

## Ch. 2 Trigonometric Functions

### 2.1 Angles and Their Measure

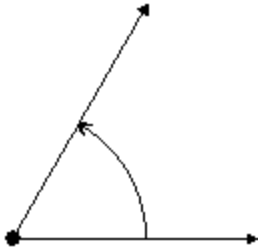
#### 1 Convert between Decimals and Degrees, Minutes, Seconds Measures for Angles

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

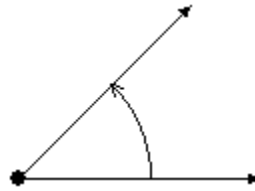
**Draw the angle.**

1)  $60^\circ$

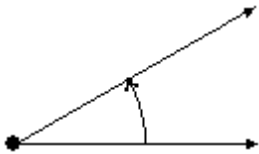
A)



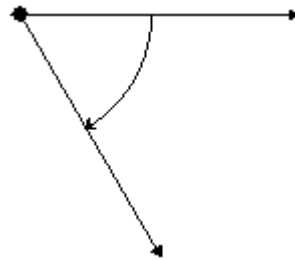
B)



C)

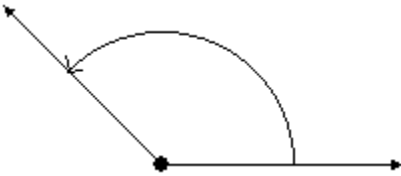


D)

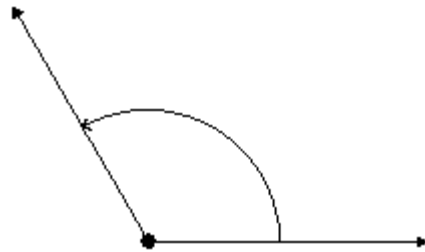


2)  $135^\circ$

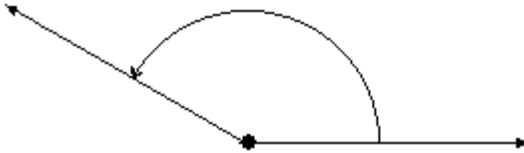
A)



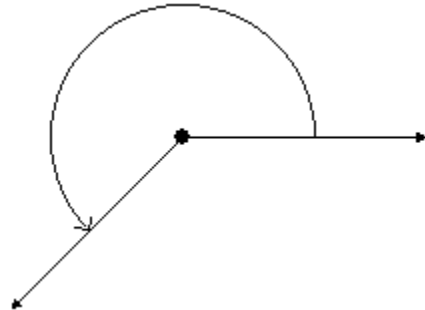
B)



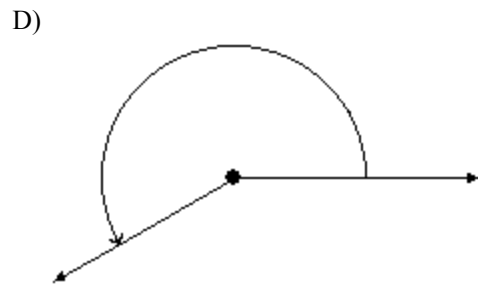
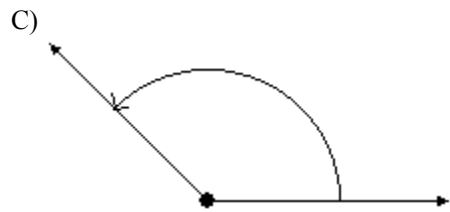
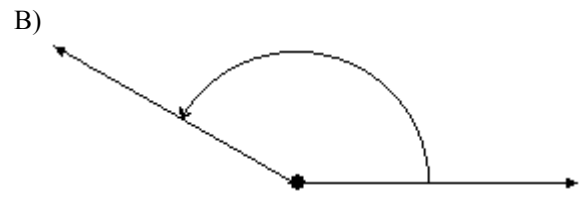
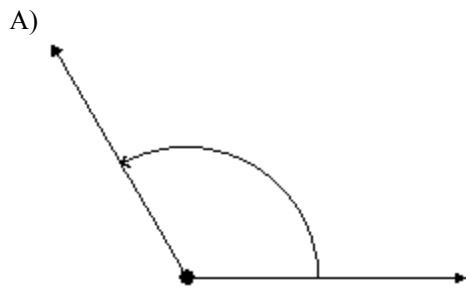
C)



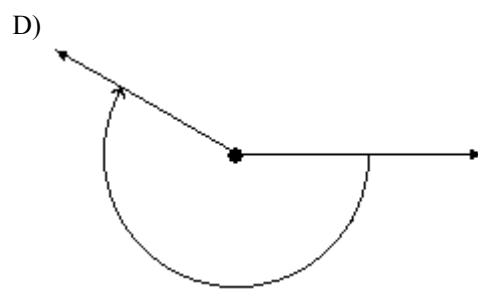
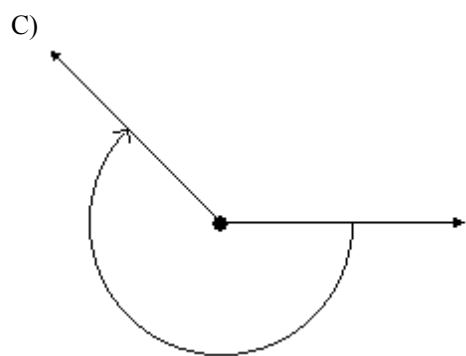
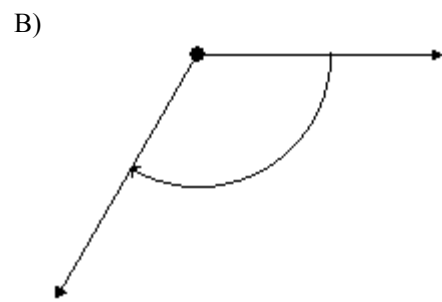
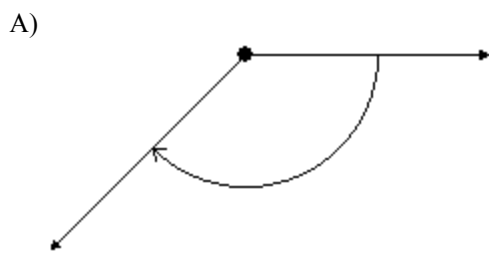
D)



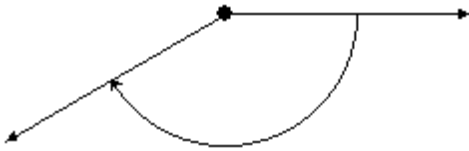
3)  $\frac{2\pi}{3}$



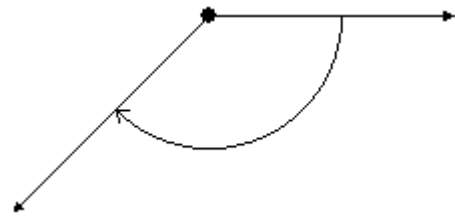
4)  $-\frac{3\pi}{4}$



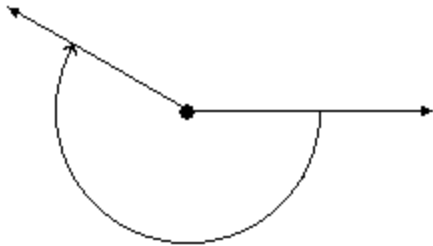
5)  $-150^\circ$   
A)



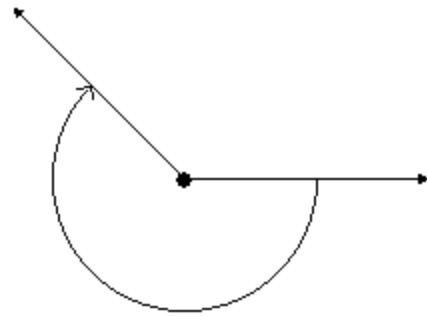
B)



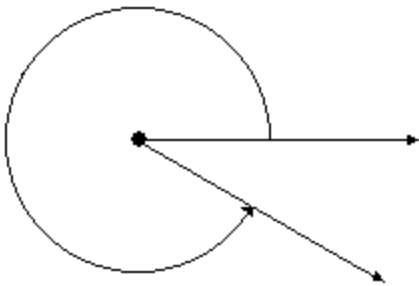
C)



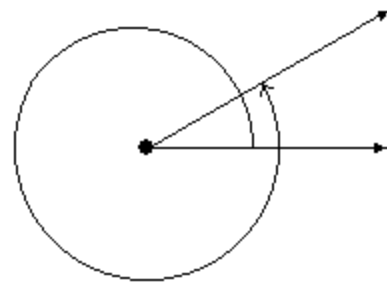
D)



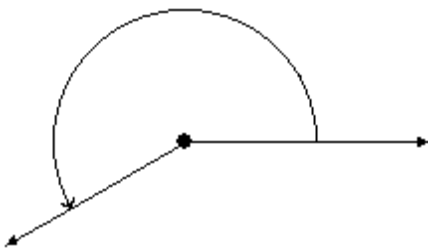
6)  $330^\circ$   
A)



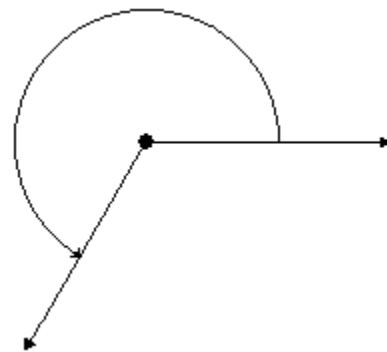
B)



C)

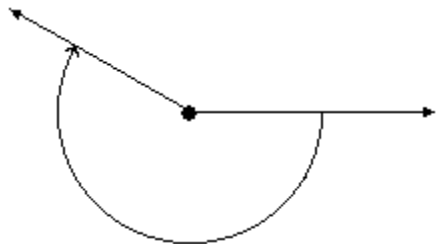


D)

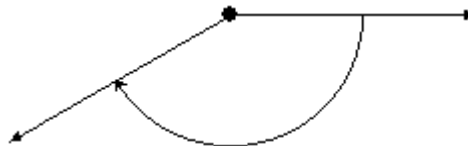


7)  $-\frac{7\pi}{6}$

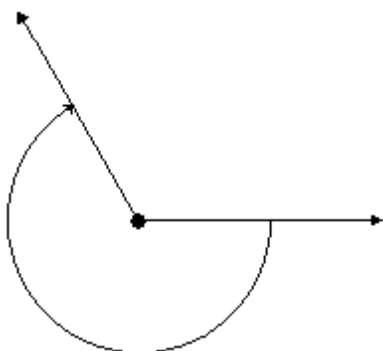
A)



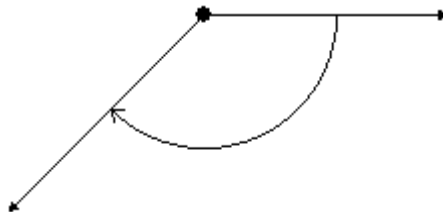
B)



C)

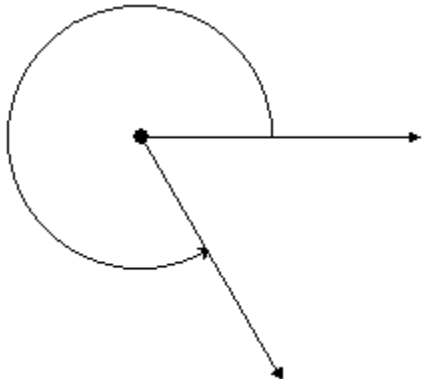


D)

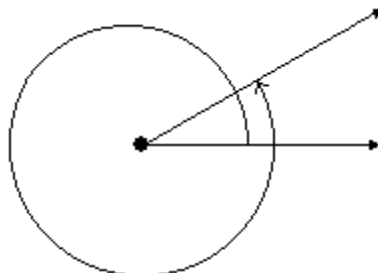


8)  $\frac{5\pi}{3}$

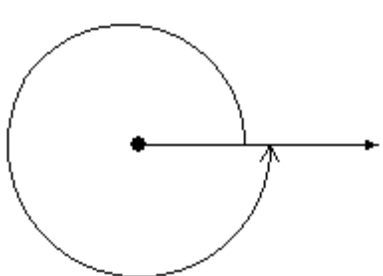
A)



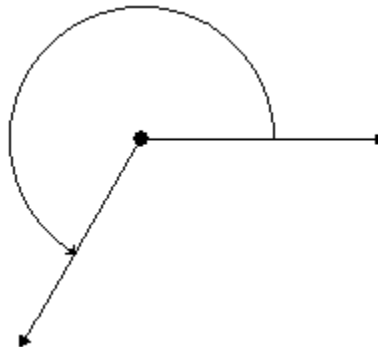
B)



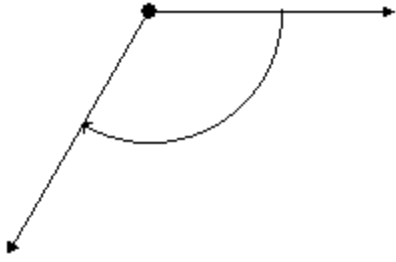
C)



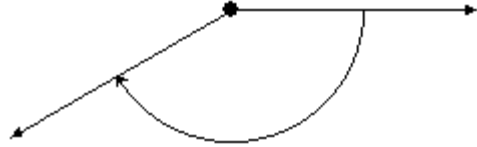
D)



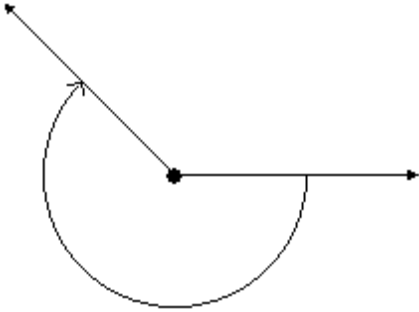
9)  $-120^\circ$   
A)



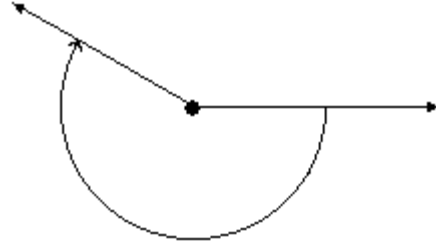
B)



C)

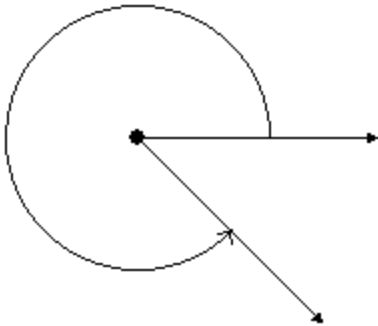


D)

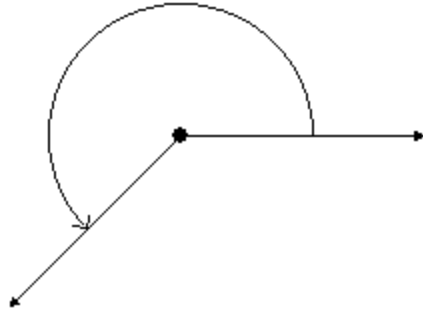


10)  $\frac{7\pi}{4}$

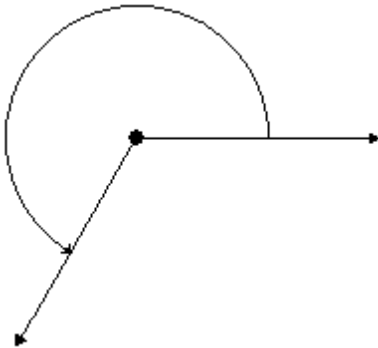
A)



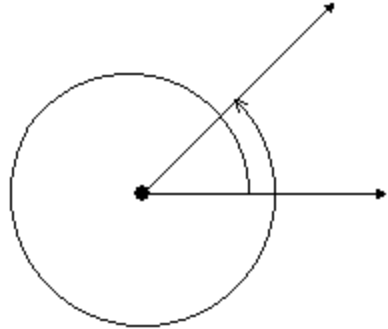
B)



C)



D)



Convert the angle to a decimal in degrees. Round the answer to two decimal places.

11)  $91^\circ 4' 21''$

A)  $91.07^\circ$

B)  $91.08^\circ$

C)  $91.03^\circ$

D)  $91.13^\circ$

- 12)  $161^{\circ}20'12''$   
 A)  $161.34^{\circ}$                       B)  $161.35^{\circ}$                       C)  $161.30^{\circ}$                       D)  $161.40^{\circ}$
- 13)  $355^{\circ}43'32''$   
 A)  $355.73^{\circ}$                       B)  $355.74^{\circ}$                       C)  $355.69^{\circ}$                       D)  $355.79^{\circ}$
- 14)  $23^{\circ}47'37''$   
 A)  $23.79^{\circ}$                       B)  $23.84^{\circ}$                       C)  $23.52^{\circ}$                       D)  $23.94^{\circ}$
- 15)  $21^{\circ}17'34''$   
 A)  $21.29^{\circ}$                       B)  $21.34^{\circ}$                       C)  $21.22^{\circ}$                       D)  $21.37^{\circ}$

**Convert the angle to D° M' S'' form. Round the answer to the nearest second.**

- 16)  $11.34^{\circ}$   
 A)  $11^{\circ}20'24''$                       B)  $11^{\circ}20'30''$                       C)  $11^{\circ}20'12''$                       D)  $11^{\circ}20'34''$
- 17)  $179.98^{\circ}$   
 A)  $179^{\circ}58'48''$                       B)  $179^{\circ}59'48''$                       C)  $179^{\circ}56'98''$                       D)  $179^{\circ}58'98''$
- 18)  $279.94^{\circ}$   
 A)  $279^{\circ}56'24''$                       B)  $279^{\circ}56'94''$                       C)  $279^{\circ}23'94''$                       D)  $279^{\circ}57'23''$

## 2 Find the Length of an Arc of a Circle

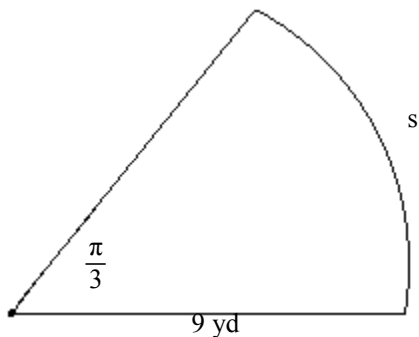
**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**If  $s$  denotes the length of the arc of a circle of radius  $r$  subtended by a central angle  $\theta$ , find the missing quantity.**

- 1)  $r = 16.07$  centimeters,  $\theta = 3.7$  radians,  $s = ?$   
 A) 59.5 cm                      B) 60.5 cm                      C) 58.5 cm                      D) 61.5 cm
- 2)  $r = 8.7$  inches,  $\theta = 225^{\circ}$ ,  $s = ?$   
 A) 34.2 in.                      B) 34.3 in.                      C) 34.4 in.                      D) 34.5 in.
- 3)  $r = \frac{2}{5}$  feet,  $s = 16$  feet,  $\theta = ?$   
 A) 40 radians                      B)  $\frac{32}{5}$  radians                      C)  $40^{\circ}$                       D)  $\frac{32}{5}^{\circ}$
- 4)  $s = 6.6$  meters,  $\theta = 3$  radians,  $r = ?$   
 A) 2.2 m                      B) 0.45 m                      C) 1.6 m                      D) 1.1 m

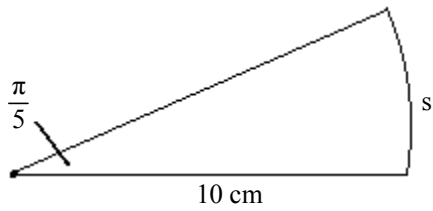
**Find the length  $s$ . Round the answer to three decimal places.**

5)



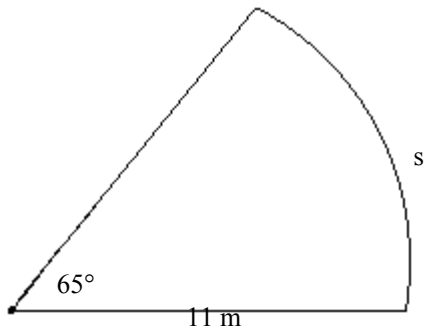
- A) 9.425 yd                      B) 1.047 yd                      C) 18.85 yd                      D) 8.594 yd

6)



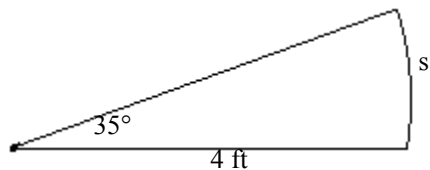
- A) 6.283 cm      B) 1.571 cm      C) 12.566 cm      D) 15.915 cm

7)



- A) 12.479 m      B) 13.727 m      C) 11.231 m      D) 9.983 m

8)



- A) 2.443 ft      B) 2.687 ft      C) 2.199 ft      D) 1.954 ft

**Solve the problem.**

- 9) For a circle of radius 4 feet, find the arc length  $s$  subtended by a central angle of  $30^\circ$ . Round to the nearest hundredth.  
 A) 376.99 ft      B) 6.28 ft      C) 2.09 ft      D) 4.19 ft
- 10) For a circle of radius 4 feet, find the arc length  $s$  subtended by a central angle of  $60^\circ$ . Round to the nearest hundredth.  
 A) 4.19 ft      B) 4.25 ft      C) 4.35 ft      D) 4.40 ft
- 11) A ship in the Atlantic Ocean measures its position to be  $21^\circ 20'$  north latitude. Another ship is reported to be due north of the first ship at  $43^\circ 21'$  north latitude. Approximately how far apart are the two ships? Round to the nearest mile. Assume that the radius of the Earth is 3960 miles.  
 A) 1522 mi      B) 87,186 mi      C) 1536 mi      D) 87,200 mi

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

- 12) Salt Lake City, Utah, is due north of Flagstaff, Arizona. Find the distance between Salt Lake City ( $40^\circ 45'$  north latitude) and Flagstaff ( $35^\circ 16'$  north latitude). Assume that the radius of the Earth is 3960 miles. Round to nearest whole mile.

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

- 13) The minute hand of a clock is 5 inches long. How far does the tip of the minute hand move in 30 minutes? If necessary, round the answer to two decimal places.  
 A) 15.71 in.      B) 18.22 in.      C) 16.94 in.      D) 13.97 in.

14) A pendulum swings through an angle of  $40^\circ$  each second. If the pendulum is 45 inches long, how far does its tip move each second? If necessary, round the answer to two decimal places.

A) 31.42 in.

B) 33.85 in.

C) 32.71 in.

D) 29.57 in.

### 3 Convert from Degrees to Radians and from Radians to Degrees

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Convert the angle in degrees to radians. Express the answer as multiple of  $\pi$ .**

1)  $90^\circ$

A)  $\frac{\pi}{2}$

B)  $\frac{\pi}{3}$

C)  $\frac{\pi}{4}$

D)  $\frac{\pi}{8}$

2)  $-90^\circ$

A)  $-\frac{\pi}{2}$

B)  $-\frac{\pi}{3}$

C)  $-\frac{\pi}{4}$

D)  $-\frac{\pi}{8}$

3)  $144^\circ$

A)  $\frac{4\pi}{5}$

B)  $\frac{5\pi}{6}$

C)  $\frac{3\pi}{4}$

D)  $\frac{5\pi}{4}$

4)  $-162^\circ$

A)  $-\frac{9\pi}{10}$

B)  $-\frac{10\pi}{11}$

C)  $-\frac{8\pi}{9}$

D)  $-\frac{10\pi}{9}$

5)  $87^\circ$

A)  $\frac{29\pi}{60}$

B)  $\frac{29\pi}{90}$

C)  $\frac{29\pi}{30}$

D)  $\frac{29\pi}{120}$

6)  $6^\circ$

A)  $\frac{\pi}{30}$

B)  $\frac{\pi}{60}$

C)  $\frac{\pi}{15}$

D)  $\frac{\pi}{18}$

**Convert the angle in radians to degrees.**

7)  $\frac{10\pi}{7}$

A)  $257.14^\circ$

B)  $258.14^\circ$

C)  $259.14^\circ$

D)  $256.14^\circ$

8)  $-\frac{5\pi}{4}$

A)  $-225^\circ$

B)  $-226^\circ$

C)  $-227^\circ$

D)  $-224^\circ$

9)  $\frac{\pi}{3}$

A)  $60^\circ$

B)  $1^\circ$

C)  $3^\circ$

D)  $60\pi^\circ$

10)  $-\frac{\pi}{4}$

A)  $-45^\circ$

B)  $-1^\circ$

C)  $1^\circ$

D)  $-45\pi^\circ$



11)  $\frac{11\pi}{4}$

A)  $495^\circ$

B)  $990^\circ$

C)  $164^\circ$

D)  $65\pi^\circ$

12)  $-\frac{59}{18}\pi$

A)  $-590^\circ$

B)  $-1180\pi^\circ$

C)  $-10^\circ$

D)  $-295^\circ$

13)  $\frac{\pi}{6}$

A)  $30^\circ$

B)  $60^\circ$

C)  $1080^\circ$

D)  $15^\circ$

14)  $\frac{11\pi}{12}$

A)  $165^\circ$

B)  $160^\circ$

C)  $150^\circ$

D)  $210^\circ$

**Convert the angle in degrees to radians. Express the answer in decimal form, rounded to two decimal places.**

15)  $345^\circ$

A) 6.02

B) 6.01

C) 6

D) 5.99

16)  $-328^\circ$

A) -5.72

B) -5.71

C) -5.7

D) -5.69

**Convert the angle in radians to degrees. Express the answer in decimal form, rounded to two decimal places.**

17) 2

A)  $114.59^\circ$

B)  $115.5^\circ$

C)  $0.03^\circ$

D)  $-0.02^\circ$

18) 8.71

A)  $499.05^\circ$

B)  $497.77^\circ$

C)  $0.15^\circ$

D)  $0.09^\circ$

19)  $\sqrt{5}$

A)  $128.12^\circ$

B)  $127.4^\circ$

C)  $0.04^\circ$

D)  $-0.11^\circ$

#### 4 Find the Area of a Sector of a Circle

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**If A denotes the area of the sector of a circle of radius r formed by the central angle  $\theta$ , find the missing quantity. If necessary, round the answer to two decimal places.**

1)  $r = 18$  inches,  $\theta = \frac{\pi}{3}$  radians,  $A = ?$

A)  $169.56 \text{ in}^2$

B)  $339.12 \text{ in}^2$

C)  $9.42 \text{ in}^2$

D)  $18.84 \text{ in}^2$

2)  $r = 14$  feet,  $A = 90$  square feet,  $\theta = ?$

A) 0.92 radians

B) 0.46 radians

C) 8820 radians

D) 17,640 radians

3)  $\theta = 5$  radians,  $A = 84$  square meters,  $r = ?$

A) 5.8 m

B) 840 m

C) 210 m

D) 14.49 m

4)  $r = 19$  inches,  $\theta = 90^\circ$ ,  $A = ?$

A)  $283.39 \text{ in}^2$

B)  $566.77 \text{ in}^2$

C)  $14.92 \text{ in}^2$

D)  $29.83 \text{ in}^2$

5)  $r = 8$  feet,  $A = 51$  square feet,  $\theta = ?$

A)  $91.36^\circ$

B)  $45.68^\circ$

C)  $93,554.14^\circ$

D)  $187,108.28^\circ$

6)  $\theta = 60^\circ$ ,  $A = 46$  square meters,  $r = ?$

A) 9.38 m

B) 96.29 m

C) 24.07 m

D) 4.91 m

7)  $r = 26.8$  centimeters,  $\theta = \frac{\pi}{10}$  radians,  $A = ?$

A) 112.8  $\text{cm}^2$

B) 4.2  $\text{cm}^2$

C) 35.9  $\text{cm}^2$

D) 225.6  $\text{cm}^2$

8)  $r = 28.8$  feet,  $\theta = 34.786^\circ$ ,  $A = ?$

A) 251.79  $\text{ft}^2$

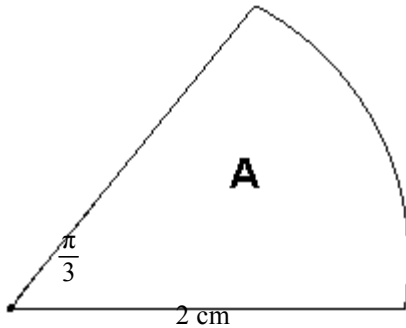
B) 254.79  $\text{ft}^2$

C) 503.58  $\text{ft}^2$

D) 506.58  $\text{ft}^2$

Find the area  $A$ . Round the answer to three decimal places.

9)



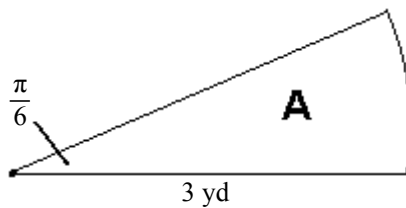
A) 2.094  $\text{cm}^2$

B) 1.047  $\text{cm}^2$

C) 4.189  $\text{cm}^2$

D) 1.333  $\text{cm}^2$

10)



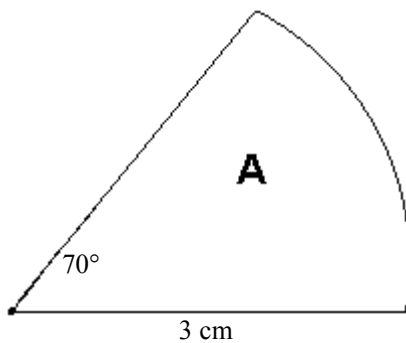
A) 2.356  $\text{yd}^2$

B) 0.785  $\text{yd}^2$

C) 4.712  $\text{yd}^2$

D) 1.5  $\text{yd}^2$

11)



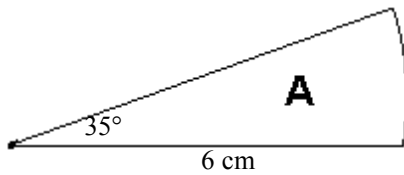
A) 5.498  $\text{cm}^2$

B) 1.833  $\text{cm}^2$

C) 10.996  $\text{cm}^2$

D) 1.75  $\text{cm}^2$

12)



- A)  $10.996\text{ cm}^2$       B)  $1.833\text{ cm}^2$       C)  $21.991\text{ cm}^2$       D)  $3.5\text{ cm}^2$

**Solve the problem.**

- 13) A circle has a radius of 11 centimeters. Find the area of the sector of the circle formed by an angle of  $60^\circ$ . If necessary, round the answer to two decimal places.  
A)  $63.36\text{ cm}^2$       B)  $5.76\text{ cm}^2$       C)  $126.71\text{ cm}^2$       D)  $20.17\text{ cm}^2$
- 14) An irrigation sprinkler in a field of lettuce sprays water over a distance of 35 feet as it rotates through an angle of  $160^\circ$ . What area of the field receives water? If necessary, round the answer to two decimal places.  
A)  $1710.42\text{ ft}^2$       B)  $48.87\text{ ft}^2$       C)  $3420.85\text{ ft}^2$       D)  $544.44\text{ ft}^2$
- 15) As part of an experiment to test different liquid fertilizers, a sprinkler has to be set to cover an area of 120 square yards in the shape of a sector of a circle of radius 40 yards. Through what angle should the sprinkler be set to rotate? If necessary, round the answer to two decimal places.  
A)  $8.59^\circ$       B)  $4.3^\circ$       C)  $6.44^\circ$       D)  $27^\circ$
- 16) The blade of a windshield wiper sweeps out an angle of  $135^\circ$  in one cycle. The base of the blade is 12 inches from the pivot point and the tip is 32 inches from the pivot point. What area does the wiper cover in one cycle? (Round to the nearest 0.1 square inch.)  
A)  $1036.7\text{ in}^2$       B)  $1105.3\text{ in}^2$       C)  $1041.8\text{ in}^2$       D)  $948.3\text{ in}^2$

**5 Find the Linear Speed of an Object Traveling in Circular Motion**

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Solve the problem.**

- 1) An object is traveling around a circle with a radius of 10 centimeters. If in 20 seconds a central angle of  $\frac{1}{3}$  radian is swept out, what is the linear speed of the object?  
A)  $\frac{1}{6}\text{ cm/sec}$       B)  $6\text{ cm/sec}$       C)  $\frac{1}{6}\text{ radians/sec}$       D)  $6\text{ radians/sec}$
- 2) An object is traveling around a circle with a radius of 20 meters. If in 10 seconds a central angle of  $\frac{1}{5}$  radian is swept out, what is the linear speed of the object?  
A)  $\frac{2}{5}\text{ m/sec}$       B)  $\frac{1}{5}\text{ m/sec}$       C)  $\frac{1}{4}\text{ m/sec}$       D)  $\frac{1}{8}\text{ m/sec}$
- 3) An object is traveling around a circle with a radius of 10 meters. If in 15 seconds a central angle of 3 radians is swept out, what is the linear speed of the object?  
A)  $2\text{ m/sec}$       B)  $3\text{ m/sec}$       C)  $\frac{2}{3}\text{ m/sec}$       D)  $\frac{1}{3}\text{ m/sec}$
- 4) A weight hangs from a rope 20 feet long. It swings through an angle of  $27^\circ$  each second. How far does the weight travel each second? Round to the nearest 0.1 foot.  
A) 9.4 feet      B) 8.7 feet      C) 9.0 feet      D) 8.1 feet

- 5) A gear with a radius of 32 centimeters is turning at  $\frac{\pi}{9}$  radians/sec. What is the linear speed at a point on the outer edge of the gear?
- A)  $\frac{32\pi}{9}$  cm/sec      B)  $\frac{9\pi}{32}$  cm/sec      C)  $288\pi$  cm/sec      D)  $\frac{\pi}{288}$  cm/sec
- 6) A wheel of radius 5.3 feet is moving forward at 11 feet per second. How fast is the wheel rotating?
- A) 2.1 radians/sec      B) 0.48 radians/sec      C) 3.5 radians/sec      D) 0.59 radians/sec
- 7) A car is traveling at 42 mph. If its tires have a diameter of 28 inches, how fast are the car's tires turning? Express the answer in revolutions per minute. If necessary, round to two decimal places.
- A) 504.2 rpm      B) 3168 rpm      C) 488.2 rpm      D) 1008.4 rpm
- 8) A pick-up truck is fitted with new tires which have a diameter of 43 inches. How fast will the pick-up truck be moving when the wheels are rotating at 345 revolutions per minute? Express the answer in miles per hour rounded to the nearest whole number.
- A) 44 mph      B) 7 mph      C) 37 mph      D) 22 mph
- 9) The Earth rotates about its pole once every 24 hours. The distance from the pole to a location on Earth  $43^\circ$  north latitude is about 2896.2 miles. Therefore, a location on Earth at  $43^\circ$  north latitude is spinning on a circle of radius 2896.2 miles. Compute the linear speed on the surface of the Earth at  $43^\circ$  north latitude.
- A) 758 mph      B) 121 mph      C) 18,197 mph      D) 725 mph
- 10) To approximate the speed of a river, a circular paddle wheel with radius 0.43 feet is lowered into the water. If the current causes the wheel to rotate at a speed of 14 revolutions per minute, what is the speed of the current? If necessary, round to two decimal places.
- A) 0.43 mph      B) 0.07 mph      C) 37.82 mph      D) 0.21 mph

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

- 11) The four Galilean moons of Jupiter have orbital periods and mean distances from Jupiter given by the following table.

|          | Distance (km)       | Period (Earth hours) |
|----------|---------------------|----------------------|
| Io       | $4.214 \times 10^5$ | 42.460               |
| Europa   | $6.709 \times 10^5$ | 85.243               |
| Ganymede | $1.070 \times 10^6$ | 171.709              |
| Callisto | $1.883 \times 10^6$ | 400.536              |

Find the linear speed of each moon. Which is the fastest (in terms of linear speed)?

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

- 12) In a computer simulation, a satellite orbits around Earth at a distance from the Earth's surface of  $2.8 \times 10^4$  miles. The orbit is circular, and one revolution around Earth takes 10.7 days. Assuming the radius of the Earth is 3960 miles, find the linear speed of the satellite. Express the answer in miles per hour to the nearest whole mile.
- A) 782 mph      B) 16,812 mph      C) 124 mph      D) 685 mph
- 13) A carousel has a radius of 18 feet and takes 35 seconds to make one complete revolution. What is the linear speed of the carousel at its outside edge? If necessary, round the answer to two decimal places.
- A) 3.23 ft/sec      B) 0.51 ft/sec      C) 12.22 ft/sec      D) 113.1 ft/sec

## 2.2 Trigonometric Functions: Unit Circle Approach

### 1 Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

In the problem,  $t$  is a real number and  $P = (x, y)$  is the point on the unit circle that corresponds to  $t$ . Find the exact value of the indicated trigonometric function of  $t$ .

1)  $(\frac{2}{9}, \frac{\sqrt{77}}{9})$  Find  $\sin t$ .

A)  $\frac{\sqrt{77}}{9}$

B)  $\frac{2}{9}$

C)  $\frac{\sqrt{77}}{2}$

D)  $\frac{2\sqrt{77}}{77}$

2)  $(\frac{3}{4}, \frac{\sqrt{7}}{4})$  Find  $\tan t$ .

A)  $\frac{\sqrt{7}}{3}$

B)  $\frac{4}{3}$

C)  $\frac{\sqrt{7}}{4}$

D)  $\frac{3\sqrt{7}}{7}$

3)  $(\frac{\sqrt{55}}{8}, \frac{3}{8})$  Find  $\sec t$ .

A)  $\frac{8\sqrt{55}}{55}$

B)  $\frac{8}{3}$

C)  $\frac{\sqrt{55}}{3}$

D)  $\frac{3\sqrt{55}}{55}$

4)  $(-\frac{\sqrt{11}}{6}, \frac{5}{6})$  Find  $\cos t$ .

A)  $-\frac{\sqrt{11}}{6}$

B)  $\frac{5}{6}$

C)  $-\frac{\sqrt{11}}{5}$

D)  $-\frac{6\sqrt{11}}{11}$

5)  $(-\frac{\sqrt{11}}{6}, \frac{5}{6})$  Find  $\cot t$ .

A)  $-\frac{\sqrt{11}}{5}$

B)  $\frac{5}{6}$

C)  $\frac{\sqrt{11}}{6}$

D)  $-\frac{6}{5}$

6)  $(-\frac{\sqrt{21}}{5}, -\frac{2}{5})$  Find  $\sin t$ .

A)  $-\frac{2}{5}$

B)  $-\frac{\sqrt{21}}{5}$

C)  $-\frac{5\sqrt{21}}{21}$

D)  $\frac{5}{2}$

7)  $(-\frac{\sqrt{33}}{7}, -\frac{4}{7})$  Find  $\cot t$ .

A)  $\frac{\sqrt{33}}{4}$

B)  $-\frac{\sqrt{33}}{4}$

C)  $-\frac{4\sqrt{33}}{33}$

D)  $\frac{\sqrt{33}}{7}$

8)  $(\frac{4}{7}, -\frac{\sqrt{33}}{7})$  Find  $\csc t$ .

A)  $-\frac{7\sqrt{33}}{33}$

B)  $-\frac{\sqrt{33}}{7}$

C)  $\frac{\sqrt{33}}{4}$

D)  $\frac{\sqrt{33}}{7}$

9)  $(\frac{4}{7}, -\frac{\sqrt{33}}{7})$  Find  $\cos t$ .

A)  $\frac{4}{7}$

B)  $-\frac{\sqrt{33}}{7}$

C)  $\frac{\sqrt{33}}{7}$

D)  $-\frac{4}{7}$

10)  $(\frac{3}{7}, -\frac{2\sqrt{10}}{7})$  Find  $\csc t$ .

A)  $-\frac{7\sqrt{10}}{20}$

B)  $\frac{7}{3}$

C)  $-\frac{\sqrt{10}}{6}$

D)  $\frac{3}{7}$

## 2 Find the Exact Values of the Trigonometric Functions of Quadrantal Angles

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Find the exact value. Do not use a calculator.**

1)  $\sin 2\pi$

A) 0

B) 1

C)  $\frac{\sqrt{2}}{2}$

D) undefined

2)  $\cos 0$

A) 1

B) 0

C)  $\frac{\sqrt{2}}{2}$

D) undefined

3)  $\tan 0$

A) 0

B) 1

C)  $\frac{\sqrt{2}}{2}$

D) undefined

4)  $\cot 0$

A) 1

B) 0

C)  $\frac{\sqrt{2}}{2}$

D) undefined

5)  $\cot \frac{\pi}{2}$

A) 0

B) 1

C) -1

D) undefined

6)  $\tan \pi$

A) 0

B) 1

C) -1

D) undefined

7)  $\cos \pi$

A) -1

B) 0

C) 1

D) undefined

8)  $\cot \frac{3\pi}{2}$

A) 0

B) 1

C) -1

D) undefined

9)  $\tan (14\pi)$

A) 0

B) 1

C) -1

D) undefined

10)  $\cot (-\frac{\pi}{2})$

A) 0

B) 1

C) -1

D) undefined

11)  $\tan(-\pi)$   
A) 0

B) 1

C) -1

D) undefined

**3 Find the Exact Values of the Trigonometric Functions of  $\pi/4 = 45^\circ$**

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Find the exact value. Do not use a calculator.**

1)  $\sec \frac{\pi}{4}$

A)  $\sqrt{2}$

B)  $-\sqrt{2}$

C)  $\frac{\sqrt{2}}{2}$

D)  $\frac{2\sqrt{3}}{3}$

2)  $\csc 45^\circ$

A)  $\sqrt{2}$

B)  $\frac{\sqrt{2}}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\sqrt{3}$

**Find the exact value of the expression if  $\theta = 45^\circ$ . Do not use a calculator.**

3)  $f(\theta) = \tan \theta$  Find  $f(\theta)$ .

A) 1

B) -1

C) 0

D)  $\frac{\sqrt{3}}{3}$

4)  $g(\theta) = \sin \theta$  Find  $[g(\theta)]^2$ .

A)  $\frac{1}{2}$

B) 2

C)  $-\frac{\sqrt{2}}{2}$

D)  $\sqrt{2}$

5)  $f(\theta) = \sin \theta$  Find  $5f(\theta)$ .

A)  $\frac{5\sqrt{2}}{2}$

B)  $-\frac{5\sqrt{2}}{2}$

C)  $-\frac{\sqrt{2}}{2}$

D)  $\frac{\sqrt{2}}{2}$

6)  $g(\theta) = \sin \theta$  Find  $10g(\theta)$ .

A)  $5\sqrt{2}$

B)  $10\sqrt{2}$

C)  $-5\sqrt{2}$

D)  $-10\sqrt{2}$

**Solve the problem.**

7) If friction is ignored, the time  $t$  (in seconds) required for a block to slide down an inclined plane is given by the formula

$$t = \sqrt{\frac{2a}{g \sin \theta \cos \theta}}$$

where  $a$  is the length (in feet) of the base and  $g \approx 32$  feet per second per second is the acceleration of gravity. How long does it take a block to slide down an inclined plane with base  $a = 12$  when  $\theta = 45^\circ$ ? If necessary, round the answer to the nearest tenth of a second.

A) 1.2 sec

B) 1.3 sec

C) 0.3 sec

D) 1.5 sec

8) The force acting on a pendulum to bring it to its perpendicular resting point is called the restoring force. The restoring force  $F$ , in Newtons, acting on a string pendulum is given by the formula

$$F = mg \sin \theta$$

where  $m$  is the mass in kilograms of the pendulum's bob,  $g \approx 9.8$  meters per second per second is the acceleration due to gravity, and  $\theta$  is angle at which the pendulum is displaced from the perpendicular. What is the value of the restoring force when  $m = 0.6$  kilogram and  $\theta = 45^\circ$ ? If necessary, round the answer to the nearest tenth of a Newton.

A) 4.2 N

B) 5 N

C) 4 N

D) 3.9 N

**4 Find the Exact Values of the Trigonometric Functions of  $\pi/6 = 30^\circ$  and  $\pi/3 = 60^\circ$**

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Find the exact value. Do not use a calculator.**

1)  $\tan 30^\circ$

A)  $\frac{\sqrt{3}}{3}$

B)  $\sqrt{3}$

C)  $\frac{\sqrt{3}}{2}$

D) 1

2)  $\sin 60^\circ$

A)  $\frac{\sqrt{3}}{2}$

B)  $\frac{1}{2}$

C)  $\frac{\sqrt{3}}{3}$

D)  $\frac{\sqrt{2}}{2}$

3)  $\csc \frac{\pi}{6}$

A) 2

B)  $\frac{2\sqrt{3}}{3}$

C)  $\frac{1}{2}$

D)  $\sqrt{2}$

4)  $\csc \frac{\pi}{3}$

A)  $\frac{2\sqrt{3}}{3}$

B) 2

C)  $\frac{\sqrt{3}}{2}$

D)  $\sqrt{2}$

**Find the exact value of the expression. Do not use a calculator.**

5)  $\cot 45^\circ - \cos 45^\circ$

A)  $\frac{2 - \sqrt{2}}{2}$

B)  $\frac{2\sqrt{3} - 3\sqrt{2}}{6}$

C)  $\frac{2 - \sqrt{3}}{2}$

D)  $-\frac{\sqrt{3}}{6}$

6)  $\cot 60^\circ - \cos 45^\circ$

A)  $\frac{2\sqrt{3} - 3\sqrt{2}}{6}$

B)  $\frac{2 - \sqrt{2}}{2}$

C)  $\frac{2 - \sqrt{3}}{2}$

D)  $\frac{2\sqrt{2} - 3\sqrt{3}}{6}$

7)  $\cos 60^\circ + \tan 60^\circ$

A)  $\frac{1 + 2\sqrt{3}}{2}$

B)  $2\sqrt{3}$

C)  $\frac{1 + \sqrt{3}}{2}$

D)  $\frac{3\sqrt{3}}{2}$

8)  $\sin \frac{\pi}{3} - \cos \frac{\pi}{6}$

A) 0

B)  $\sqrt{3}$

C)  $\frac{\sqrt{3} - 1}{2}$

D) 1

9)  $\tan \frac{\pi}{6} - \cos \frac{\pi}{6}$

A)  $-\frac{\sqrt{3}}{6}$

B)  $\sqrt{3}$

C)  $-\frac{\sqrt{6}}{2}$

D)  $\frac{2\sqrt{3} - 3\sqrt{2}}{6}$

**Find the exact value of the expression if  $\theta = 30^\circ$ . Do not use a calculator.**

10)  $f(\theta) = \cos \theta$  Find  $f(\theta)$ .

A)  $\frac{\sqrt{3}}{2}$

B)  $\sqrt{3}$

C)  $\frac{2\sqrt{3}}{3}$

D)  $\frac{\sqrt{2}}{2}$



- 11)  $g(\theta) = \cos \theta$  Find  $g(2\theta)$ .  
 A)  $\frac{1}{2}$  B)  $\frac{\sqrt{3}}{2}$  C) 1 D)  $\sqrt{3}$
- 12)  $f(\theta) = \sin \theta$  Find  $[f(\theta)]^2$ .  
 A)  $\frac{1}{4}$  B)  $\frac{1}{2}$  C) 1 D)  $\frac{3}{4}$
- 13)  $g(\theta) = \sin \theta$  Find  $12g(\theta)$ .  
 A) 6 B)  $6\sqrt{3}$  C)  $-\frac{1}{2}$  D)  $-\frac{\sqrt{3}}{2}$
- 14)  $f(\theta) = \cos \theta$  Find  $5f(\theta)$ .  
 A)  $\frac{5\sqrt{3}}{2}$  B)  $\frac{5}{2}$  C)  $-\frac{1}{2}$  D)  $-\frac{\sqrt{3}}{2}$

**Find the exact value of the expression if  $\theta = 60^\circ$ . Do not use a calculator.**

- 15)  $f(\theta) = \sin \theta$  Find  $f(\theta)$ .  
 A)  $\frac{\sqrt{3}}{2}$  B)  $\frac{1}{2}$  C)  $\frac{\sqrt{3}}{3}$  D)  $\frac{\sqrt{2}}{2}$
- 16)  $g(\theta) = \cos \theta$  Find  $[g(\theta)]^2$ .  
 A)  $\frac{1}{4}$  B)  $\frac{\sqrt{3}}{2}$  C)  $\sqrt{3}$  D)  $\frac{3}{4}$
- 17)  $f(\theta) = \sin \theta$  Find  $8f(\theta)$ .  
 A)  $4\sqrt{3}$  B) 4 C)  $-\frac{1}{2}$  D)  $-\frac{\sqrt{3}}{2}$
- 18)  $g(\theta) = \cos \theta$  Find  $11g(\theta)$ .  
 A)  $\frac{11}{2}$  B)  $\frac{11\sqrt{3}}{2}$  C)  $-\frac{1}{2}$  D)  $-\frac{\sqrt{3}}{2}$

**Solve the problem.**

- 19) If friction is ignored, the time  $t$  (in seconds) required for a block to slide down an inclined plane is given by the formula

$$t = \sqrt{\frac{2a}{g \sin \theta \cos \theta}}$$

where  $a$  is the length (in feet) of the base and  $g \approx 32$  feet per second per second is the acceleration of gravity. How long does it take a block to slide down an inclined plane with base  $a = 8$  when  $\theta = 30^\circ$ ? If necessary, round the answer to the nearest tenth of a second.

- A) 1.1 sec B) 1.8 sec C) 0.3 sec D) 1.3 sec
- 20) The force acting on a pendulum to bring it to its perpendicular resting point is called the restoring force. The restoring force  $F$ , in Newtons, acting on a string pendulum is given by the formula
- $$F = mg \sin \theta$$
- where  $m$  is the mass in kilograms of the pendulum's bob,  $g \approx 9.8$  meters per second per second is the acceleration due to gravity, and  $\theta$  is angle at which the pendulum is displaced from the perpendicular. What is the value of the restoring force when  $m = 0.8$  kilogram and  $\theta = 60^\circ$ ? If necessary, round the answer to the nearest tenth of a Newton.
- A) 6.8 N B) 2.4 N C) 3.9 N D) 6.5 N

5 Find the Exact Values for Integer Multiples of  $\pi/6 = 30^\circ$ ,  $\pi/4 = 45^\circ$ , and  $\pi/3 = 60^\circ$

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Find the exact value. Do not use a calculator.

1)  $\cos \frac{14\pi}{3}$

A)  $-\frac{1}{2}$

B)  $-\frac{\sqrt{3}}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{1}{2}$

2)  $\sec \frac{19\pi}{4}$

A)  $-\sqrt{2}$

B)  $-\frac{2\sqrt{3}}{3}$

C)  $-2$

D)  $\frac{\sqrt{2}}{2}$

3)  $\sin 495^\circ$

A)  $\frac{\sqrt{2}}{2}$

B)  $\frac{1}{2}$

C)  $-\frac{1}{2}$

D)  $-\frac{\sqrt{2}}{2}$

4)  $\cot 570^\circ$

A)  $\sqrt{3}$

B)  $-\sqrt{3}$

C)  $\frac{\sqrt{3}}{3}$

D)  $-\frac{\sqrt{3}}{3}$

Find the exact value of the expression. Do not use a calculator.

5)  $\tan \frac{7\pi}{4} + \tan \frac{5\pi}{4}$

A)  $0$

B)  $\frac{2\sqrt{2}+1}{6}$

C)  $\frac{1}{2}$

D)  $\frac{\sqrt{2}+1}{2}$

6)  $\sin 135^\circ - \sin 270^\circ$

A)  $\frac{\sqrt{2}+2}{2}$

B)  $\frac{\sqrt{2}-2}{2}$

C)  $\frac{\sqrt{2}}{2}$

D)  $2$

7)  $\cos \frac{\pi}{3} + \tan \frac{5\pi}{3}$

A)  $\frac{1-2\sqrt{3}}{2}$

B)  $\frac{\sqrt{3}+1}{2}$

C)  $\frac{2\sqrt{3}+3}{6}$

D)  $\frac{\sqrt{3}+3}{3}$

8)  $\cos 120^\circ \tan 60^\circ$

A)  $-\frac{\sqrt{3}}{2}$

B)  $\frac{3}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $-\frac{1}{4}$

9)  $\tan 150^\circ \cos 210^\circ$

A)  $-\frac{5\sqrt{3}}{6}$

B)  $\frac{\sqrt{3}+1}{2}$

C)  $\frac{3\sqrt{3}+2\sqrt{3}}{6}$

D)  $\frac{2\sqrt{3}+3}{6}$

10)  $\sin 330^\circ \sin 270^\circ$

A)  $\frac{1}{2}$

B)  $-\frac{1}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $-\frac{\sqrt{3}}{2}$

## 6 Use a Calculator to Approximate the Value of a Trigonometric Function

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use a calculator to find the approximate value of the expression rounded to two decimal places.

1)  $\sin 4^\circ$   
A) 0.07                                      B) 0.16                                      C) -0.76                                      D) -0.67

2)  $\cos 82^\circ$   
A) 0.14                                      B) 0.26                                      C) 0.95                                      D) 1.07

3)  $\tan 50^\circ$   
A) 1.19                                      B) 1.33                                      C) -0.27                                      D) -0.13

4)  $\cos \frac{2\pi}{5}$   
A) 0.31                                      B) 0.41                                      C) 1.00                                      D) 1.10

5)  $\sec \frac{\pi}{12}$   
A) 1.04                                      B) 1.17                                      C) 1.00                                      D) 0.87

6)  $\csc 58^\circ$   
A) 1.18                                      B) 1.12                                      C) 1.01                                      D) 0.95

7)  $\cot \frac{\pi}{8}$   
A) 2.41                                      B) 2.32                                      C) 145.90                                      D) 145.81

8)  $\cot 0.1239$   
A) 8.03                                      B) 0.12                                      C) 0.99                                      D) 1.01

9)  $\cos 8$   
A) -0.15                                      B) 0.99                                      C) 0.15                                      D) -0.99

10)  $\cos 9^\circ$   
A) 0.99                                      B) 0.91                                      C) -0.99                                      D) -0.91

11)  $\tan 37^\circ$   
A) 0.75                                      B) -0.84                                      C) 0.80                                      D) 0.60

**Solve the problem.**

12) If friction is ignored, the time  $t$  (in seconds) required for a block to slide down an inclined plane is given by the formula

$$t = \sqrt{\frac{2a}{g \sin \theta \cos \theta}}$$

where  $a$  is the length (in feet) of the base and  $g \approx 32$  feet per second per second is the acceleration of gravity. How long does it take a block to slide down an inclined plane with base  $a = 11$  when  $\theta = 57^\circ$ ? If necessary, round the answer to the nearest tenth of a second.

A) 1.2 sec                                      B) 1.3 sec                                      C) 0.3 sec                                      D) 1.5 sec

- 13) The force acting on a pendulum to bring it to its perpendicular resting point is called the restoring force. The restoring force  $F$ , in Newtons, acting on a string pendulum is given by the formula

$$F = mg \sin \theta$$

where  $m$  is the mass in kilograms of the pendulum's bob,  $g \approx 9.8$  meters per second per second is the acceleration due to gravity, and  $\theta$  is angle at which the pendulum is displaced from the perpendicular. What is the value of the restoring force when  $m = 0.7$  kilogram and  $\theta = 72^\circ$ ? If necessary, round the answer to the nearest tenth of a Newton.

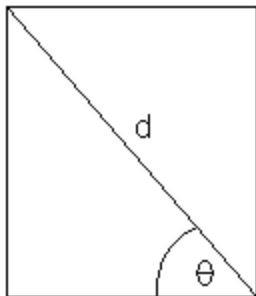
- A) 6.5 N                      B) 1.7 N                      C) 2.1 N                      D) 6.7 N

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

- 14) The strength  $S$  of a wooden beam with rectangular cross section is given by the formula

$$S = kd^3 \sin^2 \theta \cos \theta$$

where  $d$  is the diagonal length,  $\theta$  the angle illustrated, and  $k$  is a constant that varies with the type of wood used.



Let  $d = 1$  and express the strength  $S$  in terms of the constant  $k$  for  $\theta = 45^\circ, 50^\circ, 55^\circ, 60^\circ,$  and  $65^\circ$ . Does the strength always increase as  $\theta$  gets larger?

## 7 Use a Circle of Radius $r$ to Evaluate the Trigonometric Functions

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**A point on the terminal side of an angle  $\theta$  is given. Find the exact value of the indicated trigonometric function of  $\theta$ .**

- 1)  $(-3, 4)$  Find  $\sin \theta$ .  
 A)  $\frac{4}{5}$                       B)  $-\frac{3}{5}$                       C)  $-\frac{4}{5}$                       D)  $\frac{3}{5}$
- 2)  $(-4, 3)$  Find  $\cos \theta$ .  
 A)  $-\frac{4}{5}$                       B)  $\frac{3}{5}$                       C)  $\frac{4}{5}$                       D)  $-\frac{3}{5}$
- 3)  $(-\frac{1}{5}, \frac{1}{2})$  Find  $\cos \theta$ .  
 A)  $-\frac{2\sqrt{29}}{29}$                       B)  $\frac{5\sqrt{29}}{29}$                       C)  $-\frac{29}{5}$                       D)  $\frac{29}{2}$
- 4)  $(-5, -4)$  Find  $\tan \theta$ .  
 A)  $\frac{4}{5}$                       B)  $\frac{5}{4}$                       C)  $\frac{\sqrt{41}}{5}$                       D)  $-\frac{\sqrt{41}}{5}$
- 5)  $(4, -5)$  Find  $\cot \theta$ .  
 A)  $-\frac{4}{5}$                       B)  $-\frac{5}{4}$                       C)  $\frac{\sqrt{41}}{4}$                       D)  $-\frac{\sqrt{41}}{4}$

- 6)  $(-4, -1)$  Find  $\csc \theta$ .  
 A)  $-\sqrt{17}$  B)  $\sqrt{17}$  C)  $-4$  D)  $-17$
- 7)  $(-4, -1)$  Find  $\sec \theta$ .  
 A)  $-\frac{\sqrt{17}}{4}$  B)  $\frac{\sqrt{17}}{4}$  C)  $-\frac{3\sqrt{17}}{17}$  D)  $-\sqrt{17}$

**Solve the problem.**

- 8) If  $\sin \theta = 0.2$ , find  $\sin(\theta + \pi)$ .  
 A)  $-0.2$  B)  $0.2$  C)  $0.8$  D)  $-0.8$
- 9) If  $\sin \theta = \frac{1}{9}$ , find  $\csc \theta$ .  
 A)  $9$  B)  $-\frac{1}{9}$  C)  $\frac{8}{9}$  D) undefined

- 10) A racetrack curve is banked so that the outside of the curve is slightly elevated or inclined above the inside of the curve. This inclination is called the elevation of the track. The maximum speed on the track in miles per hour is given by  

$$\sqrt{r(29000 + 41000 \tan \theta)}$$
 where  $r$  is the radius of the track in miles and  $\theta$  is the elevation in degrees. Find the maximum speed for a racetrack with an elevation of  $25^\circ$  and a radius of  $0.5$  miles. Round to the nearest mile per hour.  
 A)  $155$  mph B)  $34,025$  mph C)  $16,635$  mph D)  $183$  mph

- 11) The path of a projectile fired at an inclination  $\theta$  to the horizontal with an initial speed  $v_0$  is a parabola. The range  $R$  of the projectile, the horizontal distance that the projectile travels, is found by the formula  $R = \frac{v_0^2 \sin 2\theta}{g}$  where  $g = 32.2$  feet per second per second or  $g = 9.8$  meters per second per second. Find the range of a projectile fired with an initial velocity of  $178$  feet per second at an angle of  $34^\circ$  to the horizontal. Round your answer to two decimal places.  
 A)  $912.33$  ft B)  $550.23$  ft C)  $1100.46$  ft D)  $912.23$  ft

## 2.3 Properties of the Trigonometric Functions

### 1 Determine the Domain and the Range of the Trigonometric Functions

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Solve the problem.**

- 1) What is the domain of the cosine function?  
 A) all real numbers  
 B) all real numbers from  $-1$  to  $1$ , inclusive  
 C) all real numbers, except odd multiples of  $\frac{\pi}{2}$  ( $90^\circ$ )  
 D) all real numbers, except integral multiples of  $\pi$  ( $180^\circ$ )
- 2) For what numbers  $\theta$  is  $f(\theta) = \sec \theta$  not defined?  
 A) odd multiples of  $\frac{\pi}{2}$  ( $90^\circ$ ) B) integral multiples of  $\pi$  ( $180^\circ$ )  
 C) odd multiples of  $\pi$  ( $180^\circ$ ) D) all real numbers

- 3) For what numbers  $\theta$  is  $f(\theta) = \csc \theta$  not defined?
- A) integral multiples of  $\pi$  ( $180^\circ$ )  
 B) odd multiples of  $\frac{\pi}{2}$  ( $90^\circ$ )  
 C) odd multiples of  $\pi$  ( $180^\circ$ )  
 D) all real numbers
- 4) What is the range of the cosine function?
- A) all real numbers from -1 to 1, inclusive  
 B) all real numbers  
 C) all real numbers greater than or equal to 1 or less than or equal to -1  
 D) all real numbers greater than or equal to 0
- 5) What is the range of the cotangent function?
- A) all real numbers  
 B) all real numbers from -1 to 1, inclusive  
 C) all real numbers greater than or equal to 1 or less than or equal to -1  
 D) all real numbers, except integral multiples of  $\pi(180^\circ)$
- 6) What is the range of the cosecant function?
- A) all real numbers greater than or equal to 1 or less than or equal to -1  
 B) all real numbers from -1 to 1, inclusive  
 C) all real numbers  
 D) all real numbers, except integral multiples of  $\pi(180^\circ)$

## 2 Determine the Period of the Trigonometric Functions

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use the fact that the trigonometric functions are periodic to find the exact value of the expression. Do not use a calculator.

- 1)  $\sin 405^\circ$
- A)  $\frac{\sqrt{2}}{2}$                       B)  $\frac{1}{2}$                       C)  $-\frac{1}{2}$                       D)  $-\frac{\sqrt{2}}{2}$
- 2)  $\tan 390^\circ$
- A)  $\frac{\sqrt{3}}{3}$                       B)  $\sqrt{3}$                       C)  $-\sqrt{3}$                       D)  $\frac{\sqrt{3}}{2}$
- 3)  $\csc 1020^\circ$
- A)  $-\frac{2\sqrt{3}}{3}$                       B)  $-\sqrt{2}$                       C)  $-\sqrt{3}$                       D)  $-\frac{1}{2}$
- 4)  $\cot 390^\circ$
- A)  $\sqrt{3}$                       B)  $-\sqrt{3}$                       C)  $\frac{\sqrt{3}}{3}$                       D)  $-\frac{\sqrt{3}}{3}$
- 5)  $\cot 540^\circ$
- A)  $\sqrt{3}$                       B) 0                      C) -1                      D) undefined
- 6)  $\tan 720^\circ$
- A) 0                      B)  $\frac{\sqrt{3}}{3}$                       C) -1                      D) undefined

7)  $\cos \frac{8\pi}{3}$

A)  $-\frac{1}{2}$

B)  $-\frac{\sqrt{3}}{2}$

C)  $\frac{\sqrt{3}}{2}$

D)  $\frac{1}{2}$

8)  $\sin \frac{17\pi}{3}$

A)  $-\frac{\sqrt{3}}{2}$

B)  $\frac{\sqrt{3}}{2}$

C)  $-1$

D)  $-\frac{1}{2}$

9)  $\tan \frac{21\pi}{4}$

A)  $1$

B)  $\sqrt{3}$

C)  $\frac{\sqrt{3}}{3}$

D)  $-1$

10)  $\sec \frac{11\pi}{4}$

A)  $-\sqrt{2}$

B)  $-\frac{2\sqrt{3}}{3}$

C)  $-2$

D)  $\frac{\sqrt{2}}{2}$

**Solve the problem.**

11) If  $\sin \theta = 0.8$ , find the value of  $\sin \theta + \sin (\theta + 2\pi) + \sin (\theta + 4\pi)$ .

A)  $2.4$

B)  $2.4 + 6\pi$

C)  $4.4$

D)  $0.8$

12) If  $\tan \theta = 3.8$ , find the value of  $\tan \theta + \tan (\theta + \pi) + \tan (\theta + 2\pi)$ .

A)  $11.4$

B)  $11.4 + 3\pi$

C)  $13.4$

D) undefined

13) If  $f(\theta) = \cos \theta$  and  $f(a) = \frac{1}{6}$ , find the exact value of  $f(a) + f(a + 2\pi) + f(a + 4\pi)$ .

A)  $\frac{1}{2}$

B)  $\frac{1}{2} + 6\pi$

C)  $\frac{5}{2}$

D)  $\frac{1}{6}$

14) If  $f(\theta) = \tan \theta$  and  $f(a) = 3$ , find the exact value of  $f(a) + f(a + \pi) + f(a + 3\pi)$ .

A)  $9$

B)  $9 + 4\pi$

C)  $3$

D) undefined

15) If  $f(\theta) = \cos \theta$  and  $f(a) = -\frac{1}{12}$ , find the exact value of  $f(a) + f(a - 2\pi) + f(a + 4\pi)$ .

A)  $-\frac{1}{4}$

B)  $-36$

C)  $-12$

D)  $-\frac{1}{12}$

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

16) If  $f(\theta) = \sin \theta$  and  $f(a) = -\frac{1}{9}$ , find the exact value of  $f(a) + f(a - 4\pi) + f(a - 2\pi)$ .

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

17) If  $\sin \theta = 0.5$ , find the value of  $\sin \theta + \sin (\theta + 2\pi) + \sin (\theta + 4\pi)$ .

A)  $1.5$

B)  $1.5 + 6\pi$

C)  $3.5$

D)  $0.5$

18) If  $\tan \theta = 1.0$ , find the value of  $\tan \theta + \tan (\theta + \pi) + \tan (\theta + 2\pi)$ .

A) 3

B)  $3 + 3\pi$

C) 5

D) undefined

### 3 Determine the Signs of the Trigonometric Functions in a Given Quadrant

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Name the quadrant in which the angle  $\theta$  lies.**

1)  $\tan \theta > 0$ ,  $\sin \theta < 0$

A) I

B) II

C) III

D) IV

2)  $\cos \theta < 0$ ,  $\csc \theta < 0$

A) I

B) II

C) III

D) IV

3)  $\sin \theta > 0$ ,  $\cos \theta < 0$

A) I

B) II

C) III

D) IV

4)  $\cot \theta < 0$ ,  $\cos \theta > 0$

A) I

B) II

C) III

D) IV

5)  $\csc \theta > 0$ ,  $\sec \theta > 0$

A) I

B) II

C) III

D) IV

6)  $\sec \theta < 0$ ,  $\tan \theta < 0$

A) I

B) II

C) III

D) IV

7)  $\tan \theta < 0$ ,  $\sin \theta < 0$

A) I

B) II

C) III

D) IV

8)  $\cos \theta > 0$ ,  $\csc \theta < 0$

A) I

B) II

C) III

D) IV

9)  $\cot \theta > 0$ ,  $\sin \theta < 0$

A) I

B) II

C) III

D) IV

10)  $\sin \theta > 0$ ,  $\cos \theta > 0$

A) I

B) II

C) III

D) IV

**Solve the problem.**

11) Which of the following trigonometric values are negative?

I.  $\sin(-292^\circ)$

II.  $\tan(-193^\circ)$

III.  $\cos(-207^\circ)$

IV.  $\cot 222^\circ$

A) II and III

B) III only

C) II, III, and IV

D) I and III

**SHORT ANSWER.** Write the word or phrase that best completes each statement or answers the question.

12) Determine the sign of the trigonometric values listed below.

(i)  $\sin 250^\circ$

(ii)  $\tan 330^\circ$

(iii)  $\cos(-40^\circ)$



#### 4 Find the Values of the Trigonometric Functions Using Fundamental Identities

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

In the problem,  $\sin \theta$  and  $\cos \theta$  are given. Find the exact value of the indicated trigonometric function.

1)  $\sin \theta = \frac{\sqrt{5}}{3}$ ,  $\cos \theta = \frac{2}{3}$  Find  $\tan \theta$ .

A)  $\frac{\sqrt{5}}{2}$  B)  $\frac{2\sqrt{5}}{5}$  C)  $\frac{3}{2}$  D)  $\frac{3\sqrt{5}}{5}$

2)  $\sin \theta = \frac{\sqrt{7}}{4}$ ,  $\cos \theta = \frac{3}{4}$  Find  $\cot \theta$ .

A)  $\frac{3\sqrt{7}}{7}$  B)  $\frac{\sqrt{7}}{3}$  C)  $\frac{4}{3}$  D)  $\frac{4\sqrt{7}}{7}$

3)  $\sin \theta = \frac{\sqrt{5}}{3}$ ,  $\cos \theta = \frac{2}{3}$  Find  $\sec \theta$ .

A)  $\frac{3}{2}$  B)  $\frac{\sqrt{5}}{2}$  C)  $\frac{2\sqrt{5}}{5}$  D)  $\frac{3\sqrt{5}}{5}$

4)  $\sin \theta = \frac{\sqrt{5}}{3}$ ,  $\cos \theta = \frac{2}{3}$  Find  $\csc \theta$ .

A)  $\frac{3\sqrt{5}}{5}$  B)  $\frac{\sqrt{5}}{2}$  C)  $\frac{2\sqrt{5}}{5}$  D)  $\frac{3}{2}$

Use the properties of the trigonometric functions to find the exact value of the expression. Do not use a calculator.

5)  $\sin^2 80^\circ + \cos^2 80^\circ$

A) 1 B) 0 C) 2 D) -1

6)  $\sec^2 20^\circ - \tan^2 20^\circ$

A) 1 B) 0 C) -1 D) 2

7)  $\cos 65^\circ \sec 65^\circ$

A) 1 B) 0 C) 65 D) -1

8)  $\tan 40^\circ - \frac{\sin 40^\circ}{\cos 40^\circ}$

A) 0 B) 1 C) 40 D) undefined

#### 5 Find Exact Values of the Trig Functions of an Angle Given One of the Functions and the Quadrant of the Angle

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Find the exact value of the indicated trigonometric function of  $\theta$ .

1)  $\tan \theta = -\frac{10}{7}$ ,  $\theta$  in quadrant II Find  $\cos \theta$ .

A)  $-\frac{7\sqrt{149}}{149}$  B)  $\frac{7\sqrt{149}}{149}$  C)  $\frac{\sqrt{149}}{10}$  D)  $-\frac{\sqrt{149}}{7}$

- 2)  $\csc \theta = -\frac{9}{2}$ ,  $\theta$  in quadrant III Find  $\cot \theta$ .
- A)  $\frac{\sqrt{77}}{2}$       B)  $-\frac{9\sqrt{77}}{77}$       C)  $-\frac{\sqrt{77}}{9}$       D)  $-\frac{2\sqrt{77}}{77}$
- 3)  $\sec \theta = \frac{3}{2}$ ,  $\theta$  in quadrant IV Find  $\tan \theta$ .
- A)  $-\frac{\sqrt{5}}{2}$       B)  $-\frac{\sqrt{5}}{3}$       C)  $-\frac{3}{2}$       D)  $-\sqrt{5}$
- 4)  $\tan \theta = \frac{15}{8}$ ,  $180^\circ < \theta < 270^\circ$  Find  $\cos \theta$ .
- A)  $-\frac{8}{17}$       B)  $-8$       C)  $-\frac{8\sqrt{23}}{23}$       D)  $\frac{15\sqrt{23}}{23}$
- 5)  $\cos \theta = \frac{7}{25}$ ,  $\frac{3\pi}{2} < \theta < 2\pi$  Find  $\cot \theta$ .
- A)  $-\frac{7}{24}$       B)  $\frac{-7\sqrt{2}}{6}$       C)  $-\frac{24}{7}$       D)  $\frac{25}{7}$
- 6)  $\cos \theta = \frac{2}{9}$ ,  $\tan \theta < 0$  Find  $\sin \theta$ .
- A)  $-\frac{\sqrt{77}}{9}$       B)  $-\frac{\sqrt{77}}{2}$       C)  $-\frac{9}{2}$       D)  $-\sqrt{77}$
- 7)  $\sin \theta = -\frac{2}{3}$ ,  $\tan \theta > 0$  Find  $\sec \theta$ .
- A)  $-\frac{3\sqrt{5}}{5}$       B)  $-\frac{\sqrt{5}}{3}$       C)  $\frac{\sqrt{3}}{2}$       D)  $-\frac{2\sqrt{5}}{5}$
- 8)  $\cot \theta = -\frac{7}{4}$ ,  $\cos \theta < 0$  Find  $\csc \theta$ .
- A)  $\frac{\sqrt{65}}{4}$       B)  $-\frac{7\sqrt{65}}{65}$       C)  $\frac{7\sqrt{65}}{65}$       D)  $-\frac{\sqrt{65}}{7}$
- 9)  $\sin \theta = \frac{1}{2}$ ,  $\sec \theta < 0$  Find  $\cos \theta$  and  $\tan \theta$ .
- A)  $\cos \theta = -\sqrt{3}$ ,  $\tan \theta = -\frac{10\sqrt{3}}{3}$       B)  $\cos \theta = \sqrt{\frac{3}{2}}$ ,  $\tan \theta = \frac{\sqrt{3}}{3}$
- C)  $\cos \theta = -\frac{\sqrt{3}}{2}$ ,  $\tan \theta = -\frac{\sqrt{3}}{3}$       D)  $\cos \theta = -\frac{\sqrt{3}}{2}$ ,  $\tan \theta = \frac{\sqrt{3}}{3}$

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

- 10)  $\sin \theta = \frac{1}{6}$ ,  $\sec \theta < 0$  Find  $\cos \theta$  and  $\tan \theta$ .

## 6 Use Even-Odd Properties to Find the Exact Values of the Trigonometric Functions

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use the even-odd properties to find the exact value of the expression. Do not use a calculator.

1)  $\sin(-30^\circ)$

A)  $-\frac{1}{2}$                       B)  $\frac{1}{2}$                       C)  $\frac{\sqrt{3}}{2}$                       D)  $-\frac{\sqrt{3}}{2}$

2)  $\sin(-60^\circ)$

A)  $-\frac{\sqrt{3}}{2}$                       B)  $\frac{1}{2}$                       C)  $-\frac{1}{2}$                       D)  $\frac{\sqrt{3}}{2}$

3)  $\sec(-60^\circ)$

A) 2                              B) -2                              C)  $\frac{2\sqrt{3}}{3}$                       D)  $-\frac{2\sqrt{3}}{3}$

4)  $\cot(-60^\circ)$

A)  $-\frac{\sqrt{3}}{3}$                       B)  $\frac{\sqrt{3}}{3}$                       C)  $\sqrt{3}$                       D)  $-\sqrt{3}$

5)  $\cos(-150^\circ)$

A)  $-\frac{\sqrt{3}}{2}$                       B)  $-\frac{1}{2}$                       C)  $\frac{1}{2}$                       D)  $\frac{\sqrt{3}}{2}$

6)  $\cos\left(-\frac{\pi}{4}\right)$

A)  $\frac{\sqrt{2}}{2}$                       B)  $-\frac{\sqrt{2}}{2}$                       C)  $\frac{\sqrt{3}}{2}$                       D)  $-\frac{\sqrt{3}}{2}$

7)  $\sec\left(-\frac{\pi}{6}\right)$

A)  $\frac{2\sqrt{3}}{3}$                       B) 2                              C) -2                              D)  $-\frac{2\sqrt{3}}{3}$

8)  $\cot\left(-\frac{\pi}{6}\right)$

A)  $-\sqrt{3}$                       B)  $\frac{\sqrt{3}}{3}$                       C)  $\sqrt{3}$                       D)  $-\frac{\sqrt{3}}{3}$

9)  $\csc\left(-\frac{\pi}{2}\right)$

A) -1                              B) 1                              C) 0                              D) undefined

10)  $\sec(-\pi)$

A) -1                              B) 1                              C) 0                              D) undefined

11)  $\cot\left(-\frac{\pi}{4}\right)$

A) -1

B)  $-\frac{\sqrt{3}}{3}$

C)  $-\sqrt{3}$

D) 1

**Solve the problem.**

12) If  $f(\theta) = \cos \theta$  and  $f(a) = \frac{1}{3}$ , find the exact value of  $f(-a)$ .

A)  $\frac{1}{3}$

B)  $-\frac{1}{3}$

C)  $\frac{2}{3}$

D)  $-\frac{2}{3}$

13) If  $f(\theta) = \sec \theta$  and  $f(a) = 3$ , find the exact value of  $f(-a)$ .

A) 3

B) -3

C)  $\frac{1}{3}$

D)  $-\frac{1}{3}$

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

14) Is the function  $f(\theta) = \sin \theta + \cos \theta$  even, odd, or neither?

15) Is the function  $f(\theta) = \sin \theta + \tan \theta$  even, odd, or neither?

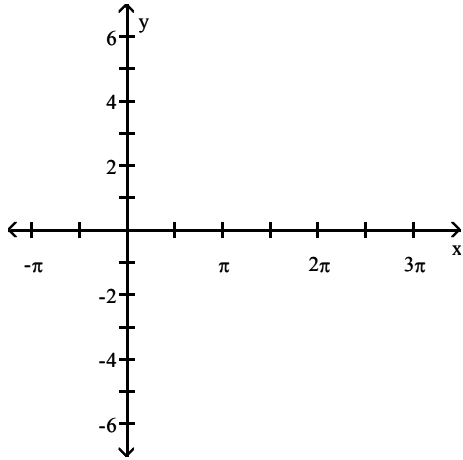
## 2.4 Graphs of the Sine and Cosine Functions

### 1 Graph Functions of the Form $y = A \sin(\omega x)$ Using Transformations

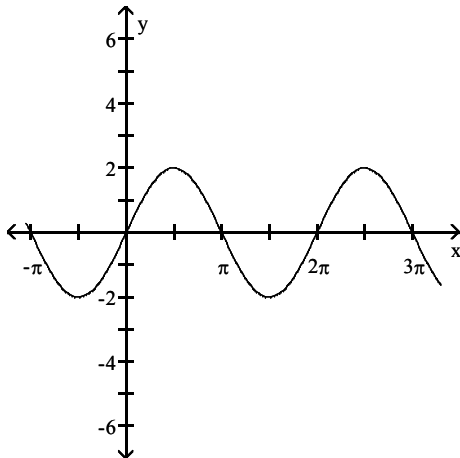
**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use transformations to graph the function.

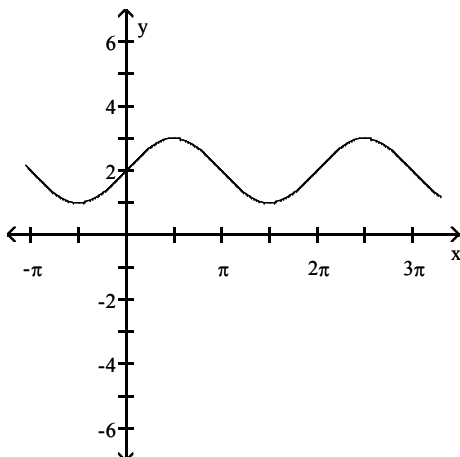
1)  $y = 2 \sin x$



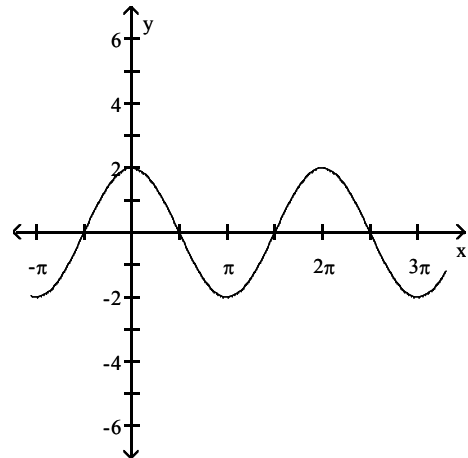
A)



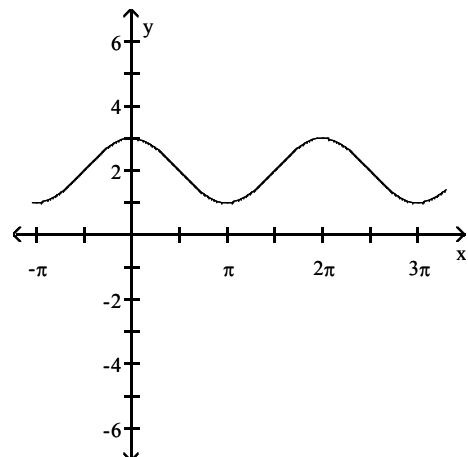
C)



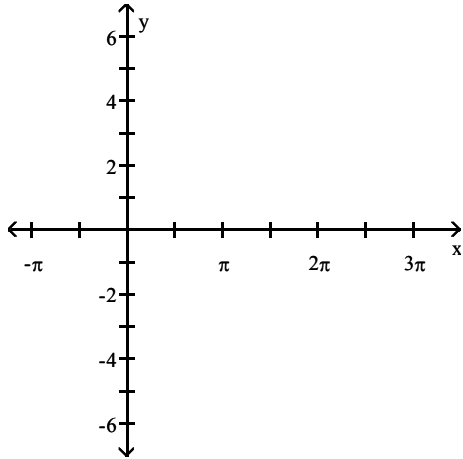
B)



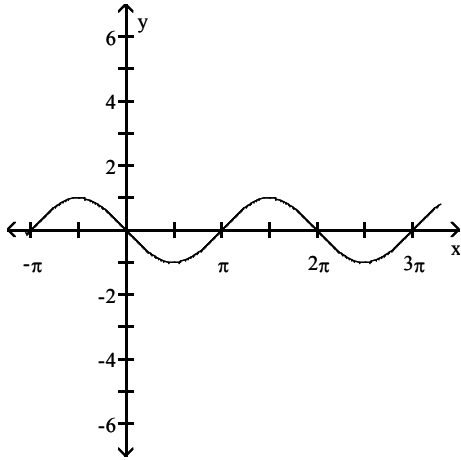
D)



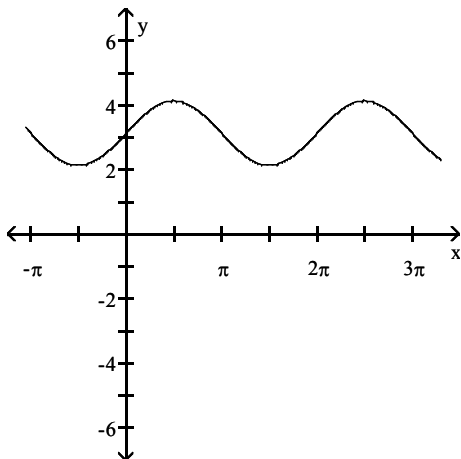
2)  $y = \sin(x + \pi)$



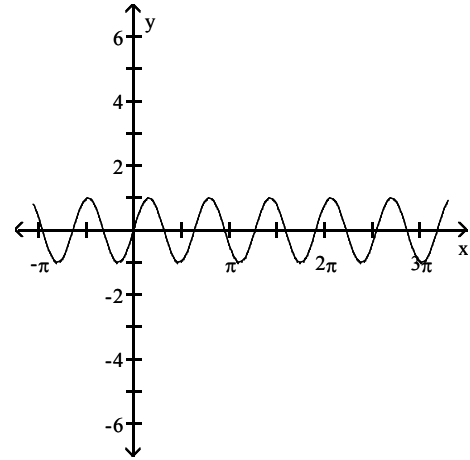
A)



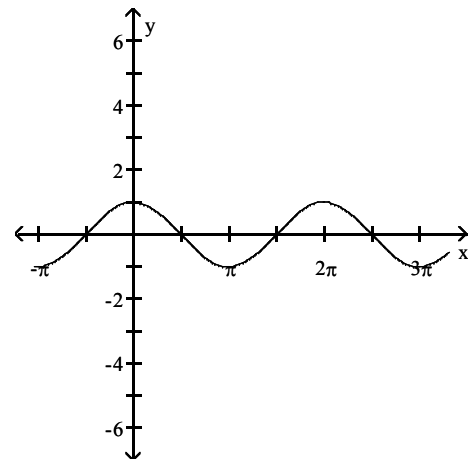
C)



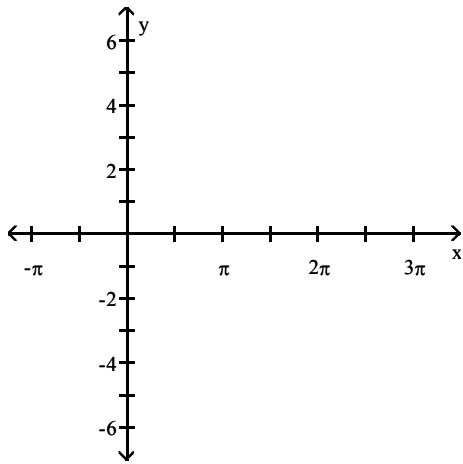
B)



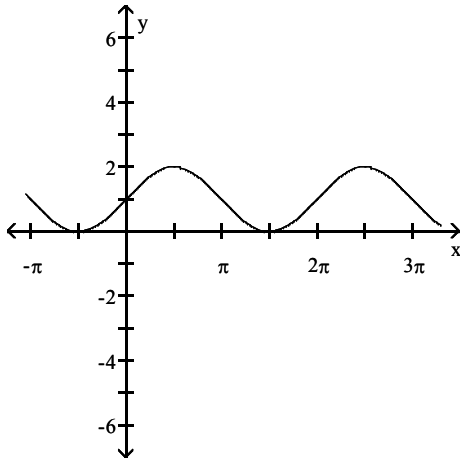
D)



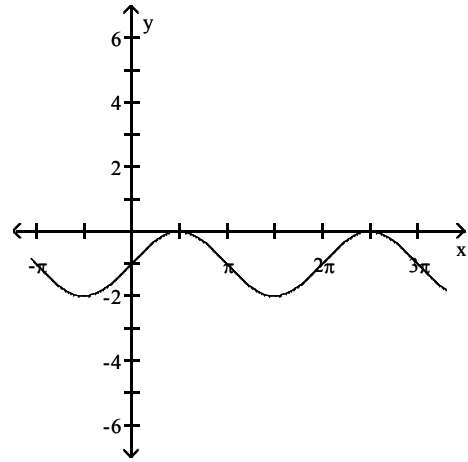
3)  $y = \sin x + 1$



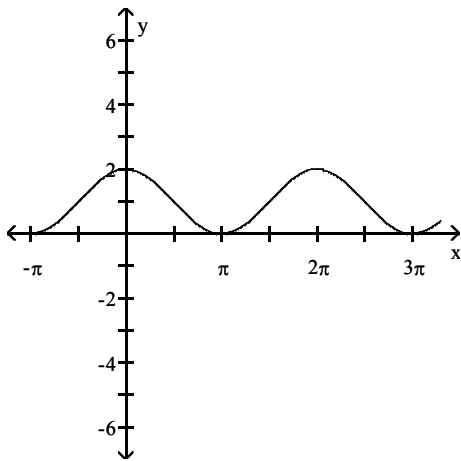
A)



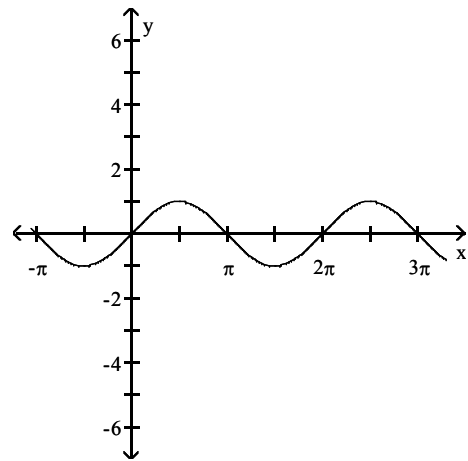
B)



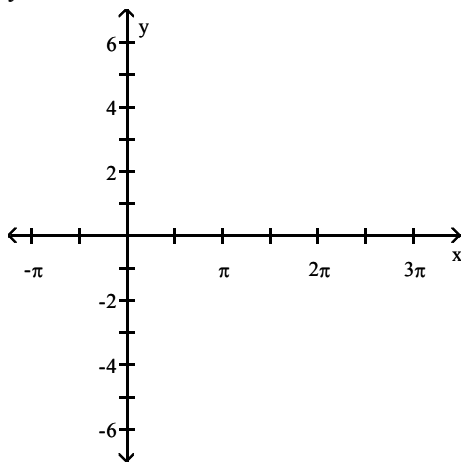
C)



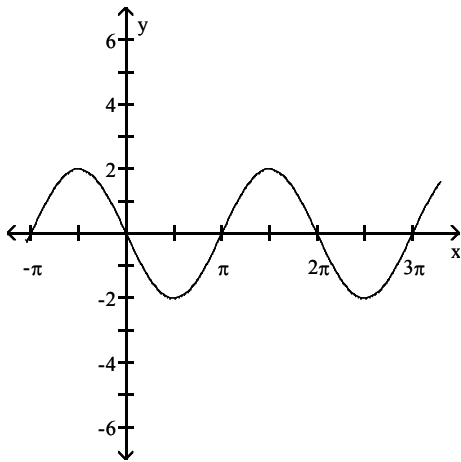
D)



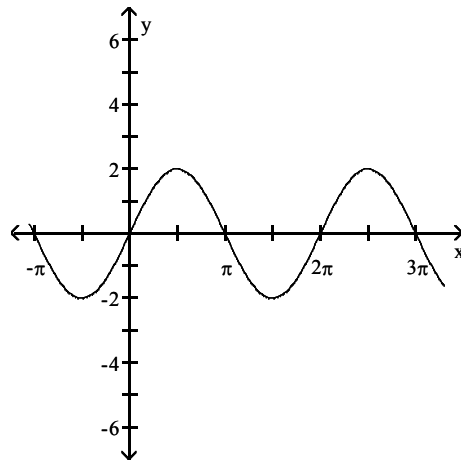
4)  $y = -2 \sin x$



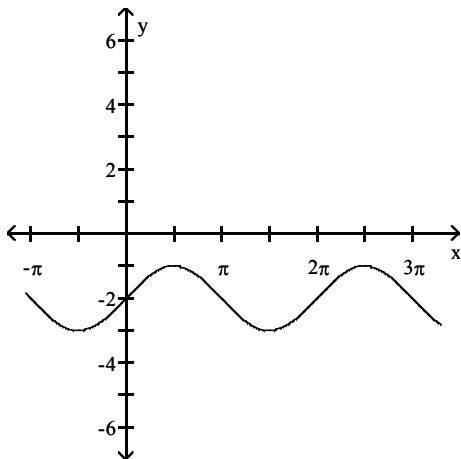
A)



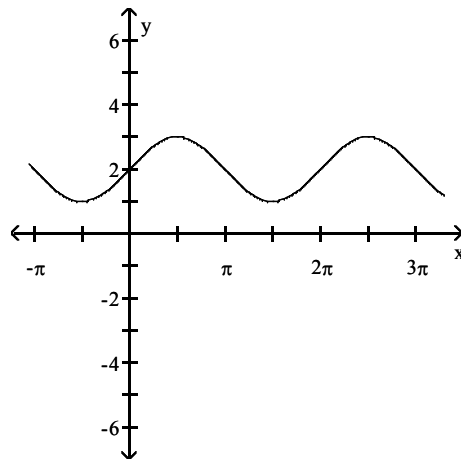
B)



C)

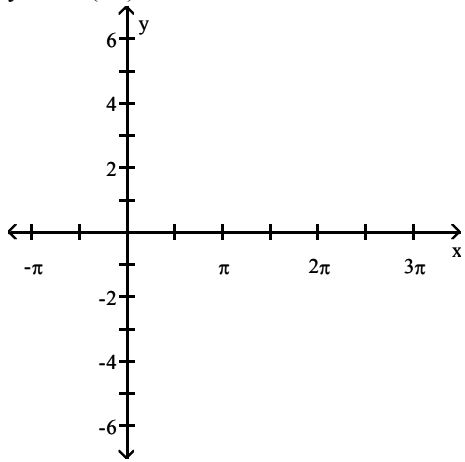


D)

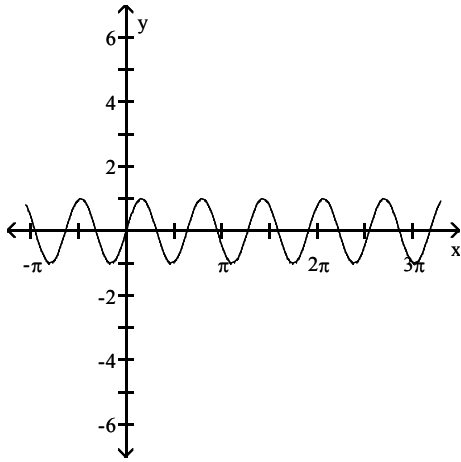




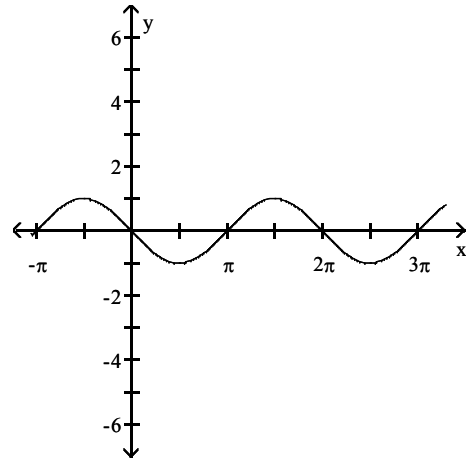
5)  $y = \sin(\pi x)$



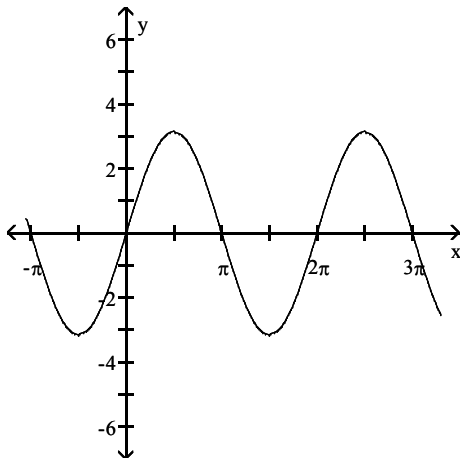
A)



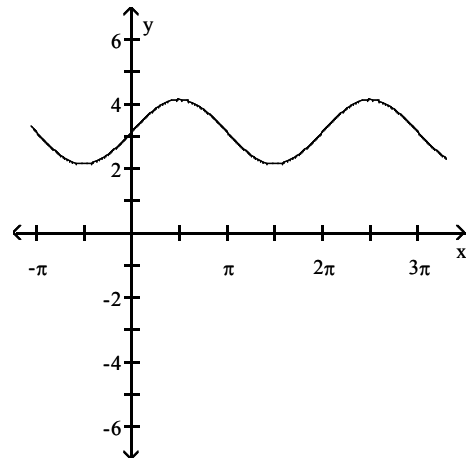
B)



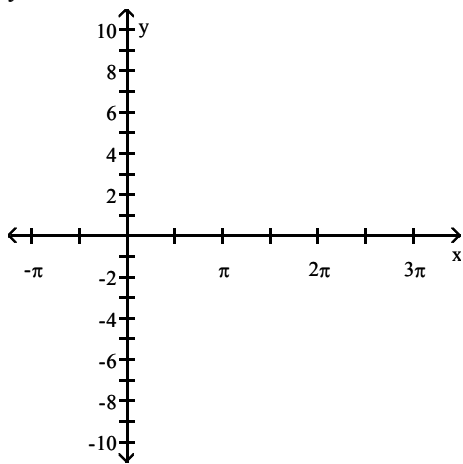
C)



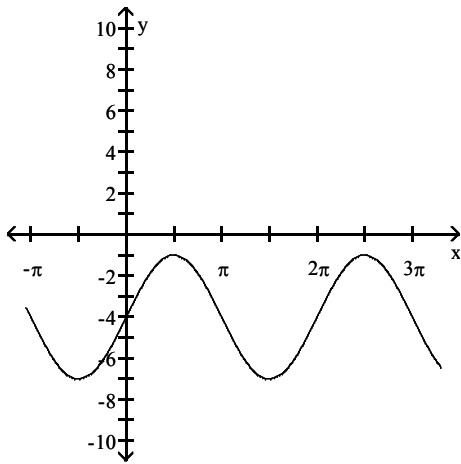
D)



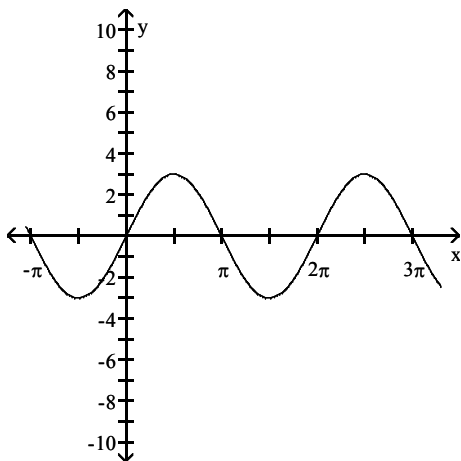
6)  $y = 3 \sin x - 4$



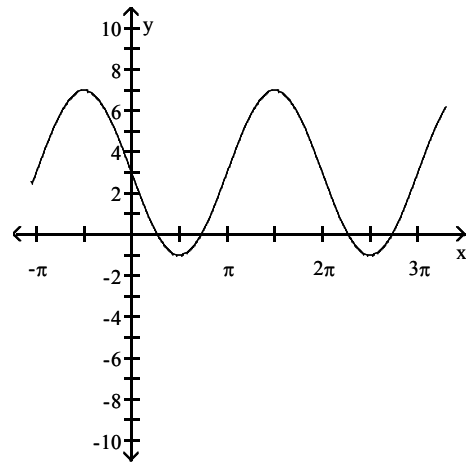
A)



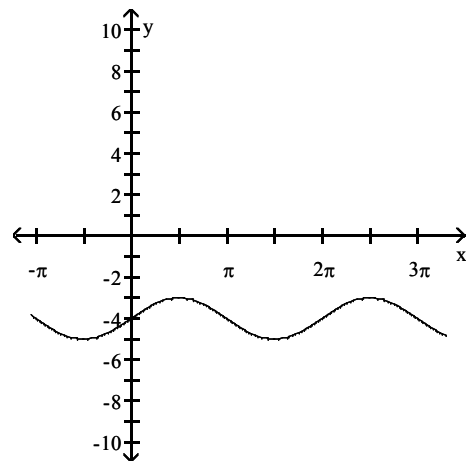
C)



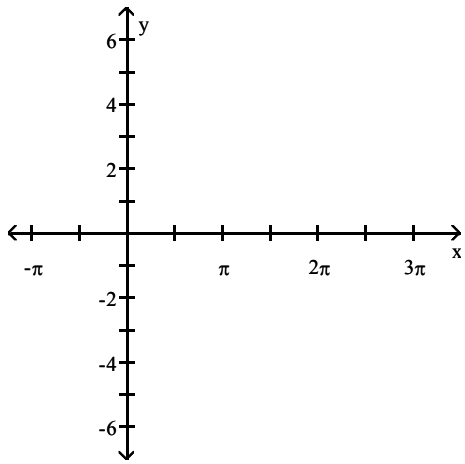
B)



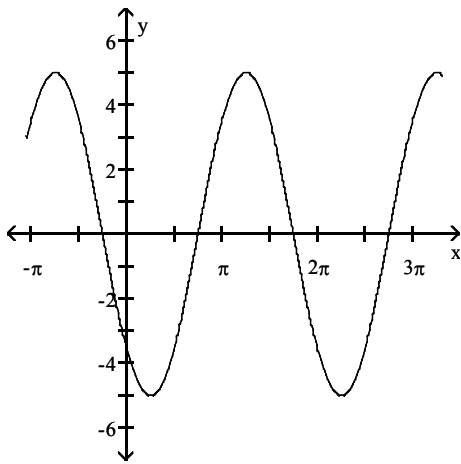
D)



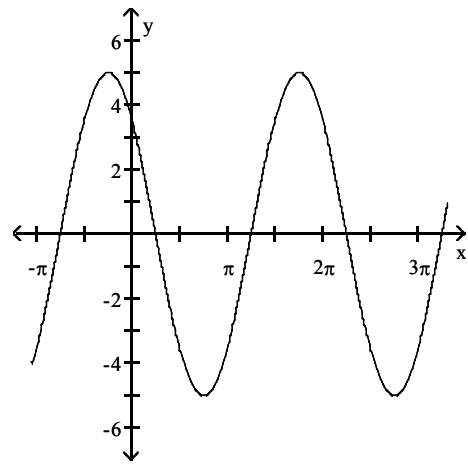
7)  $y = -5 \sin(x + \frac{\pi}{4})$



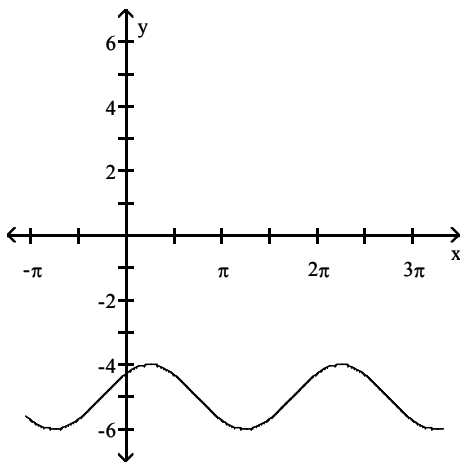
A)



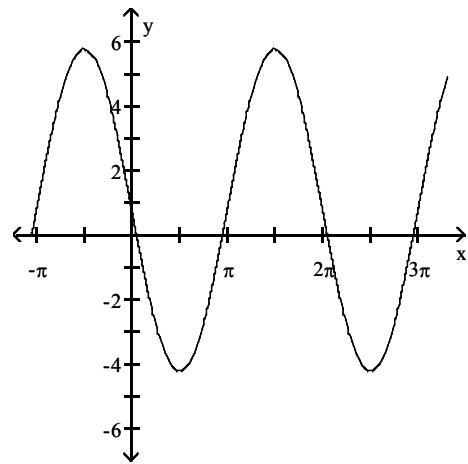
B)



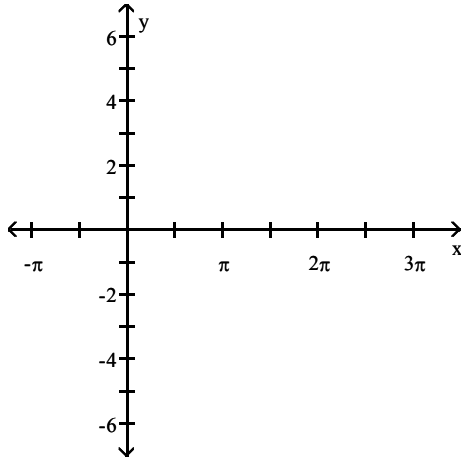
C)



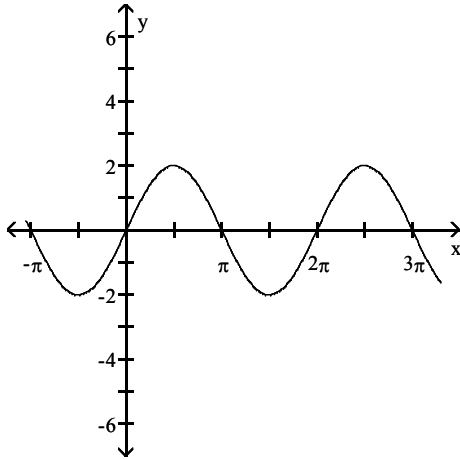
D)



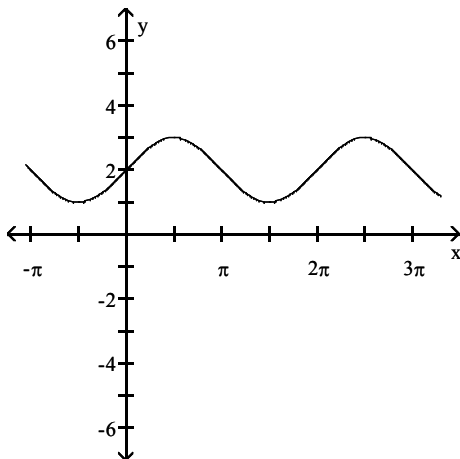
8)  $y = 2 \sin(\pi - x)$



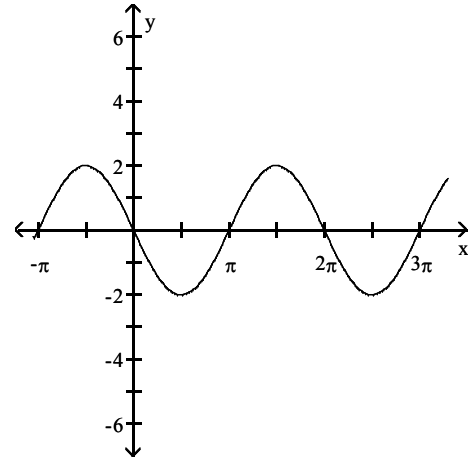
A)



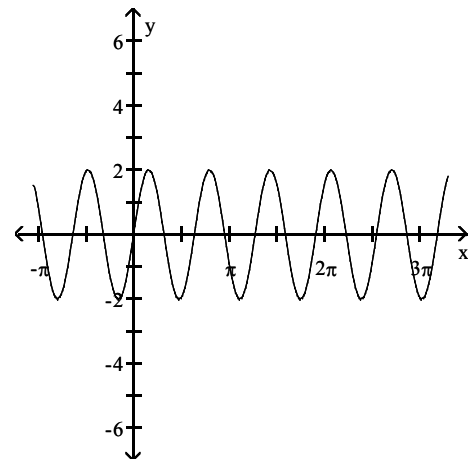
C)



B)



D)



**Solve the problem.**

9) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\sin x = 0$ ?

- A)  $0, \pi, 2\pi$                       B)  $\frac{\pi}{2}, \frac{3\pi}{2}$                       C)  $0, 1$                       D)  $0, 1, 2$

10) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\sin x = 1$ ?

- A)  $\frac{\pi}{2}$                       B)  $\frac{\pi}{2}, \frac{3\pi}{2}$                       C)  $0, 2\pi$                       D) none

11) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\sin x = -1$ ?

A)  $\frac{3\pi}{2}$

B)  $\frac{\pi}{2}, \frac{3\pi}{2}$

C)  $\pi$

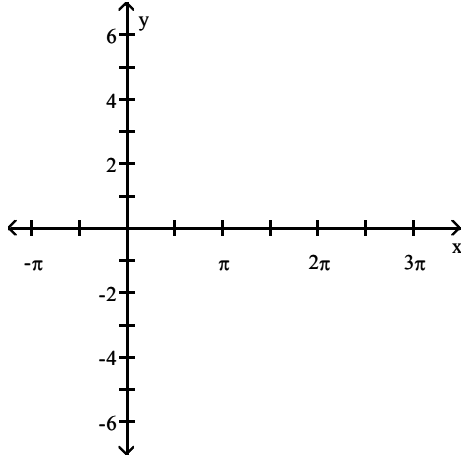
D) none

## 2 Graph Functions of the Form $y = A \cos(\omega x)$ Using Transformations

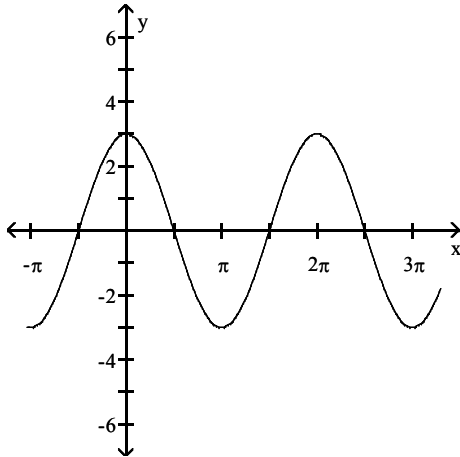
**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Use transformations to graph the function.

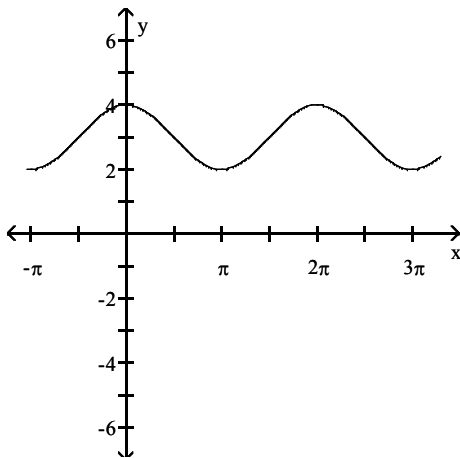
1)  $y = 3 \cos x$



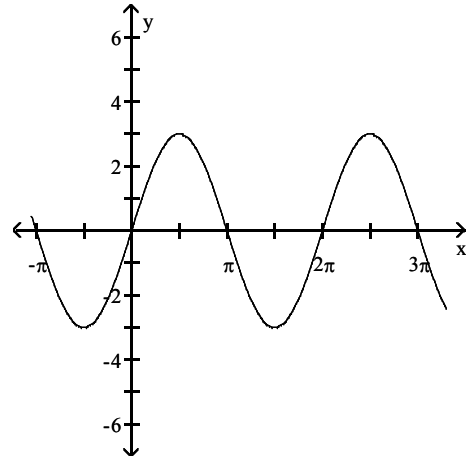
A)



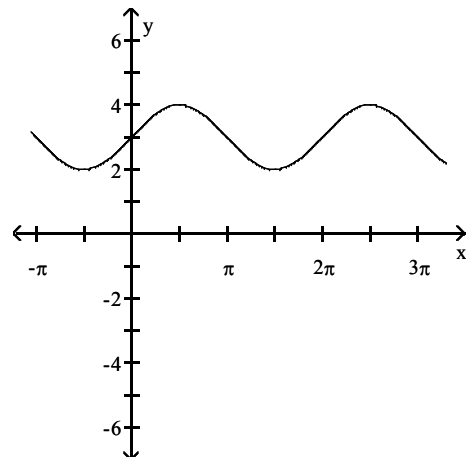
C)



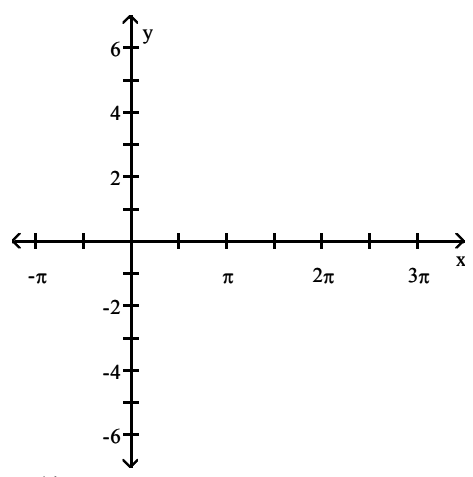
B)



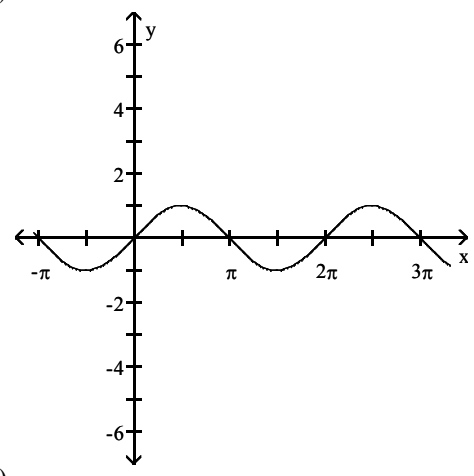
D)



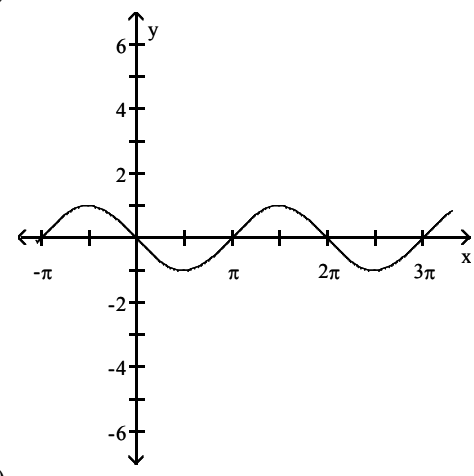
2)  $y = \cos(x - \frac{\pi}{2})$



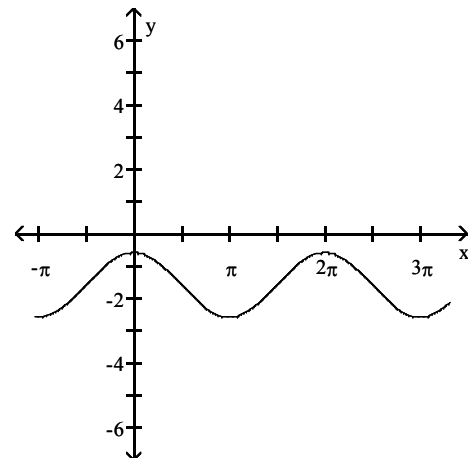
A)



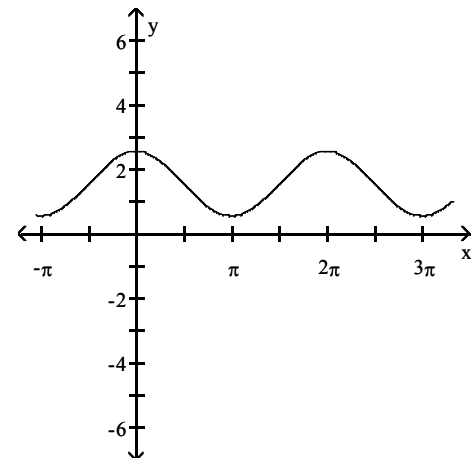
B)



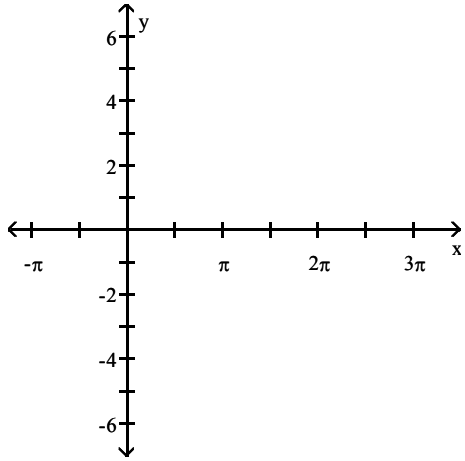
C)



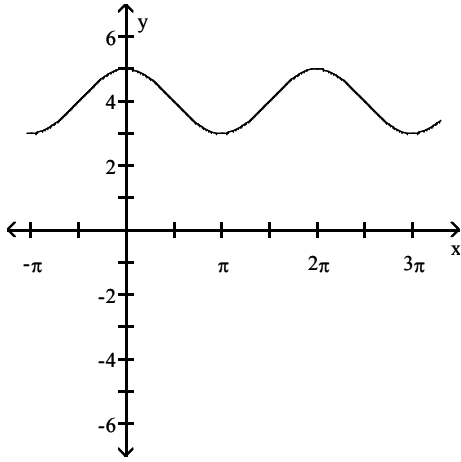
D)



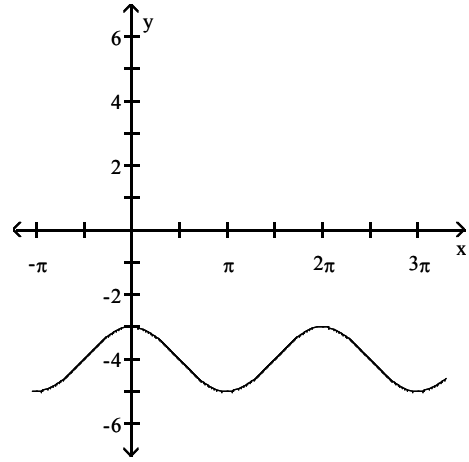
3)  $y = \cos x + 4$



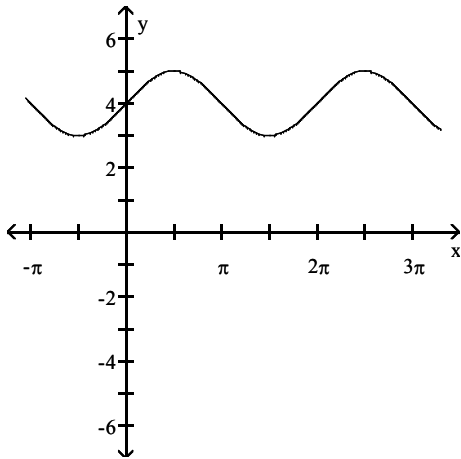
A)



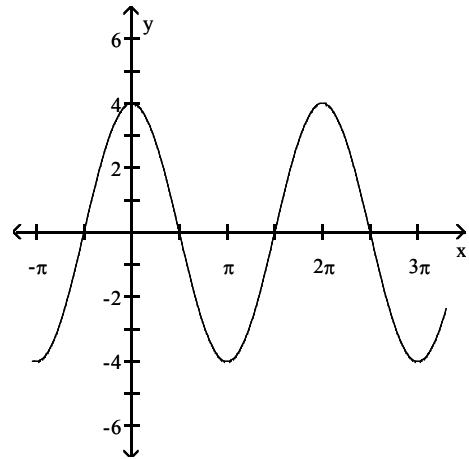
B)



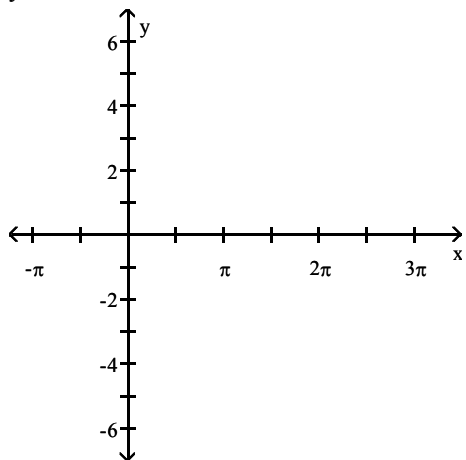
C)



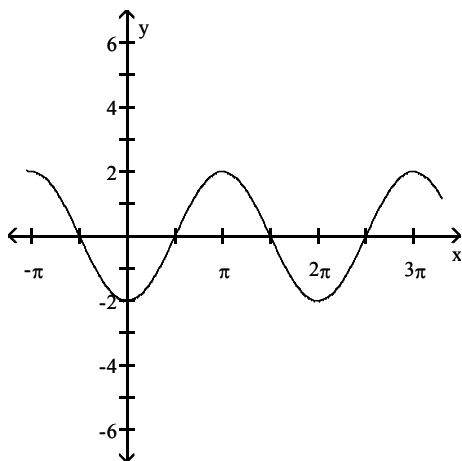
D)



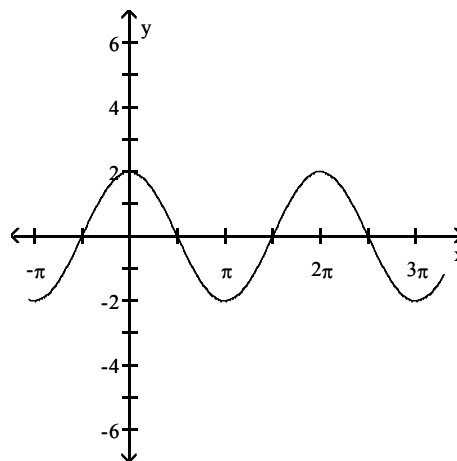
4)  $y = -2 \cos x$



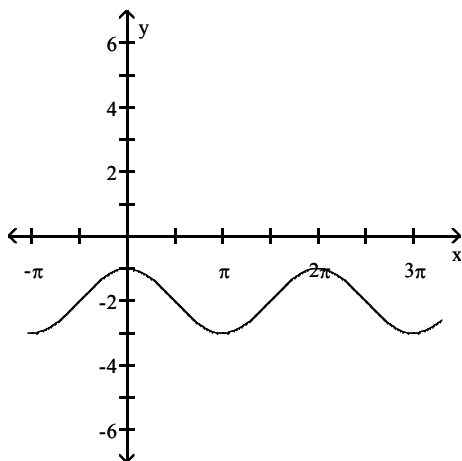
A)



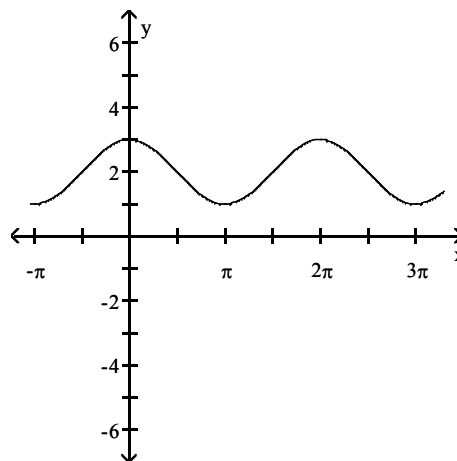
B)



C)

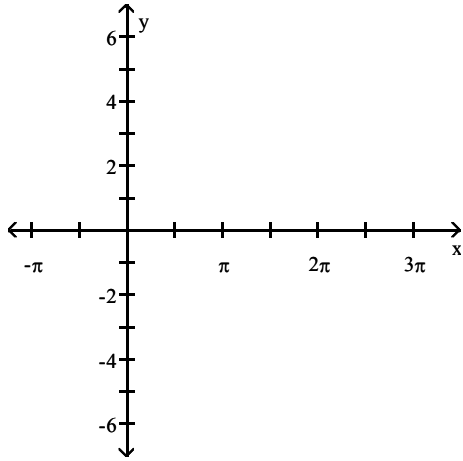


D)

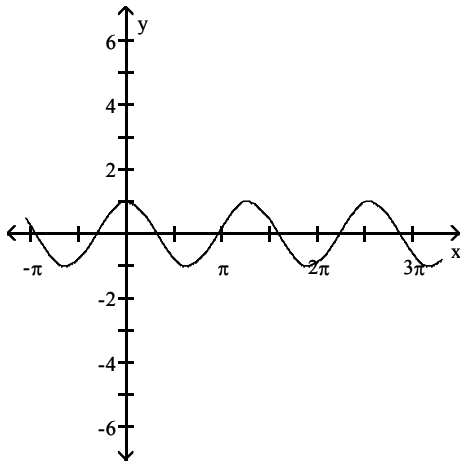




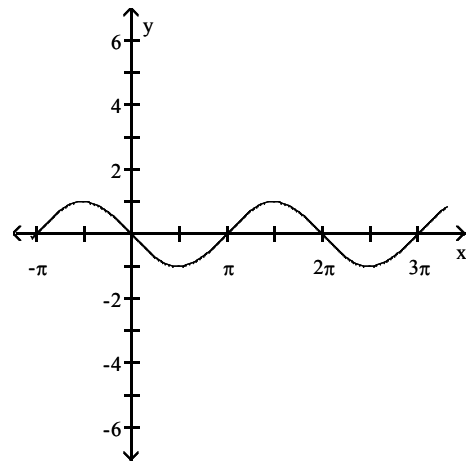
5)  $y = \cos\left(\frac{\pi}{2}x\right)$



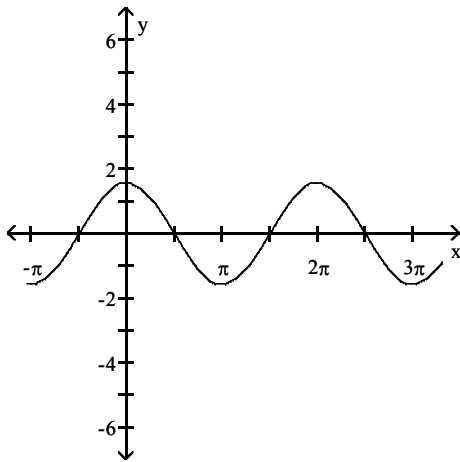
A)



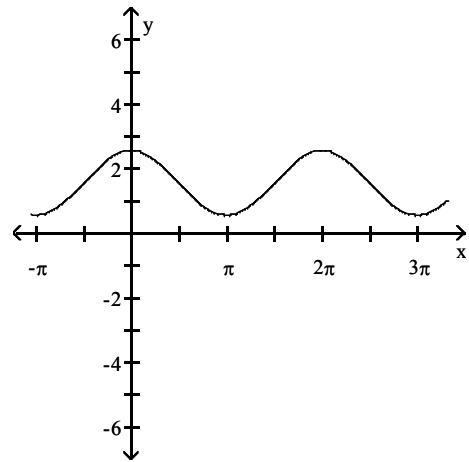
B)



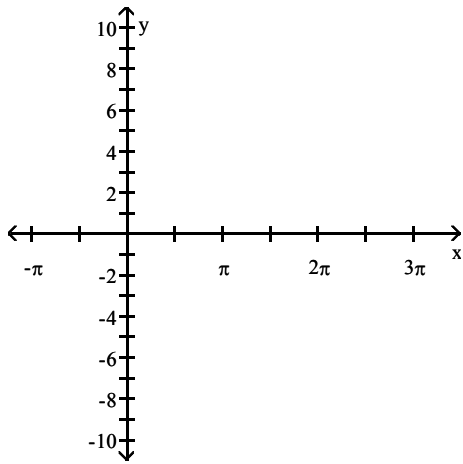
C)



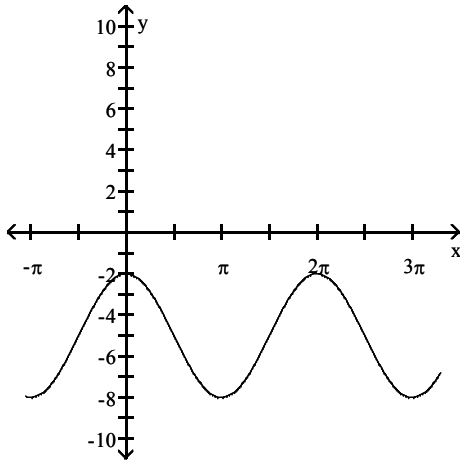
D)



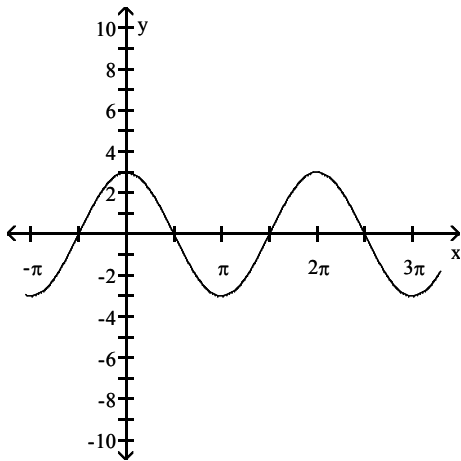
6)  $y = 3 \cos x - 5$



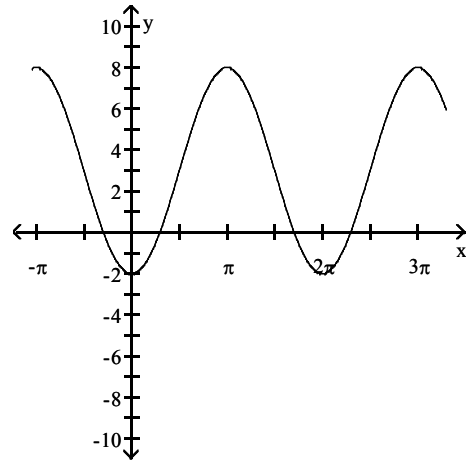
A)



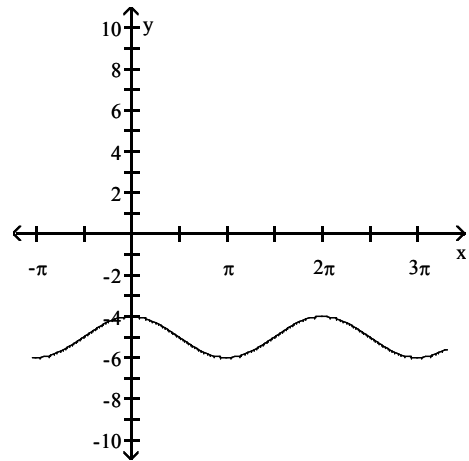
C)



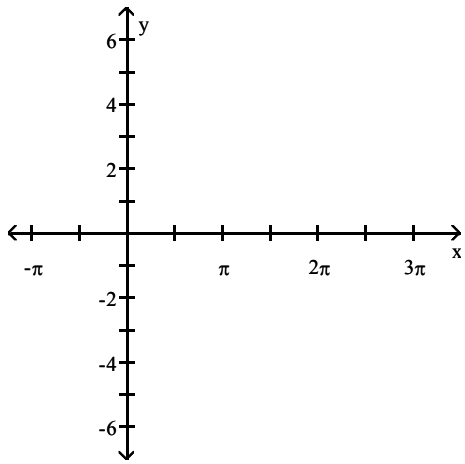
B)



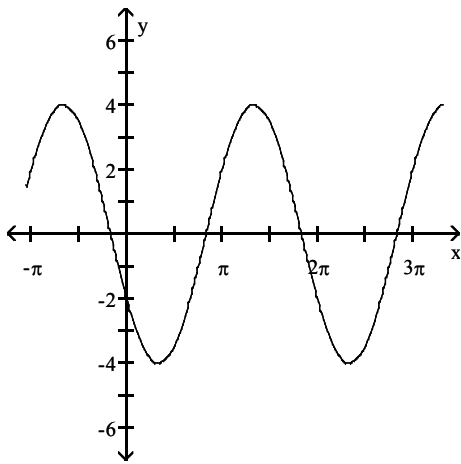
D)



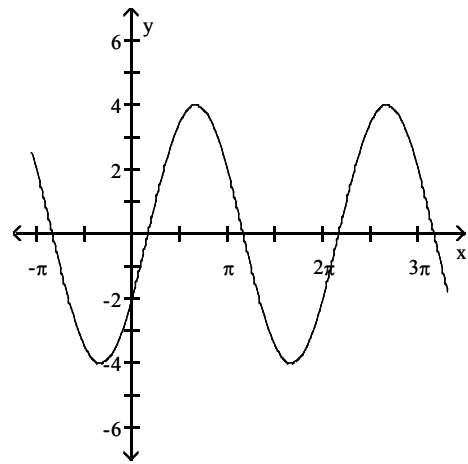
7)  $y = -4 \cos(x - \frac{\pi}{3})$



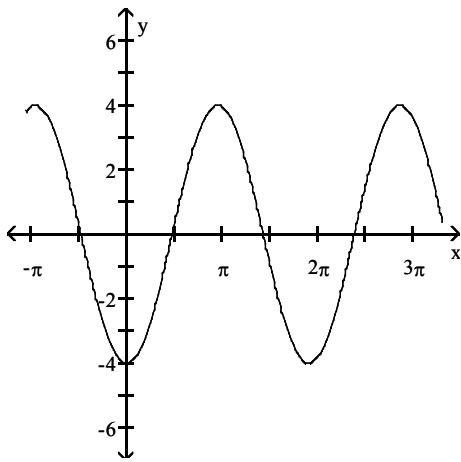
A)



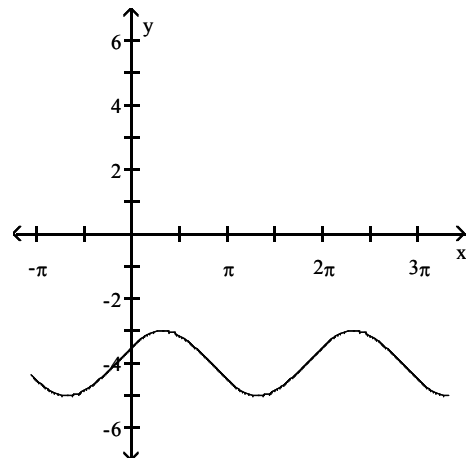
B)



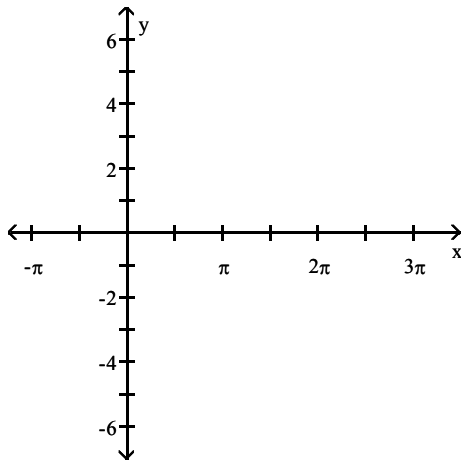
C)



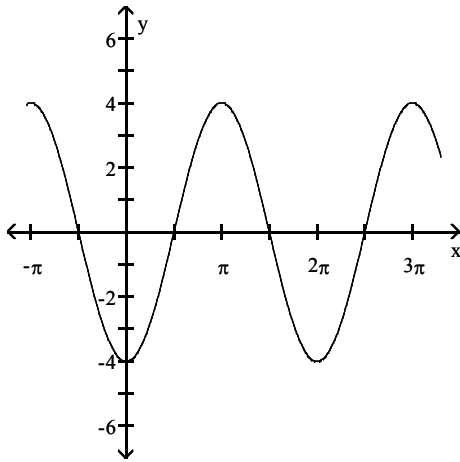
D)



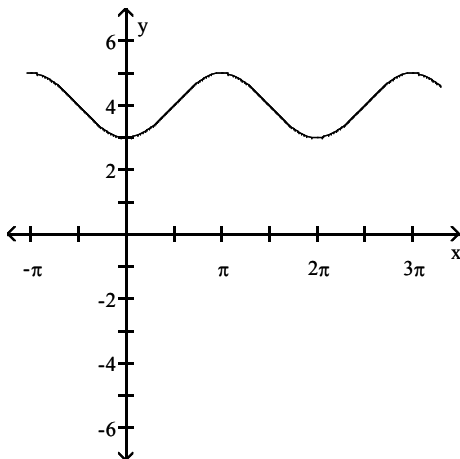
8)  $y = 4 \cos(\pi - x)$



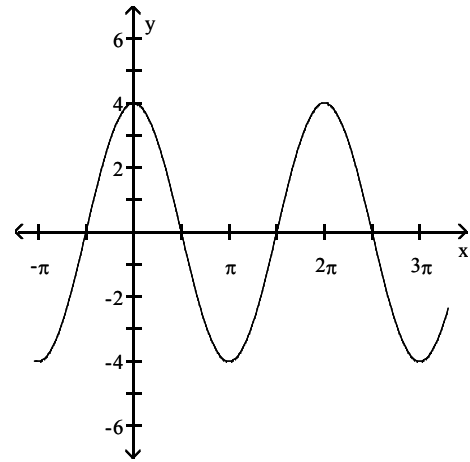
A)



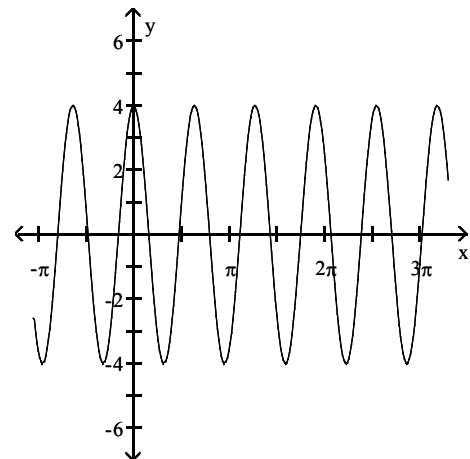
C)



B)



D)



**Solve the problem.**

9) What is the y-intercept of  $y = \cos x$ ?

A) 1

B) 0

C)  $\pi$

D)  $\frac{\pi}{2}$

10) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\cos x = 0$ ?

A)  $\frac{\pi}{2}, \frac{3\pi}{2}$

B)  $0, \pi, 2\pi$

C) 0, 1

D) 0, 1, 2

11) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\cos x = 1$ ?

A)  $0, 2\pi$

B)  $\frac{\pi}{2}, \frac{3\pi}{2}$

C)  $\frac{\pi}{2}$

D) none

12) For what numbers  $x$ ,  $0 \leq x \leq 2\pi$ , does  $\cos x = -1$ ?

A)  $\pi$

B)  $\frac{\pi}{2}, \frac{3\pi}{2}$

C)  $\frac{\pi}{2}$

D) none

### 3 Determine the Amplitude and Period of Sinusoidal Functions

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Without graphing the function, determine its amplitude or period as requested.**

1)  $y = -2 \sin x$  Find the amplitude.

A) 2

B)  $-2\pi$

C)  $\frac{\pi}{2}$

D)  $2\pi$

2)  $y = 4 \sin \frac{1}{3}x$  Find the amplitude.

A) 4

B)  $\frac{4\pi}{3}$

C)  $\frac{\pi}{4}$

D)  $6\pi$

3)  $y = -3 \sin 2x$  Find the amplitude.

A) 3

B)  $\frac{3}{2}$

C)  $\frac{\pi}{3}$

D)  $\frac{\pi}{2}$

4)  $y = \sin 3x$  Find the period.

A)  $\frac{2\pi}{3}$

B) 3

C)  $2\pi$

D) 1

5)  $y = 5 \cos \frac{1}{3}x$  Find the amplitude.

A) 5

B)  $\frac{5\pi}{3}$

C)  $\frac{\pi}{5}$

D)  $6\pi$

6)  $y = \cos 3x$  Find the period.

A)  $\frac{2\pi}{3}$

B) 3

C)  $2\pi$

D) 1

7)  $y = -2 \cos \frac{1}{3}x$  Find the period.

A)  $6\pi$

B) -2

C)  $\frac{2\pi}{3}$

D)  $\frac{\pi}{3}$

8)  $y = 5 \cos x$  Find the period.

A)  $2\pi$

B) 5

C)  $\frac{\pi}{5}$

D)  $\pi$

9)  $y = \frac{5}{4} \sin\left(-\frac{6\pi}{7}x\right)$  Find the period.

A)  $\frac{7}{3}$

B)  $\frac{12\pi}{7}$

C)  $\frac{5\pi}{2}$

D)  $\frac{2}{5}$

10)  $y = \frac{9}{8} \cos\left(-\frac{8\pi}{3}x\right)$  Find the amplitude.

A)  $\frac{9}{8}$

B)  $\frac{3}{4}$

C)  $\frac{8\pi}{9}$

D)  $\frac{8\pi}{3}$

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Solve the problem.**

- 11) Wildlife management personnel use predator-prey equations to model the populations of certain predators and their prey in the wild. Suppose the population  $M$  of a predator after  $t$  months is given by

$$M = 750 + 125 \sin \frac{\pi}{6}t$$

while the population  $N$  of its primary prey is given by

$$N = 12,250 + 3050 \cos \frac{\pi}{6}t$$

Find the period for each of these functions.

- 12) The average daily temperature  $T$  of a city in the United States is approximated by

$$T = 55 - 23 \cos \frac{2\pi}{365}(t - 30)$$

where  $t$  is in days,  $1 \leq t \leq 365$ , and  $t = 1$  corresponds to January 1. Find the period of  $T$ .

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

- 13) The current  $I$ , in amperes, flowing through a particular ac (alternating current) circuit at time  $t$  seconds is

$$I = 110 \sin (40\pi t)$$

What is the period and amplitude of the current?

A) period =  $\frac{1}{20}$  second, amplitude = 110

B) period =  $40\pi$  seconds, amplitude =  $\frac{1}{20}$

C) period =  $\frac{1}{120}$  second, amplitude = 120

D) period =  $\frac{\pi}{110}$  second, amplitude = 40

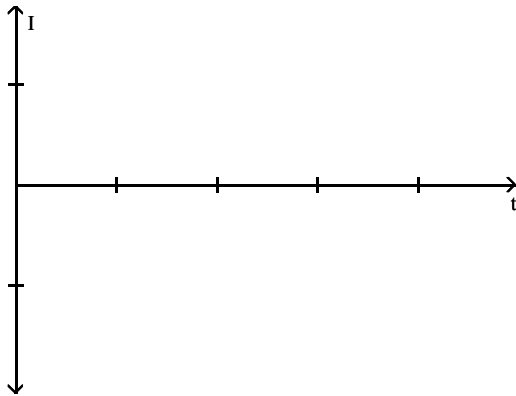
**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

- 14) The current  $I$ , in amperes, flowing through an ac (alternating current) circuit at time  $t$ , in seconds, is

$$I = 30 \sin(50\pi t)$$

What is the amplitude? What is the period?

Graph this function over two periods beginning at  $t = 0$ .

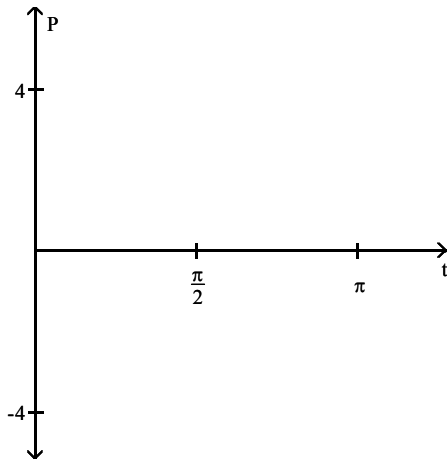


- 15) A mass hangs from a spring which oscillates up and down. The position  $P$  of the mass at time  $t$  is given by

$$P = 4 \cos(4t)$$

What is the amplitude? What is the period?

Graph this function over two periods beginning at  $t = 0$ .



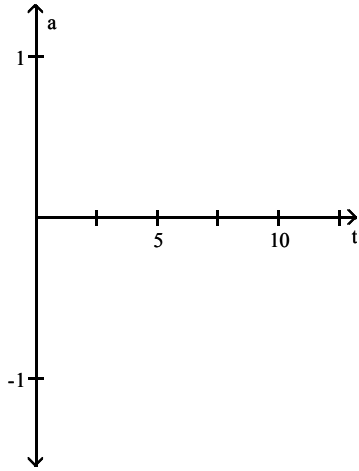
16) Before exercising, an athlete measures her air flow and obtains

$$a = 0.65 \sin\left(\frac{2\pi}{5}t\right)$$

where  $a$  is measured in liters per second and  $t$  is the time in seconds. If  $a > 0$ , the athlete is inhaling; if  $a < 0$ , the athlete is exhaling. The time to complete one complete inhalation/exhalation sequence is a respiratory cycle.

What is the amplitude? What is the period? What is the respiratory cycle?

Graph  $a$  over two periods beginning at  $t = 0$ .



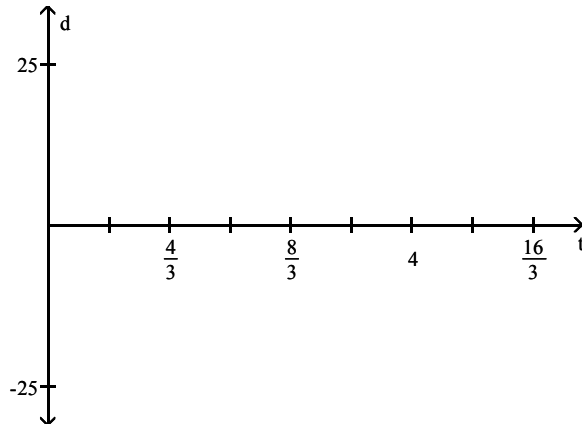
17) A boy is flying a model airplane while standing on a straight line. The plane, at the end of a twenty-five foot wire, flies in circles around the boy. The directed distance of the plane from the straight line is found to be

$$d = 25 \cos\left(\frac{3\pi}{4}t\right)$$

where  $d$  is measured in feet and  $t$  is the time in seconds. If  $d > 0$ , the plane is in front of the boy; if  $d < 0$ , the plane is behind him.

What is the amplitude? What is the period?

Graph  $d$  over two periods beginning at  $t = 0$ .





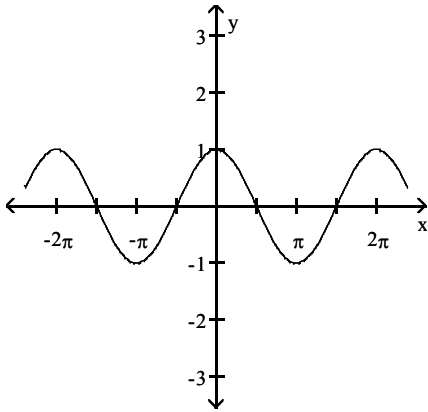
#### 4 Graph Sinusoidal Functions Using Key Points

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

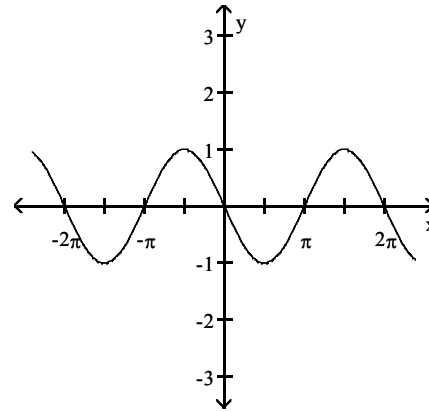
Match the given function to its graph.

- 1)  $y = \sin x$                       2)  $y = \cos x$   
 3)  $y = -\sin x$                     4)  $y = -\cos x$

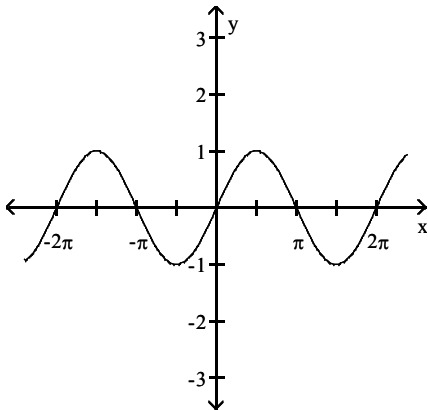
A



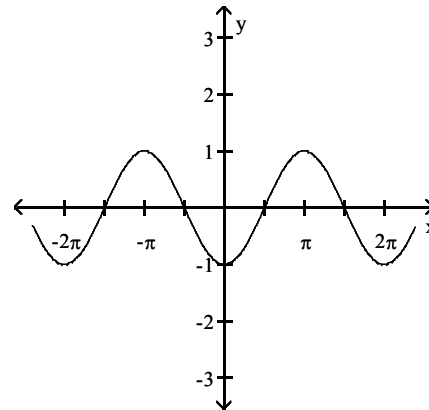
B



C



D



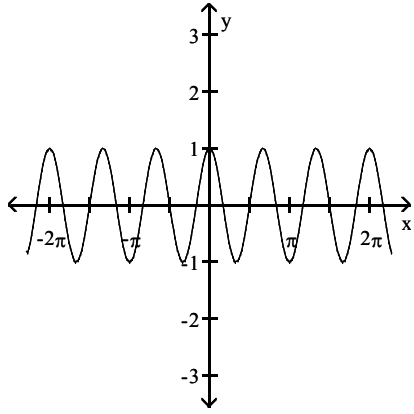
A) 1C, 2A, 3B, 4D

B) 1A, 2D, 3C, 4B

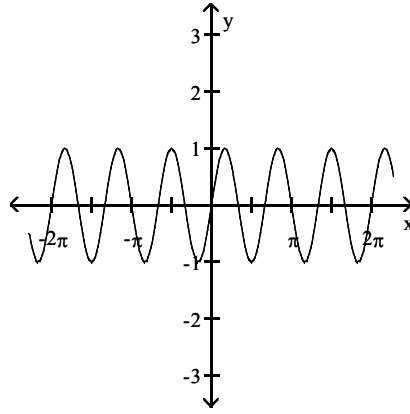
C) 1B, 2D, 3C, 4A

D) 1A, 2B, 3C, 4D

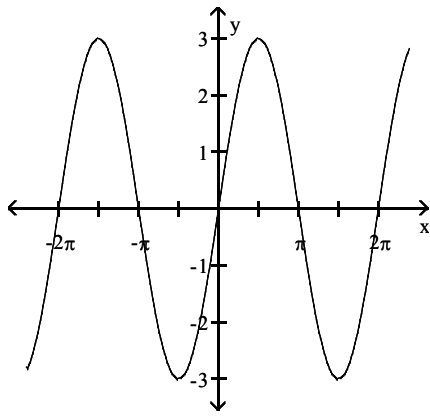
- 2) 1)  $y = \sin 3x$       2)  $y = 3 \cos x$   
 3)  $y = 3 \sin x$       4)  $y = \cos 3x$   
 A



B

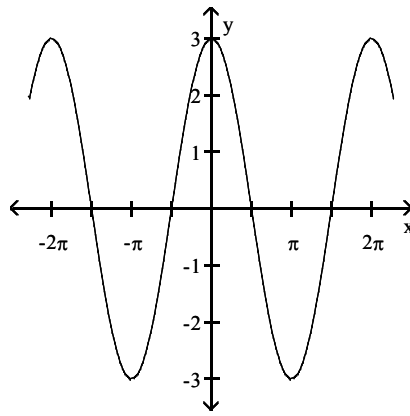


C



A) 1B, 2D, 3C, 4A

D



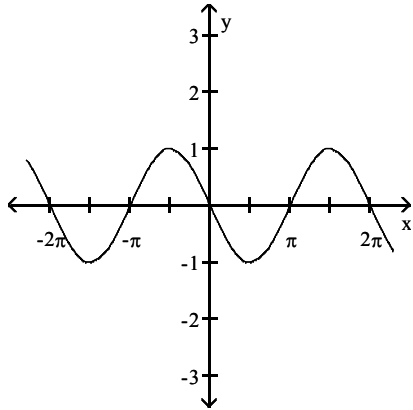
C) 1A, 2C, 3D, 4B

D) 1A, 2B, 3C, 4D

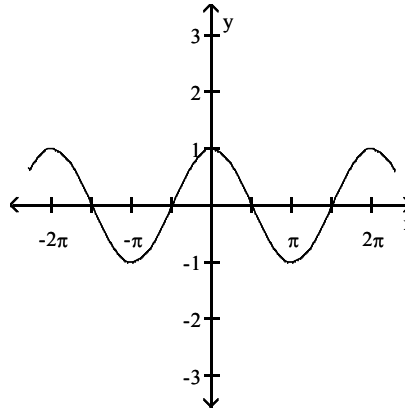
3) 1)  $y = \sin(x - \frac{\pi}{2})$     2)  $y = \cos(x + \frac{\pi}{2})$

3)  $y = \sin(x + \frac{\pi}{2})$     4)  $y = \cos(x - \frac{\pi}{2})$

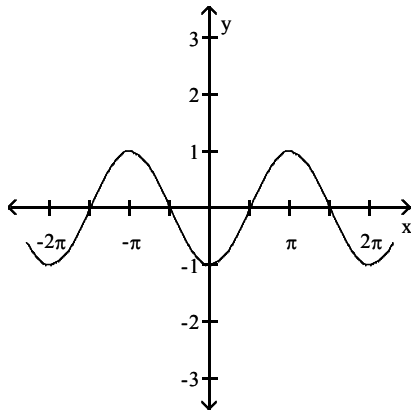
A



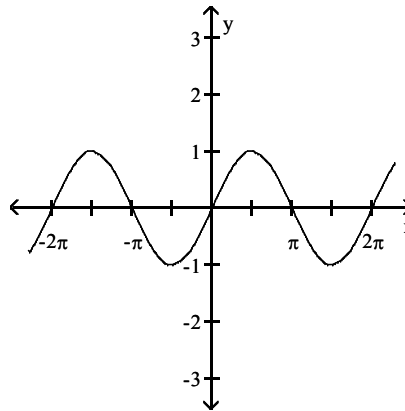
B



C



D



A) 1C, 2A, 3B, 4D

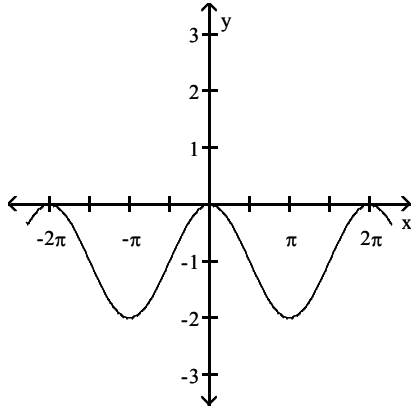
B) 1A, 2D, 3C, 4B

C) 1B, 2D, 3C, 4A

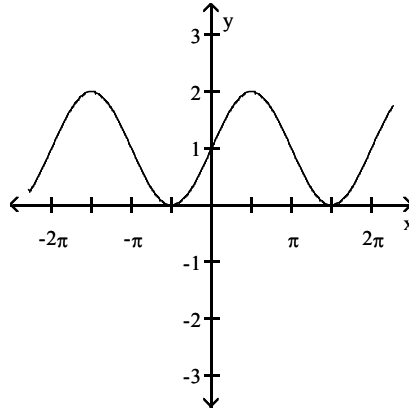
D) 1A, 2B, 3C, 4D

- 4) 1)  $y = 1 + \sin x$       2)  $y = 1 + \cos x$   
 3)  $y = -1 + \sin x$       4)  $y = -1 + \cos x$

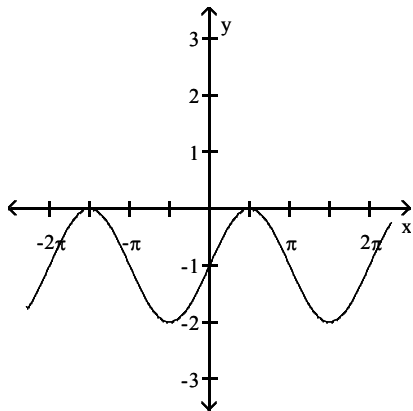
A



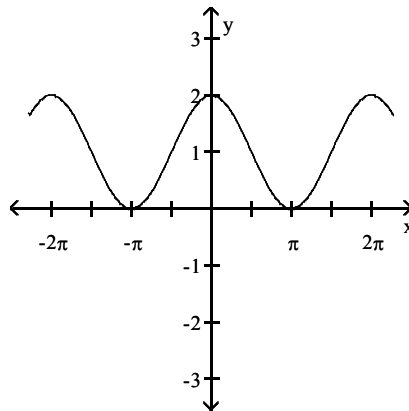
B



C



D



A) 1B, 2D, 3C, 4A

B) 1A, 2D, 3C, 4B

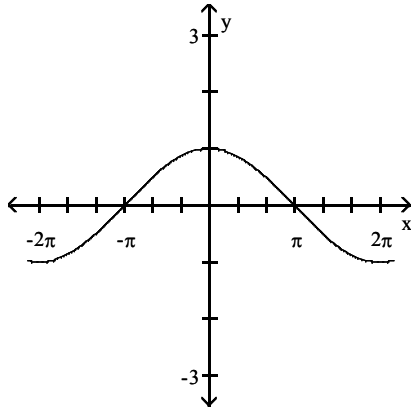
C) 1A, 2C, 3D, 4B

D) 1A, 2B, 3C, 4D

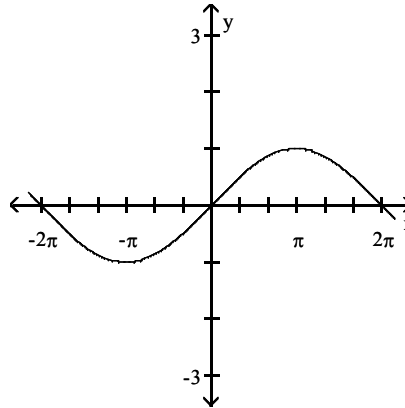
5) 1)  $y = \sin(\frac{1}{2}x)$       2)  $y = \frac{1}{2} \cos x$

3)  $y = \frac{1}{2} \sin x$       4)  $y = \cos(\frac{1}{2}x)$

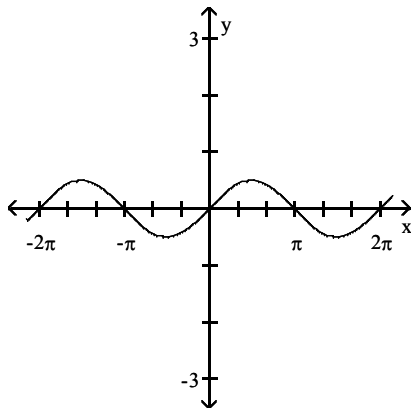
A



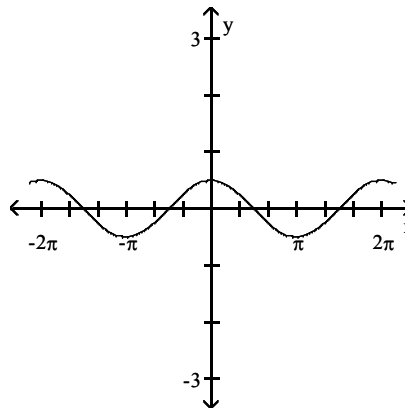
B



C



D



A) 1B, 2D, 3C, 4A

B) 1A, 2D, 3C, 4B

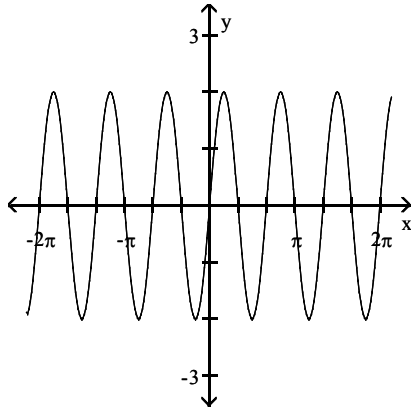
C) 1A, 2C, 3D, 4B

D) 1A, 2B, 3C, 4D

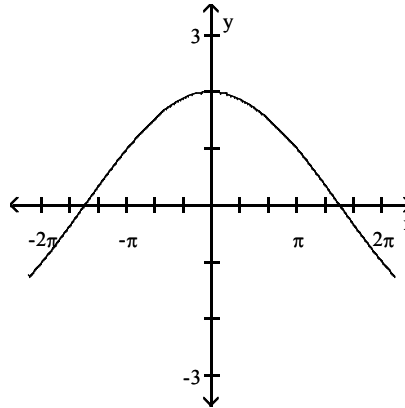
6) 1)  $y = 2 \sin(3x)$       2)  $y = 2 \sin\left(\frac{1}{3}x\right)$

3)  $y = 2 \cos(3x)$       4)  $y = 2 \cos\left(\frac{1}{3}x\right)$

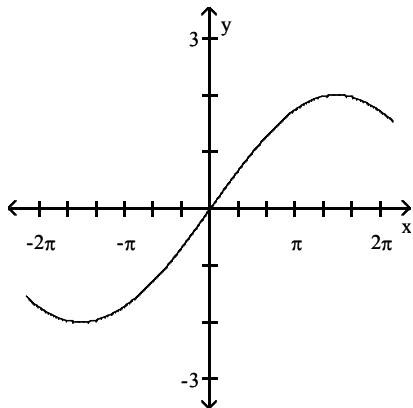
A



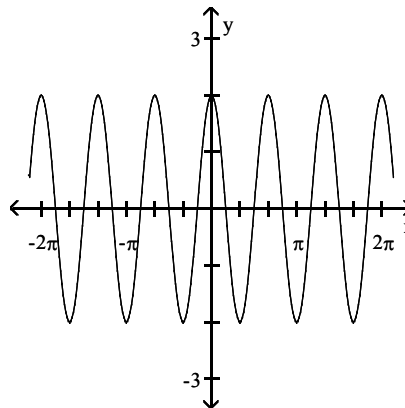
B



C



D



A) 1A, 2C, 3D, 4B

B) 1C, 2A, 3B, 4D

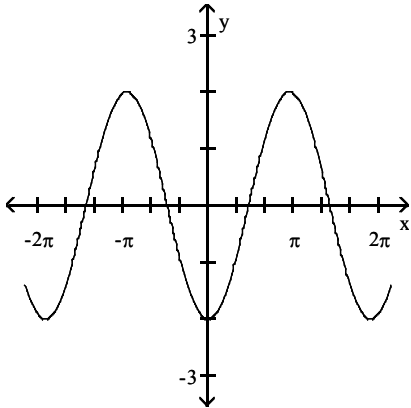
C) 1C, 2A, 3D, 4B

D) 1D, 2B, 3A, 4C

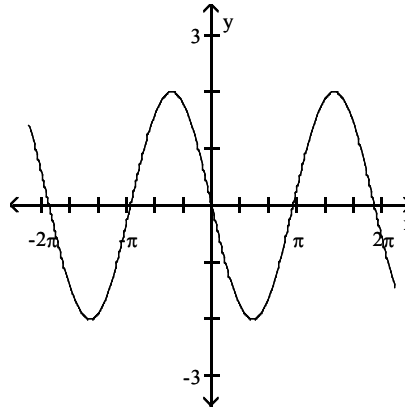
7) 1)  $y = -2 \sin\left(\frac{\pi}{3}x\right)$     2)  $y = -2 \sin\left(\frac{1}{3}x\right)$

3)  $y = -2 \cos\left(\frac{\pi}{3}x\right)$     4)  $y = -2 \cos\left(\frac{1}{3}x\right)$

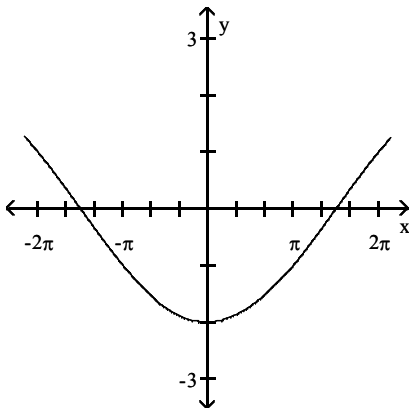
A)



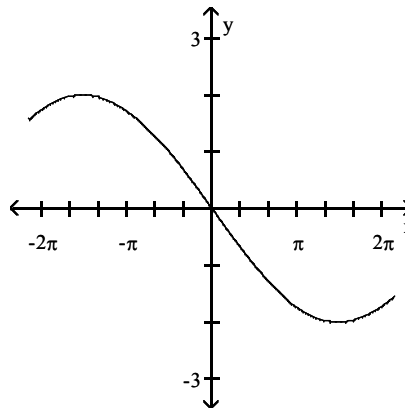
B)



C)



D)



A) 1B, 2D, 3A, 4C

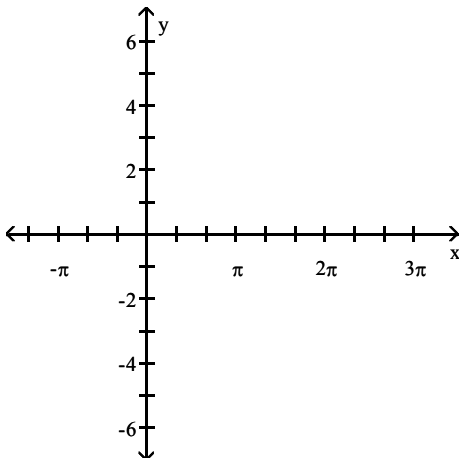
B) 1A, 2C, 3B, 4D

C) 1C, 2A, 3D, 4B

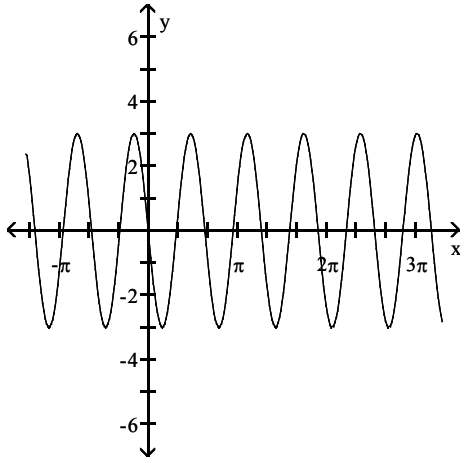
D) 1A, 2C, 3D, 4B

**Graph the sinusoidal function.**

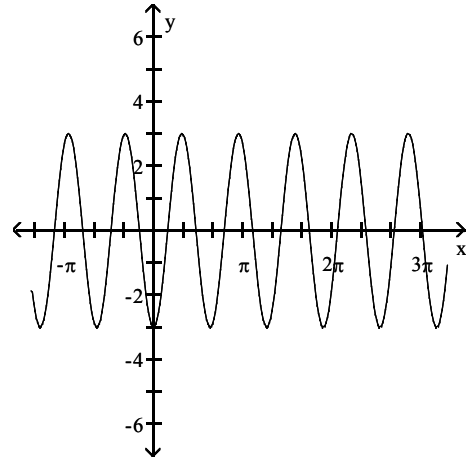
8)  $y = -3 \sin(\pi x)$



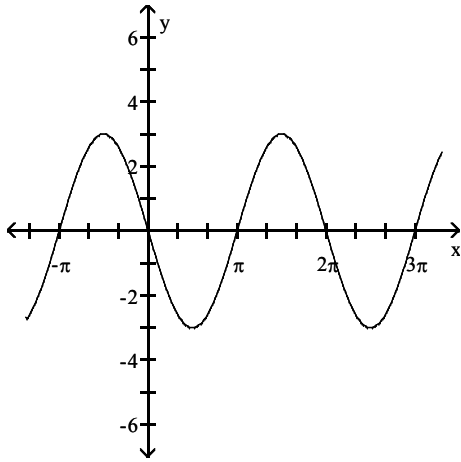
A)



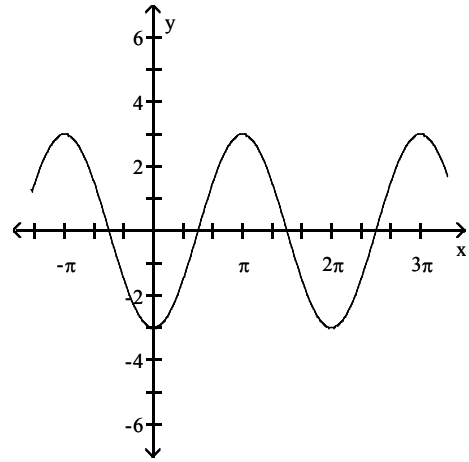
B)



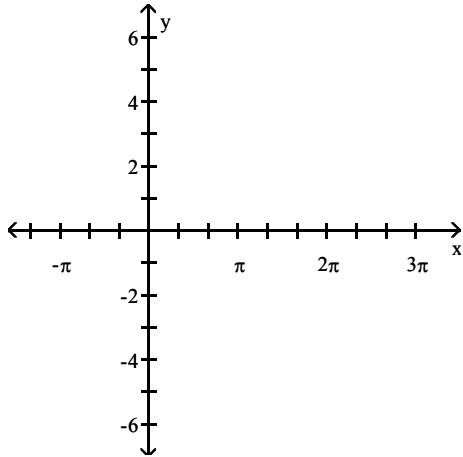
C)



D)

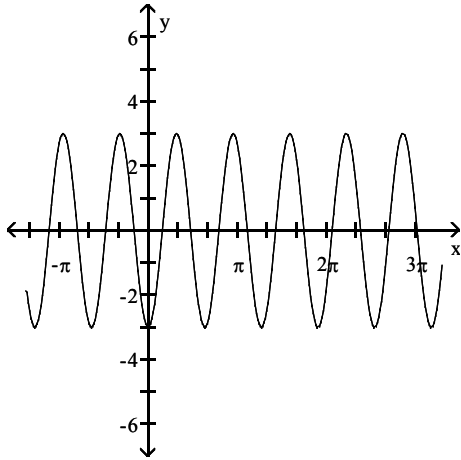


9)  $y = -3 \cos(\pi x)$

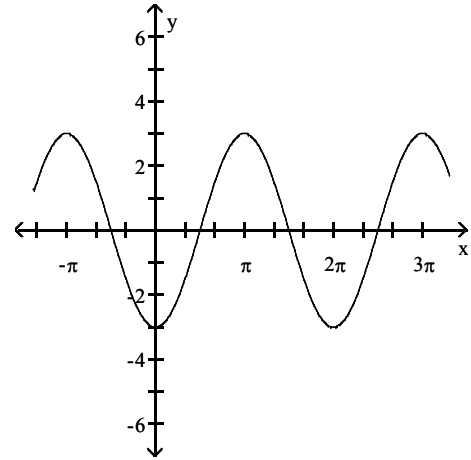




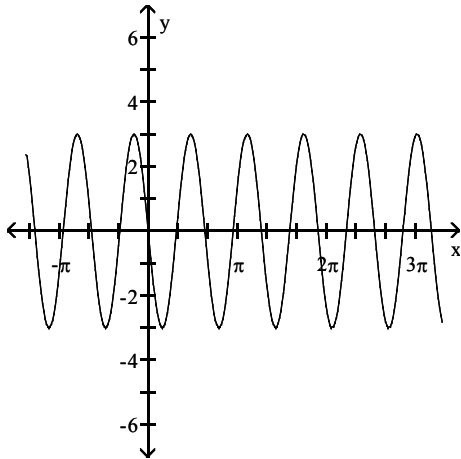
A)



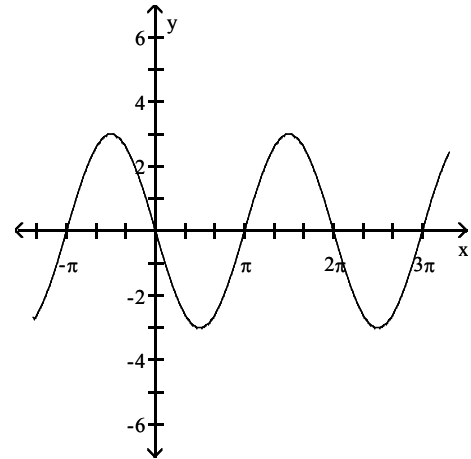
B)



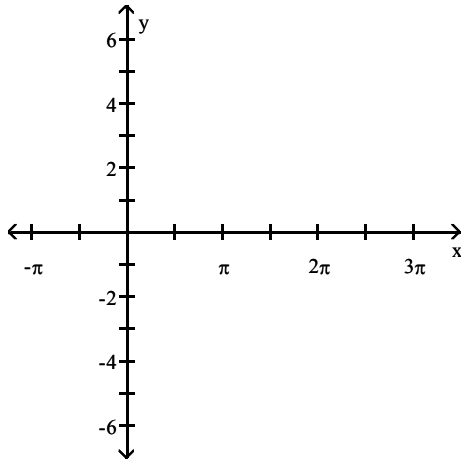
C)



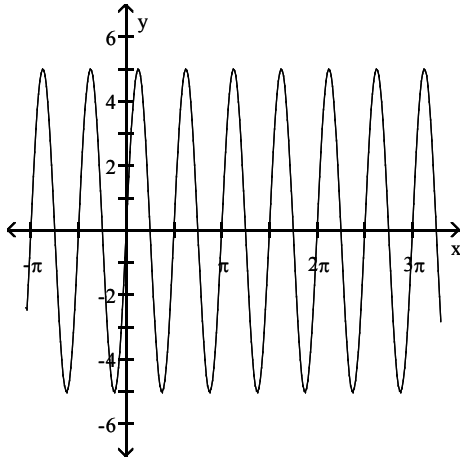
D)



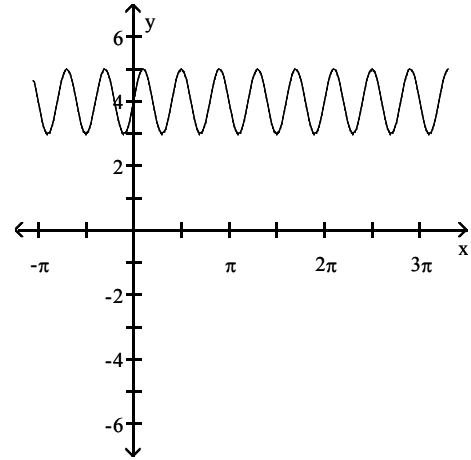
10)  $y = 5 \sin(4x)$



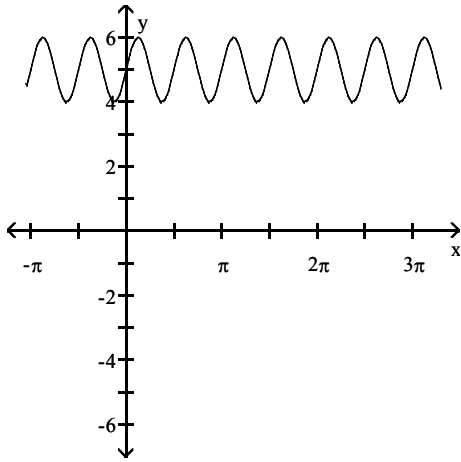
A)



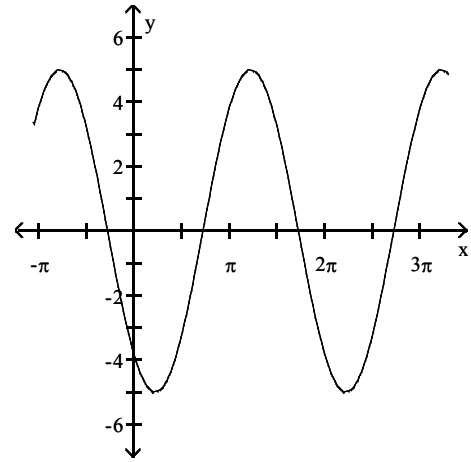
B)



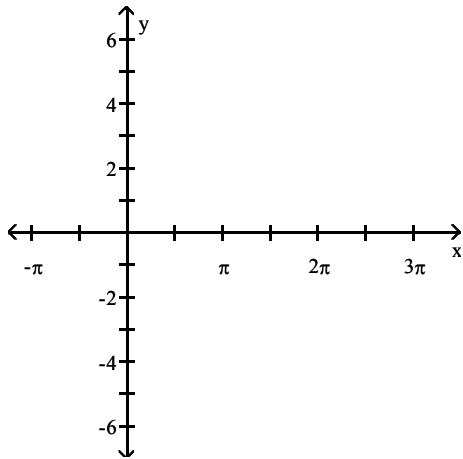
C)



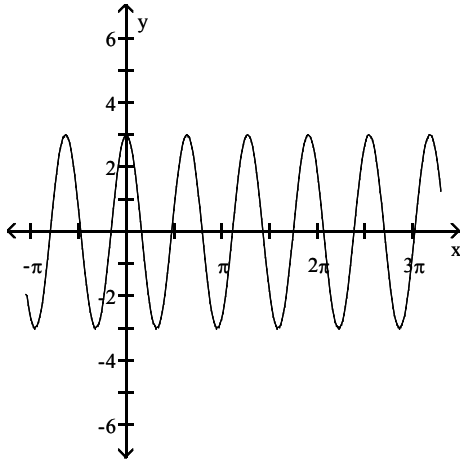
D)



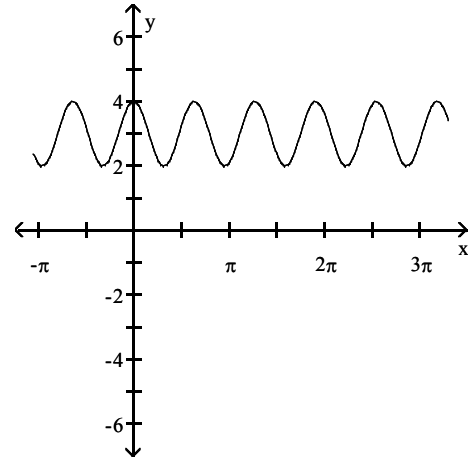
11)  $y = 3 \cos(\pi x)$



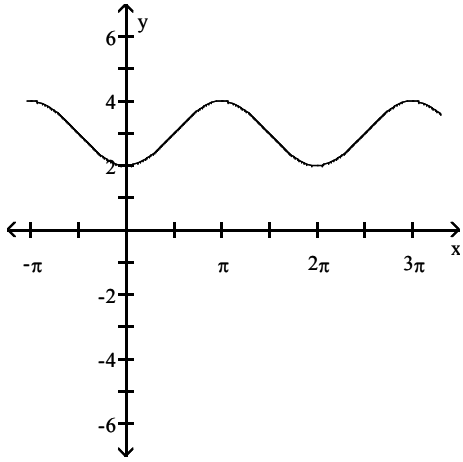
A)



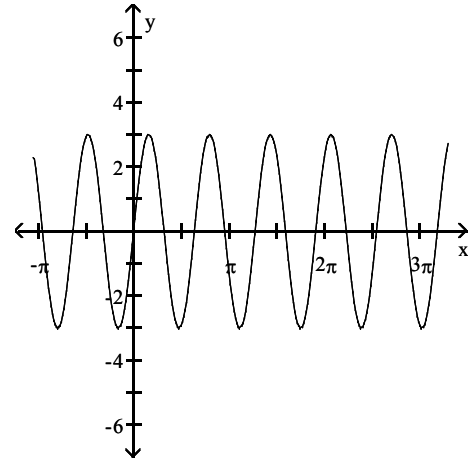
B)



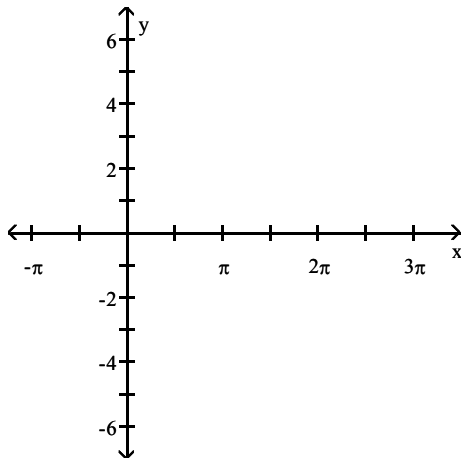
C)



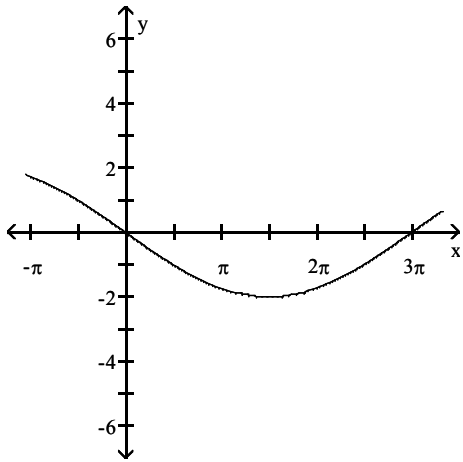
D)



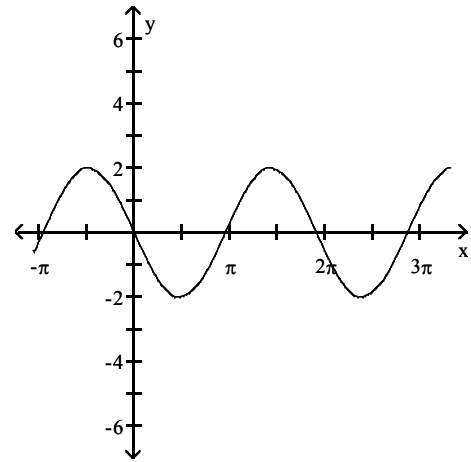
12)  $y = -2 \sin\left(\frac{1}{3}x\right)$



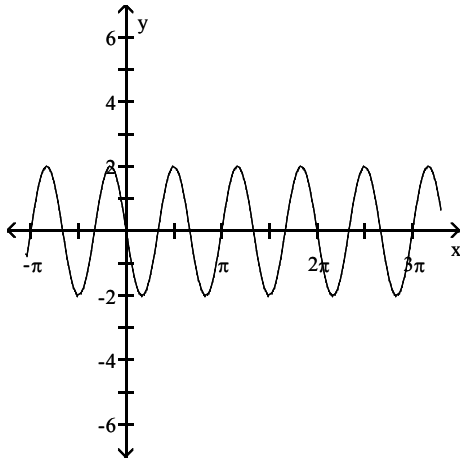
A)



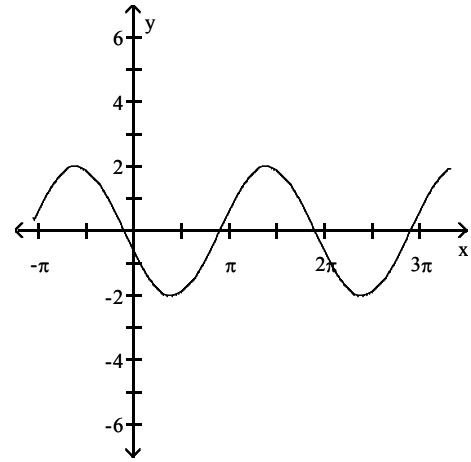
B)



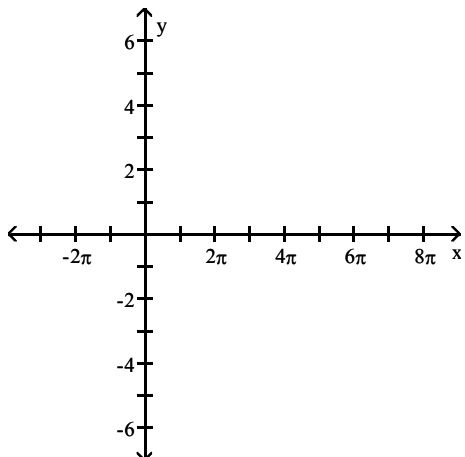
C)



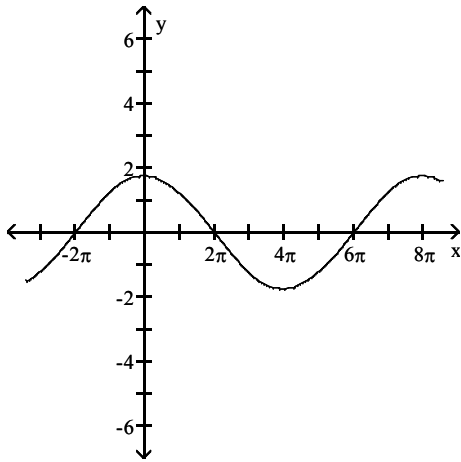
D)



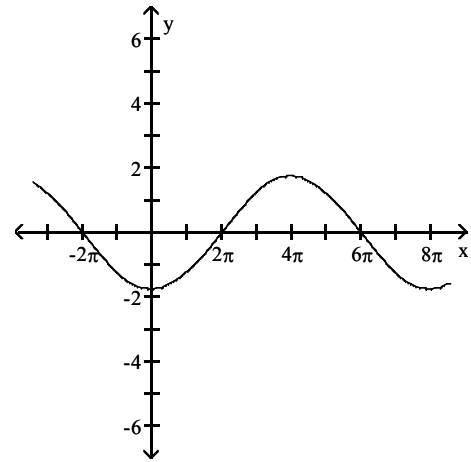
13)  $y = \frac{7}{4} \cos(-\frac{1}{4}x)$



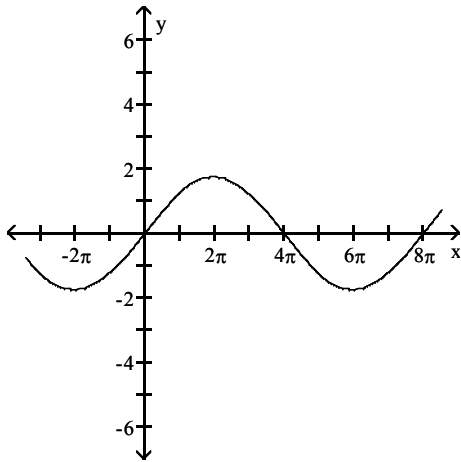
A)



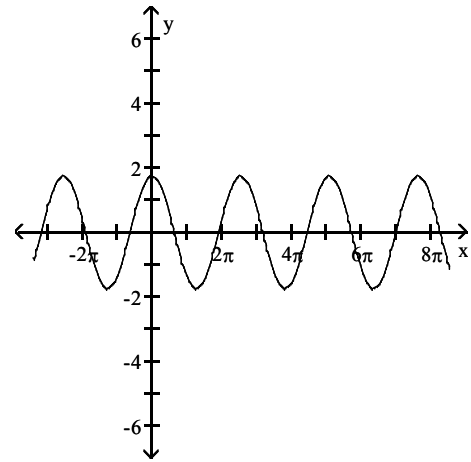
B)



C)

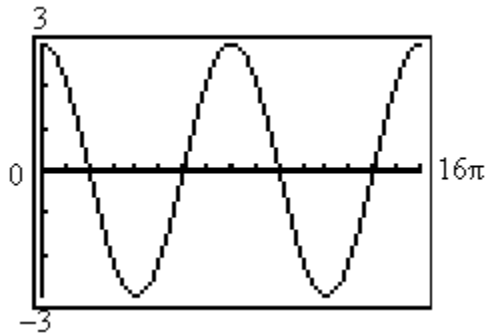


D)



**Answer the question.**

14) Which one of the equations below matches the graph?



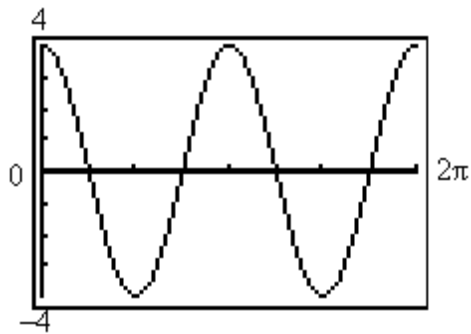
A)  $y = 3 \cos\left(\frac{1}{4}x\right)$

B)  $y = 3 \cos(4x)$

C)  $y = 3 \sin\left(\frac{1}{4}x\right)$

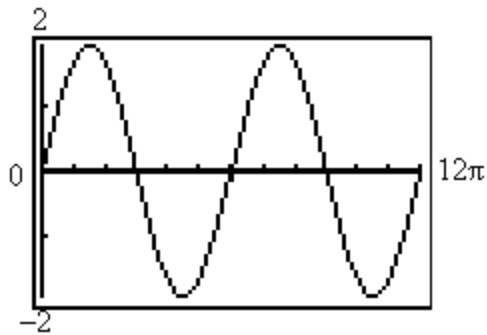
D)  $y = -3 \sin(4x)$

15) Which one of the equations below matches the graph?



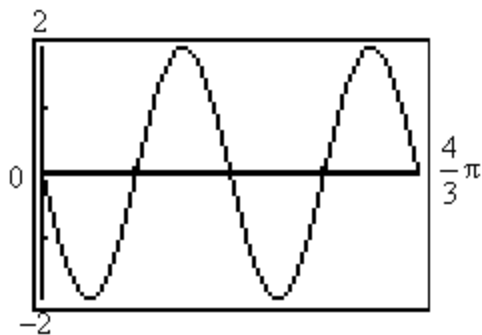
- A)  $y = 4 \cos(2x)$       B)  $y = 4 \sin\left(\frac{1}{2}x\right)$       C)  $y = 2 \cos\left(\frac{1}{4}x\right)$       D)  $y = 4 \cos\left(\frac{1}{2}x\right)$

16) Which one of the equations below matches the graph?



- A)  $y = 2 \sin\left(\frac{1}{3}x\right)$       B)  $y = -2 \sin\left(\frac{1}{3}x\right)$       C)  $y = 2 \cos\left(\frac{1}{3}x\right)$       D)  $y = 2 \cos(3x)$

17) Which one of the equations below matches the graph?



- A)  $y = -2 \sin(3x)$       B)  $y = -2 \sin\left(\frac{1}{3}x\right)$       C)  $y = 2 \sin\left(\frac{1}{3}x\right)$       D)  $y = -2 \cos(3x)$

## 5 Find an Equation for a Sinusoidal Graph

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Write the equation of a sine function that has the given characteristics.**

- 1) Amplitude: 4  
Period:  $3\pi$

- A)  $y = 4 \sin\left(\frac{2}{3}x\right)$       B)  $y = 4 \sin(3x)$       C)  $y = \sin(3x) + 4$       D)  $y = 3 \sin\left(\frac{1}{2}x\right)$

2) Amplitude: 3  
 Period: 4

A)  $y = 3 \sin\left(\frac{1}{2}\pi x\right)$

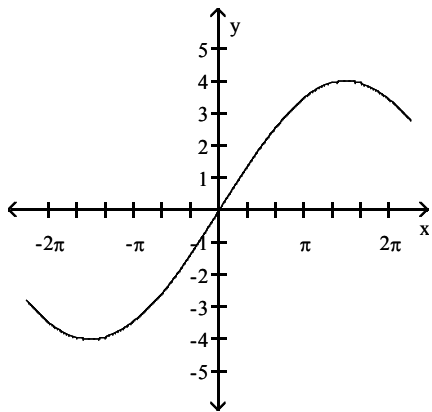
B)  $y = 3 \sin(4x)$

C)  $y = \sin(4\pi x) + 3$

D)  $y = 4 \sin\left(\frac{2}{3}\pi x\right)$

Find an equation for the graph.

3)



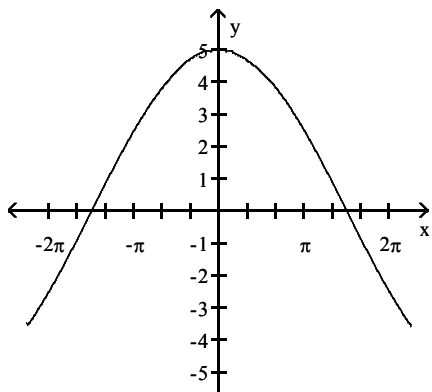
A)  $y = 4 \sin\left(\frac{1}{3}x\right)$

B)  $y = 4 \sin(3x)$

C)  $y = 3 \sin\left(\frac{1}{4}x\right)$

D)  $y = 3 \sin(4x)$

4)



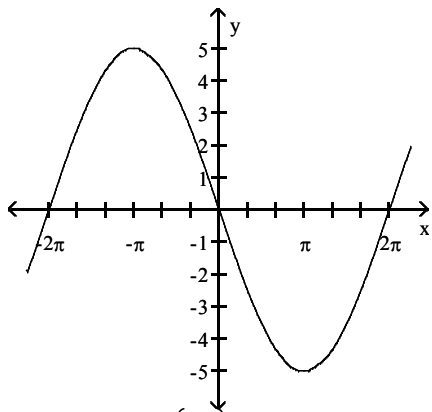
A)  $y = 5 \cos\left(\frac{1}{3}x\right)$

B)  $y = 5 \cos(3x)$

C)  $y = 3 \cos\left(\frac{1}{5}x\right)$

D)  $y = 3 \cos(5x)$

5)



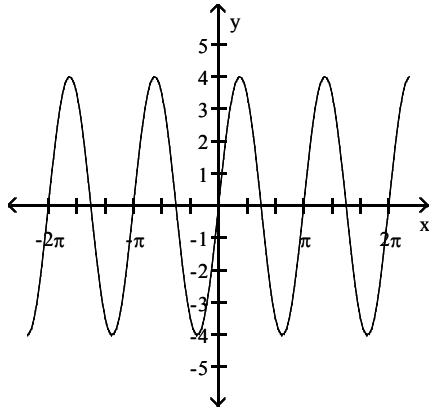
A)  $y = -5 \sin\left(\frac{1}{2}x\right)$

B)  $y = -5 \sin(2x)$

C)  $y = -5 \cos\left(\frac{1}{2}x\right)$

D)  $y = -5 \cos(2x)$

6)



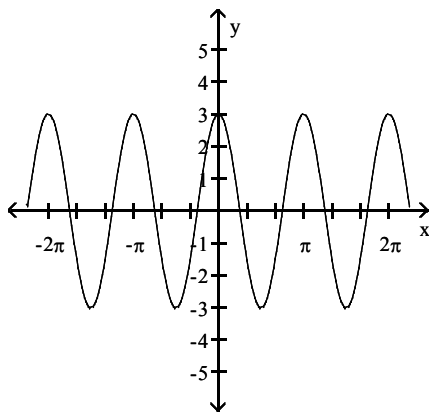
A)  $y = 4 \sin(2x)$

B)  $y = 4 \sin\left(\frac{1}{2}x\right)$

C)  $y = 2 \sin\left(\frac{1}{4}x\right)$

D)  $y = 2 \sin(4x)$

7)



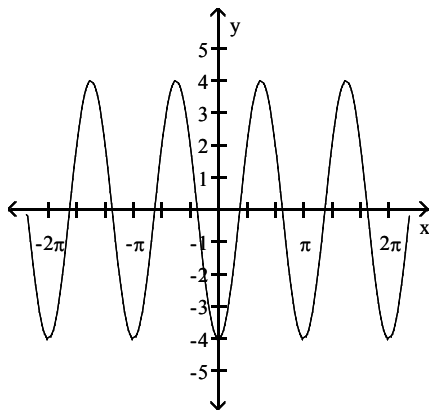
A)  $y = 3 \cos(2x)$

B)  $y = 3 \cos\left(\frac{1}{2}x\right)$

C)  $y = 2 \cos\left(\frac{1}{3}x\right)$

D)  $y = 2 \cos(3x)$

8)



A)  $y = -4 \cos(2x)$

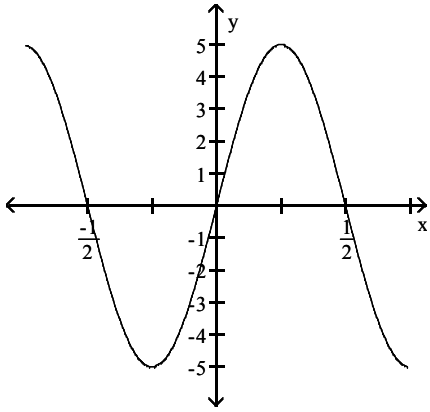
B)  $y = -4 \cos\left(\frac{1}{2}x\right)$

C)  $y = -4 \sin\left(\frac{1}{2}x\right)$

D)  $y = -4 \sin(2x)$



9)



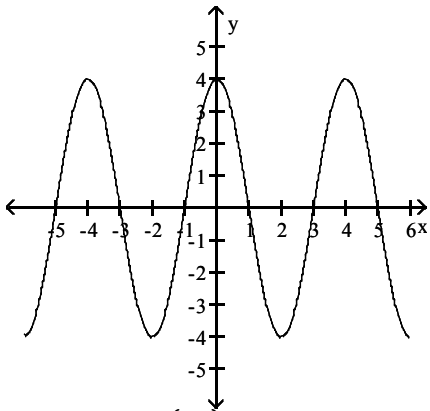
A)  $y = 5 \sin(2\pi x)$

B)  $y = 5 \sin\left(\frac{\pi}{2}x\right)$

C)  $y = 2 \sin(5\pi x)$

D)  $y = 2 \sin\left(\frac{\pi}{5}x\right)$

10)



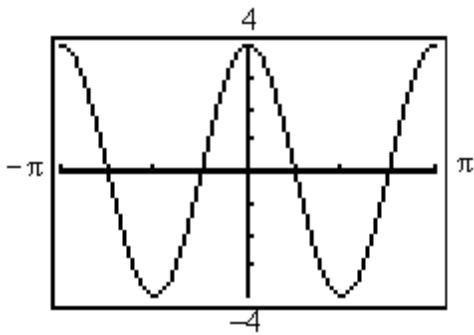
A)  $y = 4 \cos\left(\frac{\pi}{2}x\right)$

B)  $y = 4 \cos(2\pi x)$

C)  $y = 2 \cos(4\pi x)$

D)  $y = 2 \cos\left(\frac{\pi}{4}x\right)$

11)



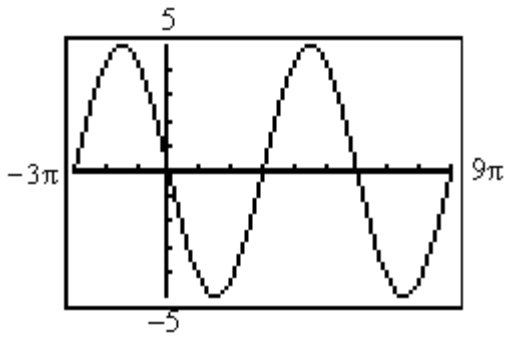
A)  $y = 4 \cos(2x)$

B)  $y = 4 \cos\left(\frac{1}{2}x\right)$

C)  $y = 4 \sin(2x)$

D)  $y = -4 \cos(2x)$

12)



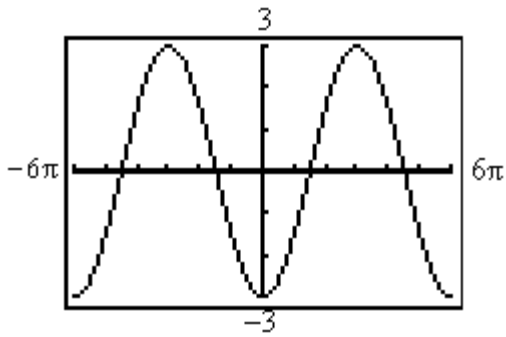
A)  $y = -5 \sin\left(\frac{1}{3}x\right)$

B)  $y = 5 \cos\left(\frac{1}{3}x\right)$

C)  $y = -5 \sin(3x)$

D)  $y = -5 \sin\left(\frac{2}{3}x\right)$

13)



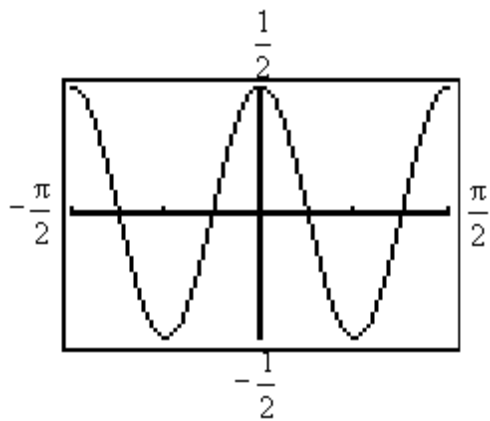
A)  $y = -3 \cos\left(\frac{1}{3}x\right)$

B)  $y = 3 \cos\left(\frac{1}{3}x\right)$

C)  $y = -3 \sin(3x)$

D)  $y = -3 \cos(3x)$

14)



A)  $y = \frac{1}{2} \cos(4x)$

B)  $y = \frac{1}{2} \cos\left(\frac{1}{2}x\right)$

C)  $y = \frac{1}{2} \cos\left(\frac{1}{4}x\right)$

D)  $y = \cos(4x)$

## 2.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

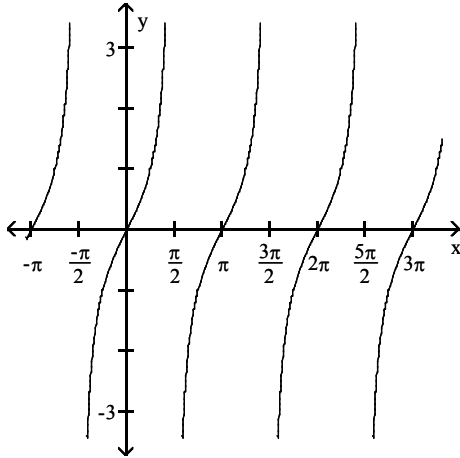
### 1 Graph Functions of the Form $y = A \tan(\omega x) + B$ and $y = A \cot(\omega x) + B$

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

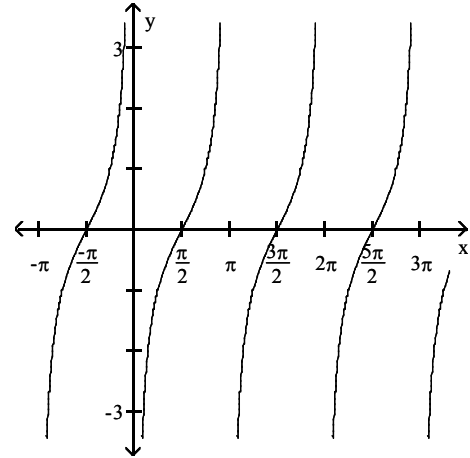
Match the function to its graph.

1)  $y = \tan x$

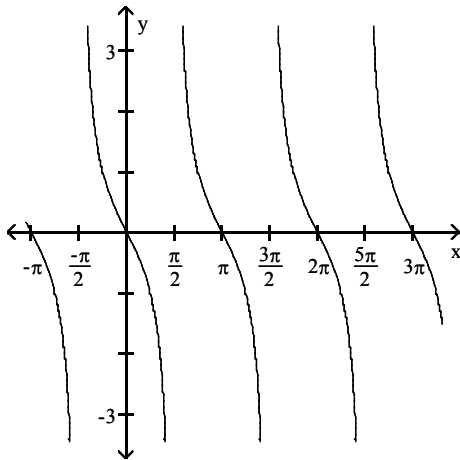
A)



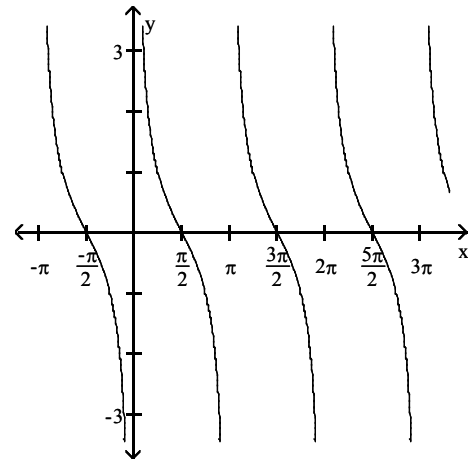
B)



C)

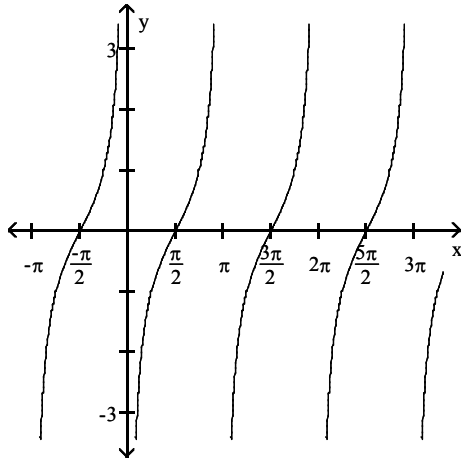


D)

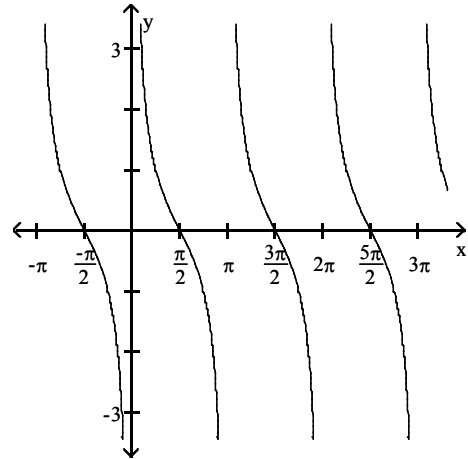


$$2) y = \tan\left(x + \frac{\pi}{2}\right)$$

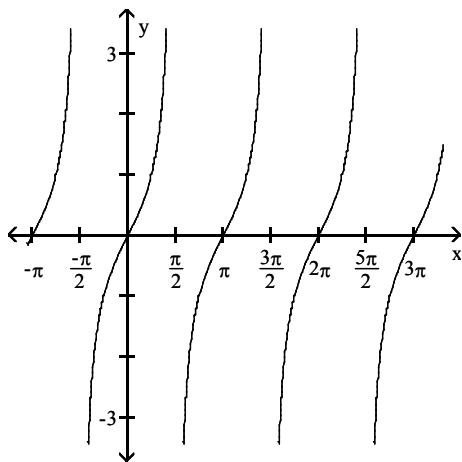
A)



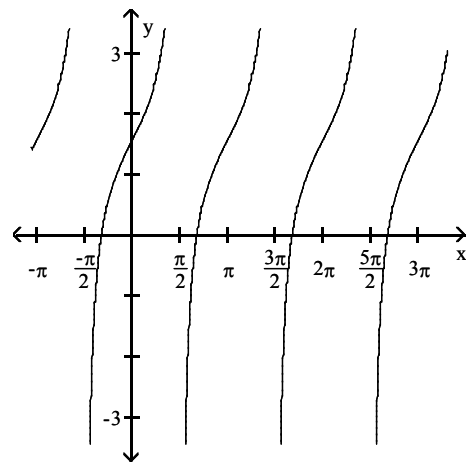
B)



C)

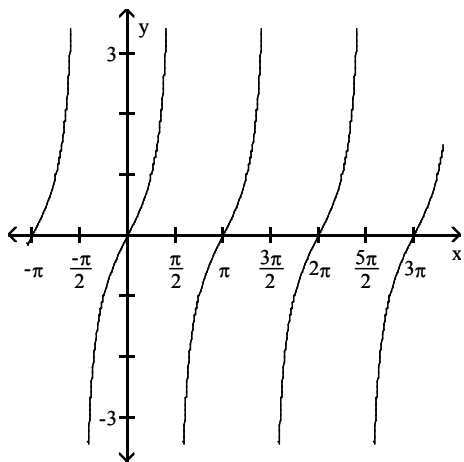


D)

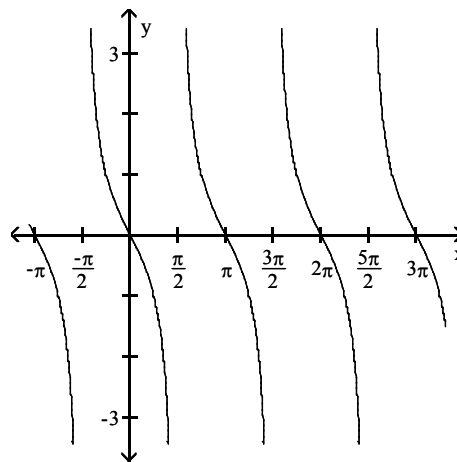


3)  $y = \tan(x + \pi)$

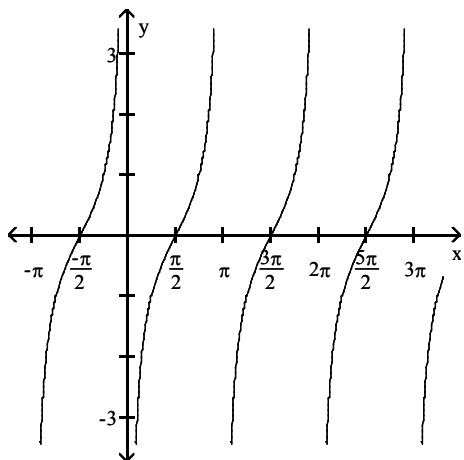
A)



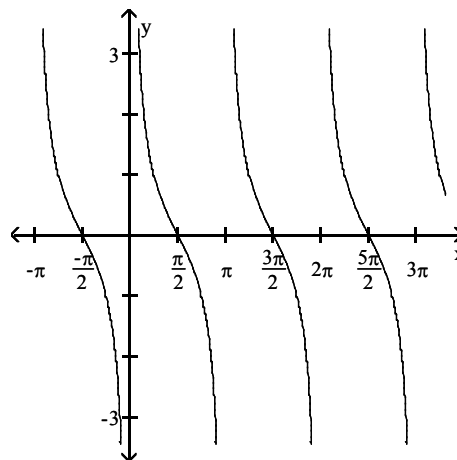
B)



C)

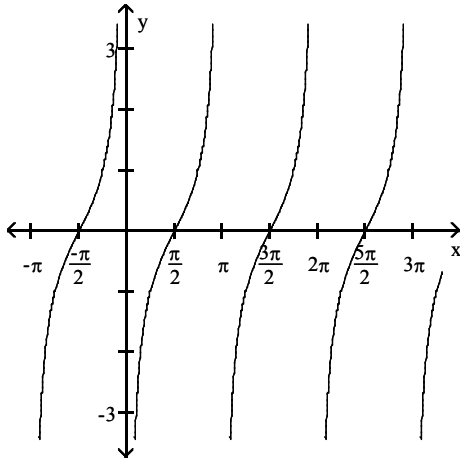


D)

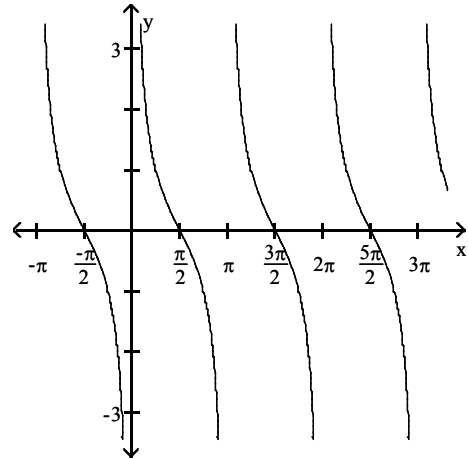


$$4) y = \tan\left(x - \frac{\pi}{2}\right)$$

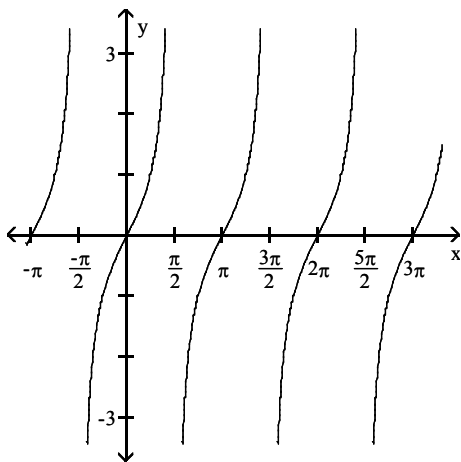
A)



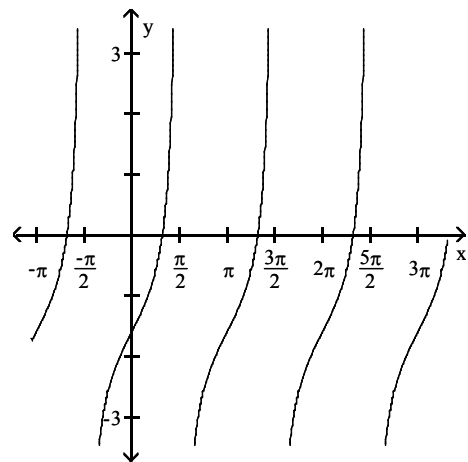
B)



C)

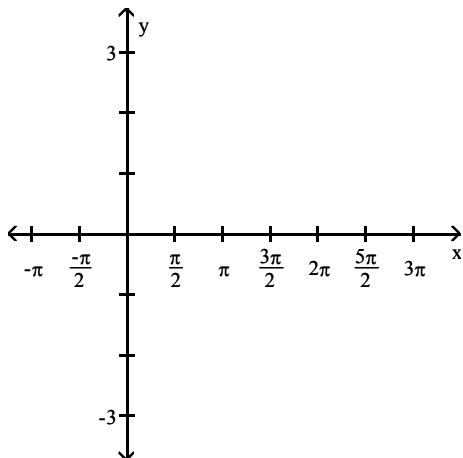


D)

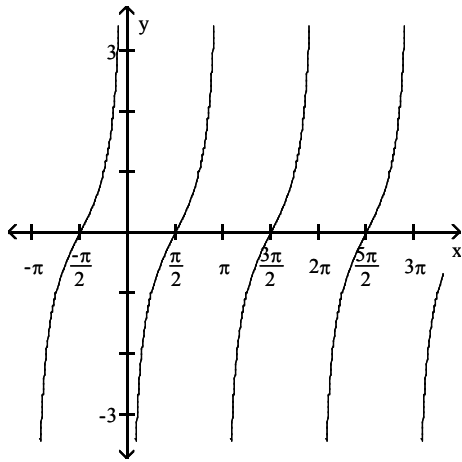


**Graph the function.**

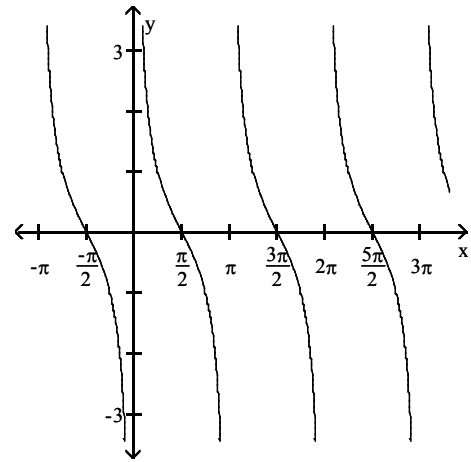
$$5) y = -\cot x$$



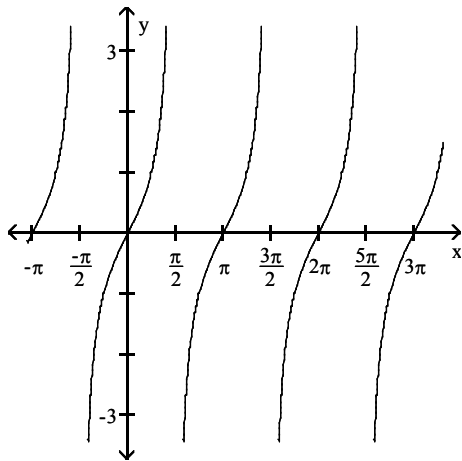
A)



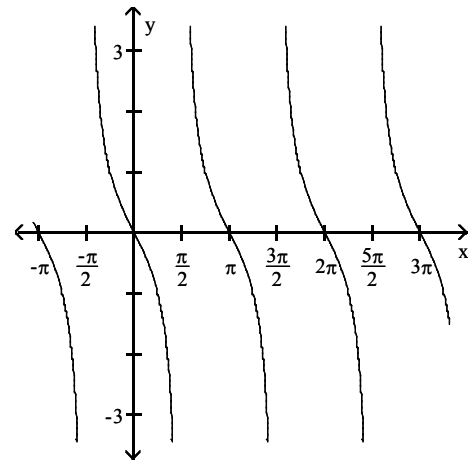
B)



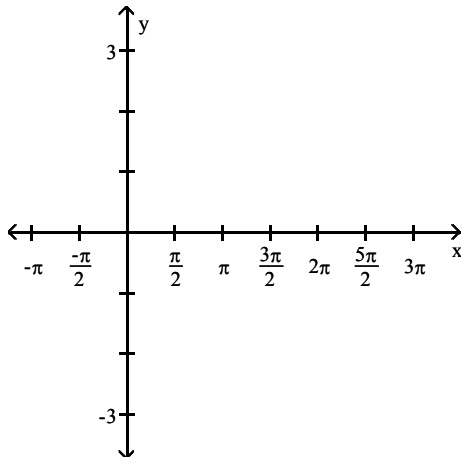
C)



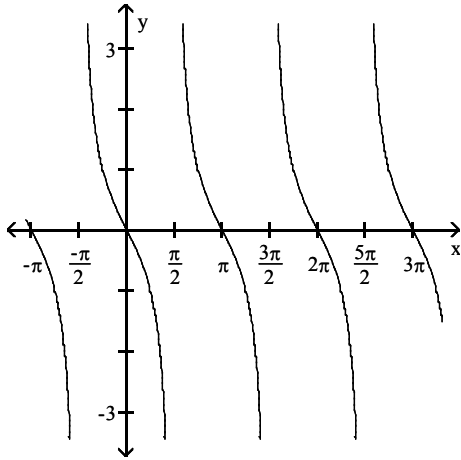
D)



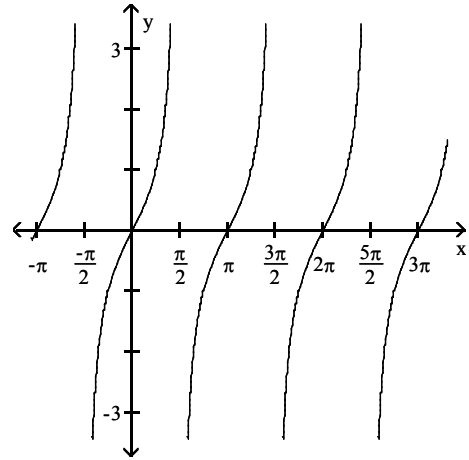
6)  $y = -\tan(x + \pi)$



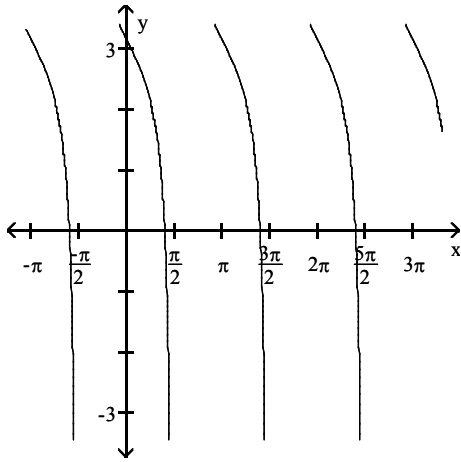
A)



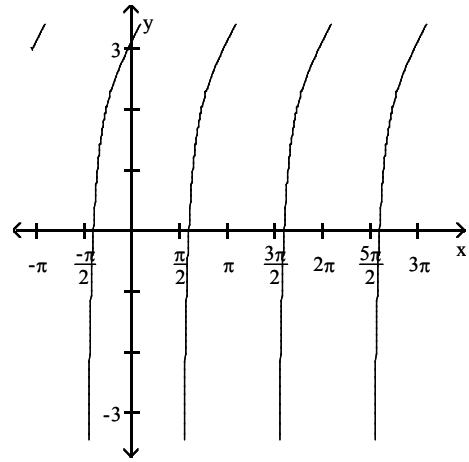
B)



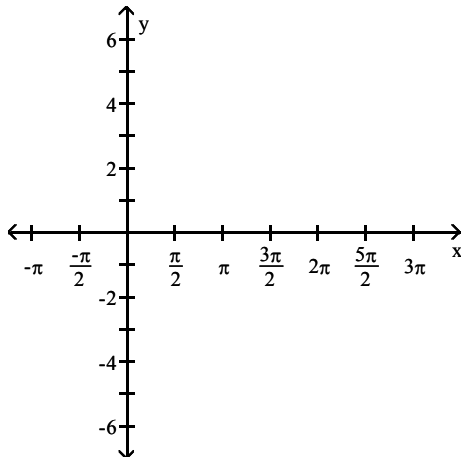
C)



D)

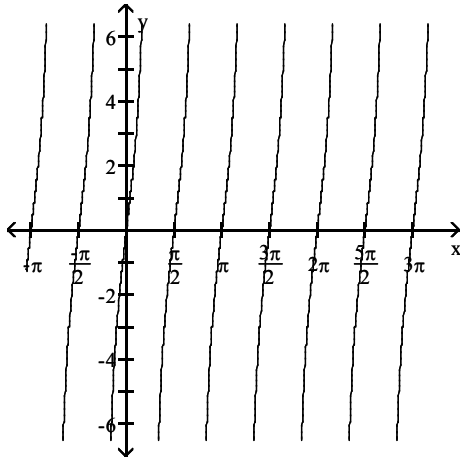


7)  $y = 4 \tan(2x)$

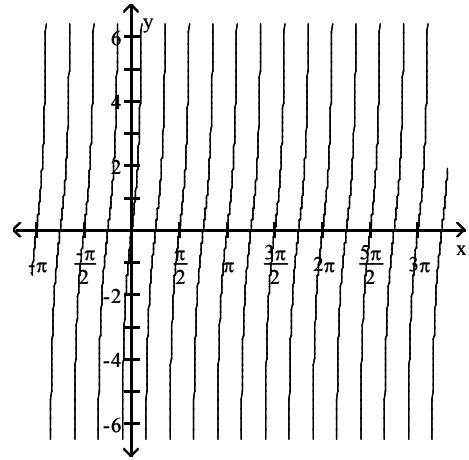




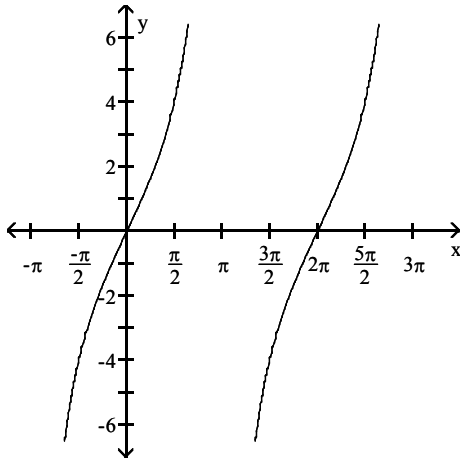
A)



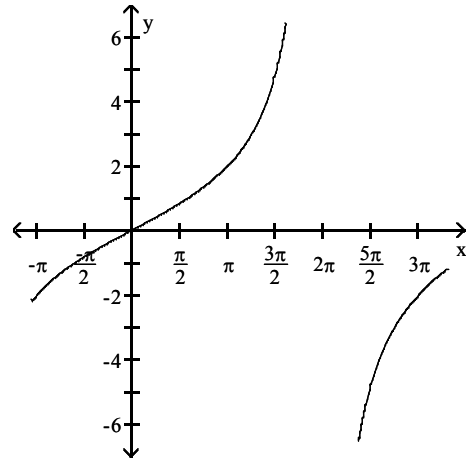
B)



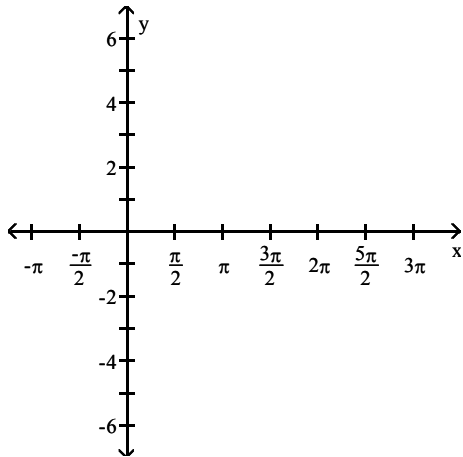
C)



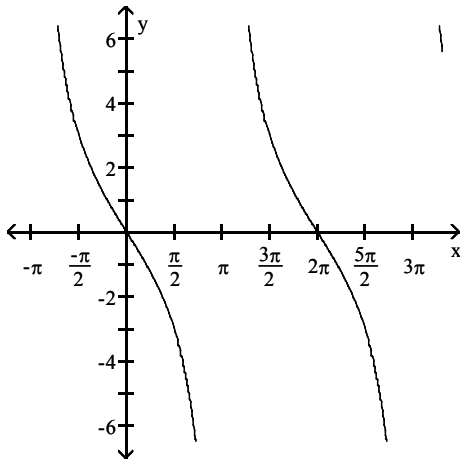
D)



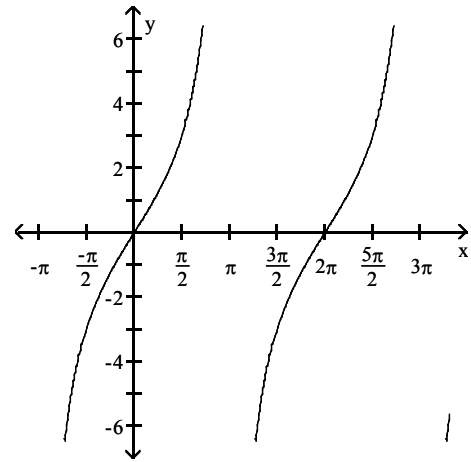
8)  $y = -3 \tan\left(\frac{1}{2}x\right)$



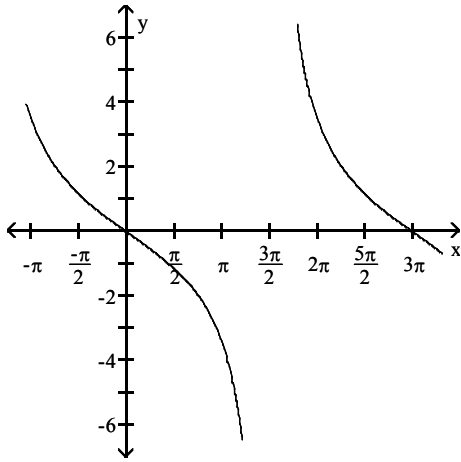
A)



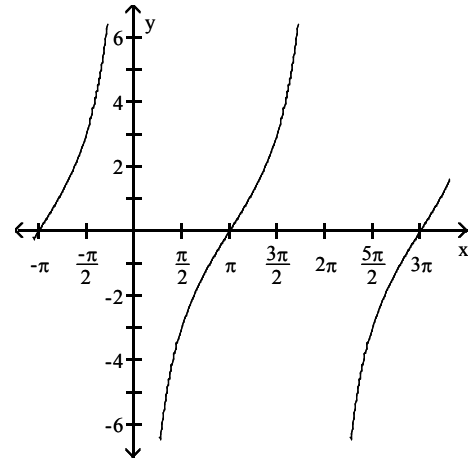
B)



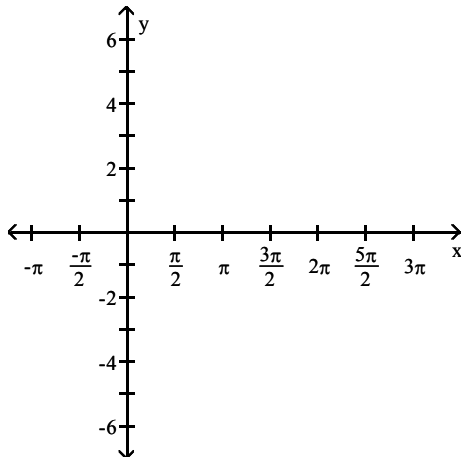
C)



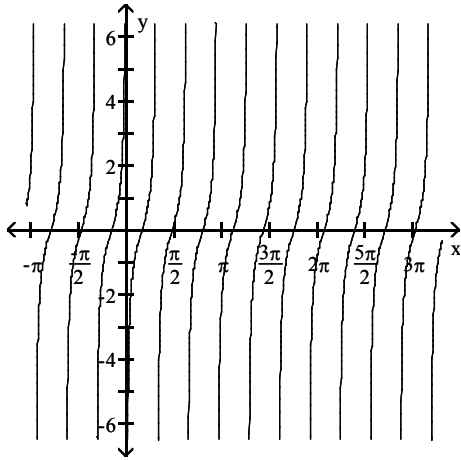
D)



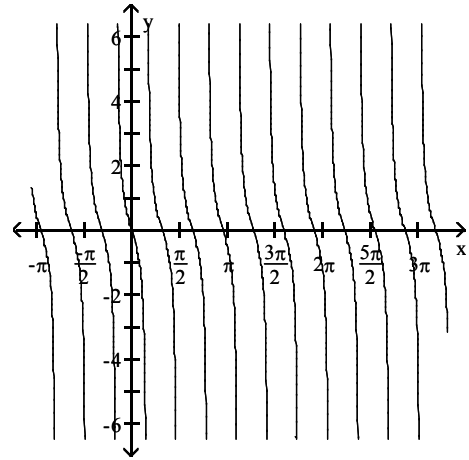
9)  $y = -\cot(\pi x)$



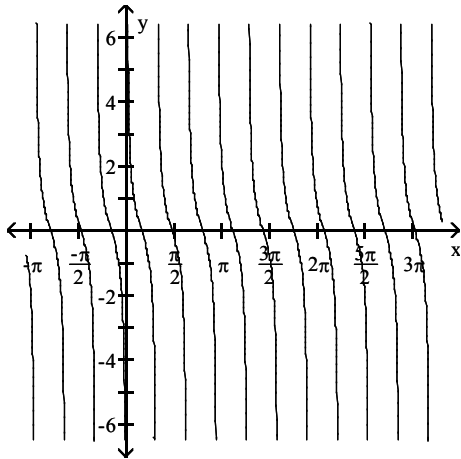
A)



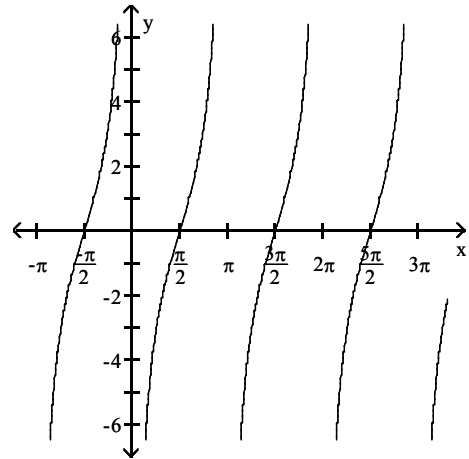
B)



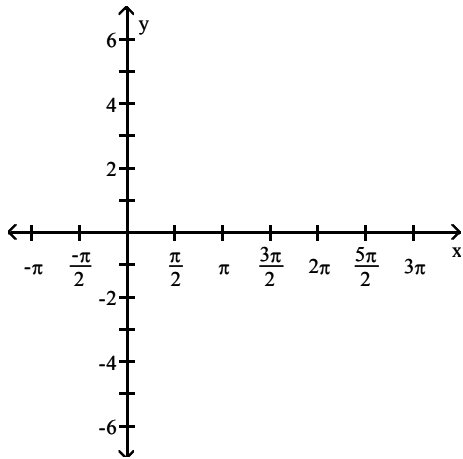
C)



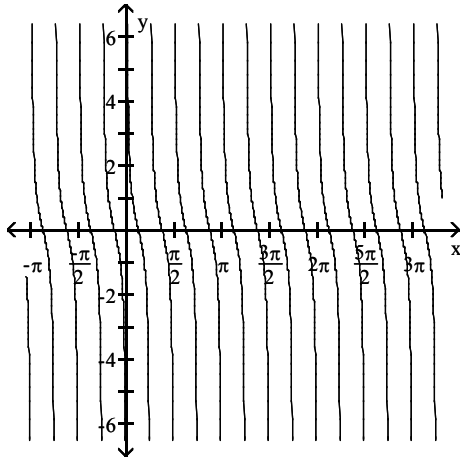
D)



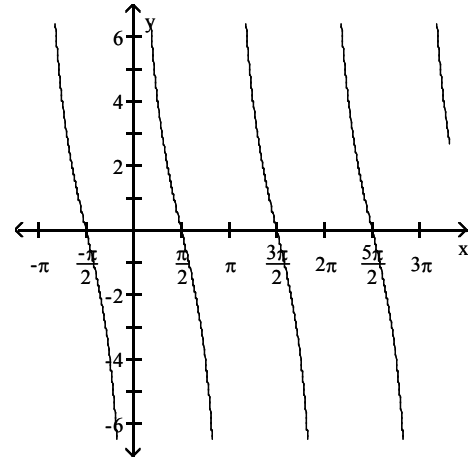
10)  $y = \cot(4x)$



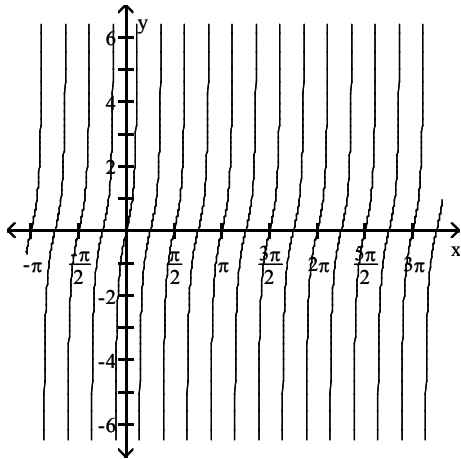
A)



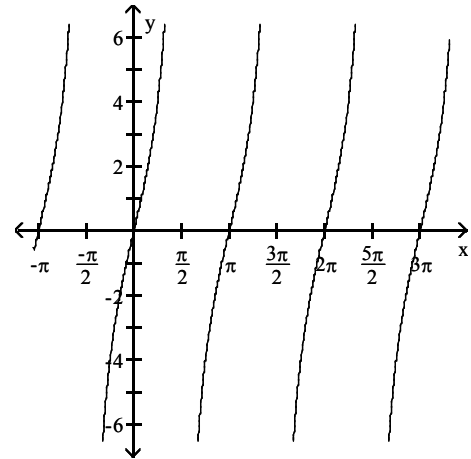
B)



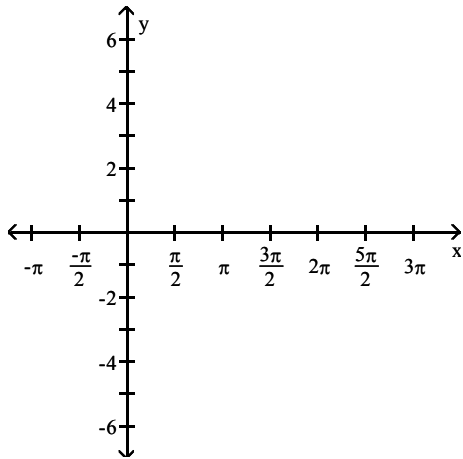
C)



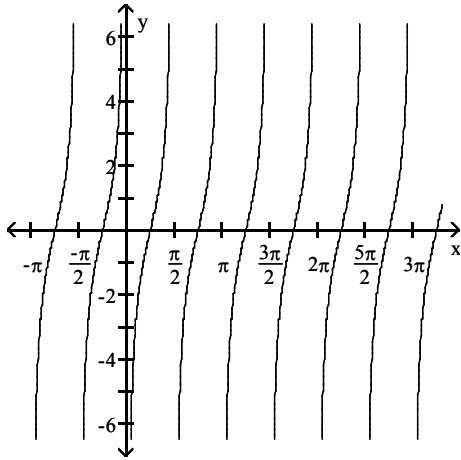
D)



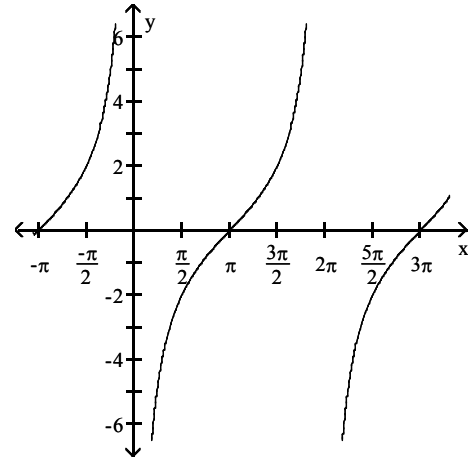
11)  $y = -2 \cot(2x)$



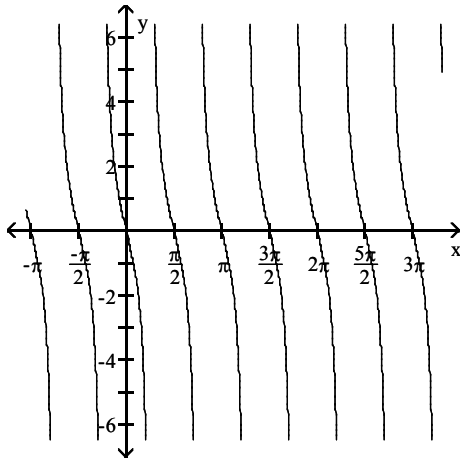
A)



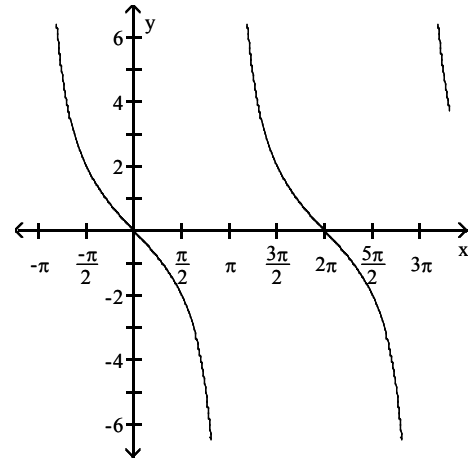
B)



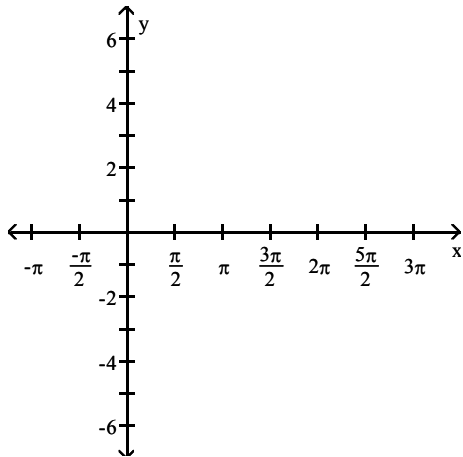
C)



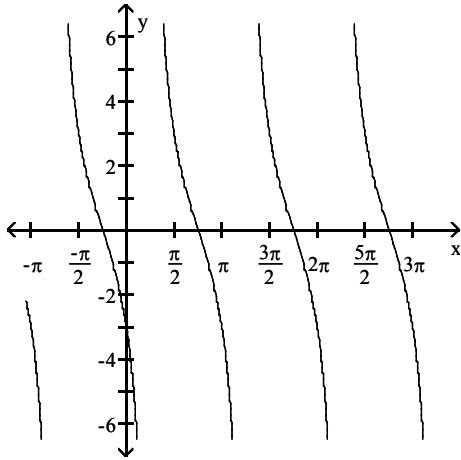
D)



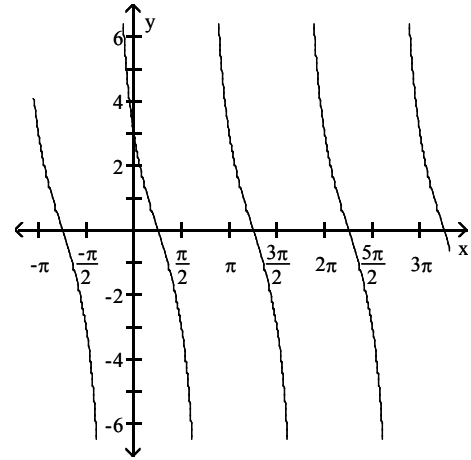
$$12) y = -3 \tan\left(x + \frac{\pi}{4}\right)$$



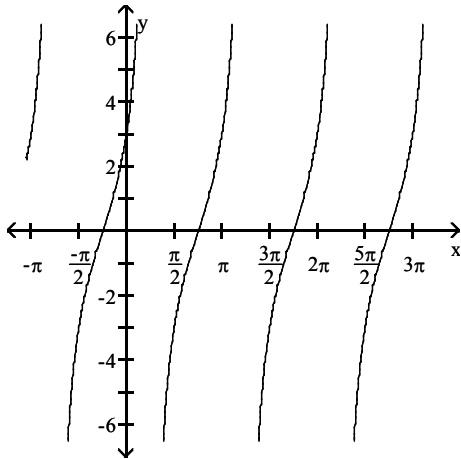
A)



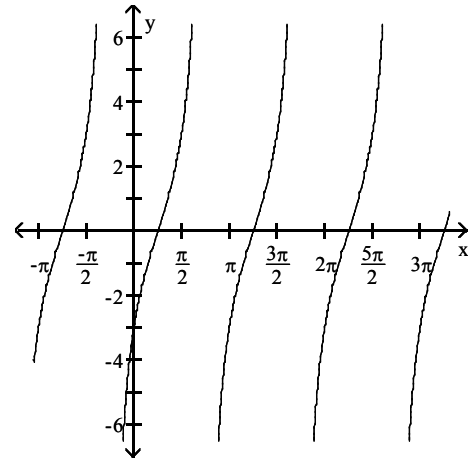
B)



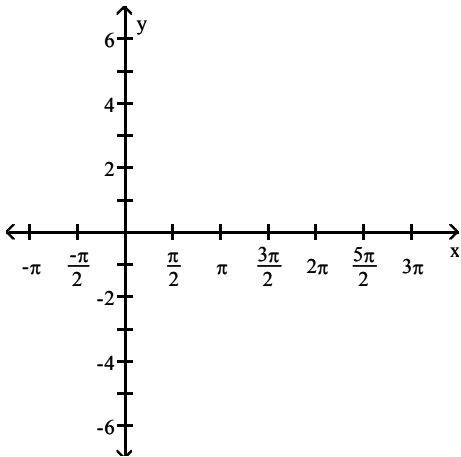
C)



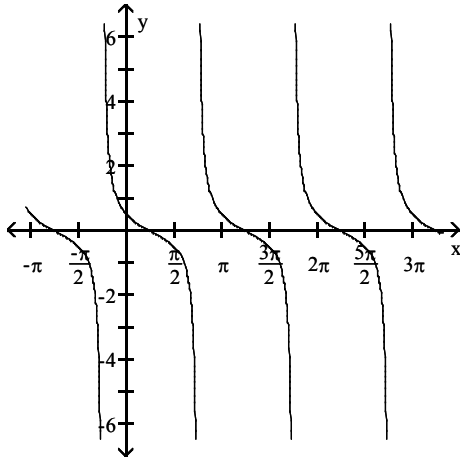
D)



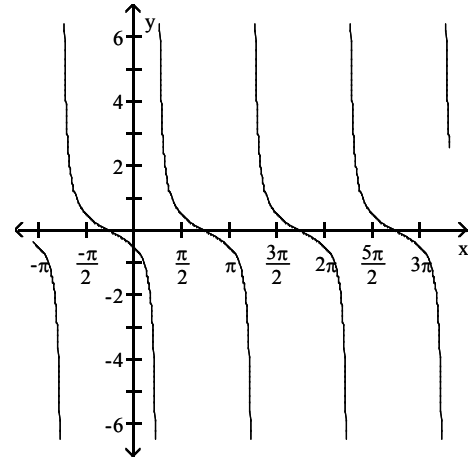
$$13) y = \frac{1}{2} \cot\left(x + \frac{\pi}{4}\right)$$



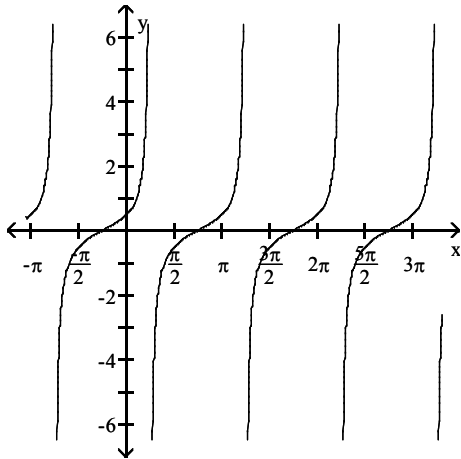
A)



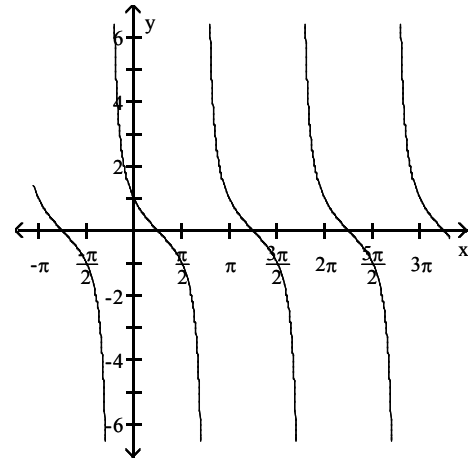
B)



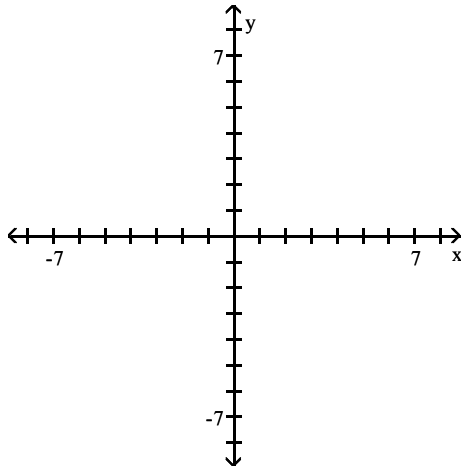
C)



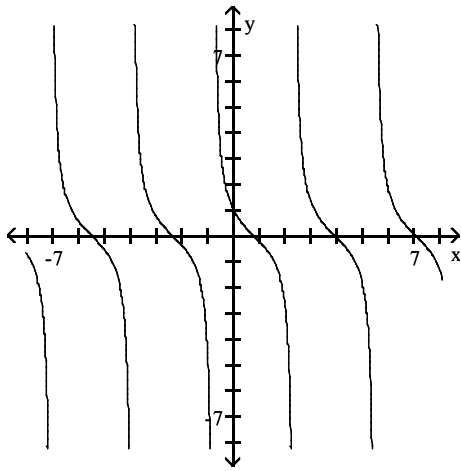
D)



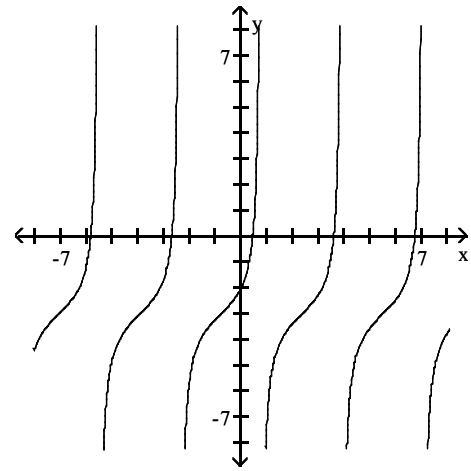
14)  $y = -\tan\left(x - \frac{\pi}{4}\right)$



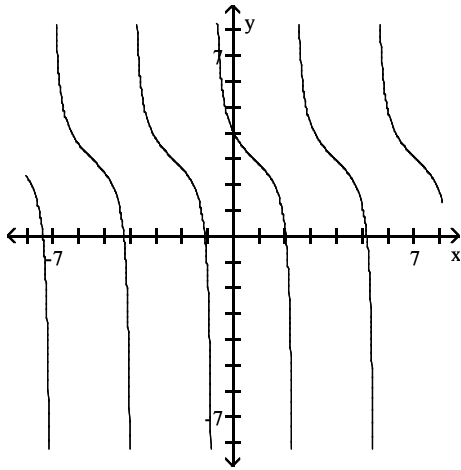
A)



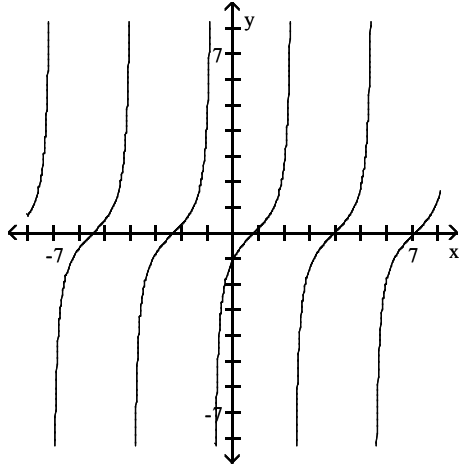
B)



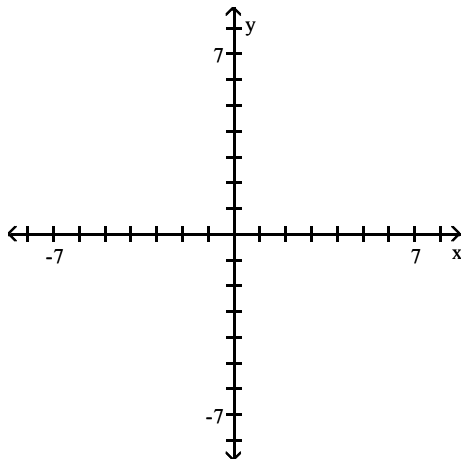
C)



D)

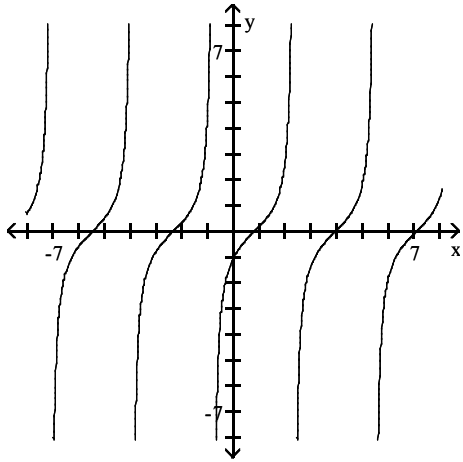


15)  $y = \tan\left(x - \frac{\pi}{4}\right)$

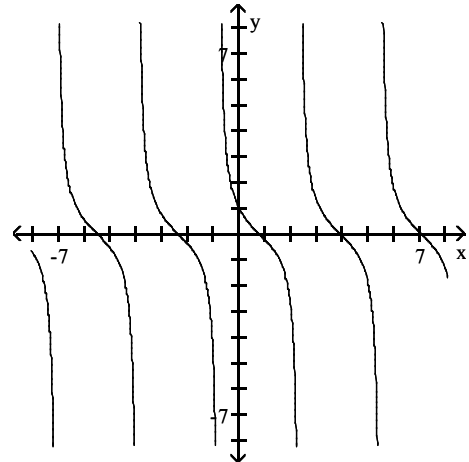




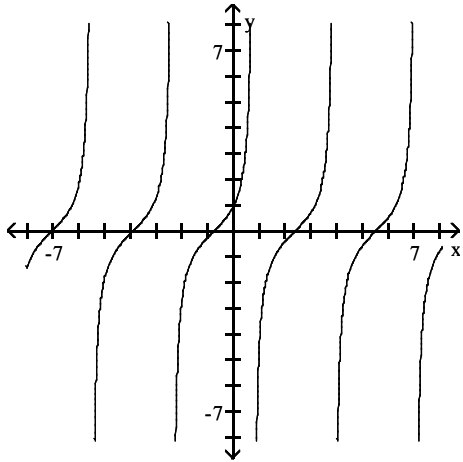
A)



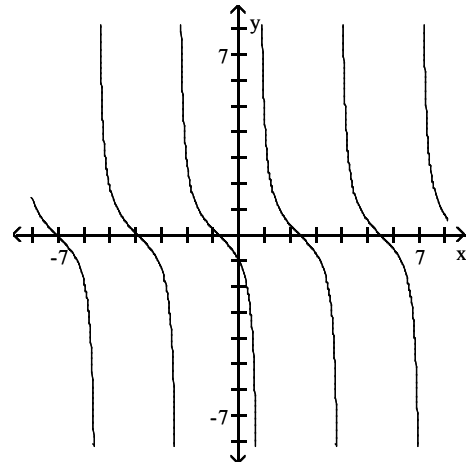
B)



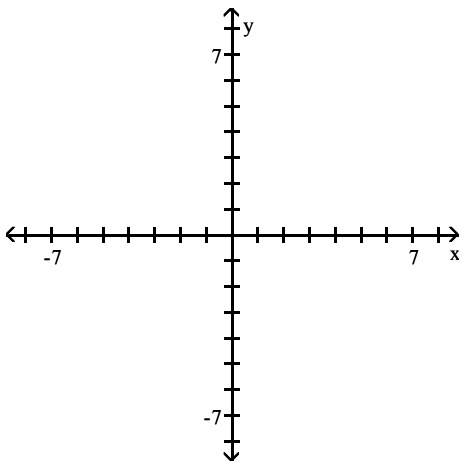
C)



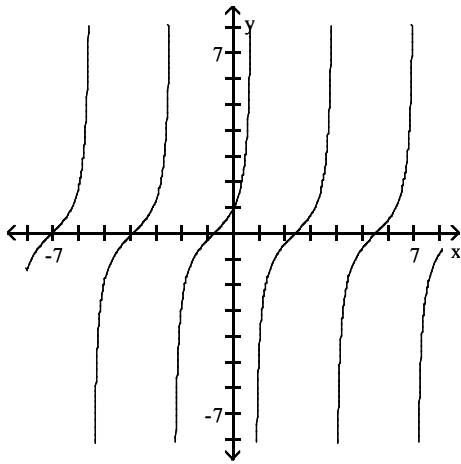
D)



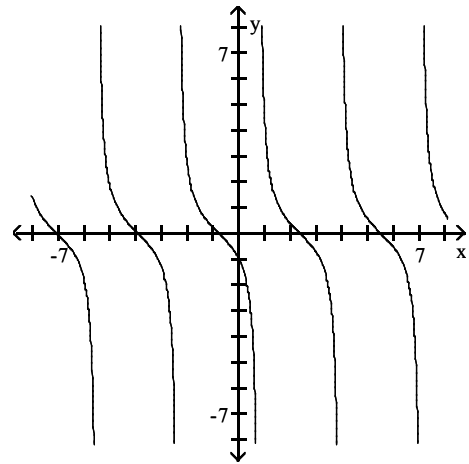
16)  $y = -\cot\left(x - \frac{\pi}{4}\right)$



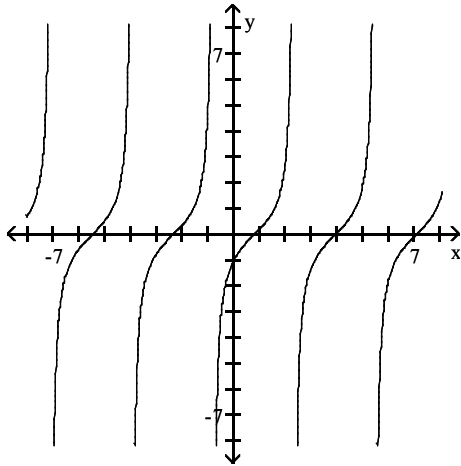
A)



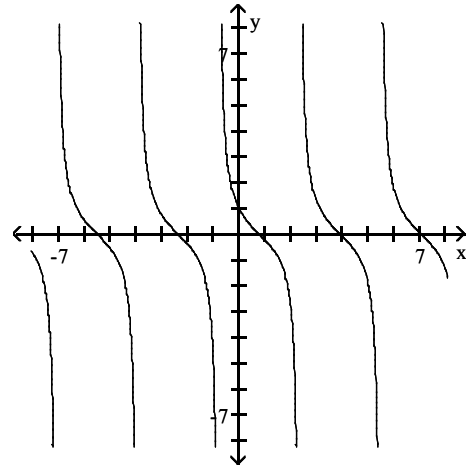
B)



C)



D)



**Solve the problem.**

17) What is the y-intercept of  $y = \sec x$ ?

A) 1

B) 0

C)  $\frac{\pi}{2}$

D) none

18) What is the y-intercept of  $y = \cot x$ ?

A)  $\frac{\pi}{2}$

B) 0

C) 1

D) none

19) For what numbers  $x$ ,  $-2\pi \leq x \leq 2\pi$ , does the graph of  $y = \tan x$  have vertical asymptotes.?

A)  $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

B)  $-2\pi, -\pi, 0, \pi, 2\pi$

C)  $-2, -1, 0, 1, 2$

D) none

20) For what numbers  $x$ ,  $-2\pi \leq x \leq 2\pi$ , does the graph of  $y = \csc x$  have vertical asymptotes.?

A)  $-2\pi, -\pi, 0, \pi, 2\pi$

B)  $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

C)  $-2, -1, 0, 1, 2$

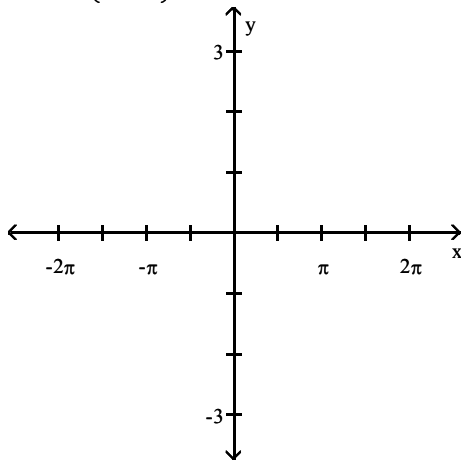
D) none

2 Graph Functions of the Form  $y = A \csc(\omega x) + B$  and  $y = A \sec(\omega x) + B$

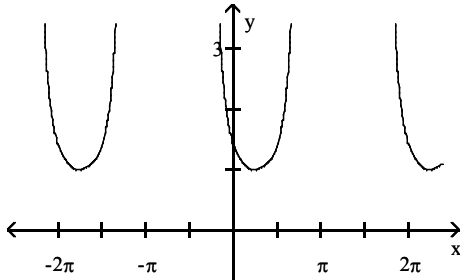
**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Graph the function.

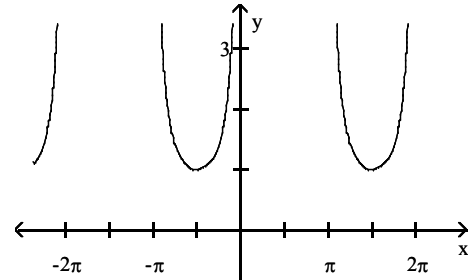
1)  $y = \csc\left(x + \frac{\pi}{4}\right)$



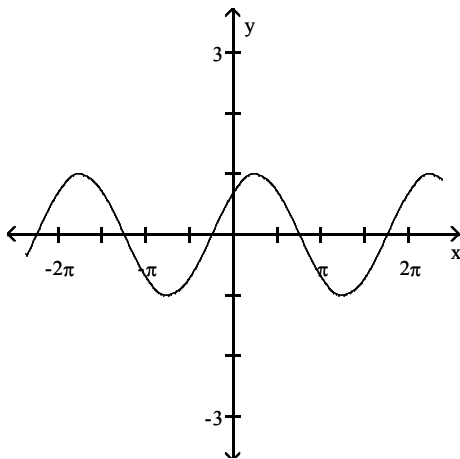
A)



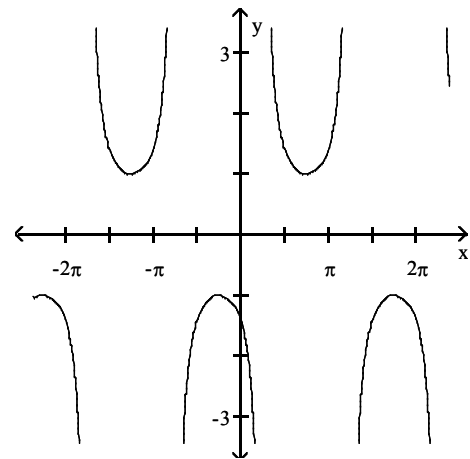
B)



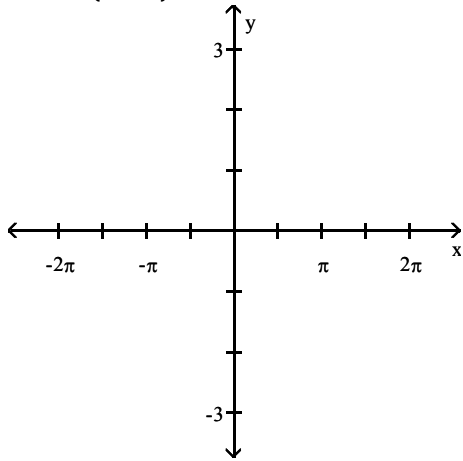
C)



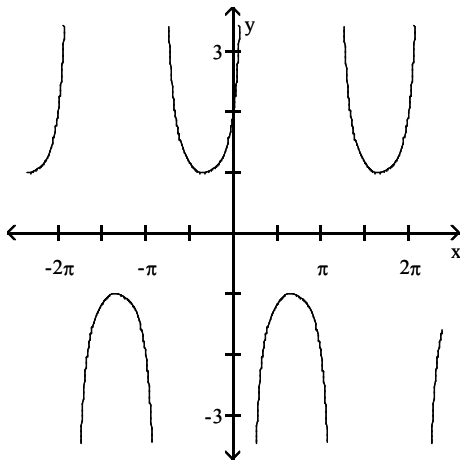
D)



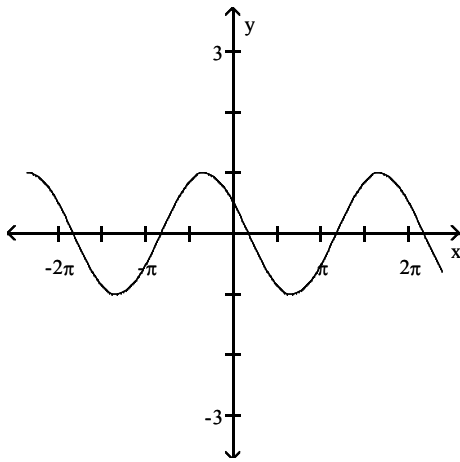
$$2) y = \sec\left(x + \frac{\pi}{3}\right)$$



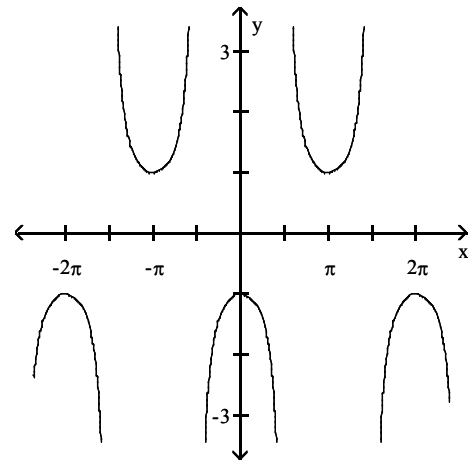
A)



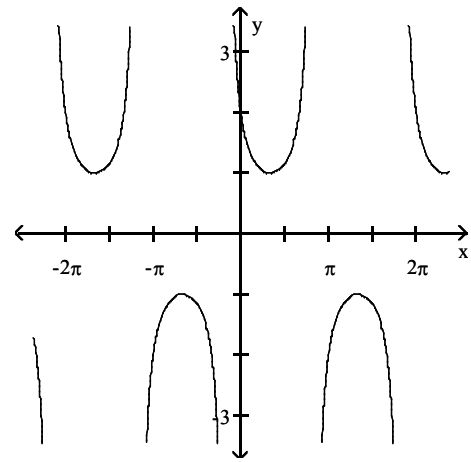
C)



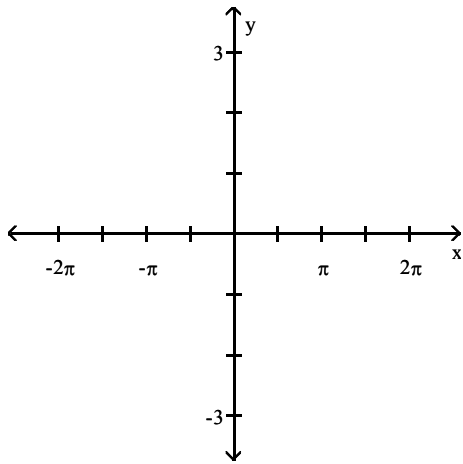
B)



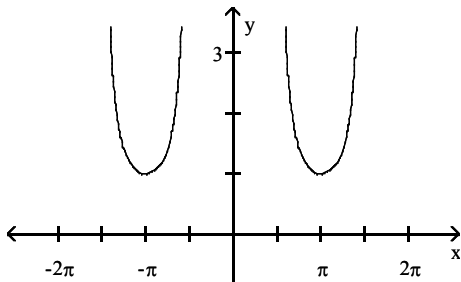
D)



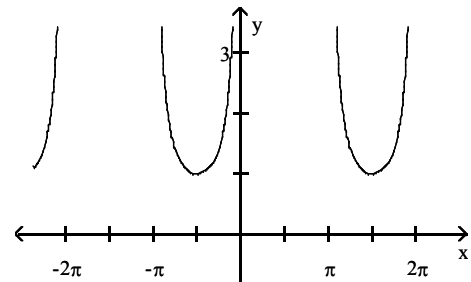
3)  $y = -\sec x$



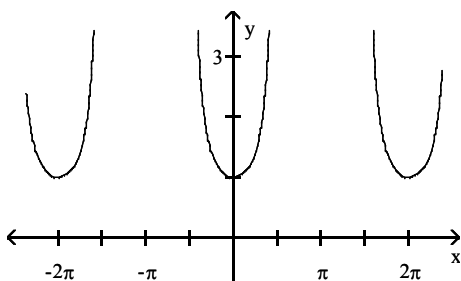
A)



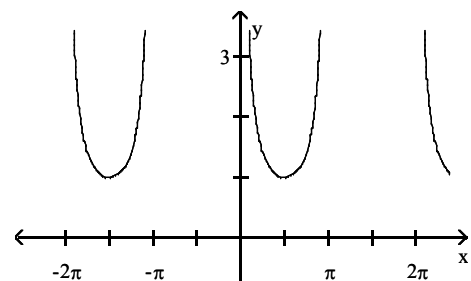
B)



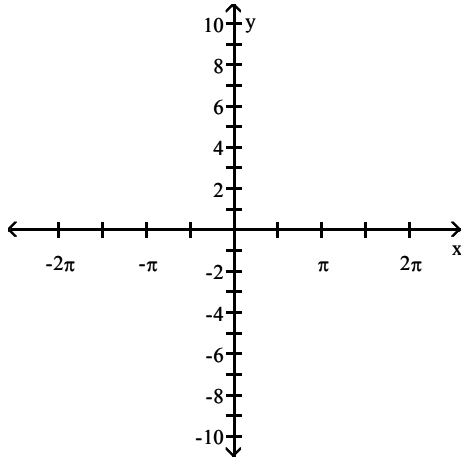
C)



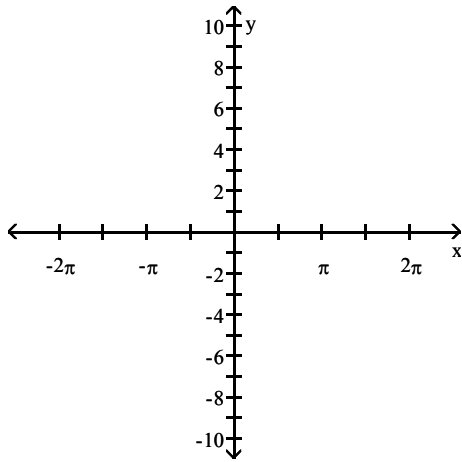
D)



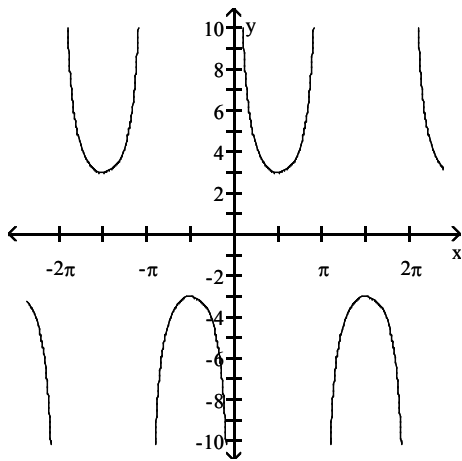
4)  $y = \csc(3x)$



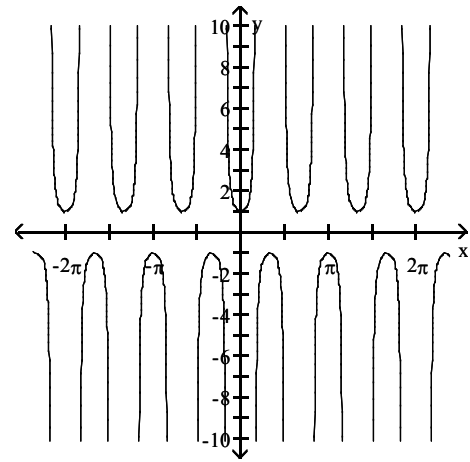
A)



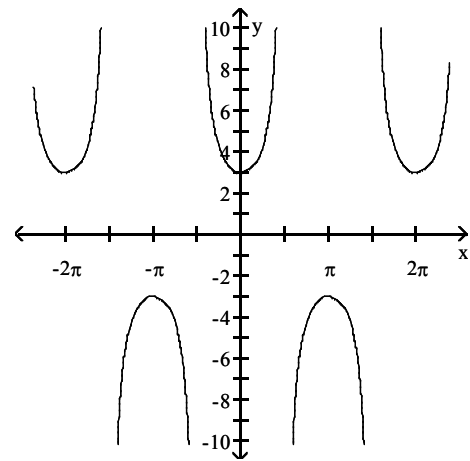
C)



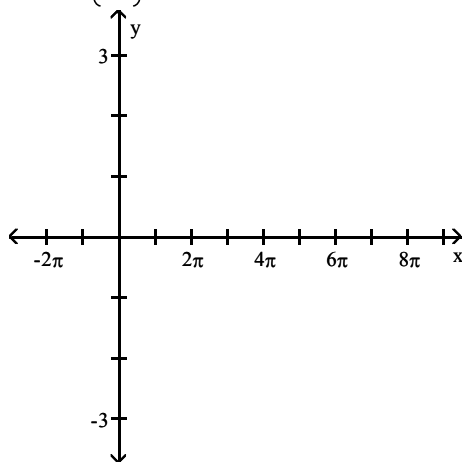
B)



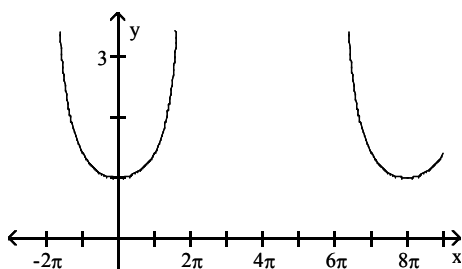
D)



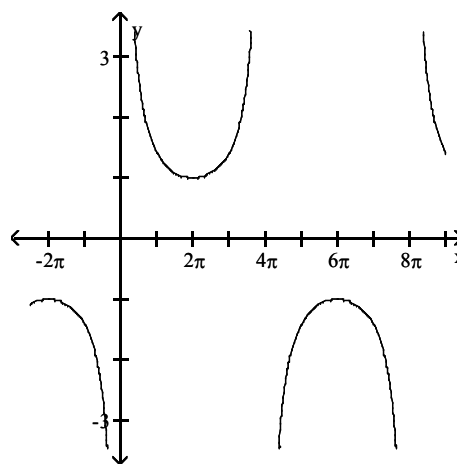
$$5) y = \sec\left(\frac{1}{4}x\right)$$



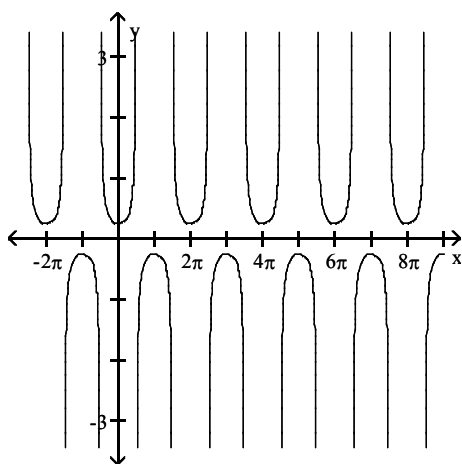
A)



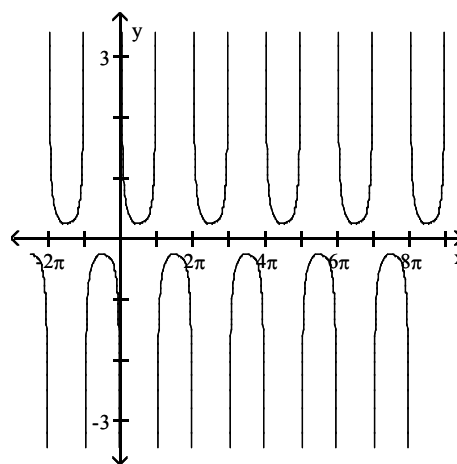
B)



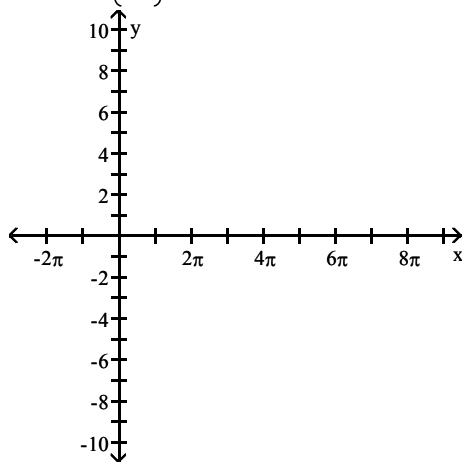
C)



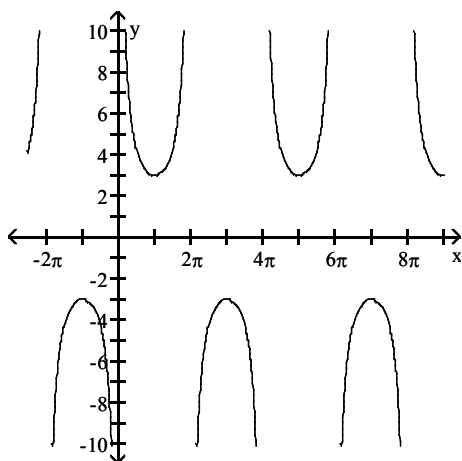
D)



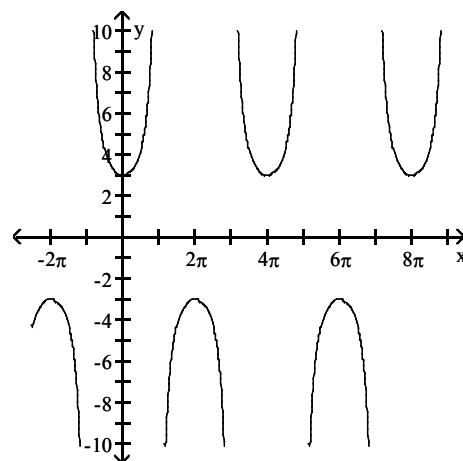
$$6) y = 3 \csc\left(\frac{1}{2}x\right)$$



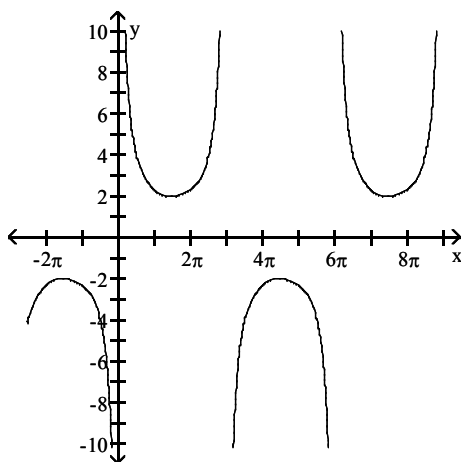
A)



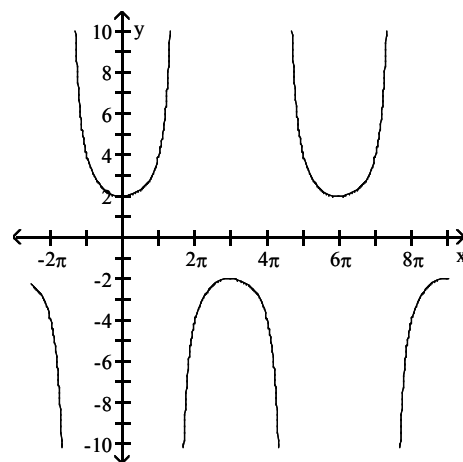
B)



C)

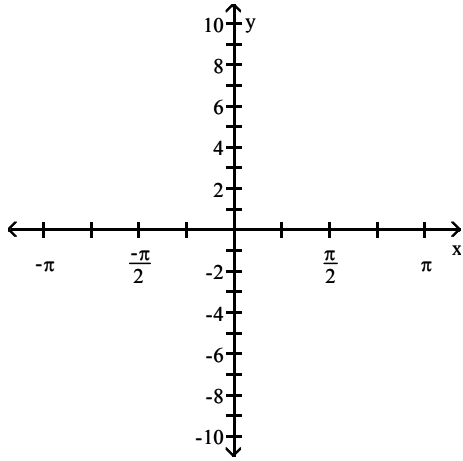


D)

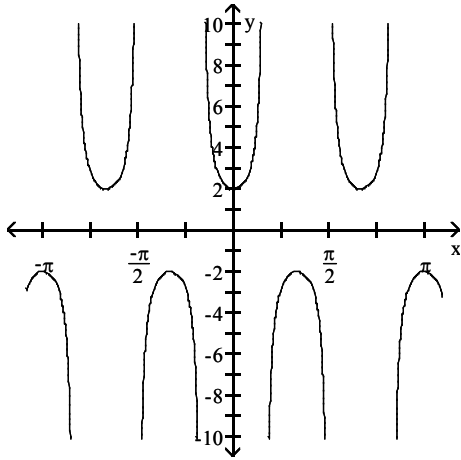




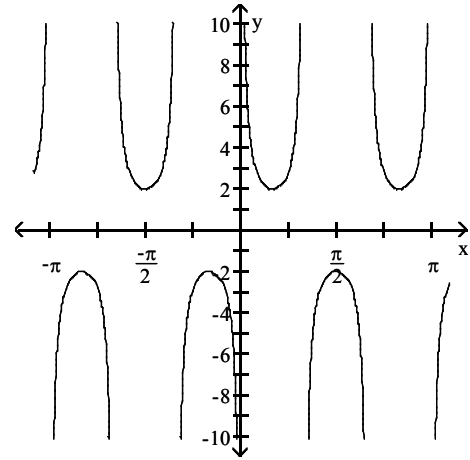
7)  $y = 2 \sec(3x)$



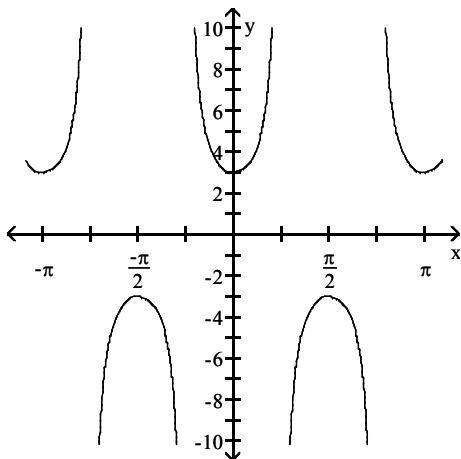
A)



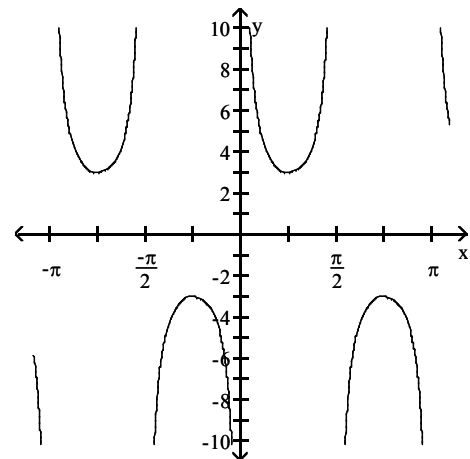
B)



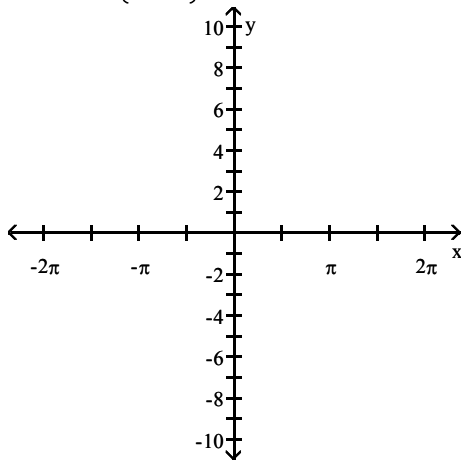
C)



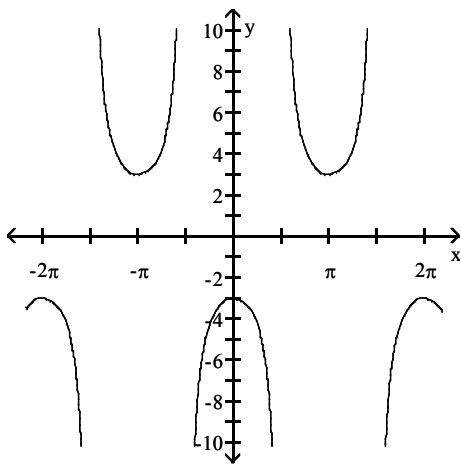
D)



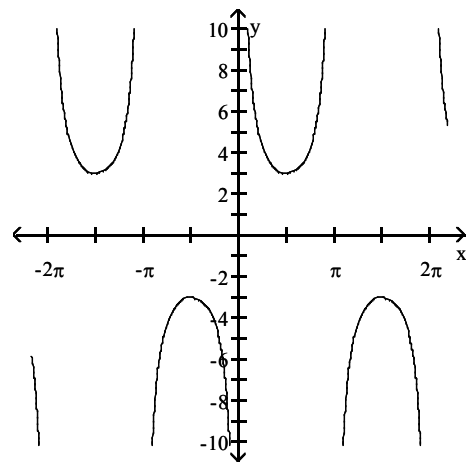
$$8) y = -3 \csc \left( x + \frac{\pi}{2} \right)$$



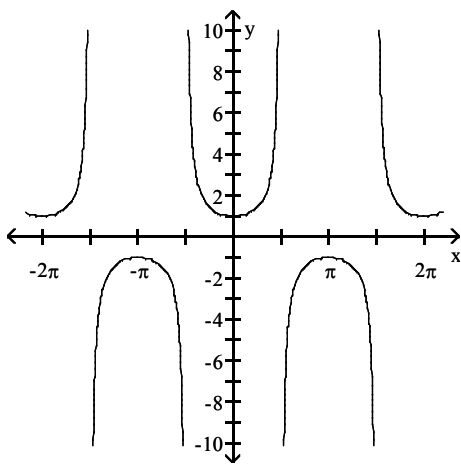
A)



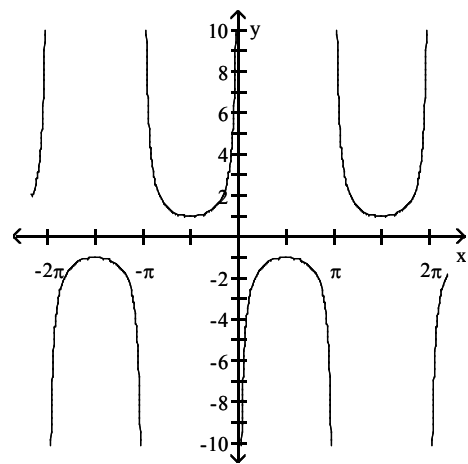
B)



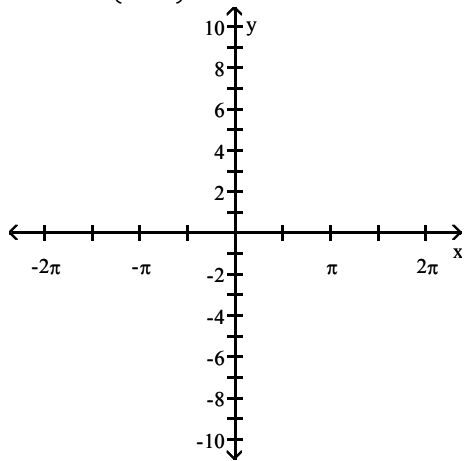
C)



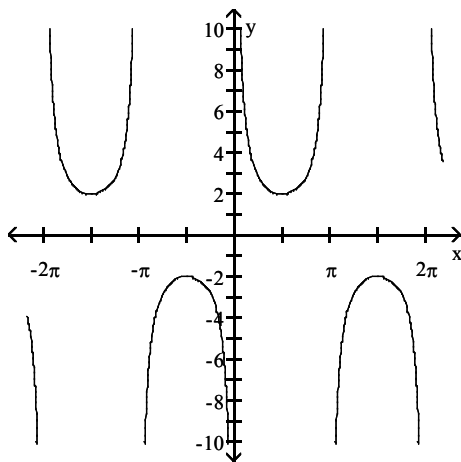
D)



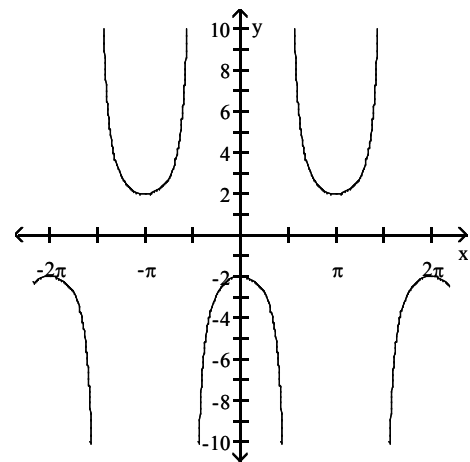
$$9) y = 2 \sec \left( x - \frac{\pi}{2} \right)$$



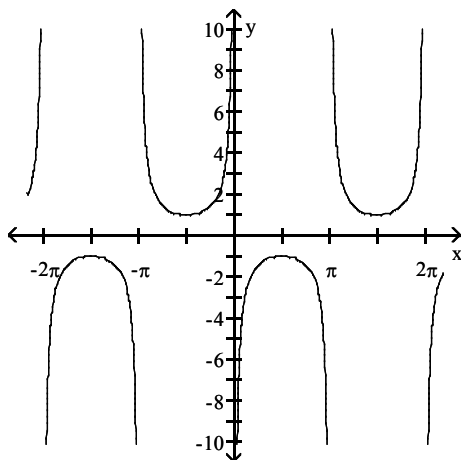
A)



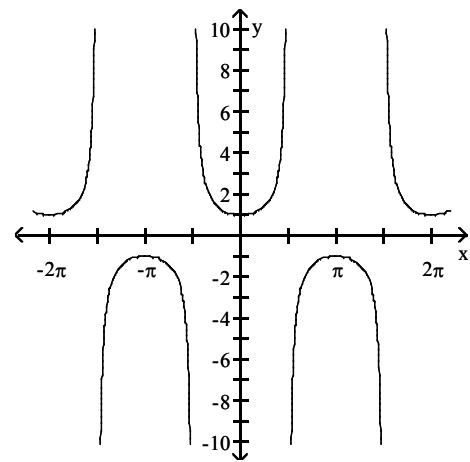
B)



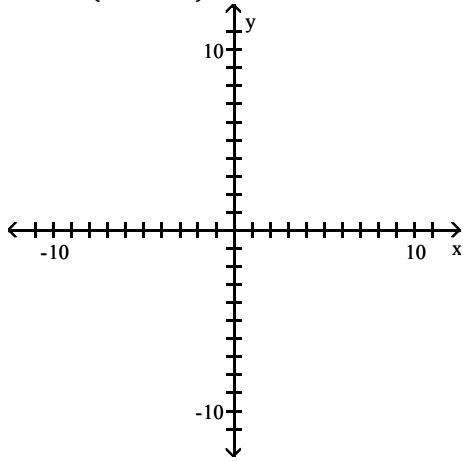
C)



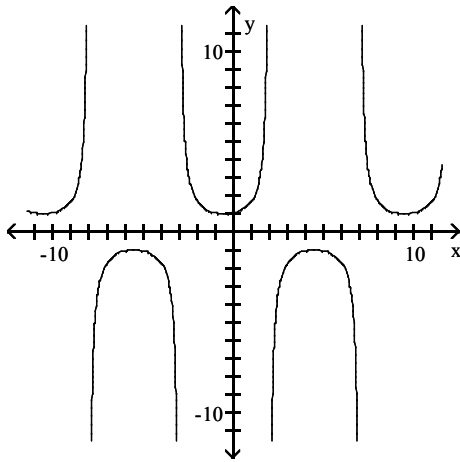
D)



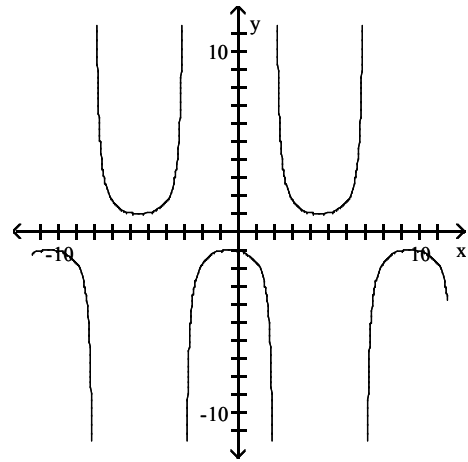
10)  $y = \csc\left(\frac{\pi x}{5} + \frac{3\pi}{5}\right)$



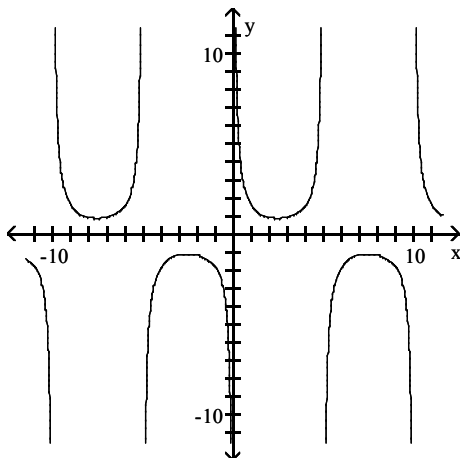
A)



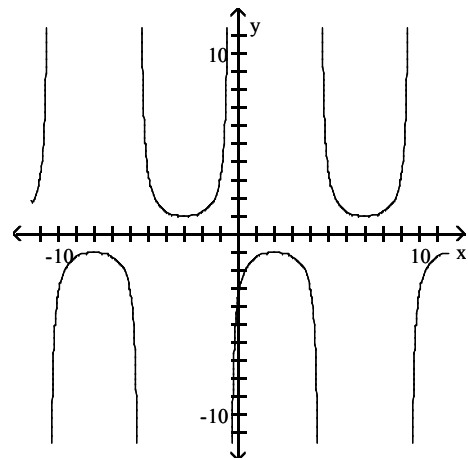
B)



C)



D)



**Solve the problem.**

11) A rotating beacon is located 14 ft from a wall. If the distance from the beacon to the point on the wall where the beacon is aimed is given by

$$a = 14|\sec 2\pi t|,$$

where  $t$  is in seconds, find  $a$  when  $t = 0.31$  seconds. Round your answer to the nearest hundredth.

A) 38.03 ft

B) 24.91 ft

C) 31.47 ft

D) -38.03 ft

## 2.6 Phase Shift; Sinusoidal Curve Fitting

### 1 Graph Sinusoidal Functions of the Form $y = A \sin(\omega x - \phi) + B$

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

Find the phase shift of the function.

1)  $y = 2 \sin\left(x - \frac{\pi}{4}\right)$

- A)  $\frac{\pi}{4}$  units to the right      B)  $\frac{\pi}{4}$  units to the left      C) 2 units up      D) 2 units down

2)  $y = 2 \cos\left(x + \frac{\pi}{2}\right)$

- A)  $\frac{\pi}{2}$  units to the left      B)  $\frac{\pi}{2}$  units to the right      C) 2 units up      D) 2 units down

3)  $y = 4 \sin\left(4x - \frac{\pi}{2}\right)$

- A)  $\frac{\pi}{8}$  units to the right      B)  $\frac{\pi}{2}$  units to the left      C)  $4\pi$  units up      D)  $4\pi$  units down

4)  $y = 5 \cos(6x + \pi)$

- A)  $\frac{\pi}{6}$  units to the left      B)  $\frac{\pi}{5}$  units to the left      C)  $5\pi$  units to the right      D)  $6\pi$  units to the right

5)  $y = 5 \sin\left(\frac{1}{4}x - \frac{\pi}{4}\right)$

- A)  $\pi$  units to the right      B)  $\frac{\pi}{4}$  units to the right      C)  $\frac{\pi}{16}$  units to the left      D)  $\frac{\pi}{5}$  units to the left

6)  $y = 4 \cos\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

- A)  $\pi$  units to the left      B)  $\frac{\pi}{2}$  units to the left      C)  $\frac{\pi}{4}$  units to the right      D)  $4\pi$  units to the right

7)  $y = 2 \sin(2\pi x - 3)$

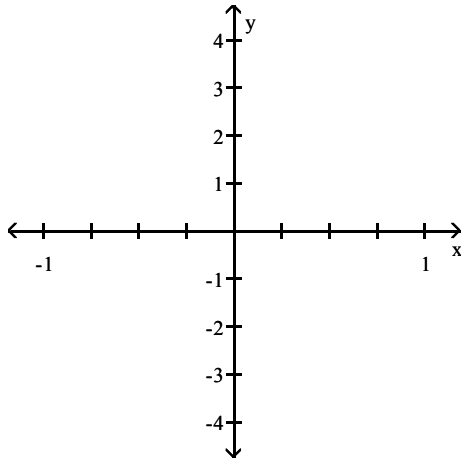
- A)  $\frac{3}{2\pi}$  units to the right      B)  $\frac{3}{2}$  units to the left      C) 3 units to the right      D) 3 units to the left

8)  $y = 4 \cos\left(-2x - \frac{\pi}{3}\right)$

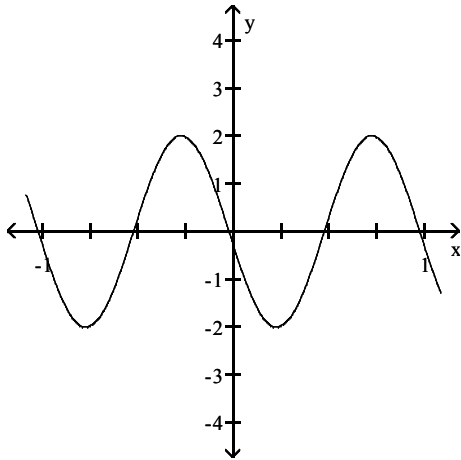
- A)  $\frac{\pi}{6}$  units to the left      B)  $\frac{\pi}{6}$  units to the right      C)  $\frac{\pi}{3}$  units to the left      D)  $\frac{\pi}{3}$  units to the right

Graph the function. Show at least one period.

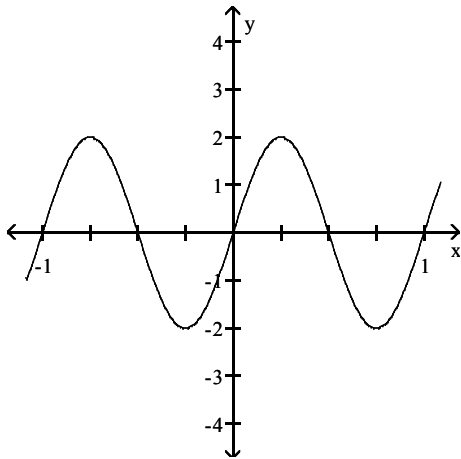
9)  $y = 2 \sin(2\pi x - 3)$



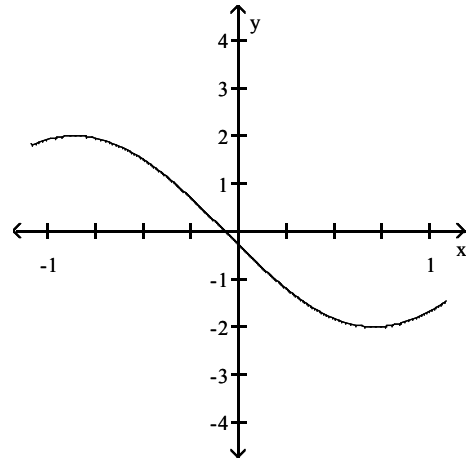
A)



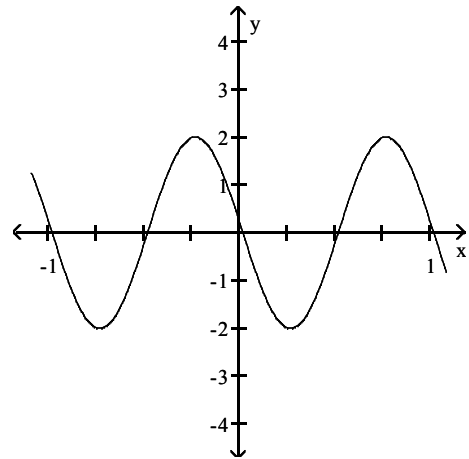
C)



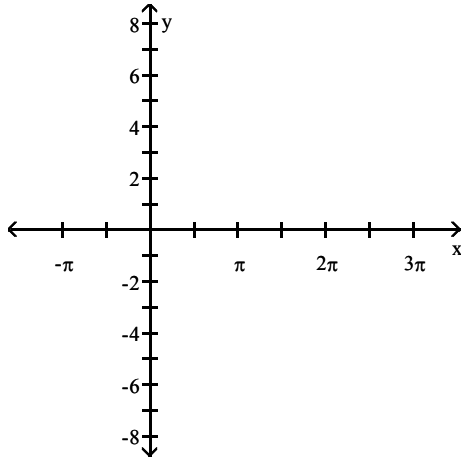
B)



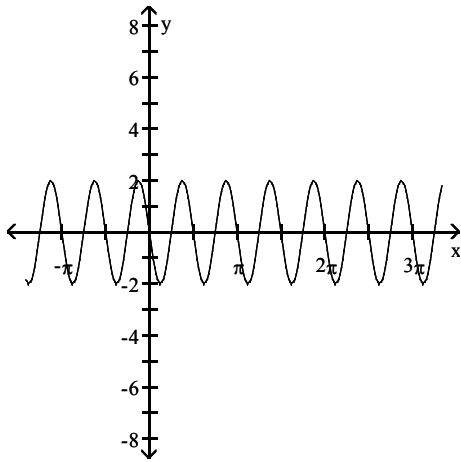
D)



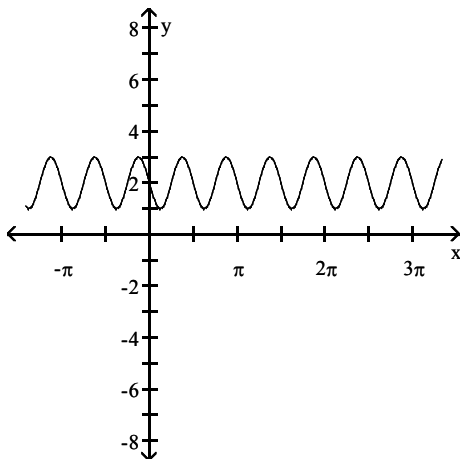
10)  $y = 2 \sin(4x - \pi)$



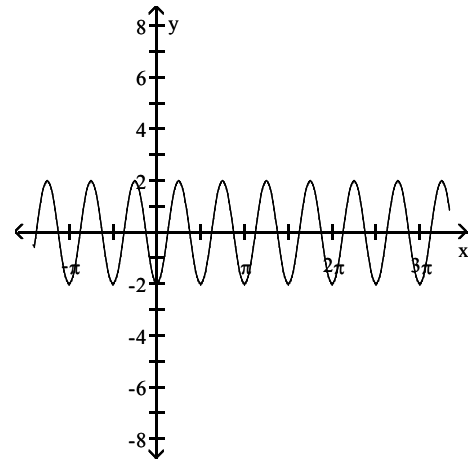
A)



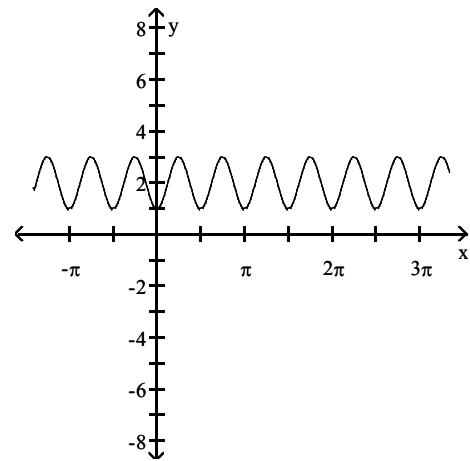
C)



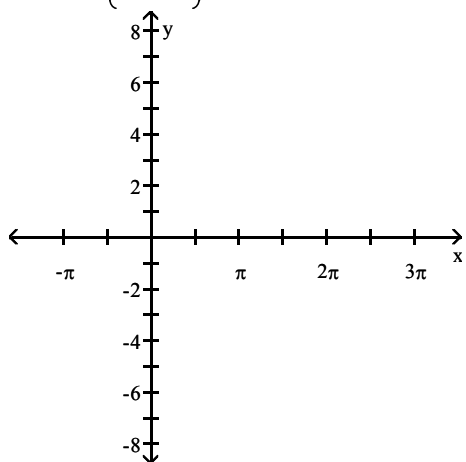
B)



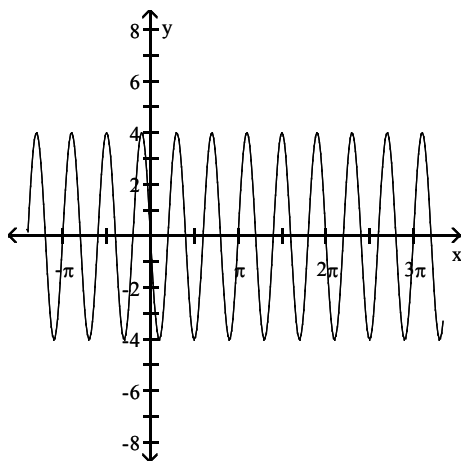
D)



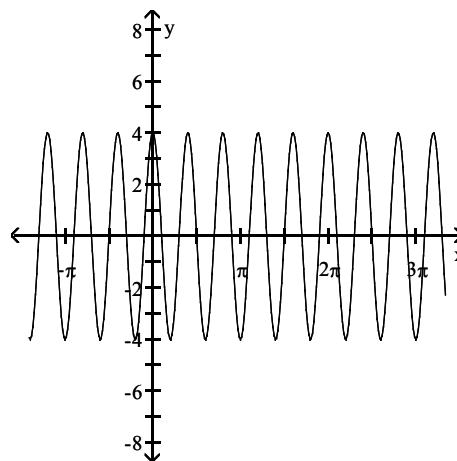
$$11) y = 4 \cos\left(5x + \frac{\pi}{2}\right)$$



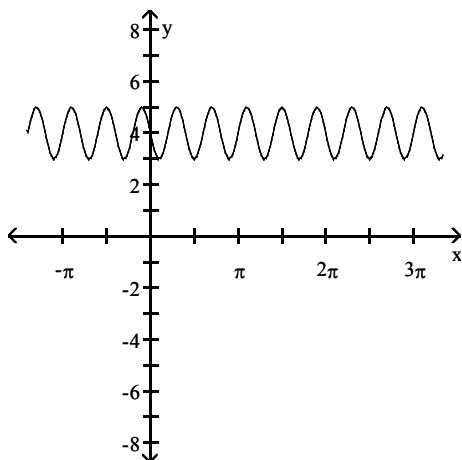
A)



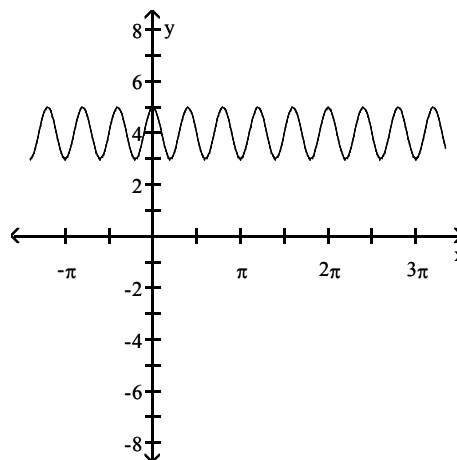
B)



C)

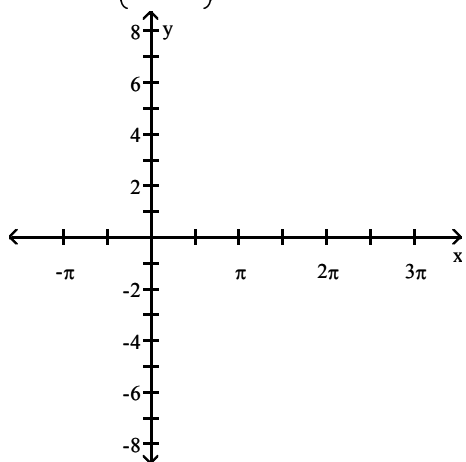


D)

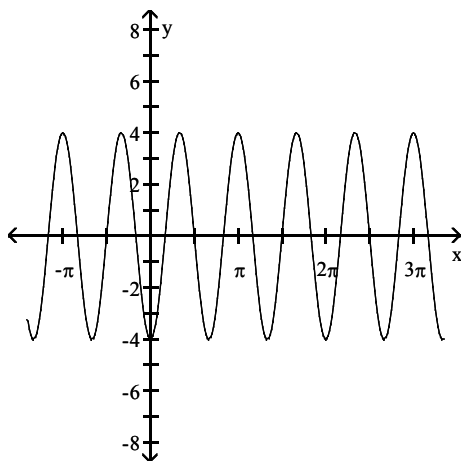




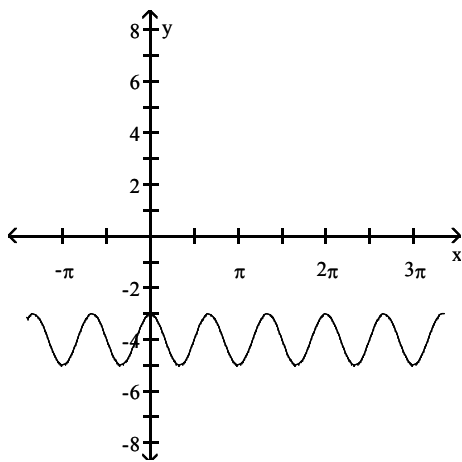
$$12) y = -4 \sin\left(3x + \frac{\pi}{2}\right)$$



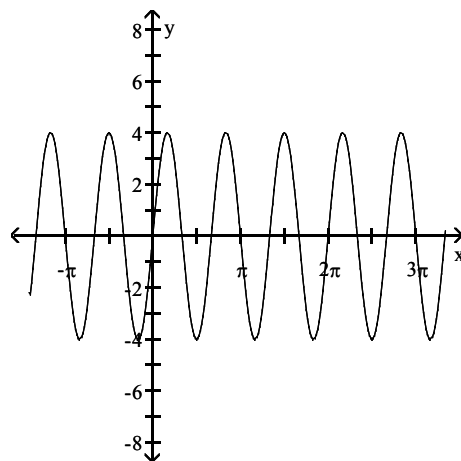
A)



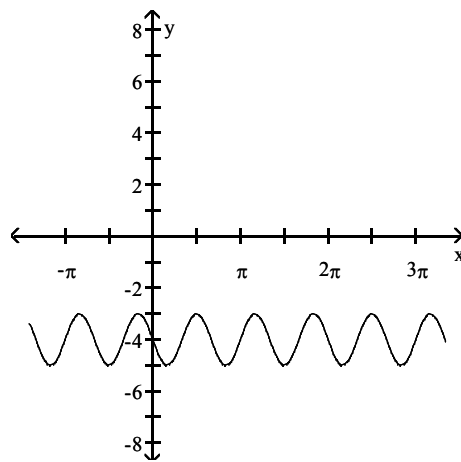
C)



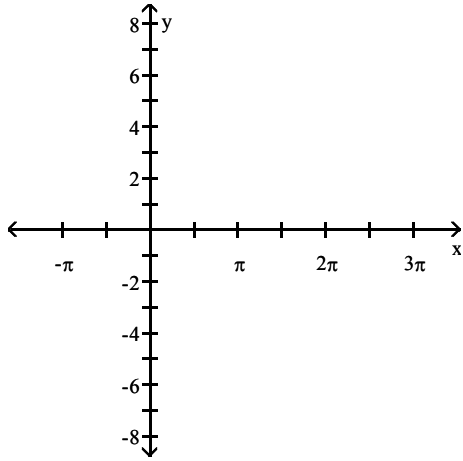
B)



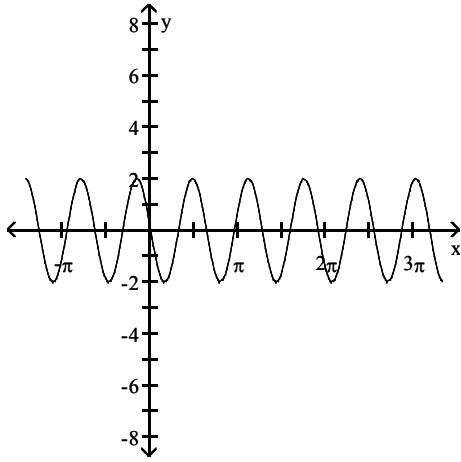
D)



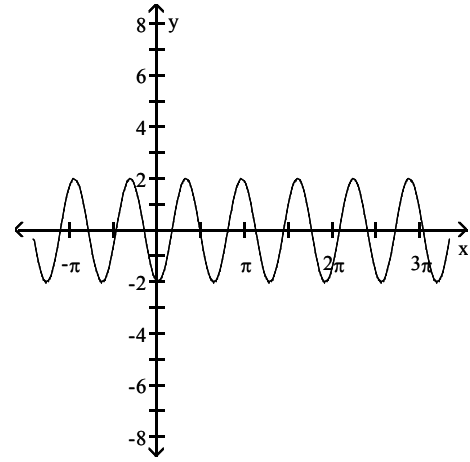
13)  $y = 2 \sin(\pi x + 3)$



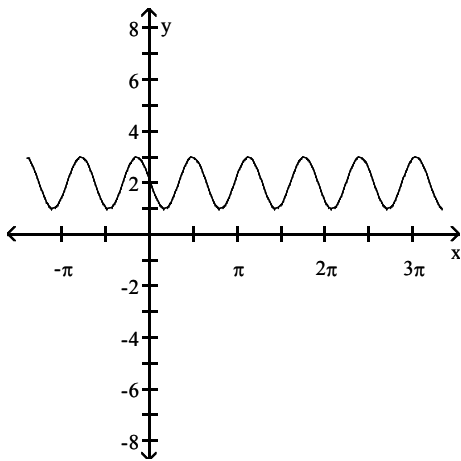
A)



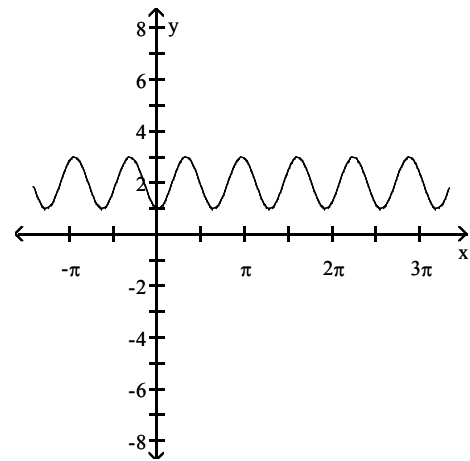
B)



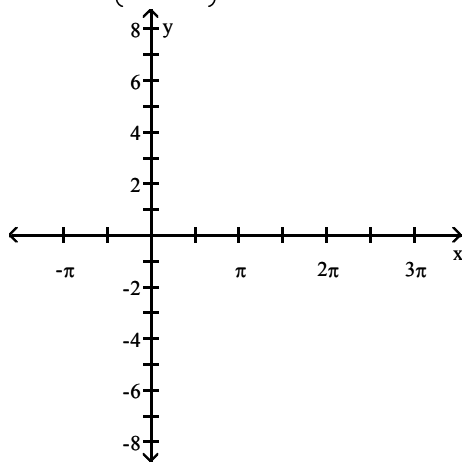
C)



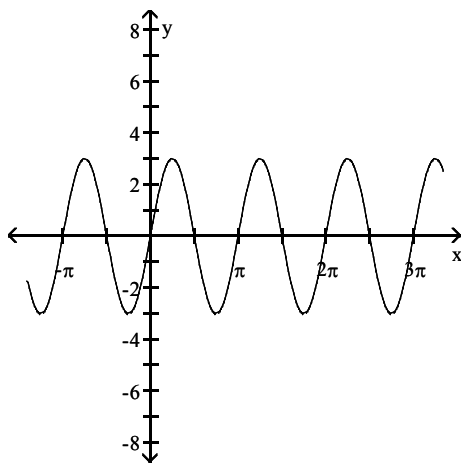
D)



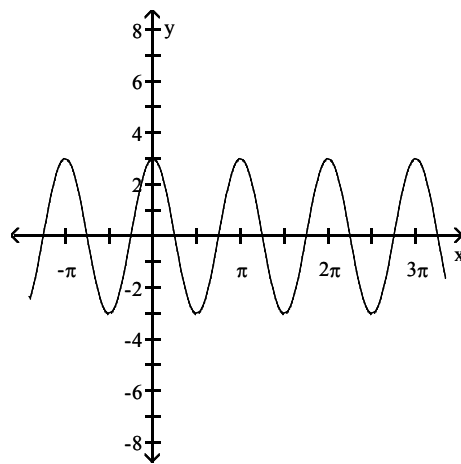
$$14) y = 3 \cos\left(-2x + \frac{\pi}{2}\right)$$



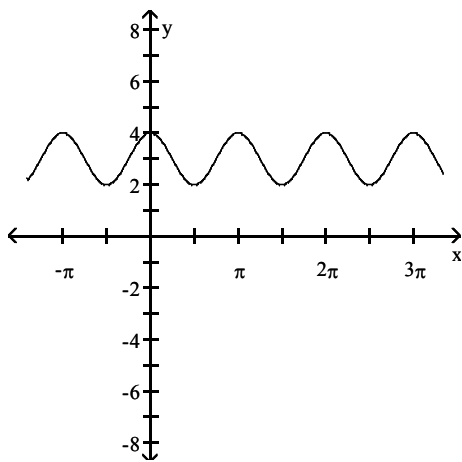
A)



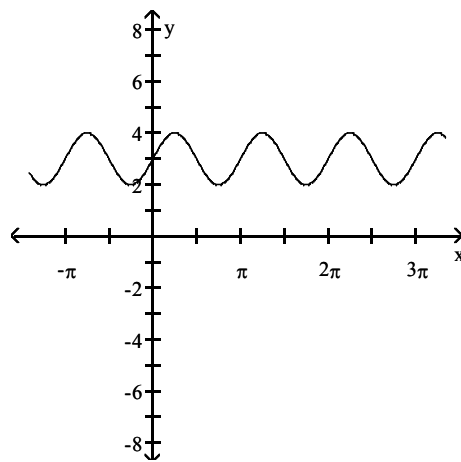
B)



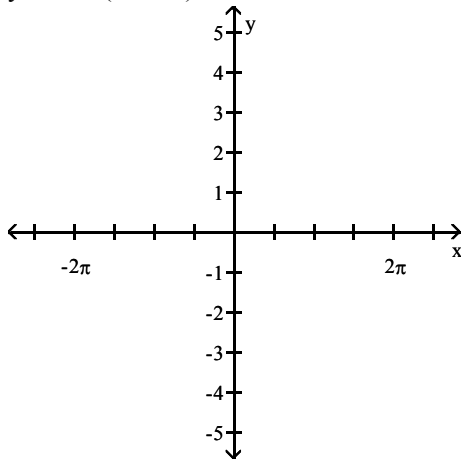
C)



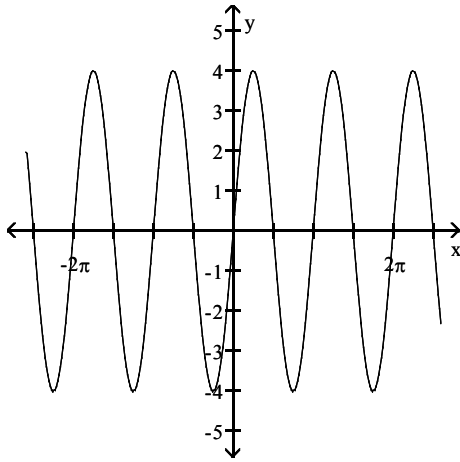
D)



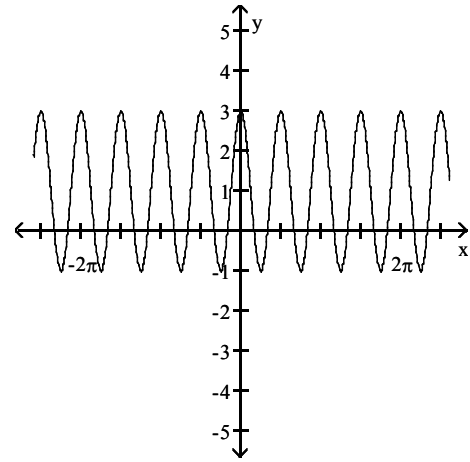
15)  $y = 4 \sin(-2x - \pi)$



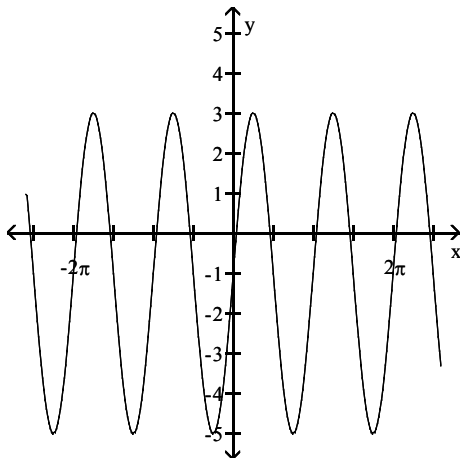
A)



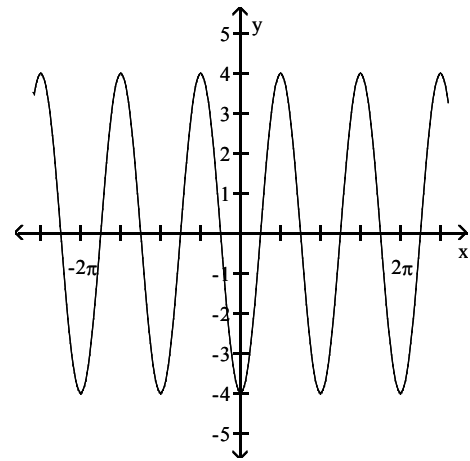
B)



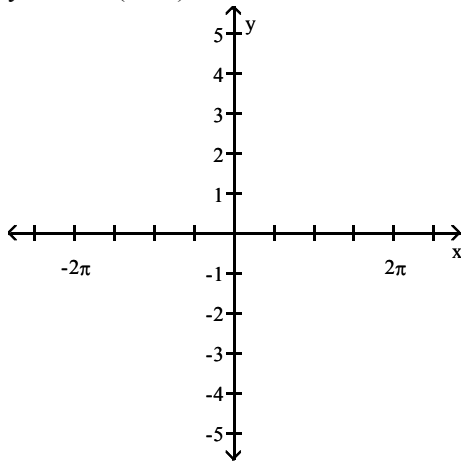
C)



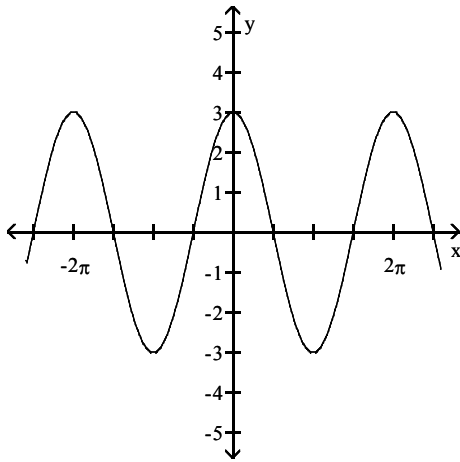
D)



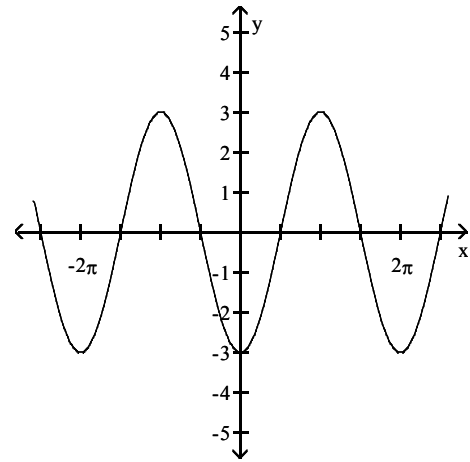
16)  $y = -3 \cos(x - \pi)$



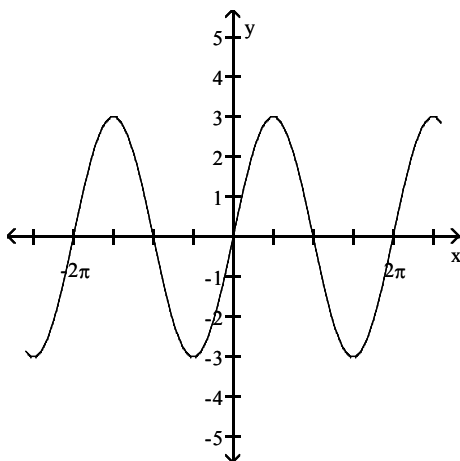
A)



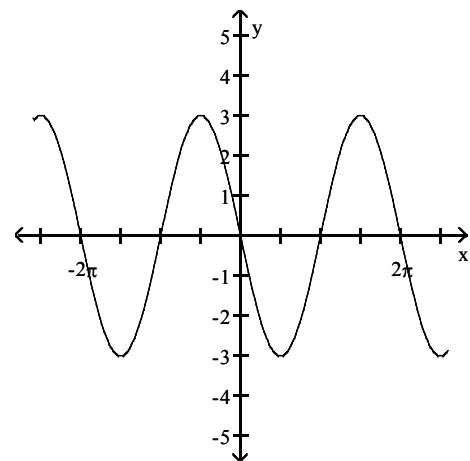
B)



C)



D)



**Solve the problem.**

17) For the equation  $y = -\frac{1}{2} \sin(4x + 3\pi)$ , identify (i) the amplitude, (ii) the phase shift, and (iii) the period.

A) (i) 2      (ii)  $3\pi$       (iii)  $\frac{\pi}{2}$

B) (i)  $\frac{1}{2}$       (ii)  $-\frac{3\pi}{4}$       (iii)  $\frac{\pi}{2}$

C) (i)  $-\frac{1}{2}$       (ii)  $-\frac{4\pi}{3}$       (iii) 4

D) (i)  $\frac{1}{2}$       (ii)  $-\frac{3\pi}{4}$       (iii) 4

18) For the equation  $y = -\frac{1}{2} \cos(2x - 2\pi)$ , identify (i) the amplitude, (ii) the phase shift, and (iii) the period.

A) (i)  $\frac{1}{2}$       (ii)  $\pi$       (iii)  $\pi$       B) (i)  $\frac{1}{2}$       (ii)  $\frac{\pi}{2}$       (iii)  $\pi$

C) (i) 2      (ii)  $2\pi$       (iii)  $2\pi$       D) (i) 2      (ii)  $\pi$       (iii)  $\pi$

**Write the equation of a sine function that has the given characteristics.**

19) Amplitude: 3

Period:  $4\pi$

Phase Shift:  $\frac{\pi}{4}$

A)  $y = 3 \sin\left(\frac{1}{2}x - \frac{1}{8}\pi\right)$       B)  $y = 3 \sin\left(2x + \frac{1}{8}\pi\right)$       C)  $y = 3 \sin\left(\frac{1}{2}x + \frac{1}{8}\pi\right)$       D)  $y = 3 \sin\left(4x + \frac{\pi}{4}\right)$

20) Amplitude: 2

Period:  $5\pi$

Phase Shift:  $-\frac{\pi}{5}$

A)  $y = 2 \sin\left(\frac{2}{5}x + \frac{2}{25}\pi\right)$       B)  $y = 2 \sin\left(\frac{5}{2}x - \frac{2}{25}\pi\right)$   
C)  $y = 2 \sin\left(\frac{2}{5}x - \frac{2}{25}\pi\right)$       D)  $y = 2 \sin\left(5x - \frac{\pi}{5}\right)$

21) Amplitude: 3

Period:  $\pi$

Phase Shift: -6

A)  $y = 3 \sin(2x + 12)$       B)  $y = 3 \sin\left(\frac{1}{2}x - 12\right)$       C)  $y = 3 \sin(x - 6)$       D)  $y = \sin(3x + 6)$

22) Amplitude: 2

Period:  $\pi$

Phase Shift:  $\frac{3}{2}$

A)  $y = 2 \sin(2x - 3)$       B)  $y = 2 \sin\left(\frac{1}{2}x - 6\right)$       C)  $y = 2 \sin\left(2x + \frac{3}{2}\right)$       D)  $y = \sin(2x + 3)$

## 2 Build Sinusoidal Models from Data

**MULTIPLE CHOICE.** Choose the one alternative that best completes the statement or answers the question.

**Solve the problem.**

- 1) An experiment in a wind tunnel generates cyclic waves. The following data is collected for 60 seconds:

| Time<br>(in seconds) | Wind speed<br>(in feet per second) |
|----------------------|------------------------------------|
| 0                    | 16                                 |
| 15                   | 37                                 |
| 30                   | 58                                 |
| 45                   | 37                                 |
| 60                   | 16                                 |

Let  $V$  represent the wind speed (velocity) in feet per second and let  $t$  represent the time in seconds. Write a sine equation that describes the wave.

- A)  $V = 21 \sin\left(\frac{\pi}{30}t - \frac{\pi}{2}\right) + 37$
- B)  $V = 58 \sin(60t - 30) + 16$
- C)  $V = 58 \sin\left(\frac{\pi}{30}t - \frac{\pi}{2}\right) + 16$
- D)  $V = 42 \sin(60t - 30) + 21$

- 2) A town's average monthly temperature data is represented in the table below:

| Month, $x$   | Average Monthly<br>Temperature, °F |
|--------------|------------------------------------|
| January, 1   | 29.6                               |
| February, 2  | 32.4                               |
| March, 3     | 45.1                               |
| April, 4     | 57.6                               |
| May, 5       | 71.5                               |
| June, 6      | 81.6                               |
| July, 7      | 84.3                               |
| August, 8    | 79.6                               |
| September, 9 | 79.6                               |
| October, 10  | 58.0                               |
| November, 11 | 41.5                               |
| December, 12 | 32.7                               |

Find a sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  that fits the data.

- A)  $y = 27.35 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 56.95$
- B)  $y = 84.3 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 29.6$
- C)  $y = 56.95 \sin\left(\frac{\pi}{6}x - \frac{\pi}{4}\right) + 27.35$
- D)  $y = 29.6 \sin\left(\frac{\pi}{6}x - \frac{\pi}{4}\right) + 84.3$

- 3) The number of hours of sunlight in a day can be modeled by a sinusoidal function. In the northern hemisphere, the longest day of the year occurs at the summer solstice and the shortest day occurs at the winter solstice. In 2000, these dates were June 22 (the 172nd day of the year) and December 21 (the 356th day of the year), respectively.

A town experiences 11.12 hours of sunlight at the summer solstice and 8.21 hours of sunlight at the winter solstice. Find a sinusoidal function  $y = A \sin(\omega x - \phi) + B$  that fits the data, where  $x$  is the day of the year. (Note: There are 366 days in the year 2000.)

A)  $y = 1.455 \sin\left(\frac{\pi}{183}x - \frac{161\pi}{366}\right) + 9.665$

B)  $y = 11.12 \sin\left(\pi x - \frac{2\pi}{3}\right) + 8.21$

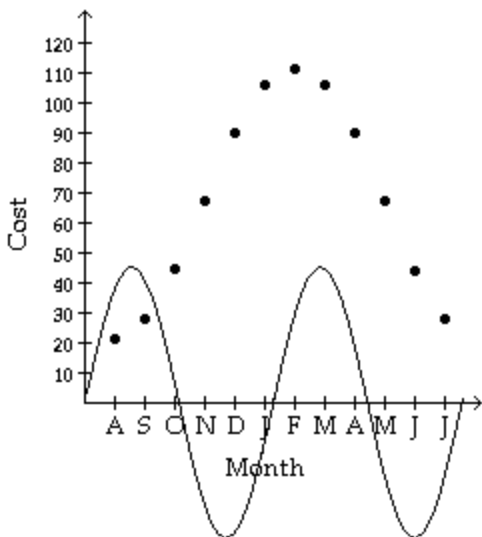
C)  $y = 1.455 \sin\left(\frac{\pi}{183}x - \frac{2\pi}{3}\right) + 9.665$

D)  $y = 11.12 \sin\left(\frac{172\pi}{356}x - \frac{2\pi}{3}\right) + 9.665$

- 4) The data below represent the average monthly cost of natural gas in an Oregon home.

| Month | Aug   | Sep   | Oct   | Nov   | Dec   | Jan    |
|-------|-------|-------|-------|-------|-------|--------|
| Cost  | 21.20 | 28.24 | 44.73 | 67.25 | 89.77 | 106.26 |

| Month | Feb    | Mar    | Apr   | May   | Jun   | Jul   |
|-------|--------|--------|-------|-------|-------|-------|
| Cost  | 111.30 | 106.26 | 89.77 | 67.25 | 43.73 | 28.24 |



Above is the graph of  $45.05 \sin x$  superimposed over a scatter diagram of the data. Find the sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  which best fits the data.

A)  $y = 45.05 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 66.25$

B)  $y = 45.05 \sin\left(\frac{\pi}{8}t + 12\right) + 21.20$

C)  $y = 45.05 \sin\left(\frac{\pi}{4}x - \frac{2\pi}{3}\right) + 21.20$

D)  $y = 45.05 \sin\left(\frac{\pi}{6}x - \frac{\pi}{12}\right) + 66.25$

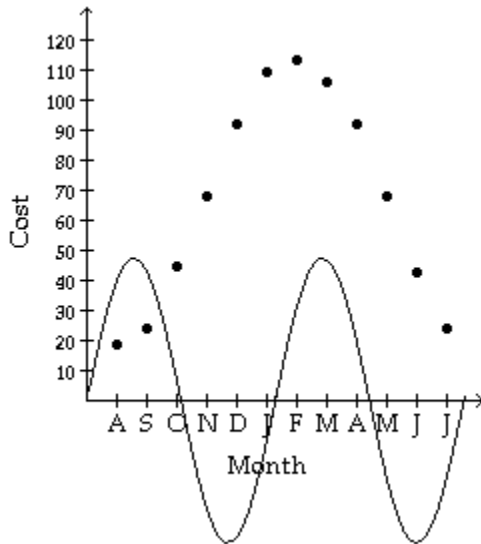


**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

5) The data below represent the average monthly cost of natural gas in an Oregon home.

| Month | Aug   | Sep   | Oct   | Nov   | Dec   | Jan    |
|-------|-------|-------|-------|-------|-------|--------|
| Cost  | 18.90 | 24.24 | 44.58 | 68.25 | 91.92 | 109.26 |

| Month | Feb    | Mar    | Apr   | May   | Jun   | Jul   |
|-------|--------|--------|-------|-------|-------|-------|
| Cost  | 113.60 | 106.26 | 91.92 | 68.25 | 42.58 | 24.24 |

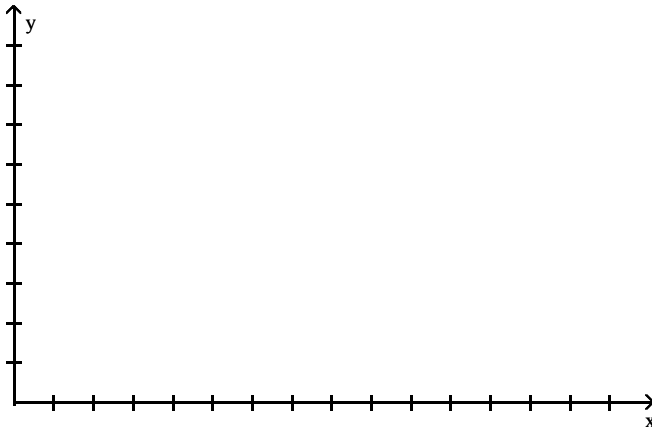


Above is the graph of  $47.35 \sin x$ . Make a scatter diagram of the data. Find the sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  which fits the data.

6) The following data represents the normal monthly precipitation for a certain city in California.

| Month, x     | Normal Monthly Precipitation, inches |
|--------------|--------------------------------------|
| January, 1   | 6.06                                 |
| February, 2  | 4.45                                 |
| March, 3     | 4.38                                 |
| April, 4     | 2.08                                 |
| May, 5       | 1.27                                 |
| June, 6      | 0.56                                 |
| July, 7      | 0.17                                 |
| August, 8    | 0.46                                 |
| September, 9 | 0.91                                 |
| October, 10  | 2.24                                 |
| November, 11 | 5.21                                 |
| December, 12 | 5.51                                 |

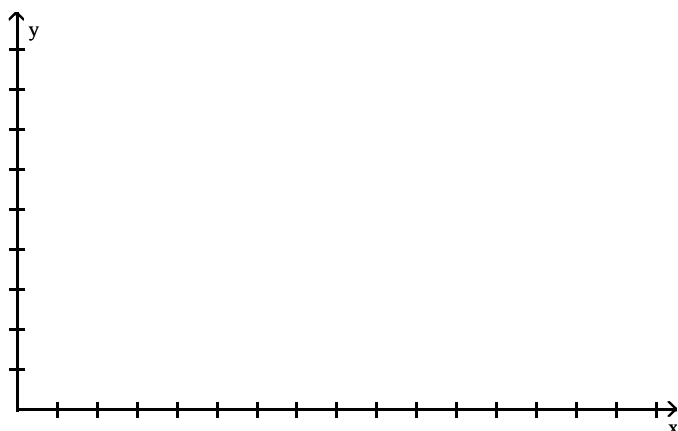
Draw a scatter diagram of the data for one period. Find a sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  that fits the data. Draw the sinusoidal function on the scatter diagram. Use a graphing utility to find the sinusoidal function of best fit. Draw the sinusoidal function of best fit on the scatter diagram.



7) The following data represents the normal monthly precipitation for a certain city in Arkansas.

| Month, x     | Normal Monthly Precipitation, inches |
|--------------|--------------------------------------|
| January, 1   | 3.91                                 |
| February, 2  | 4.36                                 |
| March, 3     | 5.31                                 |
| April, 4     | 6.21                                 |
| May, 5       | 7.02                                 |
| June, 6      | 7.84                                 |
| July, 7      | 8.19                                 |
| August, 8    | 8.06                                 |
| September, 9 | 7.41                                 |
| October, 10  | 6.30                                 |
| November, 11 | 5.21                                 |
| December, 12 | 4.28                                 |

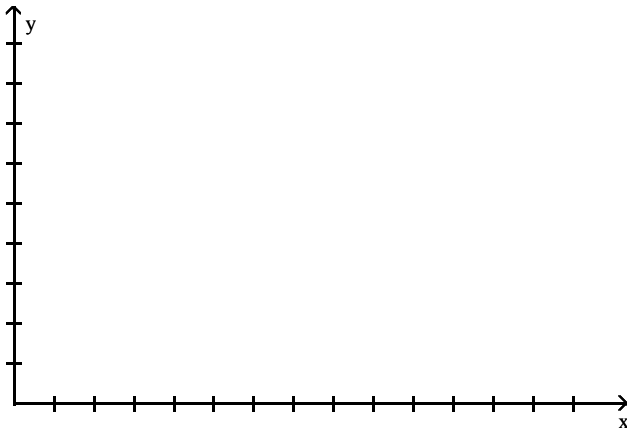
Draw a scatter diagram of the data for one period. Find the sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  that fits the data. Draw the sinusoidal function on the scatter diagram. Use a graphing utility to find the sinusoidal function of best fit. Draw the sinusoidal function of best fit on the scatter diagram.



8) The following data represents the average monthly minimum temperature for a certain city in California.

| Month, x     | Average Monthly Minimum Temperature, °F |
|--------------|---|
| January, 1   | 49.6                                    |
| February, 2  | 50.8                                    |
| March, 3     | 55.6                                    |
| April, 4     | 57.5                                    |
| May, 5       | 60.7                                    |
| June, 6      | 63.6                                    |
| July, 7      | 65.9                                    |
| August, 8    | 65.6                                    |
| September, 9 | 64.4                                    |
| October, 10  | 62.1                                    |
| November, 11 | 54.2                                    |
| December, 12 | 50.1                                    |

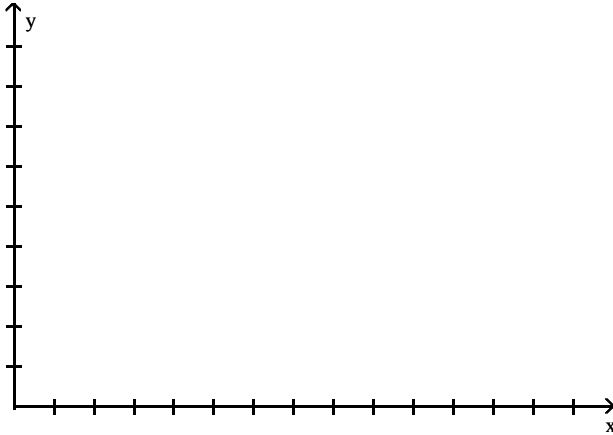
Draw a scatter diagram of the data for one period. Find a sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  that fits the data. Draw the sinusoidal function on the scatter diagram. Use a graphing utility to find the sinusoidal function of best fit. Draw the sinusoidal function of best fit on the scatter diagram.



9) The following data represents the average percent of possible sunshine for a certain city in Indiana.

| Month, x     | Average Percent of Possible Sunshine |
|--------------|--------------------------------------|
| January, 1   | 46                                   |
| February, 2  | 51                                   |
| March, 3     | 55                                   |
| April, 4     | 60                                   |
| May, 5       | 68                                   |
| June, 6      | 73                                   |
| July, 7      | 75                                   |
| August, 8    | 74                                   |
| September, 9 | 68                                   |
| October, 10  | 62                                   |
| November, 11 | 41                                   |
| December, 12 | 38                                   |

Draw a scatter diagram of the data for one period. Find the sinusoidal function of the form  $y = A \sin(\omega x - \phi) + B$  that fits the data. Draw the sinusoidal function on the scatter diagram. Use a graphing utility to find the sinusoidal function of best fit. Draw the sinusoidal function of best fit on the scatter diagram.



## Ch. 2 Trigonometric Functions

### Answer Key

#### 2.1 Angles and Their Measure

##### 1 Convert between Decimals and Degrees, Minutes, Seconds Measures for Angles

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) A
- 18) A

##### 2 Find the Length of an Arc of a Circle

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) C
- 10) A
- 11) A
- 12) 379 mi
- 13) A
- 14) A

##### 3 Convert from Degrees to Radians and from Radians to Degrees

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) A
- 18) A

19) A

**4 Find the Area of a Sector of a Circle**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A

**5 Find the Linear Speed of an Object Traveling in Circular Motion**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11)  $6.24 \times 10^4$  kmp;  $4.95 \times 10^4$  kmp;  $3.92 \times 10^4$  kmp;  $2.95 \times 10^4$  kmp; Io
- 12) A
- 13) A

**2.2 Trigonometric Functions: Unit Circle Approach**

**1 Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

**2 Find the Exact Values of the Trigonometric Functions of Quadrantal Angles**

- 1) A
- 2) A
- 3) A
- 4) D
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A

**3 Find the Exact Values of the Trigonometric Functions of  $\pi/4 = 45^\circ$**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

**4 Find the Exact Values of the Trigonometric Functions of  $\pi/6 = 30^\circ$  and  $\pi/3 = 60^\circ$**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) A
- 18) A
- 19) A
- 20) A

**5 Find the Exact Values for Integer Multiples of  $\pi/6 = 30^\circ$ ,  $\pi/4 = 45^\circ$ , and  $\pi/3 = 60^\circ$**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A

**6 Use a Calculator to Approximate the Value of a Trigonometric Function**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) 0.354k; 0.377k; 0.385k; 0.375k and 0.347k; No, it reaches a maximum near  $55^\circ$ .



**7 Use a Circle of Radius r to Evaluate the Trigonometric Functions**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A

**2.3 Properties of the Trigonometric Functions**

**1 Determine the Domain and the Range of the Trigonometric Functions**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

**2 Determine the Period of the Trigonometric Functions**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) D
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16)  $-\frac{1}{3}$
- 17) A
- 18) A

**3 Determine the Signs of the Trigonometric Functions in a Given Quadrant**

- 1) C
- 2) C
- 3) B
- 4) D
- 5) A
- 6) B
- 7) D
- 8) D
- 9) C
- 10) A
- 11) A
- 12) (i) negative  
(ii) negative  
(iii) positive

**4 Find the Values of the Trigonometric Functions Using Fundamental Identities**

- 1) A

- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

**5 Find Exact Values of the Trig Functions of an Angle Given One of the Functions and the Quadrant of the Angle**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) C

10)  $\cos \theta = -\frac{\sqrt{35}}{6}, \tan \theta = -\frac{\sqrt{35}}{35}$

**6 Use Even-Odd Properties to Find the Exact Values of the Trigonometric Functions**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) neither
- 15) odd

**2.4 Graphs of the Sine and Cosine Functions**

**1 Graph Functions of the Form  $y = A \sin(\omega x)$  Using Transformations**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A

**2 Graph Functions of the Form  $y = A \cos(\omega x)$  Using Transformations**

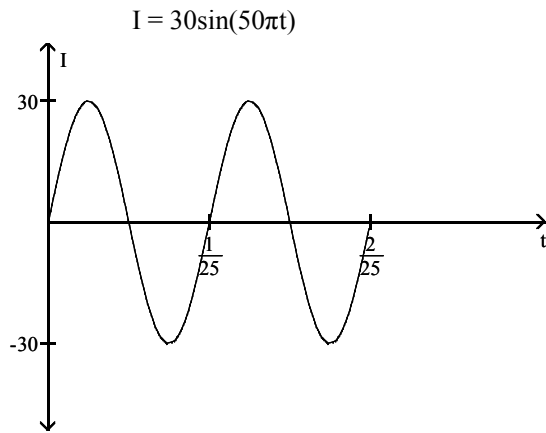
- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A

- 9) A
- 10) A
- 11) A
- 12) A

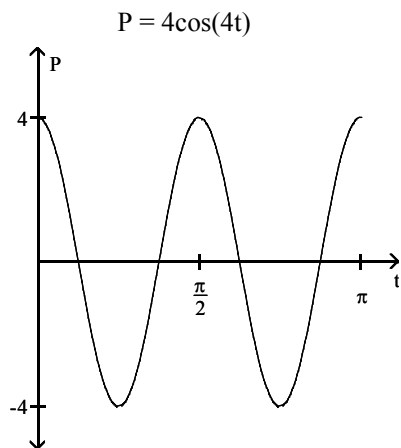
**3 Determine the Amplitude and Period of Sinusoidal Functions**

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) 12, 12
- 12) 365 days
- 13) A

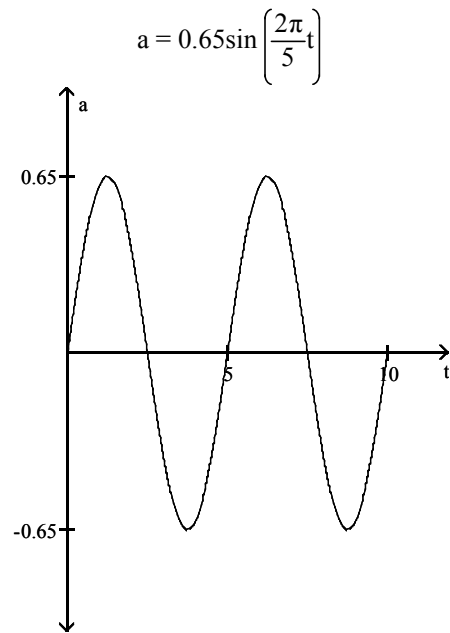
14) amplitude = 30, period =  $\frac{1}{25}$



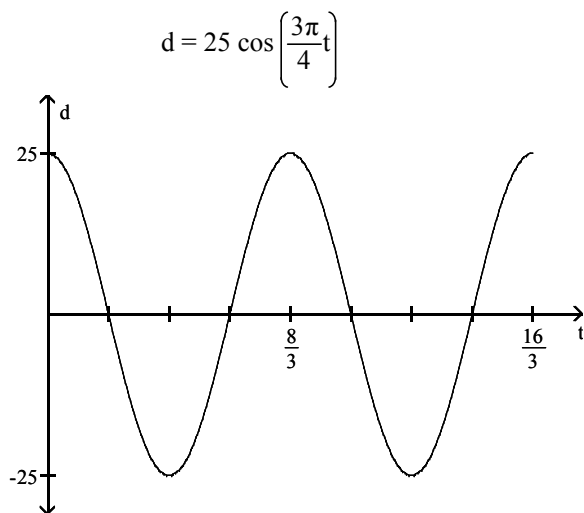
15) amplitude = 4, period =  $\frac{\pi}{2}$



16) amplitude = 0.65, period = 5, respiratory cycle = 5 seconds



17) amplitude = 25, period = 8/3



#### 4 Graph Sinusoidal Functions Using Key Points

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) A

## 5 Find an Equation for a Sinusoidal Graph

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A

## 2.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

### 1 Graph Functions of the Form $y = A \tan(\omega x) + B$ and $y = A \cot(\omega x) + B$

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) A
- 18) D
- 19) A
- 20) A

### 2 Graph Functions of the Form $y = A \csc(\omega x) + B$ and $y = A \sec(\omega x) + B$

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A
- 7) A
- 8) A
- 9) A
- 10) A
- 11) A

## 2.6 Phase Shift; Sinusoidal Curve Fitting

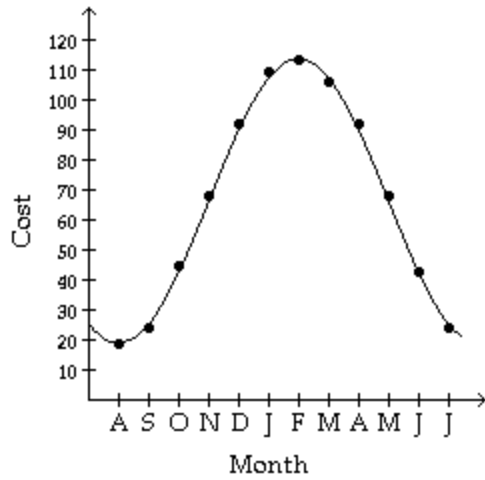
### 1 Graph Sinusoidal Functions of the Form $y = A \sin(\omega x - \phi) + B$

- 1) A
- 2) A
- 3) A
- 4) A
- 5) A
- 6) A

- 7) A
- 8) A
- 9) A
- 10) A
- 11) A
- 12) A
- 13) A
- 14) A
- 15) A
- 16) A
- 17) B
- 18) A
- 19) A
- 20) A
- 21) A
- 22) A

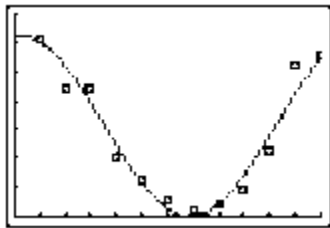
**2 Build Sinusoidal Models from Data**

- 1) A
- 2) A
- 3) A
- 4) A
- 5)

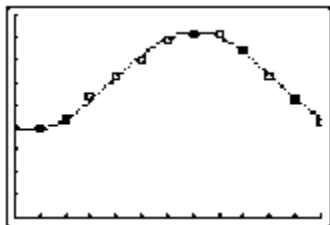


$$y = 47.35 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 66.25$$

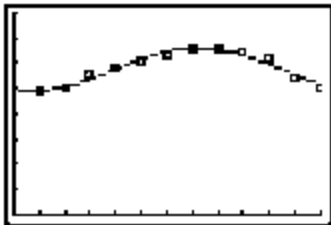
- 6)  $y = 3.14 \sin(0.46x + 1.52) + 3.16$



- 7)  $y = 2.17 \sin(0.49x - 1.88) + 6.02$



8)  $y = 8.33 \sin(0.50x - 2.06) + 57.97$



9)  $y = 15.99 \sin(0.57x - 2.29) + 60.62$

