PART II

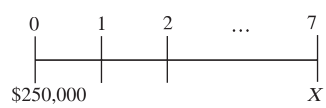
SOLUTIONS

CHAPTER 1

THE TIME VALUE OF MONEY

SOLUTIONS

1. A. Investment 2 is identical to Investment 1 except that Investment 2 has low liquidity. The difference between the interest rate on Investment 2 and Investment 1 is 0.5 percentage point. This amount represents the liquidity premium, which represents compensation for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly.
   B. To estimate the default risk premium, find the two investments that have the same maturity but different levels of default risk. Both Investments 4 and 5 have a maturity of eight years. Investment 5, however, has low liquidity and thus bears a liquidity premium. The difference between the interest rates of Investments 5 and 4 is 2.5 percentage points. The liquidity premium is 0.5 percentage point (from Part A). This leaves 2.5 – 0.5 = 2.0 percentage points that must represent a default risk premium reflecting Investment 5's high default risk.
   C. Investment 3 has liquidity risk and default risk comparable to Investment 2, but with its longer time to maturity, Investment 3 should have a higher maturity premium. The interest rate on Investment 3, r_3, should thus be above 2.5 percent (the interest rate on Investment 2). If the liquidity of Investment 3 were high, Investment 3 would match Investment 4 except for Investment 3's shorter maturity. We would then conclude that Investment 3's interest rate should be less than the interest rate on Investment 4, which is 4 percent. In contrast to Investment 4, however, Investment 3 has low liquidity. It is possible that the interest rate on Investment 3 exceeds that of Investment 4 despite 3's shorter maturity, depending on the relative size of the liquidity and maturity premiums. However, we expect r_3 to be less than 4.5 percent, the expected interest rate on Investment 4 if it had low liquidity. Thus 2.5 percent < r_3 < 4.5 percent.
2. i. Draw a time line.

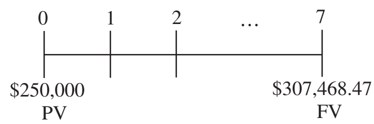


   ii. Identify the problem as the future value of a lump sum.

iii. Use the formula for the future value of a lump sum.

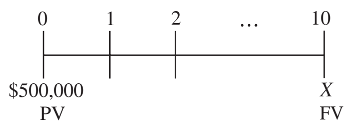
$$\text{PV} = 0.05 \times \$5{,}000{,}000 = \$250{,}000$$

$$\begin{aligned}\text{FV}_N &= \text{PV}(1+r)^N \\ &= \$250{,}000(1.03)^7 \\ &= \$307{,}468.47\end{aligned}$$



The future value in seven years of \$250,000 received today is \$307,468.47 if the interest rate is 3 percent compounded annually.

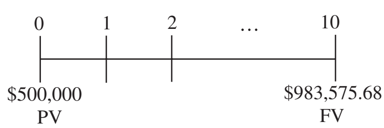
3. i. Draw a time line.



ii. Identify the problem as the future value of a lump sum.

iii. Use the formula for the future value of a lump sum.

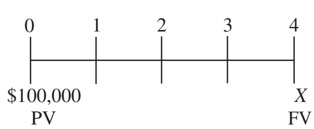
$$\begin{aligned}\text{FV}_N &= \text{PV}(1+r)^N \\ &= \$500{,}000(1.07)^{10} \\ &= \$983{,}575.68\end{aligned}$$



Your client will have \$983,575.68 in 10 years if she invests \$500,000 today and earns 7 percent annually.

4. A. To solve this problem, take the following steps:

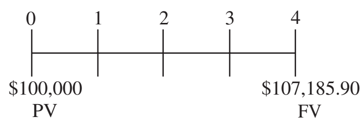
i. Draw a time line and recognize that a year consists of four quarterly periods.



ii. Recognize the problem as the future value of a lump sum with quarterly compounding.

iii. Use the formula for the future value of a lump sum with periodic compounding, where m is the frequency of compounding within a year and N is the number of years.

$$\begin{aligned} FV_N &= PV\left(1+\frac{r_s}{m}\right)^{mN} \\ &= \$100{,}000\left(1+\frac{0.07}{4}\right)^{4(1)} \\ &= \$107{,}185.90 \end{aligned}$$



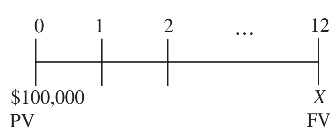
iv. As an alternative to Step iii, use a financial calculator. Most of the equations in this reading can be solved using a financial calculator. Calculators vary in the exact keystrokes required (see your calculator's manual for the appropriate keystrokes), but the following table illustrates the basic variables and algorithms. Remember, however, that a financial calculator is only a shortcut way of performing the mechanics and is not a substitute for setting up the problem or knowing which equation is appropriate.

Time Value of Money Variable	Notation Used on Most Calculators	Numerical Value for This Problem
Number of periods or payments	N	4
Interest rate per period	$\%i$	7/4
Present value	PV	$100,000
Future value	FV **compute**	X
Payment size	PMT	n/a (= 0)

In summary, your client will have $107,185.90 in one year if he deposits $100,000 today in a bank account paying a stated interest rate of 7 percent compounded quarterly.

B. To solve this problem, take the following steps:

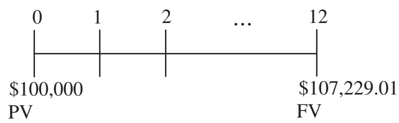
i. Draw a time line and recognize that with monthly compounding, we need to express all values in monthly terms. Therefore, we have 12 periods.



ii. Recognize the problem as the future value of a lump sum with monthly compounding.

iii. Use the formula for the future value of a lump sum with periodic compounding, where m is the frequency of compounding within a year and N is the number of years.

$$\mathrm{FV}_N = \mathrm{PV}\left(1+\frac{r_s}{m}\right)^{mN}$$
$$= \$100{,}000\left(1+\frac{0.07}{12}\right)^{12(1)}$$
$$= \$107{,}229.01$$



iv. As an alternative to Step iii, use a financial calculator.

Notation Used on Most Calculators	Numerical Value for This Problem
N	12
%i	7/12
PV	$100,000
FV **compute**	X
PMT	n/a (= 0)

Using your calculator's financial functions, verify that the future value, X, equals $107,229.01.

In summary, your client will have $107,229.01 at the end of one year if he deposits $100,000 today in his bank account paying a stated interest rate of 7 percent compounded monthly.

C. To solve this problem, take the following steps:

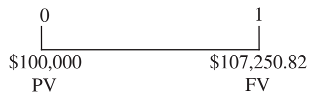
i. Draw a time line and recognize that with continuous compounding, we need to use the formula for the future value with continuous compounding.

ii. Use the formula for the future value with continuous compounding (N is the number of years in the expression).

$$\mathrm{FV}_N = \mathrm{PV}e^{r_s N}$$
$$= \$100{,}000e^{0.07(1)}$$
$$= \$107{,}250.82$$

The notation $e^{0.07(1)}$ is the exponential function, where e is a number approximately equal to 2.718282. On most calculators, this function is on the key marked e^x. First calculate the value of X. In this problem, X is 0.07(1) = 0.07.

Key 0.07 into the calculator. Next press the e^x key. You should get 1.072508. If you cannot get this figure, check your calculator's manual.



In summary, your client will have $107,250.82 at the end of one year if he deposits $100,000 today in his bank account paying a stated interest rate of 7 percent compounded continuously.

5. Stated annual interest rate = 5.89 percent.
   Effective annual rate on bank deposits = 6.05 percent.

$$1 + \text{EAR} = \left(1 + \frac{\text{Stated interest rate}}{m}\right)^m$$

$$1.0605 = \left(1 + \frac{0.0589}{m}\right)^m$$

For annual compounding, with $m = 1$, $1.0605 \neq 1.0589$.
For quarterly compounding, with $m = 4$, $1.0605 \neq 1.060214$.
For monthly compounding, with $m = 12$, $1.0605 \approx 1.060516$.
Hence, the bank uses monthly compounding.

6. A. Use the formula for the effective annual rate.
   Effective annual rate = $(1 + \text{Periodic interest rate})^m - 1$

$$\left(1 + \frac{0.08}{4}\right)^{4(1)} - 1 = 0.0824 \text{ or } 8.24\%$$

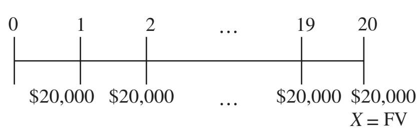
   B. Use the formula for the effective annual rate.
   Effective annual rate = $(1 + \text{Periodic interest rate})^m - 1$

$$\left(1 + \frac{0.08}{12}\right)^{12(1)} - 1 = 0.0830 \text{ or } 8.30\%$$

   C. Use the formula for the effective annual rate with continuous compounding.
   Effective annual rate = $e^{r_s} - 1$

$$e^{0.08} - 1 = 0.0833 \text{ or } 8.33\%$$

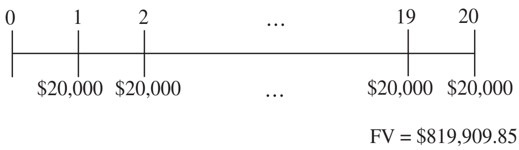
7. i. Draw a time line.



   ii. Identify the problem as the future value of an annuity.

iii. Use the formula for the future value of an annuity.

$$\begin{aligned}\mathrm{FV}_N &= A\left[\frac{(1+r)^N-1}{r}\right]\\ &= \$20{,}000\left[\frac{(1+0.07)^{20}-1}{0.07}\right]\\ &= \$819{,}909.85\end{aligned}$$



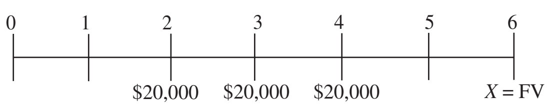
iv. Alternatively, use a financial calculator.

Notation Used on Most Calculators	Numerical Value for This Problem
N	20
%*i*	7
PV	n/a (= 0)
FV **compute**	*X*
PMT	$20,000

Enter 20 for *N*, the number of periods. Enter 7 for the interest rate and 20,000 for the payment size. The present value is not needed, so enter 0. Calculate the future value. Verify that you get $819,909.85 to make sure you have mastered your calculator's keystrokes.

In summary, if the couple sets aside $20,000 each year (starting next year), they will have $819,909.85 in 20 years if they earn 7 percent annually.

8. i. Draw a time line.



ii. Recognize the problem as the future value of a delayed annuity. Delaying the payments requires two calculations.

iii. Use the formula for the future value of an annuity (Equation 7)

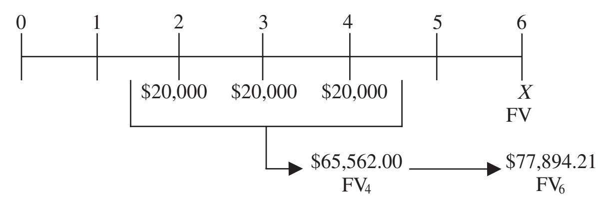
$$\mathrm{FV}_N = A\left[\frac{(1+r)^N-1}{r}\right]$$

to bring the three \$20,000 payments to an equivalent lump sum of \$65,562.00 four years from today.

Notation Used on Most Calculators	Numerical Value for This Problem
N	3
%*i*	9
PV	n/a (= 0)
FV **compute**	*X*
PMT	\$20,000

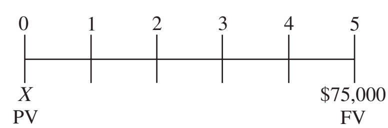
iv. Use the formula for the future value of a lump sum (Equation 2), $FV_N = PV(1 + r)^N$, to bring the single lump sum of \$65,562.00 to an equivalent lump sum of \$77,894.21 six years from today.

Notation Used on Most Calculators	Numerical Value for This Problem
N	2
%*i*	9
PV	\$65,562.00
FV **compute**	*X*
PMT	n/a (= 0)



In summary, your client will have \$77,894.21 in six years if she receives three yearly payments of \$20,000 starting in Year 2 and can earn 9 percent annually on her investments.

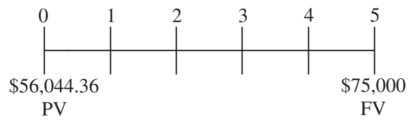
9. i. Draw a time line.



ii. Identify the problem as the present value of a lump sum.

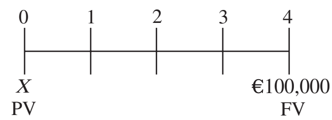
iii. Use the formula for the present value of a lump sum.

$$\begin{aligned} PV &= FV_N (1+r)^{-N} \\ &= \$75,000(1+0.06)^{-5} \\ &= \$56,044.36 \end{aligned}$$



In summary, the father will need to invest $56,044.36 today in order to have $75,000 in five years if his investments earn 6 percent annually.

10. i. Draw a time line and recognize that a year consists of four quarterly periods.



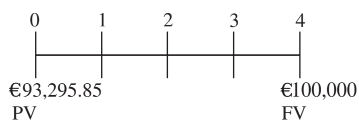
ii. Recognize the problem as the present value of a lump sum with quarterly compounding.

iii. Use the formula for the present value of a lump sum with periodic compounding, where m is the frequency of compounding within a year and N is the number of years.

$$PV = FV_N\left(1+\frac{r_s}{m}\right)^{-mN}$$

$$= €100,000\left(1+\frac{0.07}{4}\right)^{-4(1)}$$

$$= €93,295.85$$



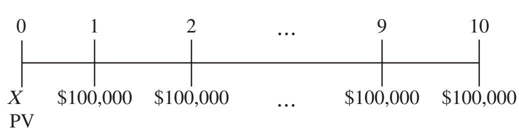
iv. Alternatively, use a financial calculator.

Notation Used on Most Calculators	Numerical Value for This Problem
N	4
%i	7/4
PV **compute**	X
FV	€100,000
PMT	n/a (= 0)

Use your calculator's financial functions to verify that the present value, X, equals €93,295.85.

In summary, your client will have to deposit €93,295.85 today to have €100,000 in one year if her bank account pays 7 percent compounded quarterly.

11. i. Draw a time line for the 10 annual payments.



ii. Identify the problem as the present value of an annuity.
iii. Use the formula for the present value of an annuity.

$$\begin{aligned} PV &= A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right] \\ &= \$100{,}000\left[\frac{1-\dfrac{1}{(1+0.05)^{10}}}{0.05}\right] \\ &= \$772{,}173.49 \end{aligned}$$

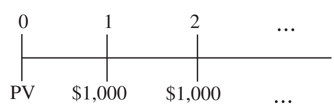
PV = $772,173.49

iv. Alternatively, use a financial calculator.

Notation Used on Most Calculators	Numerical Value for This Problem
N	10
%i	5
PV **compute**	X
FV	n/a (= 0)
PMT	$100,000

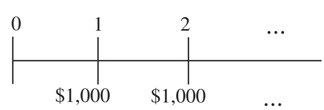
In summary, the present value of 10 payments of $100,000 is $772,173.49 if the first payment is received in one year and the rate is 5 percent compounded annually. Your client should accept no less than this amount for his lump sum payment.

12. i. Draw a time line.



ii. Recognize the problem as the present value of a perpetuity.
iii. Use the formula for the present value of a perpetuity.

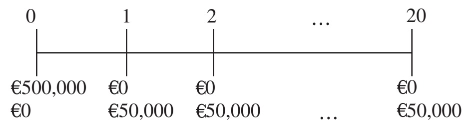
$$PV = \left(\frac{A}{r}\right) = \left(\frac{\$1{,}000}{0.03}\right) = \$33{,}333.33$$



PV = $33,333.33

The investor will have to pay $33,333.33 today to receive $1,000 per quarter forever if his required rate of return is 3 percent per quarter (12 percent per year).

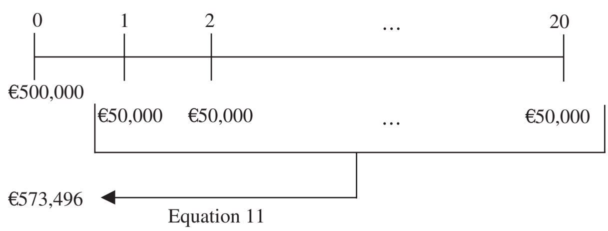
13. i. Draw a time line to compare the lump sum and the annuity.



ii. Recognize that we have to compare the present values of a lump sum and an annuity.
iii. Use the formula for the present value of an annuity (Equation 11).

$$\text{PV} = €50{,}000\left[\frac{1-\dfrac{1}{(1.06)^{20}}}{0.06}\right] = €573{,}496$$

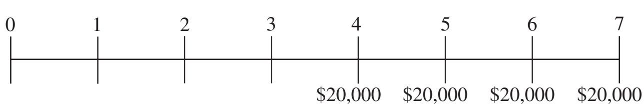
Notation Used on Most Calculators	Numerical Value for This Problem
N	20
%*i*	6
PV **compute**	*X*
FV	n/a (= 0)
PMT	$50,000



The annuity plan is better by €73,496 in present value terms (€573,496 – €500,000).

14. A. To evaluate the first instrument, take the following steps:

i. Draw a time line.



ii. $$\text{PV}_3 = A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right]$$

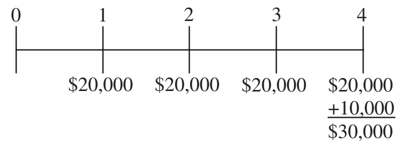
$$= \$20{,}000\left[\frac{1-\dfrac{1}{(1+0.08)^4}}{0.08}\right]$$

$$= \$66{,}242.54$$

iii. $$\text{PV}_0 = \frac{\text{PV}_3}{(1+r)^N} = \frac{\$66{,}242.54}{1.08^3} = \$52{,}585.46$$

You should be willing to pay $52,585.46 for this instrument.

B. To evaluate the second instrument, take the following steps:
   i. Draw a time line.



The time line shows that this instrument can be analyzed as an ordinary annuity of $20,000 with four payments (valued in Step ii below) and a $10,000 payment to be received at $t = 4$ (valued in Step iii below).

ii. $$\text{PV} = A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right]$$

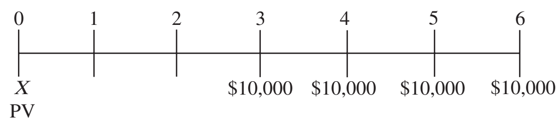
$$= \$20{,}000\left[\frac{1-\dfrac{1}{(1+0.08)^4}}{0.08}\right]$$

$$= \$66{,}242.54$$

iii. $$\text{PV} = \frac{\text{FV}_4}{(1+r)^N} = \frac{\$10{,}000}{(1+0.08)^4} = \$7{,}350.30$$

iv. Total = $66,242.54 + $7,350.30 = $73,592.84
   You should be willing to pay $73,592.84 for this instrument.

15. i. Draw a time line.



ii. Recognize the problem as a delayed annuity. Delaying the payments requires two calculations.

iii. Use the formula for the present value of an annuity (Equation 11)

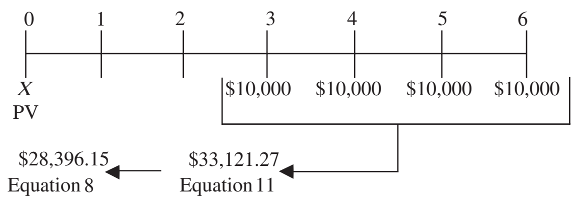
$$\text{PV} = A\left[\frac{1-\dfrac{1}{(1+r)^N}}{r}\right]$$

to bring the four payments of $10,000 back to a single equivalent lump sum of $33,121.27 at $t = 2$. Note that we use $t = 2$ because the first annuity payment is then one period away, giving an ordinary annuity.

Notation Used on Most Calculators	Numerical Value for This Problem
N	4
%*i*	8
PV **compute**	*X*
PMT	$10,000

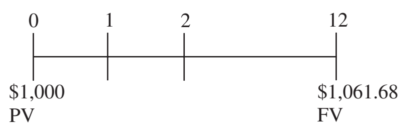
iv. Then use the formula for the present value of a lump sum (Equation 8), $PV = FV_N(1 + r)^{-N}$, to bring back the single payment of \$33,121.27 (at $t = 2$) to an equivalent single payment of \$28,396.15 (at $t = 0$).

Notation Used on Most Calculators	Numerical Value for This Problem
N	2
%*i*	8
PV **compute**	*X*
FV	\$33,121.27
PMT	n/a (= 0)



In summary, you should set aside \$28,396.15 today to cover four payments of \$10,000 starting in three years if your investments earn a rate of 8 percent annually.

16. i. Draw a time line.



ii. Recognize the problem as the future value of a lump sum with monthly compounding.

iii. Use the formula for the future value of a lump sum with periodic compounding

$$FV_N = PV[1 + (r_s/m)]^{mN}$$

and solve for r_s, the stated annual interest rate:

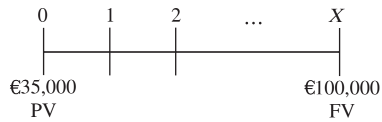
$$\$1,061.68 = \$1,000[1 + (r_s/12)]^{12(1)} \text{ so } r_s = 0.06$$

iv. Alternatively, use a financial calculator to solve for *r*.

Notation Used on Most Calculators	Numerical Value for This Problem
N	12
%*i* **compute**	*X*
PV	\$1,000
FV	\$1,061.68
PMT	n/a (= 0)

Use your calculator's financial functions to verify that the stated interest rate of the savings account is 6 percent with monthly compounding.

17. i. Draw a time line.



ii. Recognize the problem as the future value of a lump sum with monthly compounding.

iii. Use the formula for the future value of a lump sum, $FV_N = PV[1 + (r_s/m)]^{mN}$, where m is the frequency of compounding within a year and N is the number of years. Solve for mN, the number of months.

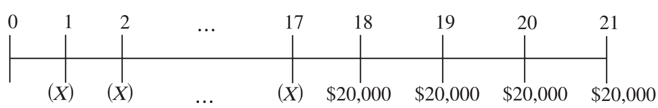
$$€100{,}000 = €35{,}000[1 + (0.05/12)]^{12N} \text{ so } 12N = 252.48 \text{ months}$$

iv. Alternatively, use a financial calculator.

Notation Used on Most Calculators	Numerical Value for This Problem
N **compute**	*X*
%*i*	5/12
PV	€35,000
FV	€100,000
PMT	n/a (= 0)

Use your calculator's financial functions to verify that your client will have to wait 252.48 months to have €100,000 if he deposits €35,000 today in a bank account paying 5 percent compounded monthly. (Some calculators will give 253 months.)

18. i. Draw a time line.



ii. Recognize that you need to equate the values of two annuities.

iii. Equate the value of the four $20,000 payments to a single payment in Period 17 using the formula for the present value of an annuity (Equation 11), with $r = 0.05$. The present value of the college costs as of $t = 17$ is $70,919.

$$PV = \$20{,}000\left[\frac{1-\dfrac{1}{(1.05)^4}}{0.05}\right] = \$70{,}919$$

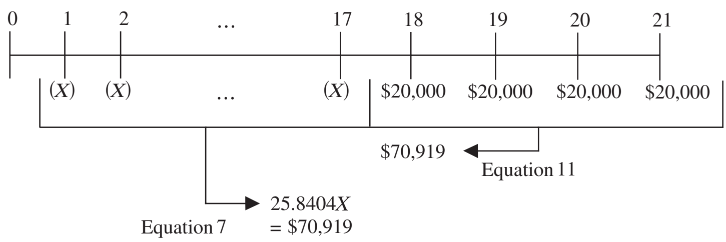
Notation Used on Most Calculators	Numerical Value for This Problem
N	4
%*i*	5
PV **compute**	*X*
FV	n/a (= 0)
PMT	$20,000

iv. Equate the value of the 17 investments of X to the amount calculated in Step iii, college costs as of $t = 17$, using the formula for the future value of an annuity (Equation 7). Then solve for X.

$$\$70{,}919 = \left[\frac{(1.05)^{17} - 1}{0.05}\right] = 25.840366X$$

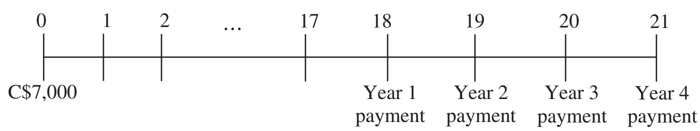
$$X = \$2{,}744.50$$

Notation Used on Most Calculators	Numerical Value for This Problem
N	17
$\%i$	5
PV	n/a (= 0)
FV	\$70,919
PMT **compute**	X



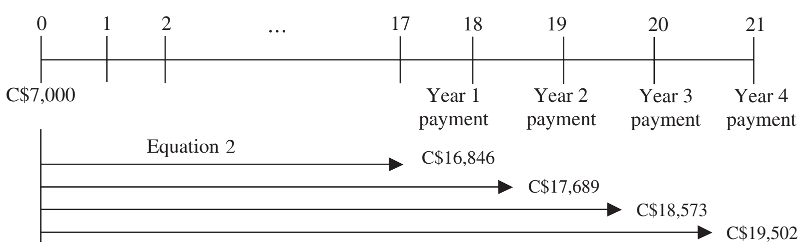
In summary, your client will have to save \$2,744.50 each year if she starts next year and makes 17 payments into a savings account paying 5 percent annually.

19. i. Draw a time line.



ii. Recognize that the payments in Years 18, 19, 20, and 21 are the future values of a lump sum of C\$7,000 in Year 0.

iii. With $r = 5\%$, use the formula for the future value of a lump sum (Equation 2), $FV_N = PV\,(1 + r)^N$, four times to find the payments. These future values are shown on the time line below.

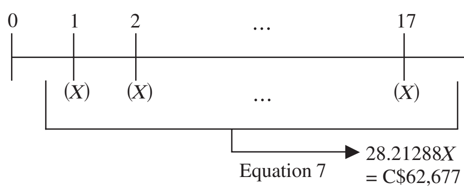


iv. Using the formula for the present value of a lump sum (r = 6%), equate the four college payments to single payments as of t = 17 and add them together. C\$16,846$(1.06)^{-1}$ + C\$17,689$(1.06)^{-2}$ + C\$18,573$(1.06)^{-3}$ + C\$19,502$(1.06)^{-4}$ = C\$62,677

v. Equate the sum of C\$62,677 at t = 17 to the 17 payments of X, using the formula for the future value of an annuity (Equation 7). Then solve for X.

$$\text{C\$62,677} = X\left[\frac{(1.06)^{17} - 1}{0.06}\right] = 28.21288X$$
$$X = \text{C\$2,221.58}$$

Notation Used on Most Calculators	Numerical Value for This Problem
N	17
%i	6
PV	n/a (= 0)
FV	C\$62,677
PMT **compute**	X



In summary, the couple will need to put aside C\$2,221.58 each year if they start next year and make 17 equal payments.

20. To compute the compound growth rate, we only need the beginning and ending EPS values of \$4.00 and \$7.00 respectively, and use the following equation:

$$\begin{aligned} FV_N &= PV(1+r)^N \\ 7 &= 4(1+r)^4 \\ 1+r &= (7/4)^{1/4} \\ r &= (7/4)^{1/4} - 1 \\ &= 0.1502 = 15.02\% \end{aligned}$$

EPS grew at an annual rate of 15.02 percent during the four years.

21. A is correct. Using the general time value of money formula, for sales, solve for r in the equation $2 = 1 \times (1 + r)^5$. For income, solve $3 = 1 \times (1 + r)^5$. Alternatively, using a financial calculator, for sales, enter N = 5, PV = 1, PMT = 0, FV = –2 and compute I/Y. For income, change the FV to –3 and again solve for I/Y. The solution for sales is 14.87%; and for income is 24.57%.