

# Chapter 2

## Past, Present and Future World Energy Use

**Problem 2.1** Locate information on the total primary energy consumption per capita and per dollar of GDP for five states from different geographical regions in the United States. Discuss any relationships between energy use and factors such as climate, population density, types of industry, and other variables that are apparent.

**Solution** Per capita energy statistics are available for all states at:

[http://www.statemaster.com/graph/ene\\_tot\\_ene\\_con\\_percap-total-electricity-consumption-per-capita](http://www.statemaster.com/graph/ene_tot_ene_con_percap-total-electricity-consumption-per-capita)

and per \$GDP energy statistics are available for all states at:

[http://www.statemaster.com/graph/ene\\_tot\\_ene\\_con\\_percgdp-energy-total-consumption-per-gdp](http://www.statemaster.com/graph/ene_tot_ene_con_percgdp-energy-total-consumption-per-gdp)

Choosing the following state the information from the web site is tabulated. Note that values in the table are in BBtu per year. These are converted to GJ as  $1055 \text{ BBtu} = 1 \text{ GJ}$ . Note that the average over all states is  $\sim 350 \text{ GJ}$  per capita per year.

| State         | E(GJ)/Capita | E(GJ)/\$GDP |
|---------------|--------------|-------------|
| Alaska        | 1171         | 0.0229      |
| Alabama       | 450          | 0.0148      |
| Maine         | 392          | 0.0120      |
| Massachusetts | 255          | 0.00514     |
| Florida       | 245          | 0.00734     |

We expect that the energy per capita will be inversely proportional to the population density and inversely proportional to the average temperature. The population density and per capita GDP are readily available on the internet (e.g. Wikipedia) as given (for Jan 2010) in the table below.

| State         | population/km <sup>2</sup> | \$GDP per capita |
|---------------|----------------------------|------------------|
| Alaska        | 0.46                       | 65,143           |
| Alabama       | 30.4                       | 36,333           |
| Maine         | 16.6                       | 40,923           |
| Massachusetts | 324                        | 58,108           |
| Florida       | 135                        | 40,106           |

Per capita energy consumption in Alaska is by far the highest. This is clearly expected on the basis of a very low population density and a very cold climate. Alabama has a warm

climate but a high per capita energy consumption. This can in fact be due to the moderate population density. This is a bit anomalous in comparison with Maine, which has a lower population density and cooler climate. Massachusetts by comparison with Maine has a slightly warmer climate but a much higher population density and a correspondingly smaller energy use. Florida has a much milder climate than Massachusetts, which compensates for its somewhat lower population density. Economic factors can be accounted for by inspecting the energy use per GDP. Note that

$$E / \text{GDP} = (E / \text{capita}) \times (\text{GDP}/\text{capita})^{-1}$$

Alaska has a high GDP/capita but not enough to compensate for other factors. Alabama has a low GDP/capita which partly accounts for its large energy use. This may be reflected by the presence of rather energy intensive industries. Maine has a higher GDP/capita which may partially explain its lower energy use than Alabama. The order of Massachusetts and Florida are reversed when considering  $E/\text{GDP}$  rather than  $E/\text{capita}$ . This is a result of its much larger GDP per capita and is reflection of the presence of more high technology industries and businesses.

**Problem 2.2** A quantity has a doubling time of 110 years. Estimate the annual percent increase in the quantity.

**Solution** From equation (2.9) the annual rate of increase  $R$  is given as

$$R = \frac{100 \ln 2}{t_D}$$

where  $t_D$  is the doubling time. If  $t_D$  is 110 years then

$$R = \frac{(100) \times (0.693)}{110 \text{ y}} = 0.63\% \text{ per year}$$

This is much less than 10% so the approximation given in equation (2.9) is valid.

**Problem 2.3** The population of a particular country has a doubling time of 45 years. When will the population be three times its present value?

**Solution** From equation (2.7) the constant  $a$  can be determined from the doubling time as

$$t_D = \frac{\ln 2}{a}$$

so

$$a = \frac{\ln 2}{t_D}$$

For  $t_D = 45$  years then

$$a = \frac{0.693}{45} = 0.0154\text{y}^{-1}$$

From equation (2.4) the quantity of any time is given in terms of the initial value as

$$N(t) = N_0 \exp(at)$$

so solving for  $t$  we get

$$t = \frac{1}{a} \ln\left(\frac{N(t)}{N_0}\right)$$

for  $N(t) = 3N_0$  then we get

$$t = \left(\frac{1}{0.0154\text{y}^{-1}}\right) \ln(3) = 71.3\text{years}$$

**Problem 2.4** Assume that the historical growth rate of the human population was constant at 1.6% per year. For a population of 7 billion in 2012, determine the time in the past when the human population was 2.

**Solution** As the annual percentage growth rate is small then we can use the approximation of equation (2.4) to get the doubling time from  $R$  so

$$t_D = \left(\frac{100 \ln 2}{R}\right) = \frac{(100) \times (0.693)}{1.6} = 43.31\text{years}$$

from equation (2.7) the constant  $a$  can be found to be

$$a = \left(\frac{\ln 2}{t_D}\right) = \frac{0.693}{43.31\text{y}} = 0.016\text{y}^{-1}$$

from equation (2.4) we start with an initial population of  $N_0 = 2$  at  $t = 0$  then  $N(t) = 6.7 \times 10^9$  then from

$$N(t) = N_0 \exp(at)$$

so

$$t = \frac{1}{a} \ln \frac{N(t)}{N_0} = \frac{1}{0.016} \ln \frac{7 \times 10^9}{2} = 1374\text{y}$$

in the past or at year 2012 – 1374 = 638 (obviously growth rate was not constant).

**Problem 2.5** What is the current average human population density (i.e., people per square kilometer) on earth?

**Solution** The radius of the Earth is 6378 km (assumed spherical). The total area (including oceans) is  $A = 4\pi r^2 = (4) \times (3.14) \times (6378 \text{ km})^2 = 5.1 \times 10^8 \text{ km}^2$ . The total current population is  $6.7 \times 10^9$ , so the population density is

$$\frac{6.7 \times 10^9}{5.1 \times 10^8 \text{ km}^2} = 13.1 \text{ people/km}^2$$

If only land area is included, the land area on Earth is from various values given on the web range from  $1.483 \times 10^8 \text{ km}^2$  to  $1.533 \times 10^8 \text{ km}^2$ . Using  $1.5 \times 10^8 \text{ km}^2$  we find

$$\frac{6.7 \times 10^9}{1.5 \times 10^8 \text{ km}^2} = 44.7 \text{ people/km}^2$$

**Problem 2.6** The total world population in 2012 was about 7 billion and Figure 2.11 shows that at that time the actual world population growth rate was about 1% per year. The figure also shows an anticipated roughly linear decrease in growth rate that extrapolates to zero growth in about the year 2080. Assuming an average growth rate of 0.5% between 2012 and 2080, what would the world population be in 2080? How does this compare with estimates discussed in the text for limits to human population?

**Solution** If  $R = 0.5\%$  per year then the doubling time is found from equation (2.9) to be

$$t_D = \frac{10 \ln 2}{R} = \frac{(100) \times (0.693)}{0.5} = 138.6 \text{ y}$$

using equation (2.7) to get the constant  $a$

$$a = \frac{\ln 2}{t_D} = \frac{0.693}{138.6 \text{ y}} = 0.005 \text{ y}^{-1}$$

then equation (2.4) gives

$$N(t) = N_0 \exp(at)$$

so from  $N_0 = 7 \times 10^9$  people and  $t = 2080 - 2012 = 68 \text{ years}$  we find

$$N(t) = (7 \times 10^9) \exp((0.005 \text{ y}^{-1}) \times (68 \text{ y})) = 9.8 \times 10^9 \text{ people}$$

This is consistent with comments in the text which suggest that the limit to human population can not be much more than 10 billion.

**Problem 2.7** The population of a state is 25,600 in the year 1800 and 218,900 in the year 1900. Calculate the expected population in the year 2000 if (a) the growth is linear and (b) the growth is exponential.

**Solution** If population growth is linear then for 100 years between 1800 and 1900 it grows by  $(218.9 - 25.6) \times 10^3 = 193.3 \times 10^3$ , so the population would grow by another  $193.3 \times 10^3$  during the 100 years from 1900 to 2000 for a total of

$$(218.9 + 193.3) \times 10^3 = 412.2 \times 10^3 \text{ people}$$

If the population growth is exponential then from equation (2.4) for  $N_0 = 25.6 \times 10^3$  in 1800 then for  $t = 100$  years,  $N(t)$  is  $218.9 \times 10^3$ . From this  $a$  can be found to be

$$a = \frac{1}{t} \ln \frac{N(t)}{N_0} = \left( \frac{1}{100\text{y}} \right) \times \ln \left( \frac{218.9 \times 10^3}{25.6 \times 10^3} \right) = 0.0215\text{y}^{-1}$$

Then using  $N_0 = 218.9 \times 10^3$  in year 1900 the population at 100y (i.e. in year 2000) is

$$N(t) = (218.9 \times 10^3) \times \exp((0.0215\text{y}^{-1}) \times (100\text{y})) = 1.87 \times 10^6$$

about 4.5 times the value for linear growth.

**Problem 2.8** The population of a country as a function of time is shown in the following table. Is the growth exponential?

| year | population (millions) |
|------|-----------------------|
| 1700 | 0.501                 |
| 1720 | 0.677                 |
| 1740 | 0.891                 |
| 1760 | 1.202                 |
| 1780 | 1.622                 |
| 1800 | 2.163                 |
| 1820 | 2.884                 |
| 1840 | 3.890                 |
| 1860 | 5.176                 |
| 1880 | 6.761                 |
| 1900 | 8.702                 |
| 1920 | 10.23                 |
| 1940 | 11.74                 |
| 1960 | 13.18                 |
| 1980 | 14.45                 |
| 2000 | 15.49                 |

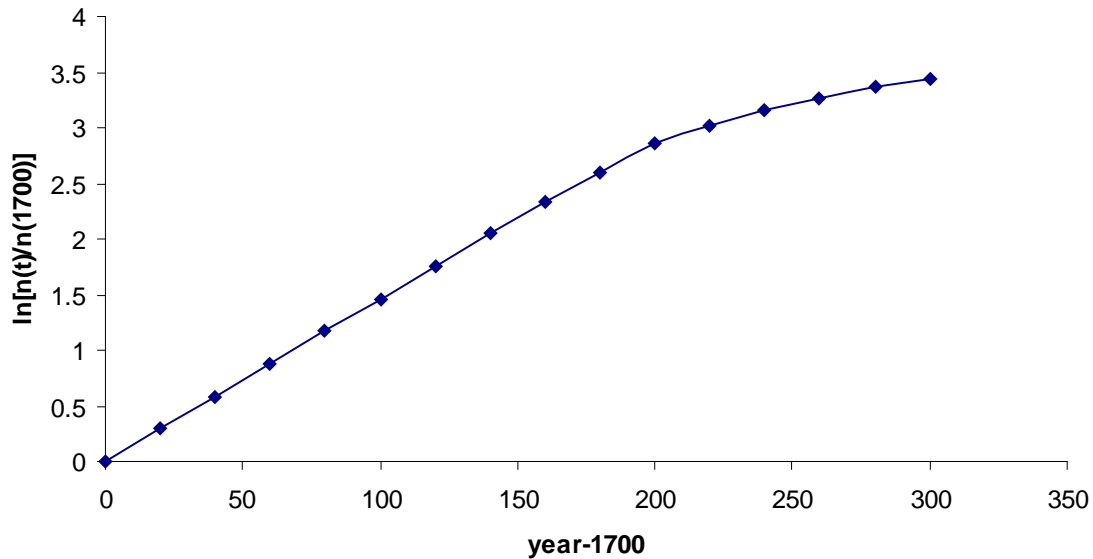
**Solution** For exponential growth

$$N(t) = N_0 \exp(a(t - t_0)) \text{ so } \ln\left(\frac{N(t)}{N_0}\right) = a(t - t_0)$$

and the ln of the related population should be linear in time. Calculating  $N(t)/N_0$  from the values above gives the tabulated values. They are plotted as a function of  $t - t_D$  as shown

| year | population (millions) | year - 1700 | $\ln[N(t)/N(1700)]$ |
|------|-----------------------|-------------|---------------------|
| 1700 | 0.501                 | 0           | 0                   |
| 1720 | 0.677                 | 20          | 0.301065            |
| 1740 | 0.891                 | 40          | 0.575738            |
| 1760 | 1.202                 | 60          | 0.875136            |
| 1780 | 1.622                 | 80          | 1.174809            |
| 1800 | 2.163                 | 100         | 1.462645            |
| 1820 | 2.884                 | 120         | 1.750327            |
| 1840 | 3.89                  | 140         | 2.049558            |
| 1860 | 5.176                 | 160         | 2.335182            |
| 1880 | 6.761                 | 180         | 2.60232             |
| 1900 | 8.702                 | 200         | 2.854702            |
| 1920 | 10.23                 | 220         | 3.016474            |
| 1940 | 11.74                 | 240         | 3.154151            |
| 1960 | 13.18                 | 260         | 3.26985             |
| 1980 | 14.45                 | 280         | 3.361844            |
| 2000 | 15.49                 | 300         | 3.431344            |

The graph shows that the  $\ln$  is linear and hence the population is exponential until ~ 1900 when the increase is less than exponential.



**Problem 2.9** Consider a solar photovoltaic system with a total rated output of  $10 \text{ MW}_e$  and a capacity factor of 29%. If the total installation cost is \$35,000,000, calculate the decrease in the cost of electricity per kilowatt-hour if the payback period is 25 years instead of 15 years. Assume a constant interest rate of 5.8%.

**Solution** From Example 2.3 the contribution to the cost of electricity per kWh due to the capital cost is

$$\frac{1}{Rf(8760\text{h/y})} \times \frac{i(1+i)^T}{((1+i)^T - 1)}$$

Using  $I = 35,000,000$ ,  $i = 0.058$ ,  $R = 10^4 \text{ kW}$ ,  $f = 0.29$ , then for a payback period of 15 years the cost per kWh is

$$\frac{3.5 \times 10^7}{10^4 \times 0.29 \times 8760} \times \frac{0.058 \times (1.058)^{15}}{((1.058)^{15} - 1)} = 1.378 \times 0.102 = \$0.140/\text{kWh}$$

For a payback period of 25 years the cost is

$$1.378 \times \frac{0.058 \times (1.058)^{25}}{((1.058)^{25} - 1)} = 1.378 \times 0.0767 = \$0.106 / \text{kWh}$$

or a decrease of  $(0.140 - 0.106) = \$0.034$  per kWh.

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