1. (10 points) The equation $f(x)=x^{2}-2 e^{x}=0$ has a solution in the interval $[-1,1]$.
(a) (5 points) With $p_{0}=-1$ and $p_{1}=1$ calculate $p_{2}$ using the Secant method.
(b) (5 points) With $p_{2}$ from part (a) calculate $p_{3}$ using Newton's method.
2. (15 points) The equation $f(x)=2-x^{2} \sin x=0$ has a solution in the interval $[-1,2]$.
(a) (5 points) Verify that the Bisection method can be applied to the function $f(x)$ on $[-1,2]$.
(b) (5 points) Using the error formula for the Bisection method find the number of iterations needed for accuracy 0.000001. Do not do the Bisection calculations.
(c) (5 points) Compute $p_{3}$ for the Bisection method.
3. (15 points) The following refer to the fixed-point problem
(a) (5 points) State the theorem which gives conditions for a fixed-point sequence to converge to a unique fixed point.
(b) (5 points) Given $g(x)=\frac{2-x^{3}+2 x}{3}$, use the theorem to show that the fixed-point sequence will converge to the unique fixed-point of $g$ for any $p_{0}$ in $[-1,1.1]$.
(c) (5 points) With $p_{0}=0.5$ generate $p_{3}$.
4. (10 points) Suppose the function $f(x)$ has a unique zero $p$ in the interval [a, b]. Further, suppose $f^{\prime \prime}(x)$ exists and is continuous on the interval $[\mathrm{a}, \mathrm{b}]$.
(a) (5 points) Under what conditions will Newton's Method give a quadratically convergent sequence to $p$ ?
(b) (5 points) Define quadratic convergence.
5. (10 points) Let $g(x)=\frac{2-x^{3}+2 x}{3}$ on the interval [-1, 1.1]. Let the initial value be 0 and compute the result of 2 iterations of Stefffensen's Method to approximate the solution of $x=$ $g(x)$.
