

# **Solar Energy Engineering**

## **Processes and Systems**

**Second Edition**

**By:**

**Soteris A. Kalogirou**

# **SOLUTIONS MANUAL**

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**for**

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## CHAPTER 2

**2.3** Calculate the solar declination for the spring and fall equinoxes and the summer and winter solstices.

Using Table 2.1 to estimate day number and Eq. (2.5) the following can be estimated: Spring equinox is on March 22, day number is 81 and from Eq. (2.5):

$$\delta = 23.45 \sin \left[ \frac{360}{365} (284 + N) \right] = 23.45 \sin \left[ \frac{360}{365} (284 + 81) \right] = 0^\circ$$

Similarly:

Fall equinox is on September 21, day number is 264 and  $\delta = 0^\circ$

Summer solstice is on June 21, day number is 172 and  $\delta = 23.45^\circ$

Winter solstice is on December 21, day number is 355 and  $\delta = -23.45^\circ$

**2.4** Calculate the sunrise and sunset times and day length for the spring and fall equinoxes and the summer and winter solstices at  $45^\circ\text{N}$  latitude and  $35^\circ\text{E}$  longitude.

The standard longitude is  $30^\circ$ , therefore longitude correction from Eq. (2.3) is  $-4(30-35) = 20$  min.

Spring equinox is on March 22, therefore from Table 2.1, day number is 81. Using Eq. (2.2):

$$B = (N - 81) \frac{360}{364} = (81 - 81) \frac{360}{364} = 0$$

From Eq. (2.1):  $ET = 9.87 \sin(2B) - 7.53 \cos(B) - 1.5 \sin(B) = 9.87 \sin(0) - 7.53 \cos(0) - 1.5 \sin(0) = -7.53 \approx -8$  min.

From Exercise 2.3,  $\delta = 0^\circ$  and from Eq. (2.17): Day Length =  $2/15 \cos^{-1}[-\tan(L) \tan(\delta)]$

Or Day Length =  $2/15 \cos^{-1}[-\tan(45) \tan(0)] = 12$  hours.

Since solar noon is at the middle of the sunrise and sunset hours, it means that they are  $12/2 = 6$  hours away from local solar noon. Therefore, sunrise is at  $12:00 - 6:00 + 0:08 - 0:20 = 5:48$ , sunset time is  $12:00 + 6:00 + 0:08 - 0:20 = 17:48$ .

Similarly:

For fall equinox  $ET = +7.9 \approx 8$  min. Day length = 12 hours. Therefore, sunrise is at  $12:00 - 6:00 - 0:08 - 0:20 = 5:32$ , sunset time is  $12:00 + 6:00 - 0:08 - 0:20 = 17:32$ .

For summer solstice  $ET = -1.5 \approx -2$  min. Day length = 15.43 hours (15:26). Therefore, sunrise is at  $12:00 - 7:43 + 0:02 - 0:20 = 3:59$ , sunset time is  $12:00 + 7:43 + 0:02 - 0:20 = 19:25$ .

Winter solstice  $ET = +1$  min. Day length = 8.57 hours (8:34). Therefore, sunrise is at  $12:00 - 4:17 - 0:01 - 0:20 = 7:22$ , sunset time is  $12:00 + 4:17 - 0:01 - 0:20 = 15:56$ .

**2.5** Determine the solar altitude and azimuth angles at 10:00 am local time for Rome, Italy on June 10.

The geographic coordinates (latitude and longitude) of Rome can be obtained from <http://www.infoplease.com/ipa/A0001769.html>. Therefore, Rome is at latitude of  $41.54^\circ$  and longitude of  $12.27^\circ\text{E}$ . From Table 2.1, on June 10, day number is 161. Therefore, from Eq. (2.5)  $\delta=23^\circ$ . From Eq. (2.1) on June 10,  $ET = 0.76 \text{ min} \approx 1 \text{ min}$ . From Eq. (2.3),  $AST = LST+ET\pm 4 (SL - LL) = 10:00+0:01-4(15-12.27) = 9:50 \text{ am} (9.83 \text{ hr})$ .

At 9:50 am the hour angle from Eq. (2.9), is:  $h=(9.83-12)*15 = -32.55^\circ$ .

From Eq. (2.12):  $\sin(\alpha) = \sin(L) \sin(\delta)+\cos(L) \cos(\delta) \cos(h) = \sin(41.54) \sin(23)+\cos(41.54) \cos(23) \cos(-32.55) = 0.8398 \rightarrow \alpha = 57.1^\circ$ .

From Eq. (2.13):  $\sin(z) = \frac{\cos(\delta) \sin(h)}{\cos(\alpha)} = \frac{\cos(23) \sin(-32.55)}{\cos(57.1)} = -0.912 \rightarrow z = -65.8^\circ$  (negative sign shows that the angle is eastward, which is obvious as 10:00 am is before noon).

**2.6 Calculate the solar zenith and azimuth angles, the sunrise and sunset times and the day length for Cairo, Egypt, at 10:30 am solar time on April 10.**

From the web site given in Exercise 2.5, Cairo is at latitude of  $30^\circ$  and longitude of  $31^\circ\text{E}$ . From Table 2.1, on April 10, day number is 100. Therefore from Eq. (2.5)  $\delta=7.5^\circ$ . At 10:30 am hour angle is from Eq. (2.9),  $h=(10.5-12)*15 = -22.5^\circ$ . From Eq. (2.12):  $\cos(\Phi) = \sin(30) \sin(7.5)+\cos(30) \cos(7.5) \cos(-22.5) = 0.858 \rightarrow \Phi = 30.8^\circ$  and from Eq. (2.11)  $\alpha = 59.2^\circ$ .

From Eq. (2.13):  $\sin(z) = \frac{\cos(7.5) \sin(-22.5)}{\cos(59.2)} = -0.741 \rightarrow z = -47.8^\circ$ .

From Eq. (2.1)  $ET = -1.6 \text{ min} \approx -2 \text{ min}$ . For Cairo the standard longitude is  $30^\circ$ , therefore longitude correction from Eq. (2.3) is  $-4(30-31) = 4 \text{ min}$ .

From Eq. (2.17) day length = 12.58 hours (12:35). Therefore, sunrise is at  $12:00-6:18-0:02+0:04 = 5:44$ , sunset time is  $12:00+6:18-0:02+0:04 = 18:20$ .

**2.7 Calculate the sunrise and sunset times and altitude and azimuth angles for London, England, on March 15 and September 15 at 10:00 am and 3:30 pm solar times.**

From the web site given in Exercise 2.5, London is at latitude of  $51^\circ$  and longitude at  $0^\circ$ . From Table 2.1, on March 15, day number is 74 and on September 15 is 258. Therefore from Eq. (2.5) on March 15,  $\delta=-2.8^\circ$  and on September 15,  $\delta=2.2^\circ$ . At 3:30 pm hour angle is from Eq. (2.9),  $h=(15.5-12)*15 = 52.5^\circ$ . Similarly at 10:00 am  $h = -30^\circ$ .

From Eq. (2.12) at 10:00 am on March 15,  $\alpha = 30.4^\circ$  and on September 15,  $\alpha = 35.1^\circ$ . At 3:30 pm on March 15,  $\alpha = 20.1^\circ$  and on September 15,  $\alpha = 24.4^\circ$ .

From Eq. (2.13) at 10:00 am on March 15,  $z = -35.4^\circ$  and on September 15,  $z = -37.6^\circ$ . At 3:30 pm on March 15,  $z = 57.6^\circ$  and on September 15,  $z = 60.5^\circ$ .

From Eq. (2.1) on March 15,  $ET = -9.7 \text{ min} \approx -10 \text{ min}$  and on September 15,  $ET = 5.7 \text{ min} \approx 6 \text{ min}$ . London is at longitude of  $0^\circ$ , therefore no longitude correction is required.

From Eq. (2.17) on March 15 day length = 11.54 hours (11:32). Therefore, sunrise is at  $12:00-5:46+0:10 = 6:24$ , sunset time is  $12:00+5:46+0:10 = 17:56$ .

From Eq. (2.17) on September 15 day length = 12.37 hours (12:22). Therefore, sunrise is at 12:00-6:11-0:06 = 5:43, sunset time is 12:00+6:11-0:06 = 18:05.

**2.8** What is the solar time in Denver, Colorado, on June 10 at 10:00 am Mountain Standard Time?

The standard meridian for Mountain Standard Time is  $105^\circ$ . From the web site given in Exercise 2.5, Denver is at latitude of  $39^\circ$  and longitude at  $105^\circ$ . So, no longitude correction is required. From Table 2.1, on June 10, day number is 161. From Eq. (2.1) on June 10,  $ET = 0.76 \text{ min} \approx 1 \text{ min}$ . Therefore, from Eq. (2.3)  $AST = LST + ET = 10:00 + 0:1 = 10:01 \text{ am}$ .

**2.9** A flat-plate collector in Nicosia, Cyprus, is tilted at  $40^\circ$  from horizontal and pointed  $10^\circ$  east of south. Calculate the solar incidence angle on the collector at 10:30 am and 2:30 pm solar times on March 10 and September 10.

Latitude of Nicosia, Cyprus is  $35^\circ\text{N}$ . From Table 2.1, on March 10, day number is 69 and on September 10 is 253. Therefore, from Eq. (2.5) on March 10,  $\delta = -4.8^\circ$  and on September 10,  $\delta = 4.2^\circ$ . At 2:30 pm hour angle is from Eq. (2.9),  $h = (14.5 - 12) * 15 = 37.5^\circ$ . Similarly at 10:30 am  $h = -22.5^\circ$ . Using Eq. (2.18):

On March 10 at 10:30 am,  $\theta = 16^\circ$ . At 2:30 pm  $\theta = 43.8^\circ$   
On September 10 at 10:30 am,  $\theta = 18.3^\circ$ . At 2:30 pm  $\theta = 44.8^\circ$

**2.10** A vertical surface in Athens, Greece, faces  $15^\circ$  west of due south. Calculate the solar incidence angle at 10:00 am and 3:00 pm solar times on January 15 and November 10.

Latitude of Athens, Greece is  $38^\circ$ . From Table 2.1, on January 15, day number is 15 and on November 10 is 314. Therefore, from Eq. (2.5) on January 15,  $\delta = -21.3^\circ$  and on November 10,  $\delta = -17.9^\circ$ . At 3:00 pm hour angle is from Eq. (2.9),  $h = (15 - 12) * 15 = 45^\circ$ . Similarly at 10:00 am,  $h = -30^\circ$ . Using Eq. (2.19) for vertical surfaces:

On January 15 at 10:00 am,  $\theta = 50.5^\circ$ . At 3:00 pm  $\theta = 33.0^\circ$   
On November 10 at 10:00 am,  $\theta = 53.1^\circ$ . At 3:00 pm  $\theta = 36.1^\circ$

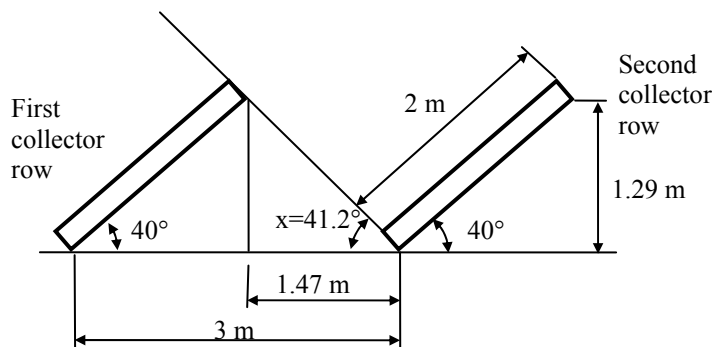
**2.11** By using the sun path diagram find the solar altitude and azimuth angles for Athens, Greece on January 20 at 10:00 am.

To use the sun path diagram the declination is required. For January 20 (day number 20)  $\delta = -20.3^\circ$ . Latitude of Athens, Greece is  $38^\circ$  so can use the sun path diagram for  $40^\circ\text{N}$  latitude (see Appendix 3, Fig. A3.3) or interpolate with that at latitude  $35^\circ\text{N}$  shown in Fig. 2.17. The former is selected here. Therefore,  $\alpha = 24^\circ$  and  $z = -30^\circ$  (actual values  $\alpha = 25^\circ$  and  $z = -31^\circ$ ).

**2.12** Two rows of 6 m wide by 2 m high flat-plate collector arrays titled at  $40^\circ$  are facing due south. If these collectors are located in  $35^\circ\text{N}$  latitude, using the sun path diagram find the months of the year and the hours of day at which the front row will cast a shadow on

the second row when the distance between the rows is 3 m. What should be the distance so there will be no shading?

For a collector length of 2 m and inclination of 40 degrees the actual height is  $2x\sin(40) = 1.29$  m. The actual distance between the collectors is  $3-2x\cos(40) = 1.47$  m. The angle  $x$  shown in Figure below is  $41.2^\circ$ . Using this value on Fig. 2.17 corresponds to a declination of  $-13^\circ$  which from Fig. 2.7 corresponds from end of October to middle of February. From sun path diagram the minimum solar altitude at noon is  $31.5^\circ$ . Therefore the distance required to avoid shading completely is  $1.29/\tan(31.5) = 2.1$  m or actual distance between collector rows is  $(3-1.47)+2.1 = 3.63$  m.



**2.13** Find the blackbody spectral emissive power at  $\lambda = 8 \mu\text{m}$  for a source at 400 K, 1000 K, and 6000 K.

Using Eq. (2.34) and the given data, we have:

- At  $T=400\text{K}$ ,  $E_{b\lambda} = 128.22 \text{ W/m}^2\text{-}\mu\text{m}$
- At  $T=1000\text{K}$ ,  $E_{b\lambda} = 2260.27 \text{ W/m}^2\text{-}\mu\text{m}$
- At  $T=6000\text{K}$ ,  $E_{b\lambda} = 32623.37 \text{ W/m}^2\text{-}\mu\text{m}$

**2.14** Assuming that the sun is a blackbody at 5777 K. At what wavelength does the maximum monochromatic emissive power occur? What fraction of energy from this source is in the visible part of the spectrum in the range  $0.38\text{--}0.78 \mu\text{m}$ ?

As is given in Chapter 2 by differentiating Eq. (2.34) and equating to zero, the maximum distribution is equal to  $\lambda_{\text{max}}T = 2897.8 \mu\text{m}\text{-K}$ . Therefore as  $T = 5777\text{K}$ ,  $\lambda_{\text{max}} = 0.502 \mu\text{m}$ . This agrees with the maximum shown in Fig. 2.26.

$$\lambda_1 T = 0.38 \times 5777 = 2195.3 \mu\text{m}\text{-K}$$

$$\lambda_2 T = 0.78 \times 5777 = 4506.1 \mu\text{m}\text{-K}$$

From Table 2.4 the fraction of energy between zero and  $2195.3 \mu\text{m}\text{-K}$  is 10.0% and the fraction from zero to  $4506.1 \mu\text{m}\text{-K}$  is 56.5%. Therefore, the fraction of energy in the visible part of the spectrum is  $56.5 - 10 = 46.5\%$ .

**2.15** What percentage of blackbody radiation for a source at 323K is in the wavelength region  $6\text{--}15 \mu\text{m}$ ?

From the given data we have:

$$\lambda_1 T = 6 \times 323 = 1938 \text{ } \mu\text{m-K}$$

$$\lambda_2 T = 15 \times 323 = 4845 \text{ } \mu\text{m-K}$$

From Table 2.4 the fraction of energy between zero and 1938  $\mu\text{m-K}$  is 5.8% and the fraction from zero to 4845  $\mu\text{m-K}$  is 61.4%. Therefore, the percentage of the blackbody radiation in the range 6 to 15  $\mu\text{m}$  is  $61.4 - 5.8 = 55.6\%$ .

**2.16** A 2 mm thick glass sheet has a refraction index of 1.526 and an extinction coefficient of  $0.2 \text{ cm}^{-1}$ . Calculate the reflectivity, transmissivity, and absorptivity of the glass sheet at  $0^\circ$ ,  $20^\circ$ ,  $40^\circ$ , and  $60^\circ$  incidence angles.

At normal incidence,  $\theta_1 = 0^\circ$  and  $\theta_2 = 0^\circ$ . From Eq. (2.51) the transmittance can be obtained as

$$\tau_a = e^{\left(-\frac{KL}{\cos(\theta_2)}\right)} = e^{\left(-\frac{0.2(0.2)}{\cos(0)}\right)} = 0.961$$

There is no polarization at normal incidence therefore from Eq. (2.49):

$$r_{(0)} = r_{\perp} = r_{\parallel} = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.526-1}{1.526+1}\right)^2 = 0.043$$

Therefore from Eqs. (2.52a)-(2.52c) we have:

$$\tau = \tau_{\alpha} \frac{1-r_{(0)}}{1+r_{(0)}} \left( \frac{1-r_{(0)}^2}{1-(r_{(0)}\tau_{\alpha})^2} \right) = 0.961 \left[ \frac{1-0.043}{1+0.043} \left( \frac{1-0.043^2}{1-(0.043 \times 0.961)^2} \right) \right] = 0.882$$

$$\rho = r_{(0)} (1 + \tau_{\alpha} \tau_{(0)}) = 0.043(1 + 0.961 \times 0.882) = 0.079$$

$$\alpha = (1 - \tau_{\alpha}) \left( \frac{1 - r_{(0)}}{1 - r_{(0)} \tau_{\alpha}} \right) = (1 - 0.961) \left( \frac{1 - 0.043}{1 - 0.043 \times 0.961} \right) = 0.039$$

At angle of incidence =  $20^\circ$ :

From Eq. (2.44) the refraction angle  $\theta_2$  is calculated as:

$$\theta_2 = \sin^{-1} \left( \frac{\sin \theta_1}{n} \right) = \sin^{-1} \left( \frac{\sin(20)}{1.526} \right) = 13^\circ$$

From Eq. (2.51) the transmittance can be obtained as:

$$\tau_a = e^{\left(-\frac{KL}{\cos \theta_2}\right)} = e^{\left(-\frac{0.2(0.2)}{\cos(13)}\right)} = 0.960$$

From Eqs. (2.45) and (2.46):

$$r_{\perp} = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} = \frac{\sin^2(13 - 20)}{\sin^2(13 + 20)} = 0.050$$

$$r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} = \frac{\tan^2(13 - 20)}{\tan^2(13 + 20)} = 0.036$$

From Eqs. (2.52a)-(2.52c) we have:

$$\begin{aligned} \tau &= \frac{0.960}{2} \left[ \frac{1-0.050}{1+0.050} \left( \frac{1-0.050^2}{1-(0.050 \times 0.960)^2} \right) + \frac{1-0.036}{1+0.036} \left( \frac{1-0.036^2}{1-(0.036 \times 0.960)^2} \right) \right] \\ &= 0.480(0.905 + 0.930) = 0.881 \\ \rho &= 0.5[0.050(1 + 0.960 \times 0.905) + 0.036(1 + 0.960 \times 0.930)] = 0.081 \end{aligned}$$

$$\alpha = \frac{(1-0.960)}{2} \left( \frac{1-0.050}{1-0.050 \times 0.960} + \frac{1-0.036}{1-0.036 \times 0.960} \right) = 0.04$$

To avoid showing all the numerical calculation only the answers are given for the other two cases. For  $\theta=40^\circ$ ;  $\theta_2=24.9^\circ$ ,  $\tau_a=0.957$ ,  $r_{\perp}=0.083$ ,  $r_{\parallel}=0.016$ ,  $\tau=0.868$ ,  $\rho=0.091$  and  $\alpha=0.043$ . For  $\theta=60^\circ$ ;  $\theta_2=34.6^\circ$ ,  $\tau_a=0.953$ ,  $r_{\perp}=0.185$ ,  $r_{\parallel}=0.001$ ,  $\tau=0.802$ ,  $\rho=0.154$  and  $\alpha=0.047$ .

**2.17** A flat-plate collector has an outer glass cover of 4 mm thick  $K = 23 \text{ m}^{-1}$  and refractive index of 1.526, and a tedlar inner cover with refractive index of 1.45. Calculate the reflectivity, transmissivity, and absorptivity of the glass sheet at a  $40^\circ$  incidence angle by considering tedlar to be of a very small thickness; i.e., absorption within the material can be neglected.

From Eq. (2.44) the refraction angle  $\theta_2$  is calculated as:

$$\theta_2 = \sin^{-1} \left( \frac{\sin \theta_1}{n} \right) = \sin^{-1} \left( \frac{\sin(40)}{1.526} \right) = 24.9^\circ$$

From Eq. (2.51) the transmittance can be obtained as:

$$\tau_a = e^{\left( -\frac{KL}{\cos \theta_2} \right)} = e^{\left( -\frac{23(0.004)}{\cos(24.9)} \right)} = 0.904$$

From Eqs. (2.45) and (2.46):

$$r_{\perp} = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} = \frac{\sin^2(24.9 - 40)}{\sin^2(24.9 + 40)} = 0.083$$

$$r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} = \frac{\tan^2(24.9 - 40)}{\tan^2(24.9 + 40)} = 0.016$$

From Eqs. (2.52a)-(2.52c) we have:

$$\tau_{\perp} = \tau_{\alpha} \frac{1-r_{\perp}}{1+r_{\perp}} \left( \frac{1-r_{\perp}^2}{1-(r_{\perp} \tau_{\alpha})^2} \right) = 0.904 \left[ \frac{1-0.083}{1+0.083} \left( \frac{1-0.083^2}{1-(0.083 \times 0.904)^2} \right) \right] = 0.765$$

$$\tau_{\parallel} = \tau_{\alpha} \frac{1-r_{\parallel}}{1+r_{\parallel}} \left( \frac{1-r_{\parallel}^2}{1-(r_{\parallel} \tau_{\alpha})^2} \right) = 0.904 \left[ \frac{1-0.016}{1+0.016} \left( \frac{1-0.016^2}{1-(0.016 \times 0.904)^2} \right) \right] = 0.875$$



$$\rho_{\perp} = r_{\perp} (1 + \tau_{\alpha} \tau_{\perp}) = 0.083(1 + 0.904 \times 0.765) = 0.140$$

$$\rho_{\parallel} = r_{\parallel} (1 + \tau_{\alpha} \tau_{\parallel}) = 0.016(1 + 0.904 \times 0.875) = 0.029$$

$$\alpha_{\perp} = (1 - \tau_{\alpha}) \left( \frac{1 - r_{\perp}}{1 - r_{\perp} \tau_{\alpha}} \right) = (1 - 0.904) \left( \frac{1 - 0.083}{1 - 0.083 \times 0.904} \right) = 0.095$$

$$\alpha_{\parallel} = (1 - \tau_{\alpha}) \left( \frac{1 - r_{\parallel}}{1 - r_{\parallel} \tau_{\alpha}} \right) = (1 - 0.904) \left( \frac{1 - 0.016}{1 - 0.016 \times 0.904} \right) = 0.096$$

To avoid repeating the same calculations the results for the tedlar cover are:  $r_{\perp} = 0.067$ ,  $r_{\parallel} = 0.011$ ,  $\tau_{\perp} = 0.875$ ,  $\tau_{\parallel} = 0.977$ ,  $\rho_{\perp} = 0.126$ ,  $\rho_{\parallel} = 0.022$ ,  $\alpha_{\perp} = 0$ ,  $\alpha_{\parallel} = 0$ . These were obtained by considering  $\tau_{\alpha} = 1.0$  as thickness is very small. The angle  $\theta_2$  is  $26.3^\circ$  estimated from Eq. (2.44) using  $n = 1.45$  (given).

Finally, Eqs. (2.56) and (2.57) can be used as follows:

$$\tau = \frac{1}{2} \left[ \left( \frac{\tau_1 \tau_2}{1 - \rho_1 \rho_2} \right)_{\perp} + \left( \frac{\tau_1 \tau_2}{1 - \rho_1 \rho_2} \right)_{\parallel} \right] = \frac{1}{2} \left[ \left( \frac{0.765 \times 0.875}{1 - 0.14 \times 0.126} \right) + \left( \frac{0.875 \times 0.977}{1 - 0.029 \times 0.022} \right) \right] = 0.768$$

$$\rho = \frac{1}{2} \left[ \left( \rho_1 + \frac{\tau \rho_2 \tau_1}{\tau_2} \right)_{\perp} + \left( \rho_1 + \frac{\tau \rho_2 \tau_1}{\tau_2} \right)_{\parallel} \right] = \frac{1}{2} \left[ \left( 0.14 + \frac{0.768 \times 0.126 \times 0.765}{0.875} \right) + \left( 0.029 + \frac{0.768 \times 0.022 \times 0.875}{0.977} \right) \right] = 0.134$$

$$\alpha = 1 - \tau - \rho = 1 - 0.768 - 0.134 = 0.098$$

**2.18** The glass plate of a solar greenhouse has a transmissivity of 0.90 for wavelengths between 0.32 and 2.8  $\mu\text{m}$  and is completely opaque at shorter and longer wavelengths. If the sun is a blackbody radiating energy to the earth's surface at an effective temperature of 5770 K and the interior of the greenhouse is at 300 K, calculate the percent of incident solar radiation transmitted through the glass and the percent of thermal radiation emitted by the interior objects that is transmitted out.

The incoming solar radiation at 5770K we have:

$$\lambda_1 T = 0.32 \times 5770 = 1846 \mu\text{m-K}$$

$$\lambda_2 T = 2.8 \times 5770 = 16156 \mu\text{m-K}$$

From Table 2.4 we get:

$$\frac{E_b(0 \rightarrow \lambda_1 T)}{\sigma T^4} = 0.453 = 45.3\%$$

$$\frac{E_b(0 \rightarrow \lambda_2 T)}{\sigma T^4} = 0.974 = 97.4\%$$

Therefore the percent of solar radiation incident on the glass in the wavelength range 0.32 to 2.8  $\mu\text{m}$  is:

$$\frac{E_b(\lambda_1 T \rightarrow \lambda_2 T)}{\sigma T^4} = 97.4 - 45.3 = 52.1\%$$

The percentage of radiation transmitted through the glass is  $0.9 \times 52.1 = 46.9\%$ .

For the outgoing infrared radiation at 300K we have:

$$\lambda_1 T = 0.32 \times 300 = 96 \text{ } \mu\text{m-K}$$

$$\lambda_2 T = 2.8 \times 300 = 840 \text{ } \mu\text{m-K}$$

From Table 2.4 we get:

$$\frac{E_b(0 \rightarrow \lambda_1 T)}{\sigma T^4} = 0.0 = 0\%$$

$$\frac{E_b(0 \rightarrow \lambda_2 T)}{\sigma T^4} = 0.0001 = 0.01\%$$

The percentage of radiation emitted is  $0.9 \times 0.01 = 0.009\%$

**2.19** A 30 m<sup>2</sup> flat plate solar collector is absorbing radiation at a rate of 900 W/m<sup>2</sup>. The environment temperature is 25°C and the collector emissivity is 0.85. Neglecting conduction and convection losses, calculate the equilibrium temperature of the collector and the net radiation exchange with the surroundings.

To be in equilibrium the collector losses energy at the rate it absorbs energy. The energy emitted from the collector can be obtained from Eq. (2.71) as:  $Q_{12} = A_1 h_r (T_1 - T_2)$ . The radiation heat transfer coefficient is given by Eq. (2.75):  $h_r = \epsilon_1 \sigma (T_1 + T_2)(T_1^2 + T_2^2)$ . So we have two equations with two unknowns  $T_1$  and  $h_r$ . Using the data given  $T_1 = 403.7\text{K} = 130.7^\circ\text{C}$  and  $h_r = 8.515 \text{ W/m}^2\text{-K}$ . The net radiation exchanged with the surroundings is  $900 \times 30 = 27 \text{ kW}$ , estimated also from  $Q = A \epsilon \sigma (T_1^4 - T_2^4)$ .

**2.20** Two large parallel plates are maintained at 500 K and 350 K, respectively. The hotter plate has an emissivity of 0.6 and the colder one 0.3. Calculate the net radiation heat transfer between the plates.

Using Eq. (2.67):

$$Q/A = \frac{\sigma (T_1^4 - T_2^4)}{(1/\epsilon_1) + (1/\epsilon_2) - 1} = \frac{5.67 \times 10^{-8} (500^4 - 350^4)}{(1/0.6) + (1/0.3) - 1} = 673.2 \text{ W/m}^2$$

**2.21** Find the direct normal and horizontal extraterrestrial radiation at 2:00 pm solar time on February 21 for 40°N latitude and the total solar radiation on an extraterrestrial horizontal surface for the day.

From Table 2.1 for February 21, day number = 52. From Eq. (2.5) on February 21,  $\delta = -11.2^\circ$ . At 2:00 pm  $h = 30^\circ$  and from Eq. (2.15)  $h_{ss} = 80.4^\circ$ . From Eq. (2.77):

$$G_{\text{on}} = G_{\text{sc}} \left[ 1 + 0.033 \cos \left( \frac{360N}{365} \right) \right] = 1366 \left[ 1 + 0.033 \cos \left( \frac{360 \times 52}{365} \right) \right] = 1394.2 \text{ W/m}^2$$

From Eq. (2.12),  $\cos(\Phi) = \sin(L) \sin(\delta) + \cos(L) \cos(\delta) \cos(h)$   
 $= \sin(40) \sin(-11.2) + \cos(40) \cos(-11.2) \cos(30) = 0.526$  or  $\Phi = 58.3^\circ$ .

From Eq. (2.78):  $G_{oH} = G_{on} \times \cos(\Phi) = 1394.2 \times 0.526 = 733.3 \text{ W/m}^2$ .

From Eq. (2.79):

$$H_o = \frac{24 \times 3600 \times 1366}{\pi} \left[ 1 + 0.033 \cos\left(\frac{360 \times 52}{365}\right) \right] \left[ \cos(40) \cos(-11.2) \sin(80.4) + \left(\frac{\pi \times 80.4}{180}\right) \sin(40) \sin(-11.2) \right] = 21.69 \frac{\text{MJ}}{\text{m}^2}$$

**2.22** Estimate the average hourly diffuse and total solar radiation incident on a horizontal surface for Rome, Italy, on March 10 at 10:00 am and 1:00 pm solar times if the monthly average daily total radiation is  $18.1 \text{ MJ/m}^2$ .

From Table 2.1 at March 10,  $N=69$ . At 10:00 am  $h=-30^\circ$  and at 1:00 pm  $h=15^\circ$ . From Eq. (2.5)  $\delta=-4.8^\circ$ . From Exercise 2.5, Rome is at latitude of  $41.54^\circ\text{N}$ . From Eq. (2.15)  $h_{ss}=85.7^\circ$ .

From Table 2.5 at  $40^\circ\text{N}$  latitude  $\bar{H}_o = 27.4 \text{ MJ/m}^2$ . From Eq. (2.82):

$$\bar{K}_T = \frac{\bar{H}}{\bar{H}_o} = \frac{18.1}{27.4} = 0.661. \text{ Using Eq. (2.105a):}$$

$$\frac{\bar{H}_D}{\bar{H}} = 1.390 - 4.027(0.661) + 5.531(0.661)^2 - 3.108(0.661)^3 = 0.247$$

Therefore:  $\bar{H}_D = 0.247 \bar{H} = 0.247(18.1) = 4.47 \text{ MJ/m}^2\text{-day}$

From Eq. (2.83) at 10:00 am:

$$r_d = \left(\frac{\pi}{24}\right) \frac{\cos(h) - \cos(h_{ss})}{\sin(h_{ss}) - \left(\frac{2\pi h_{ss}}{360}\right) \cos(h_{ss})} = \left(\frac{\pi}{24}\right) \frac{\cos(-30) - \cos(85.7)}{\sin(85.7) - \left(\frac{2\pi \times 85.7}{360}\right) \cos(85.7)} = 0.117$$

Similarly at 1:00 pm.  $r_d = 0.132$

From Eqs. (2.84b) and (2.84c):

$$\alpha = 0.409 + 0.5016 \sin(h_{ss}-60) = 0.409 + 0.5016 \sin(85.7-60) = 0.626$$

$$\beta = 0.6609 - 0.4767 \sin(h_{ss}-60) = 0.6609 - 0.4767 \sin(85.7-60) = 0.454$$

From Eq. (2.84a) at 10:00 am:

$$r = \frac{\pi}{24} (\alpha + \beta \cos(h)) \frac{\cos(h) - \cos(h_{ss})}{\sin(h_{ss}) - \left(\frac{2\pi h_{ss}}{360}\right) \cos(h_{ss})}$$

$$= \frac{\pi}{24} (0.626 + 0.454 \cos(-30)) \frac{\cos(-30) - \cos(85.7)}{\sin(85.7) - \left(\frac{2\pi \times 85.7}{360}\right) \cos(85.7)} = 0.119$$

Similarly at 1:00 pm.  $r = 0.141$

Therefore, at 10:00 am average hourly total radiation is  $0.119 \times 18.1 = 2.1539 \text{ MJ/m}^2 = 2153.9 \text{ kJ/m}^2$  and average hourly diffuse radiation is  $0.117 \times 4.47 = 0.5230 \text{ MJ/m}^2 = 523 \text{ kJ/m}^2$ . At 1:00 pm average hourly total radiation is  $0.141 \times 18.1 = 2.5521 \text{ MJ/m}^2 = 2552.1 \text{ kJ/m}^2$  and average hourly diffuse radiation is  $0.132 \times 4.47 = 0.590 \text{ MJ/m}^2 = 590 \text{ kJ/m}^2$ .

**2.23** Calculate the beam and total radiation tilt factors and the beam and total radiation incident on a surface titled at  $45^\circ$  toward the equator one hour after local solar noon on

April 15. The surface is located at 40°N latitude and the ground reflectance is 0.25. For that day, the beam radiation at normal incidence is  $G_B = 710 \text{ W/m}^2$  and diffuse radiation on horizontal is  $G_D = 250 \text{ W/m}^2$ .

From Table 2.1 for April 15,  $N=105$  and  $\delta=9.41^\circ$ . For one hour after local solar noon  $h=15^\circ$ . From Eq. (2.90a):

$$R_B = \frac{\sin(L - \beta)\sin(\delta) + \cos(L - \beta)\cos(\delta)\cos(h)}{\sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h)}$$

$$= \frac{\sin(40 - 45)\sin(9.41) + \cos(40 - 45)\cos(9.41)\cos(15)}{\sin(40)\sin(9.41) + \cos(40)\cos(9.41)\cos(15)} = 1.12$$

From the data given  $G = G_B + G_D = 710 + 250 = 960 \text{ W/m}^2$ . From Eq. (2.99):

$$R = \frac{G_B}{G} R_B + \frac{G_D}{G} \left( \frac{1 + \cos(\beta)}{2} \right) + \rho_G \left( \frac{1 - \cos(\beta)}{2} \right)$$

$$= \frac{710}{960} 1.12 + \frac{250}{960} \left( \frac{1 + \cos(45)}{2} \right) + 0.25 \left( \frac{1 - \cos(45)}{2} \right) = 1.09$$

Therefore, from Eq. (2.88) the beam radiation on tilted surface is  $G_{Bt} = G_B R_B = 710 \times 1.12 = 795 \text{ W/m}^2$ . Similarly from Eq. (2.99) the total radiation on tilted surface is  $G_t = G \cdot R = 960 \times 1.09 = 1046 \text{ W/m}^2$ .

**2.24** For a south-facing surface located at 45°N latitude and tilted at 30° from horizontal, calculate the hourly values of the beam radiation tilt factor on September 10.

From Table 2.1 for September 10,  $N=253$  and  $\delta=4.22^\circ$ . The estimation can be done using Eq. (2.90a) for each hour of the day as shown in previous Exercise. The actual estimation is done at the middle of each hour considered and the hour angle is estimated using Eq. (2.9). The results are shown in the following table.

Time	Actual time used in Eq. (2.90a)	$h$ (°)	$R_B$
7-8	7:30	-67.5	1.204
8-9	8:30	-52.5	1.258
9-10	9:30	-37.5	1.281
10-11	10:30	-22.5	1.292
11-12	11:30	-7.5	1.297
12-13	12:30	7.5	1.297
13-14	13:30	22.5	1.292
14-15	14:30	37.5	1.281
15-16	15:30	52.5	1.258
16-17	16:30	67.5	1.204
17-18	17:30	82.5	1.005
18-19	18:30	97.5	2.666

**2.25** A collector located in Berlin, Germany, is tilted at 50° and receives a monthly average daily total radiation  $\bar{H}$  equal to 17 MJ/m<sup>2</sup>-day. Determine the monthly mean beam and total radiation tilt factors for October for an area where the ground reflectance is 0.2. Estimate also the monthly average daily total solar radiation on the surface.

From the web site shown in Exercise 2.5 the latitude of Berlin Germany is  $52.5^\circ\text{N}$ . From Table 2.5 the monthly average daily total insolation on an extraterrestrial horizontal surface  $\bar{H}_o$  is equal (by interpolation) to  $15.4 \text{ MJ/m}^2$ . Therefore from Eq. (2.82):

$$\bar{K}_T = \frac{\bar{H}}{\bar{H}_o} = \frac{17}{15.4} = 1.104$$

By considering the average day of October 15 ( $N=288$ ) shown in Table 2.5 the declination is  $\delta = -9.6^\circ$ . Finally, the sunset hour angle from Eq. (2.15) is:

$$h_{ss} = \cos^{-1}[-\tan(L)\tan(\delta)] = \cos^{-1}[-\tan(52.5)\tan(-9.6)] = 77.3^\circ$$

From Eq. (2.105b):

$$\begin{aligned} \frac{\bar{H}_D}{\bar{H}} &= 0.775 + 0.00653(h_{ss} - 90) - [0.505 + 0.00455(h_{ss} - 90)]\cos(115\bar{K}_T - 103) \\ &= 0.775 + 0.00653(77.3 - 90) - [0.505 + 0.00455(77.3 - 90)]\cos(115 \times 1.104 - 103) = 0.283 \end{aligned}$$

$$\begin{aligned} \text{From Eq. (2.109): } h'_{ss} &= \min \{h_{ss}, \cos^{-1}[-\tan(L-\beta)\tan(\delta)]\} \\ &= \min \{77.3, \cos^{-1}[-\tan(52.5-50)\tan(-9.6)]\} = \min \{77.3, 89.5\} = 77.3^\circ \end{aligned}$$

From Eq. (1.108):

$$\begin{aligned} \bar{R}_B &= \frac{\cos(L-\beta)\cos(\delta)\sin(h'_{ss}) + (\pi/180)h'_{ss}\sin(L-\beta)\sin(\delta)}{\cos(L)\cos(\delta)\sin(h_{ss}) + (\pi/180)h_{ss}\sin(L)\sin(\delta)} \\ &= \frac{\cos(52.5-50)\cos(-9.6)\sin(77.3) + (\pi/180)77.3\sin(52.5-50)\sin(-9.6)}{\cos(52.5)\cos(-9.6)\sin(77.3) + (\pi/180)77.3\sin(52.5)\sin(-9.6)} = 2.337 \end{aligned}$$

Finally, from Eq. (2.107):

$$\begin{aligned} \bar{R} &= \left(1 - \frac{\bar{H}_D}{\bar{H}}\right)\bar{R}_B + \frac{\bar{H}_D}{\bar{H}}\left(\frac{1 + \cos(\beta)}{2}\right) + \rho_G\left(\frac{1 - \cos(\beta)}{2}\right) \\ &= (1 - 0.283)2.337 + 0.283\left(\frac{1 + \cos(50)}{2}\right) + 0.2\left(\frac{1 - \cos(50)}{2}\right) = 1.944 \end{aligned}$$

The average daily total radiation on the tilted surface for October is:

$$\bar{H}_t = \bar{R}\bar{H} = 1.944(17) = 33 \text{ MJ/m}^2\text{-day}$$