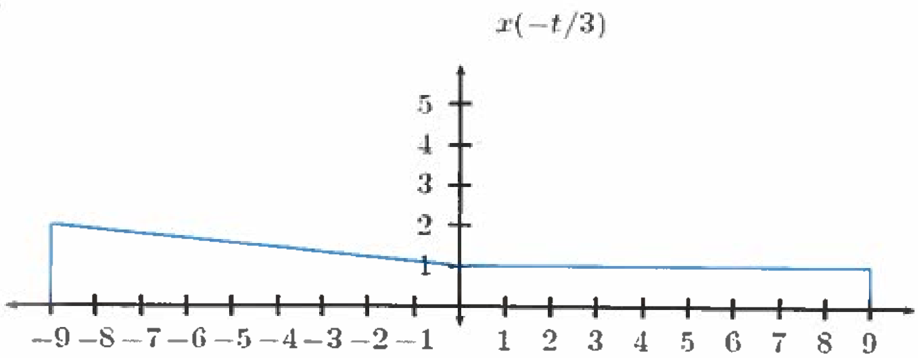
Chapter 2 solutions

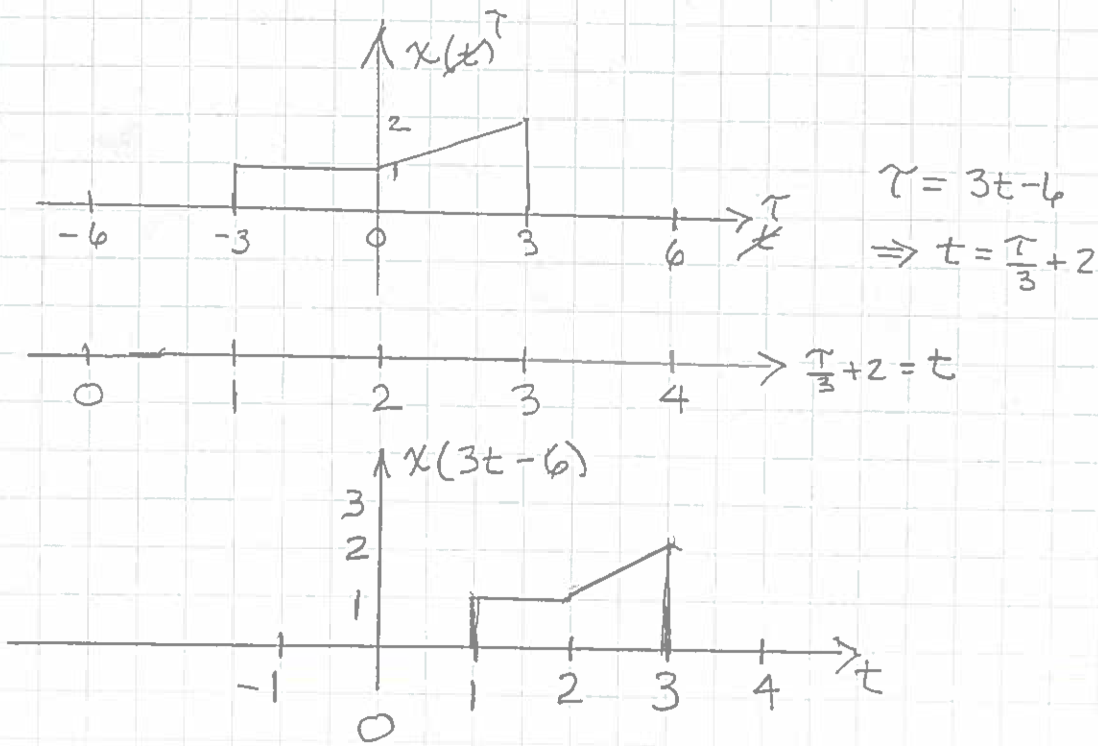
2.1

(a)

(i)

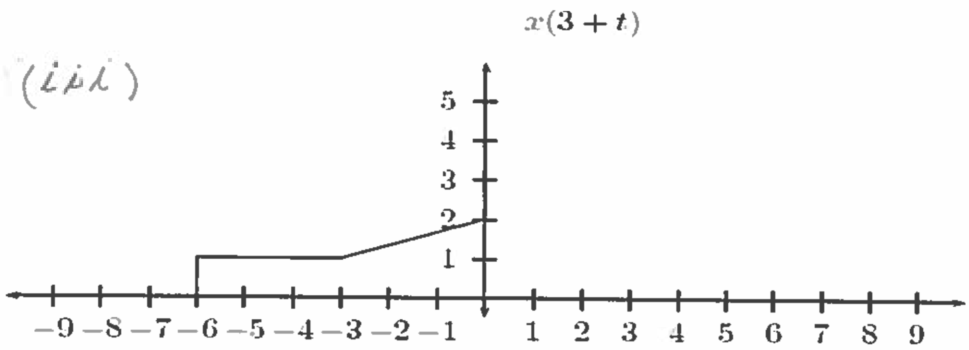


(ii)

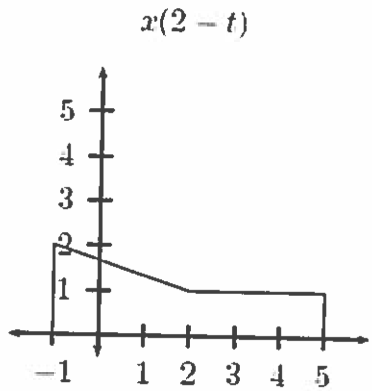


PROBLEM 2.1(a) continued

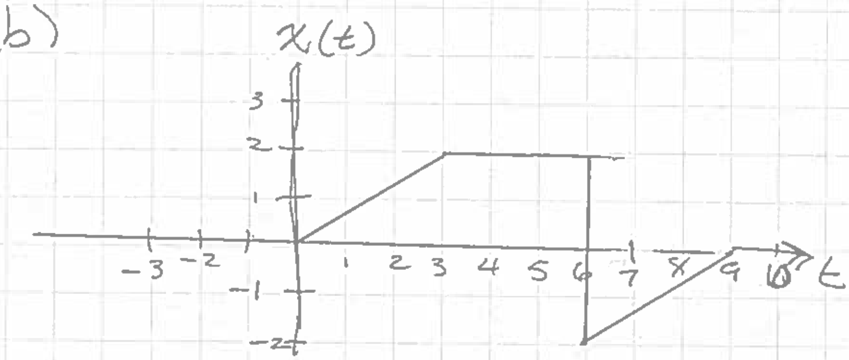
(iii)



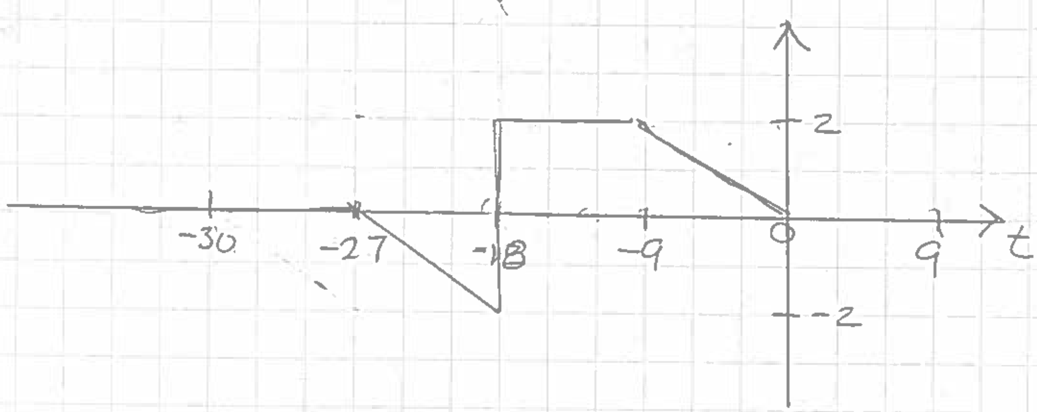
(iv)



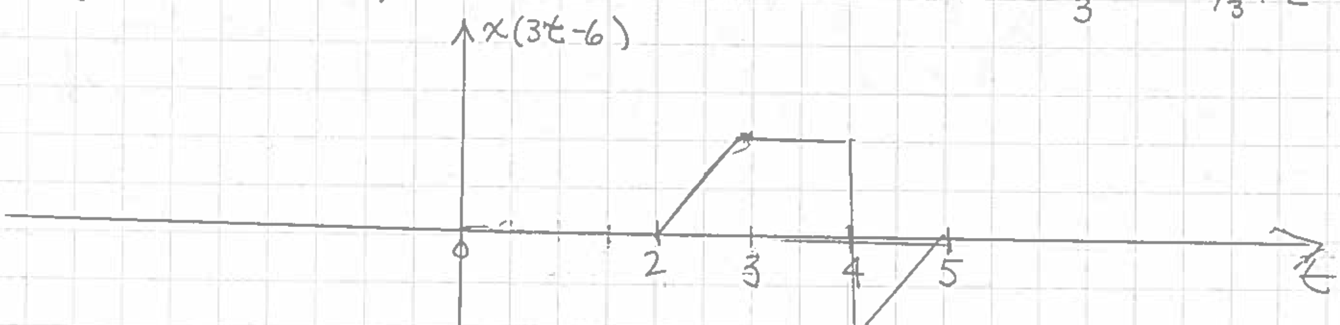
2.1(b)



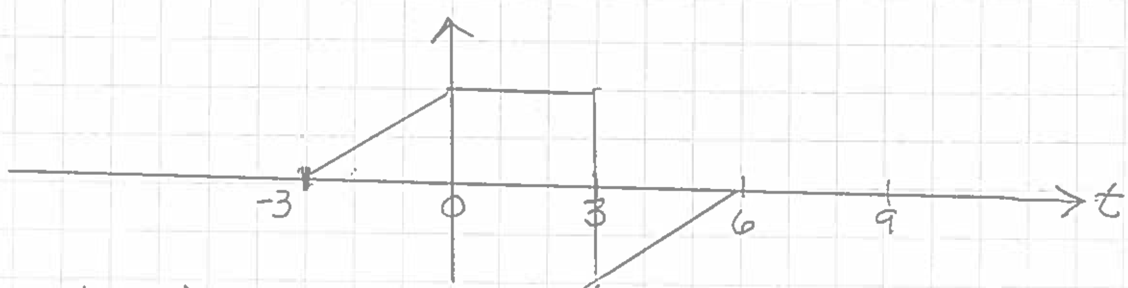
(i) $x(-t/3) = x(\tau) \Rightarrow \tau = -t/3 \Rightarrow t = -3\tau$



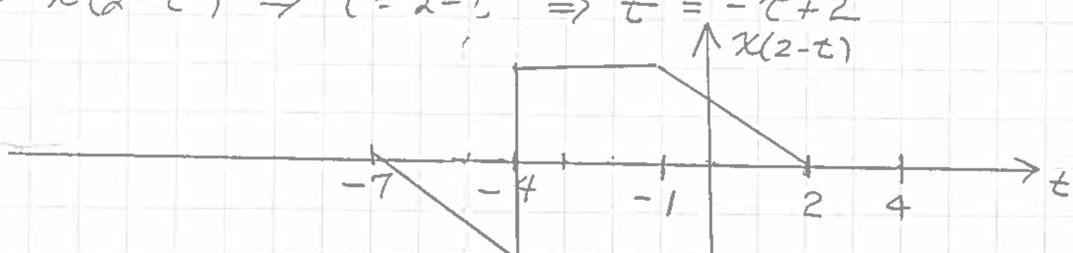
(ii) $x(3t-6) = x(\tau) \Rightarrow \tau = 3t-6 \Rightarrow t = \frac{\tau+6}{3} = \tau/3 + 2$



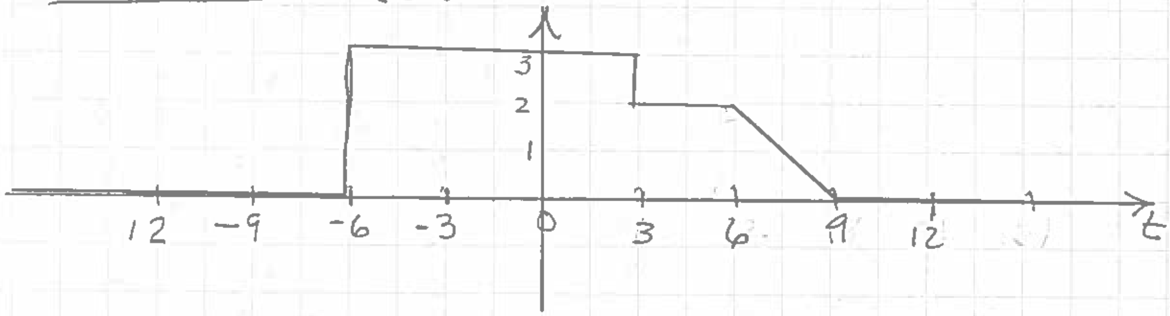
(iii) $x(3+t) = x(\tau) \Rightarrow \tau = t+3 \Rightarrow t = \tau - 3$



(iv) $x(2-t) \Rightarrow \tau = 2-t \Rightarrow t = -\tau + 2$



Problem 2.1(c)



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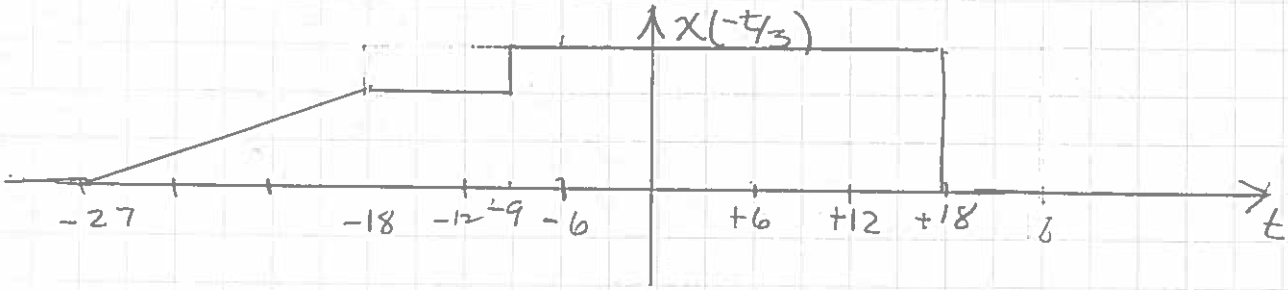

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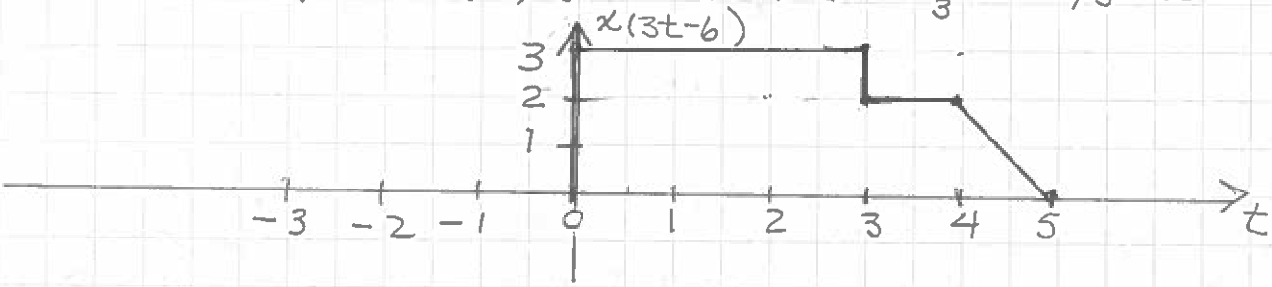

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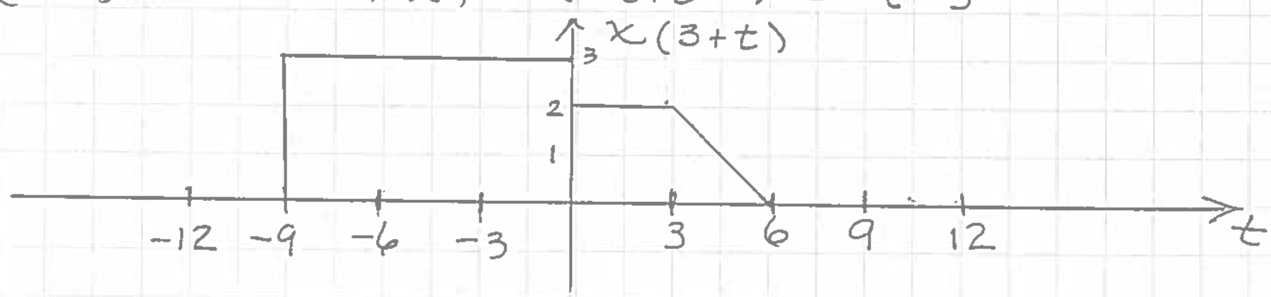

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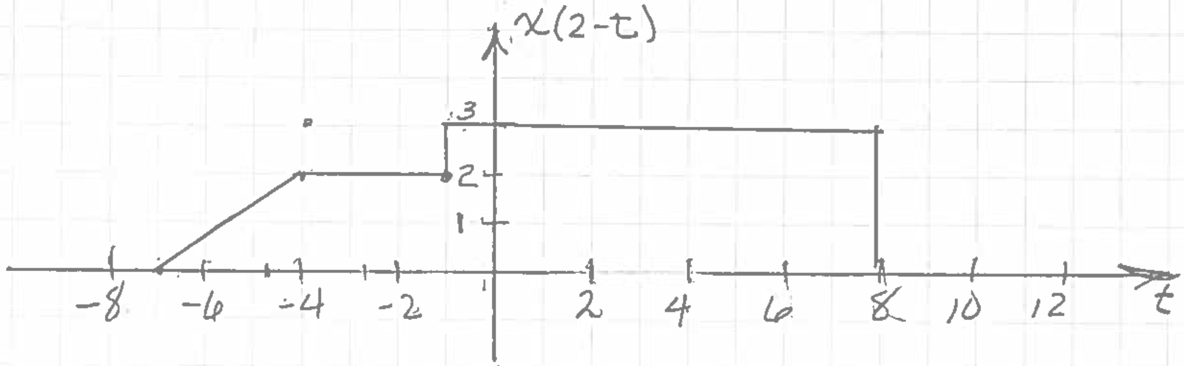
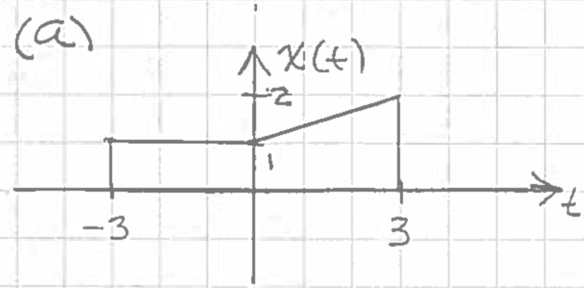
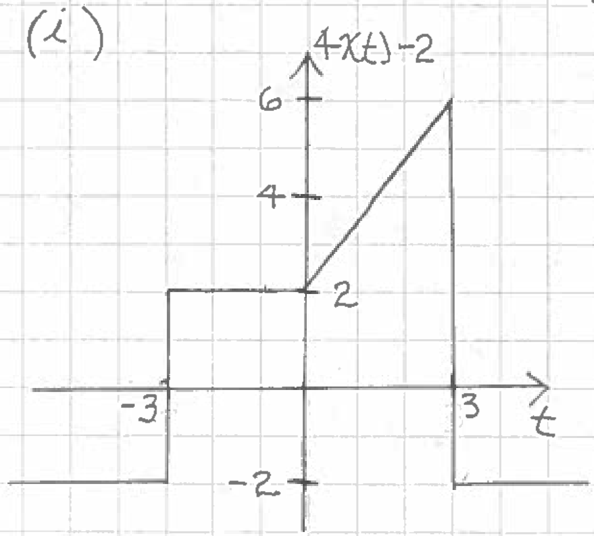


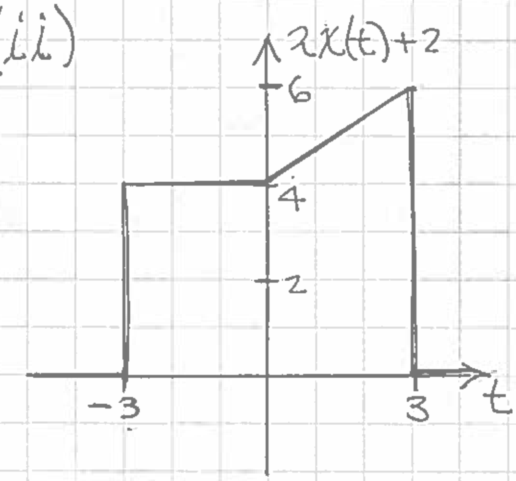
PROBLEM 2.2 (a)



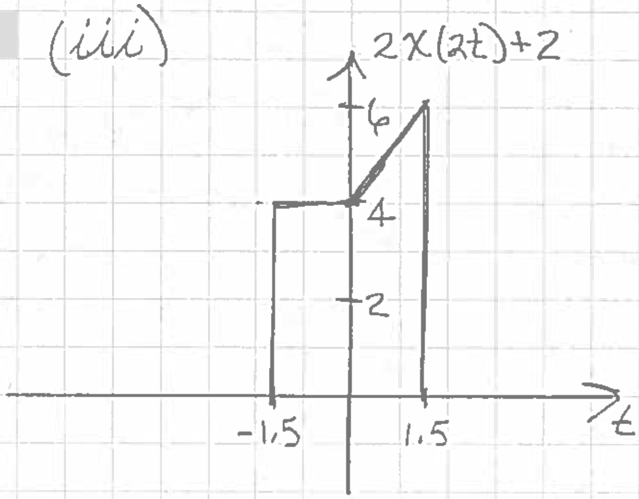
(i)



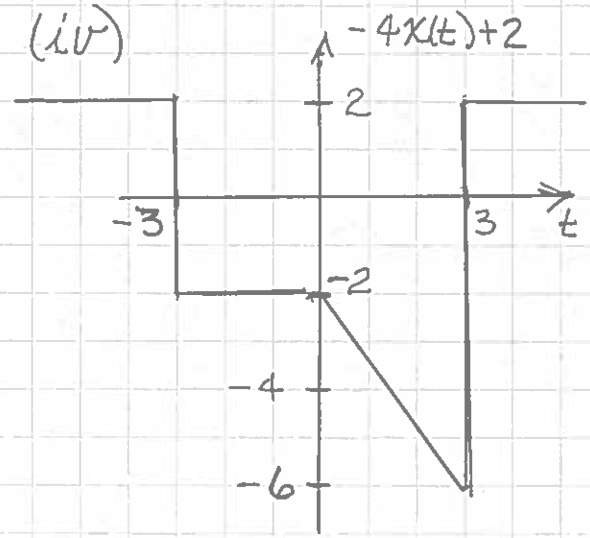
(ii)



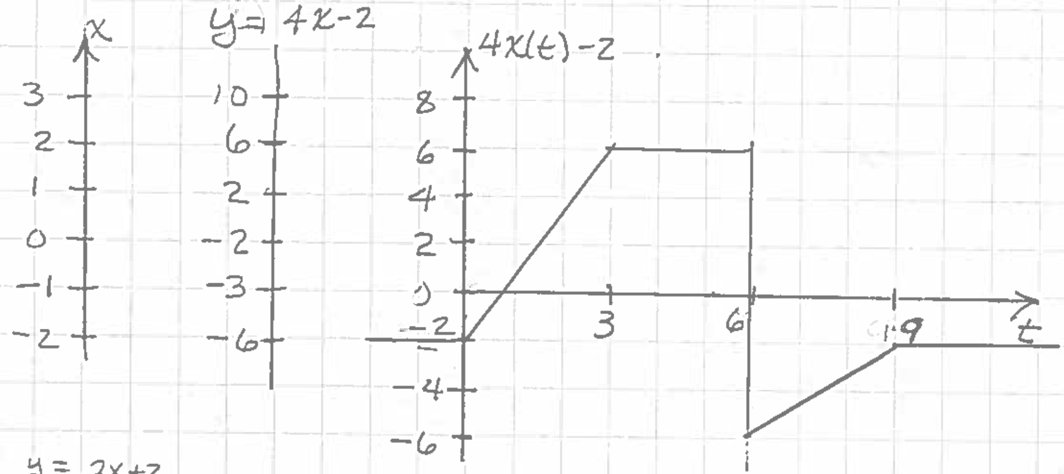
(iii)



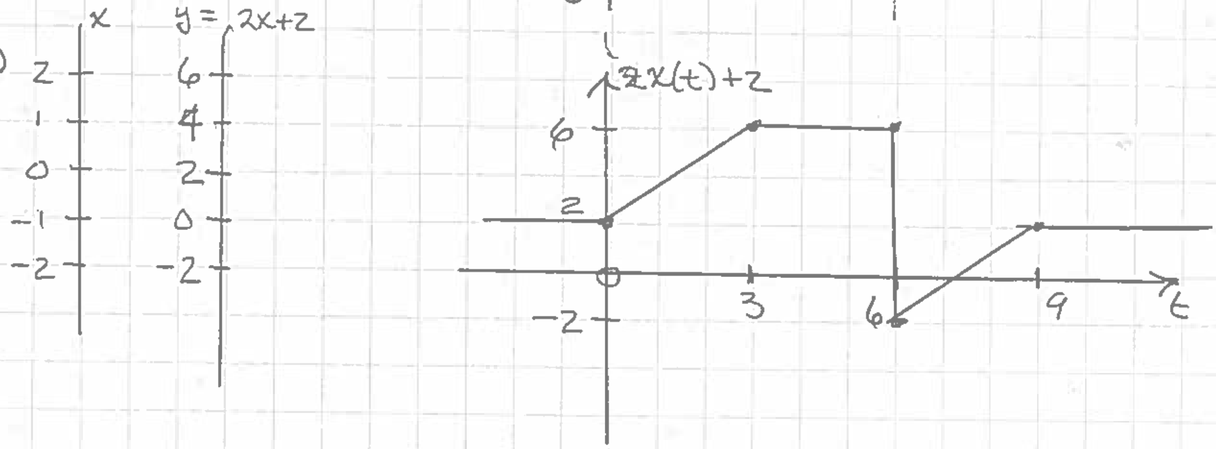
(iv)



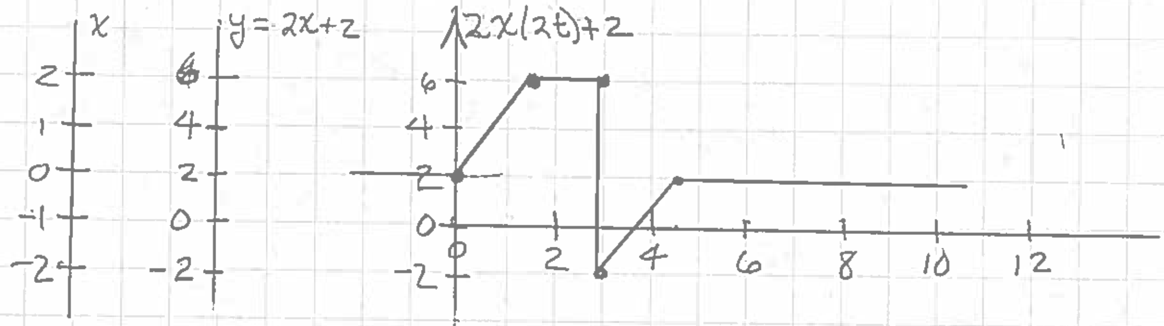
2.2(b)(i)



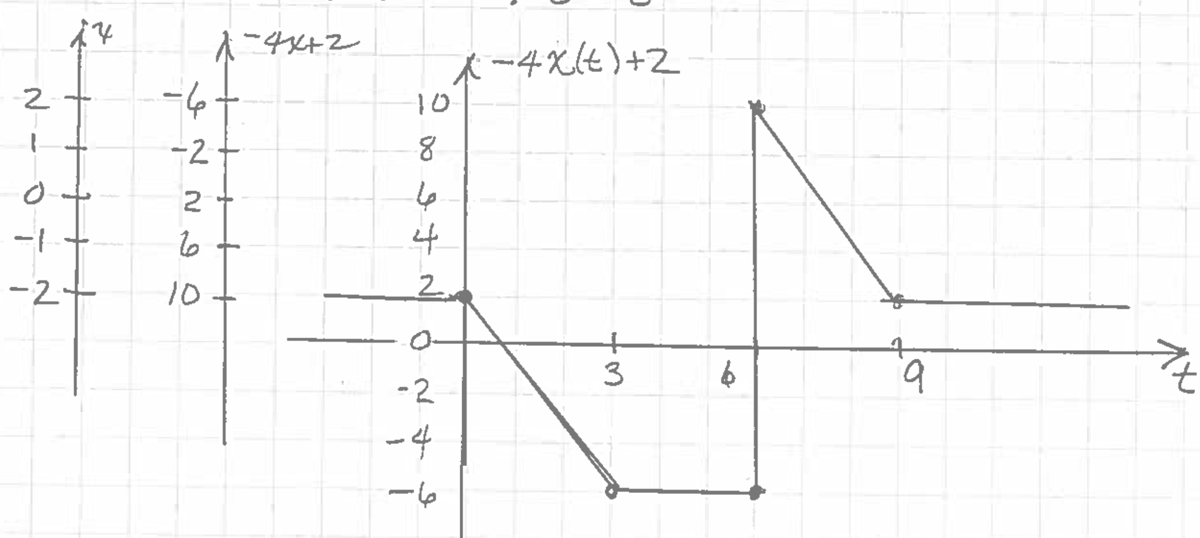
(ii)



(iii) $y(\tau) = 2x(2t)+2 \Rightarrow \tau = 2t \Rightarrow t = \tau/2$

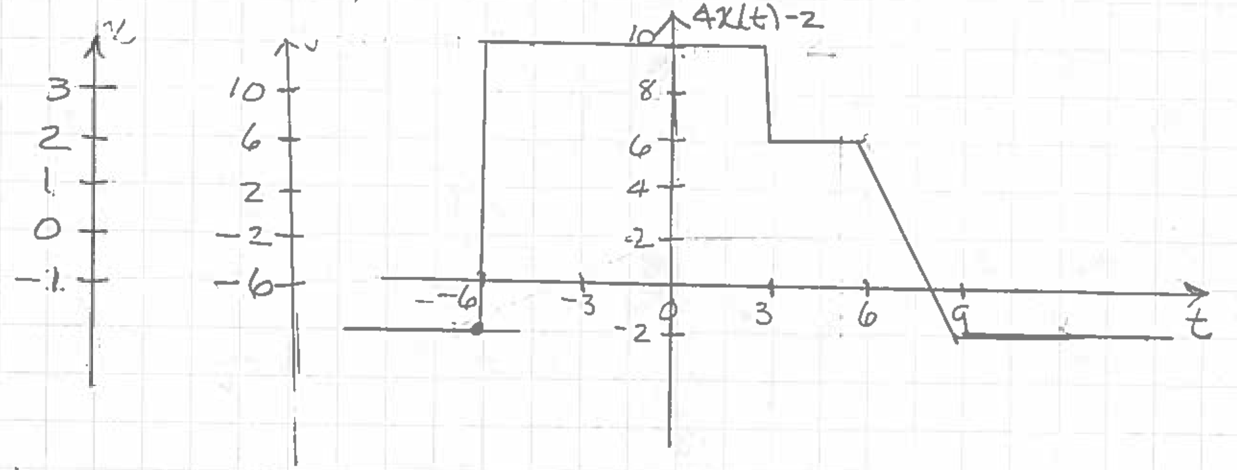


(iv) $y(\tau) = -4x(t)+2 \Rightarrow \tau = t$

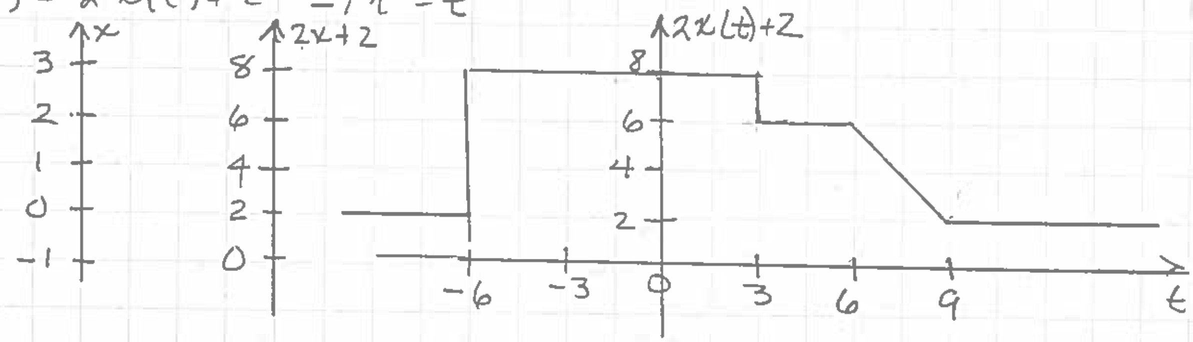


P2.2(c)

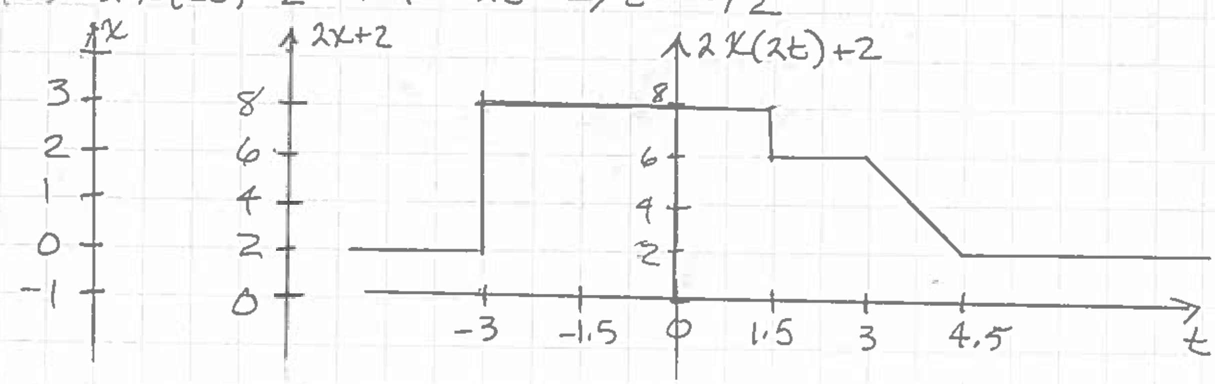
(i) $y(\tau) = 4x(t) - 2 \quad , \quad \Rightarrow \tau = t$



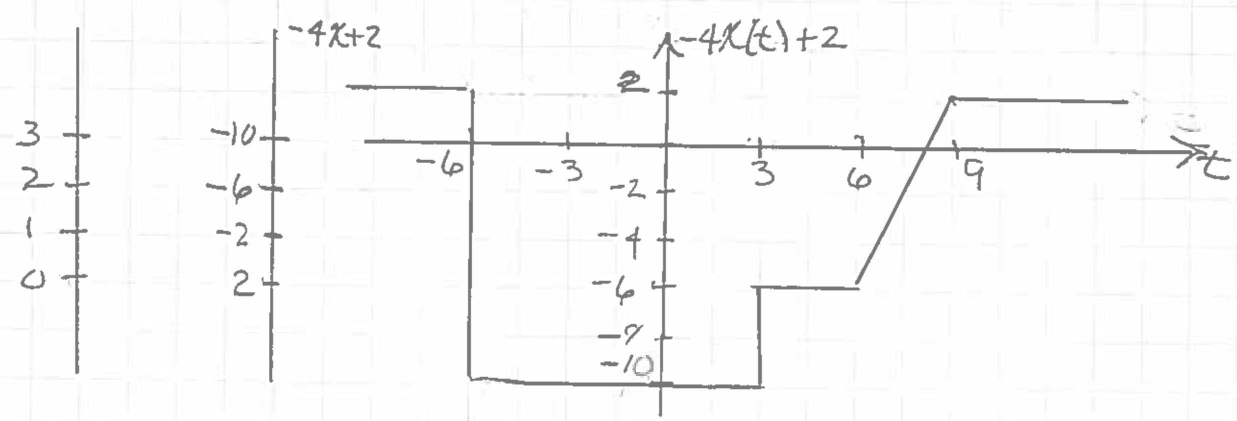
(ii) $y(\tau) = 2x(t) + 2 \quad \Rightarrow \tau = t$



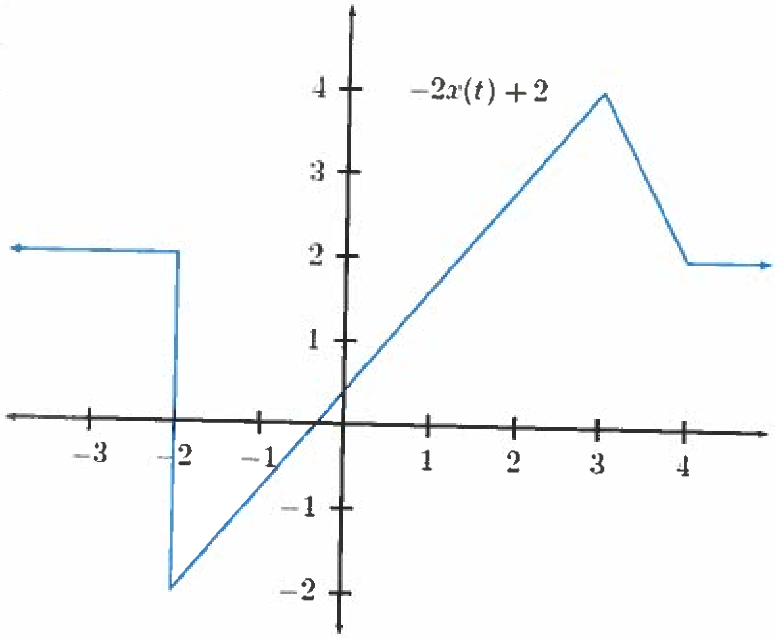
(iii) $y(\tau) = 2x(2t) + 2 \quad \Rightarrow \tau = 2t \quad \Rightarrow t = \tau/2$

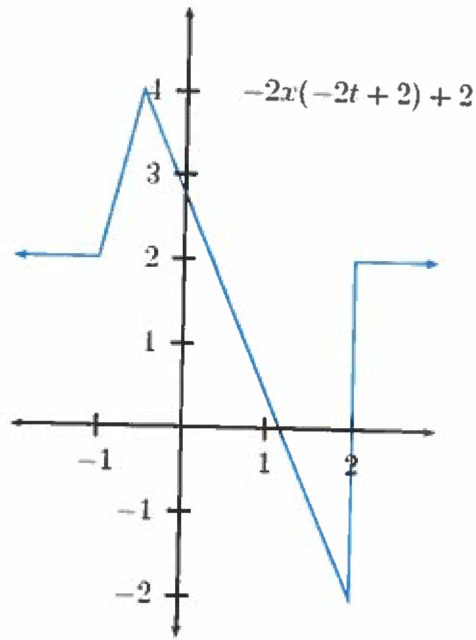


(iv) $y(\tau) = -4x(t) + 2 \quad \Rightarrow \tau = t$



PROBLEM 2.3





(a)

$$y(t) = -2(x(-2t+2))+2$$

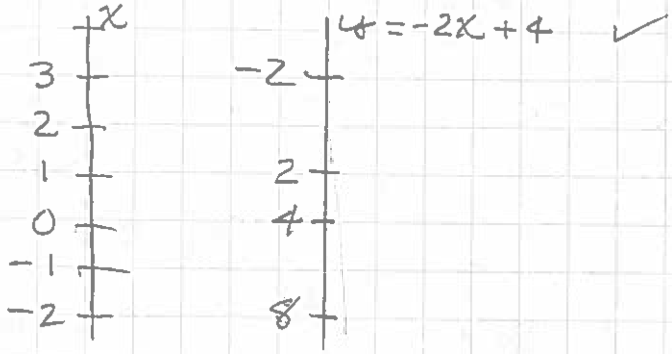
(b)

t	$y(t)$	$-2t+2$	$-2(x(-2t-1))+2$
-0.5	4	3	4
-1	2	4	2
1	0.4	0	0.4

Solution P 2.4

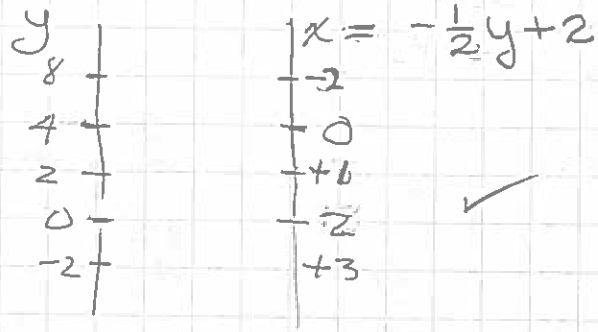
(a) $y(t) = -2x(-t/2 + 2) + 4$

$\Rightarrow \tau = -t/2 + 2 \Rightarrow t = -2\tau + 4$



(b) $x(t) = -\frac{1}{2} y(-2t + 4) + 2 \Rightarrow \tau = -2t + 4$

$\Rightarrow t = -\frac{1}{2}\tau + 2$

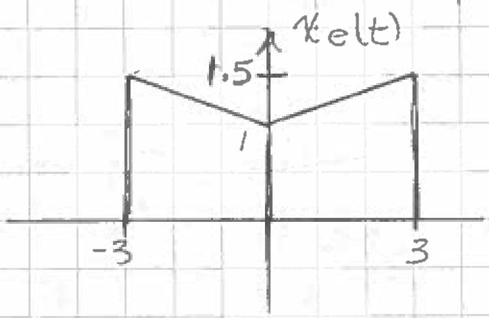


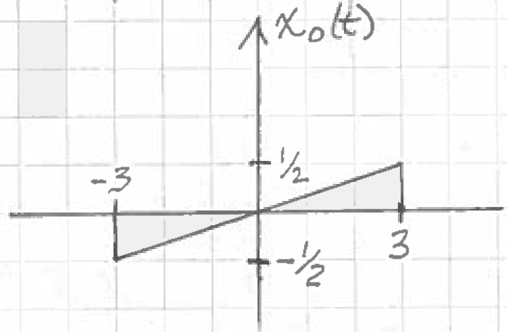
Problem 2.5

(a) $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ (2.13)

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ (2.14)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
>3	0	0	0	0
3	2	1	3/2	1/2
1.5	1.5	1	1.25	0.25
0	1	1		0
-1.5		1.5	1.25	-0.25
-3		2	3/2	$-1/2$
<-3	0	0	0	0



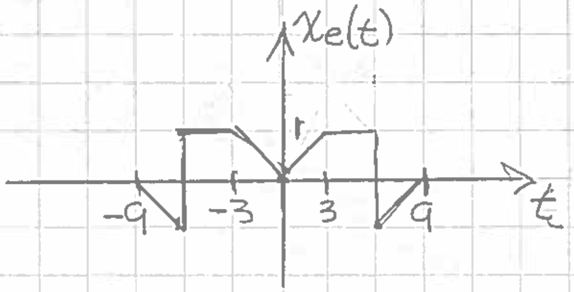


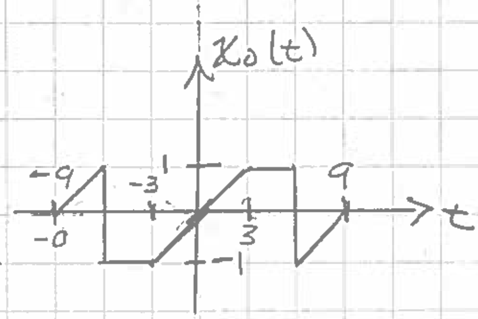
verify $x_o(t) + x_e(t) = x(t)$

Problem 2.5

(b)

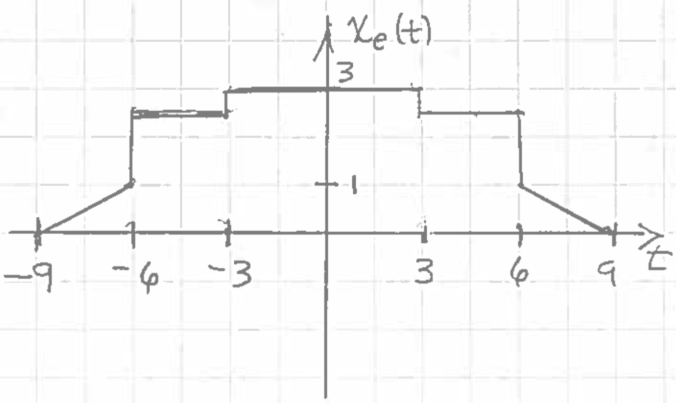
t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
>9	0	0	0	0
6^+	-2	0	-1	-1
6^-	2	0	1	1
3	2	0	1	1
0	0	0	0	0
-3	0	2	1	-1
-6^+	0	-2	-1	1

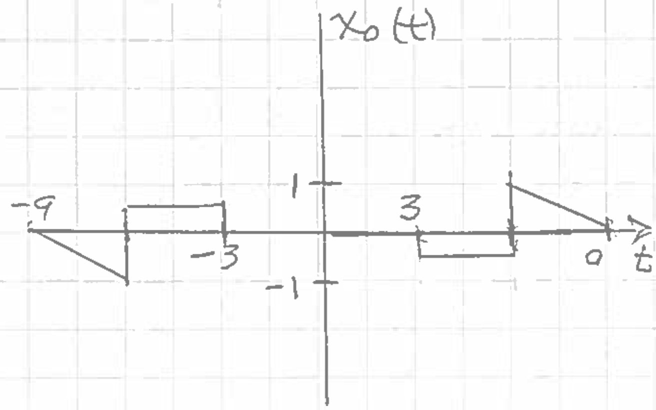




(c)

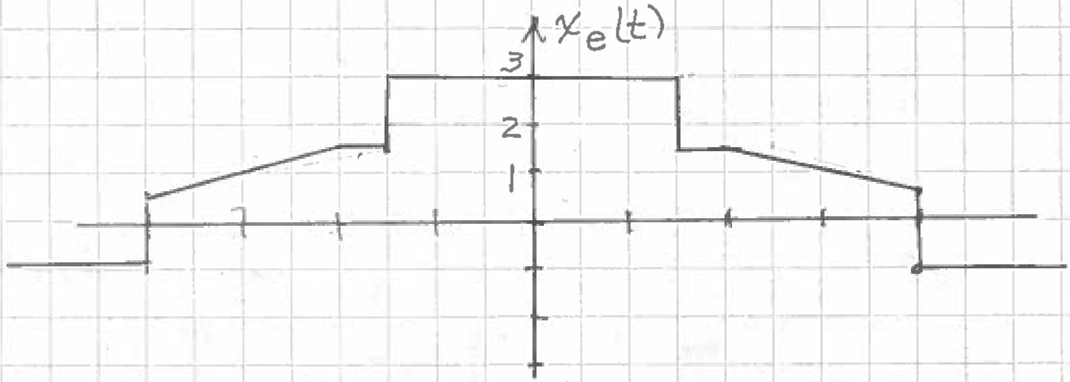
t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
≥ 9	0	0	0	0
6^+	2	0	1	1
6^-	2	3	2.5	-0.5
3^-	3	3	3	0
0	3	3	3	0
-3^+	3	3	3	0
-6^+	3	2	2.5	0.5
-6^-	0	2	1	-1

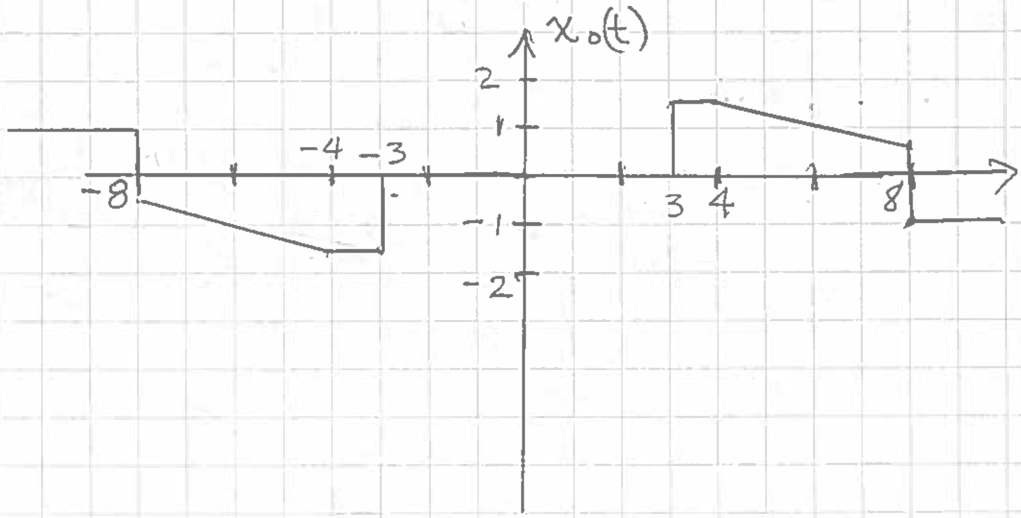




Problem 2.5 (d)

t	$x(t)$	$x(-t)$	$x_e(t)$	$x_o(t)$
$\geq 8^+$	-2	0	-1	-1
8^-	1	0	0.5	0.5
4	3	0	1.5	1.5
3^+	3	0	1.5	1.5
3^-	3	3	3	0
-3^+	3	3	3	0
-3^-	0	3	1.5	-1.5
-4	0	3	1.5	-1.5
-8^+	0	1	0.5	-0.5
$\leq 8^-$	0	-2	-1	$+1$





Problem 2.6

(a) $x(t) = -4t \Rightarrow x(-t) = 4t$

$x(t) = -x(-t) \quad \therefore \; x(t)$ is odd

(b) $x(t) = e^{-|t|} \Rightarrow x(-t) = e^{-|-t|} = e^{-|t|}$

$x(t) = x(-t) \quad \therefore \; x(t)$ is even

(c) $x(t) = 5\cos 3t \Rightarrow x(-t) = 5\cos 3(-t)$

$= 5\cos -3t = 5\cos 3t$

$x(t) = x(-t) \quad \therefore \; x(t)$ is even

(d) $x(t) = \sin(3t + \frac{3\pi}{2}) = \sin(3[t + \pi/2])$

$x(t) = -\cos(3t)$

$x(t) = x(-t) \quad \therefore \; x(t)$ is even

(e) $x(t) = u(t) - u(-t) \Rightarrow x(-t) = u(-t) - u(t)$

$= -[u(t) - u(-t)]$

$x(t) = -x(-t) \quad \therefore \; x(t)$ is odd

(f) $x(t) = -u(t-1) + u(-t-1)$

$x(-t) = -u(-t-1) + u(t-1)$

$x(-t) = -x(t), \quad \therefore \; x(t)$ is odd

PROBLEM 2.7

(a) $\int_{-T}^{T} x_o(t) = \int_{-T}^{0} x_o(t)dt + \int_0^T x_o(t)dt \quad ; \; x_o(t) = -x_o(-t)$

$\therefore \int_{-T}^{0} x_o(t)dt = -\int_{-T}^{0} x_o(-t)dt \Big|_{t=-\tau} = \int_{-T}^{0} x_o(\tau)d\tau = -\int_0^T x_o(\tau)d\tau$

$\therefore \int_{-T}^{T} x_o(t)dt = 0$

(b) $\int_{-T}^{T} x(t)dt = \int_{-T}^{T} [x_e(t) + x_o(t)]dt = \int_{-T}^{T} x_e(t)dt$

and $A_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x_e(t)d\tau$

(c) $x_o(0)=-x_o(-0)= -x_o(0)$. The only number with $a=-a$ is $a=0$ so this implies $x_o(0)=0$.
$x(0)=x_e(0)+x_o(0)=x_e(0)$.

PROBLEM 2.8

(a) Let $z(t)$ be the sum of two even functions $x_1(t)$ and $x_2(t)$. To show that $z(t)$ is even, we need to show that $z(t) = z(-t)$ for all t. This is easy to show, since $z(t) = x_1(t)+x_2(t)$ and $z(-t) = x_1(-t)+x_2(-t)$ (since to get $z(-t)$ we just plug in $-t$ everywhere for t, which amounts to just plugging in $-t$ in $x_1(t)$ and $x_2(t)$). Now since $x_1(t)$ and $x_2(t)$ are even, by definition $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$ so $x_1(t) + x_2(t) = x_1(-t) + x_2(-t)$ so $z(t) = z(-t)$.

(b) Let $x_1(t)$ and $x_2(t)$ be two odd functions. Then $x_1(-t) + x_2(-t) = -x_1(t) + (-x_2(t)) = -(x_1(t) + x_2(t))$ which shows that $x_1(t) + x_2(t)$ is odd.

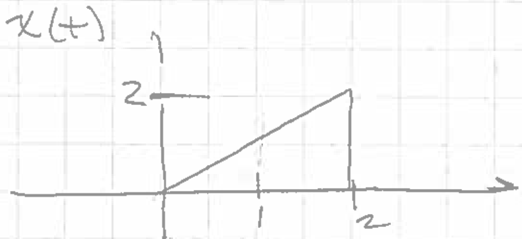
(c) Let $z(t) = x_1(t) + x_2(t)$ as in part a, where now $x_1(-t) = x_1(t)$ and $x_2(-t) = -x_2(t)$. We need to show that $z(t) \neq z(-t)$, $z(t) \neq -z(-t)$. Consider that $z(-t) = x_1(-t) + x_2(-t) = x_1(t) - x_2(t)$. In order to have $z(t)$ be even, we would therefore need to have $x_1(t) + x_2(t) = x_1(t) - x_2(t)$ for all t, which is equivalent to having $x_2(t) = -x_2(t)$ for all t, which is not possible for nonzero $x_2(t)$. Similarly, in order to have $z(t)$ be odd, we would need to have $z(t) = -z(t) \implies x_1(t) + x_2(t) = x_2(t) - x_1(t)$, which is not possible for nonzero $x_1(t)$. So the sum of an even and odd function must be neither even nor odd.

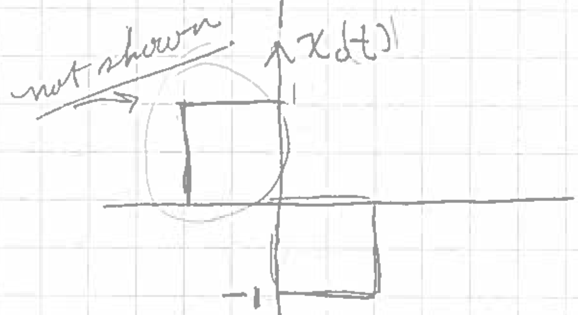
(d) Let $z(t) = x_1(t)x_2(t)$ where $x_1(t) = x_1(-t)$ and $x_2(t) = x_2(-t)$. Then $z(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = z(t)$ which shows that $z(t)$ is even.

(e) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = -x_2(-t)$. Clearly $z(t)$ is even because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))(-x_2(t)) = x_1(t)x_2(t) = z(t)$, which is the definition of evenness.

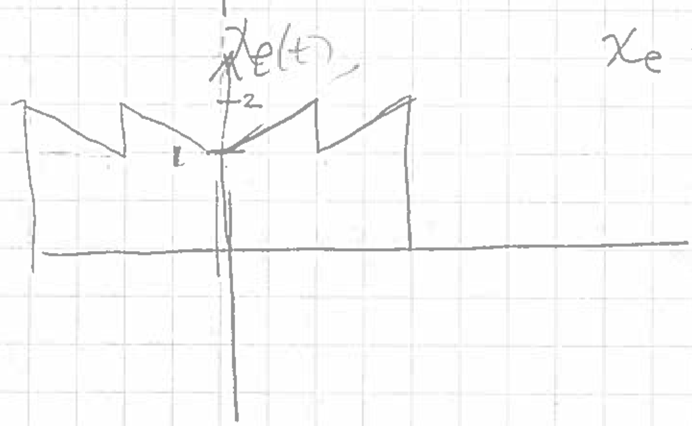
(f) Let $z(t) = x_1(t)x_2(t)$, where $x_1(t) = -x_1(-t)$ and $x_2(t) = x_2(-t)$. Clearly $z(t)$ is odd because $z(-t) = x_1(-t)x_2(-t) = (-x_1(t))\,x_2(t) = -x_1(t)x_2(t) = -z(t)$, which is the definition of oddness.

P 2.9

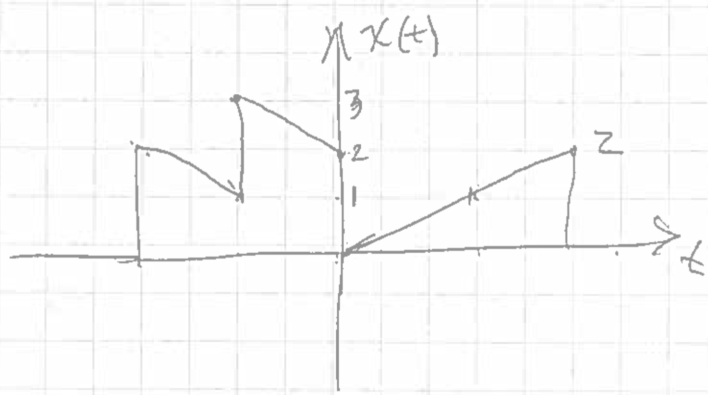




$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



$$x_e = \frac{x(t) + x(-t)}{2}$$



$$x(t) = x_o(t) + x_e(t)$$

PROBLEM 2.10

(a) $\sin(t) = \sin(t + n2\pi)$ for any integer n, so $7\sin(3t) = 7\sin(3t + n2\pi) = 7\sin\left(3(t + n\frac{2\pi}{3})\right)$; therefore $x(t)$ is periodic with fundamental period $T_0 = \frac{2\pi}{3}$ and fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = 3$.

(b) $\sin(8(t + \frac{2\pi}{8}) + 30) = \sin(8t + 2\pi + 30) = \sin(8t + 30)$.
$\omega_0 = 8$ and $T_0 = \frac{2\pi}{8} = \frac{\pi}{4}$.

(c) $e^{jt} = \cos(t) + j\sin(t)$ is periodic with fundamental period 2π, so e^{j2t} is periodic with fundamental period $\frac{2\pi}{2} = \pi$, and fundamental frequency $\omega_0 = 2$.

(d) $\cos 2t + \sin 5t$

$T_1 = \frac{2\pi}{2} = \pi$, $T_2 = \frac{2\pi}{5} \Rightarrow \frac{T_1}{T_2} = \frac{\pi}{2\pi/5} = 5/2$ ratio of integers

$T_0 = k_0 T_1 \therefore T_0 = 2\pi(s)$, (periodic) $\qquad k_0 = 2$

(e) $e^{-j(10t + \pi/3)} = e^{-j\pi/3} e^{-j10t} = (\cos\pi/3 - j\sin\pi/3)\, e^{-j10t}$

$= (0.5 + j0.866)\, e^{-j10t}$

$T_0 = \frac{2\pi}{10} = \pi/5\,(s)$; periodic

(f) $e^{j15t} - e^{j20t}$

e^{j15t} & e^{j20t} are both periodic

$T_1 = \frac{2\pi}{15}$, $T_2 = \frac{2\pi}{20} \Rightarrow \frac{T_1}{T_2} = \frac{2/15}{1/10} = \frac{20}{15}$ ratio of integers $\therefore$ periodic

$\frac{20}{15} = \frac{4}{3} \Rightarrow k_0 = 3 \therefore T_0 = 3T_1 = \frac{2\pi}{5}$ s

$\frac{2\pi/5}{2\pi/15} = 3$, $\frac{2\pi/5}{2\pi/20} = 4$

P 2.11

(a) $x(t) = \cos 3t + \sin 5t$

(b) $x(t) = \cos t + \sin \pi t$

(c) $x(t) = \cos 3t + \sin 9t$

(d) $x(t) = \cos 3\pi t + \sin 4\pi t + \cos(5t)$

(e) $x(t) = \cos 4\pi t + \sin 8\pi t + e^{j5\pi t}$

(f) $x(t) = \cos(8t + 30°) + e^{j2t} + \sin(3\pi t)$

solution

(a) $T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{5}$, $\frac{T_1}{T_2} = \frac{2\pi/3}{2\pi/5} = \frac{5}{3}$

$T_0 = 3T_1 = 2\pi$: periodic

(b) $T_1 = \frac{2\pi}{1}$, $T_2 = \frac{2\pi}{\pi} = 1$ $\quad \frac{T_1}{T_2} = 2\pi$ not a ration of integers

∴ not periodic

(c) $T_1 = \frac{2\pi}{3}$, $T_2 = \frac{2\pi}{9}$, $\frac{T_1}{T_2} = 9/3 = 3/1 \Rightarrow$ ration of integers

$T_0 = \frac{2\pi}{3}$, periodic

(d) $T_1 = \frac{2\pi}{3\pi} = 2/3$, $T_2 = \frac{2\pi}{4\pi} = \frac{1}{2}$, $T_3 = \frac{2\pi}{5}$

$\frac{T_1}{T_2} = \frac{2/3}{1/2} = \frac{4}{3}$, $\frac{T_1}{T_3} = \frac{2/3}{2\pi/5} = \frac{10}{6\pi} = \frac{5}{3\pi} \leftarrow$ NOT A RATIO OF INTEGERS ∴ SUM IS NOT PERIODIC

(e) $T_1 = \frac{2\pi}{4\pi} = 1/2$, $T_2 = \frac{2\pi}{8\pi} = 1/4$, $T_3 = \frac{2\pi}{5\pi} = \frac{2}{5}$

$\frac{T_1}{T_2} = \frac{1/2}{1/3} = \frac{3}{2}$, $\frac{T_1}{T_3} = \frac{1/2}{2/5} = 5/4$ both ratios of integers ∴ Sum periodic

lcm of denominators $= 4 \times 2 = 8 = k_0$

$T_0 = 8T_1 = 4$ s.

(f) $T_1 = 2\pi/3$, $T_2 = \frac{2\pi}{2}$, $T_3 = \frac{2\pi}{3\pi}$, $\frac{T_1}{T_3} = \frac{2\pi/3}{2/3} = \pi$ not rational ∴ Sum not periodic

2.12

(a) $x(t) = 5\sin(15t - 60^\circ) + 2\sin(7t)$

$x_1(t) = 5\sin(15t - 60^\circ)$ is periodic $\omega_1 = 15$ rad/s

$x_2(t) = 2\sin(7t)$ " " $\omega_2 = 7$ rad/s

$T_1 = \frac{2\pi}{15}$; $T_2 = \frac{2\pi}{7}$ $\frac{T_1}{T_2} = \frac{7}{15}$ ratio of integer ∴ Sum is periodic

$k_0 = 15 \Rightarrow T_0 = 15T_1 = \underline{2\pi}$ s

(b) $x_1(t) = 5\sin 5t$ is periodic $\omega_1 = 5$

$x_2(t) = 5e^{-j15t}$ is periodic $\omega_2 = 15$

$x_3(t) = \sin 7t$ " " $\omega_3 = 7$

$T_1 = 2\pi/7$, $T_2 = 2\pi/15$, $T_3 = \frac{2\pi}{5}$

$T_1/T_2 = 15/5 = 3/1$, $T_1/T_3 = 7/\underline{5}$ lcm denom = 5

$T_0 = 5T_1 = 2\pi$

(c) $x_1(t)$ is periodic $T_1 = \frac{2\pi}{\pi} = 2$

x_2 is periodic $T_2 = \frac{2\pi}{3}$

$\frac{T_1}{T_2} = \frac{2}{2\pi/3} = \frac{3}{\pi}$ not rational ∴ Sum not periodic

(d) $\cos 4\pi t$ is periodic w/ $T_1 = 2\pi/4\pi = 1/2$

$\sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t + n/2}{0.2}\right)$ is periodic with $T_1 = 0.5$ s

$4\sin\left(\frac{5\pi}{7}t + \pi/4\right)$ is periodic w/ $T_2 = \frac{2\pi}{5\pi/7} = \frac{14}{5}$

$\frac{T_1}{T_2} = \frac{1/2}{1/2} = 1$, $\frac{T_1}{T_3} = \frac{1/2}{14/5} = \frac{5}{28} \Rightarrow k_0 = 28$, $T_0 = 28T_1 = \underline{14}$ s

PROBLEM 2.13

(a) For $x_1(t) + x_2(t)$ to be periodic we need some number T such that $x_1(t+T) + x_2(t+T) = x_1(t) + x_2(t)$ for all t. This can only be true if $x_1(t+T) = x_1(t)$ and $x_2(t+T) = x_2(t)$, which can only be true if $T = k_1T_1$ and $T = k_2T_2$ (T is an integer multiple of both the periods). So we need there to be some integers k_1 and k_2 such that $k_1T_1 = k_2T_2 \implies \frac{T_1}{T_2} = \frac{k_2}{k_1}$.

(b) Put $\frac{k_2}{k_1}$ in its most reduced form $\frac{n}{m}$ by canceling any common terms in the numerator and denominator; then $T_0 = nT_2 = mT_1$.

Problem 2.14

(a)
```
>> syms t
>> xa=5*exp(-t/2);
>> ezplot(xa), grid
```

(b)
```
>> syms t
>> xb=5*exp(-2*t);
>> ezplot(xb),grid
```

(c)
```
>> syms t
>> xc=5*exp(t/2);
>> ezplot(xc),grid
```

(d)
```
>> syms t
>> xd=5*(1-exp(-t/2));
>> ezplot(xd), grid
```

(e)
```
>> syms t
>> xe=5*(1-exp(-2*t));
>> ezplot(xe), grid
```

(f)
```
>> syms t
>> xf=5*2*sin(2*t);
>> ezplot(xf),grid
```

(g)
```
>> syms t
>> xg=5*exp(-2*t)*2*sin(2*t);
>> ezplot(xg),grid
```

(h)
```
>> syms t
>> xh=5*exp(-0.5*t)*2*sin(2*t);
>> ezplot(xh),grid
```

Problem 2.15

(a)

$$\cos(\theta+\phi)=\text{Re}\{e^{j(\theta+\phi)}\}=\text{Re}\{e^{j\theta}e^{j\phi}\}$$
$$=\text{Re}\{(\cos\theta+j\sin\theta)(\cos\phi+j\sin\phi)\}$$
$$=\text{Re}\{\cos\theta\cos\phi+j\sin\theta\cos\phi$$
$$+j\cos\theta\sin\phi-\sin\theta\sin\phi\}$$
$$=\cos\theta\cos\phi-\sin\theta\sin\phi$$

(b)

$$\sin(\theta+\phi)=\text{Im}\{e^{j(\theta+\phi)}\}=\text{Im}\{e^{j\theta}e^{j\phi}\}$$
$$=\text{Im}\{(\cos\theta+j\sin\theta)(\cos\phi+j\sin\phi)\}$$
$$=\text{Im}\{(\cos\theta\cos\phi+j\sin\theta\cos\phi$$
$$+j\cos\theta\sin\phi-\sin\theta\sin\phi\}$$
$$=\cos\theta\sin\phi+\sin\theta\cos\phi$$

(c)

$$\cos\theta\cos\phi=\text{Re}\left\{e^{j\theta}\frac{e^{j\phi}+e^{-j\phi}}{2}\right\}=\text{Re}\left\{\frac{e^{j(\theta+\phi)}+e^{j(\theta-\phi)}}{2}\right\}$$
$$=\text{Re}\left\{\frac{e^{j(\theta+\phi)}}{2}+\frac{e^{j(\theta-\phi)}}{2}\right\}=\frac{\cos(\theta+\phi)}{2}+\frac{\cos(\theta-\phi)}{2}$$

(d)

$$\sin\theta\cos\phi=\text{Im}\left\{e^{j\theta}\frac{e^{j\phi}+e^{-j\phi}}{2}\right\}$$
$$=\text{Im}\left\{\frac{e^{j(\theta+\phi)}+e^{j(\theta-\phi)}}{2}\right\}$$
$$=\frac{1}{2}[\sin(\theta+\phi)+\sin(\theta-\phi)]$$

PROBLEM 2.16

(a) $x(t) = 3\cos(2t)+\sin(2t)$

$$x(t) = \frac{3}{2}\left(e^{j2t}+e^{-j2t}\right)+\frac{1}{2j}\left(e^{j2t}-e^{-j2t}\right)$$
$$=\left(\frac{3}{2}-j\tfrac{1}{2}\right)e^{j2t}+\left(\frac{3}{2}+j\frac{1}{2}\right)e^{-j2t}$$
$$=\frac{\sqrt{10}}{2}\angle\tan^{-1}(-1/3)\,e^{j2t}+\frac{\sqrt{10}}{2}\angle\tan^{-1}\left(+\frac{1}{3}\right)e^{-j2t}$$

$\tan^{-1}(-1/3)=-0.32$ rad

$\tan^{-1}(+1/3)=+0.32$ rad

$$x(t)=\frac{\sqrt{10}}{2}e^{-j0.32}e^{j2t}+\frac{\sqrt{10}}{2}e^{+j0.32}e^{-j2t}$$
$$=\sqrt{10}\,\frac{e^{j(2t-0.32)}+e^{-j(2t-0.32)}}{2}$$

$$x(t)=\sqrt{10}\cos(2t-0.32\text{ rad})$$
$$\doteq 3.16\cos(2t-18.33^\circ)$$

Problem 2.1.6 (continued)

(b) $x(t) = 4\cos(4\pi t) + 3\sin(4\pi t)$

$$= \frac{4}{2}\left(\frac{e^{j4\pi t} + e^{-j4\pi t}}{2}\right) + 3\left(\frac{e^{j4\pi t} - e^{-j4\pi t}}{2j}\right)$$

$$= \left(2 - \tfrac{3}{2}j\right)e^{j4\pi t} + \left(2 + \tfrac{3}{2}j\right)e^{-j4\pi t}$$

$$\left(2 - \tfrac{3}{2}j\right) = \sqrt{(2)^2 + (3/2)^2}\ \angle \tan^{-1}\left(\frac{-3/2}{2}\right) = \frac{5}{2}\ \angle \tan^{-1}(-3/4)$$

$$= \frac{5}{2}e^{-j0.64}$$

$$x(t) = \frac{5}{2}e^{-j0.64}e^{j4\pi t} + \frac{5}{2}e^{j0.64}e^{-j4\pi t}$$

$$= 5\ \frac{e^{j(4\pi t - 0.64)} + e^{-j(4\pi t - 0.64)}}{2}$$

$$x(t) = 5\cos(4\pi t - 0.64\ \text{rad})$$

or $5\cos(4\pi t - 36.87^\circ)$

(c) $x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

$$= A\ \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + B\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2}$$

$$= \frac{A - jB}{2}e^{j\omega_0 t} + \frac{A + jB}{2}e^{-j\omega_0 t}$$

$$= \sqrt{\frac{A^2+B^2}{4}}\ \angle \tan^{-1}\frac{-B}{A}\ e^{j\omega_0 t} + \sqrt{\frac{A^2+B^2}{2}}\ \angle \tan^{-1}\left(\frac{B}{A}\right)e^{-j\omega_0 t}$$

$$= \frac{\sqrt{A^2+B^2}}{2}e^{j\tan^{-1}\left(\frac{-B}{A}\right)}e^{j\omega_0 t} + \frac{\sqrt{A^2+B^2}}{2}e^{j\tan^{-1}\left(\frac{B}{A}\right)}e^{-j\omega_0 t}$$

$$\tan^{-1}\left(\frac{B}{A}\right) = -\tan^{-1}(-B/A)$$

$$\therefore x(t) = \sqrt{A^2+B^2}\ \frac{e^{j(\omega_0 t - \tan^{-1} B/A)} + e^{-j(\omega_0 t - \tan^{-1} B/A)}}{2}$$

$$= \sqrt{A^2+B^2}\cos\left(\omega_0 t - \tan^{-1}(B/A)\right)$$

PROBLEM 2.17

$$\int_{-\infty}^{\infty} \delta(at-b)\sin^2(t-c)\,dt$$

$$\delta(at-b) = \delta(a(t-b/a)) = \frac{1}{a}\delta\left(t-\frac{b}{a}\right)$$

$$\delta(t-b/a)\sin^2(t-c) = \sin^2\left(\frac{b}{a}-c\right)\delta(t-b/a)$$

$$\therefore \int_{-\infty}^{\infty} \delta(at-b)\sin^2(t-c)\,dt = \frac{1}{a}\sin^2(b/a-c)\int_{-\infty}^{\infty}\delta(t-b/a)\,dt$$

$$= \frac{1}{a}\sin^2(b/a-c)$$

PROBLEM 2.18

(a) $$y(t) = \int_{-\infty}^{\infty} x(\tau)\left[\delta(\tau+5)-\delta(\tau-5)\right]d\tau$$

$$x(\tau)\delta(\tau-a) = x(a)\delta(\tau-a)$$

$$\therefore y(t) = x(-5)\int_{-\infty}^{\infty}\overbrace{\delta(\tau+5)}^{1}\,d\tau - x(5)\int_{-\infty}^{\infty}\overbrace{\delta(\tau-5)}^{1}\,d\tau$$

$$y(t) = x(-5) - x(5)$$

(b) $$y(t) = \frac{1}{2}\int_{-\infty}^{\infty} x(t)e^{j\pi/2t}\,\delta(2t-3)\,dt$$

$$\delta(2t-3) = \frac{1}{2}\delta(t-3/2)$$

$$\frac{1}{2}\delta(t-3/2)\,x(t)e^{j\pi t/2} = \frac{1}{2}x(3/2)e^{j3\pi/4}$$

$$\therefore y(t) = \frac{1}{4}x(3/2)e^{j3\pi/4}\int_{-\infty}^{\infty}\overbrace{\delta(t-3/2)}^{1}\,d\tau$$

$$y(t) = \frac{1}{4}x(3/2)e^{j3\pi/4}$$

PROBEM 2.19

(a) Let $\tau = at$, then $\int_{-\infty}^{\infty} \delta(at)\,dt = \int_{-\infty}^{\infty} \delta(\tau)\frac{d\tau}{a}$

$= \frac{1}{a}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau \Rightarrow \delta(at) = \frac{1}{a}\delta(t),\ a>0$

for $a<0$, $at = \tau \Rightarrow -|a|t = \tau$

$\Rightarrow dt = \frac{-d\tau}{|a|}$

$\therefore \int_{-\infty}^{\infty} \delta(at)\,dt = \int_{\infty}^{-\infty} \delta(\tau)\frac{-d\tau}{|a|} = \frac{1}{|a|}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau$

$\therefore \delta(at) = \frac{1}{|a|}d(t)$ for the general case.

(b) $\int_{-\infty}^{t} \delta(\sigma)\,d\sigma = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases} = u(t)$

$\therefore \int_{-\infty}^{t} \delta(\tau - t_0) = \begin{cases} 1, & t>t_0 \\ 0, & t<t_0 \end{cases} = u(t-t_0)$

(c)

Recall the rules about integrating delta functions: $\delta(t)$ is nonzero only at $t=0$, so $x(t)\delta(t) = x(0)\delta(t)$, and $\int_{-\infty}^{\infty}\delta(t)dt = 1$, so $\int_{-\infty}^{\infty} x(t)\delta(t)dt = \int_{-\infty}^{\infty} x(0)\delta(t)dt = x(0)\int_{-\infty}^{\infty}\delta(t)dt = x(0)$. We can time-shift the delta function: $\delta(t-t_0)$ is nonzero only at $t=t_0$, so $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ and $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$.

i) $\int_{-\infty}^{\infty}\cos(2t)\delta(t)dt = \cos(2\cdot 0)\int_{-\infty}^{\infty}\delta(t)dt = 1.$

ii) $\delta(t-\frac{\pi}{4})$ is a time-shifted version of $\delta(t)$, and is nonzero only at $t=\frac{\pi}{4}$. So:

$$\int_{-\infty}^{\infty}\sin(2t)\delta(t-\frac{\pi}{4})dt = \int_{-\infty}^{\infty}\sin(2\cdot\tfrac{\pi}{4})\delta(t-\tfrac{\pi}{4})dt$$

$$= \sin(\tfrac{\pi}{2})\int_{-\infty}^{\infty}\delta(t-\tfrac{\pi}{4})dt = \sin(\tfrac{\pi}{2}) = 1$$

2.19 (c) (iii) $\int_{-\infty}^{\infty} \sin(2t)\,\delta(t-\pi/6)\,dt$

$$= \int_{-\infty}^{\infty} \sin(2\pi/6)\,\delta(t-\pi/6)\,dt = \sin(\pi/3)\int_{-\infty}^{\infty}\delta(t-\pi/6)\,dt$$

$$= \sin(\pi/3) = 0.866$$

(iv) $\int_{-\infty}^{\infty} \sin\left[(t-\pi/4)\right]\delta\left(t-\frac{\pi}{2}\right)dt = \int_{-\infty}^{} \sin\left(\frac{\pi}{2}-\pi/4\right)\delta(t-\pi/2)\,dt$

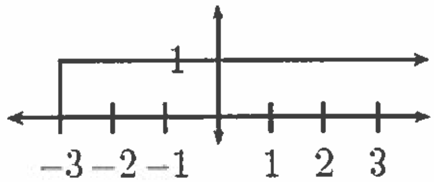
$$= \sin(\pi/4)\int_{-\infty}^{\infty}\delta(t-\pi/2)\,dt = 0.707$$

(v) $\int_{-\infty}^{\infty} \sin(t-\pi/6)\,\delta(2t-2\pi/3)\,dt = \int_{-\infty}^{\infty} \sin(t-\pi/6)\,\delta\left[2(t-\pi/3)\right]dt$

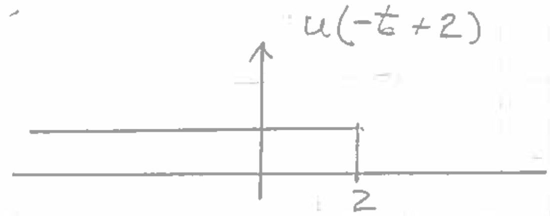
$$= \int_{-\infty}^{\infty} \sin(\pi/3-\pi/6)\,\delta\left[2(t-\pi/3)\right]dt = \frac{1}{2}\sin(\pi/6) = 0.25$$

PROBLEM 2.20

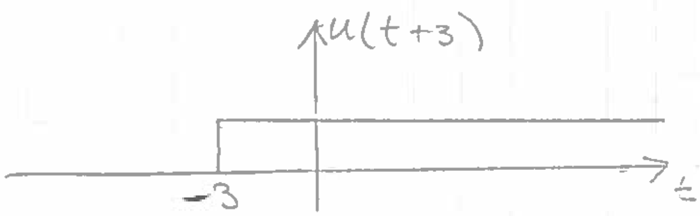
(a) $u(2t+6) = u(t+3)$



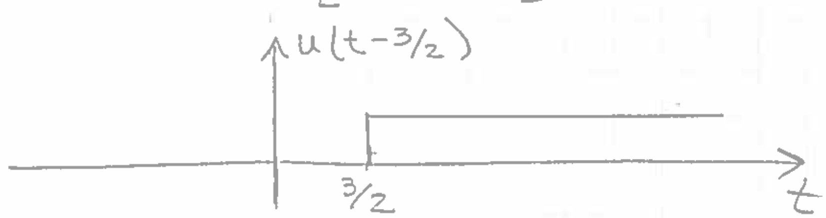
(b) $u(-3t+6) = u[-3(t-2)] = u(-t+2)$



(c) $u(t/3+1) = u\left(\frac{1}{3}(t+3)\right) = u(t+3)$

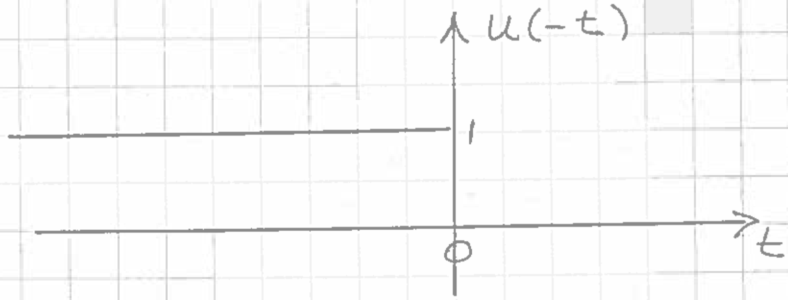


(d) $u(t/3-1/2) = u\left[\frac{1}{3}(t-3/2)\right] = u(t-3/2)$

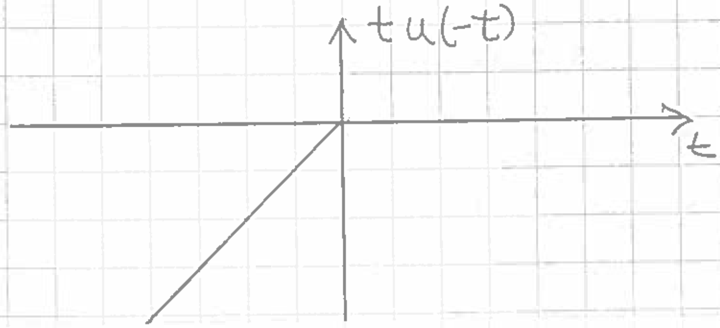


PROBLEM 2.21

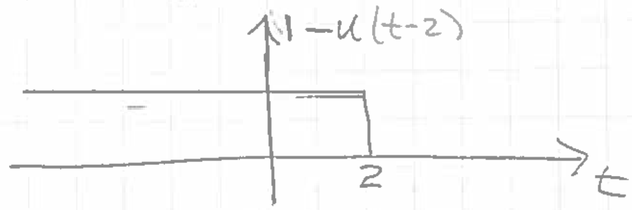
(a) $u(-t) = 1 - u(t)$



(b) $t\,u(-t) = t\,[1 - u(t)]$

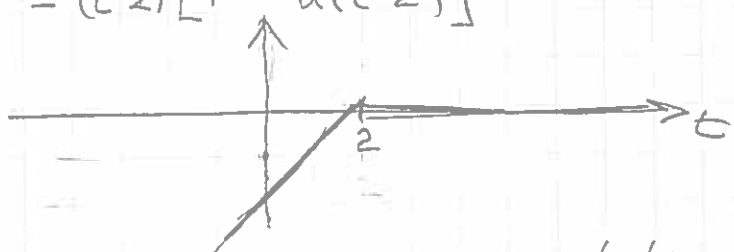


(c) $u(-t+2) = u[-1(t-2)] = 1 - u(t-2)$

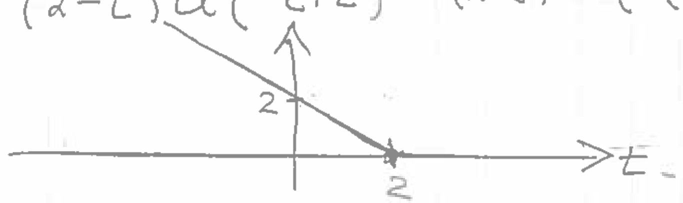


(d) $(t-2)\,u(2-t) = (t-2)\,u[-1(t-2)]$

$= (t-2)\,[1 - u(t-2)]$



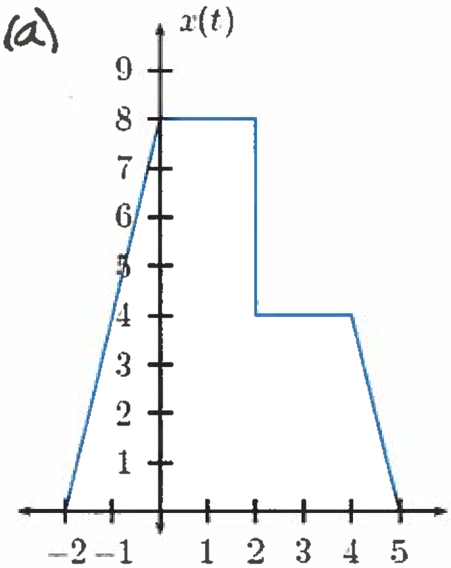
(e) $(2-t)\,u(-t+2) = (2-t)\,u(-(t-2)) = (2-t)[1-u(t-2)]$

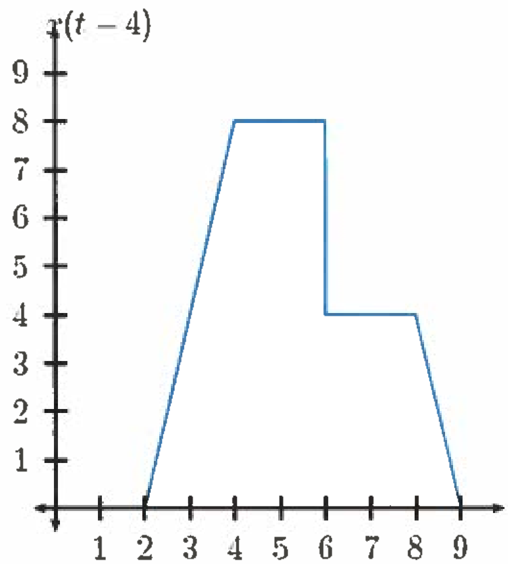


PROBLEM 2.22

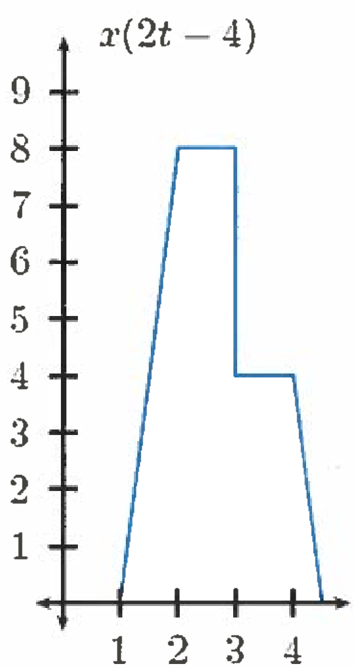
$$x(2t-4) = 4[(2t-2)u(2t-2) - (2t-4)u(2t-4) - u(2t-6) - (2t-8)u(2t-8) - (2t-9)u(2t-9)]$$
$$= 4[(2t-2)u(t-1) - (2t-4)u(t-2) - u(t-3) - (2t-8)u(t-4) - (2t-9)u(t-4.5)]$$

(a)





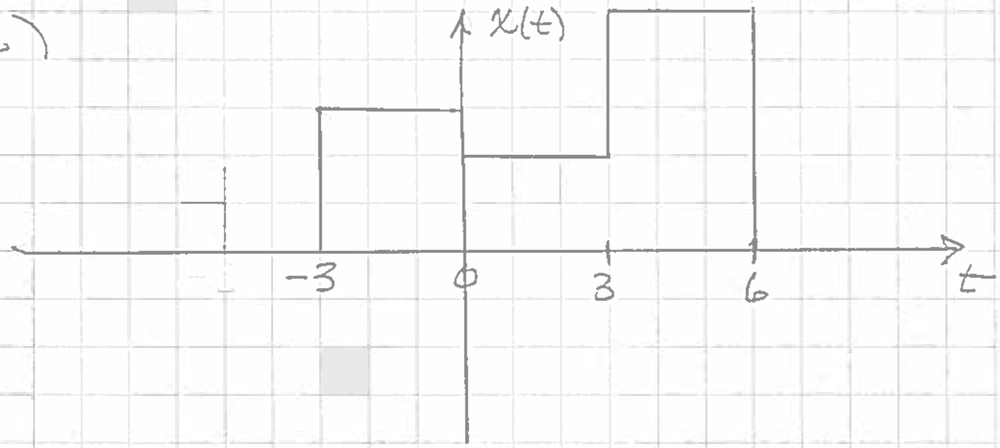
(b)



PROBLEM 2.23

$$x(t) = 3u(t+3) - u(t) + 3u(t-3) - 5u(t-6)$$

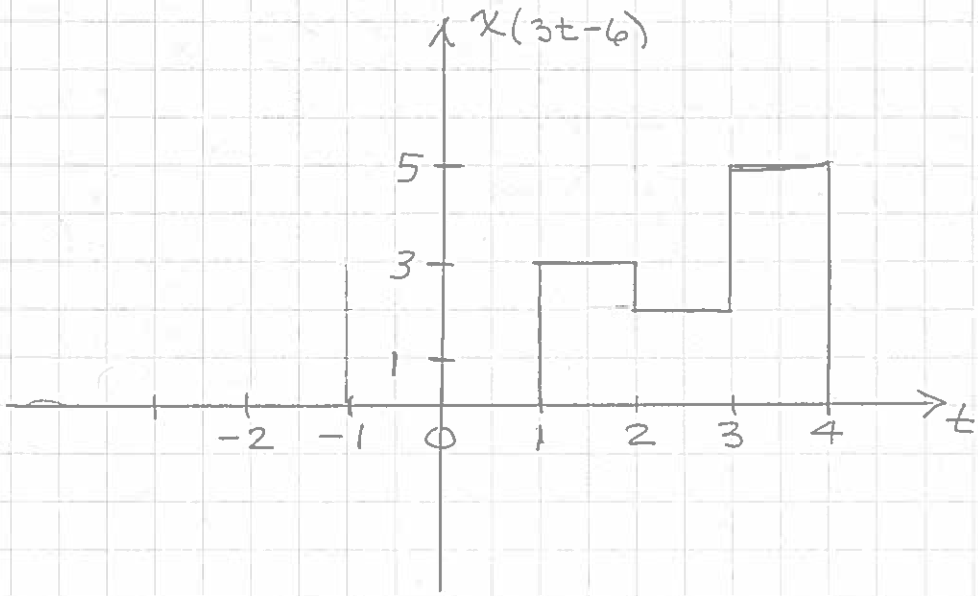
(a)



(b) $$x(3t-6) = 3u(3t-6+3) - u(3t-6) + 3u(3t-6-3) - 5u(3t-6-6)$$

$$= 3u(3t-3) - u(3t-6) + 3u(3t-9) - 5u(3t-12)$$

$$= 3u(t-1) - u(t-2) + 3u(t-3) - 5u(t-4)$$

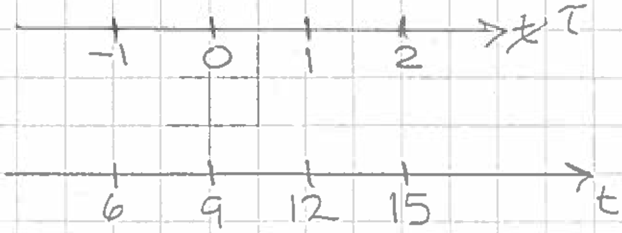


PROBLEM 2.24

(a) $$x(t) = 1 - (t+1)[u(t+1) - u(t-1)] + [-2 + 2(t-1)][u(t-1) - u(t-2)]$$

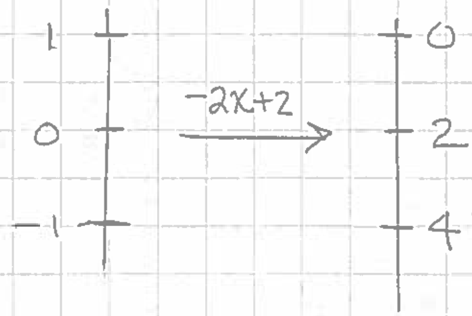
$$= 1 - (t+1)u(t+1) + 3(t-1)u(t-1) - 2(t-2)u(t-2)$$

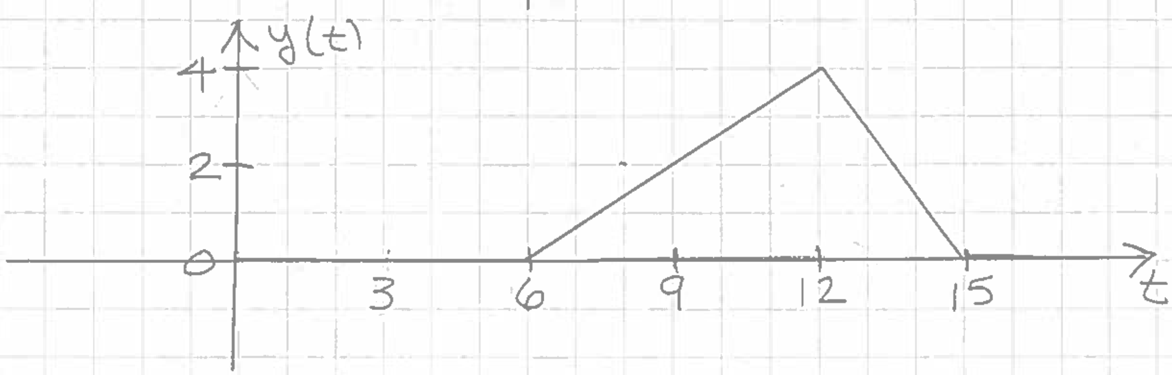
(b) use time transformation



$\tau = \frac{1}{3}t - 3 \Rightarrow \tau = 3t + 9$

and amplitude transformat





PROBLEM 2.25

(a) $x_1(t) = 2t\,u(t) - 4(t-1)\,u(t-1) + 2(t-2)\,u(t-2)$

(b) $t<0, \quad x_1(t)=0$ ✓

$0<t<1, \quad x_1(t) = 2t$ ✓

$1<t<2, \quad x_1(t) = 2t-4t+4 = 4-2t$ ✓

$2<t, \quad x_1(t) = 4-2t+2t-4=0$ ✓

(c) $x(t) = \sum_{k=-\infty}^{\infty} x_1(t-kT_0) = \sum_{k=-\infty}^{\infty} x_1(t-2k)$

PROBLEM 2.26

(a) $x_1(t) = 3\left[t\,u(t) - (t-1)u(t-1)\right] - 3\left[(t-2)\,u(t-2) - (t-3)\,u(t-3)\right]$

$(t<0), \; x_1(t) = 0$ ✓

$(0<t<1), \; x_1(t) = 3t$ ✓

$(1<t<2), \; x_1(t) = 3t-3t+3 = 3$ ✓

$(2<t<3), \; x_1(t) = \cancel{3t} \cancel{-3t}+3 -3t+6 = -3t+9$ ✓

$(t>3), \; x_1(t) = \cancel{3t}\cancel{-3t}\cancel{+3}\cancel{-3t}\cancel{+6} +\cancel{3t}-\cancel{9} = 0$ ✓

(b) $x_2(t)$ is periodic with $T_0 = 4$

$$x_2(t) = \sum_{n=-\infty}^{\infty} x_1(t+4n)$$

$$= \sum_{n=-\infty}^{\infty} 3\left[(t+4n)u(t+4n) - (t-1+4n)u(t-1+4n)\right] - 3\left[(t-2+4n)u(t-2+4n) - (t-3+4n)u(t-3+4n)\right]$$

PROBLEM 2.27

(a) $y_2(t) = T_2[T_1[x(t)]]$, $y_3(t) = T_3[T_1[x(t)]]$

$y(t) = T_2[T_1[x(t)]] + T_4\{T_3[T_1[x(t)]] + T_5[x(t)]$

(b) $y(t) = T_3\{T_2[T_1[x(t)]]\} + T_4\{T_2[T_1[x(t)]]\} + T_5[T_1[x(t)]]$

(c) $y_3(t) = T_3[T_4[T_5[x(t)]]]$

$y_4(t) = T_2[T_4[T_5[x(t)]]]$

$y_5(t) = T_1[T_5[x(t)]]$

$y(t) = y_3(t) + y_4(t) + y_5(t)$

$y(t) = T_3[T_4[T_5[x(t)]]] + T_2[T_4[T_5[x(t)]]]$

$= T + T_1[T_5[x(t)]]$

(d) $y(t) = y_3(t) \times y_4(t) \times y_5(t)$

$y(t) = T_3[T_4[T_5[x(t)]]] \times T_2[T_4[T_5[x(t)]]] \times T_1[T_5[x(t)]]$

PROBLEM 2.28

$m(t) = T_2\{x(t) - T_4[y(t)]\}$

$y(t) = T_3\{m(t) + T_1[x(t)]\}$

$y(t) = T_3\{T_2[x(t) - T_4[y(t)]] + T_1[x(t)]\}$

PROBLEM 2.29

$m(t) = T_2\{x(t) - T_3[y(t)]\} - T_4[y(t)]\}$

$y(t) = T_1[m(t)]$

$y(t) = T_1\{T_2[[x(t)] - T_3[y(t)]\} - T_4[y(t)]\}$

PROBLEM 2.30

(a) (i) The system has memory the output, $y(t)$ depends on inputs $x(t)$ over a period of time.

(ii) The system is not invertible. The input at time t_0 cannot be determined from knowledge of the output at t_0.

(iii) The system is stable. A bounded input will result in a bounded output.

(iv) The system is time-invariant.

(v) The system is linear.

(b) The system is causal if $y(t_0)$ depends on values of $x(t)$ for $t \le t_0$ only. Therefore for causality $t_0 + 1 - \alpha \le t_0 \Rightarrow \alpha \ge 1$.

PROBLEM 2.31

(a) $y(t) = \cos[x(t-2)]$

(i) The system has memory

(ii) The system is not invertible

(iii) The system is causal

(iv) $|\cos(x)| \le 1$ for all x. $\therefore$ Stable.

(v) $y(t-t_0) = x(t-t_0-2)$ $\therefore$ time invariant.

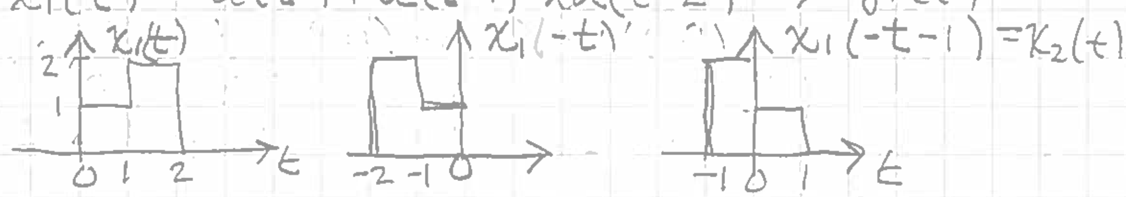
(vi) $y(t) = \cos[x_1(t) + x_2(t)] \ne \cos x_1(t) + \cos x_2(t)$,
$\therefore$ not linear.

Problem 2.32

(a)
$$\begin{aligned} x_2(t) &= 2u(t+1) - u(t) - u(t-1) \\ &= 2[u(t+1) - u(t)] + [u(t) - u(t-1)] \\ &= 2x_1(t+1) + x_1(t) \end{aligned}$$

$\therefore y_2(t) = 2y_1(t+1) + y_1(t)$

(b) $x_1(t) = u(t) + u(t-1) - 2u(t-2) \Rightarrow y_1(t)$



$\therefore y_2(t) = y_1(-t-1) = y_1[-(t+1)]$

PROBLEM 2.33

(a)

(i) The system is memoryless only if $t_0 = 0$.

(ii) The system is invertible; $x(t) = y(t + t_0)$.

(iii) The system is causal only if $t_0 \geq 0$.

(iv) The system is BIBO stable.

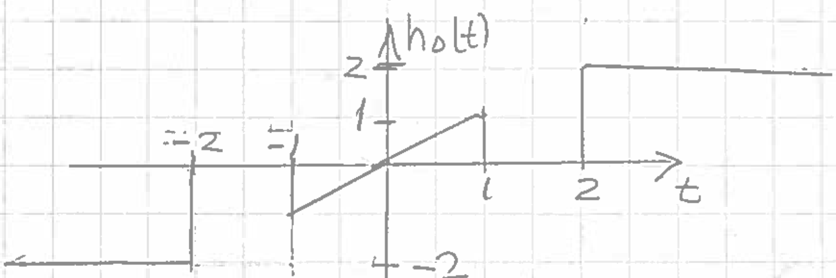
(v) The system is time invariant.

$$x(t - t_0 - t_1) \longleftrightarrow y(t - t_1)$$

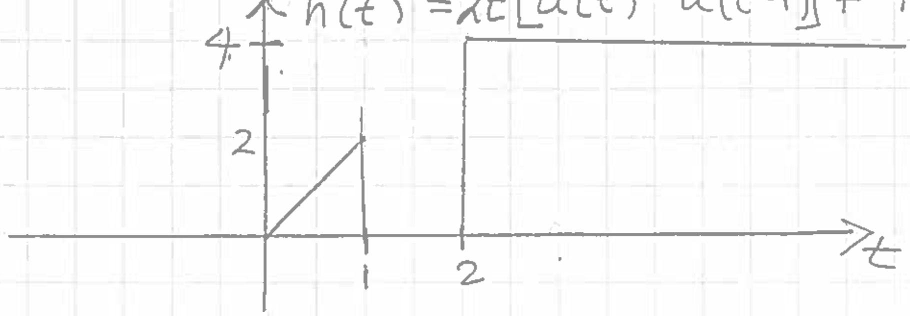
(vi) The system is linear.

$$x_1(t - t_0) \longrightarrow y_1(t)$$
$$x_2(t - t_0) \longrightarrow y_2(t)$$
$$a x_1(t - t_0) + b x_2(t - t_0) \longrightarrow a y_1(t) + b y_2(t).$$

PROBLEM 2.34



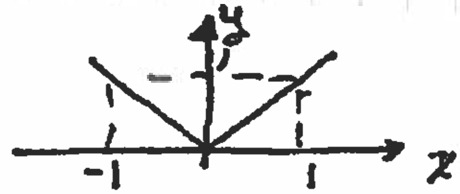
$$h(t) = 2t[u(t) - u(t-1)] + 4u(t-2)$$



PROBLEM 2.35

(a) (i) memoryless

(ii) $y=1$ for $x=\pm 1$, not invertible



(iii) causal

(iv) stable

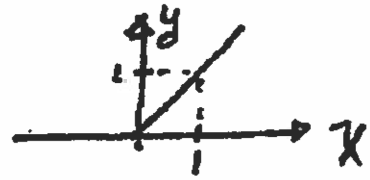
$y=|x|$

(v) time invariant

(vi) $|x_1+x_2| \neq |x_1|+|x_2|$ not linear

(b) (i) memoryless

(ii) $y=0$ for $x \le 0$, not invertible



(iii) causal

(iv) stable

(v) time invariant

(vi) $y\big|_{\substack{x_1=1 \\ x_2=-1}} \neq y\big|_{x_1+x_2}$, not linear

(c) (i) Memoryless: $y(t)$ determined by current input.

(ii) not invertible: $y(t)=1$ for all $x(t)<-1$.

(iii) Causal.

(iv) Stable: $|y(t)| \le 1$.

(v) time invariant.

(vi) not linear: $y(t)=1$ for all values of $x(t)<-1$.

(d) (i) Memoryless

(ii) not invertible: $y(t)=4$ for all $x(t)>2$.

(iii) Causal

(iv) Stable: $0 \le y(t) \le 4$

(v) time invariant

(vi) not linear: $y(t)=4$ for all $x(t)>2$