Instructor's Solutions Manual

for

Bob Dobrow's

Probability with Applications and R

Bob Dobrow, Matthew Rathkey, and Wenli Rui

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Chapter 1

1.1 (i) An experiment whose outcome is uncertain. (ii) The set of all possible outcomes. (iii) A set of outcomes. (iv) A random variable assigns numerical values to the outcomes of a random experiment.

1.2 (i) Roll four dices. (ii) $\Omega = \{1111, 1112, \ldots, 6665, 6666\}$. (iii) Event: $\{5555\}$. (iv) Let X denote the number of fives in four dice rolls. Then X is the random variable. The desired probability is $P(X = 4)$.

1.3 (i) Choosing toppings. (ii) Let a, b, c denote pineapple, peppers, and pepperoni, respectively. Then $\Omega = \{\emptyset, a, b, c, ab, ac, bc, abc\}$. (iii) Event: $\{ab, ac, bc\}$. (iv) Let X be the number of toppings. Then X is the random variable. The desired probability is $P(X = 2)$.

1.4 (i) Playing Angry Birds until you win. (ii) Let W denote winning, and L denote losing. Then $\Omega = \{W, LW, LLW, \ldots\}$. (iii) Event: $\{X < 1000\}$, where X is the number of times you play before you win. (iv) X is the random variable. The desired probability is $P(X < 1000)$.

1.5 (i) Harvesting 1000 tomatoes. (ii) Ω is the set of all 1000-element of sequences consisting of $B's$ (bad) and $G's$ (good). (iii) Event: $\{X \leq 5\}$, where X is the number of bad tomatoes. (iv) X is the random variable. The desired probability is $P(X \leq 5)$.

1.6 (a) $\{13, 22, 31\}$;

(b) $\{36, 45, 54, 63\}$;

(c) $\{13, 23, 33, 43, 53, 63\}$;

(d) $\{11, 22, 33, 44, 55, 66\}$;

(e) $\{31, 41, 51, 52, 61, 62\}$.

1.7 (a) $\{R = 0\}$;

(b) $\{R = 1, B = 2\}$;

(c) $\{R + B = 4\}$;

(d) $\{R = 2B\}$.

1.8 Let B denote a boy and G denote a girl. Then $\Omega = \{G, BG, BBG, \ldots BBBBBB\}$. The random variable is the number of boys.

1.9 $P(\omega_1) = \frac{24}{41}$; $P(\omega_2) = \frac{12}{41}$; $P(\omega_3) = \frac{4}{41}$; $P(\omega_4) = \frac{1}{41}$.

1.10 Must have $p + p^2 + p = 1$. Solve $p^2 + 2p = 1$. Since $p \geq 0$, $p = \sqrt{2} - 1 = 0.414$.

1.11 (a) $P(A) \geq 0$, since $P_1(A) \geq 0$ and $P_1(A) \geq 0$. (b)

$$\begin{aligned}
\sum_{\omega} P(\omega) &= \sum_{\omega} \frac{P_1(\omega) + P_2(\omega)}{2} \\
&= \frac{1}{2}\left(\sum_{\omega} P_1(\omega) + \sum_{\omega} P_2(\omega)\right) \\
&= \frac{1}{2}(1 + 1) = 1.
\end{aligned}$$

(c)

$$\begin{aligned}\sum_{\omega\in A} P(\omega) &= \sum_{\omega\in A} \frac{P_1(\omega)+P_2(\omega)}{2}\\ &= \frac{1}{2}\left(\sum_{\omega\in A} P_1(\omega) + \sum_{\omega\in A} P_2(\omega)\right)\\ &= \frac{1}{2}(P_1(A)+P_2(A)) = P(A).\end{aligned}$$

1.12

$$\begin{aligned}\sum_{\omega} P(\omega) &= \sum_{\omega} a_1P_1(\omega)+a_2P_2(\omega)+\cdots+a_kP_k(\omega)\\ &= a_1\sum_{\omega} P_1(\omega)+a_2\sum_{\omega} P_2(\omega)+\cdots+a_k\sum_{\omega} P_k(\omega)\\ &= a_1+a_2+\cdots+a_k.\end{aligned}$$

Thus $a_1+a_2+\cdots+a_k = 1$.

1.13

$$\begin{aligned}\sum_{\omega} Q(\omega) &= \sum_{\omega} [P(\omega)]^2\\ &= [P(a)]^2+[P(b)]^2 = 1.\end{aligned}$$

Solve $p^2+(1-p)^2 = 1$. Then $p = 0$ or 1.

1.14 (a) The number of ways to select a president is 10. The number of ways to select Tom to be the president is 1. Thus the desired probability is 1/10. (b) The number of ways to select a president and a treasurer is $10 \times 9 = 90$. The number of ways to select Brenda to be the president and Liz to be the treasurer is 1. The desired probability is 1/90.

1.15 The number of 6-element sequences with first two elements H and last two elements T is $2^2 = 4$. The number of 6-element sequences of H's and T's is $2^6 = 64$. Thus the desired probability is $4/64 = 1/16$.

1.16 (a) $\frac{1}{26^2+26^3+26^4+26^5} = 8.093 \times 10^{-8}$;
(b) $\frac{26^4}{26^2+26^3+26^4+26^5} = 0.037$;
(c) $\frac{26+2\times 26^2+26^3}{26^2+26^3+26^4+26^5} = 0.0015$;
(d) $1-\frac{25^2+25^3+25^4+25^5}{26^2+26^3+26^4+26^5} = 0.171$.

1.17 (a) $6/6^5 = 1/6^4 = 1.286 \times 10^{-4}$;
(b) $1-(5/6)^5 = 0.598$;
(c) $\frac{6\times 5\cdots\times 2}{6^5} = 0.0926$.

1.18 (a) $\frac{3\times 19\times 18}{20\times 19\times 18} = 0.15$;
(b) $\frac{6\times 18}{20\times 19\times 18} = 0.0079$;
(c) $\frac{6}{20\times 19\times 18} = 4.386 \times 10^{-4}$.

1.19 There are $k!$ orderings of which one is in increasing order. Thus, $1/k!$.

1.20 (a) 0.2; (b) 0.2; (c) 0.6.

1.21 (a) 0.9; (b) 0; (c) 0.1; (d) 0.9.

1.22 We know

$$P(A \cup B) = P(A) + P(B) - P(AB) = 0.6$$

and $P(A \cup B^c) = P(A) + P(B^c) - P(AB^c) = 0.8$.

Solving for $P(A)$ gives $P(A) = 0.4$.

1.23 (i) $A^c = \{X < 2 \text{ or } X > 4\}$; (ii) $B^c = \{X < 4\}$; (iii) $AB = \{X = 4\}$; (iv) $A \cup B = \{X \geq 2\}$.

1.24 (i) $\frac{34}{101} = 0.337$; (ii) $\frac{12}{101} = 0.119$.

1.25 We know

$$P(A \cup B \cup C) + P(A \cup B \cup C)^c = P(A \cup B \cup C) + P(A^c B^c C^c) = 1$$

Given $P(A^c B^c C^c) = 0$, if follows that $P(A \cup B \cup C) = 1$.
We also know

$$\begin{aligned} P(A \cup B \cup C) = {} & P(AB^cC^c) + P(A^cBC^c) + P(A^cB^cC) \\ & + P(AB) + P(BC) + P(AC) - 2P(ABC) = 1. \end{aligned}$$

Given

$$P(ABC) = P(AB^cC^c) = P(A^cBC^c) = P(A^cB^cC) = 0.$$

Then $P(BC) + P(AB) + P(AC) = 1$.

Thus $P(B) = P(A^cBC^c) + P(AB) + P(BC) - P(ABC) = 0.8$.

1.26 (a) h; (b) $a + c + f$; (c) $d + e + b$; (d) g; (e) $1 - h$; (f) $b + d + e + g$; (g) $a + c + f + h$; (h) $1 - g$.

1.27 (i) 1/8; (ii) 5/8; (iii) 1/8.

1.28 $P(X = k) = (2k - 1)/36$ for $k = 1, \ldots, 6$.

1.29 (a) $\Omega = \{(1,1), (1,5), (1,10), (1,25), (5,1), \ldots, (25,25)\}$.
(b) $P(X = 1) = 1/16$; $P(X = 5) = 3/16$; $P(X = 10) = 5/16$; $P(X = 25) = 7/16$.
(c) $P(\text{Judith} > \text{Joe}) = P(\{(5,1), (10,5), (10,1), (25,10), (25,5), (25,1)\}) = 3/8$.

1.30 $P(\text{At least one 2}) = 1 - P(\text{No 2's}) = 1 - (3/4)^5 = 0.7627$.

1.31 (a) Use geometric series fomular,

$$\sum_{k=0}^{\infty} Q(k) = \sum_{k=0}^{\infty} \frac{2}{3^{k+1}} = \frac{2}{3}\left(\frac{1}{1 - 1/3}\right) = 1.$$

(b) $P(X > 2) = 1 - P(X \leq 2) = 1 - \frac{2}{3} - \frac{2}{9} - \frac{2}{27} = 1/27$.

1.32 $c = e^{-3} = 0.498$.

1.33 (a) $A \cup B \cup C$
(b) $A^c B C^c$
(c) $AB^cC^c \cup A^cBC^c \cup A^cB^cC \cup A^cB^cC^c$
(d) ABC
(e) $A^cB^cC^c$.

1.34 (a) $p/(1-p) = 1/16$. Thus, $1 - p = 16/17$. (b) $p/(1-p) = 2/9$ so $p = 2/11$.

1.35 $1 - (1/5 + 1/4 + 1/3) + (1/10 + 1/10 + 1/10) = 31/60$.

1.36 (a) $P(A \cup B \cup C) = 0.95$; (b) $P(AB^cC^c \cup A^cBC^c \cup A^cB^cC \cup A^cB^cC^c) = 0.5$; (c) $P(ABC) = 0.05$; (d) $P(A^cB^cC^c) = 0.05$; (e) $P(ABC^c \cup AB^cC \cup A^cB^cC \cup ABC) = 0.5$; (f) $P((ABC)^c) = 0.95$.

1.37 By inclusion-exclusion as in Example 1.20:

$$\begin{aligned} P(D_4 \cup D_7 \cup D_{10}) &= P(D_4) + P(D_7) + P(D_{10}) \\ &\quad - P(D_{28}) - P(D_{20}) - P(D_{70}) + P(D_{140}) \\ &= \frac{1}{5000}\left[1250 + 714 + 500 - 178 - 250 - 71 + 35\right] = \frac{2}{5}. \end{aligned}$$

1.38 (a) By inclusion exclusion: $1/4 + 1/4 - 1/16 = 3/16$.
(b) By inclusion-exclusion: $1/4 + 1/4 + 1/4 - (1/16 + 1/16 + 1/16) + 1/64 = 37/64$.

1.39 Let $C = AB^c \cup A^cB$.
We have

$$\begin{aligned} P(A \cup B) &= P(AB^c \cup A^cB \cup AB) = P(C) + P(AB); \\ P(A \cup B) &= P(A) + P(B) - P(AB) \end{aligned}$$

Solving for P(C) gives the result.

1.40 Let $D = AB^cC^c \cup A^cB^cC \cup AB^cC^c$ be the event that exactly one event occurs. We have

$$\begin{aligned} P(A \cup B \cup C) &= P(D) + P(ABC^c) + P(AB^cC) + P(A^cBC) + P(ABC) \\ &= P(D) + (P(AB) + P(AC) + P(BC) - 3P(ABC)) + P(ABC). \end{aligned}$$

Also,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

Solving for P(D) gives the result.

1.41
```
n <- 10000
simlist <- vector(length=n)
for (i in 1:n): {
trial <- sample(0:1, 4, replace=TRUE)
success <- if (sum(trial) == 1) 1 else 0
simlist[i] <-success
}
mean(simlist)
```

1.42
```
simdivis<- function() {
num <- sample(1:5000, 1)
if(num%%4==0 || num%%7==0 || num%%10==0) 1 else 0
}
simlist <- replicate(1000, simdivis())
mean(simlist)
```

1.43
```
n <- 10000
simlist <- vector(length=n)
for (i in 1:n): {
trial <- sample(1:6, 2, replace=TRUE)
success <- if (sum(trial) >= 8) 1 else 0
simlist[i] <-success
}
mean(simlist)
```

1.44
```
n <- 10000
simlist <- vector(length=n)
for (i in 1:n): {
trial <- sample(1:4, 1, replace=TRUE)
success <- if (trial >= 2) 1 else 0
simlist[i] <-success
}
mean(simlist)
```

Chapter 2

2.2 We know

$$P(AB) = P(A|B)P(B) = (0.5)(0.3) = 0.15.$$

Thus, $P(A \cup B) = P(A) + P(B) - P(AB) = 0.3 + 0.3 - 0.15 = 0.45.$

2.3

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A) + P(B) - P(AB)}{P(B)} = \frac{2p_1 - p_2}{p_1}.$$

2.4 (a)

$$\begin{aligned} P(HHH|\text{First coin is } H) &= \frac{P(HHH \text{ and First coin is } H)}{P(\text{First coin is } H)} \\ &= \frac{P(HHH)}{P(\text{First coin is } H)} = \frac{1/8}{1/2} = 1/4. \end{aligned}$$

(b)

$$\begin{aligned} P(HHH|\text{One of the coins is } H) &= \frac{P(HHH \text{ and one of the coins is } H)}{P(\text{One coin is } H)} \\ &= \frac{P(HHH)}{P(\text{One coin is } H)} = \frac{1/8}{7/8} = \frac{1}{7}. \end{aligned}$$

2.5 (a) 0; (b) 1; (c) $P(A)/P(B)$; (d) 1.

2.6 (1) We know

$$\begin{aligned}
P(A > B|A = 3) &= \frac{P(A > B \text{ and } A = 3)}{P(A = 3)} = 1/3, \\
P(A > B|A = 5) &= \frac{P(A > B \text{ and } A = 5)}{P(A = 5)} = 2/3, \\
P(A > B|A = 7) &= \frac{P(A > B \text{ and } A = 7)}{P(A = 7)} = 2/3.
\end{aligned}$$

Then

$$\begin{aligned}
P(A > B) = P(A > B|A = 3)P(A = 3) \\
+ P(A > B|A = 5)P(A = 5) \\
+ P(A > B|A = 5)P(A = 5) = 5/9,
\end{aligned}$$

which is greater than 1/2.

(2) Similarly,

$$\begin{aligned}
P(B > C) =& P(B > C|B = 2)P(B = 2) \\
&+ P(B > C|B = 4)P(B = 4) \\
&+ P(B > C|B = 9)P(B = 9) = 5/9.
\end{aligned}$$

And

$$\begin{aligned}
P(C > A) =& P(C > A|C = 1)P(C = 1) \\
&+ P(C > A|C = 6)P(C = 6) \\
&+ P(C > A|C = 8)P(C = 8) = 5/9.
\end{aligned}$$

2.7 (a) False.

(b) True.

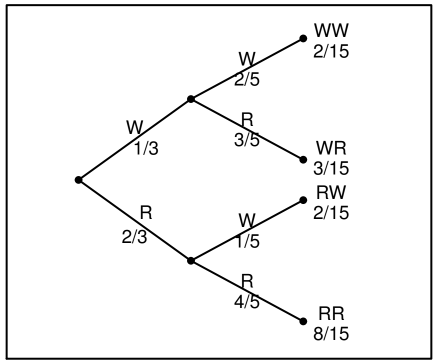
$$\begin{aligned}
P(A|B) + P(A^c|B) &= \frac{P(AB)}{P(B)} + \frac{P(A^cB)}{P(B)} \\
&= \frac{P(AB) + P(A^cB)}{P(B)} = 1.
\end{aligned}$$

2.8

$$P(\text{C-H-A-N-C-E}) = \left(\frac{6}{15}\right)\left(\frac{3}{14}\right)\left(\frac{3}{13}\right)\left(\frac{3}{12}\right)\left(\frac{5}{11}\right)\left(\frac{3}{10}\right) = \frac{27}{40040} = 0.000674.$$

2.9 The desired probability is 4 times the probability of a flush in one particular suit. This gives

$$4\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)\left(\frac{10}{49}\right)\left(\frac{9}{48}\right) = 0.001981.$$



2.10 By the tree diagram, the probability that the final ball is white is $2/15+2/15 = 4/15$.

2.11 (a) $p_1 = P(AB|A) = P(AB)/P(A)$; (b) $p_2 = P(AB|A\cup B) = P(AB)/P(A\cup B)$, since $AB \subseteq A \cup B$.

(c) Since $P(A) \le P(A \cup B)$,

$$p_1 = \frac{P(AB)}{P(A)} \ge \frac{P(AB)}{P(A \cup B)} = p_2.$$

2.12

$$\begin{aligned} P(ABC) &= P(B|AC)P(AC) = P(B|AC)P(C|A)P(A) \\ &= (1 - P(B^c|AC))P(C|A)P(A) = (2/3)(1/4)(1/2) = \frac{1}{12}. \end{aligned}$$

2.13

$$\begin{aligned} P(A \cup B|C) &= \frac{P((A\cup B)C)}{P(C)} = \frac{P(AC \cup BC)}{P(C)} \\ &= \frac{P(AC)}{P(C)} + \frac{P(BC)}{P(C)} - \frac{P(ABC)}{P(C)} \\ &= P(A|C) + P(B|C) - P(AB|C). \end{aligned}$$

2.14 We want

$$P(B) = 1 - \prod_{i=1}^{k-1} \left(1 - \frac{i}{687}\right) \ge 0.5.$$

For $k = 31$, $P(B) = 0.497$; for $k = 32$, $P(B) = 0.520$. Thus $k = 32$.

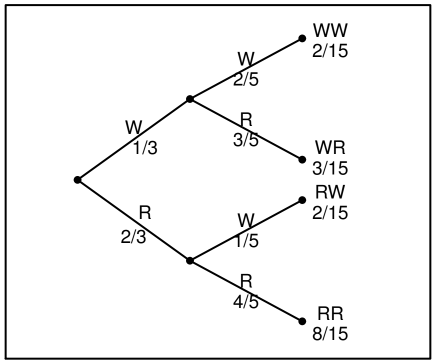
2.15 Apply the "birthday problem" with 5,000 "days" and 100 "people in the room." The desired probability is $1 - \frac{5000\times 4999 \cdots \times 4901}{5000 \times 5000 \cdots \times 5000} = 0.63088$.

2.16 (a) Let H be the event that the selected card is a heart. Let M be the event that the missing card is heart.

$$\begin{aligned} P(H) &= P(H|M)P(M) + P(H|M^c)P(M^c) \\ &= \left(\frac{12}{51}\right)\left(\frac{1}{4}\right) + \left(\frac{13}{51}\right)\left(\frac{3}{4}\right) = \frac{1}{4}. \end{aligned}$$

(b) The selected card is equally likely to be one of the four suits. Thus $P(H) = 1/4$.

2.17 (i)



The probability that Gummi Bears is chosen is $3/10 + 1/6 = 7/15$.

(ii) Let G be the event that Gummi Bears is chosen. Let A be the event that the first bag is chosen and B be the event that the second bag is chosen.

$$\begin{aligned} P(G) &= P(G|A)P(A) + P(G|B)P(B) \\ &= \left(\frac{3}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{6}\right)\left(\frac{1}{2}\right) = 7/15 = 0.467. \end{aligned}$$

2.18 (b)

$$\begin{aligned} P(A) &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) + P(A|B_4)P(B_4) \\ &= (1)\left(\frac{1}{16}\right) + \left(\frac{6}{9}\right)\left(\frac{9}{16}\right) + \left(\frac{2}{5}\right)\left(\frac{5}{16}\right) + 0 = \frac{9}{16}. \end{aligned}$$

2.19

$$P(A|B^c) = \frac{P(AB^c)}{P(B^c)} = \frac{P(A) - P(AB)}{1 - P(B)}.$$

2.20 After adding a white counter there are three equally likely states: (i) The bag initially contains a black counter B_1. A white counter W_2 is put into the bag and W_2 is picked at the first draw; (ii) The bag initially contains a white counter W_1. A white counter W_2 is put into the bag and W_2 is picked at the first draw; and (iii) The bag initially contains a white counter W_1. A white counter W_2 is put into the bag and W_1 is picked at the first draw. Thus the probability that the second draw is a white counter is $2/3$.

2.22 Let A be the event that HH first occurs, B be the event that HT first occurs, H be the event that the first coin flip is a head, and T be the event that the first coin flip is a tail. Then,

$$P(B) = P(B|H)P(H) + P(B|T)P(T) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + P(B)\frac{1}{2}.$$

That is, $P(B) = 1/2$.

2.23 Let A be the event that the youth is a smoker. Let B be the event that at least one parent is a smoker. We are given $P(A) = 0.2$, $P(B) = 0.3$, and $P(A|B) = 0.35$. Then

$$\begin{aligned} P(A|B^c) &= \frac{P(AB^c)}{P(B^c)} = \frac{P(A) - P(AB)}{1 - P(B)} = \frac{P(A) - P(A|B)P(B)}{1 - P(B)} \\ &= \frac{0.2 - (0.35)(0.3)}{0.7} = \frac{19}{140} = 0.136. \end{aligned}$$

2.24 Let B be the event that a woman has breast cancer, $+$ be the event that a mammogram gives a positive result, and $-$ be the event that a mammogram gives a negative result. We are given $P(B) = 0.0238$, $P(+|B) = 0.85$, and $P(+|B^c) = 0.05$. By Bayes Formula,

$$\begin{aligned} P(B|+) &= \frac{P(+|B)P(B)}{P(+|B)P(B) + P(+|B^c)P(B^c)} \\ &= \frac{(0.85)(0.0238)}{(0.85)(0.0238) + (0.05)(0.9762)} = 0.293. \end{aligned}$$

2.25 Let L be the event that a person is a liar, $+$ be the event that a polygraph test concludes lying, and $-$ be the event that a polygraph test concludes not lying.

We know

$$P(-|L^c) = 0.9 \text{ and } P(+|L) = 0.9.$$

(a) Given $P(L) = 0.05$,

Thus

$$\begin{aligned} P(L|+) &= \frac{P(+|L)P(L)}{P(+|L)P(L) + P(+|L^c)P(L^c)} \\ &= \frac{P(+|L)P(L)}{P(+|L)P(L) + (1 - P(-|L^c))P(L^c)} \\ &= 0.321. \end{aligned}$$

(b) Want $P(L|+) \geq 0.8$.

From (a), we know

$$P(L|+) = \frac{P(+|L)P(L)}{P(+|L)P(L) + (1 - P(-|L^c))P(L^c)} \geq 0.8.$$

Since $P(+|L) = P(-|L^c)$, solving for $P(+|L)$, we get $P(+|L) \geq 0.987$. The polygraph must be 98.7% reliable.

2.26 Let B be the event that the cab is blue. Let b be the event that a witness asserts the cab is blue.

Given $P(b|B) = P(b^c|B^c) = 0.8$ and $P(B) = 0.05$, Thus

$$\begin{aligned} P(B|b) &= \frac{P(b|B)P(B)}{P(b|B)P(B) + P(b|B^c)P(B^c)} \\ &= \frac{P(b|B)P(B)}{P(b|B)P(B) + (1 - P(b^c|B^c))P(B^c)} \\ &= 0.174. \end{aligned}$$

2.27 Let A be the event that the fair die is chosen, B be the event that the die with all $5's$ is chosen, and C be the event that the die with three $5's$ and three $4's$ is chosen.

$$\begin{aligned} P(A|5) &= \frac{P(5|A)P(A)}{P(5|A)P(A) + P(5|B)P(B) + P(5|C)P(C)} \\ &= \frac{(1/6)(1/3)}{(1/6)(1/3) + (1)(1/3) + (1/2)(1/3)} = \frac{1/18}{10/18} = \frac{1}{10}. \end{aligned}$$

2.28 (a)

```
n <- 10000
simlist <- vector(length=n)
for (i in 1:n){
trial <- sample(1:365, 23, replace=T)
success <- if (2\%in\% table(trial)) 1 else 0
simlist[i] <- success
}
mean(simlist)
```

(b) $k = 47$, (c) 0.967, (d) $k = 28$.

2.29
```
n <- 1000
simlist1 <- vector(length=n)
simlist2 <- vector(length=n)
simlist3 <- vector(length=n)
for (i in 1:n){
trialA <- sample(c(3,3,5,5,7,7), 1, replace=T)
trialB <- sample(c(2,2,4,4,9,9), 1, replace=T)
trialC <- sample(c(1,1,6,6,8,8), 1, replace=T)
success1 <- if (trialA >trialB) 1 else 0
success2 <- if (trialB >trialC) 1 else 0
success3 <- if (trialC >trialA) 1 else 0
simlist1[i] <-success1
simlist2[i] <-success2
simlist3[i] <-success3
}
mean(simlist1); mean(simlist2); mean(simlist3)
```

2.30
```
n <- 1000
envelopes <- c("A", "B", "C", "D")
```

```
noswitchwin<-vector(length=n)
switchwin<-vector(length=n)
for (i in 1:n){
         win <- sample(envelopes,1)
         pick <- sample(envelopes,1)
         remove <- sample(envelopes[which(envelopes!= pick
             & envelopes!= win)], 2)
         switchyes <- envelopes[which(envelopes!= pick&envelopes!=
                      remove[1]&envelopes!= remove[2])]
         noswitch <- if (pick==win) 1 else 0
         switch <- if (switchyes==win) 1 else 0
         noswitchwin[i] <- noswitch
         switchwin[i] <- switch
}
mean(noswitchwin); mean(switchwin)
```

After I choose an envelope, the probability that it contains the bill is 1/4. The probability that the bill is in one of the other three envelopes is 3/4. After two empty envelopes are removed, the probability that my envelope contains the bill has not changed, so the probability that the one on the table contains the bill is 3/4. Thus I should switch.

Chapter 3

3.1

$$\begin{aligned}P(A^cB^c) &= 1 - P(A \cup B) = 1 - P(A) - P(B) + P(AB)\\ &= P(A^c) - P(B) + P(A)P(B) = P(A^c) - P(B)(1 - P(A))\\ &= P(A^c) - P(B)P(A^c) = P(A^c)P(B^c).\end{aligned}$$

Thus A^c and B^c are independent.

3.2 (a) $P(ABC) = P(A)P(B)P(C) = (1/3)(1/4)(1/5) = 1/60$.

(b) $P(A \text{ or } B \text{ or } C) = 1 - P(A^cB^cC^c) = 1 - P(A^c)P(B^c)P(C^c) = 3/5$.

(c) $P(AB|C) = P(AB) = P(A)P(B) = 1/12$.

(d) $P(B|AC) = P(B) = 1/4$.

(e)

$$\begin{aligned}&P(AB^cC^c \cup A^cBC^c \cup A^cB^cC \cup A^cB^cC^c)\\ &= P(AB^cC^c) + P(A^cBC^c) + P(A^cB^cC)\\ &+ P(A^cB^cC^c) = P(A)P(B^c)P(C^c) + P(A^c)P(B)P(C^c)\\ &+ P(A^c)P(B^c)P(C) + P(A^c)P(B^c)P(C^c) = 5/6.\end{aligned}$$

3.3 Let B denote the event that a tree is infected with bark disease and R denote the event that a tree is infected with root rot.