Solutions Manual to
Optoelectronics and Photonics: Principles and Practices, Second Edition
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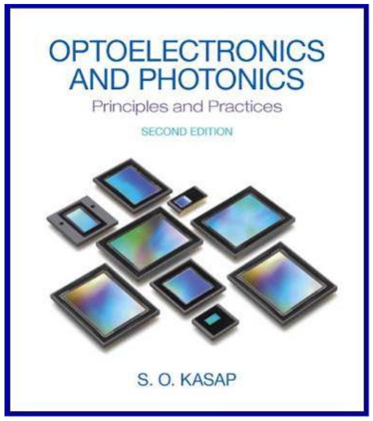
Safa Kasap



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Preliminary Solutions to Problems and Question
Chapter 2

Note: Printing errors and corrections are indicated in dark red. Currently none reported.

2.1 Symmetric dielectric slab waveguide Consider two rays such as 1 and 2′ interfering at point *P* in Figure 2.4 Both are moving with the same incidence angle but have different κ_m wavectors just before point *P*. In addition, there is a phase difference between the two due to the different paths taken to reach point *P*. We can represent the two waves as $E_1(y,z,t) = E_o\cos(\omega t - \kappa_m y - \beta_m z + \delta)$ and $E_2(y,z,t) = E_o\cos(\omega t + \kappa_m y - \beta_m z)$ where the $\kappa_m y$ terms have opposite signs indicating that the waves are traveling in opposite directions. δ has been used to indicate that the waves have a phase difference and travel different optical paths to reach point *P*. We also know that $\kappa_m = k_1\cos\theta_m$ and $\beta_m = k_1\sin\theta_m$, and obviously have the waveguide condition already incorporated into them through θ_m. Show that the superposition of E_1 and E_2 at *P* is given by

$$E(y,z,t) = 2E_o\cos(\kappa_m y - \tfrac{1}{2}\delta)\cos(\omega t - \beta_m z + \tfrac{1}{2}\delta)$$

What do the two cosine terms represent?

The planar waveguide is symmetric, which means that the intensity, E^2, must be either maximum (even m) or minimum (odd m) at the center of the guide. Choose suitable δ values and plot the relative magnitude of the electric field across the guide for $m = 0$, 1 and 2 for the following symmetric dielectric planar guide : $n_1 = 1.4550$, $n_2 = 1.4400$, $a = 10$ µm, $\lambda = 1.5$ µm (free space), the first three modes have $\theta_1 = 88.84°$, $\theta_2 = 87.67°$ $\theta_3 = 86.51°$. Scale the field values so that the maximum field is unity for $m = 0$ at the center of the guide. (Note: Alternatively, you can choose δ so that intensity (E^2) is the same at the boundaries at $y = a$ and $y = -a$; it would give the same distribution.)

Solution

$$E(y) = E_o\cos(\omega t - \kappa_m z - \beta_m z + \delta) + E_o\cos(\omega t + \kappa_m z - \beta_m z)$$

Use the appropriate trigonometric identity (see Appendix D) for $\cos A + \cos B$ to express it as a product of cosines $2\cos[(A+B)/2]\cos[(A-B)/2]$,

$$E(y,z,t) = 2E_o\cos(\kappa_m y - \tfrac{1}{2}\delta)\cos(\omega t - \beta_m z + \tfrac{1}{2}\delta)$$

The first cosine term represents the field distribution along y and the second term is the propagation of the field long the waveguide in the *z*-direction. Thus, the amplitude is

$$\text{Amplitude} = 2E_o\cos(\kappa_m y - \tfrac{1}{2}\delta)$$

The intensity is maximum or minimum at the center. We can choose $\delta = 0$ ($m = 0$), $\delta = \pi$ ($m = 1$), $\delta = 2\pi$ ($m = 2$), which would result in maximum or minimum intensity at the center. (In fact, $\delta = m\pi$). The field distributions are shown in Figure 2Q1-1.

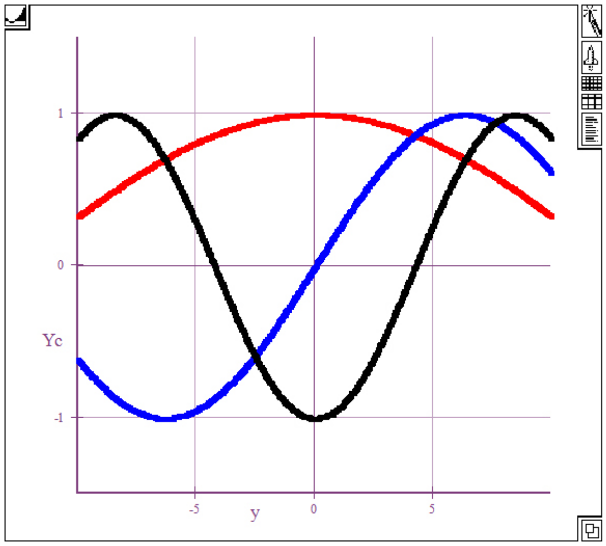


Figure 2Q1-1 Amplitude of the electric field across the planar dielectric waveguide. Red, $m = 0$; blue, $m = 1$; black, $m = 2$.

2.2 Standing waves inside the core of a symmetric slab waveguide Consider a symmetric planar dielectric waveguide. Allowed upward and downward traveling waves inside the core of the planar waveguide set-up a standing wave along y. The standing wave can only exist if the wave can be replicated after it has traveled along the y-direction over one round trip. Put differently, a wave starting at A in Figure 2.51 and traveling towards the upper face will travel along y, be reflected at B, travel down, become reflected again at A, and then it would be traveling in the same direction as it started. At this point, it must have an identical phase to its starting phase so that it can replicate itself and not destroy itself. Given that the wavevector along y is κ_m, derive the waveguide condition.

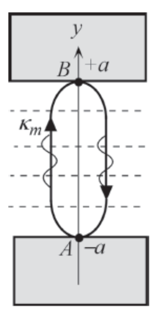


Figure 2.51 Upward and downward traveling waves along y set-up a standing wave. The condition for setting-up a standing wave is that the wave must be identical, able to replicate itself, after one round trip along y.

Solution

From Figure 2.51 it can be seen that the optical path is

$$AB + BA = 4a$$

With the ray under going a phase change ϕ with each reflection the total phase change is

$$\Delta\phi = 4a\kappa_m - 2\phi$$

The wave will replicate itself, is the phase is same after the one round-trip, thus

$$\Delta\phi = 4a\kappa_m - 2\phi = 2\pi m$$

and since $\kappa_m = k_1 \cos\theta_m = \dfrac{2\pi n_1}{\lambda}\cos\theta_m$ we get

$$\frac{2\pi n_1(2a)}{\lambda}\cos\theta_m - \phi_m = m\pi$$

as required.

2.3 Dielectric slab waveguide

(a) Consider the two parallel rays 1 and 2 in Figure 2.52. Show that when they meet at C at a distance y above the guide center, the phase difference is

$$\Phi_m = k_1 2(a - y)\cos\theta_m - \phi_m$$

(b) Using the waveguide condition, show that

$$\Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

(c) The two waves interfering at C can be most simply and conveniently represented as

$$E(y) = A\cos(\omega t) + A\cos[\omega t + \Phi_m(y)]$$

Hence find the amplitude of the field variation along y, across the guide. What is your conclusion?

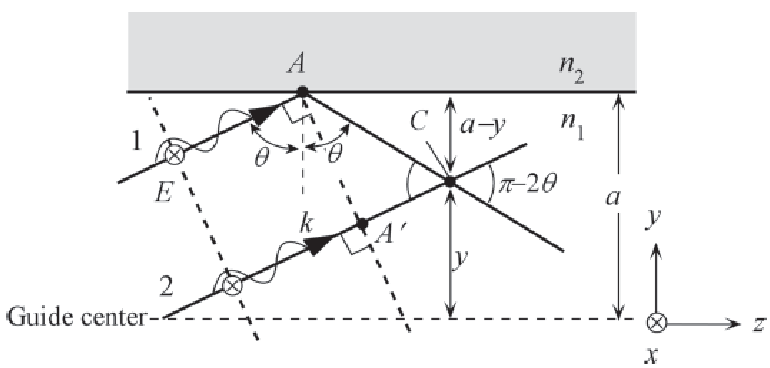


Figure 2.52 Rays 1 and 2 are initially in phase as they belong to the same wavefront. Ray 1 experiences total internal reflection at A. 1 and 2 interfere at C. There is a phase difference between the two waves.

Solution

(a) From the geometry we have the following:

$$(a - y)/AC = \cos\theta$$

and $$A'C/AC = \cos(\pi - 2\theta)$$

The phase difference between the waves meeting at C is

$$\Phi = kAC - \phi - kA'C = k_1AC - k_1AC\cos(\pi - 2\theta) - \phi$$

$$= k_1 AC[1 - \cos(\pi - 2\theta)] - \phi = k_1 AC[1 + \cos(2\theta)] - \phi$$

$$= k_1[(a - y)/\cos\theta][1 + 2\cos^2\theta - 1] - \phi$$

$$= k_1[(a - y)/\cos\theta][2\cos^2\theta] - \phi$$

$$= 2k_1(a - y)\cos\theta - \phi$$

(b) Given, $\left[\dfrac{2\pi(2a)n_1}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$

$\therefore$ $$\cos\theta_m = \frac{\lambda(m\pi + \phi_m)}{2\pi n_1(2a)} = \frac{m\pi + \phi_m}{k_1(2a)}$$

Then, $$\Phi_m = 2k_1(a - y)\cos\theta_m - \phi_m = 2k_1(a - y)\frac{m\pi + \phi_m}{k_1(2a)} - \phi_m$$

$\therefore$ $$\Phi_m = \left(1 - \frac{y}{a}\right)(m\pi + \phi_m) - \phi_m = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

$$\Phi_m = \Phi(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)\ \Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

(c) The two waves interfering at C are out phase by Φ,

$$E(y) = A\cos(\omega t) + A\cos[\omega t + \Phi_m(y)]$$

where A is an arbitrary amplitude. Thus,

$$E = 2A\cos[\omega t + \tfrac{1}{2}\Phi_m(y)]\cos[\tfrac{1}{2}\Phi_m(y)]$$

or $$E = \{2A\cos[\tfrac{1}{2}\Phi_m(y)]\}\cos(\omega t + \Phi') = E_o\cos(\omega t + \Phi')$$

in which $\Phi' = \Phi_m/2$, and $\cos(\omega t + \Phi')$ is the time dependent part that represents the wave phenomenon, and the curly brackets contain the effective amplitude. Thus, the amplitude E_o is

$$E_o = 2A\cos\left[\frac{m\pi}{2} - \frac{y}{2a}(m\pi + \phi_m)\right]$$

To plot E_o as a function of y, we need to find ϕ_m for $m = 0, 1, 2\ldots$ The variation of the field is a *truncated*) cosine function with its maximum at the center of the guide. See Figure 2Q1-1.

2.4 TE field pattern in slab waveguide Consider two parallel rays 1 and 2 interfering in the guide as in Figure 2.52. Given the phase difference

$$\Phi_m = \Phi_m(y) = m\pi - \frac{y}{a}(m\pi + \phi_m)$$

between the waves at C, distance y above the guide center, find the electric field pattern $E(y)$ in the guide. Recall that the field at C can be written as $E(y) = A\cos(\omega t) + A\cos[\omega t + \Phi_m(y)]$. Plot the field pattern for the first three modes taking a planar dielectric guide with a core thickness 20 μm, $n_1 = 1.455$ $n_2 = 1.440$, light wavelength of 1.3 μm.

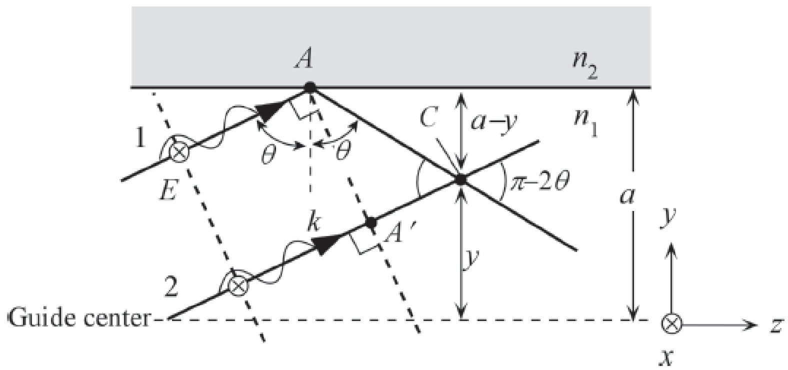


Figure 2.52 Rays 1 and 2 are initially in phase as they belong to the same wavefront. Ray 1 experiences total internal reflection at *A*. 1 and 2 interfere at *C*. There is a phase difference between the two

Solution

The two waves interfering at C are out phase by Φ,

$$E(y) = A\cos(\omega t) + A\cos[\omega t + \Phi_m(y)]$$

where *A* is an arbitrary amplitude. Thus,

$$E = 2A\cos\left[\omega t + \frac{1}{2}\Phi_m(y)\right]\cos\left[\frac{1}{2}\Phi_m(y)\right]$$

or

$$E = \left\{2A\cos\left[\frac{1}{2}\Phi_m(y)\right]\right\}\cos(\omega t + \Phi') = E_o\cos(\omega t + \Phi')$$

in which $\Phi' = \Phi_m/2$, and $\cos(\omega t + \Phi')$ is the time dependent part that represents the wave phenomenon, and the curly brackets contain the effective amplitude. Thus, the amplitude E_o is

$$E_o = 2A\cos\left[\frac{m\pi}{2} - \frac{y}{2a}(m\pi + \phi_m)\right]$$

To plot E_o as a function of y, we need to find ϕ_m for m = 0, 1 and 2 , the first three modes. From Example 2.1.1 in the textbook, the waveguide condition is

$$(2a)k_1\cos\theta_m - m\pi = \phi_m$$

we can now substitute for ϕ_m which has different forms for TE and TM waves to find,

TE waves

$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\dfrac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f_{TE}(\theta_m)$$

TM waves
$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\left(\frac{n_2}{n_1}\right)^2\cos\theta_m} = f_{TM}(\theta_m)$$

The above two equations can be solved graphically as in Example 2.1.1 to find θ_m for each choice of m. Alternatively one can use a computer program for finding the roots of a function. The above equations are functions of θ_m only for each m. Using a = 10 μm, λ = 1.3 μm, n_1 = 1.455 n_2 = 1.440, the results are:

TE Modes	$m = 0$	$m = 1$	$m = 2$
θ_m (degrees)	88.84	87.67	86.51
ϕ_m (degrees)	163.75	147.02	129.69
TM Modes	$m = 0$	$m = 1$	$m = 2$
θ_m (degrees)	88.84	87.67	86.51
ϕ_m (degrees)	164.08	147.66	130.60

There is no significant difference between the TE and TM modes (the reason is that n_1 and n_2 are very close).

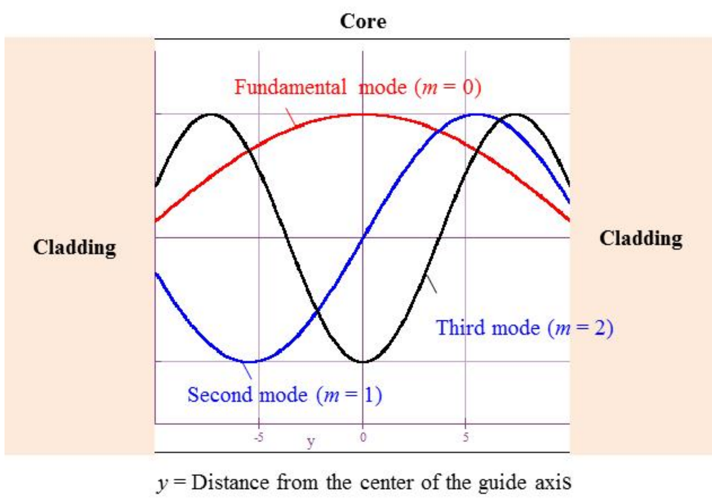


Figure 2Q4-1 Field distribution across the core of a planar dielectric waveguide

We can set A = 1 and plot E_o vs. y using

$$E_o = 2\cos\left[\frac{m\pi}{2} - \frac{y}{2a}(m\pi + \phi_m)\right]$$

with the ϕ_m and m values in the table above. This is shown in Figure 2Q4-1.

2.5 TE and TM Modes in dielectric slab waveguide Consider a planar dielectric guide with a core thickness 20 μm, n_1 = 1.455 n_2 = 1.440, light wavelength of 1.30 μm. Given the waveguide condition, and the expressions for phase changes ϕ and ϕ' in TIR for the TE and TM modes respectively,

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} \quad \text{and} \quad \tan\left(\tfrac{1}{2}\phi'_m\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\left(\frac{n_2}{n_1}\right)^2\cos\theta_m}$$

using a graphical solution find the angle θ for the fundamental TE and TM modes and compare their propagation constants along the guide.

Solution

The waveguide condition is

$$(2a)k_1\cos\theta_m - m\pi = \phi_m$$

we can now substitute for ϕ_m which has different forms for TE and TM waves to find,

TE waves $$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f_{TE}(\theta_m)$$

TM waves $$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\left(\frac{n_2}{n_1}\right)^2\cos\theta_m} = f_{TM}(\theta_m)$$

The above two equations can be solved graphically as in Example 2.1.1 to find θ_m for each choice of m. Alternatively one can use a computer program for finding the roots of a function. The above equations are functions of θ_m only for each m. Using a = 10 μm, λ = 1.3 μm, n_1 = 1.455 n_2 = 1.440, the results are:

TE Modes	$m = 0$
θ_m (degrees)	88.8361
$\beta_m = k_1\sin\theta_m$	7,030,883 m^{-1}
TM Modes	$m = 0$
θ'_m (degrees)	88.8340
$\beta'_m = k_1\sin\theta_m$	7,030,878 m^{-1}

Note that $\Delta\beta$ = 5.24 m^{-1} and the β-difference is only 7.5×10^{-5} %.

The following intuitive calculation shows how the small difference between the TE and TM waves can lead to dispersion that is time spread in the arrival times of the TE and TM optical signals.

Suppose that $\Delta\tau$ is the delay time between the TE and TM waves over a length L. Then,

$$\frac{\Delta\tau}{L} = \frac{1}{v_{\mathrm{TE}}} - \frac{1}{v_{\mathrm{TM}}} = \frac{\beta_{\mathrm{TE}}}{\omega} - \frac{\beta_{\mathrm{TM}}}{\omega} \approx \frac{\Delta\beta}{\omega} = \frac{(5.24\ \mathrm{m}^{-1})}{(1.45\times10^{15}\ \mathrm{rad/s})}$$

$$= 3.6\times10^{-15}\ \mathrm{s\ m}^{-1} = 0.0036\ \mathrm{ps\ m}^{-1}.$$

Over 1 km, the TE-TM wave dispersion is ~3.6 ps. One should be cautioned that we calculated dispersion using the phase velocity whereas we should have used the group velocity.

2.6 Group velocity We can calculate the group velocity of a given mode as a function of frequency ω using a convenient math software package. It is assumed that the math-software package can carry out symbolic algebra such as partial differentiation (the author used Livemath, , though others can also be used). The propagation constant of a given mode is $\beta = k_1\sin\theta$ where β and θ imply β_m and θ_m. The objective is to express β and ω in terms of θ. Since $k_1 = n_1\omega/c$, the waveguide condition is

$$\tan\left(a\frac{\beta}{\sin\theta}\cos\theta - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta - (n_2/n_1)^2\right]^{1/2}}{\cos\theta}$$

so that
$$\beta \approx \frac{\tan\theta}{a}\left[\arctan\left(\sec\theta\sqrt{\sin^2\theta - (n_2/n_1)^2}\right) + m(\pi/2)\right] = F_m(\theta) \qquad (1)$$

where $F_m(\theta)$ and a function of θ at a given m. The frequency ω is given by

$$\omega = \frac{c\beta}{n_1\sin\theta} = \frac{c}{n_1\sin\theta}F_m(\theta) \qquad (2)$$

Both β and ω are now a function of θ in Eqs (1) and (2). Then the group velocity is found by differentiating Eqs (1) and (2) with respect to θ, *i.e.*

$$v_g = \frac{d\omega}{d\beta} = \left[\frac{d\omega}{d\theta}\right]\times\left[\frac{d\theta}{d\beta}\right] = \frac{c}{n_1}\left[\frac{F_m'(\theta)}{\sin\theta} - \frac{\cos\theta}{\sin^2\theta}F_m(\theta)\right]\times\left[\frac{1}{F_m'(\theta)}\right]$$

i.e.
$$v_g = \frac{c}{n_1\sin\theta}\left[1 - \cot\theta\frac{F_m(\theta)}{F_m'(\theta)}\right] \qquad \textit{Group velocity, planar waveguide} \qquad (3)$$

where $F_m' = dF_m/d\theta$ is found by differentiating the second term of Eq. (1). For a given m value, Eqs (2) and (3) can be plotted parametrically, that is, for each θ value we can calculate ω and v_g and plot v_g vs. ω. Figure 2.11 shows an example for a guide with the characteristics in the figure caption. Using a convenient math-software package, or by other means, obtain the same v_g vs. ω behavior, discuss intermodal dispersion, and whether the Equation (2.2.2) is appropriate.

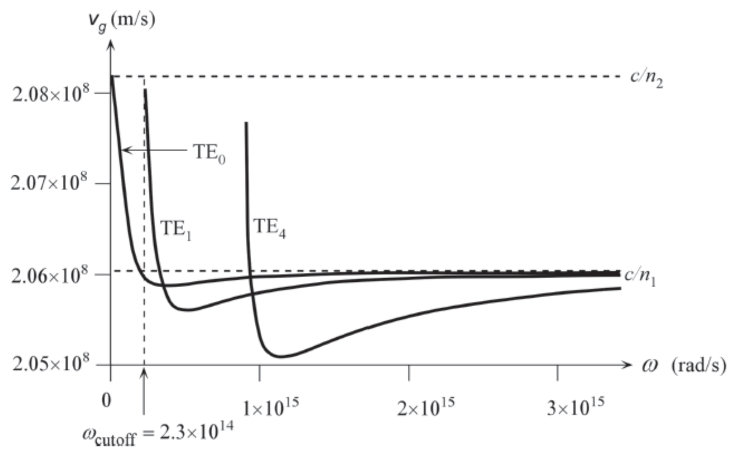


FIGURE 2.11 Group velocity v_g vs. angular frequency for three modes, m = 0, 1, and 4, for a planar dielectric waveguide that has n_1 = 1.455, n_2 = 1.440, a = 10 µm. TE$_0$ is for m = 0, *etc.* (Results calculated by using Livemath, a math-software application.)

Solution

The results shown in Figure 2.11, and Figure 2Q6-1 were generated by the author using LiveMath based on Eqs (1) and (3). Obviously other math software packages can also be used. The important conclusion from Figure 2.11 is that although the maximum group velocity is c/n_2, minimum group velocity is not c/n_1 and can be lower. Equation (2.2.2) in §2.2 is based on using $v_{g\max} = c/n_2$ and $v_{g\min} = c/n_1$, that is, taking the group velocity as the phase velocity. Thus, it is only approximate.

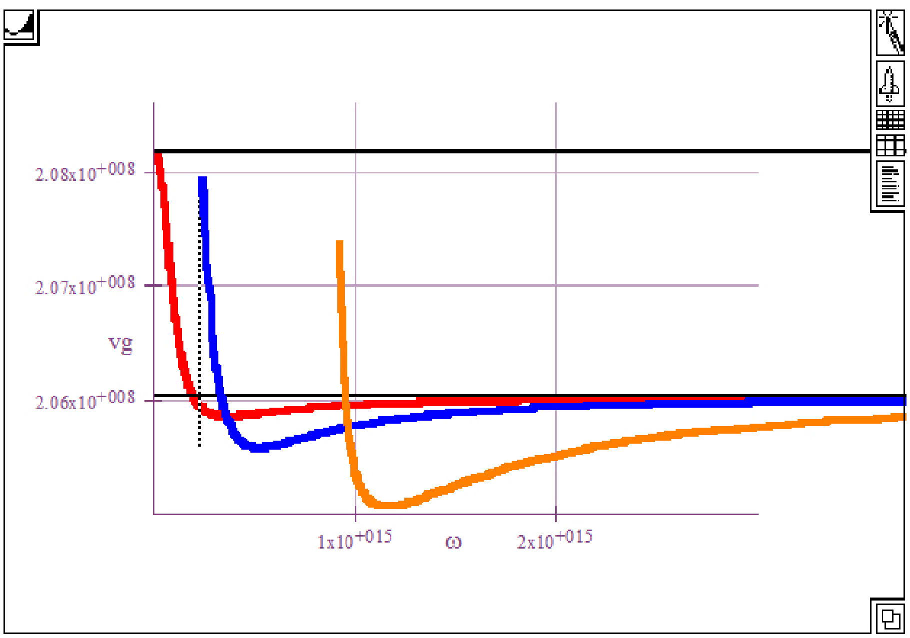


Figure 2Q6-1 Group velocity vs. angular frequency ω for three modes, TE$_0$ (red), TE$_1$ (blue) and TE$_4$ (orange) in a planar dielectric waveguide. The horizontal black lines mark the phase velocity in the core (bottom line, c/n_1) and in the cladding (top line, c/n_1). (LiveMath used)

2.7 Dielectric slab waveguide Consider a dielectric slab waveguide that has a thin GaAs layer of thickness 0.2 µm between two AlGaAs layers. The refractive index of GaAs is 3.66 and that of the AlGaAs layers is 3.40. What is the cut-off wavelength beyond which only a single mode can propagate in the waveguide, assuming that the refractive index does not vary greatly with the wavelength? If a radiation of wavelength 870 nm (corresponding to bandgap radiation) is propagating in the GaAs layer, what is the penetration of the evanescent wave into the AlGaAs layers? What is the mode field width (MFW) of this radiation?

Solution

Given n_1 = 3.66 (AlGaAs), n_2 = 3.4 (AlGaAs), $2a = 2\times10^{-7}$ m or a = 0.1 μm, for only a single mode we need

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} < \frac{\pi}{2}$$

$$\therefore \qquad \lambda > \frac{2\pi a(n_1^2 - n_2^2)^{1/2}}{\frac{\pi}{2}} = \frac{2\pi(0.1\ \mu\text{m})(3.66^2 - 3.40^2)^{1/2}}{\frac{\pi}{2}} = 0.542\ \mu\text{m}.$$

The cut-off wavelength is 542 nm.

When λ = 870 nm,

$$V = \frac{2\pi(1\ \mu\text{m})(3.66^2 - 3.40^2)^{1/2}}{(0.870\ \mu\text{m})} = 0.979 < \pi/2$$

Therefore, λ = 870 nm is a single mode operation.

For a rectangular waveguide, the fundamental mode has a *mode field width*

$$2w_o = \text{MFW} \approx 2a\frac{V+1}{V} = (0.2\ \mu\text{m})\frac{0.979+1}{0.979} = \mathbf{0.404\ \mu m.}$$

The decay constant α of the evanescent wave is given by,

$$\alpha = \frac{V}{a} = \frac{0.979}{0.1\ \mu\text{m}} = 9.79\ (\mu\text{m})^{-1} \text{ or } 9.79\times10^6\ \text{m}^{-1}.$$

The penetration depth

$$\delta = 1/\alpha = 1/\ [9.79\ (\mu\text{m})^{-1}] = 0.102\ \mu\text{m}.$$

The penetration depth is half the core thickness. The width between two e^{-1} points on the field decays in the cladding is

$$\text{Width} = 2a + 2\times\delta = 0.2\ \mu\text{m} + 2(0.102)\ \mu\text{m} = \mathbf{0.404\ \mu m.}$$

2.8 Dielectric slab waveguide Consider a slab dielectric waveguide that has a core thickness (2a) of 20 μm, n_1 = 3.00, n_2 = 1.50. Solution of the waveguide condition in Eq. (2.1.9) (in Example 2.1.1) gives the mode angles θ_0 and θ_1 for the TE_0 and TE_1 modes for selected wavelengths as summarized in Table 2.7. For each wavelength calculate ω and β_m and then plot ω vs. β_m. On the same plot show the lines with slopes c/n_1 and c/n_2. Compare your plot with the dispersion diagram in Figure 2.10

Table 2.7 The solution of the waveguide condition for a = 10 μm, n_1 = 3.00, n_2 = 1.50 gives the incidence angles θ_0 and θ_1 for modes 0 and 1 at the wavelengths shown.

λ, μm	15	20	25	30	40	45	50	70	100	150	200
θ_0°	77.8	74.52	71.5	68.7	63.9	61.7	59.74	53.2	46.4	39.9	36.45
θ_1°	65.2	58.15	51.6	45.5	35.5	32.02	30.17	-	-	-	-

Solution

Consider the case example for $\lambda = 25$ μm = 25×10^{-6} m.

The free space propagation constant $k = 2\pi/\lambda = 2\pi / 25\times10^{-6}$ m = 2.513×10^{5} m^{-1}.

The propagation constant within the core is $k_1 = n_1k$ = (3.00)(2.513×10^5 m^{-1}) = 7.540×10^5 m^{-1}.

The angular frequency $\omega = ck$ = (3×10^8 m s^{-1})(2.513×10^5 m^{-1}) = **7.54×10^{13} s^{-1}**.

Which is listed in Table 2Q8-1 in the second row under λ = 25 μm.

The propagation constant along the guide, along z is given by Eq. (2.1.4) so that

$$\beta_m = k_1\sin\theta_m$$

or $\beta_0 = k_1\sin\theta_0$ = (7.540×10^5 m^{-1})sin(71.5°) = 7.540×10^5 m^{-1} = **7.15×10^5 m^{-1}**.

which is the value listed in bold in Table 2Q8-1 for the m = 0 mode at λ = 25 μm.

Similarly $\beta_1 = k_1\sin\theta_1$ = (7.540×10^5 m^{-1})sin(51.6°) = 7.540×10^5 m^{-1} = **5.91×10^5 m^{-1}**.

which is also listed in bold in Table 2Q8-1. We now have both β_0 and β_1 at ω = **2.54×10^{13} s^{-1}**.

We can plot this 1 point for the m =0 mode at $\beta_0 = 7.15\times10^5$ m^{-1} along the x-axis, taken as the β-axis, and $\omega = 2.54\times10^{13}$ s^{-1} along the y-axis, taken as the ω-axis, as shown in Figure 2Q8-1. We can also plot the 1 point we have for the m = 1 mode.

Propagation constants (β) at other wavelengths and hence frequencies (ω) can be similarly calculated. The results are listed in Table 2Q8-1 and plotted in Figure 2Q8-1. This is the dispersion diagram. For comparison the dispersion ω vs β for the core and the cladding are also shown. They are drawn so that the slope is c/n_1 for the core and c/n_2 for the cladding.

Thus, the solutions of the waveguide condition as in Example 2.1.1 generates the data in Table 2Q8-1 for $2a$ = 10 μm, n_1 = 3; n_2 = 1.5.

Table2Q8-1 Planar dielectric waveguide with a core thickness ($2a$) of 20 μm, n_1 = 3.00, n_2 = 1.50.

λ, μm	15	20	**25**	30	40	45	50	70	100	150	200
ω $\times10^{13}$ s^{-1}	12.6	9.43	**7.54**	6.283	4.71	4.19	3.77	2.69	1.89	1.26	0.94
θ_0°	77.8	74.52	**71.5**	68.7	63.9	61.7	59.74	53.2	46.4	39.9	36.45
θ_1°	65.2	58.15	**51.6**	45.5	35.5	32.02	30.17	-	-	-	-

β_0 $\times 10^5$ 1/m	12.3	9.08	**7.15**	5.85	4.23	3.69	3.26	2.16	1.37	0.81	0.56
β_1 $\times 10^5$ 1/m	11.4	8.01	**5.91**	4.48	2.74	2.22	1.89	-	-	-	-

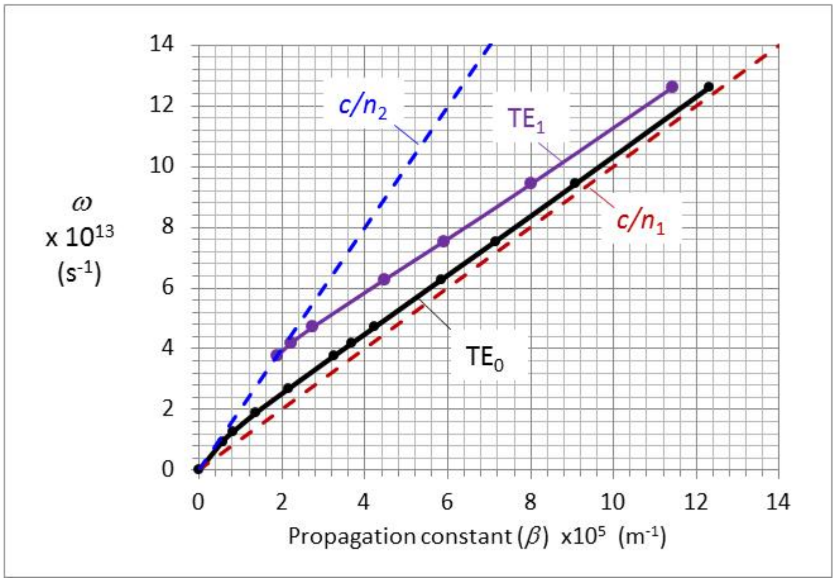


Figure 2Q8-1 Dispersion diagram for a planar dielectric waveguide that has a core thickness (2a) of 20 μm, n_1 = 3.00, n_2 = 1.50. Black, TE$_0$ mode. Purple: TE$_1$ mode. Blue: Propagation along the cladding. Red: Propagation along the core.

Author's Note: Remember that the slope at a particular frequency ω is the group velocity at that frequency. As apparent, for the TE$_0$ (m = 0) mode, this slope is initially (very long wavelengths) along the blue curve at low frequencies but then along the red curve at high frequencies (very short wavelengths). The group velocity changes from c/n_2 to c/n_1.

2.9 Dielectric slab waveguide Dielectric slab waveguide Consider a planar dielectric waveguide with a core thickness 10 μm, n_1 = 1.4446, n_2 = 1.4440. Calculate the V-number, the mode angle θ_m for m = 0 (use a graphical solution, if necessary), penetration depth, and mode field width, MFW = 2a + 2δ, for light wavelengths of 1.0 μm and 1.5 μm. What is your conclusion? Compare your MFW calculation with $2w_o = 2a(V+1)/V$. The mode angle θ_0, is given as θ_0 = 88.85° for λ = 1 μm and θ_0 = 88.72° for λ = 1.5 μm for the fundamental mode m = 0.

Solution

λ = 1 μm, n_1 = 1.4446, n_2 = 1.4440, a = 5 μm. Apply

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2}$$

to obtain **V = 1.3079**

Solve the waveguide condition

$$\tan\left(ak_1\cos\theta_m - m\frac{\pi}{2}\right) = \frac{\left[\sin^2\theta_m - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m} = f(\theta_m)$$

graphically as in Example 2.1.1 to find: θ_c = 88.35° and the mode angle (for m = 0) is θ_o = 88.85°.

Then use

$$\frac{1}{\delta_m} = \alpha_m = \frac{2\pi n_2\left[\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_m - 1\right]^{1/2}}{\lambda}$$

to calculate the penetration depth:

$\delta = 1/\alpha$ = 5.33 μm.

∴ MFW = $2a + 2\delta$ = **20.65 μm**

We can also calculate MFW from

MFW = $2a(V+1)/V$ = 2(5 μm)(1.3079+1)/(1.3079) = **17.6 μm** (Difference = 15%)

λ = 1.5 μm, **V = 0.872**, single mode. Solve waveguide condition graphically that the mode angle is θ_o = 88.72°.

$\delta = 1/\alpha$ = 9.08 μm.

∴ MFW = $2a + 2\delta$ = **28.15 μm**.

Compare with MFW = $2a(V+1)/V$ = 2(5 μm)(0.872+1)/(0.872) = **21.5 μm** (Difference = 24%)

Notice that the MFW from $2a(V+1)/V$ gets worse as V decreases. The reason for using MFW = $2a(V+1)/V$, is that this equation provides a single step calculation of MFW. The calculation of the penetration depth δ requires the calculation of the incidence angle θ and ϕ.

Author's Note: Consider a more extreme case

λ = 5 μm, V = 0.262, single mode. Solve waveguide condition graphically to find that the mode angle is θ_o = 88.40°.

$\delta = 1/\alpha$ = 77.22 μm.

∴ MFW = $2a + 2\delta$ = 164.4 μm.

Compare with MFW = $2a(V+1)/V$ = 2(5 μm)(0.262 + 1)/(0.262) = 48.2 μm (Very large difference.)

2.10 A multimode fiber Consider a multimode fiber with a core diameter of 100 μm, core refractive index of 1.4750, and a cladding refractive index of 1.4550 both at 850 nm. Consider operating this fiber at λ = 850 nm. (a) Calculate the V-number for the fiber and estimate the number of modes. (b) Calculate the wavelength beyond which the fiber becomes single mode. (c) Calculate the numerical aperture. (d) Calculate the maximum acceptance angle. (e) Calculate the modal dispersion $\Delta\tau$ and hence the bit rate × distance product.

Solution

Given $n_1 = 1.475$, $n_2 = 1.455$, $2a = 100\times10^{-6}$ m or $a = 50$ μm and $\lambda = 0.850$ μm. The V-number is,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(50\ \mu\text{m})(1.475^2 - 1.455^2)^{1/2}}{(0.850\ \mu\text{m})} = \mathbf{89.47}$$

Number of modes M,

$$M = \frac{V^2}{2} = \frac{89.47^2}{2} \approx \mathbf{4002}$$

The fiber becomes monomode when,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} < 2.405$$

or $$\lambda > \frac{2\pi a\left(n_1^2 - n_2^2\right)^{1/2}}{2.405} = \frac{2\pi(50\ \mu\text{m})(1.475^2 - 1.455^2)^{1/2}}{2.405} = \mathbf{31.6\ \mu m}$$

For wavelengths longer than 31.6 μm, the fiber is a single mode waveguide.

The numerical aperture NA is

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.475^2 - 1.455^2)^{1/2} = 0.242$$

If $\alpha_{\max}$ is the maximum acceptance angle, then,

$$\alpha_{\max} = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin(0.242/1) = \mathbf{14°}$$

Modal dispersion is given by

$$\frac{\Delta\tau_{\text{intermode}}}{L} = \frac{n_1 - n_2}{c} = \frac{1.475 - 1.455}{3\times10^8\ \ \text{m s}^{-1}}$$

$$= 66.7\ \text{ps m}^{-1} \text{ or } \mathbf{67.6\ ns\ per\ km}$$

Given that $\sigma \approx 0.29\Delta\tau$, maximum bit-rate is

$$BL = \frac{0.25L}{\sigma_{\text{total}}} \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{(0.29)(66.7\ \text{ns km}^{-1})}$$

i.e. $BL = \mathbf{13\ Mb\ s^{-1}\ km}$ (only an estimate)

We neglected material dispersion at this wavelength which would further decrease BL. Material dispersion and modal dispersion must be combined by

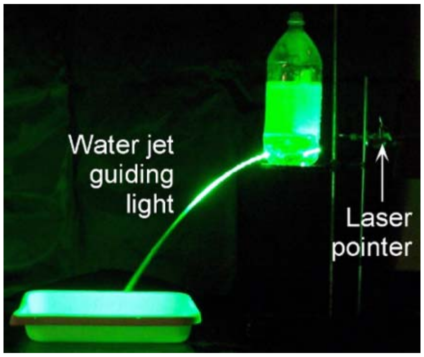
$$\sigma_{total}^2 = \sigma_{\text{intermode}}^2 + \sigma_{\text{material}}^2$$

For example, assuming an LED with a spectral rms deviation σ_λ of about 20 nm, and a $D_m \approx -200$ ps km^{-1} nm^{-1} (at about 850 nm)we would find the material dispersion as

$$\sigma_{\text{material}} = -(200 \text{ ps km}^{-1} \text{ nm}^{-1})(20 \text{ nm})(1 \text{ km}) \approx 4000 \text{ ps km}^{-1} \text{ or } 4 \text{ ns km}^{-1},$$

which is substantially smaller than the intermode dispersion and can be neglected.

2.11 A water jet guiding light One of the early demonstrations of the way in which light can be guided along a higher refractive index medium by total internal reflection involved illuminating the starting point of a water jet as it comes out from a water tank. The refractive index of water is 1.330. Consider a water jet of diameter 3 mm that is illuminated by green light of wavelength 560 nm. What is the V-number, numerical aperture, total acceptance angle of the jet? How many modes are there? What is the cut-off wavelength? The diameter of the jet increases (slowly) as the jet flows away from the original spout. However, the light is still guided. Why?



Light guided along a thin water jet. A small hole is made in a plastic soda drink bottle full of water to generate a thin water jet. When the hole is illuminated with a laser beam (from a green laser pointer), the light is guided by total internal reflections along the jet to the tray. Water with air bubbles (produced by shaking the bottle) was used to increase the visibility of light. Air bubbles scatter light and make the guided light visible. First such demonstration has been attributed to Jean-Daniel Colladon, a Swiss scientist, who demonstrated a water jet guiding light in 1841.

Solution

V-number

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi\times1.5\times10^{-3}/550\times10^{-9})(1.330^2 - 1.000^2)^{1/2} =$ **15104**

Numerical aperture

NA= $(n_1^2 - n_2^2)^{1/2} = (1.330^2 - 1.000^2)^{1/2} =$ **0.8814**

Total acceptance angle, assuming that the laser light is launched within the water medium

$\sin\alpha_{\text{max}} = \text{NA}/n_0 = 0.113/1.33$ or $\alpha_{max} = 41.4°$.

Total acceptance $2\alpha_o =$ **82.8°**

Modes $= M = V^2/2 = (15104)^2/2 =$ **1.14×10^8 modes** (~100 thousand modes)

The curoff wavelength corresponds to $V = 2.405$, that is $V = (2\pi a/\lambda)\text{NA} = 2.405$

$\lambda_c = [2\pi a\text{NA}]/2.405 = [(2\pi)(4\ \mu\text{m})(0.8814)]/2.405 =$ **3.5 mm**

The large difference in refractive indices between the water and the air ensures that total internal reflection occurs even as the width of the jet increases, which changes the angle of incidence.

2.12 Single mode fiber Consider a fiber with a 86.5%SiO_2-13.5%GeO_2 core of diameter of 8 μm and refractive index of 1.468 and a cladding refractive index of 1.464 both refractive indices at 1300 nm where the fiber is to be operated using a laser source with a half maximum width of 2 nm. (a) Calculate the V-number for the fiber. Is this a single mode fiber? (b) Calculate the wavelength below which the fiber becomes multimode. (c) Calculate the numerical aperture. (d) Calculate the maximum acceptance angle. (e) Obtain the material dispersion and waveguide dispersion and hence estimate the bit rate × distance product (B×L) of the fiber.

Solution

(a) Given $n_1 = 1.475$, $n_2 = 1.455$, $2a = 8\times10^{-6}$ m or $a = 4$ μm and $\lambda = 1.3$ μm. The V-number is,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(4\ \mu\text{m})(1.468^2 - 1.464^2)^{1/2}}{(1.3\ \mu\text{m})} = \mathbf{2.094}$$

(b) Since $V < 2.405$, this is a single mode fiber. The fiber becomes multimode when

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} > 2.405$$

or

$$\lambda < \frac{2\pi a\left(n_1^2 - n_2^2\right)^{1/2}}{2.405} = \frac{2\pi(4\ \mu\text{m})\left(1.468^2 - 1.464^2\right)^{1/2}}{2.405} = \mathbf{1.13\ \mu m}$$

For wavelengths shorter than 1.13 μm, the fiber is a multi-mode waveguide.

(c) The numerical aperture NA is

$$NA = \left(n_1^2 - n_2^2\right)^{1/2} = (1.468^2 - 1.464^2)^{1/2} = \mathbf{0.108}$$

(d) If $\alpha_{\max}$ is the maximum acceptance angle, then,

$$\alpha_{\max} = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin(0.108/1) = \mathbf{6.2°}$$

so that the *total acceptance angle* is **12.4°.**

(e) At $\lambda = 1.3$ μm, from D vs. λ, Figure 2.22, $D_m \approx -7.5$ ps km^{-1} nm^{-1}, $D_w \approx -5$ ps km^{-1} nm^{-1}.

$$\frac{\Delta\tau_{1/2}}{L} = \left|D_m + D_w\right|\Delta\lambda_{1/2}$$

$= |-7.5-5$ ps km^{-1} nm$^{-1}|(2$ nm$) = 15$ ps km^{-1} + 10 ps km^{-1}

$= 0.025$ ns km^{-1}

Obviously material dispersion is **15 ps km^{-1}** and waveguide dispersion is **10 ps km^{-1}**

The maximum bit-rate distance product is then

$$BL \approx \frac{0.59L}{\Delta\tau_{1/2}} = \frac{0.59}{0.025\ \text{ns km}^{-1}} = \mathbf{23.6\ Gb\ s^{-1}\ km.}$$

2.13 Single mode fiber Consider a step-index fiber with a core of diameter of 9 μm and refractive index of 1.4510 at 1550 nm and a normalized refractive index difference of 0.25% where the fiber is to be operated using a laser source with a half-maximum width of 3 nm. At 1.55 μm, the material and waveguide dispersion coefficients of this fiber are approximately given by D_m = 15 ps km^{-1} nm^{-1} and D_w = −5 ps km^{-1} nm^{-1}. (a) Calculate the V-number for the fiber. Is this a single mode fiber? (b) Calculate the wavelength below which the fiber becomes multimode. (c) Calculate the numerical aperture. (d) Calculate the maximum total acceptance angle. (e) Calculate the material, waveguide and chromatic dispersion per kilometer of fiber. (f) Estimate the bit rate × distance product (B×L) of this fiber. (g) What is the maximum allowed diameter that maintains operation in single mode? (h) What is the mode field diameter?

Solution

(a) The normalized refractive index difference Δ and n_1 are given.

Apply, $\Delta = (n_1 - n_2)/n_1 = (1.451 - n_2)/1.451 = 0.0025$, and solving for n_2 we find $n_2 = 1.4474$.

The V-number is given by

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(4.5\ \mu\text{m})(1.4510^2 - 1.4474^2)^{1/2}}{(1.55\ \mu\text{m})} = \mathbf{1.87;\ single\ mode\ fiber.}$$

(b) For multimode operation we need

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(4.5\ \mu\text{m})(1.4510^2 - 1.4474^2)^{1/2}}{\lambda} > 2.405$$

∴ $\lambda < 1.205\ \mu$m.

(c) The numerical aperture NA is

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.4510^2 - 1.4474^2)^{1/2} = \mathbf{0.1025.}$$

(d) If $\alpha_{\max}$ is the maximum acceptance angle, then,

$$\alpha_{\max} = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin(0.1025/1) = \mathbf{5.89°}$$

Total acceptance angle $2a_{\max}$ is **11.8 °**.

(e) Given, D_w = −5 ps km^{-1} nm^{-1} and D_m = 15 ps km^{-1} nm^{-1}.

Laser diode spectral width (FWHM) $\Delta\lambda_{1/2}$ = 3 nm

Material dispersion $\Delta\tau_{1/2}/L = |D_m|\Delta\lambda_{1/2}$ = (15 ps km^{-1} nm^{-1})(3 nm)
= 45 ps km^{-1}

Waveguide dispersion $\Delta\tau_{1/2}/L = |D_w|\Delta\lambda_{1/2}$ = (−5 ps km^{-1} nm^{-1})(3 nm)
= −15 ps km^{-1}

Chromatic dispersion, $\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2}$ = (−5 ps km^{-1} nm^{-1} + 15 ps km^{-1} nm^{-1})(3 nm)
= 30 ps km^{-1}

(f) Maximum bit-rate would be

$$BL \approx \frac{0.59L}{\Delta\tau_{1/2}} = \frac{0.59}{(\Delta\tau_{1/2}/L)} = \frac{0.59}{(30\times10^{-12}\ \text{s km}^{-1})} = 20\ \text{Gb s}^{-1}\ \text{km}$$

i.e. $BL \approx$ **20 Mb s^{-1} km** (only an estimate)

(g) To find the maximum diameter for SM operation solve,

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(a\ \mu\text{m})(1.4510^2 - 1.4474^2)^{1/2}}{(1.55\ \mu\text{m})} = 2.405$$

$\therefore$ $2a$ = **11.5 μm.**

(h) The mode filed diameter $2w$ is

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6}) = \mathbf{12.2\ \mu m}$$

2.14 Normalized propagation constant *b* Consider a weakly guiding step index fiber in which $(n_1 - n_2)/n_1$ is very small. Show that

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{(\beta/k) - n_2}{n_1 - n_2}$$

Note: Since Δ is very small, $n_2/n_1 \approx 1$ can be assumed were convenient. The first equation can be rearranged as

$$\beta/k = [n_2^2 + b(n_1^2 - n_2^2)]^{1/2} = n_2^2(1+x)^{1/2};\ x = b(n_1^2 - n_2^2)/n_2^2$$

where x is small. Taylor's expansion in x to the first linear term would then provide a linear relationship between β and b.

Solution

$$\beta/k = [n_2^2 + b(n_1^2 - n_2^2)]^{\frac{1}{2}} = n_2(1+x)^{\frac{1}{2}}$$

where $x = b\left(\frac{n_1^2}{n_2^2} - 1\right)$

Taylor expansion around $x = 0$ and truncating the expression, keeping only the linear term yields,

$$\beta/k \approx n_2 + \frac{nx}{2} = n_2\left[1 + \frac{x}{2}\right] = n_2\left[1 + \frac{b}{2}\left(\frac{n_1^2}{n_2^2} - 1\right)\right] = n_2 + \frac{b}{2\left(\frac{n_1^2 - n_2^2}{n_2}\right)}$$

then using the assumption $\frac{n_1}{n_2} \approx 1$ we get

$$\beta/k = n_2 + b(n_1 - n_2)$$

and

$$b \approx \frac{(\beta / k) - n_2}{n_1 - n_2}$$

as required.

2.15 Group velocity of the fundamental mode Reconsider Example 2.3.4, which has a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. Use the equation

$$b \approx \frac{(\beta / k) - n_2}{n_1 - n_2}; \;\; \beta = n_2 k[1 + b\Delta]$$

to recalculate the propagation constant β. Change the operating wavelength to λ' by a small amount, say 0.01%, and then recalculate the new propagation constant β'. Then determine the group velocity v_g of the fundamental mode at 1.5 μm, and the group delay τ_g over 1 km of fiber. How do your results compare with the findings in Example 2.3.4?

Solution

From example 2.3.4, we have

$$b = 0.3860859\,,\; k = 4188790\,\text{m}^{-1}\,,\; \omega = \frac{2\pi c}{\lambda} = 1.256637\times 10^{15}\,\text{s}^{-1}$$

$$\beta = n_2 k[1 + b\Delta] = (1.4400)(4188790\text{m}^{-1})\left[1 + (0.3860859)\frac{(1.4480 - 1.4400)}{1.4480}\right]$$

$$\beta = 6044795\,\text{m}^{-1}$$

$$\lambda' = 1.5\,\mu\text{m}(1 + 1.001) = 1.5015\,\mu\text{m}\,,\; b' = 0.3854382\,,\; k' = 4184606\,\text{m}^{-1}\,,\; \omega' = 1.255382\times 10^{15}\,\text{s}^{-1}$$

$$\beta' = n_2 k'[1 + b\Delta] = (1.4400)(418406\text{m}^{-1})\left[1 + (0.3854382)\frac{(1.4480 - 1.4400)}{1.4800}\right]$$

$$\beta' = 6038736\text{m}^{-1}$$

Group Velocity

$$v_g = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.255382 - 1.256637)\times 10^{15}\,\text{s}^{-1}}{(6.038736 - 6.044795)\times 10^{6}\,\text{m}^{-1}} = 2.0713\times 10^{8}\,\text{m s}^{-1}$$

$\tau_g = 4.83\,\mu\text{s}$ over 1 km.

Comparing to Example 2.3.4

$$\%\text{diff} = \frac{|2.0713 - 2.0706|}{2.0706}\times 100\% = 0.03\%$$

2.16 A single mode fiber design The Sellmeier dispersion equation provides n vs. λ for pure SiO_2 and SiO_2-13.5 mol.%GeO_2 in Table1.2 in Ch. 1. The refractive index increases linearly with the addition of GeO_2 to SiO_2 from 0 to 13.5 mol.%. A single mode step index fiber is required to have the following properties: NA = 0.10, core diameter of 9 μm, and a cladding of pure silica, and operate at 1.3 μm. What should the core composition be?

Solution

The Sellmeier equation is

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}$$

From Table1.2 in Ch.1. Sellmeier coefficients as as follows

Sellmeier	A_1	A_2	A_3	λ_1 μm	λ_2 μm	λ_3 μm
SiO_2 (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO_2-13.5%GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478

Therefore, for λ = 1.3 μm pure silica has $n(0)$ = **1.4473** and SiO_2-13.5 mol.%GeO_2 has $n(13.5)$= **1.4682**.

Confirming that for NA=0.10 we have a single mode fiber

$$n^2 = \frac{2\pi a}{\lambda} NA = \frac{2\pi(4.5\ \mu\text{m})}{(1.3\ \mu\text{m})}(0.1) = \mathbf{2.175}$$

Apply $NA = \left(n_1^2 - n_2^2\right)^{1/2}$ to obtain $n_1 = \left(NA^2 + n_2^2\right)^{1/2}$=$(0.1^2+1.4473^2)^{1/2}$ = **1.4508**

The refractive index $n(x)$ of SiO_2-x mol.%GeO_2, assuming a linear relationship, can be written as

$$n(x) = n(0)\left(1 - \frac{x}{13.5}\right) + n(13.5)\frac{x}{13.5}$$

Substituting $n(x) = n_1$ = 1.4508 gives x = **2.26**.

2.17 Material dispersion If N_{g1} is the group refractive index of the core material of a step fiber, then the propagation time (group delay time) of the fundamental mode is

$$\tau = L / v_g = LN_{g1} / c$$

Since N_g will depend on the wavelength, show that the material dispersion coefficient D_m is given approximately by

$$D_m = \frac{d\tau}{L d\lambda} \approx \frac{\lambda}{c}\frac{d^2 n}{d\lambda^2}$$

Using the Sellmeier equation and the constants in Table 1.2 in Ch. 1, evaluate the material dispersion at λ = 1.55 μm for pure silica (SiO_2) and SiO_2-13.5%GeO_2 glass.

Solution

From Ch. 1 we know that

$$N_g \approx n - \lambda \frac{dn}{d\lambda}$$

Differentiate τ with respect to wavelength λ using the above relationship between N_g and n.

$$\tau = \frac{L}{v_g} = \frac{LN_{g1}}{c}$$

$$\therefore \qquad \frac{d\tau}{d\lambda} = \frac{L}{c}\frac{dN_{g1}}{d\lambda} \approx \frac{L}{c}\left[\frac{dn}{d\lambda} - \lambda\frac{d^2n}{d\lambda^2} - \frac{dn}{d\lambda}\right] = -\frac{L}{c}\lambda\frac{d^2n}{d\lambda^2}$$

Thus, $$D_m = \left|\frac{d\tau}{Ld\lambda}\right| \approx \frac{\lambda}{c}\frac{d^2n}{d\lambda^2} \qquad (1)$$

From Ch. 1 we know that the Sellmeier equation is

$$n^2 - 1 = \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2} \qquad (2)$$

The Sellmeier coefficients for SiO_2-13.5%GeO_2.

The λ_1, λ_2, λ_3 are in μm.

	A_1	A_2	A_3	λ_1	λ_2	λ_3
SiO_2-13.5%GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478

We can use the Sellmeier coefficient in Table1.2 in Ch.1 to find n vs. λ, $dn/d\lambda$ and $d^2n/d\lambda^2$, and, from Eq. (1), D_m vs λ as in Figure 2Q17-1. At λ = 1.55 μm, D_m = **13–14 ps km^{-1} nm^{-1}**

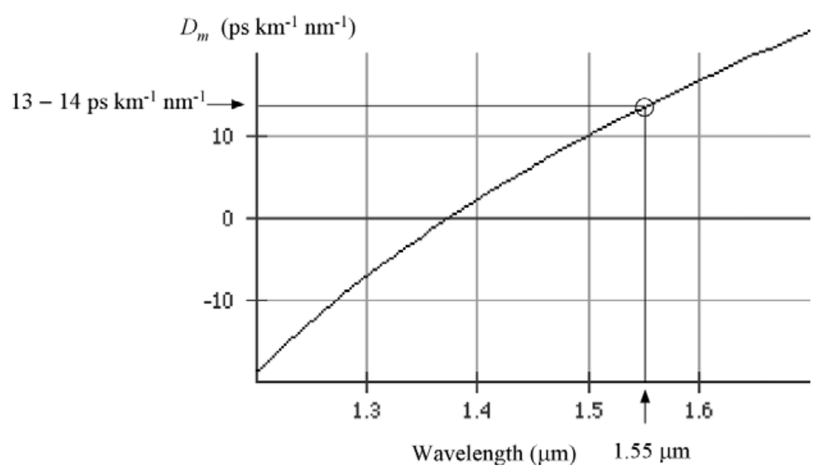


Figure 2Q17-1 Materials dispersion D_m vs. wavelength (LiveMath used). (Other math programs such as Matlab can also be used.)

2.18 Waveguide dispersion Waveguide dispersion arises as a result of the dependence of the propagation constant on the V-number, which depends on the wavelength. It is present even when the refractive index is constant; no material dispersion. Let us suppose that n_1 and n_2 are wavelength (or k) independent. Suppose that β is the propagation constant of mode lm and $k = 2\pi/\lambda$ in which λ is the free space wavelength. Then the normalized propagation constant b and propagation constant are related by

$$\beta = n_2 k[1 + b\Delta] \tag{1}$$

The group velocity is defined and given by

$$v_g = \frac{d\omega}{d\beta} = c\frac{dk}{d\beta}$$

Show that the propagation time, or the group delay time, τ of the mode is

$$\tau = \frac{L}{v_g} = \frac{Ln_2}{c} + \frac{Ln_2\Delta}{c}\frac{d(kb)}{dk} \tag{2}$$

Given the definition of V,

$$V = ka[n_1^2 - n_2^2]^{1/2} \approx kan_2(2\Delta)^{1/2} \tag{3}$$

and

$$\frac{d(Vb)}{dV} = \frac{d}{dV}\left[bkan_2(2\Delta)^{1/2}\right] = an_2(2\Delta)^{1/2}\frac{d}{dV}(bk) \tag{4}$$

Show that

$$\frac{d\tau}{d\lambda} = -\frac{Ln_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \tag{5}$$

and that the waveguide dispersion coefficient is

$$D_w = \frac{d\tau}{Ld\lambda} = -\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \tag{6}$$

Figure 2.53 shows the dependence of $V[d^2(Vb)/dV^2]$ on the V-number. In the range $1.5 < V < 2.4$,

$$V\frac{d^2(Vb)}{dV^2} \approx \frac{1.984}{V^2}$$

Show that,

$$D_w \approx -\frac{n_2\Delta}{c\lambda}\frac{1.984}{V^2} = -\frac{(n_1 - n_2)}{c\lambda}\frac{1.984}{V^2} \tag{7}$$

which simplifies to

$$D_w \approx -\frac{1.984}{c(2\pi a)^2 2n_2}\lambda \tag{8}$$

i.e.
$$D_w(\text{ps nm}^{-1}\text{ km}^{-1}) \approx -\frac{83.76\,\lambda(\mu\text{m})}{[a(\mu\text{m})]^2 n_2} \qquad \text{Waveguide dispersion coefficient} \qquad (9)$$

Consider a fiber with a core of diameter of 8 μm and refractive index of 1.468 and a cladding refractive index of 1.464, both refractive indices at 1300 nm. Suppose that a 1.3 μm laser diode with a spectral linewidth of 2 nm is used to provide the input light pulses. Estimate the waveguide dispersion per kilometer of fiber using Eqs. (6) and (8).

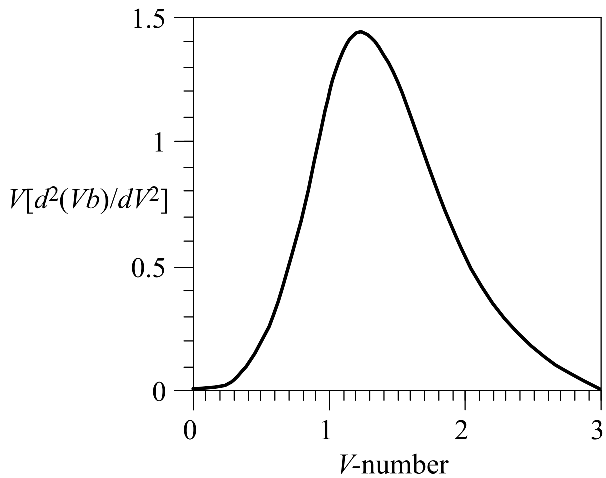


Figure 2.53 $d^2(Vb)/dV^2$ vs V-number for a step index fiber. (Data extracted from W. A. Gambling *et al. The Radio and Electronics Engineer*, *51*, 313, 1981.)

Solution

Waveguide dispersion arises as a result of the dependence of the propagation constant on the V-number which depends on the wavelength. It is present even when the refractive index is constant; no material dispersion. Let us suppose that n_1 and n_2 are wavelength (or k) independent. Suppose that β is the propagation constant of mode lm and $k = 2\pi/\lambda$ where λ is the free space wavelength. Then the normalized propagation constant b is defined as,

$$b = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2} \qquad (1)$$

Show that for small normalized index difference $\Delta = (n_1 - n_2)/n_1$, Eq. (1) approximates to

$$b = \frac{(\beta/k) - n_2}{n_1 - n_2} \qquad (2)$$

which gives β as,

$$\beta = n_2 k[1 + \Delta b] \qquad (3)$$

The group velocity is defined and given by

$$v_g = \frac{d\omega}{d\beta} = c\frac{dk}{d\beta}$$

Thus, the propagation time τ of the mode is

$$\tau = \frac{L}{v_g} = \frac{L}{c}\left(\frac{d\beta}{dk}\right) = \frac{Ln_2}{c} + \frac{Ln_2\Delta}{c}\frac{d(kb)}{dk} \quad (4)$$

where we assumed $\Delta \approx$ constant (does not depend on the wavelength). Given the definition of V,

$$\begin{aligned} V &= ka[n_1^2 - n_2^2]^{1/2} = ka[(n_1 + n_2)(n_1 - n_2)]^{1/2} \\ &\approx ka\left[(n_1 + n_2)n_1\left(\frac{n_1 - n_2}{n_1}\right)\right]^{1/2} \quad (5) \\ &\approx ka[2n_2 n_1 \Delta]^{1/2} \approx kan_2(2\Delta)^{1/2} \end{aligned}$$

From Eq. (5),

$$\frac{d(Vb)}{dV} = \frac{d}{dV}\left[bkan_2(2\Delta)^{1/2}\right] = an_2(2\Delta)^{1/2}\frac{d}{dV}(bk)$$

This means that τ depends on V as,

$$\tau = \frac{Ln_2}{c} + \frac{Ln_2\Delta}{c}\frac{d(Vb)}{dV} \quad (6)$$

Dispersion, that is, spread $\delta\tau$ in τ due to a spread $\delta\lambda$ can be found by differentiating Eq. (6) to obtain,

$$\begin{aligned} \frac{d\tau}{d\lambda} &= \frac{Ln_2\Delta}{c}\frac{dV}{d\lambda}\frac{d}{dV}\frac{d(Vb)}{dV} = \frac{Ln_2\Delta}{c}\left(-\frac{V}{\lambda}\right)\frac{d^2(Vb)}{dV^2} \\ &= -\frac{Ln_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \end{aligned} \quad (7)$$

The waveguide dispersion coefficient is defined as

$$D_w = \frac{d\tau}{Ld\lambda} = -\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} \quad (8)$$

Figure 2.53 shows the dependence of $V[d^2(Vb)/dV^2]$ on the V-number.

In the range $2 < V < 2.4$,

$$V\frac{d^2(Vb)}{dV^2} \approx \frac{1.984}{V^2}$$

so that Eq. (8) becomes,

$$D_w \approx -\frac{n_2\Delta}{c\lambda}\frac{1.984}{V^2} = -\frac{(n_1 - n_2)}{c\lambda}\frac{1.984}{V^2} \quad (9)$$

We can simplify this further by using

$$D_w \approx -\frac{n_2\Delta}{c\lambda}\frac{1.984}{V^2} \approx -\frac{1.984n_2\Delta}{c\lambda}\left[\frac{\lambda}{2\pi an_2(2\Delta)^{1/2}}\right]^{1/2}$$

$$\therefore \qquad D_w \approx -\frac{1.984}{c(2\pi a)^2 2n_2}\lambda \qquad (10)$$

Equation (6) should really have N_{g2} instead of n_2 in which case Eq. (10) would be

$$D_w \approx -\frac{1.984 N_{g2}}{c(2\pi a)^2 2n_2^2}\lambda \qquad (11)$$

Consider a fiber with a core of diameter of 8 μm and refractive index of 1.468 and a cladding refractive index of 1.464 both refractive indices at 1300 nm. Suppose that a1.3 μm laser diode with a spectral linewidth of 2 nm is used to provide the input light pulses. Estimate the waveguide dispersion per kilometer of fiber using Eqs. (8) and (11).

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(4\ \mu\text{m})(1.468^2 - 1.464^2)^{1/2}}{(1.3\ \mu\text{m})} = \mathbf{2.094}$$

and $\Delta = (n_1 - n_2)/n_1 = (1.468-1.464)/1.468 = 0.00273.$

From the graph, $Vd^2(Vb)/dV^2 = 0.45$,

$$D_w = -\frac{n_2\Delta}{c\lambda}V\frac{d^2(Vb)}{dV^2} = -\frac{(1.464)(2.73\times10^{-3})}{(3\times10^8\ \text{m s}^{-1})(1300\times10^{-9}\ \text{m})}(0.45)$$

$\therefore \qquad D_w \approx -4.6\times10^{-6}$ s m^{-2} or $\mathbf{-4.6\ ps\ km^{-1}\ nm^{-1}}$

Using Eq. (10)

$$D_w \approx -\frac{1.984}{c(2\pi a)^2 2n_2}\lambda = -\frac{1.984(1300\times10^{-9}\ \text{m})}{(3\times10^8\ \text{m s}^{-1})[2\pi\times4\times10^{-6}\ \text{m}]^2 2(1.464)]}$$

$\therefore \qquad D_w \approx -4.6\times10^{-6}$ s m^{-2} or $\mathbf{-4.6\ ps\ km^{-1}\ nm^{-1}}$

For $\Delta\lambda_{1/2}$ = 2 nm we have,

$$\Delta\tau_{1/2} = |D_w|L\,\Delta\lambda_{1/2} = (4.6\ \text{ps km}^{-1}\ \text{nm}^{-1})(2\ \text{nm}) = \mathbf{9.2\ ps/km}$$

2.19 Profile dispersion Total dispersion in a single mode, step index fiber is primarily due to material dispersion and waveguide dispersion. However, there is an additional dispersion mechanism called *profile dispersion* that arises from the propagation constant β of the fundamental mode also depending on the refractive index difference Δ. Consider a light source with a range of wavelengths $\delta\lambda$ coupled into a step index fiber. We can view this as a change $\delta\lambda$ in the input wavelength λ. Suppose that n_1, n_2, hence Δ depends on the wavelength λ. The propagation time, or the group delay time, τ_g *per unit length* is

$$\tau_g = 1/v_g = d\beta/d\omega = (1/c)(d\beta/dk) \qquad (1)$$

where k is the free space propagation constant ($2\pi/\lambda$), and we used $d\omega = cdk$. Since β depends on n_1, Δ, and V, consider τ_g as a function of n_1, Δ (thus n_2), and V. A change $\delta\lambda$ in λ will change each of these quantities. Using the partial differential chain rule,

$$\frac{\delta\tau_g}{\delta\lambda} = \frac{\partial\tau_g}{\partial n_1}\frac{\partial n_1}{\partial\lambda} + \frac{\partial\tau_g}{\partial V}\frac{\partial V}{\partial\lambda} + \frac{\partial\tau_g}{\partial\Delta}\frac{\partial\Delta}{\partial\lambda} \tag{2}$$

The mathematics turns out to be complicated but the statement in Eq. (2) is equivalent to

Total dispersion = Material dispersion (due to $\partial n_1/\partial\lambda$)

+ Waveguide dispersion (due to $\partial V/\partial\lambda$)

+ Profile dispersion (due to $\partial\Delta/\partial\lambda$)

in which the last term is due to Δ depending on λ; although small, this is not zero. Even the statement in Eq. (2) above is over simplified but nonetheless provides an insight into the problem. The total intramode (chromatic) dispersion coefficient D_{ch} is then given by

$$D_{ch} = D_m + D_w + D_p \tag{3}$$

in which D_m, D_w, D_p are material, waveguide, and profile dispersion coefficients respectively. The waveguide dispersion is given by Eq. (8) and (9) in Question 2.18, and the *profile dispersion coefficient* is (very) approximately[1],

$$D_p \approx -\frac{N_{g1}}{c}\left(V\frac{d^2(Vb)}{dV^2}\right)\left(\frac{d\Delta}{d\lambda}\right) \tag{4}$$

in which b is the normalized propagation constant and $Vd^2(Vb)/dV^2$ vs. V is shown in Figure 2.53,we can also use $V\mathrm{d}^2(Vb)/dV^2 \approx 1.984/V^2$.

Consider a fiber with a core of diameter of 8 μm. The refractive and group indices of the core and cladding at $\lambda = 1.55$ μm are $n_1 = 1.4500$, $n_2 = 1.4444$, $N_{g1} = 1.4680$, $N_{g2} = 1.4628$, and $d\Delta/d\lambda = 232$ m^{-1}. Estimate the waveguide and profile dispersion per km of fiber per nm of input light linewidth at this wavelength. (Note: The values given are approximate and for a fiber with silica cladding and 3.6% germania-doped core.)

Solution

Total dispersion in a single mode step index fiber is primarily due to material dispersion and waveguide dispersion. However, there is an additional dispersion mechanism called *profile dispersion* that arises from the propagation constant β of the fundamental mode also depending on the refractive index difference Δ. Consider a light source with a range of wavelengths $\delta\lambda$ coupled into a step index fiber. We can view this as a change $\delta\lambda$ in the input wavelength λ. Suppose that n_1, n_2, hence Δ depends on the wavelength λ. The propagation time, or the group delay time, τ_g *per unit length* is

$$\tau_g = \frac{1}{v_g} = \frac{1}{c}\left(\frac{d\beta}{dk}\right) \tag{1}$$

Since β depends on n_1, Δ and V, let us consider τ_g as a function of n_1, Δ (thus n_2) and V. A change $\delta\lambda$ in λ will change each of these quantities. Using the partial differential chain rule,

[1] J. Gowar, *Optical Communication Systems*, 2nd Edition (Prentice Hall, 1993). Ch. 8 has the derivation of this equation..

$$\frac{\delta\tau_g}{\delta\lambda} = \frac{\partial\tau_g}{\partial n_1}\frac{\partial n_1}{\partial\lambda} + \frac{\partial\tau_g}{\partial V}\frac{\partial V}{\partial\lambda} + \frac{\partial\tau_g}{\partial\Delta}\frac{\partial\Delta}{\partial\lambda} \tag{2}$$

The mathematics turns out to be complicated but the statement in Eq. (2) is equivalent to

Total dispersion = Materials dispersion (due to $\partial n_1/\partial\lambda$)

+ Waveguide dispersion (due to $\partial V/\partial\lambda$)

+ Profile dispersion (due to $\partial\Delta/\partial\lambda$)

where the last term is due Δ depending on λ; although small this is not zero. Even the above statement in Eq. (2) is over simplified but nonetheless provides an sight into the problem. The total intramode (chromatic) dispersion coefficient D_{ch} is then given by

$$D_{ch} = D_m + D_w + D_p \tag{3}$$

where D_m, D_w, D_p are material, waveguide and profile dispersion coefficients respectively. The waveguide dispersion is given by Eq. (8) in Question 2.6 and the profile dispersion coefficient away is (very) approximately,

$$D_p \approx -\frac{N_{g1}}{c}\left(V\frac{d^2(Vb)}{dV^2}\right)\left(\frac{d\Delta}{d\lambda}\right) \tag{4}$$

where b is the normalized propagation constant and $Vd^2(Vb)/dV^2$ vs. V is shown in Figure 2.53. The term $V\mathrm{d}^2(Vb)/dV^2 \approx 1.984/V^2$.

Consider a fiber with a core of diameter of 8 μm. The refractive and group indexes of the core and cladding at $\lambda = 1.55$ μm are $n_1 = 1.4504$, $n_2 = 1.4450$, $N_{g1} = 1.4676$, $N_{g2} = 1.4625$. $d\Delta/d\lambda = 161\ \mathrm{m}^{-1}$.

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(4\ \mu\mathrm{m})(1.4504^2 - 1.4450^2)^{1/2}}{(1.55\ \mu\mathrm{m})} = 2.03$$

and $\Delta = (n_1 - n_2)/n_1 = (1.4504\text{-}1.4450)/1.4504 = \mathbf{0.00372}$

From the graph in Figure 2.53, when $V = 2.03$, $Vd^2(Vb)/dV^2 \approx 0.50$,

Profile dispersion:

$$D_p = -\frac{N_{g1}}{c}\left(V\frac{d^2(Vb)}{dV^2}\right)\left(\frac{d\Delta}{d\lambda}\right) = -\frac{1.4676}{3\times10^8\ \mathrm{m\,s^{-1}}}(0.50)(161\ \mathrm{m}^{-1})$$

$\therefore$ $D_p = 3.8 \times 10^{-7}\ \mathrm{s\ m^{-1}\ m^{-1}}$ or $\mathbf{0.38\ ps\ km^{-1}\ nm^{-1}}$

Waveguide dispersion:

$$D_w \approx -\frac{1.984}{c(2\pi a)^2 2n_2}\lambda = -\frac{1.984(1500\times10^{-9}\ \mathrm{m})}{(3\times10^8\ \mathrm{m\,s^{-1}})[2\pi\times4\times10^{-6}\ \mathrm{m}]^2 2(1.4450)]}$$

$\therefore$ $D_w \approx \mathbf{-\,5.6\ ps\ km^{-1}\ nm^{-1}}$

Profile dispersion is more than 10 times smaller than waveguide dispersion.

2.20 Dispersion at zero dispersion coefficient Since the spread in the group delay $\Delta\tau$ along a fiber depends on the $\Delta\lambda$, the linewidth of the source, we can expand $\Delta\tau$ as a Taylor series in $\Delta\lambda$. Consider the expansion at $\lambda = \lambda_0$ where $D_{ch} = 0$. The first term with $\Delta\lambda$ would have $d\Delta\tau/d\lambda$ as a coefficient, that is D_{ch}, and at λ_0 this will be zero; but not the second term with $(\Delta\lambda)^2$ that has a differential, $d^2\Delta\tau/d\lambda^2$ or $dD_{ch}/d\lambda$. Thus, the dispersion at λ_0 would be controlled by the slope S_0 of D_{ch} vs. λ curve at λ_0. Show that the chromatic dispersion at λ_0 is

$$\Delta\tau = \frac{L}{2}S_0(\Delta\lambda)^2$$

A single mode fiber has a zero-dispersion at $\lambda_0 = 1310$ nm, dispersion slope $S_0 = 0.090$ ps nm^2 km. What is the dispersion for a laser with $\Delta\lambda = 1.5$ nm? What would control the dispersion?

Solution

Consider the Taylor expansion for τ, a function of wavelength, about its center around, say at λ_0, when we change the wavelength by $\Delta\lambda$. For convenience we can the absolute value of τ at λ_0 as zero since we are only interested in the spread $\Delta\tau$. Then, Taylor's expansion gives,

$$\Delta\tau = f(\Delta\lambda) = \frac{d\Delta\tau}{d\lambda}(\Delta\lambda) + \frac{1}{2!}\frac{d^2\Delta\tau}{d\lambda^2}(\Delta\lambda)^2 + \cdots$$

$$\therefore \qquad \Delta\tau = 0 + \frac{1}{2!}\frac{d^2\Delta\tau}{d\lambda^2}(\Delta\lambda)^2 = 0 + \frac{1}{2!}\frac{d}{dt}\left(\frac{d\Delta\tau}{d\lambda}\right)(\Delta\lambda)^2 = \frac{1}{2!}\frac{d}{dt}\left(D_{ch}\right)(\Delta\lambda)^2$$

$$\therefore \qquad \Delta\tau = \frac{L}{2}S_0(\Delta\lambda)^2 = \frac{1\text{ km}}{2}\left(0.090\,\text{ps}\,\text{nm}^{-2}\text{ km}^{-1}\right)\left(2\,\text{nm}\right)^2 = 1.01\ \text{ ps}$$

This can be further reduced by using a narrower laser line width since $\Delta\tau$ depends on $(\Delta\lambda)^2$

2.21 Polarization mode dispersion (PMD) A fiber manufacturer specifies a maximum value of 0.05 ps km$^{-1/2}$ for the polarization mode dispersion (PMD) in its single mode fiber. What would be the dispersion, maximum bit rate and the optical bandwidth for this fiber over an optical link that is 200 km long if the only dispersion mechanism was PMD?

Solution

Dispersion

$$\Delta\tau = D_{\text{PMD}}L^{1/2} = \left(0.05\text{ps km}^{-1/2}\right)(200\text{km})^{1/2} = \mathbf{0.707\,ps}$$

Bit rate

$$B = \frac{0.59}{\Delta\tau} = \frac{0.59}{0.707\text{ps}} = \mathbf{8.35\,Gb\,s^{-1}}$$

Optical bandwidth

$$f_{\text{op}} \approx 0.75B = (0.75)(8.35\,\text{Gb s}^{-1}) = \mathbf{6.26\ GHz}$$

2.22 Polarization mode dispersion Consider a particular single mode fiber (ITU-T G.652 compliant) that has a chromatic dispersion of 15 ps nm^{-1} km^{-1}. The chromatic dispersion is zero at 1315 nm, and the dispersion slope is 0.092 ps nm^{-2} km^{-1}. The PMD coefficient is 0.05 ps km$^{-1/2}$. Calculate the total dispersion over 100 km if the fiber is operated at 1315 nm and the source is a laser diode with a linewidth (FWHM) $\Delta\lambda$ = 1 nm. What should be the linewidth of the laser source so that over 100 km, the chromatic dispersion is the same as PMD?

Solution

Polarization mode dispersion for L = 100 km is $\Delta\tau_{\text{PMD}} = D_{\text{PMD}}L^{1/2} = 0.05\times\sqrt{100}$ ps = 0.5 ps

We need the chromatic dispersion at λ_0, where the chromatic dispersion D_{ch} = 0. For L = 100 km, the chromatic dispersion is

$$\Delta\tau_{\text{ch}} = \frac{L}{2}S_0(\Delta\lambda)^2 = 100\times0.092\times(1)^2/2 = 4.60 \text{ ps}$$

The rms dispersion is

$$\Delta\tau_{\text{rms}} = \sqrt{\Delta\tau_{\text{PMD}}^2 + \Delta\tau_{\text{ch}}^2} = \mathbf{4.63\ ps}$$

The condition for $\Delta\tau_{\text{PMD}} = \Delta\tau_{\text{ch}}$ is

$$\Delta\lambda = \sqrt{\frac{2D_{\text{PMD}}}{S_0L^{1/2}}} = \mathbf{0.33\ nm}$$

2.23 Dispersion compensation Calculate the total dispersion and the overall net dispersion coefficient when a 900 km transmission fiber with D_{ch} = +15 ps nm^{-1} km^{-1} is spliced to a compensating fiber that is 100 km long and has D_{ch} = −110 ps nm^{-1} km^{-1}. What is the overall *effective* dispersion coefficient of this combined fiber system? Assume that the input light spectral width is 1 nm.

Solution

Using Eq. (2.6.1) with $\Delta\lambda$ = 1 nm, we can find the total dispersion

$$\begin{aligned}\Delta\tau &= (D_1L_1 + D_2L_2)\Delta\lambda \\ &= [(+15 \text{ ps nm}^{-1}\text{ km}^{-1})(900\text{ km}) + (-110\text{ ps nm}^{-1}\text{ km}^{-1})(100\text{ km})](1\text{ nm}) \\ &= 2{,}500 \text{ ps nm}^{-1} \text{ for 1000 km.}\end{aligned}$$

The net or effective dispersion coefficient can be found from $\Delta\tau = D_{\text{net}}L\Delta\lambda$,

$$D_{\text{net}} = \Delta\tau/(L\Delta\lambda) = (2{,}500\text{ ps})/[(1000\text{ km})(1\text{ nm})] = 2.5 \text{ ps nm}^{-1}\text{ km}^{-1}$$

2.24 Cladding diameter A comparison of two step index fibers, one SMF and the other MMF shows that the SMF has a core diameter of 9 μm but a cladding diameter of 125 μm, while the MMF has a core diameter of 100 μm but a cladding diameter that is the same 125 μm. Discuss why the manufacturer has chosen those values.

Solution

For the single mode fiber, the small core diameter is to ensure that the V-number is below the cutoff value for singe mode operation for the commonly used wavelengths 1.1 μm and 1.5 μm. The larger total diameter is to ensure that there is enough cladding to limit the loss of light that penetrates into the cladding as an evanescent wave.

For multimode fibers, the larger core size allows multiple modes to propagate in the fiber and therefore the spectral width is not critical. Further, the larger diameter results in a greater acceptance angle. Thus, LEDs, which are cheaper and easier to use than lasers, are highly suitable. The total diameter of the core and cladding is the same because in industry it is convenient to standardize equipment and the minor losses that might accumulate from light escaping from the cladding do not matter as much over shorter distances for multimode fibers – they are short haul fibers.

2.25 Graded index fiber Consider an optimal graded index fiber with a core diameter of 30 μm and a refractive index of 1.4740 at the center of the core and a cladding refractive index of 1.4530. Find the number of modes at 1300 nm operation. What is its NA at the fiber axis, and its effective NA? Suppose that the fiber is coupled to a laser diode emitter at 1300 nm and a spectral linewidth (FWHM) of 3 nm. The material dispersion coefficient at this wavelength is about −5 ps km^{-1} nm^{-1}. Calculate the total dispersion and estimate the bit rate × distance product of the fiber. How does this compare with the performance of a multimode fiber with same core radius, and n_1 and n_2? What would the total dispersion and maximum bit rate be if an LED source of spectral width (FWHM) $\Delta\lambda_{1/2} \approx 80$ nm is used?

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.4740 - 1.453)/1.474 = 0.01425$

Modal dispersion for 1 km of graded index fiber is

$$\sigma_{\text{intermode}} \approx \frac{Ln_1}{20\sqrt{3}c}\Delta^2 = \frac{(1000)(1.474)}{20\sqrt{3}(3\times10^8)}(0.01425)^2 = 2.9\times10^{-11} \text{ s or } \mathbf{0.029\ ns}$$

The material dispersion (FWHM) is

$$\Delta\tau_{m(1/2)} = LD_m\Delta\lambda_{1/2} = (1000 \text{ m})(-5 \text{ ps ns}^{-1} \text{ km}^{-1})(3 \text{ nm}) = \mathbf{0.015\ ns}$$

Assuming a Gaussian output light pulse shape, rms material dispersion is,

$$\sigma_m = 0.425\Delta\tau_{1/2} = (0.425)(0.015 \text{ ns}) = 0.00638 \text{ ns}$$

Total dispersion is

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{intermode}}^2 + \sigma_m^2} = \sqrt{0.029^2 + 0.00638^2} = 0.0295 \text{ ns}.$$

so that $B = 0.25/\sigma_{\text{total}} = \mathbf{8.5\ Gb}$

If this were a multimode step-index fiber with the same n_1 and n_2, then the rms dispersion would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{1.474 - 1.453}{3\times10^8 \text{ m s}^{-1}} = 70 \text{ ps m}^{-1} \text{ or } \mathbf{70\ ns\ per\ km}$$

Maximum bit-rate is

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} \approx \frac{0.25L}{(0.28)\Delta\tau} = \frac{0.25}{(0.28)(70 \text{ ns km}^{-1})}$$

i.e. BL = **12.8 Mb s^{-1} km** (only an estimate!)

The corresponding B for 1 km would be around **13 Mb s^{-1}**

With LED excitation, again assuming a Gaussian output light pulse shape, rms material dispersion is

$$\sigma_m = (0.425)\Delta\tau_{m(1/2)} = (0.425)LD_m\Delta\lambda_{1/2}$$
$$= (0.425)(1000\text{ m})(-5\text{ ps ns}^{-1}\text{ km}^{-1})(80\text{ nm})$$
$$= \mathbf{0.17\ ns}$$

Total dispersion is

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_m} = \sqrt{0.029^2 + 0.17^2} = \mathbf{0.172\ ns}$$

so that $B = 0.25/\sigma_{\text{total}}$ = **1.45 Gb**

The effect of material dispersion now dominates intermode dispersion.

2.26 Graded index fiber Consider a graded index fiber with a core diameter of 62.5 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral FWHM linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient D_m at 1300 nm is −7.5 ps nm^{-1} km^{-1}. How does this compare with the performance of a multimode fiber with the same core radius, and n_1 and n_2?

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$

Modal dispersion for 1 km of graded index fiber is

$$\sigma_{\text{intermode}} \approx \frac{Ln_1}{20\sqrt{3}c}\Delta^2 = \frac{(1000)(1.474)}{20\sqrt{3}(3\times10^8)}(0.01425)^2 = 2.9\times10^{-11}\text{ s or } \mathbf{0.029\ ns.}$$

The material dispersion is

$$\Delta\tau_{m(1/2)} = LD_m\Delta\lambda_{1/2} = (1000\text{ m})(-7.5\text{ ps ns}^{-1}\text{ km}^{-1})(3\text{ nm}) = \mathbf{0.0225\ ns}$$

Assuming a Gaussian output light pulse shaper,

$$\sigma_{\text{intramode}} = 0.425\Delta\tau_{1/2} = (0.425)(0.0225\text{ ns}) = 0.0096\text{ ns}$$

Total dispersion is

$$\sigma_{rms} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_{\text{intramode}}} = \sqrt{0.029^2 + 0.0096^2} = 0.0305\text{ ns.}$$

so that $B = 0.25/\Delta\tau_{\text{rms}}$ = **8.2 Gb for 1 km**

If this were a multimode step-index fiber with the same n_1 and n_2, then the rms dispersion would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{1.474 - 1.453}{3\times10^8\text{ m s}^{-1}}$$

$= 70$ ps m^{-1} or **70 ns per km**

Maximum bit-rate would be

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} \approx \frac{0.25}{(0.28)(70\ \text{ns km}^{-1})}$$

i.e. $BL =$ **12.7 Mb s^{-1} km** (only an estimate!)

The corresponding B for 1 km would be around **13 Mb s^{-1}.**

2.27 Graded index fiber A standard graded index fiber from a particular fiber manufacturer has a core diameter of 62.5 μm, cladding diameter of 125 μm, a NA of 0.275. The core refractive index n_1 is 1.4555. The manufacturer quotes minimum optical bandwidth × distance values of 200 MHz·km at 850 nm and 500 MHz·km at 1300 nm. Assume that a laser is to be used with this fiber and the laser linewidth $\Delta\lambda = 1.5$ nm. What are the corresponding dispersion values? What type of dispersion do you think dominates? Is the graded index fiber assumed to have the ideal optimum index profile? (State your assumptions). What is the optical link distance for operation at 1 Gbs^{-1} at 850 and 1300 nm

Solution

We are given the numerical aperture NA = 0.275. Assume that this is the maximum NA at the core

$$n_2 = \left(n_1^2 - \text{NA}^2\right)^{\frac{1}{2}} = \left(1.4555^2 - 0.275^2\right)^{\frac{1}{2}} = 1.4293$$

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.4555 - 1.4293}{1.4555} = 0.018$$

We can now calculate intermodal dispersion

$$\sigma_{\text{intermodal}} = \frac{n_1}{20\sqrt{3}c}\Delta^2 = \frac{(1.4555)(0.018)^2}{20\sqrt{3(3\times10^5\,\text{km s}^{-1})}} = 45.43\ \text{ps km}^{-1}$$

The total dispersion for $\lambda = 850$ nm is

$$\sigma_T = \frac{0.19}{f_{\text{op}}} = \frac{0.19}{200\times10^6\ \text{s}^{-1}\,\text{km}} = 0.95\ \text{ns km}^{-1}$$

So the intramodal dispersion is

$$\sigma_{\text{intramodal}} = \left(\sigma_T^2 - \sigma_{\text{intermodal}}^2\right)^{\frac{1}{2}} = \left[\left(0.95\times10^{-9}\right)^2 - \left(45.43\times10^{-12}\right)^2\right]^{\frac{1}{2}} = 0.949\ \text{ns km}^{-1}$$

For $\lambda = 1300$ nm, the total dispersion is

$$\sigma_T = \frac{0.19}{f_{\text{op}}} = \frac{0.19}{500\times10^6\ \text{s}^{-1}\,\text{km}} = 0.38\ \text{ns km}^{-1}$$

and

$$\sigma_{\text{intramodal}} = \left(\sigma_T^2 - \sigma_{\text{intermodal}}^2\right)^{\frac{1}{2}} = \left[\left(0.38\times10^{-9}\right)^2 - \left(45.43\times10^{-12}\right)^2\right]^{\frac{1}{2}} = 0.377\ \text{ns km}^{-1}$$

For both 850 nm and 1300 nm intramodal dispersion dominates intermodal dispersion.

$\gamma = 2(1-\Delta) = 2(1-0.018) = 1.96$

Gamma is close to 2 so this is close to the optimal profile index.

2.28 Graded index fiber and optimum dispersion The graded index fiber theory and equations tend to be quite complicated. If γ is the profile index then the rms *intermodal dispersion* is given by[2]

$$\sigma = \frac{Ln_1\Delta}{2c}\left(\frac{\gamma}{\gamma+1}\right)\left(\frac{\gamma+2}{3\gamma+2}\right)^{1/2} \times \left[c_1^2 + \frac{4c_1c_2(\gamma+1)\Delta}{2\gamma+1} + \frac{16c_2^2\Delta^2(\gamma+1)^2}{(5\gamma+2)(3\gamma+2)}\right]^{1/2} \tag{1}$$

where c_1 and c_2 are given by

$$c_1 = \frac{\gamma-2-\delta}{\gamma+2};\ c_2 = \frac{3\gamma-2-2\delta}{2(\gamma+2)};\ \delta = -2\left(\frac{n_1}{N_{g1}}\right)\left(\frac{\lambda}{\Delta}\right)\left(\frac{d\Delta}{d\lambda}\right) \tag{2}$$

where δ is a small unitless parameter that represents the change in Δ with λ. The optimum profile coefficient γ_o is

$$\gamma_o = 2+\delta-\Delta\frac{(4+\delta)(3+\delta)}{(5+2\delta)} \tag{3}$$

Consider a graded index fiber for use at 850 nm, with $n_1 = 1.475$, $N_{g1} = 1.489$, $\Delta = 0.015$, $d\Delta/d\lambda = -683$ m^{-1}. Plot σ in ps km^{-1} vs γ from $\gamma = 1.8$ to 2.4 and find the minimum. (Consider plotting σ on a logarithmic axis.) Compare the minimum σ and the optimum γ, with the relevant expressions in §2.8. Find the percentage change in γ for a 10× increase in σ. What is your conclusion?

Solution

[2] R. Olshansky and D. Keck, *Appl. Opt.*, *15*, 483, 1976.

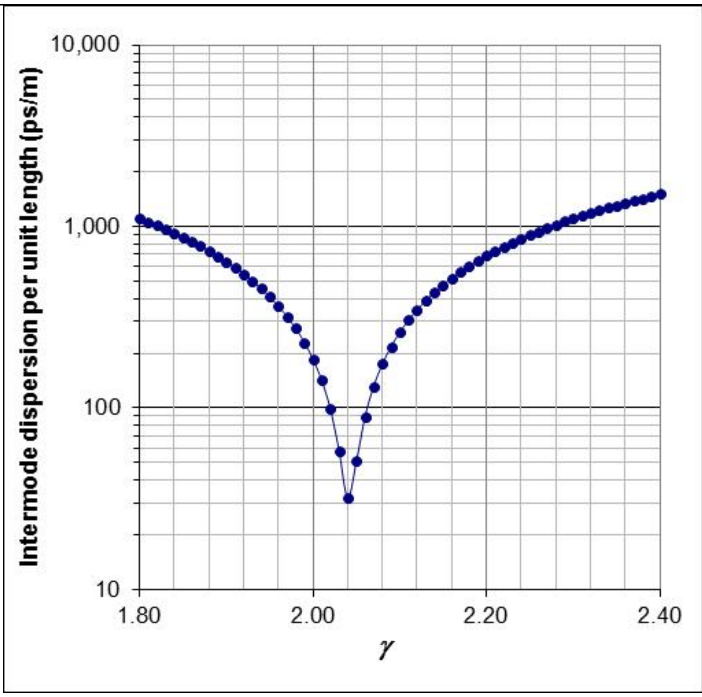


Figure 2Q28-1

From the graph in Figure 2Q28-1 γ_0 = **2.04**.

Equation (3) gives the same value γ_0 = **2.040**.

From the graph in Figure 2Q28-1

$\sigma_{\text{intermode}}/L$ = **31.87** ps km^{-1}

Consider Eq. (2.8.4) in §2.8,

$$\frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2 = \mathbf{31.93}\ \text{ps km}^{-1}.$$

From the graph in Figure 2Q28-1, a 3.4% change of γ leads to 10 times increase of dispersion. It is therefore important to control the refractive profile.

2.29 GRIN rod lenses Figure 2.32 shows graded index (GRIN) rod lenses. (a) How would you represent Figure 2.32(a) using two conventional converging lenses. What are O and O' ? (b) How would you represent Figure 2.32(b) using a conventional converging lens. What is O'? (c) Sketch ray paths for a GRIN rod with a pitch between 0.25P and 0.5P starting from O at the face center. Where is O'? (d) What use is 0.23P GRIN rod lens in Figure 2.32(c)?

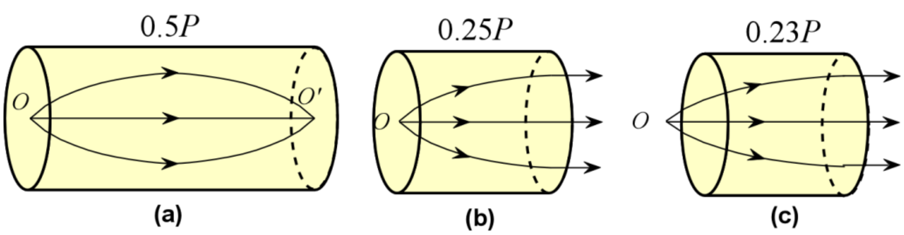


Figure 2.32 Graded index (GRIN) rod lenses of different pitches. (a) Point O is on the rod face center and the lens focuses the rays onto O' on to the center of the opposite face. (b) The rays from O on the rod face center are collimated out. (c) O is slightly away from the rod face and the rays are collimated out.

Solution

(a) and **(b)**

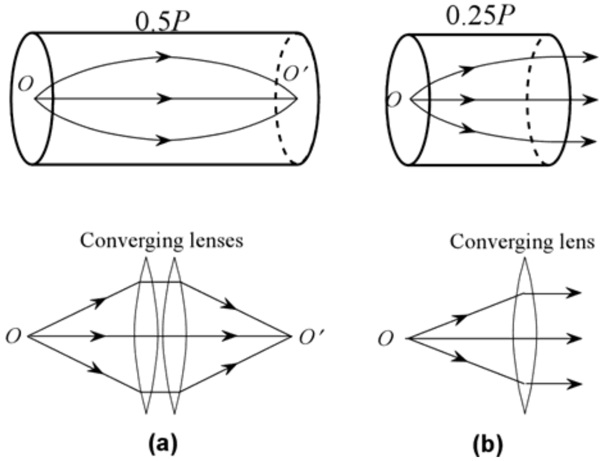


Figure 2Q29-1: (a) The beam bending from *O* to *O′* using a GRIN rod can be achieved equivalently by using two converging lenses. *O* and *O′* are the focal points of the lenses (approximately). (Schematic only). (b) The collimation of rays from a point source on the face of a GRIN rod can be equivalently achieved by a single converging lens whose focal length is 0.25*P* and *O* is the focal point. (Schematic only).

(c) Consider a GRIN rod with 0.4*P*

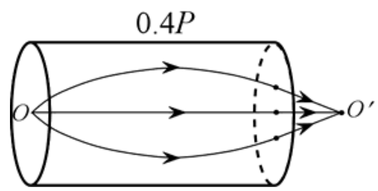


Figure 2Q29-2: Ray paths in a GRIN rod that has a pitch between 0.25P to 0.5P. (Schematic only.)

(d) Since the point *O* does not have to be right on the face of the GRIN rod, it can be used to collimate a point source *O* by bringing the rod sufficiently close to *O*; a fixed annular spacer can "fix" the required proximity of the rod to *O*. Since the source does not have to be in contact with the face of the rod, possible damage (such as scratches) to the face are avoided.

2.30 Optical Fibers Consider the manufacture of optical fibers and the materials used. (a) What factors would reduce dispersion? (b) What factors would reduce attenuation?

Possible Answers

(a) It is essential to control of refractive index profile, core radius, and minimize variations in the refractive index due to variations in doping.

(b) Minimize impurities. Reduce scattering by reducing density and hence refractive index *n* fluctuations (may not be readily possible). Use a glass material with a lower glass transition temperature so that the frozen *n*-variations are smaller.

2.31 Attenuation A laser emitter with a power 2 mW is used to send optical signals along a fiber optic link of length 170 km. Assume that all the light was launched into the fiber. The fiber is quoted as having an attenuation of 0.5 dB/km. What is the output power from the optical link that a photodetector must be able to detect?

Solution

$$P_{\text{out}} = P_{\text{in}} \exp(-\alpha L)$$

where

$$\alpha = \frac{\alpha_{\text{dB}}}{4.34} = \frac{0.5\,\text{dB km}^{-1}}{4.34} = 0.115\,\text{km}^{-1}$$

so

$$P_{\text{out}} = 2\text{ mW} \exp(-0.115\,\text{km}^{-1} \times 170\,\text{km}) = 6.24\text{ pW}$$

2.32 Cut-back method of attenuation measurement Cut-back method is a destructive measurement technique for determining the attenuation α of a fiber. The first part of the experiment involves measuring the optical power P_{far} coming out from the fiber at the far end as shown in Figure 2.54 Then, in the second part, keeping everything the same, the fiber is cut close to the launch or the source end. The output power P_{near} is measured at the near end from the short cut fiber. The attenuation is then given by

$$\alpha = (-10/L)\log(P_{\text{far}}/P_{\text{near}})$$

in which L is the separation of the measurement points, the length of the cut fiber, and α is in dB per unit length. The output P_{near} from the short cut fiber in the second measurement is actually the input into the fiber under test in the first experiment. Usually a mode scrambler (mode stripper) is used for multimode fibers before the input. The power output from a particular fiber is measured to be 13 nW. Then, 10 km of fiber is cut-out and the power output is measured again and found to be 43 nW. What is the attenuation of the fiber?

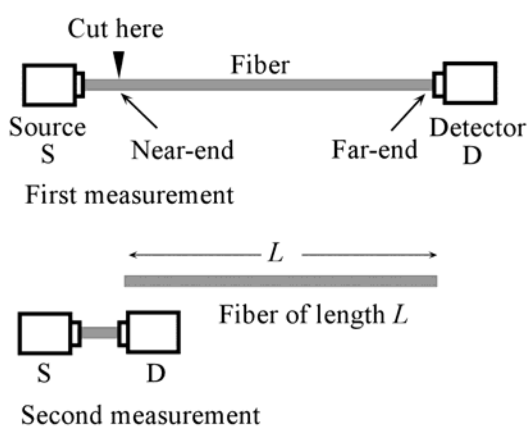


Figure 2.54 Illustration of the cut-back method for measuring the fiber attenuation. S is an optical source and D is a photodetector

Solution

$\alpha = (-10/L)\log(P_{\text{far}} / P_{\text{near}}) = (-10/10\text{ km})\log(10/43) = \mathbf{0.63\ dB\ km^{-1}}$

2.33 Intrinsic losses

(a) Consider a standard single mode fiber with a NA of 0.14. What is its attenuation at 1625 and at 1490 nm? How do these compare with the attenuation quotes for the Corning SMF-28e+, 0.20 – 0.23 dB km^{-1} at 1625 nm and 0.21 – 0.24 dB km^{-1} at 1490 nm?

(b) Consider a graded index fiber with a NA of 0.275. What would you expect for its attenuation at 850 nm and 1300 nm? How do your calculations compare with quoted maximum values of 2.9 dB km^{-1} at 850 nm and 0.6 dB km^{-1} at 1300 nm for 62.5 μm graded index fibers? Actual values would be less.

Solution

(a) When the wavelength is 1625 nm,

$$\alpha_{\text{FIR}} = A\exp(-B/\lambda)$$

$$\alpha_{\text{FIR}} = A\exp\left(-\frac{B}{\lambda}\right) = 7.8\times10^{11}\exp\left(-\frac{48.5}{1.625}\right) = 0.085\text{ dB km}^{-1}$$

$$A_R = 0.63 + 2.06NA = 0.63 + 2.06\times0.14 = 0.918\text{ dB km}^{-1}\ \mu\text{m}^4$$

$$a_R = \frac{A_R}{\lambda^4} = \frac{0.918\text{ dB km}^{-1}\ \mu\text{m}^4}{1.625\ \mu\text{m}^4} = 0.132\text{ dB km}^{-1}$$

$$\alpha_{\text{total}} = \alpha_{\text{FIR}} + \alpha_R = 0.085 + 0.132 = \mathbf{0.217\ dB\ km^{-1}}$$

When the wavelength is 1490 nm

$$\alpha_{\text{FIR}} = A\exp\left(-\frac{B}{\lambda}\right) = 7.8\times10^{11}\exp\left(-\frac{48.5}{1.490}\right) = 0.0057\text{ dB km}^{-1}$$

$$A_R = 0.63 + 2.06NA = 0.63 + 2.06\times0.14 = 0.918\text{ dB km}^{-1}\ \mu\text{m}^4$$

$$a_R = \frac{A_R}{\lambda^4} = \frac{0.918\text{ dB km}^{-1}\ \mu\text{m}^4}{1.490\ \mu\text{m}^4} = 0.1863\text{ dB km}^{-1}$$

$$\alpha_{\text{total}} = \alpha_{\text{FIR}} + \alpha_R = 0.0057 + 0.1863 = \mathbf{0.192\ dB\ km^{-1}}$$

(b) Rayleigh scattering

$A_R = 0.63 + 1.75\times\text{NA} = 0.63 + 1.75\times0.275 = 1.111$ dB km^{-1} μm^4

At 850 nm and 1300 nm the α_{FIR} term is essentially zero and does not need to be included

At 850 nm,

$$a_R = \frac{A_R}{\lambda^4} = \frac{1.111\,\text{dB km}^{-1}\,\mu\text{m}^4}{0.850\,\mu\text{m}^4} = \mathbf{2.13\,dB\,km^{-1}}$$

At 1300 nm,

$$a_R = \frac{A_R}{\lambda^4} = \frac{1.111\,\text{dB km}^{-1}\,\mu\text{m}^4}{1.300\,\mu\text{m}^4} = \mathbf{0.390\,dB\,km^{-1}}$$

2.34 Scattering losses and fictive temperature

Rayleigh scattering process decreases with wavelength, and as mentioned in Ch. 1, it is inversely proportional to λ^4. The expression for the attenuation α_R in a single component glass such as silica due to Rayleigh scattering is approximately given by two sets of different equations in the literature[3],

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_T k_B T_f \qquad \text{and} \qquad \alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2 \beta_T k_B T_f$$

in which λ is the free space wavelength, n is the refractive index at the wavelength of interest, β_T is the isothermal compressibility (at T_f) of the glass, k_T is the Boltzmann constant, and T_f is a quantity called the *fictive temperature* (or the glass transition temperature) at which the liquid structure during the cooling of the fiber is frozen to become the glass structure. Fiber is drawn at high temperatures and as the fiber cools eventually the temperature drops sufficiently for the atomic motions to be so sluggish that the structure becomes essentially "frozen-in" and remains like this even at room temperature. Thus, T_f marks the temperature below which the liquid structure is frozen and hence the density fluctuations are also frozen into the glass structure. Use these two equations and calculate the attenuation in dB/km due to Rayleigh scattering at around the λ = 1.55 μm window given that pure silica (SiO_2) has the following properties: $T_f \approx 1180$°C; $\beta_T \approx 7\times10^{-11}$ m^2 N^{-1} (at high temperatures); $n \approx 1.45$ at 1.55 μm, p = 0.28. The lowest reported attenuation around this wavelength is about 0.14 dB/km. What is your conclusion?

Solution

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_T k_B T_f = 0.0308\ \text{km}^{-1} \text{ or } 4.34\times0.0308 = 0.13\ \text{dB km}^{-1}$$

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2 \beta_T k_B T_f = 0.0245\ \text{km}^{-1} \text{ or } 4.34\times0.0245 = 0.11\ \text{dB km}^{-1}$$

The first equation appears to be the closest to the experimental value. However, note that the reported attenuation also has a contribution from the fundamental IR absorption.

$\alpha_{\text{FIR}} = A\exp(-B/\lambda)$, $A = 7.81\times10^{11}$ dB km^{-1}; B= 48.5 μm gives α_{FIR} = 0.02 dB km^{-1}.

Thus, adding α_{FIR} to α_R gives

First equation + FIR attenuation = 0.13 + 0.02 = 0.015 dB km^{-1}

Second equation + FIR attenuation = 0.11 + 0.02 = 0.013 dB km^{-1}

---

[3] For example, R. Olshansky, *Rev. Mod. Phys, 51*, 341, 1979.

The experimental value lies exactly in between.

2.35 Bending loss Bending losses always increase with the mode field diameter (MFD). Since the MFD increases for decreasing V, $2w \approx 2\times2.6a/V$, smaller V fibers have higher bending losses. How does the bending loss α vs. radius of curvature R behavior look like on a semilogarithmic plot (as in Figure 2.39(a) for two values of the V-number V_1 and V_2 if $V_2 > V_1$. It is found that for a single mode fiber with a cut-off wavelength λ_c = 1180 nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for Δ = 0.00825, 12 mm for Δ = 0.00550, and 35 mm for Δ = 0.00275. Explain these findings.

Solution

We expect the bending loss vs. R on a semilogarithmic plot to be as in Figure 2Q35-1 (schematic)

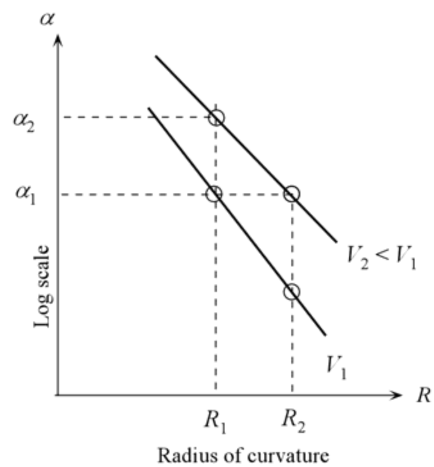


Figure 2Q35-1 Microbending loss α decreases sharply with the bend radius R. (Schematic only.)

From the figure, given $\alpha = \alpha_1$, R *increases* from R_1 to R_2 when V *decreases* from V_1 to V_2.

Expected $\qquad R\uparrow$ with $V\downarrow$ $\qquad$ (1)

Equivalently at one $R = R_1$ $\qquad \alpha\uparrow$ with $V\downarrow$ $\qquad$ (2)

We can *generalize* by noting that the penetration depth into the cladding $\delta \propto 1/V$.

<u>Expected</u> $\qquad R\uparrow$ with $\delta\uparrow$ $\qquad$ (3)

Equivalently at one $R = R_1$ $\qquad \alpha\uparrow$ with $\delta\uparrow$ $\qquad$ (4)

Eqs. (3) and (4) correspond to the general statement that microbending loss α gets worse when penetration δ into cladding increases; intuitively correct based on Figure 2.32.

Experiments show that for a given $\alpha = \alpha_1$, R increases with decreasing Δ.

<u>Observation</u> $\qquad R\uparrow$ with $\Delta\downarrow$ $\qquad$ (5)

Consider the penetration depth δ into a second medium (Example 2.1.3),

$$\delta = \frac{1}{\alpha} \approx \frac{\lambda}{2\pi}\frac{1}{(n_1^2 - n_2^2)^{1/2}} = \frac{1}{2\pi n_1}\frac{1}{(2\Delta)^{1/2}}$$

$\therefore$ $\delta\uparrow$ with $\Delta\downarrow$ (6)

Thus, δ increases with decreasing Δ.

Thus, from Eqs. (3) and (6), we expect

Expected $R\uparrow$ with $\delta\uparrow$ with $\Delta\downarrow$ (7)

Thus Eq, (7) agrees with the observation in Eq. (5).

NOTE

If we plot Δ vs. R on a log-log plot, we would find the line in Figure 2Q35-2, that is, $\Delta \propto R^x$, $x = -0.62$. Very roughly, from theoretical considerations, we expect

$$\alpha \propto \exp\left(-\frac{R}{R_c}\right) \propto \exp\left(\frac{R}{\Delta^{-3/2}}\right) \quad (8)$$

where R_c is a constant ("a critical radius type of constant") that is proportional to $\Delta^{-3/2}$. Thus, taking logs,

$$\ln\alpha = -\Delta^{3/2}R + \text{constant} \quad (9)$$

We are interested in the Δ–R behavior at a constant α. We can lump the constant into $\ln\alpha$ and obtain,

$$\Delta \propto R^{-2/3} \quad (10)$$

As shown in Figure 2Q35-2, $x = -0.62$ is close to $-2/3$ given three points and the rough derivation above.

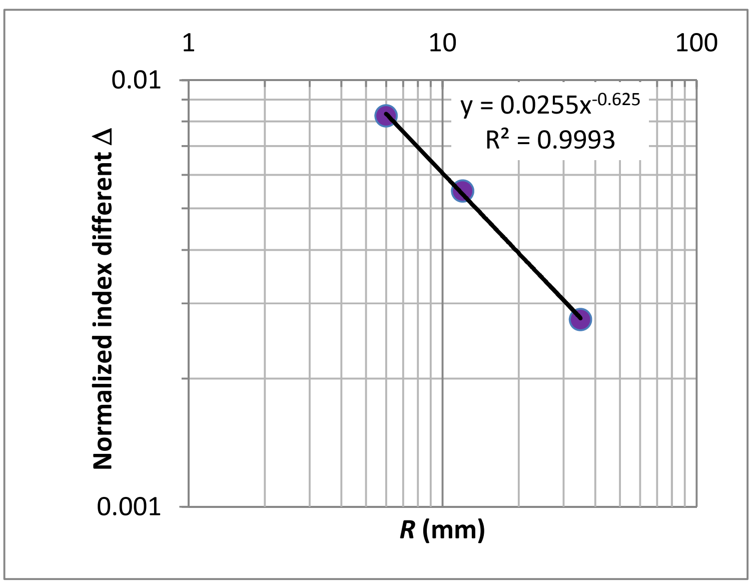


Figure 2Q35-2 The relationship between Δ and the radius of curvature R for a given amount of bending loss.

2.36 Bend loss reduced fibers Consider the bend loss measurements listed in Table 2.8 for four difference types of fiber. The trench fibers have a trench placed in the cladding where the refractive index is lowered as shown in Figure 2.39 The nanoengineered fiber is shown in the Figure 2.55. There is a ring of region in the cladding in which there are gas-filled nanoscale voids. (They are introduced during fabrication.) A void in the ring has a circular cross section but has a length along the fiber that can be a few meters. These voids occupy a volume in the ring that is only 1 - 10%. Plot the bending loss semilogarithmically (α on a log scale and R on a linear scale) and fit the data to $\alpha_{\text{micobend}} = A\exp(-R/R_c)$ and find A and R_c. What is your conclusion? Suppose that we set our maximum acceptable bending loss to 0.1 dB/turn in installation (the present goal is to bring the bending loss to below 0.1 dB/turn). What are the allowed radii of curvature for each turn?

Table 2.8 Bend radius R in mm, α in dB/turn. Data over 1.55 - 1.65 μm range. (Note, data used from a number of sources: (a) M.-J. Li *et al*. *J. Light Wave Technol.*, 27, 376, 2009; (b) K. Himeno *et al*, *J. Light Wave Technol.*, 23, 3494, 2005; (c) L.-A. de Montmorillon, *et al.* "Bend-Optimized G.652D Compatible Trench-Assisted Single-Mode Fibers" *Proceedings of the 55th IWCS/Focus*, pp. 342-347, November, 2006.)

Standard SMF[a] 1550 nm		Trench Fiber 1[b] 1650 nm		Trench Fiber 2[c] 1625 nm		Nanoengineered Fiber[a] 1550 nm	
R mm	α dB/turn	R mm	α dB/turn	R mm	α dB/turn	R mm	α dB/turn
5.0	15.0	7.50	0.354	5.0	0.178	5.0	0.031
7.0	4.00	10.0	0.135	7.5	0.0619	7.5	0.0081
10.0	0.611	15.0	0.020	10.0	0.0162	10.0	0.0030
12.5	0.124			15.0	0.00092	15	0.00018
16.0	0.0105						
17.5	0.0040						

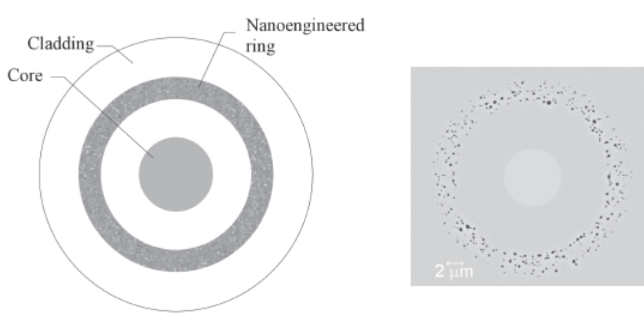


FIGURE 2.55 Left: The basic structure of bend-insensitive fiber with a nanoengineered ring in the cladding. Right: An SEM picture of the cross-section of a nanoengineered fiber with reduced bending losses. *(Courtesy of Ming-Jun Li, Corning Inc. For more information see U.S. Patent 8,055.110, 2011.)*

Solution

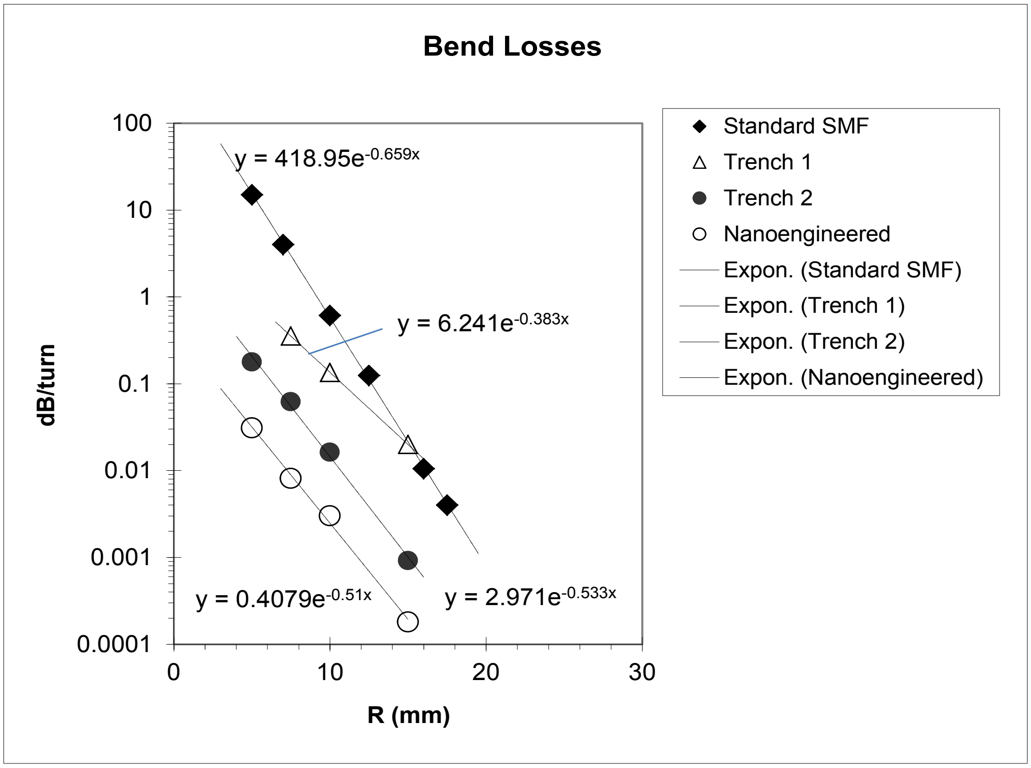


Figure 2Q36-1 Attenuation per turn as a function of bend radius

For a bending loss of 0.1 dB/turn, the allowed radii of curvature are (very roughly)

Standard SMF, 13 mm; trench 1, 10 mm; trench 2, 6 mm; nanoengineered, 3 mm.

2.37 Microbending loss Microbending loss α_B depends on the fiber characteristics and wavelength. We will calculate α_B approximately given various fiber parameters using the single mode fiber microbending loss equation (D. Marcuse, *J. Op. Soc. Am.*, *66*, 216, 1976)

$$\alpha_B \approx \frac{\pi^{1/2}\kappa^2}{2\gamma^{3/2}V^2\left[K_1(\gamma a)\right]^2} R^{-1/2} \exp(-\frac{2\gamma^3}{3\beta^2}R)$$

where R is the bend radius of curvature, a = fiber radius, β is the propagation constant, determined by b, normalized propagation constant, which is related to V, $\beta = n_2k[1 + b\Delta]$; $k = 2\pi/\lambda$ is the free-space wavevector; $\gamma = \sqrt{[\beta^2 - n_2^2k^2]}$; $\kappa = \sqrt{[n_1^2k^2 - \beta^2]}$, and $K_1(x)$ is a first-order modified Bessel function, available in math software packages. The normalized propagation constant b can be found from $b = (1.1428-0.996V^{-1})^2$. Consider a single mode fiber with $n_1 = 1.450$, $n_2 = 1.446$, $2a$ (diameter) = 3.9 μm. Plot α_B vs. R for $\lambda = 633$ nm and 790 nm from $R = 2$ mm to 15 mm. Figure 2.56 shows the experimental results on a SMF that has the same properties as the fiber above. What is your conclusion? (You might wish to compare your calculations with the experiments of A.J. Harris and P.F. Castle, *IEEE J. Light Wave Technol., LT4*, 34, 1986).

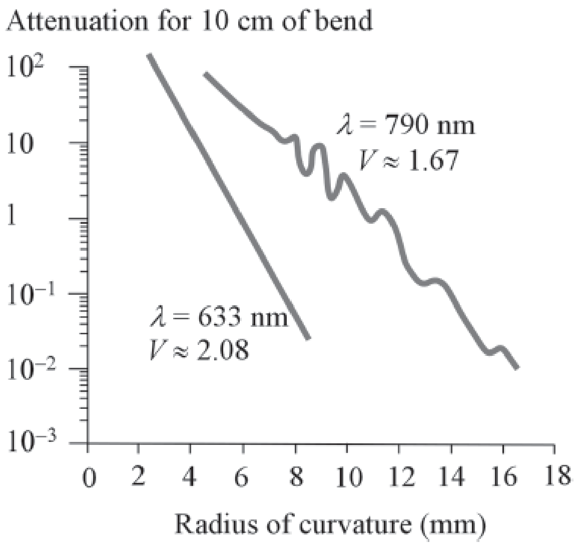


FIGURE 2.56 Measured microbending loss (attenuation) for a 10-cm fiber bent by different amounts of radius of curvature R. Single-mode fiber with a core diameter of 3.9 μm, cladding radius of 48 μm, $\Delta = 0.00275$, NA $\approx$ 0.10, $V \approx$ 1.67 and 2.08. (*Source:* Data extracted from A. J. Harris and P. F. Castle, "Bend Loss Measurements on High Aperture Single-Mode Fibers as a Function of Wavelength and Bend Radius," *IEEE J. Light Wave Technol.*, *LT14*, 34, 1986, and replotted with a smoothed curve; [see original article for the discussion of peaks in α_B vs. R at 790 nm]).

Solution

Given: $n_1 = 1.450$, $n_2 = 1.446$, $2a$ (diameter) = 3.9 μm; $\lambda = 790$ nm, we can calculate the following:

$k = 2\pi/\lambda = 7.953\times10^6$ m^{-1};

$$\Delta = \frac{(n_1^2 - n_2^2)^{1/2}}{2n_1^2} = 0.00275; \qquad V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(3.9\ \mu\text{m})(1.450^2 - 1.446^2)^{1/2}}{(0.790\ \mu\text{m})} = 1.67;$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 = 0.2977;$$

$\beta = n_2k[1 + b\Delta] = 1.1510\times10^7$ m^{-1}; $\gamma = \sqrt{}[\beta^2 - n_2^2k^2] = 4.6544\times10^5$ m^{-1};

$\kappa = \sqrt{}[\, n_1^2k^2 - \beta^2] = 7.175\times10^5$ m^{-1};

Substitute these values into

$$\alpha_B \approx \frac{\pi^{1/2}\kappa^2}{2\gamma^{3/2}V^2[K_1(\gamma a)]^2} R^{-1/2}\exp\left(-\frac{2\gamma^3}{3\beta^2}R\right)$$

to find $$\alpha_B \approx (1.03\times10^3)R^{-1/2}\exp\left(-\frac{R}{0.0020}\right)$$

which is plotted in Figure 2Q37-1 on the LHS .

Given: $n_1 = 1.450$, $n_2 = 1.446$, $2a$ (diameter) = 3.9 μm; $\lambda = 633$ nm, we can calculate the following:

$k = 2\pi/\lambda = 9.926\times10^6$ m^{-1};

$$\Delta = \frac{(n_1^2 - n_2^2)^{1/2}}{2n_1^2} = 0.00275; \qquad V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi(3.9\ \mu\text{m})(1.450^2 - 1.446^2)^{1/2}}{(0.633\ \mu\text{m})} = 2.08;$$

$$b \approx \left(1.1428 - \frac{0.996}{V}\right)^2 = 0.44127;$$

$\beta = n_2k[1 + b\Delta] = 1.437\times10^7$ m^{-1}; $\gamma = \sqrt{[\beta^2 - n_2^2k^2]} = 7.073\times10^5$ m^{-1};

$\kappa = \sqrt{[n_1^2k^2 - \beta]} = 7.99\times10^5$ m^{-1};

Substitute these values into

$$\alpha_B \approx \frac{\pi^{1/2}\kappa^2}{2\gamma^{3/2}V^2[K_1(\gamma a)]^2} R^{-1/2}\exp\left(-\frac{2\gamma^3}{3\beta^2}R\right)$$

to find $$\alpha_B \approx (2.08\times10^3)R^{-1/2}\exp\left(-\frac{R}{0.00089}\right)$$

which is plotted on the RHS of Figure 2Q37-1.

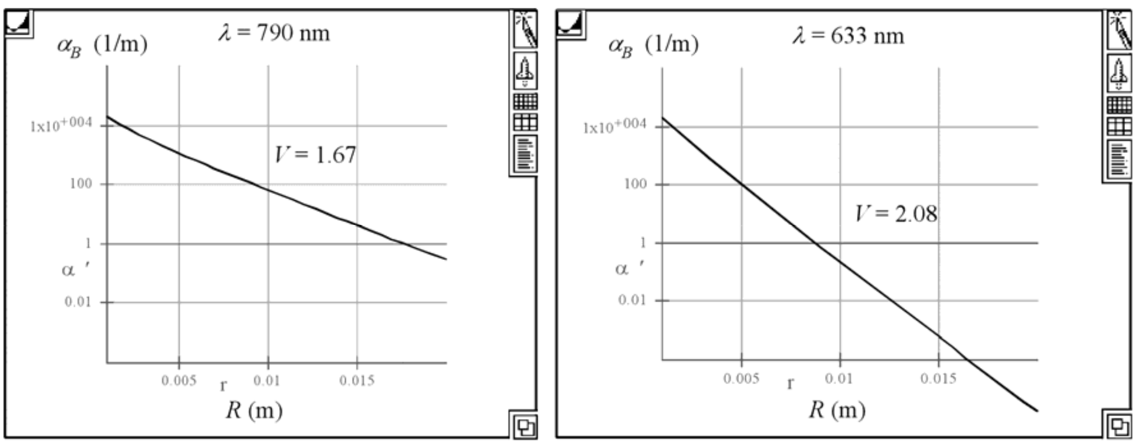


Figure 2Q37-1 Bending loss α_B vs. bend radius R (LiveMath used.)

Results compare reasonably with the experiments in Figure 2.56 given the approximate nature of the theory. Note that the calculated attenuation is per meter (for 1 meter) whereas the attenuation in Figure 2.56 is for a 10 cm fiber, so that for a 1 m of fiber, the observed attenuation will be 10 times higher.

2.38 Fiber Bragg grating A silica fiber based FBG is required to operate at 850 nm. What should be the periodicity of the grating Λ? If the amplitude of the index variation Δn is 2×10^{-5} and total length of the FBG is 5 mm, what are the maximum reflectance at the Bragg wavelength and the bandwidth of the FBG? Assume that the effective refractive index $\bar{n}$ is 1.460. What are the reflectance and the bandwidth if Δn is 2×10^{-4}?

Solution

Using equation for Bragg wavelength $\lambda_B = 2\bar{n}\Lambda$ one can get $\Lambda = \dfrac{\lambda_B}{2\bar{n}}$ = **291.1 nm**. The results of further calculations for $\Delta n = 2\times10^{-5}$ and 2×10^{-4} are collected in Table.

	FBG #1	FBG #2
Δn	2×10^{-5}	1×10^{-4}

κ (1/m)	73.92	739.2
κL	0.37	3.7
Grating is	weak	strong
$R = \tanh^2(\kappa L)$	**0.125**	**0.998**
$\Delta\lambda_{\text{strong}} = \dfrac{4\kappa\lambda_B^2}{\pi \overline{n}}$, nm	NA	**0.47**
$\Delta\lambda_{\text{weak}} = \dfrac{\lambda_B^2}{\overline{n}L}$, nm	**0.099**	N/A

The parameter κL for FBG#1 with $\Delta n = 2\times10^{-5}$ is equal to 0.37 which is a weak grating. The parameter κL for FBG#2 with $\Delta n = 2\times10^{-4}$ is equal to 3.7 which is a strong grating.

2.39 Fiber Bragg grating sensor array Consider a FBG strain sensor array embedded in a silica fiber that is used to measure strain at various locations on an object. Two neighboring sensors have grating periodicities of Λ_1 = 534.5 nm and Λ_2 = 539.7 nm. The effective refractive index is 1.450 and the photoelastic coefficient is 0.22. What is the maximum strain that can be measured assuming that (a) only one of the sensors is strained; (b) when the sensors are strained in opposite directions ? What would be the main problem with this sensor array? What is the strain at fracture if the fiber fractures roughly at an applied stress of 700 MPa and the elastic modulus is 70 GPa? What is your conclusion?

Solution

Initially the Bragg wavelengths of two sensors are $\lambda_{B1} = 2n\Lambda_1$ =1550.05 nm and $\lambda_{B2} = 2n\Lambda_2$ = 1565.13 nm, respectively. When the second sensor is stretched its effective refractive index changes due to photoelastic effect and there is also a change in the period, both of which leads to

$$\delta\lambda_{B2} = \lambda_{B2}(1-\tfrac{1}{2}n^2 p_e)\varepsilon$$

and λ_{B2} shifts towards λ_{B1}.

(a) The separation between the Bragg wavelengths is $\Delta\lambda_B = \lambda_{B2} - \lambda_{B1} =$ 1565.13 – 1550.05 = 15.08 nm

The shift due to strain is (only B is strained)

$$\delta\lambda_{B2} = = \lambda_{B2}\left(1-\tfrac{1}{2}n^2 p_e\right)\varepsilon = 2n\Lambda_2\left(1-\tfrac{1}{2}n^2 p_e\right)\varepsilon$$

Maximum strain occurs when

$$\delta\lambda_B = \lambda_{B2} - \lambda_{B1} = 2n\Lambda_2 - 2n\Lambda_1 = 2n\Lambda_2\left(1-\tfrac{1}{2}n^2 p_e\right)\varepsilon$$

$$\therefore \qquad \varepsilon = \frac{\Lambda_2 - \Lambda_1}{\Lambda_2(1-\tfrac{1}{2}n^2 p_e)} = 0.012 \text{ or } 1.2\%$$

The strain at fracture is given by strain = stress / elastic modulus = $700\times10^6 / 70\times10^9$ = 0.01 or 1%. The fiber is likely to fracture before it reaches the maximum strain.

(b) Consider the sensors strained in opposite directions. The separation between the Bragg wavelengths is still $\lambda_{B2} - \lambda_{B1}$ =1565.13 – 1550.05 = 15.08 nm. Note that $\lambda_{B2} - \lambda_{B1} = 2n(\Lambda_2 - \Lambda_2)$

Shift due to strain is now

$$\delta\lambda_B = = \lambda_{B2}\left(1 - \tfrac{1}{2}n^2 p_e\right)\varepsilon + \lambda_{B1}\left(1 - \tfrac{1}{2}n^2 p_e\right)\varepsilon = 2n(\Lambda_1 + \Lambda_2)\left(1 - \tfrac{1}{2}n^2 p_e\right)\varepsilon$$

Which must be $\lambda_{B2} - \lambda_{B1}$ so that

$$2n(\Lambda_2 - \Lambda_2) = 2n(\Lambda_1 + \Lambda_2)\left(1 - \tfrac{1}{2}n^2 p_e\right)\varepsilon$$

$\therefore$ $$\varepsilon = \frac{\Lambda_2 - \Lambda_2}{(\Lambda_1 + \Lambda_2)\left(1 - \tfrac{1}{2}n^2 p_e\right)}$$ = 0.0063 or 0.63% (about half the above value).

The main problem is precise compensation of temperature.