CHAPTER 11
Vectors and the Geometry of Space

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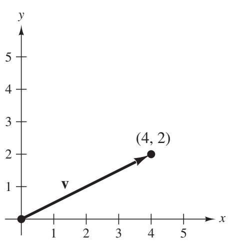
Section 11.1 Vectors in the Plane

1. Answers will vary. *Sample answer*: A scalar is a real number such as 2. A vector is represented by a directed line segment. A vector has both magnitude and direction. For example $\left\langle \sqrt{3}, 1 \right\rangle$ has direction $\dfrac{\pi}{6}$ and a magnitude of 2.

2. Notice that $\mathbf{v} = \langle 6, -7 \rangle = \langle 2 - (-4), -1 - 6 \rangle = \overrightarrow{QP}$.
Hence, Q is the initial point and P is the terminal point.

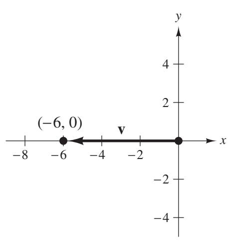
3. (a) $\mathbf{v} = \langle 5 - 1, 4 - 2 \rangle = \langle 4, 2 \rangle$

(b)



4. (a) $\mathbf{v} = \langle -4 - 2, -3 - (-3) \rangle = \langle -6, 0 \rangle$

(b)



5. $\mathbf{u} = \langle 5 - 3, 6 - 2 \rangle = \langle 2, 4 \rangle$
$\mathbf{v} = \langle 3 - 1, 8 - 4 \rangle = \langle 2, 4 \rangle$
$\mathbf{u} = \mathbf{v}$

6. $\mathbf{u} = \langle 1 - (-4), 8 - 0 \rangle = \langle 5, 8 \rangle$
$\mathbf{v} = \langle 7 - 2, 7 - (-1) \rangle = \langle 5, 8 \rangle$
$\mathbf{u} = \mathbf{v}$

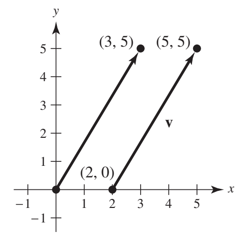
7. $\mathbf{u} = \langle 6 - 0, -2 - 3 \rangle = \langle 6, -5 \rangle$
$\mathbf{v} = \langle 9 - 3, 5 - 10 \rangle = \langle 6, -5 \rangle$
$\mathbf{u} = \mathbf{v}$

8. $\mathbf{u} = \langle 11 - (-4), -4 - (-1) \rangle = \langle 15, -3 \rangle$
$\mathbf{v} = \langle 25 - 0, 10 - 13 \rangle = \langle 15, -3 \rangle$
$\mathbf{u} = \mathbf{v}$

9. (b) $\mathbf{v} = \langle 5 - 2, 5 - 0 \rangle = \langle 3, 5 \rangle$

(c) $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$

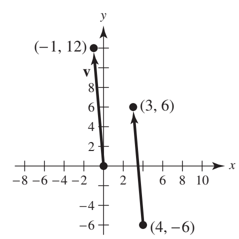
(a), (d)



10. (b) $\mathbf{v} = \langle 3 - 4, 6 - (-6) \rangle = \langle -1, 12 \rangle$

(c) $\mathbf{v} = -\mathbf{i} + 12\mathbf{j}$

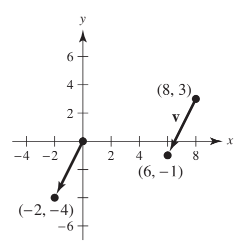
(a), (d)



11. (b) $\mathbf{v} = \langle 6 - 8, -1 - 3 \rangle = \langle -2, -4 \rangle$

(c) $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

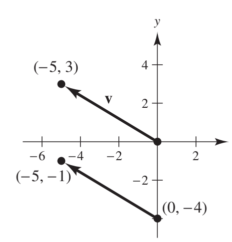
(a), (d)



12. (b) $\mathbf{v} = \langle -5 - 0, -1 - (-4) \rangle = \langle -5, 3 \rangle$

(c) $\mathbf{v} = -5\mathbf{i} + 3\mathbf{j}$

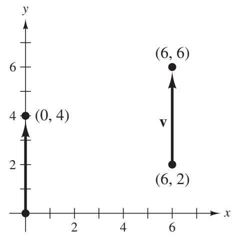
(a) and (d).



13. (b) $\mathbf{v} = \langle 6 - 6, 6 - 2 \rangle = \langle 0, 4 \rangle$

(c) $\mathbf{v} = 4\mathbf{j}$

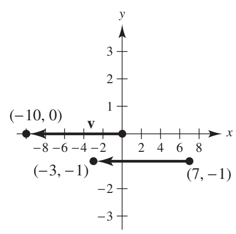
(a) and (d).



14. (b) $\mathbf{v} = \langle -3 - 7, -1 - (-1) \rangle = \langle -10, 0 \rangle$

(c) $\mathbf{v} = -10\mathbf{i}$

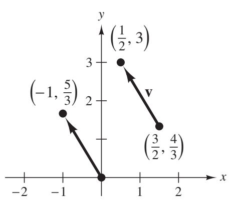
(a) and (d).



15. (b) $\mathbf{v} = \left\langle \frac{1}{2} - \frac{3}{2}, 3 - \frac{4}{3} \right\rangle = \left\langle -1, \frac{5}{3} \right\rangle$

(c) $\mathbf{v} = -\mathbf{i} + \frac{5}{3}\mathbf{j}$

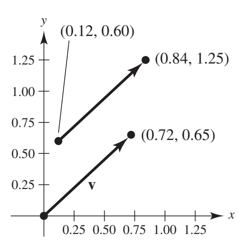
(a) and (d)



16. (b) $\mathbf{v} = \langle 0.84 - 0.12, 1.25 - 0.60 \rangle = \langle 0.72, 0.65 \rangle$

(c) $\mathbf{v} = 0.72\mathbf{i} + 0.65\mathbf{j}$

(a) and (d).



17. $u_1 - 4 = -1$ $\qquad u_1 = 3$

$u_2 - 2 = 3$ $\qquad u_2 = 5$

$Q = (3, 5)$ $\qquad$ Terminal point

18. $u_1 - 5 = 4$ $\qquad u_1 = 9$

$u_2 - 3 = -9$ $\qquad u_2 = -6$

$Q = (9, -6)$ $\qquad$ Terminal point

19. $\mathbf{v} = 4i$

$\|\mathbf{v}\| = \sqrt{4^2} = 4$

20. $\mathbf{v} = -9j$

$\|\mathbf{v}\| = \sqrt{(-9)^2} = 9$

21. $\mathbf{v} = \langle 8, 15 \rangle$

$\|\mathbf{v}\| = \sqrt{8^2 + 15^2} = 17$

22. $\mathbf{v} = \langle -24, 7 \rangle$

$\|\mathbf{v}\| = \sqrt{(-24)^2 + 7^2} = 25$

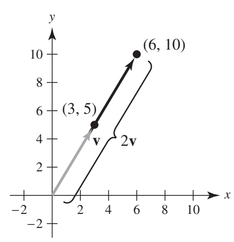
23. $\mathbf{v} = -\mathbf{i} - 5\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$

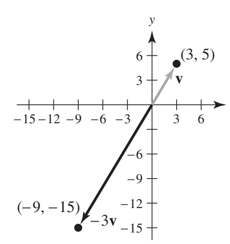
24. $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

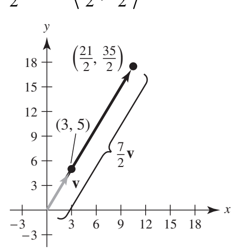
25. (a) $2\mathbf{v} = 2\langle 3, 5 \rangle = \langle 6, 10 \rangle$



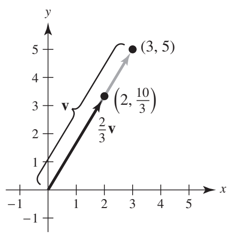
(b) $-3\mathbf{v} = \langle -9, -15 \rangle$



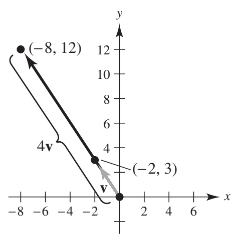
(c) $\frac{7}{2}\mathbf{v} = \left\langle \frac{21}{2}, \frac{35}{2} \right\rangle$



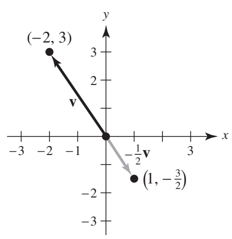
(d) $\frac{2}{3}\mathbf{v} = \left\langle 2, \frac{10}{3} \right\rangle$



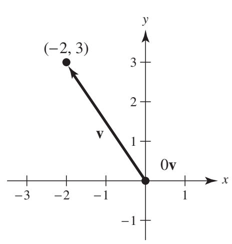
26. (a) $4\mathbf{v} = 4\langle -2, 3\rangle = \langle -8, 12\rangle$



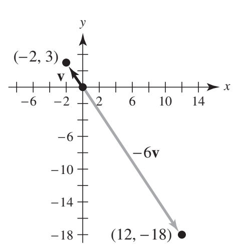
(b) $-\frac{1}{2}\mathbf{v} = \left\langle 1, -\frac{3}{2} \right\rangle$



(c) $0\,\mathbf{v} = \langle 0, 0\rangle$



(d) $-6\mathbf{u} = \langle 12, -18\rangle$



27. $\mathbf{u} = \langle 4, 9\rangle$, $\mathbf{v} = \langle 2, -5\rangle$

(a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle 4, 9\rangle = \left\langle \frac{8}{3}, 6 \right\rangle$

(b) $3\mathbf{v} = 3\langle 2, -5\rangle = \langle 6, -15\rangle$

(c) $\mathbf{v} - \mathbf{u} = \langle 2, -5\rangle - \langle 4, 9\rangle = \langle -2, -14\rangle$

(d) $2\mathbf{u} + 5\mathbf{v} = 2\langle 4, 9\rangle + 5\langle 2, -5\rangle$
$= \langle 8, 18\rangle + \langle 10, -25\rangle$
$= \langle 18, -7\rangle$

28. $\mathbf{u} = \langle -3, -8\rangle$, $\mathbf{v} = \langle 8, 7\rangle$

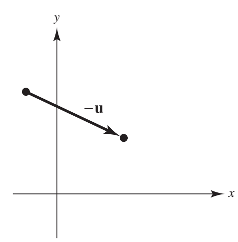
(a) $\frac{2}{3}\mathbf{u} = \frac{2}{3}\langle -3, -8\rangle = \left\langle -2, -\frac{16}{3} \right\rangle$

(b) $3\mathbf{v} = 3\langle 8, 7\rangle = \langle 24, 21\rangle$

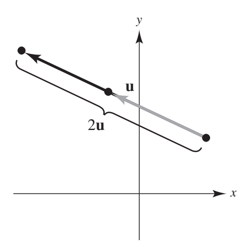
(c) $\mathbf{v} - u = \langle 8, 7\rangle - \langle -3, -8\rangle = \langle 11, 15\rangle$

(d) $2\mathbf{u} + 5\mathbf{v} = 2\langle -3, -8\rangle + 5\langle 8, 7\rangle$
$= \langle -6, -16\rangle + \langle 40, 35\rangle$
$= \langle 34, 19\rangle$

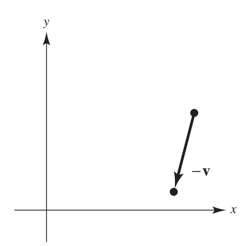
29.



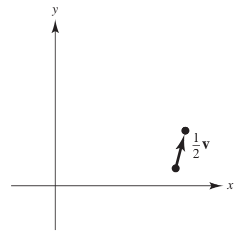
30. Twice as long as given vector **u.**



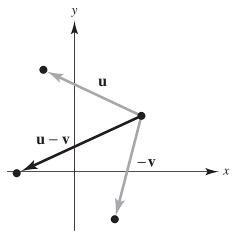
31.



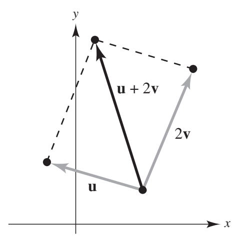
32.



33.



34.



35. $\mathbf{v} = \langle 3, 12\rangle$

$$\|\mathbf{v}\| = \sqrt{3^2 + 12^2} = \sqrt{153}$$

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\langle 3, 12\rangle}{\sqrt{153}} = \left\langle \frac{3}{\sqrt{153}}, \frac{12}{\sqrt{153}} \right\rangle$$

$$= \left\langle \frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right\rangle \text{ unit vector}$$

36. $\mathbf{v} = \langle -5, 15\rangle$

$$\|\mathbf{v}\| = \sqrt{25 + 225} = \sqrt{250} = 5\sqrt{10}$$

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\langle -5, 15\rangle}{5\sqrt{10}} = \left\langle -\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right\rangle \text{ unit vector}$$

37. $\mathbf{v} = \left\langle \frac{3}{2}, \frac{5}{2} \right\rangle$

$$\|\mathbf{v}\| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{\sqrt{34}}{2}$$

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\left\langle \left(\frac{3}{2}\right), \left(\frac{5}{2}\right) \right\rangle}{\frac{\sqrt{34}}{2}} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle$$

$$= \left\langle \frac{3\sqrt{34}}{34}, \frac{5\sqrt{34}}{34} \right\rangle \text{ unit vector}$$

38. $\mathbf{v} = \langle -6.2, 3.4\rangle$

$$\|\mathbf{v}\| = \sqrt{(-6.2)^2 + (3.4)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\langle -6.2, 3.4\rangle}{5\sqrt{2}} = \left\langle -\frac{31\sqrt{2}}{50}, \frac{17\sqrt{2}}{50} \right\rangle \text{ unit vector}$$

39. $\mathbf{u} = \langle 1, -1\rangle, \mathbf{v} = \langle -1, 2\rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + 1} = \sqrt{2}$

(b) $\|\mathbf{v}\| = \sqrt{1 + 4} = \sqrt{5}$

(c) $\mathbf{u} + \mathbf{v} = \langle 0, 1\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{0 + 1} = 1$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{1}{\sqrt{2}}\langle 1, -1\rangle$

$\left\| \dfrac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{5}}\langle -1, 2\rangle$

$\left\| \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \langle 0, 1\rangle$

$\left\| \dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

40. $\mathbf{u} = \langle 0, 1\rangle, \mathbf{v} = \langle 3, -3\rangle$

(a) $\|\mathbf{u}\| = \sqrt{0 + 1} = 1$

(b) $\|\mathbf{v}\| = \sqrt{9 + 9} = 3\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 3, -2\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + 4} = \sqrt{13}$

(d) $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \langle 0, 1\rangle$

$\left\| \dfrac{\mathbf{u}}{\|\mathbf{u}\|} \right\| = 1$

(e) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3\sqrt{2}}\langle 3, -3\rangle$

$\left\| \dfrac{\mathbf{v}}{\|\mathbf{v}\|} \right\| = 1$

(f) $\dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \dfrac{1}{\sqrt{13}}\langle 3, -2\rangle$

$\left\| \dfrac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} \right\| = 1$

41. $\mathbf{u} = \left\langle 1, \frac{1}{2} \right\rangle, \mathbf{v} = \langle 2, 3 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

(b) $\|\mathbf{v}\| = \sqrt{4 + 9} = \sqrt{13}$

(c) $\mathbf{u} + \mathbf{v} = \left\langle 3, \frac{7}{2} \right\rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{9 + \frac{49}{4}} = \frac{\sqrt{85}}{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{2}{\sqrt{5}}\left\langle 1, \frac{1}{2} \right\rangle$

$\left\|\frac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{13}}\langle 2, 3 \rangle$

$\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{2}{\sqrt{85}}\left\langle 3, \frac{7}{2} \right\rangle$

$\left\|\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\| = 1$

42. $\mathbf{u} = \langle 2, -4 \rangle, \mathbf{v} = \langle 5, 5 \rangle$

(a) $\|\mathbf{u}\| = \sqrt{4 + 16} = 2\sqrt{5}$

(b) $\|\mathbf{v}\| = \sqrt{25 + 25} = 5\sqrt{2}$

(c) $\mathbf{u} + \mathbf{v} = \langle 7, 1 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{49 + 1} = 5\sqrt{2}$

(d) $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{5}}\langle 2, -4 \rangle$

$\left\|\frac{\mathbf{u}}{\|\mathbf{u}\|}\right\| = 1$

(e) $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{5\sqrt{2}}\langle 5, 5 \rangle$

$\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$

(f) $\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|} = \frac{1}{5\sqrt{2}}\langle 7, 1 \rangle$

$\left\|\frac{\mathbf{u} + \mathbf{v}}{\|\mathbf{u} + \mathbf{v}\|}\right\| = 1$

43. $\mathbf{u} = \langle 2, 1 \rangle$

$\|\mathbf{u}\| = \sqrt{5} \approx 2.236$

$\mathbf{v} = \langle 5, 4 \rangle$

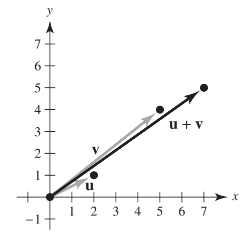
$\|\mathbf{v}\| = \sqrt{41} \approx 6.403$

$\mathbf{u} + \mathbf{v} = \langle 7, 5 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{74} \approx 8.602$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$\sqrt{74} \le \sqrt{5} + \sqrt{41}$



44. $\mathbf{u} = \langle -3, 2 \rangle$

$\|\mathbf{u}\| = \sqrt{13} \approx 3.606$

$\mathbf{v} = \langle 1, -2 \rangle$

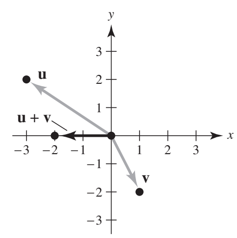
$\|\mathbf{v}\| = \sqrt{5} \approx 2.236$

$\mathbf{u} + \mathbf{v} = \langle -2, 0 \rangle$

$\|\mathbf{u} + \mathbf{v}\| = 2$

$\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$

$2 \le \sqrt{13} + \sqrt{5}$



45. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{3}\langle 0, 3 \rangle = \langle 0, 1 \rangle$

$6\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 6\langle 0, 1 \rangle = \langle 0, 6 \rangle$

$\mathbf{v} = \langle 0, 6 \rangle$

46. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{2}}\langle 1, 1 \rangle$

$4\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 2\sqrt{2}\langle 1, 1 \rangle$

$\mathbf{v} = \left\langle 2\sqrt{2}, 2\sqrt{2} \right\rangle$

47. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{\sqrt{5}}\langle -1, 2 \rangle = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

$5\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = 5\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \left\langle -\sqrt{5}, 2\sqrt{5} \right\rangle$

$\mathbf{v} = \left\langle -\sqrt{5}, 2\sqrt{5} \right\rangle$

48. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{2\sqrt{3}}\left\langle \sqrt{3}, 3 \right\rangle$

$2\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \frac{1}{\sqrt{3}}\left\langle \sqrt{3}, 3 \right\rangle$

$\mathbf{v} = \left\langle 1, \sqrt{3} \right\rangle$

49. $\mathbf{v} = 3\left[(\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j}\right] = 3\mathbf{i} = \langle 3, 0 \rangle$

50. $\mathbf{v} = 5\left[(\cos 120°)\mathbf{i} + (\sin 120°)\mathbf{j}\right]$

$= -\frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j} = \left\langle -\frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$

51. $\mathbf{v} = 2\left[(\cos 150°)\mathbf{i} + (\sin 150°)\mathbf{j}\right]$

$= -\sqrt{3}\mathbf{i} + \mathbf{j} = \left\langle -\sqrt{3}, 1 \right\rangle$

52. $\mathbf{v} = 4\left[(\cos 3.5°)\mathbf{i} + (\sin 3.5°)\mathbf{j}\right]$

$\approx 3.9925\mathbf{i} + 0.2442\mathbf{j}$

$= \langle 3.9925, 0.2442 \rangle$

53. $\mathbf{u} = (\cos 0°)\mathbf{i} + (\sin 0°)\mathbf{j} = \mathbf{i}$

$\mathbf{v} = 3(\cos 45°)\mathbf{i} + 3(\sin 45°)\mathbf{j} = \frac{3\sqrt{2}}{2}\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \left(\frac{2 + 3\sqrt{2}}{2}\right)\mathbf{i} + \frac{3\sqrt{2}}{2}\mathbf{j} = \left\langle \frac{2 + 3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$

54. $\mathbf{u} = 4(\cos 0°)\mathbf{i} + 4(\sin 0°)\mathbf{j} = 4\mathbf{i}$

$\mathbf{v} = 2(\cos 30°)\mathbf{i} + 2(\sin 30°)\mathbf{j} = \mathbf{i} + \sqrt{3}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = 5\mathbf{i} + \sqrt{3}\mathbf{j} = \left\langle 5, \sqrt{3} \right\rangle$

55. $\mathbf{u} = 2(\cos 4)\mathbf{i} + 2(\sin 4)\mathbf{j}$

$\mathbf{v} = (\cos 2)\mathbf{i} + (\sin 2)\mathbf{j}$

$\mathbf{u} + \mathbf{v} = (2\cos 4 + \cos 2)\mathbf{i} + (2\sin 4 + \sin 2)\mathbf{j}$

$= \langle 2\cos 4 + \cos 2, 2\sin 4 + \sin 2 \rangle$

56. $\mathbf{u} = 5\left[\cos(-0.5)\right]\mathbf{i} + 5\left[\sin(-0.5)\right]\mathbf{j}$

$= 5(\cos 0.5)\mathbf{i} - 5(\sin 0.5)\mathbf{j}$

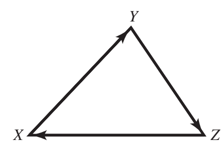
$\mathbf{v} = 5(\cos 0.5)\mathbf{i} + 5(\sin 0.5)\mathbf{j}$

$\mathbf{u} + \mathbf{v} = 10(\cos 0.5)\mathbf{i} = \langle 10\cos 0.5, 0 \rangle$

57. The forces act along the same direction. $\theta = 0°$.

58. The forces cancel out each other. $\theta = 180°$.

59.



$\overrightarrow{XY} + \overrightarrow{YZ} + \overrightarrow{ZX} = 0.$

Vectors that start and end at the same point have a magnitude of 0.

60. (a) True. **d** has the same magnitude as **a** but is in the opposite direction.

(b) True. **c** and **s** have the same length and direction.

(c) True. **a** and **u** are the adjacent sides of a parallelogram. So, the resultant vector, $\mathbf{a} + \mathbf{u}$, is the diagonal of the parallelogram, **c**.

(d) False. The negative of a vector has the opposite direction of the original vector.

(e) True. $\mathbf{a} + \mathbf{d} = \mathbf{a} + (-\mathbf{a}) = 0$

(f) False. $\mathbf{u} - \mathbf{v} = \mathbf{u} - (-\mathbf{u}) = 2\mathbf{u}$

$-2(\mathbf{b} + \mathbf{t}) = -2(\mathbf{b} + \mathbf{b}) = -2(2\mathbf{b}) = -2\left[2(-\mathbf{u})\right] = 4\mathbf{u}$

61. $\mathbf{v} = \langle 4, 5 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$4 = a + b$

$5 = 2a - b$

Adding the equations, $9 = 3a \Rightarrow a = 3$.

Then you have $b = 4 - a = 4 - 3 = 1$.

$a = 3, b = 1$

62. $\mathbf{v} = \langle -7, -2 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$-7 = a + b$

$-2 = 2a - b$

Adding the equations, $-9 = 3a \Rightarrow a = -3$.

Then you have $b = -7 - a = -7 - (-3) = -4$.

$a = -3, b = -4$

63. $\mathbf{v} = \langle -6, 0 \rangle = a\langle 1, 2 \rangle + 6\langle 1, -1 \rangle$

$-6 = a + b$

$0 = 2a - b$

Adding the equations, $-6 = 3a \Rightarrow a = -2$.

Then you have $b = -6 - a = -6 - (-2) = -4$.

$a = -2, b = -4$

64. $\mathbf{v} = \langle 0, 6 \rangle = a\langle 1, 2 \rangle + b\langle 1, -1 \rangle$

$0 = a + b$

$6 = 2a - b$

Adding the equations, $6 = 3a \Rightarrow a = 2$.

Then you have $b = -a = -2$.

$a = 2, b = -2$

65. $\mathbf{v} = \langle 1, -3\rangle = a\langle 1, 2\rangle + b\langle 1, -1\rangle$

$1 = a + b$

$-3 = 2a - b$

Adding the equations, $-2 = 3a \Rightarrow a = -\frac{2}{3}$.

Then you have $b = 1 - a = 1 - \left(-\frac{2}{3}\right) = \frac{5}{3}$.

$a = -\frac{2}{3}, b = \frac{5}{3}$

66. $\mathbf{v} = \langle -1, 8\rangle = a\langle 1, 2\rangle + b\langle 1, -1\rangle$

$-1 = a + b$

$8 = 2a - b$

Adding the equations, $7 = 3a \Rightarrow a = \frac{7}{3}$.

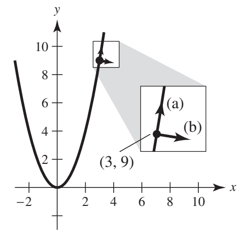
Then you have $b = -1 - a = -1 - \frac{7}{3} = -\frac{10}{3}$.

$a = \frac{7}{3}, b = -\frac{10}{3}$

67. $f(x) = x^2, f'(x) = 2x, f'(3) = 6$

(a) $m = 6$. Let $\mathbf{w} = \langle 1, 6\rangle$, $\|\mathbf{w}\| = \sqrt{37}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle 1, 6\rangle$.

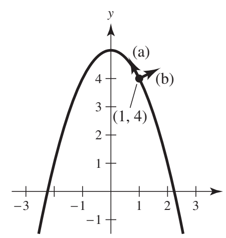
(b) $m = -\frac{1}{6}$. Let $\mathbf{w} = \langle -6, 1\rangle$, $\|\mathbf{w}\| = \sqrt{37}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{37}}\langle -6, 1\rangle$.



68. $f(x) = -x^2 + 5, f'(x) = -2x, f'(1) = -2$

(a) $m = -2$. Let $\mathbf{w} = \langle 1, -2\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 1, -2\rangle$.

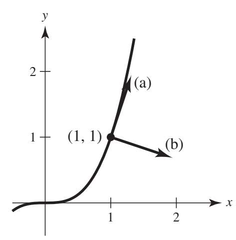
(b) $m = \frac{1}{2}$. Let $\mathbf{w} = \langle 2, 1\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\pm\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{5}}\langle 2, 1\rangle$.



69. $f(x) = x^3, f'(x) = 3x^2 = 3$ at $x = 1$.

(a) $m = 3$. Let $\mathbf{w} = \langle 1, 3\rangle$, $\|\mathbf{w}\| = \sqrt{10}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$.

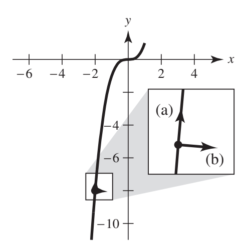
(b) $m = -\frac{1}{3}$. Let $\mathbf{w} = \langle 3, -1\rangle$, $\|\mathbf{w}\| = \sqrt{10}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{10}}\langle 3, -1\rangle$.



70. $f(x) = x^3, f'(x) = 3x^2 = 12$ at $x = -2$.

(a) $m = 12$. Let $\mathbf{w} = \langle 1, 12\rangle$, $\|\mathbf{w}\| = \sqrt{145}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 1, 12\rangle$.

(b) $m = -\frac{1}{12}$. Let $\mathbf{w} = \langle 12, -1\rangle$, $\|\mathbf{w}\| = \sqrt{145}$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{\sqrt{145}}\langle 12, -1\rangle$.

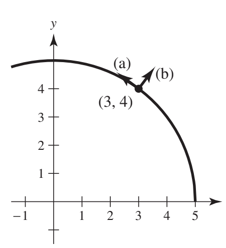


71. $f(x) = \sqrt{25 - x^2}$

$f'(x) = \frac{-x}{\sqrt{25 - x^2}} = \frac{-3}{4}$ at $x = 3$.

(a) $m = -\frac{3}{4}$. Let $\mathbf{w} = \langle -4, 3\rangle$, $\|\mathbf{w}\| = 5$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle -4, 3\rangle$.

(b) $m = \frac{4}{3}$. Let $\mathbf{w} = \langle 3, 4\rangle$, $\|\mathbf{w}\| = 5$, then $\frac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\frac{1}{5}\langle 3, 4\rangle$.

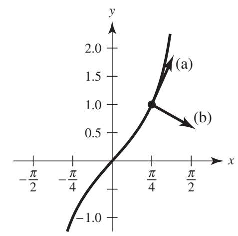


72. $f(x) = \tan x$

$f'(x) = \sec^2 x = 2$ at $x = \dfrac{\pi}{4}$

(a) $m = 2$. Let $\mathbf{w} = \langle 1, 2\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\dfrac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\dfrac{1}{\sqrt{5}}\langle 1, 2\rangle$.

(b) $m = -\dfrac{1}{2}$. Let $\mathbf{w} = \langle -2, 1\rangle$, $\|\mathbf{w}\| = \sqrt{5}$, then $\dfrac{\mathbf{w}}{\|\mathbf{w}\|} = \pm\dfrac{1}{\sqrt{5}}\langle -2, 1\rangle$.



73. $\mathbf{u} = \dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$

$\mathbf{u} + \mathbf{v} = \sqrt{2}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j} = \left\langle -\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right\rangle$

74. $\mathbf{u} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$

$\mathbf{u} + \mathbf{v} = -3\mathbf{i} + 3\sqrt{3}\mathbf{j}$

$\mathbf{v} = (\mathbf{u} + \mathbf{v}) - \mathbf{u} = \left(-3 - 2\sqrt{3}\right)\mathbf{i} + \left(3\sqrt{3} - 2\right)\mathbf{j}$

$= \left\langle -3 - 2\sqrt{3}, 3\sqrt{3} - 2\right\rangle$

75. $\mathbf{F}_1 + \mathbf{F}_2 = (500 \cos 30°\mathbf{i} + 500 \sin 30°\mathbf{j}) + \left(200 \cos(-45°)\mathbf{i} + 200 \sin(-45°)\mathbf{j}\right) = \left(250\sqrt{3} + 100\sqrt{2}\right)\mathbf{i} + \left(250 - 100\sqrt{2}\right)\mathbf{j}$

$\|\mathbf{F}_1 + \mathbf{F}_2\| = \sqrt{\left(250\sqrt{3} + 100\sqrt{2}\right)^2 + \left(250 - 100\sqrt{2}\right)^2} \approx 584.6$ lb

$\tan\theta = \dfrac{250 - 100\sqrt{2}}{250\sqrt{3} + 100\sqrt{2}} \Rightarrow \theta \approx 10.7°$

76. (a) $180(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + 275\mathbf{i} \approx 430.88\mathbf{i} + 90\mathbf{j}$

Direction: $\alpha \approx \arctan\left(\dfrac{90}{430.88}\right) \approx 0.206\ (\approx 11.8°)$

Magnitude: $\sqrt{430.88^2 + 90^2} \approx 440.18$ newtons

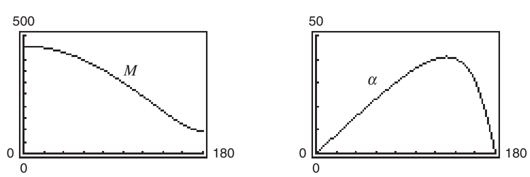
(b) $M = \sqrt{(275 + 180\cos\theta)^2 + (180\sin\theta)^2}$

$\alpha = \arctan\left[\dfrac{180\sin\theta}{275 + 180\cos\theta}\right]$

(c)

θ	0°	30°	60°	90°	120°	150°	180°
M	455	440.2	396.9	328.7	241.9	149.3	95
α	0°	11.8°	23.1°	33.2°	40.1°	37.1°	0

(d)



(e) M decreases because the forces change from acting in the same direction to acting in the opposite direction as θ increases from $0°$ to $180°$.

77. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (75 \cos 30°\mathbf{i} + 75 \sin 30°\mathbf{j}) + (100 \cos 45°\mathbf{i} + 100 \sin 45°\mathbf{j}) + (125 \cos 120°\mathbf{i} + 125 \sin 120°\mathbf{j})$

$= \left(\frac{75}{2}\sqrt{3} + 50\sqrt{2} - \frac{125}{2}\right)\mathbf{i} + \left(\frac{75}{2} + 50\sqrt{2} + \frac{125}{2}\sqrt{3}\right)\mathbf{j}$

$\|\mathbf{R}\| = \|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\| \approx 228.5$ lb

$\theta_{\mathbf{R}} = \theta_{\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3} \approx 71.3°$

78. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \left[400\left(\cos(-30°)\mathbf{i} + \sin(-30°)\mathbf{j}\right)\right] + \left[280\left(\cos(45°)\mathbf{i} + \sin(45°)\mathbf{j}\right)\right] + \left[350\left(\cos(135°)\mathbf{i} + \sin(135°)\mathbf{j}\right)\right]$

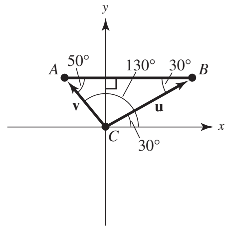
$= \left[200\sqrt{3} + 140\sqrt{2} - 175\sqrt{2}\right]\mathbf{i} + \left[-200 + 140\sqrt{2} + 175\sqrt{2}\right]\mathbf{j}$

$\|\mathbf{R}\| = \sqrt{\left(200\sqrt{3} - 35\sqrt{2}\right)^2 + \left(-200 + 315\sqrt{2}\right)^2} \approx 385.2483$ newtons

$\theta_{\mathbf{R}} = \arctan\left(\dfrac{-200 + 315\sqrt{2}}{200\sqrt{3} - 35\sqrt{2}}\right) \approx 0.6908 \approx 39.6°$

79. $\mathbf{u} = \overrightarrow{CB} = \|\mathbf{u}\|(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

$\mathbf{v} = \overrightarrow{CA} = \|\mathbf{v}\|(\cos 130°\mathbf{i} + \sin 130°\mathbf{j})$



Vertical components:
$\|\mathbf{u}\|\sin 30° + \|\mathbf{v}\|\sin 130° = 3000$

Horizontal components:
$\|\mathbf{u}\|\cos 30° + \|\mathbf{v}\|\cos 130° = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 1958.1$ pounds,

$\|\mathbf{v}\| \approx 2638.2$ pounds.

80. $\theta_1 = \arctan\left(\frac{24}{20}\right) \approx 0.8761$ or $50.2°$

$\theta_2 = \arctan\left(\frac{24}{-10}\right) + \pi \approx 1.9656$ or $112.6°$

$\mathbf{u} = \|\mathbf{u}\|(\cos\theta_1\,\mathbf{i} + \sin\theta_1\,\mathbf{j})$

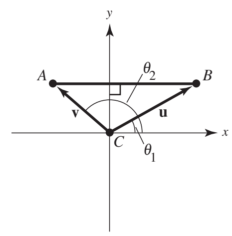
$\mathbf{v} = \|\mathbf{v}\|(\cos\theta_2\,\mathbf{i} + \sin\theta_2\,\mathbf{j})$

Vertical components:
$\|\mathbf{u}\|\sin\theta_1 + \|\mathbf{v}\|\sin\theta_2 = 5000$

Horizontal components:
$\|\mathbf{u}\|\cos\theta_1 + \|\mathbf{v}\|\cos\theta_2 = 0$

Solving this system, you obtain

$\|\mathbf{u}\| \approx 2169.4$ and $\|\mathbf{v}\| \approx 3611.2$.



81. Horizontal component $= \|\mathbf{v}\|\cos\theta$

$= 1200\cos 6° \approx 1193.43$ ft/sec

Vertical component $= \|\mathbf{v}\|\sin\theta$

$= 1200\sin 6° \approx 125.43$ ft/sec

82. To lift the weight vertically, the sum of the vertical components of **u** and **v** must be 100 and the sum of the horizontal components must be 0.

$\mathbf{u} = \|\mathbf{u}\|(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|(\cos 110°\mathbf{i} + \sin 110°\mathbf{j})$

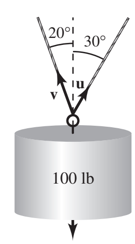
So, $\|\mathbf{u}\|\sin 60° + \|\mathbf{v}\|\sin 110° = 100$, or $\|\mathbf{u}\|\left(\frac{\sqrt{3}}{2}\right) + \|\mathbf{v}\|\sin 110° = 100$.

And $\|\mathbf{u}\|\cos 60° + \|\mathbf{v}\|\cos 110° = 0$ or $\|\mathbf{u}\|\left(\frac{1}{2}\right) + \|\mathbf{v}\|\cos 110° = 0$.

Multiplying the last equation by $\left(\sqrt{3}\right)$ and adding to the first equation gives

$\|\mathbf{u}\|\left(\sin 110° - \sqrt{3}\cos 110°\right) = 100 \Rightarrow \|\mathbf{v}\| \approx 65.27$ pounds

Then, $\|\mathbf{u}\|\left(\frac{1}{2}\right) + 65.27\cos 110° = 0$ gives $\|\mathbf{u}\| \approx 44.65$ pounds



(a) The tension in each rope: $\|\mathbf{u}\| = 44.65$ lb,

$\|\mathbf{v}\| = 65.27$ lb

(b) Vertical components: $\|\mathbf{u}\|\sin 60° \approx 38.67$ lb,

$\|\mathbf{v}\|\sin 110° \approx 61.33$ lb

83. $\mathbf{u} = 900(\cos 148°\mathbf{i} + \sin 148°\mathbf{j})$

$\mathbf{v} = 100(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$

$$\mathbf{u} + \mathbf{v} = (900\cos 148° + 100\cos 45°)\mathbf{i} + (900\sin 148° + 100\sin 45°)\mathbf{j}$$
$$\approx -692.53\mathbf{i} + 547.64\mathbf{j}$$
$$\theta \approx \arctan\left(\frac{547.64}{-692.53}\right) \approx -38.34°; \text{38.34° North of West}$$
$$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(-692.53)^2 + (547.64)^2} \approx 882.9 \text{ km/h}$$

84. $\mathbf{u} = 400\mathbf{i}$ (plane)

$\mathbf{v} = 50(\cos 135°\mathbf{i} + \sin 135°\mathbf{j}) = -25\sqrt{2}\mathbf{i} + 25\sqrt{2}\mathbf{j}$ (wind)

$\mathbf{u} + \mathbf{v} = (400 - 25\sqrt{2})\mathbf{i} + 25\sqrt{2}\mathbf{j} \approx 364.64\mathbf{i} + 35.36\mathbf{j}$

$\tan\theta = \dfrac{35.36}{364.64} \Rightarrow \theta \approx 5.54°$

Direction North of East: $\approx$ N 84.46°E

Speed: $\approx$ 336.35 mi/h

85. False. Weight has direction.

86. True

87. True

88. True

89. True

90. True

91. True

92. False

$a = b = 0$

93. False

$\|a\mathbf{i} + b\mathbf{j}\| = \sqrt{2}|a|$

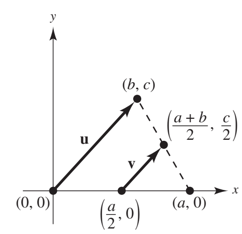
94. True

95. $\|\mathbf{u}\| = \sqrt{\cos^2\theta + \sin^2\theta} = 1,$

$\|\mathbf{v}\| = \sqrt{\sin^2\theta + \cos^2\theta} = 1$

96. Let the triangle have vertices at $(0, 0), (a, 0)$, and (b, c). Let $\mathbf{u}$ be the vector joining $(0, 0)$ and (b, c), as indicated in the figure. Then $\mathbf{v}$, the vector joining the midpoints, is

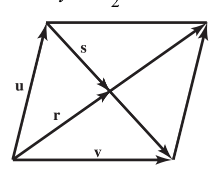
$$\mathbf{v} = \left(\frac{a+b}{2} - \frac{a}{2}\right)\mathbf{i} + \frac{c}{2}\mathbf{j}$$
$$= \frac{b}{2}\mathbf{i} + \frac{c}{2}\mathbf{j}$$
$$= \frac{1}{2}(b\mathbf{i} + c\mathbf{j}) = \frac{1}{2}\mathbf{u}.$$



97. Let $\mathbf{u}$ and $\mathbf{v}$ be the vectors that determine the parallelogram, as indicated in the figure. The two diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{v} - \mathbf{u}$. So, $\mathbf{r} = x(\mathbf{u} + \mathbf{v}), \mathbf{s} = 4(\mathbf{v} - \mathbf{u})$. But,

$$\mathbf{u} = \mathbf{r} - \mathbf{s}$$
$$= x(\mathbf{u} + \mathbf{v}) - y(\mathbf{v} - \mathbf{u}) = (x + y)\mathbf{u} + (x - y)\mathbf{v}.$$

So, $x + y = 1$ and $x - y = 0$. Solving you have $x = y = \frac{1}{2}$.



98. $\mathbf{w} = \|\mathbf{u}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{u}$

$$= \|\mathbf{u}\|\left[\|\mathbf{v}\|\cos\theta_{\mathbf{v}}\mathbf{i} + \|\mathbf{v}\|\sin\theta_{\mathbf{v}}\mathbf{j}\right] + \|\mathbf{v}\|\left[\|\mathbf{u}\|\cos\theta_{\mathbf{u}}\mathbf{i} + \|\mathbf{u}\|\sin\theta_{\mathbf{u}}\mathbf{j}\right]$$

$$= \|\mathbf{u}\|\|\mathbf{v}\|\left[(\cos\theta_{\mathbf{u}} + \cos\theta_{\mathbf{v}})\mathbf{i} + (\sin\theta_{\mathbf{u}} + \sin\theta_{\mathbf{v}})\mathbf{j}\right]$$

$$= 2\|\mathbf{u}\|\|\mathbf{v}\|\left[\cos\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)\mathbf{i} + \sin\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)\mathbf{j}\right]$$

$$\tan\theta_{\mathbf{w}} = \frac{\sin\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)}{\cos\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)\cos\left(\frac{\theta_{\mathbf{u}} - \theta_{\mathbf{v}}}{2}\right)} = \tan\left(\frac{\theta_{\mathbf{u}} + \theta_{\mathbf{v}}}{2}\right)$$

So, $\theta_{\mathbf{w}} = (\theta_{\mathbf{u}} + \theta_{\mathbf{v}})/2$ and $\mathbf{w}$ bisects the angle between $\mathbf{u}$ and $\mathbf{v}$.

99. The set is a circle of radius 5, centered at the origin.

$$\|\mathbf{u}\| = \|\langle x, y\rangle\| = \sqrt{x^2 + y^2} = 5 \Rightarrow x^2 + y^2 = 25$$

100. Let $x = v_0 t\cos\alpha$ and $y = v_0 t\sin\alpha - \frac{1}{2}gt^2$.

$$t = \frac{x}{v_0\cos\alpha} \Rightarrow y = v_0\sin\alpha\left(\frac{x}{v_0\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\alpha}\right)^2$$

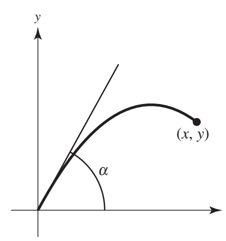
$$= x\tan\alpha - \frac{g}{2v_0^2}x^2\sec^2\alpha$$

$$= x\tan\alpha - \frac{gx^2}{2v_0^2}(1 + \tan^2\alpha)$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\tan^2\alpha + x\tan\alpha - \frac{v_0^2}{2g}$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left[\tan^2\alpha - 2\tan\alpha\left(\frac{v_0^2}{gx}\right) + \frac{v_0^4}{g^2x^2}\right]$$

$$= \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2} - \frac{gx^2}{2v_0^2}\left(\tan\alpha - \frac{v_0^2}{gx}\right)^2$$



If $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$, then α can be chosen to hit the point (x, y). To hit $(0, y)$: Let $\alpha = 90°$. Then

$$y = v_0 t - \frac{1}{2}gt^2 = \frac{v_0^2}{2g} - \frac{v_0^2}{2g}\left(\frac{g}{v_0}t - 1\right)^2, \text{ and you need } y \le \frac{v_0^2}{2g}.$$

The set H is given by $0 \le x$, $0 < y$ and $y \le \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$

Note: The parabola $y = \frac{v_0^2}{2g} - \frac{gx^2}{2v_0^2}$ is called the "parabola of safety."

Section 11.2 Space Coordinates and Vectors in Space

1. x_0 is directed distance to yz-plane.

y_0 is directed distance to xz-plane.

z_0 is directed distance to xy-plane.

2. The y-coordinate of any point in the xz-plane is 0.

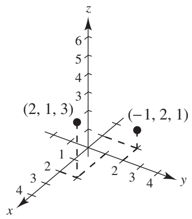
3. (a) $x = 4$ is a point on the number line.

(b) $x = 4$ is a vertical line in the plane.

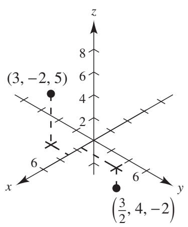
(c) $x = 4$ is a plane in space.

4. The nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel if there exists a scalar such that $\mathbf{u} = c\mathbf{v}$.

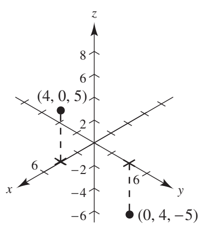
5.



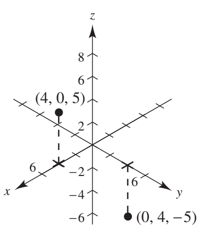
6.



7.



8.



9. $x = -3,\ y = 4,\ z = 5$: $(-3, 4, 5)$

10. $x = 7,\ y = -2,\ z = -1$:

$(7, -2, -1)$

11. $y = z = 0, x = 12$: $(12, 0, 0)$

12. $x = 0, y = 3, z = 2$: $(0, 3, 2)$

13. The point is 1 unit above the *xy*-plane.

14. The point is 6 units in front of the *xz*-plane.

15. The point is on the plane parallel to the *yz*-plane that passes through $x = -3$.

16. The point is 5 units below the *xy*-plane.

17. The point is to the left of the *xz*-plane.

18. The point more than 4 units away from the *yz*-plane.

19. The point is on or between the planes $y = 3$ and $y = -3$.

20. The point is in front of the plane $x = 4$.

21. The point (x, y, z) is 3 units below the *xy*-plane, and below either quadrant I or III.

22. The point (x, y, z) is 4 units above the *xy*-plane, and above either quadrant II or IV.

23. The point could be above the *xy*-plane and so above quadrants II or IV, or below the *xy*-plane, and so below quadrants I or III.

24. The point could be above the *xy*-plane, and so above quadrants I and III, or below the *xy*-plane, and so below quadrants II or IV.

25. $$\begin{aligned} d &= \sqrt{(8-4)^2 + (2-1)^2 + (6-5)^2} \\ &= \sqrt{16+1+1} \\ &= \sqrt{18} = 3\sqrt{2} \end{aligned}$$

26. $$\begin{aligned} d &= \sqrt{\left(-3-(-1)\right)^2 + (5-1)^2 + (-3-1)^2} \\ &= \sqrt{4+16+16} \\ &= \sqrt{36} = 6 \end{aligned}$$

27. $$\begin{aligned} d &= \sqrt{(3-0)^2 + (2-2)^2 + (8-4)^2} \\ &= \sqrt{9+0+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

28. $$\begin{aligned} d &= \sqrt{\left(-5-(-3)\right)^2 + (8-7)^2 + (-4-1)^2} \\ &= \sqrt{4+1+25} \\ &= \sqrt{30} \end{aligned}$$

29. $A(0, 0, 4), B(2, 6, 7), C(6, 4, -8)$

$$|AB| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$$

$$|AC| = \sqrt{6^2 + 4^2 + (-12)^2} = \sqrt{196} = 14$$

$$|BC| = \sqrt{4^2 + (-2)^2 + (-15)^2} = \sqrt{245} = 7\sqrt{5}$$

$$|BC|^2 = 245 = 49 + 196 = |AB|^2 + |AC|^2$$

Right triangle

30. $A(3, 4, 1), B(0, 6, 2), C(3, 5, 6)$

$$|AB| = \sqrt{9+4+1} = \sqrt{14}$$

$$|AC| = \sqrt{0+1+25} = \sqrt{26}$$

$$|BC| = \sqrt{9+1+16} = \sqrt{26}$$

Because $|AC| = |BC|$, the triangle is isosceles.

31. $A(-1, 0, -2), B(-1, 5, 2), C(-3, -1, 1)$

$|AB| = \sqrt{0 + 25 + 16} = \sqrt{41}$

$|AC| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$|BC| = \sqrt{4 + 36 + 1} = \sqrt{41}$

Because $|AB| = |BC|$, the triangle is isosceles.

32. $A(4, -1, -1), B(2, 0, -4), C(3, 5, -1)$

$|AB| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$|AC| = \sqrt{1 + 36 + 0} = \sqrt{37}$

$|BC| = \sqrt{1 + 25 + 9} = \sqrt{35}$

Neither

33. $\left(\frac{4 + 8}{2}, \frac{0 + 8}{2}, \frac{-6 + 20}{2}\right) = (6, 4, 7)$

34. $\left(\frac{7 - 5}{2}, \frac{2 - 2}{2}, \frac{2 - 3}{2}\right) = \left(1, 0, -\frac{1}{2}\right)$

35. $\left(\frac{3 + 1}{2}, \frac{4 + 8}{2}, \frac{6 + 0}{2}\right) = (2, 6, 3)$

36. $\left(\frac{5 + (-2)}{2}, \frac{-9 + 3}{2}, \frac{7 + 3}{2}\right) = \left(\frac{3}{2}, -3, 5\right)$

37. Center: $(7, 1, -2)$

Radius: 1

$(x - 7)^2 + (y - 1)^2 + (z + 2)^2 = 1$

38. Center: $(-1, -5, 8)$

Radius: 5

$(x + 1)^2 + (y + 5)^2 + (z - 8)^2 = 25$

39. Center is midpoint of diameter:

$\left(\frac{2 + 1}{2}, \frac{1 + 3}{2}, \frac{3 - 1}{2}\right) = \left(\frac{3}{2}, 2, 1\right)$

Radius is distance from center to endpoint:

$$d = \sqrt{\left(\frac{3}{2} - 1\right)^2 + (2 - 3)^2 + (1 + 1)^2} = \sqrt{\frac{1}{4} + 1 + 4} = \frac{\sqrt{21}}{2}$$

$$\left(x - \frac{3}{2}\right)^2 + (y - 2)^2 + (z - 1)^2 = \frac{21}{4}$$

40. Center is midpoint of diameter:

$\left(\frac{-2 - 4}{2}, \frac{4 + 0}{2}, \frac{-5 + 3}{2}\right) = (-3, 2, -1)$

Radius is distance from center to endpoint:

$$d = \sqrt{(-4 - (-3))^2 + (0 - 2)^2 + (3 - (-1))^2} = \sqrt{1 + 4 + 16} = \sqrt{21}$$

$$(x + 3)^2 + (y - 2)^2 + (z + 1)^2 = 21$$

41. Center: $(-7, 7, 6)$

Tangent to xy-plane

Radius is z-coordinate, 6.

$(x + 7)^2 + (y - 7)^2 + (z - 6)^2 = 36$

42. Center: $(-4, 0, 0)$

Tangent to yz-plane

Radius is distance to yz-plane, 4.

$(x + 4)^2 + y^2 + z^2 = 16$

43.

$$x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$$

$$(x^2 - 2x + 1) + (y^2 + 6y + 9) + (z^2 + 8z + 16) = -1 + 1 + 9 + 16$$

$$(x - 1)^2 + (y + 3)^2 + (z + 4)^2 = 25$$

Center: $(1, -3, -4)$

Radius: 5

44.
$$x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$$
$$\left(x^2 + 9x + \frac{81}{4}\right) + \left(y^2 - 2y + 1\right) + \left(z^2 + 10z + 25\right) = -19 + \frac{81}{4} + 1 + 25$$
$$\left(x + \frac{9}{2}\right)^2 + (y - 1)^2 + (z + 5)^2 = \frac{109}{4}$$

Center: $\left(-\frac{9}{2}, 1, -5\right)$

Radius: $\frac{\sqrt{109}}{2}$

45.
$$9x^2 + 9y^2 + 9z^2 - 6x + 18y + 1 = 0$$
$$x^2 + y^2 + z^2 - \tfrac{2}{3}x + 2y + \tfrac{1}{9} = 0$$
$$\left(x^2 - \tfrac{2}{3}x + \tfrac{1}{9}\right) + \left(y^2 + 2y + 1\right) + z^2 = -\tfrac{1}{9} + \tfrac{1}{9} + 1$$
$$\left(x - \tfrac{1}{3}\right)^2 + (y + 1)^2 + (z - 0)^2 = 1$$

Center: $\left(\tfrac{1}{3}, -1, 0\right)$

Radius: 1

46. $4x^2 + 4y^2 + 4z^2 - 24x - 4y + 8z - 23 = 0$
$$\left(x^2 - 6x + 9\right) + \left(y^2 - y + \tfrac{1}{4}\right) + \left(z^2 + 2z + 1\right) = \tfrac{23}{4} + 9 + \tfrac{1}{4} + 1$$
$$(x - 3)^2 + \left(y - \tfrac{1}{2}\right)^2 + (z + 1)^2 = 16$$

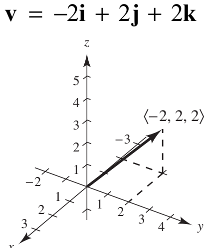
Center: $\left(3, \tfrac{1}{2}, -1\right)$

Radius: 4

47. (a) $\mathbf{v} = \langle 2 - 4, 4 - 2, 3 - 1 \rangle = \langle -2, 2, 2 \rangle$

(b) $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

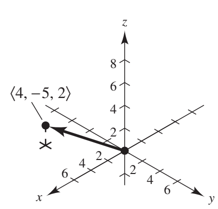
(c)



48. (a) $\mathbf{v} = \langle 4 - 0, 0 - 5, 3 - 1 \rangle = \langle 4, -5, 2 \rangle$

(b) $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{h}$

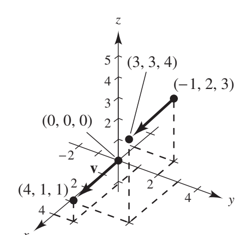
(c)



49. (b) $\mathbf{v} = \langle 3 - (-1), 3 - 2, 4 - 3 \rangle = \langle 4, 1, 1 \rangle$

(c) $\mathbf{v} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$

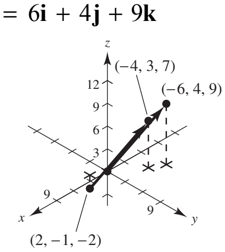
(a), (d)



50. (b) $\mathbf{v} = \langle -4 - 2, 3 - (-1), 7 - (-2) \rangle = \langle -6, 4, 9 \rangle$

(c) $\mathbf{v} = 6\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$

(a), (d)



51. $\mathbf{v} = \langle 4 - 3, 1 - 2, 6 - 0\rangle = \langle 1, -1, 6\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 1 + 36} = \sqrt{38}$

Unit vector: $\dfrac{\langle 1, -1, 6\rangle}{\sqrt{38}} = \left\langle \dfrac{1}{\sqrt{38}}, \dfrac{-1}{\sqrt{38}}, \dfrac{6}{\sqrt{38}}\right\rangle$

52. $\mathbf{v} = \langle 2 - 1, 4 - (-2), -2 - 4\rangle = \langle 1, 6, -6\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 36 + 36} = \sqrt{73}$

Unit vector: $\dfrac{\langle 1, 6, -6\rangle}{\sqrt{73}} = \left\langle \dfrac{1}{\sqrt{73}}, \dfrac{6}{\sqrt{73}}, \dfrac{-6}{\sqrt{73}}\right\rangle$

53. $\mathbf{v} = \langle 0 - 4, 5 - 2, 2 - 0\rangle = \langle -4, 3, 2\rangle$

$\|\mathbf{v}\| = \sqrt{(-4)^2 + 3^2 + 2^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

Unit vector:

$\dfrac{1}{\sqrt{29}}\langle -4, 3, 2\rangle = \left\langle -\dfrac{4}{\sqrt{29}}, \dfrac{3}{\sqrt{29}}, \dfrac{2}{\sqrt{29}}\right\rangle$

54. $\mathbf{v} = \langle 1 - 1, -2 - (-2), -3 - 0\rangle = \langle 0, 0, -3\rangle$

$\|\mathbf{v}\| = \sqrt{0^2 + 0^2 + (-3)^2} = 3$

Unit vector:

$\dfrac{1}{3}\langle 0, 0, -3\rangle = \langle 0, 0, -1\rangle$

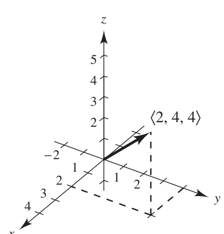
55. $(q_1, q_2, q_3) - (0, 6, 2) = (3, -5, 6)$

$Q = (3, 1, 8)$

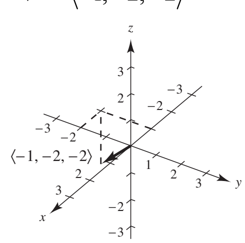
56. $(q_1, q_2, q_3) - \left(0, 2, \frac{5}{2}\right) = \left(1, -\frac{2}{3}, \frac{1}{2}\right)$

$Q = \left(1, -\frac{4}{3}, 3\right)$

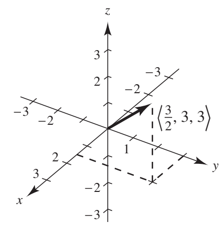
57. (a) $2\mathbf{v} = \langle 2, 4, 4\rangle$



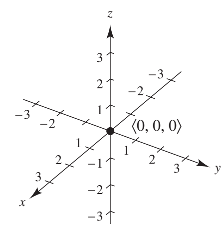
(b) $-\mathbf{v} = \langle -1, -2, -2\rangle$



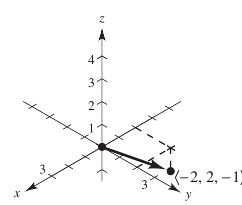
(c) $\frac{3}{2}\mathbf{v} = \left\langle \frac{3}{2}, 3, 3\right\rangle$



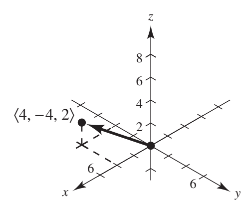
(d) $0\mathbf{v} = \langle 0, 0, 0\rangle$



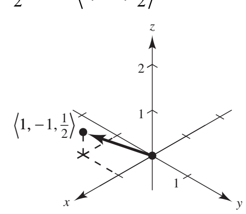
58. (a) $-\mathbf{v} = \langle -2, 2, -1\rangle$



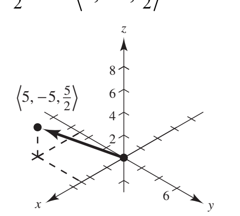
(b) $2\mathbf{v} = \langle 4, -4, 2\rangle$



(c) $\frac{1}{2}\mathbf{v} = \left\langle 1, -1, \frac{1}{2}\right\rangle$



(d) $\frac{5}{2}\mathbf{v} = \left\langle 5, -5, \frac{5}{2}\right\rangle$



59. $z = \mathbf{u} - \mathbf{v} + \mathbf{w}$
$= \langle 1, 2, 3\rangle - \langle 2, 2, -1\rangle + \langle 4, 0, -4\rangle$
$= \langle 3, 0, 0\rangle$

60. $\mathbf{z} = 5\mathbf{u} - 3\mathbf{v} - \frac{1}{2}\mathbf{w}$
$= \langle 5, 10, 15\rangle - \langle 6, 6, -3\rangle - \langle 2, 0, -2\rangle$
$= \langle -3, 4, 20\rangle$

61. $\frac{1}{3}\mathbf{z} - 3\mathbf{u} = \mathbf{w}$
$\frac{1}{3}\mathbf{z} = 3\mathbf{u} + \mathbf{w}$
$\mathbf{z} = 9\mathbf{u} + 3\mathbf{w}$
$= 9\langle 1, 2, 3\rangle + 3\langle 4, 0, -4\rangle$
$= \langle 9, 18, 27\rangle + \langle 12, 0, -12\rangle$
$= \langle 21, 18, 15\rangle$

62. $2\mathbf{u} + \mathbf{v} - \mathbf{w} + 3\mathbf{z} = 2\langle 1, 2, 3\rangle + \langle 2, 2, -1\rangle - \langle 4, 0, -4\rangle + 3\langle z_1, z_2, z_3\rangle = \langle 0, 0, 0\rangle$
$\langle 0, 6, 9\rangle + \langle 3z_1, 3z_2, 3z_3\rangle = \langle 0, 0, 0\rangle$
$0 + 3z_1 = 0 \Rightarrow z_1 = 0$
$6 + 3z_2 = 0 \Rightarrow z_2 = -2$
$9 + 3z_3 = 0 \Rightarrow z_3 = -3$
$\mathbf{z} = \langle 0, -2, -3\rangle$

63. (a) and (b) are parallel because $\langle -6, -4, 10\rangle = -2\langle 3, 2, -5\rangle$ and $\left\langle 2, \frac{4}{3}, -\frac{10}{3}\right\rangle = \frac{2}{3}\langle 3, 2, -5\rangle$.

64. (b) and (d) are parallel because $-\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{3}{2}\mathbf{k} = -2\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$ and $\frac{3}{4}\mathbf{i} - \mathbf{j} + \frac{9}{8}\mathbf{k} = \frac{3}{2}\left(\frac{1}{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{3}{4}\mathbf{k}\right)$.

65. $\mathbf{z} = -3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

(a) is parallel because $-6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k} = 2\mathbf{z}$.

66. $\mathbf{z} = \langle -7, -8, 3\rangle$

(b) is parallel because $(-z)\mathbf{z} = \langle 14, 16, -6\rangle$.

67. $P(0, -2, -5), Q(3, 4, 4), R(2, 2, 1)$
$\overrightarrow{PQ} = \langle 3, 6, 9\rangle$
$\overrightarrow{PR} = \langle 2, 4, 6\rangle$
$\langle 3, 6, 9\rangle = \frac{3}{2}\langle 2, 4, 6\rangle$

So, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel, the points are collinear.

68. $P(4, -2, 7), Q(-2, 0, 3), R(7, -3, 9)$
$\overrightarrow{PQ} = \langle -6, 2, -4\rangle$
$\overrightarrow{PR} = \langle 3, -1, 2\rangle$
$\langle 3, -1, 2\rangle = -\frac{1}{2}\langle -6, 2, -4\rangle$

So, $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are parallel. The points are collinear.

69. $P(1, 2, 4), Q(2, 5, 0), R(0, 1, 5)$
$\overrightarrow{PQ} = \langle 1, 3, -4\rangle$
$\overrightarrow{PR} = \langle -1, -1, 1\rangle$

Because $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

70. $P(0, 0, 0), Q(1, 3, -2), R(2, -6, 4)$
$\overrightarrow{PQ} = \langle 1, 3, -2\rangle$
$\overrightarrow{PR} = \langle 2, -6, 4\rangle$

Because $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are not parallel, the points are not collinear.

71. $A(2, 9, 1), B(3, 11, 4), C(0, 10, 2), D(1, 12, 5)$
$\overrightarrow{AB} = \langle 1, 2, 3\rangle$
$\overrightarrow{CD} = \langle 1, 2, 3\rangle$
$\overrightarrow{AC} = \langle -2, 1, 1\rangle$
$\overrightarrow{BD} = (-2, 1, 1)$

Because $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{AC} = \overrightarrow{BD}$, the given points form the vertices of a parallelogram.

72. $A(1, 1, 3)\ B(9, -1, -2), C(11, 2, -9), D(3, 4, -4)$
$\overrightarrow{AB} = \langle 8, -2, -5\rangle$
$\overrightarrow{DC} = \langle 8, -2, -5\rangle$
$\overrightarrow{AD} = \langle 2, 3, -7\rangle$
$\overrightarrow{BC} = \langle 2, 3, -7\rangle$

Because $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$, the given points form the vertices of a parallelogram.

73. $\|\mathbf{v}\| = \|\langle -1, 0, 1\rangle\|$

$$= \sqrt{(-1)^2 + 0^2 + 1^2}$$
$$= \sqrt{1+1} = \sqrt{2}$$

74. $\|\mathbf{v}\| = \|\langle -5, -3, -4\rangle\|$

$$= \sqrt{(-5)^2 + (-3)^2 + (-4)^2}$$
$$= \sqrt{25 + 9 + 16}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

75. $\mathbf{v} = 3\mathbf{j} - 5\mathbf{k} = \langle 0, 3, -5\rangle$

$\|\mathbf{v}\| = \sqrt{0 + 9 + 25} = \sqrt{34}$

76. $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k} = \langle 2, 5, -1\rangle$

$\|\mathbf{v}\| = \sqrt{4 + 25 + 1} = \sqrt{30}$

77. $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} = \langle 1, -2, -3\rangle$

$\|\mathbf{v}\| = \sqrt{1 + 4 + 9} = \sqrt{14}$

78. $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} = \langle -4, 3, 7\rangle$

$\|\mathbf{v}\| = \sqrt{16 + 9 + 49} = \sqrt{74}$

79. $\mathbf{v} = \langle 2, -1, 2\rangle$

$\|\mathbf{v}\| = \sqrt{4 + 1 + 4} = 3$

(a) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{3}\langle 2, -1, 2\rangle$

(b) $-\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = -\dfrac{1}{3}\langle 2, -1, 2\rangle$

80. $\mathbf{v} = \langle 6, 0, 8\rangle$

$\|\mathbf{v}\| = \sqrt{36 + 0 + 64} = 10$

(a) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{10}\langle 6, 0, 8\rangle$

(b) $-\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = -\dfrac{1}{10}\langle 6, 0, 8\rangle$

81. $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

$\|\mathbf{v}\| = \sqrt{16 + 25 + 9} = \sqrt{50} = 5\sqrt{2}$

(a) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{5\sqrt{2}}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = \dfrac{2\sqrt{2}}{5}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j} + \dfrac{3\sqrt{2}}{10}\mathbf{k}$

(b) $-\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = -\dfrac{1}{5\sqrt{2}}(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = -\dfrac{2\sqrt{2}}{5}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j} - \dfrac{3\sqrt{2}}{10}\mathbf{k}$

82. $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$\|\mathbf{v}\| = \sqrt{25 + 9 + 1} = \sqrt{35}$

(a) $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{35}}(5\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \dfrac{\sqrt{35}}{7}\mathbf{i} + \dfrac{3\sqrt{35}}{35}\mathbf{j} - \dfrac{\sqrt{35}}{35}\mathbf{k}$

(b) $-\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = -\dfrac{1}{\sqrt{35}}(5\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = -\dfrac{\sqrt{35}}{7}\mathbf{i} - \dfrac{3\sqrt{35}}{35}\mathbf{j} + \dfrac{\sqrt{35}}{35}\mathbf{k}$

83. $\mathbf{v} = 10\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 10\dfrac{\langle 0, 3, 3\rangle}{3\sqrt{2}}$

$$= 10\left\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle = \left\langle 0, \frac{10}{\sqrt{2}}, \frac{10}{\sqrt{2}}\right\rangle$$

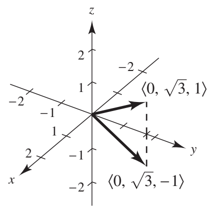
84. $\mathbf{v} = 3\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 3\dfrac{\langle 1, 1, 1\rangle}{\sqrt{3}}$

$$= 3\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle = \left\langle \frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}}, \frac{3}{\sqrt{3}}\right\rangle$$

85. $\mathbf{v} = \dfrac{3}{2}\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \dfrac{3}{2}\dfrac{(2, -2, 1)}{3} = \dfrac{3}{2}\left\langle \dfrac{2}{3}, \dfrac{-2}{3}, \dfrac{1}{3}\right\rangle = \left\langle 1, -1, \dfrac{1}{2}\right\rangle$

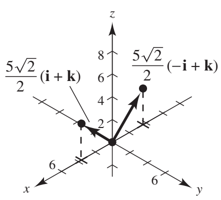
86. $\mathbf{v} = 7\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = 7\dfrac{\langle -4, 6, 2\rangle}{2\sqrt{14}} = \left\langle \dfrac{-14}{\sqrt{14}}, \dfrac{21}{\sqrt{14}}, \dfrac{7}{14}\right\rangle$

87. $\mathbf{v} = 2\left[\cos(\pm 30°)\mathbf{j} + \sin(\pm 30°)\mathbf{k}\right]$
$= \sqrt{3}\mathbf{j} \pm \mathbf{k} = \left\langle 0, \sqrt{3}, \pm 1\right\rangle$



88. $\mathbf{v} = 5(\cos 45°\mathbf{i} + \sin 45°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(\mathbf{i} + \mathbf{k})$ or

$\mathbf{v} = 5(\cos 135°\mathbf{i} + \sin 135°\mathbf{k}) = \dfrac{5\sqrt{2}}{2}(-\mathbf{i} + \mathbf{k})$



89.
$$\mathbf{v} = \langle -3, -6, 3\rangle$$
$$\tfrac{2}{3}\mathbf{v} = \langle -2, -4, 2\rangle$$
$$(4, 3, 0) + (-2, -4, 2) = (2, -1, 2)$$

90.
$$\mathbf{v} = \langle 5, 6, -3\rangle$$
$$\tfrac{2}{3}\mathbf{v} = \left\langle \tfrac{10}{3}, 4, -2\right\rangle$$
$$(1, 2, 5) + \left(\tfrac{10}{3}, 4, -2\right) = \left(\tfrac{13}{3}, 6, 3\right)$$

91. A sphere of radius 4 centered at (x_1, y_1, z_1).
$$\|\mathbf{v}\| = \|\langle x - x_2, y - y_1, z - z_1\rangle\|$$
$$= \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} = 4$$
$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = 16$$

92. $\|\mathbf{r} - \mathbf{r}_0\| = \sqrt{(x - 1)^2 + (y - 1)^2 + (z - 1)^2} = 2$
$$(x - 1)^2 + (y - 1)^2 + (z - 1)^2 = 4$$

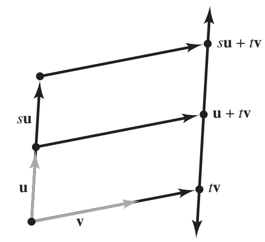
This is a sphere of radius 2 and center $(1, 1, 1)$.

93. The set of all points (x, y, z) such that $\|\mathbf{r}\| > 1$ represent outside the sphere of radius 1 centered at the origin.

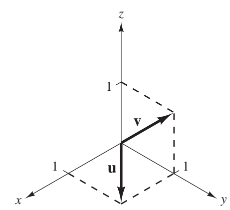
94. (a) $(x, y, z) = (3, 3, 3)$
$$\mathbf{v} = \langle 3, 3, 3\rangle - \langle 3, 0, 0\rangle$$
$$= \langle 3 - 3, 3 - 0, 3 - 0\rangle = \langle 0, 3, 3\rangle$$

(b) $(x, y, z) = (4, 4, 8)$
$$\mathbf{v} = \langle 4, 4, 8\rangle - \langle 4, 0, 0\rangle$$
$$= \langle 4 - 4, 4 - 0, 8 - 0\rangle = \langle 0, 4, 8\rangle$$

95. The terminal points of the vectors $t\mathbf{u}$, $\mathbf{u} + t\mathbf{v}$ and $s\mathbf{u} + t\mathbf{v}$ are collinear.



96. (a)

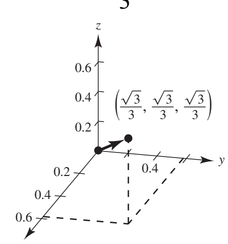


(b) $\mathbf{w} = a\mathbf{u} + b\mathbf{v} = a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = 0$
$$a = 0, a + b = 0, b = 0$$
So, a and b are both zero.

(c) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$
$$a = 1, a + b = 2, b = 1$$
$$\mathbf{w} = \mathbf{u} + \mathbf{v}$$

(d) $a\mathbf{i} + (a + b)\mathbf{j} + b\mathbf{k} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
$$a = 1, a + b = 2, b = 3$$
Not possible

97. Let α be the angle between $\mathbf{v}$ and the coordinate axes.
$$\mathbf{v} = (\cos\alpha)\mathbf{i} + (\cos\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k}$$
$$\|\mathbf{v}\| = \sqrt{3}\cos\alpha = 1$$
$$\cos\alpha = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\mathbf{v} = \frac{\sqrt{3}}{3}(\mathbf{i} + \mathbf{j} + \mathbf{k}) = \frac{\sqrt{3}}{3}\langle 1, 1, 1\rangle$$

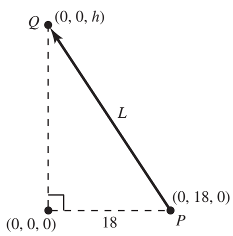


98. $$\begin{aligned} 550 &= \|c(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k})\| \\ 302{,}500 &= 18{,}125c^2 \\ c^2 &= 16.689655 \\ c &\approx 4.085 \\ \mathbf{F} &\approx 4.085(75\mathbf{i} - 50\mathbf{j} - 100\mathbf{k}) \\ &\approx 306\mathbf{i} - 204\mathbf{j} - 409\mathbf{k} \end{aligned}$$

99. (a) The height of the right triangle is $h = \sqrt{L^2 - 18^2}$.
The vector $\overline{PQ}$ is given by $\overline{PQ} = \langle 0, -18, h\rangle$.

The tension vector $\mathbf{T}$ in each wire is $\mathbf{T} = c\langle 0, -18, h\rangle$ where $ch = \dfrac{24}{3} = 8$.

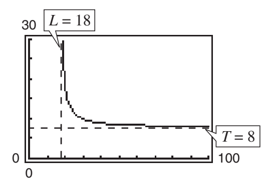
So, $\mathbf{T} = \dfrac{8}{h}\langle 0, -18, h\rangle$ and $T = \|\mathbf{T}\| = \dfrac{8}{h}\sqrt{18^2 + h^2} = \dfrac{8}{\sqrt{L^2 - 18^2}}\sqrt{18^2 + (L^2 - 18^2)} = \dfrac{8L}{\sqrt{L^2 - 18^2}}, L > 18.$



(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

(c)



$x = 18$ is a vertical asymptote and $y = 8$ is a horizontal asymptote.

(d) $\displaystyle\lim_{L\to 18^+} \frac{8L}{\sqrt{L^2 - 18^2}} = \infty$

$\displaystyle\lim_{L\to\infty} \frac{8L}{\sqrt{L^2 - 18^2}} = \lim_{L\to\infty} \frac{8}{\sqrt{1 - (18/L)^2}} = 8$

(e) From the table, $T = 10$ implies $L = 30$ inches.

100. As in Exercise 99(c), $x = a$ will be a vertical asymptote. So, $\displaystyle\lim_{r_0 \to a^-} T = \infty$.

101. $\overline{AB} = \langle 0, 70, 115\rangle, \mathbf{F}_1 = C_1\langle 0, 70, 115\rangle$

$\overline{AC} = \langle -60, 0, 115\rangle, \mathbf{F}_2 = C_2\langle -60, 0, 115\rangle$

$\overline{AD} = \langle 45, -65, 115\rangle, \mathbf{F}_3 = C_3\langle 45, -65, 115\rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \langle 0, 0, 500\rangle$

So: $$\begin{aligned} -60C_2 + 45C_3 &= 0 \\ 70C_1 - 65C_3 &= 0 \\ 115(C_1 + C_2 + C_3) &= 500 \end{aligned}$$

Solving this system yields $C_1 = \frac{104}{69}, C_2 = \frac{28}{23}$, and $C_3 = \frac{112}{69}$. So:

$\|\mathbf{F}_1\| \approx 202.919\text{N}$

$\|\mathbf{F}_2\| \approx 157.909\text{N}$

$\|\mathbf{F}_3\| \approx 226.521\text{N}$

102. Let A lie on the y-axis and the wall on the x-axis. Then $A = (0, 10, 0)$, $B = (8, 0, 6)$, $C = (-10, 0, 6)$ and

$\overline{AB} = \langle 8, -10, 6\rangle$, $\overline{AC} = \langle -10, -10, 6\rangle$.

$\|AB\| = 10\sqrt{2}$, $\|AC\| = 2\sqrt{59}$

Thus, $\mathbf{F}_1 = 420\dfrac{\overline{AB}}{\|\overline{AB}\|}$, $\mathbf{F}_2 = 650\dfrac{\overline{AC}}{\|\overline{AC}\|}$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \approx \langle 237.6, -297.0, 178.2\rangle + \langle -423.1, -423.1, 253.9\rangle \approx \langle -185.5, -720.1, 432.1\rangle$$

$\|\mathbf{F}\| \approx 860.0$ lb

103. $d(AP) = 2d(BP)$

$$\sqrt{x^2 + (y+1)^2 + (z-1)^2} = 2\sqrt{(x-1)^2 + (y-2)^2 + z^2}$$

$$x^2 + y^2 + z^2 + 2y - 2z + 2 = 4(x^2 + y^2 + z^2 - 2x - 4y + 5)$$

$$0 = 3x^2 + 3y^2 + 3z^2 - 8x - 18y + 2z + 18$$

$$-6 + \frac{16}{9} + 9 + \frac{1}{9} = \left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) + \left(y^2 - 6y + 9\right) + \left(z^2 + \frac{2}{3}z + \frac{1}{9}\right)$$

$$\frac{44}{9} = \left(x - \frac{4}{3}\right)^2 + (y-3)^2 + \left(z + \frac{1}{3}\right)^2$$

Sphere; center: $\left(\dfrac{4}{3}, 3, -\dfrac{1}{3}\right)$, radius: $\dfrac{2\sqrt{11}}{3}$

Section 11.3 The Dot Product of Two Vectors

1. The vectors are orthogonal (perpendicular) if the dot product of the vectors is zero.

2. If $\arccos\dfrac{v^2}{\|\mathbf{v}\|} = 30°$, then $\cos 30° = \dfrac{v^2}{\|\mathbf{v}\|}$.

So, the angle between $\mathbf{v}$ and $\mathbf{j}$ is 30°.

3. $\mathbf{u} = \langle 3, 4\rangle$, $\mathbf{v} = \langle -1, 5\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 3(-1) + 4(5) = 17$

(b) $\mathbf{u}\cdot\mathbf{u} = 3(3) + 4(4) = 25$

(c) $\|\mathbf{v}\|^2 = (-1)^2 + 5^2 = 26$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 17\langle -1, 5\rangle = \langle -17, 85\rangle$

(e) $\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(17) = 51$

4. $\mathbf{u} = \langle 4, 10\rangle$, $\mathbf{v} = \langle -2, 3\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 4(-2) + 10(3) = 22$

(b) $\mathbf{u}\cdot\mathbf{u} = 4(4) + 10(10) = 116$

(c) $\|\mathbf{v}\|^2 = (-2)^2 + 3^2 = 13$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 22\langle -2, 3\rangle = \langle -44, 66\rangle$

(e) $\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(22) = 66$

5. $\mathbf{u} = \langle 6, -4\rangle$, $\mathbf{v} = \langle -3, 2\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 6(-3) + (-4)(2) = -26$

(b) $\mathbf{u}\cdot\mathbf{u} = 6(6) + (-4)(-4) = 52$

(c) $\|\mathbf{v}\|^2 = (-3)^2 + 2^2 = 13$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = -26\langle -3, 2\rangle = \langle 78, -52\rangle$

(e) $\mathbf{u}\cdot(3\mathbf{v}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(-26) = -78$

6. $\mathbf{u} = \langle -7, -1\rangle$, $\mathbf{v} = \langle -4, -1\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = -7(-4) + -1(-1) = 29$

(b) $\mathbf{u}\cdot\mathbf{u} = -7(-7) + -1(-1) = 50$

(c) $\|\mathbf{v}\|^2 = (-4)^2 + (-1)^2 = 17$

(d) $(\mathbf{u}\cdot\mathbf{v})\mathbf{v} = 29\langle -4, -1\rangle = \langle -116, -29\rangle$

(e) $\mathbf{u}\cdot(3\mathbf{u}) = 3(\mathbf{u}\cdot\mathbf{v}) = 3(29) = 87$

7. $\mathbf{u} = \langle 2, -3, 4\rangle, \mathbf{v} = \langle 0, 6, 5\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(0) + (-3)(6) + (4)(5) = 2$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + (-3)(-3) + 4(4) = 29$

(c) $\|\mathbf{v}\|^2 = 0^2 + 6^2 + 5^2 = 61$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 2\langle 0, 6, 5\rangle = \langle 0, 12, 10\rangle$

(e) $\mathbf{u} \cdot (3\mathbf{v}) = 3(\mathbf{u} \cdot \mathbf{v}) = 3(2) = 6$

8. $\mathbf{u} = \langle -5, 0, 5\rangle, \mathbf{v} = \langle -1, 2, 1\rangle$

(a) $\mathbf{u} \cdot \mathbf{v} = -5(-1) + 0(2) + 5(1) = 10$

(b) $\mathbf{u} \cdot \mathbf{u} = (-5)(-5) + (0)(0) + 5(5) = 50$

(c) $\|\mathbf{v}\|^2 = (-1)^2 + 2^2 + 1^2 = 6$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = 10\langle -1, 2, 1\rangle = \langle -10, 20, 10\rangle$

(e) $\mathbf{u} \cdot (3\mathbf{v}) = 3(\mathbf{u} \cdot \mathbf{v}) = 3(10) = 30$

9. $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - \mathbf{k}$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(1) + (-1)(0) + 1(-1) = 1$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + (-1)(-1) + (1)(1) = 6$

(c) $\|\mathbf{v}\|^2 = 1^2 + (-1)^2 = 2$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = \mathbf{v} = \mathbf{i} - \mathbf{k}$

(e) $\mathbf{u} \cdot (3\mathbf{v}) = 3(\mathbf{u} \cdot \mathbf{v}) = 3(1) = 3$

10. $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}, \mathbf{v} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

(a) $\mathbf{u} \cdot \mathbf{v} = 2(1) + 1(-3) + (-2)(2) = -5$

(b) $\mathbf{u} \cdot \mathbf{u} = 2(2) + 1(1) + (-2)(-2) = 9$

(c) $\|\mathbf{v}\|^2 = 1^2 + (-3)^2 + 2^2 = 14$

(d) $(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = -5(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -5\mathbf{i} + 15\mathbf{j} - 10\mathbf{k}$

(e) $\mathbf{u} \cdot (3\mathbf{v}) = 3(\mathbf{u} \cdot \mathbf{v}) = 3(-5) = -15$

11. $\mathbf{u} = \langle 1, 1\rangle, \mathbf{v} = \langle 2, -2\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{0}{\sqrt{2}\sqrt{8}} = 0$$

(a) $\theta = \dfrac{\pi}{2}$ (b) $\theta = 90°$

12. $\mathbf{u} = \langle 3, 1\rangle, \mathbf{v} = \langle 2, -1\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{5}{\sqrt{10}\sqrt{5}} = \frac{1}{\sqrt{2}}$$

(a) $\theta = \dfrac{\pi}{4}$ (b) $\theta = 45°$

13. $\mathbf{u} = 3\mathbf{i} + \mathbf{j}, \mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-2}{\sqrt{10}\sqrt{20}} = \frac{-1}{5\sqrt{2}}$$

(a) $\theta = \arccos\left(-\dfrac{1}{5\sqrt{2}}\right) \approx 1.713$

(b) $\theta \approx 98.1°$

14. $\mathbf{u} = \cos\left(\dfrac{\pi}{6}\right)\mathbf{i} + \sin\left(\dfrac{\pi}{6}\right)\mathbf{j} = \dfrac{\sqrt{3}}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j}$

$\mathbf{v} = \cos\left(\dfrac{3\pi}{4}\right)\mathbf{i} + \sin\left(\dfrac{3\pi}{4}\right)\mathbf{j} = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

$$= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)$$

(a) $\theta = \arccos\left[\dfrac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right)\right] = \dfrac{7\pi}{2}$

(b) $\theta = 105°$

15. $\mathbf{u} = \langle 1, 1, 1\rangle, \mathbf{v} = \langle 2, 1, -1\rangle$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{2}{\sqrt{3}\sqrt{6}} = \frac{\sqrt{2}}{3}$$

(a) $\theta = \arccos\dfrac{\sqrt{2}}{3} \approx 1.080$

(b) $\theta \approx 61.9°$

16. $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{3(2) + 2(-3) + 0}{\|\mathbf{u}\|\|\mathbf{v}\|} = 0$$

(a) $\theta = \dfrac{\pi}{2}$

(b) $\theta = 90°$

17. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{j} + 3\mathbf{k}$

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{-8}{5\sqrt{13}} = \frac{-8\sqrt{13}}{65}$$

(a) $\theta = \arccos\left(-\dfrac{8\sqrt{13}}{65}\right) \approx 2.031$

(b) $\theta \approx 116.3°$

18. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{9}{\sqrt{14}\sqrt{6}} = \frac{9}{2\sqrt{21}} = \frac{3\sqrt{21}}{14}$$

(a) $\theta = \arccos\left(\frac{3\sqrt{21}}{14}\right) \approx 0.190$

(b) $\theta \approx 10.9°$

19. $\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \cos\theta$

$\mathbf{u}\cdot\mathbf{v} = (8)(5)\cos\frac{\pi}{3} = 20$

20. $\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \cos\theta$

$\mathbf{u}\cdot\mathbf{v} = (40)(25)\cos\frac{5\pi}{6} = -500\sqrt{3}$

21. $\mathbf{u} = \langle 4, 3\rangle, \mathbf{v} = \left\langle \frac{1}{2}, -\frac{2}{3}\right\rangle$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

22. $\mathbf{u} = -\frac{1}{3}(\mathbf{i} - 2\mathbf{j}), \mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$

$\mathbf{u} = -\frac{1}{6}\mathbf{v} \Rightarrow$ parallel

23. $\mathbf{u} = \mathbf{j} + 6\mathbf{k}, \mathbf{v} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = -8 \neq 0 \Rightarrow$ not orthogonal

Neither

24. $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

25. $\mathbf{u} = \langle 2, -3, 1\rangle, \mathbf{v} = \langle -1, -1, -1\rangle$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

26. $\mathbf{u} = \langle \cos\theta, \sin\theta, -1\rangle,$

$\mathbf{v} = \langle \sin\theta, -\cos\theta, 0\rangle$

$\mathbf{u} \neq c\mathbf{v} \Rightarrow$ not parallel

$\mathbf{u}\cdot\mathbf{v} = 0 \Rightarrow$ orthogonal

27. The vector $\langle 1, 2, 0\rangle$ joining $(1, 2, 0)$ and $(0, 0, 0)$ is perpendicular to the vector $\langle -2, 1, 0\rangle$ joining $(-2, 1, 0)$ and $(0, 0, 0)$: $\langle 1, 2, 0\rangle \cdot \langle -2, 1, 0\rangle = 0$

The triangle has a right angle, so it is a right triangle.

28. Consider the vector $\langle -3, 0, 0\rangle$ joining $(0, 0, 0)$ and $(-3, 0, 0)$, and the vector $\langle 1, 2, 3\rangle$ joining $(0, 0, 0)$ and $(1, 2, 3)$: $\langle -3, 0, 0\rangle \cdot \langle 1, 2, 3\rangle = -3 < 0$

The triangle has an obtuse angle, so it is an obtuse triangle.

29. $A(2, 0, 1), B(0, 1, 2), C\left(-\frac{1}{2}, \frac{3}{2}, 0\right)$

$\overline{AB} = \langle -2, 1, 1\rangle \qquad \overline{BA} = \langle 2, -1, -1\rangle$

$\overline{AC} = \left\langle -\frac{5}{2}, \frac{3}{2}, -1\right\rangle \qquad \overline{CA} = \left\langle \frac{5}{2}, -\frac{3}{2}, 1\right\rangle$

$\overline{BC} = \left\langle -\frac{1}{2}, \frac{1}{2}, -2\right\rangle \qquad \overline{CB} = \left\langle \frac{1}{2}, -\frac{1}{2}, 2\right\rangle$

$\overline{AB}\cdot\overline{AC} = 5 + \frac{3}{2} - 1 > 0$

$\overline{BA}\cdot\overline{BC} = -1 - \frac{1}{2} + 2 > 0$

$\overline{CA}\cdot\overline{CB} = \frac{5}{4} + \frac{3}{4} + 2 > 0$

The triangle has three acute angles, so it is an acute triangle.

30. $A(2, -7, 3), B(-1, 5, 8), C(4, 6, -1)$

$\overline{AB} = \langle -3, 12, 5\rangle \qquad \overline{BA} = \langle 3, -12, -5\rangle$

$\overline{AC} = \langle 2, 13, -4\rangle \qquad \overline{CA} = \langle -2, -13, 4\rangle$

$\overline{BC} = \langle 5, 1, -9\rangle \qquad \overline{CB} = \langle -5, -1, 9\rangle$

$\overline{AB}\cdot\overline{AC} = -6 + 156 - 20 > 0$

$\overline{BA}\cdot\overline{BC} = 15 - 12 + 45 > 0$

$\overline{CA}\cdot\overline{CB} = 10 + 13 + 36 > 0$

The triangle has three acute angles, so it is an acute triangle.

31. $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \|\mathbf{u}\| = \sqrt{1 + 4 + 4} = 3$

$\cos\alpha = \frac{1}{3} \Rightarrow \alpha \approx 1.2310$ or $70.5°$

$\cos\beta = \frac{2}{3} \Rightarrow \beta \approx 0.8411$ or $48.2°$

$\cos\gamma = \frac{2}{3} \Rightarrow \gamma \approx 0.8411$ or $48.2°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = 1$

32. $\mathbf{u} = 5\mathbf{i} + 3\mathbf{j} - \mathbf{k}, \quad \|\mathbf{u}\| = \sqrt{25 + 9 + 1} = \sqrt{35}$

$\cos\alpha = \frac{5}{\sqrt{35}} \Rightarrow \alpha \approx 0.5639$ or $32.3°$

$\cos\beta = \frac{3}{\sqrt{35}} \Rightarrow \beta \approx 1.0390$ or $59.5°$

$\cos\gamma = \frac{-1}{\sqrt{35}} \Rightarrow \gamma \approx 1.7406$ or $99.7°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{25}{35} + \frac{9}{35} + \frac{1}{35} = 1$

33. $\mathbf{u} = 7\mathbf{i} + \mathbf{j} - \mathbf{k},\ \|\mathbf{u}\| = \sqrt{49 + 1 + 1} = \sqrt{51}$

$$\cos\alpha = \frac{7}{\sqrt{51}} \Rightarrow \alpha \approx 11.4°$$

$$\cos\beta = \frac{1}{\sqrt{51}} \Rightarrow \beta \approx 82.0°$$

$$\cos\gamma = -\frac{1}{\sqrt{51}} \Rightarrow \gamma \approx 98.0°$$

34. $\mathbf{u} = -4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k},\ \|\mathbf{u}\| = \sqrt{16 + 9 + 25} = \sqrt{50} = 5\sqrt{2}$

$$\cos\alpha = \frac{-4}{5\sqrt{2}} \Rightarrow \alpha \approx 2.1721 \text{ or } 124.4°$$

$$\cos\beta = \frac{3}{5\sqrt{2}} \Rightarrow \beta \approx 1.1326 \text{ or } 64.9°$$

$$\cos\gamma = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \gamma \approx \frac{\pi}{4} \text{ or } 45°$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{16}{50} + \frac{9}{50} + \frac{25}{50} = 1$$

35. $\mathbf{u} = \langle 0, 6, -4\rangle,\ \|\mathbf{u}\| = \sqrt{0 + 36 + 16} = \sqrt{52} = 2\sqrt{13}$

$$\cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2} \text{ or } 90°$$

$$\cos\beta = \frac{3}{\sqrt{13}} \Rightarrow \beta \approx 0.5880 \text{ or } 33.7°$$

$$\cos\gamma = -\frac{2}{\sqrt{13}} \Rightarrow \gamma \approx 2.1588 \text{ or } 123.7°$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 0 + \frac{9}{13} + \frac{4}{13} = 1$$

36. $\mathbf{u} = \langle -1, 5, 2\rangle,\ \|\mathbf{u}\| = \sqrt{1 + 25 + 4} = \sqrt{30}$

$$\cos\alpha = \frac{-1}{\sqrt{30}} \Rightarrow \alpha \approx 1.7544 \text{ or } 100.5°$$

$$\cos\beta = \frac{5}{\sqrt{30}} \Rightarrow \beta \approx 0.4205 \text{ or } 24.1°$$

$$\cos\gamma = \frac{2}{\sqrt{30}} \Rightarrow \gamma \approx 1.1970 \text{ or } 68.6°$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{1}{30} + \frac{25}{30} + \frac{4}{30} = 1$$

37. $\mathbf{u} = \langle 6, 7\rangle,\ \mathbf{v} = \langle 1, 4\rangle$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{6(1) + 7(4)}{1^2 + 4^2}\langle 1, 4\rangle = \frac{34}{17}\langle 1, 4\rangle = \langle 2, 8\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 6, 7\rangle - \langle 2, 8\rangle = \langle 4, -1\rangle$

38. $\mathbf{u} = \langle 9, 7\rangle,\ \mathbf{v} = \langle 1, 3\rangle$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{9(1) + 7(3)}{1 + 3^2}\langle 1, 3\rangle = \frac{30}{10}\langle 1, 3\rangle = \langle 3, 9\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 9, 7\rangle = \langle 3, 9\rangle = \langle 6, -2\rangle$

39. $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} = \langle 2, 3\rangle,\ \mathbf{v} = 5\mathbf{i} + \mathbf{j} = \langle 5, 1\rangle$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{2(5) + 3(1)}{5^2 + 1}\langle 5, 1\rangle = \frac{13}{26}\langle 5, 1\rangle = \left\langle \frac{5}{2}, \frac{1}{2}\right\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, 3\rangle - \left\langle \frac{5}{2}, \frac{1}{2}\right\rangle = \left\langle -\frac{1}{2}, \frac{5}{2}\right\rangle$

40. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} = \langle 2, -3\rangle,\ \mathbf{v} = 3\mathbf{i} + 2\mathbf{j} = \langle 3, 2\rangle$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{2(3) + (-3)(2)}{3^2 + 2^2}\langle 3, 2\rangle = 0\langle 3, 2\rangle = \langle 0, 0\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 2, -3\rangle$

41. $\mathbf{u} = \langle 0, 3, 3\rangle, \mathbf{v} = \langle -1, 1, 1\rangle$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{0(-1) + 3(1) + 3(1)}{1 + 1 + 1}\langle -1, 1, 1\rangle = \frac{6}{3}\langle -1, 1, 1\rangle = \langle -2, 2, 2\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3, 3\rangle - \langle -2, 2, 2\rangle = \langle 2, 1, 1\rangle$

42. $\mathbf{u} = \langle 8, 2, 0\rangle, \mathbf{v} = (2, 1, -1)$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{8(2) + 2(1) + 0(-1)}{2^2 + 1 + 1}\langle 2, 1, -1\rangle = \frac{18}{6}\langle 2, 1, -1\rangle = \langle 6, 3, -3\rangle$$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 8, 2, 0\rangle - \langle 6, 3, -3\rangle = \langle 2, -1, 3\rangle$

43. $\mathbf{u} = -9\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}, \mathbf{v} = 4\mathbf{j} + 4\mathbf{k}$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{(-2)(4) + (-4)(4)}{4^2 + 4^2}\right)\langle 0, 4, 4\rangle = -\frac{3}{4}\langle 0, 4, 4\rangle = \langle 0, -3, -3\rangle$$

(b) $$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -9, -2, -4\rangle - \langle 0, -3, -3\rangle = \langle -9, 1, -1\rangle$$

44. $\mathbf{u} = 5\mathbf{i} - \mathbf{j} - \mathbf{k},\ \mathbf{v} = -\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{5(-1) + (-1)(5) + (-1)(8)}{(-1)^2 + 5^2 + 8^2}\right)\langle -1, 5, 8\rangle = \frac{-18}{90}\langle -1, 5, 8\rangle = \frac{-1}{5}\langle -1, 5, 8\rangle = \left\langle \frac{1}{5}, -1, -\frac{8}{5}\right\rangle$$

(b) $$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 5, -1, -1\rangle - \left\langle \frac{1}{5}, -1, -\frac{8}{5}\right\rangle = \left\langle \frac{24}{5}, 0, \frac{3}{5}\right\rangle$$

45. $\mathbf{u}$ is a vector and $\mathbf{v}\cdot\mathbf{w}$ is a scalar. You cannot add a vector and a scalar.

46. $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \mathbf{u} \Rightarrow \mathbf{u} = c\mathbf{v} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

47. Yes,
$$\left\|\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}\right\| = \left\|\frac{\mathbf{v}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u}\right\|$$
$$|\mathbf{u}\cdot\mathbf{v}|\frac{\|\mathbf{v}\|}{\|\mathbf{v}\|^2} = |\mathbf{v}\cdot\mathbf{u}|\frac{\|\mathbf{u}\|}{\|\mathbf{u}\|^2}$$
$$\frac{1}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{u}\|}$$
$$\|\mathbf{u}\| = \|\mathbf{v}\|$$

48. (a) Orthogonal, $\theta = \frac{\pi}{2}$

(b) Acute, $0 < \theta < \frac{\pi}{2}$

(c) Obtuse, $\frac{\pi}{2} < \theta < \pi$

49. $\mathbf{u} = \langle 3240, 1450, 2235\rangle$

$\mathbf{v} = \langle 2.25, 2.95, 2.65\rangle$

$$\mathbf{u}\cdot\mathbf{v} = 3240(2.25) + 1450(2.95) + 2235(2.65) = \$17{,}490.25$$

This represents the total revenue the restaurant earned on its three products.

50. $\mathbf{u} = \langle 3240, 1450, 2235 \rangle$

$\mathbf{v} = \langle 2.25, 2.95, 2.65 \rangle$

Decrease prices by 2%: $0.98\mathbf{v}$

New total revenue:

$$0.98\langle 3240, 1450, 2235 \rangle \cdot \langle 2.25, 2.95, 2.65 \rangle = 0.98(17490.25)$$
$$= \$17{,}140.45$$

51. Answers will vary. *Sample answer:*

$\mathbf{u} = -\dfrac{1}{4}\mathbf{i} + \dfrac{3}{2}\mathbf{j}$. Want $\mathbf{u} \cdot \mathbf{v} = 0$.

$\mathbf{v} = 12\mathbf{i} + 2\mathbf{j}$ and $-\mathbf{v} = -12\mathbf{i} - 2\mathbf{j}$ are orthogonal to **u.**

52. Answers will vary. *Sample answer:*

$\mathbf{u} = 9\mathbf{i} - 4\mathbf{j}$. Want $\mathbf{u} \cdot \mathbf{v} = 0$.

$\mathbf{v} = 4\mathbf{i} + 9\mathbf{j}$ and $-\mathbf{v} = -4\mathbf{i} - 9\mathbf{j}$

are orthogonal to **u**.

53. Answers will vary. *Sample answer:*

$\mathbf{u} = \langle 3, 1, -2 \rangle$. Want $\mathbf{u} \cdot \mathbf{v} = 0$.

$\mathbf{v} = \langle 0, 2, 1 \rangle$ and $-\mathbf{v} = \langle 0, -2, -1 \rangle$ are orthogonal to **u**.

54. Answers will vary. *Sample answer:*

$\mathbf{u} = \langle 4, -3, 6 \rangle$. Want $\mathbf{u} \cdot \mathbf{v} = 0$

$\mathbf{v} = \langle 0, 6, 3 \rangle$ and $-\mathbf{v} = \langle 0, -6, -3 \rangle$

are orthogonal to **u**.

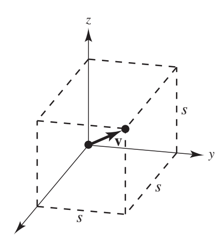
55. Let s = length of a side.

$\mathbf{v} = \langle s, s, s \rangle$

$\|\mathbf{v}\| = s\sqrt{3}$

$$\cos\alpha = \cos\beta = \cos\gamma = \frac{s}{s\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha = \beta = \gamma = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7^\circ$$



56. $\mathbf{v}_1 = \langle s, s, s \rangle$

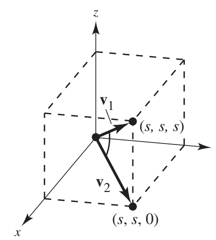
$\|\mathbf{v}_1\| = s\sqrt{3}$

$\mathbf{v}_2 = \langle s, s, 0 \rangle$

$\|\mathbf{v}_2\| = s\sqrt{2}$

$$\cos\theta = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\theta = \arccos\frac{\sqrt{6}}{3} \approx 35.26^\circ$$



57. (a) Gravitational Force $\mathbf{F} = -48{,}000\mathbf{j}$

$\mathbf{v} = \cos 10^\circ \mathbf{i} + \sin 10^\circ \mathbf{j}$

$$\mathbf{w}_1 = \frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F} \cdot \mathbf{v})\mathbf{v}$$
$$= (-48{,}000)(\sin 10^\circ)\mathbf{v}$$
$$\approx -8335.1(\cos 10^\circ \mathbf{i} + \sin 10^\circ \mathbf{j})$$

$\|\mathbf{w}_1\| \approx 8335.1$ lb

(b) $\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$

$$= -48{,}000\mathbf{j} + 8335.1(\cos 10^\circ \mathbf{i} + \sin 10^\circ \mathbf{j})$$
$$= 8208.5\mathbf{i} - 46{,}552.6\mathbf{j}$$

$\|\mathbf{w}_2\| \approx 47{,}270.8$ lb

58. (a) Gravitational Force $\mathbf{F} = -5400\mathbf{j}$

$\mathbf{v} = \cos 18^\circ \mathbf{i} + \sin 18^\circ \mathbf{j}$

$$\mathbf{w}_1 = \frac{\mathbf{F} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F} \cdot \mathbf{v})\mathbf{v}$$
$$= (-5400)(\sin 18^\circ)\mathbf{v}$$
$$\approx -1668.7(\cos 18^\circ \mathbf{i} + \sin 18^\circ \mathbf{j})$$

$\|\mathbf{w}_1\| = 1668.7$ lb

(b) $\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$

$$= -5400\mathbf{j} + 1668.7(\cos 18^\circ \mathbf{i} + \sin 18^\circ \mathbf{j})$$
$$\approx 1587.0\mathbf{i} - 4884.3\mathbf{j}$$

$\|\mathbf{w}_2\| \approx 5135.7$ lb

59. $\mathbf{F} = 85\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)$

$\mathbf{v} = 10\mathbf{i}$

$W = \mathbf{F} \cdot \mathbf{v} = 425$ ft-lb

60. $W = \left\|\text{proj}_{\overrightarrow{PQ}}\mathbf{F}\right\|\left\|\overrightarrow{PQ}\right\|$

$= \cos 20°\|\mathbf{F}\|\left|\overrightarrow{PQ}\right|$

$= (\cos 20°)(65)(50)$

≈ 3054.0 ft-lb

61. $\mathbf{F} = 1600(\cos 25°\,\mathbf{i} + \sin 25°\,\mathbf{j})$

$\mathbf{v} = 2000\mathbf{i}$

$W = \mathbf{F} \cdot \mathbf{v} = 1600(2000)\cos 25°$

$\approx 2{,}900{,}184.9$ Newton meters (Joules)

≈ 2900.2 km-N

62. $W = \left\|\text{proj}_{\overrightarrow{PQ}}\mathbf{F}\right\|\left\|\overrightarrow{PQ}\right\|$

$= (\cos 60°)\|\mathbf{F}\|\left\|\overrightarrow{PQ}\right\|$

$= \frac{1}{2}(400)(40)$

$= 8000$ Joules

63. False.

For example, let $\mathbf{u} = \langle 1, 1\rangle$, $\mathbf{v} = \langle 2, 3\rangle$ and $\mathbf{w} = \langle 1, 4\rangle$.

Then $\mathbf{u} \cdot \mathbf{v} = 2 + 3 = 5$ and $\mathbf{u} \cdot \mathbf{w} = 1 + 4 = 5$.

64. True

$\mathbf{w} \cdot (\mathbf{u} + \mathbf{v}) = \mathbf{w} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{v} = 0 + 0 = 0$ so, $\mathbf{w}$ and $\mathbf{u} + \mathbf{v}$ are orthogonal.

65. (a) The graphs $y_1 = x^2$ and $y_2 = x^{1/3}$ intersect at $(0, 0)$ and $(1, 1)$.

(b) $y_1' = 2x$ and $y_2' = \frac{1}{3x^{2/3}}$.

At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0, 1\rangle$ is tangent to y_2.

At $(1, 1)$, $y_1' = 2$ and $y_2' = \frac{1}{3}$.

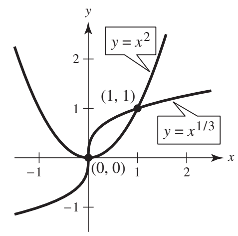
$\pm\frac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle$ is tangent to y_2.

(c) At $(0, 0)$, the vectors are perpendicular $(90°)$.

At $(1, 1)$,

$$\cos\theta = \frac{\frac{1}{\sqrt{5}}\langle 1, 2\rangle \cdot \frac{1}{\sqrt{10}}\langle 3, 1\rangle}{(1)(1)} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$$

$\theta = 45°$



66. (a) The graphs $y_1 = x^3$ and $y_2 = x^{1/3}$ intersect at $(-1, -1)$, $(0, 0)$ and $(1, 1)$.

(b) $y_1' = 3x^2$ and $y_2' = \frac{1}{3x^{2/3}}$.

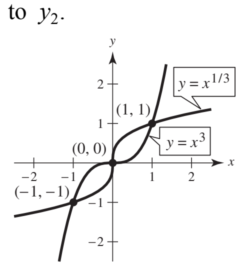
At $(0, 0)$, $\pm\langle 1, 0\rangle$ is tangent to y_1 and $\pm\langle 0, 1\rangle$ is tangent to y_2.

At $(1, 1)$, $y_1' = 3$ and $y_2' = \frac{1}{3}$.

$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle$ is tangent to y_2.

At $(-1, -1)$, $y_1' = 3$ and $y_2' = \frac{1}{3}$.

$\pm\frac{1}{\sqrt{10}}\langle 1, 3\rangle$ is tangent to y_1, $\pm\frac{1}{\sqrt{10}}\langle 3, 1\rangle$ is tangent to y_2.



(c) At $(0, 0)$, the vectors are perpendicular $(90°)$.

At $(1, 1)$,

$$\cos\theta = \frac{\frac{1}{\sqrt{10}}\langle 1, 3\rangle \cdot \frac{1}{\sqrt{10}}\langle 3, 1\rangle}{(1)(1)} = \frac{6}{10} = \frac{3}{5}.$$

$\theta \approx 0.9273$ or $53.13°$

By symmetry, the angle is the same at $(-1, -1)$.

67. (a) The graphs of $y_1 = 1 - x^2$ and $y^2 = x^2 - 1$ intersect at $(1, 0)$ and $(-1, 0)$.

(b) $y_1' = -2x$ and $y_2' = 2x$.

At $(1, 0)$, $y_1' = -2$ and $y_2' = 2$. $\pm\dfrac{1}{\sqrt{5}}\langle 1, -2\rangle$ is tangent to y_1, $\pm\dfrac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_2.

At $(-1, 0)$, $y_1' = 2$ and $y_2' = -2$. $\pm\dfrac{1}{\sqrt{5}}\langle 1, 2\rangle$ is tangent to y_1, $\pm\dfrac{1}{\sqrt{5}}\langle 1, -2\rangle$ is tangent to y_2.

(c) At $(1, 0)$, $\cos\theta = \dfrac{1}{\sqrt{5}}\langle 1, -2\rangle \cdot \dfrac{-1}{\sqrt{5}}\langle 1, -2\rangle = \dfrac{3}{5}$.

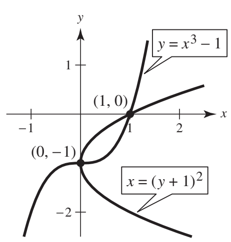
$\theta \approx 0.9273$ or $53.13°$

By symmetry, the angle is the same at $(-1, 0)$.

68. (a) To find the intersection points, rewrite the second equation as $y + 1 = x^3$. Substituting into the first equation

$(y + 1)^2 = x \Rightarrow x^6 = x \Rightarrow x = 0, 1$.

There are two points of intersection, $(0, -1)$ and $(1, 0)$, as indicated in the figure.



(b) First equation:

$(y + 1)^2 = x \Rightarrow 2(y + 1)y' = 1 \Rightarrow y' = \dfrac{1}{2(y + 1)}$

At $(1, 0)$, $y' = \dfrac{1}{2}$.

Second equation: $y = x^3 - 1 \Rightarrow y' = 3x^2$. At $(1, 0)$, $y' = 3$.

$\pm\dfrac{1}{\sqrt{5}}\langle 2, 1\rangle$ unit tangent vectors to first curve,

$\pm\dfrac{1}{\sqrt{10}}\langle 1, 3\rangle$ unit tangent vectors to second curve

At $(0, 1)$, the unit tangent vectors to the first curve are $\pm\langle 0, 1\rangle$, and the unit tangent vectors to the second curve are $\pm\langle 1, 0\rangle$.

(c) At $(1, 0)$,

$\cos\theta = \dfrac{1}{\sqrt{5}}\langle 2, 1\rangle \cdot \dfrac{1}{\sqrt{10}}\langle 1, 3\rangle = \dfrac{5}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$.

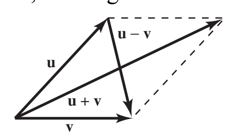
$\theta \approx \dfrac{\pi}{4}$ or $45°$

At $(0, -1)$ the vectors are perpendicular, $\theta = 90°$.

69. In a rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

$$\begin{aligned}(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) &= (\mathbf{u} + \mathbf{v}) \cdot \mathbf{u} - (\mathbf{u} + \mathbf{v}) \cdot \mathbf{v} \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} \\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0\end{aligned}$$

So, the diagonals are orthogonal.



70. If $\mathbf{u}$ and $\mathbf{v}$ are the sides of the parallelogram, then the diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$, as indicated in the figure.

the parallelogram is a rectangle.

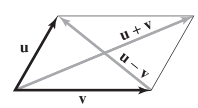
$\Leftrightarrow \mathbf{u} \cdot \mathbf{v} = 0$

$\Leftrightarrow 2\mathbf{u} \cdot \mathbf{v} = -2\mathbf{u} \cdot \mathbf{v}$

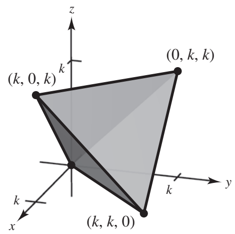
$\Leftrightarrow (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$

$\Leftrightarrow \|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u} - \mathbf{v}\|^2$

$\Leftrightarrow$ The diagonals are equal in length.



71. (a)



(b) Length of each edge: $\sqrt{k^2 + k^2 + 0^2} = k\sqrt{2}$

(c) $\cos\theta = \dfrac{k^2}{\left(k\sqrt{2}\right)\left(k\sqrt{2}\right)} = \dfrac{1}{2}$

$$\theta = \arccos\left(\frac{1}{2}\right) = 60°$$

(d) $\vec{r_1} = \langle k, k, 0\rangle - \left\langle \dfrac{k}{2}, \dfrac{k}{2}, \dfrac{k}{2}\right\rangle = \left\langle \dfrac{k}{2}, \dfrac{k}{2}, -\dfrac{k}{2}\right\rangle$

$$\vec{r_2} = \langle 0, 0, 0\rangle - \left\langle \frac{k}{2}, \frac{k}{2}, \frac{k}{2}\right\rangle = \left\langle -\frac{k}{2}, -\frac{k}{2}, -\frac{k}{2}\right\rangle$$

$$\cos\theta = \frac{-\dfrac{k^2}{4}}{\left(\dfrac{k}{2}\right)^2 \cdot 3} = -\frac{1}{3}$$

$$\theta = 109.5°$$

72. $\mathbf{u} = \langle \cos\alpha, \sin\alpha, 0\rangle$, $\mathbf{v} = \langle \cos\beta, \sin\beta, 0\rangle$

The angle between $\mathbf{u}$ and $\mathbf{v}$ is $\alpha - \beta$. (Assuming that $\alpha > \beta$). Also,

$$\begin{aligned}\cos(\alpha - \beta) &= \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}\\ &= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{(1)(1)}\\ &= \cos\alpha\cos\beta + \sin\alpha\sin\beta.\end{aligned}$$

73.
$$\begin{aligned}\|\mathbf{u} + \mathbf{v}\|^2 &= (\mathbf{u} + \mathbf{v})\cdot(\mathbf{u} + \mathbf{v})\\ &= (\mathbf{u} + \mathbf{v})\cdot\mathbf{u} + (\mathbf{u} + \mathbf{v})\cdot\mathbf{v}\\ &= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}\\ &= \|\mathbf{u}\|^2 + 2\mathbf{u}\cdot\mathbf{v} + \|\mathbf{v}\|^2\\ &\le \|\mathbf{u}\|^2 + 2\|\mathbf{u}\|\|\mathbf{v}\| + \|\mathbf{v}\|^2 \le \left(\|\mathbf{u}\| + \|\mathbf{v}\|\right)^2\end{aligned}$$

So, $\|\mathbf{u} + \mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

74. Let $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u}$, as indicated in the figure. Because $\mathbf{w}_1$ is a scalar multiple of $\mathbf{v}$, you can write

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = c\mathbf{v} + \mathbf{w}_2$.

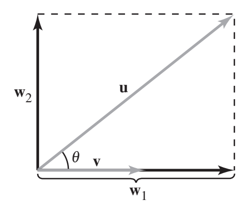
Taking the dot product of both sides with $\mathbf{v}$ produces

$\mathbf{u}\cdot\mathbf{v} = (c\mathbf{v} + \mathbf{w}_2)\cdot\mathbf{v} = c\mathbf{v}\cdot\mathbf{v} + \mathbf{w}_2\cdot\mathbf{v}$

$= c\|\mathbf{v}\|^2$, because $\mathbf{w}_2$ and $\mathbf{v}$ are orthogonol.

So, $\mathbf{u}\cdot\mathbf{v} = c\|\mathbf{v}\|^2 \Rightarrow c = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}$ and

$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = c\mathbf{v} = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}$.



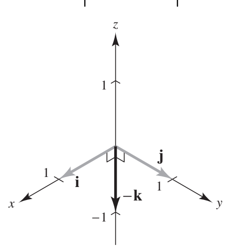
75.
$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\\ |\mathbf{u}\cdot\mathbf{v}| &= \left|\|\mathbf{u}\|\|\mathbf{v}\|\cos\theta\right|\\ &= \|\mathbf{u}\|\|\mathbf{v}\||\cos\theta|\\ &\le \|\mathbf{u}\|\|\mathbf{v}\| \text{ because } |\cos\theta| \le 1.\end{aligned}$$

Section 11.4 The Cross Product of Two Vectors in Space

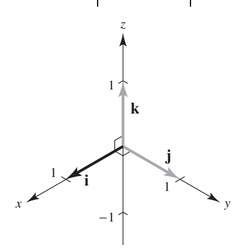
1. $\mathbf{u}\times\mathbf{v}$ is a vector that is perpendicular (orthogonal) to both $\mathbf{u}$ and $\mathbf{v}$.

2. If $\mathbf{u}$ and $\mathbf{v}$ are the adjacent sides of a parallelogram, then $A = \|\mathbf{u}\times\mathbf{v}\|$.

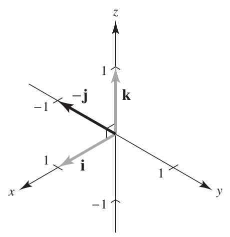
3. $\mathbf{j}\times\mathbf{i} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & 0\\ 1 & 0 & 0\end{vmatrix} = -\mathbf{k}$



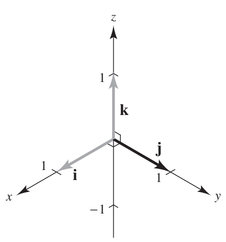
4. $\mathbf{j}\times\mathbf{k} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} = \mathbf{i}$



5. $\mathbf{i} \times \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\mathbf{j}$



6. $\mathbf{k} \times \mathbf{i} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \mathbf{j}$



7. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & 0 \\ 3 & 2 & 5 \end{vmatrix} = 20\mathbf{i} + 10\mathbf{j} - 16\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -20\mathbf{i} - 10\mathbf{j} + 16\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

8. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 5 \\ 2 & 3 & -2 \end{vmatrix} = -15\mathbf{i} + 16\mathbf{j} + 9\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = 15\mathbf{i} - 16\mathbf{j} - 9\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

9. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 3 & 2 \\ 1 & -1 & 5 \end{vmatrix} = 17\mathbf{i} - 33\mathbf{j} - 10\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -17\mathbf{i} + 33\mathbf{j} + 10\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

10. (a) $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -9 \\ -6 & -2 & -1 \end{vmatrix}$

$= -19\mathbf{i} + 56\mathbf{j} + 2\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = 19\mathbf{i} - 56\mathbf{j} - 2\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = 0$

11. $\mathbf{u} = \langle 4, -1, 0 \rangle$, $\mathbf{v} = \langle -6, 3, 0 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 0 \\ -6 & 3 & 0 \end{vmatrix} = 6\mathbf{k} = \langle 0, 0, 6 \rangle$

$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 4(0) + (-1)(0) + 0(6) = 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v}$

$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = -6(0) + 3(0) + 0(6) = 0 \Rightarrow \mathbf{v} \perp \mathbf{u} \times \mathbf{v}$

12. $\mathbf{u} = \langle -5, 2, 2 \rangle$, $\mathbf{v} = \langle 0, 1, 8 \rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 2 & 2 \\ 0 & 1 & 8 \end{vmatrix} = 14\mathbf{i} + 40\mathbf{j} - 5\mathbf{k} = \langle 14, 40, -5 \rangle$

$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = (-5)(14) + 2(40) + 2(-5) = 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v}$

$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = (0)(14) + 1(40) + 8(-5) = 0 \Rightarrow \mathbf{v} \perp \mathbf{u} \times \mathbf{v}$

13. $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \ \mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k} = \langle -2, 3, -1 \rangle$$

$$\begin{aligned} \mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) &= 1(-2) + 1(3) + 1(-1) \\ &= 0 \Rightarrow \mathbf{u} \perp \mathbf{u} \times \mathbf{v} \\ \mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) &= 2(-2) + 1(3) + (-1)(-1) \\ &= 0 \Rightarrow \mathbf{v} \perp \mathbf{u} \times \mathbf{v} \end{aligned}$$

14. $\mathbf{u} = \mathbf{i} + 6\mathbf{j}, \ \mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 0 \\ -2 & 1 & 1 \end{vmatrix} = 6\mathbf{i} - \mathbf{j} + 13\mathbf{k}$$

$$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = 1(6) + 6(-1) = 0 \Rightarrow \mathbf{u} \perp (\mathbf{u} \times \mathbf{v})$$

$$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = -2(6) + 1(-1) + 1(13) = 0 \Rightarrow \mathbf{v} \perp (\mathbf{u} \times \mathbf{v})$$

15.

$$\begin{aligned} \mathbf{u} &= \langle 4, -3, 1 \rangle \\ \mathbf{v} &= \langle 2, 5, 3 \rangle \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & 1 \\ 2 & 5 & 3 \end{vmatrix} = -14\mathbf{i} - 10\mathbf{j} + 26\mathbf{k}$$

$$\begin{aligned} \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} &= \frac{1}{\sqrt{972}} \langle -14, -10, 26 \rangle \\ &= \frac{1}{18\sqrt{3}} \langle -14, -10, 26 \rangle \\ &= \left\langle -\frac{7}{9\sqrt{3}}, -\frac{5}{9\sqrt{3}}, \frac{13}{9\sqrt{3}} \right\rangle \end{aligned}$$

16.

$$\begin{aligned} \mathbf{u} &= \langle -8, -6, 4 \rangle \\ \mathbf{v} &= \langle 10, -12, -2 \rangle \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & -6 & 4 \\ 10 & -12 & -2 \end{vmatrix} = 60\mathbf{i} + 24\mathbf{j} + 156\mathbf{k}$$

$$\begin{aligned} \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} &= \frac{1}{36\sqrt{22}} \langle 60, 24, 156 \rangle \\ &= \left\langle \frac{5}{3\sqrt{22}}, \frac{2}{3\sqrt{22}}, \frac{13}{3\sqrt{22}} \right\rangle \end{aligned}$$

17.

$$\begin{aligned} \mathbf{u} &= -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} \\ \mathbf{v} &= \mathbf{i} - \mathbf{j} + 4\mathbf{k} \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -5 \\ 1 & -1 & -4 \end{vmatrix} = 3\mathbf{i} + 7\mathbf{j} + \mathbf{k}$$

$$\begin{aligned} \frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} &= \frac{1}{\sqrt{59}} \langle 3, 7, 1 \rangle \\ &= \left\langle \frac{3}{\sqrt{59}}, \frac{7}{\sqrt{59}}, \frac{1}{\sqrt{59}} \right\rangle \end{aligned}$$

18.

$$\begin{aligned} \mathbf{u} &= 2\mathbf{k} \\ \mathbf{v} &= 4\mathbf{i} + 6\mathbf{k} \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2 \\ 4 & 0 & 6 \end{vmatrix} = 8\mathbf{j}$$

$$\frac{\mathbf{u} \times \mathbf{v}}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{1}{8}(8\mathbf{j}) = \mathbf{j} = \langle 0, 1, 0 \rangle$$

19.

$$\begin{aligned} \mathbf{u} &= \mathbf{j} \\ \mathbf{v} &= \mathbf{j} + \mathbf{k} \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i}$$

$$A = \|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{i}\| = 1$$

20.

$$\begin{aligned} \mathbf{u} &= \mathbf{i} + \mathbf{j} + \mathbf{k} \\ \mathbf{v} &= \mathbf{j} + \mathbf{k} \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -\mathbf{j} + \mathbf{k}$$

$$A = \|\mathbf{u} \times \mathbf{v}\| = \|-\mathbf{j} + \mathbf{k}\| = \sqrt{2}$$

21.

$$\begin{aligned} \mathbf{u} &= \langle 3, 2, -1 \rangle \\ \mathbf{v} &= \langle 1, 2, 3 \rangle \end{aligned}$$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 8, -10, 4 \rangle$$

$$A = \|\mathbf{u} \times \mathbf{v}\| = \|\langle 8, -10, 4 \rangle\| = \sqrt{180} = 6\sqrt{5}$$

22. $\mathbf{u} = \langle 2, -1, 0\rangle$

$\mathbf{v} = \langle -1, 2, 0\rangle$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \langle 0, 0, 3\rangle$$

$A = \|\mathbf{u}\times\mathbf{v}\| = \|\langle 0, 0, 3\rangle\| = 3$

23. $A(0, 3, 2), B(1, 5, 5), C(6, 9, 5), D(5, 7, 2)$

$\overrightarrow{AB} = \langle 1, 2, 3\rangle$

$\overrightarrow{DC} = \langle 1, 2, 3\rangle$

$\overrightarrow{BC} = \langle 5, 4, 0\rangle$

$\overrightarrow{AD} = \langle 5, 4, 0\rangle$

Because $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{BC} = \overrightarrow{AD}$, the figure $ABCD$ is a parallelogram.

$\overrightarrow{AB}$ and $\overrightarrow{AD}$ are adjacent sides

$$\overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 4 & 0 \end{vmatrix} = \langle -12, 15, -6\rangle$$

$A = \left\|\overrightarrow{AB}\times\overrightarrow{AD}\right\| = \sqrt{144 + 225 + 36} = 9\sqrt{5}$

24. $A(2, -3, 1), B(6, 5, -1), C(7, 2, 2), D(3, -6, 4)$

$\overrightarrow{AB} = \langle 4, 8, -2\rangle$

$\overrightarrow{DC} = \langle 4, 8, -2\rangle$

$\overrightarrow{BC} = \langle 1, -3, 3\rangle$

$\overrightarrow{AD} = \langle 1, -3, 3\rangle$

Because $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{BC} = \overrightarrow{AD}$, the figure $ABCD$ is a parallelogram.

$\overrightarrow{AB}$ and $\overrightarrow{AD}$ are adjacent sides

$$\overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 8 & -2 \\ 1 & -3 & 3 \end{vmatrix} = \langle 18, -14, -20\rangle$$

$A = \left\|\overrightarrow{AB}\times\overrightarrow{AD}\right\| = \sqrt{324 + 196 + 400} = 2\sqrt{230}$

25. $A(0, 0, 0), B(1, 0, 3), C(-3, 2, 0)$

$\overrightarrow{AB} = \langle 1, 0, 3\rangle,\ \overrightarrow{AC} = \langle -3, 2, 0\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 3 \\ -3 & 2 & 0 \end{vmatrix} = \langle -6, -9, 2\rangle$$

$A = \frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\| = \frac{1}{2}\sqrt{36 + 81 + 4} = \frac{11}{2}$

26. $A(2, -3, 4), B(0, 1, 2), C(-1, 2, 0)$

$\overrightarrow{AB} = \langle -2, 4, -2\rangle, \overrightarrow{AC} = \langle -3, 5, -4\rangle$

$$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4 & -2 \\ -3 & 5 & -4 \end{vmatrix} = -6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$$

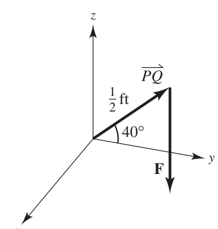
$A = \frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\| = \frac{1}{2}\sqrt{44} = \sqrt{11}$

27. $\mathbf{F} = -20\mathbf{k}$

$\overrightarrow{PQ} = \frac{1}{2}(\cos 40°\mathbf{j} + \sin 40°\mathbf{k})$

$$\overrightarrow{PQ}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \cos 40°/2 & \sin 40°/2 \\ 0 & 0 & -20 \end{vmatrix} = -10\cos 40°\mathbf{i}$$

$\left\|\overrightarrow{PQ}\times\mathbf{F}\right\| = 10\cos 40° \approx 7.66$ ft-lb



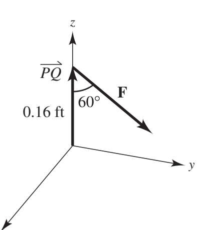
28. $\mathbf{F} = -2000(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) = -1000\sqrt{3}\mathbf{j} - 1000\mathbf{k}$

$\overrightarrow{PQ} = 0.16\mathbf{k}$

$$\overrightarrow{PQ}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.16 \\ 0 & -1000\sqrt{3} & -1000 \end{vmatrix}$$

$= 160\sqrt{3}\mathbf{i}$

$\left\|\overrightarrow{PQ}\times\mathbf{F}\right\| = 160\sqrt{3}$ ft-lb

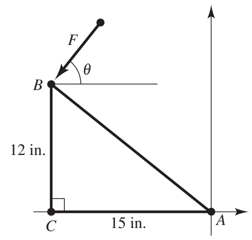


29. (a) $AC = 15 \text{ inches} = \dfrac{5}{4} \text{ feet}$

$BC = 12 \text{ inches} = 1 \text{ foot}$

$\overrightarrow{AB} = -\dfrac{5}{4}\mathbf{j} + \mathbf{k}$

$\mathbf{F} = -180(\cos\theta\,\mathbf{j} + \sin\theta\,\mathbf{k})$



(b) $\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -\frac{5}{4} & 1 \\ 0 & -180\cos\theta & -180\sin\theta \end{vmatrix}$

$= (225\sin\theta + 180\cos\theta)i$

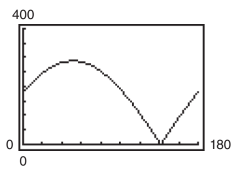
$\left\|\overrightarrow{AB} \times \mathbf{F}\right\| = \left|225\sin\theta + 180\cos\theta\right|$

(c) When $\theta = 30°$, $\left\|\overrightarrow{AB} \times \mathbf{F}\right\| = 225\left(\dfrac{1}{2}\right) + 180\left(\dfrac{\sqrt{3}}{2}\right) \approx 268.38$

(d) If $T = \left|225\sin\theta + 180\cos\theta\right|$, $T = 0$ for $225\sin\theta = -180\cos\theta \Rightarrow \tan\theta = -\dfrac{4}{5} \Rightarrow \theta \approx 141.34°$.

For $0 < \theta < 141.34$, $T'(\theta) = 225\cos\theta - 180\sin\theta = 0 \Rightarrow \tan\theta = \dfrac{5}{4} \Rightarrow \theta \approx 51.34°$. $\overrightarrow{AB}$ and $\mathbf{F}$ are perpendicular.

(e)



From part (d), the zero is $\theta \approx 141.34°$, when the vectors are parallel.

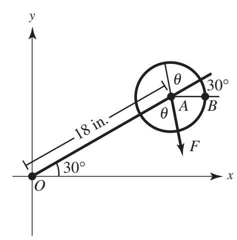
30. (a) Place the wrench in the xy-plane, as indicated in the figure.

The angle from $\overrightarrow{AB}$ to $\mathbf{F}$ is $30° + 180° + \theta = 210° + \theta$

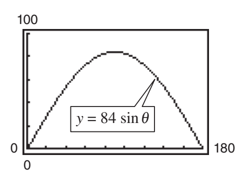
$\left\|\overrightarrow{OA}\right\| = 18 \text{ inches} = 1.5 \text{ feet}$

$\overrightarrow{OA} = 1.5\left[\cos(30°)\mathbf{i} + \sin(30°)\mathbf{j}\right] = \dfrac{3\sqrt{3}}{4}\mathbf{i} + \dfrac{3}{4}\mathbf{j}$

$\mathbf{F} = 56\left[\cos(210° + \theta)\mathbf{i} + \sin(210° + \theta)\mathbf{j}\right]$



$$\overrightarrow{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{3\sqrt{3}}{4} & \dfrac{3}{4} & 0 \\ 56\cos(210° + \theta) & 56\sin(210° + \theta) & 0 \end{vmatrix}$$



$$= \left[42\sqrt{3}\sin(210° + \theta) - 42\cos(210° + \theta)\right]\mathbf{k}$$

$$= \left[42\sqrt{3}(\sin 210°\cos\theta + \cos 210°\sin\theta) - 42(\cos 210°\cos\theta - \sin 210°\sin\theta)\right]\mathbf{k}$$

$$= \left[42\sqrt{3}\left(-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) - 42\left(-\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)\right]\mathbf{k} = (-84\sin\theta)\mathbf{k}$$

$\left\|\overrightarrow{OA} \times \mathbf{F}\right\| = 84\sin\theta,\ 0 \le \theta \le 180°$

(b) When $\theta = 45°$, $\left\|\overrightarrow{OA} \times \mathbf{F}\right\| = 84\dfrac{\sqrt{2}}{2} = 42\sqrt{2} \approx 59.40$

(c) Let $T = 84\sin\theta$

$\dfrac{dT}{d\theta} = 84\cos\theta = 0$ when $\theta = 90°$.

This is reasonable. When $\theta = 90°$, the force is perpendicular to the wrench.

31. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$

32. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1$

33. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6$

34. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 0$

35. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$

$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = 2$

36. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix} = -72$

$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = 72$

37. $\mathbf{u} = \langle 3, 0, 0\rangle$

$\mathbf{v} = \langle 0, 5, 1\rangle$

$\mathbf{w} = \langle 2, 0, 5\rangle$

$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 5 & 1 \\ 2 & 0 & 5 \end{vmatrix} = 75$

$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = 75$

38. $\mathbf{u} = \langle 0, 4, 0\rangle$

$\mathbf{v} = \langle -3, 0, 0\rangle$

$\mathbf{w} = \langle -1, 1, 5\rangle$

$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 0 & 4 & 0 \\ -3 & 0 & 0 \\ -1 & 1 & 5 \end{vmatrix} = -4(-15) = 60$

$V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})| = 60$

39. (a) $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = (\mathbf{v}\times\mathbf{w})\cdot\mathbf{u}$ (b)

$= \mathbf{w}\cdot(\mathbf{u}\times\mathbf{v}) = (\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$ (c)

$= \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = (\mathbf{u}x-\mathbf{w})\cdot\mathbf{v}$ (d)

$= \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = (\mathbf{w}\times\mathbf{u})\cdot\mathbf{v}$ (h)

(e) $\mathbf{u}\cdot(\mathbf{w}\times\mathbf{v}) = \mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$ (f)

$= \mathbf{w}\cdot(\mathbf{v}\times\mathbf{u}) = (-\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}$ (g)

So, $a = b = c = d = h$ and $e = f = g$

40. $\mathbf{u}\times\mathbf{v} = \mathbf{0} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are parallel.

$\mathbf{u}\cdot\mathbf{v} = \mathbf{0} \Rightarrow \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

So, $\mathbf{u}$ or $\mathbf{v}$ (or both) is the zero vector.

41. The cross product is orthogonal to the two vectors, so it is orthogonal to the yz-plane. It lies on the x-axis, since it is of the form $\langle k, 0, 0\rangle$.

42. Form the vectors for two sides of the triangle, and compute their cross product.

$\langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle \times \langle x_3 - x_1, y_3 - y_1, z_3 - z_1\rangle$

43. False. If the vectors are ordered pairs, then the cross product does not exist.

44. False. The cross product is zero if the given vectors are parallel.

45. False. Let $\mathbf{u} = \langle 1, 0, 0\rangle, \mathbf{v} = \langle 1, 0, 0\rangle, \mathbf{w} = \langle -1, 0, 0\rangle$.

Then, $\mathbf{u}\times\mathbf{v} = \mathbf{u}\times\mathbf{w} = \mathbf{0}$, but $\mathbf{v} \neq \mathbf{w}$.

46. True

47. $\mathbf{u} = \langle u_1, u_2, u_3\rangle,\ \mathbf{v} = \langle v_1, v_2, v_3\rangle,\ \mathbf{w} = \langle w_1, w_2, w_3\rangle$

$$\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1+w_1 & v_2+w_2 & v_3+w_3 \end{vmatrix}$$

$$= [u_2(v_3+w_3) - u_3(v_2+w_2)]\mathbf{i} - [u_1(v_3+w_3) - u_3(v_1+w_1)]\mathbf{j} + [u_1(v_2+w_2) - u_2(v_1+w_1)]\mathbf{k}$$

$$= (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k} + (u_2w_3 - u_3w_2)\mathbf{i} - (u_1w_3 - u_3w_1)\mathbf{j} + (u_1w_2 - u_2w_1)\mathbf{k}$$

$$= (\mathbf{u}\times\mathbf{v}) + (\mathbf{u}\times\mathbf{w})$$

48. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle, \mathbf{v} = \langle v_1, v_2, v_3 \rangle$, c is a scalar:

$$(c\mathbf{u}) \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ cu_1 & cu_2 & cu_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= (cu_2v_3 - cu_3v_2)\mathbf{i} - (cu_1v_3 - cu_3v_1)\mathbf{j} + (cu_1v_2 - cu_2v_1)\mathbf{k}$$

$$= c\left[(u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}\right] = c(\mathbf{u} \times \mathbf{v})$$

49. $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$

$$\mathbf{u} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = (u_2u_3 - u_3u_2)\mathbf{i} - (u_1u_3 - u_3u_1)\mathbf{j} + (u_1u_2 - u_2u_1)\mathbf{k} = \mathbf{0}$$

50. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{w} \cdot (\mathbf{u} \times \mathbf{v}) = \begin{vmatrix} w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= w_1(u_2v_3 - v_2u_3) - w_2(u_1v_3 - v_1u_3) + w_3(u_1v_2 - v_1u_2)$$

$$= u_1(v_2w_3 - w_2v_3) - u_2(v_1w_3 - w_1v_3) + u_3(v_1w_2 - w_1v_2) = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

51. $\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2)\mathbf{i} - (u_1v_3 - u_3v_1)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = (u_2v_3 - u_3v_2)u_1 + (u_3v_1 - u_1v_3)u_2 + (u_1v_2 - u_2v_1)u_3 = \mathbf{0}$$

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = (u_2v_3 - u_3v_2)v_1 + (u_3v_1 - u_1v_3)v_2 + (u_1v_2 - u_2v_1)v_3 = \mathbf{0}$$

So, $\mathbf{u} \times \mathbf{v} \perp \mathbf{u}$ and $\mathbf{u} \times \mathbf{v} \perp \mathbf{v}$.

52. If **u** and **v** are scalar multiples of each other, $\mathbf{u} = c\mathbf{v}$ for some scalar c.

$$\mathbf{u} \times \mathbf{v} = (c\mathbf{v}) \times \mathbf{v} = c(\mathbf{v} \times \mathbf{v}) = c(\mathbf{0}) = \mathbf{0}$$

If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\|\mathbf{u}\|\|\mathbf{v}\| \sin\theta = 0$. (Assume $\mathbf{u} \neq \mathbf{0}, \mathbf{v} \neq \mathbf{0}$.) So, $\sin\theta = 0, \theta = 0$, and **u** and **v** are parallel. So, $\mathbf{u} = c\mathbf{v}$ for some scalar c.

53. $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$

If **u** and **v** are orthogonal, $\theta = \pi/2$ and $\sin\theta = 1$. So, $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\|$.

54. $\mathbf{u} = \langle a_1, b_1, c_1 \rangle, \mathbf{v} = \langle a_2, b_2, c_2 \rangle, \mathbf{w} = \langle a_3, b_3, c_3 \rangle$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (a_2c_3 - a_3c_2)\mathbf{j} + (a_2b_3 - a_3b_2)\mathbf{k}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ (b_2c_3 - b_3c_2) & (a_3c_2 - a_2c_3) & (a_2b_3 - a_3b_2) \end{vmatrix}$$

$$\begin{aligned}\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) &= \left[b_1(a_2b_3 - a_3b_2) - c_1(a_3c_2 - a_2c_3)\right]\mathbf{i} - \left[a_1(a_2b_3 - a_3b_2) - c_1(b_2c_3 - b_3c_2)\right]\mathbf{j} \\ &\quad + \left[a_1(a_3c_2 - a_2c_3) - b_1(b_2c_3 - b_3c_2)\right]\mathbf{k} \\ &= \left[a_2(a_1a_3 + b_1b_3 + c_1c_3) - a_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{i} + \left[b_2(a_1b_3 + b_1b_3 + c_1c_3) - b_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{j} \\ &\quad + \left[c_2(a_1a_3 + b_1b_3 + c_1c_3) - c_3(a_1a_2 + b_1b_2 + c_1c_2)\right]\mathbf{k} \\ &= (a_1a_3 + b_1b_3 + c_1c_3)\langle a_2, b_2, c_2 \rangle - (a_1a_2 + b_1b_2 + c_1c_2)\langle a_3, b_3, c_3 \rangle = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}\end{aligned}$$

55. $\mathbf{u} = \mathbf{u}_1\mathbf{i} + \mathbf{u}_2\mathbf{j} + \mathbf{u}_3\mathbf{k}$, $\mathbf{v} = \mathbf{v}_1\mathbf{i} + \mathbf{v}_2\mathbf{j} + \mathbf{v}_3\mathbf{k}$, $\mathbf{w} = \mathbf{w}_1\mathbf{i} + \mathbf{w}_2\mathbf{j} + \mathbf{w}_3\mathbf{k}$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \langle v_2w_3 - w_2v_3, -(v_1w_3 - w_1v_3), v_1w_2 - w_1v_2 \rangle$$

$$\begin{aligned} \mathbf{u} \times (\mathbf{v} \times \mathbf{w}) &= \langle u_1, u_2, u_3 \rangle \cdot \langle v_2w_3 - w_2v_3, -(v_1w_3 - w_1v_3), v_1w_2 - w_1v_2 \rangle \\ &= u_1v_2w_3 - u_1v_3w_2 - u_2v_1w_3 + u_2v_3w_1 + u_3v_1w_2 - u_3v_2w_1 \\ &= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \end{aligned}$$

Section 11.5 Lines and Planes in Space

1. The parametric equations of a line L parallel to $\mathbf{v} = \langle a, b, c, \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are
$x = x_1 + at$, $y = y_1 + bt$, $z = z_1 + ct$.
The symmetric equations are
$\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$.

2. In the equation of the plane $2(x - 1) + 4(y - 3) - (z + 5) = 0$, $a = 2, b = 4$, and $c = -1$. Therefore, the normal vector is $\langle 2, 4, -1 \rangle$.

3. Answers will vary. Any plane that has a missing x-variable in its equation is parallel to the x-axis.

Sample answer: $3y - z = 5$

4. First choose a point Q in one plane. Then use Theorem 11.13:

$$D = \frac{\left|\overrightarrow{PQ} \cdot n\right|}{\|\mathbf{n}\|}$$

where P is a point in the other plane and $\mathbf{n}$ is normal to that plane.

5. $x = -2 + t, y = 3t, z = 4 + t$

(a) $(0, 6, 6)$: For $x = 0 = -2 + t$, you have $t = 2$.
Then $y = 3(2) = 6$ and $z = 4 + 2 = 6$. Yes, $(0, 6, 6)$ lies on the line.

(b) $(2, 3, 5)$: For $x = 2 = -2 + t$, you have $t = 4$.
Then $y = 3(4) = 12 \neq 3$. No, $(2, 3, 5)$ does not lie on the line.

(c) $(-4, -6, 2)$: For $x = -4 = -2 + t$, you have $t = -2$.
Then $y = 3(-2) = -6$ and $z = 4 - 2 = 2$. Yes, $(-4, -6, 2)$ lies on the line.

6. $\dfrac{x - 3}{2} = \dfrac{y - 7}{8} = z + 2$

(a) $(7, 23, 0)$: Substituting, you have

$$\frac{7 - 3}{2} = \frac{23 - 7}{8} = 0 + 2$$
$$2 = 2 = 2$$

Yes, $(7, 23, 0)$ lies on the line.

(b) $(1, -1, -3)$: Substituting, you have

$$\frac{1 - 3}{2} = \frac{-1 - 7}{8} = -3 + 2$$
$$-1 = -1 = -1$$

Yes, $(1, -1, -3)$ lies on the line.

(c) $(-7, 47, -7)$: Substituting, you have

$$\frac{-7 - 3}{2} = \frac{47 - 7}{8} = -7 + 2$$
$$-5 \neq 5 \neq -5$$

No, $(-7, 47, -7)$ does not lie on the line.

7. Point: $(0, 0, 0)$

Direction vector: $\langle 3, 1, 5 \rangle$

Direction numbers: 3, 1, 5

(a) Parametric: $x = 3t, y = t, z = 5t$

(b) Symmetric: $\dfrac{x}{3} = y = \dfrac{z}{5}$

8. Point: $(0, 0, 0)$

Direction vector: $\mathbf{v} = \left\langle -2, \frac{5}{2}, 1 \right\rangle$

Direction numbers: $-4, 5, 2$

(a) Parametric: $x = -4t, y = 5t, z = 2t$

(b) Symmetric: $\frac{x}{-4} = \frac{y}{5} = \frac{z}{2}$

9. Point: $(-2, 0, 3)$

Direction vector: $\mathbf{v} = \langle 2, 4, -2 \rangle$

Direction numbers: $2, 4, -2$

(a) Parametric: $x = -2 + 2t, y = 4t, z = 3 - 2t$

(b) Symmetric: $\frac{x+2}{2} = \frac{y}{4} = \frac{z-3}{-2}$

10. Point: $(-3, 0, 2)$

Direction vector: $\mathbf{v} = \langle 0, 6, 3 \rangle$

Direction numbers: $0, 2, 1$

(a) Parametric: $x = -3, y = 2t, z = 2 + t$

(b) Symmetric: $\frac{y}{2} = z - 2, x = -3$

11. Point: $(1, 0, 1)$

Direction vector: $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Direction numbers: $3, -2, 1$

(a) Parametric: $x = 1 + 3t, y = -2t, z = 1 + t$

(b) Symmetric: $\frac{x-1}{3} = \frac{y}{-2} = \frac{z-1}{1}$

12. Point: $(-3, 5, 4)$

Directions numbers: $3, -2, 1$

(a) Parametric: $x = -3 + 3t, y = 5 - 2t, z = 4 + t$

(b) Symmetric: $\frac{x+3}{3} = \frac{y-5}{-2} = z - 4$

13. Points: $(5, -3, -2), \left(-\frac{2}{3}, \frac{2}{3}, 1\right)$

Direction vector: $\mathbf{v} = \frac{17}{3}\mathbf{i} - \frac{11}{3}\mathbf{j} - 3\mathbf{k}$

Direction numbers: $17, -11, -9$

(a) Parametric:
$x = 5 + 17t, y = -3 - 11t, z = -2 - 9t$

(b) Symmetric: $\frac{x-5}{17} = \frac{y+3}{-11} = \frac{z+2}{-9}$

14. Points: $(0, 4, 3), (-1, 2, 5)$

Direction vector: $\langle 1, 2, -2 \rangle$

Direction numbers: $1, 2, -2$

(a) Parametric: $x = t, y = 4 + 2t, z = 3 - 2t$

(b) Symmetric: $x = \frac{y-4}{2} = \frac{z-3}{-2}$

15. Points: $(7, -2, 6), (-3, 0, 6)$

Direction vector: $\langle -10, 2, 0 \rangle$

Direction numbers: $-10, 2, 0$

(a) Parametric: $x = 7 - 10t, y = -2 + 2t, z = 6$

(b) Symmetric: Not possible because the direction number for z is 0. But, you could describe the line as $\frac{x-7}{10} = \frac{y+2}{-2}, z = 6$.

16. Points: $(0, 0, 25), (10, 10, 0)$

Direction vector: $\langle 10, 10, -25 \rangle$

Direction numbers: $2, 2, -5$

(a) Parametric: $x = 2t, y = 2t, z = 25 - 5t$

(b) Symmetric: $\frac{x}{2} = \frac{y}{2} = \frac{z-25}{-5}$

17. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = \mathbf{k}$

Direction numbers: $0, 0, 1$

Parametric: $x = 2, y = 3, z = 4 + t$

18. Point: $(-4, 5, 2)$

Direction vector: $\mathbf{v} = \mathbf{j}$

Direction numbers: $0, 1, 0$

Parametric: $x = -4, y = 5 + t, z = 2$

19. Point: $(2, 3, 4)$

Direction vector: $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Direction numbers: $3, 2, -1$

Parametric: $x = 2 + 3t, y = 3 + 2t, z = 4 - t$

20. Point $(-4, 5, 2)$

Direction vector: $\mathbf{v} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Direction numbers: $-1, 2, 1$

Parametric: $x = -4 - t, y = 5 + 2t, z = 2 + t$

21. Point: $(5, -3, -4)$

Direction vector: $\mathbf{v} = \langle 2, -1, 3 \rangle$

Direction numbers: $2, -1, 3$

Parametric: $x = 5 + 2t, y = -3 - t, z = -4 + 3t$

22. Point: $(-1, 4, -3)$

Direction vector: $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$

Direction numbers: $5, -1, 0$

Parametric: $x = -1 + 5t, y = 4 - t, z = -3$

23. Point: $(2, 1, 2)$

Direction vector: $\langle -1, 1, 1 \rangle$

Direction numbers: $-1, 1, 1$

Parametric: $x = 2 - t, y = 1 + t, z = 2 + t$

24. Point: $(-6, 0, 8)$

Direction vector: $\langle -2, 2, 0 \rangle$

Direction numbers: $-2, 2, 0$

Parametric: $x = -6 - 2t, y = 2t, z = 8$

25. Let $t = 0$: $P = (3, -1, -2)$ (other answers possible)

$\mathbf{v} = \langle -1, 2, 0 \rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

26. Let $t = 0$: $P = (0, 5, 4)$ (other answers possible)

$\mathbf{v} = \langle 4, -1, 3 \rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

27. Let each quantity equal 0:

$P = (7, -6, -2)$ (other answers possible)

$\mathbf{v} = \langle 4, 2, 1 \rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

28. Let each quantity equal 0:

$P = (-3, 0, 3)$ (other answers possible)

$\mathbf{v} = \langle 5, 8, 6 \rangle$ (any nonzero multiple of $\mathbf{v}$ is correct)

29. L_1: $\mathbf{v}_1 = \langle -3, 2, 4 \rangle$ and $P = (6, -2, 5)$ on L_1

L_2: $\mathbf{v}_2 = \langle 6, -4, -8 \rangle$ and $P = (6, -2, 5)$ on L_2

The lines are identical.

30. L_1: $\mathbf{v}_1 = \langle 2, -1, 3 \rangle$ and $P = (1, -1, 0)$ on L_1

L_2: $\mathbf{v}_2 = \langle 2, -1, 3 \rangle$ and P not on L_1

The lines are parallel.

31. L_1: $\mathbf{v}_1 = \langle 4, -2, 3 \rangle$ and $P = (8, -5, -9)$ on L_1

L_2: $\mathbf{v}_2 = \langle -8, 4, -6 \rangle$ and $P = (8, -5, -9)$ on L_2

The lines are identical.

32. L_1: $\mathbf{v}_1 = \langle 4, 2, 4 \rangle$ and $P = (1, 1, -3)$ on L_1

L_2: $\mathbf{v}_2 = \langle 1, 0.5, 1 \rangle$ and P not on L_2

The lines are parallel.

33. At the point of intersection, the coordinates for one line equal the corresponding coordinates for the other line. So,

(i) $4t + 2 = 2s + 2$, (ii) $3 = 2s + 3$, and
(iii) $-t + 1 = s + 1$.

From (ii), you find that $s = 0$ and consequently, from (iii), $t = 0$. Letting $s = t = 0$, you see that equation (i) is satisfied and so the two lines intersect. Substituting zero for s or for t, you obtain the point $(2, 3, 1)$.

$\mathbf{u} = 4\mathbf{i} - \mathbf{k}$ (First line)

$\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (Second line)

$$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{8 - 1}{\sqrt{17}\sqrt{9}} = \frac{7}{3\sqrt{17}} = \frac{7\sqrt{17}}{51}$$

$\theta \approx 55.5^\circ$

34. By equating like variables, you have

(i) $-3t + 1 = 3s + 1$, (ii) $4t + 1 = 2s + 4$, and
(iii) $2t + 4 = -s + 1$.

From (i) you have $s = -t$, and consequently from (ii), $t = \frac{1}{2}$ and from (iii), $t = -3$. The lines do not intersect.

35. Writing the equations of the lines in parametric form you have

$x = 3t \qquad y = 2 - t \qquad z = -1 + t$

$x = 1 + 4s \qquad y = -2 + s \qquad z = -3 - 3s.$

For the coordinates to be equal, $3t = 1 + 4s$ and $2 - t = -2 + s$. Solving this system yields $t = \frac{17}{7}$ and $s = \frac{11}{7}$. When using these values for s and t, the z coordinates are not equal. The lines do not intersect.

36. Writing the equations of the lines in parametric form you have

$x = 2 - 3t \quad y = 2 + 6t \quad z = 3 + t$

$x = 3 + 2s \quad y = -5 + s \quad z = -2 + 4s.$

By equating like variables, you have $2 - 3t = 3 + 2s$, $2 + 6t = -5 + s$, $3 + t = -2 + 4s$. So, $t = -1, s = 1$ and the point of intersection is $(5, -4, 2)$.

$\mathbf{u} = \langle -3, 6, 1 \rangle$ (First line)

$\mathbf{v} = \langle 2, 1, 4 \rangle$ (Second line)

$$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{4}{\sqrt{46}\sqrt{21}} = \frac{4}{\sqrt{966}} = \frac{2\sqrt{966}}{483}$$

$\theta \approx 82.6°$

37. $x + 2y - 4z - 1 = 0$

(a) $(-7, 2, -1)$: $(-7) + 2(2) - 4(-1) - 1 = 0$

Point is in plane.

(b) $(5, 2, 2)$: $5 + 2(2) - 4(2) - 1 = 0$

Point is in plane.

(c) $(-6, 1, -1)$: $-6 + 2(1) - 4(-1) - 1 = -1 \neq 0$

Point is not in plane.

38. $2x + y + 3z - 6 = 0$

(a) $(3, 6, -2)$: $2(3) + 6 + 3(-2) - 6 = 0$

Point is in plane.

(b) $(-1, 5, -1)$: $2(-1) + 5 + 3(-1) - 6 = -6 \neq 0$

Point is not in plane.

(c) $(2, 1, 0)$: $2(2) + 1 + 3(0) - 6 = -1 \neq 0$

Point is not in plane.

39. Point: $(1, 3, -7)$

Normal vector: $\mathbf{n} = \mathbf{j} = \langle 0, 1, 0 \rangle$

$$0(x - 1) + 1(y - 3) + 0(z - (-7)) = 0$$
$$y - 3 = 0$$

40. Point: $(0, -1, 4)$

Normal vector: $\mathbf{n} = \mathbf{k} = \langle 0, 0, 1 \rangle$

$$0(x - 0) + 0(y + 1) + 1(z - 4) = 0$$
$$z - 4 = 0$$

41. Point: $(3, 2, 2)$

Normal vector: $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$$2(x - 3) + 3(y - 2) - 1(z - 2) = 0$$
$$2x + 3y - z - 10 = 0$$

42. Point: $(0, 0, 0)$

Normal vector: $\mathbf{n} = -3\mathbf{i} + 2\mathbf{k}$

$$-3(x - 0) + 0(y - 0) + 2(z - 0) = 0$$
$$-3x + 2z = 0$$

43. Point: $(-1, 4, 0)$

Normal vector: $\mathbf{v} = \langle 2, -1, -2 \rangle$

$$2(x + 1) - 1(y - 4) - 2(z - 0) = 0$$
$$2x - y - 2z + 6 = 0$$

44. Point: $(3, 2, 2)$

Normal vector: $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$

$$4(x - 3) + (y - 2) - 3(z - 2) = 0$$
$$4x + y - 3z - 8 = 0$$

45. Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(2, 0, 3)$: $\mathbf{u} = \langle 2, 0, 3 \rangle$

Let $\mathbf{u}$ be the vector from $(0, 0, 0)$ to $(-3, -1, 5)$: $\mathbf{v} = \langle -3, -1, 5 \rangle$

Normal vectors: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 3 \\ -3 & -1 & 5 \end{vmatrix} = \langle 3, -19, -2 \rangle$

$$3(x - 0) - 19(y - 0) - 2(z - 0) = 0$$
$$3x - 19y - 2z = 0$$

46. Let $\mathbf{u}$ be the vector from $(3, -1, 2)$ to $(2, 1, 5)$:

$\mathbf{u} = \langle -1, 2, 3 \rangle$

Let $\mathbf{u}$ be the vector from $(3, -1, 2)$ to $(1, -2, -2)$:

$\mathbf{v} = \langle -2, -1, -4 \rangle$

Normal vector:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 3 \\ -2 & -1 & -4 \end{vmatrix} = \langle -5, -10, 5 \rangle = -5\langle 1, 2, -1 \rangle$$

$$1(x - 3) + 2(y + 1) - (z - 2) = 0$$
$$x + 2y - z + 1 = 0$$

47. Let $\mathbf{u}$ be the vector from $(1, 2, 3)$ to $(3, 2, 1)$: $\mathbf{u} = 2\mathbf{i} - 2\mathbf{k}$

Let $\mathbf{v}$ be the vector from $(1, 2, 3)$ to $(-1, -2, 2)$: $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$

Normal vector:

$$\left(\tfrac{1}{2}\mathbf{u}\right) \times (-\mathbf{v}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 2 & 4 & 1 \end{vmatrix} = 4\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$$

$$4(x - 1) - 3(y - 2) + 4(z - 3) = 0$$
$$4x - 3y + 4z - 10 = 0$$

48. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{i}$, $1(x - 1) = 0$, $x - 1 = 0$

49. $(1, 2, 3)$, Normal vector: $\mathbf{v} = \mathbf{k}$, $1(z - 3) = 0$, $z - 3 = 0$

50. The plane passes through the three points $(0, 0, 0)$, $(0, 1, 0)$, $\left(\sqrt{3}, 0, 1\right)$.

The vector from $(0, 0, 0)$ to $(0, 1, 0)$: $\mathbf{u} = \mathbf{j}$

The vector from $(0, 0, 0)$ to $\left(\sqrt{3}, 0, 1\right)$: $\mathbf{v} = \sqrt{3}\mathbf{i} + \mathbf{k}$

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ \sqrt{3} & 0 & 1 \end{vmatrix} = \mathbf{i} - \sqrt{3}\mathbf{k}$$

$$x - \sqrt{3}z = 0$$

51. The direction vectors for the lines are $\mathbf{u} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$.

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = -5(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Point of intersection of the lines: $(-1, 5, 1)$

$$(x + 1) + (y - 5) + (z - 1) = 0$$
$$x + y + z - 5 = 0$$

52. The direction of the line is $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Choose any point on the line, $[(0, 4, 0)$, for example$]$, and let $\mathbf{v}$ be the vector from $(0, 4, 0)$ to the given point $(2, 2, 1)$:

$$\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\text{Normal vector: } \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \mathbf{i} - 2\mathbf{k}$$

$$(x - 2) - 2(z - 1) = 0$$
$$x - 2z = 0$$

53. Let $\mathbf{v}$ be the vector from $(-1, 1, -1)$ to $(2, 2, 1)$: $\mathbf{v} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

Let $\mathbf{n}$ be a vector normal to the plane $2x - 3y + z = 3$: $\mathbf{n} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

Because $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane P, the normal vector to P is

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = 7\mathbf{i} - \mathbf{j} - 11\mathbf{k}$$

$$7(x - 2) + 1(y - 2) - 11(z - 1) = 0$$
$$7x + y - 11z - 5 = 0$$

54. Let $\mathbf{v}$ be the vector from $(3, 2, 1)$ to $(3, 1, -5)$: $\mathbf{v} = -\mathbf{j} - 6\mathbf{k}$

Let $\mathbf{n}$ be the normal to the given plane: $\mathbf{n} = 6\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$

Because $\mathbf{v}$ and $\mathbf{n}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & -6 \\ 6 & 7 & 2 \end{vmatrix} = 40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}$$
$$= 2(20\mathbf{i} - 18\mathbf{j} + 3\mathbf{k})$$

$$20(x - 3) - 18(y - 2) + 3(z - 1) = 0$$
$$20x - 18y + 3z - 27 = 0$$

55. Let $\mathbf{u} = \mathbf{i}$ and let $\mathbf{v}$ be the vector from $(1, -2, -1)$ to $(2, 5, 6)$: $\mathbf{v} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$

Because $\mathbf{u}$ and $\mathbf{v}$ both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 1 & 7 & 7 \end{vmatrix} = -7\mathbf{j} + 7\mathbf{k} = -7(\mathbf{j} - \mathbf{k})$$

$$[y - (-2)] - [z - (-1)] = 0$$
$$y - z + 1 = 0$$

56. Let $\mathbf{u} = \mathbf{k}$ and let $\mathbf{v}$ be the vector from $(4, 2, 1)$ to $(-3, 5, 7)$: $\mathbf{v} = -7\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

Because **u** and **v** both lie in the plane P, the normal vector to P is:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ -7 & 3 & 6 \end{vmatrix} = -3\mathbf{i} - 7\mathbf{j} = -(3\mathbf{i} + 7\mathbf{j})$$

$$3(x - 4) + 7(y - 2) = 0$$
$$3x + 7y - 26 = 0$$

57. Let (x, y, z) be equidistant from $(2, 2, 0)$ and $(0, 2, 2)$.

$$\sqrt{(x-2)^2 + (y-2)^2 + (z-0)^2} = \sqrt{(x-0)^2 + (y-2)^2 + (z-2)^2}$$
$$x^2 - 4x + 4 + y^2 - 4y + 4 + z^2 = x^2 + y^2 - 4y + 4 + z^2 - 4z + 4$$
$$-4x + 8 = -4z + 8$$
$$x - z = 0 \text{ Plane}$$

58. Let (x, y, z) be equidistant from $(1, 0, 2)$ and $(2, 0, 1)$.

$$\sqrt{(x-1)^2 + (y-0)^2 + (z-2)^2} = \sqrt{(x-2)^2 + (y-0)^2 + (z-1)^2}$$
$$x^2 - 2x + 1 + y^2 + z^2 - 4z + 4 = x^2 - 4x + 4 + y^2 + z^2 - 2z + 1$$
$$-2x - 4z + 5 = -4x - 2z + 5$$
$$2x - 2z = 0$$
$$x - z = 0 \text{ Plane}$$

59. Let (x, y, z) be equidistant from $(-3, 1, 2)$ and $(6, -2, 4)$.

$$\sqrt{(x+3)^2 + (y-1)^2 + (z-2)^2} = \sqrt{(x-6)^2 + (y+2)^2 + (z-4)^2}$$
$$x^2 + 6x + 9 + y^2 - 2y + 1 + z^2 - 4z + 4 = x^2 - 12x + 36 + y^2 + 4y + 4 + z^2 - 8z + 16$$
$$6x - 2y - 4z + 14 = -12x + 4y - 8z + 56$$
$$18x - 6y + 4z - 42 = 0$$
$$9x - 3y + 2z - 21 = 0 \text{ Plane}$$

60. Let (x, y, z) be equidistant from $(-5, 1, -3)$ and $(2, -1, 6)$

$$\sqrt{(x+5)^2 + (y-1)^2 + (z+3)^2} = \sqrt{(x-2)^2 + (y+1)^2 + (z-6)^2}$$
$$x^2 + 10x + 25 + y^2 - 2y + 1 + z^2 + 6z + 9 = x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 12z + 36$$
$$10x - 2y + 6z + 35 = -4x + 2y - 12z + 41$$
$$14x - 4y + 18z - 6 = 0$$
$$7x - 2y + 9z - 3 = 0 \text{ Plane}$$

61. First plane: $\mathbf{n}_1 = \langle -5, 2, -8\rangle$ and $P = (0, 3, 0)$ on plane

Second plane: $\mathbf{n}_2 = \langle 15, -6, 24\rangle = -3\mathbf{n}_1$ and P not on plane

Parallel planes

(Note: The equations are not equivalent.)

62. First plane: $\mathbf{n}_1 = \langle 2, -1, 3\rangle$ and $P = (4, 0, 0)$ on plane

Second plane: $\mathbf{n}_2 = \langle 8, -4, 12\rangle = 4\mathbf{n}_1$ and P not on plane.

Parallel planes

(Note: The equations are not equivalent.)

63. First plane: $\mathbf{n}_1 = \langle 3, -2, 5\rangle$ and $P = (0, 0, 2)$ on plane

Second plane: $\mathbf{n}_2 = \langle 75, -50, 125\rangle = 25\mathbf{n}_1$ and P on plane

Planes are identical.

(Note: The equations are equivalent.)

64. First plane: $\mathbf{n}_1 = \langle -1, 4, -1\rangle$ and $P = (-6, 0, 0)$ on plane

Second plane: $\mathbf{n}_2 = \left\langle -\frac{5}{2}, 10, -\frac{5}{2}\right\rangle = \frac{5}{2}\mathbf{n}_1$ and P on plane

Planes are identical.

(Note: The equations are equivalent.)

65. (a) $\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-7|}{\sqrt{14}\sqrt{21}} = \frac{\sqrt{6}}{6}$$

$\Rightarrow \theta \approx 65.91°$

(b) The direction vector for the line is

$$\mathbf{n}_2 \times \mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 7(\mathbf{j} + 2\mathbf{k}).$$

Find a point of intersection of the planes.

$$\begin{aligned} 6x + 4y - 2z &= 14 \\ x - 4y + 2z &= 0 \\ \hline 7x \qquad\qquad &= 14 \\ x &= 2 \end{aligned}$$

Substituting 2 for x in the second equation, you have $-4y + 2z = -2$ or $z = 2y - 1$. Letting $y = 1$, a point of intersection is $(2, 1, 1)$.

$x = 2, y = 1 + t, z = 1 + 2t$

66. (a) $\mathbf{n}_1 = \langle -2, 1, 1\rangle$ and $\mathbf{n}_2 = \langle 6, -3, 2\rangle$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-13|}{\sqrt{6}\sqrt{49}} = \frac{13\sqrt{6}}{42}$$

$\Rightarrow \theta \approx 40.70°$

(b) The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 6 & -3 & 2 \end{vmatrix} = 5(\mathbf{i} + 2\mathbf{j}).$$

Find a point of intersection of the planes.

$$\begin{aligned} -6x + 3y + 3z &= 6 \\ 6x - 3y + 2z &= 4 \\ \hline 5z &= 10 \\ z &= 2 \end{aligned}$$

Substituting 2 for z in the first equation, you have $-2x + y = 0$ or $y = 2x$. Letting $x = 0$, a point of intersection is $(0, 0, 2)$.

$x = 5t, y = 10t, z = 2$ or $x = t, y = 2t, z = 2$

67. (a) $\mathbf{n}_1 = \langle 3, -1, 1\rangle$ and $\mathbf{n}_2 = \langle 4, 6, 3\rangle$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|9|}{\sqrt{11}\sqrt{61}} = \frac{9\sqrt{671}}{671}$$

$\Rightarrow \theta \approx 69.67°$

(b) The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ 4 & 6 & 3 \end{vmatrix} = -9\mathbf{i} - 5\mathbf{j} + 22\mathbf{k}.$$

Find a point of intersection of the planes.

$$\begin{aligned} 18x - 6y + 6z &= 42 \\ 4x + 6y + 3z &= 2 \\ \hline 22x \qquad + 9z &= 44 \end{aligned}$$

Let $z = 0$, $22x = 44 \Rightarrow x = 2$ and $3(2) - y + 0 = 7 \Rightarrow y = -1$.

A point of intersection is $(2, -1, 0)$.

$x = 2 - 9t, y = -1 - 5t, z = 22t$

68. (a) $\mathbf{n}_1 = 6\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{n}_2 = -\mathbf{i} + \mathbf{j} + 5\mathbf{k}$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|-4|}{\sqrt{46}\sqrt{27}} = \frac{2\sqrt{138}}{207}$$

$\theta \approx 1.6845 \approx 96.52°$

(b) The direction vector for the line is

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -3 & 1 \\ -1 & 1 & 5 \end{vmatrix} = \langle -16, -31, 3\rangle.$$

Find a point of intersection of the planes.

$$\begin{aligned} 6x - 3y + z = 5 &\Rightarrow \quad 6x - 3y + z = 5 \\ -x + y + 5z = 5 &\Rightarrow \underline{-6x + 6y + 30z = 30} \\ &\qquad\qquad 3y + 31z = 35 \end{aligned}$$

Let $y = -9, z = 2 \Rightarrow x = -4 \Rightarrow (-4, -9, 2)$.

$x = -4 - 16t, y = -9 - 31t, z = 2 + 3t$

69. The normal vectors to the planes are

$$\mathbf{n}_1 = \langle 5, -3, 1\rangle,\ \mathbf{n}_2 = \langle 1, 4, 7\rangle,\ \cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = 0.$$

So, $\theta = \pi/2$ and the planes are orthogonal.

70. The normal vectors to the planes are

$\mathbf{n}_1 = \langle 3, 1, -4\rangle,\ \mathbf{n}_2 = \langle -9, -3, 12\rangle.$

Because $\mathbf{n}_2 = -3\mathbf{n}_1$, the planes are parallel, but not equal.

71. The normal vectors to the planes are

$\mathbf{n}_1 = \mathbf{i} - 3\mathbf{j} + 6\mathbf{k},\ \mathbf{n}_2 = 5\mathbf{i} + \mathbf{j} - \mathbf{k},$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|5 - 3 - 6|}{\sqrt{46}\sqrt{27}} = \frac{4\sqrt{138}}{414} = \frac{2\sqrt{138}}{207}.$$

So, $\theta = \arccos\left(\dfrac{2\sqrt{138}}{207}\right) \approx 83.5°.$

72. The normal vectors to the planes are

$\mathbf{n}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k},\ \mathbf{n}_2 = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k},$

$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = \frac{|3 - 8 - 2|}{\sqrt{14}\sqrt{21}} = \frac{7\sqrt{6}}{42} = \frac{\sqrt{6}}{6}.$$

So, $\theta = \arccos\left(\dfrac{\sqrt{6}}{6}\right) \approx 65.9°.$

73. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -5, -1\rangle$ and $\mathbf{n}_2 = \langle 5, -25, -5\rangle$. Because $\mathbf{n}_2 = 5\mathbf{n}_1$, the planes are parallel, but not equal.

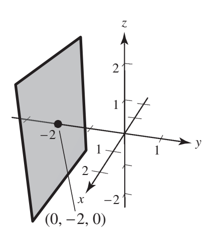
74. The normal vectors to the planes are

$\mathbf{n}_1 = \langle 2, 0, -1\rangle, \mathbf{n}_2 = \langle 4, 1, 8\rangle,$

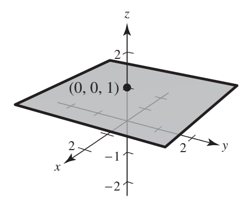
$$\cos\theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} = 0$$

So, $\theta = \dfrac{\pi}{2}$ and the planes are orthogonal.

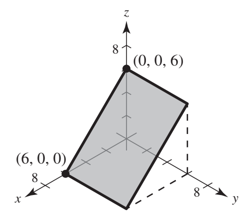
75. $y \le -2$



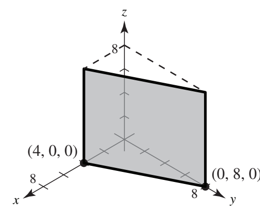
76. $z = 1$



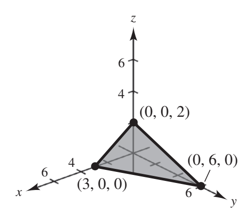
77. $x + z = 6$



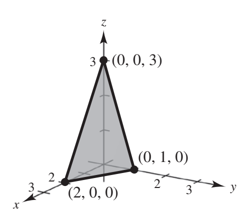
78. $2x + y = 8$



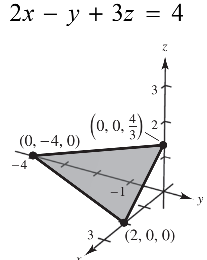
79. $4x + 2y + 6z = 12$



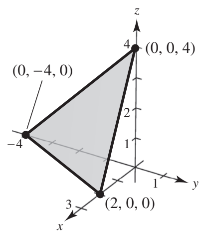
80. $3x + 6y + 2z = 6$



81. $2x - y + 3z = 4$



82. $2x - y + z = 4$



83. Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$x = -7 + 2t,\ y = 4 + t,\ z = -1 + 5t$

$$(-7 + 2t) + 3(4 + t) - (-1 + 5t) = 6$$
$$6 = 6$$

The equation is valid for all t.

The line lies in the plane.

84. Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$x = 1 + 4t,\ y = 2t,\ z = 3 + 6t$

$2(1 + 4t) + 3(2t) = -5,\ t = \dfrac{-1}{2}$

Substituting $t = -\frac{1}{2}$ into the parametric equations for the line you have the point of intersection $(-1, -1, 0)$.

The line does not lie in the plane.

85. Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$x = 1 + 3t,\ y = -1 - 2t,\ z = 3 + t$

$2(1 + 3t) + 3(-1 - 2t) = 10, -1 = 10$, contradiction

So, the line does not intersect the plane.

86. Writing the equation of the line in parametric form and substituting into the equation of the plane you have:

$x = 4 + 2t,\ y = -1 - 3t,\ z = -2 + 5t$

$5(4 + 2t) + 3(-1 - 3t) = 17,\ t = 0$

Substituting $t = 0$ into the parametric equations for the line you have the point of intersection $(4, -1, -2)$.

The line does not lie in the plane.

87. Point: $Q(0, 0, 0)$

Plane: $2x + 3y + z - 12 = 0$

Normal to plane: $\mathbf{n} = \langle 2, 3, 1\rangle$

Point in plane: $P(6, 0, 0)$

Vector $\overrightarrow{PQ} = \langle -6, 0, 0\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|-12|}{\sqrt{14}} = \frac{6\sqrt{14}}{7}$$

88. Point: $Q(0, 0, 0)$

Plane: $5x + y - z - 9 = 0$

Normal to plane: $\mathbf{n} = \langle 5, 1, -1\rangle$

Point in plane: $P(0, 9, 0)$

Vector $\overrightarrow{PQ} = \langle 0, -9, 0\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|-9|}{\sqrt{27}} = \sqrt{3}$$

89. Point: $Q(2, 8, 4)$

Plane: $2x + y + z = 5$

Normal to plane: $\mathbf{n} = \langle 2, 1, 1\rangle$

Point in plane: $P\langle 0, 0, 5\rangle$

Vector: $\overrightarrow{PQ} = \langle 2, 8, -1\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{11}{\sqrt{6}} = \frac{11\sqrt{6}}{6}$$

90. Point: $Q(1, 3, -1)$

Plane: $3x - 4y + 5z - 6 = 0$

Normal to plane: $\mathbf{n} = \langle 3, -4, 5\rangle$

Point in plane: $P(2, 0, 0)$

Vector $\overrightarrow{PQ}$: $\langle -1, 3, -1\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|-20|}{\sqrt{50}} = 2\sqrt{2}$$

91. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, -3, 4\rangle$ and $\mathbf{n}_2 = \langle 1, -3, 4\rangle$. Because $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(10, 0,0)$ is a point in $x - 3y + 4z = 10$.

$Q(6, 0, 0)$ is a point in $x - 3y + 4z = 6$.

$$\overrightarrow{PQ} = \langle -4, 0, 0\rangle,\ D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}_1\right|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{26}} = \frac{2\sqrt{26}}{13}$$

92. The normal vectors to the planes are $\mathbf{n}_1 = \langle 2, 7, 1\rangle$ and $\mathbf{n}_2 = \langle 2, 7, 1\rangle$. Because $\mathbf{n}_1 = \mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(0, 0, 13)$ is a point in $2x + 7y + z = 13$.

$Q(0, 0, 9)$ is a point in $2x + 7y + z = 9$.

$\overrightarrow{PQ} = \langle 0, 0, 4\rangle$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}_1\right|}{\|\mathbf{n}_1\|} = \frac{4}{\sqrt{54}} = \frac{2\sqrt{6}}{9}$$

93. The normal vectors to the planes are $\mathbf{n}_1 = \langle -3, 6, 7\rangle$ and $\mathbf{n}_2 = \langle 6, -12, -14\rangle$. Because $\mathbf{n}_2 = -2\mathbf{n}_1$, the planes are parallel. Choose a point in each plane.

$P(0, -1, 1)$ is a point in $-3x + 6y + 7z = 1$.

$Q\left(\frac{25}{6}, 0, 0\right)$ is a point in $6x - 12y - 14z = 25$.

$$\overrightarrow{PQ} = \left\langle \frac{25}{6}, 1, -1\right\rangle$$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}_1\right|}{\|\mathbf{n}_1\|} = \frac{|-27/2|}{\sqrt{94}} = \frac{27}{2\sqrt{94}} = \frac{27\sqrt{94}}{188}$$

94. The normal vectors to the planes are $\mathbf{n}_1 = \langle -1, 6, 2\rangle$ and $\mathbf{n}_2 = \left\langle -\frac{1}{2}, 3, 1\right\rangle$. Because $\mathbf{n}_1 = 2\mathbf{n}_2$, the planes are parallel. Choose a point in each plane.

$P(-3, 0, 0)$ is a point in $-x + 6y + 2z = 3$.

$Q(0, 0, 4)$ is a point in $-\frac{1}{2}x + 3y + z = 4$.

$$\overrightarrow{PQ} = \langle 3, 0, 4\rangle$$

$$D = \frac{\left|\overrightarrow{PQ}\cdot\mathbf{n}_1\right|}{\|\mathbf{n}_1\|} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$$

95. $\mathbf{u} = \langle 4, 0, -1\rangle$ is the direction vector for the line.

$Q(1, 5, -2)$ is the given point, and $P(-2, 3, 1)$ is on the line.

$$\overrightarrow{PQ} = \langle 3, 2, -3\rangle$$

$$\overrightarrow{PQ}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -3 \\ 4 & 0 & -1 \end{vmatrix} = \langle -2, -9, -8\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\|\mathbf{u}\|} = \frac{\sqrt{149}}{\sqrt{17}} = \frac{\sqrt{2533}}{17}$$

96. $\mathbf{u} = \langle 2, 1, 2\rangle$ is the direction vector for the line.

$Q(1, -2, 4)$ is the given point, and $P(0, -3, 2)$ is a point on the line (let $t = 0$).

$$\overrightarrow{PQ} = \langle 1, 1, 2\rangle$$

$$\overrightarrow{PQ}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix} = \langle 0, 2, -1\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\|\mathbf{u}\|} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$$

97. $\mathbf{u} = \langle -1, 1, -2\rangle$ is the direction vector for the line.

$Q(-2, 1, 3)$ is the given point, and $P(1, 2, 0)$ is on the line (let $t = 0$ in the parametric equations for the line).

$$\overrightarrow{PQ} = \langle -3, -1, 3\rangle$$

$$\overrightarrow{PQ}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -1 & 3 \\ -1 & 1 & -2 \end{vmatrix} = \langle -1, -9, -4\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\|\mathbf{u}\|} = \frac{\sqrt{1+81+16}}{\sqrt{1+1+4}} = \frac{\sqrt{98}}{\sqrt{6}} = \frac{7}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

98. $\mathbf{u} = \langle 0, 3, 1\rangle$ is the direction vector for the line.

$Q(4, -1, 5)$ is the given point, and $P(3, 1, 1)$ is on the line.

$$\overrightarrow{PQ} = \langle 1, -2, 4\rangle$$

$$\overrightarrow{PQ}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 4 \\ 0 & 3 & 1 \end{vmatrix} = \langle -14, -1, 3\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ}\times\mathbf{u}\right\|}{\|\mathbf{u}\|}$$

$$= \frac{\sqrt{14^2+1+9}}{\sqrt{9+1}} = \sqrt{\frac{206}{10}} = \sqrt{\frac{103}{5}} = \frac{\sqrt{515}}{5}$$

99. The direction vector for L_1 is $\mathbf{v}_1 = \langle -1, 2, 1\rangle$.

The direction vector for L_2 is $\mathbf{v}_2 = \langle 3, -6, -3\rangle$.

Because $\mathbf{v}_2 = -3\mathbf{v}_1$, the lines are parallel.

Let $Q(2, 3, 4)$ to be a point on L_1 and $P(0, 1, 4)$ a point on L_2. $\overrightarrow{PQ} = \langle 2, 2, 0\rangle$.

$\mathbf{u} = \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ}\times\mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ 3 & -6 & -3 \end{vmatrix} = \langle -6, 6, -18\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ}\times\mathbf{v}_2\right\|}{\|\mathbf{v}_2\|}$$

$$= \frac{\sqrt{36+36+324}}{\sqrt{9+36+9}} = \sqrt{\frac{396}{54}} = \sqrt{\frac{22}{3}} = \frac{\sqrt{66}}{3}$$

100. The direction vector for L_1 is $\mathbf{v}_1 = \langle 6, 9, -12\rangle$.

The direction vector for L_2 is $\mathbf{v}_2 = \langle 4, 6, -8\rangle$.

Because $\mathbf{v}_1 = \frac{3}{2}\mathbf{v}_2$, the lines are parallel.

Let $Q(3, -2, 1)$ to be a point on L_1 and $P(-1, 3, 0)$ a point on L_2. $\overrightarrow{PQ} = \langle 4, -5, 1\rangle$.

$\mathbf{u} = \mathbf{v}_2$ is the direction vector for L_2.

$$\overrightarrow{PQ} \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 1 \\ 4 & 6 & -8 \end{vmatrix} = \langle 34, 36, 44\rangle$$

$$D = \frac{\left\|\overrightarrow{PQ} \times \mathbf{v}_2\right\|}{\left\|\mathbf{v}_2\right\|} = \frac{\sqrt{34^2 + 36^2 + 44^2}}{\sqrt{16 + 36 + 64}} = \frac{\sqrt{4388}}{\sqrt{116}} = \sqrt{\frac{1097}{29}} = \frac{\sqrt{31813}}{29}$$

101. Exactly one plane contains the point and line. Select two points on the line and observe that three noncolinear points determine a unique plane.

102. There are an infinite number of planes orthogonal to a given plane in space.

103. Yes, Consider two points on one line, and a third distinct point on another line. Three distinct points determine a unique plane.

104. (a) $ax + by + d = 0$ matches (iv). The plane is parallel to the z-axis.

(b) $ax + d = 0$ matches (i). The plane is parallel to the yz-plane.

(c) $cz + d = 0$ matches (ii). The plane is parallel to the xy-plane.

(d) $ax + cz + d = 0$ matches (iii). The plane is parallel to the y-axis.

105. $z = 0.23x + 0.14y + 6.85$

(a)

Year	2009	2010	2011	2012	2013	2014
z(Approx)	18.93	19.46	20.31	21.10	21.58	22.62

The approximations are close to the actual values.

(b) If x and y both increase, then so does z.

106. On one side you have the points $(0, 0, 0), (6, 0, 0)$, and $(-1, -1, 8)$.

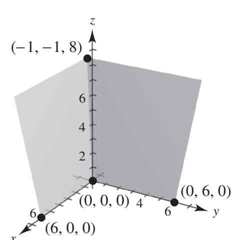
$$\mathbf{n}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -1 & -1 & 8 \end{vmatrix} = -48\mathbf{j} - 6\mathbf{k}$$

On the adjacent side you have the points $(0, 0, 0), (0, 6, 0)$, and $(-1, -1, 8)$.

$$\mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ -1 & -1 & 8 \end{vmatrix} = 48\mathbf{i} + 6\mathbf{k}$$

$$\cos\theta = \frac{\left|\mathbf{n}_1 \cdot \mathbf{n}_2\right|}{\left\|\mathbf{n}_1\right\|\left\|\mathbf{n}_2\right\|} = \frac{36}{2340} = \frac{1}{65}$$

$$\theta = \arccos\frac{1}{65} \approx 89.1°$$



107. L_1: $x_1 = 6 + t$; $y_1 = 8 - t$, $z_1 = 3 + t$

L_2: $x_2 = 1 + t$, $y_2 = 2 + t$, $z_2 = 2t$

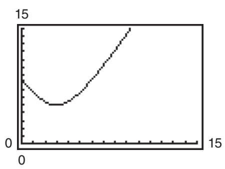
(a) At $t = 0$, the first insect is at $P_1(6, 8, 3)$ and the second insect is at $P_2(1, 2, 0)$.

$$\text{Distance} = \sqrt{(6-1)^2 + (8-2)^2 + (3-0)^2} = \sqrt{70} \approx 8.37 \text{ inches}$$

(b) $\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = \sqrt{5^2 + (6-2t)^2 + (3-t)^2} = \sqrt{5t^2 - 30t + 70},\ 0 \le t \le 10$

(c) The distance is never zero.

(d) Using a graphing utility, the minimum distance is 5 inches when $t = 3$ minutes.



108. First find the distance D from the point $Q(-3, 2, 4)$ to the plane. Let $P(4, 0, 0)$ be on the plane.

$\mathbf{n} = \langle 2, 4, -3\rangle$ is the normal to the plane.

$$D = \frac{|\overline{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|\langle -7, 2, 4\rangle \cdot \langle 2, 4, -3\rangle|}{\sqrt{4+16+9}} = \frac{|-14 + 8 - 12|}{\sqrt{29}} = \frac{18}{\sqrt{29}} = \frac{18\sqrt{29}}{29}$$

The equation of the sphere with center $(-3, 2, 4)$ and radius $18\sqrt{29}/29$ is $(x+3)^2 + (y-2)^2 + (z-4)^2 = \dfrac{324}{29}$.

109. The direction vector $\mathbf{v}$ of the line is the normal to the plane, $\mathbf{v} = \langle 3, -1, 4\rangle$.

The parametric equations of the line are $x = 5 + 3t$, $y = 4 - t$, $z = -3 + 4t$.

To find the point of intersection, solve for t in the following equation:

$$\begin{aligned} 3(5+3t) - (4-t) + 4(-3+4t) &= 7 \\ 26t &= 8 \\ t &= \tfrac{4}{13} \end{aligned}$$

Point of intersection:

$$\left(5 + 3\left(\tfrac{4}{13}\right), 4 - \tfrac{4}{13}, -3 + 4\left(\tfrac{4}{13}\right)\right) = \left(\tfrac{77}{13}, \tfrac{48}{13}, -\tfrac{23}{13}\right)$$

110. The normal to the plane, $\mathbf{n} = \langle 2, -1, -3\rangle$ is perpendicular to the direction vector $\mathbf{v} = (2, 4, 0)$ of the line because $\langle 2, -1, -3\rangle \cdot \langle 2, 4, 0\rangle = 0$.

So, the plane is parallel to the line. To find the distance between them, let $Q(-2, -1, 4)$ be on the line and $P(2, 0, 0)$ on the plane. $\overline{PQ} = \langle -4, -1, 4\rangle$.

$$\begin{aligned} D &= \frac{|\overline{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} \\ &= \frac{|\langle -4, -1, 4\rangle \cdot \langle 2, -1, -3\rangle|}{\sqrt{4+1+9}} = \frac{19}{\sqrt{14}} = \frac{19\sqrt{14}}{14} \end{aligned}$$

111. $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 1 \\ -3 & 1 & 4 \end{vmatrix} = -21\mathbf{i} - 11\mathbf{j} - 13\mathbf{k}$$

Direction numbers: 21, 11, 13

$x = 21t$, $y = 1 + 11t$, $z = 4 + 13t$

112. The unknown line L is perpendicular to the normal vector $\mathbf{n} = \langle 1, 1, 1\rangle$ of the plane, and perpendicular to the direction vector $\mathbf{u} = \langle 1, 1, -1\rangle$. So, the direction vector of L is

$$\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \langle -2, 2, 0\rangle.$$

The parametric equations for L are $x = 1 - 2t$, $y = 2t$, $z = 2$.

113. True

114. False. They may be skew lines. (See Section Project.)

115. True

116. False. For example, the lines $x = t$, $y = 0$, $z = 1$ and $x = 0$, $y = t$, $z = 1$ are both parallel to the plane $z = 0$, but the lines are not parallel.

117. False. For example, planes $7x + y - 11z = 5$ and $5x + 2y - 4z = 1$ are both perpendicular to plane $2x - 3y + z = 3$, but are not parallel.

118. True

Section 11.6 Surfaces in Space

1. Quadric surfaces are the three-dimensional analogs of conic sections.

2. In the xz-plane, $z = x^2$ is a parabola.

In three-space, $z = x^2$ is a cylinder.

3. The trace of a surface is the intersection of the surface with a plane. You find a trace by setting one variable equal to a constant, such as $x = 0$ or $z = 2$.

4. No. For example, $x^2 + y^2 + z^2 = 0$ is a single point and $x^2 + y^2 = 1$ is a right circular cylinder.

5. Ellipsoid

Matches graph (c)

6. Hyperboloid of two sheets

Matches graph (e)

7. Hyperboloid of one sheet

Matches graph (f)

8. Elliptic cone

Matches graph (b)

9. Elliptic paraboloid

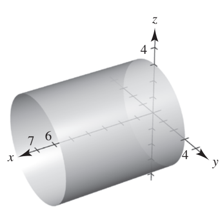
Matches graph (d)

10. Hyperbolic paraboloid

Matches graph (a)

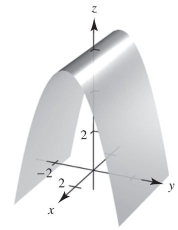
11. $y^2 + z^2 = 9$

The x-coordinate is missing so you have a right circular cylinder with rulings parallel to the x-axis. The generating curve is a circle.



12. $y^2 + z = 6$

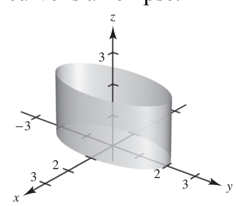
The x-coordinate is missing so you have a parabolic cylinder with the rulings parallel to the x-axis. The generating curve is a parabola.



13. $4x^2 + y^2 = 4$

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$

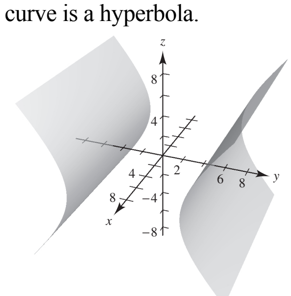
The z-coordinate is missing so you have an elliptic cylinder with rulings parallel to the z-axis. The generating curve is an ellipse.



14. $y^2 - z^2 = 25$

$$\frac{y^2}{25} - \frac{z^2}{25} = 1$$

The x-coordinate is missing so you have a hyperbolic cylinder with rulings parallel to the x-axis. The generating curve is a hyperbola.



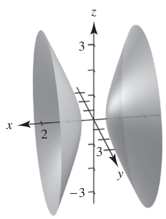
15. $4x^2 - y^2 - z^2 = 1$

Hyperboloid of two sheets

xy-trace: $4x^2 - y^2 = 1$ hyperbola

yz-trace: none

xz-trace: $4x^2 - z^2 = 1$ hyperbola



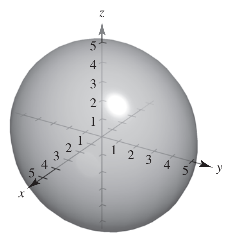
16. $\dfrac{x^2}{16} + \dfrac{y^2}{25} + \dfrac{z^2}{25} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ ellipse

xz-trace: $\dfrac{x^2}{16} + \dfrac{z^2}{25} = 1$ ellipse

yz-trace: $y^2 + z^2 = 25$ circle



17. $16x^2 - y^2 + 16z^2 = 4$

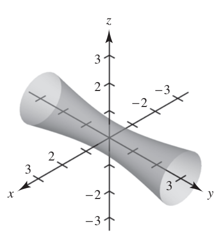
$4x^2 - \dfrac{y^2}{4} + 4z^2 = 1$

Hyperboloid of one sheet

xy-trace: $4x^2 - \dfrac{y^2}{4} = 1$ hyperbola

xz-trace: $4\left(x^2 + z^2\right) = 1$ circle

yz-trace: $\dfrac{-y^2}{4} + 4z^2 = 1$ hyperbola



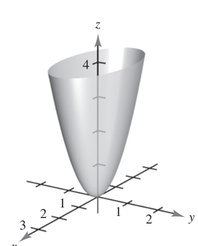
18. $z = x^2 + 4y^2$

Elliptic paraboloid

xy-trace: point $(0, 0, 0)$

xz-trace: $z = x^2$ parabola

yz-trace: $z = 4y^2$ parabola



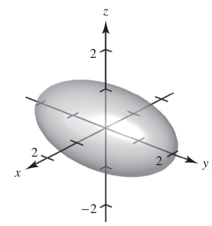
19. $\dfrac{x^2}{1} + \dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$

Ellipsoid

xy-trace: $\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$ ellipse

xz-trace: $x^2 + z^2 = 1$ circle

yz-trace: $\dfrac{y^2}{4} + \dfrac{z^2}{1} = 1$ ellipse



20. $z^2 - x^2 - \dfrac{y^2}{4} = 1$

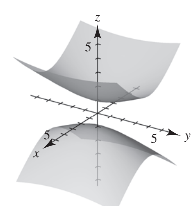
Hyperboloid of two sheets

xy-trace: none

xz-trace: $z^2 - x^2 = 1$ hyperbola

yz-trace: $z^2 - \dfrac{y^2}{4} = 1$ hyperbola

$z = \pm\sqrt{10}$: $\dfrac{x^2}{9} + \dfrac{y^2}{36} = 1$ ellipse



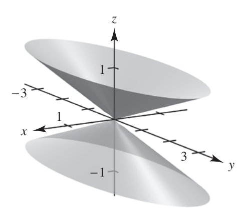
21. $z^2 = x^2 + \dfrac{y^2}{9}$

Elliptic cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm x$

yz-trace: $z = \pm\dfrac{y}{3}$



When $z = \pm 1$, $x^2 + \dfrac{y^2}{9} = 1$ ellipse

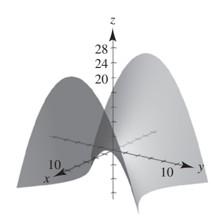
22. $3z = -y^2 + x^2$

Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = \frac{1}{3}x^2$

yz-trace: $z = -\frac{1}{3}y^2$



23. $x^2 - y^2 + z = 0$

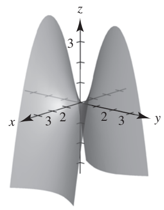
Hyperbolic paraboloid

xy-trace: $y = \pm x$

xz-trace: $z = -x^2$

yz-trace: $z = y^2$

$y = \pm 1$: $z = 1 - x^2$



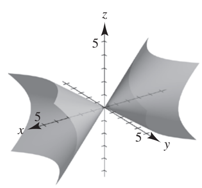
24. $x^2 = 2y^2 + 2z^2$

Elliptic Cone

xy-trace: $x = \pm\sqrt{2}y$

xz-trace: $x = \pm\sqrt{2}z$

yz-trace: point: $(0, 0, 0)$



25. $x^2 - y + z^2 = 0$

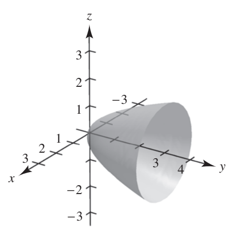
Elliptic paraboloid

xy-trace: $y = x^2$

xz-trace: $x^2 + z^2 = 0$, point $(0, 0, 0)$

yz-trace: $y = z^2$

$y = 1$: $x^2 + z^2 = 1$



26. $-8x^2 + 18y^2 + 18z^2 = 2$

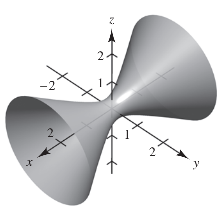
$9y^2 + 9z^2 - 4x^2 = 1$

Hyperboloid of one sheet

xy-trace: $9y^2 - 4x^2 = 1$ hyperbola

yz-trace: $9y^2 + 9z^2 = 1$ circle

xz-trace: $9z^2 - 4x^2 = 1$ hyperbola



27. These have to be two minus signs in order to have a hyperboloid of two sheets. The number of sheets is the same as the number of minus signs.

28. Yes. Every trace is an ellipse (or circle or point).

29. No. See the table on pages 800 and 801.

30. $z = x^2 + y^2$

(a) You are viewing the paraboloid from the x-axis: $(20, 0, 0)$

(b) You are viewing the paraboloid from above, but not on the z-axis: $(10, 10, 20)$

(c) You are viewing the paraboloid from the z-axis: $(0, 0, 20)$

(d) You are viewing the paraboloid from the y-axis: $(0, 20, 0)$

31. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) = 5y$, so

$x^2 + z^2 = 25y^2$.

32. $x^2 + z^2 = [r(y)]^2$ and $z = r(y) \pm 3\sqrt{y}$, so

$x^2 + z^2 = 9y$.

33. $x^2 + y^2 = [r(z)]^2$ and $y = r(z) = 2z^{1/3}$, so

$x^2 + y^2 = 4z^{2/3}$.

34. $x^2 + y^2 = [r(z)]^2$ and $x = r(z) = e^z$, so

$x^2 + y^2 = e^{2z}$.

35. $y^2 + z^2 = [r(x)]^2$ and $y = r(x) = \dfrac{2}{x}$, so

$y^2 + z^2 = \left(\dfrac{2}{x}\right)^2 \Rightarrow y^2 + z^2 = \dfrac{4}{x^2}$.

36. $y^2 + z^2 = [r(x)]^2$ and $z = r(x) = \frac{1}{2}\sqrt{4 - x^2}$, so

$y^2 + z^2 = \frac{1}{4}(4 - x^2) \Rightarrow x^2 + 4y^2 + 4z^2 = 4$.

37. $x^2 + y^2 - 2z = 0$

$x^2 + y^2 = \left(\sqrt{2z}\right)^2$

Equation of generating curve: $y = \sqrt{2z}$ or $x = \sqrt{2z}$

38. $x^2 + z^2 = \cos^2 y$

Equation of generating curve: $x = \cos y$ or $z = \cos y$

39. $y^2 + z^2 = 5 - 8x^2 = \left(\sqrt{5 - 8x^2}\right)^2$

Equation of generating curve: $y = \sqrt{5 - 8x^2}$ or $z = \sqrt{5 - 8x^2}$

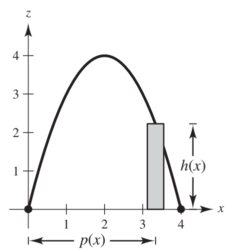
40. $6x^2 + 2y^2 + 2z^2 = 1$

$$y^2 + z^2 = \frac{1}{2} - 3x^2 = \left(\sqrt{\frac{1}{2} - 3x^2}\right)^2$$

Equation of generating curve: $y = \sqrt{\frac{1}{2} - 3x^2}$ or

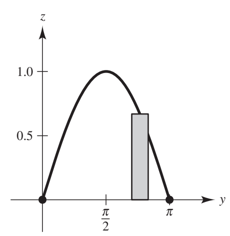
$z = \sqrt{\frac{1}{2} - 3x^2}$

41. $V = 2\pi\int_0^4 x(4x - x^2)\,dx = 2\pi\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 = \frac{218\pi}{3}$



42. $V = 2\pi\int_0^{\pi} y \sin y\,dy$

$= 2\pi[\sin y - y\cos y]_0^{\pi} = 2\pi^2$



43. $z = \frac{x^2}{2} + \frac{y^2}{4}$

(a) When $z = 2$ we have $2 = \frac{x^2}{2} + \frac{y^2}{4}$, or

$1 = \frac{x^2}{4} + \frac{y^2}{8}$

Major axis: $2\sqrt{8} = 4\sqrt{2}$

Minor axis: $2\sqrt{4} = 4$

$c^2 = a^2 - b^2, c^2 = 4, c = 2$

Foci: $(0, \pm 2, 2)$

(b) When $z = 8$ we have $8 = \frac{x^2}{2} + \frac{y^2}{4}$, or

$1 = \frac{x^2}{16} + \frac{y^2}{32}$.

Major axis: $2\sqrt{32} = 8\sqrt{2}$

Minor axis: $2\sqrt{16} = 8$

$c^2 = 32 - 16 = 16, c = 4$

Foci: $(0, \pm 4, 8)$

44. $z = \frac{x^2}{2} + \frac{y^2}{4}$

(a) When $y = 4$ you have $z = \frac{x^2}{2} + 4$,

$4\left(\frac{1}{2}\right)(z - 4) = x^2$.

Focus: $\left(0, 4, \frac{9}{2}\right)$

(b) When $x = 2$ you have

$z = 2 + \frac{y^2}{4}, 4(z - 2) = y^2$.

Focus: $(2, 0, 3)$

45. If (x, y, z) is on the surface, then

$$(y + 2)^2 = x^2 + (y - 2)^2 + z^2$$
$$y^2 + 4y + 4 = x^2 + y^2 - 4y + 4 + z^2$$
$$x^2 + z^2 = 8y$$

Elliptic paraboloid

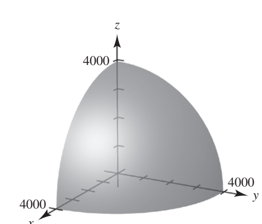
Traces parallel to *xz*-plane are circles.

46. If (x, y, z) is on the surface, then

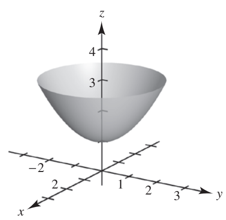
$$z^2 = x^2 + y^2 + (z - 4)^2$$
$$z^2 = x^2 + y^2 + z^2 - 8z + 16$$
$$8z = x^2 + y^2 + 16 \Rightarrow z = \frac{x^2}{8} + \frac{y^2}{8} + 2$$

Elliptic paraboloid shifted up 2 units. Traces parallel to *xy*-plane are circles.

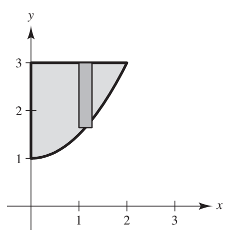
47. $\frac{x^2}{3963^2} + \frac{y^2}{3963^2} + \frac{z^2}{3950^2} = 1$



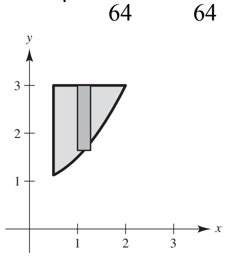
48. (a)
$$\begin{aligned} x^2 + y^2 &= \left[r(z)\right]^2 \\ &= \left[\sqrt{2(z-1)}\right]^2 \end{aligned}$$
$$x^2 + y^2 - 2z + 2 = 0$$



(b)
$$\begin{aligned} V &= 2\pi\int_0^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx \\ &= 2\pi\int_0^2 \left(2x - \frac{1}{2}x^3\right)dx \\ &= 2\pi\left[x^2 - \frac{x^4}{8}\right]_0^2 = 4\pi \approx 12.6 \text{ cm}^3 \end{aligned}$$



(c)
$$\begin{aligned} V &= 2\pi\int_{1/2}^2 x\left[3 - \left(\frac{1}{2}x^2 + 1\right)\right]dx \\ &= 2\pi\int_{1/2}^2 \left(2x - \frac{1}{2}x^3\right)dx \\ &= 2\pi\left[x^2 - \frac{x^4}{8}\right]_{1/2}^2 \\ &= 4 - \frac{31\pi}{64} = \frac{225\pi}{64} \approx 11.04 \text{ cm}^3 \end{aligned}$$



49. $z = \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2}, z = bx + ay$

$$\begin{aligned} bx + ay &= \frac{y^2}{b^2} - \frac{x^2}{a^2} \\ \frac{1}{a^2}\left(x^2 + a^2bx + \frac{a^4b^2}{4}\right) &= \frac{1}{b^2}\left(y^2 - ab^2y + \frac{a^2b^4}{4}\right) \\ \frac{\left(x + \frac{a^2b}{2}\right)^2}{a^2} &= \frac{\left(y - \frac{ab^2}{2}\right)^2}{b^2} \\ y &= \pm\frac{b}{a}\left(x + \frac{a^2b}{2}\right) + \frac{ab^2}{2} \end{aligned}$$

Letting $x = at$, you obtain the two intersecting lines $x = at$, $y = -bt$, $z = 0$ and $x = at$, $y = bt + ab^2$, $z = 2abt + a^2b^2$.

50. Equating twice the first equation with the second equation:

$$\begin{aligned} 2x^2 + 6y^2 - 4z^2 + 4y - 8 &= 2x^2 + 6y^2 - 4z^2 - 3x - 2 \\ 4y - 8 &= -3x - 2 \\ 3x + 4y &= 6, \text{ a plane} \end{aligned}$$

51. The Klein bottle *does not* have both an "inside" and an "outside." It is formed by inserting the small open end through the side of the bottle and making it contiguous with the top of the bottle.

Section 11.7 Cylindrical and Spherical Coordinates

1. The cylindrical coordinate system is an extension of the polar coordinate system. In this system, a point P in space is represented by an ordered triple (r, θ, z). (r, θ) is a polar representation of the projection of P in the xy-plane, and z is the directed distance from (r, θ) to P.

2. The point is 2 units from the origin, in the xz-plane, and makes an angle of $30°$ with the z-axis.

3. $(-7, 0, 5)$, cylindrical

$$\begin{aligned} x &= r\cos\theta = -7\cos 0 = -7 \\ y &= r\sin\theta = -7\sin 0 = 0 \\ z &= 5 \end{aligned}$$

$(-7, 0, 5)$, rectangular

4. $(2, -\pi, -4)$, cylindrical

$x = r\cos\theta = 2\cos(-\pi) = -2$

$y = r\sin\theta = 2\sin(-\pi) = 0$

$z = -4$

$(-2, 0, -4)$, rectangular

5. $\left(3, \frac{\pi}{4}, 1\right)$, cylindrical

$x = r\cos\theta = 3\cos\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta = 3\sin\frac{\pi}{4} = \frac{3\sqrt{2}}{2}$

$z = 1$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, 1\right)$, rectangular

6. $\left(6, -\frac{3\pi}{2}, 2\right)$, cylindrical

$x = r\cos\theta = 6\cos\left(-\frac{3\pi}{2}\right) = 0$

$y = r\sin\theta = 6\sin\left(-\frac{3\pi}{2}\right) = 6$

$z = 2$

$(0, 6, 2)$, rectangular

7. $\left(4, \frac{7\pi}{6}, -3\right)$, cylindrical

$x = r\cos\theta = 4\cos\frac{7\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y = r\sin\theta = 4\sin\frac{7\pi}{6} = 4\left(-\frac{1}{2}\right) = -2$

$z = -3$

$\left(-2\sqrt{3}, -2, -3\right)$, rectangular

8. $\left(-\frac{2}{3}, \frac{4\pi}{3}, 8\right)$, cylindrical

$x = r\cos\theta = -\frac{2}{3}\cos\frac{4\pi}{3} = \left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right) = \frac{1}{3}$

$y = r\sin\theta = -\frac{2}{3}\sin\frac{4\pi}{3} = \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{3}$

$z = 8$

$\left(\frac{1}{3}, \frac{\sqrt{3}}{3}, 8\right)$, rectangular

9. $(0, 5, 1)$, rectangular

$r = \sqrt{(0)^2 + (5)^2} = 5$

$\tan\theta = \frac{5}{2} \Rightarrow \theta = \arctan\frac{5}{0} = \frac{\pi}{2}$

$z = 1$

$\left(5, \frac{\pi}{2}, 1\right)$, cylindrical

10. $\left(6, 2\sqrt{3}, -1\right)$, rectangular

$r = \sqrt{6^2 + \left(2\sqrt{3}\right)^2} = \sqrt{36 + 12} = \sqrt{48} = 4\sqrt{3}$

$\tan\theta = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3} \Rightarrow \theta = \arctan\frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$z = -1$

$\left(4\sqrt{3}, \frac{\pi}{6}, -1\right)$, cylindrical

11. $(2, -2, -4)$, rectangular

$r = \sqrt{2^2 + (-2)^2} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{2} \Rightarrow \theta = \arctan(-1) = -\frac{\pi}{4}$

$z = -4$

$\left(2\sqrt{2}, -\frac{\pi}{4}, -4\right)$, cylindrical

12. $(3, -3, 7)$, rectangular

$r = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \frac{-3}{3} \Rightarrow \theta = \arctan(-1) = -\frac{\pi}{4}$

$z = 7$

$\left(3\sqrt{2}, -\frac{\pi}{4}, 7\right)$, cylindrical

13. $\left(1, \sqrt{3}, 4\right)$, rectangular

$r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$

$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = \arctan\sqrt{3} = \frac{\pi}{3}$

$z = 4$

$\left(2, \frac{\pi}{3}, 4\right)$, cylindrical

14. $\left(2\sqrt{3}, -2, 6\right)$, rectangular

$$r = \sqrt{12 + 4} = 4$$
$$\theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = \frac{5\pi}{6}$$
$$z = 6$$
$\left(4, -\dfrac{\pi}{6}, 6\right)$, cylindrical

15. $z = 4$ is the equation in cylindrical coordinates. (plane)

16. $x = 9$, rectangular equation

$r \cos \theta = 9$

$r = 9 \sec \theta$, cylindrical equation

17. $x^2 + y^2 - 2z^2 = 5$, rectangular equation

$r^2 - 2z^2 = 5$, cylindrical equation

18. $z = x^2 + y^2 - 11$, rectangular equation

$z = r^2 - 11$, cylindrical equation

19. $y = x^2$, rectangular equation

$r \sin \theta = (r \cos \theta)^2$

$\sin \theta = r \cos^2 \theta$

$r = \sec \theta \cdot \tan \theta$, cylindrical equation

20. $x^2 + y^2 = 8x$, rectangular equation

$r^2 = 8r \cos \theta$

$r = 8 \cos \theta$, cylindrical equation

21. $y^2 = 10 - z^2$, rectangular equation

$(r \sin \theta)^2 = 10 - z^2$

$r^2 \sin^2 \theta + z^2 = 10$, cylindrical equation

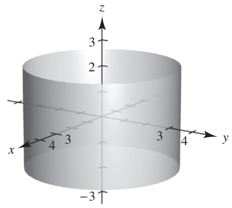
22. $x^2 + y^2 + z^2 - 3z = 0$, rectangular equation

$r^2 + z^2 - 3z = 0$, cylindrical equation

23. $r = 3$, cylindrical equation

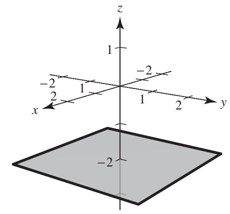
$\sqrt{x^2 + y^2} = 3$

$x^2 + y^2 = 9$, rectangular equation



24. $z = -2$, cylindrical equation

$z = -2$, rectangular equation



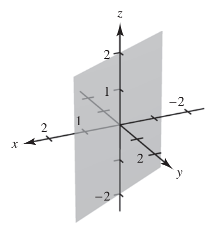
25. $\theta = \dfrac{\pi}{6}$, cylindrical equation

$\tan \dfrac{\pi}{6} = \dfrac{y}{x}$

$\dfrac{1}{\sqrt{3}} = \dfrac{y}{x}$

$x = \sqrt{3}y$

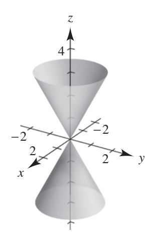
$x - \sqrt{3}y = 0$, rectangular equation



26. $r = \dfrac{z}{2}$, cylindrical equation

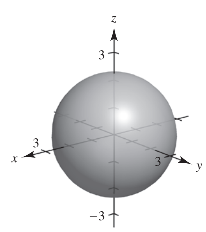
$\sqrt{x^2 + y^2} = \dfrac{z}{2}$

$x^2 + y^2 - \dfrac{z^2}{4} = 0$, rectangular equation



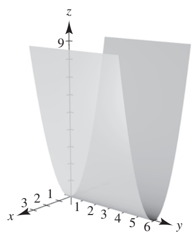
27. $r^2 + z^2 = 5$, cylindrical equation

$x^2 + y^2 + z^2 = 5$, rectangular equation

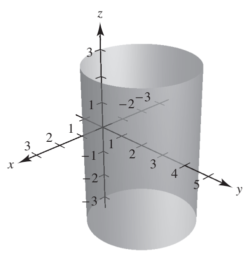


28. $z = r^2 \cos^2 \theta$, cylindrical equation

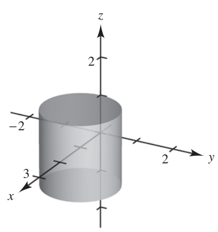
$z = x^2$, rectangular equation



29.
$$\begin{aligned} r &= 4 \sin \theta, \text{ cylindrical equation} \\ r^2 &= 4r \sin \theta \\ x^2 + y^2 &= 4y \\ x^2 + y^2 - 4y + 4 &= 4 \\ x^2 + (y - 2)^2 &= 4, \text{ rectangular equation} \end{aligned}$$



30.
$$\begin{aligned} r &= 2 \cos \theta, \text{ cylindrical equation} \\ r^2 &= 2r \cos \theta \\ x^2 + y^2 &= 2x \\ x^2 + y^2 - 2x &= 0 \\ (x - 1)^2 + y^2 &= 1, \text{ rectangular equation} \end{aligned}$$



31. $(4, 0, 0)$, rectangular

$\rho = \sqrt{4^2 + 0^2 + 0^2} = 4$

$\tan \theta = \dfrac{y}{x} = 0 \Rightarrow \theta = 0$

$\phi = \arccos 0 = \dfrac{\pi}{2}$

$\left(4, 0, \dfrac{\pi}{2}\right)$, spherical

32. $(-4, 0, 0)$, rectangular

$\rho = \sqrt{(-4)^2 + 0^2 + 0^2} = 4$

$\tan \theta = \dfrac{y}{x} = 0 \Rightarrow \theta = 0$

$\phi = \arccos \dfrac{z}{\rho} = \arccos 0 = \dfrac{\pi}{2}$

$\left(4, 0, \dfrac{\pi}{2}\right)$, spherical

33. $\left(-2, 2\sqrt{3}, 4\right)$, rectangular

$\rho = \sqrt{(-2)^2 + \left(2\sqrt{3}\right)^2 + 4^2} = 4\sqrt{2}$

$\tan \theta = \dfrac{y}{x} = \dfrac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\theta = \dfrac{2\pi}{3}$

$\phi = \arccos \dfrac{z}{\rho} = \arccos \dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$

$\left(4\sqrt{2}, \dfrac{2\pi}{3}, \dfrac{\pi}{4}\right)$, spherical

34. $\left(-5, -5, \sqrt{2}\right)$, rectangular

$\rho = \sqrt{(-5)^2 + (-5)^2 + \left(\sqrt{2}\right)^2} = \sqrt{52} = 2\sqrt{13}$

$\tan \theta = \dfrac{y}{x} = \dfrac{-5}{-5} = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$\phi = \arccos \dfrac{z}{\rho} = \arccos \dfrac{\sqrt{2}}{2\sqrt{13}} = \arccos \dfrac{\sqrt{26}}{26}$

$\left(2\sqrt{13}, \dfrac{\pi}{4}, \arccos \dfrac{\sqrt{26}}{26}\right)$, spherical

35. $\left(\sqrt{3}, 1, 2\sqrt{3}\right)$, rectangular

$\rho = \sqrt{3 + 1 + 12} = 4$

$\tan \theta = \dfrac{y}{x} = \dfrac{1}{\sqrt{3}}$

$\theta = \dfrac{\pi}{6}$

$\phi = \arccos \dfrac{z}{\rho} = \arccos \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{6}$

$\left(4, \dfrac{\pi}{6}, \dfrac{\pi}{6}\right)$, spherical

36. $(-1, 2, 1)$, rectangular

$\rho = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$

$\tan\theta = \dfrac{y}{x} = -2 \Rightarrow \theta = \arctan(-2) + \pi$

$\phi = \arccos\dfrac{z}{\rho}\arccos\dfrac{1}{\sqrt{6}}$

$\left(\sqrt{6}, \arctan(-2) + \pi, \arccos\dfrac{1}{\sqrt{6}}\right)$, spherical

37. $\left(4, \dfrac{\pi}{6}, \dfrac{\pi}{4}\right)$, spherical

$x = \rho\sin\phi\cos\theta = 4\sin\dfrac{\pi}{4}\cos\dfrac{\pi}{6} = \sqrt{6}$

$y = \rho\sin\phi\sin\theta = 4\sin\dfrac{\pi}{4}\sin\dfrac{\pi}{6} = \sqrt{2}$

$z = \rho\cos\phi = 4\cos\dfrac{\pi}{4} = 2\sqrt{2}$

$\left(\sqrt{6}, \sqrt{2}, 2\sqrt{2}\right)$, rectangular

38. $\left(6, \pi, \dfrac{\pi}{2}\right)$, spherical

$x = \rho\sin\phi\cos\theta = 6\sin\dfrac{\pi}{2}\cos\pi = -6$

$y = \rho\sin\phi\sin\theta = 6\sin\dfrac{\pi}{2}\sin\pi = 0$

$z = \rho\cos\phi = 6\cos\dfrac{\pi}{2} = 0$

$(-6, 0, 0)$, rectangular

39. $\left(12, -\dfrac{\pi}{4}, 0\right)$, spherical

$x = \rho\sin\phi\cos\theta = 12\sin 0\cos\left(-\dfrac{\pi}{4}\right) = 0$

$y = \rho\sin\phi\sin\theta = 12\sin 0\sin\left(-\dfrac{\pi}{4}\right) = 0$

$z = \rho\cos\phi = 12\cos 0 = 12$

$(0, 0, 12)$, rectangular

40. $\left(9, \dfrac{\pi}{4}, \pi\right)$, spherical

$x = \rho\sin\phi\cos\theta = 9\sin\pi\cos\dfrac{\pi}{4} = 0$

$y = \rho\sin\phi\sin\theta = 9\sin\pi\sin\dfrac{\pi}{4} = 0$

$z = \rho\cos\phi = 9\cos\pi = -9$

$(0, 0, -9)$, rectangular

41. $\left(5, \dfrac{\pi}{4}, \dfrac{\pi}{12}\right)$, spherical

$x = \rho\sin\phi\cos\theta = 5\sin\dfrac{\pi}{12}\cos\dfrac{\pi}{4} \approx 0.915$

$y = \rho\sin\phi\sin\theta = 5\sin\dfrac{\pi}{12}\sin\dfrac{\pi}{4} \approx 0.915$

$z = \rho\cos\theta = 5\cos\dfrac{\pi}{12} \approx 4.830$

$(0.915, 0.915, 4.830)$, rectangular

42. $\left(7, \dfrac{3\pi}{4}, \dfrac{\pi}{9}\right)$, spherical

$x = \rho\sin\phi\cos\theta = 7\sin\dfrac{\pi}{9}\cos\dfrac{3\pi}{4} \approx -1.693$

$y = \rho\sin\phi\sin\theta = 7\sin\dfrac{\pi}{9}\sin\dfrac{3\pi}{4} \approx 1.693$

$z = \rho\cos\phi = 7\cos\dfrac{\pi}{9} \approx 6.578$

$(-1.693, 1.693, 6.578)$, rectangular

43. $y = 2$, rectangular equation

$\rho\sin\phi\sin\theta = 2$

$\rho = 2\csc\phi\csc\theta$, spherical equation

44. $z = 6$, rectangular equation

$\rho\cos\phi = 6$

$\rho = 6\sec\phi$, spherical equation

45. $x^2 + y^2 + z^2 = 49$, rectangular equation

$\rho^2 = 49$

$\rho = 7$, spherical equation

46. $x^2 + y^2 - 3z^2 = 0$, rectangular equation

$$
\begin{aligned}
x^2 + y^2 + z^2 &= 4z^2 \\
\rho^2 &= 4\rho^2\cos^2\phi \\
1 &= 4\cos^2\phi \\
\cos\phi &= \frac{1}{2} \\
\phi &= \frac{\pi}{3}, \text{(cone) spherical equation}
\end{aligned}
$$

47. $x^2 + y^2 = 16$, rectangular equation

$$\rho^2 \sin^2 \phi \sin^2 \theta + \rho^2 \sin^2 \phi \cos^2 \theta = 16$$
$$\rho^2 \sin^2 \phi(\sin^2 \theta + \cos^2 \theta) = 16$$
$$\rho^2 \sin^2 \phi = 16$$
$$\rho \sin \phi = 4$$
$$\rho = 4 \csc \phi, \text{ spherical equation}$$

48. $x = 13$, rectangular equation

$$\rho \sin \phi \cos \theta = 13$$
$$\rho = 13 \csc \phi \sec \theta, \text{ spherical equation}$$

49. $x^2 + y^2 = 2z^2$, rectangular equation

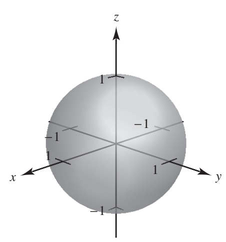
$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 2\rho^2 \cos^2 \phi$$
$$\rho^2 \sin^2 \phi[\cos^2 \theta + \sin^2 \theta] = 2\rho^2 \cos^2 \phi$$
$$\rho^2 \sin^2 \phi = 2\rho^2 \cos^2 \theta$$
$$\frac{\sin^2 \phi}{\cos^2 \phi} = 2$$
$$\tan^2 \phi = 2$$
$$\tan \phi = \pm\sqrt{2}, \text{ spherical equation}$$

50. $x^2 + y^2 + z^2 - 9z = 0$, rectangular equation

$$\rho^2 - 9\rho \cos \phi = 0$$
$$\rho = 9 \cos \phi, \text{ spherical equation}$$

51. $\rho = 1$, spherical equation

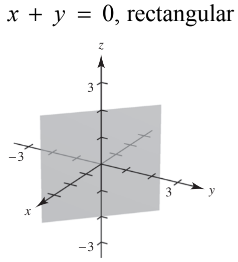
$x^2 + y^2 + z^1 = 1$, rectangular equation



52. $\theta = \dfrac{3\pi}{4}$, spherical equation

$$\tan \theta = \frac{y}{x}$$
$$-1 = \frac{y}{x}$$

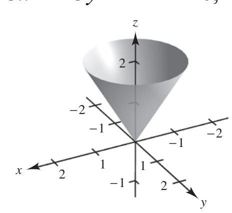
$x + y = 0$, rectangular equation



53. $\phi = \dfrac{\pi}{6}$, spherical equation

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$
$$\frac{\sqrt{3}}{2} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$
$$\frac{3}{4} = \frac{z^2}{x^2 + y^2 + z^2}$$

$3x^2 + 3y^2 - z^2 = 0$, $z \geq 0$, rectangular equation



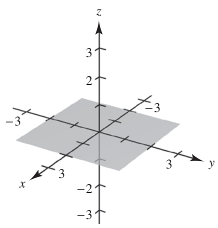
54. $\phi = \dfrac{\pi}{2}$, spherical equation

$$\cos\phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$0 = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$z = 0$, rectangular equation

xy-plane

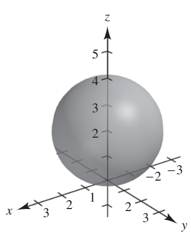


55. $\rho = 4\cos\phi$, spherical equation

$$\sqrt{x^2 + y^2 + z^2} = \frac{4z}{\sqrt{x^2 + y^2 + z^2}}$$

$$x^2 + y^2 + z^2 - 4z = 0$$

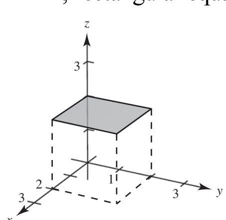
$x^2 + y^2 + (z - 2)^2 = 4$, $z \ge 0$, rectangular equation



56. $\rho = 2\sec\phi$, spherical equation

$$\rho\cos\phi = 2$$

$z = 2$, rectangular equation

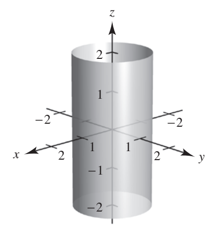


57. $\rho = \csc\phi$, spherical equation

$$\rho\sin\phi = 1$$

$$\sqrt{x^2 + y^2} = 1$$

$x^2 + y^2 = 1$, rectangular equation

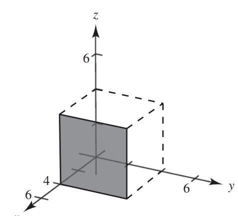


58. $\rho = 4\csc\phi\sec\phi$, spherical equation

$$= \frac{4}{\sin\phi\cos\theta}$$

$$\rho\sin\phi\cos\theta = 4$$

$x = 4$, rectangular equation



59. $\left(4, \dfrac{\pi}{4}, 0\right)$, cylindrical

$$\rho = \sqrt{4^2 + 0^2} = 4$$

$$\theta = \frac{\pi}{4}$$

$$\phi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos 0 = \frac{\pi}{2}$$

$\left(4, \dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$, spherical

60. $\left(3, -\dfrac{\pi}{4}, 0\right)$, cylindrical

$$\rho = \sqrt{3^2 + 0^2} = 3$$

$$\theta = -\frac{\pi}{4}$$

$$\phi = \arccos\frac{z}{\sqrt{r^2 + z^2}} = \arccos\frac{0}{9} = \frac{\pi}{2}$$

$\left(3, -\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$, spherical

61. $\left(6, \dfrac{\pi}{2}, -6\right)$, cylindrical

$$\rho = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$$

$$\theta = \frac{\pi}{2}$$

$$\phi = \arccos \frac{z}{\sqrt{r^2 + z^2}} = \arccos\left(\frac{-6}{6\sqrt{2}}\right) = \arccos\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$\left(6\sqrt{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}\right)$, spherical

62. $\left(-4, \dfrac{\pi}{3}, 4\right)$, cylindrical

$$\rho = \sqrt{(-4)^2 + 4^2} = 4\sqrt{2}$$

$$\theta = \frac{\pi}{3}$$

$$\phi = \arccos \frac{z}{\sqrt{r^2 + z^2}} = \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$\left(4\sqrt{2}, \dfrac{\pi}{3}, \dfrac{\pi}{4}\right)$, spherical

63. $(12, \pi, 5)$, cylindrical

$$\rho = \sqrt{12^2 + 5^2} = 13$$

$$\theta = \pi$$

$$\phi = \arccos \frac{z}{\sqrt{r^2 + z^2}} = \arccos \frac{5}{13}$$

$\left(13, \pi, \arccos \dfrac{5}{13}\right)$, spherical

64. $\left(4, \dfrac{\pi}{2}, 3\right)$, cylindrical

$$\rho = \sqrt{4^2 + 3^2} = 5$$

$$\theta = \frac{\pi}{2}$$

$$\phi = \arccos \frac{z}{\sqrt{r^2 + z^2}} = \arccos \frac{3}{5}$$

$\left(5, \dfrac{\pi}{2}, \arccos \dfrac{3}{5}\right)$, spherical

65. $\left(10, \dfrac{\pi}{6}, \dfrac{\pi}{2}\right)$, spherical

$$r = 10 \sin \frac{\pi}{2} = 10$$

$$\theta = \frac{\pi}{6}$$

$$z = 10 \cos \frac{\pi}{2} = 0$$

$\left(10, \dfrac{\pi}{6}, 0\right)$, cylindrical

66. $\left(4, \dfrac{\pi}{18}, \dfrac{\pi}{2}\right)$, spherical

$$r = 4 \sin \frac{\pi}{2} = 4$$

$$\theta = \frac{\pi}{18}$$

$$z = 4 \cos \frac{\pi}{2} = 0$$

$\left(4, \dfrac{\pi}{18}, 0\right)$, cylindrical

67. $\left(6, -\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$, spherical

$$r = 6 \sin \frac{\pi}{3} = 3\sqrt{3}$$

$$\theta = -\frac{\pi}{6}$$

$$z = 6 \cos \frac{\pi}{3} = 3$$

$\left(3\sqrt{3}, -\dfrac{\pi}{6}, 3\right)$, cylindrical

68. $\left(5, -\dfrac{5\pi}{6}, \pi\right)$, spherical

$$r = 5 \sin \pi = 0$$

$$\theta = -\frac{5\pi}{6}$$

$$z = 5 \cos \pi = -5$$

$\left(0, -\dfrac{5\pi}{6}, -5\right)$, cylindrical

69. $\left(8, \dfrac{7\pi}{6}, \dfrac{\pi}{6}\right)$, spherical

$$r = 8 \sin \frac{\pi}{6} = 4$$

$$\theta = \frac{7\pi}{6}$$

$$z = 8 \cos \frac{\pi}{6} = \frac{8\sqrt{3}}{2}$$

$\left(4, \dfrac{7\pi}{6}, 4\sqrt{3}\right)$, cylindrical

70. $\left(7, \frac{\pi}{4}, \frac{3\pi}{4}\right)$, spherical

$r = 7 \sin \frac{3\pi}{4} = \frac{7\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$z = 7 \cos \frac{3\pi}{4} = -\frac{7\sqrt{2}}{2}$

$\left(\frac{7\sqrt{2}}{2}, \frac{\pi}{4}, -\frac{7\sqrt{2}}{2}\right)$, cylindrical

71. $r = 5$

Cylinder

Matches graph (d)

72. $\theta = \frac{\pi}{4}$

Plane

Matches graph (e)

73. $\rho = 5$

Sphere

Matches graph (c)

74. $\phi = \frac{\pi}{4}$

Cone

Matches graph (a)

75. $r^2 = z, x^2 + y^2 = z$

Paraboloid

Matches graph (f)

76. $\rho = 4 \sec \phi, z = \rho \cos \phi = 4$

Plane

Matches graph (b)

77. $\theta = c$ is a half-plane because of the restriction $r \geq 0$.

78. (a) The surface is a cone. The equation is (i) $x^2 + y^2 = \frac{4}{9}z^2$.

In cylindrical coordinates, the equation is

$$x^2 + y^2 = \tfrac{4}{9}z^2$$
$$r^2 = \tfrac{4}{9}z^2$$
$$r = \tfrac{2}{3}z.$$

(b) The surface is a hyperboloid of one sheet. The equation is (ii) $x^2 + y^2 - z^2 = 2$.

In cylindrical coordinates, the equation is

$$x^2 + y^2 - z^2 = 2$$
$$r^2 - z^2 = 2$$
$$r^2 = z^2 + 2.$$

79. $x^2 + y^2 + z^2 = 27$

(a) $r^2 + z^2 = 27$

(b) $\rho^2 = 27 \Rightarrow \rho = 3\sqrt{3}$

80. $4\left(x^2 + y^2\right) = z^2$

(a) $4r^2 = z^2 \Rightarrow 2r = z$

(b) $4\left(\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta\right) = \rho^2 \cos^2 \phi$

$$4 \sin^2 \phi = \cos^2 \phi,$$
$$\tan^2 \phi = \frac{1}{4},$$
$$\tan \phi = \frac{1}{2} \Rightarrow \phi = \arctan \frac{1}{2}$$

81. $x^2 + y^2 + z^2 - 2z = 0$

(a) $r^2 + z^2 - 2z = 0 \Rightarrow r^2 + (z - 1)^2 = 1$

(b) $\rho^2 - 2\rho \cos \phi = 0$

$$\rho(\rho - 2 \cos \phi) = 0$$
$$\rho = 2 \cos \phi$$

82. $x^2 + y^2 = z$

(a) $r^2 = z$

(b) $\rho^2 \sin^2 \phi = \rho \cos \phi$

$$\rho \sin^2 \phi = \cos \phi$$
$$\rho = \frac{\cos \phi}{\sin^2 \phi}$$
$$\rho = \csc \phi \cot \phi$$

83. $x^2 + y^2 = 4y$

(a) $r^2 = 4r \sin \theta, r = 4 \sin \theta$

(b) $\rho^2 \sin^2 \phi = 4\rho \sin \phi \sin \theta$

$$\rho \sin \phi(\rho \sin \phi - 4 \sin \theta) = 0$$
$$\rho = \frac{4 \sin \theta}{\sin \phi}$$
$$\rho = 4 \sin \theta \csc \phi$$

84. $x^2 + y^2 = 45$

(a) $r^2 = 45$ or $r = 3\sqrt{5}$

(b) $\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 45$

$$\rho^2 \sin^2 \phi = 45$$

$$\rho = 3\sqrt{5} \csc \phi$$

85. $x^2 - y^2 = 9$

(a) $r^2 \cos^2 \theta - r^2 \sin^2 \theta = 9$

$$r^2 = \frac{9}{\cos^2 \theta - \sin^2 \theta}$$

(b) $\rho^2 \sin^2 \phi \cos^2 \theta - \rho^2 \sin^2 \phi \sin^2 \theta = 9$

$$\rho^2 \sin^2 \phi = \frac{9}{\cos^2 \theta - \sin^2 \theta}$$

$$\rho^2 = \frac{9 \csc^2 \phi}{\cos^2 \theta - \sin^2 \theta}$$

86. $y = 4$

(a) $r \sin \theta = 4 \Rightarrow r = 4 \csc \theta$

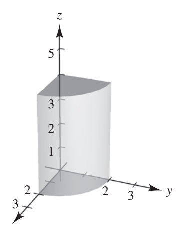
(b) $\rho \sin \phi \sin \theta = 4,$

$\rho = 4 \csc \phi \csc \theta$

87. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 2$

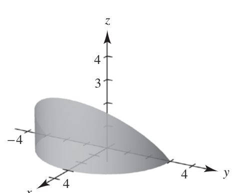
$0 \le z \le 4$



88. $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$0 \le r \le 3$

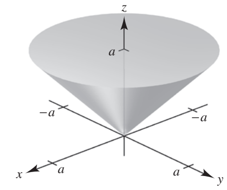
$0 \le z \le r \cos \theta$



89. $0 \le \theta \le 2\pi$

$0 \le r \le a$

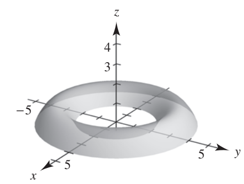
$r \le z \le a$



90. $0 \le \theta \le 2\pi$

$2 \le r \le 4$

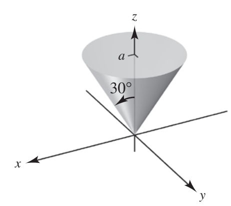
$z^2 \le -r^2 + 6r - 8$



91. $0 \le \theta \le 2\pi$

$0 \le \phi \le \dfrac{\pi}{6}$

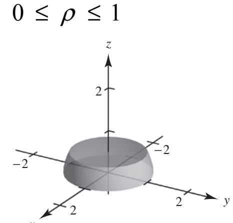
$0 \le \rho \le a \sec \phi$



92. $0 \le \theta \le 2\pi$

$\dfrac{\pi}{4} \le \phi \le \dfrac{\pi}{2}$

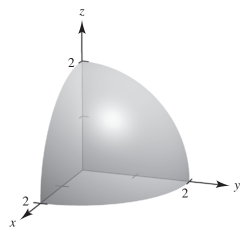
$0 \le \rho \le 1$



93. $0 \le \theta \le \dfrac{\pi}{2}$

$0 \le \phi \le \dfrac{\pi}{2}$

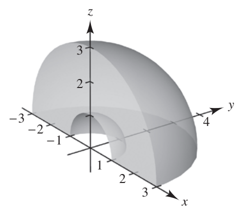
$0 \le \rho \le 2$



94. $0 \le \theta \le \pi$

$0 \le \phi \le \dfrac{\pi}{2}$

$1 \le \rho \le 3$

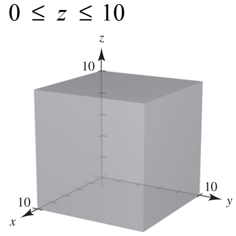


95. Rectangular

$0 \le x \le 10$

$0 \le y \le 10$

$0 \le z \le 10$



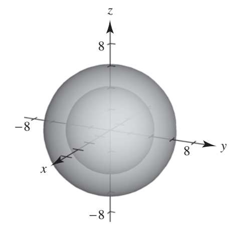
96. Cylindrical:

$0.75 \le r \le 1.25$

$0 \le z \le 8$

97. Spherical

$4 \le \rho \le 6$

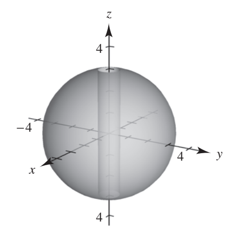


98. Cylindrical

$\frac{1}{2} \le r \le 3$

$0 \le \theta \le 2\pi$

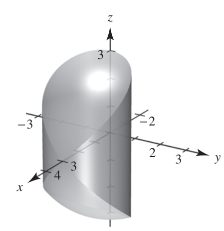
$-\sqrt{9 - r^2} \le z \le \sqrt{9 - r^2}$



99. Cylindrical coordinates:

$r^2 + z^2 \le 9,$

$r \le 3\cos\theta, 0 \le \theta \le \pi$

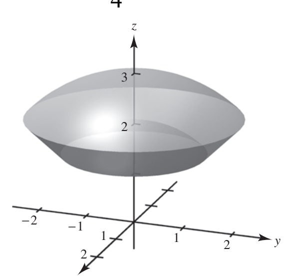


100. Spherical coordinates:

$\rho \ge 2$

$\rho \le 3$

$0 \le \phi \le \dfrac{\pi}{4}$



101. False. $(r, \theta, z) = (0, 0, 1)$ and $(r, \theta, z) = (0, \pi, 1)$ represent the same point $(x, y, z) = (0, 0, 1)$.

102. True (except for the origin).

103. $z = \sin\theta, r = 1$

$$z = \sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

The curve of intersection is the ellipse formed by the intersection of the plane $z = y$ and the cylinder $r = 1$.

104. $\rho = 2\sec\phi \Rightarrow \rho\cos\phi = 2 \Rightarrow z = 2$ plane

$\rho = 4$ sphere

The intersection of the plane and the sphere is a circle.

Review Exercises for Chapter 11

1. $P = (1, 2), Q = (4, 1), R = (5, 4)$

(a) $\mathbf{u} = \overline{PQ} = \langle 4 - 1, 1 - 2\rangle = \langle 3, -1\rangle$

$\mathbf{v} = \overline{PR} = \langle 5 - 1, 4 - 2\rangle = \langle 4, 2\rangle$

(b) $\mathbf{u} = 3\mathbf{i} - \mathbf{j}, \mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$

(c) $\|\mathbf{u}\| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$ $\|\mathbf{v}\| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

(d) $-3\mathbf{u} + \mathbf{v} = -3\langle 3, -1\rangle + \langle 4, 2\rangle = \langle -5, 5\rangle$

2. $P = (-2, -1), Q = (5, -1), R = (2, 4)$

(a) $\mathbf{u} = \overline{PQ} = \langle 5 - (-2), -1 - (-1)\rangle = \langle 7, 0\rangle$

$\mathbf{v} = \overline{PR} = \langle 2 - (-2), 4 - (-1)\rangle = \langle 4, 5\rangle$

(b) $\mathbf{u} = 7\mathbf{i}, \mathbf{v} = 4\mathbf{i} + 5\mathbf{j}$

(c) $\|\mathbf{u}\| = \sqrt{7^2 + 0^2} = \sqrt{49} = 7$ $\|\mathbf{v}\| = \sqrt{4^2 + 5^2} = \sqrt{41}$

(d) $-3\mathbf{u} + \mathbf{v} = -3\langle 7, 0\rangle + \langle 4, 5\rangle = \langle -17, 5\rangle$

3. $\mathbf{v} = \|\mathbf{v}\|(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$

$= 8(\cos 60^\circ\,\mathbf{i} + \sin 60^\circ\,\mathbf{j})$

$= 8\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = 4\mathbf{i} + 4\sqrt{3}\,\mathbf{j} = \langle 4, 4\sqrt{3}\rangle$

4. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j}$

$= \frac{1}{2}\cos 225^\circ\mathbf{i} + \frac{1}{2}\sin 225^\circ\mathbf{j}$

$= -\frac{\sqrt{2}}{4}\mathbf{i} - \frac{\sqrt{2}}{4}\mathbf{j} = \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}\right\rangle$

5. $z = 0, y = 4, x = -5$: $(-5, 4, 0)$

6. $y = 3$ describes a plane parallel to the xz-plane and passing through $(0, 3, 0)$.

7. $d = \sqrt{(-2 - 1)^2 + (3 - 6)^2 + (5 - 3)^2}$

$= \sqrt{9 + 9 + 4} = \sqrt{22}$

8. $d = \sqrt{(4 - (-2))^2 + (-1 - 1)^2 + (-1 - (-5))^2}$

$= \sqrt{36 + 4 + 16} = \sqrt{56} = 2\sqrt{14}$

9. $(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = 4^2$

$(x - 3)^2 + (y + 2)^2 + (z - 6)^2 = 16$

10. Center: $\left(\frac{0 + 4}{2}\right), \left(\frac{0 + 6}{2}\right), \left(\frac{4 + 0}{2}\right) = (2, 3, 2)$

Radius:

$\sqrt{(2 - 0)^2 + (3 - 0)^2 + (2 - 4)^2} = \sqrt{4 + 9 + 4} = \sqrt{17}$

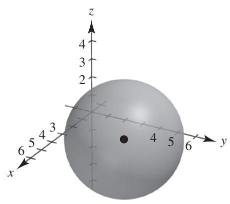
$(x - 2)^2 + (y - 3)^2 + (z - 2)^2 = 17$

11. $\left(x^2 - 4x + 4\right) + \left(y^2 - 6y + 9\right) + z^2 = -4 + 4 + 9$

$$(x - 2)^2 + (y - 3)^3 + z^2 = 9$$

Center: $(2, 3, 0)$

Radius: 3

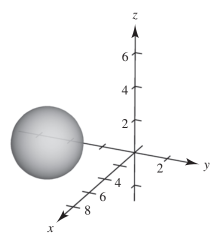


12. $\left(x^2 - 10x + 25\right) + \left(y^2 + 6y + 9\right) + \left(z^2 - 4z + 4\right) = -34 + 25 + 9 + 4$

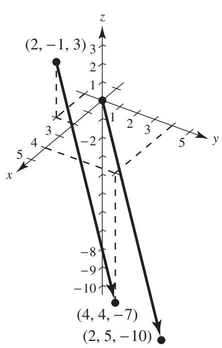
$$(x - 5)^2 + (y + 3)^2 + (z - 2)^2 = 4$$

Center: $(5, -3, 2)$

Radius: 2



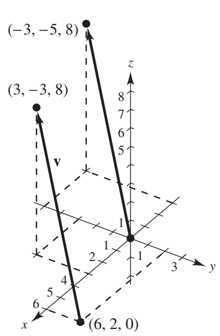
13. (a), (d)



(b) $\mathbf{v} = \langle 4 - 2, 4 - (-1), -7 - 3\rangle = \langle 2, 5, -10\rangle$

(c) $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}$

14. (a), (d)



(b) $\mathbf{v} = \langle 3 - 6, -3 - 2, 8 - 0\rangle = \langle -3, -5, 8\rangle$

(c) $\mathbf{v} = -3\mathbf{i} - 5\mathbf{j} + 8\mathbf{k}$

15. $z = -\mathbf{u} + 3\mathbf{v} + \frac{1}{2}\mathbf{w}$

$$= -\langle 5, -2, 3\rangle + 3\langle 0, 2, 1\rangle + \frac{1}{2}\langle -6, -6, 2\rangle$$
$$= \langle -5, 2, -3\rangle + \langle 0, 6, 3\rangle + \langle -3, -3, 1\rangle$$
$$= \langle -8, 5, 1\rangle$$

16. $\mathbf{u} - \mathbf{v} + \mathbf{w} - 2\mathbf{z} = 0$

$$z = \frac{1}{2}(\mathbf{u} - \mathbf{v} + \mathbf{w})$$
$$= \frac{1}{2}\left(\langle 5, -2, 3\rangle - \langle 0, 2, 1\rangle + \langle -6, -6, 2\rangle\right)$$
$$= \frac{1}{2}\langle -1, -10, 4\rangle$$
$$= \left\langle -\frac{1}{2}, -5, 2\right\rangle$$

17. $\mathbf{v} = \langle -1 - 3, 6 - 4, 9 + 1\rangle = \langle -4, 2, 10\rangle$

$\mathbf{w} = \langle 5 - 3, 3 - 4, -6 + 1\rangle = \langle 2, -1, -5\rangle$

Because $-2\mathbf{w} = \mathbf{v}$, the points lie in a straight line.

18. $\mathbf{v} = \langle 8 - 5, -5 + 4, 5 - 7\rangle = \langle 3, -1, -2\rangle$

$\mathbf{w} = \langle 11 - 5, 6 + 4, 3 - 7\rangle = \langle 6, 10, -4\rangle$

Because $\mathbf{v}$ and $\mathbf{w}$ are not parallel, the points do not lie in a straight line.

19. Unit vector: $\dfrac{\mathbf{u}}{\|\mathbf{u}\|} = \left\langle \dfrac{2, 3, 5}{\sqrt{38}}\right\rangle = \left\langle \dfrac{2}{\sqrt{38}}, \dfrac{3}{\sqrt{38}}, \dfrac{5}{\sqrt{38}}\right\rangle$

20. $8\dfrac{\langle 6, -3, 2\rangle}{\sqrt{49}} = \dfrac{8}{7}\langle 6, -3, 2\rangle = \left\langle \dfrac{48}{7}, -\dfrac{24}{7}, \dfrac{16}{7}\right\rangle$

21. $P = \langle 5, 0, 0\rangle, Q = \langle 4, 4, 0\rangle, R = \langle 2, 0, 6\rangle$

(a) $\mathbf{u} = \overline{PQ} = \langle -1, 4, 0\rangle$

$\mathbf{v} = \overline{PR} = \langle -3, 0, 6\rangle$

(b) $\mathbf{u} \cdot \mathbf{v} = (-1)(-3) + 4(0) + 0(6) = 3$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 = 45$

22. $P = \langle 2, -1, 3\rangle, Q = \langle 0, 5, 1\rangle, R = \langle 5, 5, 0\rangle$

(a) $\mathbf{u} = \overline{PQ} = \langle -2, 6, -2\rangle$

$\mathbf{v} = \overline{PR} = \langle 3, 6, -3\rangle$

(b) $\mathbf{u} \cdot \mathbf{v} = (-2)(3) + (6)(6) + (-2)(-3) = 36$

(c) $\mathbf{v} \cdot \mathbf{v} = 9 + 36 + 9 = 54$

23. $\mathbf{u} = 5\left(\cos\frac{3\pi}{4}\mathbf{i} + \sin\frac{3\pi}{4}\mathbf{j}\right) = \frac{5\sqrt{2}}{2}[-\mathbf{i} + \mathbf{j}]$

$\mathbf{v} = 2\left(\cos\frac{2\pi}{3}\mathbf{i} + \sin\frac{2\pi}{3}\mathbf{j}\right) = -\mathbf{i} + \sqrt{3}\,\mathbf{j}$

$\mathbf{u} \cdot \mathbf{v} = \frac{5\sqrt{2}}{2}\left(1 + \sqrt{3}\right)$

$\|\mathbf{u}\| = \sqrt{\frac{25}{2} + \frac{25}{2}} = 5 \qquad \|\mathbf{v}\| = \sqrt{1 + 3} = 2$

$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\left(5\sqrt{2}/2\right)\left(1 + \sqrt{3}\right)}{5(2)} = \frac{\sqrt{2} + \sqrt{6}}{4}$

(a) $\theta = \arccos\frac{\sqrt{2} + \sqrt{6}}{4} = \frac{\pi}{12} \approx 0.262$

(b) $\theta \approx 15^\circ$

24. $\mathbf{u} = \langle 1, 0, -3\rangle$

$\mathbf{v} = (2, -2, 1)$

$\mathbf{u} \cdot \mathbf{v} = -1$

$\|\mathbf{u}\| = \sqrt{1 + 9} = \sqrt{10}$

$\|\mathbf{v}\| = \sqrt{4 + 4 + 1} = 3$

$\cos\theta = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{1}{3\sqrt{10}}$

(a) $\theta = \arccos\left(\frac{1}{3\sqrt{10}}\right) \approx 1.465$

(b) $\theta = 83.9^\circ$

25. $\mathbf{u} = \langle 7, -2, 3\rangle$, $\mathbf{v} = \langle -1, 4, 5\rangle$

Because $\mathbf{u} \cdot \mathbf{v} = 0$, the vectors are orthogonal.

26. $\mathbf{u} = \langle -3, 0, 9\rangle = -3\langle 1, 0, -3\rangle = -3\mathbf{v}$

The vectors are parallel.

27. $\mathbf{u} = \langle 4, 2\rangle$, $\mathbf{v} = \langle 3, 4\rangle$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \left(\frac{20}{25}\right)\langle 3, 4\rangle$

$= \frac{4}{5}\langle 3, 4\rangle = \left\langle \frac{12}{5}, \frac{16}{5}\right\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2\rangle - \left\langle \frac{12}{5}, \frac{16}{5}\right\rangle = \left\langle \frac{8}{5}, -\frac{6}{5}\right\rangle$

28. $\mathbf{u} = \langle 1, -1, 1\rangle$, $\mathbf{v} = \langle 2, 0, 2\rangle$

(a) $\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

$= \frac{4}{8}\langle 2, 0, 2\rangle = \langle 1, 0, 1\rangle$

(b) $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 1, -1, 1\rangle - \langle 1, 0, 1\rangle = \langle 0, -1, 0\rangle$

29. There are many correct answers.

For example: $\mathbf{v} = \pm\langle 6, -5, 0\rangle$.

30. $W = \mathbf{F} \cdot \overline{PQ} = \|\mathbf{F}\|\|\overline{PQ}\|\cos\theta = (75)(8)\cos 30^\circ$

$= 300\sqrt{3}$ ft-lb

31. (a) $\mathbf{u} \times \mathbf{v} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 6 \\ 5 & 2 & 1 \end{bmatrix} = -9\mathbf{i} + 26\mathbf{j} - 7\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = 9\mathbf{i} - 26\mathbf{j} + 7\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

32. (a) $\mathbf{u} \times \mathbf{v} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 1 \\ 1 & -3 & 4 \end{bmatrix} = 11\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

(b) $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -11\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

(c) $\mathbf{v} \times \mathbf{v} = \mathbf{0}$

33. $\mathbf{u} \times \mathbf{v} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -10 & 8 \\ 4 & 6 & -8 \end{bmatrix} = 32\mathbf{i} + 48\mathbf{j} + 52\mathbf{k}$

$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{6032} = 4\sqrt{377}$

Unit vector: $\frac{1}{\sqrt{377}}\langle 8, 12, 13\rangle$

34. $\mathbf{u} = \langle 3, -1, 5\rangle$, $\mathbf{v} = \langle 2, -4, 1\rangle$

$\mathbf{u} \times \mathbf{v} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 2 & -4 & 1 \end{bmatrix} = 19\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$

$A = \|\mathbf{u} \times \mathbf{v}\| = \sqrt{19^2 + 7^2 + (-10)^2}$

$= \sqrt{510}$

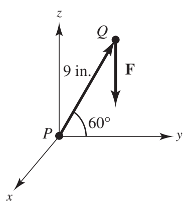
35. $\mathbf{F} = -40\mathbf{k}$ $\quad\left(9 \text{ in.} = \frac{3}{4} \text{ ft}\right)$

$$\overrightarrow{PQ} = \frac{3}{4}(\cos 60°\mathbf{j} + \sin 60°\mathbf{k}) = \frac{3}{8}\mathbf{j} + \frac{3\sqrt{3}}{8}\mathbf{k}$$

The moment of **F** about P is

$$M = \overrightarrow{PQ} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \frac{3}{8} & \frac{3\sqrt{3}}{8} \\ 0 & 0 & -40 \end{vmatrix} = -15\mathbf{i}$$

Torque $= 15$ ft-lb



36. $$V = |\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})| = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix} = 2(5) = 10$$

37. $\mathbf{v} = \langle 9 - 3, 11 - 0, 6 - 2 \rangle = \langle 6, 11, 4 \rangle$

(a) Parametric equations:
$x = 3 + 6t,\ y = 11t,\ z = 2 + 4t$

(b) Symmetric equations: $\dfrac{x - 3}{6} = \dfrac{y}{11} = \dfrac{z - 2}{4}$

38. $\mathbf{v} = \langle 8 + 1, 10 - 4, 5 - 3 \rangle = \langle 9, 6, 2 \rangle$

(a) Parametric equations:
$x = -1 + 9t,\ y = 4 + 6t,\ z = 3 + 2t$

(b) Symmetric equations: $\dfrac{x + 1}{9} = \dfrac{y - 4}{6} = \dfrac{z - 3}{2}$

39. $P = (-6, -8, 2)$

$\mathbf{v} = \mathbf{j} = \langle 0, 1, 0 \rangle$

$x = -6,\ y = -8 + t,\ z = 2$

40. Direction numbers: $1, 1, 1$, $\mathbf{v} = \langle 1, 1, 1 \rangle$

$P(1, 2, 3)$

$x = 1 + t,\ y = 2 + t,\ z = 3 + t$

41. $P = (-3, -4, 2),\ Q = (-3, 4, 1),\ R = (1, 1, -2)$

$\overrightarrow{PQ} = \langle 0, 8, -1 \rangle,\ \overrightarrow{PR} = [4, 5, -4]$

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = -27\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$$

$$-27(x + 3) - 4(y + 4) - 32(z - 2) = 0$$
$$27x + 4y + 32z = -33$$

42. $\mathbf{n} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$

$3(x + 2) - 1(y - 3) + 1(z - 1) = 0$

$3x - y + z + 8 = 0$

43. The two lines are parallel as they have the same direction numbers, $-2, 1, 1$. Therefore, a vector parallel to the plane is $\mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$. A point on the first line is $(1, 0, -1)$ and a point on the second line is $(-1, 1, 2)$. The vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ connecting these two points is also parallel to the plane. Therefore, a normal to the plane is

$$\mathbf{v} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$
$$= -2\mathbf{i} - 4\mathbf{j} = -2(\mathbf{i} + 2\mathbf{j}).$$

Equation of the plane: $(x - 1) + 2y = 0$

$$x + 2y = 1$$

44. Let $\mathbf{v} = \langle 5 - 2, 1 + 2, 3 - 1 \rangle = \langle 3, 3, 2 \rangle$ be the direction vector for the line through the two points. Let $\mathbf{n} = \langle 2, 1, -1 \rangle$ be the normal vector to the plane. Then

$$\mathbf{v} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 2 \\ 2 & 1 & -1 \end{vmatrix} = \langle -5, 7, -3 \rangle$$

is the normal to the unknown plane.

$-5(x - 5) + 7(y - 1) - 3(z - 3) = 0$

$-5x + 7y - 3z + 27 = 0$

45. $Q(1, 0, 2)$ point

$2x - 3y + 6z = 6$

A point P on the plane is $(3, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, 0, 2 \rangle$

$\mathbf{n} = \langle 2, -3, 6 \rangle$ normal to plane

$$D = \frac{\left|\overrightarrow{PQ} \cdot \mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{8}{7}$$

46. $Q(3, -2, 4)$ point

$2x - 5y + z = 10$

A point P on the plane is $(5, 0, 0)$.

$\overrightarrow{PQ} = \langle -2, -2, 4 \rangle$

$\mathbf{n} = \langle 2, -5, 1 \rangle$ normal to plane

$$D = \frac{\left|\overrightarrow{PQ} \cdot \mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{10}{\sqrt{30}} = \frac{\sqrt{30}}{3}$$

47. The normal vectors to the planes are the same, $\mathbf{n} = (5, -3, 1)$.

Choose a point in the first plane $P(0, 0, 2)$. Choose a point in the second plane, $Q(0, 0, -3)$.

$\overrightarrow{PQ} = \langle 0, 0, -5 \rangle$

$$D = \frac{\left|\overrightarrow{PQ} \cdot \mathbf{n}\right|}{\|\mathbf{n}\|} = \frac{|-5|}{\sqrt{35}} = \frac{5}{\sqrt{35}} = \frac{\sqrt{35}}{7}$$

48. $Q(-5, 1, 3)$ point

$\mathbf{u} = \langle 1, -2, -1 \rangle$ direction vector

$P(1, 3, 5)$ point on line

$\overrightarrow{PQ} = \langle -6, -2, -2 \rangle$

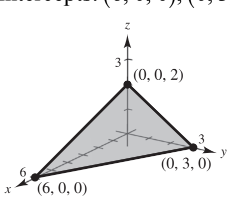
$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & -2 & -2 \\ 1 & -2 & -1 \end{vmatrix} = \langle -2, -8, 14 \rangle$$

$$D = \frac{\left\|\overrightarrow{PQ} \times \mathbf{u}\right\|}{\|\mathbf{u}\|} = \frac{\sqrt{264}}{\sqrt{6}} = 2\sqrt{11}$$

49. $x + 2y + 3z = 6$

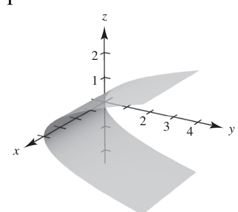
Plane

Intercepts: $(6, 0, 0), (0, 3, 0), (0, 0, 2),$



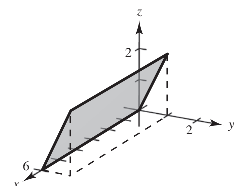
50. $y = z^2$

Because the x-coordinate is missing, you have a cylindrical surface with rulings parallel to the x-axis. The generating curve is a parabola in the yz-coordinate plane.



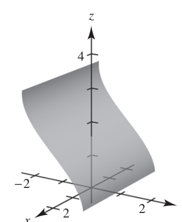
51. $y = \frac{1}{2}z$

Plane with rulings parallel to the x-axis.



52. $y = \cos z$

Because the x-coordinate is missing, you have a cylindrical surface with rulings parallel to the x-axis. The generating curve is $y = \cos z$.



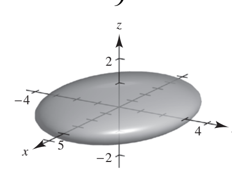
53. $\frac{x^2}{16} + \frac{y^2}{9} + z^2 = 1$

Ellipsoid

xy-trace: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

xz-trace: $\frac{x^2}{16} + z^2 = 1$

yz-trace: $\frac{y^2}{9} + z^2 = 1$



54. $16x^2 + 16y^2 - 9z^2 = 0$

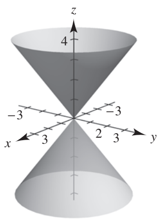
Cone

xy-trace: point $(0, 0, 0)$

xz-trace: $z = \pm\dfrac{4x}{3}$

yz-trace: $z = \pm\dfrac{4y}{3}$

$z = 4, x^2 + y^2 = 9$



55. $\dfrac{x^2}{16} - \dfrac{y^2}{9} + z^2 = -1$

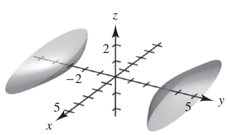
$\dfrac{y^2}{9} - \dfrac{x^2}{16} - z^2 = 1$

Hyperboloid of two sheets

xy-trace: $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$

xz-trace: None

yz-trace: $\dfrac{y^2}{9} - z^2 = 1$



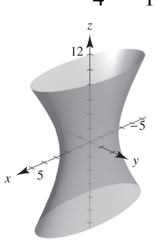
56. $\dfrac{x^2}{25} + \dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$

Hyperboloid of one sheet

xy-trace: $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$

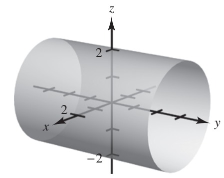
xz-trace: $\dfrac{x^2}{25} - \dfrac{z^2}{100} = 1$

yz-trace: $\dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$



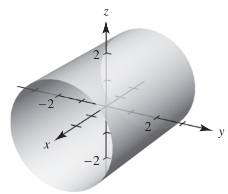
57. $x^2 + z^2 = 4.$

Cylinder of radius 2 about y-axis



58. $y^2 + z^2 = 16.$

Cylinder of radius 4 about x-axis



59. $z^2 = 2y$ revolved about y-axis

$z = \pm\sqrt{2y}$

$x^2 + z^2 = [r(y)]^2 = 2y$

$x^2 + z^2 = 2y$

60. $2x + 3z = 1$ revolved about the x-axis

$z = \dfrac{1 - 2x}{3}$

$y^2 + z^2 = [r(x)]^2 = \left(\dfrac{1 - 2x}{3}\right)^2$, Cone

61. $\left(-\sqrt{3}, 3, -5\right)$, rectangular

(a) $r = \sqrt{\left(-\sqrt{3}\right)^2 + 3^2} = \sqrt{12} = 2\sqrt{3}$

$\tan\theta = \dfrac{-3}{\sqrt{3}} \Rightarrow \theta = -\dfrac{\pi}{3}$

$z = -5$

$\left(2\sqrt{3}, -\dfrac{\pi}{3}, -5\right)$, cylindrical

(b) $\rho = \sqrt{\left(-\sqrt{3}\right)^2 + 3^2 + (-5)^2} = \sqrt{37}$

$\tan\theta = -\dfrac{3}{\sqrt{3}} \Rightarrow \theta = -\dfrac{\pi}{3}$

$\phi = \arccos\dfrac{z}{\rho} = \arccos\left(\dfrac{-5}{\sqrt{37}}\right)$

$\left(\sqrt{37}, -\dfrac{\pi}{3}, \arccos\left(-\dfrac{5\sqrt{37}}{37}\right)\right)$, spherical

62. $(8, 8, 1)$, rectangular

(a) $r = \sqrt{8^2 + 8^2} = 8\sqrt{2}$

$\tan\theta = \dfrac{8}{8} = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$z = 1$

$\left(8\sqrt{2}, \dfrac{\pi}{4}, 1\right)$, cylindrical

(b) $\rho = \sqrt{8^2 + 8^2 + 1^2} = \sqrt{129}$

$\tan\theta = \dfrac{8}{8} = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$\phi = \arccos\dfrac{z}{\rho} = \arccos\dfrac{1}{\sqrt{129}}$

$\left(\sqrt{129}, \dfrac{\pi}{4}, \arccos\dfrac{\sqrt{129}}{129}\right)$, spherical

63. $(5, \pi, 1)$, cylindrical

$x = r\cos\theta = 5\cos\pi = -5$

$y = r\sin\theta = 5\sin\pi = 0$

$z = 1$

$(-5, 0, 1)$, rectangular

64. $\left(-2, \dfrac{\pi}{3}, 3\right)$, cylindrical

$x = r\cos\theta = -2\cos\dfrac{\pi}{3} = -1$

$y = r\sin\theta = -2\sin\dfrac{\pi}{3} = -\sqrt{3}$

$z = 3$

$\left(-1, -\sqrt{3}, 3\right)$, rectangular

65. $\left(4, \pi, \dfrac{\pi}{4}\right)$, spherical

$x = \rho\sin\phi\cos\theta = 4\sin\dfrac{\pi}{4}\cos\pi = -2\sqrt{2}$

$y = \rho\sin\phi\sin\theta = 4\sin\dfrac{\pi}{4}\sin\pi = 0$

$z = \rho\cos\phi = 4\cos\dfrac{\pi}{4} = 2\sqrt{2}$

$\left(-2\sqrt{2}, 0, 2\sqrt{2}\right)$, rectangular

66. $\left(8, -\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$, spherical

$$x = \rho\sin\phi\cos\theta = 8\sin\frac{\pi}{3}\cos\left(-\frac{\pi}{6}\right) = 8\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 6$$

$$y = \rho\sin\phi\sin\theta = 8\sin\frac{\pi}{3}\sin\left(-\frac{\pi}{6}\right) = 8\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) = -2\sqrt{3}$$

$$z = \rho\cos\phi = 8\cos\frac{\pi}{3} = 4$$

$\left(6, -2\sqrt{3}, 4\right)$, rectangular

67. $x^2 - y^2 = 2z$

(a) Cylindrical:

$r^2\cos^2\theta - r^2\sin^2\theta = 2z \Rightarrow r^2\cos 2\theta = 2z$

(b) Spherical:

$\rho^2\sin^2\phi\cos^2\theta - \rho^2\sin^2\phi\sin^2\theta = 2\rho\cos\phi$

$\rho\sin^2\phi\cos 2\theta - 2\cos\phi = 0$

$\rho = 2\sec 2\theta\cos\phi\csc^2\phi$

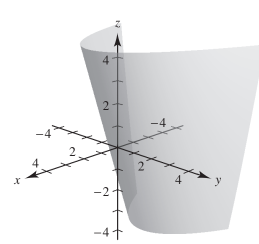
68. $x^2 + y^2 + z^2 = 16$

(a) Cylindrical: $r^2 + z^2 = 16$

(b) Spherical: $\rho = 4$

69. $z = r^2\sin^2\theta + 3r\cos\theta$, cylindrical equation

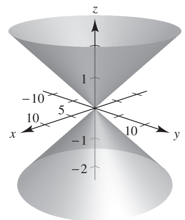
$z = y^2 + 3x$, rectangular equation



70. $r = -5z$, cylindrical equation

$$\sqrt{x^2 + y^2} = -5z$$

$x^2 + y^2 - 25z^2 = 0$, rectangular equation



71. $\phi = \frac{\pi}{4}$, spherical equation

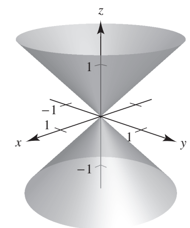
$$\phi = \arccos \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{\pi}{4}$$

$$\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$z^2 = \frac{1}{2}\left(x^2 + y^2 + z^2\right)$$

$$2z^2 = x^2 + y^2 + z^2$$

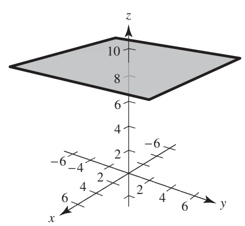
$x^2 + y^2 - z^2 = 0$, rectangular equation



72. $\rho = 9 \sec \theta$, spherical equation

$\rho \cos \theta = 9$

$z = 9$, rectangular equation

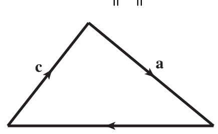


Problem Solving for Chapter 11

1.
$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$$
$$\mathbf{b} \times (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{0}$$
$$(\mathbf{b} \times \mathbf{a}) + (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$$
$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{b} \times \mathbf{c}\|$$
$$\|\mathbf{b} \times \mathbf{c}\| = \|\mathbf{b}\|\|\mathbf{c}\| \sin A$$
$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin C$$

Then,

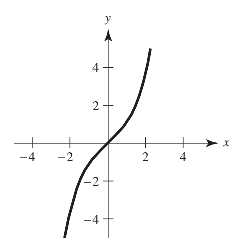
$$\frac{\sin A}{\|\mathbf{a}\|} = \frac{\|\mathbf{b} \times \mathbf{c}\|}{\|\mathbf{a}\|\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\|\|\mathbf{b}\|\|\mathbf{c}\|} = \frac{\sin C}{\|\mathbf{c}\|}.$$



The other case, $\frac{\sin A}{\|\mathbf{a}\|} = \frac{\sin B}{\|\mathbf{b}\|}$ is similar.

2. $f(x) = \int_0^x \sqrt{t^4 + 1}\, dt$

(a)



(b) $f'(x) = \sqrt{x^4 + 1}$

$f'(0) = 1 = \tan \theta$

$\theta = \frac{\pi}{4}$

$\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

(c) $\pm\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

(d) The line is $y = x$: $x = t$, $y = t$.

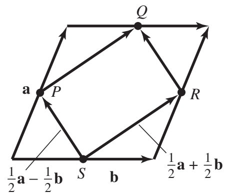
3. Label the figure as indicated.

From the figure, you see that

$\overrightarrow{SP} = \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} = \overrightarrow{RQ}$ and $\overrightarrow{SR} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \overrightarrow{PQ}$.

Because $\overrightarrow{SP} = \overrightarrow{RQ}$ and $\overrightarrow{SR} = \overrightarrow{PQ}$,

$PSRQ$ is a parallelogram.



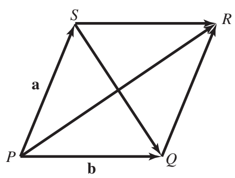
4. Label the figure as indicated.

$\overrightarrow{PR} = \mathbf{a} + \mathbf{b}$

$\overrightarrow{SQ} = \mathbf{b} - \mathbf{a}$

$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = \|\mathbf{b}\|^2 - \|\mathbf{a}\|^2 = 0$, because

$\|\mathbf{a}\| = \|\mathbf{b}\|$ in a rhombus.

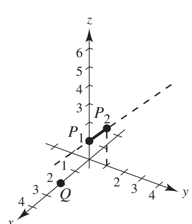


5. (a) $\mathbf{u} = \langle 0, 1, 1\rangle$ is the direction vector of the line determined by P_1 and P_2.

$$D = \frac{\|\overrightarrow{P_1Q} \times \mathbf{u}\|}{\|\mathbf{u}\|}$$

$$= \frac{\|\langle 2, 0, -1\rangle \times \langle 0, 1, 1\rangle\|}{\sqrt{2}}$$

$$= \frac{\|\langle 1, -2, 2\rangle\|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

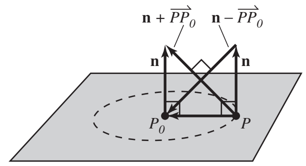


(b) The shortest distance to the line **segment** is $\|P_1Q\| = \|\langle 2, 0, -1\rangle\| = \sqrt{5}$.

6. $\left(\mathbf{n} + \overrightarrow{PP_0}\right) \perp \left(\mathbf{n} - \overrightarrow{PP_0}\right)$

Figure is a square.

So, $\|\overrightarrow{PP_0}\| = \|\mathbf{n}\|$ and the points P form a circle of radius $\|\mathbf{n}\|$ in the plane with center at P_0.



7. (a) $V = \pi\int_0^1 \left(\sqrt{z}\right)^2 dz = \left[\pi\frac{z^2}{2}\right]_0^1 = \frac{1}{2}\pi$

Note: $\frac{1}{2}$(base)(altitude) $= \frac{1}{2}\pi(1) = \frac{1}{2}\pi$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$: (slice at $z = c$)

$$\frac{x^2}{\left(\sqrt{c}a\right)^2} + \frac{y^2}{\left(\sqrt{c}b\right)^2} = 1$$

At $z = c$, figure is ellipse of area

$\pi\left(\sqrt{c}a\right)\left(\sqrt{c}b\right) = \pi abc.$

$$V = \int_0^k \pi abc \cdot dc = \left[\frac{\pi abc^2}{2}\right]_0^k = \frac{\pi abk^2}{2}$$

(c) $V = \frac{1}{2}(\pi abk)k = \frac{1}{2}$(area of base)(height)

8. (a) $V = 2\int_0^r \pi\left(r^2 - x^2\right)dx = 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r = \frac{4}{3}\pi r^3$

(b) At height $z = d > 0$,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{d^2}{c^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{d^2}{c^2} = \frac{c^2 - d^2}{c^2}$$

$$\frac{x^2}{\frac{a^2\left(c^2 - d^2\right)}{c^2}} + \frac{y^2}{\frac{b^2\left(c^2 - d^2\right)}{c^2}} = 1.$$

$$\text{Area} = \pi\sqrt{\left(\frac{a^2\left(c^2 - d^2\right)}{c^2}\right)\left(\frac{b^2\left(c^2 - d^2\right)}{c^2}\right)} = \frac{\pi ab}{c^2}\left(c^2 - d^2\right)$$

$$V = 2\int_0^c \frac{\pi ab}{c^2}\left(c^2 - d^2\right)dd$$

$$= \frac{2\pi ab}{c^2}\left[c^2d - \frac{d^3}{3}\right]_0^c = \frac{4}{3}\pi abc$$

9. From Exercise 54, Section 11.4,

$$(\mathbf{u} \times \mathbf{v}) \times (\mathbf{w} \times \mathbf{z}) = \left[(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{z}\right]\mathbf{w} - \left[(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}\right]\mathbf{z}.$$

10. $x = -t + 3, y = \frac{1}{2}t + 1, z = 2t - 1; Q = (4, 3, s)$

(a) $\mathbf{u} = \langle -2, 1, 4\rangle$ direction vector for line

$P = (3, 1, -1)$ point on line

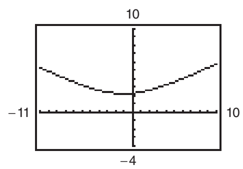
$\overrightarrow{PQ} = \langle 1, 2, s + 1\rangle$

$$\overrightarrow{PQ} \times \mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & s+1 \\ -2 & 1 & 4 \end{vmatrix}$$

$$= (7 - s)\mathbf{i} + (-6 - 2s)\mathbf{j} + 5\mathbf{k}$$

$$D = \frac{\left\|\overrightarrow{PQ} \times \mathbf{u}\right\|}{\|\mathbf{u}\|} = \frac{\sqrt{(7 - s)^2 + (-6 - 2s)^2 + 25}}{\sqrt{21}}$$

(b)



The minimum is $D \approx 2.2361$ at $s = -1$.

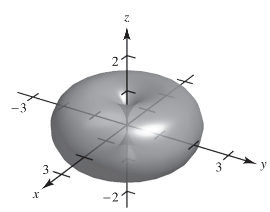
(c) Yes, there are slant asymptotes. Using $s = x$, you have

$$D(s) = \frac{1}{\sqrt{21}}\sqrt{5x^2 + 10x + 110} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{x^2 + 2x + 22} = \frac{\sqrt{5}}{\sqrt{21}}\sqrt{(x + 1)^2 + 21} \to \pm\sqrt{\frac{5}{21}}(x + 1)$$

$y = \pm\frac{\sqrt{105}}{21}(s + 1)$ slant asymptotes.

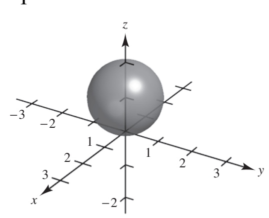
11. (a) $\rho = 2 \sin \phi$

Torus



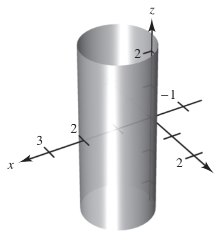
(b) $\rho = 2 \cos \phi$

Sphere



12. (a) $r = 2\cos\theta$

Cylinder



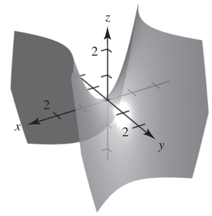
(b) $z = r^2 \cos 2\theta$

$= r^2\left(\cos^2\theta - \sin^2\theta\right)$

$= \left(r\cos\theta\right)^2 - \left(r\sin\theta\right)^2$

$= x^2 - y^2$

Hyperbolic paraboloid



13. (a) $\mathbf{u} = \|\mathbf{u}\|(\cos 0\,\mathbf{i} + \sin 0\,\mathbf{j}) = \|\mathbf{u}\|\mathbf{i}$

Downward force $\mathbf{w} = -\mathbf{j}$

$\mathbf{T} = \|\mathbf{T}\|\left(\cos(90° + \theta)\mathbf{i} + \sin(90° + \theta)\mathbf{j}\right)$

$= \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\mathbf{0} = \mathbf{u} + \mathbf{w} + \mathbf{T} = \|\mathbf{u}\|\mathbf{i} - \mathbf{j} + \|\mathbf{T}\|(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$\|\mathbf{u}\| = \sin\theta\|\mathbf{T}\|$

$1 = \cos\theta\|\mathbf{T}\|$

If $\theta = 30°$, $\|\mathbf{u}\| = (1/2)\|\mathbf{T}\|$ and $1 = \left(\sqrt{3}/2\right)\|\mathbf{T}\| \Rightarrow \|\mathbf{T}\| = \dfrac{2}{\sqrt{3}} \approx 1.1547$ lb and $\|\mathbf{u}\| = \dfrac{1}{2}\left(\dfrac{2}{\sqrt{3}}\right) \approx 0.5774$ lb

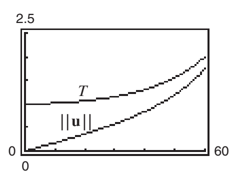
(b) From part (a), $\|\mathbf{u}\| = \tan\theta$ and $\|\mathbf{T}\| = \sec\theta$.

Domain: $0 \le \theta \le 90°$

(c)

θ	0°	10°	20°	30°	40°	50°	60°
$\mathbf{T}$	1	1.0154	1.0642	1.1547	1.3054	1.5557	2
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774	0.8391	1.1918	1.7321

(d)



(e) Both are increasing functions.

(f) $\lim_{\theta \to \pi/2^-} T = \infty$ and $\lim_{\theta \to \pi/2^-} \|\mathbf{u}\| = \infty$.

Yes. As θ increases, both T and $\|\mathbf{u}\|$ increase.

14. (a) The tension T is the same in each tow line.

$$6000\mathbf{i} = T(\cos 20° + \cos(-20))\mathbf{i} + T(\sin 20° + \sin(-20°))\mathbf{j}$$
$$= 2T\cos 20°\mathbf{i}$$
$$\Rightarrow T = \frac{6000}{2\cos 20°} \approx 3192.5 \text{ lb}$$

(b) As in part (a), $6000\mathbf{i} = 2T\cos\theta$

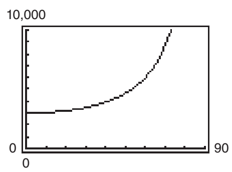
$$\Rightarrow T = \frac{3000}{\cos\theta}$$

Domain: $0 < \theta < 90°$

(c)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(d)



(e) As θ increases, there is less force applied in the direction of motion.

15. Let $\theta = \alpha - \beta$, the angle between **u** and **v.** Then

$$\sin(\alpha - \beta) = \frac{\|\mathbf{u}\times\mathbf{v}\|}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\|\mathbf{v}\times\mathbf{u}\|}{\|\mathbf{u}\|\|\mathbf{v}\|}.$$

For $\mathbf{u} = \langle\cos\alpha, \sin\alpha, 0\rangle$ and $\mathbf{v} = \langle\cos\beta, \sin\beta, 0\rangle$, $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$ and

$$\mathbf{v}\times\mathbf{u} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\mathbf{k}.$$

So, $\sin(\alpha - \beta) = \|\mathbf{v}\times\mathbf{u}\| = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$

16. (a) Los Angeles: $(4000, -118.24°, 55.95°)$ Rio de Janeiro: $(4000, -43.23°, 112.90°)$

(b) Los Angeles: $x = 4000\sin(55.95°)\cos(-118.24°)$

$y = 4000\sin(55.95°)\sin(-118.24°)$

$z = 4000\cos(55.95°)$

$(x, y, z) \approx (-1568.2, -2919.7, 2239.7)$

Rio de Janeiro: $x = 4000\sin(112.90°)\cos(-43.23°)$

$y = 4000\sin(112.90°)\sin(-43.23°)$

$z = 4000\cos(112.90°)$

$(x, y, z) \approx (2684.7, -2523.8, -1556.5)$

(c) $$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(-1568.2)(2684.7) + (-2919.7)(-2523.8) + (2239.7)(-1556.5)}{(4000)(4000)} \approx -0.02047$$

$\theta \approx 91.17°$ or 1.59 radians

(d) $s = r\theta = 4000(1.59) \approx 6360$ miles

(e) For Boston and Honolulu:

a. Boston: $(4000, -71.06°, 47.64°)$ Honolulu: $(4000, -157.86°, 68.69°)$

b. Boston: $x = 4000 \sin 47.64° \cos(-71.06°)$ Honolulu: $x = 4000 \sin 68.69° \cos(-157.86°)$

$y = 4000 \sin 47.64° \sin(-71.06°)$ $y = 4000 \sin 68.69° \sin(-157.86°)$

$z = 4000 \cos 47.64°$ $z = 4000 \cos 68.69°$

$(959.4, -2795.7, 2695.1)$ $(-3451.7, -1404.4, 1453.7)$

c. $$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{(959.4)(-3451.7) + (-2795.7)(-1404.4) + (2695.1)(1453.7)}{(4000)(4000)} \approx 0.28329$$

$\theta \approx 73.54°$ or 1.28 radians

d. $s = r\theta = 4000(1.28) \approx 5120$ miles

17. From Theorem 11.13 and Theorem 11.7 (6) you have

$$D = \frac{|\overline{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|}$$

$$= \frac{|\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|}{\|\mathbf{u} \times \mathbf{v}\|} = \frac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{\|\mathbf{u} \times \mathbf{v}\|}.$$

18. Assume one of a, b, c, is not zero, say a. Choose a point in the first plane such as $(-d_1/a, 0, 0)$. The distance between this point and the second plane is

$$D = \frac{|a(-d_1/a) + b(0) + c(0) + d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

19. $x^2 + y^2 = 1$ cylinder

$z = 2y$ plane

Introduce a coordinate system in the plane $z = 2y$.

The new u-axis is the original x-axis.

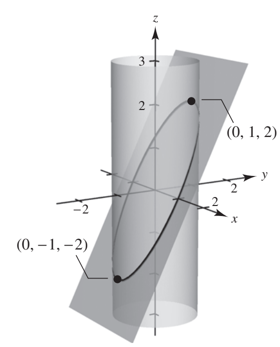
The new v-axis is the line $z = 2y, x = 0$.

Then the intersection of the cylinder and plane satisfies the equation of an ellipse:

$$x^2 + y^2 = 1$$

$$x^2 + \left(\frac{z}{2}\right)^2 = 1$$

$$x^2 + \frac{z^2}{4} = 1 \quad \text{ellipse}$$



20. Essay

Chapter 2 Differentiation

Chapter Comments

The material presented in Chapter 2 forms the basis for the remainder of calculus. Much of it needs to be memorized, beginning with the definition of a derivative of a function found on page 103. Students need to have a thorough understanding of the tangent line problem and they need to be able to find an equation of a tangent line. Frequently, students will use the function $f'(x)$ as the slope of the tangent line. They need to understand that $f'(x)$ is the formula for the slope and the actual value of the slope can be found by substituting into $f'(x)$ the appropriate value for x. On pages 105–106 of Section 2.1, you will find a discussion of situations where the derivative fails to exist. These examples (or similar ones) should be discussed in class.

As you teach this chapter, vary your notations for the derivative. One time write y'; another time write dy/dx or $f'(x)$. Terminology is also important. Instead of saying "find the derivative," sometimes say, "differentiate." This would be an appropriate time, also, to talk a little about Leibnitz and Newton and the discovery of calculus.

Sections 2.2, 2.3, and 2.4 present a number of rules for differentiation. Have your students memorize the Product Rule and the Quotient Rule (Theorems 2.7 and 2.8) in words rather than symbols. Students tend to be lazy when it comes to trigonometry and therefore, you need to impress upon them that the formulas for the derivatives of the six trigonometric functions need to be memorized also. You will probably not have enough time in class to prove every one of these differentiation rules, so choose several to do in class and perhaps assign a few of the other proofs as homework.

The Chain Rule, in Section 2.4, will require two days of your class time. Students need a lot of practice with this and the algebra involved in these problems. Many students can find the derivative of $f(x) = x^2\sqrt{1 - x^2}$ without much trouble, but simplifying the answer is often difficult for them. Insist that they learn to factor and write the answer without negative exponents. Strive to get the answer in the form given in the back of the book. This will help them later on when the derivative is set equal to zero.

Implicit differentiation is often difficult for students. Have students think of y as a function of x and therefore y^3 is $[f(x)]^3$. This way they can relate implicit differentiation to the Chain Rule studied in the previous section.

Try to get your students to see that related rates, discussed in Section 2.6, are another use of the Chain Rule.

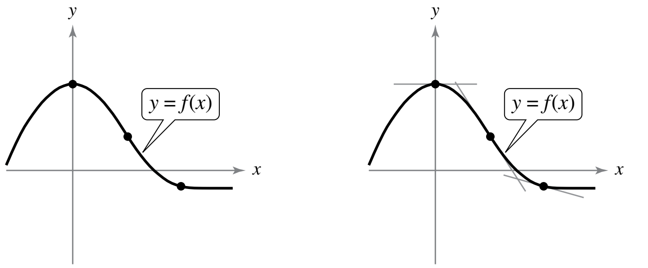
Section 2.1 The Derivative and the Tangent Line Problem

Section Comments

2.1 **The Derivative and the Tangent Line Problem**—Find the slope of the tangent line to a curve at a point. Use the limit definition to find the derivative of a function. Understand the relationship between differentiability and continuity.

Teaching Tips

Ask students what they think "the line tangent to a curve" means. Draw a curve with tangent lines to show a visual picture of tangent lines. For example:



When talking about the tangent line problem, use the suggested example of finding the equation of the tangent line to the parabola $y = x^2$ at the point $(1, 1)$.

Compute an approximation of the slope m by choosing a nearby point $Q(x, x^2)$ on the parabola and computing the slope m_{PQ} of the secant line PQ.

After going over Examples 1–3, return to Example 2 where $f(x) = x^2 + 1$ and note that $f'(x) = 2x$. How can we find the equation of the line tangent to f and parallel to $4x - y = 0$? Because the slope of the line is 4,

$$2x = 4$$

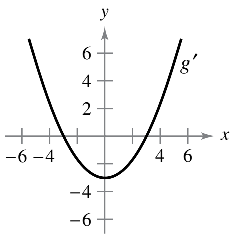
$$x = 2.$$

So, at the point $(2, 5)$, the tangent line is parallel to $4x - y = 0$. The equation of the tangent line is $y - 5 = 4(x - 2)$ or $y = 4x - 3$.

Be sure to find the derivatives of various types of functions to show students the different types of techniques for finding derivatives. Some suggested problems are $f(x) = 4x^3 - 3x^2$, $g(x) = 2/(x - 1)$, and $h(x) = \sqrt{2x + 5}$.

How Do You See It? Exercise

Page 108, Exercise 64 The figure shows the graph of g'.



(a) $g'(0) =$

(b) $g'(3) =$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $g(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

Solution

(a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$ and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

Suggested Homework Assignment

Pages 107–109: 1, 3, 7, 11, 21–27 odd, 37, 43–47 odd, 53, 57, 61, 77, 87, 93, and 95.

Section 2.2 Basic Differentiation Rules and Rates of Change

Section Comments

2.2 **Basic Differentiation Rules and Rates of Change**—Find the derivative of a function using the Constant Rule. Find the derivative of a function using the Power Rule. Find the derivative of a function using the Constant Multiple Rule. Find the derivative of a function using the Sum and Difference Rules. Find the derivatives of the sine function and of the cosine function. Use derivatives to find rates of change.

Teaching Tips

Start by showing proofs of the Constant Rule and the Power Rule. Students who are mathematics majors need to start seeing proofs early on in their college careers as they will be taking Functions of a Real Variable at some point.

Go over an example in class like $f(x) = \dfrac{5x^2 + x}{x}$. Show students that before differentiating they can rewrite the function as $f(x) = 5x + 1$. Then they can differentiate to obtain $f'(x) = 5$. Use this example to emphasize the prudence of examining the function first before differentiating. Rewriting the function in a simpler, equivalent form can expedite the differentiating process.

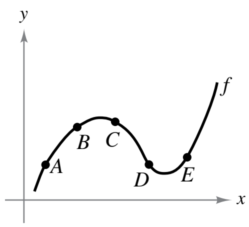
Give mixed examples of finding derivatives. Some suggested examples are:

$f(x) = 3x^6 - x^{2/3} + 3\sin x$ and $g(x) = \dfrac{4}{\sqrt[3]{x}} + \dfrac{2}{(3x)^2} - 3\cos x + 7x + \pi^3$.

This will test students' understanding of the various differentiation rules of this section.

How Do You See It? Exercise

Page 119, Exercise 76 Use the graph of f to answer each question. To print an enlarged copy of the graph, go to *MathGraphs.com*.



(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

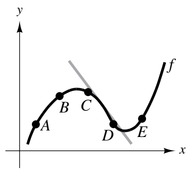
(c) Sketch a tangent line to the graph between C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

Solution

(a) The slope appears to be steepest between A and B.

(b) The average rate of change between A and B is **greater** than the instantaneous rate of change at B.

(c)



Suggested Homework Assignment

Pages 118–120: 1, 3, 5, 7–29 odd, 35, 39–53 odd, 55, 59, 65, 75, 85–89 odd, 91, 95, and 97.

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

Section Comments

2.3 **Product and Quotient Rules and Higher-Order Derivatives**—Find the derivative of a function using the Product Rule. Find the derivative of a function using the Quotient Rule. Find the derivative of a trigonometric function. Find a higher-order derivative of a function.

Teaching Tips

Some students have difficulty simplifying polynomial and rational expressions. Students should review these concepts by studying Appendices A.2–A.4 and A.7 in *Precalculus*, 10th edition, by Larson.

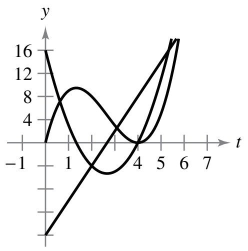
When teaching the Product and Quotient Rules, give proofs of each rule so that students can see where the rules come from. This will provide mathematics majors a tool for writing proofs, as each proof requires subtracting and adding the same quantity to achieve the desired results. For the Project Rule, emphasize that there are many ways to write the solution. Remind students that there must be one derivative in each term of the solution. Also, the Product Rule can be extended to more that just the product of two functions. Simplification is up to the discretion of the instructor. Examples such as $f(x) = (2x^2 - 3x)(5x^3 + 6)$ can be done with or without the Product Rule. Show the class both ways.

After the Quotient Rule has been proved to the class, give students the memorization tool of LO d HI – HI d LO. This will give students a way to memorize what goes in the numerator of the Quotient Rule.

Some examples to use are $f(x) = \dfrac{2x - 1}{x^2 + 7x}$ and $g(x) = \dfrac{4 - (1/x)}{3 - x^2}$. Save $f(x)$ for the next section as this will be a good example for the Chain Rule. $g(x)$ is a good example for first finding the least common denominator.

How Do You See It? Exercise

Page 132, Exercise 120 The figure shows the graphs of the position, velocity, and acceleration functions of a particle.

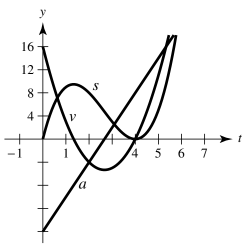


(a) Copy the graphs of the functions shown. Identify each graph. Explain your reasoning. To print an enlarged copy of the graph, go to *MathGraphs.com*.

(b) On your sketch, identify when the particle speeds up and when it slows down. Explain your reasoning.

Solution

(a)

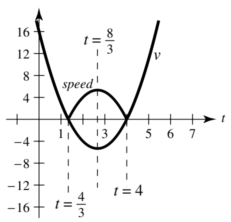


s position function

v velocity function

a acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$, and $(8/3, 4)$ and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.



Suggested Homework Assignment

Pages 129–132: 1, 3, 9, 13, 19, 23, 29–55 odd, 59, 61, 63, 75, 77, 91–107 odd, 111, 113, 117, and 131–135 odd.

Section 2.4 The Chain Rule

Section Comments

2.4 **The Chain Rule**—Find the derivative of a composite function using the Chain Rule. Find the derivative of a function using the General Power Rule. Simplify the derivative of a function using algebra. Find the derivative of a trigonometric function using the Chain Rule.

Teaching Tips

Begin this section by asking students to consider finding the derivative of $F(x) = \sqrt{x^2 + 1}$. F is a composite function. Letting $y = f(u) = \sqrt{u}$ and $u = g(x) = x^2 + 1$, then $y = F(x) = f(g(x))$ or $F = f \circ g$. When stating the Chain Rule, be sure to state it using function notation and using Leibniz notation as students will see both forms when studying other courses with other texts. Following the definition, be sure to prove the Chain Rule as done on page 134.

Be sure to give examples that involve all rules discussed so far. Some examples include:
$f(x) = (\sin(6x))^4$, $g(x) = \left(\dfrac{3 + \sin(2x)}{\sqrt[3]{x + 3}}\right)^2$, and $h(x) = \left(\sqrt{x - \dfrac{2}{x}}\right) \cdot [8x + \cos(x^2 + 1)]^3$.

You can use Exercise 98 on page 141 to review the following concepts:

- Product Rule
- Chain Rule
- Quotient Rule
- General Power Rule

Students need to understand these rules because they are the foundation of the study of differentiation.

Use the solution to show students how to solve each problem. As you apply each rule, give the definition of the rule verbally. Note that part (b) is not possible because we are not given $g'(3)$.

Solution

(a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible. You need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(x) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$$

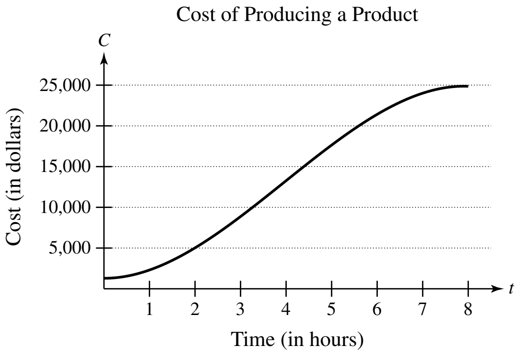
(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

How Do You See It? Exercise

Page 142, Exercise 106 The cost C (in dollars) of producing x units of a product is $C = 60x + 1350$. For one week, management determined that the number of units produced x at the end of t hours can be modeled by $x = -1.6t^3 + 19t^2 - 0.5t - 1$. The graph shows the cost C in terms of the time t.



(a) Using the graph, which is greater, the rate of change of the cost after 1 hour or the rate of change of the cost after 4 hours?

(b) Explain why the cost function is not increasing at a constant rate during the eight-hour shift.

Solution

(a) According to the graph, $C'(4) > C'(1)$.

(b) Answers will vary.

Suggested Homework Assignment

Pages 140–143: 1–53 odd, 63, 67, 75, 81, 83, 91, 97, 121, and 123.

Section 2.5 Implicit Differentiation

Section Comments

2.5 **Implicit Differentiation**—Distinguish between functions written in implicit form and explicit form. Use implicit differentiation to find the derivative of a function.

Teaching Tips

Material learned in this section will be vital for students to have for related rates. Be sure to ask students to find $\frac{dy}{dx}$ when $x = c$.

You can use the exercise below to review the following concepts:

- Finding derivatives when the variables agree and when they disagree
- Using implicit differentiation to find the derivative of a function

Determine if the statement is true. If it is false, explain why and correct it. For each statement, assume y is a function of x.

(a) $\dfrac{d}{dx}\cos(x^2) = -2x\sin(x^2)$

(b) $\dfrac{d}{dy}\cos(y^2) = 2y\sin(y^2)$

(c) $\dfrac{d}{dx}\cos(y^2) = -2y\sin(y^2)$

Implicit differentiation is often difficult for students, so as you review this concept remind students to think of y as a function of x. Part (a) is true, and part (b) can be corrected as shown below. Part (c) requires implicit differentiation. Note that the result can also be written as $-2y\sin(y^2)\dfrac{dy}{dx}$.

Solution

(a) True

(b) False. $\dfrac{d}{dy}\cos(y^2) = -2y\sin(y^2)$.

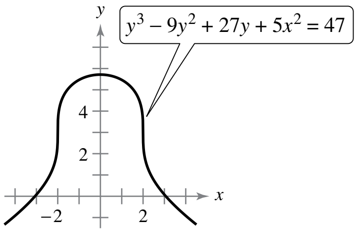
(c) False. $\dfrac{d}{dx}\cos(y^2) = -2yy'\sin(y^2)$.

A good way to teach students how to understand the differentiation of a mix of variables in part (c) is to let $g = y$. Then $g' = y'$. So,

$$\begin{aligned}\frac{d}{dx}\cos(y^2) &= \frac{d}{dx}\cos(g^2)\\ &= -\sin(g^2)\cdot 2gg'\\ &= -\sin(y^2)\cdot 2yy'\end{aligned}$$

How Do You See It? Exercise

Page 151, Exercise 70 Use the graph to answer the questions.



(a) Which is greater, the slope of the tangent line at $x = -3$ or the slope of the tangent line at $x = -1$?

(b) Estimate the point(s) where the graph has a vertical tangent line.

(c) Estimate the point(s) where the graph has a horizontal tangent line.

Solution

(a) The slope is greater at $x = -3$.

(b) The graph has vertical tangent lines at about $(-2, 3)$ and $(2, 3)$.

(c) The graph has a horizontal tangent line at about $(0, 6)$.

Suggested Homework Assignment

Pages 149–150: 1–17 odd, 25–35 odd, 53, and 61.

Section 2.6 Related Rates

Section Comments

2.6 **Related Rates**—Find a related rate. Use related rates to solve real-life problems.

Teaching Tips

Begin this lesson with a quick review of implicit differentiation with an implicit function in terms of x and y differentiated with respect to time. Follow this with an example similar to Example 1 on page 152, outlining the step-by-step procedure at the top of page 153 along with the guidelines at the bottom of page 153. Be sure to tell students, that for every related rate problem, to write down the given information, the equation needed, and the unknown quantity. A suggested problem to work out with the students is as follows:

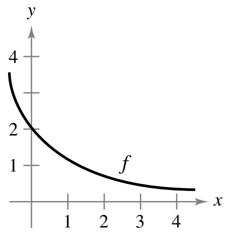
A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?

Be sure to go over a related rate problem similar to Example 5 on page 155 so that students are exposed to working with related rate problems involving trigonometric functions.

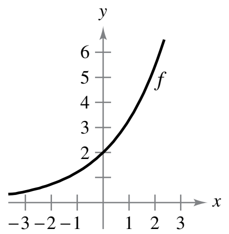
How Do You See It? Exercise

Page 159, Exercise 34 Using the graph of f, (a) determine whether dy/dt is positive or negative given that dx/dt is negative, and (b) determine whether dx/dt is positive or negative given that dy/dt is positive. Explain.

(i)



(ii)



Solution

(i) (a) $\dfrac{dx}{dt}$ negative $\implies$ $\dfrac{dy}{dt}$ positive

(b) $\dfrac{dy}{dt}$ positive $\implies$ $\dfrac{dx}{dt}$ negative

(ii) (a) $\dfrac{dx}{dt}$ negative $\implies$ $\dfrac{dy}{dt}$ negative

(b) $\dfrac{dy}{dt}$ positive $\implies$ $\dfrac{dx}{dt}$ positive

Suggested Homework Assignment

Pages 157–160: 1, 7, 11, 13, 15, 17, 21, 25, 29, and 41.

Chapter 2 Project

Timing a Handoff

You are a competitive bicyclist. During a race, you bike at a constant velocity of k meters per second. A chase car waits for you at the ten-mile mark of a course. When you cross the ten-mile mark, the car immediately accelerates to catch you. The position function of the chase car is given by the equation $s(t) = \frac{15}{4}t^2 - \frac{5}{12}t^3$, for $0 \le t \le 6$, where t is the time in seconds and s is the distance traveled in meters. When the car catches you, you and the car are traveling at the same velocity, and the driver hands you a cup of water while you continue to bike at k meters per second.

Exercises

1. Write an equation that represents your position s (in meters) at time t (in seconds).
2. Use your answer to Exercise 1 and the given information to write an equation that represents the velocity k at which the chase car catches you in terms of t.
3. Find the velocity function of the car.
4. Use your answers to Exercises 2 and 3 to find how many seconds it takes the chase car to catch you.
5. What is your velocity when the car catches you?
6. Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.
7. Find the point of intersection of the two graphs in Exercise 6. What does this point represent in the context of the problem?
8. Describe the graphs in Exercise 6 at the point of intersection. Why is this important for a successful handoff?
9. Suppose you bike at a constant velocity of 9 meters per second and the chase car's position function is unchanged.
   (a) Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.
   (b) In this scenario, how many times will the chase car be in the same position as you after the 10-mile mark?
   (c) In this scenario, would the driver of the car be able to successfully handoff a cup of water to you? Explain.
10. Suppose you bike at a constant velocity of 8 meters per second and the chase car's position function is unchanged.
   (a) Use a graphing utility to graph the chase car's position function and your position function in the same viewing window.
   (b) In this scenario, how many times will the chase car be in the same position as you after the ten-mile mark?
   (c) In this scenario, why might it be difficult for the driver of the chase car to successfully handoff a cup of water to you? Explain.

P Preparation for Calculus





P.2 Linear Models and Rates of Change

Objectives

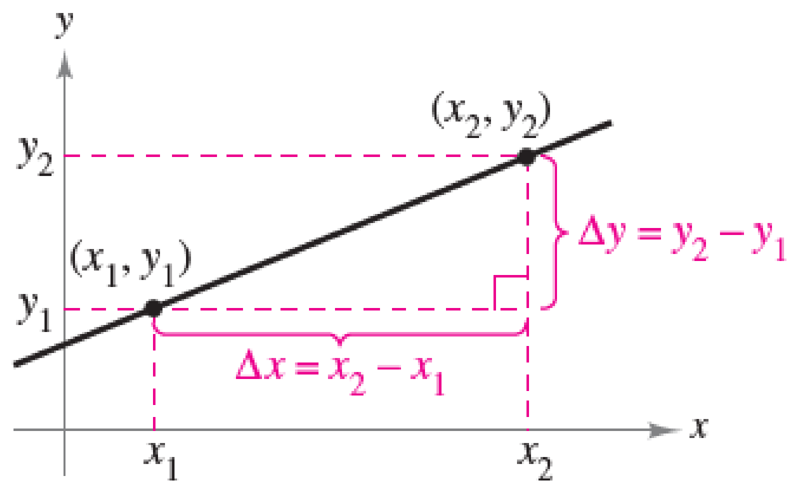
- Find the slope of a line passing through two points.
- Write the equation of a line with a given point and slope.
- Interpret slope as a ratio or as a rate in a real-life application.
- Sketch the graph of a linear equation in slope-intercept form.
- Write equations of lines that are parallel or perpendicular to a given line.

The Slope of a Line

The Slope of a Line

The **slope** of a nonvertical line is a measure of the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right.

Consider the two points (x_1, y_1) and (x_2, y_2) on the line in Figure P.12.



$$\Delta y = y_2 - y_1 = \text{change in } y$$
$$\Delta x = x_2 - x_1 = \text{change in } x$$

Figure P.12

The Slope of a Line

As you move from left to right along this line, a vertical change of

$$\Delta y = y_2 - y_1 \qquad \text{Change in } y$$

units corresponds to a horizontal change of

$$\Delta x = x_2 - x_1 \qquad \text{Change in } x$$

units. (The symbol Δ is the uppercase Greek letter delta, and the symbols Δy and Δx are read "delta y" and "delta x.")

The Slope of a Line

Definition of the Slope of a Line

The **slope** m of the nonvertical line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2.$$

Slope is not defined for vertical lines.

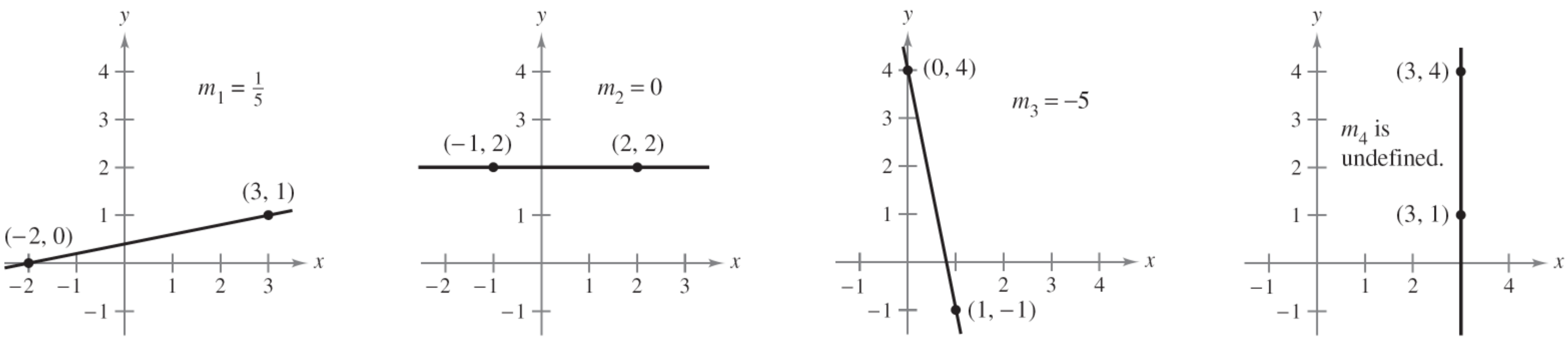
The Slope of a Line

When using the formula for slope, note that

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{-(x_1 - x_2)} = \frac{y_1 - y_2}{x_1 - x_2}.$$

So, it does not matter in which order you subtract *as long as* you are consistent and both “subtracted coordinates” come from the same point.

The Slope of a Line



If m is positive, then the line rises from left to right.

If m is zero, then the line is horizontal.

If m is negative, then the line falls from left to right.

If m is undefined, then the line is vertical.

Figure P.13

Figure P.13 shows four lines: one has a positive slope, one has a slope of zero, one has a negative slope, and one has an “undefined” slope.

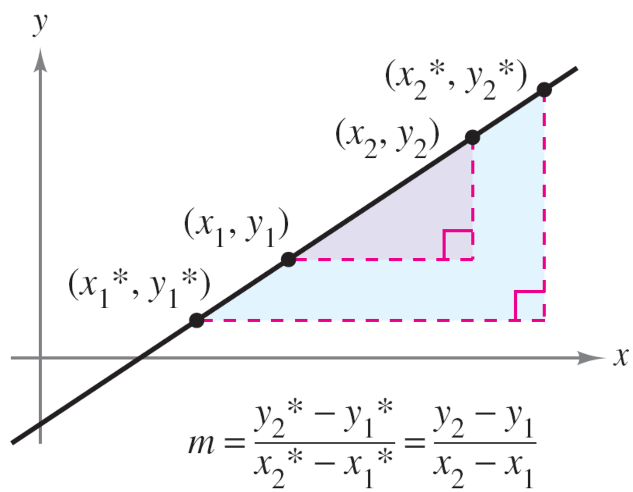
In general, the greater the absolute value of the slope of a line, the steeper the line.

Equations of Lines

Equations of Lines

Any two points on a nonvertical line can be used to calculate its slope.

This can be verified from the similar triangles shown in Figure P.14.



Any two points on a nonvertical line can be used to determine its slope.

Figure P.14

Equations of Lines

If (x_1, y_1) is a point on a nonvertical line that has a slope of m and (x, y) is *any other* point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the form

$$y - y_1 = m(x - x_1)$$

which is called the **point-slope form** of the equation of a line.

Equations of Lines

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

Example 1 – *Finding an Equation of a Line*

Find an equation of the line that has a slope of 3 and passes through the point (1, –2). Then sketch the line.

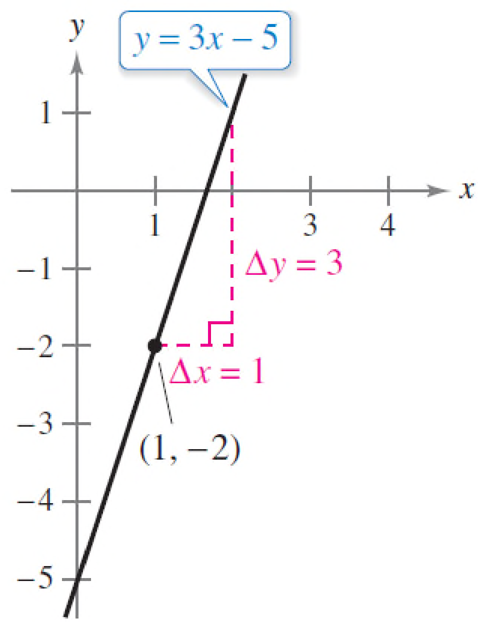
Solution:

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 3(x - 1)$	Substitute -2 for y_1, 1 for x_1, and 3 for m.
$y + 2 = 3x - 3$	Simplify.
$y = 3x - 5$	Solve for y.

Example 1 – *Solution*

cont'd

To sketch the line, first plot the point (1, –2). Then, because the slope is m = 3, you can locate a second point on the line by moving one unit to the right and three units upward, as shown in Figure P.15.



The line with a slope of 3 passing through the point (1, −2)

Figure P.15

Ratios and Rates of Change

Ratios and Rates of Change

The slope of a line can be interpreted as either a *ratio* or a *rate*.

If the x- and y-axes have the same unit of measure, then the slope has no units and is a **ratio.**

If the x- and y-axes have different units of measure, then the slope is a rate or **rate of change.**

Example 2 – *Using Slope as a Ratio*

The maximum recommended slope of a wheelchair ramp is 1/12. A business installs a wheelchair ramp that rises to a height of 22 inches over a length of 24 feet, as shown in Figure P.16. Is the ramp steeper than recommended?

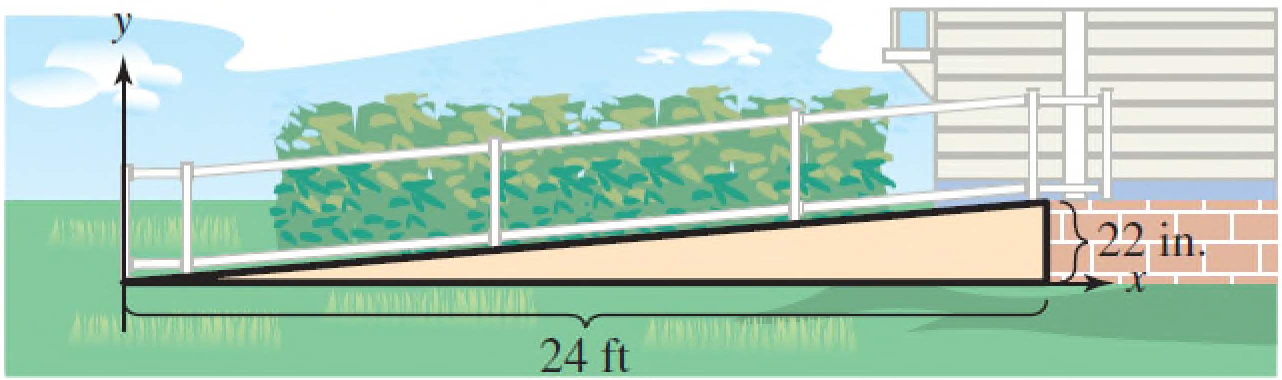


Figure P.16

Example 2 – *Solution*

The length of the ramp is 24 feet or 12 (24) = 288 inches. The slope of the ramp is the ratio of its height (the rise) to its length (the run).

$$\text{Slope of ramp} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{22 \text{ in.}}{288 \text{ in.}}$$

$$\approx 0.076$$

Because the slope of the ramp is less than ½ ≈ 0.083, the ramp is not steeper than recommended. Note that the slope is a ratio and has no units.

Example 3 – *Using Slope as a Rate of Change*

The population of Oregon was about 3,831,000 in 2010 and about 3,970,000 in 2014. Find the average rate of change of the population over this four-year period. What will the population of Oregon be in 2024?

Solution:

Over this four-year period, the average rate of change of the population of Oregon was

$$\text{Rate of change} = \frac{\text{change in population}}{\text{change in years}}$$

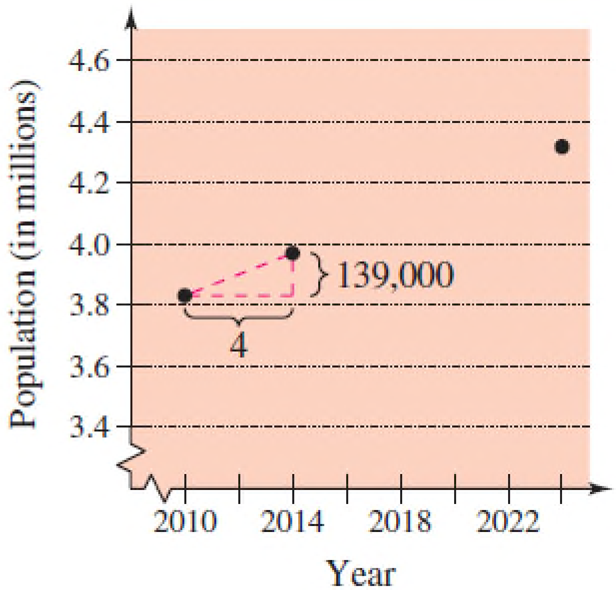
Example 3 – *Solution*

cont'd

$$= \frac{3{,}970{,}000 - 3{,}831{,}000}{2014 - 2010}$$

$$= 34{,}750 \text{ people per year.}$$

Assuming that Oregon's population continues to increase at this same rate for the next 10 years, it will have a 2024 population of about 4,318,000.
(See Figure P.17.)



Population of Oregon

Figure P.17

Ratios and Rates of Change

The rate of change found in Example 3 is an **average rate of change**. An average rate of change is always calculated over an interval.

Graphing Linear Models

Graphing Linear Models

Many problems in analytic geometry can be classified in two basic categories:

1. Given a graph (or parts of it), find its equation.

2. Given an equation, sketch its graph.

For lines, problems in the first category can be solved by using the point-slope form. The point-slope form, however, is not especially useful for solving problems in the second category.

Graphing Linear Models

The form that is better suited to sketching the graph of a line is the **slope-intercept** form of the equation of a line.

The Slope-Intercept Form of the Equation of a Line

The graph of the linear equation

$$y = mx + b \qquad \text{Slope-intercept form}$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

Example 4 – *Sketching Lines in the Plane*

Sketch the graph of each equation.

a. $y = 2x + 1$

b. $y = 2$

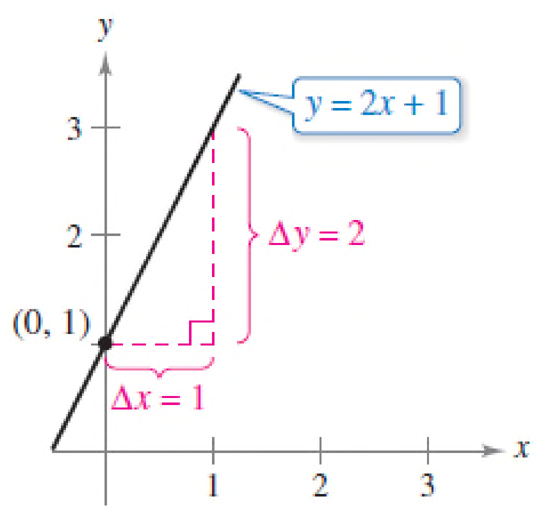
c. $3y + x - 6 = 0$

Example 4(a) – *Solution*

cont'd

Because $b = 1$, the y-intercept is $(0, 1)$.

Because the slope is $m = 2$, you know that the line rises two units for each unit it moves to the right, as shown in Figure P.18(a).



(a) $m = 2$; line rises

Figure P.18(a)

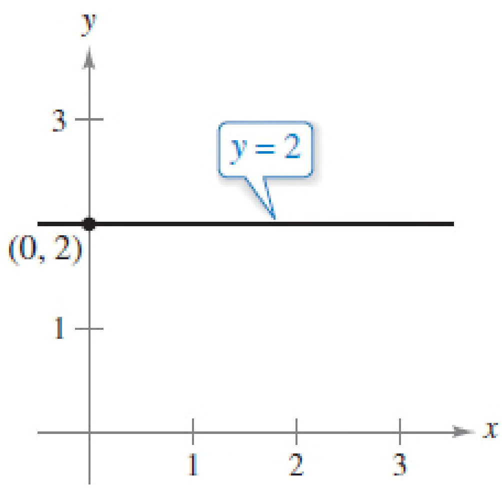
Example 4(b) – *Solution*

cont'd

By writing the equation $y = 2$ in slope-intercept form

$$y = (0)x + 2$$

you can see that the slope is $m = 0$ and the y-intercept is $(0,2)$. Because the slope is zero, you know that the line is horizontal, as shown in Figure P.18(b).



(b) $m = 0$; line is horizontal

Figure P.18(b)

Example 4(c) – *Solution*

cont'd

Begin by writing the equation in slope-intercept form.

$$3y + x - 6 = 0 \qquad \text{Write original equation.}$$

$$3y = -x + 6 \qquad \text{Isolate } y\text{-term on the left.}$$

$$y = -\frac{1}{3}x + 2 \qquad \text{Slope-intercept form}$$

In this form, you can see that the *y*-intercept is (0, 2) and the slope is $m = -\frac{1}{3}$. This means that the line falls one unit for every three units it moves to the right.

Example 4(c) – *Solution*

cont'd

This is shown in Figure P.18(c).

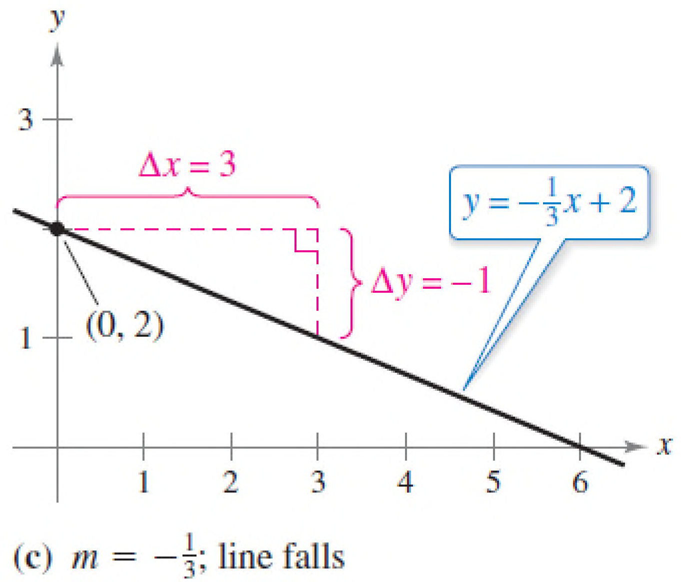


Figure P.18(c)

Graphing Linear Models

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of any line can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

where *A* and *B* are not *both* zero. For instance, the vertical line

$$x = a \qquad \text{Vertical line}$$

can be represented by the general form

$$x - a = 0. \qquad \text{General form}$$

Graphing Linear Models

SUMMARY OF EQUATIONS OF LINES

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure P.19.

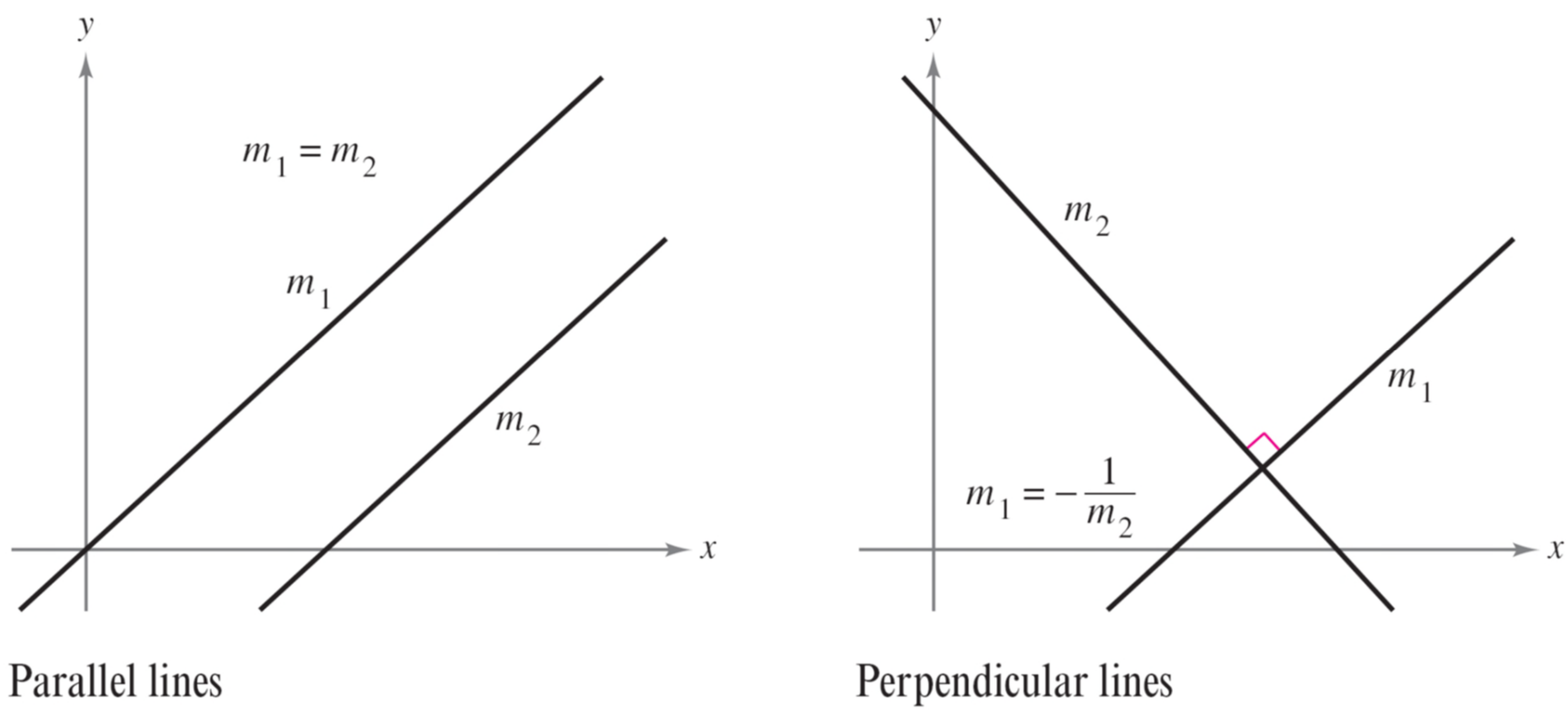


Figure P.19

Parallel and Perpendicular Lines

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if

$$m_1 = m_2.$$

Parallel $\Leftrightarrow$ Slopes are equal.

2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

$$m_1 = -\frac{1}{m_2}.$$

Perpendicular $\Leftrightarrow$ Slopes are negative reciprocals.

Example 5 – *Finding Parallel and Perpendicular Lines*

Find the general forms of the equations of the lines that pass through the point (2, –1) and are

(a) parallel to the line
$2x - 3y = 5$

(b) perpendicular to the line
$2x - 3y = 5$.

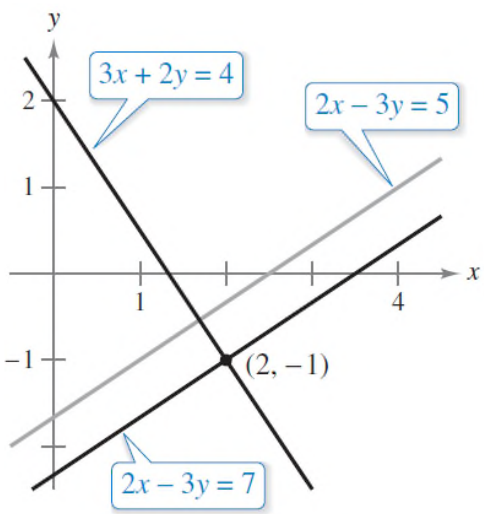
Example 5 – *Solution*

Begin by writing the linear equation $2x - 3y = 5$ in slope-intercept form.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$

$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Slope-intercept form}$$

So, the given line has a slope of $m = \frac{2}{3}$. (See Figure P.20.)



Lines parallel and perpendicular to $2x - 3y = 5$

Figure P.20

Example 5 – *Solution*

cont'd

a. The line through (2, –1) that is parallel to the given line also has a slope of 2/3.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-1) = \tfrac{2}{3}(x - 2) \qquad \text{Substitute.}$$

$$3(y + 1) = 2(x - 2) \qquad \text{Simplify.}$$

$$3y + 3 = 2x - 4 \qquad \text{Distributive Property}$$

$$2x - 3y - 7 = 0 \qquad \text{General form}$$

Note the similarity to the equation of the given line, $2x - 3y = 5$.

Example 5 – *Solution*

cont'd

b. Using the negative reciprocal of the slope of the given line, you can determine that the slope of a line perpendicular to the given line is –3/2.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-1) = -\frac{3}{2}(x - 2)$	Substitute.
$2(y + 1) = -3(x - 2)$	Simplify.
$2y + 2 = -3x + 6$	Distributive Property
$3x + 2y - 4 = 0$	General form