CHAPTER 2
PROPERTIES OF REINFORCED CONCRETE

2.1- 2.8 Refer to the relative section in text

2.3 Estimate the modulus of elasticity and shear modulus of concrete
Dry density= 150 pcf
Compressive strength= 4500 psi
Poisson's ratio, μ= 0.18

Modulus of elasticity

$E_c = 33w^{1.5}\sqrt{f_c'}$ psi

$E_c = 33(150^{1.5})\sqrt{(4500)}$

$\mathbf{E_c = 4.07 \times 10^6}$ **psi**

Shear Modulus

$G_c = E_c/(2(1+\mu)$

$G_c = (4.07\times10^6)/(2(1+0.18)$

$\mathbf{G_c = 1.72\times10^6}$ **psi**

2.9 Calculate the modulus of elasticity; E_c (see the table below)

$E_c = 33w^{1.5}\sqrt{f_c'}$ psi $\qquad\qquad E_c = 0.043w^{1.5}\sqrt{f_c'}$ MPa

Density	f_c'	E_c
160 pcf	5000 psi	4,723,000 psi
145 pcf	4000 psi	3,644,000 psi
125 pcf	2500 psi	2,306,000 psi
2400 kg/m^3	35 MPa	29,910 MPa
2300 kg/m^3	30 MPa	25,980 MPa
2100 kg/m^3	25 MPa	20,690 MPa

2.10 Determine the modular ratio; n and modulus of rupture; f_r: for each case in Example 2.9

$$f_r = 7.5\lambda\sqrt{f_c'}\ \text{psi} \qquad\qquad f_r = 0.62\lambda\sqrt{f_c'}\ \text{N/mm}^2$$

Where: λ is a modification factor for type of concrete (ACI 19.2.4)
= 1.0 Normal-weight concrete
= 0.85 Sand-lightweight concrete
= 0.75 for all-lightweight concrete

$$n = \frac{29000\,(\text{ksi})}{E_c\ (\text{ksi})} \qquad\qquad n = \frac{2{,}000{,}000\ (\text{MPa})}{E_c\ (\text{MPa})}$$

Density	f_c'	E_c	n	f_r
160 pcf	5000 psi	4,723,000 psi	6.14	530.3 psi
145 pcf	4000 psi	3,644,000 psi	7.96	474.3 psi
125 pcf	2500 psi	2,306,000 psi	12.58	375.0 psi
2400 kg/m^3	35 MPa	29,910 MPa	6.69	3.668 MPa
2300 kg/m^3	30 MPa	25,980 MPa	7.70	3.396 MPa
2100 kg/m^3	25 MPa	20,690 MPa	9.67	3.10 MPa

2.11 a.) Draw the stress-strain diagram.
b.) Determine the secant modulus and initial modulus.
c.) Calculate E_c using ACI formula and compare results.
Area of 6 in. diameter cylinder = 28.274 in.^{2}. Stress = load / area

Solution:

Maximum f_c'= 3820 psi at a strain = 0.003.

a.) See figure 2.1

b.) Secant modulus (at $f_c'/2$ = 1910 psi)
E_c = 1910 / 6.10×10^{-4} = 3130 ksi
Approximate Initial Modulus = 2.6(ksi) / 5.45×10^{-4} = 4771 ksi
(Possible range 4600 – 5200)

c.) E_c (ACI formula) = $57000\sqrt{f_c'} = 57000\sqrt{3820} = 3523$ ksi.

Approximately percentage change from test = (3523-3130) / 3523 = 11.15% (from secant modulus).

Load (kip)	Stress (psi)	Strain×10^-4^
0	0.00	0
12	424.4	1.2
24	848.8	2.0
36	1273.3	3.2
48	1697.7	5.2
60	2122.1	7.2
72	2546.5	10.0
84	2970.9	13.6
96	3395.4	18.0
108	3819.8	30.0
95	3360.0	39.0
82	2900.2	42.0

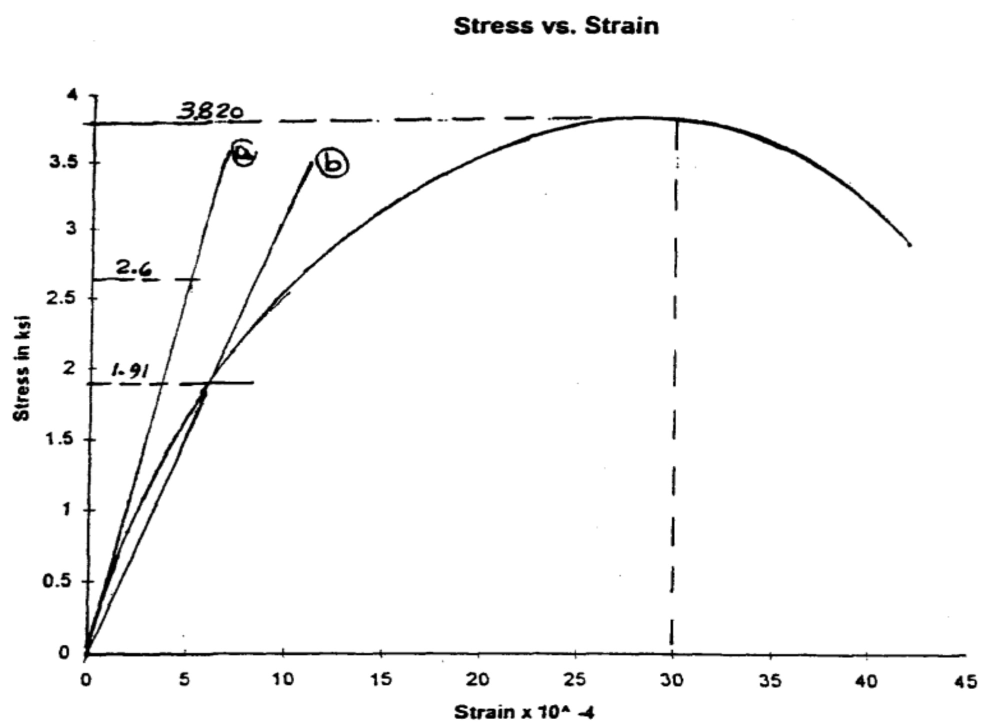


Figure 2.1

2.12 Calculate shrinkage strain, creep compliance, and creep coefficient for a 6x12 in. steam-cured concrete cylinder with cement type III and the following properties. Use ACI 209R-92

Given:

H	90	%
h_e =		
2V/S	6	in.
f_{cm28}	4021	psi
w	345	lb/yd^3
w/c	0.4	
a/c	3.25	
t	400	days
t_0	28	days
t_c	1	days
ɣ	146	lb/ft^3

Solution:

Shrinkage Calculation

$$\varepsilon_{sh}(t, t_c) = \frac{t - t_c}{f + (t - t_c)} K_{ss} K_{sh} \varepsilon_{shu}$$

$$\varepsilon_{shu} = 780 \times 10^{-6} \frac{in.}{in.}$$

According to Table 2.5, f = 55

$$\frac{V}{S} = \frac{6}{2} = 3\ in.$$

$$K_{ss} = 1.17 - 0.116\left(\frac{V}{S}\right) = 1.23 - 0.116\ (3) = 0.822$$

For H = 90%,

$$K_{sh} = 3.00 - 0.03H = 3.00 - 0.03\ (90) = 0.30$$

$$\varepsilon_{sh}(t, t_c) = \frac{t - t_c}{f + (t - t_c)} K_{ss} K_{sh} \varepsilon_{shu}$$

$$\varepsilon_{sh}(t, t_c) = \frac{400 - 1}{55 + (400 - 1)} (0.882)(0.30)(780 \times 10^{-6}) = \mathbf{169 \times 10^{-6}} \frac{\boldsymbol{in.}}{\boldsymbol{in.}}$$

Creep Calculation

$$J(t,t_0) = \frac{1+\ C_c(t)}{E_{cmt_0}}$$

Determination of E_{cmt_0}:

a = 0.70 b = 0.98 (Table 2.5)

$$f_c'(t_0) = f_{cm_{28}}\ \frac{t_0}{a+bt_0} = 4021\ \ \frac{28}{0.70+0.98(28)} = 4001\, psi$$

$$E_{cmt_0} = 33\ (\gamma)^{3/2}\sqrt{f_c'(t_0)} = 33\ (146)^{3/2}\sqrt{4001} = 3{,}682{,}368\, psi$$

Determination of C_c (t):

$$C_{cu} = 2.35$$

$$K_{ch} = 1.27 - 0.0067H = 1.27 - 0.006\ (90) = 0.667$$

$$K_{ca} = 1.13(t_0)^{-0.095} = 1.13(28)^{-0.095} = 0.823$$

$$K_{cs} = 1.10 - 0.068\left(\frac{V}{S}\right) = 1.10 - 0.068\ (3) = 0.896$$

$$C_c(t) = \frac{(t-\ t_0)^{0.60}}{10+(t-\ t_0)^{0.60}}\ C_{cu}K_{ch}K_{ca}K_{cs}$$

$$= \frac{(400-\ 28)^{0.60}}{10+(400-\ 28)^{0.60}}\ (2.35)(0.667)(0.823)(0.896) = \mathbf{0.899}$$

$$J(t,t_0) = \frac{1+\ C_c(t)}{E_{cmt_0}} = \frac{1+\ 0.899}{3{,}682{,}368} = \mathbf{0.516\times 10^{-6}}\ \boldsymbol{psi^{-1}}$$

2.13 Calculate shrinkage strain, creep compliance, and creep coefficient for problem 2.12 using the GL 2000 Model.

Solution:

Shrinkage Calculation

Calculation of ε_{shu}:
$K = 1.15\ (Table\ 2.12)$

$$\varepsilon_{shu} = (900)\ K\ (\frac{4350}{f_{cm28}})^{\frac{1}{2}} \times 10^{-6}$$

$$= (900)\ (1.15)\ (\frac{4350}{4021})^{\frac{1}{2}} \times 10^{-6}$$

$$= 1076 \times 10^{-6}\ \frac{in.}{in.}$$

Calculation of $\beta(h)$:

$$\beta(h) = 1 - 1.18\ \left(\frac{H}{100}\right)^4$$

$$= 1 - 1.18\ \left(\frac{90}{100}\right)^4$$

$$= 0.226$$

Calculation of $\beta(t - t_c)$:

$$\beta(t - t_c) = \left(\frac{t - t_c}{t - t_c + 77\ \left(\frac{V}{S}\right)^2}\right)^{1/2}$$

$$= \left(\frac{400 - 1}{400 - 1 + 77\ \left(\frac{6}{2}\right)^2}\right)^{\frac{1}{2}}$$

$$= 0.604$$

$$\varepsilon_s(t) = \varepsilon_{shu}\ \beta(h)\ \beta(t - t_c)$$

$$= (1076 \times 10^{-6})(0.226)(0.604)$$

$$= \mathbf{147 \times 10^{-6}\ \frac{in.}{in.}}$$

Creep Calculation

$$J(t, t_0) = \frac{1}{E_{cmt_0}} + \frac{\emptyset_{28}\,(t, t_0)}{E_{cm28}}$$

Calculation of E_{cmt_0} *and* E_{cm28}:

$$t_0 = 28\ days\ \ => \ E_{cmt_0} = E_{cm28}$$

$$E_{cm28} = 500{,}000 + 52{,}000\sqrt{f_{cm_{28}}}$$

$$= 500{,}000 + 52{,}000\sqrt{4021}$$

$$= 3{,}797{,}390\ psi = E_{cmt_0}$$

Calculation of $\emptyset_{28}(t, t_0)$:

$$t_0 = 28 > \ t_c = 1, then\ \emptyset(t_c) = \left(1 - \left(\frac{t_0 - t_c}{t_0 - t_c + 77\left(\frac{V}{S}\right)^2}\right)^{0.5}\right)^{0.5}$$

$$= \left(1 - \left(\frac{28 - 1}{28 - 1 + 77\left(\frac{6}{2}\right)^2}\right)^{0.5}\right)^{0.5}$$

$$= 0.898$$

$$h = \frac{H}{100} = \frac{90}{100} = 0.90$$

$$\emptyset_{28}(t, t_0) = \ \emptyset(t_c)\left[2\left(\frac{(t - t_0)^{0.3}}{(t - t_0)^{0.3}}\right) + \left(\frac{7}{t_0}\right)^{0.5}\left(\frac{t - t_0}{t - t_0 + 7}\right)^{0.5} + 2.5\left(1 - 1.086\,(h^2)\right)\left(\frac{t - t_0}{t - t_0 + 77\left(\frac{V}{S}\right)^2}\right)^{0.5}\right]$$

$$= \ 0.898\left[2\left(\frac{(400 - 28)^{0.3}}{(400 - 28)^{0.3}}\right) + \left(\frac{7}{28}\right)^{0.5}\left(\frac{400 - 28}{400 - 28 + 7}\right)^{0.5} + 2.5\left(1 - 1.086\,(0.90^2)\right)\left(\frac{400 - 28}{400 - 28 + 77\left(\frac{6}{2}\right)^2}\right)^{0.5}\right]$$

$$= \mathbf{1.137}$$

$$J(t, t_0) = \ \frac{1}{3{,}797{,}390} + \frac{1.137}{3{,}797{,}390}$$

$$= \mathbf{0.563 \times 10^{-6}}\ \boldsymbol{psi^{-1}}$$

2.14 Calculate shrinkage strain, creep compliance, and creep coefficient for problem 2.12 using the fib MC 2010 Model.

fib MC 2010:

<u>*Shrinkage Calculation*</u>

$$\varepsilon_s(t,t_c) = \varepsilon_{as}(t) + \varepsilon_{ds}(t,t_c)$$

$$\varepsilon_{as}(t) = \varepsilon_{as0}(f_{cm28})\,\beta_{as}(t)$$

$$\varepsilon_{as0}(f_{cm28}) = -\alpha_{as}\left(\frac{\frac{f_{cm28}}{1450}}{6+\frac{f_{cm28}}{1450}}\right)^{2.5}\times10^{-6}$$

$\alpha_{as} = 600$, Type III cement

$$\varepsilon_{as0}(f_{cm28}) = -600\left(\frac{\frac{4021}{1450}}{6+\frac{4021}{1450}}\right)^{2.5}\times10^{-6} = -33.7\times10^{-6}$$

$$\beta_{as}(t) = 1 - exp[-0.2(t)^{0.5}]$$

$$\beta_{as}(t) = 1 - exp[-0.2(400)^{0.5}] = 0.982$$

$$\varepsilon_{as}(t) = (-33.7\times10^{-6})(\,0.982) = -33.1\times10^{-6}$$

$$\varepsilon_{ds}(t,t_c) = \varepsilon_{ds0}(f_{cm28})\,\beta_{RH}(H)\,\beta_{ds}(t-t_c)$$

$$\varepsilon_{ds0}(f_{cm28}) = \left[(220+110\alpha_{ds1})\,exp\left(-\frac{\alpha_{ds2}f_{cm28}}{1450}\right)\right]\times10^{-6}$$

$$\varepsilon_{ds0}(f_{cm28}) = \left[(220+110(6))\,exp\left(-\frac{(0.12)(4021)}{1450}\right)\right]\times10^{-6} = 630.9\times10^{-6}$$

$$\beta_{RH}(H) = -1.55\left[1-\left(\frac{H}{100}\right)^3\right]\ for\ 40\% \le H < 99\%\times\beta_{s1}$$

$$\beta_{s1} = \left(\frac{3.5\times1450}{f_{cm28}}\right)^{0.1} \le 1.0$$

$$\beta_{s1} = \left(\frac{3.5\times1450}{4021}\right)^{0.1} = 1.02, use\ \beta_{s1} = 1.0$$

$$\beta_{RH}(H) = -1.55\left[1 - \left(\frac{90}{100}\right)^3\right] = -0.420$$

$$\beta_{ds}(t - t_c) = \left(\frac{t - t_c}{350\left(\frac{h_e}{4}\right)^2 + (t - t_c)}\right)^{0.5}$$

$$\beta_{ds}(t - t_c) = \left(\frac{400 - 1}{350\left(\frac{6}{4}\right)^2 + (400 - 1)}\right)^{0.5} = 0.580$$

$$\varepsilon_{ds}(t, t_c) = (630.9\times10^{-6})(-0.420)(0.580) = -153.7\times10^{-6}$$

$$\varepsilon_s(t, t_c) = \varepsilon_{as}(t) + \varepsilon_{ds}(t, t_c)$$

$$\varepsilon_s(t, t_c) = (-33.1\times10^{-6}) + (-153.7\times10^{-6}) = \mathbf{-187\times10^{-6}}\ \frac{\boldsymbol{in.}}{\boldsymbol{in.}}$$

Creep Calculation

$$J(t, t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\emptyset\,(t, t_0)}{E_{ci}}$$

$$E_{ci} = 3{,}118{,}310\times\left(\frac{f_{cm28}}{1450}\right)^{\frac{1}{3}}$$

$$E_{ci} = 3{,}118{,}310\times\left(\frac{4021}{1450}\right)^{\frac{1}{3}} = 4{,}381{,}014\ psi$$

$$E_{ci}\,(t_0) = E_{ci}\exp\left\{0.5S\left[1 - \left(\frac{28}{t_0}\right)\right]\right\}$$

$S = 0.20$, Type III cement

$$E_{ci}\,(t_0) = 4{,}381{,}014\exp\left\{0.5(0.20)\left[1 - \left(\frac{28}{28}\right)\right]\right\}$$

$$E_{ci}(t_0) = 4{,}381{,}014\ psi$$

$$\varphi(t, t_0) = \varphi_{bc}(t, t_0) + \varphi_{dc}(t, t_0)$$

$$\varphi_{bc}(t, t_0) = \beta_{bc}(f_{cm})\times\beta_{bc}(t, t_0)$$

$$\beta_{bc}(f_{cm}) = \frac{58.6}{(f_{cm})^{0.7}} = \frac{58.6}{(4021)^{0.7}} = 0.176$$

$$t_{0,adj} = t_{0,T}\left[\frac{9}{2 + t_{0,T}^{1.2}} + 1\right]^{\alpha} \geq 0.5\ days, \alpha = 1$$

$$t_{0,adj} = 28\left[\frac{9}{2 + 28^{1.2}} + 1\right]^{1} = 32.5 \geq 0.5\ days$$

$$\therefore\ t_{0,adj} = 32.5\ days$$

$$\beta_{bc}(t, t_0) = ln\left[\left(\frac{30}{t_{0,adj}} + 0.035\right)^2 \times (t - t_0) + 1\right]$$

$$\beta_{bc}(t, t_0) = ln\left[\left(\frac{30}{32.5} + 0.035\right)^2 \times (400 - 28) + 1\right] = 5.839$$

$$\varphi_{bc}(t, t_0) = (0.176)(5.839) = 1.026$$

$$\varphi_{dc} = \beta_{dc}(f_{cm}) \times \beta(RH) \times \beta_{dc}(t_0) \times \beta_{dc}(t, t_0)$$

$$\beta_{dc}(f_{cm}) = \frac{437{,}333}{(f_{cm})^{1.4}} = \frac{437{,}333}{(4021)^{1.4}} = 3.933$$

$$\beta(RH) = \frac{1 - \frac{RH}{100}}{\sqrt[3]{0.1 \times \frac{h}{4}}} = \frac{1 - \frac{90}{100}}{\sqrt[3]{0.1 \times \frac{6}{4}}} = 0.188$$

$$\beta_{dc}(t_0) = \frac{1}{0.1 + (t_{0,adj})^{0.2}} = \frac{1}{0.1 + (32.5)^{0.2}} = 0.475$$

$$\beta_{dc}(t, t_0) = \left[\frac{(t - t_0)}{\beta_h + (t - t_0)}\right]^{\gamma(t_0)}$$

$$\alpha_{fcm} = \left[\frac{5075}{f_{cm}}\right]^{0.5} = \left[\frac{5075}{4021}\right]^{0.5} = 1.123$$

$\beta_h = 38.1 \times h + 250\ \alpha_{fcm} \leq 1500\ \alpha_{fcm}$
$= 38.1 \times 6 + 250\ (1.123) \leq 1500\ (1.123)$
$= 509.4 \leq 1684.5$

$$\gamma(t_0) = \frac{1}{2.3 + \frac{3.5}{\sqrt{t_{0,adj}}}} = \frac{1}{2.3 + \frac{3.5}{\sqrt{32.5}}} = 0.343$$

$$\beta_{dc}(t, t_0) = \left[\frac{(400 - 28)}{509.4 + (400 - 28)}\right]^{0.343} = 0.744$$

$$\varphi_{dc}(t, t_0) = 3.933 \times 0.188 \times 0.475 \times 0.744 = 0.261$$

$$\varphi(t, t_0) = 1.028 + 0.261 = \mathbf{1.288}$$

$$J(t, t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\varphi(t, t_0)}{E_{ci}}$$

$$= \frac{1}{4{,}381{,}014} + \frac{1.288}{4{,}381{,}014} = \mathbf{0.522 \times 10^{-6}}\ \boldsymbol{psi^{-1}}$$

2.15 A concrete specimen has the following properties: Humidity = 50%; $h_e = 2V/S = 35$ mm; $f_{cm28} = 33.9$ MPa; cement content (c) = 350 kg/m^3; w/c = 0.49; a/c = 4.814; $t_0 = 7$ days; $\gamma = 2296.74$ kg/m^3; the specimen is Type I cement; and it was moist-cured. Use the ACI 209R-92 model and the fib MC 2010 model to answer the following:

a. Predict the creep compliance of the concrete specimen for ages: 14; 90; 365; 2,190; and 3,650 days.

b. Create a graph showing the predictions versus loading duration.

c. Comment on the trend of each model and how the models compare with each other.

Solution

Part a

ACI 209R-92 model:

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}}$$

$$a = 4 \qquad b = 0.85 \quad \text{(Table2.5)}$$

$$f_c'(t_0) = f_{\text{cm28}} \frac{t_0}{a+bt_0} = 33.9 \frac{7}{4+0.85\times 7} = 23.8 \text{ MPa}$$

$$E_{\text{cmt}_0} = 0.043(\gamma)^{3/2}\sqrt{f_c'(t_0)} = 0.043(2296.74\ )^{3/2}\sqrt{23.8} = 23{,}113.9 \text{ MPa}$$

$$C_{\text{cu}} = 2.35$$

$$K_{\text{ca}} = 1.25(t_0)^{-0.118} = 1.25(7)^{-0.118} = 0.994$$

$$K_{\text{ch}} = 1.27 - 0.0067(H) = 1.27 - 0.0067(50) = 0.935$$

$$K_{\text{cs}} = 1.14 - 0.00363\left(\frac{V}{S}\right) = 1.14 - 0.00363(17.5) = 1.076$$

For t = 14 days

$$C_c(t) = \frac{(t-t_0)^{0.60}}{10+(t-t_0)^{0.60}} C_{\text{cu}} K_{\text{ch}} K_{\text{ca}} K_{\text{cs}} = \frac{(14-7)^{0.60}}{10+(14-7)^{0.60}} 2.35\times 0.935\times 0.994\times 1.076 = 0.572$$

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}} = \frac{1+0.572}{23{,}113.9} = 68.0\times 10^{-6}\,\text{MPa}^{-1}$$

For t = 90 days

$$C_c(t) = \frac{(t-t_0)^{0.60}}{10+(t-t_0)^{0.60}} C_{\text{cu}} K_{\text{ch}} K_{\text{ca}} K_{\text{cs}} = \frac{(90-7)^{0.60}}{10+(90-7)^{0.60}} 2.35\times 0.935\times 0.994\times 1.076 = 1.378$$

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}} = \frac{1+1.378}{23{,}113.9} = 102.9\times 10^{-6}\,\text{MPa}^{-1}$$

For t = 365 days

$$K_{\text{cs}} = 1.10 - 0.00268\left(\frac{V}{S}\right) = 1.10 - 0.00268(17.5) = 1.053$$

$$C_c(t) = \frac{(t-t_0)^{0.60}}{10+(t-t_0)^{0.60}} C_{\text{cu}} K_{\text{ch}} K_{\text{ca}} K_{\text{cs}} = \frac{(365-7)^{0.60}}{10+(365-7)^{0.60}} 2.35\times 0.935\times 0.994\times 1.053 = 1.778$$

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}} = \frac{1+1.778}{23{,}113.9} = 120.2\times 10^{-6}\,\text{MPa}^{-1}$$

For t = 2,190 days

$$C_c(t) = \frac{(t-t_0)^{0.60}}{10+(t-t_0)^{0.60}} C_{\text{cu}} K_{\text{ch}} K_{\text{ca}} K_{\text{cs}} = \frac{(2{,}190-7)^{0.60}}{10+(2{,}190-7)^{0.60}} 2.35\times 0.935\times 0.994\times 1.053 = 2.092$$

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}} = \frac{1+2.092}{23{,}113.9} = 133.8\times 10^{-6}\,\text{MPa}^{-1}$$

For t = 3,650 days

$$C_c(t) = \frac{(t-t_0)^{0.60}}{10+(t-t_0)^{0.60}} C_{\text{cu}} K_{\text{ch}} K_{\text{ca}} K_{\text{cs}} = \frac{(3{,}650-7)^{0.60}}{10+(3{,}650-7)^{0.60}} 2.35\times 0.935\times 0.994\times 1.053 = 2.143$$

$$J(t,t_0) = \frac{1+C_c(t)}{E_{\text{cmt}_0}} = \frac{1+2.143}{23{,}113.9} = 136.0\times 10^{-6}\,\text{MPa}^{-1}$$

fib MC 2010:

$$J(t,t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\phi(t,t_0)}{E_{\text{ci}}}$$

$$E_{\text{ci}} = 21{,}500\sqrt[3]{\frac{f_{\text{cm}_{28}}}{10}} = 21{,}500\sqrt[3]{\frac{33.9}{10}} = 32{,}297.7\text{ MPa}$$

$$S = 0.25$$

$$E_{ci}(t_0) = E_{\text{ci}}\exp\left[0.5S\left(1-\sqrt{\left(\frac{28}{t_0}\right)}\right)\right]$$

$$= (32{,}297.7)\exp\left[0.5\cdot 0.25\left(1-\sqrt{\left(\frac{28}{7}\right)}\right)\right] = 28{,}502.6\text{ MPa}$$

$$\beta_{bc}(f_{cm}) = \frac{1.8}{(f_{cm})^{0.7}} = \frac{1.8}{(33.9)^{0.7}} = 0.153$$

$\alpha = 0$ for type I cement

$$t_{0,adj} = t_{0,T}\left[\frac{9}{2+t_{0,T}^{\ 1.2}}+1\right]^{\alpha} \geq 0.5 \text{ days}$$

$$= 7\cdot\left[\frac{9}{2+7^{1.2}}+1\right]^{0} = 7 \text{ days} \geq 0.5 \text{ days}$$

$$\beta_{dc}(f_{cm}) = \frac{412}{\left(f_{cm}\right)^{1.4}} = \frac{412}{\left(33.9\right)^{1.4}} = 2.969$$

$$\beta(RH) = \frac{\left(1-\frac{RH}{100}\right)}{\sqrt[3]{0.1\cdot\frac{h}{100}}} = \frac{\left(1-\frac{50}{100}\right)}{\sqrt[3]{0.1\cdot\frac{35}{100}}} = 1.53$$

$$\beta_{dc}(t_0) = \frac{1}{0.1+t_{0,adj}^{\ 0.2}} = \frac{1}{0.1+7^{0.2}} = 0.635$$

$$\gamma(t_0) = \frac{1}{2.3+\frac{3.5}{\sqrt{t_{0,adj}}}} = \frac{1}{2.3+\frac{3.5}{\sqrt{7}}} = 0.276$$

$$\alpha_{fcm} = \left(\frac{35}{f_{cm}}\right)^{0.5} = \left(\frac{35}{33.9}\right)^{0.5} = 1.016$$

$$\beta_h = 1.5\cdot h + 250\cdot\alpha_{fcm} \leq 1500\cdot\alpha_{fcm}$$

$$= 1.5\cdot(35)+250\cdot(1.016) \leq 1500\cdot(1.016)$$

$$\Rightarrow 306.5 \leq 1524$$

For t = 14 days

$$\beta_{bc}(t,t_0) = \ln\left[\left(\frac{30}{t_{0,adj}}+0.035\right)^2\cdot(t-t_0)+1\right]$$

$$= \ln\left[\left(\frac{30}{7}+0.035\right)^2\cdot(14-7)+1\right] = 4.88$$

$$\phi_{bc}(t,t_0) = \beta_{bc}(f_{cm})\cdot\beta_{bc}(t,t_0) = (0.153)(4.88) = 0.747$$

$$\beta_{dc}(t,t_0) = \left[\frac{(t-t_0)}{\beta_h+(t-t_0)}\right]^{\gamma(t_0)} = \left[\frac{(14-7)}{306.5+(14-7)}\right]^{0.276} = 0.350$$

$$\phi_{dc}(t,t_0) = \beta_{dc}(f_{cm})\cdot\beta(RH)\cdot\beta_{dc}(t_0)\cdot\beta_{dc}(t,t_0)$$

$$= (2.969)(1.53)(0.635)(0.350) = 1.01$$

$$\phi(t,t_0) = \phi_{bc}(t,t_0)+\phi_{dc}(t,t_0)$$

$$= 0.747+1.01 = 1.757$$

$$J(t,t_0)=\frac{1}{E_{ci}\left(t_0\right)}+\frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$=\frac{1}{28{,}502.6}+\frac{1.757}{32{,}297.7}=89.5\text{x}10^{-6}\,\text{MPa}^{-1}$$

For t = 157 days

$$\beta_{bc}(t,t_0)=\ln\left[\left(\frac{30}{t_{0,adj}}+0.035\right)^2\cdot\left(t-t_0\right)+1\right]$$
$$=\ln\left[\left(\frac{30}{7}+0.035\right)^2\cdot\left(90-7\right)+1\right]=7.346$$
$$\phi_{bc}(t,t_0)=\beta_{bc}(f_{cm})\cdot\beta_{bc}(t,t_0)=\left(0.153\right)\left(7.346\right)=1.124$$
$$\beta_{dc}(t,t_0)=\left[\frac{\left(t-t_0\right)}{\beta_h+\left(t-t_0\right)}\right]^{\gamma(t_0)}=\left[\frac{\left(90-7\right)}{306.5+\left(90-7\right)}\right]^{0.276}=0.653$$
$$\phi_{dc}(t,t_0)=\beta_{dc}(f_{cm})\cdot\beta(RH)\cdot\beta_{dc}(t_0)\cdot\beta_{dc}(t,t_0)$$
$$=\left(2.969\right)\left(1.53\right)\left(0.635\right)\left(0.653\right)=1.883$$
$$\phi(t,t_0)=\phi_{bc}(t,t_0)+\phi_{dc}(t,t_0)$$
$$=1.124+1.883=3.007$$
$$J(t,t_0)=\frac{1}{E_{ci}\left(t_0\right)}+\frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$=\frac{1}{28{,}502.6}+\frac{3.007}{32{,}297.7}=128.2\text{x}10^{-6}\,\text{MPa}^{-1}$$

For t = 365 days

$$\beta_{bc}(t,t_0)=\ln\left[\left(\frac{30}{t_{0,adj}}+0.035\right)^2\cdot\left(t-t_0\right)+1\right]$$
$$=\ln\left[\left(\frac{30}{7}+0.035\right)^2\cdot\left(365-7\right)+1\right]=8.807$$
$$\phi_{bc}(t,t_0)=\beta_{bc}(f_{cm})\cdot\beta_{bc}(t,t_0)=\left(0.153\right)\left(8.807\right)=1.347$$
$$\beta_{dc}(t,t_0)=\left[\frac{\left(t-t_0\right)}{\beta_h+\left(t-t_0\right)}\right]^{\gamma(t_0)}=\left[\frac{\left(365-7\right)}{306.5+\left(365-7\right)}\right]^{0.276}=0.843$$
$$\phi_{dc}(t,t_0)=\beta_{dc}(f_{cm})\cdot\beta(RH)\cdot\beta_{dc}(t_0)\cdot\beta_{dc}(t,t_0)$$
$$=\left(2.969\right)\left(1.53\right)\left(0.635\right)\left(0.843\right)=2.432$$

$$\phi(t,t_0)=\phi_{bc}(t,t_0)+\phi_{dc}(t,t_0)$$
$$=1.347+2.432=3.779$$
$$J(t,t_0)=\frac{1}{E_{ci}(t_0)}+\frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$=\frac{1}{28{,}502.6}+\frac{3.779}{32{,}297.7}=152.1\text{x}10^{-6}\,\text{MPa}^{-1}$$

For t = 2,190 days

$$\beta_{bc}(t,t_0)=\ln\left[\left(\frac{30}{t_{0,adj}}+0.035\right)^2\cdot(t-t_0)+1\right]$$
$$=\ln\left[\left(\frac{30}{7}+0.035\right)^2\cdot(2{,}190-7)+1\right]=10.615$$
$$\phi_{bc}(t,t_0)=\beta_{bc}(f_{cm})\cdot\beta_{bc}(t,t_0)=(0.153)(10.615)=1.624$$
$$\beta_{dc}(t,t_0)=\left[\frac{(t-t_0)}{\beta_h+(t-t_0)}\right]^{\gamma(t_0)}=\left[\frac{(2{,}190-7)}{306.5+(2{,}190-7)}\right]^{0.276}=0.964$$
$$\phi_{dc}(t,t_0)=\beta_{dc}(f_{cm})\cdot\beta(RH)\cdot\beta_{dc}(t_0)\cdot\beta_{dc}(t,t_0)$$
$$=(2.969)(1.53)(0.635)(0.964)=2.782$$
$$\phi(t,t_0)=\phi_{bc}(t,t_0)+\phi_{dc}(t,t_0)$$
$$=1.624+2.782=4.406$$
$$J(t,t_0)=\frac{1}{E_{ci}(t_0)}+\frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$=\frac{1}{28{,}502.6}+\frac{4.406}{32{,}297.7}=171.5\text{x}10^{-6}\,\text{MPa}^{-1}$$

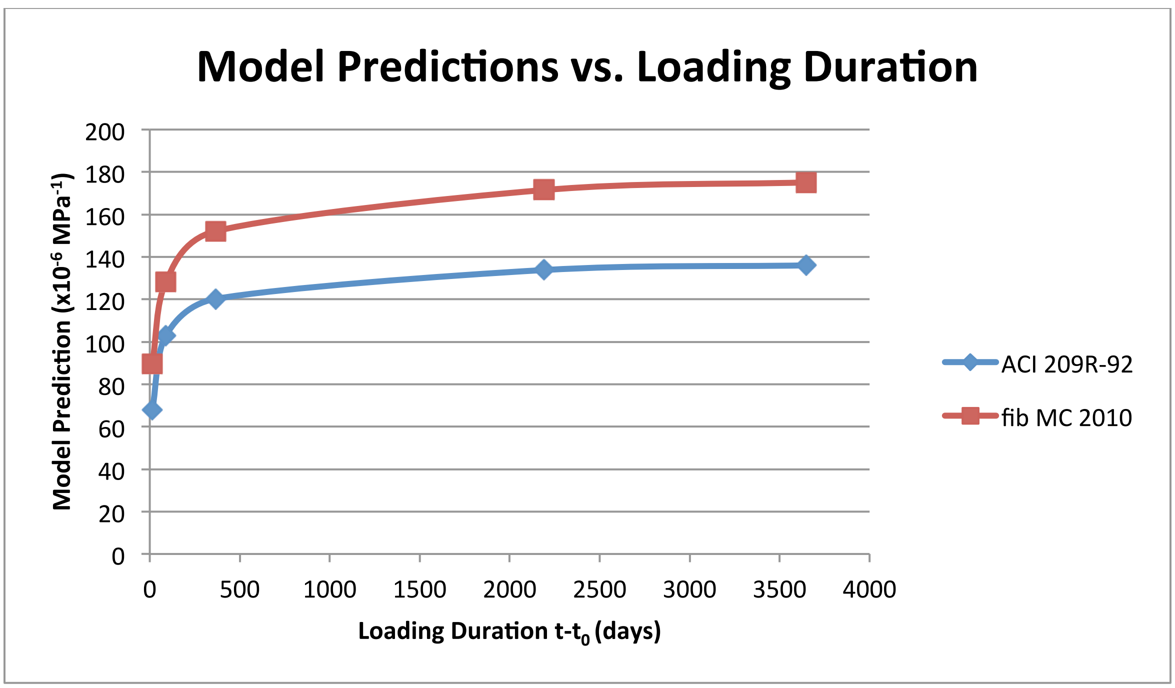
For t = 3,650 days

$$\beta_{bc}(t,t_0)=\ln\left[\left(\frac{30}{t_{0,adj}}+0.035\right)^2\cdot(t-t_0)+1\right]$$
$$=\ln\left[\left(\frac{30}{7}+0.035\right)^2\cdot(3{,}650-7)+1\right]=11.127$$
$$\phi_{bc}(t,t_0)=\beta_{bc}(f_{cm})\cdot\beta_{bc}(t,t_0)=(0.153)(11.127)=1.702$$
$$\beta_{dc}(t,t_0)=\left[\frac{(t-t_0)}{\beta_h+(t-t_0)}\right]^{\gamma(t_0)}=\left[\frac{(3{,}650-7)}{306.5+(3{,}650-7)}\right]^{0.276}=0.978$$

$$\phi_{dc}(t,t_0) = \beta_{dc}(f_{cm}) \cdot \beta(RH) \cdot \beta_{dc}(t_0) \cdot \beta_{dc}(t,t_0)$$
$$= (2.969)(1.53)(0.635)(0.978) = 2.821$$
$$\phi(t,t_0) = \phi_{bc}(t,t_0) + \phi_{dc}(t,t_0)$$
$$= 1.702 + 2.821 = 4.523$$
$$J(t,t_0) = \frac{1}{E_{ci}(t_0)} + \frac{\phi(t,t_0)}{E_{\text{ci}}}$$
$$= \frac{1}{28,502.6} + \frac{4.523}{32,297.7} = 175.1\text{x}10^{-6}\,\text{MPa}^{-1}$$

Part b

$t\text{-}t_0$ (days)	ACI 209R-92 Predictions ($\text{x}10^{-6}$ MPa^{-1})	fib MC 2010 Predictions ($\text{x}10^{-6}$ MPa^{-1})
14	68.0	89.5
90	102.9	128.2
365	120.2	152.1
2,190	133.8	171.5
3,650	136	175.1



Part C

The ACI 209R-92 and fib MC 2010 models both predict rapid change in creep compliance for the first few days of loading. At later loading ages the models seem to be reaching a plateau. The difference between the models is that the fib MC 2010 model predicts higher levels of creep compliance.

2.16 A concrete specimen has the following properties: Humidity = 50%; $h_e = 2V/S = 51$ mm; $f_{cm28} = 16.5$ MPa; cement content (c) = 320 kg/m^3; w/c = 0.59; a/c = 5.669; $t_c = 28$ days; $\gamma = 2296.74$ kg/m^3; the specimen is Type I cement; and it was moist-cured. Use the B3 model and the GL 2000 model to answer the following:

a. Predict the amount of shrinkage the concrete specimen will undergo for ages: 41; 118; 2,010; 8,988; and 10,028 days.

b. Create a graph showing the predictions versus drying duration and discuss the results

Solution

Part a
B3:

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t)$$

Determination of εshu:

$$\alpha_1 = 1.0 \text{ (Table 2.8)}$$

$$\alpha_2 = 1.0 \text{ (Table 2.9)}$$

$$\varepsilon_{\text{shu}} = -\varepsilon_{\text{su}} \frac{E_{\text{cm607}}}{E_{\text{cm}(t_c+\tau_{\text{sh}})}}$$

$$w = \frac{w}{c} \cdot c = 0.59 \cdot 320 \text{ kg/m}^3 = 188.8 \text{ kg/m}^3$$

$$\varepsilon_{\text{su}} = -\alpha_1\alpha_2[0.019(w)^{2.1}(f_{\text{cm28}})^{-0.28} + 270] \times 10^{-6}$$

$$= -(1.0)(1.0)[0.019(188.8)^{2.1}(16.5)^{-0.28} + 270] \times 10^{-6} = -791.7 \times 10^{-6} \text{ mm/mm}$$

$$E_{\text{cm28}} = 4735\sqrt{f_{\text{cm28}}} = 4735\sqrt{16.5} = 19{,}233.7 \text{ MPa}$$

$$k_s = 1.0 \text{ (Since the type of member is not defined)}$$

$$T_{\text{sh}} = 0.085(t_c)^{-0.08}(f_{\text{cm}28})^{-0.25}\left[2k_s\left(\frac{V}{S}\right)\right]^2$$

$$T_{\text{sh}} = 0.085(28)^{-0.08}(16.5)^{-0.25}\left[2(1)(25.5)\right]^2$$

$$= 84.0 \text{ days}$$

$$E_{\text{cm}607} = (1.167)^{1/2} E_{\text{cm}28}$$

$$=(1.167)^{1/2}(19{,}233.7)=20{,}777.7 \text{ MPa}$$

$$E_{\text{cm}(t_c+\tau_{\text{sh}})} = \left(\frac{t_c+\tau_{\text{sh}}}{4+0.85(t_c+\tau_{\text{sh}})}\right)^{1/2} E_{\text{cm}28} = \left(\frac{28+84.0}{4+0.85(28+84.0)}\right)^{1/2}(19{,}233.7)$$

$$= 20{,}436.9 \text{ MPa}$$

$$\varepsilon_{\text{shu}} = -\varepsilon_{\text{su}}\frac{E_{\text{cm}607}}{E_{\text{cm}(t_c+\tau_{\text{sh}})}} = -(-791.7\times10^{-6})\frac{20{,}777.7}{20{,}436.9} = 804.9\times10^{-6}\text{ mm/mm}$$

Determination of Kh:
According to the Table 2.10, for H = 50%

$$K_h = 1-\left(\frac{H}{100}\right)^3 = 1-\left(\frac{50}{100}\right)^3 = 0.875$$

For t = 41 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{41-28}{84.0}} = 0.374$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.374) = 263.4\times10^{-6}\text{ mm/mm}$$

For t = 118 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{118-28}{84.0}} = 0.776$$.

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.776) = 546.5\times10^{-6}\text{ mm/mm}$$

For t = 2,010 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{2{,}010-28}{84.0}} = 0.999$$.

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(0.999) = 704.2\times10^{-6}\text{ mm/mm}$$

For t = 8,988 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{8{,}988-28}{84.0}} = 1$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(1) = 704.3\times10^{-6}\ \text{mm/mm}$$

For t = 10,028 days:

$$S(t) = \tanh\sqrt{\frac{t-t_c}{T_{\text{sh}}}} = \tanh\sqrt{\frac{10{,}028-28}{84.0}} = 1$$

$$\varepsilon_s(t) = (\varepsilon_{\text{shu}})(K_h)S(t) = (804.9\times10^{-6})(0.875)(1) = 704.3\times10^{-6}\ \text{mm/mm}$$

GL 2000:

$$\varepsilon_s(t) = \varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)$$

$K = 1.00$ (Table 2.12)

$$\varepsilon_{\text{shu}} = (900)K\left(\frac{30}{f_{\text{cm28}}}\right)^{1/2}\times10^{-6} = (900)(1.00)\left(\frac{30}{16.5}\right)^{1/2}\times10^{-6} = 1{,}213.6\times10^{-6}\ \text{mm/mm}$$

$$\beta(h) = 1-1.18\left(\frac{H}{100}\right)^4 = 1-1.18\left(\frac{50}{100}\right)^4 = 0.926$$

For t = 41 days:

$$\beta(t-t_c) = \left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2} = \left(\frac{41-28}{41-28+0.12(25.5)^2}\right)^{1/2} = 0.3779$$

$$\varepsilon_s(t) = \varepsilon_{\text{shu}}\beta(h)\beta(t-t_c) = (1{,}213.6\times10^{-6})(0.926)(0.3779) = 424.7\times10^{-6}\ \text{mm/mm}$$

For t = 118 days:

$$\beta(t-t_c) = \left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2} = \left(\frac{118-28}{118-28+0.12(25.5)^2}\right)^{1/2} = 0.732$$

$$\varepsilon_s(t) = \varepsilon_{\text{shu}}\beta(h)\beta(t-t_c) = (1{,}213.6\times10^{-6})(0.926)(0.732) = 822.6\times10^{-6}\ \text{mm/mm}$$

For t = 2,010 days:

$$\beta(t-t_c) = \left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2} = \left(\frac{2{,}010-28}{2{,}010-28+0.12(25.5)^2}\right)^{1/2} = 0.981$$

$$\varepsilon_s(t) = \varepsilon_{\text{shu}}\beta(h)\beta(t-t_c) = (1{,}213.6\times10^{-6})(0.926)(0.981) = 1{,}102.4\times10^{-6}\ \text{mm/mm}$$

For t = 8,988 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{8{,}988\ -28}{8{,}988\ -28+0.12(25.5)^2}\right)^{1/2}=0.996$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1{,}213.6\times10^{-6})(0.926)(0.996)=1{,}119.3\times10^{-6}\,\text{mm/mm}$$

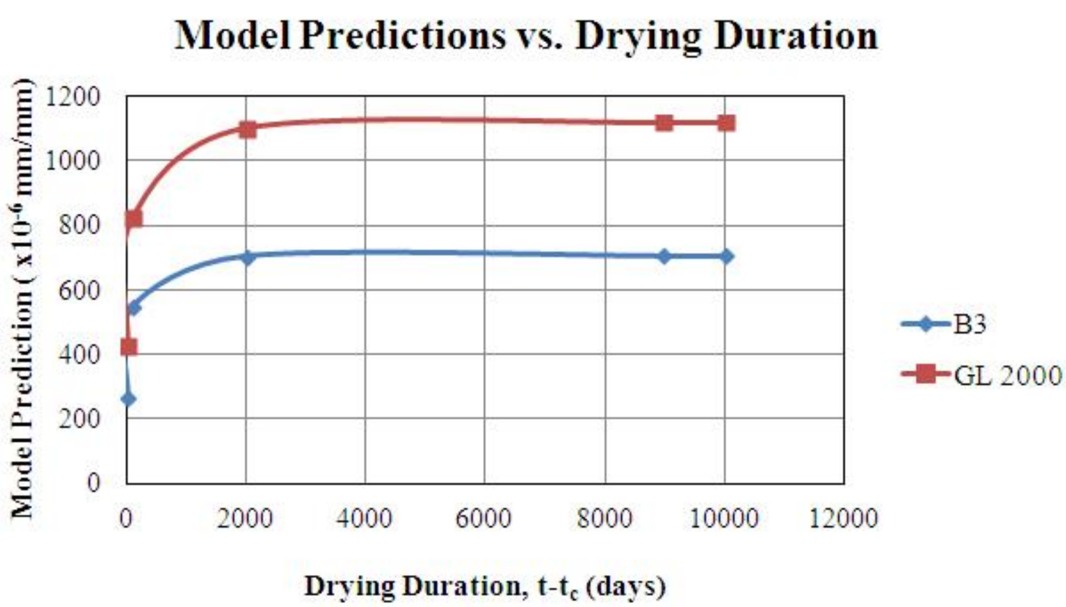
For t = 10,028 days:

$$\beta(t-t_c)=\left(\frac{t-t_c}{t-t_c+0.12(V/S)^2}\right)^{1/2}=\left(\frac{10{,}028-28}{10{,}028-28+0.12(25.5)^2}\right)^{1/2}=0.996$$

$$\varepsilon_s(t)=\varepsilon_{\text{shu}}\beta(h)\beta(t-t_c)=(1{,}213.6\times10^{-6})(0.926)(0.996)=1{,}119.3\times10^{-6}\,\text{mm/mm}$$

Part b

$t\text{-}t_c$ (days)	B3 Prediction ($\text{x}10^{-6}$ mm/mm)	GL 2000 Predictions ($\text{x}10^{-6}$ mm/mm)
13	263.4	424.7
90	546.5	822.6
1982	704.2	1102.4
8960	704.3	1119.3
10000	704.3	1119.3



The GL 2000 model predicts higher levels of shrinkage for the concrete specimen compared to the B3 model. In addition, the B3 and GL 2000 models both reach a plateau at later drying durations.