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4. Cannons The velocity of a projectile depends upon many factors, in particular, the weight of the ammunition.

(a) Plot a scatter diagram of the data in the table below. Let x be the weight in kilograms and let y be the velocity in meters per second.

Type	Weight (kg)	Initial Velocity (m/sec)
MG 17	10.2	905
MG 131	19.7	710
MG 151	41.5	850
MG 151/20	42.3	695
MG/FF	35.7	575
MK 103	145	860
MK 108	58	520
WGr 21	111	315

(Data and information taken from "Flugzeug-Handbuch, Ausgabe Dezember 1996: Guns and Cannons of the Jagdwaffe" at www.xs4all.nl/~rhorta/jgguns.htm*)*

(b) Determine which type of function would fit this data the best: linear or quadratic. Use a graphing utility to find the function of best fit. Are the results reasonable?

(c) Based on velocity, we can determine how high a projectile will travel before it begins to come back down. If a cannon is fired at an angle of 45° to the horizontal, then the function for the height of the projectile is given by $s(t) = -16t^2 + \frac{\sqrt{2}}{2}v_0 t + s_0$, where v_0 is the velocity at which the shell leaves the cannon (initial velocity), and s_0 is the initial height of the nose of the cannon (because cannons are not very long, we may assume that the nose and the firing pin at the back are at the same height for simplicity). Graph the function $s = s(t)$ for each of the guns described in the table. Which gun would be the best for anti-aircraft if the gun were sitting on the ground? Which would be the best to have mounted on a hilltop or on the top of a tall building? If the guns were on the turret of a ship, which would be the most effective?

3. Suppose $f(x) = \sin x$.

(a) Build a table of values for $f(x)$ where $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi$. Use exact values.

(b) Find the **first differences** for each consecutive pair of values in part (a). That is, evaluate $g(x_i) = \frac{\Delta f(x_i)}{\Delta x_i} = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots, x_{17} = 2\pi$. Use your calculator to approximate each value rounded to three decimal places.

(c) Plot the points $(x_i, g(x_i))$ for $i = 1, \ldots, 16$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(d) Find the first differences for each consecutive pair of values in part (b). That is, evaluate $h(x_i) = \frac{\Delta g(x_i)}{\Delta x_i} = \frac{g(x_{i+1}) - g(x_i)}{x_{i+1} - x_i}$ where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots, x_{16} = \frac{11\pi}{6}$. This is the set of **second differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, h(x_i))$ for $i = 1, \ldots, 15$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(e) Find the first differences for each consecutive pair of values in part (d). That is, evaluate $k(x_i) = \dfrac{\Delta h(x_i)}{\Delta x_i} = \dfrac{h(x_{i+1}) - h(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0,\ x_2 = \dfrac{\pi}{6}, \ldots, x_{15} = \dfrac{7\pi}{4}$. This is the set of **third differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, k(x_i))$ for $i = 1, \ldots, 14$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(f) Find the first differences for each consecutive pair of values in part (e). That is, evaluate $m(x_i) = \dfrac{\Delta k(x_i)}{\Delta x_i} = \dfrac{k(x_{i+1}) - k(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0,\ x_2 = \dfrac{\pi}{6}, \ldots, x_{14} = \dfrac{5\pi}{3}$. This is the set of **fourth differences** of $f(x)$.

Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, m(x_i))$ for $i = 1, \ldots, 13$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(g) What pattern do you notice about the curves that you found? What happened in part (f)? Can you make a generalization about what happened as you computed the differences? Explain your answers.

7. **CBL Experiment** Locate the motion detector on a Calculator Based Laboratory (CBL) or a Calculator Based Ranger (CBR) above a bouncing ball.

(a) Plot the data collected in a scatter diagram with time as the independent variable.

(b) Find the quadratic function of best fit for the second bounce.

(c) Find the quadratic function of best fit for the third bounce.

(d) Find the quadratic function of best fit for the fourth bounce.

(e) Compute the maximum height for the second bounce.

(f) Compute the maximum height for the third bounce.

(g) Compute the maximum height for the fourth bounce.

(h) Compute the ratio of the maximum height of the third bounce to the maximum height of the second bounce.

(i) Compute the ratio of the maximum height of the fourth bounce to the maximum height of the third bounce.

(j) Compare the results from parts (h) and (i). What do you conclude?