = 100$ m, which checks.

(c) IDENTIFY and **SET UP:** $a_{\text{av-}x} = \dfrac{v_x - v_{0x}}{t}$, where now the time interval is the full 9.1 s of the race.

We have calculated the final speed to be 14.08 m/s, so

$$a_{\text{av-}x} = \frac{14.08 \text{ m/s}}{9.1 \text{ s}} = 1.5 \text{ m/s}^2.$$

EVALUATE: The acceleration is zero for the last 5.1 s, so it makes sense for the answer in part **(c)** to be less than half the answer in part (a).

(d) The runner spends different times moving with the average accelerations of parts (a) and (b).

2.66. **IDENTIFY:** Apply the constant acceleration equations to the motion of the sled. The average velocity for a time interval Δt is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be parallel to the incline and directed down the incline. The problem doesn't state how much time it takes the sled to go from the top to 14.4 m from the top.

EXECUTE: **(a)** 14.4 m to 25.6 m: $v_{\text{av-}x} = \dfrac{25.6 \text{ m} - 14.4 \text{ m}}{2.00 \text{ s}} = 5.60 \text{ m/s}$. 25.6 to 40.0 m:

$v_{\text{av-}x} = \dfrac{40.0 \text{ m} - 25.6 \text{ m}}{2.00 \text{ s}} = 7.20 \text{ m/s}$. 40.0 m to 57.6 m: $v_{\text{av-}x} = \dfrac{57.6 \text{ m} - 40.0 \text{ m}}{2.00 \text{ s}} = 8.80 \text{ m/s}$.

(b) For each segment we know $x - x_0$ and t but we don't know v_{0x} or v_x. Let $x_1 = 14.4 \text{ m}$ and $x_2 = 25.6 \text{ m}$. For this interval $\left(\dfrac{v_1 + v_2}{2}\right) = \dfrac{x_2 - x_1}{t}$ and $at = v_2 - v_1$. Solving for v_2 gives $v_2 = \frac{1}{2}at + \dfrac{x_2 - x_1}{t}$. Let $x_2 = 25.6 \text{ m}$ and $x_3 = 40.0 \text{ m}$. For this second interval, $\left(\dfrac{v_2 + v_3}{2}\right) = \dfrac{x_3 - x_2}{t}$ and $at = v_3 - v_2$. Solving for v_2 gives $v_2 = -\frac{1}{2}at + \dfrac{x_3 - x_2}{t}$. Setting these two expressions for v_2 equal to each other and solving for a gives $a = \dfrac{1}{t^2}[(x_3 - x_2) - (x_2 - x_1)] = \dfrac{1}{(2.00 \text{ s})^2}[(40.0 \text{ m} - 25.6 \text{ m}) - (25.6 \text{ m} - 14.4 \text{ m})] = 0.80 \text{ m/s}^2$.

Note that this expression for a says $a = \dfrac{v_{\text{av-23}} - v_{\text{av-12}}}{t}$, where $v_{\text{av-12}}$ and $v_{\text{av-23}}$ are the average speeds for successive 2.00 s intervals.

(c) For the motion from $x = 14.4 \text{ m}$ to $x = 25.6 \text{ m}$, $x - x_0 = 11.2 \text{ m}$, $a_x = 0.80 \text{ m/s}^2$ and $t = 2.00 \text{ s}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_{0x} = \dfrac{x - x_0}{t} - \frac{1}{2}a_x t = \dfrac{11.2 \text{ m}}{2.00 \text{ s}} - \dfrac{1}{2}(0.80 \text{ m/s}^2)(2.00 \text{ s}) = 4.80 \text{ m/s}$.

(d) For the motion from $x = 0$ to $x = 14.4 \text{ m}$, $x - x_0 = 14.4 \text{ m}$, $v_{0x} = 0$, and $v_x = 4.8 \text{ m/s}$.

$x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives $t = \dfrac{2(x - x_0)}{v_{0x} + v_x} = \dfrac{2(14.4 \text{ m})}{4.8 \text{ m/s}} = 6.0 \text{ s}$.

(e) For this 1.00 s time interval, $t = 1.00 \text{ s}$, $v_{0x} = 4.8 \text{ m/s}$, $a_x = 0.80 \text{ m/s}^2$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.8 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(0.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 5.2 \text{ m}$.

EVALUATE: With $x = 0$ at the top of the hill, $x(t) = v_{0x}t + \frac{1}{2}a_x t^2 = (0.40 \text{ m/s}^2)t^2$. We can verify that $t = 6.0 \text{ s}$ gives $x = 14.4 \text{ m}$, $t = 8.0 \text{ s}$ gives 25.6 m, $t = 10.0 \text{ s}$ gives 40.0 m, and $t = 12.0 \text{ s}$ gives 57.6 m.

2.67. **IDENTIFY:** When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0 \text{ s}$ segment, $v_{0x} = 4.00 \text{ m/s}$ and $v_x = 12.0 \text{ m/s}$. For the $t = 10.0 \text{ s}$ to 12.0 s segment, $v_{0x} = 12.0 \text{ m/s}$ and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to $t = 10.0 \text{ s}$, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00 \text{ m/s} + 12.0 \text{ m/s}}{2}\right)(10.0 \text{ s}) = 80.0 \text{ m}$.

For $t = 10.0 \text{ s}$ to $t = 12.0 \text{ s}$, $x - x_0 = \left(\dfrac{12.0 \text{ m/s} + 0}{2}\right)(2.00 \text{ s}) = 12.0 \text{ m}$. The total distance traveled is 92.0 m.

(b) $x - x_0 = 80.0 \text{ m} + 12.0 \text{ m} = 92.0 \text{ m}$

(c) For $t = 0$ to 10.0 s, $a_x = \dfrac{12.0 \text{ m/s} - 4.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$. For $t = 10.0 \text{ s}$ to 12.0 s, $a_x = \dfrac{0 - 12.0 \text{ m/s}}{2.00 \text{ s}} = -6.00 \text{ m/s}^2$. The graph of a_x versus t is given in Figure 2.67.

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

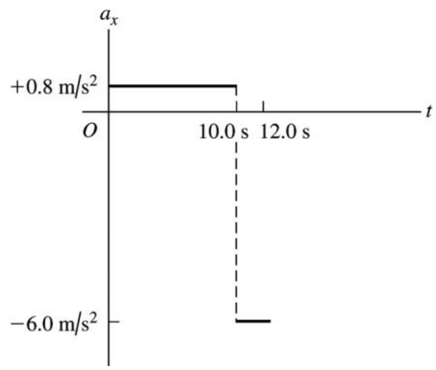


Figure 2.67

2.68. **IDENTIFY:** When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. For $t = 0$ to 5.0 s, v_x is positive and the ball moves in the $+x$ direction. For $t = 5.0$ s to 20.0 s, v_x is negative and the ball moves in the $-x$ direction. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 5.0$ s segment, $v_{0x} = 0$ and $v_x = 30.0$ m/s. For the $t = 5.0$ s to $t = 20.0$ s segment, $v_{0x} = 2$ 20.0 m/s and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to 5.0 s, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 30.0\text{ m/s}}{2}\right)(5.0\text{ m/s}) = 75.0\text{ m}.$ The ball travels a distance of 75.0 m. For $t = 5.0$ s to 20.0 s, $x - x_0 = \left(\dfrac{-20.0\text{ m/s} + 0}{2}\right)(15.0\text{ m/s}) = -150.0\text{ m}.$ The total distance traveled is $75.0\text{ m} + 150.0\text{ m} = 225.0\text{ m}.$

(b) The total displacement is $x - x_0 = 75.0\text{ m} + (-150.0\text{ m}) = -75.0\text{ m}.$ The ball ends up 75.0 m in the negative x-direction from where it started.

(c) For $t = 0$ to 5.0 s, $a_x = \dfrac{30.0\text{ m/s} - 0}{5.0\text{ s}} = 6.00\text{ m/s}^2.$ For $t = 5.0$ s to 20.0 s, $a_x = \dfrac{0 - (-20.0\text{ m/s})}{15.0\text{ s}} = +1.33\text{ m/s}^2.$ The graph of a_x versus t is given in Figure 2.68.

(d) The ball is in contact with the floor for a small but nonzero period of time and the direction of the velocity doesn't change instantaneously. So, no, the actual graph of $v_x(t)$ is not really vertical at 5.00 s.

EVALUATE: For $t = 0$ to 5.0 s, both v_x and a_x are positive and the speed increases. For $t = 5.0$ s to 20.0 s, v_x is negative and a_x is positive and the speed decreases. Since the direction of motion is not the same throughout, the displacement is not equal to the distance traveled.

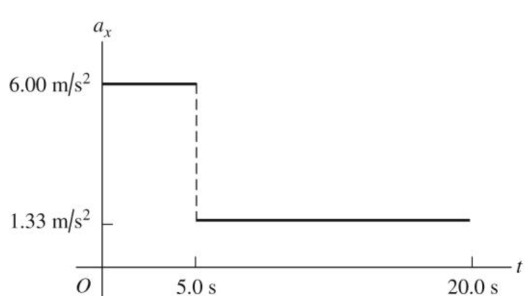


Figure 2.68

2.69. **IDENTIFY** and **SET UP:** Apply constant acceleration equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0,\ t = 5.0$ s.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\text{ s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:
$v_{0x} = a_x(5.0\text{ s}),\ t = 5.0\text{ s},\ x - x_0 = 150\text{ m}.$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $150\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ and $a_x = 4.0\text{ m/s}^2$

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(4.0\text{ m/s}^2)(5.0\text{ s})^2 = 50.0\text{ m}.$

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first. In fact, $x - x_0$ is proportional to t^2 since it starts from rest. If it goes 50.0 m in 5.0 s, in twice the time (10.0 s) it should go four times as far. In 10.0 s we calculated it went $50\text{ m} + 150\text{ m} = 200\text{ m}$, which is four times 50 m.

2.70. **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.
SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200$ m. For the freight train $v_F = 15.0$ m/s and $a_F = 0$. For the passenger train $v_P = 25.0$ m/s and $a_P = -0.100\text{ m/s}^2$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200\text{ m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200\text{ m} + v_F t$. $(0.0500\text{ m/s}^2)t^2 - (10.0\text{ m/s})t + 200\text{ m} = 0$. The quadratic formula gives $t = \frac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right)\text{ s} = (100 \pm 77.5)\text{ s}$. The collision occurs at $t = 100\text{ s} - 77.5\text{ s} = 22.5\text{ s}$. The equations that specify a collision have a physical solution (real, positive *t*), so a collision does occur.

(b) $x_P = (25.0\text{ m/s})(22.5\text{ s}) + \frac{1}{2}(-0.100\text{ m/s}^2)(22.5\text{ s})^2 = 537\text{ m}$. The passenger train moves 537 m before the collision. The freight train moves $(15.0\text{ m/s})(22.5\text{ s}) = 337\text{ m}$.

(c) The graphs of x_F and x_P versus *t* are sketched in Figure 2.70.

EVALUATE: The second root for the equation for *t*, $t = 177.5$ s is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

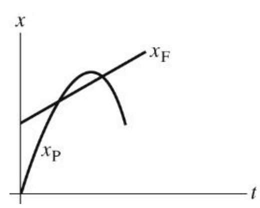


Figure 2.70

2.71. **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, you and the cockroach. You catch up with the roach when both objects are at the same place at the same time. Let *T* be the time when you catch up with the cockroach.
SET UP: Take $x = 0$ to be at the $t = 0$ location of the roach and positive *x* to be in the direction of motion of the two objects.
roach:
$v_{0x} = 1.50\text{ m/s},\ a_x = 0,\ x_0 = 0,\ x = 1.20\text{ m},\ t = T$

you:
$v_{0x} = 0.80$ m/s, $x_0 = 2\ 0.90$ m, $x = 1.20$ m, $t = T$, $a_x = ?$

Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to both objects:

EXECUTE: roach: 1.20 m = (1.50 m/s)T, so $T = 0.800$ s.

you: 1.20 m − (–0.90 m) = (0.80 m/s)$T + \frac{1}{2}a_x T^2$

$2.10 \text{ m} = (0.80 \text{ m/s})(0.800 \text{ s}) + \frac{1}{2}a_x(0.800 \text{ s})^2$

$2.10 \text{ m} = 0.64 \text{ m} + (0.320 \text{ s}^2)a_x$

$a_x = 4.6 \text{ m/s}^2$.

EVALUATE: Your final velocity is $v_x = v_{0x} + a_x t = 4.48$ m/s. Then $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 2.10$ m, which checks. You have to accelerate to a speed greater than that of the roach so you will travel the extra 0.90 m you are initially behind.

2.72. **IDENTIFY:** The insect has constant speed 15 m/s during the time it takes the cars to come together.
SET UP: Each car has moved 100 m when they hit.

EXECUTE: The time until the cars hit is $\frac{100 \text{ m}}{10 \text{ m/s}} = 10$ s. During this time the grasshopper travels a distance of $(15 \text{ m/s})(10 \text{ s}) = 150$ m.

EVALUATE: The grasshopper ends up 100 m from where it started, so the magnitude of his final displacement is 100 m. This is less than the total distance he travels since he spends part of the time moving in the opposite direction.

2.73. **IDENTIFY:** Apply constant acceleration equations to each object.
Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.73a.
Let d be the distance that the auto initially is behind the truck, so $x_0(\text{auto}) = -d$ and $x_0(\text{truck}) = 0$. Let T be the time it takes the auto to catch the truck. Thus at time T the truck has undergone a displacement $x - x_0 = 40.0$ m, so is at $x = x_0 + 40.0 \text{ m} = 40.0$ m. The auto has caught the truck so at time T is also at $x = 40.0$ m.

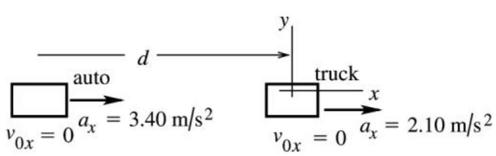


Figure 2.73a

(a) SET UP: Use the motion of the truck to calculate T:
$x - x_0 = 40.0$ m, $v_{0x} = 0$ (starts from rest), $a_x = 2.10 \text{ m/s}^2$, $t = T$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Since $v_{0x} = 0$, this gives $t = \sqrt{\frac{2(x - x_0)}{a_x}}$

EXECUTE: $T = \sqrt{\frac{2(40.0 \text{ m})}{2.10 \text{ m/s}^2}} = 6.17$ s

(b) SET UP: Use the motion of the auto to calculate d:
$x - x_0 = 40.0 \text{ m} + d$, $v_{0x} = 0$, $a_x = 3.40 \text{ m/s}^2$, $t = 6.17$ s

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $d + 40.0\text{ m} = \frac{1}{2}(3.40\text{ m/s}^2)(6.17\text{ s})^2$

$d = 64.8\text{ m} - 40.0\text{ m} = 24.8\text{ m}$

(c) auto: $v_x = v_{0x} + a_x t = 0 + (3.40\text{ m/s}^2)(6.17\text{ s}) = 21.0\text{ m/s}$

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\text{ m/s}^2)(6.17\text{ s}) = 13.0\text{ m/s}$

(d) The graph is sketched in Figure 2.73b.

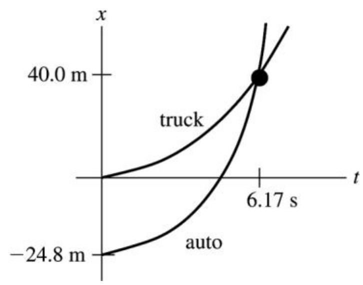


Figure 2.73b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they came abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the *x-t* curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.74. **IDENTIFY:** Apply the constant acceleration equations to the motion of each car. The collision occurs when the cars are at the same place at the same time.

SET UP: Let $+x$ be to the right. Let $x = 0$ at the initial location of car 1, so $x_{01} = 0$ and $x_{02} = D$. The cars collide when $x_1 = x_2$. $v_{0x1} = 0$, $a_{x1} = a_x$, $v_{0x2} = 2\,v_0$ and $a_{x2} = 0$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_1 = \frac{1}{2}a_x t^2$ and $x_2 = D - v_0 t$. $x_1 = x_2$ gives $\frac{1}{2}a_x t^2 = D - v_0 t$.

$\frac{1}{2}a_x t^2 + v_0 t - D = 0$. The quadratic formula gives $t = \frac{1}{a_x}\left(-v_0 \pm \sqrt{v_0^2 + 2a_x D}\right)$. Only the positive root is physical, so $t = \frac{1}{a_x}\left(-v_0 + \sqrt{v_0^2 + 2a_x D}\right)$.

(b) $v_1 = a_x t = \sqrt{v_0^2 + 2a_x D} - v_0$

(c) The *x-t* and v_x-*t* graphs for the two cars are sketched in Figure 2.74.

EVALUATE: In the limit that $a_x = 0$, $D - v_0 t = 0$ and $t = D/v_0$, the time it takes car 2 to travel distance *D*. In the limit that $v_0 = 0$, $t = \sqrt{\frac{2D}{a_x}}$, the time it takes car 1 to travel distance *D*.

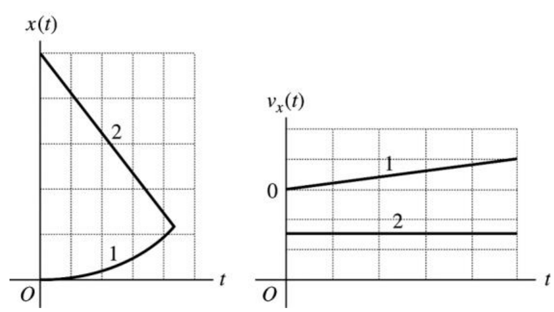


Figure 2.74

2.75. **IDENTIFY:** The average speed is the distance traveled divided by the time. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: The distance the ball travels is half the circumference of a circle of diameter 50.0 cm so is $\frac{1}{2}\pi d = \frac{1}{2}\pi(50.0\ \text{cm}) = 78.5\ \text{cm}$. Let $+x$ be horizontally from the starting point toward the ending point, so Δx equals the diameter of the bowl.

EXECUTE: **(a)** The average speed is $\dfrac{\frac{1}{2}\pi d}{t} = \dfrac{78.5\ \text{cm}}{10.0\ \text{s}} = 7.85\ \text{cm/s}$.

(b) The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{50.0\ \text{cm}}{10.0\ \text{s}} = 5.00\ \text{cm/s}$.

EVALUATE: The average speed is greater than the magnitude of the average velocity, since the distance traveled is greater than the magnitude of the displacement.

2.76. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \dfrac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt$.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$ for $n \geq 0$.

EXECUTE: **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \frac{1}{3}\beta t^3$. $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \frac{1}{3}\beta t^3 = (4.00\ \text{m/s})t - (0.667\ \text{m/s}^3)t^3$. $a_x(t) = \dfrac{dv_x}{dt} = -2\beta t = -(4.00\ \text{m/s}^3)t$.

(b) The maximum positive x is when $v_x = 0$ and $a_x < 0$. $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\dfrac{\alpha}{\beta}} = \sqrt{\dfrac{4.00\ \text{m/s}}{2.00\ \text{m/s}^3}} = 1.41\ \text{s}$. At this t, a_x is negative. For $t = 1.41$ s,

$x = (4.00\ \text{m/s})(1.41\ \text{s}) - (0.667\ \text{m/s}^3)(1.41\ \text{s})^3 = 3.77\ \text{m}$.

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty$.

2.77. **IDENTIFY:** Apply constant acceleration equations to each vehicle.

SET UP: **(a)** It is very convenient to work in coordinates attached to the truck.
Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.

EXECUTE: First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.77.

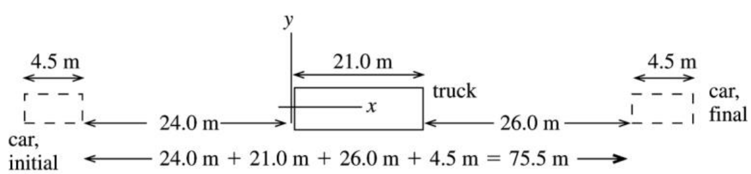


Figure 2.77

The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$a_x = 0.600\ \text{m/s}^2$, $v_{0x} = 0$, $x - x_0 = 75.5$ m, $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$t = \sqrt{\dfrac{2(x - x_0)}{a_x}} = \sqrt{\dfrac{2(75.5\ \text{m})}{0.600\ \text{m/s}^2}} = 15.86\ \text{s}$

It takes the car 15.9 s to pass the truck.

(b) Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0\text{ m/s}$ for the car. Take the origin to be at the initial position of the car.

$v_{0x} = 20.0\text{ m/s},\ a_x = 0.600\text{ m/s}^2,\ t = 15.86\text{ s},\ x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (20.0\text{ m/s})(15.86\text{ s}) + \frac{1}{2}(0.600\text{ m/s}^2)(15.86\text{ s})^2$

$x - x_0 = 317.2\text{ m} + 75.5\text{ m} = 393\text{ m}.$

(c) In coordinates fixed to the earth:

$v_x = v_{0x} + a_x t = 20.0\text{ m/s} + (0.600\text{ m/s}^2)(15.86\text{ s}) = 29.5\text{ m/s}$

EVALUATE: In 15.86 s the truck travels $x - x_0 = (20.0\text{ m/s})(15.86\text{ s}) = 317.2\text{ m}$. The car travels $392.7\text{ m} - 317.2\text{ m} = 75\text{ m}$ farther than the truck, which checks with part (a). In coordinates attached to the truck, for the car $v_{0x} = 0,\ v_x = 9.5\text{ m/s}$ and in 15.86 s the car travels $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 75\text{ m},$ which checks with part (a).

2.78. **IDENTIFY:** Use a velocity-time graph to find the acceleration of a stone. Then use that information to find how long it takes the stone to fall through a known distance and how fast you would have to throw it upward to reach a given height and the time to reach that height.

SET UP: Take $+y$ to be downward. The acceleration is the slope of the v_y versus t graph.

EXECUTE: **(a)** Since v_y is downward, it is positive and equal to the speed v. The v versus t graph has slope $a_y = \frac{30.0\text{ m/s}}{2.0\text{ s}} = 15\text{ m/s}^2$. The formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2,\ v_y^2 = v_{0y}^2 + 2a_y(y - y_0),$ and $v_y = v_y + a_y t$ apply.

(b) $v_{0y} = 0$ and let $y_0 = 0$. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(3.5\text{ m})}{15\text{ m/s}^2}} = 0.68\text{ s}.$

$v_y = v_{0y} + a_y t = (15\text{ m/s}^2)(0.68\text{ s}) = 10.2\text{ m/s}.$

(c) At the maximum height, $v_y = 0$. Let $y_0 = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-15\text{ m/s}^2)(18.0\text{ m})} = 23\text{ m/s}.$ $v_y = v_y + a_y t$ gives

$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 23\text{ m/s}}{-15\text{ m/s}^2} = 1.5\text{ s}.$

EVALUATE: The acceleration is 9.80 m/s^2, downward, throughout the motion. The velocity initially is upward, decreases to zero because of the downward acceleration and then is downward and increasing in magnitude because of the downward acceleration.

2.79. $a(t) = \alpha + \beta t,$ with $\alpha = -2.00\text{ m/s}^2$ and $\beta = 3.00\text{ m/s}^3$

(a) IDENTIFY and **SET UP:** Integrage $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

EXECUTE: $v_x = v_{0x} + \int_0^t a_x\, dt = v_{0x} + \int_0^t (\alpha + \beta t)\, dt = v_{0x} + \alpha t + \frac{1}{2}\beta t^2$

$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t (v_{0x} + \alpha t + \frac{1}{2}\beta t^2)\, dt = x_0 + v_{0x}t + \frac{1}{2}\alpha t^2 + \frac{1}{6}\beta t^3$

At $t = 0,\ x = x_0$.

To have $x = x_0$ at $t_1 = 4.00\text{ s}$ requires that $v_{0x}t_1 + \frac{1}{2}\alpha t_1^2 + \frac{1}{6}\beta t_1^3 = 0.$

Thus $v_{0x} = -\frac{1}{6}\beta t_1^2 - \frac{1}{2}\alpha t_1 = -\frac{1}{6}(3.00\text{ m/s}^3)(4.00\text{ s})^2 - \frac{1}{2}(-2.00\text{ m/s}^2)(4.00\text{ s}) = -4.00\text{ m/s}.$

(b) With v_{0x} as calculated in part (a) and $t = 4.00\text{ s},$

$v_x = v_{0x} + \alpha t + \frac{1}{2}\beta t^2 = -4.00\text{ s} + (-2.00\text{ m/s}^2)(4.00\text{ s}) + \frac{1}{2}(3.00\text{ m/s}^3)(4.00\text{ s})^2 = +12.0\text{ m/s}.$

EVALUATE: $a_x = 0$ at $t = 0.67$ s. For $t > 0.67$ s, $a_x > 0$. At $t = 0$, the particle is moving in the $-x$-direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

2.80. **IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.

SET UP: Let $+y$ be downward. The egg has $v_{0y} = 0$ and $a_y = 9.80\ \text{m/s}^2$. At the height of the professor's head, the egg has $y - y_0 = 44.2$ m.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(44.2\ \text{m})}{9.80\ \text{m/s}^2}} = 3.00$ s. The professor walks a distance $x - x_0 = v_{0x}t = (1.20\ \text{m/s})(3.00\ \text{s}) = 3.60$ m. Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80\ \text{m/s}^2)(3.00\text{s}) = 29.4$ m/s. It is traveling much faster than the professor.

2.81. **IDENTIFY:** Use the constant acceleration equations to establish a relationship between maximum height and acceleration due to gravity and between time in the air and acceleration due to gravity.

SET UP: Let $+y$ be upward. At the maximum height, $v_y = 0$. When the rock returns to the surface, $y - y_0 = 0$.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y H = -\frac{1}{2}v_{0y}^2$, which is constant, so $a_\text{E}H_\text{E} = a_\text{M}H_\text{M}$.

$$H_\text{M} = H_\text{E}\left(\frac{a_\text{E}}{a_\text{M}}\right) = H\left(\frac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right) = 2.64H.$$

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $a_y t = -2v_{0y}$, which is constant, so $a_\text{E}T_\text{E} = a_\text{M}T_\text{M}$.

$$T_\text{M} = T_\text{E}\left[\frac{a_\text{E}}{a_\text{M}}\right] = 2.64T.$$

EVALUATE: On Mars, where the acceleration due to gravity is smaller, the rocks reach a greater height and are in the air for a longer time.

2.82. **IDENTIFY:** Calculate the time it takes her to run to the table and return. This is the time in the air for the thrown ball. The thrown ball is in free-fall after it is thrown. Assume air resistance can be neglected.

SET UP: For the thrown ball, let $+y$ be upward. $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = 0$ when the ball returns to its original position.

EXECUTE: **(a)** It takes her $\frac{5.50\ \text{m}}{2.50\ \text{m/s}} = 2.20$ s to reach the table and an equal time to return. For the ball, $y - y_0 = 0$, $t = 4.40$ s and $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = -\tfrac{1}{2}a_y t = -\tfrac{1}{2}(-9.80\ \text{m/s}^2)(4.40\ \text{s}) = 21.6\ \text{m/s}.$$

(b) Find $y - y_0$ when $t = 2.20$ s.

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = (21.6\ \text{m/s})(2.20\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.20\ \text{s})^2 = 23.8\ \text{m}$$

EVALUATE: It takes the ball the same amount of time to reach its maximum height as to return from its maximum height, so when she is at the table the ball is at its maximum height. Note that this large maximum height requires that the act either be done outdoors, or in a building with a very high ceiling.

2.83. **(a) IDENTIFY:** Use constant acceleration equations, with $a_y = g$, downward, to calculate the speed of the diver when she reaches the water.

SET UP: Take the origin of coordinates to be at the platform, and take the $+y$-direction to be downward.

$y - y_0 = +21.3$ m, $a_y = +9.80\ \text{m/s}^2$, $v_{0y} = 0$ (since diver just steps off), $v_y = ?$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

EXECUTE: $v_y = +\sqrt{2a_y(y-y_0)} = +\sqrt{2(9.80\text{ m/s}^2)(21.3\text{ m})} = +20.4\text{ m/s}.$

We know that v_y is positive because the diver is traveling downward when she reaches the water.

The announcer has exaggerated the speed of the diver.

EVALUATE: We could also use $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to find $t = 2.085\text{ s}.$ The diver gains 9.80 m/s of speed each second, so has $v_y = (9.80\text{ m/s}^2)(2.085\text{ s}) = 20.4\text{ m/s}$ when she reaches the water, which checks.

(b) IDENTIFY: Calculate the initial upward velocity needed to give the diver a speed of 25.0 m/s when she reaches the water. Use the same coordinates as in part (a).

SET UP: $v_{0y} = ?,\ v_y = +25.0\text{ m/s},\ a_y = +9.80\text{ m/s}^2,\ y - y_0 = +21.3\text{ m}$

$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$

EXECUTE: $v_{0y} = 2\sqrt{v_y^2 - 2a_y(y-y_0)} = -\sqrt{(25.0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(21.3\text{ m})} = -14.4\text{ m/s}$

(v_{0y} is negative since the direction of the initial velocity is upward.)

EVALUATE: One way to decide if this speed is reasonable is to calculate the maximum height above the platform it would produce:

$v_{0y} = -14.4\text{ m/s},\ v_y = 0$ (at maximum height), $a_y = +9.80\text{ m/s}^2,\ y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0-(-14.4\text{ s})^2}{2(+9.80\text{ m/s})} = -10.6\text{ m}$$

This is not physically attainable; a vertical leap of 10.6 m upward is not possible.

2.84. **IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2.$

EXECUTE: Motion past the window: $y - y_0 = 1.90\text{ m},\ t = 0.420\text{ s},\ a_y = +9.80\text{ m/s}^2.$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = \frac{y-y_0}{t} - \frac{1}{2}a_y t = \frac{1.90\text{ m}}{0.420\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.420\text{ s}) = 2.466\text{ m/s}.$ This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0,\ v_y = 2.466\text{ m/s},\ a_y = +9.80\text{ m/s}^2.$

$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(2.466\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.310\text{ m}.$ The top of the window is 0.310 m below the windowsill.

EVALUATE: It takes the flowerpot $t = \frac{v_y - v_{0y}}{a_y} = \frac{2.466\text{ m/s}}{9.80\text{ m/s}^2} = 0.252\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.310\text{ m} + 1.90\text{ m} = 2.21\text{ m}$ in $0.252\text{ s} + 0.420\text{ s} = 0.672\text{ s}.$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.672\text{ s})^2 = 2.21\text{ m},$ which checks.

2.85. **(a) IDENTIFY:** Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: Take positive *y* to be upward. $v_{0y} = 0,\ v_y = ?,\ a_y = 35.0\text{ m/s}^2,\ y - y_0 = 0.640\text{ m},$

$v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$

EXECUTE: $v_y = \sqrt{2a_y(y-y_0)} = \sqrt{2(35.0\text{ m/s}^2)(0.640\text{ m})} = 6.69\text{ m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0.$ Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = ?$, $a_y =$ 2 9.80 m/s^2 (free fall), $v_{0y} = 6.69$ m/s (from part (a), $v_y = 0$ at maximum height), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (6.69 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 2.29 \text{ m}$, $y = 2.20 \text{ m} + 2.29 \text{ m} = 4.49 \text{ m}.$

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = 1.83$ m, $a_y =$ 2 9.80 m/s^2, $v_{0y} = +6.69$ m/s, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83 \text{ m} - 2.20 \text{ m} = (6.69 \text{ m/s})t + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2 = (6.69 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$,

$4.90t^2 - 6.69t - 0.37 = 0$, with t in seconds. Use the quadratic formula to solve for t:

$t = \dfrac{1}{9.80}\left(6.69 \pm \sqrt{(6.69)^2 - 4(4.90)(-0.37)}\right) = 0.6830 \pm 0.7362.$ Since t must be positive,

$t = 0.6830 \text{ s} + 0.7362 \text{ s} = 1.42 \text{ s}.$

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so $t = (v_y - v_{0y})/a_y =$ $-(6.69 \text{ m/s})/(-9.80 \text{ m/s}^2) = 0.68 \text{ s}.$ It also takes 0.68 s to return to 2.2 m above the ground, for a total time of 1.36 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.36 s after being thrown; the answer of 1.42 s in part (c) makes sense.

2.86. **IDENTIFY:** The motion of the rocket can be broken into 3 stages, each of which has constant acceleration, so in each stage we can use the standard kinematics formulas for constant acceleration. But the acceleration is not the same throughout all 3 stages.

SET UP: The formulas $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$, and $v_y = v_{0y} + a_y t$ apply.

EXECUTE: **(a)** Let +y be upward. At $t = 25.0$ s, $y - y_0 = 1094$ m and $v_y = 87.5$ m/s. During the next 10.0 s the rocket travels upward an additional distance $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t = \left(\dfrac{87.5 \text{ m/s} + 132.5 \text{ m/s}}{2}\right)(10.0 \text{ s}) = 1100 \text{ m}.$

The height above the launch pad when the second stage quits therefore is $1094 \text{ m} + 1100 \text{ m} = 2194 \text{ m}.$

For the free-fall motion after the second stage quits: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (132.5 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 896 \text{ m}.$

The maximum height above the launch pad that the rocket reaches is $2194 \text{ m} + 896 \text{ m} = 3090 \text{ m}.$

(b) $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$ gives $-2194 \text{ m} = (132.5 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2.$ From the quadratic formula the positive root is $t = 38.6$ s.

(c) $v_y = v_{0y} + a_y t = 132.5 \text{ m/s} + (-9.8 \text{ m/s}^2)(38.6 \text{ s}) = -246 \text{ m/s}.$ The rocket's speed will be 246 m/s just before it hits the ground.

EVALUATE: We cannot solve this problem in a single step because the acceleration, while constant in each stage, is not constant over the entire motion. The standard kinematics equations apply to each stage but not to the motion as a whole.

2.87. **IDENTIFY** and **SET UP:** Let $+y$ be upward. Each ball moves with constant acceleration $a_y = -9.80 \text{ m/s}^2$. In parts (c) and (d) require that the two balls be at the same height at the same time.

EXECUTE: **(a)** At ceiling, $v_y = 0$, $y - y_0 = 3.0$ m, $a_y = -9.80 \text{ m/s}^2$. Solve for v_{0y}.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = 7.7$ m/s.

(b) $v_y = v_{0y} + a_y t$ with the information from part (a) gives $t = 0.78$ s.

(c) Let the first ball travel downward a distance d in time t. It starts from its maximum height, so $v_{0y} = 0$.

$y - y_0 = v_{0y}t = \frac{1}{2}a_y t^2$ gives $d = (4.9 \text{ m/s}^2)t^2$

The second ball has $v_{0y} = \frac{2}{3}(7.7\text{ m/s}) = 5.1\text{ m/s}.$ In time t it must travel upward $3.0\text{ m} - d$ to be at the same place as the first ball.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $3.0\text{ m} - d = (5.1\text{ m/s})t - (4.9\text{ m/s}^2)t^2.$

We have two equations in two unknowns, d and t. Solving gives $t = 0.59\text{ s}$ and $d = 1.7\text{ m}.$

(d) $3.0\text{ m} - d = 1.3\text{ m}$

EVALUATE: In 0.59 s the first ball falls $d = (4.9\text{ m/s}^2)(0.59\text{ s})^2 = 1.7\text{ m},$ so is at the same height as the second ball.

2.88. **IDENTIFY:** The teacher is in free-fall and falls with constant acceleration $9.80\text{ m/s}^2,$ downward. The sound from her shout travels at constant speed. The sound travels from the top of the cliff, reflects from the ground and then travels upward to her present location. If the height of the cliff is h and she falls a distance y in 3.0 s, the sound must travel a distance $h + (h - y)$ in 3.0 s.

SET UP: Let $+y$ be downward, so for the teacher $a_y = 9.80\text{ m/s}^2$ and $v_{0y} = 0.$ Let $y = 0$ at the top of the cliff.

EXECUTE: **(a)** For the teacher, $y = \frac{1}{2}(9.80\text{ m/s}^2)(3.0\text{ s})^2 = 44.1\text{ m}.$ For the sound, $h + (h - y) = v_s t.$

$h = \frac{1}{2}(v_s t + y) = \frac{1}{2}\left([340\text{ m/s}][3.0\text{ s}] + 44.1\text{ m}\right) = 532\text{ m},$ which rounds to 530 m.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80\text{ m/s}^2)(532\text{ m})} = 102\text{ m/s}.$

EVALUATE: She is in the air for $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{102\text{ m/s}}{9.80\text{ m/s}^2} = 10.4\text{ s}$ and strikes the ground at high speed.

2.89. **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of $2.0\text{ m/s}^2.$

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t = (5.0\text{ m/s}^2)(10.0\text{ s}) = 50.0\text{ m/s}$ and are at $y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0\text{ m/s}^2)(10.0\text{ s})^2 = 250\text{ m}.$

For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0\text{ m/s},$ $y_0 = 250\text{ m},$ and $a_y = -9.80\text{ m/s}^2.$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0 - (50.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} + 250\text{ m} = 378\text{ m},$ which rounds to 380 m.

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0\text{ m/s},$ $a_y = -9.80\text{ m/s}^2,$ and $y - y_0 = -250\text{ m}.$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-250\text{ m} = (50.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$ $(4.90\text{ m/s}^2)t^2 - (50.0\text{ m/s})t - 250\text{ m} = 0.$ The quadratic formula gives

$t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right)$ s. Only the positive solution is physical, so $t = 13.9\text{ s}.$ Powers therefore has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9\text{ s} - 7.0\text{ s} = 6.9\text{ s}.$ After 7.0 s of free-fall he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 250\text{ m} + (50.0\text{ m/s})(7.0\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(7.0\text{ s})^2 = 360\text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0\text{ m/s} + (-9.80\text{ m/s}^2)(7.0\text{ s}) = -18.6\text{ m/s}.$ After the next 6.9 s he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 360\text{ m} + (-18.6\text{ m/s})(6.9\text{ s}) + \frac{1}{2}(-2.00\text{ m/s}^2)(6.9\text{ s})^2 = 184\text{ m}.$ Powers is 184 m above the ground when the helicopter crashes.

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

2.90. **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_{fall} be the time for the rock to fall to the ground and let t_{s} be the time it takes the sound to travel from the impact point back to you. $t_{\text{fall}} + t_{\text{s}} = 10.0\text{ s}$. Both the rock and sound travel a distance d that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = 9.80\text{ m/s}^2$.

EXECUTE: **(a)** For the rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t_{\text{fall}} = \sqrt{\dfrac{2d}{9.80\text{ m/s}^2}}$.

For the sound, $t_{\text{s}} = \dfrac{d}{330\text{ m/s}}$. Let $\alpha^2 = d$. $0.00303\alpha^2 + 0.4518\alpha - 10.0 = 0$. $\alpha = 19.6$ and $d = 384\text{ m}$.

(b) You would have calculated $d = \frac{1}{2}(9.80\text{ m/s}^2)(10.0\text{ s})^2 = 490\text{ m}$. You would have overestimated the height of the cliff. It actually takes the rock less time than 10.0 s to fall to the ground.

EVALUATE: Once we know d we can calculate that $t_{\text{fall}} = 8.8\text{ s}$ and $t_{\text{s}} = 1.2\text{ s}$. The time for the sound of impact to travel back to you is 12% of the total time and cannot be neglected. The rock has speed 86 m/s just before it strikes the ground.

2.91. **(a) IDENTIFY:** Let $+y$ be upward. The can has constant acceleration $a_y = -g$. The initial upward velocity of the can equals the upward velocity of the scaffolding; first find this speed.

SET UP: $y - y_0 = -15.0\text{ m}$, $t = 3.25\text{ s}$, $a_y = 2\ 9.80\text{ m/s}^2$, $v_{0y} = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 11.31\text{ m/s}$

Use this v_{0y} in $v_y = v_{0y} + a_y t$ to solve for v_y: $v_y = -20.5\text{ m/s}$

(b) IDENTIFY: Find the maximum height of the can, above the point where it falls from the scaffolding:

SET UP: $v_y = 0$, $v_{0y} = +11.31\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = ?$

EXECUTE: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 6.53\text{ m}$

The can will pass the location of the other painter. Yes, he gets a chance.

EVALUATE: Relative to the ground the can is initially traveling upward, so it moves upward before stopping momentarily and starting to fall back down.

2.92. **IDENTIFY:** Both objects are in free-fall. Apply the constant acceleration equations to the motion of each person.

SET UP: Let $+y$ be downward, so $a_y = +9.80\text{ m/s}^2$ for each object.

EXECUTE: **(a)** Find the time it takes the student to reach the ground: $y - y_0 = 180\text{ m}$, $v_{0y} = 0$, $a_y = 9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(180\text{ m})}{9.80\text{ m/s}^2}} = 6.06\text{ s}$. Superman must reach the ground in $6.06\text{ s} - 5.00\text{ s} = 1.06\text{ s}$: $t = 1.06\text{ s}$, $y - y_0 = 180\text{ m}$, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{180\text{ m}}{1.06\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(1.06\text{ s}) = 165\text{ m/s}$. Superman must have initial speed $v_0 = 165\text{ m/s}$.

(b) The graphs of y-t for Superman and for the student are sketched in Figure 2.92.

(c) The minimum height of the building is the height for which the student reaches the ground in 5.00 s, before Superman jumps. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(5.00\text{ s})^2 = 122\text{ m}$. The skyscraper must be at least 122 m high.

EVALUATE: $165\text{ m/s} = 369\text{ mi/h}$, so only Superman could jump downward with this initial speed.

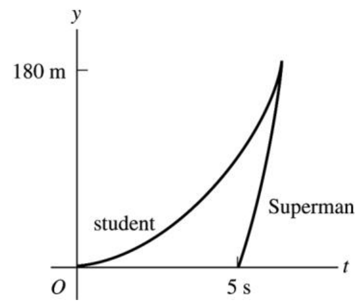


Figure 2.92

2.93. **IDENTIFY:** Apply constant acceleration equations to the motion of the rocket and to the motion of the canister after it is released. Find the time it takes the canister to reach the ground after it is released and find the height of the rocket after this time has elapsed. The canister travels up to its maximum height and then returns to the ground.

SET UP: Let $+y$ be upward. At the instant that the canister is released, it has the same velocity as the rocket. After it is released, the canister has $a_y = -9.80\text{ m/s}^2$. At its maximum height the canister has $v_y = 0$.

EXECUTE: **(a)** Find the speed of the rocket when the canister is released: $v_{0y} = 0$, $a_y = 3.30\text{ m/s}^2$, $y - y_0 = 235\text{ m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(3.30\text{ m/s}^2)(235\text{ m})} = 39.4\text{ m/s}$.

For the motion of the canister after it is released, $v_{0y} = +39.4\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -235\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-235\text{ m} = (39.4\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 12.0\text{ s}$ as the positive solution. Then for the motion of the rocket during this 12.0 s,
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 235\text{ m} + (39.4\text{ m/s})(12.0\text{ s}) + \frac{1}{2}(3.30\text{ m/s}^2)(12.0\text{ s})^2 = 945\text{ m}$.

(b) Find the maximum height of the canister above its release point: $v_{0y} = +39.4\text{ m/s}$, $v_y = 0$, $a_y = -9.80\text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (39.4\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 79.2\text{ m}$. After its release the canister travels upward 79.2 m to its maximum height and then back down $79.2\text{ m} + 235\text{ m}$ to the ground. The total distance it travels is 393 m.

EVALUATE: The speed of the rocket at the instant that the canister returns to the launch pad is $v_y = v_{0y} + a_y t = 39.4\text{ m/s} + (3.30\text{ m/s}^2)(12.0\text{ s}) = 79.0\text{ m/s}$. We can calculate its height at this instant by $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_{0y} = 0$ and $v_y = 79.0\text{ m/s}$. $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(79.0\text{ m/s})^2}{2(3.30\text{ m/s}^2)} = 946\text{ m}$, which agrees with our previous calculation.

2.94. **IDENTIFY:** Both objects are in free-fall and move with constant acceleration 9.80 m/s^2, downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = 2\ 9.80\text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball *A* be the one thrown straight up and ball *B* be the one dropped from rest at height *H*. $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$. $y_A = y_B$ gives $v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

(b) For ball *A* at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_0}{g}$. Setting this equal to the time in part (a) gives $\dfrac{H}{v_0} = \dfrac{v_0}{g}$ and $H = \dfrac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \dfrac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H\left(1 - \dfrac{gH}{2v_0^2}\right)$.

H must be less than $\dfrac{2v_0^2}{g}$ in order for the balls to collide before ball *A* returns to the ground. This is because it takes ball *A* time $t = \dfrac{2v_0}{g}$ to return to the ground and ball *B* falls a distance $\frac{1}{2}gt^2 = \dfrac{2v_0^2}{g}$ during this time. When $H = \dfrac{2v_0^2}{g}$ the two balls collide just as ball *A* reaches the ground and for *H* greater than this ball *A* reaches the ground before they collide.

2.95. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \dfrac{dv_{Bx}}{dt} - 2\gamma - 6\delta t$

(a) IDENTIFY and **SET UP:** The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car *A* moves ahead.

(b) IDENTIFY and **SET UP:** Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by *t*: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{2\delta}\left(-(\beta-\gamma) \pm \sqrt{(\beta-\gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00\text{ s} \pm 1.73\text{ s}$$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

EVALUATE: Car *A* has constant, positive a_x. Its v_x is positive and increasing. Car *B* has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car *B* initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car *B* has overtaken car *A* and then passes it. At $t = 5.73$ s, car *B* is moving in the $-x$-direction as it passes car *A* again.

(c) IDENTIFY: The distance from *A* to *B* is $x_B - x_A$. The rate of change of this distance is $\dfrac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\dfrac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between *A* and *B* is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\text{ s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00$ s and $t = 4.33$ s.

EVALUATE: At $t = 1.00$ s, $v_{Ax} = v_{Bx} = 5.00$ m/s. At $t = 4.33$ s, $v_{Ax} = v_{Bx} = 13.0$ m/s. Now car *B* is slowing down while *A* continues to speed up, so their velocities aren't ever equal again.

(d) IDENTIFY and **SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \dfrac{\gamma - \beta}{3\delta} = \dfrac{2.80\text{ m/s}^2 - 1.20\text{ m/s}^2}{3(0.20\text{ m/s}^3)} = 2.67$ s.

EVALUATE: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67$ s but for all times after that $a_{Bx} < a_{Ax}$.

2.96. IDENTIFY: Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\max}/2$ on the way down equals the time spent above $y_{\max}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\max}/2$ when he is a distance $y_{\max}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\max}/2$ above the floor is given by $y_{\max}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\max} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\max}/2$ to the floor is $t_2 = t_{tot} - t_1$.

$$\frac{2t_1}{t_2} = \frac{\sqrt{y_{\max}/2}}{\sqrt{y_{\max}} - \sqrt{y_{\max}/2}} = \frac{1}{\sqrt{2} - 1} = 4.8.$$

EVALUATE: The person spends over twice as long above $y_{\max}/2$ as below $y_{\max}/2$. His average speed is less above $y_{\max}/2$ than it is when he is below this height.

2.97. IDENTIFY: Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times *t* gives $t = \dfrac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{(0.170\text{ m/s}^2)}\left((5.0\text{ m/s}) \pm \sqrt{(5.0\text{ m/s})^2 - 2(0.170\text{ m/s}^2)(40.0\text{ m})}\right) = 9.55\text{ s and } 49.3\text{ s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\text{ m/s})(9.55\text{ s}) = 47.8$ m.

(b) The speed of the bus is $(0.170\text{ m/s}^2)(9.55\text{ s}) = 1.62$ m/s.

(c) The results can be verified by noting that the *x* lines for the student and the bus intersect at two points, as shown in Figure 2.97a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\text{ m/s}^2)(49.3\text{ s}) = 8.38$ m/s.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at 3.5 m/s, Figure 2.97b shows that the two lines do *not* intersect:

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. And so the minimum speed is $\sqrt{2(0.170\text{ m/s}^2)(40\text{ m/s})} = 3.688$ m/s. She would be running for a time $\dfrac{3.69\text{ m/s}}{0.170\text{ m/s}^2} = 21.7$ s, and covers a distance $(3.688\text{ m/s})(21.7\text{ s}) = 80.0$ m.

However, when the student runs at 3.688 m/s, the lines intersect at *one* point, at $x = 80$ m, as shown in Figure 2.97c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{\text{tot}} - t_1$

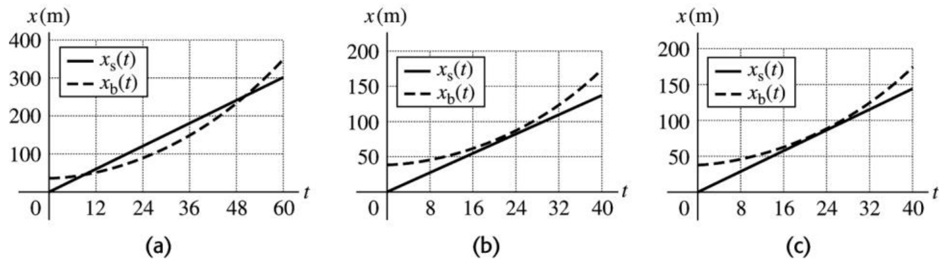


Figure 2.97

2.98. **IDENTIFY:** Apply constant acceleration equations to the motion of the boulder.

SET UP: Let $+y$ be downward, so $a_y = +g$.

EXECUTE: **(a)** Let the height be h and denote the 1.30-s interval as Δt; the simultaneous equations $h = \frac{1}{2}gt^2, \frac{2}{3}h = \frac{1}{2}g(t-\Delta t)^2$ can be solved for t. Eliminating h and taking the square root, $\frac{t}{t-\Delta t} = \sqrt{\frac{3}{2}}$, and $t = \frac{\Delta t}{1-\sqrt{2/3}}$, and substitution into $h = \frac{1}{2}gt^2$ gives $h = 246$ m.

(b) The above method assumed that $t > 0$ when the square root was taken. The negative root (with $\Delta t = 0$) gives an answer of 2.51 m, clearly not a "cliff." This would correspond to an object that was initially near the bottom of this "cliff" being thrown upward and taking 1.30 s to rise to the top and fall to the bottom. Although physically possible, the conditions of the problem preclude this answer.

EVALUATE: For the first two-thirds of the distance, $y - y_0 = 164$ m, $v_{0y} = 0$, and $a_y = 9.80\text{ m/s}^2$. $v_y = \sqrt{2a_y(y-y_0)} = 56.7$ m/s. Then for the last third of the distance, $y - y_0 = 82.0$ m, $v_{0y} = 56.7$ m/s and $a_y = 9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $(4.90\text{ m/s}^2)t^2 + (56.7\text{ m/s})t - 82.0\text{ m} = 0$.

$t = \frac{1}{9.8}\left(-56.7 + \sqrt{(56.7)^2 + 4(4.9)(82.0)}\right)\text{ s} = 1.30\text{ s}$, as required.

2.99. **IDENTIFY:** Apply constant acceleration equations to both objects.

SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t-t_0)^2$. In this case, $t_0 = 1.00$ s.

EXECUTE: **(a)** Setting $y_1 = y_2 = 0$, expanding the binomial $(t-t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0t = gt_0t - \frac{1}{2}gt_0^2$. Solving for t: $t = \frac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \frac{t_0}{2}\left(\frac{1}{1-v_0/(gt_0)}\right)$.

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for h as a function of v_0 yields, after some algebra, $h = \frac{1}{2}gt_0^2\frac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}$. Using the given value $t_0 = 1.00$ s and $g = 9.80\text{ m/s}^2$,

$h = 20.0\text{ m} = (4.9\text{ m})\left(\frac{4.9\text{ m/s} - v_0}{9.8\text{ m/s} - v_0}\right)^2$.

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields 8.2 m/s. The graph of *y* versus *t* for each ball is given in Figure 2.99.

(b) The above expression gives for (i), 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches 9.8 m/s, the height *h* becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\text{ m/s}$, the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9\text{ m/s}$, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than v_{min} and less than v_{max}.

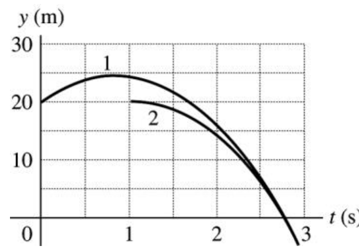


Figure 2.99