INSTRUCTOR'S SOLUTIONS MANUAL

PROBABILITY AND STATISTICAL INFERENCE

NINTH EDITION

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PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
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Reproduced by Pearson from electronic files supplied by the author.

Publishing as Pearson, 75 Arlington Street, Boston, MA 02116.

ISBN-13: 978-0-321-91379-1
ISBN-10: 0-321-91379-5

www.pearsonhighered.com

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Preface

This solutions manual provides answers for the even-numbered exercises in *Probability and Statistical Inference*, 9th edition, by Robert V. Hogg, Elliot A. Tanis, and Dale L. Zimmerman. Complete solutions are given for most of these exercises. You, the instructor, may decide how many of these solutions and answers you want to make available to your students. Note that the answers for the odd-numbered exercises are given in the textbook.

All of the figures in this manual were generated using *Maple*, a computer algebra system. Most of the figures were generated and many of the solutions, especially those involving data, were solved using procedures that were written by Zaven Karian from Denison University. We thank him for providing these. These procedures are available free of charge for your use. They are available for down load at http://www.math.hope.edu/tanis/. Short descriptions of these procedures are provided on the "Maple Card." Complete descriptions of these procedures are given in *Probability and Statistics: Explorations with MAPLE*, second edition, 1999, written by Zaven Karian and Elliot Tanis, published by Prentice Hall (ISBN 0-13-021536-8). You can download a copy of this manual at http://www.math.hope.edu/tanis/MapleManual.pdf.

Our hope is that this solutions manual will be helpful to each of you in your teaching.

If you find an error or wish to make a suggestion, send these to Elliot Tanis, `tanis@hope.edu`, and he will post corrections on his web page, http://www.math.hope.edu/tanis/.

R.V.H.
E.A.T.
D.L.Z.

Chapter 1

Probability

1.1 Properties of Probability

1.1-2 Sketch a figure and fill in the probabilities of each of the disjoint sets.

Let $A = \{\text{insure more than one car}\}$, $P(A) = 0.85$.

Let $B = \{\text{insure a sports car}\}$, $P(B) = 0.23$.

Let $C = \{\text{insure exactly one car}\}$, $P(C) = 0.15$.

It is also given that $P(A \cap B) = 0.17$. Since $A \cap C = \phi$, $P(A \cap C) = 0$. It follows that $P(A \cap B \cap C') = 0.17$. Thus $P(A' \cap B \cap C') = 0.06$ and $P(A' \cap B' \cap C) = 0.09$.

1.1-4 **(a)** $S = \{$HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT$\}$;

(b) **(i)** 5/16, **(ii)** 0, **(iii)** 11/16, **(iv)** 4/16, **(v)** 4/16, **(vi)** 9/16, **(vii)** 4/16.

1.1-6 **(a)** $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

(b)
$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B') &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

1.1-8 Let $A =$\{lab work done\}, $B =$\{referral to a specialist\},

$P(A) = 0.41$, $P(B) = 0.53$, $P([A \cup B]') = 0.21$.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.79 &= 0.41 + 0.53 - P(A \cap B) \\ P(A \cap B) &= 0.41 + 0.53 - 0.79 \;=\; 0.15. \end{aligned}$$

1.1-10
$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.1-12 **(a)** 1/3; **(b)** 2/3; **(c)** 0; **(d)** 1/2.

1.1-14 $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}.$

1.1-16 Note that the respective probabilities are $p_0,\ p_1 = p_0/4,\ p_2 = p_0/4^2, \cdots$.

$$\begin{aligned} \sum_{k=0}^{\infty} \frac{p_0}{4^k} &= 1 \\ \frac{p_0}{1 - 1/4} &= 1 \\ p_0 &= \frac{3}{4} \end{aligned}$$

$$1 - p_0 - p_1 = 1 - \frac{15}{16} = \frac{1}{16}.$$

1.2 Methods of Enumeration

1.2-2 **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8$.

1.2-4 **(a)** $4\dbinom{6}{3} = 80$;

(b) $4(2^6) = 256$;

(c) $\dfrac{(4-1+3)!}{(4-1)!3!} = 20$.

1.2-6 $S =${ DDD, DDFD, DFDD, FDDD, DDFFD, DFDFD, FDDFD, DFFDD, FDFDD, FFDDD, FFF, FFDF, FDFF, DFFF FFDDF, FDFDF, DFFDF, FDDFF, DFDFF, DDFFF } so there are 20 possibilities.

1.2-8 $3 \cdot 3 \cdot 2^{12} = 36{,}864$.

1.2-10
$$\begin{aligned} \binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}. \end{aligned}$$

1.2-12
$$\begin{aligned} 0 &= (1-1)^n = \sum_{r=0}^{n} \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^{n} (-1)^r \binom{n}{r}. \\ 2^n &= (1+1)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^{n} \binom{n}{r}. \end{aligned}$$

1.2-14 $\dbinom{10-1+36}{36} = \dfrac{45!}{36!9!} = 886{,}163{,}135.$

1.2-16 **(a)** $\dfrac{\dbinom{19}{3}\dbinom{52-19}{6}}{\dbinom{52}{9}} = \dfrac{102{,}486}{351{,}325} = 0.2917;$

(b) $\dfrac{\dbinom{19}{3}\dbinom{10}{2}\dbinom{7}{1}\dbinom{3}{0}\dbinom{5}{1}\dbinom{2}{0}\dbinom{6}{2}}{\dbinom{52}{9}} = \dfrac{7{,}695}{1{,}236{,}664} = 0.00622.$

1.3 Conditional Probability

1.3-2 **(a)** $\frac{1041}{1456}$;

(b) $\frac{392}{633}$;

(c) $\frac{649}{823}$.

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.3-4 **(a)** $P(\text{HH}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$;

(b) $P(\text{HC}) = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204}$;

(c) $P(\text{Non-Ace Heart, Ace}) + P(\text{Ace of Hearts, Non-Heart Ace})$

$$= \frac{12}{52} \cdot \frac{4}{51} + \frac{1}{52} \cdot \frac{3}{51} = \frac{51}{52 \cdot 51} = \frac{1}{52}.$$

1.3-6 Let $H = \{\text{died from heart disease}\}$; $P = \{\text{at least one parent had heart disease}\}$.

$$P(H \mid P') = \frac{N(H \cap P')}{N(P')} = \frac{110}{648}.$$

1.3-8 **(a)** $\frac{3}{20} \cdot \frac{2}{19} \cdot \frac{1}{18} = \frac{1}{1140}$;

(b) $\dfrac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} \cdot \frac{1}{17} = \frac{1}{380}$;

(c) $\displaystyle\sum_{k=1}^{9} \frac{\binom{3}{2}\binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605.$

(d) Draw second. The probability of winning is $1 - 0.4605 = 0.5395$.

1.3-10 **(a)** $P(A) = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = \frac{8{,}808{,}975}{11{,}881{,}376} = 0.74141$;

(b) $P(A') = 1 - P(A) = 0.25859$.

1.3-12 **(a)** It doesn't matter because $P(B_1) = \frac{1}{18}$, $P(B_5) = \frac{1}{18}$, $P(B_{18}) = \frac{1}{18}$;

(b) $P(B) = \frac{2}{18} = \frac{1}{9}$ on each draw.

1.3-14 **(a)** $P(A_1) = 30/100$;

(b) $P(A_3 \cap B_2) = 9/100$;

(c) $P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$;

(d) $P(A_1 \mid B_2) = 11/41$;

(e) $P(B_1 \mid A_3) = 13/29$.

1.3-16 $\frac{3}{5} \cdot \frac{5}{8} + \frac{2}{5} \cdot \frac{4}{8} = \frac{23}{40}$.

1.4 Independent Events

1.4-2 **(a)**
$$\begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0$.

1.4-4 Proof of **(b)**:
$$\begin{aligned} P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

Proof of **(c)**:
$$\begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

1.4-6
$$\begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') \mid B] \\ &= P(B)[1 - P(A \cup C \mid B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C') \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\qquad P(B)P(C) - PA)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

1.4-8 $\frac{1}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} + \frac{1}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} + \frac{5}{6} \cdot \frac{2}{6} \cdot \frac{3}{6} = \frac{2}{9}$.

1.4-10 **(a)** $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$;

(b) $\frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} = \frac{9}{16}$;

(c) $\frac{2}{4} \cdot \frac{1}{4} + \frac{2}{4} \cdot \frac{4}{4} = \frac{10}{16}$.

1.4-12 **(a)** $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$;

(b) $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$;

(c) $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$;

(d) $\frac{5!}{3!\,2!} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$.

1.4-14 **(a)** $1 - (0.4)^3 = 1 - 0.064 = 0.936$;

(b) $1 - (0.4)^8 = 1 - 0.00065536 = 0.99934464$.

1.4-16 **(a)** $\sum_{k=0}^{\infty} \frac{1}{5}\left(\frac{4}{5}\right)^{2k} = \frac{5}{9}$;

(b) $\frac{1}{5} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{3}{5}$.

1.4-18 **(a)** 7; **(b)** $(1/2)^7$; **(c)** 63; **(d)** No! $(1/2)^{63} = 1/9{,}223{,}372{,}036{,}854{,}775{,}808$.

1.4-20 No.

1.5 Bayes' Theorem

1.5-2 **(a)**
$$\begin{aligned} P(G) &= P(A \cap G) + P(B \cap G) \\ &= P(A)P(G \,|\, A) + P(B)P(G \,|\, B) \\ &= (0.40)(0.85) + (0.60)(0.75) = 0.79; \end{aligned}$$

(b)
$$\begin{aligned} P(A \,|\, G) &= \frac{P(A \cap G)}{P(G)} \\ &= \frac{(0.40)(0.85)}{0.79} = 0.43. \end{aligned}$$

1.5-4 Let event B denote an accident and let A_1 be the event that age of the driver is 16–25. Then

$$\begin{aligned} P(A_1 \,|\, B) &= \frac{(0.1)(0.05)}{(0.1)(0.05) + (0.55)(0.02) + (0.20)(0.03) + (0.15)(0.04)} \\ &= \frac{50}{50 + 110 + 60 + 60} = \frac{50}{280} = 0.179. \end{aligned}$$

1.5-6 Let B be the event that the policyholder dies. Let A_1, A_2, A_3 be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned} P(A_1 \mid B) &= \frac{(0.60)(0.01)}{(0.60)(0.01) + (0.30)(0.008) + (0.10)(0.007)} \\ &= \frac{60}{60+24+7} = \frac{60}{91} = 0.659; \\ P(A_2 \mid B) &= \frac{24}{91} = 0.264; \\ P(A_3 \mid B) &= \frac{7}{91} = 0.077. \end{aligned}$$

1.5-8 Let A be the event that the tablet is under warranty.

$$\begin{aligned} P(B_1 \mid A) &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\ &= \frac{40}{40+15+6+2} = \frac{40}{63} = 0.635; \\ P(B_2 \mid A) &= \frac{15}{63} = 0.238; \\ P(B_3 \mid A) &= \frac{6}{63} = 0.095; \\ P(B_4 \mid A) &= \frac{2}{63} = 0.032. \end{aligned}$$

1.5-10 **(a)** $P(D^+) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674$;

(b) $P(A^- \mid D^+) = \dfrac{0.0490}{0.0674} = 0.727$; $P(A^+ \mid D^+) = \dfrac{0.0184}{0.0674} = 0.273$;

(c) $P(A^- \mid D^-) = \dfrac{(0.98)(0.95)}{(0.02)(0.08) + (0.98)(0.95)} = \dfrac{9310}{16 + 9310} = 0.998$;

$P(A^+ \mid D^-) = 0.002$.

(d) Yes, particularly those in part (b).

1.5-12 Let $D = \{\text{has the disease}\}$, $DP = \{\text{detects presence of disease}\}$. Then

$$\begin{aligned} P(D \mid DP) &= \frac{P(D \cap DP)}{P(DP)} \\ &= \frac{P(D) \cdot P(DP \mid D)}{P(D) \cdot P(DP \mid D) + P(D') \cdot P(DP \mid D')} \\ &= \frac{(0.005)(0.90)}{(0.005)(0.90) + (0.995)(0.02)} \\ &= \frac{0.0045}{0.0045 + 0.199} = \frac{0.0045}{0.0244} = 0.1844. \end{aligned}$$

1.5-14 Let $D = \{\text{defective roll}\}$. Then

$$\begin{aligned} P(I \mid D) &= \frac{P(I \cap D)}{P(D)} \\ &= \frac{P(I) \cdot P(D \mid I)}{P(I) \cdot P(D \mid I) + P(II) \cdot P(D \mid II)} \\ &= \frac{(0.60)(0.03)}{(0.60)(0.03) + (0.40)(0.01)} \\ &= \frac{0.018}{0.018 + 0.004} = \frac{0.018}{0.022} = 0.818. \end{aligned}$$

Chapter 2

Discrete Distributions

2.1 Random Variables of the Discrete Type

2.1-2 **(a)**

$$f(x) = \begin{cases} 0.6, & x = 1, \\ 0.3, & x = 5, \\ 0.1, & x = 10, \end{cases}$$

(b)

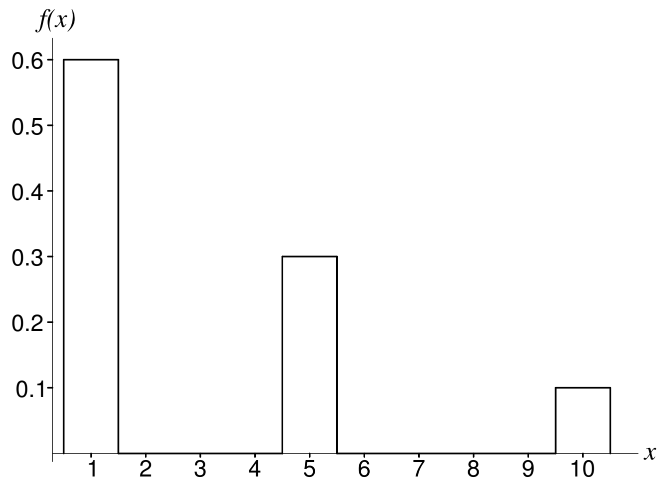

Figure 2.1–2: A probability histogram

2.1-4 **(a)** $f(x) = \dfrac{1}{10}, \qquad x = 0, 1, 2, \cdots, 9;$

(b)
$\mathcal{N}(\{0\})/150 = 11/150 = 0.073;$ $\mathcal{N}(\{5\})/150 = 13/150 = 0.087;$
$\mathcal{N}(\{1\})/150 = 14/150 = 0.093;$ $\mathcal{N}(\{6\})/150 = 22/150 = 0.147;$
$\mathcal{N}(\{2\})/150 = 13/150 = 0.087;$ $\mathcal{N}(\{7\})/150 = 16/150 = 0.107;$
$\mathcal{N}(\{3\})/150 = 12/150 = 0.080;$ $\mathcal{N}(\{8\})/150 = 18/150 = 0.120;$
$\mathcal{N}(\{4\})/150 = 16/150 = 0.107;$ $\mathcal{N}(\{9\})/150 = 15/150 = 0.100.$

(c)

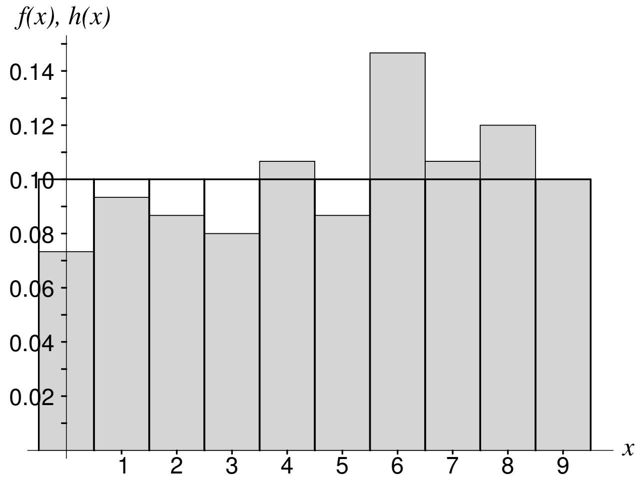

Figure 2.1–4: Michigan daily lottery digits

2.1-6 **(a)** $f(x) = \dfrac{6 - |\,7 - x\,|}{36}, \;\; x = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.$

(b)

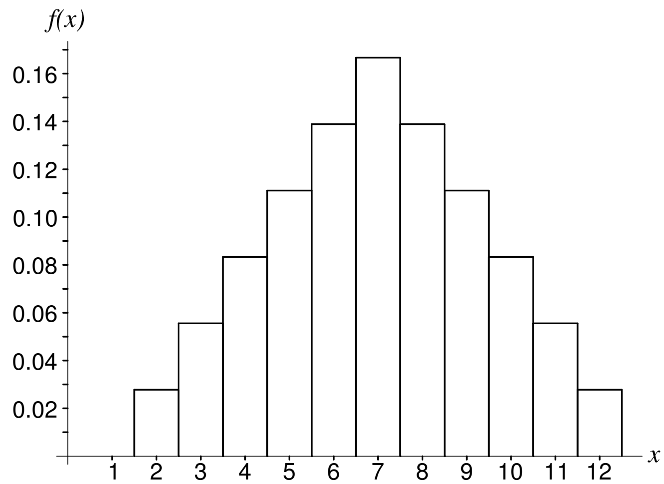

Figure 2.1–6: Probability histogram for the sum of a pair of dice

2.1-8 **(a)** The space of W is $S = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

$$P(W=0) = P(X=0, Y=0) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}, \text{ assuming independence.}$$
$$P(W=1) = P(X=0, Y=1) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=2) = P(X=2, Y=0) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=3) = P(X=2, Y=1) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=4) = P(X=0, Y=4) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=5) = P(X=0, Y=5) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=6) = P(X=2, Y=4) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8},$$
$$P(W=7) = P(X=2, Y=5) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}.$$

That is, $f(w) = P(W=w) = \dfrac{1}{8}, \quad w \in S.$

(b)

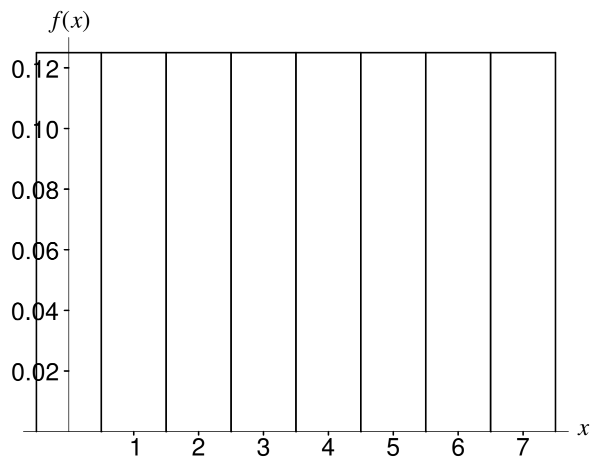

Figure 2.1–8: Probability histogram of sum of two special dice

2.1-10 **(a)** $$\frac{\binom{3}{1}\binom{47}{9}}{\binom{50}{10}} = \frac{39}{98};$$

(b) $$\sum_{x=0}^{1} \frac{\binom{3}{x}\binom{47}{10-x}}{\binom{50}{10}} = \frac{221}{245}.$$

2.1-12
$$P(X \ge 4 \,|\, X \ge 1) = \frac{P(X \ge 4)}{P(X \ge 1)} = \frac{1 - P(X \le 3)}{1 - P(X = 0)}$$
$$= \frac{1 - [1 - 1/2 + 1/2 - 1/3 + 1/3 - 1/4 + 1/4 - 1/5]}{1 - [1 - 1/2]} = \frac{2}{5}.$$

2.1-14 $P(X \ge 1) = 1 - P(X = 0) = 1 - \dfrac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}} = 1 - \dfrac{91}{228} = \dfrac{137}{228} = 0.60.$

2.1-16 **(a)** $P(2, 1, 6, 10)$ means that 2 is in position 1 so 1 cannot be selected. Thus
$$P(2, 1, 6, 10) = \frac{\binom{1}{0}\binom{1}{1}\binom{8}{5}}{\binom{10}{6}} = \frac{56}{210} = \frac{4}{15};$$

(b) $P(i, r, k, n) = \dfrac{\binom{i-1}{r-1}\binom{1}{1}\binom{n-i}{k-r}}{\binom{n}{k}}.$

2.2 Mathematical Expectation

2.2-2 $E(X) = (-1)\left(\dfrac{4}{9}\right) + (0)\left(\dfrac{1}{9}\right) + (1)\left(\dfrac{4}{9}\right) = 0;$

$E(X^2) = (-1)^2\left(\dfrac{4}{9}\right) + (0)^2\left(\dfrac{1}{9}\right) + (1)^2\left(\dfrac{4}{9}\right) = \dfrac{8}{9};$

$E(3X^2 - 2X + 4) = 3\left(\dfrac{8}{9}\right) - 2(0) + 4 = \dfrac{20}{3}.$

2.2-4
$$1 = \sum_{x=0}^{6} f(x) = \frac{9}{10} + c\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right)$$
$$c = \frac{2}{49};$$
$$E(\text{Payment}) = \frac{2}{49}\left(1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{1}{6}\right) = \frac{71}{490} \text{ units.}$$

2.2-6 Note that $\displaystyle\sum_{x=1}^{\infty} \frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x^2} = \frac{6}{\pi^2}\frac{\pi^2}{6} = 1$, so this is a pdf

$$E(X) = \sum_{x=1}^{\infty} x \,\frac{6}{\pi^2 x^2} = \frac{6}{\pi^2} \sum_{x=1}^{\infty} \frac{1}{x} = +\infty$$

and it is well known that the sum of this harmonic series is not finite.

2.2-8 $E(|X - c|) = \dfrac{1}{7} \displaystyle\sum_{x \in S} |x - c|$, where $S = \{1, 2, 3, 5, 15, 25, 50\}$.

When $c = 5$,

$$E(|X - 5|) = \frac{1}{7}\left[(5 - 1) + (5 - 2) + (5 - 3) + (5 - 5) + (15 - 5) + (25 - 5) + (50 - 5)\right].$$

If c is either increased or decreased by 1, this expectation is increased by 1/7. Thus $c = 5$, the median, minimizes this expectation while $b = E(X) = \mu$, the mean, minimizes $E[(X-b)^2]$. You could also let $h(c) = E(\,|\,X-c\,|\,)$ and show that $h'(c) = 0$ when $c = 5$.

2.2-10 $(1)\cdot\dfrac{15}{36} + (-1)\cdot\dfrac{21}{36} = \dfrac{-6}{36} = \dfrac{-1}{6};$

$$(1)\cdot\frac{15}{36} + (-1)\cdot\frac{21}{36} = \frac{-6}{36} = \frac{-1}{6};$$

$$(4)\cdot\frac{6}{36} + (-1)\cdot\frac{30}{36} = \frac{-6}{36} = \frac{-1}{6}.$$

2.2-12 **(a)** The average class size is $\dfrac{(16)(25)+(3)(100)+(1)(300)}{20} = 50;$

(b)

$$f(x) = \begin{cases} 0.4, & x = 25, \\ 0.3, & x = 100, \\ 0.3, & x = 300, \end{cases}$$

(c) $E(X) = 25(0.4) + 100(0.3) + 300(0.3) = 130.$

2.3 Special Mathematical Expectations

2.3-2 **(a)**

$$\begin{aligned}
\mu &= E(X) \\
&= \sum_{x=1}^{3} x\,\frac{3!}{x!\,(3-x)!}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{3-x} \\
&= 3\left(\frac{1}{4}\right)\sum_{k=0}^{2}\frac{2!}{k!\,(2-k)!}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{2-k} \\
&= 3\left(\frac{1}{4}\right)\left(\frac{1}{4}+\frac{3}{4}\right)^{2} = \frac{3}{4}; \\
E[X(X-1)] &= \sum_{x=2}^{3} x(x-1)\,\frac{3!}{x!\,(3-x)!}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{3-x} \\
&= 2(3)\left(\frac{1}{4}\right)^{2}\frac{3}{4} + 6\left(\frac{1}{4}\right)^{3} \\
&= 6\left(\frac{1}{4}\right)^{2} = 2\left(\frac{1}{4}\right)\left(\frac{3}{4}\right); \\
\sigma^2 &= E[X(X-1)] + E(X) - \mu^2 \\
&= (2)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^{2} \\
&= (2)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = 3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right);
\end{aligned}$$

(b)
$$\begin{aligned}\mu &= E(X)\\ &= \sum_{x=1}^{4} x\,\frac{4!}{x!\,(4-x)!}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}\\ &= 4\left(\frac{1}{2}\right)\sum_{k=0}^{3}\frac{3!}{k!\,(3-k)!}\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{3-k}\\ &= 4\left(\frac{1}{2}\right)\left(\frac{1}{2}+\frac{1}{2}\right)^3 = 2;\end{aligned}$$

$$\begin{aligned}E[X(X-1)] &= \sum_{x=2}^{4} x(x-1)\,\frac{4!}{x!\,(4-x)!}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}\\ &= 2(6)\left(\frac{1}{2}\right)^4 + (6)(4)\left(\frac{1}{2}\right)^4 + (12)\left(\frac{1}{2}\right)^4\\ &= 48\left(\frac{1}{2}\right)^4 = 12\left(\frac{1}{2}\right)^2;\\ \sigma^2 &= (12)\left(\frac{1}{2}\right)^2 + \frac{4}{2} - \left(\frac{4}{2}\right)^2 = 1.\end{aligned}$$

2.3-4 $E[(X-\mu)/\sigma] = (1/\sigma)[E(X)-\mu] = (1/\sigma)(\mu-\mu) = 0;$

$E\{[(X-\mu)/\sigma]^2\} = (1/\sigma^2)E[(X-\mu)^2] = (1/\sigma^2)(\sigma^2) = 1.$

2.3-6 $f(1) = \dfrac{3}{8},\ f(2) = \dfrac{2}{8},\ f(3) = \dfrac{3}{8}$

$\mu = 1\cdot\dfrac{3}{8} + 2\cdot\dfrac{2}{8} + 3\cdot\dfrac{3}{8} = 2,$

$\sigma^2 = 1^2\cdot\dfrac{3}{8} + 2^2\cdot\dfrac{2}{8} + 3^2\cdot\dfrac{3}{8} - 2^2 = \dfrac{3}{4}.$

2.3-8
$$\begin{aligned}E(X) &= \sum_{x=1}^{4} x\cdot\frac{2x-1}{16}\\ &= \frac{50}{16} = 3.125;\\ E(X^2) &= \sum_{x=1}^{4} x^2\cdot\frac{2x-1}{16}\\ &= \frac{85}{8};\\ \mathrm{Var}(X) &= \frac{85}{8} - \left(\frac{25}{8}\right)^2 = \frac{55}{64} = 0.8594;\\ \sigma &= \frac{\sqrt{55}}{8} = 0.9270.\end{aligned}$$

2.3-10 We have $N = N_1 + N_2$. Thus

$$\begin{aligned}
E[X(X-1)] &= \sum_{x=0}^{n} x(x-1)f(x) \\
&= \frac{\displaystyle\sum_{x=2}^{n} x(x-1)\,\frac{N_1!}{x!\,(N_1-x)!}\cdot\frac{N_2!}{(n-x)!\,(N_2-n+x)!}}{\dbinom{N}{n}} \\
&= N_1(N_1-1)\,\frac{\displaystyle\sum_{x=2}^{n} \frac{(N_1-2)!}{(x-2)!\,(N_1-x)!}\cdot\frac{N_2!}{(n-x)!\,(N_2-n+x)!}}{\dbinom{N}{n}}.
\end{aligned}$$

In the summation, let $k = x - 2$, and in the denominator, note that

$$\binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{N(N-1)}{n(n-1)}\binom{N-2}{n-2}.$$

Thus

$$\begin{aligned}
E[X(X-1)] &= \frac{N_1(N_1-1)}{\dfrac{N(N-1)}{n(n-1)}}\sum_{k=0}^{n-2}\frac{\dbinom{N_1-2}{k}\dbinom{N_2}{n-2-k}}{\dbinom{N-2}{n-2}} \\
&= \frac{N_1(N_1-1)(n)(n-1)}{N(N-1)}.
\end{aligned}$$

2.3-12 **(a)** $f(x) = \left(\dfrac{364}{365}\right)^{x-1}\left(\dfrac{1}{365}\right), \qquad x = 1, 2, 3, \ldots,$

(b)
$$\begin{aligned}
\mu &= \frac{1}{\dfrac{1}{365}} = 365, \\
\sigma^2 &= \frac{\dfrac{364}{365}}{\left(\dfrac{1}{365}\right)^2} = 132{,}860, \\
\sigma &= 364.500;
\end{aligned}$$

(c)
$$\begin{aligned}
P(X > 400) &= \left(\frac{364}{365}\right)^{400} = 0.3337, \\
P(X < 300) &= 1 - \left(\frac{364}{365}\right)^{299} = 0.5597.
\end{aligned}$$

2.3-14 $P(X \ge 100) = P(X > 99) = (0.99)^{99} = 0.3697.$

2.3-16 **(a)** $f(x) = (1/2)^{x-1}, \quad x = 2, 3, 4, \ldots;$

(b)
$$\begin{aligned} M(t) &= E[e^{tx}] = \sum_{x=2}^{\infty} e^{tx}(1/2)^{x-1} \\ &= 2\sum_{x=2}^{\infty}(e^t/2)^x \\ &= \frac{2(e^t/2)^2}{1-e^t/2} = \frac{e^{2t}}{2-e^t}, \qquad t < \ln 2; \end{aligned}$$

(c)
$$\begin{aligned} M'(t) &= \frac{4e^{2t}-e^{3t}}{(2-e^t)^2} \\ \mu &= M'(0) = 3; \\ M''(t) &= \frac{(2-e^t)^2(8e^{2t}-3e^{3t})-(4e^{2t}-e^{3t})2*(2-e^t)(-e^t)}{(2-e^t)^4} \\ \sigma^2 &= M''(0)-\mu^2 = 11-9 = 2; \end{aligned}$$

(d) **(i)** $P(X \le 3) = 3/4$; **(ii)** $P(X \ge 5) = 1/8$; **(iii)** $P(X = 3) = 1/4$.

2.3-18
$$\begin{aligned} P(X > k+j \mid X > k) &= \frac{P(X > k+j)}{P(X > k)} \\ &= \frac{q^{k+j}}{q^k} = q^j = P(X > j). \end{aligned}$$

2.4 The Binomial Distribution

2.4-2 $f(-1) = \dfrac{11}{18}, \qquad f(1) = \dfrac{7}{18};$

$$\mu = (-1)\frac{11}{18} + (1)\frac{7}{18} = -\frac{4}{18};$$

$$\sigma^2 = \left(-1 + \frac{4}{18}\right)^2\left(\frac{11}{18}\right) + \left(1 + \frac{4}{18}\right)^2\left(\frac{7}{18}\right) = \frac{77}{81}.$$

2.4-4 **(a)** X is $b(7,\ 0.15)$;

(b) **(i)** $P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.7166 = 0.2834;$

(ii) $P(X = 1) = P(X \le 1) - P(X \le 0) = 0.7166 - 0.3206 = 0.3960;$

(iii) $P(X \le 3) = 0.9879.$

2.4-6 **(a)** X is $b(15, 0.75)$; $\qquad 15 - X$ is $b(15, 0.25)$;

(b) $P(X \ge 10) = P(15 - X \le 5) = 0.8516;$

(c) $P(X \le 10) = P(15 - X \ge 5) = 1 - P(15 - X \le 4) = 1 - 0.6865 = 0.3135;$

(d) $P(X = 10) = P(X \ge 10) - P(X \ge 11)$
$= P(15 - X \le 5) - P(15 - X \le 4) = 0.8516 - 0.6865 = 0.1651;$

(e) $\mu = (15)(0.75) = 11.25, \quad \sigma^2 = (15)(0.75)(0.25) = 2.8125; \quad \sigma = \sqrt{2.8125} = 1.67705.$

2.4-8 **(a)** $1 - 0.01^4 = 0.99999999;$ **(b)** $0.99^4 = 0.960596.$

2.4-10 **(a)** X is $b(8,\ 0.90)$;

(b) **(i)** $P(X = 8) = P(8 - X = 0) = 0.4305;$

(ii) $P(X \le 6) = P(8 - X \ge 2)$
$= 1 - P(8 - X \le 1) = 1 - 0.8131 = 0.1869;$

(iii) $P(X \ge 6) = P(8 - X \le 2) = 0.9619.$

2.4-12 **(a)**

$$f(x) = \begin{cases} 125/216, & x = -1, \\ 75/216, & x = 1, \\ 15/216, & x = 2, \\ 1/216, & x = 3; \end{cases}$$

(b)

$$\begin{aligned} \mu &= (-1)\cdot\frac{125}{216} + (1)\cdot\frac{75}{216} + (2)\cdot\frac{15}{216} + (3)\cdot\frac{1}{216} = -\frac{17}{216}; \\ \sigma^2 &= E(X^2) - \mu^2 = \frac{269}{216} - \left(-\frac{17}{216}\right)^2 = 1.2392; \\ \sigma &= 1.11; \end{aligned}$$

(c) See Figure 2.4-12.

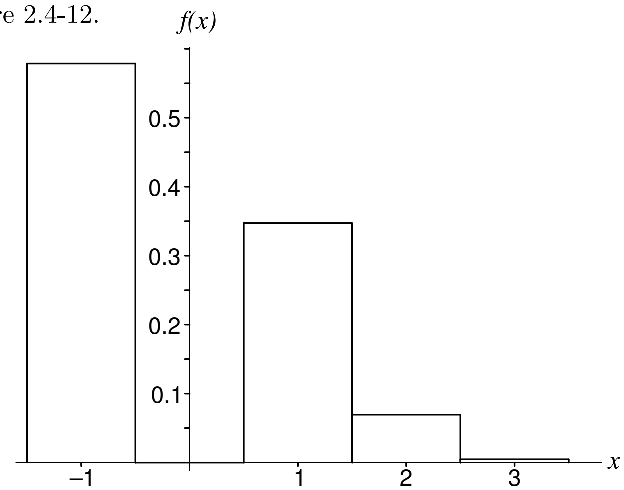

Figure 2.4–12: Losses in chuck-a-luck

2.4-14 Let X equal the number of winning tickets when n tickets are purchased. Then

$$\begin{aligned} P(X \ge 1) &= 1 - P(X = 0) \\ &= 1 - \left(\frac{9}{10}\right)^n. \end{aligned}$$

(a)

$$\begin{aligned} 1 - (0.9)^n &= 0.50 \\ (0.9)^n &= 0.50 \\ n \ln 0.9 &= \ln 0.5 \\ n &= \frac{\ln 0.5}{\ln 0.9} = 6.58 \end{aligned}$$

so $n = 7$.

(b)

$$\begin{aligned} 1 - (0.9)^n &= 0.95 \\ (0.9)^n &= 0.05 \\ n &= \frac{\ln 0.05}{\ln 0.09} = 28.43 \end{aligned}$$

so $n = 29$.

2.4-16 It is given that X is $b(10,\ 0.10)$. We are to find M so that

$P(1000X \le M) \ge 0.99)$ or $P(X \le M/1000) \ge 0.99$. From Appendix Table II,

$P(X \le 4) = 0.9984 > 0.99$. Thus $M/1000 = 4$ or $M = 4000$ dollars.

2.4-18 X is $b(5,\ 0.05)$. The expected number of tests is

$1\,P(X = 0) + 6\,P(X > 0) = 1\,(0.7738) + 6\,(1 - 0.7738) = 2.131.$

2.4-20 **(a)** **(i)** $b(5,\ 0.7)$; **(ii)** $\mu = 3.5, \sigma^2 = 1.05$; **(iii)** 0.1607;

(b) **(i)** geometric, $p = 0.3$; **(ii)** $\mu = 10/3, \sigma^2 = 70/9$; **(iii)** 0.51;

(c) **(i)** Bernoulli, $p = 0.55$; **(ii)** $\mu = 0.55, \sigma^2 = 0.2475$; **(iii)** 0.55;

(d) **(ii)** $\mu = 2.1, \sigma^2 = 0.89$; **(iii)** 0.7;

(e) **(i)** discrete uniform on $1, 2, \ldots, 10$; **(ii)** 5.5, 8.25; **(iii)** 0.2.

2.5 The Negative Binomial Distribution

2.5-2 $\displaystyle \binom{10-1}{5-1}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 = \frac{126}{1024} = \frac{63}{512}.$

2.5-4 Let "being missed" be a success and let X equal the number of trials until the first success. Then $p = 0.01$.

$P(X \le 50) = 1 - 0.99^{50} = 1 - 0.605 = 0.395.$

2.5-6 **(a)**

$$\begin{aligned}
R(t) &= \ln(1 - p + pe^t),\\
R'(t) &= \left[\frac{pe^t}{1 - p + pe^t}\right]_{t=0} = p,\\
R''(t) &= \left[\frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2}\right]_{t=0} = p(1 - p);
\end{aligned}$$

(b)

$$\begin{aligned}
R(t) &= n\ln(1 - p + pe^t),\\
R'(t) &= \left[\frac{npe^t}{1 - p + pe^t}\right]_{t=0} = np,\\
R''(t) &= n\left[\frac{(1 - p + pe^t)(pe^t) - (pe^t)(pe^t)}{(1 - p + pe^t)^2}\right]_{t=0} = np(1 - p);
\end{aligned}$$

(c)

$$\begin{aligned}
R(t) &= \ln p + t - \ln[1 - (1 - p)e^t],\\
R'(t) &= \left[1 + \frac{(1 - p)e^t}{1 - (1 - p)e^t}\right]_{t=0} = 1 + \frac{1 - p}{p} = \frac{1}{p},\\
R''(t) &= \left[(-1)\{1 - (1 - p)e^t\}^2\{-(1 - p)e^t\}\right]_{t=0} = \frac{1 - p}{p};
\end{aligned}$$

(d)

$$\begin{aligned}
R(t) &= r\left[\ln p + t - \ln\{1 - (1 - p)e^t\}\right],\\
R'(t) &= r\left[\frac{1}{1 - (1 - p)e^t}\right]_{t=0} = \frac{r}{p},\\
R''(t) &= r\left[(-1)\{1 - (1 - p)e^t\}^{-2}\{-(1 - p)e^t\}\right]_{t=0} = \frac{r(1 - p)}{p^2}.
\end{aligned}$$

2.5-8 $(0.7)(0.7)(0.3) = 0.147.$

2.5-10 **(a)** Let X equal the number of boxes that must be purchased. Then

$$E(X) = \sum_{x=1}^{12} \frac{1}{(13-x)/12} = \frac{86021}{2310} = 37.2385;$$

(b) $\dfrac{100 \cdot E(X)}{365} \approx 10.2.$

2.6 The Poisson Distribution

2.6-2 $\lambda = \mu = \sigma^2 = 3$ so $P(X = 2) = 0.423 - 0.199 = 0.224.$

2.6-4

$$\begin{aligned} 3\,\frac{\lambda^1 e^{-\lambda}}{1!} &= \frac{\lambda^2 e^{-\lambda}}{2!} \\ e^{-\lambda}\lambda(\lambda - 6) &= 0 \\ \lambda &= 6 \end{aligned}$$

Thus $P(X = 4) = 0.285 - 0.151 = 0.134.$

2.6-6 $\lambda = (1)(50/100) = 0.5$, so $P(X = 0) = e^{-0.5}/0! = 0.607.$

2.6-8 $np = 1000(0.005) = 5;$

(a) $P(X \le 1) \approx 0.040;$

(b) $P(X = 4, 5, 6) = P(X \le 6) - P(X \le 3) \approx 0.762 - 0.265 = 0.497.$

2.6-10 $\sigma = \sqrt{9} = 3,$

$P(3 < X < 15) = P(X \le 14) - P(X \le 3) = 0.959 - 0.021 = 0.938.$

2.6-12 Since $E(X) = 0.2$, the expected loss is $(0.02)(\$10{,}000) = \$2{,}000.$

Chapter 3

Continuous Distributions

3.1 Random Variables of the Continuous Type

3.1–2 $\mu = 0, \quad \sigma^2 = (1+1)^2/12 = 1/3.$

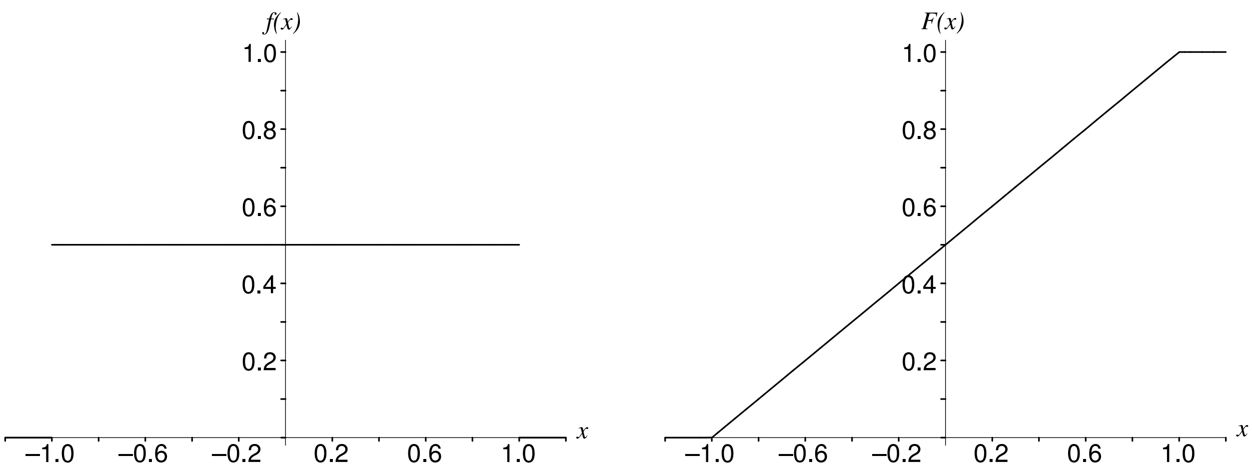

Figure 3.1–2: $f(x) = 1/2$ and $F(x) = (x+1)/2$

3.1–4 X is $U(4, 5)$;

(a) $\mu = 9/2$; **(b)** $\sigma^2 = 1/12$; **(c)** 0.5.

3.1–6

$$\begin{aligned}
E(\text{profit}) &= \int_0^n [x - 0.5(n-x)]\frac{1}{200}\,dx + \int_n^{200} [n - 5(x-n)]\frac{1}{200}\,dx \\
&= \frac{1}{200}\left[\frac{x^2}{2} + \frac{(n-x)^2}{4}\right]_0^n + \frac{1}{200}\left[6nx - \frac{5x^2}{2}\right]_n^{200} \\
&= \frac{1}{200}\left[-3.25n^2 + 1200n - 100000\right] \\
\text{derivative} &= \frac{1}{200}[-6.5n + 1200] \;=\; 0 \\
n &= \frac{1200}{6.5} \;\approx\; 185.
\end{aligned}$$

3.1–8 **(a)** **(i)**

$$\begin{aligned} \int_0^c x^3/4\,dx &= 1 \\ c^4/16 &= 1 \\ c &= 2; \end{aligned}$$

(ii)

$$\begin{aligned} F(x) &= \int_{-\infty}^x f(t)\,dt \\ &= \int_0^x t^3/4\,dt \\ &= x^4/16, \end{aligned}$$

$$F(x) = \begin{cases} 0, & -\infty < x < 0, \\ x^4/16, & 0 \le x < 2, \\ 1, & 2 \le x < \infty. \end{cases}$$

(iii)

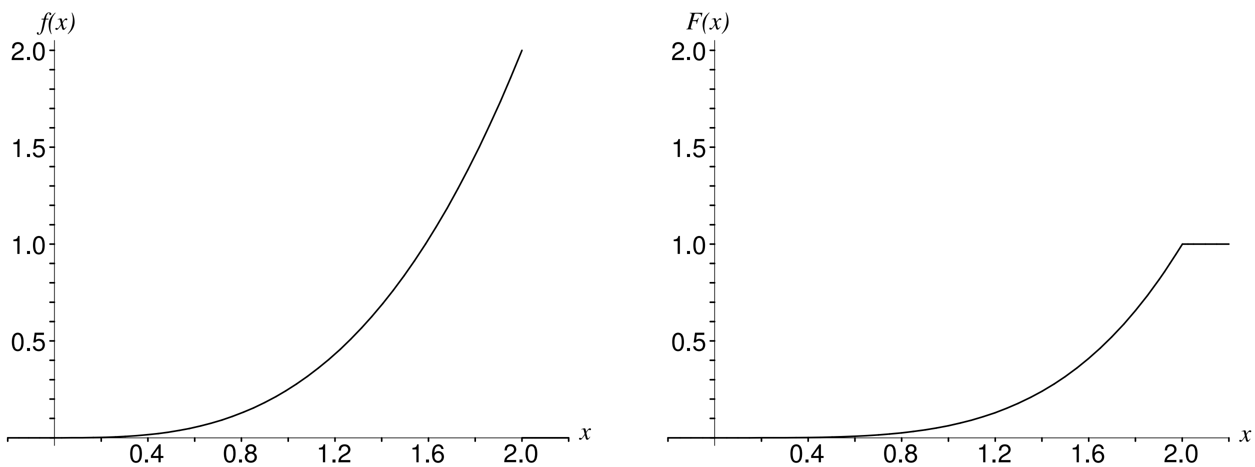

Figure 3.1–8: **(a)** Continuous distribution pdf and cdf

(iv)

$$\begin{aligned} \mu &= \int_0^2 (x)(x^3/4)\,dx = \frac{8}{5}; \\ E(X^2) &= \int_0^2 (x^2)(x^3/4)\,dx = \frac{8}{3}; \\ \sigma^2 &= \frac{8}{3} - \left(\frac{8}{5}\right)^2 = \frac{8}{75}. \end{aligned}$$

(b) **(i)**

$$\begin{aligned}\int_{-c}^{c}(3/16)x^2\,dx &= 1\\ c^3/8 &= 1\\ c &= 2;\end{aligned}$$

(ii)

$$\begin{aligned}F(x) &= \int_{-\infty}^{x} f(t)\,dt\\ &= \int_{-2}^{x}(3/16)t^2\,dt\\ &= \left[\frac{t^3}{16}\right]_{-2}^{x}\\ &= \frac{x^3}{16}+\frac{1}{2},\end{aligned}$$

$$F(x)=\begin{cases}0, & -\infty < x < -2,\\ \dfrac{x^3}{16}+\dfrac{1}{2}, & -2\le x<2,\\ 1, & 2\le x<\infty.\end{cases}$$

(iii)

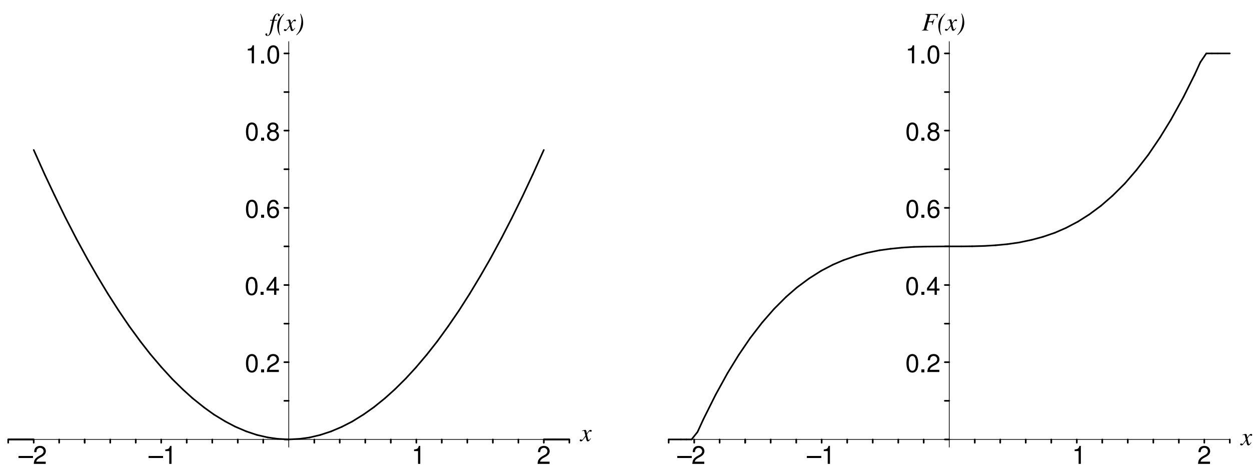

Figure 3.1–8: **(b)** Continuous distribution pdf and cdf

(iv)

$$\begin{aligned}\mu &= \int_{-2}^{2}(x)(3/16)(x^2)\,dx \;=\; 0;\\ \sigma^2 &= \int_{-2}^{2}(x^2)(3/16)(x^2)\,dx \;=\; \frac{12}{5}.\end{aligned}$$

(c) **(i)**
$$\begin{aligned}\int_0^1 \frac{c}{\sqrt{x}}\,dx &= 1\\ 2c &= 1\\ c &= 1/2.\end{aligned}$$

The pdf in part (c) is unbounded.

(ii)
$$\begin{aligned}F(x) &= \int_{-\infty}^{x} f(t)\,dt\\ &= \int_0^x \frac{1}{2\sqrt{t}}\,dt\\ &= \Big[\sqrt{t}\Big]_0^x \;=\; \sqrt{x},\end{aligned}$$

$$F(x) = \begin{cases} 0, & -\infty < x < 0,\\ \sqrt{x}, & 0 \le x < 1,\\ 1, & 1 \le x < \infty.\end{cases}$$

(iii)

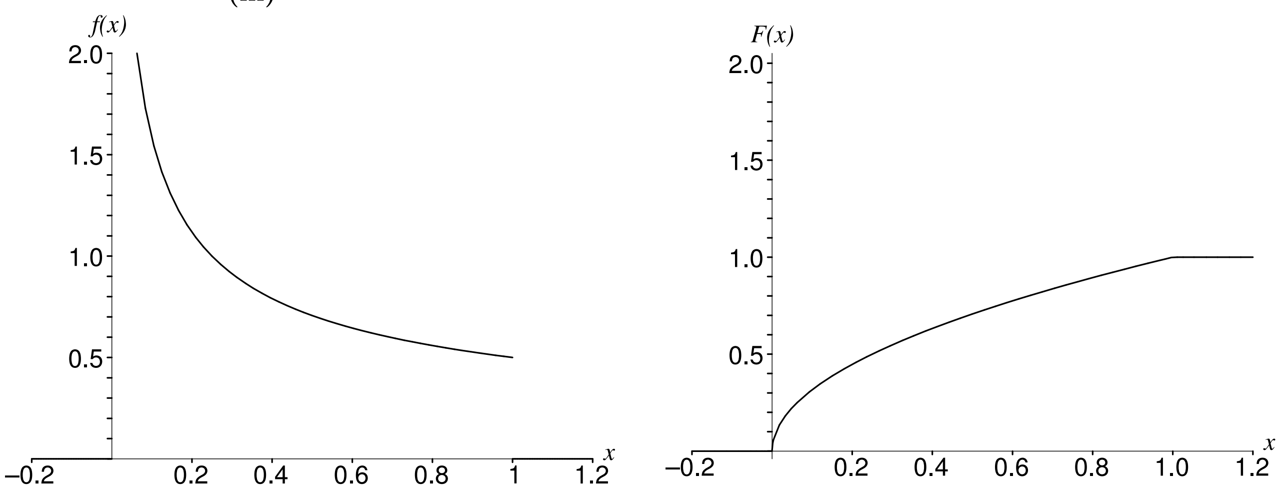

Figure 3.1–8: **(c)** Continuous distribution pdf and cdf

(iv)
$$\begin{aligned}\mu &= \int_0^1 (x)(1/2)/\sqrt{x}\,dx \;=\; \frac{1}{3};\\ E(X^2) &= \int_0^1 (x^2)(1/2)/\sqrt{x}\,dx \;=\; \frac{1}{5};\\ \sigma^2 &= \frac{1}{5} - \left(\frac{1}{3}\right)^2 \;=\; \frac{4}{45}.\end{aligned}$$

3.1–10 **(a)**
$$\begin{aligned}\int_1^\infty \frac{c}{x^2}\,dx &= 1\\ \left[\frac{-c}{x}\right]_1^\infty &= 1\\ c &= 1;\end{aligned}$$

(b) $E(X) = \displaystyle\int_1^\infty \frac{x}{x^2}\,dx = [\ln x]_1^\infty$, which is unbounded.

3.1–12 **(a)**

$$F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x^3+1)/2, & -1 \le x < 1, \\ 1, & 1 \le x < \infty. \end{cases}$$

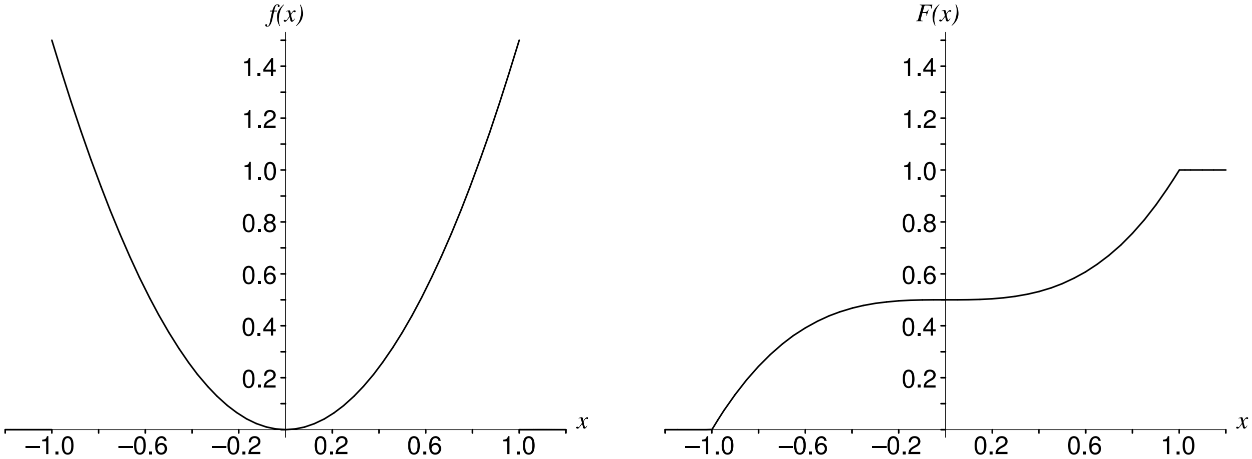

Figure 3.1–12: **(a)** $f(x) = (3/2)x^2$ and $F(x) = (x^3+1)/2$

(b)

$$F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x+1)/2, & -1 \le x < 1, \\ 1, & 1 \le x < \infty. \end{cases}$$

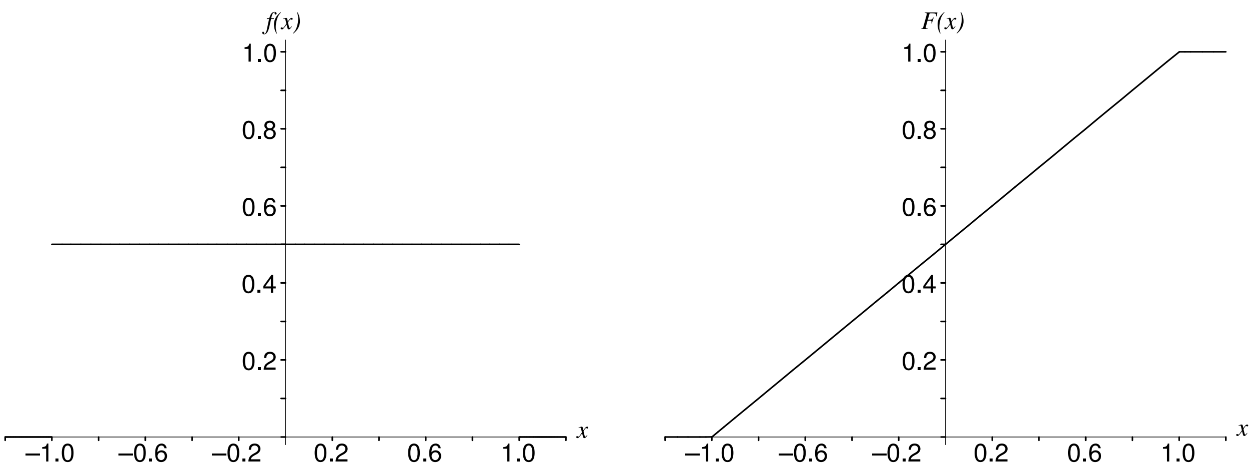

Figure 3.1–12: **(b)** $f(x) = 1/2$ and $F(x) = (x+1)/2$

(c)

$$F(x) = \begin{cases} 0, & -\infty < x < -1, \\ (x+1)^2/2, & -1 \le x < 0, \\ 1-(1-x)^2/2, & 0 \le x < 1, \\ 1, & 1 \le x < \infty. \end{cases}$$

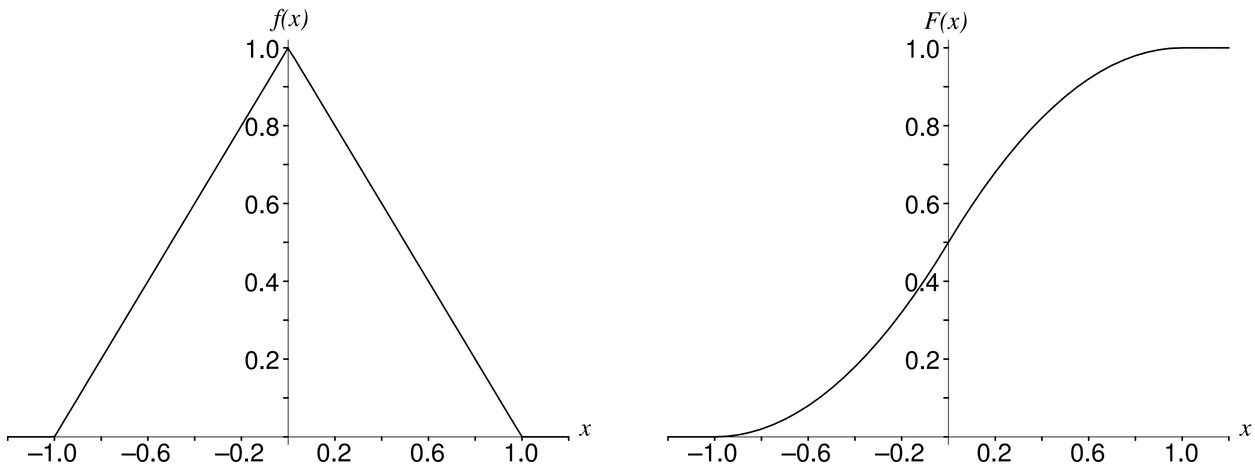

Figure 3.1–12: **(c)** $f(x)$ and $F(x)$ for Exercise **3.1-12(c)**

3.1–14 (b)

$$F(x) = \begin{cases} 0, & -\infty < x \le 0, \\ \dfrac{x}{2}, & 0 < x \le 1, \\ \dfrac{1}{2}, & 1 < x \le 2, \\ \dfrac{x}{2} - \dfrac{1}{2} & 2 \le x < 3, \\ 1, & 3 \le x < \infty; \end{cases}$$

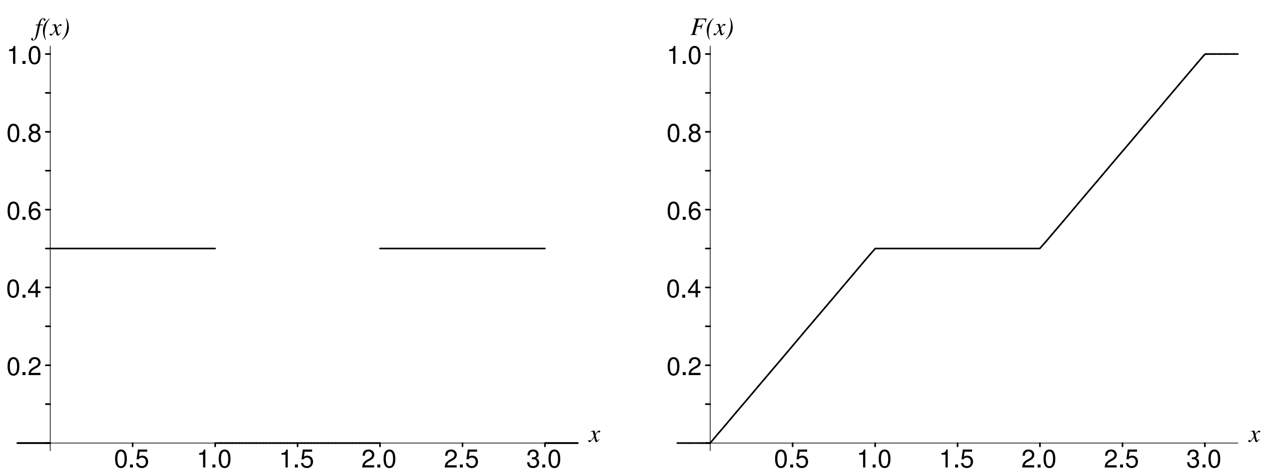

Figure 3.1–14: $f(x)$ and $F(x)$ for Exercise **3.1-14(a)** and **(b)**