Chapter 2

2-1 Explain, in your own words, the distinction between *average path loss, shadow fading,* and *multipath fading*. How are they related to one another?

Average path loss is the decrease of the far-field average power of the transmitted EM wave over distance at a rate of *1/(distance)*^{n}, where $n \geq 2$, $n = 2$ being the free-space case, $n = 4$ if we assume a two-ray propagation model, with the received signal composed of a direct line-of-sight component and an indirect component by perfect reflection from a flat ground surface.

Shadow fading is the long-term variations in radio signal power about the average power due to terrain obstructions such as hills or buildings. This type of fading is slowly varying, being manifested over relatively long distances (many wavelengths), from tens to hundreds of meters. A good approximation to the effect of shadow fading is to assume that the power measured in decibels (dB) follows a Gaussian or Normal distribution centered about its average value, with some standard deviation ranging, typically, from 6 – 10 dB. The power probability distribution is thus commonly called a log-normal distribution.

Multipath fading is the small-scale variation of the received signal attributed to the destructive/constructive phase interference of many received signal paths. The signal power fluctuates substantially on the order of wavelengths. For macrocellular systems, the amplitude of the received signal due to multipath fading is often modeled as varying randomly according to a Rayleigh distribution. For microcellular systems, with the existence of a direct line-of-sight signal, the small-scale variation is better approximated by a Ricean distribution.

Putting these three phenomena together, the statistically-varying received signal power P_R may be modeled by the following equation:

$$P_R = \alpha^2 10^{\frac{x}{10}} g(d) P_T G_T G_R$$

where P_T is the transmitted signal power, G_T is the transmitter gain, G_R is the receiver gain and *g(d)* is the *1/(distance)*^{n} relation which models the effect of average path loss. The average received power is given by $\overline{P}_R = g(d) P_T G_T G_R$. The terms $10^{x/10}$ and α^2 represent the shadow fading and multipath fading effects respectively. The shadow-fading random variable *x*, measured in dB, is Gaussian-distributed with a zero mean. The multipath-fading random variable α is either Rayleigh-distributed for macrocellular systems, or Ricean-distributed for microcellular systems.

2-2 Using Table 2-1, plot $\overline{P}_{R,db} / \overline{P}_{T,db}$ for Orlando as a function of distance *d*, in meters, with 0 < *d* < 200m. Assume transmitter and receiver antenna gains are both 1.

From (2-5),

$$\overline{P}_R = g(d)\overline{P}_T G_T G_R$$

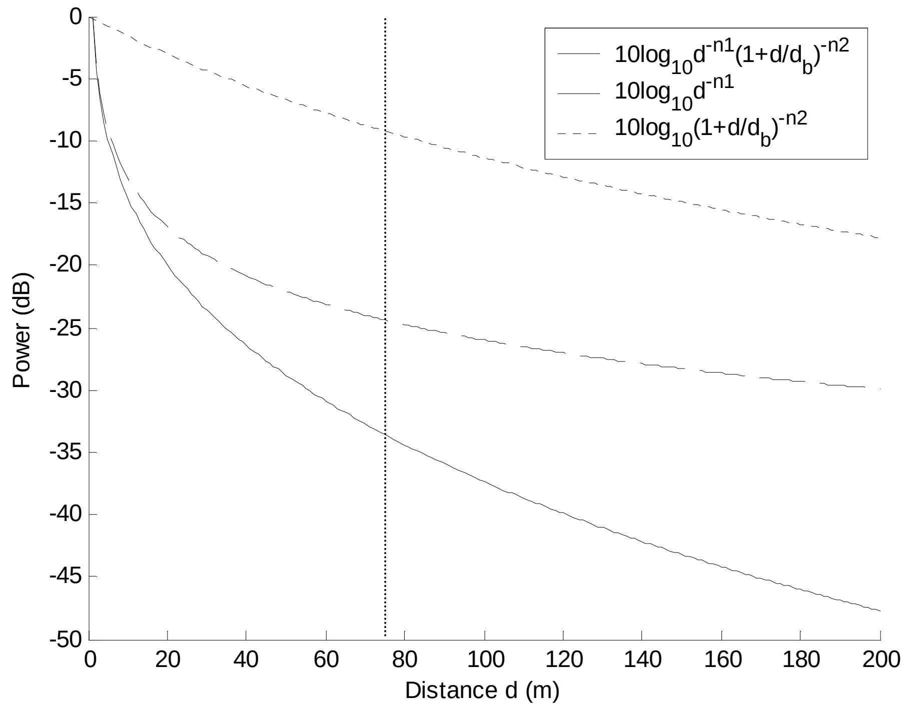
Assume $G_T = G_R = 1$,

$$\overline{P}_R / \overline{P}_T = g(d)$$

$$10\log_{10}(\overline{P}_R / \overline{P}_T) = 10\log_{10} g(d)$$

$$\overline{P}_{R,db} - \overline{P}_{T,db} = 10\log_{10}[d^{-n1}(1+\frac{d}{d_b})^{-n2}]$$

Plot of $\overline{P}_{R,db} - \overline{P}_{T,db}$ with respect to Distance for Orlando



2-3 Determine the shadow-fading parameter σ for each of the four measured curves of Figs. 2-4 and 2-5, and compare. *Hint:* First calculate the average value of each curve and then the root mean-squared value about these averages.

For each curve, we choose 30 data points to compute the shadow-fading parameter σ.

Case (i) 836 MHz; winter values:

-40	-42	-39	-35	-29	-22	-25	-27	-24	-30	-30	-32	-25	-20	-15
-20	-25	-20	-9	-4	0	-10	0	-5	-7	-11	-17	-11	-6	-12

Average received signal level = -19.7 dB; σ = 12 dB

Case (ii) 836 MHz; summer values:

-44	-45	-46	-42	-33	-28	-28	-36	-26	-35	-39	-40	-30	-30	-24
-25	-28	-28	-12	-14	-4	-10	-8	-3	-8	-20	-21	-17	-13	-20

Average received signal level = -25.2 dB; σ = 12.6 dB

Case (iii) 11.2 GHz; winter values:

-46	-45	-37	-30	-30	-29	-39	-28	-42	-42	-35	-30	-28	-26	-28
-34	-28	-15	-23	-2	-20	-6	-6	-4	-10	-16	-17	-15	-16	-20

Average received signal level = -25.9 dB; σ = 12.4 dB

Case (iv) 11.2 GHz; summer values:

-55	-57	-57	-54	-48	-42	-48	-50	-44	-57	-57	-55	-54	-44	-40
-46	-50	-45	-32	-16	-15	-15	-15	-17	-21	-30	-34	-24	-29	-38

Average received signal level = -39.6 dB; σ = 14.8 dB

2-4 The average power received at mobiles 100 m from a base station is 1 mW. Log-normal, shadow, fading is experienced at that distance.

a. What is the probability that the received power at a mobile at that distance from the base station will exceed 1 mW? Be less than 1 mW?

0.5

b. The log-normal standard deviation σ is 6 dB. An acceptable received signal is 10 mW or higher. What is the probability that a mobile will have an acceptable signal? Repeat for σ = 10 dB. Repeat both cases for an acceptable received signal of 6 mW. *Note:* Solutions here require the integration of the Gaussian function. Most mathematical software packages contain the means to do this. Most books on probability and statistics have tables of the error function used for just this purpose. The error function is defined in chapter 3 of this text. See (3-12).

Case (i) $p_0 \geq 10\ mW$; $\sigma = 6\ dB$

$$P(p \geq p_0) = \frac{1}{2} - \frac{1}{2} erf(\frac{10\log_{10} 10}{\sqrt{2}\times 6}) = 0.0478$$

Case (ii) $p_0 \geq 10\ mW$; $\sigma = 10\ dB$

$$P(p \geq p_0) = \frac{1}{2} - \frac{1}{2} erf(\frac{10\log_{10} 10}{\sqrt{2}\times 10}) = 0.1587$$

Case (iii) $p_0 \geq 6\ mW$; $\sigma = 6\ dB$

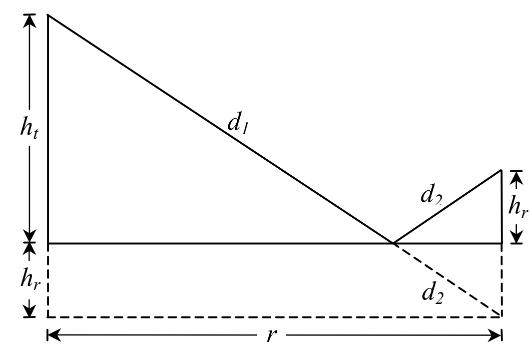
$$P(p \geq p_0) = \frac{1}{2} - \frac{1}{2} erf(\frac{10\log_{10} 6}{\sqrt{2}\times 6}) = 0.0973$$

Case (iv) $p_0 \geq 6\ mW$; $\sigma = 10\ dB$

$$P(p \geq p_0) = \frac{1}{2} - \frac{1}{2} erf(\frac{10\log_{10} 6}{\sqrt{2}\times 10}) = 0.2182$$

2-5 a. Fill in the details of the derivation of the two-ray average received power result given by (2-13a).

Step 1: Derivation of $\Delta\phi \approx 4\pi h_t h_r / \lambda d$

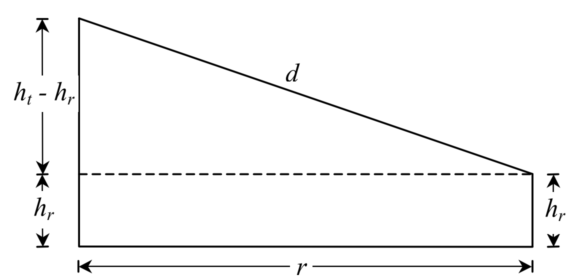


With reference to the first diagram on the right, we can prove by Pythagoras' Theorem that

$$d_1 + d_2 = \sqrt{r^2 + (h_t + h_r)^2} \qquad (1)$$

With reference to the second diagram on the right, we have

$$r^2 = d^2 - (h_t - h_r)^2 \qquad (2)$$



Substitute (2) into (1), we have

$$d_1 + d_2 = \sqrt{d^2 - (h_t - h_r)^2 + (h_t + h_r)^2}$$

$$d_1 + d_2 = \sqrt{d^2 + 4h_t h_r} \qquad (3)$$

Assume $d^2 >> 4h_t h_r$,

$$\Delta d = (d_1 + d_2) - d = \sqrt{d^2 + 4h_t h_r} - d$$

$$\approx \sqrt{d^2 + 4h_t h_r + (2h_t h_r / d)^2} - d$$
$$= (d + 2h_t h_r / d) - d = 2h_t h_r / d \qquad (4)$$

Substitute (4) into $\Delta\phi \equiv 2\pi\Delta d / \lambda$, we have $\Delta\phi \equiv 4\pi h_t h_r / \lambda d$.

Step 2: Derivation of $\left|1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi}\right|^2 \approx (\Delta\phi)^2$

Assume $\Delta\phi$ is small and $d/(d_1 + d_2) \approx 1$

$$\left|1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi}\right|^2 \approx \left|(1 - \cos\Delta\phi)^2 + j\sin\Delta\phi\right|^2 \approx \left|j\sin\Delta\phi\right|^2 = (\sin\Delta\phi)^2 \approx (\Delta\phi)^2 \qquad (5)$$

Finally, we can obtain (2-13a) by substituting the results in Steps 1 and 2:

$$\overline{P}_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \left|1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi}\right|^2 \approx P_T G_T G_R \left(\frac{\lambda\Delta\phi}{4\pi d}\right)^2 \approx P_T G_T G_R \frac{(h_t h_r)^2}{d^4}$$

b. Superimpose a $1/d^4$ curve on the measured curves of Figs. 2-7 and 2-8, and compare with the measured curves.

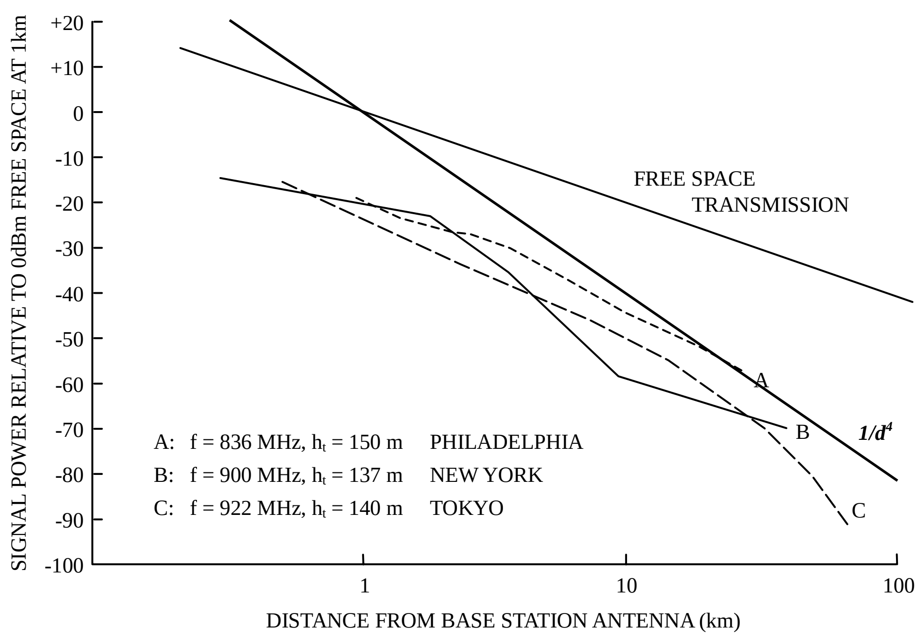


Fig. 2-7 Received signal power as function of distance

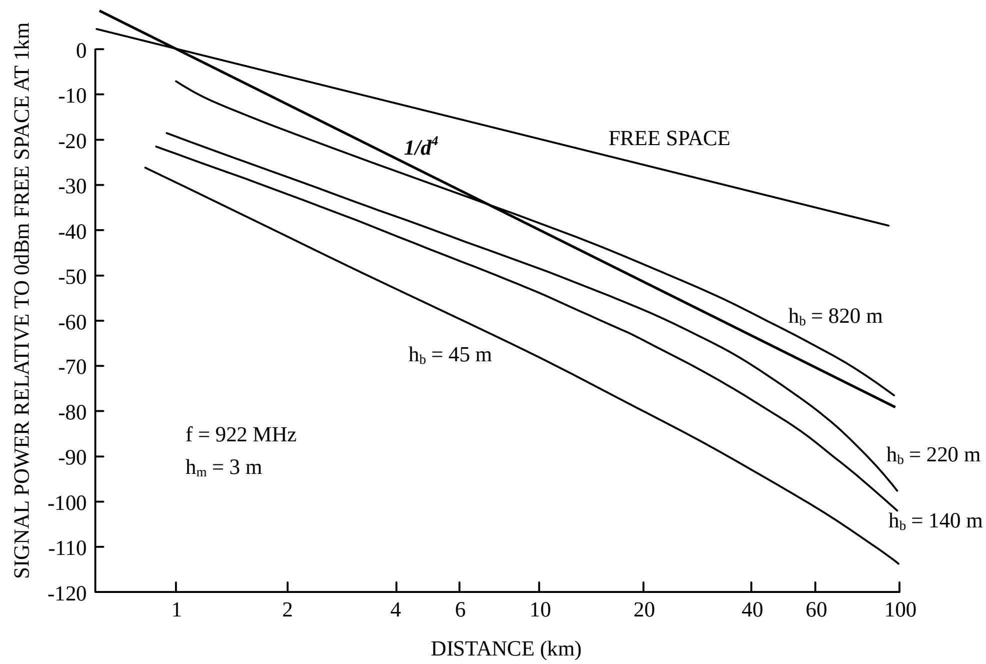


Fig. 2-8 Effect of base station antenna height on received power

In Fig. 2-7, we can see that for all the measured curves, the rates of decrease of the received signal power only come close to that of the $1/d^4$ curve when the distance from the base station antenna is large enough. This is due to the fact that in the derivation of (2-13a), $\Delta\phi \equiv 4\pi h_t h_r / \lambda d$ is assumed to be small. For instance, take $\Delta\phi \leq 0.6\, radian$, signal frequency $f_c = 836MHz$ ($\lambda = 0.36m$), $h_t = 150m$ and $h_r = 1m$, we have $d \geq 4\pi \times 150/(0.36 \times 0.6) = 8.7km$. In Fig. 2-8, all the measured curves have slopes similar to that of the $1/d^4$ curve, and raising the base station antenna tends to increase the strength of the received signal.

c. Do the results of Fig. 2-8 validate (2-13a)? Explain.

Yes. First, all the measured curves in Fig. 2-8 have the received signal powers decrease with distance at a rate similar to that of the $1/d^4$ curve. Second, as the height of the base station antenna h_b increases, the received signal power increases. Both observations agree with (2-13a).

2-6 a. Verify, as indicated in the text, that, for the Rayleigh-distributed random variable α in (2-15), σ_r^2 must equal 1/2.

From (2-4), the instantaneous received signal power $P_R = \alpha^2 10^{\frac{x}{10}} g(d) P_T G_T G_R$. Taking expectation on both sides, we have

$$\overline{P}_R = E[P_R] = E[\alpha^2]E[10^{\frac{x}{10}}]g(d)P_T G_T G_R$$

Since the average received power is just $P_T g(d) G_T G_R$ and $E[10^{\frac{x}{10}}]=1$, this implies $E[\alpha^2]=1$. From (2-15),

$$E[\alpha^2] = \int_0^\infty \frac{\alpha^3}{\sigma_r^2} e^{-\alpha^2/2\sigma_r^2} d\alpha$$

Substitute $y=\alpha^2$, $dy = 2\alpha d\alpha$, we have

$$E[\alpha^2] = \int_0^\infty \frac{y}{2\sigma_r^2} e^{-y/2\sigma_r^2} dy = -\int_0^\infty y d(e^{-y/2\sigma_r^2}) = -[ye^{-y/2\sigma_r^2} + 2\sigma_r^2 e^{-y/2\sigma_r^2}]_0^\infty = 2\sigma_r^2$$

Therefore, $E[\alpha^2] = 2\sigma_r^2 = 1$ or $\sigma_r^2 = 1/2$

b. Derive (2-17a) from (2-17) and show that x and y are zero-mean random variables, each with variance $\sigma_R{}^2$ as defined.

The actual received normalized signal $S_R(t)$ is given by (2-17):

$$S_R(t) = \sum_{k=1}^{L} a_k \cos[\omega_c(t-t_0)+\phi_k]$$

Expanding (2-17) by trigonometry, we get

$$\begin{aligned} S_R(t) &= \sum_{k=1}^{L} a_k[\cos\phi_k \cos\omega_c(t-t_0) - \sin\phi_k \sin\omega_c(t-t_0)] \\ &= \sum_{k=1}^{L} a_k \cos\phi_k \cos\omega_c(t-t_0) - \sum_{k=1}^{L} a_k \sin\phi_k \sin\omega_c(t-t_0) \\ &= x\cos\omega_c(t-t_0) - y\sin\omega_c(t-t_0) \end{aligned}$$

where $x \equiv \sum_{k=1}^{L} a_k \cos\phi_k$ and $y \equiv \sum_{k=1}^{L} a_k \sin\phi_k$.

Consider the means of x and y.

$$E[x] = E[\sum_{k=1}^{L} a_k \cos\phi_k] = L\cdot E[a_k]E[\cos\phi_k] \qquad E[y] = E[\sum_{k=1}^{L} a_k \sin\phi_k] = L\cdot E[a_k]E[\sin\phi_k]$$

Since ϕ_k is uniformly distributed between 0 and 2π, $E[\cos\phi_k] = E[\sin\phi_k] = 0$. Hence,

$$E[x] = E[y] = 0$$

Now, consider the variances of x and y.

$$\begin{aligned} \sigma_x^2 &= E[(x-\overline{x})^2] = E[(\sum_{k=1}^{L} a_k \cos\phi_k - 0)^2] \\ &= E[\sum_{k=1}^{L} a_k^2 \cos^2\phi_k] + 2E[\sum_{i\neq j} a_i a_j \cos\phi_i \cos\phi_j] \end{aligned}$$

$$= E[\sum_{k=1}^{L} a_k^2 (\frac{1+\cos 2\phi_k}{2})] + 0 = \frac{1}{2}\sum_{k=1}^{L} E(a_k^2) = \sigma_R^2$$

$$\sigma_y^2 = E[(y-\overline{y})^2] = E[(\sum_{k=1}^{L} a_k \sin\phi_k - 0)^2]$$

$$= E[\sum_{k=1}^{L} a_k^2 \sin^2\phi_k] + 2E[\sum_{i\neq j} a_i a_j \sin\phi_i \sin\phi_j]$$

$$= E[\sum_{k=1}^{L} a_k^2 (\frac{1-\cos 2\phi_k}{2})] + 0 = \frac{1}{2}\sum_{k=1}^{L} E(a_k^2) = \sigma_R^2$$

Therefore, both x and y have the same variance σ_R^2.

c. Starting with the Rayleigh distribution (2-21) for the received signal envelope a, show the instantaneous received power P_R obeys the exponential distribution of (2-23).

From (2-22), $P_R = ca^2/2 \quad \Rightarrow \quad a^2 = 2P_R/c$

Differentiate (2-22) with respect to P_R on both sides, we have $\frac{da}{dP_R} = \frac{1}{ca}$

From simple probability theory,

$$f_{P_R}(P_R) = f_a(a)\left|\frac{da}{dP_R}\right|$$

Substitute (2-21) and $a^2 = 2P_R/c$, we have

$$f_{P_R}(P_R) = \frac{1}{c\sigma_R^2} e^{-a^2/2\sigma_R^2} = \frac{1}{c\sigma_R^2} e^{-P_R/c\sigma_R^2}$$

From (2-20), $c\sigma_R^2 = p$. Put it into the above equation and we can obtain the exponential distribution of (2-23):

$$f_{P_R}(P_R) = \frac{1}{p} e^{-P_R/p}$$

2-7 a. Show, following the hints provided in the text, that the Ricean distribution (2-25) approaches a Gaussian distribution centered about A for $A^2/2\sigma_R^2 \gg 1$.

From (2-25), $f_a(a) = \frac{a}{\sigma_R^2} e^{-(\frac{a^2+A^2}{2\sigma_R^2})} I_0(\frac{aA}{\sigma_R^2})$

According to the hints, $I_0(z) \to e^z/\sqrt{2\pi z}$ when $z \gg 1$. This corresponds to the case $A^2/2\sigma_R^2 \gg 1$. With this assumption, (2-25) becomes

$$f_a(a) = \frac{a}{\sigma_R^2} e^{-(\frac{a^2+A^2}{2\sigma_R^2})} \cdot e^{\frac{aA}{\sigma_R^2}} \cdot \frac{\sigma_R}{\sqrt{2\pi aA}} = \frac{1}{\sigma_R\sqrt{2\pi}} \cdot \sqrt{\frac{a}{A}} e^{-(\frac{a^2-2aA+A^2}{2\sigma_R^2})} = \frac{1}{\sigma_R\sqrt{2\pi}} \cdot \sqrt{\frac{a}{A}} e^{-\frac{(a-A)^2}{2\sigma_R^2}}$$

Therefore, it is readily seen that $f_a(a)$ peaks at about A, and, in the vicinity of that value of the amplitude, is closely Gaussian.

b. Verify that the instantaneous received power distribution in the case of a direct ray is given by (2-27). Show that, as the K-factor gets smaller (the direct line-of-sight ray decreases relative to the scattered signal terms), the fading distribution of (2-27) approaches a Rayleigh distribution.

From (2-22), $P_R = ca^2/2 \quad \Rightarrow \quad a^2 = 2P_R/c$ ....(i)

Take expectation on both sides of (2-22),

$$p = E[P_R] = cE[a^2]/2$$

Since

$$E[a^2] = E[(A+x)^2 + y^2] = A^2 + 2\sigma_R^2 = 2\sigma_R^2(1+K)$$

we have

$$p = c\sigma_R^2(1+K) \qquad \text{or} \qquad c = p/\sigma_R^2(1+K) \quad \text{.............................(ii)}$$

Put (ii) into (i)

$$a = \sqrt{2P_R\sigma_R^2(1+K)/p} \quad \text{..(iii)}$$

Differentiate (2-22) with respect to P_R on both sides, we have $\frac{da}{dP_R} = \frac{1}{ca}$ ....(iv)

From simple probability theory,

$$f_{P_R}(P_R) = f_a(a)\left|\frac{da}{dP_R}\right|$$

Substitute (iii), (iv) and (2-25) into the above expression,

$$f_{P_R}(P_R) = \frac{1}{c\sigma_R^2} e^{-(\frac{1+K}{p}\cdot P_R + \frac{A^2}{2\sigma_R^2})} I_0(\sqrt{\frac{4(1+K)}{p}\cdot P_R \cdot \frac{A^2}{2\sigma_R^2}})$$

Put $K \equiv A^2/2\sigma_R^2$ and $c\sigma_R^2 = p/(1+K)$, we have

$$f_{P_R}(P_R) = \frac{(1+K)e^{-K}}{p} e^{-\frac{1+K}{p}\cdot P_R} I_0(\sqrt{\frac{4K(1+K)}{p}\cdot P_R})$$

Put *K=0*, the expression for the instantaneous received power distribution becomes

$$f_{P_R}(P_R) = \frac{1}{p} e^{-\frac{P_R}{p}}$$

Therefore, it is shown that as the K-factor gets smaller, $f_{P_R}(P_R)$ approaches an exponential distribution.

2-8 As will be seen throughout the text, simulation is commonly used to determine the performance, as well as verify analysis, of cellular systems. Most critical here is the simulation of fading conditions. This problem provides an introduction to the simulation of Rayleigh fading.

a. Consider a sequence of *n* random numbers x_i, j =1 to *n*, uniformly distributed from 0 to 1. (Pseudo-random number generators are often available in mathematical software packages.) Let $x = (b/n)\sum_{j=1}^{n}(x_j - 1/2)$. Show x approximates a Gaussian random variable of zero average value and variance $\sigma^2 = b^2/12n$. Repeat for another set of *n* (independent) uniformly-distributed random numbers, calling the sum obtained in this case *y*. Using *x* and *y*, generate a Rayleigh-distributed random variable. Comparing with (2-21), what is the Rayleigh parameter σ_R^2 in this case? *Hint*: Consider the derivation of (2-21) starting with (2-19) and the discussion in the text following.

By the Central Limit Theorem of probability, for large *n*, the random variable *x*, defined as the sum of *n* random variables, becomes approximately Gaussian-distributed. Now we will derive the mean and variance of *x*.

Mean: $E(x) = (b/n)\sum_{j=1}^{n}[E(x_j) - 1/2] = 0$

Variance: $\sigma^2 = E[(x-0)^2] = (b/n)^2 \sum_{j=1}^{n} E[(x_j - 1/2)^2]$

$$= (\frac{b}{n})^2 \cdot n \cdot \int_{-1/2}^{1/2} y_j^2 dy_j = \frac{b^2}{n} \cdot \left[\frac{y_j^3}{3}\right]_{-1/2}^{1/2} = \frac{b^2}{12n}$$

We can generate a Rayleigh-distributed random variable *a* from *x* and *y* by the formula $a = \sqrt{x^2 + y^2}$ and in this case the Rayleigh parameter $\sigma_R^2 = b^2/12n$.

b. A different method of obtaining the Rayleigh distribution directly from a uniformly-distributed random variable *x* is to write the expression

$$a = \sqrt{-2\sigma^2 \log_e x}$$

Show the variable *a* is Rayleigh-distributed. How would you now use a sequence of uniformly-distributed random numbers to generate a Rayleigh distribution?

Rewrite the expression of *a*, we get

$$x = \exp(-\frac{a^2}{2\sigma^2})$$

As a increases from 0 to ∞, x decreases from 1 to 0. Since x is bounded by 0 and 1, the probability density function $f_x(x) = 1$.

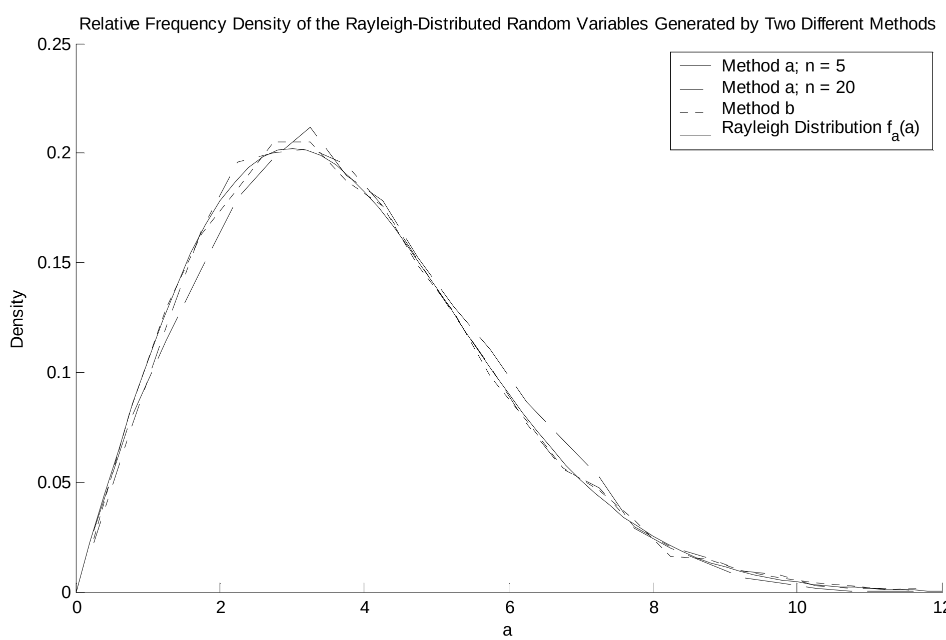
$$P(a < a_0) = \int_{\exp(-\frac{a_0^2}{2\sigma^2})}^{\exp(0)} dx = 1 - \exp(-\frac{a_0^2}{2\sigma^2})$$

$$f_a(a) = \frac{d}{da}[1 - \exp(-\frac{a^2}{2\sigma^2})] = \frac{a}{\sigma^2}\exp(-\frac{a^2}{2\sigma^2})$$

Therefore, a is Rayleigh-distributed. We can obtain Rayleigh-distributed random numbers a_i from a sequence of uniformly-distributed random numbers x_i by directly applying the formula $a_i = \sqrt{-2\sigma^2 \log_e x_i}$.

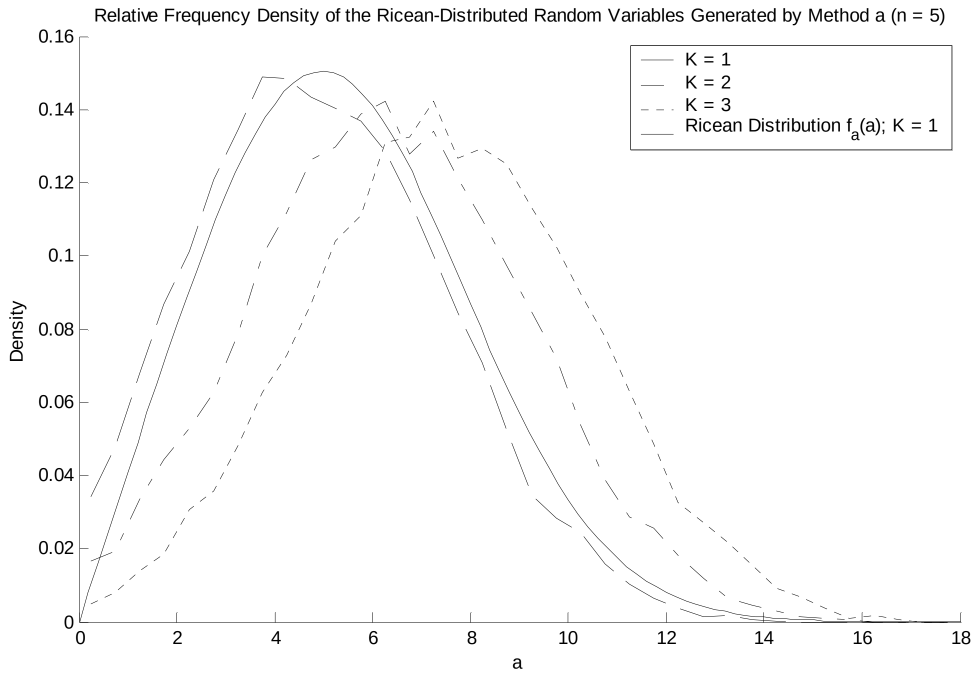
c. Choosing various values for n, generate the Rayleigh distribution using the two methods of **a.** and **b.**, and compare, both with each other and with a plot of the Rayleigh distribution. What is the effect on these results of varying n?

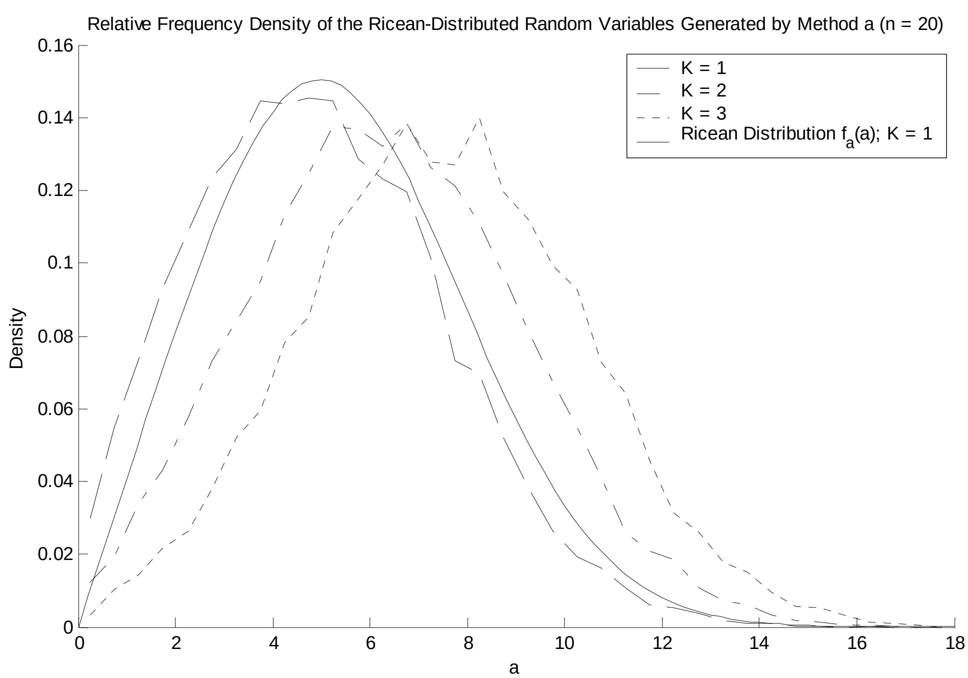
Choose $\sigma_R^2 = 9$. Hence, in method **a.**, the value of the parameter b is $\sqrt{108n}$. Each plot is based on the statistics of $k = 10,000$ data points generated by the approximation methods. To obtain a plot of the relative frequency density, we first divide the x-axis into equal intervals each of width w. Then we count the number of data points c_i in each interval. The relative frequency density of interval i is then given by $c_i/(k \times w)$. From the plots we find that by increasing the value of n, method **a.** gives a better approximation to the Rayleigh distribution.



d. Show how you would extend the method of **a.** to generate the Ricean distribution. Generate and plot this distribution using the pseudo-random numbers generated in **c.** for various values of the Ricean *K*-factor.

We can extend the method of **a.** to generate the Ricean distribution by the formula $a = \sqrt{(A+x)^2 + y^2}$, where A is a positive constant.





2-9 Consider the average fade duration equation (2-39). Take the case of a vehicle moving at a speed of 100 km/hr. The system frequency of operation is 1 GHz. Say the ratio ρ = 1. Show the average fade duration is 8 msec, as noted in the text. Now let the received signal amplitude be 0.3 of the rms value. Show the average fade duration is now 1 msec.

Case (i) $\rho = 1$, $f_m = \dfrac{v}{\lambda} = \dfrac{100\times 10^3/(60\times 60)}{3\times 10^8/10^9} = 92.59 Hz$ $\quad \tau_f = \dfrac{e-1}{92.59\times\sqrt{2\pi}} = 7.4 m\sec.$

Case (ii) $\rho = 0.3$, $f_m = 92.59 Hz$ $\quad \tau_f = \dfrac{e^{0.3^2}-1}{0.3\times 92.59\times\sqrt{2\pi}} = 1.35 m\sec.$

2-10 a. Summarize, in your own words, the discussion in the text on time dispersion and frequency-selective fading.

In a wireless medium, due to reflection, scattering, diffraction and refraction, a signal usually reaches a destination through multiple paths of various distances. Thus, a receiver, instead of getting a copy of the signal, often obtains replicas of a signal which arrive at different times. The superposition of the delayed replicas results in the broadening of the signal. This is known as time dispersion, which corresponds to a non-flat frequency response of the channel. For relatively large signal bandwidth, one encounters frequency-selective fading, with different frequency components of the signal being handled differently over the channel, leading to signal distortion. For the case of digital signals, this distortion introduced by frequency-selective fading manifests itself in intersymbol interference (ISI), with successive digital symbols overlapping into adjacent symbol intervals.

b. Consider several cases: a delay spread of 0.5 μsec, one of 1 μsec, and a third one of 6 μsec. Determine whether individual multipath rays are resolvable for the two transmission bandwidths, 1.25 MHz used in IS-95 and cdma2000, and 5 MHz used in WCDMA. (See Chapter 10)

For IS-95 and cdma2000, multipath echoes appearing much greater than $1/(2\pi\times 1.25)\mu\sec \approx 0.13\mu\sec$ apart will be resolvable. Therefore, multipath rays should be resolvable for delay spreads of $1\mu\sec$ and $6\mu\sec$, but probably not for the delay spread of $0.5\mu\sec$.

For WCDMA, multipath echoes appearing much greater than $1/(2\pi\times 5)\mu\sec$ $\approx 0.032\mu\sec$ apart will be resolvable, which is valid for all the three cases of delay spread.

2-11 Indicate the condition for flat fading for each of the following data rates:

8 kbps, 40 kbps, 100 kbps, 6 Mbps.

Indicate which, if any, radio environments would result in flat fading for each of these data rates.

Data Rate	Symbol Interval	Flat Fading Condition	Radio Environment for Flat Fading
8 kbps	125 μsec	$\tau_{av} < 25$ μsec	Most environments
40 kbps	25 μsec	$\tau_{av} < 5$ μsec	Some urban environments and most suburban and rural environments
100 kbps	10 μsec	$\tau_{av} < 2$ μsec	Most suburban and rural environments
6 Mbps	0.167 μsec	$\tau_{av} < 0.033$ μsec	Some indoor picocellular environments

2-12 **a.** Consider the transversal filter equalizer of Fig. 2-20. A training sequence of K binary digits is used to determine the $2N + 1$ tap gains, as described in the text. Show that, under a minimum mean-squared performance objective, the optimum choice of tap gains is given by (2-53).

Under a minimum mean-squared performance objective, we want to choose the tap gains h_n, $\quad -N \le n \le N$, such that $\sum_{j=1}^{K}(s_j - \hat{s}_j)^2$ is minimum.

To obtain the optimum choice of tap gains, we find the solution of

$$\frac{\partial}{\partial h_l}\sum_{j=1}^{K}(s_j - \hat{s}_j)^2 = \sum_{j=1}^{K}2(s_j - \hat{s}_j)\frac{\partial \hat{s}_j}{\partial h_l} = 0$$

Since $\hat{s}_j = \sum_{n=-N}^{N} h_n r_{j-n}$ and $\frac{\partial \hat{s}_j}{\partial h_l} = r_{j-l}$, the above equation becomes

$$\sum_{j=1}^{K}2(s_j - \hat{s}_j)r_{j-l} = 0$$

$$\Rightarrow \qquad \sum_{j=1}^{K}s_j r_{j-l} = \sum_{j=1}^{K}\hat{s}_j r_{j-l}$$

$$\Rightarrow \qquad \sum_{j=1}^{K}s_j r_{j-l} = \sum_{j=1}^{K}\sum_{n=-N}^{N}h_n r_{j-n} r_{j-l} \qquad\qquad -N \le l \le N$$

b. Show the vector form of (2-53) is given by (2-56), with the solution given by (2-57).

Define $R_{l,n} \equiv \sum_{j=1}^{K} r_{j-n} r_{j-l}$ and $g_l \equiv \sum_{j=1}^{K} s_j r_{j-l}$, $-N \le l, n \le N$. Equation (2-53) then takes on the simpler looking form

$$g_l = \sum_{n=-N}^{N} h_n R_{l,n} \qquad -N \le l \le N$$

In matrix notation,

$$\begin{pmatrix} g_{-N} \\ g_{-N+1} \\ \vdots \\ g_N \end{pmatrix} = \begin{pmatrix} R_{-N,-N} & R_{-N,-N+1} & \cdots & R_{-N,N} \\ R_{-N+1,-N} & R_{-N+1,-N+1} & \cdots & R_{-N+1,N} \\ \vdots & & & \vdots \\ R_{N,-N} & R_{N,-N+1} & \cdots & R_{N,N} \end{pmatrix} \begin{pmatrix} h_{-N} \\ h_{-N+1} \\ \vdots \\ h_N \end{pmatrix} \qquad \text{or} \qquad \boldsymbol{g} = \boldsymbol{Rh}$$

Multiply $\boldsymbol{R}^{-1}$ to both sides, we have

$$\boldsymbol{R}^{-1}\boldsymbol{g} = (\boldsymbol{R}^{-1}\boldsymbol{R})\boldsymbol{h} \qquad \text{or} \qquad \boldsymbol{h} = \boldsymbol{R}^{-1}\boldsymbol{g}$$

2-13 a. Work out a simple example of the transversal filter equalizer: Say the equalizer has three taps to be found using the minimum mean-squared performance objective. Choose a set of K = 10 arbitrarily-chosen binary digits as the training sequence and then let some of these digits be received in "error", i.e. some are converted to the opposite polarity. Find the "best" set of taps in this case. Try to choose the training sequence so that there are equal numbers of +1 and –1 digits. Compare the tap coefficients with those found using the approximation of (2-58).

Choose the training sequence $\{s_j\}$ = {1 1 -1 1 -1 -1 -1 1 1 -1}

And the received sequence $\{r_j\}$ = {1 1 1 1 -1 -1 -1 -1 1 -1}
with the third and eighth bits received in errors.

From equations (2-54), (2-55) and (2-57), we find that

$$\boldsymbol{g} = \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix} \quad \boldsymbol{R} = \begin{pmatrix} 10 & 3 & 2 \\ 3 & 10 & 3 \\ 2 & 3 & 10 \end{pmatrix} \quad \text{and} \quad \boldsymbol{h} = \boldsymbol{R}^{-1}\boldsymbol{g} = \frac{1}{816}\begin{pmatrix} 91 & -24 & -11 \\ -24 & 96 & -24 \\ -11 & -24 & 91 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.1716 \\ 0.6471 \\ -0.3284 \end{pmatrix}$$

By using the approximation of (2-58), the tap coefficients

$$\boldsymbol{h}_{approx} = \frac{1}{10} \cdot \boldsymbol{g} = \frac{1}{10} \cdot \begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.6 \\ -0.1 \end{pmatrix}$$

The approximated coefficients obtained by (2-58) are very close to the exact coefficients obtained by (2-57). The mean-squared difference given by the approximated tap coefficients is just 0.0763 higher than the optimum value.

b. Repeat this example for a different set of transmitted digits and errors in reception.

`{`s_j`} = {1 -1 -1 1 -1 1 -1 -1 1 1}`

`{`r_j`} = {1 -1 1 1 1 1 -1 -1 1 1}` (third and fifth bits received in errors)

$$\boldsymbol{g} = \begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix} \qquad \boldsymbol{R} = \begin{pmatrix} 10 & 1 & -2 \\ 1 & 10 & 1 \\ -2 & 1 & 10 \end{pmatrix} \qquad \boldsymbol{h} = \frac{1}{936}\begin{pmatrix} 99 & -12 & 21 \\ -12 & 96 & -12 \\ 21 & -12 & 99 \end{pmatrix}\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -0.3718 \\ 0.6410 \\ -0.0385 \end{pmatrix}$$

$$\boldsymbol{h}_{approx} = \frac{1}{10}\cdot\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -0.3 \\ 0.6 \\ 0.1 \end{pmatrix}$$

c. Choose a larger example of a transversal filter equalizer and repeat **a.** and **b.**, comparing with the results obtained there.

In the following two examples, we use transversal filter equalizers of 11 taps (N = 5) adapted with 30-bit training sequences (K = 30). Since the matrix manipulation is cumbersome, a computer program was written to solve the optimum tap coefficients $\boldsymbol{h}$.

Example 1

{s_j} = `{1 1 -1 1 -1 -1 -1 1 1 -1 1 1 -1 1 -1 -1 -1 1 1 -1 1 1 -1 1 -1 -1 -1 1 1 -1}`

{r_j} = `{1 1 -1 1 1 1 1 1 1 -1 1 1 -1 1 -1 -1 -1 1 1 -1 1 1 -1 1 -1 -1 -1 1 1 -1}` (fifth to seventh bits received in error)

$$\boldsymbol{g} = \begin{pmatrix} -1 \\ -2 \\ 5 \\ -8 \\ -7 \\ 24 \\ -7 \\ -4 \\ 5 \\ -2 \\ -3 \end{pmatrix} \quad \boldsymbol{R} = \begin{pmatrix} 30 & -1 & -2 & 7 & 0 & 1 & -2 & 9 & -6 & -7 & 14 \\ -1 & 30 & -1 & -2 & 7 & 0 & 1 & -2 & 9 & -6 & -7 \\ -2 & -1 & 30 & -1 & -2 & 7 & 0 & 1 & -2 & 9 & -6 \\ 7 & -2 & -1 & 30 & -1 & -2 & 7 & 0 & 1 & -2 & 9 \\ 0 & 7 & -2 & -1 & 30 & -1 & -2 & 7 & 0 & 1 & -2 \\ 1 & 0 & 7 & -2 & -1 & 30 & -1 & -2 & 7 & 0 & 1 \\ -2 & 1 & 0 & 7 & -2 & -1 & 30 & -1 & -2 & 7 & 0 \\ 9 & -2 & 1 & 0 & 7 & -2 & -1 & 30 & -1 & -2 & 7 \\ -6 & 9 & -2 & 1 & 0 & 7 & -2 & -1 & 30 & -1 & -2 \\ -7 & -6 & 9 & -2 & 1 & 0 & 7 & -2 & -1 & 30 & -1 \\ 14 & -7 & -6 & 9 & -2 & 1 & 0 & 7 & -2 & -1 & 30 \end{pmatrix} \quad \boldsymbol{h} = \begin{pmatrix} 0.0080 \\ -0.0472 \\ -0.0598 \\ -0.1492 \\ -0.2215 \\ 0.7994 \\ -0.1814 \\ -0.0115 \\ -0.0245 \\ -0.0222 \\ -0.1230 \end{pmatrix}$$

$$\boldsymbol{h}_{approx} = (-0.0333 \;\; -0.0667 \;\; 0.1667 \;\; -0.2667 \;\; -0.2333 \;\; 0.8000 \;\; -0.2333 \;\; -0.1333 \;\; 0.1667 \;\; -0.0667 \;\; -0.1000)^T$$

Example 2

$\{s_j\}$ = {1 -1 -1 1 -1 1 -1 -1 1 1 1 -1 -1 1 -1 1 -1 -1 1 1 1 -1 -1 1 -1 1 -1 -1 1 1}

$\{r_j\}$ = {1 -1 -1 1 -1 1 -1 -1 -1 -1 -1 -1 -1 1 -1 1 -1 -1 1 1 1 -1 -1 1 -1 1 -1 -1 1 1} (ninth to eleventh bits received in error)

$$\boldsymbol{g} = \begin{pmatrix} 13 \\ -4 \\ -1 \\ -4 \\ -9 \\ 24 \\ -9 \\ -4 \\ -1 \\ -4 \\ 13 \end{pmatrix} \quad \boldsymbol{R} = \begin{pmatrix} 30 & -3 & 2 & 1 & -2 & 11 & 0 & -3 & 0 & -5 & 12 \\ -3 & 30 & -3 & 2 & 1 & -2 & 11 & 0 & -3 & 0 & -5 \\ 2 & -3 & 30 & -3 & 2 & 1 & -2 & 11 & 0 & -3 & 0 \\ 1 & 2 & -3 & 30 & -3 & 2 & 1 & -2 & 11 & 0 & -3 \\ -2 & 1 & 2 & -3 & 30 & -3 & 2 & 1 & -2 & 11 & 0 \\ 11 & -2 & 1 & 2 & -3 & 30 & -3 & 2 & 1 & -2 & 11 \\ 0 & 11 & -2 & 1 & 2 & -3 & 30 & -3 & 2 & 1 & -2 \\ -3 & 0 & 11 & -2 & 1 & 2 & -3 & 30 & -3 & 2 & 1 \\ 0 & -3 & 0 & 11 & -2 & 1 & 2 & -3 & 30 & -3 & 2 \\ -5 & 0 & -3 & 0 & 11 & -2 & 1 & 2 & -3 & 30 & -3 \\ 12 & -5 & 0 & -3 & 0 & 11 & -2 & 1 & 2 & -3 & 30 \end{pmatrix} \quad \boldsymbol{h} = \begin{pmatrix} 0.1146 \\ 0.0548 \\ 0.0016 \\ -0.2065 \\ -0.2417 \\ 0.7024 \\ -0.2417 \\ -0.2065 \\ 0.0016 \\ 0.0548 \\ 0.1146 \end{pmatrix}$$

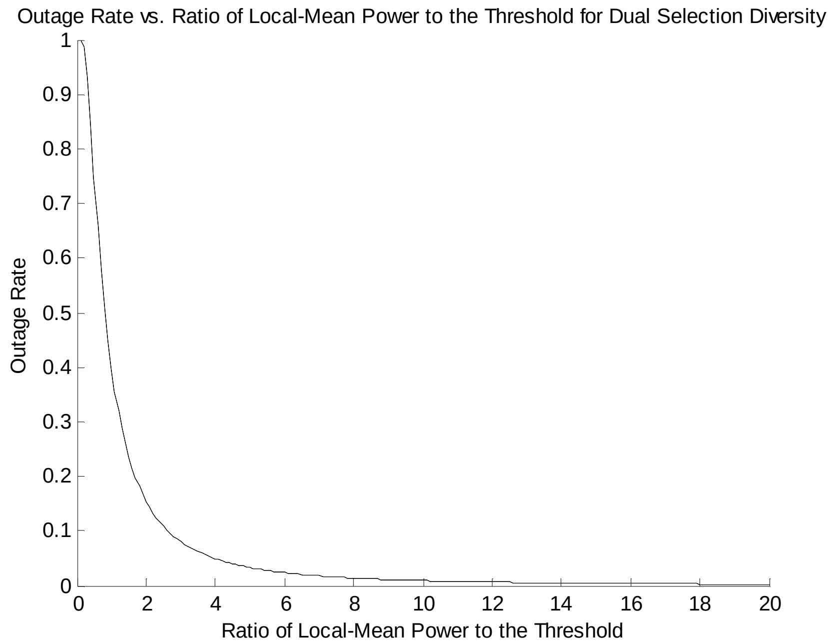
$$\boldsymbol{h}_{approx} = (0.4333\ \ -0.1333\ \ -0.0333\ \ -0.1333\ \ -0.3000\ \ 0.8000\ \ -0.3000\ \ -0.1333\ \ -0.0333\ \ -0.1333\ \ 0.4333)^T$$

A comparison of the four examples of traversal filter equalizers is summarized in the table below.

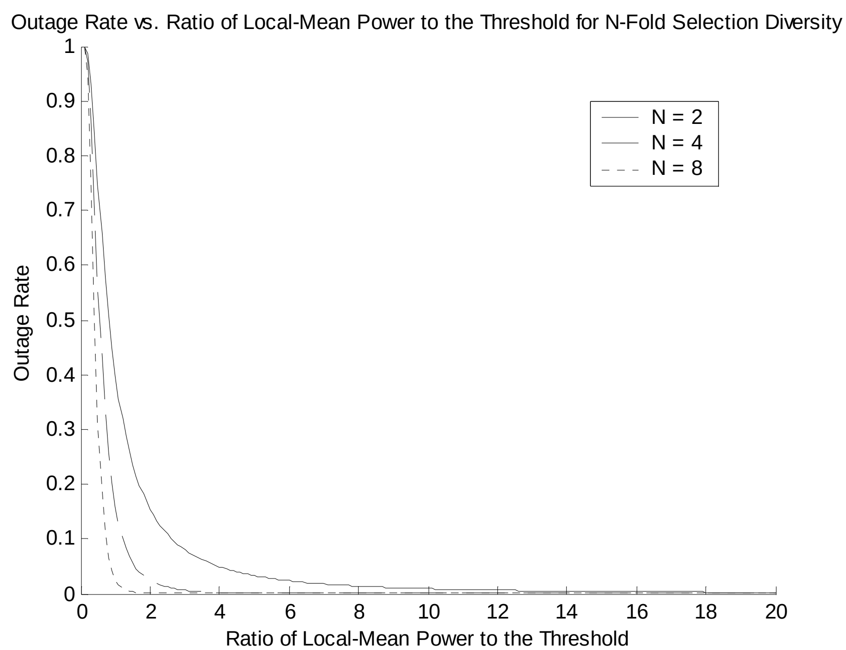
	K	$2N$+1	Avg. distance between optimum and approx. h_j's $\frac{1}{2N+1}\sum_{j=-N}^{N}\left\|h(j)-h_{approx}(j)\right\|$	Optimum ms diff. $\frac{1}{K}\sum_{j=1}^{K}(s_j-\hat{s}_j)^2$ (s_j - training seq.)	Increase in ms diff. with approx. $\boldsymbol{h}$
1.	10	3	0.1346	0.5137	0.0763
2.	10	3	0.0838	0.4937	0.0243
3.	30	11	0.0772	0.2128	0.0917
4.	30	11	0.1313	0.1397	0.4456

In general, the approximated tap coefficients are very close to the exact optimum coefficients obtained by (2-57). Larger traversal filter equalizers with more tap coefficients and adapted by longer training sequences usually give better estimated sequences, i.e. resulting in a smaller mean square difference between the transmitted and the estimated signal sequences.

2-14 **a.** Plot the improvement in performance obtained with the use of dual selection diversity as the ratio of local-mean power to the threshold varies. Use at least the following cases: (1) the local-mean power 20 times the threshold; (2) local-mean power 10 times the threshold; (3) local-mean power equal to the threshold; (4) local-mean power 0.1 of the threshold. *Note:* Performance may be defined as outage rate or the probability that at least one of the channels has an instantaneous power greater than the threshold.



b. Repeat **a.** for four- and eight-fold diversity and compare all three orders of diversity.



In general, the higher the order of diversity, the better the performance. However, as the ratio of local-mean power to threshold increases, the improvement becomes less significant. For a ratio larger than three, there is no much difference between four- and eight-fold diversities. Besides, if the order of diversity is high enough, it will be hard to improve the performance further by more diversity branches.

2-15 Show the optimum maximal-ratio combining gain for the *k*th diversity branch is given by (2-64). Explain the statement that the SIR is then the sum of the SIRs, summed over the N diversity branches.

From (2-61) $$SIR \equiv |s|^2 / I_0 = \left| \sum_{k=1}^{N} g_k a_k \right|^2 / \sum_{k=1}^{N} |g_k|^2 |n'_k|^2$$

According to Schwartz's inequality for complex numbers,

$$\left| \sum_{k=1}^{N} c_k^* d_k \right| \le \left(\sum_{k=1}^{N} |c_k|^2 \right) \cdot \left(\sum_{k=1}^{N} |d_k|^2 \right)$$

Choose $c_k = a_k^* / |n'_k|$ and $d_k = g_k |n'_k|$, we have

$$\left| \sum_{k=1}^{N} a_k g_k \right| \le \left(\sum_{k=1}^{N} |a_k|^2 / |n'_k|^2 \right) \cdot \left(\sum_{k=1}^{N} |g_k|^2 |n'_k|^2 \right)$$

$$\Rightarrow \qquad SIR \equiv \left| \sum_{k=1}^{N} a_k g_k \right| / \sum_{k=1}^{N} |g_k|^2 |n'_k|^2 \le \sum_{k=1}^{N} |a_k|^2 / |n'_k|^2$$

SIR is maximized when equality is obtained with $d_k = Kc_k$, i.e. $g_k |n'_k| = Ka_k^* / |n'_k|$ or $g_k = Ka_k^* / |n'_k|^2$, which is the expression in (2-64).

Since the SIR in this case is $\sum_{k=1}^{N} |a_k|^2 / |n'_k|^2$, and the SIR on the *k*th diversity branch is just given by $|a_k|^2 / |n'_k|^2$, we can state that the resultant SIR is the sum of the SIRs over the N diversity branches.

2-16 **a.** Explain how equal-gain combining differs from maximal-ratio combining. In particular, write an expression for the SIR in the case of equal-gain combining. *Hint*: How would this expression compare with (2-61)?

Equal-gain combining differs from maximal-ratio combining in that the gains g_k over all diversity branches are the same, rather than being adjusted according to the SIR on the respective branch. Hence, in equal-gain combining the diversity channel outputs are simply added together. The SIR in this case is

$$SIR = \left|\sum_{k=1}^{N} a_k\right|^2 / \sum_{k=1}^{N} |n'_k|^2$$

b. Why would you expect the performance of diversity schemes to be ranked in the order maximal-ratio combining best, equal-gain next best, selection diversity last?

We can rank the performance of the three diversity schemes by comparing their respective SIR's. First, the SIR of the maximal-ratio scheme must excel that of the equal-gain scheme, based on the fact that the branch gains of the maximal-ratio schemes are adapted to maximize the SIR. Next, we will show that the SIR of the equal-gain scheme should in general be larger than that of the selection scheme. Consider the SIR of the selection scheme. Suppose the signal on the *i*th branch has the largest SIR, then

$$SIR_{selection} = \max_k\{\frac{|a_k|^2}{|n'_k|^2}\} = \frac{|a_i|^2}{|n'_i|^2}$$

Since the SIR on each branch $|a_k|^2 / |n'_k|^2$ is usually greater than one, we can write the following inequality:

$$SIR_{selection} = \frac{|a_i|^2}{|n'_i|^2} < \frac{\left|a_i + \sum_{k \neq i} a_k\right|^2}{|n'_i|^2 + \sum_{k \neq i} |n'_k|^2} = \frac{\left|\sum_{k=1}^{N} a_k\right|^2}{\sum_{k=1}^{N} |n'_k|^2} = SIR_{equal-gain}$$

Therefore, it can be concluded that the performance of maximal-ratio combining is the best, equal-gain combining is the next best, and selection combining is the last.

2-17 a. Explain the operation of the RAKE receiver in your own words.

A RAKE receiver boosts up the signal reception performance by combining separately-arriving rays of a signal transmitted over a fading channel. The technique can only be applied to very wideband wireless systems, such as the CDMA, in which the delay spread over a fading channel is greater than the symbol period, and as a result individual components of the multipath signal can be separately distinguished. The differential delays as well as relative phases and amplitudes of the individual multipath components need to be estimated accurately, so that the different received rays can be shifted in time, compensating for the differential delays, and then be combined using the maximal-ratio scheme. In practice, only a few of the earliest arriving rays, normally the strongest in power, are used to carry out the RAKE processing.

b. Two third-generation CDMA systems are discussed in chapter 10. The first, W-CDMA, uses a chip rate of 3.84 Mcps (million chips per second), with a corresponding chip duration of 0.26 μsec; the second system, cdma2000, uses a chip rate of 1.2288 Mcps, with a chip duration of 0.81 μsec. (This is the same chip rate used by the second-generation CDMA system IS-95 discussed in chapters 6 and 8.) Explain the statements made in chapter 10 that RAKE receivers can be used to provide multipath time-diversity for paths differing in time by at least those two chip durations, respectively. Which system potentially provides better RAKE performance?

In CDMA, multipath components separated in time over a chip duration are orthogonal to each other and hence can be individually received by applying the same pseudorandom code at different times which correspond to the arrivals of the delayed signals. The receiving system can rapidly scan through different delay values of the received rays, searching for the sequence corresponding to the one transmitted. Once a number of such differentially-delayed received signals are identified, delay compensation can be carried out and maximal-ratio combining used to recover the transmitted sequence.

W-CDMA, having a shorter chip duration, should potentially provide a better RAKE performance.