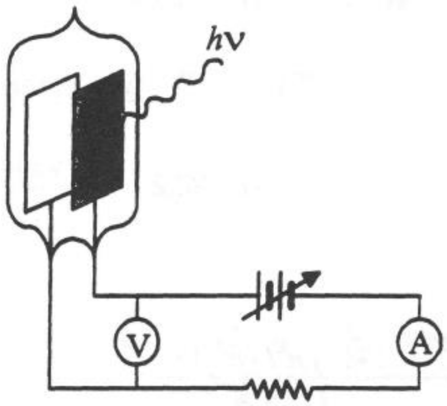
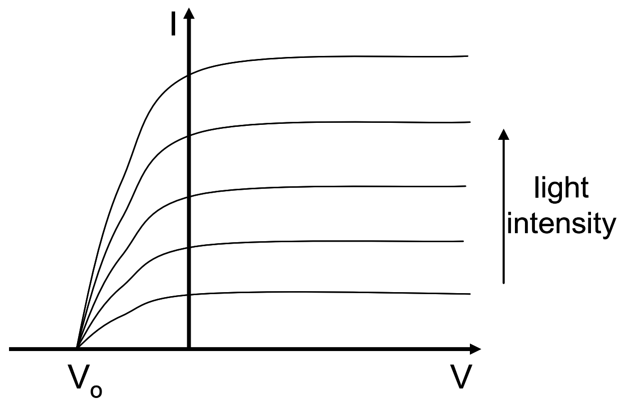
# Chapter 2 Solutions

**Prob. 2.1**

*(a&b) Sketch a vacuum tube device. Graph photocurrent I versus retarding voltage V for several light intensities.*




Note that $V_o$ remains same for all intensities.

*(c) Find retarding potential.*

$\lambda$=2440Å=0.244μm $\Phi$=4.09eV

$$V_o = h\nu - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{\lambda(\mu\text{m})} - \Phi = \frac{1.24\text{eV}\cdot\mu\text{m}}{0.244\mu\text{m}} - 4.09\text{eV} = 5.08\text{eV} - 4.09\text{eV} \approx 1\text{eV}$$

**Prob. 2.2**

*Show third Bohr postulate equates to integer number of DeBroglie waves fitting within circumference of a Bohr circular orbit.*

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} \text{ and } \frac{q^2}{4\pi\epsilon_o r^2} = \frac{mv^2}{r} \text{ and } p_\theta = mvr$$

$$r_n = \frac{4\pi\epsilon_o n^2\hbar^2}{mq^2} = \frac{n^2\hbar^2}{mr_B{}^2}\cdot\frac{4\pi\epsilon_o r_n{}^2}{q^2} = \frac{n^2\hbar^2}{mr_n{}^2}\cdot\frac{r_n}{mv^2} = \frac{n^2\hbar^2}{m^2v^2r_n}$$

$$m^2v^2r_n{}^2 = n^2\hbar^2$$

$$mvr_n = n\hbar$$

$p_\theta = n\hbar$ is the third Bohr postulate

**Prob. 2.3**

*(a) Find generic equation for Lyman, Balmer, and Paschen series.*

$$\Delta E = \frac{hc}{\lambda} = \frac{mq^4}{32\pi^2\epsilon_o^{\,2}n_1^{\,2}\hbar^2} - \frac{mq^4}{32\pi^2\epsilon_o^{\,2}n_2^{\,2}\hbar^2}$$

$$\frac{hc}{\lambda} = \frac{mq^4(n_2^{\,2}-n_1^{\,2})}{32\epsilon_o^{\,2}n_1^{\,2}n_2^{\,2}\hbar^2\pi^2} = \frac{mq^4(n_2^{\,2}-n_1^{\,2})}{8\epsilon_o^{\,2}n_1^{\,2}n_2^{\,2}h^2}$$

$$\lambda = \frac{8\epsilon_o^{\,2}n_1^{\,2}n_2^{\,2}h^2\cdot hc}{mq^4(n_2^{\,2}-n_1^{\,2})} = \frac{8\varepsilon_o^{\,2}h^3c}{mq^4}\cdot\frac{n_1^{\,2}n_2^{\,2}}{n_2^{\,2}-n_1^{\,2}}$$

$$\lambda = \frac{8(8.85\cdot10^{-12}\,\frac{F}{m})^2\cdot(6.63\cdot10^{-34}\,J{\cdot}s)^3\cdot2.998\cdot10^8\,\frac{m}{s}}{9.11\cdot10^{-31}kg\cdot(1.60\cdot10^{-19}C)^4}\cdot\frac{n_1^{\,2}n_2^{\,2}}{n_2^{\,2}-n_1^{\,2}}$$

$$\lambda = 9.11\cdot10^{8}m\cdot\frac{n_1^{\,2}n_2^{\,2}}{n_2^{\,2}-n_1^{\,2}} = 9.11\mathring{A}\cdot\frac{n_1^{\,2}n_2^{\,2}}{n_2^{\,2}-n_1^{\,2}}$$

$n_1$=1 for Lyman, 2 for Balmer, and 3 for Paschen

*(b) Plot wavelength versus n for Lyman, Balmer, and Paschen series.*

| LYMAN SERIES | | | | |
|---|---|---|---|---|
| n | n^2 | n^2-1 | n^2/(n^2-1) | 911*n^2/(n^2-1) |
| 2 | 4 | 3 | 1.33 | 1215 |
| 3 | 9 | 8 | 1.13 | 1025 |
| 4 | 16 | 15 | 1.07 | 972 |
| 5 | 25 | 24 | 1.04 | 949 |

LYMAN LIMIT 911Å

| BALMER SERIES | | | | |
|---|---|---|---|---|
| n | n^2 | n^2-4 | 4n^2/(n^2-4) | 911*4*n^2/(n^2-4) |
| 3 | 9 | 5 | 7.20 | 6559 |
| 4 | 16 | 12 | 5.33 | 4859 |
| 5 | 25 | 21 | 4.76 | 4338 |
| 6 | 36 | 32 | 4.50 | 4100 |
| 7 | 49 | 45 | 4.36 | 3968 |

BALMER LIMIT 3644Å

| PASCHEN SERIES | | | | |
|---|---|---|---|---|
| n | n^2 | n^2-9 | 9*n^2/(n^2-9) | 911*9*n^2/(n^2-9) |
| 4 | 16 | 7 | 20.57 | 18741 |
| 5 | 25 | 16 | 14.06 | 12811 |
| 6 | 36 | 27 | 12.00 | 10932 |
| 7 | 49 | 40 | 11.03 | 10044 |
| 8 | 64 | 55 | 10.47 | 9541 |
| 9 | 81 | 72 | 10.13 | 9224 |
| 10 | 100 | 91 | 9.89 | 9010 |

PASCHEN LIMIT 8199Å

**Prob. 2.4**

*(a) Find $\Delta p_x$ for $\Delta x = 1\acute{\mathring{A}}$.*

$$\Delta p_x \cdot \Delta x = \frac{h}{4\pi} \rightarrow \Delta p_x = \frac{h}{4\pi \cdot \Delta x} = \frac{6.63 \cdot 10^{-34} \mathrm{J \cdot s}}{4\pi \cdot 10^{-10} \mathrm{m}} = 5.03 \cdot 10^{-25} \, \tfrac{\mathrm{kg \cdot m}}{\mathrm{s}}$$

*(b) Find $\Delta t$ for $\Delta E = 1eV$.*

$$\Delta E \cdot \Delta t = \frac{h}{4\pi} \rightarrow \Delta t = \frac{h}{4\pi \cdot \Delta E} = \frac{4.14 \cdot 10^{-15} \mathrm{eV \cdot s}}{4\pi \cdot 1\mathrm{eV}} = 3.30 \cdot 10^{-16} \mathrm{s}$$

**Prob. 2.5**

*Find wavelength of 100eV and 12keV electrons. Comment on electron microscopes compared to visible light microscopes.*

$$E = \tfrac{1}{2} m v^2 \rightarrow v = \sqrt{\frac{2 \cdot E}{m}}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2 \cdot E \cdot m}} = \frac{6.63 \cdot 10^{-34} \mathrm{J \cdot s}}{\sqrt{2 \cdot 9.11 \cdot 10^{-31} \mathrm{kg}}} \cdot E^{-\frac{1}{2}} = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m}$$

For 100eV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (100\mathrm{eV} \cdot 1.602 \cdot 10^{-19} \tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.23 \cdot 10^{-10} \mathrm{m} = 1.23\text{Å}$$

For 12keV,

$$\lambda = E^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = (1.2 \cdot 10^{4} \mathrm{eV} \cdot 1.602 \cdot 10^{-19} \tfrac{\mathrm{J}}{\mathrm{eV}})^{-\frac{1}{2}} \cdot 4.91 \cdot 10^{-19} \mathrm{J}^{\frac{1}{2}} \cdot \mathrm{m} = 1.12 \cdot 10^{-11} \mathrm{m} = 0.112\text{Å}$$

The resolution on a visible microscope is dependent on the wavelength of the light which is around 5000Å; so, the much smaller electron wavelengths provide much better resolution.



**Prob. 2.6**

Which of the following could NOT possibly be wave functions ***and why*****?** Assume 1-D in each case. (Here i= imaginary number, C is a normalization constant)

A) $\Psi$ (x) = C for all x.

B) $\Psi$ (x) = C for values of x between 2 and 8 cm, and $\Psi$ (x) = 3.5 C for values of x between 5 and 10 cm. $\Psi$ (x) is zero everywhere else.

C) $\Psi$ (x) = i C for x= 5 cm, and linearly goes down to zero at x= 2 and x = 10 cm from this peak value, and is zero for all other x.

If any of these are valid wavefunctions, calculate C for those case(s). What potential energy for x $\leq$ 2 and x $\geq$ 10 is consistent with this?

A) For a wavefunction $\Psi(x)$, we know $P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=1$

$$P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=c^2\int_{-\infty}^{\infty}dx \rightarrow P=\begin{cases}0 & c=0\\ \infty & c\neq 0\end{cases} \Rightarrow \Psi(x) \text{ cannot be a wave function}$$

B) For $5\le x\le 8$, $\Psi(x)$ has two values, C and 3.5C. For $c\neq 0$, $\Psi(x)$ is not a function and for $c=0$: $P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=0 \Rightarrow \Psi(x)$ cannot be a wave function.

C) $\Psi(x)=\begin{cases}\frac{iC}{3}(x\text{-}2) & 2\le x\le 5\\ -\frac{iC}{5}(x\text{-}10) & 5\le x\le 10\end{cases}$

$$P=\int_{-\infty}^{\infty}\Psi^*(x)\Psi(x)dx=\int_{2}^{5}\frac{c^2}{9}(x\text{-}2)^2\,dx+\int_{5}^{10}\frac{c^2}{25}(x\text{-}10)^2\,dx$$

$$=\frac{c^2}{3\times 9}(x\text{-}2)^3\Big]_2^5+\frac{c^2}{3\times 25}(x\text{-}10)^3\Big]_5^{10}$$

$$=c^2\left[\frac{27}{27}+\frac{125}{3\times 25}\right]=\frac{8c^2}{3}$$

$$P=1 \Rightarrow \frac{8c^2}{3}=1 \Rightarrow c=0.612 \rightarrow \Psi(x) \text{ can be a wave function}$$

Since $\Psi(x)=0$ for $x\le 2$ and $x\ge 10$, the potential energy should be infinite in these two regions.

**Prob. 2.7**

A particle is described in 1D by a wavefunction:
$\Psi = Be^{-2x}$ for $x \geq 0$ and $Ce^{+4x}$ for $x<0$, and B and C are real constants. Calculate B and C to make $\Psi$ a valid wavefunction. Where is the particle most likely to be?

A valid wavefunction must be continuous, and normalized.

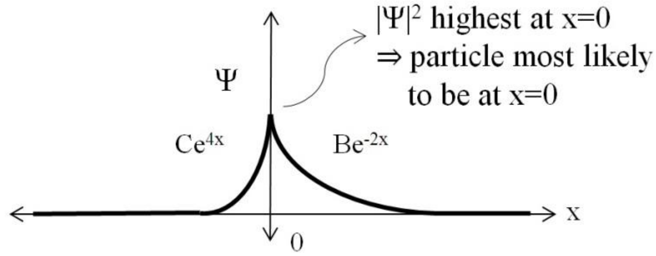
For $\Psi(0) = C = B$

To normalize $\Psi$, $\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$

$$\int_{-\infty}^{0} C^2 e^{8x} dx + \int_{0}^{\infty} C^2 e^{-4x} dx = 1$$

$$\frac{C^2}{8}\left[e^{8x}\right]_{-\infty}^{0} + C^2\left(\frac{-1}{4}\right)\left[e^{-4x}\right]_{0}^{\infty} = 1$$

$$\frac{C^2}{8} + \frac{C^2}{4} = 1 \Rightarrow C = \sqrt{\frac{8}{3}}$$



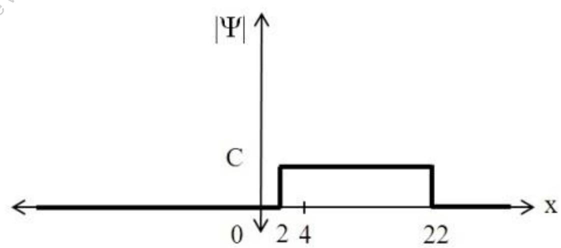

**Prob. 2.8**

The electron wavefunction is $Ce^{ikx}$ between x=2 and 22 cm, and zero everywhere else. What is the value of C? What is the probability of finding the electron between x=0 and 4 cm?

$$\Psi = Ce^{ikx}$$

$$\int_{2}^{22} \Psi^* \Psi dx = C^2(20) = 1 \Rightarrow C = \frac{1}{\sqrt{20}} \text{cm}^{-1}$$

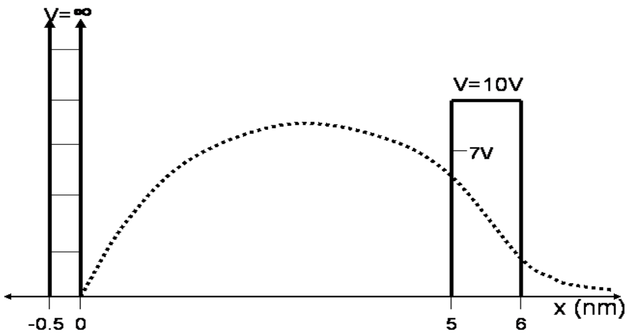
$$\text{Probability} = \int_{0}^{4} |\Psi|^2 dx = \left(\frac{1}{\sqrt{20}}\right)^2 (2) = \frac{1}{10}$$




**Prob. 2.9**

*Find the probability of finding an electron at x<0. Is the probability of finding an electron at x>0 zero or non-zero? Is the classical probability of finding an electron at x>6 zero or non?*

The energy barrier at x=0 is infinite; so, there is zero probability of finding an electron at x<0 ($|\psi|^2=0$). However, it is possible for electrons to tunnel through the barrier at 5<x<6; so, the probability of finding an electron at x>6 would be quantum mechanically greater than zero ($|\psi|^2>0$) and classical mechanically zero.




**Prob. 2.10**

*Find* $4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+7mE$ *for* $\Psi(x,y,z,t) = A\cdot e^{j(10\cdot x+3\cdot y\text{-}4\cdot t)}$.

$$\langle p_x^{\,2}\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial x}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dx}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dx} = 100\cdot\hbar^2$$

$$\langle p_z^{\,2}\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(\frac{\hbar}{j}\frac{\partial}{\partial z}\right)^2 A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dz}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dz} = 0$$

$$\langle E\rangle = \frac{\int_{-\infty}^{\infty} A^*\cdot e^{-j(10\cdot x+3\cdot y-4\cdot t)}\left(-\frac{\hbar}{j}\frac{\partial}{\partial t}\right) A\cdot e^{j(10\cdot x+3\cdot y-4\cdot t)}dt}{\int_{-\infty}^{\infty}|A|^2\, e^{-j(10\cdot x+3\cdot y-4\cdot t)}e^{j(10\cdot x+3\cdot y-4\cdot t)}dt} = 4\cdot\hbar$$

$$4\cdot p_x^{\,2}+2\cdot p_z^{\,2}+7mE = 400\hbar^2+28(9.11\cdot 10^{-31}\text{kg})\,\hbar$$

**Prob. 2.11**

*Find the uncertainty in position (Δx) and momentum (Δp).*

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{\pi x}{L}\right) \cdot e^{-2\pi j E t/h} \text{ and } \int_0^L \Psi^* \cdot \Psi dx = 1$$

$$\langle x \rangle = \int_0^L \Psi^* \cdot x \cdot \Psi dx = \frac{2}{L}\int_0^L x \cdot \sin^2\left(\frac{\pi x}{L}\right) dx = 0.5L \text{ (from problem note)}$$

$$\langle x^2 \rangle = \int_0^L \Psi^* \cdot x \cdot \Psi dx = \frac{2}{L}\int_0^L x^2 \cdot \sin^2\left(\frac{\pi x}{L}\right) dx = 0.28L^2 \text{ (from problem note)}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{0.28L^2 - (0.5L)^2} = 0.17L$$

$$\Delta p \geq \frac{h}{4\pi \cdot \Delta x} = 0.47 \cdot \frac{h}{L}$$

**Prob. 2.12**

*Calculate the first three energy levels for a 10Å quantum well with infinite walls.*



$$E_n = \frac{n^2 \cdot \pi^2 \cdot \hbar^2}{2 \cdot m \cdot L^2} = \frac{(6.63 \cdot 10^{-34})^2}{8 \cdot 9.11 \cdot 10^{-31} \cdot (10^{-9})^2} \cdot n^2 = 6.03 \cdot 10^{-20} \cdot n^2$$

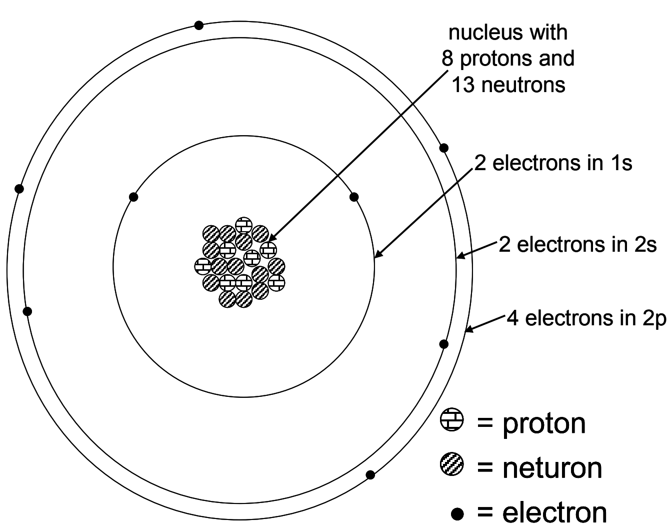
$$E_1 = 6.03 \cdot 10^{-20} J = 0.377 eV$$

$$E_2 = 4 \cdot 0.377 eV = 1.508 eV$$

$$E_3 = 9 \cdot 0.377 eV = 3.393 eV$$

**Prob. 2.13**

*Show schematic of atom with $1s^2 2s^2 2p^4$ and atomic weight 21. Comment on its reactivity.*




This atom is chemically reactive because the outer 2p shell is not full. It will tend to try to add two electrons to that outer shell.