2.1a) $\quad L[u(t)\sin(2t+\frac{\pi}{4})]=\int_0^\infty e^{-st}\sin(2t+\frac{\pi}{4})dt=I$

Integrating by parts:

$$I=\left|e^{-st}(-\frac{1}{2})\cos(2t+\frac{\pi}{4})\right|_0^\infty-\int_0^\infty -se^{-st}(-\frac{1}{2})\cos(2t+\frac{\pi}{4})dt$$

$$I=-\frac{1}{2}[0-\frac{1}{\sqrt{2}}]-\frac{s}{2}\int_0^\infty e^{-st}\cos(2t+\frac{\pi}{4})dt$$

$$I=\frac{1}{2\sqrt{2}}-\frac{s}{2}(\frac{1}{2})(0-\frac{1}{\sqrt{2}})-\frac{s^2}{4}I$$

$$I(1+\frac{s^2}{4})=\frac{2+s}{4\sqrt{2}}$$

$$\boxed{I=\frac{\frac{1}{2}s\sqrt{2}+\sqrt{2}}{(s^2+4)}}$$

2.1 a) MATLAB Solution

```
syms x y z t s k

laplace(sin(2*t+pi/4))

ans =
(1/2*s*2^(1/2)+2^(1/2))/(s^2+4)


pretty (ans)
                        1/2       1/2
            1/2 s 2        + 2
            -----------------
                    2
                  s    + 4
```

2.1b)

$$L[u(t)e^{-t}\cos 2t] = \int_0^\infty \cos 2t e^{-t} e^{-st} dt = \int_0^\infty \cos 2t e^{-t(s+1)} dt = I$$

Integrating by parts:

$$I = \left| e^{-t(s+1)}(\frac{1}{2})\sin(2t) \right|_0^\infty - \int_0^\infty -(s+1)e^{-t(s+1)}(\frac{1}{2})\sin(2t)dt$$

$$I = (s+1)\int_0^\infty e^{-t(s+1)}\sin(2t)dt$$

$$I = \frac{(s+1)}{4}\left| \frac{\cos 2t}{e^{-t(s+1)}} \right|_0^\infty - \frac{(s+1)^2}{4}I$$

$$I(1+\frac{(s+1)^2}{4}) = \frac{s+1}{4}$$

$$\boxed{I = \frac{s+1}{(s+1)^2+4}}$$

2.1 b) MATLAB Solution

```
syms x y z t s k

laplace(exp(-t)*cos(2*t))

ans =
(s+1)/((s+1)^2+4)

pretty(ans)
```

```
                    s + 1
                ------------
                     2
                (s + 1)  + 4
```


2.2a) $\dfrac{3}{s} = 3$

MATLAB Solution

```
ilaplace(3/s)
```

2.2b) $\dfrac{3}{(s+2)} = 3e^{-2t}$

MATLAB Solution
```
ilaplace (3/(s+2))
```

2.2c) $\dfrac{3}{(s+2)^2} = 3\dfrac{1}{(s+2)^{1+1}} = 3te^{-2t}$

MATLAB Solution
```
ilaplace(3/(s+2)^2)
```

2.2d) $\dfrac{3}{s^3} = \dfrac{3}{2}\dfrac{2}{s^{2+1}} = \dfrac{3}{2}t^2$

MATLAB Solution
```
ilaplace(3/s^3)
```

2.2e) $\dfrac{\frac{1}{2}}{(s^2+9)} = \dfrac{1}{6}\dfrac{3}{s^2+3^2} = \dfrac{1}{6}\sin 3t$

MATLAB Solution
```
ilaplace(0.5/(s^2+9))
```

```
simplify(ans)
```

2.2f) $\dfrac{3}{s^2+4s+8} = \dfrac{3}{(s+2)^2+2^2} = \dfrac{3}{2}\dfrac{2}{(s+2)^2+2^2} = \dfrac{3}{2}e^{-2t}\sin 2t$

MATLAB Solution
```
ilaplace(3/(s^2+4*s+8))
```

ans =
-3/16*(-16)^(1/2)*(exp((-2+1/2*(-16)^(1/2))*t)-exp((-2-1/2*(-
16)^(1/2))*t))
```
simplify(ans)
```

ans =
3/4*i*(-exp((-2+2*i)*t)+exp((-2-2*i)*t))

```
pretty(ans)
```

> 3/4 i (-exp((-2 + 2 i) t) + exp((-2 - 2 i) t))

How does this compare with the hand solution? Recalling the following relations:

$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \quad , \quad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2} \quad , \quad e^{i\alpha} = \cos\alpha + i\sin\alpha \text{ , let's work with the}$$

MATLAB solution and see if we can derive the hand-solution. The MATLAB solution cleaned up a bit is:

$$\frac{3i}{4}(-e^{-2t}e^{(2t)i} + e^{-2t}e^{-(2t)i}) = i(\frac{3}{4})e^{-2t}(-e^{(2t)i} + e^{-(2t)i})$$

From the above relations, $2i(\sin\alpha) = e^{i\alpha} - e^{-i\alpha}$, if we let $\alpha = 2t$, then the MATLAB

solution becomes: $i\left(\frac{3}{4}\right)e^{-2t}(-2i\sin 2t) = i^2\left(\frac{6}{4}\right)e^{-2t}\sin 2t = \frac{3}{2}e^{-2t}\sin 2t$ (check)

2.2g) $\dfrac{s+4}{s^2+4s+8} = \dfrac{(s+2)+2}{(s+2)^2+2^2} = \dfrac{(s+2)}{(s+2)^2+2^2} + \dfrac{2}{(s+2)^2+2} = e^{-2t}(\cos 2t + \sin 2t)$

MATLAB Solution
```
ilaplace((s+4)/(s^2+4*s+8))
```

ans = exp(-2*t)*cos(2*t)+exp(-2*t)*sin(2*t)

2.2h) $\dfrac{1}{(s+2)^2} = \dfrac{1}{(s+2)^{1+1}} = te^{-2t}$

MATLAB Solution
```
ilaplace(1/(s+2)^2)
```

ans = t*exp(-2*t)

2.3)   Find x(s) for the following differential equations:
2.3a)

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = u(t) \qquad x(0) = x^1(0) = 0$$

$$x(s) = \frac{1}{s(s^2 + 4s + 3)} = \frac{1}{s(s+3)(s+1)}$$

$$x(s) = \frac{c1}{s} + \frac{c2}{s+3} + \frac{c3}{s+1}$$

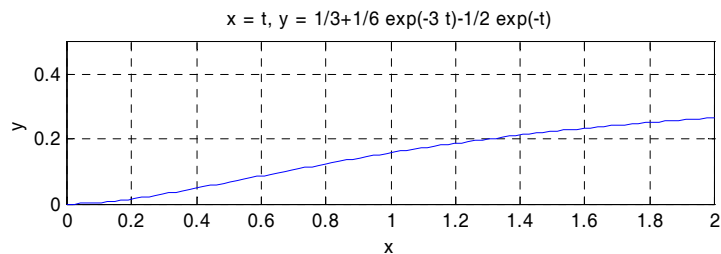$$c1 = \frac{1}{3}, c2 = \frac{1}{6}, c3 = -\frac{1}{2}$$

$$\boxed{x(t) = \frac{1}{3} + \frac{e^{-3t}}{6} - \frac{e^{-t}}{2}}$$

MATLAB Solution
```
x=dsolve('D2x+4*Dx+3*x=1','x(0)=0','Dx(0)=0')

x =
1/3+1/6*exp(-3*t)-1/2*exp(-t)

ezplot(x,[0,2])
```


x = t, y = 1/3+1/6 exp(-3 t)-1/2 exp(-t)

2.3b)

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = u(t) \qquad x(0) = x^1(0) = 0$$

$$x(s) = \frac{1}{s(s^2 + 2s + 1)} = \frac{c1}{s} + \frac{c2}{(s+1)^2} + \frac{c3}{s+1}$$

$$c1 = 1, c2 = -1$$

$$1 = (s^2 + 2s + 1) - s + c3(s^2 + s)$$
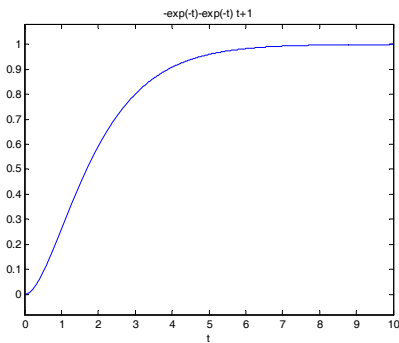
$$1 = (1 + c3)s^2 + (1 + c3)s + 1$$

$$c3 = -1$$

$$x(t) = 1 - (1+t)e^{-t}$$

MATLAB Solution

```
x=dsolve('D2x+2*Dx+x=1','x(0)=0','Dx(0)=0')
```

```
x =
-exp(-t)-exp(-t)*t+1
```


2.3c)

$$2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = u(t) \qquad x(0) = x^1(0) = 0$$

$$x(s) = \frac{1}{s(2s^2 + 2s + 1)} = \frac{A}{s} + \frac{Bs + C}{(2s^2 + 2s + 1)}$$

$$2As^2 + 2As + A + Bs^2 + Cs = 1$$

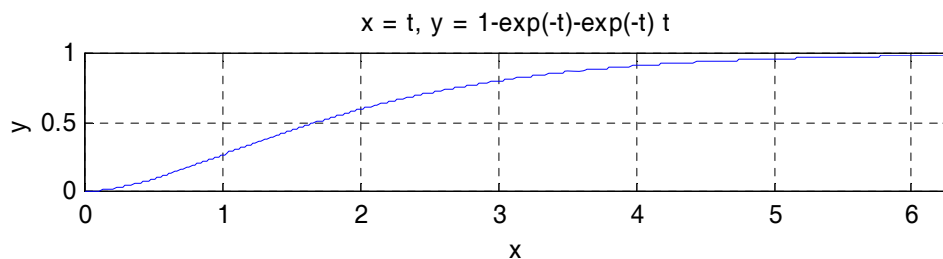$$(2A + B)s^2 + (2A + C)s + A = 1$$

$$A = 1, B = -2, C = -2$$

$$x(s) = \frac{1}{s} - \frac{s+1}{(s+\frac{1}{2})^2 + \frac{1}{4}} = \frac{1}{s} - \frac{(s+\frac{1}{2}) + \frac{1}{2}}{(s+\frac{1}{2})^2 + \frac{1}{4}}$$

$$\boxed{x(t) = 1 - e^{-\frac{t}{2}}[\cos(\frac{t}{2}) + \sin(\frac{t}{2})]}$$

MATLAB Solution
```
x=dsolve('D2x+2*Dx+x=1','x(0)=0','Dx(0)=0')

x =
1-exp(-t)-exp(-t)*t
```


x = t, y = 1-exp(-t)-exp(-t) t

2.4 See solution to problem 2.1. The MATLAB solutions are interspersed with the hand solutions.

2.5 See solution to problem 2.2. The MATLAB solutions are interspersed with the hand solutions.

2.6 See solution to problem 2.3. The MATLAB solutions are interspersed with the hand solutions.

2.7 `Ca=dsolve('5*DCa+Ca=2','Ca(0)=3')`

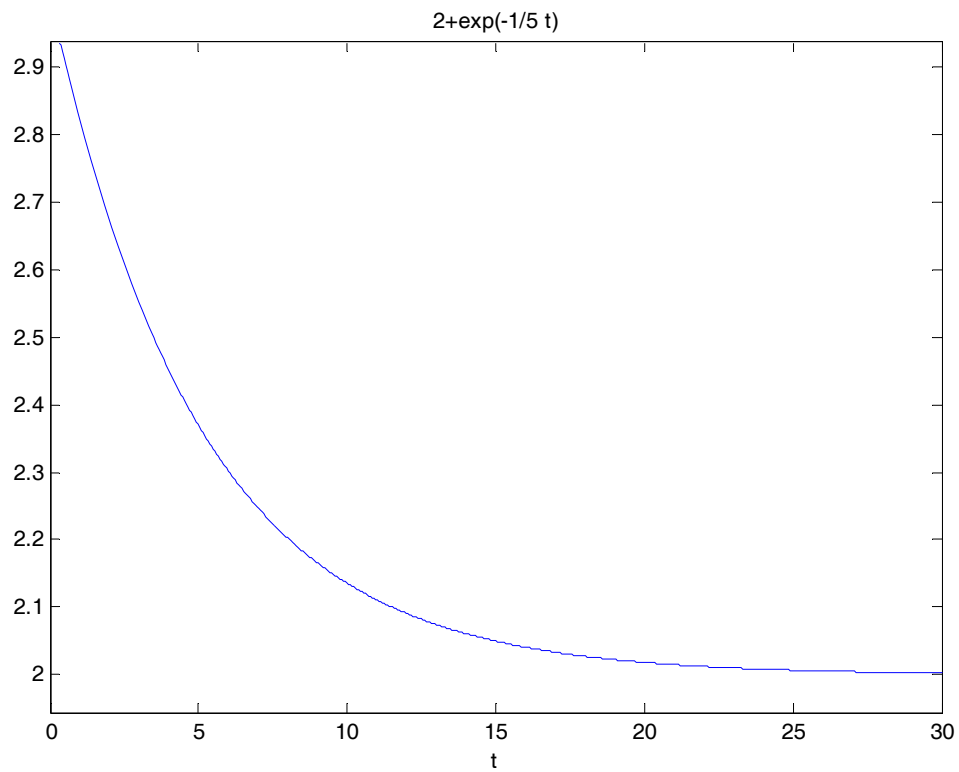`Ca =`

`2+exp(-1/5*t)`

`T=dsolve('5*DT+T=70','T(0)=80')`

`T =`

`70+10*exp(-1/5*t)`

Graphing the solutions:

For the mass balance,

`ezplot(Ca,[0,30])`
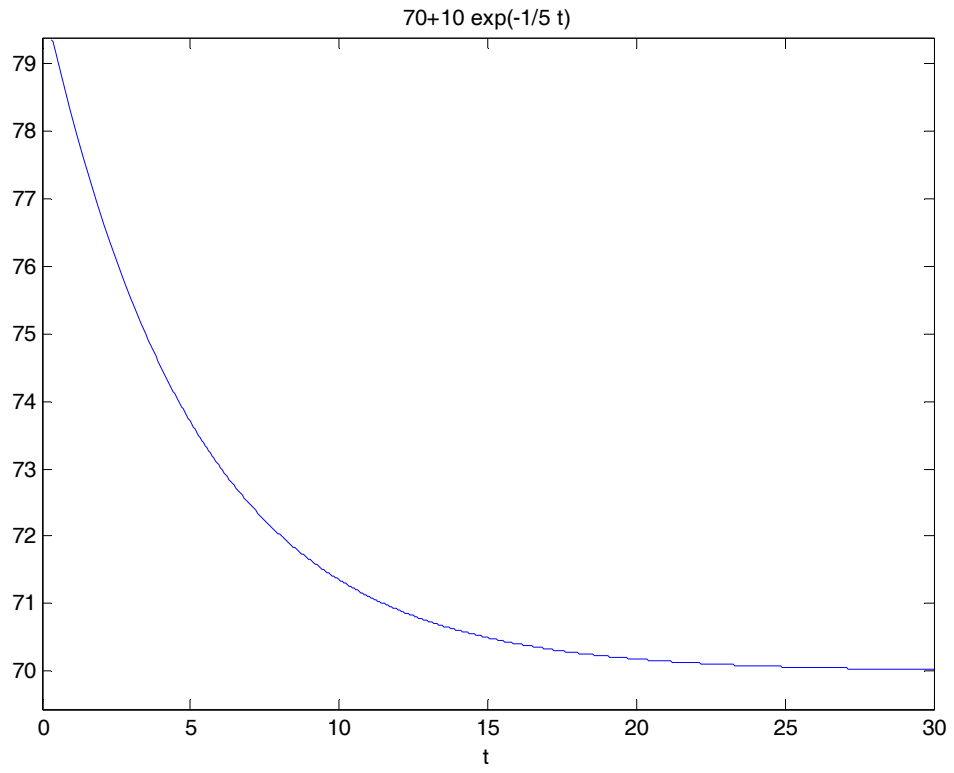

2+exp(-1/5 t)

This figure matches Figure 2-3.

For the energy balance,

`ezplot(T,[0,30])`
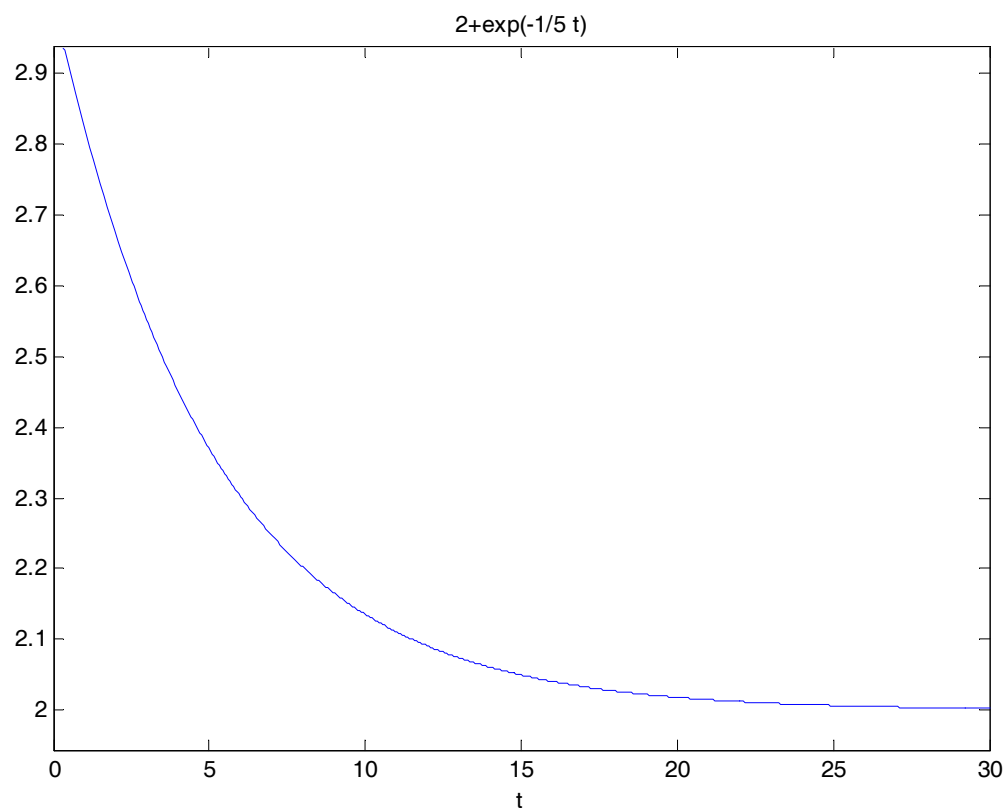
70+10 exp(-1/5 t)

This figure matches Figure 2-7.

2.8 From Eq. 2.10, $C_a(s) = \dfrac{\dfrac{2}{s}+5(3)}{5s+1} = \dfrac{2}{s(5s+1)}+\dfrac{15}{5s+1}$

`ilaplace (2/s/(5*s+1)+15/(5*s+1))`

`ans =`

`2+exp(-1/5*t)`

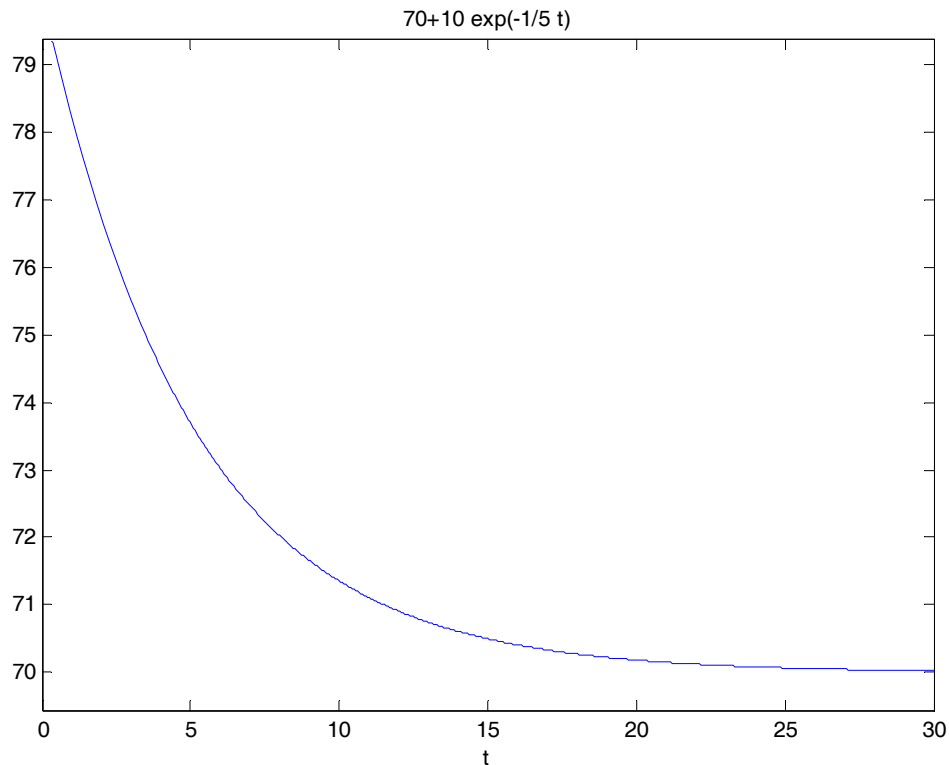**`ezplot(ans,[0,30])`**

Same result as before for the mass balance.

From Equation 2.11, $T(s) = \dfrac{\dfrac{70}{s}+5(80)}{5s+1} = \dfrac{70}{s(5s+1)}+\dfrac{400}{5s+1}.$

`ilaplace(70/s/(5*s+1)+400/(5*s+1))`

`ans =`

`70+10*exp(-1/5*t)`

```
ezplot(ans,[0,30])
```


70+10 exp(-1/5 t)

Same result for the temperature as before.

2.9 If the inlet flow of stream 1 goes from 10 liters/min to 20 liters/min and the rest of the input parameters stay the same, the following changes occur in the inlet occurs at 3PM:

$$C_{a3} \quad 3\frac{g}{L} \rightarrow 2.5\frac{g}{L} \quad \left(20L/\min\right)(1g/L)+(20L/\min)(4g/L)=(40g/L)(C_{a3})$$

$$T_3 \quad 45C \rightarrow 40C \quad \left(20L/\min\right)(25C)+(20L/\min)(55C)=(40L/\min)(T_3)$$

The new flow rate of 40 L/min means that the new time constant will be:

$$\tau = \frac{150L}{40L/\min} = 3.75\,\text{min}\,,\text{ assuming that the outlet flow increases correspondingly to}$$
40L/min (this will be the case if the outlet from the tank is an overflow exit stream), otherwise the tank will eventually overflow.

The new mass balance, from Eq. 2.1 becomes:
$$3.75\frac{dC_a}{dt}+C_a=2.5 \qquad C_a(0)=3$$

and the new energy balance from Eq. 2.3 becomes:

$$3.75\frac{dT}{dt}+T=40+\frac{1}{(1000)(40)(1)}(1.05\times10^{6})=66.25 \qquad T(0)=80C$$

Solving these two equations in a similar manner to that in the text yields:

$$C_a=2.5+0.5e^{-\frac{t}{3.75}}$$

$$T=66.25+13.75e^{-\frac{t}{3.75}}$$