

# Chapter 2

## Past, Present and Future World Energy Use

**2.1** A quantity increases at a rate of 1.5% per year. What is its doubling time?

**Solution** The doubling time  $t_D$  is related to the growth rate  $R$  (for small  $R$ ) by

$$t_D = \frac{100 \ln 2}{R}$$

If  $R = 1.5\%$  then the doubling time (in years) is given as

$$t_D = 100 \times \ln(2)/(1.5) = 46.2 \text{ years.}$$

**2.2** The population of a particular country was 1.1 million in 1940 and 3.4 million in 2010. Calculate the growth rate (in % per year). The growth rate was constant over that period of time.

**Solution** For constant growth the population at a time  $t$  relative to  $t=0$  is given by

$$N(t) = N_0 \exp(at)$$

In this problem  $N(t)/N_0 = 3.4 \times 10^6 / 1.1 \times 10^6 = 3.1$  and for a time period of 2010-1940 = 70 years we solve for  $a$  as

$$a = \frac{1}{t} \ln \left( \frac{N(t)}{N_0} \right)$$

or  $a = (1/70) \times (\ln(3.1)) = 0.0162 \text{ y}^{-1}$ . The doubling time is

$$t_D = \frac{\ln 2}{a} = (\ln 2)/(0.0162 \text{ y}^{-1}) = 42.9 \text{ y.}$$

From this the growth rate may be estimated to be

$$R = \frac{100 \ln 2}{t_D} = 100 \times (\ln 2)/(42.9 \text{ y}^{-1}) = 1.62\%$$

**2.3** Consider the earth to be a sphere with a radius of 6378 km. 71% of its surface area is covered with water. The population density in Japan is currently 337 people per km<sup>2</sup>. What would the population of the earth be if the population density on land was, on the average, the same as in Japan. Compare this with a current actual world population of about 7 billion.

**Solution** The total area of the earth (including oceans) is

$$A = 4\pi r^2 = (4) \times (3.14) \times (6378 \text{ km})^2 = 5.1 \times 10^8 \text{ km}^2.$$

If 71% is water then the remaining land area is

$$(5.1 \times 10^8 \text{ km}^2) \times (0.29) = 1.48 \times 10^8 \text{ km}^2.$$

To attain a population density of 337 people per km<sup>2</sup> will, therefore, require a total population of

$$(1.48 \times 10^8 \text{ km}^2) \times (337 \text{ km}^{-2}) = 49.9 \text{ billion.}$$

This is 7 times the current world population and well above virtually all estimates of a maximum sustainable population.

**2.4** A country has a constant annual growth rate of 5%. How long will it take for the population to increase by a factor of 10?

**Solution** The population as a function of time is be given by

$$N(t) = N_0 \exp(at)$$

Solving for t gives

$$t = \frac{1}{a} \ln\left(\frac{N(t)}{N_0}\right)$$

The constant  $a$  is related to the growth rate as

$$R = 100 \times (\exp(a) - 1)$$

Solving this for the constant  $a$  in terms of the given growth rate gives

$$a = \ln(1+R/100) = \ln(1.05) = 0.0488 \text{ y}^{-1}$$

Substituting this value and  $N(t)/N_0 = 10$  in the above gives the time as

$$t = (1/0.0488 \text{ y}^{-1}) \times \ln(10) = 47.2 \text{ years.}$$

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