1. Is the old Greek numeration system ($\alpha = 1$, $\beta = 2$, $\gamma = 3$, etc.) a place-value system? Explain.

2. Most present-day societies use the Hindu-Arabic numeration system.
   A) True
   B) False

3. How many tens are in 7654? How many whole tens are in 7654?

4. How many hundreds are in 23? How many whole hundreds?

5. How many tenths are in 1.03? How many whole tenths?

6. How many ones are in 4352.678? How many whole ones?

7. In base ten, 3421 is exactly _____ ones, exactly _____ tens, exactly _____ hundreds, and exactly _____ thousands; also, 3421 is exactly _____ tenths and exactly _____ hundredths.

8. In base ten, 215.687 is exactly _____ ones, exactly _____ tens, exactly _____ hundreds, and exactly _____ thousands; also, 3421 is exactly _____ tenths and exactly _____ hundredths.

9. (Roman numerals) IX = _____$_{\text{ten}}$ and XI = _____ $_{\text{ten}}$.

10. 34,597 has 345 whole thousands in it.
    A) True
    B) False

11. 34.597 has 345 whole tenths in it.
    A) True
    B) False

12. 56 has 560 tenths in it.
    A) True
    B) False

13. 23 has 230 hundredths in it.
    A) True
    B) False

14. 45 has 4500 hundredths in it.
    A) True
    B) False

15. 632.1 has 632.1 ones in it.
    A) True
    B) False

16. A soap factory packs 100 bars of soap in each box for shipment. If the factory makes 15,287 bars of soap, how many *full* boxes will they have for shipment? Explain.

17. How many $10 bills could one get for $10 million?
    A) 10,000,000
    B) 1,000,000
    C) 100,000
    D) 10,000
    E) 1000

18. How many $100 bills could one get for a billion dollars?
    A) 100,000,000
    B) 10,000,000
    C) 1,000,000
    D) 100,000
    E) 10,000

19. How many $100 bills would make $45 billion?

20. Judy says, “Well, hundredths are smaller than tenths. So 0.36 is smaller than 0.4.” Comment on Judy's reasoning.

21. Tyler thinks that 0.36 is bigger than 0.4 because 36 is bigger than 4. Comment on Tyler's reasoning.

22. Four children are talking about different ways to think about the number 196.83. Following are their answers. For each answer, tell whether it is correct or incorrect. If it is incorrect, please explain.

    Kim's answer196.83 could be thought about as 190 tens and 683 hundredths.
    A) Is Kim's answer correct or incorrect?
    B) If Kim's answer is incorrect, please explain the error.

    Laurie's answer: 196.83 could be thought about as 1 hundred, 9 tens, and 6.83 hundredths.
    A) Is Laurie's answer correct or incorrect?
    B) If Laurie's answer is incorrect, please explain the error.

    Mick's answer: 196.83 could be thought about as 1 hundred and 9683 hundredths.
    A) Is Mick's answer correct or incorrect?
    B) If Mick's answer is incorrect, please explain the error.

    Nola's answer: 196.83 could be thought about as 18 tens, 15 ones, 17 tenths, and 13 hundredths.
    A) Is Nola's answer correct or incorrect?
    B) If Nola's answer is incorrect, please explain the error.

23. For whole numbers, any two-digit numeral in base five represents a smaller number than the *same* two-digit numeral in base twenty.
    A) True
    B) False

24. In base b, there are $b - 1$ different digits.
    A) True
    B) False

25. These are the digits that are needed for a base *seven* place-value system: 0, 1, 2, 3, 4, 5, 6, 7.
    A) True
    B) False

26. In base b, $3 + 2b^3 + b$ would be written _____.

27. A place-value, base-twenty system would require _____ digits.

28. 524_{eight} = _____ten.

29. 287_{ten} = _____four.

30. 1012_{five} = _____ in base ten.

31. 32_{ten} = _____ in base four.

32. 2.31_{four} = _____ as a mixed number in base ten.

33. $6\frac{2}{3}$ in base ten = _____ in base three.

34. 1_{ten} = _____twelve.

35. 214.3_{five} = _____ in base ten.

36. 29_{ten} = _____ in base three.

37. 7_{ten} = _____ in base nine.

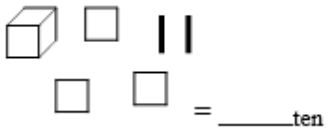
38. 203.6_{ten} = _____five.

39. 2003_{five} = _____ten.

40. 200.3_{five} = _____ten.

41. Write 49_{ten} in base seven.

42. Do the "translations" in parts A–D. Show your work.
    A) 3102_{five} = _____ten
    B) 310.2_{five} = _____ten
    C) 203.6_{ten} = _____five
    D) (base-six pieces with small block as the unit)



43. You are living and working on a planet that uses only base five. How many $5 bills can you get for $\$1234.20_{five}$? Write your answer in base five because you are living on the planet. Write enough (numbers, words, etc.) to make your thinking clear.

44. In base five, the two whole numbers immediately BEFORE 2001_{five} are _____five and _____five.

45. If you are counting in base five, what would be the next six numerals AFTER 2314_{five}?

46. If you have been counting in base five, what would have been the five numerals BEFORE 2314_{five}?

47. Write how many fingers you have in base five. In base two. In base ten.

48. Which is LARGER? 2_{four} or 2_{five}? Explain.

49. Consider: $x = 81765_{fifteen}$ and $y = 81765_{thirteen}$. Which of x and y is GREATER? Explain.

50. Consider: $x = 74213_{sixteen}$ and $y = 74213_{fourteen}$. Which is GREATER, x or y (or are they equal)? Explain.

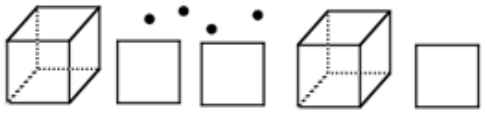
51. Consider $x = 0.3147_{\text{eight}}$ and $y = 0.3147_{\text{nine}}$. Which of x and y is GREATER? Explain. (Be careful.)

52. Write an algebraic expression for 204_b.

53. If a base-*eight* flat represents 1, the numeral _____ would give the numerical value of the small cube. (You may give your answer either in base eight or in base ten—just make clear which.)

54. Write the base-ten numeral for the following base-*eight* pieces.

    Base-*eight* pieces, with the small cube (a dot here) as the unit:



55. Sketch the base blocks that show 1203_{seven}, and give the English *words* for the base-*ten* value of each different-sized piece.

56. Write the base b numeral for $2b^4 + b^2 + 3b + 1$.

57. Write out 32004_m in the algebraic form of the last item.

58. The best coins to use in thinking about the first three whole-number place values in base five would be the penny, the nickel, and the quarter.
    A) True
    B) False

59. The best coins to use in thinking about the first three whole-number place values in base ten would be the penny, the dime, and the half-dollar.
    A) True
    B) False

60. If $10000_{\text{ten}} + 10_b = 10023_{\text{ten}}$, what is base b?

61. Define your unit and sketch base blocks to represent 32.67_{eight}.

62. Define your unit, sketch the base blocks that show 1203_{nine}, and give the English *words* for the base-*ten* value of each piece of wood.

63. 53_{six} is the same number as which of these base-ten numerals?
    A) 18
    B) 183
    C) 12
    D) 85
    E) 33

64. In base ten, 111_{five} would be written:
    A) 421.
    B) 155.
    C) 31.
    D) 21.
    E) 555.

65. The base *b* numeral 321_b means:
    A) $3 \cdot b^2 + 2 \cdot b^1 + 1$.
    B) $3 \cdot b^3 + 2 \cdot b^2 + 1 \cdot b^1$.
    C) $6b$.
    D) $3 \cdot b + 2 \cdot b + 1$.

66. In base five, 32_{ten} would be written:
    A) 152_{five}.
    B) 112_{five}.
    C) 62_{five}.
    D) 17_{five}.

67. The base two numeral 100_{two} equals the base ten numeral:
    A) 1100100.
    B) 1011100.
    C) 8.
    D) 4.

68. In base ten, 32_{four} would be written:
    A) 400.
    B) 200.
    C) 122.
    D) 14.
    E) 8.

69. The base-four numeral 11.1_{four} could be written in base ten as:
    A) $33\frac{1}{4}$.
    B) $33\frac{1}{10}$.
    C) $11\frac{1}{4}$.
    D) $5\frac{1}{4}$.

70. The base-ten decimal 18.5 could be written in base six as:
    A) 10.5_{six}.
    B) 20.3_{six}.
    C) 30.3_{six}.
    D) 128.5_{six}.

71. The base-ten fraction 1/4 equals which base-eight numeral?
    A) 0.2_{eight}
    B) 0.14_{eight}
    C) 0.02_{eight}
    D) 1.4_{eight}

72. If $31_b = 28_{ten}$, then b =
    A) 4.
    B) 5.
    C) 7.
    D) 9.
    E) This is impossible for any whole number b.

73. In what base does the following counting work:
    1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, …
    A) base two
    B) base four
    C) base five
    D) base six

74. What is the base-ten fraction representation for 1.21_{four}?
    A) $1\frac{9}{16}$
    B) $1\frac{3}{4}$
    C) $1\frac{21}{100}$
    D) $1\frac{3}{5}$

75. Write an addition equation for (# fingers) + (# toes) = (answer) in some base other than base ten.

76. Three-fourths in base ten is what in base two?

77. Seven-eighths in base ten is what in base two?

78. Suppose you visited an alien planet and the creatures wrote numbers in order as follows: "obi, fin, mus, obi-na, obi-obi, obi-fin, obi-mus…" These creatures must be using which base?
    A) base ten
    B) base three
    C) base four
    D) base nine
    E) base twelve

79. A) Add $24_{five} + 33_{five}$ in base five. (The numbers are already written in base five, so no conversions are necessary.)
    B) How would you illustrate this with the base-five blocks using drawings and showing the intermediate steps?

80. A) 0.5_{ten} = _____$_{eight}$.
    B) $312.2_{four} + 22.3_{four}$ = _____$_{four}$
    C) 84_{ten} = _____$_{three}$
    D) 33.3_{six} = _____$_{ten}$

81. Determine the possible value(s) for base *b*:

$$\begin{array}{r} 321_b \\ -\ \underline{234_b} \\ 43_b \end{array}$$

82. Below is a partially completed addition, written in connection with base five pieces. *At the time of the work below*, what base pieces would be displayed if the small piece were the unit? (Drawings or word descriptions are okay.)
Finish the numerical calculation. (You do not have to draw the base pieces for the rest of the work.)

$$\begin{array}{r} {\scriptstyle 1} \\ 2\ 1\ 4_{\text{five}} \\ +\ \underline{\ 3\ 3_{\text{five}}} \\ {\scriptstyle 2} \end{array}$$

83. $$\begin{array}{r} 241_{\text{six}} \\ +\ \underline{135_{\text{six}}} \end{array}$$

84. $$\begin{array}{r} 127_{\text{nine}} \\ -\ \underline{\ \ 58_{\text{nine}}} \end{array}$$

85. $$\begin{array}{r} 4.4_{\text{five}} \\ +\ \underline{3.3_{\text{five}}} \end{array}$$

86. $$\begin{array}{r} 0.24_{\text{seven}} \\ -\ \underline{0.15_{\text{seven}}} \end{array}$$

87. $$\begin{array}{r} 21_{\text{six}} \\ +\ \underline{35.2_{\text{six}}} \end{array}$$

88. Use drawings of multibase blocks to illustrate $231_{\text{ten}} + 87_{\text{ten}}$.

89. Use drawings of multibase blocks to illustrate $32_{\text{five}} + 23_{\text{five}}$.

90. In what base does the following addition NOT work: 13 + 13 = 26?
    A) base six
    B) base seven
    C) base eight
    D) base ten
    E) It works in all of these bases.

91. Two-thirds in base ten is what in base twelve?
    A) 0.8
    B) 0.4
    C) 0.2
    D) 0.12
    E) 0.3

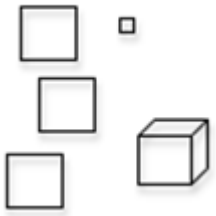
92. A) Subtract the following in base five. Show all your work.

$$\begin{array}{r} 2\,2\,1_{\text{five}} \\ -\quad 4\,2_{\text{five}} \\ \hline \end{array}$$

    B) Use your work in part A to explain how the way we regroup in base-five subtraction is similar to the way that we regroup in base-ten subtraction.

93. 221.2_{three} = _____$_{\text{ten}}$.

94. The blocks are base four.
    A) What base-four numeral is illustrated with the long as the unit?
    B) Give the English *words* for the base-*ten* value.
    C) Suppose the small block is the unit. Give the English *words* for the base-*ten* value.



95. A teacher challenged her class to find out how many ways the number 345.23 could be thought about. Macy said 345.23 could be thought of as 340 ones and 52.3 tenths. Is Macy correct or incorrect? Explain how you know.

96. Write the “basimal” (expanded) place values and then the usual base-ten fraction or mixed number for 20.11_{three}.

Answer Key

1.
2. A
3.
4.
5.
6.
7.
8.
9.
10. B
11. A
12. A
13. B
14. A
15. A
16.
17. B
18. B
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23. A
24. B
25. B
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56.
57.
58. A
59. B
60.
61.
62.
63. E
64. C
65. A
66. B
67. D
68. D
69. D
70. C
71. A
72. D
73. C
74. A
75.
76.
77.
78. C
79.
80.
81.
82.
83.
84.
85.
86.
87.
88.
89.
90. A

91. A
92.
93.
94.
95.
96.