1 FUNDAMENTALS

1.1 *Real Numbers*

1. State the property of real numbers being used.

$x+4=4+x$

(a) Commutative Property for addition
(b) Commutative Property for multiplication
(c) Associative Property for addition
(d) Associative Property for multiplication
(e) Distributive Property

Answer: (a)

2. State the property of real numbers being used.

$2(x+5y)=2x+10y$

(a) Commutative Property for addition
(b) Commutative Property for multiplication
(c) Associative Property for addition
(d) Associative Property for multiplication
(e) Distributive Property

Answer: (e)

3. State the property of real numbers being used.

$3xy = yx3$

(a) Commutative Property for addition
(b) Commutative Property for multiplication
(c) Associative Property for addition
(d) Associative Property for multiplication
(e) Distributive Property

Answer: (b)

4. Use properties of real numbers to write $3(2a+b)$ without parentheses.

(a) $6a+6b$ (b) $3+2ab$ (c) $6a-b$ (d) $6a+3b$ (e) $6a-3b$

Answer: (d)

5. Use properties of real numbers to write $12(z/4)$ without parentheses.

(a) $\frac{1}{4}z$ (b) $\frac{1}{12}z$ (c) $48z$ (d) $12z$ (e) $3z$

Answer: (e)

6. Use properties of real numbers to write $-r(s-2)$ without parentheses.

(a) $-2r+2s$ (b) $-rs+2$ (c) $rs+2r$ (d) $-rs+2r$ (e) $-rs-2r$

Answer: (d)

7. Use the properties of real numbers to write the expression without parentheses.

$$2x\left(a-b-2c+\frac{d}{2}\right)$$

(a) $xa-xb-2xc+xd$
(b) $2xa-2xb-xc+4xd$
(c) $xa-2xb-4xc+xd$
(d) $2xa-2xb-4xc+xd$
(e) $2xa-xb-2xc+2xd$

Answer: (d)

8. Perform the indicated operation.

$$\frac{1}{4}+\frac{1}{12}$$

(a) $\frac{1}{16}$ (b) $\frac{1}{48}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) 3

Answer: (d)

9. Perform the indicated operation.

$$\frac{1}{6}\div\frac{2}{3}$$

(a) $-\frac{1}{2}$ (b) $\frac{2}{18}$ (c) $\frac{1}{18}$ (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

Answer: (e)

10. Perform the indicated operations.

$$\frac{\frac{2}{3}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{5}}$$

Answer: $5/3$

11. Perform the indicated operations.

$$\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{5}}$$

Answer: $9/7$

12. Perform the indicated operations.

$$\frac{\frac{1}{12}}{\frac{1}{8}-\frac{1}{12}}$$

Answer: 2

13. Perform the indicated operations.

$$\frac{\frac{1}{10}}{\frac{1}{3}-\frac{1}{5}}$$

Answer: $3/4$

14. Perform the indicated operation(s) and simplify.

$$\frac{1}{4}+5\left(\frac{1}{4}\cdot\frac{1}{5}\right)-\frac{3}{4}$$

(a) $-\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{25}$ (d) $-\frac{1}{4}$ (e) $\frac{1}{4}$

Answer: (d)

15. Perform the indicated operation(s) and simplify.

$$\frac{1}{5}\div\left[\frac{1}{4}\left(\frac{1}{2}+\frac{1}{3}\right)\right]$$

(a) $\frac{3}{2}$ (b) $\frac{25}{24}$ (c) 1 (d) $\frac{2}{3}$ (e) $\frac{24}{25}$

Answer: (e)

16. State whether the inequality is true or false.

$$-\sqrt{2}<-1.41$$

Answer: True

17. State whether the inequality is true or false.

$$-\frac{1}{10}>-\frac{1}{100}$$

Answer: False

18. State whether the inequality is true or false.

$\frac{22}{7} > \pi$

Answer: True

19. Write the following statement in terms of inequalities.

x is negative.

(a) $x > 0$ (b) $x < 0$ (c) $x \geq 0$ (d) $x \leq 0$ (e) $x = 0$

Answer: (b)

20. Write the following statement in terms of inequalities.

z is greater than or equal to -1.

(a) $z \leq -1$ (b) $z \geq -1$ (c) $z < -1$ (d) $z > -1$ (e) $z = 0$

Answer: (b)

21. Write the statement in terms of inequalities.

The distance from x to 3 is at most 6 .

(a) $|x-3| \leq 6$ (b) $|x-3| \geq 6$ (c) $|x-3| < 6$ (d) $|x-6| \leq 3$ (e) $|x-6| \geq 3$

Answer: (a)

22. Find the set $A \cup B$ if $A = \left\{-3, -2, 0, \frac{1}{3}, \frac{2}{3}, 6, 9\right\}$ and $B = \left\{0, \frac{1}{3}, \frac{2}{3}\right\}$.

(a) $\left\{0, -2, -3, 6, 9, \frac{1}{3}, \frac{2}{3}\right\}$

(b) $\left\{0, -2, 3, 6, 9, \frac{-1}{3}, \frac{2}{3}\right\}$

(c) $\left\{0, 2, -3, 6, 9, \frac{1}{3}, \frac{-2}{3}\right\}$

(d) $\left\{0, -2, -3, 6, -9\right\}$

(e) $\left\{0, -2, -3, 6, 9, \frac{2}{3}\right\}$

Answer: (a)

23. Find the set $A \cup B \cap C$ if $A = \{0\}$ and $B = \{1, 2\}$ and $C = \{-1, 0, 1, 2\}$.

(a) $\{0, 2\}$ (b) $\{-1, 2\}$ (c) $\{1, 2\}$ (d) $\{-1, 0, 1, 2\}$ (e) $\{0, 1, 2\}$

Answer: (e)

24. Find $A \cap C$ if $A = \{x \mid x < 4\}$ and $C = \{x \mid -2 < x \le 6\}$.

(a) $\{x \mid -2 < x < 4\}$
(b) $\{x \mid -2 < x < 6\}$
(c) $\{x \mid 4 < x < 6\}$
(d) $\{x \mid -2 < x \le 6\}$
(e) $\varnothing$

Answer: (a)

25. Find $A \cup B$ if $A = \{x \mid x > \pi\}$ and $B = \{x \mid -1 < x < \pi\}$.

(a) $\varnothing$
(b) $\{x \mid x > -1 \text{ and } x \ne \pi\}$
(c) $\{x \mid -1 < x < \pi\}$
(d) $\{x \mid -\pi < x \le 1\}$
(e) $\{x \mid x > -\pi\}$

Answer: (b)

26. Evaluate the expression.

$$\left|2 - |-3| - 3\right|$$

Answer: $\left|2 - |-3| + (-3)\right| = |2 - 3 - 3| = |-4| = 4$

27. Evaluate the expression.

$$-\left|-|(-2)| - |(-2)|\right|$$

Answer: $-\left|-|(-2)| - |(-2)|\right| = -|-2 - 2| = -|-4| = -4$

28. Evaluate the expression.

$$\left|\frac{25 - 52}{52 - 25}\right|$$

Answer: $\left|\frac{25 - 52}{52 - 25}\right| = \left|\frac{-27}{27}\right| = |-1| = 1$

29. Find the distance between $-1/15$ and $-3/25$.

Answer: $\left|-\frac{1}{15} - \left(-\frac{3}{25}\right)\right| = \left|\frac{4}{75}\right| = \frac{4}{75}$

30. Find the distance between π and . $\pi - 1$

Answer: $\left|(\pi - 1) - \pi\right| = \left|-1\right| = 1$

31. Express the repeating decimal as a fraction.

$0.\overline{8}$

Answer: $8/9$

32. Express the repeating decimal $2.\overline{45}$ as a fraction.

Answer: $2.\overline{45} = \frac{27}{11}$

33. Express the repeating decimal as a fraction.

$1.1\overline{35}$

Answer: $562/495$

1. Evaluate -2^3

(a) -16 (b) -8 (c) -4 (d) 8 (e) 16

Answer: (b) $-2^3 = -(2^3) = -8$

2. Evaluate the expression.

4^{-3}

(a) $-\frac{1}{256}$ (b) $-\frac{1}{64}$ (c) $\frac{1}{64}$ (d) $\frac{1}{256}$ (e) $\frac{1}{512}$

Answer: (c) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$

3. Evaluate $\left(-\frac{2}{3}\right)^{-2}$

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{-9}{4}$ (d) $\frac{1}{4}$ (e) 1

Answer: (a) $\left(-\frac{2}{3}\right)^{-2} = \frac{9}{4}$

4. Evaluate $\left(\sqrt{\frac{36}{9}}\right)^{-2}$

(a) $\frac{9}{2}$ (b) $\frac{-2}{9}$ (c) $\frac{-3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{1}{12}$

Answer: (d) $\left(\sqrt{\frac{36}{9}}\right)^{-2} = \frac{1}{4}$

5. Evaluate $\left(\frac{1}{4}\right)^{-2}\left(\frac{1}{2}\right)^{-4}$

(a) $\frac{1}{64}$ (b) $\frac{1}{256}$ (c) 64 (d) $\frac{-1}{4}$ (e) 256

Answer: (e) $\left(\frac{1}{4}\right)^{-2}\left(\frac{1}{2}\right)^{-4} = 256$

6. Evaluate each expression.

(a) $\left(\frac{7}{3}\right)^0 2^{-1}$ (b) $\frac{3^{-3}}{4^0}$ (c) $\left(\frac{1}{5}\right)^{-2}$

Answer: (a) $\left(\frac{7}{3}\right)^0 2^{-1} = \frac{1}{2}$ (b) $\frac{3^{-3}}{4^0} = \frac{1}{27}$ (c) $\left(\frac{1}{5}\right)^{-2} = 25$

7. Evaluate each expression.

(a) $\left(\frac{5}{3}\right)^0 3^{-1}$ (b) $\frac{3^{-3}}{5^0}$ (c) $\left(\frac{1}{3}\right)^{-2}$

Answer: (a) $\left(\frac{5}{3}\right)^0 3^{-1} = \frac{1}{3}$ (b) $\frac{3^{-3}}{5^0} = \frac{1}{27}$ (c) $\left(\frac{1}{3}\right)^{-2} = 9$

8. Evaluate $(512)^{-\frac{1}{9}}$

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 16 (d) $\frac{1}{16}$ (e) 256

Answer: (b) $(512)^{-1/9} = \sqrt[9]{\frac{1}{512}} = \frac{1}{2}$

9. Evaluate $(32)^{-0.2}$

(a) 2 (b) $\frac{1}{2}$ (c) 16 (d) $\frac{1}{4}$ (e) $\frac{1}{1024}$

Answer: (b) $(32)^{-0.2} = (32)^{-1/5} = \left(\frac{1}{\sqrt[5]{32}}\right) = \frac{1}{2}$

10. Simplify the expression $\sqrt{27} - \sqrt{48}$.

(a) $-\sqrt{21}$ (b) -1 (c) $4\sqrt{3}$ (d) $-3\sqrt{3}$ (e) $-\sqrt{3}$

Answer: (e) $\sqrt{27} - \sqrt{48} = 3\sqrt{3} - 4\sqrt{3} = -\sqrt{3}$

11. Simplify $\frac{x^{1/3}y^{1/3}}{(xy)^{-2/3}}$ and eliminate any negative exponents.

(a) xy (b) $(xy)^{-2/3}$ (c) $(xy)^{1/3}$ (d) y (e) $x^{1/3}y$

Answer: (a) $\frac{x^{1/3}y^{1/3}}{(xy)^{-2/3}} = \frac{(xy)^{1/3}}{(xy)^{-2/3}} = (xy)^{(1/3)-(-2/3)} = xy$

12. Simplify $\left(x^2y^{-5}z^5\right)^{-2/5}$ and eliminate any negative exponents.

(a) $\frac{y^2}{z^2\left(\sqrt{x}\right)^5}$ (b) $\frac{y^2}{z^2\left(\sqrt[5]{x}\right)^4}$ (c) $\frac{y}{z^2\left(\sqrt[4]{x}\right)^4}$ (d) $\frac{y^2}{z\sqrt{x}^2}$ (e) $\frac{y^4}{z^2\left(\sqrt[5]{x}\right)}$

Answer: (b) $\left(x^2y^{-5}z^5\right)^{-2/5} = \left(x^{-4/5}y^2z^{-2}\right) = \frac{y^2}{z^2\left(\sqrt[5]{x}\right)^4}$

13. Simplify $ab^3c^2\left(\dfrac{2a^2b}{c^4}\right)^{-1}$ and eliminate any negative exponents.

(a) b^2c^6 (b) $\dfrac{b^6c^2}{2a^2}$ (c) $\dfrac{2b^2c^6}{3a}$ (d) $\dfrac{b^2c^6}{2a}$ (e) $2b^2c^6$

Answer: (d) $ab^3c^2\left(\dfrac{2a^2b}{c^4}\right)^{-1} = ab^3c^2\left(\dfrac{c^4}{2a^2b}\right) = \dfrac{b^2c^6}{2a}$

14. Simplify $\left[\left(a^{-3}b^{-4}c^2\right)^{-2}(ab)^{-3}b^{-1}c\right]^{-2}$ and eliminate any negative exponents.

Answer:

$$\left[\left(a^{-3}b^{-4}c^2\right)^{-2}(ab)^{-3}b^{-1}c\right]^{-2} = \left[\left(\frac{a^6b^8}{c^4}\right)\left(\frac{1}{(ab)^3}\right)\frac{c}{b}\right]^{-2} = \left(\frac{a^3b^4}{c^3}\right)^{-2} = \frac{1}{\left(\dfrac{a^3b^4}{c^3}\right)^2} = \frac{c^6}{a^6b^8}$$

15. Simplify the expression $\sqrt[3]{\dfrac{y^3z^3}{64}}$.

Answer: $\sqrt[3]{\dfrac{y^3z^3}{64}} = \sqrt[3]{\dfrac{(yz)^3}{64}} = \dfrac{yz}{4}$

16. Simplify the expression. Assume the letters denote any real numbers.

$\sqrt[4]{x^4y^4}$

Answer: $\sqrt[4]{x^4y^4} = |xy|$

17. Simplify the expression. Assume the letters denote any real numbers.

$\sqrt[5]{u^6v^7}$

Answer: $\sqrt[5]{u^6v^7} = uv\sqrt[5]{uv^2}$

18. Simplify the expression. Assume the letters denote any real numbers.

$\sqrt{36a^6b^6}$

Answer: $\sqrt{36a^6b^6} = 6\left|a^3b^3\right|$

19. Simplify the expression. Assume the letters denote any real numbers.

$\sqrt[3]{\sqrt{64x^6y^6}}$

Answer: $\sqrt[3]{\sqrt{64x^6y^6}} = 2|xy|$

20. Simplify the expression and eliminate any negative exponents(s).
Assume that all letters denote positive numbers.

$\sqrt[3]{u\sqrt{u}}$

Answer: $\sqrt[3]{u\sqrt{u}} = \left(u\sqrt{u}\right)^{1/3} = \left(u^{2/2}u^{1/2}\right)^{1/3} = \left(u^{3/2}\right)^{1/3} = u^{1/2} = \sqrt{u}$

21. Simplify the expression and eliminate any negative exponents(s).
Assume that all letters denote positive numbers.

$\sqrt[3]{r^2s}\sqrt[3]{64r^4s^2}$

Answer: $4r^2s$

22. Simplify the expression and eliminate any negative exponents(s).
Assume that all letters denote positive numbers.

$\sqrt[3]{r^2s}\sqrt[3]{81r^4s^2}$

Answer: $3r^2s\sqrt[3]{3}$

23. Write 0.000101010 in scientific notation.

Answer: $0.000101010 = 1.0101\times10^{-4}$

24. Write $1{,}010{,}100{,}000{,}000$ in scientific notation.

Answer: $1{,}010{,}100{,}000{,}000 = 1.0101\times10^{12}$

25. Write the number in the statement in scientific notation:

The mass of a proton is $0.00000000000000000000000000016726231$ kg.

Answer: $0.00000000000000000000000000016726231 = 1.6726\times10^{-27}$ kg

26. Write the number in the statement in scientific notation. The distance to the edge of the observable universe is about $100{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ m.

Answer: $100{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 = 1\times10^{26}$ m

27. Use scientific notation, laws of exponents, and a calculator to perform the indicated operations. State your answer correct to the number of significant digits indicated by the data.

$(4.01\times10^{16})(8.11\times10^{10})$

Answer: $(4.01\times10^{16})(8.11\times10^{10}) = (4.01)(8.11)\times10^{16+10} \approx 3.25\times10^{27}$

28. A sealed warehouse measuring 12 m wide, 18 m long and 6 m high, is filled with pure oxygen. One cubic meter contains 1000 L, and 22.4 L of any gas contains 6.02×10^{23} molecules. How many molecules of oxygen are there in the room?

Answer: Volume $=12\times18\times6\times1000=1,296,000$ L $\Rightarrow \left(\frac{1{,}296{,}000}{22.4}\right)\times6.02\times10^{23}=3.483\times10^{28}$ molecules

29. Which of the numbers $\frac{10}{0.001}$ and $\frac{10}{0.01}$ is smaller?

Answer: $\frac{10}{0.01}$ is smaller because $\frac{10}{0.01}=\frac{10}{10^{-2}}=10^3=1000$, whereas $\frac{10}{0.001}=\frac{10}{10^{-3}}=10^4=10,000$.

30. Which of the numbers $-\left(\frac{10}{0.001}\right)^{-1}$ and $-\left(\frac{10}{0.01}\right)^{-2}$ is larger?

Answer: $-\left(\frac{10}{0.01}\right)^{-2}$ is larger because $-\left(\frac{10}{0.01}\right)^{-2}=-\left(10^{-6}\right)=-\frac{1}{1{,}000{,}000}$, $-\left(\frac{10}{0.001}\right)^{-1}=-\left(10^{-4}\right)=-\frac{1}{10{,}000}$.

31. Rationalize the denominator.

$\sqrt{\frac{x}{6}}$

Answer: $\sqrt{\frac{x}{6}}=\frac{\sqrt{6x}}{6}$

32. Rationalize the denominator.

$\frac{1}{s^{3/7}}$

Answer: $\frac{1}{s^{3/7}}=\frac{\sqrt[7]{s^4}}{s}$

1. Perform the indicated operations and simplify.

$3(3x-4)-5(x-1)$

(a) $4x-13$ (b) $2x-13$ (c) $4x-7$ (d) $2x+13$ (e) $2x+29$

Answer: (c) $3(3x-4)-5(x-1)=9x-12-5x+5=4x-7$

2. Perform the indicated operations and simplify.

$\left(-2x^3+6x^2-4x+8\right)+\left(3x^3+x-4\right)$

(a) $2x^3+6x^2+3x+5$
(b) $2x^3+6x^2+3x+12$
(c) $2x^3+6x^2+3x-1$
(d) x^3+6x^2+3x-4
(e) x^3+6x^2-3x+4

Answer: (e) $\left(-2x^3+6x^2-4x+8\right)+\left(3x^3+x-4\right)=x^3+6x^2-3x+4$

3. Find the sum, difference, or product.

$2(2-5t)-t^2(t-1)+(t^4-1)$

Answer: $2(2-5t)-t^2(t-1)+(t^4-1)=3-10t-t^3+t^2+t^4$

4. Find the sum, difference, or product.

$2(2-5t)-t^2(t-2)+(t^4-1)$

Answer: $2(2-5t)-t^2(t-2)+(t^4-1)=3-10t-t^3+2t^2+t^4$

5. Multiply the algebraic expressions using the FOIL method and simplify.

$(4x-5y)(2x-4y)$

Answer: $(4x-5y)(2x-4y)=8x^2-26xy+20y^2$

6. Multiply the algebraic expressions using a Special Product Formula and simplify.

$\left(3x+4y\right)^2$

Answer: $\left(3x+4y\right)^2=9x^2+24xy+16y^2$

7. Multiply the algebraic expressions using a Special Product Formula and simplify.

$(2x-5y)^2 = 4x^2 - 20xy + 25y^2$

Answer: $(2x-5y)^2 = 4x^2 - 20xy + 25y^2$

8. Multiply the algebraic expressions using a Special Product Formula and simplify.

$\left(\sqrt{y}+\sqrt{3}\right)\left(\sqrt{y}-\sqrt{3}\right)$

Answer: $\left(\sqrt{y}+\sqrt{3}\right)\left(\sqrt{y}-\sqrt{3}\right) = y-3$

9. Multiply the algebraic expressions using a Special Product Formula and simplify.

$\left(\sqrt{y}+\sqrt{5}\right)\left(\sqrt{y}-\sqrt{5}\right)$

Answer: $\left(\sqrt{y}+\sqrt{5}\right)\left(\sqrt{y}-\sqrt{5}\right) = y-5$

10. Perform the indicated operations and simplify.

$\left(ax^2 - y^2\right)\left(ax^2 + y^2\right)$

(a) $a^2x^2 - y^2$ (b) $a^2x^4 + ax^2y^2 - y^4$ (c) $a^2x^4 - y^4$ (d) $a^2x^4 - ax^2y^2 - y^4$ (e) $a^2x^4 + y^4$

Answer: (c) $\left(ax^2 - y^2\right)\left(ax^2 + y^2\right) = a^2x^4 - y^4$

11. Perform the indicated operations and simplify.

$\left(x^2 - b^2y^2\right)\left(x^2 - cxy + ay^2\right)$

(a) $x^4 - cx^3y + ax^2y^2 - b^2y^2x^2 + b^2cy^3x - ab^2y^4$
(b) $x^4 - x^3cy + x^2ay^2 + b^2y^3cx - b^2y^4a$
(c) $x^2 - x^3y + x^2ay^2 - b^2y^3cx - ab^2y^4$
(d) $a^2x^4 - ax^2y^2 - y^4$
(e) $x^4 - cx^3y + ax^2y^2 - cy^3x - ab^2y^4$

Answer: (a) $\left(x^2 - b^2y^2\right)\left(x^2 - cxy + ay^2\right) = x^4 - cx^3y + ax^2y^2 - b^2y^2x^2 + b^2cy^3x - ab^2y^4$

12. Perform the indicated operations and simplify.

$$\left(x+\frac{1}{y^2}\right)^3$$

(a) $x^3+\frac{3x^2}{y^2}+\frac{3x}{y^4}+\frac{1}{y^6}$

(b) $x^3+\frac{3x^2}{y^2}+\frac{x}{y^2}+\frac{1}{y^6}$

(c) $x^3+\frac{3x^2}{y}+\frac{3x^2}{y^4}+\frac{3}{y^3}$

(d) $x^3+\frac{3x^2}{y}+\frac{1}{y^6}$

(e) $x^3+\frac{1}{y^6}$

Answer: (a) $\left(x+\frac{1}{y^2}\right)^3=x^3+\frac{3x^2}{y^2}+\frac{3x}{y^4}+\frac{1}{y^6}$

13. Perform the indicated operations and simplify.

$$\left(1-x^{2/3}\right)\left(2+x^{1/3}\right)$$

(a) $2+\sqrt[3]{x}-2\left(\sqrt[3]{x}\right)^3+x$

(b) $1+\sqrt[3]{x}-\left(\sqrt[3]{x}\right)^2-x$

(c) $2+\sqrt[3]{x}-\sqrt[3]{x}-x^2$

(d) $2+\sqrt[3]{x}-2\left(\sqrt[3]{x}\right)^2-x$

(e) $2+\sqrt[3]{x}+2\left(\sqrt[3]{x}\right)^2$

Answer: (d) $\left(1-x^{2/3}\right)\left(2+x^{1/3}\right)=2+\sqrt[3]{x}-2\left(\sqrt[3]{x}\right)^2-x$

14. Perform the indicated operations and simplify.

$$(t+3)(2t-1)-3(t+2)$$

(a) $2t^2+6t+4$ (b) $2t^2+2t-9$ (c) $2t^2-2t-10$ (d) $2t^2-3t-5$ (e) $2t^2+5t+4$

Answer: (b) $(t+3)(2t-1)-3(t+2)=2t^2-t+6t-3-3t-6=2t^2+2t-9$

15. Perform the indicated operations and simplify.

$(2u-v)(2u+v)$

(a) $4u^2-v^2$ (b) $8u^2+4v^2$ (c) $16u^2+4v^2$ (d) $16u^2+2v^2$ (e) $16u^2-4v^2$

Answer: (a) $(2u-v)(2u+v)=4u^2+2uv-2uv-v^2=4u^2-v^2$

16. Factor the expression.

$y^2+xy-2x^2$

(a) $-(2x+xy)(y-2x)$
(b) $-(2x+y)(x-y)$
(c) $(2x-y)(x-y)$
(d) $(2x+y)(x+y)$
(e) $-(2x+y)(x-2y)$

Answer: (b) $y^2+xy-2x^2=-(2x+y)(x-y)$

17. Factor the expression.

$6x^2+13x+6$

(a) $(2x+3)^2$
(b) $(x+3)(6x+2)$
(c) $(2x+1)(3x+6)$
(d) $(6x+1)(x+6)$
(e) $(2x+3)(3x+2)$

Answer: (e) $6x^2+13x+6=(2x+3)(3x+2)$

18. Factor the expression.

$x(n-y)+(x-1)(y-n)$

(a) $xn-y$
(b) $x+(x-1)(y-n)$
(c) $n-y$
(d) $x+y$
(e) xy

Answer: (c) $x(n-y)+(x-1)(y-n)=n-y$

19. Factor the expression.

$x^2(x-5)^3+x^3(x-5)^2$

(a) $x(2x+5)(x-5)^2$
(b) $x^2(2x+5)(x-5)^2$
(c) $x(2x-5)^2(x-5)^2$
(d) $x^2(5x-5)(2x-5)$
(e) $x^2(2x-5)(x-5)^2$

Answer: (e) $x^2(x-5)^3+x^3(x-5)^2=x^2(2x-5)(x-5)^2$

20. Factor the expression.

x^2+7x-8

(a) $(x-4)(x+2)$ (b) $(x+4)(x+2)$ (c) $(x-8)(x-1)$ (d) $(x+8)(x-1)$ (e) $(x+4)(x-2)$

Answer: (d) $x^2+7x-8=(x+8)(x-1)$

21. Factor the expression.

$x^2+8x+15$

(a) $(x+3)(x+5)$ (b) $(x-3)(x+5)$ (c) $(x-3)(x-5)$ (d) $(x+3)(x-5)$ (e) $(-x-3)(x-5)$

Answer: (a) $x^2+8x+15=(x+3)(x+5)$

22. Factor the expression.

$x^3+6x^2+12x+8$

(a) $(x+2)^3$ (b) x^3+8 (c) $(x-2)^3$ (d) $2(x+2)(x^2+2x+4)$ (e) $2(x+4)^2$

Answer: (a) $x^3+6x^2+12x+8=(x+2)^3$

23. Factor the expression completely.

$2x^3+x+6x^2+3$

Answer: $2x^3+x+6x^2+3=(x+3)(2x^2+1)$

24. Factor the expression completely.

$2x^3 + x + 10x^2 + 5$

Answer: $2x^3 + x + 10x^2 + 5 = (x+5)(2x^2+1)$

25. Factor out the common factor.

$-14x^4y^2 + 7xy^3 - 21x^4$

Answer: $-14x^4y^2 + 7xy^3 - 21x^4 = -7x(2x^3y^2 - y^3 + 3x^3)$

26. Use a Factoring Formula to factor the expression.

$16x^2 - 56x + 49$

Answer: $16x^2 - 56x + 49 = (4x-7)^2$

27. Use a Factoring Formula to factor the expression.

$125s^3 - 216t^3$

Answer: $(5s-6t)(25s^2 + 30st + 36t^2)$

28. Factor the expression.

$64s^2 - y^2$

Answer: $64s^2 - y^2 = (8s-y)(8s+y)$

29. Factor the expression.

$25s^2 - 4r^2$

Answer: $25s^2 - 4r^2 = (5s-2r)(5s+2r)$

30. Factor the expression.

$64x^3 - 1$

Answer: $64x^3 - 1 = (4x-1)(16x^2 + 4x + 1)$

31. Factor the expression completely.

$x^2(x^2-1)-25(x^2-1)$

Answer: $x^2(x^2-1)-25(x^2-1)=(x-1)(x+1)(x-5)(x+5)$

32. Factor the expression completely.

$x^2(x^2-4)-16(x^2-4)$

Answer: $x^2(x^2-4)-16(x^2-4)=(x-2)(x+2)(x-4)(x+4)$

33. Factor the expression.

$2y^2x+xy+5y+10y^2$

(a) $xy(2y-1)(x+5)$
(b) $y^2(y+1)(5x+5)$
(c) $y(2y+1)(x+5)$
(d) $y(y+2)(5x+1)$
(e) $(y+1)^2(x+5)$

Answer: (c) $2y^2x+xy+5y+10y^2 = y(2y+1)(x+5)$

34. Factor the expression.

$(x-2)^{7/2}-(x-2)^{3/2}$

(a) $(x-2)^2(x-1)(x-3)$
(b) $(x-2)(x-1)(x+3)$
(c) $(x-2)^{3/2}(x+1)(x-3)$
(d) $(x-2)^{1/2}(x-1)(x-3)$
(e) $(x-2)^{3/2}(x-1)(x-3)$

Answer: (e) $(x-2)^{3/2}(x-1)(x-3)$

35. Factor the expression completely. Begin by factoring out the lowest power of each common factor.

$(x-1)^{5/2}-(x-1)^{3/2}$

Answer: $(x-1)^{5/2}-(x-1)^{3/2}=(x-1)^{3/2}(x-2)$

36. Factor the expression.

$$\left(n^2+1\right)^2-15\left(n^2+1\right)+50$$

(a) $(n-2)(n-3)(n+3)(n+2)$

(b) $(n-2)^2(n+3)(n-3)$

(c) $(n+1)^2(n-3)(n+3)$

(d) $(n+1)(n-1)(n+2)$

(e) $(n+2)^4$

Answer: (a) $\left(n^2+1\right)^2-15\left(n^2+1\right)+50=(n-2)(n-3)(n+3)(n+2)$

37. Factor the expression.

$$x\left(x-\tfrac{1}{2}\right)^3+y\left(x-\tfrac{1}{2}\right)^2$$

Answer: $x\left(x-\tfrac{1}{2}\right)^3+y\left(x-\tfrac{1}{2}\right)^2=\left(x-\tfrac{1}{2}\right)^2\left(x^2-\tfrac{1}{2}x+y\right)$

1. Simplify the expression.

$$\frac{x^2+x-2}{x^2+2x-3}$$

(a) $\frac{x+1}{x+2}$ (b) $\frac{x+2}{x+3}$ (c) $\frac{x-3}{x-2}$ (d) $\frac{x+3}{x+2}$ (e) $\frac{x-2}{x-2}$

Answer: (b) $\frac{x^2+x-2}{x^2+2x-3}=\frac{(x-1)(x+2)}{(x-1)(x+3)}=\frac{x+2}{x+3}$

2. Simplify the expression

$$\frac{y^2-25}{2y^2-4y-30}$$

(a) $\frac{2y-3}{y-5}$ (b) $\frac{y-6}{2y+5}$ (c) $\frac{y+6}{2y+6}$ (d) $\frac{y+5}{2y+6}$ (e) $\frac{y-3}{y+6}$

Answer: (d) $\frac{y^2-25}{2y^2-4y-30}=\frac{(y-5)(y+5)}{(y-5)(2y+6)}=\frac{y+5}{2y+6}$

3. Simplify the expression.

$$\frac{-x^2+1}{x^3-x-1}$$

(a) $\frac{-x+1}{x^2-x-1}$ (b) $\frac{-1+x}{-x^2-x+1}$ (c) $\frac{x-1}{x^2+2x+1}$ (d) $\frac{x+1}{x^2+x+1}$ (e) $\frac{x+1}{x^2-x-1}$

Answer: (a) $\frac{-x^2+1}{x^3-x-1}=\frac{(x+1)(-x+1)}{(x+1)(x^2+x+1)}=\frac{-x+1}{x^2-x-1}$

4. Simplify the expression.

$$\frac{x^2-x-6}{x^2+2x}\cdot\frac{x^2+x}{x^2-2x-3}$$

(a) $\frac{x+2}{x^2}$ (b) $\frac{x}{2-x}$ (c) 1 (d) x (e) x^2

Answer: (c) $\frac{x^2-x-6}{x^2+2x}\cdot\frac{x^2+x}{x^2-2x-3}=\frac{(x+2)(x-3)x(x+1)}{x(x+2)(x-3)(x+1)}=1$

5. Simplify the expression.

$$\frac{6x^2-xy-y^2}{3x^2-5yx-2y^2}\cdot\frac{x^2-3xy+2y^2}{x^2+xy-2y^2}$$

(a) $\frac{2x+y}{x-2y}$ (b) $\frac{x+2y}{2x-y}$ (c) $\frac{2x-y}{x+2y}$ (d) $\frac{2-xy}{2x+y}$ (e) $\frac{x+y+2}{x-2y-2}$

Answer: (c) $\frac{6x^2-xy-y^2}{3x^2-5yx-2y^2}\cdot\frac{x^2-3xy+2y^2}{x^2+xy-2y^2}=\frac{(3x+y)(2x-y)\cdot(x-2y)(x-y)}{(x-2y)(3x+y)\cdot(x+2y)(x-y)}=\frac{2x-y}{x+2y}$

6. Simplify the expression.

$$\frac{y^2-\frac{1}{4}}{2y^2+4y+\frac{3}{2}}$$

(a) $\frac{2y-1}{2y+3}$ (b) $\frac{3}{4}\left(\frac{y-1}{2y+3}\right)$ (c) $\frac{1}{2}\left(\frac{2y+3}{2y-1}\right)$ (d) $\frac{1}{2}\left(\frac{2y-1}{2y+3}\right)$ (e) $2\left(\frac{2y+1}{2y-3}\right)$

Answer: (d) $\frac{y^2-\frac{1}{4}}{2y^2+4y+\frac{3}{2}}=\frac{\frac{1}{4}(2y-1)(2y+1)}{\frac{1}{2}(2y+3)(2y+1)}=\frac{1}{2}\left(\frac{2y-1}{2y+3}\right)$

7. Simplify the expression.

$$\frac{x^4}{x+2}\div\frac{x^3}{x^2+6x+8}$$

(a) $x(x+2)^2$
(b) $x^2(x-4)$
(c) $x(x+4)$
(d) $-x(x+2)$
(e) $x^3(x+4)$

Answer: (c) $\frac{x^4}{x+2}\cdot\frac{(x+4)(x+2)}{x^3}=(x+4)x$

8. Perform the division and simplify.

$$\frac{x+4}{9x^2-4}\div\frac{x^2+8x+16}{3x^2+13x-10}$$

Answer: $\frac{x+4}{9x^2-4}\div\frac{x^2+8x+16}{3x^2+13x-10}=\frac{(x+5)}{(x+4)(3x+2)}$

9. Perform the division and simplify.

$$\frac{x+2}{9x^2-4} \div \frac{x^2+6x+8}{3x^2+13x-10}$$

Answer: $\frac{x+2}{9x^2-4} \div \frac{x^2+6x+8}{3x^2+13x-10} = \frac{(x+5)}{(x+4)(3x+2)}$

10. Simplify the expression.

$$\frac{2x}{x-3} - \frac{2}{x+1}$$

(a) $\frac{x^2+2x+6}{(x-2)(x+3)}$ (b) $\frac{2x^2+6}{(x-3)(x+1)}$ (c) $\frac{2x^2+6}{(x-3)(x+1)}$ (d) $\frac{x^2+3}{(x-2)(x+3)}$ (e) $\frac{x^2+1}{(x+2)(x-3)}$

Answer: (b) $\frac{2x}{x-3} - \frac{2}{x+1} = \frac{2x^2+2x-2x+6}{(x-3)(x+1)} = \frac{2x^2+6}{(x-3)(x+1)}$

11. Simplify the expression.

$$\frac{-1}{x-1} - \frac{4}{(x-2)^2}$$

(a) $\frac{x^2-18}{(2x-3)^2}$ (b) $\frac{-x^2-7}{(x-2)^2}$ (c) $\frac{-x^2}{(x-1)(x-2)^2}$ (d) $\frac{10x-2}{(2x+1)^2}$ (e) $\frac{2x^2+9}{(2x-1)^2}$

Answer: (c) $\frac{-1}{x-1} - \frac{4}{(x-2)^2} = \frac{-x^2+4x-4-4x+4}{(x-1)(x-2)^2} = \frac{-x^2}{(x-1)(x-2)^2}$

12. Perform the subtraction and simplify.

$$\frac{2}{x+3} - \frac{1}{x^2+8x+15}$$

Answer: $\frac{2}{x+3} - \frac{1}{x^2+8x+15} = \frac{2x+9}{x^2+8x+15}$

13. Perform the subtraction and simplify.

$$\frac{2}{x+3} - \frac{1}{x^2+5x+6}$$

Answer: $\frac{2}{x+3} - \frac{1}{x^2+5x+6} = \frac{2x+3}{x^2+5x+6}$

14. Simplify the expression.

$$\frac{1}{a}-\frac{2}{a^2b}+\frac{1}{b^2}$$

(a) $\dfrac{b^2-3ab+4a^2}{2a^2b^2}$

(b) $\dfrac{b-3ab+2a}{2ab}$

(c) $\dfrac{b^2-3ab+6a^2}{3a^2b^2}$

(d) $\dfrac{ab^2-2b+a^2}{a^2b^2}$

(e) $\dfrac{ab^2-3ab+2a^2b}{a^3b^3}$

Answer: (d) $\dfrac{1}{a}-\dfrac{2}{a^2b}+\dfrac{1}{b^2}=\dfrac{ab^2-2b+a^2}{a^2b^2}$

15. Simplify the expression.

$$\frac{2y}{y^2-1}+\frac{1}{(y-1)}$$

(a) $\dfrac{3y+1}{(y-1)^2}$ (b) $3\dfrac{y+1}{(3y-1)(3y+1)}$ (c) $\dfrac{3y+1}{(y-1)(y+1)}$ (d) $\dfrac{y+1}{(y-1)(y+1)}$ (e) $\dfrac{(3y+1)(y+1)}{(y-1)}$

Answer: (c) $\dfrac{2y}{y^2-1}+\dfrac{1}{(y-1)}=\dfrac{2y}{y^2-1}+\dfrac{(y-1)}{y^2-1}=\dfrac{3y+1}{(y-1)(y+1)}$

16. Simplify the expression.

$$\frac{\dfrac{x}{x-1}+\dfrac{1}{x+1}}{\dfrac{5}{x-1}-\dfrac{2}{x+1}}$$

(a) $\dfrac{x^2+2x-1}{x-1}$ (b) $\dfrac{3x+7}{x^2+2x-1}$ (c) $\dfrac{3x+7}{x-1}$ (d) $\dfrac{x^2-2x+1}{x-1}$ (e) $\dfrac{x^2+2x-1}{3x+7}$

Answer: (e) $\dfrac{\dfrac{x}{x-1}+\dfrac{1}{x+1}}{\dfrac{5}{x-1}-\dfrac{2}{x+1}}=\dfrac{\dfrac{x^2+2x-1}{(x-1)(x+1)}}{\dfrac{3x+7}{(x-1)(x+1)}}=\dfrac{x^2+2x-1}{(x-1)(x+1)}\cdot\dfrac{(x-1)(x+1)}{3x+7}=\dfrac{x^2+2x-1}{3x+7}$

17. Simplify the compound fractional expression.

$$\frac{2+\dfrac{1}{x-3}}{2-\dfrac{1}{x-3}}$$

Answer: $\dfrac{2+\dfrac{1}{x-3}}{2-\dfrac{1}{x-3}}=\dfrac{2x-5}{2x-7}$

18. Simplify the compound fractional expression.

$$x+\frac{y}{\dfrac{x}{y}-\dfrac{y}{x}}$$

Answer: $x+\dfrac{y}{\dfrac{x}{y}-\dfrac{y}{x}}=\dfrac{x^3}{x^2-y^2}$

19. Simplify the compound fractional expression.

$$\frac{x^{-1}+y^{-1}}{x^{-2}+y^{-1}}$$

Answer: $\dfrac{x^{-1}+y^{-1}}{x^{-2}+y^{-1}}=\dfrac{x(x+y)}{y+x^2}$

20. Rationalize the denominator of the expression.

$$\frac{1}{2+\sqrt{5}}$$

Answer: $\dfrac{1}{2+\sqrt{5}}=\dfrac{2-\sqrt{5}}{4-5}=\sqrt{5}-2$

21. Rationalize the denominator of the expression.

$$\frac{\sqrt{y}}{1-\sqrt{y}}$$

Answer: $\dfrac{\sqrt{y}}{1-\sqrt{y}}=\dfrac{\sqrt{y}+y}{1-y}$

22. Rationalize the numerator of the expression.

$\sqrt{x^2-2x+1}-\sqrt{x}$

Answer: $\sqrt{x^2-2x+1}-\sqrt{x}=\dfrac{\left(\sqrt{x^2-2x+1}-\sqrt{x}\right)\left(\sqrt{x^2-2x+1}+\sqrt{x}\right)}{\sqrt{x^2-2x+1}+\sqrt{x}}=\dfrac{x^2-2x+1-x}{\sqrt{x^2-2x+1}+\sqrt{x}}$

23. Rationalize the numerator of the expression.

$\sqrt{x-1}+\sqrt{x}$

Answer: $\sqrt{x-1}+\sqrt{x}=\dfrac{\left(\sqrt{x-1}+\sqrt{x}\right)\left(\sqrt{x-1}-\sqrt{x}\right)}{\left(\sqrt{x-1}-\sqrt{x}\right)}=\dfrac{x-1-x}{\left(\sqrt{x-1}-\sqrt{x}\right)}=\dfrac{1}{\sqrt{x}-\sqrt{x-1}}$

24. Rationalize the denominator of the expression.

$\dfrac{5-x}{\sqrt{5}+\sqrt{x}}$

Answer: $\dfrac{5-x}{\sqrt{5}+\sqrt{x}}=\dfrac{(5-x)}{\sqrt{5}+\sqrt{x}}\cdot\dfrac{\sqrt{5}-\sqrt{x}}{\sqrt{5}-\sqrt{x}}=\dfrac{(5-x)\left(\sqrt{5}-\sqrt{x}\right)}{\left(\sqrt{5}+\sqrt{x}\right)\left(\sqrt{5}-\sqrt{x}\right)}=\dfrac{(5-x)\left(\sqrt{5}-\sqrt{x}\right)}{5-x}=\sqrt{5}-\sqrt{x}$

25. Rationalize the numerator of the expression.

$\sqrt{x^2+5x+6}-\sqrt{x}$

Answer: $\dfrac{x^2+4x+6}{\sqrt{x^2+5x+6}+\sqrt{x}}$

1. Determine whether the given values of the variables are solutions of the equation.

$(x-3)+(2-3x)=4(x-2)+1$ (a) $x=0$ (b) $x=1$

Answer: (a) LHS $=(0-3)+(2-0)=-3+2=-1$. RHS $=4(0-2)+1=-8+1=-7\neq$ LHS. So $x=0$ is not a solution. (b) LHS $=(1-3)+(2-3)=-2+-1=-3$. RHS $=4(1-2)+1=-4+1=-3=$ LHS. So $x=1$ is a solution.

2. Determine whether the given values of the variables are solutions of the equation.

$2-[2+(2-x)]=x-(x+1)$ (a) $x=1$ (b) $x=1000$

Answer: (a) LHS $=2-[2+(2-1)]=-1$, RHS $=1-(1+1)=-1=$ LHS. So $x=1$ is a solution.
(b) LHS $=2-[2+(2-1000)]=998$, RHS $=1000-(1000+1)=-1\neq$ LHS. So $x=1000$ is not a solution.

3. Determine whether the given values of the variables are solutions of the equation.

$\dfrac{ax-bx}{bx-a}=\dfrac{bx+a}{-a-b}$ (a) $x=1$ (b) $x=-a/b$

Answer: (a) LHS $=\dfrac{a-b}{b-a}=-1$, RHS $=\dfrac{b+a}{-a-b}=-1=$ LHS. So $x=1$ is a solution.
(b) LHS $=\dfrac{(a-b)(-a/b)}{b(-a/b)-a}=\dfrac{-a^2/b+a}{-2a}=\dfrac{a(b-a)}{-2ab}\neq 0$, RHS $=\dfrac{b(-a/b)+a}{-a-b}=\dfrac{-a+a}{-a-b}=0$.
So $x=-a/b$ is not a solution.

4. Determine whether the given values of the variables are solutions of the equation.

$\left(x-\frac{1}{2}\right)+(2x-1)=2\left(x-\frac{1}{2}\right)-(1-x)+\frac{1}{2}$ (a) $x=\frac{5}{2}$ (b) $x=\frac{1}{2}$

Answer: (a) LHS $=\left(\frac{5}{2}-\frac{1}{2}\right)+\left(2\left(\frac{5}{2}\right)-1\right)=6$; RHS $=2\left(\frac{5}{2}-\frac{1}{2}\right)-\left(1-\frac{5}{2}\right)+\frac{1}{2}=6=$ LHS. Thus $x=\frac{5}{2}$ is a solution. (b) LHS $=\left(\frac{1}{2}-\frac{1}{2}\right)+\left(2\left(\frac{1}{2}\right)-1\right)=0$; RHS $=2\left(\frac{1}{2}-\frac{1}{2}\right)-\left(1-\frac{1}{2}\right)+\frac{1}{2}=0=$ LHS. Thus $x=\frac{1}{2}$ is a solution.

5. Solve the equation.

$2x-4=15$

(a) $x=\dfrac{15}{4}$ (b) $x=11$ (c) $x=\dfrac{35}{3}$ (d) $x=\dfrac{19}{2}$ (e) $x=13$

Answer: (d) $2x-4=15 \Leftrightarrow 2x=19 \Leftrightarrow x=\dfrac{19}{2}$

6. Solve the equation.

$4x-9=23$

(a) -8 (b) 9 (c) -6 (d) 6 (e) 8

Answer: (e)

7. Solve the equation.

$z=\frac{5z}{7}-7$

(a) $z=23$ (b) $z=\frac{47}{2}$ (c) $z=-\frac{49}{2}$ (d) $z=-\frac{70}{3}$ (e) $z=-\frac{47}{2}$

Answer: (c) $z=\frac{5z}{7}-7 \Leftrightarrow 7z=5z-49 \Leftrightarrow 2z=-49 \Leftrightarrow z=-\frac{49}{2}$

8. Solve the equation.

$2(x-1)-7=-2(x+2)-9$

(a) $x=4$ (b) $x=-11$ (c) $x=-\frac{67}{2}$ (d) $x=-3$ (e) $x=-1$

Answer: (e) $2(x-1)-7=-2(x+2)-9 \Leftrightarrow 2x-9=-2x-13 \Leftrightarrow 4x=-4 \Leftrightarrow x=-1$

9. Solve the equation.

$\frac{x-1}{4x+7}=\frac{1}{5}$

(a) $x=-7$ (b) $x=10$ (c) $x=\frac{19}{3}$ (d) $x=12$ (e) $x=-21$

Answer: (d)

10. Solve the equation.

$\frac{3}{2}\left(t-\frac{2}{3}\right)+7=\frac{t+1}{3}-1$

(a) $-\frac{40}{7}$ (b) 2 (c) $-\frac{36}{7}$ (d) $\frac{40}{7}$ (e) $\frac{7}{36}$

Answer: (a) $\frac{3}{2}\left(t-\frac{2}{3}\right)+7=\frac{t+1}{3}-1 \Rightarrow \frac{3}{2}t+6=\frac{1}{3}t-\frac{2}{3} \Rightarrow t=-\frac{40}{7}$

11. Solve the equation by completing the square.

$5x^2 - x = 0$

Answer: $x = 0,\ x = \frac{1}{5}$

12. Solve the equation by completing the square.

$x^2 - 16x = 17$

Answer: $x = -1,\ x = 17$

13. Solve the equation by completing the square.

$x^2 + 4x - \frac{9}{4} = 0$

Answer: $x = -\frac{9}{2},\ x = \frac{1}{2}$

14. Find all real solutions of the equation.

$x^2 - 6x - 12 = 0$

Answer: $x = 3 \pm \sqrt{21}$

15. Solve the equation.

$x^2 + 4x - 4 = 0$

(a) $x = -2 \pm 2\sqrt{2}$ (b) $x = 2 \pm \sqrt{2}$ (c) $x = 1 \pm \sqrt{3}$ (d) $x = -1 \pm \sqrt{2}$ (e) $x = \pm 2\sqrt{2}$

Answer: (a) $x^2 + 4x - 4 = 0 \Leftrightarrow x = -2 \pm 2\sqrt{2}$

16. Solve the equation.

$x^2 - 12x + 32 = 0$

(a) $x = 4$, $x = 3$
(b) $x = 6$, $x = 2$
(c) $x = -4$, $x = 8$
(d) $x = -4$, $x = 4$
(e) $x = 4$, $x = 8$

Answer: (e) $x^2 - 12x + 32 = 0 \Leftrightarrow (x-4)(x-8) = 0 \Leftrightarrow x = 4$ or $x = 8$

17. Solve the equation.

$3x^2 - 6x - 2 = 0$

(a) $x = -6$, $x = -4$
(b) $x = 4$, $x = -6$
(c) $x = 1 \pm \frac{\sqrt{15}}{3}$
(d) $x = 1 \pm \frac{\sqrt{5}}{3}$
(e) $x = -1 \pm \frac{3\sqrt{5}}{3}$

Answer: (c) $3x^2 - 6x - 2 = 0 \iff x = 1 \pm \frac{\sqrt{15}}{3}$

18. Solve the equation.

$10x^2 = 13x - 4$

(a) $x = 1 \pm 5\sqrt{2}$
(b) $x = \frac{5}{2}$ or 6
(c) $x = \frac{1}{2}$ or $\frac{4}{5}$
(d) $x = -\frac{11}{2}$ or $\frac{1}{2}$
(e) $x = \frac{5}{2}$ or 1

Answer: (c) $10x^2 = 13x - 4 \iff 10x^2 - 13x + 4 = 0 \iff (2x-1)(5x-4) = 0 \iff x = \frac{1}{2}$ or $x = \frac{4}{5}$

19. Find all real solutions of the quadratic equation.

$$z^2 - \frac{6}{5}z + \frac{9}{25} = 0$$

Answer: $z = 3/5$

20. Find all real solutions of the quadratic equation.

$$z^2 - \frac{8}{5}z + \frac{16}{25} = 0$$

Answer: $z = 4/5$

21. Solve the equation.

$$\frac{3x-2}{1-x} = \frac{14}{3}$$

(a) $x = \frac{-20}{23}$ (b) $x = \frac{20}{23}$ (c) $x = \frac{3}{4}$ (d) $x = \frac{4}{3}$ (e) $x = 23$

Answer: (b) $\frac{3x-2}{1-x} = \frac{14}{3} \iff 3(3x-2) = 14(1-x) \Rightarrow 9x - 6 = 14 - 14x \iff x = \frac{20}{23}$

22. Solve the equation.

$$(y-3)^2=(y+3)^2-6y-9$$

(a) 3 (b) 2 (c) $-\frac{3}{2}$ (d) $\frac{2}{3}$ (e) $\frac{3}{2}$

Answer: (e) $(y-3)^2=(y+3)^2-6y-9 \;\Rightarrow\; y^2-6y+9=y^2 \;\Rightarrow\; y=\frac{3}{2}$

23. Solve the equation.

$$\frac{1}{y}-\frac{1}{5y}+\frac{1}{y+1}=\frac{5}{2y}$$

(a) $\frac{10}{7}$ (b) 17 (c) 7 (d) $-\frac{7}{17}$ (e) $\frac{7}{17}$

Answer: (d) $\frac{1}{y}-\frac{1}{5y}+\frac{1}{y+1}=\frac{5}{2y} \;\Rightarrow\; \frac{1}{5}\frac{9y+4}{y(y+1)}=\frac{5}{2y} \;\Rightarrow\; 2y(9y+4)=y(y+1) \;\Rightarrow$
$17y^2=-7y \;\Rightarrow\; y=-\frac{7}{17}$.

24. Solve the equation.

$$\frac{x}{\frac{x}{2}+\frac{2(x+3)}{5}}=2$$

(a) –4 (b) –3 (c) 3 (d) 12 (e) 4

Answer: (b) $\frac{x}{\frac{9}{10}x+\frac{6}{5}}=2 \Rightarrow x=2\left(\frac{9}{10}x+\frac{6}{5}\right) \Rightarrow -\frac{4}{5}x=\frac{12}{5} \Rightarrow x=-3$

25. Solve the equation.

$$\frac{4}{x-7}+\frac{1}{x+7}+\frac{3}{2(x^2-49)}=0$$

(a) $\frac{9}{2}$ (b) $-\frac{2}{9}$ (c) –4 (d) –5 (e) $-\frac{9}{2}$

Answer: (e) $2(x^2-49)\left[\frac{4}{x-7}+\frac{1}{x+7}+\frac{3}{2(x^2-49)}\right]=(0)2(x^2-49) \Leftrightarrow 8(x+7)+2(x-7)+3=0$
$\Leftrightarrow\; 10x+45=0 \;\Leftrightarrow\; x=-\frac{9}{2}$

26. Solve the equation.

$$\frac{x}{4x^2+x}+\frac{2}{x-1}=\frac{3}{4x+1}$$

(a) -1 (b) $-\frac{3}{2}$ (c) 1 (d) $-\frac{2}{3}$ (e) $\frac{2}{3}$

Answer: (d) $x(9x+1)=3x(x-1) \Leftrightarrow 9x^2+x=3x^2-3x \Leftrightarrow x=-\frac{2}{3}$

27. Find all real solutions of the equation.

$$7(x-1)^3-2401=0$$

(a) No real solution (b) -8 (c) 8 (d) ± 8 (e) ± 7

Answer: (c) $7(x-1)^3-2401=0 \Leftrightarrow (x-1)^3=343 \Leftrightarrow x-1=7 \Leftrightarrow x=8$

28. Find all real solutions of the equation.

$$8x^{2/3}-250=6+4x^{2/3}$$

(a) 64 (b) ± 64 (c) 256 (d) ± 512 (e) No real solution

Answer: (d) $8x^{2/3}-250=6+4x^{2/3} \Leftrightarrow 4x^{2/3}=256 \Leftrightarrow x=\pm\sqrt{(64)^3}=\pm 512$

29. Find all real solutions of the equation.

$$|5x|=9$$

(a) $x=\frac{9}{5}$, $x=-\frac{9}{5}$ (b) $x=\frac{9}{5}$ (c) $x=-\frac{4}{5}$, $x=\frac{4}{5}$ (d) $x=5$, $x=9$ (e) no solution

Answer: (a) $|5x|=9 \Leftrightarrow x=\frac{9}{5}$ or $x=-\frac{9}{5}$

30. Find all real solutions of the equation.

$$|7x+1|=7$$

(a) $x=-8$, $x=8$
(b) $x=-\frac{8}{7}$, $x=\frac{6}{7}$
(c) $x=-\frac{8}{7}$, $x=-1$
(d) $x=-\frac{7}{8}$, $x=\frac{7}{6}$
(e) no solution

Answer: (b) $|7x+1|=7 \Leftrightarrow x=\frac{6}{7}$ or $x=-\frac{8}{7}$

31. Find all real solutions of the equation.

$|x-1|=-1$

(a) $x=-1, x=0$ (b) $x=0$ (c) $x=0, x=2$ (d) $x=-1, x=1$ (e) no solution

Answer: (b) $|x-1|=-1$ has no solution since the absolute value is always non-negative.

32. Solve the equation $P=a+art$ for t.

Answer: $P=a+art \Leftrightarrow P-a=art \Leftrightarrow t=\dfrac{P-a}{ar}$

33. Solve the equation $S=\dfrac{a}{1-r}$ for r.

Answer: $S=\dfrac{a}{1-r} \Rightarrow S-Sr=a \Rightarrow r=\dfrac{S-a}{S}$

34. Solve the equation $A=2lw+2lh+2w$ for h.

Answer: $A=2lw+2lh+2w \Leftrightarrow A-2lw-2w=2lh \Leftrightarrow h=\dfrac{A-2lw-2w}{2l}$

35. Solve the equation $A=2\pi rh+2\pi r^2$ for h.

Answer: $A=2\pi rh+2\pi r^2 \Leftrightarrow A-2\pi r^2=2\pi rh \Leftrightarrow h=\dfrac{A-2\pi r^2}{2\pi r}$

36. Solve the equation $t(x-a)+x(2t-1)-t=t(x-a)$ for t.

Answer: $t=\dfrac{x}{2x-1}$

37. Solve the equation for the indicated variable.

$F=k\left(\dfrac{q_1q_2}{r^2}\right)$; for r

Answer: $r=\pm\dfrac{1}{F}\sqrt{(Fkq_1q_2)}$

38. Solve the equation for the indicated variable.

$A=2y^2+4yh$; for y

Answer: $y=-h\pm\dfrac{1}{2}\sqrt{4h^2+2A}$

39. Solve the equation $\frac{1}{s+a}+\frac{2}{s-a}=\frac{5}{s+c}$ for c.

Answer $c=\frac{2s^2-as-5a^2}{3s+a}$

40. Solve the equation for x.

$b^2x^2-4bx+3=0 \quad (b\neq 0)$

Answer: $x=\frac{1}{b},\ x=\frac{3}{b}$

41. Solve the equation for x.

$ax^2-(2a-1)x+(a-1)=0 \quad (a\neq 0)$

Answer: $x=1,\ x=\frac{a-1}{a}$

42. The height h of a ball after t seconds dropped above ground is given by the formula $h=-16t^2+h_0$, where h_0 is the initial height of the ball. Suppose a ball is dropped form the top of a 256 - foot building.

(a) After 2 seconds what is the height of the ball?
(b) Find the time it takes for the ball to fall half the distance to ground level.
(c) Find the time it takes for the ball to reach the ground.

Answer: (a) Since the initial height is $h_0=256$ and $t=2$, the height of the ball is $h=-16(2)^2+256=192$ feet. (b) Half the distance is 128 feet, so we have $128=-16(t)^2+256$ or $t\approx 2.8$ seconds. (c) The ball will hit the ground when $h=0$ or $0=-16(t)^2+256$, so $t=4$ seconds.

43. A large pond is stocked with fish. The fish population P is modeled by the formula $P=3t+10\sqrt{t}+140$, where t is the number of days since the fish were first introduced into the pond. How many days will it take for the fish population to reach 400?

Answer: $t=\frac{830}{9}-\frac{10}{9}\sqrt{805}\approx 61$ days

44. A large pond is stocked with fish. The fish population P is modeled by the formula $P=3t+10\sqrt{t}+140$, where t is the number of days since the fish were first introduced into the pond. How many days will it take for the fish population to reach 300?

Answer: $t=\frac{530}{9}-\frac{10}{9}\sqrt{505}\approx 34$ days

1. Express the interest obtained after a year on an investment of x dollars at 12% simple interest per year.

Answer: $0.12x$

2. Find the sum of five consecutive even integers, if n is the first integer of the five.

(a) $2n+4$ (b) $3n+4$ (c) $5n+20$ (d) $4n+16$ (e) $6n-12$

Answer: (c) $n+(n+2)+(n+4)+(n+6)+(n+8)=5n+20$, n is the first even integer.

3. Find the area of a rectangle whose base is twice its height if its base is b.

(a) $\frac{b^2}{4}$ (b) $\frac{b^2}{8}$ (c) $\frac{b^4}{16}$ (d) $\frac{b^2}{2}$ (e) b^2

Answer: (d) $b\cdot\frac{1}{2}b=\frac{1}{2}b^2$

4. Find the distance (in miles) traveled by a car which moves at speed s (in mi/h) for two hours, then moves 10 mi / h slower for another two hours.

(a) $\frac{14}{3}s-\frac{15}{2}$ (b) $4s-20$ (c) $13s-15$ (d) $\frac{17}{2}s-\frac{227}{2}$ (e) $12s-1$

Answer: (b) $2s+2(s-10)=4s-20$; s is the initial speed, in mi / hr.

5. Express the value (in cents) of the change in a purse that contains half as many dimes as pennies, four more nickels than dimes, and as many quarters as dimes and pennies combined, if the number of pennies in the purse is p.

(a) $46p+20$ (b) $27p-13$ (c) $100p-81$ (d) $32p-14$ (e) $107p-27$

Answer: (a) $1(p)+10(p/2)+5(p/2+4)+25(p+p/2)=46p+20$, p is the number of pennies.

6. Caitlin's science professor gives an A for an average of 85 or better, and a B for an average of 75 or better in her course. Caitlin has obtained scores of $84, 71$, and 77 on her midterm examinations. If the final exam counts for as much as the three midterms together, what score must she get on her final exam to receive a B? To receive an A? (Assume that the maximum possible score on each test is 100.)

Answer: Let x be Caitlin's final exam score. Since the final counts for the same as the three midterms, her average score will be $\frac{84+71+77+3x}{6}$. For her to get a B, this average must be at least 75, so $\frac{84+71+77+3x}{6}=75 \Rightarrow x\approx 73$. Thus she must get at least 73 on the final for a B. To get an A, she must average at least 85. So $\frac{84+72+74+3x}{6}=85 \Rightarrow x\approx 93$. Thus she would have to get at least 93 on the final exam to get an A.

7. Find three consecutive integers whose sum is 57.

Answer: Let n be the lowest of the three. Then $n+(n+1)+(n+2)=57 \Leftrightarrow 3n+3=57 \Leftrightarrow n=18$.

8. The difference of the squares of two consecutive integers is 1307. Find the integers.

Answer: Let the first integer be x, so the next consecutive integer is $(x+1)$. We know $(x+1)^2-(x)^2=1307 \Leftrightarrow x^2-x^2+2x+1=1307 \Leftrightarrow 2x+1=1307 \Leftrightarrow x=653$. Thus the two integers are 653 and 654.

9. If Arnold invests $\$6500$ at 4% per year, how much additional money must he invest at 10% to ensure that the interest he receives each year is 8% of the total invested?

(a) $\$13,000$ (b) $\$12,375$ (c) $\$18,000$ (d) $\$21,000$ (e) $\$22,500$

Answer: (a) Let x be the additional amount of money that he invests at 10%. Hence the total amount invested is $x+6500$. Thus $0.1x+0.04(6500)=0.08(x+6500) \Leftrightarrow 0.02x=260 \Leftrightarrow x=13,000$.

10. What annual rate of interest would you have to earn on an investment of $\$1500$ to ensure receiving $\$187.50$ interest after one year?

Answer: Let x be the interest, then $\frac{x}{100}\cdot 1500=187.5 \Leftrightarrow x=\frac{187.5}{15} \Leftrightarrow x=12.5$. So you must earn 12.5%.

11. There are 63 coins in Taylor's pockets, all quarters and dimes. The total value of his change is $\$9.60$. Find the number of each type of coin.

Answer: If x is the number of quarters, then there are $(63-x)$ dimes. The total value is $0.25x+0.10(63-x)=9.60 \Leftrightarrow$ Thus, there are $x=22$ quarters and $(63-22)=41$ dimes.

12. A money manager invests $\$1250$ at $4\frac{1}{2}\%$ simple interest and $\$1850$ at $5\frac{1}{2}\%$ simple interest. How much money must the manager invest at $6\frac{1}{2}\%$ in order to get a total yield of 6% of the total investment for his clients?

Answer: Let x be the amount he needs to invest at $6\frac{1}{2}\%$, and the total investment is $1250+1850+x$. So, $0.065(x)+0.045(1250)+0.055(1850)=0.06(1250+1850+x) \Leftrightarrow$ $158.0+0.065x=186.0+0.06x \Leftrightarrow x=\$5600.$ The manager must invest $\$5600$ at $6\frac{1}{2}\%$ in order to get a total yield of 6%.

13. A rectangular garden is surrounded by 300 ft of chain-link fence. Find the length and width of the garden if the area is $3600\ \text{ft}^2$.

Answer: Perimeter $=2l+2w=300 \Rightarrow l=150-w$, area $=lw=3600 \Rightarrow (150-w)w=3600 \Rightarrow$ width $=30$, length $=120$.

14. A cylindrical can has a volume of $36\pi\ \text{cm}^3$ and a diameter of 3 cm. What is its height?

Answer: Let h be the height of the cylindrical can. The volume is $V = \pi r^2 h \iff 36\pi = \pi\left(\frac{3}{2}\right)^2 h \Rightarrow h = 16$ cm.

15. A box with a square base and no top is to be made from a square piece of cardboard by cutting 4.5 inch squares from each corner and folding up the sides. The box is to hold $450\ \text{in}^3$. How big a piece of cardboard is needed?

Answer: $V = lwh = 450 = 4.5(x-9)(x-9) \Rightarrow 4.5x^2 - 81.0x + 364.5 - 450 = 0 \Rightarrow x = -1,\ x = 19$. So the piece of cardboard must be 19 in $\times 19$ in.

16. The owner of a tow truck wishes to change the mixture of antifreeze in his radiator. The radiator contains 15.5 quarts of 35% antifreeze solution. How much would he need to drain and replace with pure antifreeze in order to get a 55% antifreeze solution?

Answer: Let $15.5 - x$ be the amount of solution to drain. Then $0.35(15.5-x) + x = 0.55(15.5) \iff 5.425 + 0.65x = 8.525 \iff x \approx 4.7$. The driver needs to drain 4.7 quarts from the current system, then replace the same amount with pure antifreeze in order to achieve a 55% antifreeze solution.

17. Carolyn and Brian work in an accounting office. If they work together they can complete a corporate tax return in 2.5 hours. Working alone it would take Brian 35 minutes longer than Carolyn to complete the return. How long does it take each person working alone to finish the job? If a third person is used to help complete the task, and works with the same efficiency as Brian, what is the total time it would take to complete the return if all three work together?

Answer: Let t be the time it takes Carolyn to work. Then Brian's time would be $t + 35$ min. Thus

$$\frac{1}{t} + \frac{1}{t+35} = \frac{1}{150} \iff 150(2t+35) = t^2 + 36t \iff t^2 - 265t - 5250 = 0 \iff t = \pm\frac{\sqrt{(-265)^2 - 4(1)(-5250)}}{2}$$

≈ 283.5 minutes or about 4.7 hours. Brian takes 318.5 minutes or 5.3 hours.

With the third person helping, the total time is $\frac{1}{283.5} + \frac{2}{318.5} = \frac{1}{T} \iff T \approx 101.1$ min or 1.7 hours.

18. A certain board game requires a two-toned wood playing surface. The central portion is made of a rectangular piece of pine, measuring 4 in by 12 in. The outer border is made of redwood, and has equal width on all sides. The area of the redwood border must be one third the area of the central pine portion. What must the width of the redwood border be?

Answer: Let the width of the border be x in. The area of the border is $\frac{1}{3}(4)(12) = 16\ \text{in}^2$. The area of the pine surface is $48\ \text{in}^2$. Hence, the board has a total area of $64\ \text{in}^2$. Then

$$(4+2x)(12+2x) = 64 \iff (2+x)(6+x) = 16 \iff x^2 + 8x - 4 = 0 \iff x = \frac{-8 \pm \sqrt{64 + 4\cdot 4}}{2}.$$

But $x > 0$ so $x = -4 + \sqrt{20} \approx 0.47$ in.

19. A running track has straight sides and semicircular ends. If the length of the track is 440 yd and the two straight parts are each 110 yd long, what is the radius of the semicircular part, to the nearest yard?

Answer: Let the radius of the semicircular part be x yd. Thus $2\pi x + 2(110) = 440 \iff$ $x = \dfrac{440 - 220}{2\pi}$. So $x \approx 35$ and the radius of the semicircular part is about 35 yd.

20. A circle has a radius of 4 cm. By how much should the radius be reduced so that the area is reduced by a third?

Answer: The area of the original circle is $\pi(4)^2 = 16\pi$. If its radius is reduced by x cm, its area becomes $\pi(4-x)^2 = 16\pi - 8\pi x + \pi x^2$. We have $16\pi - 8\pi x + \pi x^2 = \frac{2}{3}16\pi \iff \frac{1}{3}16\pi - 8\pi x + \pi x^2 = 0$ $\iff x = \dfrac{8\pi \pm \sqrt{64\pi^2 - 64\pi^2/3}}{2\pi} \iff x = \dfrac{8\pi \pm 8\pi\sqrt{2/3}}{2\pi} \iff x = 4 \pm 4\sqrt{2/3}.$ But $x < 4$, thus the radius must be reduced by $\left(4 - 4\sqrt{2/3}\right)$ cm.

21. A cylindrical tin can has a volume of 972π cm^3 and a diameter of 18 cm. What is its height?

Answer: Let x be the height, in cm, of the cylindrical tin, then the volume V is $\pi r^2 x$.

Thus $972\pi = \pi\left(\dfrac{18}{2}\right)^2(x) \iff x = 12$, so the height is 12 cm.

22. Peter drove to the beach at 40 mi / h. Rock left the house at the same time but drove at 69 mi / h. When Rock arrived at the beach, Peter had another 30 mi to drive. Approximately how far is it from their house to the beach?

Answer: Let x be the distance, in miles, from their house to the beach.

Thus $\dfrac{x}{69} = \dfrac{x-30}{40} \iff 40x = 69x - 2070 \iff x = \dfrac{2070}{29}$ so $x \approx 71.4$ mi.

23. Alyson drove from Bluesville to Greensburg at a speed of 60 mi/h. On the way back, she drove at 45 mi/h. The total trip took $5\frac{3}{5}$ h of driving time. Find the distance between these two cities.

Answer: 144 mi

24. Caitlin drove from Greensville to Bluesburg at a speed of 50 mi/h. On the way back, she drove at 75 mi/h. The total trip took $7\frac{1}{2}$ h of driving time. Find the distance between these two cities.

Answer: 225 mi

25. A running track has the shape of a rectangle with a semicircle on both ends. If the length of the track is 550 yards and the length of the side of the rectangle is 125 yards, what is the radius of the of the semicircular parts?

Answer: If you form a circle from the two semicircles, its circumference would be $P_C = 2\pi r$. The perimeter of the track is therefore $P = 2l + P_C$ or $P = 2l + 2\pi r$. So $P = 2l + 2\pi r \iff$
$550 = 2(125) + 2\pi r \iff r = \dfrac{150}{\pi}$ yd.

26. A wire 45 in long is cut into two pieces. One piece is bent into the shape of a circle and the other a square. If the two figures have the same area, what are the lengths of the two pieces of wire (to the nearest tenth of an inch)?

Answer: The circumference of a circle is $2\pi r$, and the perimeter of a square is $4s$. Thus $2\pi r + 4s = 45$. Also the areas are equal, so $\pi r^2 = s^2 \Rightarrow s = \sqrt{\pi} r \Rightarrow 2\pi r + 4\left(\sqrt{\pi} r\right) = 45 \Rightarrow$ $r \approx 3.37.$ Therefore one piece is $2\pi(3.37) \approx 21.2$ in. long (the circle), and the other is 23.8 inches long (the square).

1. Let $S=\left\{-1,0,\frac{1}{2},\sqrt{2},2\right\}$. Use substitution to determine which of the elements of S satisfy the inequality $x-1\le 0$.

(a) $\{-1,0\}$ (b) $\left\{-1,0,\frac{1}{2}\right\}$ (c) $\left\{0,\frac{1}{2},\sqrt{2}\right\}$ (d) $\left\{\frac{1}{2},\sqrt{2},2\right\}$ (e) $\left\{\sqrt{2},2\right\}$

Answer: (b) $\left\{-1,0,\frac{1}{2}\right\}$ satisfy $x-1\le 0$.

2. Let $S=\left\{-1,0,\frac{1}{2},\sqrt{2},2\right\}$. Use substitution to determine which of the elements of S satisfy the inequality $2x-1\ge x$.

(a) $\{-1\}$ (b) $\left\{-1,0,\frac{1}{2}\right\}$ (c) $\left\{0,\frac{1}{2},\sqrt{2}\right\}$ (d) $\left\{0,\frac{1}{2},\sqrt{2},2\right\}$ (e) $\left\{\sqrt{2},2\right\}$

Answer: (e) $\left\{\sqrt{2},2\right\}$ satisfy $2x-1\ge x$.

3. Let $S=\left\{-2,-\sqrt{3},1,\frac{5}{3},2\right\}$. Use substitution to determine which of the elements of S satisfy the inequality $10\le\dfrac{5}{2x}$.

(a) $\varnothing$ (b) $\{-2\}$ (c) $\left\{-2,-\sqrt{3}\right\}$ (d) $\left\{1,\frac{5}{3},2\right\}$ (e) S

Answer: (a) No element of S satisfies $10\le\dfrac{5}{2x}$.

4. Find the solution set for the linear inequality $11-5x<-2$.

(a) $\left(\frac{18}{5},\infty\right)$ (b) $\left(\frac{13}{5},\infty\right)$ (c) $\left(-\infty,-\frac{5}{13}\right)$ (d) $\left(-\infty,\frac{13}{5}\right)$ (e) No solution

Answer: (b) $11-5x<-2 \Leftrightarrow -5x<-13 \Leftrightarrow x>\frac{13}{5} \Leftrightarrow x\in\left(\frac{13}{5},\infty\right)$

5. Find the solution set for the linear inequality $\frac{22}{3}x-5>\frac{2}{3}x$.

(a) $\left[-\frac{4}{3},\infty\right)$ (b) $\left(-\frac{3}{4},\frac{3}{4}\right]$ (c) $\left(\frac{3}{4},\infty\right)$ (d) $(-\infty,5)$ (e) $\frac{3}{4}$

Answer: (c) $\frac{22}{3}x-5>\frac{2}{3}x \Leftrightarrow \frac{20}{3}x>5 \Leftrightarrow x>\frac{3}{4} \Leftrightarrow x\in\left(\frac{3}{4},\infty\right)$

6. Find the solution set for the linear inequality $4\left(2-\frac{1}{2}x\right)\le 6-\frac{3}{4}x$.

(a) $\left[\frac{4}{5},\infty\right)$ (b) $\left[-\frac{5}{8},\infty\right)$ (c) $\left(-\infty,-\frac{8}{5}\right)$ (d) $\left(-\frac{8}{5},\infty\right)$ (e) $\left[\frac{8}{5},\infty\right)$

Answer: (e) $4\left(2-\frac{1}{2}x\right)\le 6-\frac{3}{4}x \Leftrightarrow 8-2x\le 6-\frac{3}{4}x \Leftrightarrow x\ge\frac{8}{5} \Leftrightarrow x\in\left[\frac{8}{5},\infty\right)$

7. Find the solution set for the linear inequality $3 \le x-4 < 14$.

(a) $[7,18)$ (b) $[7,18]$ (c) $[-7,18)$ (d) $\left(-\frac{18}{7},\infty\right)$ (e) $\left[-\frac{18}{7},\frac{18}{7}\right)$

Answer: (a) $3 \le x-4 < 14 \iff 7 \le x < 18 \iff x \in [7,18)$

8. Find the solution set for the linear inequality $\frac{2}{3}x-2 \le 2x-3 \le 6+\frac{2}{3}x$.

(a) $\frac{3}{4}$ (b) $\left[-\frac{27}{4},\frac{3}{4}\right]$ (c) $\left(\frac{4}{3},\frac{27}{4}\right]$ (d) $\left[\frac{3}{4},\frac{27}{4}\right]$ (e) $\left[\frac{-3}{4},\frac{9}{4}\right]$

Answer: (d) $\frac{2}{3}x-2 \le 2x-3 \le 6+\frac{2}{3}x \iff 1 \le 2x-\frac{2}{3}x \le 9 \iff 1 \le \frac{4}{3}x \le 9 \iff \frac{3}{4} \le x \le \frac{27}{4} \Rightarrow x \in \left[\frac{3}{4},\frac{27}{4}\right]$

9. Find the solution set for the linear inequality $-\frac{3}{10} < \frac{2x-3}{5} \le \frac{6}{5}$.

(a) $\left(-\frac{9}{2},\frac{3}{4}\right)$ (b) $\left[\frac{3}{2},\frac{9}{4}\right]$ (c) $\left(\frac{3}{2},\frac{9}{4}\right]$ (d) $\left(\frac{-3}{2},9\right]$ (e) $\left(\frac{3}{4},\frac{9}{2}\right]$

Answer: (e) $-\frac{3}{10} < \frac{2x-3}{5} \le \frac{6}{5} \iff -\frac{15}{10} < 2x-3 \le 6 \iff \frac{3}{2} < 2x \le 9 \iff \frac{3}{4} < x \le \frac{9}{2} \Rightarrow x \in \left(\frac{3}{4},\frac{9}{2}\right]$

10. Find the solution set for the nonlinear inequality $x^2+x-20>0$.

(a) $(-\infty,4)\cup(5,\infty)$
(b) $(-\infty,-5)\cup(4,\infty)$
(c) $(-\infty,-1)\cup(5,\infty)$
(d) $(-\infty,-5)\cup(-4,\infty)$
(e) $(-5,-4)$

Answer: (b) $x^2+x-20>0 \Rightarrow x \in (-\infty,-5)\cup(4,\infty)$

11. Find the solution set for the nonlinear inequality $2x^2+5x>-3$.

(a) $(-\infty,\infty)$
(b) $\left(-\infty,\frac{3}{2}\right)\cup(1,\infty)$
(c) $\left(-\infty,-\frac{3}{2}\right)\cup(-1,\infty)$
(d) $\left(-\infty,-\frac{3}{2}\right)$
(e) $(-1,\infty)$

Answer: (c) $2x^2+5x>-3 \Rightarrow x \in \left(-\infty,-\frac{3}{2}\right)\cup(-1,\infty)$

12. Find the solution set for the nonlinear inequality $6x^2 < -11x-3$.

(a) $(-1,3)$
(b) $\left(-\infty,\frac{1}{3}\right)\cup\left(\frac{3}{2},\infty\right)$
(c) $\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{1}{3},\infty\right)$
(d) $\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{3}{2},\infty\right)$
(e) $\left(-\frac{3}{2},-\frac{1}{3}\right)$

Answer: (e) $6x^2 < -11x-3 \Rightarrow \left(-\frac{3}{2},-\frac{1}{3}\right)$

13. Find the solution set for the nonlinear inequality $4x^2 \le 256$.

(a) $[-8,8]$
(b) $[-16,16]$
(c) $[-4,4]$
(d) $(-\infty,-4]\cup[4,\infty)$
(e) $\left[-\frac{1}{8},\frac{1}{8}\right]$

Answer: (a) $4x^2 < 256 \Rightarrow x\in[-8,8]$

14. Find the solution set for the nonlinear inequality $\dfrac{x-2}{x+1}\ge 0$.

(a) $[-1,2]$ (b) $(0,2)$ (c) $(-\infty,-2)\cup[1,\infty)$ (d) $(-\infty,-1)\cup[2,\infty)$ (e) $(-\infty,-1)$

Answer: (d) $\dfrac{x-2}{x+1}\ge 0 \Rightarrow x\in(-\infty,-1)\cup[2,\infty)$

15. Find the solution set for the nonlinear inequality $\dfrac{2x+1}{x-4}\le 5$.

(a) $(-\infty,-4)\cup(7,\infty)$
(b) $(-\infty,-7)\cup[4,\infty)$
(c) $(-\infty,4)\cup[7,\infty)$
(d) $(-\infty,4]$
(e) $(-\infty,7)$

Answer: (c) $\dfrac{2x+1}{x-4}\le 5 \Rightarrow x\in(-\infty,4)\cup[7,\infty)$

16. Find the solution set for the nonlinear inequality $\dfrac{1}{x-3}-\dfrac{2}{x-2}>0$.

(a) $(-\infty,-3)\cup(2,4)$ (b) $(-\infty,2]\cup[3,4]$ (c) $(2,3)$ (d) $(-\infty,2)$ (e) $(-\infty,2)\cup(3,4)$

Answer: (e) $\frac{1}{x-3}-\frac{2}{x-2}>0 \quad\Rightarrow\quad x\in(-\infty,2)\cup(3,4)$

17. Find the solution set for the nonlinear inequality $x^4 < x^2$.

(a) $(-\infty,-1)\cup(0,1)$
(b) $(-1,0)\cup(0,1)$
(c) $(-\infty,-1)\cup(1,\infty)$
(d) $(-1,1)$
(e) $(-\infty,0)\cup(0,1)$

Answer: (b) $x^4 < x^2 \iff x\in(-1,0)\cup(0,1)$

18. Solve the absolute value inequality. Express the answer using interval notation.

$|8x+5|>15$

Answer: $(-\infty,-5/2)\cup(5/4,\infty)$

19. Solve the absolute value inequality. Express the answer using interval notation.

$|8x+7|>14$

Answer: $(-\infty,-21/8)\cup(7/8,\infty)$

20. Solve the inequality $\left|t+\frac{1}{2}\right|\geq 2$.

(a) $t\in(-\infty,-2]\cup[3,\infty)$
(b) $t\in\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{3}{2},\infty\right)$
(c) $t\in\left(-\infty,-\frac{3}{2}\right]\cup\left[\frac{3}{2},\infty\right)$
(d) $t\in\left(-\infty,-\frac{5}{2}\right]\cap\left[\frac{3}{2},\infty\right)$
(e) $t\in\left(-\infty,-\frac{5}{2}\right]\cup\left[\frac{3}{2},\infty\right)$

Answer: (e) $\left|t+\frac{1}{2}\right|\geq 2 \iff t+\frac{1}{2}\geq 2$ or $t+\frac{1}{2}\leq -2 \iff t\geq\frac{3}{2}$ or $t\leq-\frac{5}{2}$

21. Solve the inequality $\left|\frac{2x-0.1}{0.01}\right|\leq 2$.

(a) $x \in [0.4, 0.6] \cup [3, \infty)$
(b) $x \in (-\infty, -0.06] \cup (0.04, \infty)$
(c) $x \in [0.04, 0.06]$
(d) $x \in [0.4, 0.6]$
(e) $x \in (-\infty - 0.02) \cup [0.04, 0.06]$

Answer: (c) $\left|\dfrac{2x-0.1}{0.01}\right| \le 2 \Leftrightarrow -2 \le \dfrac{2x-0.1}{0.01} \le 2 \Leftrightarrow 0.04 \le x \le 0.06 \Leftrightarrow x \in [0.04, 0.06]$

22. Solve the inequality $-2\left|7x - \frac{1}{2}\right| + 4 \le -17$.

(a) $x \in \left(-\infty, -\frac{10}{7}\right] \cup \left[\frac{11}{7}, \infty\right)$
(b) $x \in \left(-\infty, \frac{10}{7}\right) \cup \left[\frac{11}{7}, \infty\right)$
(c) $x \in \left(-\infty, -\frac{10}{7}\right]$
(d) $x \in \left(\frac{11}{7}, \infty\right)$
(e) $x \in \left[-\frac{10}{7}, \frac{11}{7}\right]$

Answer: (a) $\left|7x - \frac{1}{2}\right| \ge \frac{21}{2} \Leftrightarrow 7x - \frac{1}{2} \ge \frac{21}{2}$ or $7x - \frac{1}{2} \le -\frac{21}{2}$; so, $x \in \left(-\infty, -\frac{10}{7}\right] \cup \left[\frac{11}{7}, \infty\right)$

23. A phone company offers two monthly phone packages. The company charges \$19.99 per month and 10 cents per minute for Plan A and \$39.99 per month and 5 cents per minute for plan B. How many minutes per month would you need to use in order that plan B be more economical than plan A?

Answer: $39.99 + 0.05x < 19.99 + 0.10x \Leftrightarrow x > 400$. If you use more than 400 minutes per month, Plan B is more economical.

24. A prescription label for a certain type of medicine indicates that the medicine needs to be stored in the refrigerator at a temperature between 5° C and 10° C. What range of temperatures does this correspond to on the Fahrenheit scale?

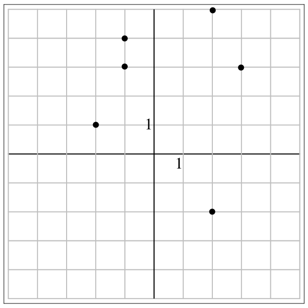
Answer: $5 < C < 10 \Rightarrow 5 < \frac{5}{9}(F - 32) < 10 \Rightarrow \frac{9}{5}(5) < F - 32 < \frac{9}{5}(10) \Rightarrow 41 < F < 50$

25. Suppose that Alyson scores 67, 87, 93, 95 on her algebra tests, and the only test left to take is the final exam. Her final exam however, accounts for one-half of her overall course average. What range of scores on the final exam would she need in order to achieve an overall course grade of between 80 and 85?

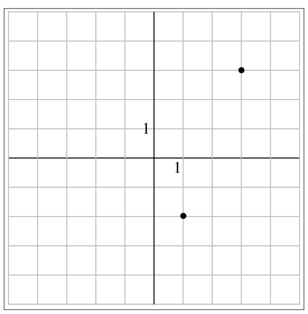
Answer: Since there are four exams, the final exam counts four times. Thus
$80 < \dfrac{67+87+93+95+4x}{8} < 85 \Leftrightarrow 80 < \dfrac{342+4x}{8} < 85 \Leftrightarrow 640 < 342 + 4x < 680 \Rightarrow$
$289 < 4x < 338 \Leftrightarrow 72.25 < x < 84.5$. She needs to achieve between 72.25 and 84.5 on her final exam in order to have an overall course grade of between 80 and 85.

1. Plot the following points in a coordinate plane: $(2,5)$, $(-1,3)$, $(3,3)$, $(2,-2)$, $(-1,4)$, $(-2,1)$

Answer:



2. Find the coordinates of the points shown in the figure.



(a) $(1,2)$, $(3,3)$
(b) $(1,-2)$, $(3,3)$
(c) $(-1,2)$, $(3,-3)$
(d) $(1,-2)$, $(3,-3)$
(e) $(1,-2)$, $(-3,3)$

Answer: (b)

3. For the points $(3,1)$ and $(2,5)$:

(a) Find the distance between them.
(b) Find the midpoint of the line segment that joins them.

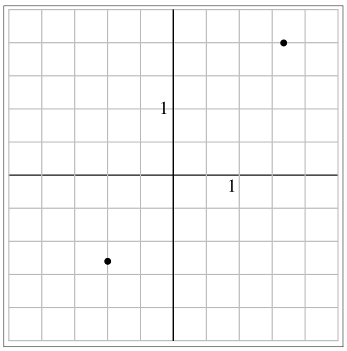
Answer:(a) $P_1 = (3,1)$ and $P_2 = (2,5)$ so $|P_1P_2| = \sqrt{(3-2)^2 + (1-5)^2} = \sqrt{17}$

(b) The midpoint is $\left(\frac{3+2}{2}, \frac{1+5}{2}\right) = (5/2,\ 3)$.

4. For the points $\left(-1,-\frac{4}{3}\right)$ and $\left(\frac{5}{3},2\right)$:

(a) Plot the points on a coordinate plane.
(b) Find the distance between them.
(c) Find the midpoint of the line segment that joins them.

Answer:(a)

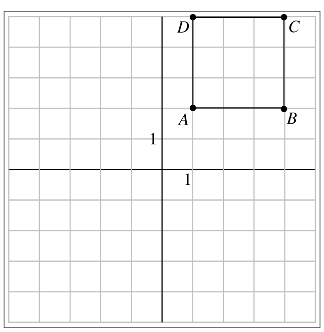


(b) $P_1=\left(-1,-\frac{4}{3}\right)$ and $P_2=\left(\frac{5}{3},2\right)$ so $\left|P_1P_2\right|=\sqrt{\left(-1-\frac{5}{3}\right)^2+\left(-\frac{4}{3}-2\right)^2}=\sqrt{\frac{164}{9}}=\frac{2}{3}\sqrt{41}$

(c) The midpoint is $\left(\frac{-1+\frac{5}{3}}{2},\frac{-\frac{4}{3}+2}{2}\right)=\left(\frac{1}{3},\frac{1}{3}\right)$

5. Draw the square with vertices $A(1,2)$, $B(4,2)$, $C(4,5)$, and $D(1,5)$ on a coordinate plane. Find the area of the square.

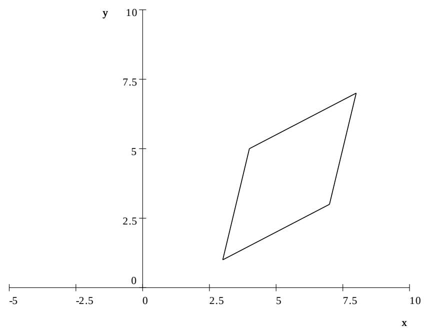
Answer:



The area of the square is $3^2=9$.

6. Plot the points $P(3,1)$, $Q(4,5)$, and $R(8,7)$ on a coordinate plane. Where should the point S be located so that $PQRS$ is a parallelogram?

Answer: The midpoint of PR is $\left(\frac{3+8}{2},\frac{1+7}{2}\right)=\left(\frac{11}{2},4\right)$. The midpoint of QS is $\left(\frac{4+x}{2},\frac{5+y}{2}\right)$.
The two diagonals must have the same midpoint, therefore $\left(\frac{4+x}{2},\frac{5+y}{2}\right)=\left(\frac{11}{2},4\right) \Rightarrow \frac{4+x}{2}=\frac{11}{2}$
$\Rightarrow x=7$, $\frac{5+y}{2}=4 \Rightarrow y=3$. Thus the point is $S(7,3)$.



7. Which of the points $C(-3,2)$ or $D(5,1)$ is closer to the point $E(-2,4)$?

Answer: $C(-3,2)$, $D(5,1)$ and $E(-2,4)$. $|CE|=\sqrt{(-3+2)^2+(2-4)^2}=\sqrt{5^2+2^2}=\sqrt{29}$.
$|DE|=\sqrt{(5+2)^2+(1-4)^2}=\sqrt{7^2+3^2}=\sqrt{58}$. Because $|CE|<|DE|$, C is closer to E.

8. Find the area of the right triangle with base AB, where the vertices are $A=(-3,0)$, $B=(2,0)$ and $C=(2,4)$.

Answer: Area $=\frac{1}{2}|AB||BC|=\frac{1}{2}\sqrt{(-3-2)^2+(0-0)^2}\sqrt{(2-2)^2+(0-4)^2}=\frac{1}{2}\cdot 5\cdot 4=10$

9. Find y such that the distance between the points $P(-2,5)$ and $Q(-3,y)$ is 4 units.

Answer: $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \Rightarrow 4=\sqrt{(-3-(-2))^2+((y)-5)^2} \Rightarrow 16=1+((y)-5)^2 \Rightarrow$
$16=1+((y)-5)^2 \Rightarrow y-5=\pm\sqrt{15} \Rightarrow y=5\pm\sqrt{15}$

10. If $M(2,1)$ is the midpoint of the line segment AB, and if A has coordinates $\left(-\frac{1}{2},6\right)$, find the coordinates of B.

Answer: $\frac{-\frac{1}{2}+x}{2}=2 \Rightarrow x=\frac{9}{2}$, $\frac{6+y}{2}=1 \Rightarrow y=-4$. Thus the coordinates of B are $\left(\frac{9}{2},-4\right)$.

11. Determine which of the points $(-5,0)$, $(2,23)$, and $\left(-\sqrt{2},\sqrt{23}\right)$ are on the graph of the equation

$x^2 + y^2 = 25$.

(a) $(2,23)$, $(-\sqrt{2},\sqrt{23})$
(b) $(-5,0)$, $(-\sqrt{2},\sqrt{23})$
(c) $(-5,0)$, $(2,23)$, $(-\sqrt{2},\sqrt{23})$
(d) $(-5,0)$, $(2,23)$
(e) None

Answer: (b) Substitute to find that $(-5)^2 + 0^2 = 25$ and $(-\sqrt{2})^2 + (\sqrt{23})^2 = 25$, but $(2)^2 + (23)^2 \neq 25$.

12. Find the x and y - intercepts of the graph of $x^2 + 2xy + 4y^2 = 4$.

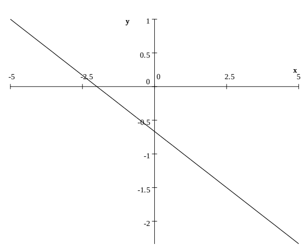
(a) x - intercept 2, y - intercepts ± 1
(b) x - intercepts ± 2, y - intercepts ± 2
(c) x - intercept 2, y - intercept 1
(d) x - intercepts ± 1, y - intercepts $\pm \frac{1}{2}$
(e) x - intercepts ± 2, y - intercepts ± 1

Answer: (e) When $y = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$. When $x = 0$, $4y^2 = 4 \Rightarrow y = \pm 1$.

13. Make a table of values and sketch the graph of the equation $x + 3y + 2 = 0$. Find the x - and y - intercepts and test for symmetry.

Answer: No symmetry

x	$y = -\frac{1}{3}x - \frac{2}{3}$	(x, y)
-2	0	$(-2,0)$
-1	$-\frac{1}{3}$	$(-1,-\frac{1}{3})$
0	$-\frac{2}{3}$	$(0,-\frac{2}{3})$
1	-1	$(1,-1)$

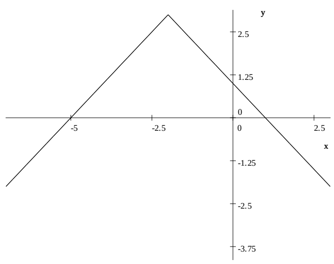


14. Make a table of values and sketch the graph of the equation $y = -|x+2| + 3$. Find the x - and y -

intercepts and test for symmetry.

Answer: No Symmetry

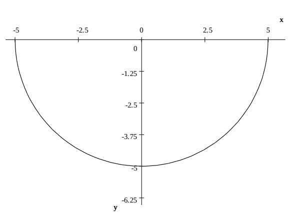
x	$y=-\lvert x+2\rvert+3$	(x,y)
-5	0	$(-5,0)$
-3	2	$(-3,2)$
-2	3	$(-2,3)$
-1	2	$(-1,2)$
0	1	$(0,1)$
1	0	$(1,0)$
2	-1	$(2,-1)$



15. Make a table of values and sketch the graph of the equation $y=-\sqrt{25-x^2}$. Find the x - and y - intercepts and test for symmetry.

Answer: Symmetry with respect to y -axis.

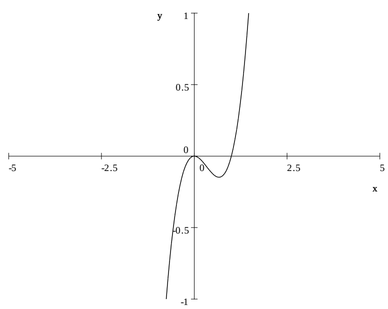
x	$y=-\sqrt{25-x^2}$	(x,y)
-5	0	$(-5,0)$
-4	-3	$(-4,-3)$
-3	-4	$(-3,-4)$
0	-5	$(0,-5)$
3	-4	$(3,-4)$
4	-3	$(4,-3)$
5	0	$(5,0)$



16. Make a table of values and sketch the graph of the equation $y=x^3-x^2$. Find the x - and y -intercepts and test for symmetry.

Answer: No symmetry.

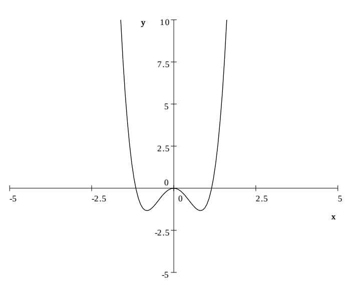
x	$y=x^3-x^2$	(x,y)
-2	-12	$(-2,-12)$
-1	-2	$(-1,-2)$
0	0	$(0,0)$
$\frac{1}{2}$	$-\frac{1}{8}$	$(\frac{1}{2},-\frac{1}{8})$
1	0	$(1,0)$
2	4	$(2,4)$
3	18	$(3,18)$



17. Make a table of values and sketch the graph of the equation $y=3x^4-4x^2$. Find the x - and y - intercepts and test for symmetry.

Answer: Symmetry with respect to the y - axis.

x	$y=3x^4-4x^2$	(x,y)
-2	32	$(-2,32)$
-1	-1	$(-1,-1)$
$-\frac{1}{2}$	$-\frac{13}{16}$	$(-\frac{1}{2},-\frac{13}{16})$
0	0	$(0,0)$
1	-1	$(1,-1)$
$\frac{1}{2}$	$-\frac{13}{16}$	$(\frac{1}{2},-\frac{13}{16})$
2	32	$(2,32)$



18. Find the equation of the circle with center $(-1,7)$ and radius $\sqrt{2}$.

(a) $x^2+2x+y^2-28y+50=0$
(b) $x^2-2x-y^2-14y-48=0$
(c) $x^2+14x+48+y^2-2y=0$
(d) $x^2-14x+48+y^2-2y=0$
(e) $x^2+2x+y^2-14y+48=0$

Answer: (e) $(h,k)=(-1,7)$, $r=\sqrt{2}$ $\Rightarrow$ $(x-(-1))^2+(y-7)^2=(\sqrt{2})^2$ $\Rightarrow$ $(x+1)^2+(y-7)^2=2$ $\Rightarrow$ $x^2+2x+y^2-14y+48=0$

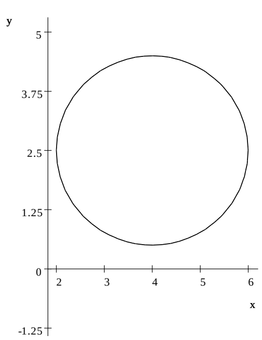
19. Find the equation of the circle with center $(0,-6)$ and tangent to the x - axis.

(a) $(x+6)^2+(y-6)^2=9$
(b) $(x)^2+(y-6)^2=6$
(c) $(x)^2+(y+6)^2=36$
(d) $(x)^2+(y+6)^2=9$
(e) $(x-6)^2+(y)^2=36$

Answer: (c) $(h,k)=(0,-6)$. The radius must be $r=6$, thus $(x)^2+(y+6)^2=36$.

20. Show that $x^2-8x+\frac{89}{4}+y^2-5y=4$ represents a circle, find its center and radius, and sketch its graph.

Answer: $x^2-8x+\frac{89}{4}+y^2-5y=4$ $\Leftrightarrow$ $x^2-8x+y^2-5y=\frac{-73}{4}$ $\Leftrightarrow$ $(x-4)^2+(y-\frac{5}{2})^2=16+\frac{25}{4}-\frac{73}{4}$ $\Leftrightarrow$ $(x-4)^2+(y-\frac{5}{2})^2=4$. Center $(4,\frac{5}{2})$, $r=2$.



1. Use a graphing calculator or computer to decide which one of the viewing rectangles produces the most appropriate graph of the equation $y = x^4 + 3$.

(a) $[-2,2]$ by $[-2,2]$
(b) $[-1,2]$ by $[-12,3]$
(c) $[2,6]$ by $[-10,4]$
(d) $[-1,1]$ by $[0,4]$
(e) $[-5,5]$ by $[-10,-6]$

Answer: (d)

2. Use a graphing calculator or computer to decide which one of the viewing rectangles produces the most appropriate graph of the equation $y = 4x^2 - 12x + 6$.

(a) $[2,10]$ by $[-10,10]$
(b) $[-1,4]$ by $[-3,20]$
(c) $[5,10]$ by $[-10,50]$
(d) $[-10,3]$ by $[0,50]$
(e) $[-10,10]$ by $[-20,20]$

Answer: (b)

3. Use a graphing calculator or computer to determine which of the viewing rectangles produces the most appropriate graph of the equation $y = 5x^5 - 4x^4$.

(a) $[-1,1]$ by $[-1,1]$
(b) $[-10,10]$ by $[-10,10]$
(c) $[-10,10]$ by $[-1,1]$
(d) $[-20,20]$ by $[-1,1]$
(e) $[-5,2]$ by $[-1,1]$

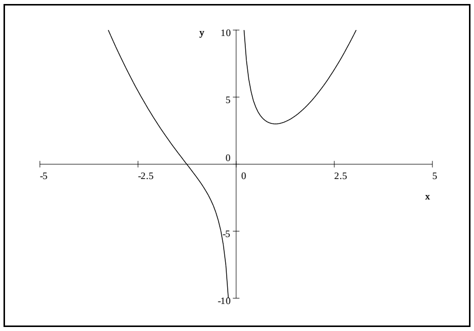
Answer: (a)

4. Use a graphing calculator or computer to determine which of the viewing rectangles produces the most appropriate graph of the equation $y = -|x+2| - 3$.

(a) $[-1,1]$ by $[-1,1]$
(b) $[-6,1]$ by $[-8,0]$
(c) $[6,1]$ by $[-8,8]$
(d) $[-6,8]$ by $[8,0]$
(e) $[-15,2]$ by $[-1,1]$
Answer: (b)

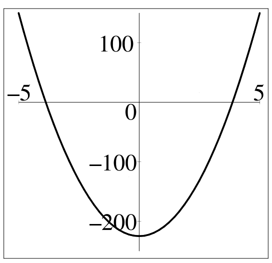
5. Determine an appropriate viewing rectangle for the equation $y = x^2 + \frac{2}{x}$.

Answer: $[-5, 5]$ by $[-10, 10]$



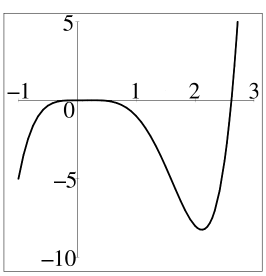
6. Determine an appropriate viewing rectangle for the equation $y = 15x^2 - 225$ and use it to draw the graph.

Answer: $[-5, 5]$ by $[-250, 150]$



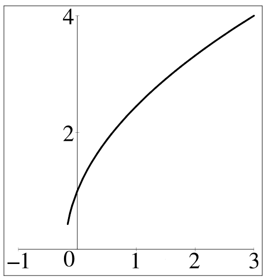
7. Determine an appropriate viewing rectangle for the equation $y = x^5 - 3x^4 + x^3$ and use it to draw the graph.

Answer: $[-1, 3]$ by $[-10, 5]$



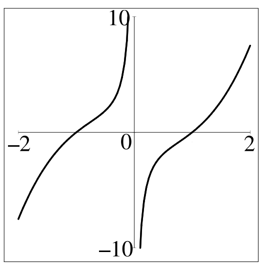
8. Determine an appropriate viewing rectangle for the equation $y = \sqrt{5x+1}$ and use it to draw the graph.

Answer: $[-1, 3]$ by $[0, 4]$



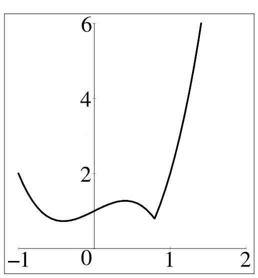
9. Determine an appropriate viewing rectangle for the equation $y = x^3 - \frac{1}{x}$ and use it to draw the graph.

Answer: $[-2, 2]$ by $[-10, 10]$



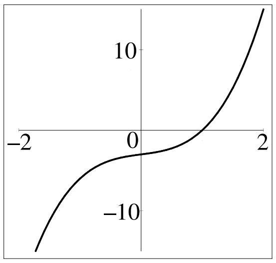
10. Determine an appropriate viewing rectangle for the equation $y = |2x^3 - 1| + x$ and use it to draw the graph.

Answer: $[-1, 2]$ by $[0, 6]$



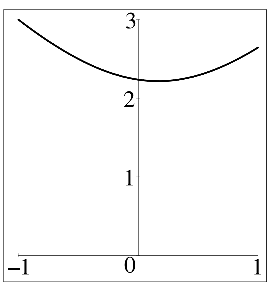
11. Determine an appropriate viewing rectangle for the equation $y = 2x^3 + x - 3$ and use it to draw the graph.

Answer: $[-2, 2]$ by $[-15, 15]$



12. Determine an appropriate viewing rectangle for the equation $y = \sqrt{3x^2 - x + 5}$ and use it to draw the graph.

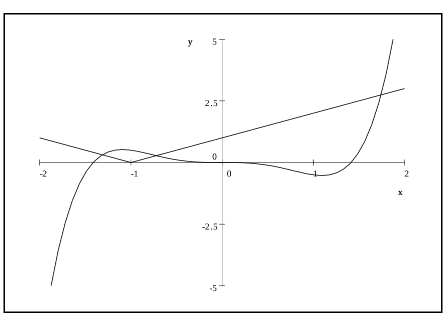
Answer: $[-1, 1]$ by $[0, 3]$



13. In how many distinct points does the graph of $y = \frac{1}{2}x^5 - x^3$ intersect the graph of $y = |x + 1|$ in the viewing rectangle $[-2, 2]$ by $[-5, 5]$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

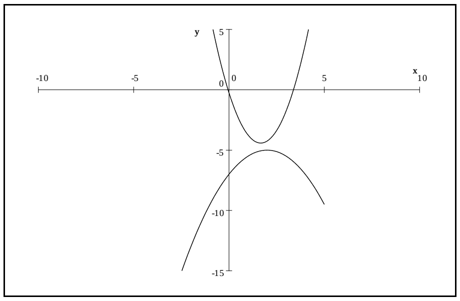
Answer: (c)



14. In how many distinct points does the graph of $y = \frac{3}{2}x^2 - 5x - \frac{1}{4}$ intersect the graph of $y = -\frac{1}{2}x^2 + 2x - 7$ in the viewing rectangle $[-10,10]$ by $[-15,5]$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

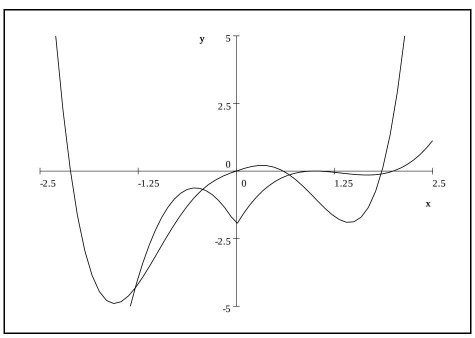
Answer: (a)



15. In how many distinct points does the graph of $x^3 - 4x^2 + 5|x| - 2$ intersect the graph of $y = 2x^4 - 4|x^3| + x$ in the viewing rectangle $[-2.5, 2.5]$ by $[-5,5]$?

(a) 6 (b) 1 (c) 3 (d) 4 (e) 5

Answer: (d)



16. Find the approximate solutions of the equation $\sqrt{3}x + \sqrt{5} = 0$ in the interval $[-3,0]$ by drawing a graph in an appropriate viewing rectangle.

(a) -1.3 (b) -2.9 (c) 1.4 (d) 3.6 (e) 3.4

Answer: (a)

17. Find the approximate solutions of the equation $2x^2 + 3x - 8 = 0$ in the interval $[-3,3]$ by drawing a graph in an appropriate viewing rectangle.

(a) $-2.9, 3.6$ (b) $-2.9, 2.1, 3.6$ (c) $-2.9, 1.4$ (d) $-2.9, 1.4, 3.6$ (e) $-2.4, 3.4$

Answer: (c)

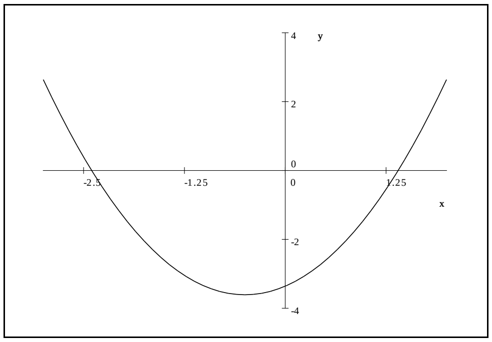
18. Find the approximate solutions of the equation $2x^3 - 3x + 2 = 0$ in the interval $[-3, 2]$ by drawing a graph in an appropriate viewing rectangle.

(a) $-1.5, 3.6$ (b) $-1.5, 2.1, 3.6$ (c) $-1.5, 1.4$ (d) -1.5 (e) $-2.4, 3.4$

Answer: (d)

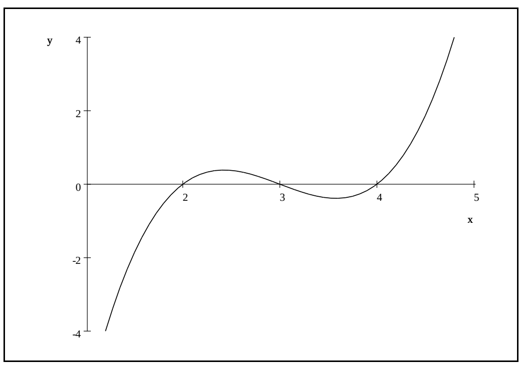
19. Find the approximate solutions of the equation $x^2 + x - 3.36 = 0$ by drawing a graph in the interval $[-3, 2]$.

Answer: $x = -2.4$, $x = 1.4$



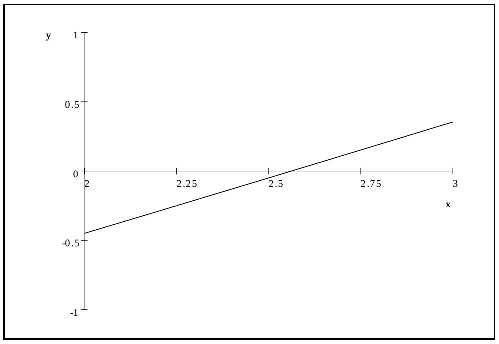
20. Find the approximate solutions of the equation $x^3 - 9x^2 = 24 - 26x$ by drawing a graph in the interval $[1, 5]$.

Answer: $x = 2$, $x = 3$, $x = 4$



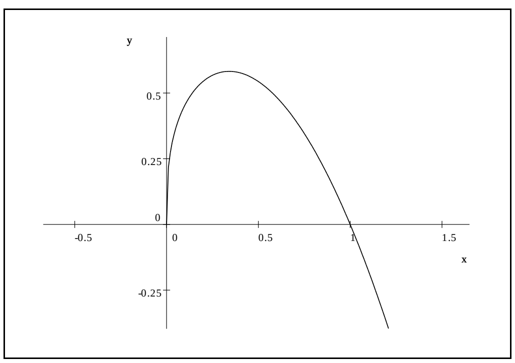
21. Find the approximate solutions of the equation $x-\sqrt{x+4}=0$ by drawing a graph in the interval $[2,3]$.

Answer: $x = 2.56$



22. Find the approximate solutions of the equation $x^{1/3}-x^{2}=0$ by drawing a graph in the interval $[-0.5,1.5]$.

Answer: $x=0$, $x=1$



1. Find the slope of the line through $P(2,-1)$ and $Q(0,3)$.

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

Answer: (a)

2. Find the slope of the line through $P(-1,0)$ and $Q(3,2)$.

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{4}{3}$ (e) $\frac{1}{2}$

Answer: (e)

3. Find the slope of the line through $P(5,1)$ and $Q(7,6)$.

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{11}{2}$

Answer: (b)

4. Find the slope of the line through $P(-1,1)$ and $Q(-2,10)$.

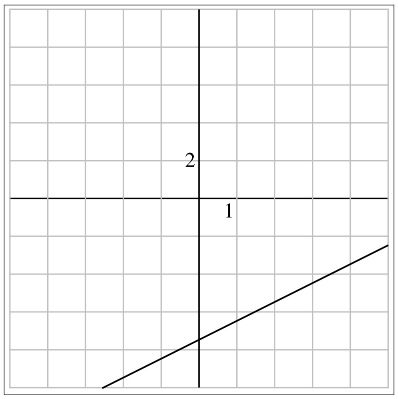
(a) -9 (b) $\frac{-1}{9}$ (c) Undefined (d) 10 (e) -8

Answer: (a)

5. Find the slope and y - intercept of the line $-2x+2y+15=0$ and draw its graph.

Answer:

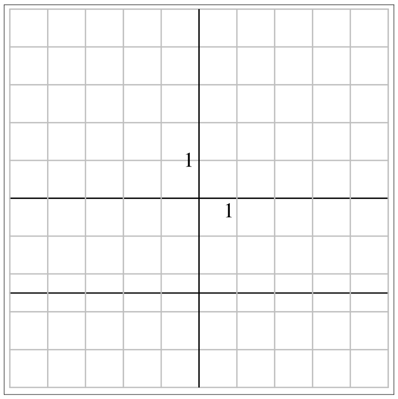
$$-2x+2y+15=0 \quad \Leftrightarrow \quad y = x-\frac{15}{2} \quad \Rightarrow \quad m=1 \text{ and } b=-\frac{15}{2}$$



6. Find the slope and y - intercept of the line $2y+5=0$ and draw its graph.

Answer:

$2y+5=0 \;\Leftrightarrow\; y=-\frac{5}{2} \;\Rightarrow\; m=0$ and $b=-\frac{5}{2}$



7. Find an equation of the line with slope -1 that passes through $(-1,3)$.

(a) $2x+2y-1=0$
(b) $x-2y+4=0$
(c) $x+y-2=0$
(d) $x+y+4=0$
(e) $3x-y-4=0$

Answer: (c) $y-3=-1(x+1) \;\Leftrightarrow\; x+y-2=0$

8. Find an equation of the line with slope $-\frac{1}{3}$ that passes through $(-7,3)$.

(a) $3x+3y-2=0$
(b) $x+y-2=0$
(c) $x+3y-4=0$
(d) $x+3y-2=0$
(e) $2x+3y-1=0$

Answer: (d) $y-3=-\frac{1}{3}(x+7) \;\Leftrightarrow\; x+3y-2=0$

9. Find an equation of the line that passes through $(2,-1)$ and $(4,1)$.

(a) $2y-x+3=0$ (b) $y-x+3=0$ (c) $y-2x+3=0$ (d) $y-4x+3=0$ (e) $y-x=0$

Answer: (b) $y+1=(x-2) \;\Leftrightarrow\; y-x+3=0$

10. Find an equation of the line with slope $2/5$ and y - intercept 1.

(a) $2x-5y+5=0$
(b) $x-5y+5=0$
(c) $2x+5y+5=0$
(d) $x-5y-5=0$
(e) $2x-5y=0$

Answer: (a) ; $m=\frac{2}{5}$, $b=1$; Substituting into $y=mx+b$ gives $y=\frac{2}{5}x+1 \Leftrightarrow 2x-5y+5=0$

11. Find the equation of a vertical line passing through the point $(-1,-2)$.

(a) $y=-x-2$ (b) $x=-y$ (c) $y=-1$ (d) $x=-1$ (e) Undefined

Answer: (d); The vertical line passing through the point $(-1,y)$ is $x=-1$.

12. Find the equation of a line that is parallel to $3x+11y+14=0$ and passes through the point $\left(-\frac{1}{2},6\right)$.

(a) $y=\frac{3}{11}x+33$
(b) $y=22-\frac{3}{11}x$
(c) $y=\frac{64}{11}-x$
(d) $y=\frac{3}{11}x+\frac{129}{22}$
(e) $y=\frac{129}{22}-\frac{3}{11}x$

Answer: (e) $3x+11y+14=0 \Leftrightarrow y=-\frac{3}{11}x-\frac{14}{11} \Rightarrow m=-\frac{3}{11}$. The equation of the line passing through the point $\left(-\frac{1}{2},6\right)$ and parallel to $3x+11y+14=0$ has the same slope $m=-\frac{3}{11} \Rightarrow$ $y-6=-\frac{3}{11}\left(x+\frac{1}{2}\right) \Leftrightarrow y=\frac{129}{22}-\frac{3}{11}x$.

13. Find the equation of a line that is perpendicular to $4x+12y-3=0$ and passes through the point $(2,3)$.

(a) $y=3x-\frac{3}{2}$
(b) $y=3x-3$
(c) $y=2x-3$
(d) $y=\frac{1}{3}x-2$
(e) $y=-\frac{1}{3}x+\frac{1}{4}$

Answer: (b); $4x+12y-3=0 \Leftrightarrow y=-\frac{1}{3}x+\frac{1}{4} \Rightarrow m=-\frac{1}{3}$. The equation of the line passing through the point $(2,3)$ and perpendicular to $4x+12y-3=0$ has slope $m_p=-\frac{1}{\left(-\frac{1}{3}\right)}=3$. Thus $y-3=3(x-2) \Leftrightarrow y=3x-3$.

14. Find the equation of a line that passes through the point $\left(-7,\frac{7}{2}\right)$ and the midpoint of $(-2,4)$ and $(3,4)$.

(a) $30y-2x-119=0$
(b) $2y-30x=0$
(c) $40y-120x-119=0$
(d) $3y-2x-19=0$
(e) $30y-40x-119=0$

Answer: (a) ; Midpoint $=\left(\frac{-2+3}{2},\frac{4+4}{2}\right)=\left(\frac{1}{2},4\right)$. The slope of the line passing through $\left(\frac{1}{2},4\right)$ and $\left(-7,\frac{7}{2}\right)$ is $m=\left(\frac{\frac{7}{2}-4}{-7-\frac{1}{2}}\right)=\frac{1}{15}$. Therefore the equation is $y-\frac{7}{2}=\frac{1}{15}(x+7) \Leftrightarrow y=\frac{1}{15}x+\frac{119}{30}$ or $30y-2x-119=0$.

15. Find a perpendicular line that passes through the midpoint of $(8,-12)$ and $(7,10)$.

(a) $4y-22x+59=0$
(b) $2y-4x=59$
(c) $44y-2x+59=0$
(d) $x-22y=164$
(e) $22x-y-164=0$

Answer: (c) ; The midpoint of $(8,-12)$ and $(7,10)$ is $\left(\frac{8+7}{2},\frac{-12+10}{2}\right)=\left(\frac{15}{2},-1\right)$. The slope of the line that passes through $(8,-12)$ and $(7,10)$ is $m=\left(\frac{10+12}{7-8}\right)=-22$. Therefore the perpendicular line that passes through $\left(\frac{15}{2},-1\right)$ is $(y-(-1))=\left(\frac{1}{22}\right)\left(x-\frac{15}{2}\right) \Leftrightarrow y=\frac{1}{22}x-\frac{59}{44}$ or $44y-2x+59=0$.

16. Use slopes to determine whether the points $(4,-1)$, $(3,6)$, and $(10,3)$ are collinear (lie on a line).

Answer: $P_1=(4,-1)$ $P_2=(3,6)$ $P_3=(10,3)$. Then $m_{12}=\frac{6-(-1)}{3-4}=-7$, $m_{23}=\frac{3-6}{10-3}=-\frac{3}{7}$, and $m_{13}=\frac{3+1}{10-4}=\frac{2}{3}$. Since $m_{12}\neq m_{23}$, the points are not collinear.

17. Use slopes to determine whether the points $(1,1)$, $\left(3,-\frac{1}{3}\right)$, and $\left(0,\frac{5}{3}\right)$ are collinear (lie on a line).

Answer: $P_1=(1,1)$ $P_2=\left(3,-\frac{1}{3}\right)$ $P_3=\left(0,\frac{5}{3}\right)$. Then $m_{12}=\frac{-\frac{1}{3}+1}{3-1}=-\frac{2}{3}$, $m_{23}=\frac{\frac{5}{3}+\frac{1}{3}}{0-3}=-\frac{2}{3}$ and $m_{13}=\frac{\frac{5}{3}-1}{0-1}=-\frac{2}{3}$. Since $m_{12}=m_{23}=m_{13}$, the points are collinear.

18. (a) Find an equation for the line tangent to the circle $x^2+y^2=4$ at the point $\left(1,-\sqrt{3}\right)$.
(b) At what other point on the circle will the tangent line be parallel to the line in part (a)?

Answer:(a) Since the circle $x^2+y^2=4$ has center $(0,\ 0)$ and radius 2, the slope of the line passing through the center $(0,\ 0)$ and the point $\left(1,-\sqrt{3}\right)$ is $\dfrac{-\sqrt{3}-0}{1-0}=-\sqrt{3}$. Hence the slope of the tangent line at $\left(1,-\sqrt{3}\right)$ is $\dfrac{\sqrt{3}}{3}\ \Rightarrow\ y+\sqrt{3}=\dfrac{\sqrt{3}}{3}(x-1)\ \Leftrightarrow\ \sqrt{3}x-3y-4\sqrt{3}=0$
(b) Since diametrically opposite points on the circle have parallel tangent lines, the other point is $\left(-1,\sqrt{3}\right)$.

19. Find the area of a triangle formed by the coordinate axes and the line $\frac{1}{2}x+2y-5=0$.

Answer: Since the base of the triangle is 10 units (the distance from the origin to its x - intercept), and its height is $\frac{5}{2}$ (the distance from the origin to its y - intercept), its area is $A=\frac{1}{2}bh\ \Rightarrow$
$A=\frac{1}{2}(10)\left(\frac{5}{2}\right)=\frac{25}{2}$ units ^{2}.

20. A particular mountain highway has a 5% grade. While making your accent to the top of the mountain, you notice that an elevation sign indicates that you are at an elevation of 2342 ft. (assume this means a vertical distance from the base where you started and not from sea level). Find the change in your horizontal distance in miles. Round to two significant digits.

Answer: Slope of the mountain $=\dfrac{\text{change in vertical distance}}{\text{change in horizontal distance}}\Rightarrow\dfrac{5}{100}=\left(\dfrac{2342}{\Delta x}\right)\Leftrightarrow\Delta x=46{,}840$ ft or $46{,}840$ ft $\times\left(\dfrac{1\text{ mile}}{5280\text{ ft}}\right)=8.8$ miles.

21. If the recommended adult dosage for a drug is *D* (in mg), then to determine the appropriate dosage *c* for a child of age *a*, pharmacists use the equation $c=0.0417D(a+1)$.
Suppose the dosage for an adult is 300 mg.

(a) Find the slope. What does it represent?
(b) What is the dosage for a newborn?

Answer: (a) 12.51; the slope represents the increase in dosage for a one-year increase in age. (b) 12.51 mg

22. If the recommended adult dosage for a drug is *D* (in mg), then to determine the appropriate dosage *c* for a child of age *a*, pharmacists use the equation $c=0.0417D(a+1)$.
Suppose the dosage for an adult is 400 mg.

(c) Find the slope. What does it represent?
(d) What is the dosage for a newborn?

Answer: (a) 16.68; the slope represents the increase in dosage for a one-year increase in age. (b) 16.68 mg

1. Write an equation that expresses the statement.

Q varies directly as x.

Answer: $Q = kx$

2. Write an equation that expresses the statement.

V is inversely proportional to y.

Answer: $V = \frac{k}{y}$

3. Write an equation that expresses the statement.

J is proportional to s and inversely proportional to t.

Answer: $J = \frac{ks}{t}$

4. Write an equation that expresses the statement.

D is proportional to the square root of x.

Answer: $D = k\sqrt{x}$

5. Write an equation that expresses the statement that V is proportional to the square root x and inversely proportional to the square of y.

Answer: $V = \frac{k\sqrt{x}}{y^2}$, where k is constant.

6. Write an equation that expresses the statement that A is proportional to the square of x and inversely proportional to t and the cube root of y.

Answer: $A = \frac{kx^2}{t\sqrt[3]{y}}$, where k is constant.

7. Write an equation that expresses the statement that T varies jointly as s and the square of r and inversely as the cube root of c.

Answer: $T = \frac{kr^2 s}{\sqrt[3]{c}}$, where k is constant.

8. Express the statement as a formula and use the information to find the constant of proportionality:

P is jointly proportional to the squares of s and t. If $s = 4$ and $t = 8$ then $P = 5120$.

Answer: $P = ks^2t^2 \Rightarrow 5120 = k(4)^2(8)^2 \Rightarrow k = 5$, so $P = 5s^2t^2$.

9. Express the statement as a formula and use the information to find the constant of proportionality:

Z is jointly proportional to a, b, and c and inversely proportional to d. If c and d have the same value, and if a and b are both 6, then $Z = 360$.

Answer: $Z = k\dfrac{abc}{d}$, and since $c = d$ then $Z = kab \Rightarrow 360 = k(6)(6) \Rightarrow k = 10$, so $Z = 10ab$.

10. The pressure of a sample of gas is directly proportional to the temperature and inversely proportional to the volume.

(a) Write an equation that expresses this fact if 50 L of gas exerts a pressure of 14 kPa at a temperature of 350° K (absolute temperature measured on the Kelvin scale).
(b) If the temperature is increased to 400° K and the volume is decreased to 40 L, what is the pressure of the gas?

Answer: Let P, T and V be the pressure, temperature and volume of the sample of gas, respectively.(a) $P = \dfrac{kT}{V} \Rightarrow 14 = \dfrac{k \cdot 350}{50} \Rightarrow k = 2 \Rightarrow P = \dfrac{2T}{V}$

(b) $P = \dfrac{2 \cdot 400}{40} = 200$ kPa. Hence the pressure of the sample of gas is 200 kPa.

11. The resistance of a wire varies directly as its length and inversely as the square of its diameter. A wire 50 m long and 0.01 m in diameter has a resistance of $25\ \Omega$. Write an equation to describe its resistance, and find the resistance of a wire made of the same material that is 20 m long and has diameter 0.02 m.

Answer: $R = kL/d^2$; $25 = k \cdot 50/(0.01)^2 \Rightarrow k = \dfrac{1}{20{,}000}$. The resistance of the wire is

$R = \dfrac{1}{20{,}000} \cdot 20/(0.02)^2 = 2.5$.

12. The gravitational force F between two objects is inversely proportional to the square of the distance between the objects. If the force F is 1.62×10^{-6} N when the two objects are 25 cm apart, what is the gravitational force when the objects are 35 cm apart?

Answer: $F = \dfrac{k}{r^2} \Rightarrow 1.62 \times 10^{-6} = \dfrac{k}{25^2} \Rightarrow k = 1.0125 \times 10^{-3} \Rightarrow F = \dfrac{1.0125 \times 10^{-3}}{35^2} = 8.2653 \times 10^{-7}$ N

13. The resistance R of a wire varies directly as its length L and inversely as the square of twice its radius. (a) Write an equation that expresses this direct variation. (b) Find the constant of proportionality if a wire 2.4 m long and 0.004 m in diameter has a resistance of 150 ohms. (c) Using the constant of proportionality from part (b), find the diameter of a wire that has a resistance of 90 ohms and is 1.2 m long.

Answer:(a) $R = \dfrac{kL}{(2r)^2} = \dfrac{kL}{4r^2}$ (b) $R = \dfrac{kL}{4r^2}$, and since $2r = d$ we have $R = \dfrac{kL}{d^2} \Rightarrow 150 = \dfrac{k2.4}{0.004}$

$\Rightarrow k = \dfrac{1}{4}$. (c) $R = \dfrac{kL}{d^2} \Rightarrow d^2 = \dfrac{1}{4}\dfrac{L}{R} \Rightarrow d^2 = \dfrac{1}{4}\left(\dfrac{1.2}{90}\right) \Rightarrow d \approx 5.78 \times 10^{-2}$ m

1 FUNDAMENTALS

1.11 *Modeling Variation*

14. The cost C of printing a magazine is jointly proportional to the number of pages p in the magazine and the number of magazines printed m.

(a) Write an equation that expresses this joint variation.
(b) Find the constant of proportionality if the printing cost is \$120,000 for 4000 copies of a 120-page magazine.
(c) How much would the printing cost be for 6000 copies of a 95-page magazine?

Answer: (a) $C = kpm$; (b) $k = \frac{1}{4}$; (c) \$142,500

15. The cost C of printing a magazine is jointly proportional to the number of pages p in the magazine and the number of magazines printed m.

(a) Write an equation that expresses this joint variation.
(b) Find the constant of proportionality if the printing cost is \$120,000 for 4000 copies of a 120-page magazine.
(c) How much would the printing cost be for 2500 copies of an 85-page magazine?

Answer: (a) $C = kpm$; (b) $k = \frac{1}{4}$; (c) \$53,125

16. The loudness L of a sound (measured in decibels, dB) is inversely proportional to the square of the distance d from the source of the sound. A person who is 10 ft from a lawn mower experiences a sound level of 70 dB. How loud is the lawn mower when the person is 50 ft away?

Answer: 2.8 dB

17. The loudness L of a sound (measured in decibels, dB) is inversely proportional to the square of the distance d from the source of the sound. A person who is 10 ft from a lawn mower experiences a sound level of 70 dB. How loud is the lawn mower when the person is 25 ft away?

Answer: 11.2 dB

18. The stopping distance D of a car after the brakes have been applied varies directly as the square of the speed s. A certain car traveling at 50 mi/h can stop in 240 ft. What is the maximum speed it can be traveling if it needs to stop in 150 ft? Round your answer to one decimal place.

Answer: 39.5 mi/hr

19. The stopping distance D of a car after the brakes have been applied varies directly as the square of the speed s. A certain car traveling at 50 mi/h can stop in 240 ft. What is the maximum speed it can be traveling if it needs to stop in 180 ft? Round your answer to one decimal place.

Answer: 43.3 mi/h