CHAPTER ONE

Physics and the Life Sciences

MULTIPLE CHOICE QUESTIONS

Multiple Choice 1.1

Correct Answer (c). m_{brain} directly proportional to M_{body} means that b = +1 in:

$$m_{brain} = a \cdot M_{body}^{b} \quad \textbf{(1)}$$

Note that the coefficient *b* is the slope of the curve after the natural logarithm is taken on both sides of Eq. [1].

Multiple Choice 1.2

Correct Answer (c). We can argue in two ways: physically, we note that the slope represents an actual physical relation. Replotting data by using another unit system cannot change the physical facts. The original relationship is given by:

$$m_{brain} = a \cdot M_{body}^{b} \quad \textbf{(1)}$$

Mathematically, plotting m_{brain} in unit g means that we use values that are larger by a factor of 1000 on the left side in Eq. [1]. Thus, for Eq. [1] to remain correct, the prefactor must also be larger by a factor of 1000. In Eq. [2] we take the natural logarithm on both sides of Eq. [1], with the brain mass in unit *kg* on the right–hand side:

$$\ln\ m_{brain}(kg) = \ln\ a + b \cdot \ln\ M_{body}(kg) \quad \textbf{(2)}$$

In Eq. [3] we rewrite Eq. [1] once more with natural logarithms, but use the brain mass in unit *g*:

$$\ln\ m_{brain}(g) = \ln(1000a) + b \cdot \ln\ M_{body}(kg) \quad \textbf{(3)}$$

in which ln (1000 · *a*) = ln 1000 + ln *a*.
Eqs. [2] and [3] differ only in that a constant term, ln1000 = 6.908, is added in the second case. This represents a vertical shift of the curve, but not a change in its slope.

Multiple Choice 1.3

Correct Answer (e). The precision of each number is represented by the smaller power of ten in the number. Smaller powers of ten indicate more precise numbers. (d) is the least precise number since the smallest power of ten is 10^{11} for the last digit. (a) follows with a precision of 10^{6}, (b) with 10^{-2}, (c) with 10^{-6}, and (e) is the most precise with 10^{-18} being the smallest power of ten in the number.

Multiple Choice 1.4

Correct Answer (a). The number of significant figures in the number represents its accuracy. The more significant figures a number has, the more accurate the number is. In order of increasing accuracy, (e) has only one significant figure, (d) has two, both (b) and (c) have four, and (a) is the most accurate with five significant figures.

CONCEPTUAL QUESTIONS

Conceptual Question 1.1

No. We build physical models to describe observations of the world around us. These observations face limitations that are then unavoidably transferred to the physical model. Upon improving on our observations, the model might still be valid, it might need corrections, or might be wrong in a fundamental way. Similarly, to build physical models we are required to make some assumptions as a starting point, these assumptions form the basis for the physical model. However, further observations might confirm or deny the initial assumptions and thus validate or invalidate the physical model. Physical models are therefore under constant revision and continued testing.

Conceptual Question 1.2

(a) The number 11 is represented by the digit ♥. Note that 10 in base-12 actually represents the number 12 in base-10. This is because the sequence of digits one-zero in base-12 means 0 $\times 12^{0} + 1 \times 12^{1}$ when expanded in base-10.
(b) Repeated integer division of 3498572 by 12 will yield the various digits in base-12: 3498572 / 12 = 291547 with 8 as residue, so that 8 is the digit for 12^{0}; 291547 / 12 = 24298 with 7 as residue, so that 7 is the digit for 12^{1}.

Continuing this division we find the sequence of residues: $8 \times 12^0 + 7 \times 12^1 + 7 \times 12^2 + 8 \times 12^3 + 0 \times 12^4 + 2 \times 12^5 + 1 \times 12^6 = 3498572$. Thus, 3498572 in base 12 is represented a 1208778.

(c) The number ♦6♥3 expands as $10 \times 12^3 + 6 \times 12^2 + 11 \times 12^1 + 3 \times 12^0$, where we have converted from ♦ to 10, ♥ to 11, and multiplied each digit by the respective power of 12 for the place in the number. The result is 18279 in base-10.

Conceptual Question 1.3

This office is roughly 4 m wide by 5 m long by 3 m high. Therefore, in m^3 the volume is 60 m^3. Since 1 m = 10^2 cm = 10^3 mm = 10^{-3} km we can express the volume of the office as 60 $m^3 = 6 \times 10^7$ $cm^3 = 6 \times 10^{10}$ $mm^3 = 6 \times 10^{-8}$ km^3. Although all these values are correct expressing volumes of this order of magnitude would be simpler in m^3.

Conceptual Question 1.4

To convert a temperature T_F in Fahrenheit to T_C in Celsius we use the expression:

$$T_C = (T_F - 32) \times \left(5/9\right)$$

Therefore, in terms of the chirps per minute, the temperature in degrees Celsius is:

$$T_C = \left(18 + \left(\frac{N - 40}{4}\right)\right) \times \left(5/9\right)$$

Although the formula is dimensionally incorrect as N is expressed in chirps per minute and all other numbers are dimensionless, it can still be used to correctly predict the temperature. Simplifications have been to make it easier to write the formula by omitting all the units. If we wanted to include the units, the correct values would be 18 °F, 40 chirps/min, 4 min/(chirp × °F), and 5/9 °C/°F.

Conceptual Question 1.5

We can expect a smaller slope of the graph for the birds as compared to the mammals. A smaller slope on this log-log graph represents a smaller ratio of m_{brain} to M_{body} and thus smaller brain size relative to the size of the subjects in the class. From the data discussed in the chapter, on average and insofar as we can infer the intelligence of non-human subjects, larger brain mass to body mass ratio seems to be correlated to intelligence.

Conceptual Question 1.6

In my household we average around 1400 kWh per month of electricity consumption. Converting kWh/month we find an average power consumption of 1900 W and thus an average of 1900 joules of electrical energy used each second. There are seven individuals living in my household so each person uses 270 W or about 300 joules every second. If we are considering a city of population 10^6 individuals, then the power consumed will be:

$$10^6 \times 300\ \text{J/s} = 3 \times 10^8\ \text{J/s}$$

Since the bomb released 10^6 Joules of energy, it will power the city for:

$$\text{Time} = \frac{10^{14}\ \text{J}}{3 \times 10^8\ \text{J/s}} \cong 3 \times 10^5\ \text{s} \cong 4\ \text{days}$$

Note that this result is of the same order of magnitude as the result found in Example 1.11. Averaging first the electric bill from a number of households we could do a better estimate. We could also add the electric bills from a number of businesses to add it to the total. However, we would expect the result to be still within the same order of magnitude.

ANALYTICAL PROBLEMS

Problem 1.1

(a) 1.23×10^2

(b) 1.230×10^3

(c) 1.23000 × 104 since the last zero is significant it must be expressed

(d) 1.23×10^{-1}

(e) 1.23×10^{-3} the zeros between the decimal point and the 1 are not significant

(f) 1.23000×10^{-6} the zeros between the decimal point and the 1 are not significant, however the three zeros to the right of the 3 are significant

Problem 1.2

(a) 5 significant figures

(b) 4 significant figures

(c) 4 significant figures; the zero to the right of the decimal point and before the first non-zero digit is not a significant figure

(d) 4 significant figures; when a number is written in scientific notation, all digits expressed are significant figures

Problem 1.3

The product must be given to the number of significant figures of the least accurate number, that is the number with the smallest number if significant figures:

(a) 5.61×10^{-1} both numbers have three significant figures since the zeros in 0.00456 after the decimal point and before the 4 are not significant

(b) 5.61×10^{2} note that the last zero in 1230 is not significant

(c) $5.6088 \times 10^{0} = 5.6088$ both numbers have five significant figures

(d) $5.609 \times 10^{0} = 5.609$ note that the zero in 0.01230 after the decimal point and before the 1 is not significant while the last zero is significant

Problem 1.4

The quotient follows the same rules as the multiplication; the result must be given to the number of significant figures of the least accurate number:

(a) 2.70×10^{4} both numbers have three significant figures since the zeros in 0.00456 after the decimal point and before the 4 are not significant

(b) 2.70×10^{3} note that the last zero in 1230 is not significant

(c) 2.6974×10^{-7} both numbers have five significant figures

(d) 2.697×10^{-5} note that the zero in 0.01230 after the decimal point and before the 1 is not significant while the last zero is significant

Problem 1.5

Sums and differences must be given with the precision of the least precise number, where the precision is found by the smallest power of ten present in each number:

(a) 5.79×10^{2} both numbers are precise to 10^{0} so the result must be quoted to that precision as 579 and then expressed in scientific notation

(b) 1.23×10^{3} while 0.456 is precise to 10^{-3}, 1230 is only precise to 10^{1} as the last zero is not significant and the result includes significant figures up to 10^{1}

(c) 3.33×10^{-1} both numbers are precise to 10^{-3} so the result is 0.333 which is then written in scientific notation

(d) 3.34×10^{-1} the number 123.123 is the least precise of the two to only 10-3 so the result is 0.334 which is then written in scientific notation

Problem 1.6

The standard precedence of operations means quotients and multiplications must be performed first computing values with the result quoted to the significant figures of the least accurate number involved. Then differences and sums are performed with the result quoted to the significant figures of the least precise number involved:

(a) 1.27×10^{2} the multiplication requires three significant figures and the sum must be precise to 10^{0} which is the precision of both numbers

(b) 5.62×10^{4} the multiplication requires three significant figures so the partial result is precise only to 10^{1} and thus the sum will be quoted to a precision of 10^{1}

(c) -3.71×10^{5} the quotient requires three significant figures and the partial result is precise only to 10^{3} and thus the difference will be quoted to a precision of 10^{3}

(d) 6.85×10^{-6} the multiplication requires three significant figures and the partial result is precise to 10^{-8}, while the number to be added is precise to 10^{-10}. Therefore the final result must be quoted to a precision of 10^{-8}

Problem 1.7

My height is 165 cm or 1.65 m, written in scientific notation it is:

(a) 1.65 m × (10^9 nm / 1 m) = 1.65 × 10^9 nm
(b) 1.65 m × (10^3 mm / 1 m) = 1.65 × 10^3 mm
(c) 1.65 m × (10^2 cm / 1 m) = 1.65 × 10^2 cm
(d) 1.65 × 100 m = 1.65 m
(e) 1.65 m × (1 km / 10^3 m) = 1.65 × 10^{-3} km

Representing the length in m (d) is best suited for lengths of the order of my height.

Problem 1.8

My mass is approximately 75 kg so my weight would be:

(a) 75 kg × 10 m/s^2 × (10^6 μN / 1 N) = 7.5 × 10^8 μN
(b) 75 kg × 10 m/s^2 × (10^3 mN / 1 N) = 7.5 × 10^5 mN
(c) 75 kg × 10 m/s^2 = 7.5 × 10^2 N
(d) 75 kg × 10 m/s^2 × (1 kN / 10^3 N) = 7.5 × 10^{-1} kN
(e) 75 kg × 10 m/s^2 × (1 GN / 10^9 N) = 7.5 × 10^{-7} GN

Representing the weight as 750 N as done in part (c) or 0.75 kN as done in part (d) would be well suited for forces of the order of my weight.

Problem 1.9

We start with a total time of 7 × 10^6 s so that we can calculate:

(a) 7 × 10^6 s × (1 min / 60 s) = 10^5 min
(b) 10^5 min × (1 h / 60 min) = 2 × 10^3 h
(c) 2 × 10^3 h × (1 day / 24 h) = 8 × 10^1 day
(d) 8 × 10^1 day × (1 year / 365 day) = 2 × 10^{-1} year

Representing the total time as 80 days (c) or 0.2 years (d) would be well suited for times of the order of the time spent brushing your teeth.

Problem 1.10

We have a distance of 42.195 km, and a time of 2 h 2 min 11 s. A suitable combination of distance and time that leads to a quantity with the dimensions of speed $[L]/[T]$ would be the ratio of the distance to the time. For the calculations, we can convert the time into seconds as 7331 s. The average speed is thus:

(a) (42.195 km / 7331 s) × (3600 s / 1 h) = 2.072 × 10^1 km/h
(b) (42.195 km / 7331 s) × (10^3 m / 1 km) = 5.756 × 10^0 m/s = 5.756 m/s
(c) (42.195 km / 7331 s) = 5.756 × 10^{-3} km/s
(d) (42.195 km / 7331 s) × (3600 s / 1 h) × (10^3 m / 1 km) = 2.072 × 104 m/h
(e) (42195 m / 7331 s) × (1 s / 10^9 ns) × (10^6 μm / 1 m) = 5.756 × 10^{-3} μm/ns

Problem 1.11

You must be careful when converting units to any power other than one. For example, 1 s^2 = 1 s^2 × (10^3 ms / 1 s)^2 = 10^6 ms^2. With this in mind:

(a) 10 m/s^2 × (10^3 mm / 1 m) = 10^4 mm/s^2
(b) 10 m/s^2 × (1 s / 10^3 ms)^2 = 10^{-5} m/ms^2
(c) 10 m/s^2 × (1 km / 10^3 m) × (3600 s / 1 h)^2 = 10^5 km/h^2
(d) 10 m/s^2 × (1 Mm / 10^6 m) × [(3600 s / 1 h) × (24 × 365 h / 1 yr)]^2 = 10^{10} Mm/yr^2
(e) 10 m/s^2 × (10^6 μm / 1 m) × (1 s / 10^3 ms)^2 = 10^1 μm/ms^2 = 10 μm/ms^2

Problem 1.12

The area will be height × width. Since 1 cm = 10 mm = 10^4 μm = 10^{-2} m = 10^{-5} km, the area in various units is:

(a) 30 cm × 20 cm = 6 × 10^2 cm^2; note the number of significant figures
(b) 300 mm × 200 mm = 6 × 10^4 mm^2
(c) (30 × 10^4 μm) × (20 × 10^4 μm) = 6 × 10^{10} $μm^2$
(d) (30 × 10^{-2} m) × (20 × 10^{-2} m) = 6 × 10^{-2} m^2
(c) (30 × 10^{-5} km) × (20 × 10^{-5} km) = 6 × 10^{-8} km^2

Problem 1.13

For conversion purposes, first note that 1 km = 10^3 m = 10^4 dm = 10^5 cm, and for litres 1 L = 10^3 cm^3. Furthermore, although the radius given has four significant figures, we the multiplicative factor for the volume has been approximated to 4 with only one significant figure. With these in mind:

(a) 4 × (6378 km)^3 × (10^5 cm / 1 km)^3 = 1.038 × 10^{27} cm^3 = 10^{27} cm^3
(b) 4 × (6378 km)^3 × (10^3 m / 1 km)^3 = 1.038 × 10^{21} m^3 = 10^{21} m^3
(c) 4 × (6378 km)^3 = 1.038 × 10^{11} km^3 = 10^{11} km^3
(d) 4 × (6378 km)^3 × (10^4 dm / 1 km)^3 = 1.038 × 10^{24} dm^3 = 10^{24} dm^3
(e) Using (a) 1.038 × 10^{27} cm^3 × (1 L / 10^3 cm^3) = 1.038 × 10^{24} L = 10^{24} L

Problem 1.14

The density of an object measures the mass per unit volume. Since in Problem 1.13 we are told that the volume of a sphere is approximately $V = 4\,R^3$, the density should be:

$$\text{Density} = \frac{m}{V} = \frac{m}{4R^3}$$

We can now use this formula with R = 6378 km and $m = 5.9742 \times 10^{24}$ kg to find the required densities. Note that although the radius has four significant figures and the mass has five significant figures, we were told to approximate the volume by multiplying by 4, a number with only one significant figure. Note that 1 km = 10^3 m = 10^4 dm = 10^5 cm = 10^9 μm = 10^{15} pm and 1 kg = 10^3 g = 10^9 μg = 10^{12} ng = 10^{-3} Mg. The densities are:

(a) $(5.9742 \times 10^{24} \times 10^3\text{ g}) / [4 \times (6378 \times 10^5\text{ cm})^3] = 5.757 \times 10^0\text{ g/cm}^3 = 6\text{ g/cm}^3$

(b) $(5.9742 \times 10^{24}\text{ kg}) / [4 \times (6378 \times 10^3\text{ m})^3] = 5.757 \times 10^3\text{ kg/m}^3 = 6 \times 10^3\text{ kg/m}^3$

(c) $(5.9742 \times 10^{24} \times 10^{-3}\text{ Mg}) / [4 \times (6378\text{ km})^3] = 5.757 \times 10^9\text{ Mg/km}^3 = 6 \times 10^9\text{ Mg/km}^3$

(d) $(5.9742 \times 10^{24} \times 10^9\ \mu\text{g}) / [4 \times (6378 \times 10^{15}\text{ pm})^3] = 5.757 \times 10^{-24}\ \mu\text{g/pm}^3 = 6 \times 10^{-24}\ \mu\text{g/pm}^3$

(e) $(5.9742 \times 10^{24} \times 10^{12}\text{ ng}) / [4 \times (6378 \times 10^9\ \mu\text{m})^3] = 5.757 \times 10^{-3}\text{ ng}/\mu\text{m}^3 = 6 \times 10^{-24}\text{ ng}/\mu\text{m}^3$

Problem 1.15

An equation cannot be both dimensionally correct and wrong. It is important to note that the sum or difference of two terms is dimensionally correct only if both terms have the same dimensions.

(a) Wrong. On the left-hand side we have $[A] = [L^2]$, while on the right-hand side we have $[4\pi] \times [R] = 1 \times [L]$ since 4π is a dimensionless quantity. Since $[L^2] \neq [L]$ the equation is dimensionally wrong.

(b) Wrong. On the right-hand side we have $[x_1] = [L]$ being added to $[v_1\, t^2] = ([L]/[T]) \times [T^2] = [L] \times [T]$. Since $[L] \neq [L] \times [T]$, the two quantities cannot be added and the formula is dimensionally wrong.

(c) Correct. On the left-hand side we have $[V] = [L^3]$, and on the right-hand side we have $[xyz] = [L] \times [L] \times [L] = [L^3]$. Since both sides match the equation is dimensionally correct.

(d) Wrong. On the right-hand side we have $[m/2] = [T]/[2] = [T]$ being added to $[7] = 1$. Since $[T] \neq 1$, and a quantity with dimensions cannot be added to a dimensionless quantity the formula is dimensionally wrong.

Problem 1.16

We are told that $v \propto t^{-2}$, where v is the speed of the object, and t is the time it has travelled. Let t_1 = 4 s and t_2 = 8s so that v_1 = 10 m/s and v_2 is the speed we want to find.
From the proportionality relationship:

$$v_1 \propto \frac{1}{t_1^2} \quad \text{and} \quad v_2 \propto \frac{1}{t_2^2}$$

Taking the ratio v_2/v_1 we find:

$$\frac{v_2}{v_1} = \frac{1/t_2^2}{1/t_1^2} = \frac{t_1^2}{t_2^2}$$

With the given values:

$$v_2 = v_1 \frac{t_1^2}{t_2^2} = 10\,\text{m/s}\left(\frac{4\text{s}}{8\text{s}}\right)^2 = 2.5\,\text{m/s}$$

The proportionality between v and t described in this problem is often called an Inverse-Square Law. In our case, doubling the time reduces the speed by a factor of four; halving the time would increase the speed by a factor of four.

Problem 1.17

From Figure 1.9 we derived the relationship:

$$m_{\text{brain}} \propto M_{\text{body}}^{0.68}$$

We can use one of the data points listed in Table 1.5 alongside the proportionality relationship to find out the mass of the brain for the pygmy sloth. I will use line 12 in the table that lists values for the mainland three-toed sloth with brain mass m_s = 15.1 g and body mass M_s = 3.121 kg. We know that the pygmy sloth has a body mass of M_p = 3 kg and we want to find its brain mass m_p. From the proportionality relationship:

$$m_s \propto M_s^{0.68} \quad \text{and} \quad m_p \propto M_p^{0.68}$$

We can then solve for m_m:

$$m_p = m_s \times \left(\frac{M_p}{M_s}\right)^{0.68}$$

$$= (15.1\,\text{g}) \times \left(\frac{3}{3.121}\right)^{0.68} = 14.7\,\text{g}$$

Thus, a mass of approximately 15 g is what we would expect for the brain of the pigmy three-toed sloth. A brain mass of 200 g would put in question the measurement for the body mass, highlight an experimental error, an anomalous sample of the species (maybe with a severe brain defect), or simply an error in the reported data or calculations (possibly by one order of magnitude).

Problem 1.18

(a) The resulting double–logarithmic plot is shown in Figure 1. An organized approach to plotting these data is based on extending Table 1.6 to include the logarithm values of wingspan and mass, as shown here in Table 1.

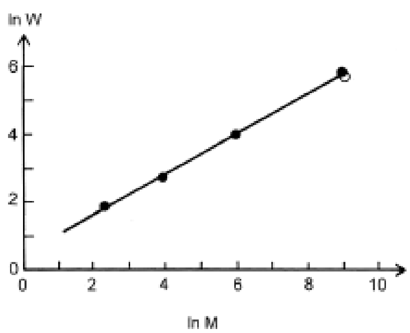

Figure 1

Using these logarithmic data, the given power law $W = a \cdot M^b$ is rewritten in the form $\ln W = b \cdot \ln M + \ln a$.

Table 1

Bird	W (cm)	ln W	M (g)	ln M
Hummingbird	7	1.95	10	2.30
Sparrow	15	2.71	50	3.91
Dove	50	3.91	400	5.99
Andean condor	320	5.77	11500	9.35
California condor	290	5.67	12000	9.39

The constants a and b are determined from this equation in the manner described in the Appendix *Graph analysis methods* on p. 9 – 12 in Chapter 1. For the analysis we do not choose data pairs from Table 1. As the graph in Fig. 1 illustrates, actual data points deviate from the line that best fits the data (represented by the solid line). To avoid that the deviation of actual data affects our results, the two data pairs used in the analysis are obtained directly from the solid line in Fig. 1. We choose $\ln W_1 = 2$ with $\ln M_1 = 2.6$ and $\ln W_2 = 6$ with $\ln M_2 = 9.6$. This leads to:

$$\begin{array}{ll} (I) & 2.0 = \ln(a) + 2.6 \cdot b \\ (II) & 6.0 = \ln(a) + 9.6 \cdot b \\ \hline (II)-(I) & 4.0 = (9.6-2.6)b \end{array} \qquad \textbf{(1)}$$

Thus, $b = 0.57$. Due to the fluctuations of the original data and the systematic errors you commit when reading data off a given plot, values in the interval $0.5 \le b \le 0.6$ may have been obtained.

Substituting the value we found for b in formula (I) of Eq. [1] yields: $2 = 1.48 + \ln a$, i.e., $\ln a = 0.52$ which corresponds to a value of $a = 1.7$.

(b) We use the given value for the pterosaurs' wingspan: $W = 11$ m = 1100 cm. The value has been converted to unit cm since that is the unit used when we developed our formula in part **(a).** We first rewrite the formula for the wingspan with the mass as the dependent variable:

$$W = a \cdot M^b \quad \Rightarrow \quad M = \left(\frac{W}{a}\right)^{1/b}$$

Entering the given value for the wingspan then leads to:

$$M = \left(\frac{1100}{1.7}\right)^{1/0.57} = 85400\ g = 85.4\ kg$$

The mass of pterosaurs did not exceed 85 kg.

(c) We use again the power law relation we found in part (a) and insert the given value for the person's mass:

$$W = 1.7 \cdot 70000^{0.57} = 580\ cm$$

A person of mass 70 kg would need a 5.8 m wing span. Notice that a pterosaur has a only 20 % larger mass than a human, but requires a 90 % increase in wingspan in order to achieve flight. Whereas a sparrow has a 500 % larger mass than a hummingbird, yet it requires only a 100 % increase in wingspan. This shows the exponential nature of this relationship.

Problem 1.19

Both the car and the cow produce the same mass, the cow producing methane, the car producing carbon dioxide. The molar mass of methane (CH_4) is 16 g/mol while the molar mass of carbon dioxide (CO_2) is 44 g/mol. This means in one gram of CH_4 you will find 1/16 mol, while in one gram of CO_2 you will find 1/44 mol. Therefore, if you have the same mass of CH_4 and CO_2, there will be 44/16 more moles of CH_4 than of CO_2. Since per mole CH_4 has 3.7 times the global warming potential of CO_2, the cow will have 3.7 × (44/16) = 10 times the global warming potential of the car.

Problem 1.20

According to the *Fuel Consumption Guide* (2011) published by Natural Resources Canada, the compact car with the best fuel efficiency in the city consumes on average 4.5 L of regular gas to travel a distance of 100 km. In downtown Toronto, I have noticed gas stations being somewhere between 10 to 30 blocks apart. In contrast, gas stations seem to be closer to each other in the suburban areas roughly 5 to 20 blocks apart. If a block is about 100 m it seems a good average distance between gas stations to be 2 km. This means that on average you will spend 90 mL of gas to go from one station to the next. Last week, the price of regular gas in Toronto oscillated between 120 ¢/L and 130 ¢/L. If the starting station had a price at 125 ¢/L then it would cost (125 ¢/L) × 0.09 L = 11.3 ¢ to drive to the next station. Therefore, to break even as you go searching for another gas station, the cost at the next station should be about 11 ¢/L cheaper.

Problem 1.21

To compare and rank the relative magnitudes we need to establish the point of comparison. Regardless of the comparison standard, the relative magnitudes and thus the ranking should come up the same. I will use the force exerted by my mother as the point of comparison and will call that force F_m. We will use the proportionality relationship:

$$F \propto \frac{M}{R^2},$$

where M is the mass of the object exerting the gravitational force, and R is the distance between that object and the baby. My mother's mass was about 70 kg and the moment I was born I was really close to my mother. The distance between our centers was probably around 10 cm.

The mass of the Moon is 7×10^{22} kg and it is at a distance of 4×10^{8} m from Earth. So the force exerted by the Moon F_M will be related to the force exerted by my mother F_m according to:

$$\frac{F_M}{F_m} = \frac{m_M / R_M^2}{m_m / R_m^2} = \left(\frac{m_M}{m_m}\right) \times \left(\frac{R_m^2}{R_M^2}\right) = \left(10^{21}\right) \times \left(6 \times 10^{-20}\right)$$

Thus, $F_M = 60\ F_m$.

The mass of the Sun is 2×10^{30} kg and it is at a distance of 2×10^{11} m from Earth. So the force exerted by the Sun F_S will be related to the force exerted by my mother F_m according to:

$$\frac{F_S}{F_m} = \frac{m_S / R_S^2}{m_m / R_m^2} = \left(\frac{m_S}{m_m}\right) \times \left(\frac{R_m^2}{R_S^2}\right) = \left(3 \times 10^{28}\right) \times \left(3 \times 10^{-25}\right)$$

Thus, $F_S = 10^4\ F_m$.

The mass of Jupiter is 2×10^{27} kg and it is on average at a distance of 8×10^{11} m from Earth. So the force exerted by Jupiter F_J will be related to the force exerted by my mother F_m according to:

$$\frac{F_J}{F_m} = \frac{m_J / R_J^2}{m_m / R_m^2} = \left(\frac{m_J}{m_m}\right) \times \left(\frac{R_m^2}{R_J^2}\right).$$

$$= \left(3 \times 10^{25}\right) \times \left(2 \times 10^{-26}\right)$$

Thus, $F_J = 0.6\ F_m$.

The mass of Mars is 6×10^{23} kg and it is on average at a distance of 2×10^{11} m from Earth. So the force exerted by Mars F_{Mars} will be related to the force exerted by my mother F_m according to:

$$\frac{F_{Mars}}{F_m} = \frac{m_{Mars} / R_{Mars}^2}{m_m / R_m^2} = \left(\frac{m_{Mars}}{m_m}\right) \times \left(\frac{R_m^2}{R_{Mars}^2}\right)$$

$$= \left(10^{23}\right) \times \left(3 \times 10^{-25}\right)$$

Thus, $F_{Mars} = 0.03\ F_m$.

The correct ranking from smallest to largest is:

$$F_{Mars} < F_J < F_m < F_M < F_S$$

Problem 1.22

Assume you do not need any breaks, for resting or eating. Furthermore, assume that you are moving the dirt just to your back so that you do not take time taking the dirt to another place. Furthermore, assume that you have an unlimited air supply, which is not realistic if you are taking the dirt from the front of the tunnel and placing it at your back. For your body to fit through the tunnel must have a diameter of at least your shoulder width, but you will need extra room to maneuver. Let's say the tunnel needs to be 1 m in diameter. The total volume of the tunnel is then the volume of a cylinder of radius R = 0.5 m and total length L = 100 m:

$$V = \pi R^2 L = 78.5\,\mathrm{m}^3 \cong 80\ \mathrm{m}^3$$

Since each spoonful is about 5 cm^3, it will take 80 m^3 / 5 cm^3 ~ 10^9 spoonfuls of dirt to dig the tunnel. If it takes you about one second per spoonful, then you will need 10^9 s ~ 40 years to dig the full volume of the tunnel. This result quite clearly negates all the previous assumptions about breaks for rest and food, and you will definitely need more time to find a place for the removed dirt.

Problem 1.23

The current life expectancy of a human is about 70 years ~ 2×10^9 s. Although the heart rate changes with age and with the level of activity, on average we could use the rest heart rate of an adult, which is around 70 bpm, or 70 beats per minute. Since 70 bpm = 1.2 beats/s, throughout your life your heart will beat:

$$\left(2 \times 10^9\ \mathrm{s}\right) \times \left(1.2\ ^{\mathrm{beats}}\!/_{\mathrm{s}}\right) \cong 2 \times 10^9\ \mathrm{beats}$$

In other words, your heart will beat about 2 billion times.

Problem 1.24

I trim my fingernails about once a week and the piece cut is about 1 mm wide, so the speed of growth is about 1 mm/week:

(a) 1 mm/week = (10^{-3} m)/($7 \times 24 \times 3600$ s) = 2×10^{-9} m/s

(b) 1 mm/week = (10^3 μm)/(7 day) = 1×10^2 μm/day = 100 μm/day

(c) 1 mm/week = (10^{-1} cm)/(7/365 year) = 5 cm/year

Problem 1.25

The number of grains of sand will be V_b/V_g, where V_b is the volume of the beach, and V_g is the volume of a grain of sand. Since the beach is box-shaped, $V_b = l \times w \times d$, where l is the length, w is the width, and d is the depth.

(a) Since 1 mm^3 = 1 mm^3 × (1 m / 10^3 mm)^3 = 10^{-9} m^3, with h = 4 m we have:

$$\frac{V_b}{V_g} = \frac{100\,\mathrm{m} \times 10\,\mathrm{m} \times 4\,\mathrm{m}}{10^{-9}\,\mathrm{m}^3} = 4 \times 10^{12}\ \mathrm{grains}$$

(b) Since the grains have the same volume of 10^{-9} m^3 but h = 2 m we have:

$$\frac{V_b}{V_g} = \frac{100\,\mathrm{m} \times 10\,\mathrm{m} \times 2\,\mathrm{m}}{10^{-9}\,\mathrm{m}^3} = 2 \times 10^{12}\ \mathrm{grains}$$

(c) With h = 4 m and the grains with average volume 2 mm^3 = 2 × 10^{-9} m^3 we have:

$$\frac{V_b}{V_g} = \frac{100\,\mathrm{m} \times 10\,\mathrm{m} \times 4\,\mathrm{m}}{2 \times 10^{-9}\,\mathrm{m}^3} = 2 \times 10^{12}\ \text{grains}$$

From (b) to (c) we have doubled the depth of the box. Doubling one of the dimensions of the box will only double the total volume of the box. If we had instead doubled each one of the dimensions of the box, the volume would have increased by a factor of 2^3 = 8. We also doubled the total volume of a grain of sand and this therefore compensates exactly the doubling of the depth of the box and the doubling of the volume of the box. This is different from our conversions of cubed units because in the conversions all three dimensions are being converted. Therefore, the final effect is the cube of the conversion in one of the linear dimensions.

Problem 1.26

Earth is roughly a sphere of radius 6400 km. The surface area of a sphere of radius R is given by $4\pi R^2$. However, about ¾ of the surface of the Earth is covered by water.
Therefore the dry surface of the Earth is an area of:

$$A_{dry} = \frac{1}{4} \times 4\pi R^2 = \pi\left(6400\,\mathrm{km}\right)^2 \cong 10^{14}\ \mathrm{m}^2$$

If a person stretches his/her arms out, the distance between the fingertips will be about the same as the height of the person. This distance is known as the arm span. If we assume and average height of 1.70 m then the person can reach around in a circle of diameter 1.70 m. However, if the person leans, the reach might be extended. We assume the reach of the person extends to a circle 2 m in diameter. The area for the person not to touch another person is then:

$$A_{person} = \pi\left(\frac{d}{2}\right)^2 \cong 3\,\mathrm{m}^2$$

The number of people able to stand without touching each other is then:

$$\frac{A_{dry}}{A_{person}} = \frac{10^{14}}{3} \cong 3 \times 10^{13} = 30\ \text{trillion}$$

This is about four thousand times the current world population.

Problem 1.27

Earth is roughly a sphere of radius 6400 km. The surface area of a sphere of radius R is given by $4\pi R^2$. However, about ¾ of the surface of the Earth is covered by water. Therefore the dry surface of the Earth is an area of:

$$A_{dry} = \frac{1}{4} \times 4\pi R^2 = \pi\left(6400\,\mathrm{km}\right)^2 \cong 10^{14}\ \mathrm{m}^2$$

An average person is about 1.70 m tall and about 60 cm wide at the shoulders. To simplify the estimate, a coffin would then be 2 m long by 1 m wide so each dead person will take up A_{person} = 2 m^2. There is enough room to bury:

$$\frac{A_{dry}}{A_{person}} = \frac{10^{14}}{2} \cong 5 \times 10^{13}\ \text{people}$$

According to an estimate done by the *Population Reference Bureau*, about 100 billion people (10^{11}) have lived and died on Earth. To the significant figures we are estimating, this number is not relevant. The current population on Earth is close to 7 billion (7 × 10^9), and according to the *United Nations* the current death rate is about 8.4 deaths per 1000 people per year. If the population on Earth remains stable at 7 billion, each year about (8.4/1000) × (7 × 10^9) = 6 × 10^7 people die. Therefore to cover the dry surface of Earth with graves we will have to wait:

$$\frac{\left(5 \times 10^{13}\ \text{people}\right)}{6 \times 10^7\ {}^{\text{people}}\!/\!_{\text{yr}}} = 8 \times 10^5\ \text{yr} \sim 1\,\text{Myr}$$

However, this estimate ignores the changes in birth rate as well as death rate. Both these quantities are quite hard to predict.

Problem 1.28

If two mammals *A* and *B* have the same densities then:

$$\frac{M_A}{V_A} = \frac{M_B}{V_B},$$

where *M* is the mass and *V* the volume of the mammal. Therefore we can reorganize for the ratio of the volumes:

$$\frac{V_B}{V_A} = \frac{M_B}{M_A}$$

That is, if all mammals have approximately the same density, the ratio of the volumes of two mammals is the same as the ratio of their masses. We can use Table 1.5 to find the required ratios:

$$\frac{V_{Hippopotamus}}{V_{Sheep}} = \frac{M_{Hippopotamus}}{M_{Sheep}} = \frac{1351}{52.1} \cong 26$$

and:

$$\frac{V_{Lion}}{V_{Chipmunk}} = \frac{M_{Lion}}{M_{Chipmunk}} = \frac{190.8}{0.075} \cong 2500$$

The volume of the ark is $V_{Ark} = (300 \times 50 \times 30)$ cubit$^3 \times (0.45$ m / cubit$)^3 \sim 4 \times 10^4$ m^3. Since the density of water is 1000 kg/m^3 we can use Table 1.5 to find the volume of each and all the animals according to:

$$V_{Animal} = \frac{M_{Animal}}{D_{Water}} = \frac{M_{Animal}}{1000\ \mathrm{kg/m^3}}$$

Adding the masses of all land mammals in Table 1.5 (all except 34, 40, 43 and 49), we get 1.41×10^4 kg but since we need two of each animal $M_{Total} = 2.82 \times 10^4$ kg. Therefore:

$$V_{Total} = \frac{M_{Total}}{D_{Water}} = \frac{2.82 \times 10^4\ \mathrm{kg}}{1000\ \mathrm{kg/m^3}} \cong 30\,\mathrm{m}^3$$

Since $V_{Total} < V_{Ark}$, two of every land mammal from Table 1.5 will fit in the ark. For the last part of the question, we can calculate the volume of the blue whale:

$$V_{BlueWhale} = \frac{M_{BlueWhale}}{D_{water}} = \frac{58059\,kg}{1000\ \mathrm{kg/m^3}} \cong 60\,\mathrm{m}^3$$

Since $V_{Total} < V_{Blue\ Whale}$, two of every land mammal from Table 1.5 could be housed in the volume equivalent to that of a blue whale.

