CHAPTER TWO

SETS

Exercise Set 2.1

1. Set
2. Ellipsis
3. Description, Roster form, Set-builder notation
4. Finite
5. Infinite
6. Equal
7. Equivalent
8. Cardinal
9. Empty or null
10. $\{\ \}, \varnothing$
11. Universal
12. One-to-one
13. Not well defined, "best" is interpreted differently by different people.
14. Not well defined, "most interesting" is interpreted differently by different people.
15. Well defined, the contents can be clearly determined.
16. Well defined, the contents can be clearly determined.
17. Well defined, the contents can be clearly determined.
18. Not well defined, "most interesting" is interpreted differently by different people.
19. Infinite, the number of elements in the set is not a natural number.
20. Finite, the number of elements in the set is a natural number.
21. Infinite, the number of elements in the set is not a natural number.
22. Infinite, the number of elements in the set is not a natural number.
23. Infinite, the number of elements in the set is not a natural number.
24. Finite, the number of elements in the set is a natural number.
25. {Maine, Maryland, Massachusetts, Michigan, Minnesota, Misssissippi, Missouri, Montana}
26. {San Marino, Scotland, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland}
27. $\{11, 12, 13, 14, \ldots, 177\}$
28. $C = \{4\}$
29. $B = \{2, 4, 6, 8, \ldots\}$
30. $\{\ \}$ or $\varnothing$

31. $\{\ \}$ or $\varnothing$

32. {Idaho, Oregon}

33. $E = \{14, 15, 16, 17, \ldots, 84\}$

34. { Alaska, Hawaii }

35. {Metropolitan Museum of Art, Tate Modern, National Gallery of Art, British Museum, Louvre Museum}

36. { Musee d'Art Moderne Prado, Museum of Modern Art }

37. { Museum of Modern Art, Musee d'Art Moderne Prado, Musee d'Orsay }

38. { National Gallery, Vatican Museums, Metropolitan Museum of Art, Tate Modern, National Gallery of Art }

39. { 2007, 2008 }

40. { 2003 }

41. { 2004, 2005, 2006, 2007 }

42. $\{\ \}$ or $\varnothing$

43. $B = \{x \mid x \in N \text{ and } 6 < x < 15\}$ or $B = \{x \mid x \in N \text{ and } 7 \le x \le 14\}$

44. $A = \{x \mid x \in N \text{ and } x < 10\}$ or $A = \{x \mid x \in N \text{ and } x \le 9\}$

45. $C = \{x \mid x \in N \text{ and } x \text{ is a multiple of } 3\}$

46. $D = \{x \mid x \in N \text{ and } x \text{ is a multiple of } 5\}$

47. $E = \{x \mid x \in N \text{ and } x \text{ is odd}\}$

48. $A = \{x \mid x \text{ is Independence Day}\}$

49. $C = \{x \mid x \text{ is February}\}$

50. $F = \{x \mid x \in N \text{ and } 14 < x < 101\}$ or $F = \{x \mid x \in N \text{ and } 15 \le x \le 100\}$

51. Set A is the set of natural numbers less than or equal to 7.
52. Set D is the set of natural numbers that are multiples of 3.
53. Set V is the set of vowels in the English alphabet.
54. Set S is the set of the seven dwarfs in *Snow White and the Seven Dwarfs*.
55. Set T is the set of species of trees.
56. Set E is the set of natural numbers greater than or equal to 4 and less than 11.
57. Set S is the set of seasons.
58. Set B is the set of members of the Beatles.

59. { China, India, United States }

60. { Pakistan, United Kingdom }

61. { Russia, Brazil, Indonesia, Japan, Germany }

62. { India, United States }

63. { 2008, 2009, 2010, 2011 }

64. { 1996, 1997, 1998, 1999 }

65. {2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007}

66. {2008, 2010}

67. False; $\{e\}$ is a set, and not an element of the set.

68. True; b is an element of the set.

69. False; h is not an element of the set.

70. True; Mickey Mouse is an element of the set.

71. False; 3 is an element of the set.

72. False; the Amazon is a river in South America.

73. True; *Titanic* is an element of the set.

74. False; 2 is an even natural number.

75. $n(A) = 4$

76. $n(B) = 6$

77. $n(C) = 0$

78. $n(D) = 5$

79. Both; A and B contain exactly the same elements.
80. Equivalent; both sets contain the same number of elements, 3.
81. Neither; the sets have a different number of elements.

82. Neither; not all cats are Siamese
83. Equivalent; both sets contain the same number of elements, 4.
84. Equivalent; both sets contain the same number of elements, 50.
85. a) Set A is the set of natural numbers greater than 2. Set B is the set of all numbers greater than 2.
    b) Set A contains only natural numbers. Set B contains other types of numbers, including fractions and decimal numbers.
    c) $A = \{3, 4, 5, 6, \ldots\}$
    d) No

86. a) Set A is the set of natural numbers greater than 2 and less than or equal to 5. Set B is the set of numbers greater than 2 and less than or equal to 5.
    b) Set A contains only natural numbers. Set B contains other types of numbers, including fractions and decimal numbers.
    c) $A = \{3, 4, 5\}$
    d) No; because there are an infinite number of elements between any two elements in set B, we cannot write set B in roster form.

87. Cardinal; 7 tells how many.
88. Ordinal; 25 tells the relative position of the chart.
89. Ordinal; sixteenth tells Lincoln's relative position.
90. Cardinal; 35 tells how many dollars she spent.

91. Answers will vary.
92. Answers will vary. Examples: the set of people in the class who were born on the moon, the set of automobiles that get 400 miles on a gallon of gas, the set of fish that can talk
93. Answers will vary.
94. Answers will vary. Here are some examples.
    a) The set of men. The set of actors. The set of people over 12 years old. The set of people with two legs. The set of people who have been in a movie.
    b) The set of all the people in the world.

<u>Exercise Set 2.2</u>

1. Subset
2. Proper
3. 2^n, where n is the number of elements in the set.
4. $2^n - 1$, where n is the number of elements in the set.
5. True; $\{$book$\}$ is a subset of $\{$ magazine, newspaper, book $\}$.
6. True; $\{$Italy$\}$ is a subset of $\{$Italy, Spain, France, Switzerland, Austria$\}$.
7. False; McIntosh is not in the second set.
8. False; pepper is not in the second set.
9. True; $\{$ motorboat, kayak $\}$ is a proper subset of $\{$ kayak, fishing boat, sailboat, motorboat $\}$.
10. True; $\{$polar bear, tiger, lion$\}$ is a proper subset of $\{$tiger, lion, polar bear, penguin$\}$.
11. False; no subset is a proper subset of itself.
12. False; no set is a proper subset of itself.
13. True; Xbox 360 is an element of $\{$ PSIII, Wii, Xbox 360 $\}$.
14. True; LaGuardia is an element of $\{$ JFK, LaGuardia, Newark $\}$.
15. False; $\{$swimming$\}$ is a set, not an element.
16. False; $\{\ \}$ is a set, not an element.

17. True; 5 is not an element of $\{2,4,6\}$.

18. True; the empty set is a subset of every set, including itself.

19. True; $\{\text{red}\}$ is a proper subset of $\{\text{red, blue, green}\}$.

20. True; $\{3,5,9\}=\{3,9,5\}$.

21. False; the set $\{\varnothing\}$ contains the element $\varnothing$.

22. True; $\{\ \}$ and $\varnothing$ each represent the empty set.

23. False; the set $\{0\}$ contains the element 0.

24. True; the empty set is a subset of every set, including itself.

25. False; 0 is a number and $\{\ \}$ is a set.

26. True; the elements of the set are themselves sets.

27. $B\subseteq A, B\subset A$

28. $A=B, A\subseteq B, B\subseteq A$

29. $A\subseteq B, A\subset B$

30. None

31. $B\subseteq A, B\subset A$

32. $B\subseteq A, B\subset A$

33. $A=B, A\subseteq B, B\subseteq A$

34. $B\subseteq A, B\subset A$

35. $\{\ \}$ is the only subset.

36. $\{\ \},\{\bigcirc\}$

37. $\{\ \},\{\text{cow}\},\{\text{horse}\},\{\text{cow, horse}\}$

38. $\{\ \},\{\text{steak}\},\{\text{pork}\},\{\text{chicken}\},\{\text{steak, pork}\},$
$\{\text{steak, chicken}\},\{\text{pork, chicken}\},$
$\{\text{steak, pork, chicken}\}$

39.a) $\{\ \},\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},$
$\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},$
$\{a,c,d\},\{b,c,d\},\{a,b,c,d\}$

b) All the sets in part (a) are proper subsets of A except $\{a,b,c,d\}$.

40. a) $2^9=2\times2\times2\times2\times2\times2\times2\times2\times2=512$ subsets

b) $2^9-1=512-1=511$ proper subsets

41. False; A could be equal to B.

42. True; every proper subset is a subset.

43. True; every set is a subset of itself.

44. False; no set is a proper subset of itself.

45. True; $\varnothing$ is a proper subset of every set except itself.

46. True; $\varnothing$ is a subset of every set.

47. True; every set is a subset of the universal set.

48. False; a set cannot be a proper subset of itself.

49. True; $\varnothing$ is a proper subset of every set except itself and $U\neq\varnothing$.

50. False; the only subset of $\varnothing$ is itself and $U\neq\varnothing$.

51. True; $\varnothing$ is a subset of every set.
52. False; U is not a subset of $\varnothing$.
53. The number of different variations is equal to the number of subsets of $\{\text{cheese, pepperoni, sausage, onions, green peppers, mushrooms, anchovies, ham}\}$, which is $2^8=2\times2\times2\times2\times2\times2\times2\times2=256$.
54. The number of different variations of the house is equal to the number of subsets of $\{\text{deck, jacuzzi, security system, hardwood flooring}\}$, which is $2^4=2\times2\times2\times2=16$..

55. The number of options is equal to the number of subsets of
$\{$cucumber, onion, tomato, carrot, green pepper, olive, mushroom$\}$, which is
$2^7 = 2\times2\times2\times2\times2\times2\times2 = 128$

56. The number of different variations is equal to the number of subsets of
$\{$call waiting, call forwarding, caller identification, three way calling, voice mail, fax line$\}$,
which is $2^6 = 2\times2\times2\times2\times2\times2 = 64$.

57. $E = F$ since they are both subsets of each other.

58. If there is a one-to-one correspondence between boys and girls, then the sets are equivalent.

59. a) Yes.
b) No, c is an element of set D.
c) Yes, each element of $\{a,b\}$ is an element of set D.

60. a) Each person has 2 choices, namely yes or no. $2\times2\times2\times2 = 16$
b) YYYY, YYYN, YYNY, YNYY, NYYY, YYNN, YNYN, YNNY, NYNY, NNYY, NYYN, YNNN, NYNN,
NNYN, NNNY, NNNN
c) 5 out of 16

61. A one element set has one proper subset, namely the empty set. A one element set has two subsets, namely itself and the empty set. One is one-half of two. Thus, the set must have one element.

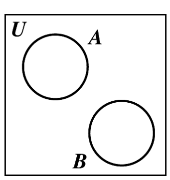
62. Yes

63. Yes

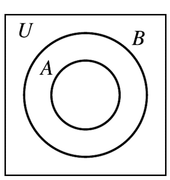
64. No

Section 2.3

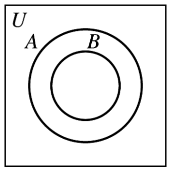
1. Complement
2. Union
3. Intersection
4. Difference
5. Cartesian
6. $m\times n$
7. Disjoint
8. Four
9.



10.



11.



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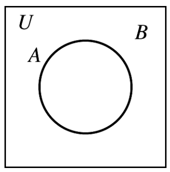
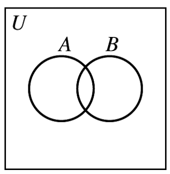


13.

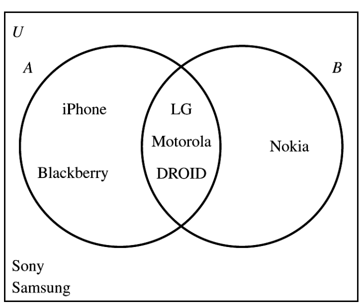


14. *Or* is generally interpreted to mean *union*.

15. *And* is generally interpreted to mean *intersection*.

16. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

17.



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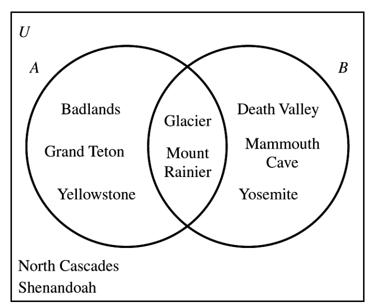

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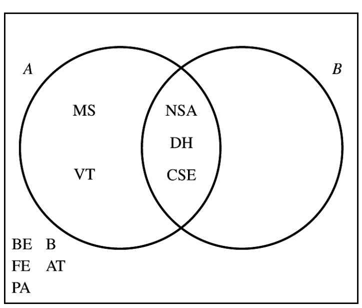
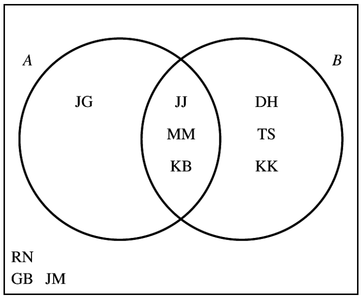


20.



21. The set of animals in U.S. zoos that are not in the San Diego Zoo
22. The set of U.S. colleges and universities that are not in the state of Mississippi
23. The set of farms in the U.S. that do not produce corn
24. The set of farms in the U.S. that do not produce tomatoes
25. The set of farms in the U.S. that produce corn or tomatoes
26. The set of farms in the U.S. that produce corn and tomatoes
27. The set of farms in the U.S. that produce corn and do not produce tomatoes
28. The set of farms in the U.S. that produce corn or do not produce tomatoes
29. The set of furniture stores in the U.S. that sell mattresses or leather furniture
30. The set of furniture stores in the U.S. that sell mattresses and outdoor furniture
31. The set of furniture stores in the U.S. that do not sell outdoor furniture and sell leather furniture
32. The set of furniture stores in the U.S. that sell mattresses, outdoor furniture, and leather furniture
33. The set of furniture stores in the U.S. that sell mattresses or outdoor furniture or leather furniture
34. The set of furniture stores in the U.S. that do not sell mattresses or do not sell leather furniture
35. $A = \{ b, c, t, w, a, h \}$
36. $B = \{ a, d, f, g, h, r \}$
37. $A \cap B = \{ w, b, c, t, a, h \} \cap \{ a, h, f, r, d, g \} = \{ a, h \}$
38. $U = \{ c, w, b, t, a, h, d, f, g, r, p, m, z \}$
39. $A \cup B = \{ w, b, c, t, a, h \} \cup \{ a, h, f, r, d, g \} = \{ w, b, c, t, a, h, f, r, d, g \}$
40. $(A \cup B)'$: From #39, $A \cup B = \{ w, b, c, t, a, h, f, r, d, g \}$. $(A \cup B)' = \{ w, b, c, t, a, h, f, r, d, g \}' = \{ p, m, z \}$
41. $A' \cap B' = \{ w, c, b, t, a, h \} \{ a, h, f, r, d, g \} \cap \{ w, c, b, t, p, m, z \} = \{ p, m, z \}$
42. $(A \cap B)'$: From #37, $A \cap B = \{ a, h \}$. $(A \cap B)' = \{ a, h \}' = \{ w, c, b, t, f, r, d, g, p, m, z \}$
43. $A = \{ \text{L}, \Delta, @, *, \$ \}$
44. $B = \{ *, \$, \text{R}, \square, \alpha \}$
45. $U = \{ \text{L}, \Delta, @, *, \$, \text{R}, \square, \alpha, \infty, \text{Z}, \Sigma \}$
46. $A \cap B = \{\text{L}, \Delta, @, *, \$\} \cap \{*, \$, \text{R}, \square, \alpha\} = \{*, \$\}$
47. $A' \cup B = \{\text{R}, \square, \alpha, \infty, \text{Z}, \Sigma\} \cup \{*, \$, \text{R}, \square, \alpha\} = \{\text{R}, \square, \alpha, \infty, \text{Z}, \Sigma, *, \$ \}$
48. $A \cup B' = \{\text{L}, \Delta, @, *, \$\} \cup \{*, \$, \text{R}, \square, \alpha\}' = \{\text{L}, \Delta, @, *, \$\} \cup \{\text{L}, \Delta, @, \infty, \Sigma, \text{Z}\} = \{\text{L}, \Delta, @, *, \$, \infty, \Sigma, \text{Z}\}$
49. $A' \cap B = \{\text{L}, \Delta, @, *, \$\}' \cap \{ *, \$, \text{R}, \square, \alpha \} = \{ \text{R}, \square, \alpha, \infty, \text{Z}, \Sigma\} \cap \{ *, \$, \text{R}, \square, \alpha \} = \{ \text{R}, \square, \alpha \}$
50. $(A \cup B)'$: From the diagram, $(A \cup B)' = \{ \infty, \text{Z}, \Sigma \}$
51. $A \cup B = \{ 1, 2, 4, 5, 7 \} \cup \{ 2, 3, 5, 6 \} = \{ 1, 2, 3, 4, 5, 6, 7 \}$

52. $A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 3, 5, 6\} = \{2, 5\}$

53. $B' = \{2, 3, 5, 6\}' = \{1, 4, 7, 8\}$

54. $A \cup B' = \{1, 2, 4, 5, 7\} \cup \{2, 3, 5, 6\}' = \{1, 2, 4, 5, 7\} \cup \{1, 4, 7, 8\} = \{1, 2, 4, 5, 7, 8\}$

55. $(A \cup B)'$ From #51, $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$. $(A \cup B)' = \{1, 2, 3, 4, 5, 6, 7\}' = \{8\}$

56. $A' \cap B' = \{1, 2, 4, 5, 7\}' \cap \{2, 3, 5, 6\}' = \{3, 6, 8\} \cap \{1, 4, 7, 8\} = \{8\}$

57. $(A \cup B)' \cap B$: From #55, $(A \cup B)' = \{8\}$. $(A \cup B)' \cap B = \{8\} \cap \{2, 3, 5, 6\} = \{\ \}$

58. $(A \cup B) \cap (A \cup B)' = \{\ \}$ (The intersection of a set and its complement is always empty.)

59. $(B \cup A)' \cap (B' \cup A')$: From #55, $(A \cup B)' = (B \cup A)' = \{8\}$.

$(B \cup A)' \cap (B' \cup A') = \{8\} \cap \left(\{2, 3, 5, 6\}' \cup \{1, 2, 4, 5, 7\}'\right) = \{8\} \cap (\{1, 4, 7, 8\} \cup \{3, 6, 8\})$

$= \{8\} \cap \{1, 3, 4, 6, 7, 8\} = \{8\}$

60. $A' \cup (A \cap B)$: From #52, $A \cap B = \{2, 5\}$.

$A' \cup (A \cap B) = \{1, 2, 4, 5, 7\}' \cup \{2, 5\} = \{3, 6, 8\} \cup \{2, 5\} = \{2, 3, 5, 6, 8\}$

61. $B' = \{b, c, d, f, g\}' = \{a, e, h, i, j, k\}$

62. $B \cup C = \{b, c, d, f, g\} \cup \{a, b, f, i, j\} = \{a, b, c, d, f, g, i, j\}$

63. $A \cap C = \{a, c, d, f, g, i\} \cap \{a, b, f, i, j\} = \{a, f, i\}$

64. $A' \cup B'$: $A' = \{b, e, h, j, k\}$, $B' = \{a, e, h, i, j, k\}$.

$A' \cup B' = \{b, e, h, j, k\} \cup \{a, e, h, i, j, k\} = \{a, b, e, h, i, j, k\}$

65. $(A \cap C)'$: From #63, $A \cap C = \{a, f, i\}$. $(A \cap C)' = \{a, f, i\}' = \{b, c, d, e, g, h, j, k\}$

66. $(A \cap B) \cup C = (\{a, c, d, f, g, i\} \cap \{b, c, d, f, g\}) \cup \{a, b, f, i, j\} = \{c, d, f, g\} \cup \{a, b, f, i, j\}$

$= \{a, b, c, d, f, g, i, j\}$

67. $A \cup (C \cap B)' = \{a, c, d, f, g, i\} \cup (\{a, b, f, i, j\} \cap \{b, c, d, f, g\})' = \{a, c, d, f, g, i\} \cup \{b, f\}'$

$= \{a, c, d, f, g, i\} \cup \{a, c, d, e, g, h, i, j, k\} = \{a, c, d, e, f, g, h, i, j, k\}$

68. $A \cup (C' \cup B') = \{a, c, d, f, g, i\} \cup \left(\{a, b, f, i, j\}' \cup \{b, c, d, f, g\}'\right)$

$= \{a, c, d, f, g, i\} \cup (\{c, d, e, g, h, k\} \cup \{a, e, h, i, j, k\}) = \{a, c, d, f, g, i\} \cup \{a, c, d, e, g, h, i, j, k\}$

$= \{a, c, d, e, f, g, h, i, j, k\}$

69. $(A' \cup C) \cup (A \cap B) = \left(\{a, c, d, f, g, i\}' \cup \{a, b, f, i, j\}\right) \cup (\{a, c, d, f, g, i\} \cap \{b, c, d, f, g\})$

$= (\{b, e, h, j, k\} \cup \{a, b, f, i, j\}) \cup \{c, d, f, g\} = \{a, b, e, f, h, i, j, k\} \cup \{c, d, f, g\}$

$= \{a, b, c, d, e, f, g, h, i, j, k\}$, or U

70.

$(C\cap B)\cap(A'\cap B)$: From #67, $C\cap B=\{b,f\}$.

$(C\cap B)\cap(A'\cap B)=\{b,f\}\cap\left(\{a,c,d,f,g,i\}'\cap\{b,c,d,f,g\}\right)=\{b,f\}\cap(\{b,e,h,j,k\}\cap\{b,c,d,f,g\})$

$=\{b,f\}\cap\{b\}=\{b\}$

For exercises 71-78: $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,2,4,6,9\}$, $B=\{1,3,4,5,8\}$, $C=\{4,5,9\}$

71. $A-B=\{1,2,4,6,9\}-\{1,3,4,5,8\}=\{2,6,9\}$
72. $A-C=\{1,2,4,6,9\}-\{4,5,9\}=\{1,2,6\}$
73. $A-B'$: This leaves only $A\cap B$, which is $\{1,4\}$
74. $A'-C=\{3,5,7,8,10\}-\{4,5,9\}=\{3,7,8,10\}$
75. $(A-B)'=\{2,6,9\}'=\{1,3,4,5,7,8,10\}$
76. $(A-B)'-C=\{1,3,4,5,7,8,10\}-\{4,5,9\}=\{1,3,7,8,10\}$
77. $C-A'=\{4,5,9\}-\{3,5,7,8,10\}=\{4,9\}$
78. $(C-A)'-B=\{5\}'-\{1,3,4,5,8\}=\{2,6,7,9,10\}$

For exercises 79-84: $A=\{a,b,c\}$ and $B=\{1,2\}$

79. $\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$
80. $\{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$
81. No; the ordered pairs are not the same.
82. 6
83. 6
84. Yes
85. $A\cap B=\{1,3,5,7,9\}\cap\{2,4,6,8\}=\{\ \}$
86. $A\cup B=\{1,3,5,7,9\}\cup\{2,4,6,8\}=\{1,2,3,4,5,6,7,8,9\}$, or U
87. $A'\cup B=\{1,3,5,7,9\}'\cup\{2,4,6,8\}=\{2,4,6,8\}\cup\{2,4,6,8\}=\{2,4,6,8\}$, or B
88. $(B\cup C)'=(\{2,4,6,8\}\cup\{1,2,3,4,5\})'=\{1,2,3,4,5,6,8\}'=\{7,9\}$
89. $A\cap C'=\{1,3,5,7,9\}\cap\{1,2,3,4,5\}'=\{1,3,5,7,9\}\cap\{6,7,8,9\}=\{7,9\}$
90. $A\cap B'=\{1,3,5,7,9\}\cap\{2,4,6,8\}'=\{1,3,5,7,9\}\cap\{1,3,5,7,9\}=\{1,3,5,7,9\}$, or A
91. $(B\cap C)'=(\{2,4,6,8\}\cap\{1,2,3,4,5\})'=\{2,4\}'=\{1,3,5,6,7,8,9\}$
92. $(A\cup C)\cap B=(\{1,3,5,7,9\}\cup\{1,2,3,4,5\})\cap\{2,4,6,8\}=\{1,2,3,4,5,7,9\}\cap\{2,4,6,8\}=\{2,4\}$
93. $(C'\cup A)\cap B=\left(\{1,2,3,4,5\}'\cup\{1,3,5,7,9\}\right)\cap\{2,4,6,8\}=(\{6,7,8,9\}\cup\{1,3,5,7,9\})\cap\{2,4,6,8\}$

    $=\{1,3,5,6,7,8,9\}\cap\{2,4,6,8\}=\{6,8\}$
94. $(C\cap B)\cup A$: From #91, $C\cap B=\{2,4\}$. $(C\cap B)\cup A=\{2,4\}\cup\{1,3,5,7,9\}=\{1,2,3,4,5,7,9\}$

95. $(A\cap B)'\cup C$: From #85, $A\cap B=\{\ \}$.

$(A\cap B)'\cup C=\{\ \}'\cup\{1,2,3,4,5\}=\{1,2,3,4,5,6,7,8,9\}\cup\{1,2,3,4,5\}=\{1,2,3,4,5,6,7,8,9\}$, or U

96. $(A'\cup C)\cap B=\left(\{1,3,5,7,9\}'\cup\{1,2,3,4,5\}\right)\cap\{2,4,6,8\}=\left(\{2,4,6,8\}\cup\{1,2,3,4,5\}\right)\cap\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}\cap\{2,4,6,8\}=\{2,4,6,8\}$, or B

97. $(A'\cup B')\cap C=\left(\{1,3,5,7,9\}'\cup\{2,4,6,8\}'\right)\cap\{1,2,3,4,5\}$

$=\left(\{2,4,6,8\}\cup\{1,3,5,7,9\}\right)\cap\{1,2,3,4,5\}=\{1,2,3,4,5,6,7,8,9\}\cap\{1,2,3,4,5\}=\{1,2,3,4,5\}$, or C

98. $(A'\cap C)\cup(A\cap B)$: From #83, $A\cap B=\{\ \}$.

$(A'\cap C)\cup(A\cap B)=\left(\{1,3,5,7,9\}'\cap\{1,2,3,4,5\}\right)\cup\{\ \}=\left(\{2,4,6,8\}\cap\{1,2,3,4,5\}\right)\cup\{\ \}$

$=\{2,4\}\cup\{\ \}=\{2,4\}$

99. A set and its complement will always be disjoint since the complement of a set is all of the elements in the universal set that are not in the set. Therefore, a set and its complement will have no elements in common. For example, if $A\cap B=\{\ \}$. $n(A\cap B)=0$

100. $n(A\cap B)=0$ when A and B are disjoint sets. For example, if $U=\{1,2,3,4,5,6\}$, $A=\{1,3\}$, $B=\{2,4\}$, then $A\cap B=\{\ \}$. $n(A\cap B)=0$

101. Let $A=\{$customers who owned dogs$\}$ and $B=\{$customers who owned cats$\}$.

$n(A\cup B)=n(A)+n(B)-n(A\cap B)=27+38-16=49$

102. Let $A=\{$students who sang in the chorus$\}$ and $B=\{$students who played in the stage band$\}$.

$$n(A\cup B)=n(A)+n(B)-n(A\cap B)$$
$$46=n(A)+30-4$$
$$46=n(A)+26$$
$$20=n(A)$$

103. a) $A\cup B=\{a,b,c,d\}\cup\{b,d,e,f,g,h\}=\{a,b,c,d,e,f,g,h\}$, $n(A\cup B)=8$,

$A\cap B=\{a,b,c,d\}\cap\{b,d,e,f,g,h\}=\{b,d\}, n(A\cap B)=2$.

$n(A)+n(B)-n(A\cap B)=4+6-2=8$

Therefore, $n(A\cup B)=n(A)+n(B)-n(A\cap B)$.

b) Answers will vary.

c) Elements in the intersection of A and B are counted twice in $n(A)+n(B)$.

104. $A\cap B'$ defines Region I. $A\cap B$ defines Region II. $A'\cap B$ defines Region III.

$A'\cap B'$ or $(A\cup B)'$ defines Region IV.

105. $A\cup B=\{1,2,3,4,\ldots\}\cup\{4,8,12,16,\ldots\}=\{1,2,3,4,\ldots\}$, or A

106. $A\cap B=\{1,2,3,4,\ldots\}\cap\{4,8,12,16,\ldots\}=\{4,8,12,16,\ldots\}$, or B

107. $B\cup C=\{4,8,12,16,\ldots\}\cup\{2,4,6,8,\ldots\}=\{2,4,6,8,\ldots\}$, or C

108. $B\cap C=\{4,8,12,16,\ldots\}\cap\{2,4,6,8,\ldots\}=\{4,8,12,16,\ldots\}$, or B

109. $A \cap C = \{1, 2, 3, 4, \ldots\} \cap \{2, 4, 6, 8, \ldots\} = \{2, 4, 6, 8, \ldots\}$, or C

110. $A' \cap C = \{1, 2, 3, 4, \ldots\}' \cap \{2, 4, 6, 8, \ldots\} = \{0\} \cap \{2, 4, 6, 8, \ldots\} = \{\ \}$

111. $B' \cap C = \{4, 8, 12, 16, \ldots\}' \cap \{2, 4, 6, 8, \ldots\} = \{0, 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, \ldots\} \cap \{2, 4, 6, 8, \ldots\}$
$= \{2, 6, 10, 14, 18, \ldots\}$

112. $(B \cup C)' \cup C$: From #107, $B \cup C = C$. $(B \cup C)' \cup C = C' \cup C = \{2, 4, 6, 8, \ldots\}' \cup \{2, 4, 6, 8, \ldots\}$
$= \{0, 1, 2, 3, 4, \ldots\}$, or U

113. $(A \cap C) \cap B'$: From #109, $A \cap C = C$. $(A \cap C) \cap B' = C \cap B'$.
From #111, $B' \cap C = C \cap B' = \{2, 6, 10, 14, 18, \ldots\}$

114. $U' \cap (A \cup B)$: From #103, $A \cup B = A$. $U' \cap (A \cup B) = U' \cap A = \{\ \} \cap \{1, 2, 3, 4, \ldots\} = \{\ \}$

115. $A \cap A' = \{\ \}$

116. $A \cup A' = U$

117. $A \cup \varnothing = A$

118. $A \cap \varnothing = \varnothing$

119. $A' \cup U = U$

120. $A \cap U = A$

121. $A \cup U = U$

122. $A \cap A = A$

123. If $A \cap B = B$, then $B \subseteq A$.

124. If $A \cup B = B$, then $A \subseteq B$.

125. If $A \cap B = \varnothing$, then A and B are disjoint sets.

126. If $A \cup B = A$, then $B \subseteq A$.

127. If $A \cap B = A$, then $A \subseteq B$.

128. If $A \cup B = \varnothing$, then $A = \varnothing$ and $B = \varnothing$. Therefore, they are equal sets.

<u>Exercise Set 2.4</u>

1. 8

2.a) V

b) VI

3.a) $A' \cap B'$

b) $A' \cup B'$

4.Deductive

5. $A' \cap B'$ is represented by regions V and VI. If $B \cap C$ contains 12 elements and region V contains 4 elements, then region VI contains $12 - 4 = 8$ elements.

6. $A \cap B$ is represented by regions II and V. If $A \cap B$ contains 9 elements and region V contains 4 elements, then region II contains $9 - 4 = 5$ elements.

7. a) Yes

$A \cup B = \{1,4,5\} \cup \{1,4,5\} = \{1,4,5\}$

$A \cap B = \{1,4,5\} \cap \{1,4,5\} = \{1,4,5\}$

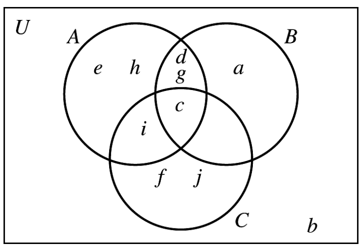
b) No, one specific case cannot be used as proof.

c) No, not equal

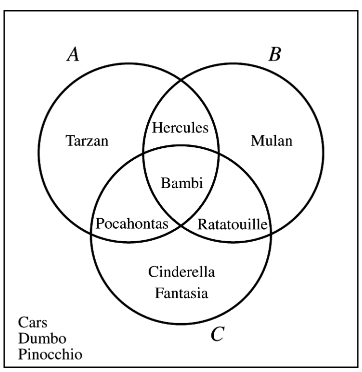
$A \cup B$		$A \cap B$	
Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	B	II, III
$A \cup B$	I, II, III	$A \cap B$	II

Since the two statements are not represented by the same regions, $A \cup B \neq A \cap B$ for all sets A and B.

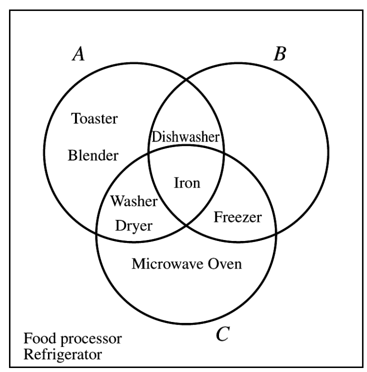
8.



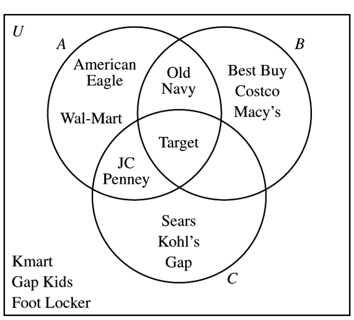
9.



10.



11.



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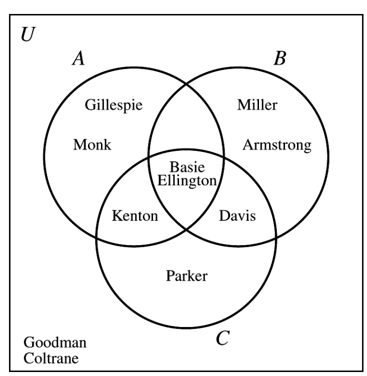
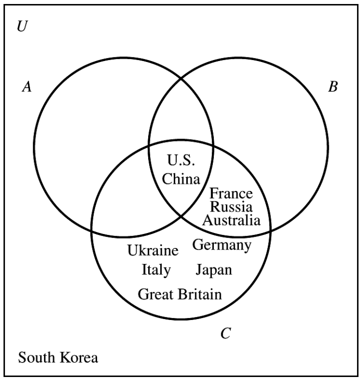
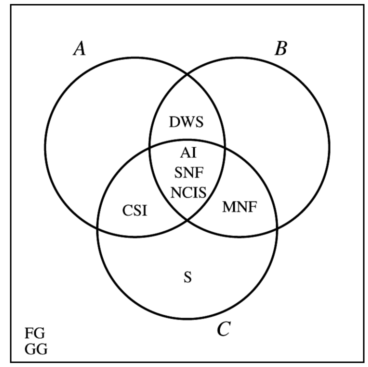


13.



14.



15. Italy, II
16. United States, V
17. Canada, VIII
18. Portugal, VII
19. Spain, III
20. Mexico, VI
21. VI
22. VIII
23. III
24. IV
25. III
26. I
27. V
28. III

29. II
30. VIII
31. VII
32. VI
33. I
34. VII
35. VIII
36. V
37. VI
38. III
39. $A = \{1, 2, 3, 4, 5, 7\}$
40. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$
41. $B = \{3, 4, 5, 6, 8, 9, 12, 14\}$
42. $C = \{4, 5, 6, 7, 8, 11\}$
43. $A \cap B = \{3, 4, 5\}$
44. $A \cap C = \{4, 5, 7\}$
45. $(B \cap C)' = \{1, 2, 3, 7, 9, 10, 11, 12, 13, 14\}$
46. $A \cap B \cap C = \{4, 5\}$

47. $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 14\}$
48. $B \cup C = \{3, 4, 5, 6, 7, 8, 9, 11, 12, 14\}$
49. $(A \cup C)' = \{9, 10, 12, 13, 14\}$
50. $A \cap (B \cup C) = \{3, 4, 5, 7\}$
51. $A' = \{6, 8, 9, 10, 11, 12, 13, 14\}$
52. $(A \cup B \cup C)' = \{10, 13\}$

53. $(A\cap B)'$ $A'\cup B'$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	A'	III, IV
$A\cap B$	II	B	II, III
$(A\cap B)'$	I, III, IV	B'	I, IV
		$A'\cup B'$	I, III, IV

Both statements are represented by the same regions, I, III, IV, of the Venn diagram. Therefore, $(A\cap B)' = A'\cup B'$ for all sets A and B.

54. $(A\cap B)'$ $A'\cup B$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	A'	III, IV
$A\cap B$	II	B	II, III
$(A\cap B)'$	I, III, IV	$A'\cup B$	II, III, IV

Since the two statements are not represented by the same regions, it is not true that $(A\cap B)' = A'\cup B$ for all sets A and B.

55. $A'\cup B'$ $A\cap B$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	$A\cap B$	II
B'	I, IV		
$A'\cup B'$	I, III, IV		

Since the two statements are not represented by the same regions, it is not true that $A'\cup B' = A\cap B$ for all sets A and B.

56. $(A\cup B)'$ $(A\cap B)'$

Set	Regions	Set	Regions
A	I, II	A	I, II
B	II, III	B	II, III
$A\cup B$	I, II, III	$A\cap B$	II
$(A\cup B)'$	IV	$(A\cap B)'$	I, III, IV

Since the two statements are not represented by the same regions, it is not true that $(A\cup B)' = (A\cap B)'$ for all sets A and B.

57. $A'\cup B'$ $(A\cup B)'$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	$(A\cup B)'$	I, II, IV
B'	I, IV		
$A'\cup B'$	I, III, IV		

Since the two statements are not represented by the same regions, it is not true that $A'\cup B' = (A\cup B)'$ for all sets A and B.

58. $A'\cap B'$ $A\cup B'$

Set	Regions	Set	Regions
A	I, II	A	I, II
A'	III, IV	B	II, III
B	II, III	B'	I, IV
B'	I, IV	$A\cup B'$	I, II, IV
$A'\cap B'$	IV		

Since the two statements are not represented by the same regions, it is not true that $A'\cap B' = A\cup B'$ for all sets A and B.

59. $(A'\cap B)'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
$A'\cap B$	III
$(A'\cap B)'$	I, II, IV

$A\cup B'$

Set	Regions
A	I, II
B	II, III
B'	I, IV
$A\cup B'$	I, II, IV

Both statements are represented by the same regions, I, II, IV, of the Venn diagram. Therefore, $(A'\cap B)' = A\cup B'$ for all sets A and B.

60. $A'\cap B'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A'\cap B'$	IV

$(A'\cap B')'$

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A'\cap B'$	IV
$(A'\cap B')'$	I, II, III

Since the two statements are not represented by the same regions, it is not true that $A'\cap B' = (A'\cap B')'$ for all sets A and B.

61. $A\cap(B\cup C)$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B\cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$A\cap(B\cup C)$	II, IV, V

$(A\cap B)\cup C$

Set	Regions
A	I, II, IV, V
B	II, III, V, VI
$A\cap B$	II, V
C	IV, V, VI, VII
$(A\cap B)\cup C$	II, IV, V, VI, VII

Since the two statements are not represented by the same regions, it is not true that $A\cap(B\cup C) = (A\cap B)\cup C$ for all sets $A, B,$ and C.

62. $A\cup(B\cap C)$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B\cap C$	V, VI
A	I, II, IV, V
$A\cup(B\cap C)$	I, II, IV, V, VI

$(B\cap C)\cup A$

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B\cap C$	V, VI
A	I, II, IV, V
$(B\cap C)\cup A$	I, II, IV, V, VI

Both statements are represented by the same regions, I, II, IV, V, VI, of the Venn diagram. Therefore, $A\cup(B\cap C) = (B\cap C)\cup A$ for all sets $A, B,$ and C.

63. $A\cap(B\cup C)$

Set	Regions
B	II, III, V, VI
C	IV , V, VI, VII
$B\cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$A\cap(B\cup C)$	II, IV, V

$(B\cup C)\cap A$

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B\cup C$	II, III, IV, V, VI, VII
A	I, II, IV, V
$(B\cup C)\cap A$	II, IV, V

Both statements are represented by the same regions, II, IV, V, of the Venn diagram. Therefore, $A\cap(B\cup C) = (B\cup C)\cap A$ for all sets $A, B,$ and C.

64. $A\cup(B\cap C)'$ $A'\cap(B'\cup C)$

Set	Regions	Set	Regions
B	II, III, V, VI	B'	I, IV, VII, VIII
C	IV , V, VI, VII	C	IV, V, VI, VII
$B\cap C$	V, VI	$B'\cup C$	I, IV, V, VI, VII, VIII
$(B\cap C)'$	I, II, III, IV, VII, VIII	A	I, II, IV, V
A	I, II, IV, V	A'	III, VI, VII, VIII
$A\cup(B\cap C)'$	I, II, III, IV, V, VII, VIII	$A'\cap(B'\cup C)$	VI, VII, VIII

Since the two statements are not represented by the same regions, it is not true that $A\cup(B\cap C)' = A'\cap(B'\cup C)$ for all sets $A, B,$ and C.

65. $A\cap(B\cup C)$ $(A\cap B)\cup(A\cap C)$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cup C$	II, III, IV, V, VI, VII	$A\cap B$	II, V
A	I, II, IV, V	C	IV, V, VI, VII
$A\cap(B\cup C)$	II, IV, V	$A\cap C$	IV, V
		$(A\cap B)\cup(A\cap C)$	II, IV, V

Both statements are represented by the same regions, II, IV, V, of the Venn diagram.
Therefore, $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$ for all sets $A, B,$ and C.

66. $A\cup(B\cap C)$ $(A\cup B)\cap(A\cup C)$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cap C$	V, VI	$A\cup B$	I, II, III, IV, V, VI
A	I, II, IV, V	C	IV, V, VI, VII
$A\cup(B\cap C)$	I, II, IV, V, VI	$A\cup C$	I, II, IV, V, VI, VII
		$(A\cup B)\cap(A\cup C)$	I, II, IV, V, VI

Both statements are represented by the same regions, I, II, IV, V, VI, of the Venn diagram.
Therefore, $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ for all sets $A, B,$ and C.

67. $A\cup(B\cup C)'$ $A\cup(B'\cap C')$

Set	Regions	Set	Regions
B	II, III, V, VI	B	II, III, V, VI
C	IV, V, VI, VII	B'	I, IV, VII, VIII
$B\cup C$	II, III, IV, V, VI, VII	C	IV, V, VI, VII
$(B\cup C)'$	I, VIII	C'	I, II, III, VIII
A	I, II, IV, V	$B'\cap C'$	I, VIII
$A\cup(B\cup C)'$	I, II, IV, V, VIII	A	I, II, IV, V
		$A\cup(B'\cap C')$	I, II, IV, V, VIII

Both statements are represented by the same region, I, II, IV, V, VIII of the Venn diagram.

Therefore, $A\cup(B\cup C)'=A\cup(B'\cap C')$ for all sets $A, B,$ and C.

68. $(A\cup B)\cap(B\cup C)$ $B\cup(A\cap C)$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
B	II, III, V, VI	C	IV, V, VI, VII
$A\cup B$	I, II, III, IV, V, VI	$A\cap C$	IV, V
C	IV, V, VI, VII	B	II, III, V, VI
$B\cup C$	II, III, IV, V, VI, VII	$B\cup(A\cap C)$	II, III, IV, V, VI
$(A\cup B)\cap(B\cup C)$	II, III, IV, V, VI		

Both statements are represented by the same regions, II, III, IV, V, VI, of the Venn diagram.
Therefore, $(A\cup B)\cap(B\cup C)=B\cup(A\cap C)$ for all sets $A, B,$ and C.

69. $(A\cup B)'\cap C$ $(A'\cup C)\cap(B'\cup C)$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
B	II, III, V, VI	A'	III, VI, VII, VIII
$A\cup B$	I, II, III, IV, V, VI	C	IV, V, VI, VII
$(A\cup B)'$	VII, VIII	$A'\cup C$	III, IV, V, VI, VII, VIII
C	IV, V, VI, VII	B	II, III, V, VI
$(A\cup B)'\cap C$	VII	B'	I, IV, VII, VIII
		$B'\cup C$	I, IV, V, VI, VII, VIII
		$(A'\cup C)\cap(B'\cup C)$	IV, V, VI, VII, VIII

Since the two statements are not represented by the same regions, it is not true that $(A\cup B)'\cap C=(A'\cup C)\cap(B'\cup C)$ for all sets $A, B,$ and C.

70. $(C\cap B)'\cup(A\cap B)'$

Set	Regions
C	IV, V, VI, VII
B	II, III, V, VI
$C\cap B$	V, VI
$(C\cap B)'$	I, II, III, IV, VII, VIII
A	I, II, IV, V
$A\cap B$	II, V
$(A\cap B)'$	I, III, IV, VI, VII, VIII
$(C\cap B)'\cup(A\cap B)'$	I, II, III, IV, VI, VII, VIII

$A\cap(B\cap C)$

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B\cap C$	V, VI
A	I, II, IV, V
$A\cap(B\cap C)$	V

Since the two statements are not represented by the same regions, it is not true that $(C\cap B)'\cup(A\cap B)'=A\cap(B\cap C)$ for all sets $A, B,$ and C.

71. $(A\cup B)'$

72. $(A\cap B)'$

73. $(A\cup B)\cap C'$

74. $(A\cap B)\cup(B\cap C)$

75. a) $(A\cup B)\cap C=(\{1,2,3,4\}\cup\{3,6,7\})\cap\{6,7,9\}=\{1,2,3,4,6,7\}\cap\{6,7,9\}=\{6,7\}$

$(A\cap C)\cup(B\cap C)=(\{1,2,3,4\}\cap\{6,7,9\})\cup(\{3,6,7\}\cap\{6,7,9\})=\varnothing\cup\{6,7\}=\{6,7\}$

Therefore, for the specific sets, $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$.

b) Answers will vary.

c) $(A\cup B)\cap C$

Set	Regions
A	I, II, IV, V
B	II, III, V, VI
$A\cup B$	I, II, III, IV, V, VI
C	IV, V, VI, VII
$(A\cup B)\cap C$	IV, V, VI

$(A\cap C)\cup(B\cap C)$

Set	Regions
A	I, II, IV, V
C	IV, V, VI, VII
$A\cap C$	IV, V
B	II, III, V, VI
$B\cap C$	V, VI
$(A\cap C)\cup(B\cap C)$	IV, V, VI

Both statements are represented by the same regions, IV, V, VI, of the Venn diagram.
Therefore, $(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$ for all sets $A, B,$ and C.

76. . a) $(A \cup C)' \cap B = (\{a, c, d, e, f\} \cup \{a, b, c, d, e\})' \cap \{c, d\} = \{a, b, c, d, e, f\}' \cap \{c, d\}$

$= \{g, h, i\} \cap \{c, d\} = \varnothing$

$(A \cap C)' \cap B = (\{a, c, d, e, f\} \cap \{a, b, c, d, e\})' \cap \{c, d\} = \{a, c, d, e\}' \cap \{c, d\} = \{b, f, g, h, i\} \cap \{c, d\} = \varnothing$

Therefore, for the specific sets, $(A \cup C)' \cap B = (A \cap C)' \cap B$.

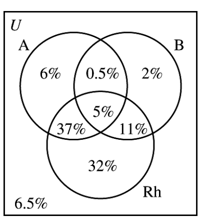
b) Answers will vary.

c) $(A \cup C)' \cap B$ $(A \cap C)' \cap B$

Set	Regions	Set	Regions
A	I, II, IV, V	A	I, II, IV, V
C	IV, V, VI, VII	C	IV, V, VI, VII
$A \cup C$	I, II, IV, V, VI, VII	$A \cap C$	IV, V
$(A \cup C)'$	III, VIII	$(A \cap C)'$	I, II, III, VI, VII, VIII
B	II, III, V, VI	B	II, III, V, VI
$(A \cup C)' \cap B$	III	$(A \cap C)' \cap B$	II, III, VI

Since the two statements are not represented by the same regions, $(A \cup C)' \cap B \neq (A \cap C)' \cap B$ for all sets $A, B,$ and C.

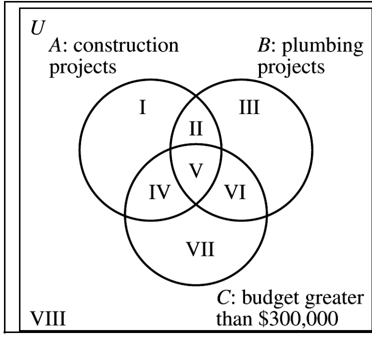
77.



78.

Region	Set	Region	Set
I	$A \cap B' \cap C'$	V	$A \cap B \cap C$
II	$A \cap B \cap C'$	VI	$A' \cap B \cap C$
III	$A' \cap B \cap C'$	VII	$A' \cap B' \cap C$
IV	$A \cap B' \cap C$	VIII	$A' \cap B' \cap C'$

79. a) A : Office Building Construction Projects, B : Plumbing Projects, C : Budget Greater Than \$300,000



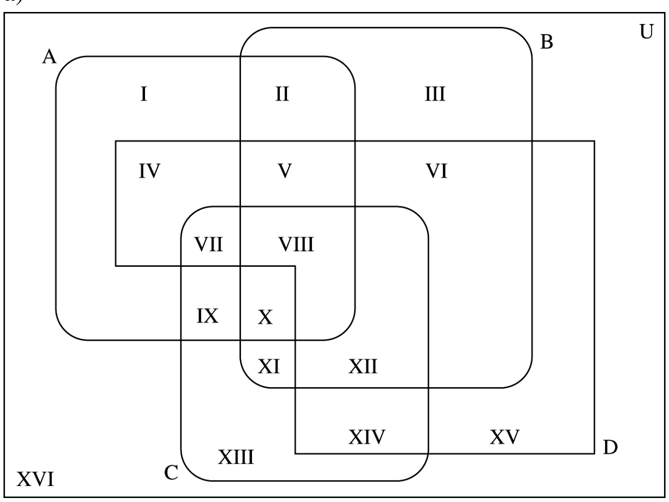
b) Region V; $A \cap B \cap C$

c) Region VI; $A' \cap B \cap C$

d) Region I; $A \cap B' \cap C'$

80. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - 2n(A \cap B \cap C) - n(A \cap B \cap C') - n(A \cap B' \cap C) - n(A' \cap B \cap C)$

81. a)

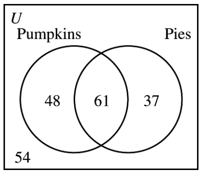


b)

Region	Set	Region	Set
I	$A \cap B' \cap C' \cap D'$	IX	$A \cap B' \cap C \cap D'$
II	$A \cap B \cap C' \cap D'$	X	$A \cap B \cap C \cap D'$
III	$A' \cap B \cap C' \cap D'$	XI	$A' \cap B \cap C \cap D'$
IV	$A \cap B' \cap C' \cap D$	XII	$A' \cap B \cap C \cap D$
V	$A \cap B \cap C' \cap D$	XIII	$A' \cap B' \cap C \cap D'$
VI	$A' \cap B \cap C' \cap D$	XIV	$A' \cap B' \cap C \cap D$
VII	$A \cap B' \cap C \cap D$	XV	$A' \cap B' \cap C' \cap D$
VIII	$A \cap B \cap C \cap D$	XVI	$A' \cap B' \cap C' \cap D'$

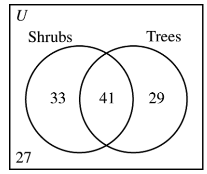
Exercise Set 2.5

1.



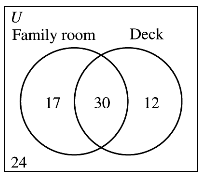
a) 48
b) 37
c) $200-(48+61+37)$, or 54

2.



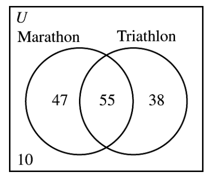
a) 33
b) 29
c) 27

3.



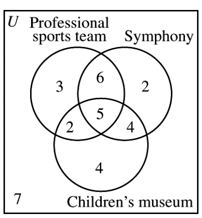
a) 17 b) 12
c) 59, the sum of the numbers in Regions I, II, III

4.



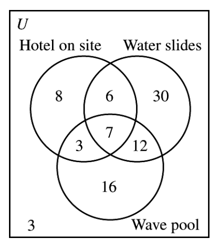
a) 47 b) 38
c) 140, the sum of the numbers in Regions I, II, III
d) $150-140$, or 10.

5.



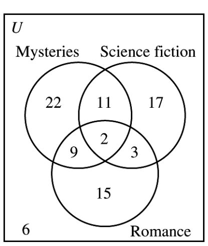
a) 3 b) 6
c) $3+2+6+5+2+4$, or 22
d) $3+6+2$, or 11
e) $2+6+4$, or 12

6.



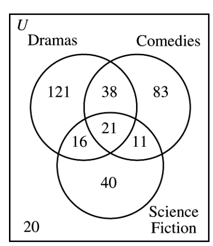
a) 30
b) $8+30+16$, or 54
c) $85-3$, or 82
d) $3+6+12$, or 21 e) 3

7.



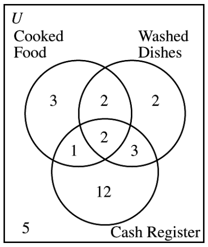
a) 22 b) 11
c) $85-15-6$, or 64
d) $22+11+17$, or 50
e) $9+11+3$, or 23

8.



a) 20 b) 121
c) $121+83+40$, or 244
d) $16+38+11$, or 65
e) $350-20-40$, or 290

9.

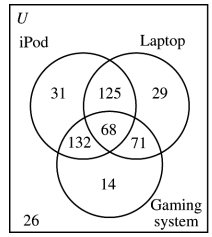


a) 3   b) 12

c) 3

d) $12+3+2$, or 17   e) 8

10.

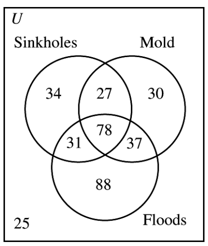


a) 496, the sum of the numbers in all the regions

b) 132   c) 29

d) $132+125+71$, , or 328   e) $496-26$, or 470

11.

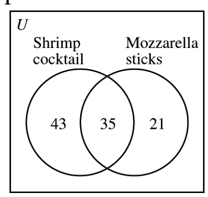


a) $30+37$, or 67

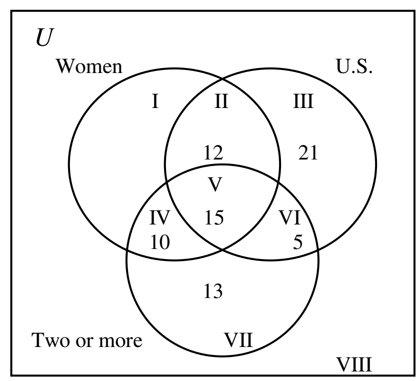
b) $350-25-88$, or 237

c) 37   d) 25

12. No. The sum of the numbers in the Venn diagram is 99. Dennis claims he surveyed 100 people.



13. The Venn diagram shows the number of cars driven by women is 37, the sum of the numbers in Regions II, IV, V. This exceeds the 35 women the agent claims to have surveyed.



14. First fill in 15, 20 and 35 on the Venn diagram. Referring to the labels in the Venn diagram and the given information, we see that

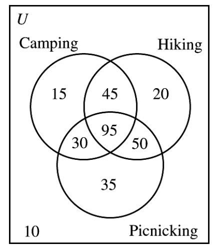
$a+c=140$

$b+c=125$

$a+b+c=185-15=170$

Adding the first two equations and subtracting the third from this sum gives $c=125+140-170=95$. Then $a=45$ and $b=30$. Then $d=210-45-95-20=50$. We now have labeled all the regions except the region outside the three circles, so the number of parks with at least one of the features is $15+45+20+30+95+50+35$, or 290. Thus the number with none of the features is $300-290$, or 10.

a) 290
b) 95
c) 10
d) $30+45+50$, or 125.



15. First fill in 15, 20 and 35 on the Venn diagram. Referring to the labels in the Venn diagram and the given information, we see that

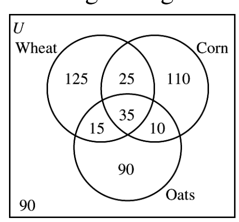
$a+c=60$

$b+c=50$

$a+b+c=200-125=75$

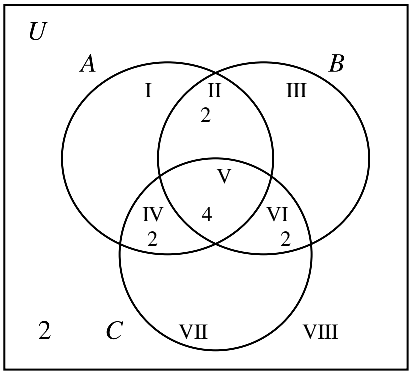
Adding the first two equations and subtracting the third from this sum gives $c=60+50-75=35$. Then $a=25$ and $b=15$. Then $d=180-110-25-35=10$. We now have labeled all the regions except the region outside the three circles, so the number of farmers growing at least one of the crops is $125+25+110+15+35+10+90$, or 410. Thus the number growing none of the crops is $500-410$, or 90.

a) 410
b) 35
c) 90
d) $15+25+10$, or 50



16. 16

17. From the given information we can generate the Venn diagram. First fill in 4 for Region V. Then since the intersections in pairs all have 6 elements, we can fill in 2 for each of Regions II, IV, and VI. This already accounts for the 10 elements $A \cup B \cup C$, so the remaining 2 elements in U must be in Region VIII.



a) 10, the sum of the numbers in Regions I, II, III, IV, V, VI
b) 10, the sum of the numbers in Regions III, IV, V, VI, VIII
c) 6, the sum of the numbers in Regions I, III, IV, VI, VII

Exercise Set 2.6

1. Infinite
2. Countable
3. $\{3, 4, 5, 6, 7, \ldots, n+2, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{4, 5, 6, 7, 8, \ldots, n+3, \ldots\}$
4. $\{30, 31, 32, 33, 34, \ldots, n+29, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{31, 32, 33, 34, 35, \ldots, n+30, \ldots\}$
5. $\{3, 5, 7, 9, 11, \ldots, 2n+1, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{5, 7, 9, 11, 13, \ldots, 2n+3, \ldots\}$
6. $\{20, 22, 24, 26, 28, \ldots, 2n+18, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{22, 24, 26, 28, 30, \ldots, 2n+20, \ldots\}$
7. $\{5, 9, 13, 17, 21 \ldots, 4n\ 1, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{9, 13, 17, 21, 25, \ldots, 4n+5, \ldots\}$
8. $\{6, 11, 16, 21, 26, \ldots, 5n+1, \ldots\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\{11, 16, 21, 26, 31, \ldots, 5n+6, \ldots\}$
9. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n}, \ldots\right\}$
   $\downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n+2}, \ldots\right\}$
10. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n}, \ldots\right\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n+1}, \ldots\right\}$
11. $\left\{\frac{4}{11}, \frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \ldots, \frac{n+3}{11}, \ldots\right\}$
   $\downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\left\{\frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \ldots, \frac{n+4}{11}, \ldots\right\}$
12. $\left\{\frac{6}{13}, \frac{7}{13}, \frac{8}{13}, \frac{9}{13}, \frac{10}{13}, \ldots, \frac{n+5}{13}, \ldots\right\}$
   $\downarrow \downarrow \downarrow \downarrow \downarrow \quad \downarrow$
   $\left\{\frac{7}{13}, \frac{8}{13}, \frac{9}{13}, \frac{10}{13}, \frac{11}{13}, \ldots, \frac{n+6}{13}, \ldots\right\}$

13. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{3, 6, 9, 12, 15, …, 3n, …}

14. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{40, 41, 42, 43, 44, …, n + 39, …}

15. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{4, 6, 8, 10, 12, …, 2n + 2, …}

16. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{0, 2, 4, 6, 8, …, 2n - 2, …}

17. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{2, 5, 8, 11, 14, …, 3n˜ 1, …}

18. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{7, 11, 15, 19, 23, …, 4n + 3, …}

19. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
$\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \ldots, \frac{1}{3n}, \ldots\right\}$

20. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots, \frac{1}{2n}, \ldots\right\}$

21. {1, 2, 3, 4, 7, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
$\left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \ldots, \frac{1}{n+2}, \ldots\right\}$

22. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots, \frac{n}{n+1}, \ldots\right\}$

23. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{1, 4, 9, 16, 25, …, n^2, …}

24. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{2, 4, 8, 16, 32, …, 2^n, …}

25. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
{3, 9, 27, 81, 243, …, 3^n, …}

26. {1, 2, 3, 4, 5, …, n, …}
↓ ↓ ↓ ↓ ↓ ↓
$\left\{\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \frac{1}{48}, \ldots, \frac{1}{3 \times 2^{n-1}}, \ldots\right\}$

27. =

28. =

29. =

30. =

31. =

32. a) Answers will vary.
b) No

Review Exercises

1. True
2. False; the word *best* makes the statement not well defined.
3. True
4. False; no set is a proper subset of itself.
5. False; the elements 6, 12, 18, 24, … are members of both sets.
6. True
7. False; the two sets do not contain exactly the same elements.
8. True
9. True
10. True

11. True

12. True

13. True

14. True

15. $A = \{ 7, 9, 11, 13, 15 \}$

16. $B = \{$ Colorado, Nebraska, Missouri, Oklahoma $\}$

17. $C = \{ 1, 2, 3, 4, \ldots, 161 \}$

18. $D = \{ 9, 10, 11, 12, \ldots, 80 \}$

19. $A = \{ x | x \in N \text{ and } 50 < x < 150 \}$

20. $B = \{ x | x \in N \text{ and } x > 42 \}$

21. $C = \{ x | x \in N \text{ and } x < 7 \}$

22. $D = \{ x | x \in N \text{ and } 27 \le x \le 51 \}$

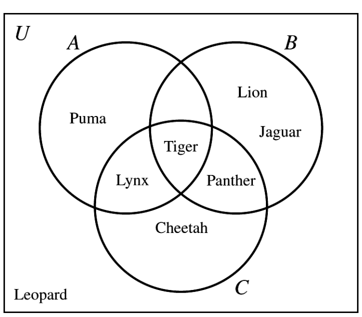
23. A is the set of capital letters in the English alphabet from E through M, inclusive.
24. B is the set of U.S. coins with a value of less than one dollar.
25. C is the set of the first three lowercase letters in the English alphabet.
26. D is the set of numbers greater than or equal to 3 and less than 9.
27. $A \cap B = \{ 1, 3, 5, 7 \} \cap \{ 3, 7, 9, 10 \} = \{ 3, 7 \}$
28. $A \cup B' = \{ 1, 3, 5, 7 \} \cup \{ 3, 7, 9, 10 \}' = \{ 1, 3, 5, 7 \} \cup \{ 1, 2, 4, 5, 6, 8 \} = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$
29. $A' \cap B = \{ 1, 3, 5, 7 \}' \cap \{ 3, 7, 9, 10 \} = \{ 2, 4, 6, 8, 9, 10 \} \cap \{ 5, 7, 9, 10 \} = \{ 9, 10 \}$
30. $(A \cup B)' \cup C = (\{ 1, 3, 5, 7 \} \cup \{ 3, 7, 9, 10 \})' \cup \{ 1, 7, 10 \} = \{ 1, 3, 5, 7, 9, 10 \}' \cup \{ 1, 7, 10 \}$
    $= \{ 2, 4, 6, 8 \} \cup \{ 1, 7, 10 \} = \{ 1, 2, 4, 6, 7, 8, 10 \}$
31. $A - B = \{ 1, 3, 5, 7 \} - \{ 3, 7, 9, 10 \} = \{ 1, 5 \}$
32. $A - C' = \{ 1, 3, 5, 7 \} - \{ 1, 7, 10 \}' = \{ 1, 3, 5, 7 \} - \{ 2, 3, 4, 5, 6, 8, 9 \} = \{ 1, 7 \}$

33. $\{ (1,1), (1,7), (1,10), (3,1), (3,7), (3,10), (5,1), (5,7), (5,10), (7,1), (7,7), (7,10) \}$
34. $\{ (3,1), (3,3), (3,5), (3,7), (7,1), (7,3), (7,5), (7,7), (9,1), (9,3), (9,5), (9,7), (10,1), (10,3), (10,5), (10,7) \}$

35. $2^4 = 2 \times 2 \times 2 \times 2 = 16$

36. $2^4 - 1 = (2 \times 2 \times 2 \times 2) - 1 = 16 - 1 = 15$

37.



38. $A \cup B = \{ a, c, d, f, g, i, k, l \}$

39. $A \cap B' = \{ i, k \}$

40. $A \cup B \cup C = \{ a, b, c, d, f, g, h, i, k, l \}$

41. $A \cap B \cap C = \{ f \}$

42. $(A \cup B) \cap C = \{ a, f, i \}$

43. $(A\cap B)\cup C=\{\text{a, b, d, f, h, i, l}\}$

44. $(A'\cup B')'$ $\qquad$ $A\cap B$

Set	Regions
A	I, II
A'	III, IV
B	II, III
B'	I, IV
$A'\cup B'$	I, III, IV
$(A'\cup B')'$	II

Set	Regions
A	I, II
B	II, III
$A\cap B$	II

Both statements are represented by the same region, II, of the Venn diagram. Therefore, $(A'\cup B')'=A\cap B$ for all sets A and B.

45. $(A\cup B')\cup(A\cup C')$ $\qquad$ $A\cup(B\cap C)'$

Set	Regions
A	I, II, IV, V
B	II, III, V, VI
B'	I, IV, VII, VIII
$A\cup B'$	I, II, IV, V, VII, VIII
C	IV, V, VI, VII
C'	I, II, III, VIII
$A\cup C'$	I, II, III, IV, V, VIII
$(A\cup B')\cup(A\cup C')$	I, II, III, IV, V, VII, VIII

Set	Regions
B	II, III, V, VI
C	IV, V, VI, VII
$B\cap C$	V, VI
$(B\cap C)'$	I, II, III, IV, VII, VIII
A	I, II, IV, V
$A\cup(B\cap C)'$	I, II, III, IV, V, VII, VIII

Both statements are represented by the same regions, I, II, III, IV, V, VII, VIII, of the Venn diagram.

Therefore, $(A\cup B')\cup(A\cup C')=A\cup(B\cap C)'$ for all sets $A, B,$ and C.

46. II

47. III

48. I

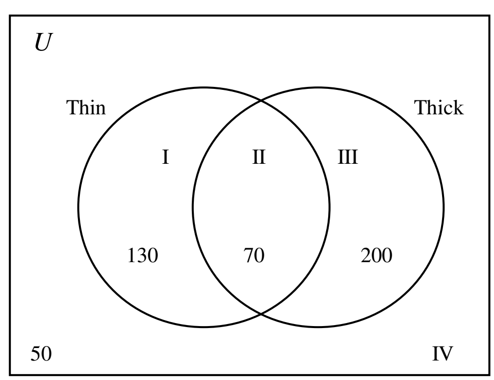
49. IV

50. IV

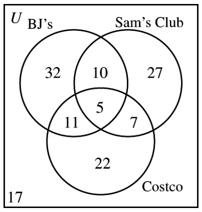
51. II

52. II

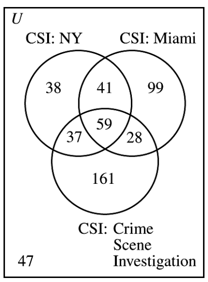
53. The company paid \$450 since the sum of the numbers in Regions I through IV is 450.



54. a) 131, the sum of the numbers in Regions I through VIII
b) 32, Region I
c) 10, Region II
d) 65, the sum of the numbers in Regions I, IV, VII



55. a) 38, Region I
b) 298, the sum of the numbers in Regions I, III, VII
c) 28, Region VI
d) 236, the sum of the numbers in Regions I, IV, VII
e) 106, the sum of the numbers in Regions II, IV, VI



56. $\{2, 4, 6, 8, 10, \ldots, 2n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{4, 6, 8, 10, 12, \ldots, 2n+2, \ldots\}$

57. $\{3, 5, 7, 9, 11, \ldots, 2n+1, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{5, 7, 9, 11, 13, \ldots, 2n+3, \ldots\}$

58. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{5, 8, 11, 14, 17, \ldots, 3n+2, \ldots\}$

59. $\{1, 2, 3, 4, 5, \ldots, n, \ldots\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\qquad\downarrow$
$\{4, 9, 14, 19, 24, \ldots, 5n - 1, \ldots\}$

Chapter Test

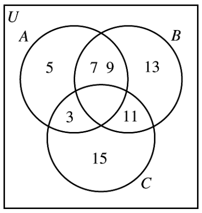
1. True
2. False; the sets do not contain exactly the same elements.
3. True
4. False; the second set has no subset that contains the element 7.
5. False; the set has $2^4 = 2\times2\times2\times2 = 16$ subsets.
6. True
7. False; for any set A, $A \cup A' = U$, not $\{\ \}$.
8. True
9. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
10. Set A is the set of natural numbers less than 10.
11. $A \cap B = \{3, 5, 7, 9\} \cap \{7, 9, 11, 13\} = \{7, 9\}$
12. $A \cup C' = \{3, 5, 7, 9\} \cup \{3, 11, 15\}' = \{3, 5, 7, 9\} \cup \{5, 7, 9, 13\} = \{3, 5, 7, 9, 13\}$
13. $A \cap \left(B \cap C'\right) = \{3, 5, 7, 9\} \cap \left(\{7, 9, 11, 13\} \cap \{5, 7, 9, 13\}\right) = \{3, 5, 7, 9\} \cap \{7, 9\} = \{7, 9\}$

14. $n(A\cap B')=n\left(\{3,5,7,9\}\cap\{7,9,11,13\}'\right)=n(\{3,5,7,9\}\cap\{3,5,15\})=n(\{3,5\})=2$

15. $A-B=\{3,5,7,9\}-\{7,9,11,13\}=\{3,5\}$

16. $A\times C=\{(3,3),(3,11),(3,15),(5,3),(5,11),(5,15),(7,3),(7,11),(7,15),(9,3),(9,11),(9,15)\}$

17.

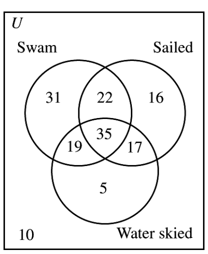


18. $A\cap(B\cup C')$ $\qquad$ $(A\cap B)\cup(A\cap C')$

Set	Regions	Set	Regions
B	II, III, V, VI	A	I, II, IV, V
C	IV, V, VI, VII	B	II, III, V, VI
C'	I, II, III, VIII	$A\cap B$	II, V
$B\cup C'$	I, II, III, V, VI, VIII	C	IV, V, VI, VII
A	I, II, IV, V	C'	I, II, III, VIII
$A\cap(B\cup C')$	I, II, V	$A\cap C'$	I, II
		$(A\cap B)\cup(A\cap C')$	I, II, V

Both statements are represented by the same regions, I, II, V, of the Venn diagram.
Therefore, $A\cap(B\cup C')=(A\cap B)\cup(A\cap C')$ for all sets $A, B,$ and C.

19.

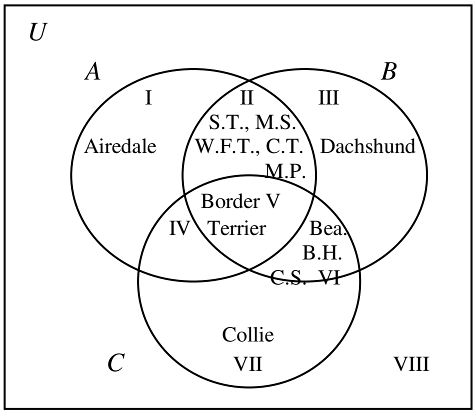


a) 52, the sum of the numbers in Regions I, III, VII
b) 10, Region VIII
c) 93, the sum of the numbers in Regions II, IV, V, VI
d) 22, Region II
e) 69, the sum of the numbers in Regions I, II, III
f) 5, Region VII

20. {7, 8, 9, 10, 11, …, $n+6$, …}
↓ ↓ ↓ ↓ ↓ ↓
{8, 9, 10, 11, 12, …, $n+7$, …}

Group Projects

1. a) A : Does not shed, B : Less than 16 in. tall, C : Good with kids



b) Border terrier, Region V

2. a) Animal b) Chordate c) Mammalia d) Carnivore
e) Felidae f) Felis g) Catus

3.

	First	Second	Third	Fourth	Fifth
a) Color	yellow	blue	red	ivory	green
b) Nationality	Norwegian	Afghan.	Senegalese	Spanish	Japanese
c) Food	apple	cheese	banana	peach	fish
d) Drink	vodka	tea	milk	whiskey	ale
e) Pet	fox	horse	snail	dog	zebra

f) Ale