

Solutions Manual

B. Wayne Bequette

Process Control

Modeling,
Design,
and Simulation

Prentice Hall International Series
in the Physical and Chemical
Engineering Sciences



This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted. The work and materials from it should never be made available to students except by instructors using the accompanying text in their classes. All recipients of this work are expected to abide by these restrictions and to honor the intended pedagogical purposes and the needs of other instructors who rely on these materials.

Table of Contents

Solutions for Chapters 1-14

Solutions for Module 5.4 and Module 5.5

Solution for Module 16, Discrete PI Example

Chapter 1 Solutions

1.1

i. *Driving a car*

Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

ii. Two sample favorite activities:

Jogging

(a) Objectives:

- Jog intensely (heart rate at 180bpm) for 30 min.
- Smooth changes in jogging intensity and speed.

(b) Input Variables:

- Jogging Rate – Manipulated input
- Shocking surprises (dogs, cars, etc.)– Disturbance

(c) Output Variables:

- Blood Oxygen level – unmeasured
- Heart beat – measured
- Breathing rate – unmeasured

(d) Constraints:

- Hard: Max Heart Rate (to avoid heart attack → death)
- Hard: Blood oxygen minimum and maximum
- Soft: Time spent jogging

(e) Operating characteristics: Continuous during period, Semi Batch when viewed over larger time periods.

(f) Safety, environmental, economic factors: Potential for injury, overexertion

(g) Control: Feedback/Feedforward system. Oxygen level, heartbeat, fatigue all part of determining action after the fact. Path, weather are part of feedforward system

Cycling

(a) Objectives:

- Ensure stability (don't crash)
- Enjoy ride
- Prevent mechanical failure

(b) Input Variables – Manipulated:

- Body Position
- Steering

- Braking Force
- Gear Selection

Input Variables – Disturbances:

- Weather
- Path Conditions
- Other people, animals

(c) Output Variables – Measured:

- Speed
- Direction
- Caloric Output (via electronic monitor)

Output Variables – Unmeasured:

- Level of enjoyment
- Mechanical integrity of person and bicycle
- Aesthetics (smoothness of ride)

(d) Constraints – Hard:

- Turning radius
- Mechanical limits of bike and person
- Maximum fatigue limit of person

Constraints – Soft:

- Steering dynamics that lead to instability before mechanical failure (i.e. you crash, the bike doesn't break)
- Terrain and weather can limit enjoyment level.

(e) Operation: Continuous: Steering, weight distribution, terrain selection within a path, pedal force Semi batch: Gear selection, braking force Batch: Tire pressure, bike selection, path selection

(f) Safety, environment, economics: Safety: Stability and mechanical limits prevent injury to rider and others Environment: Trail erosion, noise Economics: Health costs, maintenance costs

(g) Control Structure: Feedback: Levels of exertion, bike performance are monitored and ride is adjusted after the fact FeedForward: Path is seen ahead and ride is adjusted accordingly.

iii. A stirred tank heater

(a) Objectives

- Maintain Operating Temperature
- Maintain flow rate at desired level

(b) Input Variables:

- Manipulated: Added heat to system
 - Disturbance: Upstream flow rate and conditions
- (c) Output Variables – Measured: Tank fluid temperature, Outflow
- (d) Constraints:
- Hard: Max inflow and outflow as per pipe size and valve limitations
 - Soft: Fluid temperature for operating objective
- (e) Operating conditions: Continuous fluid flow adjustment, continuous heating adjustment
- (f) Safety, Environmental, Economic considerations: Safety: Tank overflow, failure could cause injury Economics: Heating costs, spill costs, process quality costs Environmental: Energy consumption, contamination due to spills of hot water
- (g) Control System: Feedback: Temperature is monitored, heating rate is adjusted Feed-forward: Upstream flow velocity is used to predict future tank state and input is adjusted accordingly.
- iv. *Beer fermentation*
Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.
- v. *An activated sludge process*
Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.
- vi. A household thermostat
- (a) Objectives:
- Maintain comfortable temperature
 - Minimize energy consumption
- (b) Input Variables:
- Manipulated: Temperature setting
 - Disturbance: Outside temperature, energy transmission between house and environment
- (c) Output Variables:
- Measured: Thermostat reading
 - Unmeasured: Comfort level
- (d) Constraints:
- Hard: Max heating or cooling duty of system
 - Soft: Max or minimum temperature for comfort
- (e) Operating conditions: Continuous heating adjustment, continuous temperature reading.
- (f) Safety, Environmental, Economic considerations: Safety: heater may be an electrical or burning hazard Economics: Heating costs Environmental: Energy consumption.
- (g) Control System: Feedback; temperature is monitored, heating rate is adjusted after the fact.
- vii. *Air traffic control*
Please see either jogging, cycling, stirred tank heater, or household thermostat for a representative answer.

1.2

a. Fluidized Catalytic Cracking Unit

i. Summary of paper:

A fluidized catalytic cracking unit (FCCU) is one of the typical and complex processes in petroleum refining. Its principal components are a reactor and a generator. The reactor executes catalytic cracking to produce lighter petro-oil products. The regenerator recharges the catalyst and feeds it back to the reactor. In this paper, the authors test their control schemes on a FCCU model. The model is a nonlinear multi-input/multi-output (MIMO) which couples time varying and stochastic processes. Considerable computation is needed to use model predictive process control algorithms (MPC). Standard PID control gives inferior performance. A simplified MPC algorithm is able to reduce the number of parameters and computational load while still performing better than a PID control method.

ii. Familiar Terms: Constraint, nonlinearity, control performance, MPC, unmeasured disturbance rejection, modeling, simulation.

b. *Reactive Ion Etching*

Please see FCCU for a representative answer.

c. *Rotary Lime Kiln*

Please see FCCU for a representative answer.

d. *Continuous Drug Infusion*

Please see FCCU for a representative answer.

e. Anaerobic Digester

Please see FCCU for a representative answer.

f. Distillation

Please see FCCU for a representative answer.

g. Polymerization Reactor

Please see FCCU for a representative answer.

h. pH

Please see FCCU for a representative answer.

i. Beer Production

Please see FCCU for a representative answer.

j. Paper Machine Headbox

Please see FCCU for a representative answer.

k. Batch Chemical Reactor

Please see FCCU for a representative answer.

1.3

a. Vortex-shedding flow meters

The principal of vortex shedding can be seen in the curling motion of a flag waving in the breeze, or the eddies created by a fast moving stream. The flag outlines the shape of air vortices as the flow past the pole. Van Karman produced a formula describing the phenomena in 1911. In the late 1960's the first vortex shedding meters appeared on the market. Turbulent flow causes vortex formation in a fluid. The frequency of vortex detachment is directly proportional to fluid velocity in moderate to high flow regions. At low velocity, algorithms exist to account for nonlinearity. Vortex frequency is an input, fluid velocity is an output.

b. Orifice-plate flow meters

Please see vortex-shedding flow meters for a representative answer.

c. Mass flow meters

Please see vortex-shedding flow meters for a representative answer.

d. Thermocouple based temperature measurements

Please see vortex-shedding flow meters for a representative answer.

e. Differential pressure measurements

Please see vortex-shedding flow meters for a representative answer.

f. Control valves

Please see vortex-shedding flow meters for a representative answer.

representative answer.

g. pH

Please see vortex-shedding flow meters for a representative answer.

1.4

No solutions are required to work through Module 1

1.5

a.

The main objective is to maintain the process fluid outlet temperature at a desired setpoint of 300 C.

b.

The measured output is the process fluid outlet temperature.

c.

The manipulated input is the fuel gas flowrate, specifically the valve position of the fuel gas control valve.

d.

Possible disturbances include: process fluid flowrate, process fluid inlet temperature, fuel gas quality, and fuel gas upstream pressure.

e.

This is a continuous process.

f.

This is a feedback controller.

g.

The control valve should be *fail-closed*. Increasing air pressure to the valve will then increase the valve position and lead to an increase in flowrate. Loss of air to the valve will cause it to close. The gain of the valve is positive, because an increase in the signal to the valve results in an increase in flow.

h.

It is important from a safety perspective to have a fail-closed valve. If the valve failed open, there might not be enough combustion air, causing a loss of the flame - this could cause the furnace firebox to fill with fuel gas, which could then re-ignite under certain conditions. Although the combustion air is not shown, it should be supplied with a small stoichiometric excess. If there is too much excess combustion air, energy is wasted in heating up air that is not combusted. If there is too little excess air, combustion will not be complete, causing fuel gas to be wasted and pollution to the atmosphere. The process fluid is flowing

to another unit. If the process fluid is not at the desired setpoint temperature, the performance of the unit (reactor, etc) may not be as good as desired, and therefore not as profitable.

i.

The control block diagram for the process furnace is shown below in Figure 1. Where the the signals as

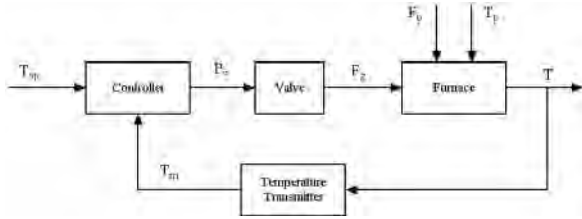


Figure 1-1: Control block diagram of process furnace follows:

- T_{sp} : Temperature setpoint
- p_v : Valve top pressure
- F_g : Fuel gas flowrate
- F_p : Process fluid flowrate
- T_p : Process fluid inlet temperature
- T : Temperature of process fluid outlet
- T_m : Measured temperature

1.6

The problem statement tells us that the gasoline is worth \$500,000 a day. A 2% increase in value is:

$$\frac{\$500,000}{day} \times 0.02 = \frac{\$10,000}{day}$$

We are also given that the revamp will cost \$2,000,000. We can now calculate the time required to payback the control system investment.

$$\frac{\$2,000,000}{\frac{\$10,000}{days}} = 200days$$

Therefore, we know it will take 200 days to pay off the investment.

1.7

$$2yrs \cdot 4.4 \text{ million } \$/yr \cdot 0.2\% = \$176,000$$

1.8

We know from the problem statement that the inlet (F_{in}) and outlet (F_{out}) flowrates can be represented with the following equations:

$$F_{in} = 50 + 10 \sin(0.1t)$$

$$F_{out} = 50$$

The change in volume as a function of time is

$$\frac{dV}{dt} = F_{in} - F_{out}$$

substituting what we know:

$$\frac{dV}{dt} = 50 + 10 \sin(0.1t) - 50$$

Simplifying

$$\frac{dV}{dt} = 10 \sin(0.1t)$$

Rearranging the equation

$$dV = 10 \sin(0.1t) dt$$

Taking the integral of both sides

$$\int dV = \int 10 \sin(0.1t) dt$$

Using basic calculus to solve

$$V - V_0 = \frac{-10}{0.1} \cos(0.1t) \Big|_{t=0}^t$$

We know the initial tank volume is 500 liters

$$V - 500 = -100 \cos(0.1t) - 100 \cos(0)$$

$$V - 500 = -100 \cos(0.1t) + 100$$

$$V(t) = 600 - 100 \cos(0.1t)$$

The equation above tells us how the volume of the tank will vary with time. This can also be seen visually in Figure 2 below.

1.9

a.

The objective is to maintain a desired blood glucose concentration by insulin injection. Insulin is the manipulated input and blood glucose is the measured output. As performed by injection, the input is really discrete and not continuous. Also, glucose is not continuously measured, so the measured output is discrete. Disturbances include meal consumption and exercise. Feedforward action is used when a diabetic administers an injection to compensate for a

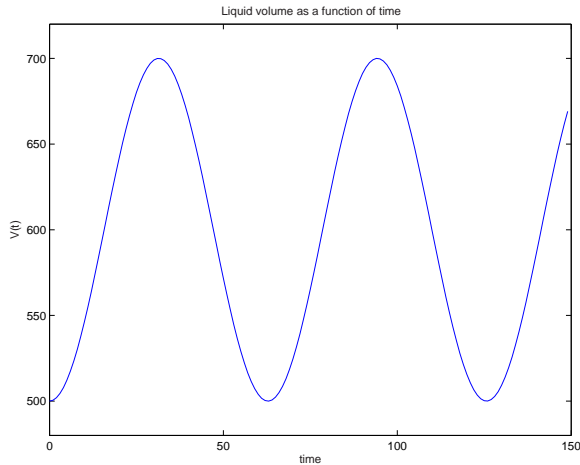


Figure 1-2: Liquid volume as a function of time

meal. Feedback action occurs when a diabetic administers more or less insulin based on a blood glucose measurement. It is important not to administer too much insulin, because this could lead to too low of a blood glucose level, resulting in hypoglycemia.

b.

A process and instrumentation diagram of an automated closed-loop system is shown in Figure 3 below. For simplicity, this is shown as a pump and valve

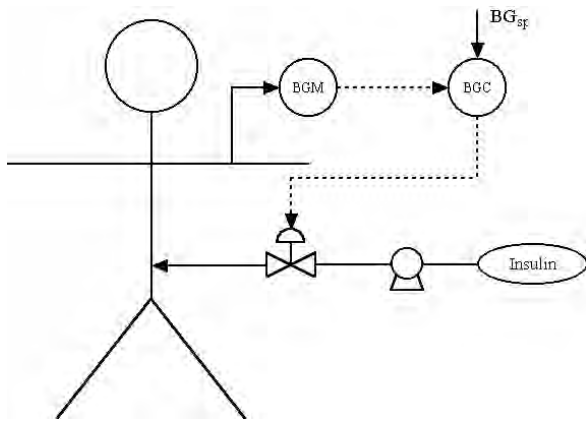


Figure 1-3: P&ID of closed-loop insulin infusion

arrangement. In practice, the pump speed would be varied. The associated control block diagram is shown in Figure 4 below.

1.10

a.

An increase in the hot stream flowrate leads to an increase in the cold stream outlet temperature, so the

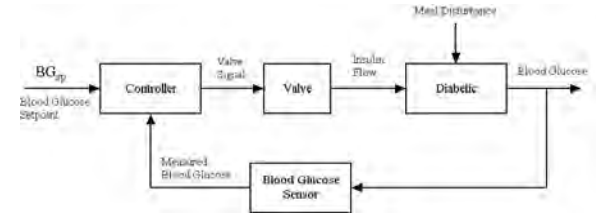


Figure 1-4: control block diagram of closed-loop insulin infusion

gain is positive. A *fail-closed* valve should be specified. If the valve failed-open, the cold stream outlet temperature could become too high.

b.

An increase in the hot by-pass flow leads to a decrease in the cold stream outlet temperature, so the gain is negative. A *fail-open* valve should be specified.

c.

An increase in the cold by-pass flowrate leads to a decrease in the outlet temperature, so the gain is negative. A *fail-open* valve should be specified, so that the outlet temperature is not too high or the air pressure is lost.

d.

Strategy (c), cold by-pass, will have the fastest dynamic behavior because the effect of changing the by-pass flow will be almost instantaneous. The other strategies have a dynamic lag through the heat exchanger.

1.11

The anesthesiologist attempts to maintain a desired setpoint for blood pressure. This is done by manipulating the drug flowrate. A major disturbance is the effect of an anesthetic on blood pressure.

The control block diagram for the automated system is shown below in Figure 5, where, for simplicity, the drug is shown being changed by a valve.

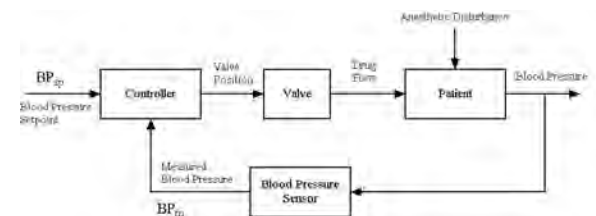


Figure 1-5: Control block diagram of drug delivery

Chapter 2 Solutions

2.1

The modeling equation is

$$\frac{dP}{dt} = \frac{RT}{V} q_i - \frac{RT}{V} \beta \sqrt{P - P_h}$$

At steady state

$$\begin{aligned} \frac{dP}{dt} &= \frac{RT}{V} q_i - \frac{RT}{V} \beta \sqrt{P - P_h} = 0 \\ \frac{RT}{V} \beta \sqrt{P_s - P_{hs}} &= \frac{RT}{V} q_{is} \\ P_s &= P_{hs} + \frac{q_{is}^2}{\beta^2} \end{aligned}$$

Thus we can conclude that it is a self-regulating system, as for a change in input it will attain a new steady-state.

The sketch of the steady-state input-output curve should look like figure 2-1.

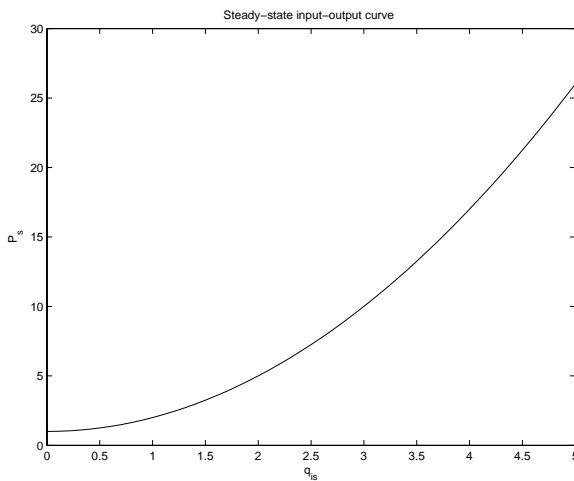


Figure 2-1: Plot for 2.1

2.2

The model equations are

$$\begin{aligned} \frac{dV}{dt} &= F_i - F \\ \frac{dT}{dt} &= \frac{F_i}{V} (T_i - T) + Q \end{aligned}$$

- a. At steady-state, the volume will not change, as

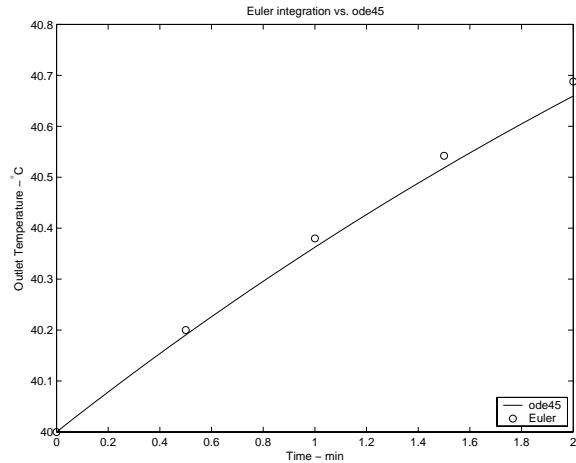


Figure 2-2: Plot for 2.2

the inlet and outlet flow rates are the same. Thus

$$\begin{aligned} \frac{F_i}{V} (T_i - T) + Q &= 0 \\ \frac{100}{500} (20 - 40) + Q &= 0 \\ Q &= 4^\circ\text{C}/\text{min} \end{aligned}$$

- b. For this part we need to integrate

$$\frac{dT}{dt} = \frac{100}{500} (22 - T) + 4$$

from an initial state of $T = 40^\circ\text{C}$. The Euler formula is

$$\begin{aligned} x_{k+1} &= x_k + \Delta t x'_k \\ x'_k &= f(x_k) \end{aligned}$$

where $f(\cdot)$ is the right hand side of the differential equation, and x is the state, in this case T . Using $\Delta t = 0.5$, and for a total of 2 minutes, we have

$$x_0 = 40$$

$$x_1 = 40 + 0.5 \left(\frac{1}{5} (22 - 40) + 4 \right) = 40.2$$

$$x_2 = 40.2 + 0.5 \left(\frac{1}{5} (22 - 40.2) + 4 \right) = 40.38$$

$$x_3 = 40.38 + 0.5 \left(\frac{1}{5} (22 - 40.38) + 4 \right) = 40.542$$

$$x_4 = 40.542 + 0.5 \left(\frac{1}{5} (22 - 40.542) + 4 \right) = 40.6878$$

Figure 2-2 shows the curve of the solution found using MATLAB's `ode45`, with the circles marking the points of the Euler solution.

2.3

Since the model equations have only two states, V_l and P , we have to assume the following are constant: density of the liquid (ρ), temperature (T), the ideal gas constant (R) and the molecular weight of the gas (MW).

Starting with the balance of the liquid mass in the system, we have

$$\begin{aligned}\frac{dM_l}{dt} &= \frac{dV_l \rho}{dt} = F_f \rho - F \rho \\ \frac{dV_l}{dt} &= F_f - F\end{aligned}$$

For the balance of the mass of gas

$$\begin{aligned}\frac{dnMW}{dt} &= q_i MW_i - q MW \\ \frac{dn}{dt} &= q_i - q\end{aligned}$$

From the ideal gas law $PV_g = nRT$, where the volume of gas is $V_g = V - V_l$. Then,

$$\begin{aligned}\frac{d(PV_g)}{dt} &= \frac{nRT}{dt} \\ P \frac{d(V - V_l)}{dt} + (V - V_l) \frac{dP}{dt} &= RT \frac{dn}{dt}\end{aligned}$$

and using the previously derived expressions for $\frac{dn}{dt}$ and $\frac{dV_l}{dt}$

$$\begin{aligned}P \frac{d(V - V_l)}{dt} + (V - V_l) \frac{dP}{dt} &= RT \frac{dn}{dt} \\ -P \frac{dV_l}{dt} + (V - V_l) \frac{dP}{dt} &= RT(q_i - q) \\ \frac{dP}{dt} &= \frac{P}{V - V_l}(F_f - F) + \frac{RT}{V - V_l}(q_i - q)\end{aligned}$$

Thus our model equations are

$$\begin{aligned}\frac{dV_l}{dt} &= F_f - F \\ \frac{dP}{dt} &= \frac{P}{V - V_l}(F_f - F) + \frac{RT}{V - V_l}(q_i - q)\end{aligned}$$

2.4

Since we have a larger volume than the example, we have to calculate the flow rate for a single reactor as well. Our volume is $V = 106.9444\text{ft}^3$. The equations for the first tank are

$$\begin{aligned}\frac{dC_{A1}}{dt} &= \frac{F}{V}(C_{A_i} - C_{A1}) - kC_{A1} \\ \frac{dC_{P1}}{dt} &= -\frac{F}{V}C_{P1} + kC_{A1}\end{aligned}$$

solving at steady-state, we get

$$C_{P1_s} = \frac{C_{A_{i_s}}}{\frac{F_s}{kV} + 1}$$

We need to meet a yearly production, so our final constraint is

$$F_s C_{P1_s} S = 100x10^6 \text{lb/yr}$$

Where $S = 62\text{lb/lbmol} \cdot 504000\text{min/yr}$ is our conversion factor, assuming 350 days of operation in a year. Then,

$$F_s \frac{C_{A_{i_s}}}{\frac{F_s}{kV} + 1} S = 100x10^6 \text{lb/yr}$$

solving for the flowrate, we get $F_s = 7.9256\text{ft}^3/\text{min}$.

Now we need to consider the second reactor in series, which will also change the flowrate needed to meet production levels. The equations for the second reactor are

$$\begin{aligned}\frac{dC_{A2}}{dt} &= \frac{F}{V}(C_{A1} - C_{A2}) - kC_{A2} \\ \frac{dC_{P1}}{dt} &= \frac{F}{V}(C_{P1} - C_{P2}) + kC_{A2}\end{aligned}$$

Solving at steady-state, we get

$$C_{P2_s} = \frac{\left(\frac{kV}{F_s}\right) \left(\frac{kV}{F_s} + 2\right) C_{A_{i_s}}}{\left(\frac{kV}{F_s} + 1\right)^2}$$

Again, we need to meet production levels, so

$$\begin{aligned}F_s C_{P2_s} S &= 100x10^6 \text{lb/yr} \\ \frac{kV \left(\frac{kV}{F_s} + 2\right) C_{A_{i_s}}}{\left(\frac{kV}{F_s} + 1\right)^2} S &= 100x10^6 \text{lb/yr}\end{aligned}$$

Solving for the flowrate, we get $F_s = 6.5799\text{ft}^3/\text{min}$. Thus we have a savings of 16.98% using the two reactors in series over a single one.

2.5

The resulting graph should be the same as figure 2-5, except that the time range from -1 to 0 will not appear.

2.6

$$\begin{aligned}\frac{d(VC_a)}{dt} &= F_{in} C_{A_{in}} - kVC_a \\ \frac{dV}{dt} &= F_{in}\end{aligned}$$

We need equations whose states are V and C_A , then

$$\begin{aligned}\frac{d(VC_a)}{dt} &= F_{in}C_{A_{in}} - kVC_A \\ V\frac{dC_a}{dt} + C_A\frac{d(V)}{dt} &= F_{in}C_{A_{in}} - kVC_A \\ V\frac{dC_a}{dt} &= -C_A\frac{d(V)}{dt} + F_{in}C_{A_{in}} - kVC_A \\ V\frac{dC_a}{dt} &= -C_AF_{in} + F_{in}C_{A_{in}} - kVC_A \\ \frac{dC_a}{dt} &= \frac{F_{in}}{V}(C_{A_{in}} - C_A) - kC_A\end{aligned}$$

our two equations are

$$\begin{aligned}\frac{dV}{dt} &= F_{in} \\ \frac{dC_a}{dt} &= \frac{F_{in}}{V}(C_{A_{in}} - C_A) - kC_A\end{aligned}$$

2.7

a. The modeling equations are

$$\begin{aligned}\frac{dC_{w1}}{dt} &= \frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \\ \frac{dC_{w2}}{dt} &= \frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2\end{aligned}$$

b. At steady-state we can solve the following equations

$$\begin{aligned}\frac{F_s}{V_1}(C_{wis} - C_{w1s}) - kC_{w1s}^2 &= 0 \\ \frac{F_s}{V_2}(C_{w1s} - C_{w2s}) - kC_{w2s}^2 &= 0\end{aligned}$$

Rearranging the first equation, we have the quadratic

$$kC_{w1s}^2 + \frac{F_s}{V_1}C_{w1s} - \frac{F_s}{V_1}C_{wis} = 0$$

the positive root gives us

$$C_{w1s} = 0.33333 \text{ mol/liter}$$

Rearranging the second equation, we have the quadratic

$$kC_{w2s}^2 + \frac{F_s}{V_2}C_{w2s} - \frac{F_s}{V_2}C_{w1s} = 0$$

the positive root gives us

$$C_{w2s} = 0.0900521 \text{ mol/liter}$$

c. To linearize, we have the functions

$$\begin{aligned}f_1 &= \frac{dC_{w1}}{dt} = \frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \\ f_2 &= \frac{dC_{w2}}{dt} = \frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2\end{aligned}$$

and using the state and input variables as defined, we have

$$\begin{aligned}a_{11} &= \left. \frac{\delta f_1}{\delta x_1} \right|_{ss} = \left. \frac{\delta}{\delta C_{w1}} \left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \right) \right|_{ss} \\ &= -\frac{F_s}{V_1} - 2kC_{w1s} \\ a_{12} &= \left. \frac{\delta f_1}{\delta x_2} \right|_{ss} = \left. \frac{\delta}{\delta C_{w2}} \left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \right) \right|_{ss} \\ &= 0 \\ a_{21} &= \left. \frac{\delta f_2}{\delta x_1} \right|_{ss} = \left. \frac{\delta}{\delta C_{w1}} \left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2 \right) \right|_{ss} \\ &= \frac{F_s}{V_2} \\ a_{22} &= \left. \frac{\delta f_2}{\delta x_2} \right|_{ss} = \left. \frac{\delta}{\delta C_{w2}} \left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2 \right) \right|_{ss} \\ &= -\frac{F_s}{V_2} - 2kC_{w2s} \\ b_{11} &= \left. \frac{\delta f_1}{\delta u_1} \right|_{ss} = \left. \frac{\delta}{\delta F} \left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \right) \right|_{ss} \\ &= \frac{1}{V_1}(C_{wis} - C_{w1s}) \\ b_{12} &= \left. \frac{\delta f_1}{\delta u_2} \right|_{ss} = \left. \frac{\delta}{\delta C_{wi}} \left(\frac{F}{V_1}(C_{wi} - C_{w1}) - kC_{w1}^2 \right) \right|_{ss} \\ &= \frac{F_s}{V_1} \\ b_{21} &= \left. \frac{\delta f_2}{\delta u_1} \right|_{ss} = \left. \frac{\delta}{\delta F} \left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2 \right) \right|_{ss} \\ &= \frac{1}{V_2}(C_{w1s} - C_{w2s}) \\ b_{22} &= \left. \frac{\delta f_2}{\delta u_2} \right|_{ss} = \left. \frac{\delta}{\delta C_{wi}} \left(\frac{F}{V_2}(C_{w1} - C_{w2}) - kC_{w2}^2 \right) \right|_{ss} \\ &= 0\end{aligned}$$

d. Evaluating these coefficients at our steady-state, we have

$$\begin{aligned}A &= \begin{bmatrix} -1.25 \text{ hr}^{-1} & 0 \\ 0.05 \text{ hr}^{-1} & -0.320156 \text{ hr}^{-1} \end{bmatrix} \\ B &= \begin{bmatrix} 0.0016667 \text{ mol/l}^2 & 0.25 \text{ hr}^{-1} \\ 0.000121641 \text{ mol/l}^2 & 0 \end{bmatrix}\end{aligned}$$

e. Since $\vec{y} = \vec{x}$, it is straightforward to show that

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

f. Using MATLAB, the eigenvalues are -0.320156 and -1.25 .

analytically, we have

$$\det(\lambda I - A) = 0$$

$$\det \begin{bmatrix} \lambda + 1.25 & 0 \\ -0.05 & \lambda + 0.320156 \end{bmatrix} = 0$$

$$(\lambda + 1.25)(\lambda + 0.320156) = 0$$

thus the eigenvalues are $\lambda_1 = -1.25$ and $\lambda_2 = -0.320156$

- g. Figure 2-3 shows the plot for the linear and non-linear responses; as it can be seen, the extraction requirements are still met.

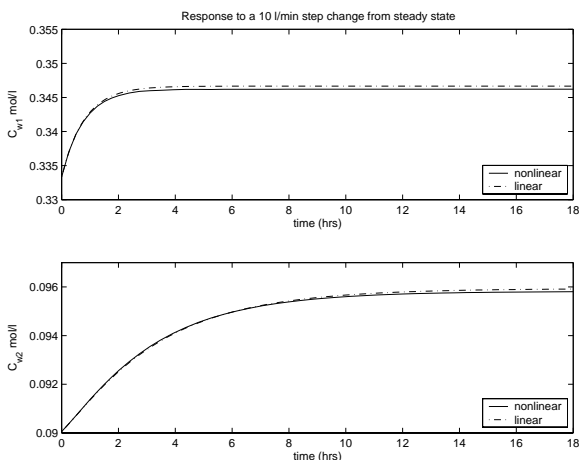


Figure 2-3: Plot for 2.7g

- h. If the order of the reaction vessels is reversed, the steady-state equations we have to solve are

$$\frac{F_s}{V_2} (C_{w1s} - C_{w2s}) - kC_{w1s}^2 = 0$$

$$\frac{F_s}{V_1} (C_{w1s} - C_{w2s}) - kC_{w2s}^2 = 0$$

solving then we get $C_{w1s} = \frac{1}{6}$ mol/l and $C_{w2s} = 0.10301$ mol/l. Thus, the extraction requirements are no longer met.

2.8

- a. Solve the following two simultaneous equations using the parameters and steady-state values provided:

$$\frac{F_s}{V} (T_{is} - T_s) + \frac{UA}{V\rho c_p} (T_{js} - T_s) = 0$$

$$\frac{F_{js}}{V_j} (T_{jins} - T_{js}) - \frac{UA}{V_j \rho_j c_{pj}} (T_{js} - T_s) = 0$$

then

$$UA = 183.9 \text{ Btu}/(^{\circ}\text{F}\cdot\text{min})$$

$$F_{js} = 1.5 \text{ ft}^3/\text{min}$$

- b. Applying the equations for the elements of the linearization matrices

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.3 \\ 1.2 & -1.8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -7.5 & 0.1 & 0 \\ 20 & 0 & 0 & 0.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- c. `heater.m` file should be like the example in the book (p. 73)
- d. Using `delJ = 0`, run `ode45` to solve the equations defined in `heater.m`, then plot the two states vs. time. The result should be constant values that match the steady states for all time.
- e. To get the desired plots for the two step responses, the m-file shown in pages 74-75 can be used, starting with the definition of the state space linear model. Since the model is linear, the output of the step response command can be scaled accordingly for steps of different sizes by just multiplying by `delFj`. Figures 4(a) and 4(b) show the responses for a small (0.2% change in F_j) and a large (10% change in F_j) steps, respectively.

- f. Since we know UA for the small vessel, and we are assuming that U remains constant, we can find the value of UA for a larger volume as

$$UA_{small} \cdot \frac{A_{large}}{A_{small}} = UA_{large}$$

Modeling the vessel as a cylinder, the volume is $V = \frac{\pi}{2}D^3$, and the area is $A = 2.25\pi D^2$. We can then calculate the area of the small vessel as a function of its volume, for which we get

$$A_{small} = 2.25\pi \left(\frac{20}{\pi} \right)^{\frac{2}{3}}$$

Similarly, we have the area of the larger vessel in terms of its volume

$$A_{large} = 2.25\pi \left(\frac{2V}{\pi} \right)^{\frac{2}{3}}$$

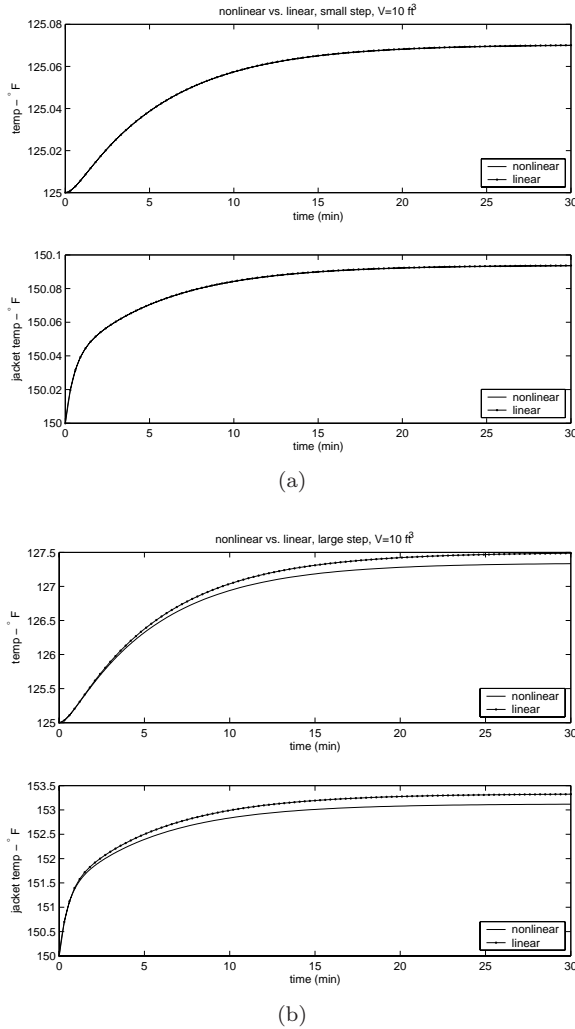


Figure 2-4: Plot for 2.8e (a) small step of 0.2% (b) large step of 10%

And the ratio of the areas is

$$\frac{A_{large}}{A_{small}} = \left(\frac{V}{10}\right)^{\frac{2}{3}}$$

Applying this, we find that

$$UA_{large} = 853.588 \text{ Btu}/(^{\circ}\text{F}\cdot\text{min})$$

- g. Solve the following two simultaneous equations using the value of UA calculated for the large vessel

$$\begin{aligned} \frac{F_s}{V} (T_{is} - T_s) + \frac{UA}{V\rho c_p} (T_{js} - T_s) &= 0 \\ F_{js} (T_{jins} - T_{js}) - \frac{UA}{\rho_j c_{pj}} (T_{js} - T_s) &= 0 \end{aligned}$$

then

$$\begin{aligned} T_{js} &= 178.86 \text{ }^{\circ}\text{F} \\ F_{js} &= 35.478 \text{ ft}^3/\text{min} \end{aligned}$$

We can already see that the steady-state jacket temperature has gone up, so we want to know how large we can make the vessel before the jacket temperature approaches the inlet jacket temperature. We can solve the following equation, using $\frac{F_s}{V} = 0.1 \text{ min}^{-1}$, as we want to maintain the residence time. We also use the expression in terms of the volume that we found in part f

$$\begin{aligned} \frac{F_s}{V} (T_{is} - T_s) + \frac{UA}{V\rho c_p} (T_{js} - T_s) &= 0 \\ 0.1 (T_{is} - T_s) + \frac{UA_{small} \left(\frac{V}{10}\right)^{\frac{2}{3}}}{V\rho c_p} (T_{js} - T_s) &= 0 \end{aligned}$$

then $V = 270 \text{ ft}^3$.

- h. Applying the equations for the elements of the linearization matrices using the steady state for the larger vessel, we have

$$\begin{aligned} A &= \begin{bmatrix} -0.2392 & 0.1392 \\ 0.5570 & -1.9761 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & -0.75 & 0.1 & 0 \\ 0.8456 & 0 & 0 & 1.4191 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ D &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

The eigenvalues for the system with the large vessel are at $\lambda_1 = -0.1957$ and $\lambda_2 = -2.0197$. While for the system with the smaller vessel, they are $\lambda_1 = -0.1780$ and $\lambda_2 = -2.0220$. They are very close to each other, thus the speed of the response will be similar for both vessels.

- i. Figure 2-5 shows the response to a step of $0.1 \text{ ft}^3/\text{min}$. Comparing this to figure 4(a), we can see that both linear model responses are practically the same as the nonlinear model response. The change in temperatures is also minor, as the change in jacket flow rate is small (0.2% of the steady state value in both cases).
- j. Figure 2-6 shows the response to a step of 10% of the steady state jacket flow rate. Comparing this to figure 4(b), we can see that both linear model

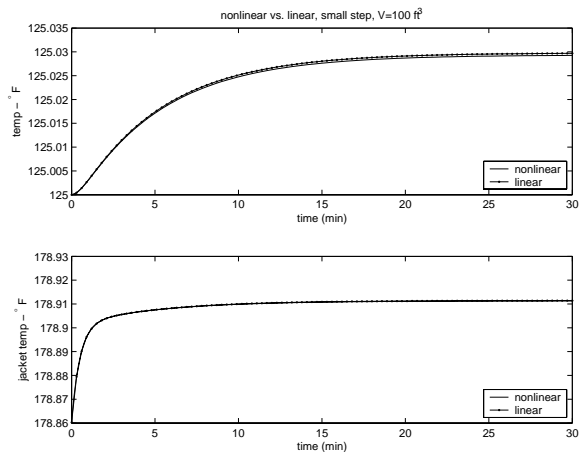


Figure 2-5: Plot for 2.8i

responses are close to the nonlinear model response, but there is an offset in the steady state. The change in temperatures is also more marked, and larger in the case of the smaller reactor, as would be expected due to the smaller volume.

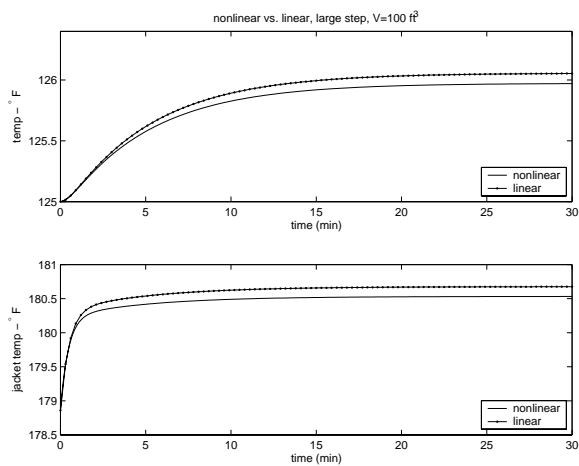


Figure 2-6: Plot for 2.8j

Chapter 4 Solutions

4.1

The maximum rate-of-change means taking a derivative. The output is:

$$y(t) = k_p \Delta u \left\{ 1 - \exp\left(-\frac{t - \theta}{\tau_p}\right) \right\}$$

The derivative with respect to time is:

$$\frac{dy}{dt} = \frac{k_p \Delta u}{\tau_p} \exp\left(-\frac{t - \theta}{\tau_p}\right)$$

Due to the negative sign, the largest that $\exp\left\{-\frac{t - \theta}{\tau_p}\right\}$ can be is 1. An exponential will give a value of 1 when the argument is 0. That means the following is true:

$$\frac{t - \theta}{\tau_p} = 0$$

Solving for θ gives

$$t = \theta$$

To find the maximum slope, plug $t = \theta$ into the slope equation:

$$\frac{dy}{dt} = \frac{k_p \Delta u}{\tau_p} \exp\left(-\frac{\theta - \theta}{\tau_p}\right)$$

Which simplifies down to a maximum slope of:

$$\frac{dy}{dt} = \frac{k_p \Delta u}{\tau_p}$$

4.2

The output response $y(t)$ for a first order plus dead time (FOPDT) model to a step change is:

$$y(t) = k_p \Delta u \left\{ 1 - \exp\left(-\frac{t - \theta}{\tau_p}\right) \right\}$$

Recognizing that $\Delta y = k_p \Delta u$ gives:

$$y(t) = \Delta y \left\{ 1 - \exp\left(-\frac{t - \theta}{\tau_p}\right) \right\}$$

Substituting $t_1 = \frac{\tau_p}{3} + \theta$ into the output equation:

$$y(t) = \Delta y \left\{ 1 - \exp\left(-\frac{\frac{\tau_p}{3} + \theta - \theta}{\tau_p}\right) \right\}$$

Cancelling out terms gives:

$$y(t) = \Delta y \left\{ 1 - \exp\left(-\frac{1}{3}\right) \right\}$$

Which simplifies down to:

$$y(t_1) = 0.238 \Delta y$$

Substituting $t_2 = \tau_p + \theta$ into the output equation:

$$y(t) = \Delta y \left\{ 1 - \exp\left(-\frac{\tau_p + \theta - \theta}{\tau_p}\right) \right\}$$

Cancelling out terms gives:

$$y(t) = \Delta y \{ 1 - \exp(-1) \}$$

Which simplifies down to:

$$y(t_1) = 0.632 \Delta y$$

Use the equations for t_1 and t_2 to solve for θ and τ_p

$$t_1 = \frac{\tau_p}{3} + \theta$$

$$t_2 = \tau_p + \theta$$

Solving for θ in the t_2 equation gives:

$$\theta = t_2 - \tau_p$$

Plugging that into the t_1 equation:

$$t_1 = \frac{\tau_p}{3} + t_2 - \tau_p$$

Solving for τ_p yields:

$$\tau_p = \frac{3}{2}(t_2 - t_1)$$

4.3

The first technique to estimate the parameters in a first order plus dead time (FOPDT) model is the 63.2% method. Find the gain using the formula:

$$k_p = \frac{\Delta y}{\Delta u}$$

$$k_p = \frac{68 - 50 \text{ }^\circ\text{C}}{28 - 25 \text{ psig}} = 6 \frac{^\circ\text{C}}{\text{psig}}$$

The time delay can be “eye balled” by looking at when the output begins to change significantly, and when the input change was applied. For this process:

$$\theta = 5 \text{ min} - 1 \text{ min} = 4 \text{ min}$$

The time constant is estimated using the 63.2% method. First, calculate what 63.2% of the output change is:

$$0.632 \Delta y = 0.632(68 - 50) = 11.376 \text{ }^\circ\text{C}$$

Looking at the output response, the time for the response to reach 11.376 °C is 15 minutes. The time constant can then be calculated using the formula given in the chapter.

$$t_{63.2\%} = \tau + \theta$$

$$\tau = t_{63.2\%} - \theta = 15 - 4 = 11 \text{ min}$$

The FOPDT can then be expressed as:

$$g_p(s) = \frac{6e^{-4s}}{10s + 1}$$

The second technique to estimate the parameters in a first order plus dead time (FOPDT) model is the Maximum Slope method. Find the gain using the formula:

$$k_p = \frac{\Delta y}{\Delta u}$$

$$k_p = \frac{68 - 50 \text{ }^\circ\text{C}}{28 - 25 \text{ psig}} = 6 \frac{^\circ\text{C}}{\text{psig}}$$

The time delay can be “eye balled” by looking at when the output begins to change significantly, and when the input change was applied. For this process:

$$\theta = 5 \text{ min} - 1 \text{ min} = 4 \text{ min}$$

The time constant can be estimated using the maximum slope of the output response. Looking at the response, we can see that the maximum slope can be calculated using:

$$\text{slope} = \frac{58 - 50}{15 - 5} = 0.8 \frac{^\circ\text{C}}{\text{min}}$$

The time constant can now be calculated using:

$$\tau_p = \frac{\Delta y}{\text{slope}}$$

$$\tau_p = \frac{6 \text{ }^\circ\text{C}}{0.8 \frac{^\circ\text{C}}{\text{min}}} = 7.5 \text{ min}$$

The FOPDT can then be expressed as:

$$g_p(s) = \frac{6e^{-4s}}{7.5s + 1}$$

The second technique to estimate the parameters in a first order plus dead time (FOPDT) model is the Two Point method. Find the gain using the formula:

$$k_p = \frac{\Delta y}{\Delta u}$$

$$k_p = \frac{68 - 50 \text{ }^\circ\text{C}}{28 - 25 \text{ psig}} = 6 \frac{^\circ\text{C}}{\text{psig}}$$

The time delay can be “eye balled” by looking at when the output begins to change significantly, and when the input change was applied. For this process:

$$\theta = 5 \text{ min} - 1 \text{ min} = 4 \text{ min}$$

The time constant can be estimated by first determining what 63.2% and 28.3% of the output is.

$$0.632\Delta y = (0.632)(18) = 11.376$$

$$0.283\Delta y = (0.283)(18) = 5.094$$

Looking at the response, the respective times to reach those output values are $t_{63.2} = 15$ and $t_{28.3} = 8$. The time constant can then be calculated using the formula:

$$\tau_p = 1.5(t_{63.2} - t_{28.3})$$

$$\tau_p = 1.5(15 - 8) = 10.5 \text{ min}$$

The FOPDT can then be expressed as:

$$g_p(s) = \frac{6e^{-4s}}{10.5s + 1}$$

Note that all three methods give similar, but not identical FOPDT models.

4.4

An integrator plus dead time model has the form:

$$g_p(s) = \frac{k_p e^{-\theta s}}{s}$$

We therefore need to estimate a gain and a time delay. The time delay is estimated by looking at when the output begins to change significantly, and subtracting the time when the input change was made. For this process:

$$\theta = 3 \text{ min} - 1 \text{ min} = 2 \text{ min}$$

To get the gain, we need to find both the slope of the output and the change in input. Looking at the figure, we see that:

$$\Delta u = 9.5 - 10.0 \text{ lps} = -0.5 \text{ lps}$$

$$\text{slope} = \frac{0.3 - 1 \text{ m}}{10 - 3 \text{ min}} = -0.1 \frac{\text{m}}{\text{min}}$$

We can then calculate the gain using the formula:

$$k_p = \frac{\text{slope}}{\Delta u}$$

$$k_p = \frac{-0.1 \frac{\text{m}}{\text{min}}}{-0.5 \text{ lps}} = 0.2 \frac{\text{m}}{\text{min} \cdot \text{lps}}$$

The integrator plus dead time model is thus:

$$g_p(s) = \frac{0.2e^{-2s}}{s}$$