Solutions Manual
for
Microwave Engineering 3/e

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3/23/04

3/04

Solutions Manual

for

Microwave Engineering
Third Edition

Contained here are solutions for all of the end-of-chapter problems in the third edition of Microwave Engineering. Some of these problems require the derivation of theoretical results, but many are design oriented. Some of these problems are easy, while others are lengthy and challenging. Many of the matching, coupler, filter, and amplifier design problems ask for a CAD analysis of the final circuit, where it is presumed that the student has access to a microwave CAD software tool, such as Ansoft's Serenade, or similar. There are several such packages that are available for free download on the Internet. The Wiley web site contains Serenade files for the problems and examples amenable to CAD analysis.

Working problems is a critical part of the learning process for engineering students, and these problems have been developed to give students practice in applying the basic concepts of microwave engineering, as well as practice in the analysis and design of practical microwave circuits and components. These problems can be used as assigned homework problems, exam problems, or as supplemental problems for students to work out on their own. The present edition features many new and revised problems, but if additional problems are needed, it should be easy for the instructor to derive new problems from those given in the text. Also new to this edition is the inclusion of short answers to many of the problems at the back of the text.

The majority of these solutions have been checked with known results, compared with independent solutions by others or, in the case of design problems, verified by computer simulation. Such results usually have a check mark to indicate that they have a high (but not perfect!) likelihood of correctness. Nevertheless, there are undoubtedly some errors that remain, and the author will be grateful if such mistakes are brought to his attention.

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Chapter 1

1.1 As in Example 1.3, assume outgoing plane wave fields in each region. To get J_{sx}, we need H_y, since $\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s$ ($\hat{n} = \hat{z}$). Then we must have E_x to get $\bar{S} = \bar{E} \times \bar{H}^* = \pm S \hat{z}$. So the form of the fields must be,

for $z<0$, $\bar{E}_1 = \hat{x} A e^{jk_o z}$ $\quad$ for $z>0$, $\bar{E}_2 = \hat{x} B e^{-jkz}$

$\bar{H}_1 = \frac{-\hat{y}}{\eta_o} A e^{jk_o z}$ $\quad$ $\bar{H}_2 = \hat{y} \frac{B}{\eta} e^{-jkz}$

with $k_o = \omega\sqrt{\mu_o \epsilon_o}$, $k = \omega\sqrt{\mu_o \epsilon_o \epsilon_r}$, $\eta_o = \sqrt{\mu_o/\epsilon_o}$, $\eta = \sqrt{\mu_o/\epsilon_o \epsilon_r}$, and A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = 0 \quad \Rightarrow \quad A = B$$

$$\hat{z} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s \quad \Rightarrow \quad -\left(\frac{B}{\eta} + \frac{A}{\eta_o}\right) = J_o$$

$$\therefore \quad A = B = \frac{-J_o \eta \eta_o}{\eta + \eta_o}$$

(1.2)
$$\nabla\times\bar{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial\rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right)$$

$$\nabla\times\nabla\times\bar{E} = \hat{\rho}\left[\frac{-1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi}\right]$$
$$+\hat{\phi}\left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} - \frac{\partial^2 E_\phi}{\partial\rho^2} - \frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\phi\partial\rho}\right]$$
$$+\hat{z}\left[\frac{-\partial^2 E_z}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z} - \frac{1}{\rho}\frac{\partial E_z}{\partial\rho}\right]$$

$$\nabla(\nabla\cdot\bar{E}) = \hat{\rho}\left[\frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} - \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi} - \frac{E_\rho}{\rho^2}\right]$$
$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi}\right]$$
$$+\hat{z}\left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z}\right]$$

If we apply ∇^2 to the cylindrical components of $\bar{E}$ we get:

$$\nabla^2\bar{E} \overset{?}{=} \hat{\rho}\nabla^2 E_\rho + \hat{\phi}\nabla^2 E_\phi + \hat{z}\nabla^2 E_z \qquad \text{(THIS IS NOT A VALID STEP!)}$$
$$= \hat{\rho}\left[\frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} + \frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial z^2}\right]$$
$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{\partial^2 E_\phi}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{\partial^2 E_\phi}{\partial z^2}\right]$$
$$+\hat{z}\left[\frac{1}{\rho}\frac{\partial E_z}{\partial\rho} + \frac{\partial^2 E_z}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_z}{\partial z^2}\right]$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla\times\nabla\times\bar{E}$ and $\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2\bar{E} \neq \hat{\rho}\nabla^2 E_\rho + \hat{\phi}\nabla^2 E_\phi + \hat{z}\nabla^2 E_z$. The $\hat{z}$ components are equal.

(1.3) $\bar{S} = \bar{E} \times \bar{H}^* = E_0 H_0 \hat{z} \neq 0$

The problem here is that Poynting's theorem requires a closed surface integral for a meaningful interpretation in terms of power flow. If we calculate $\oint \bar{S} \cdot d\bar{s}$ over the closed surface of a cube bounded by the magnet faces and the capacitor plates, we will get zero, since $\hat{n} = \hat{z}$ on one side of the cube, while $\hat{n} = -\hat{z}$ on the other opposite side. Since S is a constant, these terms cancel.

(1.4) $E_y = E_0 \cos(\omega t - kz)$, $f = 2.4\,GHz$, $E_0 = 30\ V/m$

a) $\eta = 377/\sqrt{2.55} = 236\ \Omega$

$H_x = -E_y/\eta = -0.127 \cos(\omega t - kz)$

b) $v_p = c/\sqrt{\epsilon_r} = 1.88 \times 10^8$ m/sec

c) $k = \omega/v_p = 80.2\ m^{-1}$

$\Delta\phi = k\Delta z = 5514° = 114°$

(1.5) Let $\bar{E} = A(\hat{x} - j\hat{y}) e^{-jk_0 z} + B(\hat{x} + j\hat{y}) e^{-jk_0 z}$

where A is the amplitude of the RHCP component, and B is the amplitude of the LHCP component. Equating this expression to the given linearly polarized field gives,

$\hat{x}: \quad A + B = E_0$

$\hat{y}: \quad -jA + jB = 2E_0$

Solving for A, B gives

$A = (\frac{1}{2} + j) E_0$

$B = (\frac{1}{2} - j) E_0$

Any linearly polarized wave (in any direction) can be decomposed into the sum of two circularly polarized waves.

(1.6) From eq. (1.76),

$$\bar{H} = \frac{1}{\eta_0}\hat{n}\times\bar{E} \quad , \quad \bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$$

$$\bar{S} = \bar{E}\times\bar{H}^* = \frac{1}{\eta_0}\bar{E}\times\hat{n}\times\bar{E}^*$$

$$= \frac{1}{\eta_0}\left[(\bar{E}\cdot\bar{E}^*)\hat{n} - (\bar{E}\cdot\hat{n})\bar{E}^*\right] \qquad \text{(from B.5)}$$

Since $\bar{k}\cdot\bar{E}_0 = k_0\hat{n}\cdot\bar{E}_0 = 0$ from (1.69) and (1.74), we have

$$\bar{S} = \frac{\hat{n}}{\eta_0}\bar{E}\cdot\bar{E}^* = \frac{\hat{n}}{\eta_0}|E_0|^2 \quad W/m^2 \quad ✓$$

(1.7) Writing general plane wave fields in each region:

$\bar{E}^i = \hat{x}\, e^{-jk_0 z}$	$\bar{H}^i = \frac{\hat{y}}{\eta_0} e^{-jk_0 z}$	for $z<0$
$\bar{E}^r = \hat{x}\,\Gamma e^{jk_0 z}$	$\bar{H}^r = \frac{-\hat{y}}{\eta_0}\Gamma e^{jk_0 z}$	for $z<0$
$\bar{E}^s = \hat{x}(A e^{-jkz} + B e^{jkz})$	$\bar{H}^s = \frac{\hat{y}}{\eta}(A e^{-jkz} - B e^{jkz})$	for $0<z<d$
$\bar{E}^t = \hat{x}\, T\, e^{-jk_0(z-d)}$	$\bar{H}^t = \frac{\hat{y}}{\eta_0} T e^{-jk_0(z-d)}$	for $z>d$

Now match E_x and H_y at $z=0$ and $z=d$ to obtain four equations for Γ, T, A, B:

$z=0$: $\quad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma) = \frac{1}{\eta}(A-B)$

$z=d$: $\quad j(-A+B) = T \qquad \frac{j}{\eta}(-A-B) = \frac{T}{\eta_0} \qquad$ (since $d = \lambda_0/4\sqrt{\epsilon_r}$)

Solving for Γ gives

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2} \quad ✓$$

CHECK:

$\lambda/4$ TRANSFORMER $\Rightarrow Z_{in} = \eta^2/\eta_0$, $\quad \Gamma = \dfrac{\eta^2/\eta_0 - \eta_0}{\eta^2/\eta_0 + \eta_0} = \dfrac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2}$.

(1.8) The incident, reflected, and transmitted fields can be written as,

$$\bar{E}^i = E_0(\hat{x} - j\hat{y})e^{-jk_0 z} \qquad \bar{H}^i = j\frac{E_0}{\eta_0}(\hat{x}-j\hat{y})e^{-jk_0 z} \qquad \text{(RHCP)}$$

$$\bar{E}^r = E_0\Gamma(\hat{x} - j\hat{y})e^{jk_0 z} \qquad \bar{H}^r = -j\frac{E_0}{\eta_0}\Gamma(\hat{x}-j\hat{y})e^{jk_0 z} \qquad \text{(LHCP)}$$

$$\bar{E}^t = E_0 T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \bar{H}^t = j\frac{E_0}{\eta}T(\hat{x}-j\hat{y})e^{-\gamma z} \qquad \text{(RHCP)}$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}, \qquad T = \frac{2\eta}{\eta+\eta_0}$$

The Poynting vectors are: $\qquad (\hat{x}-j\hat{y})\times(\hat{x}-j\hat{y})^* = 2j\hat{z}$

For $z<0$: $\bar{S}^- = (\bar{E}^i + \bar{E}^r)\times(\bar{H}^i+\bar{H}^r)^* = \dfrac{2\hat{z}|E_0|^2}{\eta_0}\left(1-|\Gamma|^2 + \Gamma e^{2jk_0 z} + \Gamma^* e^{-2jk_0 z}\right)$ ✓

For $z>0$: $\bar{S}^+ = \bar{E}^t \times \bar{H}^{t*} = \dfrac{2\hat{z}|E_0|^2|T|^2}{\eta^*}e^{-2\alpha z}$ ✓

at $z=0$,

$$\bar{S}^- = \frac{2\hat{z}|E_0|^2}{\eta_0}\left(1-|\Gamma|^2+\Gamma-\Gamma^*\right) = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

$$\bar{S}^+ = 2\hat{z}|E_0|^2\frac{4\eta}{|\eta+\eta_0|^2} \qquad \left(\text{using } T = \frac{2\eta}{\eta+\eta_0}\right)$$

$$= \frac{2\hat{z}|E_0|^2}{\eta_0}\left(\frac{2\eta}{\eta+\eta_0}\right)\left(\frac{2\eta_0}{\eta+\eta_0}\right)^* = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

Thus $\bar{S}^- = \bar{S}^+$ at $z=0$, and power is conserved.

1.9 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = 2\pi j f\sqrt{\mu_0\epsilon_0}\sqrt{5-j2} = j\frac{2\pi(1000)}{300}\sqrt{5.385\angle{-22^\circ}}$$

$$= 48.5\angle 79^\circ = 9.25 + j47.6 = \alpha + j\beta \quad \text{(neper/m, rad/m)}$$

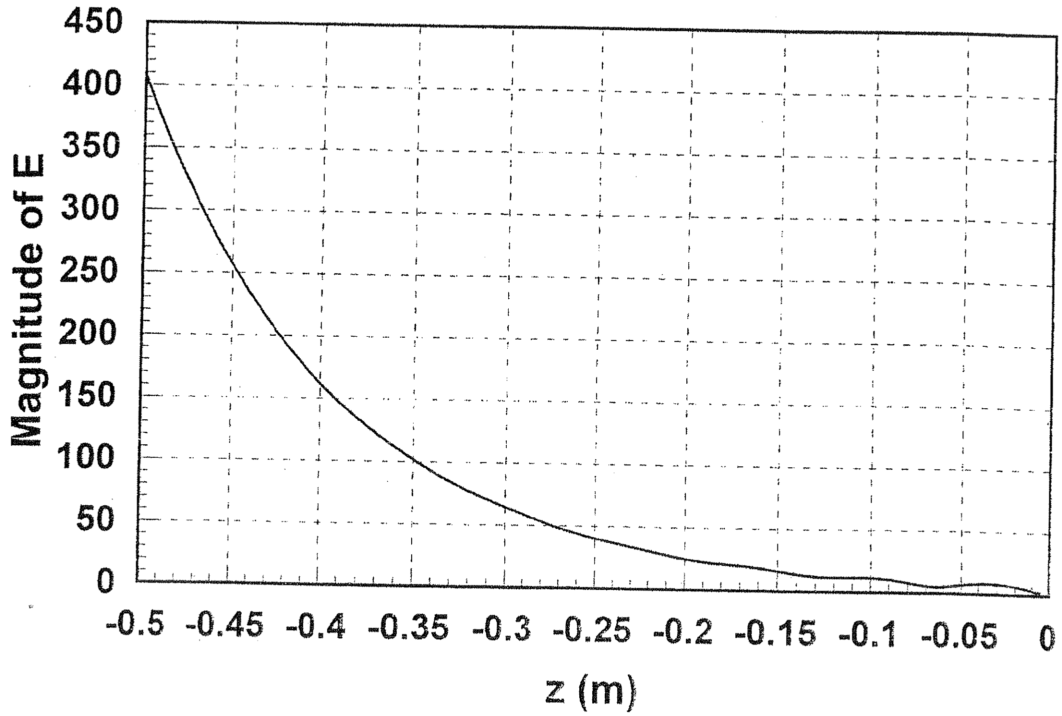
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{\cancel{j\omega}\sqrt{\cancel{\mu_0}\epsilon_0}}{\cancel{j\omega}\sqrt{\cancel{\mu_0}\epsilon}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32\angle{-11^\circ}} = 163\angle 11^\circ \ \Omega$$

$$\Gamma = -1$$

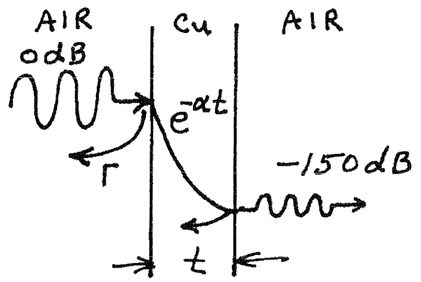
For $z<0$, $\bar{E} = \bar{E}^i + \bar{E}^r = 4\hat{x}\,(e^{-\gamma z} - e^{\gamma z})$

$$|\bar{E}| = 4\left|e^{-\alpha z}e^{-j\beta z} - e^{\alpha z}e^{j\beta z}\right|$$

$|\bar{E}|$ vs z is plotted below.



(1.10) The total loss through the sheet is the product of the transmission losses at the air-copper and copper-air interfaces, and the exponential loss through the sheet.



$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 2.09\times10^{-6}\,m = \frac{1}{\alpha}$$

$$\eta_c = \frac{(1+j)}{\sigma\delta_s} = 8.2\times10^{-3}(1+j)\,\Omega$$

a) Power transfer from air into copper is given by,

$$1-|\Gamma|^2 \quad , \quad \Gamma = \frac{\eta_c-\eta_0}{\eta_c+\eta_0} \simeq \frac{8.2\times10^{-3}(1+j)-377}{377} = -0.999956 + j\,4.35E{-5}$$

This yields a power transfer of -40.6 dB into the copper. By symmetry, the same transfer occurs for the copper-air interface.

b) the attenuation within the copper sheet is,

$$\text{copper att.} = 150\,dB - 40.6\,dB - 40.6\,dB = 68.8\,dB$$

$$= -20\log e^{-t/\delta_s} \Rightarrow \underline{t = 0.017\ mm}\ ✓$$

(J. Mead provided this correction on 9/04)

1.11 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_o \epsilon} = j\frac{2\pi(3000)}{300}\sqrt{3(1-j.1)} = 5.435 + j108.964 = \alpha + j\beta \quad m^{-1}$$

$$\eta = \frac{\eta_o}{\sqrt{\epsilon_r(1-j.1)}} = 217.121\angle 2.855°$$

(a) $$P_i = Re\left\{\frac{|\bar{E}_i(z=0)|^2}{\eta^*}\right\} = 46.000 \quad W/m^2 \quad ✓$$

$\Gamma = -1$ at $z = l = 20$ cm

$$\bar{E}_r = \Gamma \bar{E}_i(z=l)\, e^{\gamma(z-l)} = -100\hat{x}\, e^{-2\gamma l} e^{\gamma z}$$

$$P_r = Re\left\{\frac{|\bar{E}_r(z=0)|^2}{\eta^*}\right\} = 0.595 \quad W/m^2 \quad ✓$$

(b) $\bar{E}_t = \bar{E}_i + \bar{E}_r$

$$\bar{E}_t(z=0) = 100\hat{x}(1 - e^{-2\gamma l}) \qquad H_t(z=0) = \frac{100\hat{y}}{\eta}(1 + e^{-2\gamma l})$$

$$P_{in} = Re\{\bar{E}_t \times \bar{H}_t^* \cdot \hat{z}\} = 45.584 \quad W/m^2$$

But $P_i - P_r = 45.405\ W/m^2 \neq P_{in}$. This is because P_i and P_r individually are not physically meaningful in a lossy medium.

The above values were computed entirely using a FORTRAN program, with 6-digit precision. The error between $P_i - P_r$ and P_{in} is only about 0.4% – this could be made more significant if the loss were increased.

(1.12) This current sheet will generate obliquely propagating plane waves. From (1.132)–(1.133), assume

$$\left.\begin{aligned} \bar{E}_1 &= A(\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1)\, e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)} \\ \bar{H}_1 &= \frac{-A}{\eta_o}\,\hat{y}\, e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)} \end{aligned}\right\} \text{for } z<0$$

$$\left.\begin{aligned} \bar{E}_2 &= B(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \\ \bar{H}_2 &= \frac{B}{\eta}\,\hat{y}\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \end{aligned}\right\} \text{for } z>0$$

with $k_o = \omega\sqrt{\mu_o\epsilon_o}$, $k = \sqrt{\epsilon_r}\,k_o$, $\eta_o = \sqrt{\mu_o/\epsilon_o}$, $\eta = \eta_o/\sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z}\times(\bar{E}_2 - \bar{E}_1) = 0 \implies A\cos\theta_1 e^{-jk_o x\sin\theta_1} - B\cos\theta_2 e^{-jkx\sin\theta_2} = 0$$

$$\hat{z}\times(\bar{H}_2 - \bar{H}_1) = J_s \implies \frac{A}{\eta_o} e^{-jk_o x\sin\theta_1} + \frac{B}{\eta} e^{-jkx\sin\theta_2} = -J_o e^{-j\beta x}$$

For phase matching we must have $k_o\sin\theta_1 = k\sin\theta_2 = \beta$

$\therefore\ \theta_1 = \sin^{-1}\beta/k_o$ ✓ $\quad \theta_2 = \sin^{-1}\beta/k$ (must have $\beta < k_o$)

Then,

$$A\cos\theta_1 = B\cos\theta_2, \quad \frac{A}{\eta_o} + \frac{B}{\eta} = -J_o$$

$$A = \frac{-J_o\eta\eta_o\cos\theta_2}{\eta\cos\theta_2 + \eta_o\cos\theta_1}, \quad B = \frac{-J_o\eta\eta_o\cos\theta_1}{\eta\cos\theta_2 + \eta_o\cos\theta_1}$$

Check: If $\beta = 0$, then $\theta_1 = \theta_2 = 0$, and $A = B = \dfrac{-J_o\eta\eta_o}{\eta + \eta_o}$,

which agrees with Problem 1.1 ✓

(1.13) This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1, k_2, η_1, and η_2. Thus,

$$k_0 \sin\theta_i = k_0 \sin\theta_r = k \sin\theta_t \quad ; \; k = k_0\sqrt{\mu_r}$$

$$\Gamma = \frac{\eta\cos\theta_t - \eta_0\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i} \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

$$\eta = \eta_0\sqrt{\mu_r}$$

There will be a Brewster angle if $\Gamma = 0$. This requires that,

$$\eta\cos\theta_t = \eta_0\cos\theta_i$$

$$\sqrt{\mu_r}\sqrt{1-\left(\frac{k_0}{k}\right)^2\sin^2\theta_i} = \cos\theta_i = \sqrt{1-\sin^2\theta_i}$$

$$\mu_r\left(1-\frac{1}{\mu_r}\cancel{\sin^2\theta_i}\right) = 1-\cancel{\sin^2\theta_i}$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

(1.14) Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1, k_2, η_1, η_2. Thus,

$$\Gamma = \frac{\eta\cos\theta_i - \eta_0\cos\theta_t}{\eta\cos\theta_i + \eta_0\cos\theta_t}, \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

A Brewster angle exists if

$$\eta\cos\theta_i = \eta_0\cos\theta_t$$

$$\sqrt{\mu_r}\sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{1}{\mu_r}\sin^2\theta_i}$$

$$\mu_r^2 - \mu_r^2\sin^2\theta_i = \mu_r - \sin^2\theta_i$$

$$\mu_r = (\mu_r+1)\sin^2\theta_i$$

$$\sin\theta_i = \sin\theta_b = \sqrt{\frac{\mu_r}{1+\mu_r}} < 1 \quad ✓$$

Thus, a Brewster angle does exist for this case.

(1.15) $$\bar{E} = 2\hat{x} + 3\hat{y} + 4\hat{z}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 1 & -2j & 0 \\ 2j & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2-6j \\ 9+4j \\ 16 \end{bmatrix}$$

(1.16) $$D_x = \epsilon_0(\epsilon_r E_x + jKE_y)$$
$$D_y = \epsilon_0(-jKE_x + \epsilon_r E_y)$$
$$D_z = \epsilon_0 E_z$$

Then, $$D_+ = D_x - jD_y = \epsilon_0(\epsilon_r - K)E_x - j\epsilon_0(\epsilon_r - K)E_y = \epsilon_0(\epsilon_r - K)E_+$$
$$D_- = D_x + jD_y = \epsilon_0(\epsilon_r + K)E_x + j\epsilon_0(\epsilon_r+K)E_y = \epsilon_0(\epsilon_r + K)E_-$$

OR, $$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - K) & 0 & 0 \\ 0 & (\epsilon_r+K) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,

$$\left.\begin{aligned} \nabla\times\bar{E} &= -j\omega\mu\bar{H} \\ \nabla\times\bar{H} &= j\omega[\epsilon]\bar{E} \end{aligned}\right\} \quad \nabla\times\nabla\times\bar{E} = -j\omega\mu\,\nabla\times\bar{H} = \omega^2\mu[\epsilon]\bar{E}$$
$$\nabla^2\bar{E} + \omega^2\mu[\epsilon]\bar{E} = 0 \quad \text{(CARTESIAN)}$$

Expanding this wave equation gives,

$$\nabla^2 E_x + \omega^2\mu\epsilon_0(\epsilon_r E_x + jKE_y) = 0 \qquad (1)$$
$$\nabla^2 E_y + \omega^2\mu\epsilon_0(-jKE_x + \epsilon_r E_y) = 0 \qquad (2)$$
$$\nabla^2 E_z + k_0^2 E_z = 0 \qquad (3)$$

adding (1) +j(2) gives $$\nabla^2(E_x + jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r+K)E_x + j(\epsilon_r+K)E_y] = 0$$
$$\nabla^2 E^+ + \omega^2\mu\epsilon_0(\epsilon_r+K)E^+ = 0$$
$$\therefore \beta_+ = k_0\sqrt{\epsilon_r + K} \quad ✓$$

adding (1) $-j$(2) gives $\nabla^2(E_x - jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r-\kappa)E_x - j(\epsilon_r-\kappa)E_y] = 0$

$$\nabla^2 E^- + \omega^2\mu\epsilon_0(\epsilon_r - \kappa)E^- = 0$$

$$\therefore \beta_- = k_0\sqrt{\epsilon_r - \kappa} \checkmark$$

Note that the wave equations for E^+, E^- must be satisfied simultaneously. Thus, for E^+ we must have $E^- = 0$. This implies that $E_y = jE_x = jE_0$. The actual electric field is then,

$$\bar{E}^+ = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} + j\hat{y})\, e^{-j\beta_+ z} \quad \text{(LHCP)}$$

This is a LHCP wave. Similarly for $\bar{E}^-$ we must have $E^+ = 0$:

$$\bar{E}^- = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} - j\hat{y})\, e^{-j\beta_- z} \quad \text{(RHCP)}$$

(1.17) Comparing (1.118), (1.125), and (1.129) shows that

$$E_t = \frac{J_t}{\sigma} = \frac{J_s}{\sigma\delta_s} = R_s J_s.$$

Thus $\bar{E}_t = R_s\bar{J}_s = R_s\,\hat{n}\times\bar{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S,

$$[(\bar{E}_1\times\bar{H}_2) - (\bar{E}_2\times\bar{H}_1)]\cdot\hat{n} = R_s[(\hat{n}\times\bar{H}_{1t})\times\bar{H}_{2t} - (\hat{n}\times\bar{H}_{2t})\times\bar{H}_{1t}]$$

(USING B.5)
$$= R_s[(\cancel{\bar{H}_{2t}\cdot\hat{n}})\bar{H}_{1t} - (\cancel{\bar{H}_{2t}\cdot\bar{H}_{1t}})\hat{n} - (\cancel{\bar{H}_{1t}\cdot\hat{n}})H_{2t} + (\cancel{\bar{H}_{1t}\cdot\bar{H}_{2t}})\hat{n}]$$

$$= 0$$

So (1.157) is obtained.