Chapter 2 Solutions

Problem 2.2-1

$L = 3m \qquad q_0 = 30 \cdot \frac{kN}{m} \qquad k = 700 \frac{kN}{m}$

$\Sigma M_B = 0 \quad A_y = \frac{1}{L} \cdot \left(\frac{1}{2} \cdot q_0 \cdot L \cdot \frac{L}{3} \right) = 15 \cdot kN \qquad \delta_A = \frac{A_y}{k} = 21.429 \cdot mm \qquad \frac{\delta_A}{L} = 7.143 \times 10^{-3}$

Problem 2.2-2

$E = 200GPa$ $d_r = 25mm$ $q = 5\frac{kN}{m}$ $L_r = 0.75m$ $P = 10kN$ $a = 2.5m$ $b = 0.75m$

$A_r = \frac{\pi}{4}\cdot d_r^{\ 2} = 0.761\cdot in^2$

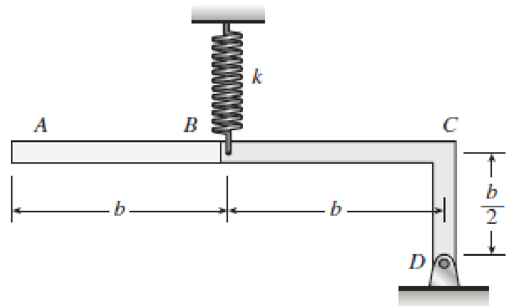
Force in rod $\Sigma M_A = 0$ $F_r = \frac{1}{a}\cdot\left[q\cdot a\cdot\frac{a}{2} + P\cdot(a+b)\right] = 19.25\cdot kN$

Change in length of rod $\delta_{rod} = \frac{F_r\cdot L_r}{E\cdot A_r} = 0.1471\cdot mm$

Displacement at B using similar triangles $\delta_B = \frac{a+b}{a}\cdot\delta_{rod} = 0.1912\cdot mm$

Problem 2.2-3

(a) Sum moments about A



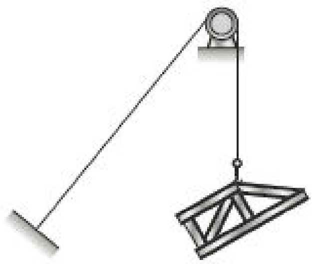
(b)

$$\Sigma M_A = 0 \qquad \frac{2b}{\frac{5}{2}b}Wb + \frac{\frac{b}{2}}{\frac{5}{2}b}W(2b) = k\delta b$$

$$\boxed{\delta = \frac{\frac{2b}{\frac{5}{2}b}Wb + \frac{\frac{b}{2}}{\frac{5}{2}b}W(2b)}{kb} = \frac{6W}{5k}}$$

(b) $\Sigma M_D = 0 \quad kb\delta = \frac{2b}{\frac{5}{2}b}Wb = \frac{4Wb}{5}$ so $\boxed{\delta = \frac{\frac{2b}{\frac{5}{2}b}Wb}{kb} = \frac{4W}{5k}}$

Problem 2.2-4



$A = 304\ \text{mm}^2$ (from Table 2-1)

$W = 38\ \text{kN}$

$E = 140\ \text{GPa}$

$L = 14\ \text{m}$

(b) Factor of safety

$P_{ULT} = 406\ \text{kN}$ (from Table 2-1)

$P_{max} = 70\ \text{kN}$

$$n = \frac{P_{ULT}}{P_{max}} = \frac{406\ \text{kN}}{70\ \text{kN}} = 5.8 \quad \leftarrow$$

(a) Stretch of cable

$$\delta = \frac{WL}{EA} = \frac{(38\ \text{kN})(14\ \text{m})}{(140\ \text{GPa})(304\ \text{mm}^2)}$$

$$= 12.5\ \text{mm} \quad \leftarrow$$

Problem 2.2-5

(a) $\dfrac{\delta_a}{\delta_s} = \dfrac{\dfrac{PL}{E_a A}}{\left(\dfrac{PL}{E_s A}\right)} \rightarrow \dfrac{E_s}{E_a}$

$E_s = 206\text{ GPa} \quad E_a = 76\text{ GPa}$

$\boxed{\dfrac{E_s}{E_a} = 2.711} \quad \dfrac{206}{76} \rightarrow \dfrac{103}{38} = 2.711$

(b) $\delta_a = \delta_s$ so $\dfrac{PL}{E_a A_a} = \dfrac{PL}{E_s A_s}$ so $\dfrac{A_a}{A_s} = \dfrac{E_s}{E_a}$ and $\boxed{\dfrac{d_a}{d_s} = \sqrt{\dfrac{E_s}{E_a}} = 1.646}$

(c) SAME DIAM., SAME LOAD, FIND RATIO OF LENGTH OF ALUM. TO STEEL WIRE IF ELONG. OF ALUM. IS 1.5 TIMES THAT OF STEEL WIRE

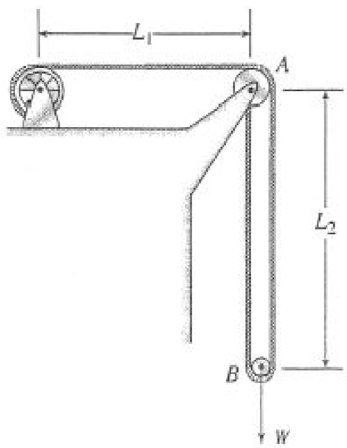
$$\frac{\delta_a}{\delta_s} = \frac{\dfrac{PL_a}{E_a A}}{\left(\dfrac{PL_s}{E_s A}\right)} \qquad \frac{\dfrac{PL_a}{E_a A}}{\left(\dfrac{PL_s}{E_s A}\right)} = 1.5 \qquad \boxed{\frac{L_a}{L_s} = 1.5\frac{E_a}{E_s} = 0.553}$$

(d) SAME DIAM., SAME LENGTH, SAME LOAD—BUT WIRE 1 ELONGATES 1.7 TIMES THE STEEL WIRE > WHAT IS WIRE 1 MATERIAL?

$$\frac{\delta_1}{\delta_s} = \frac{\dfrac{PL}{E_1 A}}{\left(\dfrac{PL}{E_s A}\right)} \qquad \frac{\dfrac{PL}{E_1 A}}{\left(\dfrac{PL}{E_s A}\right)} = 1.7 \qquad E_1 = \frac{E_s}{1.7} = 121\text{ GPa}$$

<cast iron or copper alloy (see App. I)

Problem 2.2-6



$d_A = 300$ mm
$d_B = 150$ mm
$L_1 = 4.6$ m
$L_2 = 10.5$ m
$EA = 10{,}700$ kN
$W = 22$ kN

TENSILE FORCE IN CABLE

$$T = \frac{W}{2} = 11 \text{ kN}$$

LENGTH OF CABLE

$$L = L_1 + 2L_2 + \frac{1}{4}(\pi d_A) + \frac{1}{2}(\pi d_B)$$

$$= 4{,}600 \text{ mm} + 21{,}000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm}$$

$$= 26{,}072 \text{ mm}$$

ELONGATION OF CABLE

$$\delta = \frac{TL}{EA} = \frac{(11 \text{ kN})(26{,}072 \text{ mm})}{(10{,}700 \text{ kN})} = 26.8 \text{ mm}$$

LOWERING OF THE CAGE

h = distance the cage moves downward

$$h = \frac{1}{2}\delta = 13.4 \text{ mm} \quad \leftarrow$$

Problem 2.2-7

$d_o = 380\text{mm}$ $\quad d_i = 365\text{mm}$ $\quad E = 200\text{GPa}$ $\quad P = 22\text{kN}$

$L_{DC} = \sqrt{(0.9\text{m})^2 + (1.2\text{m})^2} = 1.5\cdot\text{m}$ $\qquad A_{DC} = \frac{\pi}{4}\cdot\left(d_o^{\,2} - d_i^{\,2}\right) = 8.777 \times 10^3\cdot\text{mm}^2$

Find force in DC - use FBD of ACB

$\Sigma M_A = 0$ $\qquad \frac{3}{5}F_{DC}\cdot 1.2\text{m} = P\cdot(2.7\text{m})$ so $\qquad F_{DC} = \frac{5}{3}\cdot P\cdot\left(\frac{9}{4}\right) = 82.5\cdot\text{kN}$ compression

Change in length of strut

$$\Delta_{DC} = \frac{F_{DC}\cdot L_{DC}}{E\cdot A_{DC}} = 7.05 \times 10^{-2}\cdot\text{mm}$$ shortening

Vertical displacement at C (see Example 2-7) and at B

$\delta_C = \frac{\Delta_{DC}}{\sin(ACD)}$ $\qquad \delta_C = \frac{\Delta_{DC}}{\frac{3}{5}} = 0.117\cdot\text{mm}$ $\qquad \delta_B = \frac{9}{4}\cdot\delta_C = 2.644 \times 10^{-1}\cdot\text{mm}$ downward

Problem 2.2-8

$L_{BD} = 350\text{mm} \qquad L_{CE} = 450\text{mm} \qquad A = 720\text{mm}^2 \qquad E = 200\text{GPa} \qquad P = 20\text{kN}$

Statics - find axial forces in BD and CE - remove pins at B and E, use FBD of beam ABC - assume beam is rigid

$\Sigma M_B = 0 \qquad CE = \frac{1}{350\text{mm}} \cdot [P \cdot (600\text{mm})] = 34.286 \cdot \text{kN}$ CE is in tension; force CE acts downward on ABC

$\Sigma F_y = 0 \qquad BD = P + CE = 54.286 \cdot \text{kN}$ BD is in compression; force BD acts upward on ABC

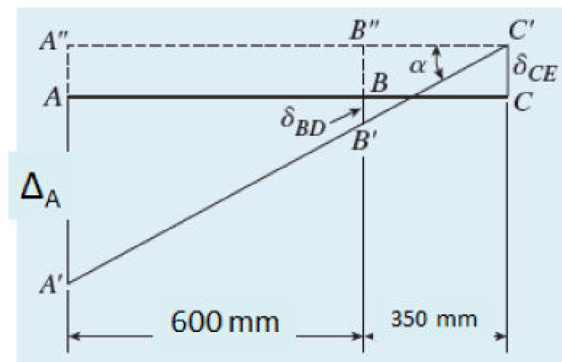
Use force-displacement relation to find change in lengths of CE and BD and vertical displacements at B and C

$\delta_{BD} = \frac{BD \cdot L_{BD}}{E \cdot A} = 0.13194 \cdot \text{mm}$ shortening $\qquad \delta_{CE} = \frac{CE \cdot L_{CE}}{E \cdot A} = 0.10714 \cdot \text{mm}$ elongation

Use geometry to find downward displacement at A

$$\alpha = \text{atan}\left(\frac{|\delta_{BD}| + \delta_{CE}}{350\text{mm}}\right) = 0.03914 \cdot \text{deg}$$

$$\Delta_A = 950\text{mm} \cdot \tan(\alpha) - \delta_{CE} = 0.542 \cdot \text{mm} \quad \text{downward}$$

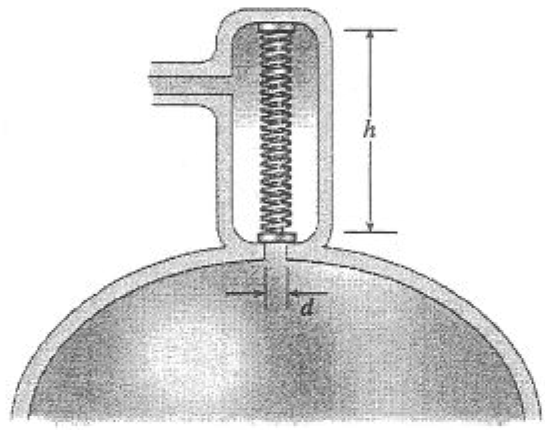


$$\frac{\delta_A + \delta_{CE}}{600 + 350} = \frac{\delta_{BD} + \delta_{CE}}{350}$$

or similar triangles (see figure) $\qquad \frac{\Delta_A + \delta_{CE}}{600 + 350} = \frac{|\delta_{BD}| + \delta_{CE}}{350}$

$$\Delta_A = (|\delta_{BD}| + \delta_{CE}) \cdot \left(\frac{950}{350}\right) - \delta_{CE} = 0.542 \cdot \text{mm} \quad \text{downward}$$

Problem 2.2-9



h = height of valve (compressed length of the spring)

d = diameter of discharge hole

p = pressure in tank

p_{max} = pressure when valve opens

L = natural length of spring ($L > h$)

k = stiffness of spring

FORCE IN COMPRESSED SPRING

$F = k(L - h)$ (From Eq. 2-1a)

PRESSURE FORCE ON SPRING

$$P = p_{max}\left(\frac{\pi d^2}{4}\right)$$

EQUATE FORCES AND SOLVE FOR h:

$$F = P \quad k(L - h) = \frac{\pi p_{max} d^2}{4}$$

$$h = L - \frac{\pi p_{max} d^2}{4k} \quad \leftarrow$$

Problem 2.2-10

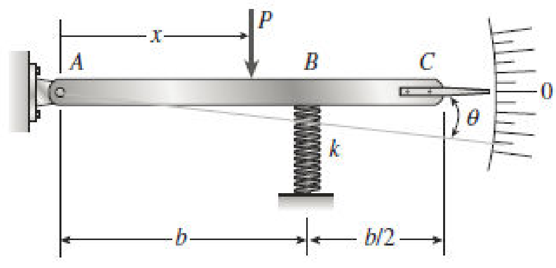
NUMERICAL DATA $k = 950\text{ N/m}$ $b = 165\text{ mm}$ $P = 11\text{ N}$ $\theta = 2.5°$ $\theta_{max} = 2°$

$W_p = 3\text{N}$ $W_s = 2.75\text{N}$

(a) If the load $P = 11$ N, at what distance x should the load be placed so that the pointer will read $\theta = 2.5°$ on the scale (see Fig. a)?

Sum moments about A, then solve for x:

$$x = \frac{k\theta b^2}{P} = 102.6\text{ mm} \quad \boxed{x = 102.6\text{ mm}}$$



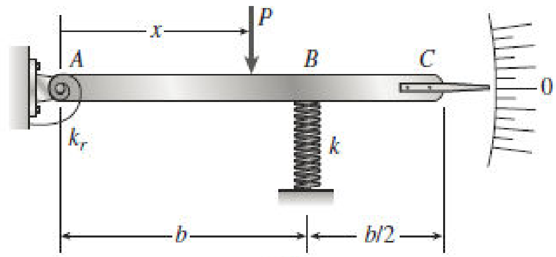
(a)

(b) Repeat (a) if a rotational spring $k_r = kb^2$ is added at A (see Fig. b).

$k_r = k\,b^2 = 25864\text{ N·mm}$

Sum moments about A, then solve for x:

$$x = \frac{k\theta b^2 + k_r\theta}{P} = 205\text{ mm} \quad \frac{x}{b} = 1.244 \quad \boxed{x = 205\text{ mm}}$$



(b)

(c) Now if $x = 7b/8$, what is P_{max} (N) if θ cannot exceed $2°$? $x = \frac{7}{8}b = 144.375\text{ mm}$

Sum moments about A, then solve for P: $$P_{max} = \frac{k\theta_{max}b^2 + k_r\theta_{max}}{\frac{7}{8}b} = 12.51\text{N} \quad \boxed{P_{max} = 12.51\text{N}}$$

(d) Now, if the weight of the pointer ABC is known to be $W_p = 3$ N and the weight of the spring is $W_s = 2.75$ N, what initial angular position (i.e., θ in degrees) of the pointer will result in a zero reading on the angular scale once the pointer is released from rest? Assume $P = k_r = 0$.

Deflection at spring due to W_p:

$$\delta_{Bp} = \frac{W_p\left(\frac{3}{4}b\right)}{kb} = 2.368\text{ mm}$$

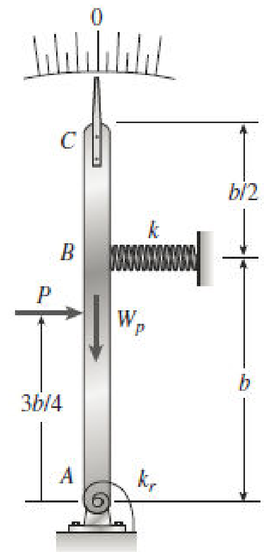
Deflection at B due to self weight of spring:

$$\delta_{Bk} = \frac{W_s}{2k} = 1.447\text{ mm}$$

$$\delta_B = \delta_{Bp} + \delta_{Bk} = 3.816\text{ mm} \qquad \theta_{init} = \frac{\delta_B}{b} = 1.325°$$

$$\text{OR} \quad \theta_{init} = \arctan\left(\frac{\delta_B}{b}\right) = 1.325° \quad \boxed{\theta_{init} = 1.325°}$$

(e) If the pointer is rotated to a vertical position (figure part c), find the required load P, applied at mid-height of the pointer that will result in a pointer reading of $\theta = 2.5^\circ$ on the scale. Consider the weight of the pointer, W_p, in your analysis.



(c)

$$k = 950 \text{ N/m} \quad b = 165 \text{ mm} \quad W_p = 3\,\text{N}$$

$$k_r = kb^2 = 25.864\,\text{N·m} \quad \theta = 2.5^\circ$$

Sum moments about A to get P:

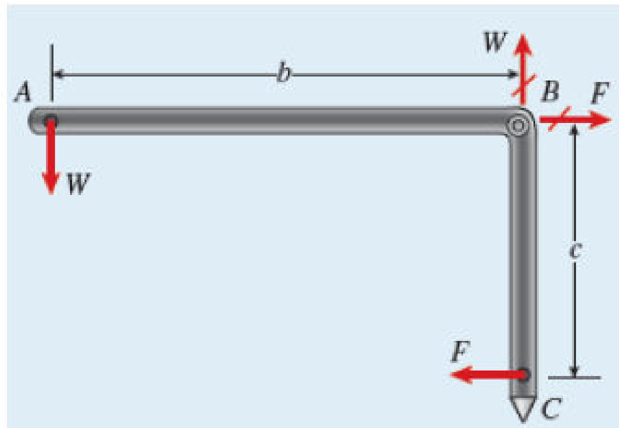
$$P = \frac{\theta}{\left(\frac{3b}{4}\right)}\left[k_r + k\left(\frac{5}{4}b^2\right) - W_p\left(\frac{3b}{4}\right)\right] = 20.388 \text{ N} \qquad \boxed{P = 20.4 \text{ N}}$$

Problem 2.2-11

$b = 250mm \quad c = 175mm \quad k = 875\frac{N}{m} \quad p = 1.6mm \quad n = 12$

Use FBD of ABC (pin forces B_x = F and B_y = W at B; see fig.); sum moments about B s.t. Wb = Fc, F = force in spring

$\Sigma M_B = 0 \qquad W = F \cdot \frac{c}{b}$

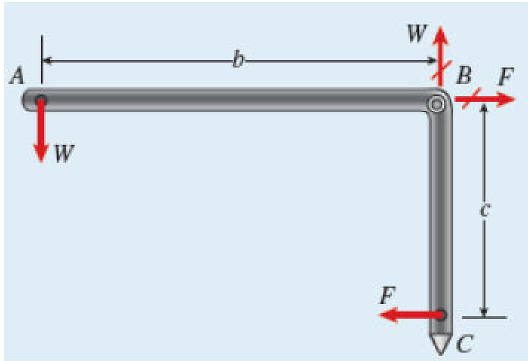


Force in spring is $\quad F = k \cdot (n \cdot p) = 16.8 \cdot N \quad$ so $\quad \boxed{W = F \cdot \frac{c}{b} = 11.76 \cdot N}$

Problem 2.2-12

$b = 30\text{cm} \qquad c = 20\text{cm} \qquad k = 3650\dfrac{N}{m} \qquad p = 1.5\text{mm} \qquad W = 65N$

Force in parallel springs is $\quad F = 2 \cdot k \cdot (n \cdot p)$



Sum moments about B (see FBD) to find F in terms of weight W

$$Wb = Fc \qquad \text{so} \qquad F = W \cdot \frac{b}{c}$$

Substitute expression for F and solve for n

$$n = \frac{W \cdot \frac{b}{c}}{2 \cdot k \cdot p} = 8.904$$

Problem 2.2-13

(a) Derive a formula for the displacement δ_4 at point 4 when the load P is applied at joint 3 and moment PL is applied at joint 1, as shown.

Cut horizontally through both springs to create upper and lower FBD's. Sum moments about joint 1 for upper FBD and also sum moments about joint 6 for lower FBD to get two equations of equilibrium; assume both springs are in tension.

Note that $\delta_2 = \frac{2}{3}\delta_3$ and $\delta_5 = \frac{3}{4}\delta_4$ Force in left spring: $k\left(\delta_4 - \frac{2}{3}\delta_3\right)$

Force in right spring: $2k\left(\frac{3}{4}\delta_4 - \delta_3\right)$

Summing moments about joint 1 (upper FBD) and about joint 6 (lower FBD) then dividing through by k gives

$$\begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{17}{6} \end{pmatrix}\begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{17}{6} \end{pmatrix}^{-1}\begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{17P}{2k} \\ \frac{26P}{3k} \end{pmatrix} \quad \begin{matrix} \frac{17}{2} = 8.5 \\ \frac{26}{3} = 8.667 \end{matrix} \quad \boxed{\delta_4 = \frac{26P}{3k}}$$

^ deltas are **positive downward**

(b) Repeat part (a) if a rotational spring $k_r = kL^2$ is now added at joint 6. What is the ratio of the deflection $\delta 4$ in part (a) to that in (b)?

Upper FBD—sum moments about joint 1:

$$k\left(\delta_4 - \frac{2}{3}\delta_3\right)\frac{2L}{3} + 2k\left(\frac{3}{4}\delta_4 - \delta_3\right)L = -2PL \quad \text{OR} \quad \left(\frac{22Lk}{9}\right)\delta_3 + \frac{13Lk}{6}\delta_4 = -2PL$$

Lower FBD—sum moments about joint 6:

$$k\left(\delta_4 - \frac{2}{3}\delta_3\right)\frac{4L}{3} + 2k\left(\frac{3}{4}\delta_4 - \delta_3\right)L - k_r\theta_6 = 0$$

$$\left[k\left(\delta_4 - \frac{2}{3}\delta_3\right)\frac{4L}{3} + 2k\left(\frac{3}{4}\delta_4 - \delta_3\right)L\right] + (kL^2)\left(\frac{\delta_4}{\frac{4}{3}L}\right) = 0 \quad \text{OR} \quad \left(\frac{26Lk}{9}\right)\delta_3 + \frac{43Lk}{12}\delta_4 = 0$$

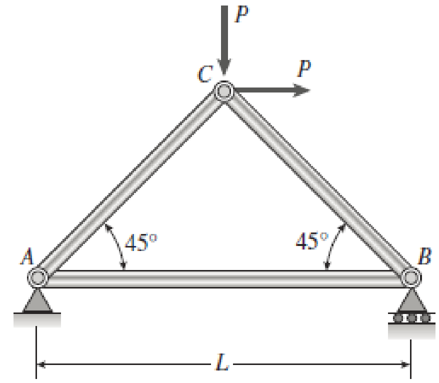
Divide matrix equilibrium equations through by k to get the following displacement equations:

$$\begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{43}{6} \end{pmatrix}\begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{43}{12} \end{pmatrix}^{-1}\begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{43P}{15k} \\ \frac{104P}{45k} \end{pmatrix} \quad \begin{matrix} \frac{43}{15} = 2.867 \\ \frac{104}{45} = 2.311 \end{matrix} \quad \boxed{\delta_4 = \frac{104P}{45k}}$$

^ deltas are positive downward

Ratio of the deflection δ_4 in part (a) to that in (b): $\dfrac{\frac{26}{3}}{\frac{104}{45}} = \dfrac{15}{4}$ $\boxed{\text{Ratio} = \frac{15}{4} = 3.75}$

Problem 2.2-14



(a)

NUMERICAL DATA

$A = 3900\ \text{mm}^2 \qquad E = 200\ \text{GPa}$

$P = 475\ \text{kN} \qquad L = 3000\ \text{mm}$

$\delta_{B\max} = 1.5\ \text{mm}$

(a) FIND HORIZONTAL DISPLACEMENT OF JOINT B

STATICS TO FIND SUPPORT REACTIONS AND THEN MEMBER FORCES:

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L}\left(2P\frac{L}{2}\right)$$

$$B_y = P$$

$$\Sigma F_H = 0 \qquad A_x = -P$$

$$\Sigma F_V = 0 \qquad A_y = P - B_y \qquad A_y = 0$$

METHOD OF JOINTS: $AC_V = A_Y \quad AC_V = 0 \quad$ Force in $AC = 0$

$AB = A_X$

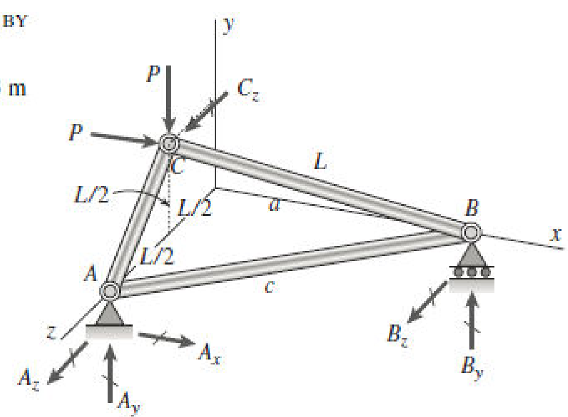
Force in AB is P (tension) so elongation of AB is the horizontal displacement of joint B.

$$\delta_B = \frac{F_{AB}L}{EA} \qquad \delta_B = \frac{PL}{EA} \qquad \delta_B = 1.82692\ \text{mm} \qquad \boxed{\delta_B = 1.827\ \text{mm}}$$

(b) FIND $P_{\max}$ IF DISPLACEMENT OF JOINT $B = \delta_{B\max} = 1.5$ mm $\quad P_{\max} = \frac{EA}{L}\delta_{B\max} \quad \boxed{P_{\max} = 390\ \text{kN}}$

(c) REPEAT PARTS (a) AND (b) IF THE PLANE TRUSS IS REPLACED BY A SPACE TRUSS (SEE FIGURE PART b).

FIND MISSING DIMENSIONS a AND c: $\quad P = 475\ \text{kN} \quad L = 3\ \text{m}$



(b)

$$a = \sqrt{L^2 - 2\left(\frac{L}{2}\right)^2} = 2.12132\ \text{m} \qquad \frac{a}{L} = 0.707 \qquad a = \frac{L}{\sqrt{2}} = 2.12132\ \text{m}$$

$$c = \sqrt{L^2 + a^2} = 3.67423\ \text{m} \qquad c = \sqrt{L^2 + \left(\frac{L}{\sqrt{2}}\right)^2} = 3.67423\ \text{m} \qquad c = L\sqrt{\frac{3}{2}} = 3.67423\ \text{m}$$

(1) SUM MOMENTS ABOUT A LINE THRU A WHICH IS PARALLEL TO THE y-AXIS

$$B_z = -P\frac{L}{a} = -671.751\ \text{kN}$$

(2) Sum moments about the z-axis

$$B_y = \frac{P\left(\frac{L}{2}\right)}{a} = 335.876\text{ kN} \qquad \text{SO} \qquad A_y = P - B_y = 139.124\text{ kN}$$

(3) Sum moments about the x-axis

$$C_z = \frac{A_y L - P\frac{L}{2}}{\frac{L}{2}} = -196.751\text{ kN}$$

(4) Sum forces in the x- and z-directions $\quad A_x = -P = -475\text{ kN} \qquad A_z = -C_z - B_z = 868.503\text{ kN}$

(5) Use method of joints to find member forces

Sum forces in x-direction at joint A: $\frac{a}{c}F_{AB} + A_x = 0 \qquad F_{AB} = \frac{-c}{a}A_x = 823\text{ kN}$

Sum forces in y-direction at joint A: $\frac{\frac{L}{2}}{\sqrt{2}\frac{L}{2}}F_{AC} + A_y = 0 \qquad F_{AC} = \sqrt{2}(-A_y) = -196.8\text{ kN}$

Sum forces in y-direction at joint B: $\frac{\frac{L}{2}}{L}F_{BC} + B_y = 0 \qquad F_{BC} = -2B_y = -672\text{ kN}$

(6) Find displacement along x-axis at joint B

Find change in length of member AB then find its projection along x axis:

$$\delta_{AB} = \frac{F_{AB}c}{EA} = 3.875\text{ mm} \quad \beta = \arctan\left(\frac{L}{a}\right) = 54.736° \quad \delta_{Bx} = \frac{\delta_{AB}}{cos(\beta)} = 6.713\text{ mm} \quad \boxed{\delta_{Bx} = 6.71\text{ mm}}$$

(7) Find P_{max} for space truss if δ_{Bx} must be limited to 1.5 mm

Displacements are linearly related to the loads for this linear elastic small displacement problem, so reduce load variable P from 475 kN to

$$\frac{1.5}{6.71254}475 = 106.145\text{ kN} \qquad \boxed{P_{max} = 106.1\text{ kN}}$$

Repeat space truss analysis using vector operations $\quad a = 2.121\text{ m} \quad L = 3\text{ m} \quad P = 475\text{ kN}$

Position and unit vectors:

$$r_{AB} = \begin{pmatrix} a \\ 0 \\ -L \end{pmatrix} \quad e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \begin{pmatrix} 0.577 \\ 0 \\ -0.816 \end{pmatrix} \qquad r_{AC} = \begin{pmatrix} 0 \\ \frac{L}{2} \\ \frac{-L}{2} \end{pmatrix} \qquad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0 \\ 0.707 \\ -0.707 \end{pmatrix}$$

Find moment at A:

$$M_A = r_{AB} \times R_B + r_{AC} \times R_C$$

$$M_A = r_{AB} \times \begin{pmatrix} 0 \\ RB_y \\ RB_z \end{pmatrix} + r_{AC} \times \begin{pmatrix} 2.P \\ -P \\ RC_z \end{pmatrix} = \begin{pmatrix} 3.0\text{ m } RB_y + 1.5\text{ m } RC_z - 712.5\text{ kN·m} \\ -2.1213\text{ m } RB_Z - 1425.0\text{ kN·m} \\ 2.1213\text{ m } RB_y - 1425.0\text{ kN·m} \end{pmatrix}$$

FIND MOMENTS ABOUT LINES OR AXES:

$M_A e_{AB} = -1.732 \text{ m } RB_y + 1.7321 \text{ m } RB_y + 0.86603 \text{ m } RC_z + 752.15 \text{ kN·m}$

$$RC_z = \frac{-244.12}{0.72169} = -338.262 \qquad C_z = -196.751 \text{ kN}$$

$M_A e_{AC} = -1.5 \text{ m } RB_y + -1.5 \text{ m } RB_z \qquad \text{So} \qquad RB_y = -RB_z$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -2.1213 \text{ m } RB_z + -1425.0 \text{ kN·m} \quad \text{So} \quad RB_z = \frac{462.5}{-1.7678} = -261.625 \quad B_z = -671.75 \text{ kN}$$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2.1213 \text{ m } RB_y + -1425.0 \text{ kN·m} \quad \text{So} \quad RB_y = -RB_z = 261.625 \quad B_y = -335.876 \text{ kN}$$

$\sum F_y = 0 \qquad A_y = P - B_y = 139.124 \text{ kN}$

Reactions obtained using vector operations agree with those based on scalar operations.

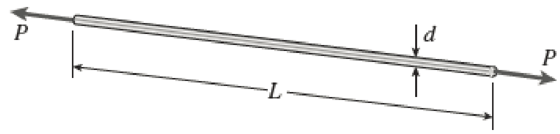
Problem 2.2-15

$d = 2\text{ mm} \quad L = 3.8\text{ m} \quad E = 75\text{ GPa}$

$\delta_a = 3\text{ mm} \quad \sigma_a = 60\text{ MPa}$

$A = \dfrac{\pi d^2}{4} \quad A = 3.142 \times 10^{-6}\text{ m}^2$

$EA = 2.356 \times 10^5\text{ N}$



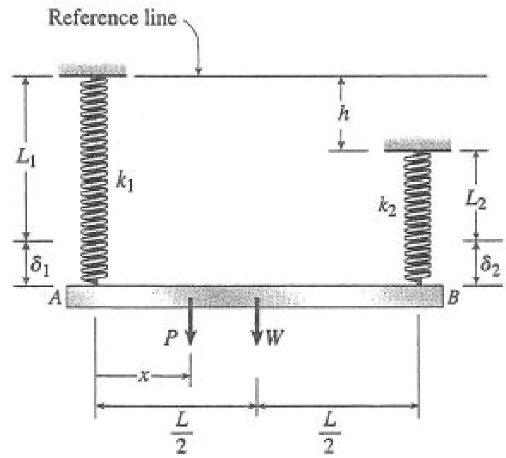
Maximum load based on elongation:

$P_{max1} = \dfrac{EA}{L}\delta_a \quad P_{max1} = 186.0\text{ N} \leftarrow$ controls

Maximum load based on stress:

$P_{max2} = \sigma_a A \quad P_{max2} = 188.496\text{ N}$

Problem 2.2-16



NUMERICAL DATA

$W = 25\text{ N} \quad k_1 = 0.300\text{ N/mm} \quad L_1 = 250\text{ mm}$

$k_2 = 0.400\text{ N/mm} \quad L_2 = 200\text{ mm}$

$L = 350\text{ mm} \quad h = 80\text{ mm} \quad P = 18\text{ N}$

(a) LOCATION OF LOAD P TO BRING BAR TO HORIZONTAL POSITION

Use statics to get forces in both springs:

$$\sum M_A = 0 \qquad F_2 = \frac{1}{L}\left(W\frac{L}{2} + Px\right)$$

$$F_2 = \frac{W}{2} + P\frac{x}{L}$$

$$\sum F_V = 0 \qquad F_1 = W + P - F_2$$

$$F_1 = \frac{W}{2} + P\left(1 - \frac{x}{L}\right)$$

Use constraint equation to define horizontal position, then solve for location x:

$$L_1 + \frac{F_1}{k_1} = L_2 + h + \frac{F_2}{k_2}$$

Substitute expressions for F_1 and F_2 above into constraint equilibrium and solve for x:

$$x = \frac{-2L_1\,L\,k_1\,k_2 - k_2WL - 2k_2\,P\,L + 2L_2\,L\,k_1\,k_2 + 2\,h\,L\,k_1\,k_2 + k_1WL}{-2P(k_1 + k_2)}$$

$$x = 134.7\text{ mm} \quad \leftarrow$$

(b) NEXT REMOVE P AND FIND NEW VALUE OF SPRING CONSTANT k_1 SO THAT BAR IS HORIZONTAL UNDER WEIGHT W

Now, $F_1 = \frac{W}{2} \qquad F_2 = \frac{W}{2}$ since $P = 0$

Same constraint equation as above but now $P = 0$:

$$L_1 + \frac{\frac{W}{2}}{k_1} - (L_2 + h) - \frac{\left(\frac{W}{2}\right)}{k_2} = 0$$

Solve for k_1:

$$k_1 = \frac{-Wk_2}{[2k_2[L_1 - (L_2 + h)]] - W}$$

$$k_1 = 0.204\text{ N/mm} \quad \leftarrow$$

PART (C)—CONTINUED (from page below)

STATICS

$$\sum M_{k_1} = 0 \qquad F_2 = \frac{w\left(\frac{L}{2} - b\right)}{L - b}$$

$$\sum F_V = 0$$

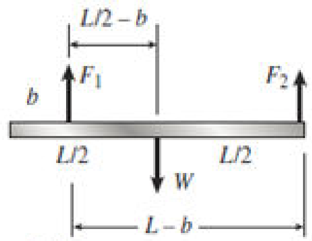
$$F_1 = W - F_2$$

$$F_1 = W - \frac{W\left(\frac{L}{2} - b\right)}{L - b}$$

$$F_1 = \frac{WL}{2(L - b)}$$

Part (c) continued in right column below

(c) USE $k_1 = 0.300$ N/mm BUT RELOCATE SPRING k_1 ($x = b$) SO THAT BAR ENDS UP IN HORIZONTAL POSITION UNDER WEIGHT W



FBD

$$b = \frac{2L_1k_1k_2L + WLk_2 - 2L_2k_1k_2L - 2hk_1k_2L - Wk_1L}{(2L_1k_1k_2) - 2L_2k_1k_2 - 2hk_1k_2 - 2Wk_1}$$

Part (c) continued on page above

Constraint equation—substitute above expressions for F_1 and F_2 and solve for b:

$$L_1 + \frac{F_1}{k_1} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

Use the following data:

$k_1 = 0.300$ N/mm $\quad k_2 = 0.4$ N/mm $\quad L_1 = 250$ mm

$L_2 = 200$ mm $\quad L = 350$ mm

$b = 74.1$ mm $\leftarrow$

(d) REPLACE SPRING k_1 WITH SPRINGS IN SERIES: $k_1 = 0.3$ N/mm, $L_1/2$, AND k_3, $L_1/2$. FIND k_3 SO THAT BAR HANGS IN HORIZONTAL POSITION

STATICS $\quad F_1 = \frac{W}{2} \quad F_2 = \frac{W}{2}$

$$k_3 = \frac{Wk_1k_2}{-2L_1k_1k_2 - Wk_2 + 2L_2k_1k_2 + 2hk_1k_2 + Wk_1}$$

NOTE—equivalent spring constant for series springs:

$$k_e = \frac{k_1k_3}{k_1 + k_3}$$

Part (d) continued from left column

New constraint equation; solve for k_3:

$$L_1 + \frac{F_1}{k_1} + \frac{F_1}{k_3} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

$$L_1 + \frac{W/2}{k_1} + \frac{W/2}{k_3} - (L_2 + h) - \frac{W/2}{k_2} = 0$$

$k_3 = 0.638$ N/mm $\leftarrow$

$k_e = 0.204$ N/mm $\leftarrow$ checks—same as (b) above

Problem 2.2-17

The figure shows a section cut through the pipe, cap, and rod.

NUMERICAL DATA

$E_c = 83\text{ GPa} \quad E_b = 96\text{ GPa}$

$W = 9\text{ kN} \quad d_c = 150\text{ mm} \quad d_r = 12\text{ mm}$

$\sigma_a = 35\text{ MPa} \quad \delta_a = 0.5\text{ mm}$

Unit weights (see Table I-1): $\gamma_s = 77\dfrac{\text{kN}}{\text{m}^3}$

$\gamma_b = 82\dfrac{\text{kN}}{\text{m}^3}$

$L_c = 1.25\text{ m} \quad L_r = 1.1\text{ m}$

$t_s = 25\text{ mm}$

(a) MIN. REQ'D WALL THICKNESS OF CI PIPE, t_{cmin}

First check allowable stress, then allowable shortening.

$W_{cap} = \gamma_s\left(\dfrac{\pi}{4}d_c^2 t_s\right)$

$W_{cap} = 34.018\text{ N}$

$W_{rod} = \gamma_b\left(\dfrac{\pi}{4}d_r^2 L_r\right)$

$W_{rod} = 10.201\text{ N}$

$W_t = W + W_{cap} + W_{rod} \qquad W_t = 9.044 \times 10^3\text{ N}$

$A_{min} = \dfrac{W_t}{\sigma_a} \qquad A_{min} = 258.406\text{ mm}^2$

$A_{pipe} = \dfrac{\pi}{4}[d_c^2 - (d_c - 2t_c)^2]$

$A_{pipe} = \pi t_c(d_c - t_c)$

$t_c(d_c - t_c) = \dfrac{W_t}{\pi\sigma_a}$

Let $\alpha = \dfrac{W_t}{\pi\sigma_a} \qquad \alpha = 8.225 \times 10^{-5}\text{ m}^2$

$t_c^2 - d_c t_c + \alpha = 0$

$t_c = \dfrac{d_c - \sqrt{d_c^2 - 4\alpha}}{2} \qquad t_c = 0.55\text{ mm}$

^ min. based on σ_a

Now check allowable shortening requirement.

$\delta_{pipe} = \dfrac{W_t L_c}{E_c A_{min}} \qquad A_{min} = \dfrac{W_t L_c}{E_c \delta_a}$

$A_{min} = 272.416\text{ mm}^2$ < larger than value based on σ_a above

$\pi t_c(d_c - t_c) = \dfrac{W_t L_c}{E_c \delta_a}$

$t_c^2 - d_c t_c + \beta = 0 \quad \beta = \dfrac{W_t L_c}{\pi E_c \delta_a}$

$\beta = 8.671 \times 10^{-5}\text{ m}^2$

$t_c = \dfrac{d_c - \sqrt{d_c^2 - 4\beta}}{2}$

$t_c = 0.580\text{ mm} \quad \leftarrow$ min. based on δ_a controls

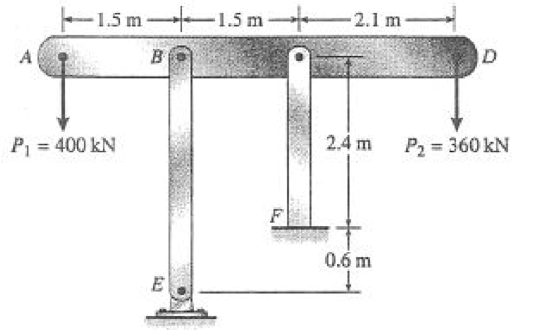
(b) ELONGATION OF ROD DUE TO SELF-WEIGHT AND ALSO WEIGHT W

$\delta_r = \dfrac{\left(W + \dfrac{W_{rod}}{2}\right)L_r}{E_b\left(\dfrac{\pi}{4}d_r^2\right)} \qquad \delta_r = 0.912\text{ mm} \quad \leftarrow$

(c) MIN. CLEARANCE h

$h_{min} = \delta_a + \delta_r \qquad h_{min} = 1.412\text{ mm} \quad \leftarrow$

Problem 2.2-18



$A_{BE} = 11{,}100\ \text{mm}^2$

$A_{CF} = 9{,}280\ \text{mm}^2$

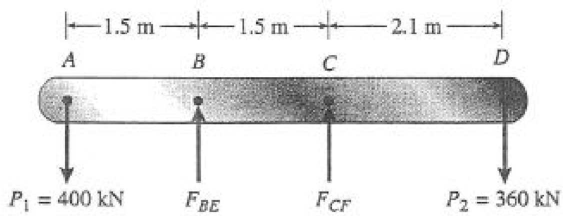
$E = 200\ \text{GPa}$

$L_{BE} = 3.0\ \text{m}$

$L_{CF} = 2.4\ \text{m}$

$P_1 = 400\ \text{kN};\ P_2 = 360\ \text{kN}$

FREE-BODY DIAGRAM OF BAR *ABCD*



$\Sigma M_B = 0$

$(400\ \text{kN})(1.5\ \text{m}) + F_{CF}(1.5\ \text{m}) - (360\ \text{kN})(3.6\ \text{m}) = 0$

$F_{CF} = 464\ \text{kN}$

$\Sigma M_C = 0$

$(400\ \text{kN})(3.0\ \text{m}) - F_{BE}(1.5\ \text{m}) - (360\ \text{kN})(2.1\ \text{m}) = 0$

$F_{BE} = 296\ \text{kN}$

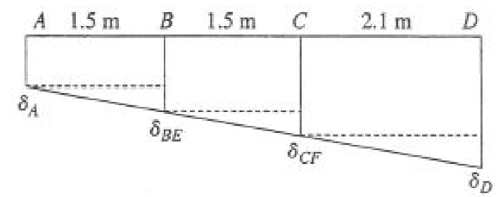
SHORTENING OF BAR *BE*

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}} = \frac{(296\ \text{kN})(3.0\ \text{m})}{(200\ \text{GPa})(11{,}100\ \text{mm}^2)} = 0.400\ \text{mm}$$

SHORTENING OF BAR *CF*

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA_{CF}} = \frac{(464\ \text{kN})(2.4\ \text{m})}{(200\ \text{GPa})(9{,}280\ \text{mm}^2)} = 0.600\ \text{mm}$$

DISPLACEMENT DIAGRAM



$\delta_{BE} - \delta_A = \delta_{CF} - \delta_{BE}$ or $\delta_A = 2\delta_{BE} - \delta_{CF}$

$\delta_A = 2(0.400\ \text{mm}) - 0.600\ \text{m}$

$= 0.200\ \text{mm} \quad \leftarrow$

(Downward)

$$\delta_D - \delta_{CF} = \frac{2.1}{1.5}(\delta_{CF} - \delta_{BE})$$

or $$\delta_D = \frac{12}{5}\delta_{CF} - \frac{7}{5}\delta_{BE} = \frac{12}{5}(0.600\ \text{mm}) - \frac{7}{5}(0.400\ \text{mm}) = 0.880\ \text{mm} \quad \leftarrow$$

(Downward)

Problem 2.2-19

(a) DISPLACEMENT δ_D

Use *FBD* of beam *BCD* $\quad \Sigma M_B = 0 \quad R_C = \frac{1}{L}\left[\left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right)\left(\frac{3}{8}L\right) + \frac{P}{4}\left(L + \frac{3}{4}L\right)\right] = P$ <compression force in column *CF*

$$\Sigma F_V = 0 \quad R_B = \left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right) + \frac{P}{4} - R_C = \frac{3P}{4}$$ <compression force in column *BA*

Downward displacements at *B* and *C*: $\quad \delta_B = R_B f_1 = \frac{3Pf_1}{4} \qquad \delta_C = R_C f_2 = Pf_2$

Geometry: $\delta_D = \delta_B + (\delta_C - \delta_B)\left(\frac{L + \frac{3}{4}L}{L}\right) = \frac{7Pf_2}{4} - \frac{9Pf_1}{16} \qquad \delta_D = \frac{7Pf_2}{4} - \frac{9Pf_1}{16} = \boxed{\frac{P}{16}(28f_2 - 9f_1)}$

(b) DISPLACEMENT TO HORIZONTAL POSITION, SO $\delta_C = \delta_B$ and $\frac{3Pf_1}{4} = Pf_2$ or $\frac{f_1}{f_2} = \frac{4}{3}$

$$\frac{\frac{L_1}{EA_1}}{\frac{L_2}{EA_2}} = \frac{4}{3} \quad \text{or} \quad \frac{L_1}{L_2} = \frac{4}{3}\left(\frac{A_1}{A_2}\right) \quad \frac{L_1}{L_2} = \frac{4}{3}\left(\frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}\right) = \frac{4d_1^2}{3d_2^2} \quad \frac{L_1}{L_2} = \frac{4}{3}\left(\frac{d_1}{d_2}\right)^2 \quad \text{with} \quad \frac{d_1}{d_2} = \frac{9}{8}$$

$$\frac{L_1}{L_2} = \frac{4}{3}\left(\frac{9}{8}\right)^2 = \frac{27}{16} \qquad \boxed{\frac{L_1}{L_2} = \frac{27}{16}}$$

(c) IF $L_1 = 2\,L_2$, FIND THE d_1/d_2 RATIO SO THAT BEAM *BCD* DISPLACES DOWNWARD TO A HORIZONTAL POSITION

$\frac{L_1}{L_2} = 2$ and $\delta_C = \delta_B$ from part (b). $\left(\frac{d_1}{d_2}\right)^2 = \frac{3}{4}\left(\frac{L_1}{L_2}\right)$ so $\boxed{\frac{d_1}{d_2} = \sqrt{\frac{3}{4}(2)} = 1.225}$

(d) IF $d_1 = (9/8)\,d_2$ AND $L_1/L_2 = 1.5$, AT WHAT HORIZONTAL DISTANCE *x* FROM *B* SHOULD LOAD *P*/4 AT *D* BE PLACED?

Given $\frac{d_1}{d_2} = \frac{9}{8}$ and $\frac{L_1}{L_2} = 1.5$ or $\frac{f_1}{f_2} = \frac{L_1}{L_2}\left(\frac{A_2}{A_1}\right) \quad \frac{f_1}{f_2} = \frac{L_1}{L_2}\left(\frac{d_2}{d_1}\right)^2 = \frac{3}{2}\left(\frac{8}{9}\right)^2 = \frac{32}{27}$

Recompute column forces R_B and R_C but now with load *P*/4 positioned at distance *x* from *B*.

Use *FBD* of beam *BCD*: $\quad \Sigma M_B = 0 \qquad R_C = \frac{1}{L}\left[\left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right)\left(\frac{3}{8}L\right) + \frac{P}{4}(x)\right] = \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$

$$\Sigma F_V = 0 \qquad R_B = \left(2\frac{P}{L}\right)\left(\frac{3}{4}L\right) + \frac{P}{4} - R_C = \frac{7P}{4} - \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$$

Horizontal displaced position under load *q* and load *P*/4 so $\quad \delta_C = \delta_B \quad$ or $\quad R_C f_2 = R_B f_1$.

$$\left(\frac{\frac{9LP}{16}+\frac{Px}{4}}{L}\right)f_2=\left(\frac{7P}{4}-\frac{\frac{9LP}{16}+\frac{Px}{4}}{L}\right)f_1 \text{ solve, } x=-\frac{9Lf_2-19Lf_1}{4f_1+4f_2}=-\frac{L(9f_2-19f_1)}{4(f_1+f_2)}$$

$$x=-\frac{L(9f_2-19f_1)}{4(f_1+f_2)} \text{ or } \quad x=L\left[\frac{19\frac{f_1}{f_2}-9}{4\left(\frac{f_1}{f_2}+1\right)}\right]$$

Now substitute f_1/f_2 ratio from above: $\boxed{x=L\left[\frac{19\frac{32}{27}-9}{4\left(\frac{32}{27}+1\right)}\right]=\frac{365L}{236}}$ $\quad \frac{365}{236}=1.547$

Problem 2.2-20

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below).

OVERALL FBD: $\Sigma F_H = 0 \quad H_A - k_1\delta = 0 \quad \text{so} \quad H_A = k_1\delta$

$\Sigma F_V = 0 \quad R_A + R_C = P$

$$\Sigma M_A = 0 \quad k_r(\alpha - \theta) - P\frac{L_2}{2} + R_C L_2 = 0 \qquad R_C = \frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right]$$

LHFB: $$\Sigma M_B = 0 \quad H_A h + k\frac{\delta}{2}\left(\frac{h}{2}\right) - R_A\left(\frac{L_2}{2}\right) + k_r(\alpha - \theta) = 0$$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right]$$

RHFB: $$\Sigma M_B = 0 \quad -k\frac{\delta}{2}\left(\frac{h}{2}\right) - k_1\delta h + R_C\frac{L_2}{2} = 0 \qquad R_C = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right]$$

Equate the two expressions for R_C then substitute expressions for L_2, k_r, k_1, h and δ

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right] \quad \text{OR}$$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$b = 200\text{ mm} \quad k = 3.2\text{ kN/m} \quad \alpha = 45° \quad P = 50\text{ N} \quad k_1 = 0 \quad k_r = 0$

$L_2 = 2b\cos(\theta) \quad L_1 = 2b\cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b\sin(\theta)$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{1}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 35.1°}$ $\boxed{\delta = 44.6\text{ mm}}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right] = 25\text{ N} \qquad R_C = \frac{1}{L_C}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = 25\text{ N}$$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right] = 25\text{ N} \qquad M_A = k_r(\alpha - \theta) = 0$$

$R_A + R_C = 50\text{ N}$ < check $\qquad \boxed{R_A = 25\text{ N}}$ $\boxed{R_C = 25\text{ N}}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$b = 200\text{ mm} \quad k = 3.2\text{ kN/m} \quad \alpha = 45° \quad P = 50\text{ N} \quad k_1 = \frac{k}{2} \quad k_r = \frac{k}{2}b^2$

$L_2 = 2b\cos(\theta) \quad L_1 = 2b\cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b\sin(\theta)$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 43.3°}$ $\boxed{\delta = 8.19\text{ mm}}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right] = 18.5\text{ N} \quad R_2 = \frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = 18.5\text{ N}$$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right] = 31.5\text{ N} \quad M_A = k_r(\alpha - \theta) = 1.882\text{ N·m}$$

$R_A + R_C = 50\text{ N}$ < check $\quad \boxed{R_A = 31.5\text{ N}}$ $\boxed{R_C = 18.5\text{ N}}$ $\boxed{M_A = 1.882\text{ N·m}}$

Problem 2.2-21

Apply the laws of statics to the structure in its displaced position; also use FBDs of the left and right bars alone (referred to as LHFB and RHFB below).

OVERALL FBD $\Sigma F_H = 0 \quad H_A - k_1\delta = 0 \quad \text{so} \quad H_A = k_1\delta$

$\Sigma F_V = 0 \quad R_A + R_C = P$

$\Sigma M_A = 0 \quad k_r(\alpha - \theta) - P\dfrac{L_2}{2} + R_C L_2 = 0 \qquad R_C = \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right]$

LHFB: $\Sigma M_B = 0 \quad H_A h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - R_A\dfrac{L_2}{2} + k_r(\alpha - \theta) = 0$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right]$$

RHFB: $\Sigma M_B = 0 \quad -k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) - k_1\delta h + R_C\dfrac{L_2}{2} = 0 \qquad R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right]$

Equate the two expressions above for R_C, then substitute expressions for L_2, k_r, k_1, h, and δ

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right] \qquad \text{OR}$$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE θ AND DISTANCE INCREASE δ

$b = 300\text{ mm} \qquad k = 7.8\,\dfrac{\text{kN}}{\text{m}} \qquad \alpha = 55° \qquad P = 100\text{ N} \qquad k_1 = 0 \qquad k_r = 0$

$L_2 = 2b\cos(\theta) \qquad L_1 = 2b\cos(\alpha) \qquad \delta = L_2 - L_1 \qquad \delta = 2b(\cos(\theta) - \cos(\alpha)) \qquad h = b\sin(\theta)$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 52.7°}$ $\boxed{\delta = 19.54\text{ mm}}$

COMPUTE REACTIONS

$R_C = \dfrac{2}{L_2}\left[k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_1\delta h\right] = 49.99\text{ N} \qquad R_C = \dfrac{1}{L_2}\left[P\dfrac{L_2}{2} - k_r(\alpha - \theta)\right] = 50\text{ N}$

$R_A = \dfrac{2}{L_2}\left[k_1\delta h + k\dfrac{\delta}{2}\left(\dfrac{h}{2}\right) + k_r(\alpha - \theta)\right] = 50\text{ N} \qquad M_A = k_r(\alpha - \theta) = 0$

$R_A + R_C = 100\text{ N} \qquad <\text{ check} \qquad \boxed{R_A = 50\text{ N}}$ $\boxed{R_C = 50\text{ N}}$

(b) REPEAT PART (A) BUT SPRING k_1 AT C AND SPRING k_r AT A

$$b = 300\ \text{mm} \quad k = 7.8\,\frac{\text{kN}}{\text{m}} \quad \alpha = 55^\circ \quad P = 100\ \text{N} \quad k_1 = \frac{k}{2} \quad k_r = \frac{k}{2}b^2$$

$$L_2 = 2b\cos(\theta) \quad L_1 = 2b\cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b\sin(\theta)$$

$$\frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] - \left[\frac{2}{L_2}\left[k\frac{2b(\cos(\theta) - \cos(\alpha))}{2}\frac{b\sin(\theta)}{2} + k_1[2b(\cos(\theta) - \cos(\alpha))](b\sin(\theta))\right]\right] = 0$$

Solving above equation numerically gives $\boxed{\theta = 54.4^\circ}$ $\boxed{\delta = 4.89\ \text{mm}}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2}\left[k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_1\delta h\right] = 39.95\ \text{N} \qquad R_C = \frac{1}{L_2}\left[P\frac{L_2}{2} - k_r(\alpha - \theta)\right] = 39.97\ \text{N}$$

$$R_A = \frac{2}{L_2}\left[k_1\delta h + k\frac{\delta}{2}\left(\frac{h}{2}\right) + k_r(\alpha - \theta)\right] = 60.02\ \text{N} \qquad M_A = k_r(\alpha - \theta) = 3.504\ \text{N}\cdot\text{m}$$

$R_A + R_C = 99.99\ \text{N}$ $<$ check $\boxed{R_A = 60\text{N}}$ $\boxed{R_C = 40\text{N}}$ $\boxed{M_A = 3.5\ \text{N}\cdot\text{m}}$

Problem 2.3-1

NUMERICAL DATA

$P = 14\text{ kN} \qquad L_1 = 500\text{ mm} \qquad L_2 = 1250\text{ mm} \qquad d_A = 12\text{ mm} \qquad d_B = 24\text{ mm} \qquad E = 120\text{ GPa}$

(a) TOTAL ELONGATION

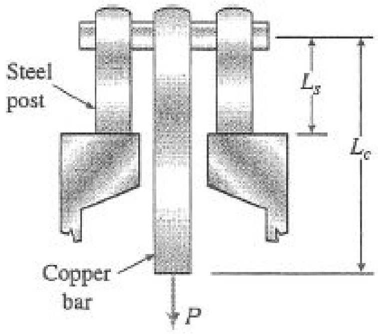
$$\delta_1 = \frac{4PL_1}{\pi E d_A d_B} = 0.25789\text{ mm} \qquad \delta_2 = \frac{PL_2}{E\frac{\pi}{4}{d_B}^2} = 0.32236\text{ mm}$$

$$\delta = 2\delta_1 + \delta_2 = 0.8381\text{ mm} \qquad \boxed{\delta = 0.838\text{ mm}}$$

(b) FIND NEW DIAMETERS AT B AND C IF TOTAL ELONGATION CANNOT EXCEED 0.635 mm

$$2\left(\frac{4PL_1}{\pi E d_A d_B}\right) + \frac{PL_2}{E\frac{\pi}{4}d_B^2} = 0.635\text{ mm} \qquad \text{Solving for } d_B\text{:} \quad \boxed{d_B = 29.4\text{ mm}}$$

Problem 2.3-2



$L_c = 2.0$ m

$A_c = 4800$ mm^2

$E_c = 120$ GPa

$L_s = 0.5$ m

$A_s = 4500$ mm^2

$E_s = 200$ GPa

(a) Downward displacement δ ($P = 180$ kN)

$$\delta_c = \frac{PL_c}{E_cA_c} = \frac{(180\text{ kN})(2.0\text{ m})}{(120\text{ GPa})(4800\text{ mm}^2)} = 0.625\text{ mm}$$

$$\delta_s = \frac{(P/2)L_s}{E_sA_s} = \frac{(90\text{ kN})(0.5\text{ m})}{(200\text{ GPa})(4500\text{ mm}^2)} = 0.050\text{ mm}$$

$$\delta = \delta_c + \delta_s = 0.625\text{ mm} + 0.050\text{ mm} = 0.675\text{ mm} \quad \leftarrow$$

(b) Maximum load P_{max} ($\delta_{max} = 1.0$ mm)

$$\frac{P_{max}}{P} = \frac{\delta_{max}}{\delta} \qquad P_{max} = P\left(\frac{\delta_{max}}{\delta}\right)$$

$$P_{max} = (180\text{ kN})\left(\frac{1.0\text{ mm}}{0.675\text{ mm}}\right) = 267\text{ kN} \quad \leftarrow$$

Problem 2.3-3

NUMERICAL DATA

$A = 250\ \text{mm}^2 \qquad P_1 = 7560\ \text{N}$

$P_2 = 5340\ \text{N} \qquad P_3 = 5780\ \text{N}$

$E = 72\ \text{GPa}$

$a = 1525\ \text{mm} \qquad b = 610\ \text{mm} \qquad c = 910\ \text{mm}$

(a) TOTAL ELONGATION

$$\delta = \frac{1}{EA}\left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c\right] = 0.2961\ \text{mm} \quad \boxed{\delta = 0.296\ \text{mm}} \quad \text{elongation} \quad \leftarrow$$

(b) INCREASE P_3 SO THAT BAR DOES NOT CHANGE LENGTH

$$\frac{1}{EA}\left[(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c\right] = 0 \quad \text{solving, } P_3 = \frac{218{,}380\ \text{N}}{29} = 7530\ \text{N} \quad \leftarrow$$

$$\boxed{\text{So new value of } P_3 \text{ is 7530 N, an increase of 1750 N}}$$

(c) NOW CHANGE CROSS-SECTIONAL AREA OF AB SO THAT BAR DOES NOT CHANGE LENGTH $\quad P_3 = 5780\ \text{N}$

$$\frac{1}{E}\left[(P_1 + P_2 - P_3)\frac{a}{A_{AB}} + (P_2 - P_3)\frac{b}{A} + (-P_3)\frac{c}{A}\right] = 0$$

Solving for A_{AB}: $\boxed{A_{AB} = 491\ \text{mm}^2}$ $\qquad \dfrac{A_{AB}}{A} = 1.964$

Problem 2.3-4

$E = 200GPa$

$A_1 = 6000mm^2$ $\quad A_2 = 5000mm^2$ $\quad A_3 = 4000mm^2$ $\quad L_1 = 500mm$ $\quad L_2 = L_1$ $\quad L_3 = L_1$

$P_B = 50N$ $\quad P_C = 250N$ $\quad P_E = 350N$

Internal forces in each segment (tension +) - cut bar and use lower FBD

$N_{AB} = -P_B + P_C + P_E = 550\,N$ $\quad N_{BC} = P_C + P_E = 600\,N$ $\quad N_{CD} = P_E = 350\,N$ $\quad N_{DE} = P_E = 350\,N$

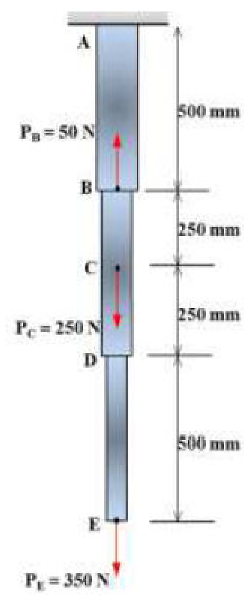
Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

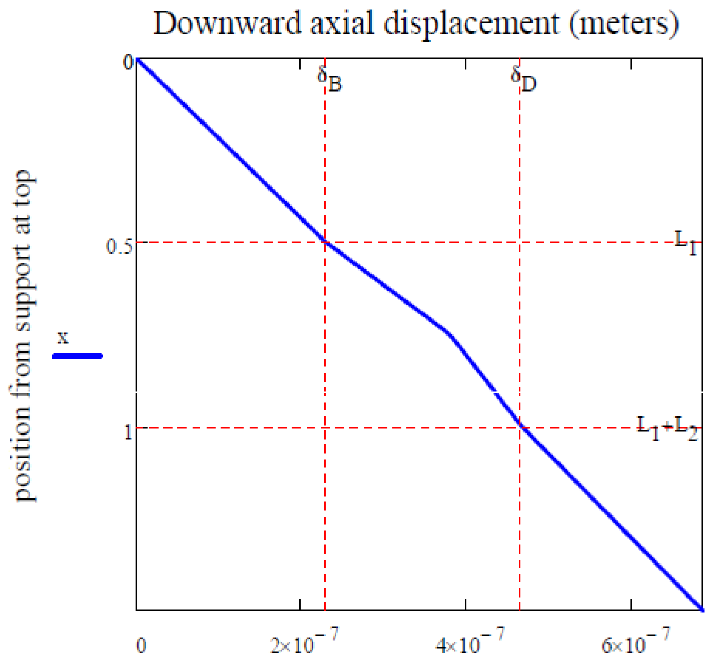
$$\delta_B = \frac{N_{AB} \cdot L_1}{E \cdot A_1} = 2.292 \times 10^{-4} \cdot mm \quad \text{downward} \qquad \delta_C = \delta_B + \frac{N_{BC} \cdot \frac{L_2}{2}}{E \cdot A_2} = 3.792 \times 10^{-4} \cdot mm$$

$$\delta_D = \delta_C + \frac{N_{CD} \cdot \frac{L_2}{2}}{E \cdot A_2} = 4.667 \times 10^{-4} \cdot mm \qquad \delta_E = \delta_D + \frac{N_{DE} \cdot L_3}{E \cdot A_3} = 6.854 \times 10^{-4} \cdot mm$$

Axial displacement diagram - x origin at A, positive downward

$$\delta(x) = \begin{cases} \delta_B \cdot \dfrac{x}{L_1} & \text{if } x \le L_1 \\ \delta_B + (\delta_C - \delta_B) \cdot \left(\dfrac{x - L_1}{\frac{L_2}{2}} \right) & \text{if } L_1 \le x \le L_1 + \dfrac{L_2}{2} \\ \delta_C + (\delta_D - \delta_C) \cdot \left[\dfrac{x - \left(L_1 + \frac{L_2}{2}\right)}{\frac{L_2}{2}} \right] & \text{if } L_1 + \dfrac{L_2}{2} \le x \le L_1 + L_2 \\ \delta_D + (\delta_E - \delta_D) \cdot \left[\dfrac{x - (L_1 + L_2)}{L_3} \right] & \text{otherwise} \end{cases}$$





Downward axial displacement (meters)
position from support at top
x
0
0.5
1
δB
δD
L1
L1+L2
0
2×10^-7
4×10^-7
6×10^-7

Problem 2.3-5

$E = 200GPa \qquad A = 5300mm^2 \qquad L_1 = 500mm \qquad L_2 = 500mm \qquad L_3 = 1000mm$

$P_B = 225N \qquad P_C = 450N \qquad P_D = 900N$

Internal forces in each segment (tension +) - cut bar and use lower FBD

$N_{AB} = -P_B + P_C - P_D = -675 \cdot N \qquad N_{BC} = P_C - P_D = -450 \cdot N \qquad N_{CD} = -P_D = -900 \cdot N$

Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

$$\delta_B = \frac{N_{AB} \cdot L_1}{E \cdot A} = -3.184 \times 10^{-4} \cdot mm$$

upward

$$\delta_C = \delta_B + \frac{N_{BC} \cdot L_2}{E \cdot A} = -5.307 \times 10^{-4} \cdot mm$$

upward

$$\delta_D = \delta_C + \frac{N_{CD} \cdot L_3}{E \cdot A} = -1.38 \times 10^{-3} \cdot mm$$

upward

Problem 2.3-6

$\gamma = 77.0\frac{kN}{m^3}$ from Table I-1

$E = 200GPa$

$A_1 = 6000mm^2$ $A_2 = 5000mm^2$ $A_3 = 4000mm^2$ $L_1 = 500mm$ $L_2 = L_1$ $L_3 = L_1$

$P_B = 50N$ $P_C = 250N$ $P_E = 350N$

Internal forces in each segment (tension +) - cut bar and use lower FBD - weight per unit length = γA_i

$P_{AB} = -P_B + P_C + P_E = 550\,N$ $P_{BC} = P_C + P_E = 600\,N$ $P_{CD} = P_E = 350\,N$ $P_{DE} = P_E = 350\,N$

Now add weight per unit length - x origin at A, positive downward

$$N_{AB}(x) = P_{AB} + \gamma \cdot A_1 \cdot (L_1 - x) + \gamma \cdot A_2 \cdot L_2 + \gamma \cdot A_3 \cdot L_3 \qquad N_{BC}(x) = P_{BC} + \gamma \cdot A_2 \cdot \left(L_1 + \frac{L_2}{2} - x\right) + \gamma \cdot A_2 \cdot \frac{L_2}{2} + \gamma \cdot A_3 \cdot L_3$$

$$N_{CD}(x) = P_{CD} + \gamma \cdot A_2 \cdot (L_1 + L_2 - x) + \gamma \cdot A_3 \cdot L_3 \qquad N_{DE}(x) = P_{DE} + \gamma \cdot A_3 \cdot (L_1 + L_2 + L_3 - x)$$

Note that total bar weight is not small compared to applied loads $W = \gamma \cdot (A_1 \cdot L_1 + A_2 \cdot L_2 + A_3 \cdot L_3) = 577.5\,N$

Use force-displacement relation to find segment elongations then sum elongations to find displacements

$$\Delta_B = \int_0^{L_1} \frac{N_{AB}(x)}{E \cdot A_1}\,dx = 4.217 \times 10^{-4} \cdot mm \qquad \Delta_C = \Delta_B + \int_{L_1}^{L_1 + \frac{L_2}{2}} \frac{N_{BC}(x)}{E \cdot A_2}\,dx = 6.463 \times 10^{-4} \cdot mm$$

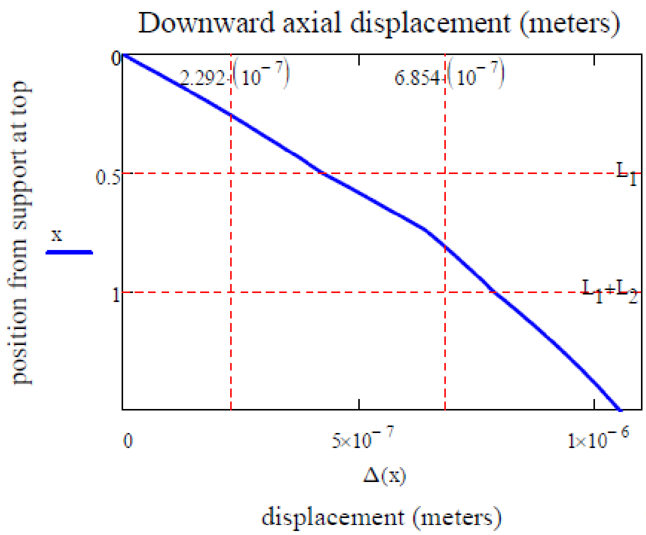
$$\Delta_D = \Delta_C + \int_{L_1 + \frac{L_2}{2}}^{L_1 + L_2} \frac{N_{CD}(x)}{E \cdot A_2}\,dx = 7.843 \times 10^{-4} \cdot mm \qquad \Delta_E = \Delta_D + \int_{L_1 + L_2}^{L_1 + L_2 + L_3} \frac{N_{DE}(x)}{E \cdot A_3}\,dx = 1.051 \times 10^{-3} \cdot mm$$

Compare to Prob. 2.3-4

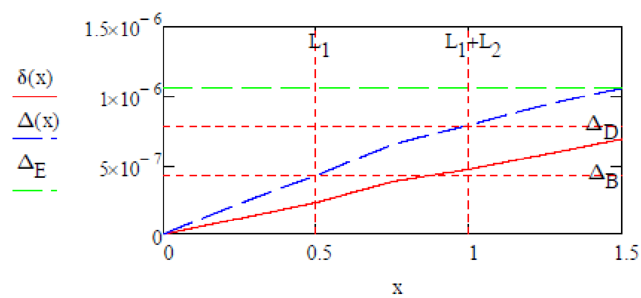
$$\frac{\Delta_B}{2.292 \cdot (10^{-4}) \cdot mm} = 1.84 \qquad \frac{\Delta_C}{3.792 \cdot (10^{-4}) \cdot mm} = 1.704 \qquad \frac{\Delta_D}{4.667 \cdot (10^{-4}) \cdot mm} = 1.681 \qquad \frac{\Delta_E}{6.854 \cdot (10^{-4}) \cdot mm} = 1.534$$

Axial displacement diagram including weight of bar - x origin at A, positive downward

$$\Delta(x) = \begin{cases} \displaystyle\int_0^x \frac{N_{AB}(x)}{E \cdot A_1}\,dx & \text{if } x \le L_1 \\ \Delta_B + \displaystyle\int_{L_1}^x \frac{N_{BC}(x)}{E \cdot A_2}\,dx & \text{if } L_1 \le x \le L_1 + \frac{L_2}{2} \\ \Delta_C + \displaystyle\int_{L_1+\frac{L_2}{2}}^x \frac{N_{CD}(x)}{E \cdot A_2}\,dx & \text{if } L_1 + \frac{L_2}{2} \le x \le L_1 + L_2 \\ \Delta_D + \displaystyle\int_{L_1+L_2}^x \frac{N_{DE}(x)}{E \cdot A_3}\,dx & \text{if } x \ge L_1 + L_2 \end{cases}$$



Compare ADD's without (δ) & with (Δ) weight of bar (plotted horizontally)



Problem 2.3-7

$E = 200\text{GPa} \qquad A = 5300\text{mm}^2$

$L_1 = 500\text{mm} \qquad L_2 = 500\text{mm} \qquad L_3 = 1000\text{mm}$

$P_B = 225\text{N} \qquad P_C = 450\text{N} \qquad P_D = 900\text{N} \qquad \gamma = 77\frac{\text{kN}}{\text{m}^3}$

<u>Internal forces in each segment (tension +) - cut bar and use lower FBD</u>
<u>x origin at A, positive downward</u>

$$P_{AB} = -P_B + P_C - P_D = -675\cdot\text{N} \qquad P_{BC} = P_C - P_D = -450\cdot\text{N} \qquad P_{CD} = -P_D = -900\cdot\text{N}$$

$$N_{AB}(x) = P_{AB} + \gamma\cdot A\cdot(L_1 - x) + \gamma\cdot A\cdot(L_2 + L_3) \qquad N_{BC}(x) = P_{BC} + \gamma\cdot A\cdot(L_1 + L_2 - x) + \gamma\cdot A\cdot L_3$$

$$N_{CD}(x) = P_{CD} + \gamma\cdot A\cdot(L_1 + L_2 + L_3 - x)$$

Note that total bar weight is not small compared to applied loads $W = \gamma\cdot A\cdot(L_1 + L_2 + L_3) = 816.2\cdot\text{N}$

<u>Use force-displacement relation to find segment elongations then sum elongations to find downward displacements</u>

$$\delta_B = \int_0^{L_1} \frac{N_{AB}(x)}{E\cdot A}\,dx = 1.848\times 10^{-5}\cdot\text{mm} \qquad \delta_C = \delta_B + \int_{L_1}^{L_1+L_2} \frac{N_{BC}(x)}{E\cdot A}\,dx = 4.684\times 10^{-5}\cdot\text{mm}$$

downward — downward

$$\delta_D = \delta_C + \int_{L_1+L_2}^{L_1+L_2+L_3} \frac{N_{CD}(x)}{E\cdot A}\,dx = -6.097\times 10^{-4}\cdot\text{mm}$$

upward

Problem 2.3-8

(a) $$\delta = \frac{P}{E}\left(\frac{2\frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt}\right) = \frac{7LP}{6\,Ebt} \qquad \boxed{\delta = \frac{7PL}{6\,Ebt}}$$

(b) Numerical data $E = 210$ GPa $\quad L = 750$ mm $\quad \sigma_{mid} = 160$ MPa

so $$\sigma_{mid} = \frac{P}{\frac{3}{4}bt} \quad \text{and} \quad \frac{P}{bt} = \frac{3}{4}\sigma_{mid}$$

$$\delta = \frac{7LP}{6Ebt} \quad \text{or} \quad \delta = \frac{7L}{6E}\left(\frac{3}{4}\sigma_{mid}\right) = 0.5\text{ mm} \qquad \boxed{\delta = 0.5\text{ mm}}$$

(c) $$\delta_{max} = \frac{P}{E}\left(\frac{L - L_{slot}}{bt} + \frac{L_{slot}}{\frac{3}{4}bt}\right) \quad \text{or} \quad \delta_{max} = \left(\frac{P}{bt}\right)\left(\frac{1}{E}\right)\left(L - L_{slot} + \frac{4}{3}L_{slot}\right)$$

or $$\delta_{max} = \left(\frac{3}{4}\sigma_{mid}\right)\left(\frac{1}{E}\right)\left(L + \frac{L_{slot}}{3}\right)$$ Solving for L_{slot} with $\delta_{max} = 0.475$ mm

$$L_{slot} = \frac{4E\delta_{max} - 3L\sigma_{mid}}{\sigma_{mid}} = 244\text{ mm} \qquad \boxed{L_{slot} = 244\text{ mm}} \qquad \frac{L_{slot}}{L} = 0.325$$

Problem 2.3-9

(a) $$\delta = \frac{P}{E}\left(\frac{2\frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt}\right) \qquad \text{simplifying gives} \qquad \delta = \frac{7LP}{6Ebt}$$

(b) $E = 207\text{ GPa} \qquad L = 760\text{ mm} \qquad \sigma_{\text{mid}} = 165\text{ MPa} \qquad \delta_{\max} = 0.5\text{ mm}$

So $$\sigma_{\text{mid}} = \frac{P}{\frac{3}{4}bt} \qquad \text{and} \qquad \frac{P}{bt} = \frac{3}{4}\sigma_{\text{mid}}$$

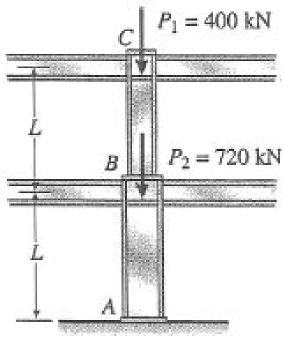
$$\delta = \frac{7LP}{6Ebt} \qquad \text{or} \qquad \delta = \frac{7L}{6E}\left(\frac{3}{4}\sigma_{\text{mid}}\right) = 0.53007\text{ mm} \qquad \boxed{\delta = 0.53\text{ mm}}$$

(c) $$\delta_{\max} = \frac{P}{E}\left(\frac{L - L_{\text{slot}}}{bt} + \frac{L_{\text{slot}}}{\frac{3}{4}bt}\right) \qquad \text{or} \qquad \delta_{\max} = \left(\frac{P}{bt}\right)\left(\frac{1}{E}\right)\left(L - L_{\text{slot}} + \frac{4}{3}L_{\text{slot}}\right)$$

or $$\delta_{\max} = \left(\frac{3}{4}\sigma_{\text{mid}}\right)\left(\frac{1}{E}\right)\left(L + \frac{L_{\text{slot}}}{3}\right)$$ Solving for L_{slot} with $\delta_{\max} = 0.5\text{ mm}$

$$L_{\text{slot}} = \frac{4E\delta_{\max} - 3L\sigma_{\text{mid}}}{\sigma_{\text{mid}}} = 229.09\text{ mm} \qquad \boxed{L_{\text{slot}} = 229\text{ mm}} \qquad \frac{L_{\text{slot}}}{L} = 0.301$$

Problem 2.3-10



L = length of each column = 3.75 m

E = 206 GPa

$A_{AB} = 11{,}000 \text{ mm}^2$

$A_{BC} = 3{,}900 \text{ mm}^2$

(a) SHORTENING δ_{AC} OF THE TWO COLUMNS

$$\delta_{AC} = \Sigma\frac{N_iL_i}{E_iA_i} = \frac{N_{AB}L}{EA_{AB}} + \frac{N_{BC}L}{EA_{BC}}$$

$$= \frac{(1120 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(11{,}000 \text{ mm}^2)} + \frac{(400 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(3{,}900 \text{ mm}^2)}$$

$$= 1.8535 \text{ mm} + 1.8671 \text{ mm} = 3.7206 \text{ mm}$$

$\delta_{AC} = 3.72$ mm ←

(b) ADDITIONAL LOAD P_0 AT POINT C

$(\delta_{AC})_{max} = 4.0$ mm

δ_0 = additional shortening of the two columns due to the load P_0

$\delta_0 = (\delta_{AC})_{max} - \delta_{AC} = 4.0 \text{ mm} - 3.7206 \text{ mm}$

$= 0.2794$ mm

Also, $\delta_0 = \frac{P_0L}{EA_{AB}} + \frac{P_0L}{EA_{BC}} = \frac{P_0L}{E}\left(\frac{1}{A_{AB}} + \frac{1}{A_{BC}}\right)$

Solve for P_0:

$$P_0 = \frac{E\delta_0}{L}\left(\frac{A_{AB}A_{BC}}{A_{AB} + A_{BC}}\right)$$

SUBSTITUTE NUMERICAL VALUES:

$E = 206 \times 10^9 \text{ N/m}^2 \quad \delta_0 = 0.2794 \times 10^{-3} \text{ m}$

$L = 3.75 \text{ m} \quad A_{AB} = 11{,}000 \times 10^{-6} \text{ m}^2$

$A_{BC} = 3{,}900 \times 10^{-6} \text{ m}^2$

$P_0 = 44{,}200 \text{ N} = 44.2 \text{ kN}$ ←

Problem 2.3-11

NUMERICAL DATA $E = 205\ \text{GPa}$ $P = 22\ \text{kN}$ $L = 2.4\ \text{m}$ $d_1 = 20\ \text{mm}$ $d_2 = 12\ \text{mm}$

(a) $$\delta_a = \frac{PL}{E}\left(\frac{1}{\frac{\pi}{4}d_1^2} + \frac{1}{\frac{\pi}{4}d_2^2}\right) = 3.0972\ \text{mm} \qquad \boxed{\delta_a = 3.1\ \text{mm}}$$

(b) $$\text{Vol}_a = \left(\frac{\pi}{4}d_1^2 + \frac{\pi}{4}d_2^2\right)L = 1.025 \times 10^6\ \text{mm} \quad d = \sqrt{\frac{\text{Vol}_a}{\frac{\pi}{4}(2L)}} = 16.492\ \text{mm} \quad A = \frac{\pi}{4}d^2 = 213.6283\ \text{mm}$$

$$\delta_b = \frac{P(2L)}{EA} = 2.4113\ \text{mm} \qquad \boxed{\delta_b = 2.41\ \text{mm}}$$

(c) $q = 18.33\,\frac{\text{kN}}{\text{m}}$ $L = 2.4\ \text{m}$

$$\delta_c = \frac{qL^2}{2E\left(\frac{\pi}{4}d_1^2\right)} + \frac{PL}{E\left(\frac{\pi}{4}d_2^2\right)} = 2.0253\ \text{mm} \qquad \boxed{\frac{\delta_c}{\delta_a} = 1.0} \qquad \boxed{\frac{\delta_c}{\delta_b} = 1.284}$$

Problem 2.3-12

NUMERICAL DATA

$d_1 = 100$ mm $\quad d_2 = 60$ mm

$L = 1200$ mm $\quad E = 4.0$ GPa $\quad P = 110$ kN

$\delta_a = 8.0$ mm

(a) FIND d_{max} IF SHORTENING IS LIMITED TO δ_a

$$A_1 = \frac{\pi}{4}{d_1}^2 \quad A_2 = \frac{\pi}{4}{d_2}^2$$

$$\delta = \frac{P}{E}\left[\frac{\frac{L}{4}}{\frac{\pi}{4}({d_1}^2 - {d_{max}}^2)} + \frac{\frac{L}{4}}{A_1} + \frac{\frac{L}{2}}{A_2}\right]$$

Set δ to δ_a, and solve for d_{max}:

$$d_{max} = d_1\sqrt{\frac{E\delta_a \pi {d_1}^2{d_2}^2 - 2PL{d_2}^2 - 2PL{d_1}^2}{E\delta_a\pi {d_1}^2{d_2}^2 - PL{d_2}^2 - 2PL{d_1}^2}}$$

$d_{max} = 23.9$ mm $\quad\leftarrow$

(b) NOW, IF d_{max} IS INSTEAD SET AT $d_2/2$, AT WHAT DISTANCE b FROM END C SHOULD LOAD P BE APPLIED TO LIMIT THE BAR SHORTENING TO $\delta_a = 8.0$ mm?

$$A_0 = \frac{\pi}{4}\left[d_1^2 - \left(\frac{d_2}{2}\right)^2\right]$$

$$A_1 = \frac{\pi}{4}{d_1}^2 \qquad A_2 = \frac{\pi}{4}{d_2}^2$$

$$\delta = \frac{P}{E}\left[\frac{L}{4A_0} + \frac{L}{4A_1} + \frac{\left(\frac{L}{2} - b\right)}{A_2}\right]$$

No axial force in segment at end of length b; set $\delta = \delta_a$ and solve for b:

$$b = \left[\frac{L}{2} - A_2\left[\frac{E\delta_a}{P} - \left(\frac{L}{4A_0} + \frac{L}{4A_1}\right)\right]\right]$$

$b = 4.16$ mm $\quad\leftarrow$

(c) FINALLY IF LOADS P ARE APPLIED AT THE ENDS AND $d_{max} = d_2/2$, WHAT IS THE PERMISSIBLE LENGTH x OF THE HOLE IF SHORTENING IS TO BE LIMITED TO $\delta_a = 8.0$ mm?

$$\delta = \frac{P}{E}\left[\frac{x}{A_0} + \frac{\left(\frac{L}{2} - x\right)}{A_1} + \frac{\left(\frac{L}{2}\right)}{A_2}\right]$$

Set $\delta = \delta_a$ and solve for x:

$$x = \frac{\left[A_0A_1\left(\frac{E\delta_a}{P} - \frac{L}{2A_2}\right)\right] - \frac{1}{2}A_0L}{A_1 - A_0}$$

$x = 183.3$ mm $\quad\leftarrow$

Problem 2.3-13

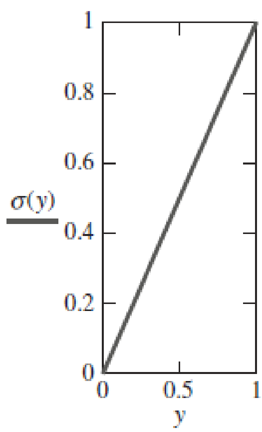
AFD LINEAR

(a) $N(y) = fy \qquad \delta = \int_0^L \frac{(fy)}{EA}\,dy = \frac{L^2 f}{2AE} \qquad \boxed{\delta = \frac{PL}{2EA}}$

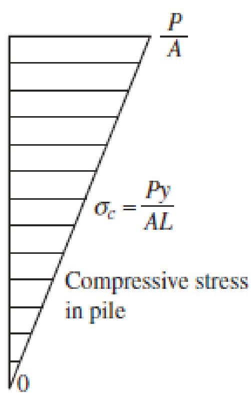
(b) $\sigma(y) = \frac{N(y)}{A} \qquad \sigma(y) = \frac{fy}{A} \qquad \sigma(L) = \frac{fL}{A} = \frac{P}{A}$

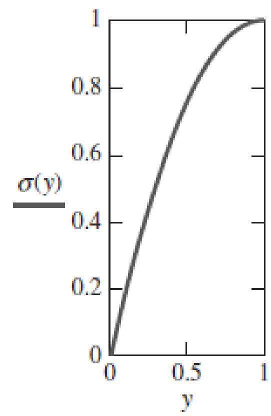
$\sigma(0) = 0$ So linear variation, zero at bottom, P/A at top (i.e., at ground surface)

$N(L) = f \qquad \boxed{\sigma(y) = \frac{P}{A}\left(\frac{y}{L}\right)}$



$f(y)$ is constant and AFD is linear





$f(y)$ is linear and AFD quadratic

(c) $N(y) = f(y)y$

$$N(y) = \int_0^y f_0\left(1 - \frac{\zeta}{L}\right)d\zeta = \frac{f_0 y(y-2)}{2} \qquad N(L) = \frac{f_0}{2} \qquad N(0) = 0$$

$$\delta = \frac{\left(\frac{f_0 L}{2}\right)}{\frac{3}{2}EA} \qquad P = \frac{1}{2}f_0 L \qquad \boxed{\delta = \frac{PL}{EA}\left(\frac{2}{3}\right)} \qquad \boxed{\sigma(y) = \frac{P}{A}\left[\frac{y}{L}\left(2 - \frac{y}{L}\right)\right]} \qquad \sigma(0) = 0 \qquad \sigma(L) = \frac{f_0}{2} = P/A$$

Problem 2.3-14

NUMERICAL DATA

$P = 5 \text{ kN} \qquad E_c = 120 \text{ GPa}$

$L_2 = 18 \text{ mm} \qquad L_4 = L_2$

$L_3 = 40 \text{ mm}$

$d_{o3} = 22.2 \text{ mm} \qquad t_3 = 1.65 \text{ mm}$

$d_{o5} = 18.9 \text{ mm} \qquad t_5 = 1.25 \text{ mm}$

$\tau_Y = 30 \text{ MPa} \qquad \sigma_Y = 200 \text{ MPa}$

$\text{FS}_\tau = 2 \qquad \text{FS}_\sigma = 1.7$

$$\tau_a = \frac{\tau_Y}{\text{FS}_\tau} \qquad \tau_a = 15 \text{ MPa}$$

$$\sigma_a = \frac{\sigma_Y}{\text{FS}_\sigma} \qquad \sigma_a = 117.6 \text{ MPa}$$

(a) ELONGATION OF SEGMENT 2-3-4

$$A_2 = \frac{\pi}{4}[d_{o3}^2 - (d_{o5} - 2t_5)^2]$$

$$A_3 = \frac{\pi}{4}[d_{o3}^2 - (d_{o3} - 2t_3)^2]$$

$A_2 = 175.835 \text{ mm}^2 \qquad A_3 = 106.524 \text{ mm}^2$

$$\delta_{24} = \frac{P}{E_c}\left(\frac{L_2 + L_4}{A_2} + \frac{L_3}{A_3}\right)$$

$\delta_{24} = 0.024 \text{ mm} \quad \leftarrow$

(b) MAXIMUM LOAD P_{max} THAT CAN BE APPLIED TO THE JOINT

First check normal stress:

$$A_1 = \frac{\pi}{4}[d_{o5}^2 - (d_{o5} - 2t_5)^2]$$

$A_1 = 69.311 \text{ mm}^2$ < smallest cross-sectional area controls normal stress

$P_{max\sigma} = \sigma_a A_1 \qquad P_{max\sigma} = 8.15 \text{ kN} \quad \leftarrow$ smaller than P_{max} based on shear below so normal stress controls

Next check shear stress in solder joint:

$A_{sh} = \pi d_{o5} L_2 \qquad A_{sh} = 1.069 \times 10^3 \text{ mm}^2$

$P_{max\tau} = \tau_a A_{sh} \qquad P_{max\tau} = 16.03 \text{ kN}$

(c) FIND THE VALUE OF L_2 AT WHICH TUBE AND SOLDER *CAPACITIES* ARE EQUAL

Set P_{max} based on shear strength equal to P_{max} based on tensile strength and solve for L_2:

$$L_2 = \frac{\sigma_a A_1}{\tau_a(\pi d_{o5})} \qquad L_2 = 9.16 \text{ mm} \quad \leftarrow$$

Problem 2.3-15

(a) STATICS $\sum F_H = 0 \quad R_1 = -P - \frac{P}{2}$

$$R_1 = \frac{-3}{2}P \quad \leftarrow$$

(b) DRAW FBD'S CUTTING THROUGH SEGMENT 1 AND AGAIN THROUGH SEGMENT 2

$N_1 = \frac{3P}{2}$ < tension $\quad N_2 = \frac{P}{2}$ < tension

(c) FIND x REQUIRED TO OBTAIN AXIAL DISPLACEMENT AT JOINT 3 OF $\delta_3 = PL/EA$

Add axial deformations of segments 1 and 2, then set to δ_3; solve for x:

$$\frac{N_1 x}{E\frac{3}{4}A} + \frac{N_2(L-x)}{EA} = \frac{PL}{EA}$$

$$\frac{\frac{3P}{2}x}{E\frac{3}{4}A} + \frac{\frac{P}{2}(L-x)}{EA} = \frac{PL}{EA}$$

$$\frac{3}{2}x = \frac{L}{2} \quad x = \frac{L}{3} \quad \leftarrow$$

(d) WHAT IS THE DISPLACEMENT AT JOINT 2, δ_2?

$$\delta_2 = \frac{N_1 x}{E\frac{3}{4}A} \quad \delta_2 = \frac{\left(\frac{3P}{2}\right)\frac{L}{3}}{E\frac{3}{4}A}$$

$$\delta_2 = \frac{2}{3}\frac{PL}{EA}$$

(e) IF $x = 2L/3$ AND $P/2$ AT JOINT 3 IS REPLACED BY βP, FIND β SO THAT $\delta_3 = PL/EA$

$$N_1 = (1+\beta)P \quad N_2 = \beta P \quad x = \frac{2L}{3}$$

substitute in axial deformation expression above and solve for β

$$\frac{[(1+\beta)P]\frac{2L}{3}}{E\frac{3}{4}A} + \frac{\beta P\left(L - \frac{2L}{3}\right)}{EA} = \frac{PL}{EA}$$

$$\frac{1}{9}PL\frac{8+11\beta}{EA} = \frac{PL}{EA}$$

$$(8 + 11\beta) = 9$$

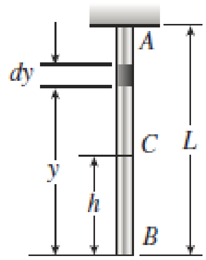
$$\beta = \frac{1}{11} \quad \leftarrow$$

$$\beta = 0.091$$

(f) Draw AFD, ADD—see plots for $x = \frac{L}{3}$

No plots provided here

Problem 2.3-16



W = Weight of bar

(a) Downward displacement δ_C
Consider an element at distance y from the lower end.

$$N(y) = \frac{Wy}{L} \quad d\delta = \frac{N(y)\,dy}{EA} = \frac{Wydy}{EAL}$$

$$\delta_C = \int_h^L d\delta = \int_h^L \frac{Wydy}{EAL} = \frac{W}{2EAL}(L^2 - h^2)$$

$$\delta_C = \frac{W}{2EAL}(L^2 - h^2) \quad \leftarrow$$

(b) Elongation of bar ($h = 0$)

$$\delta_B = \frac{WL}{2EA} \quad \leftarrow$$

(c) Ratio of elongations

Elongation of upper half of bar $\left(h = \frac{L}{2}\right)$:

$$\delta_{\text{upper}} = \frac{3WL}{8EA}$$

Elongation of lower half of bar:

$$\delta_{\text{lower}} = \delta_B - \delta_{\text{upper}} = \frac{WL}{2EA} - \frac{3WL}{8EA} = \frac{WL}{8EA}$$

$$\beta = \frac{\delta_{\text{upper}}}{\delta_{\text{lower}}} = \frac{3/8}{1/8} = 3 \quad \leftarrow$$

(d) Numerical data

$\gamma_s = 77\ \text{kN/m}^3 \qquad \gamma_w = 10\ \text{kN/m}^3 \qquad L = 1500\ \text{m} \qquad A = 0.0157\ \text{m}^2 \qquad E = 210\ \text{GPa}$

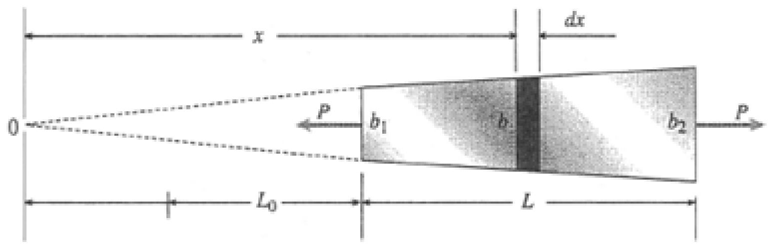
In sea water:

$$W = (\gamma_s - \gamma_w)AL = 1577.85\,\text{kN} \qquad \boxed{\delta = \frac{WL}{2EA} = 359\ \text{mm}} \qquad \frac{\delta}{L} = 2.393 \times 10^{-4}$$

In air:

$$W = (\gamma_s)AL = 1813.35\,\text{kN} \qquad \boxed{\delta = \frac{WL}{2EA} = 412\ \text{mm}} \qquad \frac{\delta}{L} = 2.75 \times 10^{-4}$$

Problem 2.3-17



t = thickness (constant)

$$b = b_1\left(\frac{x}{L_0}\right) \quad b_2 = b_1\left(\frac{L_0 + L}{L_0}\right) \qquad \text{(Eq. 1)}$$

$$A(x) = bt = b_1 t\left(\frac{x}{L_0}\right)$$

(a) ELONGATION OF THE BAR

$$d\delta = \frac{P\,dx}{EA(x)} = \frac{PL_0\,dx}{Eb_1 tx}$$

$$\delta = \int_{L_0}^{L_0+L} d\delta = \frac{PL_0}{Eb_1 t}\int_{L_0}^{L_0+L}\frac{dx}{x}$$

$$= \frac{PL_0}{Eb_1 t}\ln x\Big|_{L_0}^{L_0+L} = \frac{PL_0}{Eb_1 t}\ln\frac{L_0 + L}{L_0} \qquad \text{(Eq. 2)}$$

From Eq. (1): $\dfrac{L_0 + L}{L_0} = \dfrac{b_2}{b_1}$ (Eq. 3)

Solve Eq. (3) for L_0: $L_0 = L\left(\dfrac{b_1}{b_2 - b_1}\right)$ (Eq. 4)

Substitute Eqs. (3) and (4) into Eq. (2):

$$\delta = \frac{PL}{Et\,(b_2 - b_1)}\ln\frac{b_2}{b_1} \qquad \text{(Eq. 5)}$$

(b) SUBSTITUTE NUMERICAL VALUES:

$L = 1.5$ m $\quad t = 25$ mm

$P = 125$ kN $\quad b_1 = 100$ mm

$b_2 = 150$ mm $\quad E = 200$ GPa

From Eq. (5): $\delta = 0.304$ mm ←

Problem 2.3-18

$P = 200kN$ $\quad L = 2m$ $\quad t = 20mm$ $\quad b_1 = 100mm$ $\quad b_2 = 115mm$ $\quad E = 96GPa$

Bar width at B at L/2 $\quad b_B = \frac{b_1 + b_2}{2} = 107.5 \cdot mm$

Axial forces in bar segments (use RHFB) $\quad N_{AB} = 2 \cdot P - P = 200 \cdot kN$ $\quad N_{BC} = 2 \cdot P = 400 \cdot kN$

Axial displacement at B $\quad \delta_B = \frac{N_{AB} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_B)} \cdot \ln\left(\frac{b_2}{b_B}\right) = 0.937 \cdot mm$

Axial displacement at C $\quad \delta_C = \delta_B + \frac{N_{BC} \cdot \frac{L}{2}}{E \cdot t \cdot (b_B - b_1)} \cdot \ln\left(\frac{b_B}{b_1}\right) = 2.946 \cdot mm$

Problem 2.3-19

$P = 225kN \quad L = 1.5m \quad t = 10mm \quad b_1 = 75mm \quad b_2 = 70mm \quad E = 110GPa$

$N_{AB} = 2 \cdot P - P = 225 \cdot kN \qquad N_{BC} = 2 \cdot P = 450 \cdot kN$

Axial displacement at B $\quad \delta_B = \dfrac{N_{AB} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = 2.117 \cdot mm$

Axial displacement at C $\quad \delta_C = \delta_B + \dfrac{N_{BC} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\dfrac{b_2}{b_1}\right) = 6.35 \cdot mm$

Problem 2.3-20

$E = 72GPa \qquad P_2 = 200kN \qquad L = 2m \qquad t = 20mm \qquad b_1 = 100mm \qquad b_2 = 115mm$

$A_{BC} = b_1 \cdot t = 2 \times 10^3 \cdot mm^2$

If only load P_2 is applied at C

$$\delta_B = \frac{P_2 \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 1.294 \cdot mm \qquad \delta_C = \delta_B + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = 2.683 \cdot mm$$

Now apply both P_1 (to the left) and P_2 at C and solve for P_1 s.t. axial displacement at C = 0

Given

$$\frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = 0 \qquad \text{Find}(P_1) = 414.651 \cdot kN$$

Axial displacement at B with both loads applied as shown

Let $P_1 = 414.651kN$

$$\delta_B = \frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = -1.389 \cdot mm$$

leftward

Check

$$\frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = -0\,m$$

Problem 2.3-21

$d_A = 100mm \qquad d_B = 200mm \qquad P = 200kN \qquad \delta_A = 0.5mm \qquad d(x) = d_A + \left(\frac{d_B - d_A}{L}\right) \cdot x$

$A(x) = \frac{\pi}{4} \cdot d(x)^2 \qquad E = 72GPa$

$\int_0^L \frac{P}{E \cdot A(x)} dx = \delta_A$ expand integral to obtain following expression $\frac{4 \cdot P \cdot L}{\pi \cdot E \cdot d_A \cdot d_B} = \delta_A$

Solving for L $\qquad L = \frac{\pi \cdot E \cdot d_A \cdot d_B}{4 \cdot P} \cdot \delta_A = 2.827 \cdot m$

Problem 2.3-22

$L = 1.8m \qquad r = 36mm \qquad E = 72GPa \qquad a = \frac{r}{8} = 4.5 \cdot mm \qquad \sigma_2 = 180MPa$

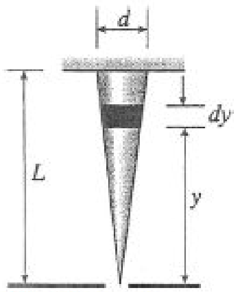
$A_1 = \pi \cdot r^2 = 4071.504 \cdot mm^2$ Use formulas in ***Appendix E***, ***Case 15*** for area of slotted segment

$\alpha = acos\left(\frac{a}{r}\right) = 1.445 \qquad b = \sqrt{r^2 - a^2} = 35.718 \cdot mm \qquad A_2 = 2 \cdot r^2 \cdot \left(\alpha - \frac{a \cdot b}{r^2}\right) = 3425.196 \cdot mm^2 \qquad \frac{A_2}{A_1} = 0.841$

Stress in middle half is known so use to find force P $\quad P = \sigma_2 \cdot A_2 = 616.535 \cdot kN$

Compute bar elongation now that P is known $\quad \delta = 2 \cdot \frac{P \cdot \frac{L}{4}}{E \cdot A_1} + \frac{P \cdot \frac{L}{2}}{E \cdot A_2} = 4.143 \cdot mm$

Problem 2.3-23



TERMINOLOGY

N_y = axial force acting on element dy

A_y = cross-sectional area at element dy

A_B = cross-sectional area at base of cone

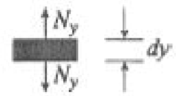
$= \frac{\pi d^2}{4}$ $\quad V$ = volume of cone

$= \frac{1}{3} A_B L$ $\quad V_y$ = volume of cone below element dy

$= \frac{1}{3} A_y y$ $\quad W_y$ = weight of cone below element dy

$= \frac{V_y}{V}(W) = \frac{A_y y W}{A_B L}$ $\quad N_y = W_y$

ELEMENT OF BAR



W = weight of cone

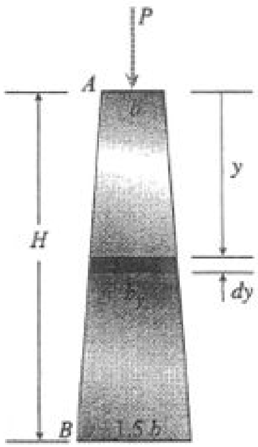
ELONGATION OF ELEMENT dy

$$d\delta = \frac{N_y\, dy}{E A_y} = \frac{W y\, dy}{E A_B L} = \frac{4W}{\pi d^2 E L} y\, dy$$

ELONGATION OF CONICAL BAR

$$\delta = \int d\delta = \frac{4W}{\pi d^2 EL} \int_0^L y\, dy = \frac{2WL}{\pi d^2 E} \quad \leftarrow$$

Problem 2.3-24



Square cross sections:

b = width at A

$1.5b$ = width at B

b_y = width at distance y

$$= b + (1.5b - b)\frac{y}{H}$$

$$= \frac{b}{H}(H + 0.5y)$$

A_y = cross-sectional area at distance y

$$= (b_y)^2 = \frac{b^2}{H^2}(H + 0.5y)^2$$

Shortening of element dy

$$d\delta = \frac{Pdy}{EA_y} = \frac{Pdy}{E\left(\frac{b^2}{H^2}\right)(H + 0.5y)^2}$$

Shortening of entire post

$$\delta = \int d\delta = \frac{PH^2}{Eb^2}\int_0^H \frac{dy}{(H + 0.5y)^2}$$

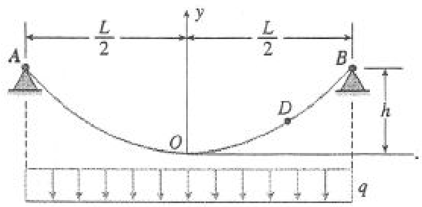
From Appendix C: $\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$

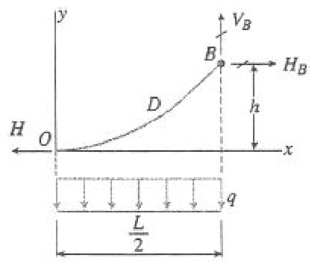
$$\delta = \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(H + 0.5y)}\right]_0^H$$

$$= \frac{PH^2}{Eb^2}\left[-\frac{1}{(0.5)(1.5H)} + \frac{1}{0.5H}\right]$$

$$= \frac{2PH}{3Eb^2} \quad \leftarrow$$

Problem 2.3-25





Equation of parabolic curve:

$$y = \frac{4hx^2}{L^2}$$

$$\frac{dy}{dx} = \frac{8hx}{L^2}$$

FREE-BODY DIAGRAM OF HALF OF CABLE

$$\Sigma M_B = 0$$

$$-Hh + \frac{qL}{2}\left(\frac{L}{4}\right) = 0$$

$$H = \frac{qL^2}{8h}$$

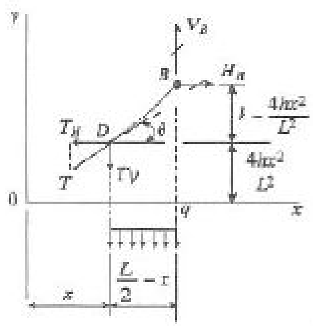
$$\Sigma F_{\text{horizontal}} = 0$$

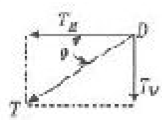
$$H_B = H = \frac{qL^2}{8h} \qquad \text{(Eq. 1)}$$

$$\Sigma F_{\text{vertical}} = 0$$

$$V_B = \frac{qL}{2} \qquad \text{(Eq. 2)}$$

FREE-BODY DIAGRAM OF SEGMENT DB OF CABLE





$$\Sigma F_{\text{horiz}} = 0 \qquad T_H = H_B = \frac{qL^2}{8h} \qquad \text{(Eq. 3)}$$

$$\Sigma F_{\text{vert}} = 0 \quad V_B - T_v - q\left(\frac{L}{2} - x\right) = 0$$

$$T_v = V_B - q\left(\frac{L}{2} - x\right) = \frac{qL}{2} - \frac{qL}{2} + qx$$

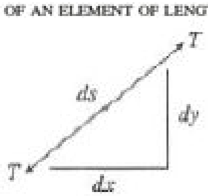
$$= qx \qquad \text{(Eq. 4)}$$

TENSILE FORCE T IN CABLE

$$T = \sqrt{T_H^2 + T_v^2} = \sqrt{\left(\frac{qL^2}{8h}\right)^2 + (qx)^2}$$

$$= \frac{qL^2}{8h}\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 5)}$$

ELONGATION $d\delta$ OF AN ELEMENT OF LENGTH ds



$$d\delta = \frac{T\,ds}{EA}$$

$$ds = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1 + \left(\frac{8hx}{L^2}\right)^2}$$

$$= dx\sqrt{1 + \frac{64h^2x^2}{L^4}} \qquad \text{(Eq. 6)}$$

(a) ELONGATION δ OF CABLE AOB

$$\delta = \int d\delta = \int \frac{T\,ds}{EA}$$

Substitute for T from Eq. (5) and for ds from Eq. (6):

$$\delta = \frac{1}{EA}\int \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

For both halves of cable:

$$\delta = \frac{2}{EA}\int_0^{L/2} \frac{qL^2}{8h}\left(1 + \frac{64h^2x^2}{L^4}\right)dx$$

$$\delta = \frac{qL^3}{8hEA}\left(1 + \frac{16h^2}{3L^4}\right) \quad \leftarrow \qquad \text{(Eq. 7)}$$

(b) GOLDEN GATE BRIDGE CABLE

$L = 1300$ m $\qquad h = 140$ m

$q = 185$ kN/m $\qquad E = 200$ GPa

27,572 wires of diameter $d = 5$ mm

$$A = (27{,}572)\left(\frac{\pi}{4}\right)(5 \text{ mm})^2 = 541.375 \text{ mm}^2$$

Substitute into Eq. (7):

$\delta = 3.55$ m $\leftarrow$

Problem 2.3-26

(a) ELONGATION δ FOR CASE OF CONSTANT DIAMETER HOLE

$$d(\zeta) = d_A\left(1 + \frac{\zeta}{L}\right) \qquad A(\zeta) = \frac{\pi}{4}d(\zeta)^2 \quad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4}(d(\zeta)^2 - {d_A}^2) \quad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E}\left(\int \frac{1}{A(\zeta)}\,d\zeta\right) \qquad \delta = \frac{P}{E}\left[\int_0^{L-x} \frac{4}{\pi d(\zeta)^2}\,d\zeta + \int_{L-x}^{L} \frac{4}{\pi(d(\zeta)^2 - {d_A}^2)}\,d\zeta\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x} \frac{1}{\left[\frac{\pi}{4}\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^2\right]}\,d\zeta + \int_{L-x}^{L} \frac{1}{\left[\frac{\pi}{4}\left[\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^2 - {d_A}^2\right]\right]}\,d\zeta\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2+x)\pi {d_A}^2} + \left[\left[4\frac{L}{\pi d_A^2} + \int_{L-x}^{L} \frac{1}{\left[\frac{\pi}{4}\left[\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^2 - {d_A}^2\right]\right]}\,d\zeta\right]\right]\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2+x)\pi {d_A}^2} + \left(4\frac{L}{\pi {d_A}^2} - 2L\frac{\ln(3)}{\pi {d_A}^2} + 2L\frac{-\ln(L-x) + \ln(3L-x)}{\pi {d_A}^2}\right)\right]$$

$$\text{if } x = L/2 \quad \delta = \frac{P}{E}\left(\frac{4}{3}\frac{L}{\pi d_A^2} - 2L\frac{\ln(3)}{\pi d_A^2} + 2L\frac{-\ln\left(\frac{1}{2}L\right) + \ln\left(\frac{5}{2}L\right)}{\pi d_A^2}\right)$$

Substitute numerical data:

$\delta = 2.18$ mm $\leftarrow$

(b) ELONGATION δ FOR CASE OF VARIABLE DIAMETER HOLE BUT CONSTANT WALL THICKNESS $t = d_A/20$ OVER SEGMENT x

$$d(\zeta) = d_A\left(1 + \frac{\zeta}{L}\right) \qquad A(\zeta) = \frac{\pi}{4}d(\zeta)^2 \quad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4}\left[d(\zeta)^2 - \left(d(\zeta) - 2\frac{d_A}{20}\right)^2\right] \quad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E}\left(\int \frac{1}{A(\zeta)}\,d\zeta\right) \qquad \delta = \frac{P}{E}\left[\int_0^{L-x} \frac{4}{\pi d(\zeta)^2}\,d\zeta + \int_{L-x}^{L} \frac{4}{\pi\left[d(\zeta)^2 - \left(d(\zeta) - 2\frac{d_A}{20}\right)^2\right]}\,d\zeta\right]$$

$$\delta = \frac{P}{E}\left[\int_0^{L-x} \frac{4}{\pi\left[d_A\left(1+\frac{\zeta}{L}\right)\right]} d\zeta + \int_{L-x}^{L} \frac{4}{\pi\left[\left[d_A\left(1+\frac{\zeta}{L}\right)\right]^2 - \left[d_A\left(1+\frac{\zeta}{L}\right) - 2\frac{d_A}{20}\right]^2\right]} d\zeta\right]$$

$$\delta = \frac{P}{E}\left[4\frac{L^2}{(-2L+x)\pi d_A{}^2} + 4\frac{L}{\pi d_A{}^2} + 20L\frac{\ln(3)+\ln(13)+2\ln(d_A)+\ln(L)}{\pi d_A{}^2} - 20L\frac{2\ln(d_A)+\ln(39L-20x)}{\pi d_A{}^2}\right]$$

if $x = L/2$

$$\delta = \frac{P}{E}\left(\frac{4}{3}\frac{L}{\pi d_A{}^2} + 20L\frac{\ln(3)+\ln(13)+2\ln(d_A)+\ln(L)}{\pi d_A{}^2} - 20L\frac{2\ln(d_A)+\ln(29L)}{\pi d_A{}^2}\right)$$

Substitute numerical data:

$\delta = 6.74$ mm $\leftarrow$

Problem 2.3-27

$P_1 = 11\text{kN} \quad P_2 = 4.5\text{kN} \quad M = 2.8\text{kN}\cdot\text{m} \quad E = 200\text{GPa} \quad A_1 = 160\text{mm}^2 \quad A_2 = 100\text{mm}^2$

Find pin force at B - use FBD of bar BDE

$$\Sigma M_D = 0 \qquad B_y = \frac{1}{625\text{mm}}\cdot\left[P_2\cdot(625\text{mm}) - M\right] = 20\cdot\text{N}$$

No pin force at B so bar ABC is subjected force P_1 at C only

$$\delta_C = \frac{P_1}{E}\cdot\left(\frac{500\text{mm}}{A_1} + \frac{875\text{mm}}{A_2}\right) = 0.653\cdot\text{mm} \ \text{downward}$$

Problem 2.3-28

Find pin force at B - use FBD of bar ABC $\Sigma M_A = 0$ $B_y = \frac{1}{\frac{L}{3}} \cdot (3 \cdot P \cdot L)$ $B_y \to 9\ P$ acts upward on ABC so downward on DBF

Vertical displacements at B and F

$N_{BD} = P - 9 \cdot P \to -8\ P$ $\delta_B = \frac{N_{BD} \cdot L}{2 \cdot EA}$ $\delta_B \to -4\ \frac{PL}{EA}$ downward

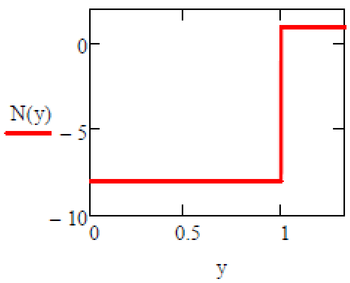
$N_{BF} = P$ $\delta_C = \delta_B + \frac{N_{BF} \cdot \frac{L}{3}}{EA}$ $\delta_C \to -\frac{11}{3}\ \frac{PL}{EA}$ downward

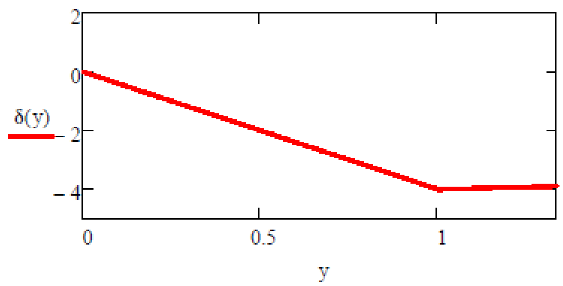
Axial force (N(y)) and displacement (δ(y)) diagrams - origin of y at D, positive upward (rotated CW to horiz. position below)

$N(y) = \begin{cases} N_{BD} & \text{if } y \le L \\ N_{BF} & \text{otherwise} \end{cases}$ $\delta(y) = \begin{cases} \left[N_{BD} \cdot y \cdot \left(\frac{L}{2 \cdot EA}\right)\right] & \text{if } y \le L \\ \left[\delta_B + N_{BF} \cdot (y - L) \cdot \left(\frac{L}{3 \cdot EA}\right)\right] & \text{otherwise} \end{cases}$

$\delta(0) \to 0$ $\delta(L) \to -4$

$\delta\left(\frac{4 \cdot L}{3}\right) \to -\frac{35}{9} = -3.889$ times PL/EA





Problem 2.3-29

Find pin force at B - use FBD of bar ABC $\Sigma F_y = 0$ $B_y = 2P$ upward at B on ABC so downward on DBF

Axial forces in column segments (tension is positive)

$N_{DB} = -P$ $N_{BF} = -P - B_y \rightarrow -3 \cdot P$ so AFD is constant and compressive over each column segment

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow -\frac{3 \cdot L \cdot P}{4 \cdot EA} \qquad \delta_D = \delta_B + \frac{N_{DB} \cdot \frac{L}{2}}{EA} \rightarrow -\frac{5 \cdot L \cdot P}{4 \cdot EA}$$

so ADD is linear and downward over each column segment

Problem 2.3-30

Use FBD of beam ABC - find pin force at B $\Sigma F_y = 0$ $B_y = 2 \cdot P$ upward on ABC so downward on DBF

Axial forces in column segments (tension is positive)

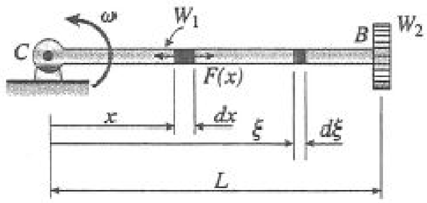
$N_{DB} = 0$ $N_{BF} = -B_y \rightarrow -2 \cdot P$ so AFD is 0 over DB and constant and compressive over column segment BF

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow -\frac{L \cdot P}{2 \cdot EA} \qquad \delta_D = \delta_B \rightarrow -\frac{L \cdot P}{2 \cdot EA}$$

so ADD is linear over BF and constant over column segment DB, both downward

Problem 2.3-31



ω = angular speed

A = cross-sectional area

E = modulus of elasticity

g = acceleration of gravity

$F(x)$ = axial force in bar at distance x from point C

Consider an element of length dx at distance x from point C.

To find the force $F(x)$ acting on this element, we must find the inertia force of the part of the bar from distance x to distance L, plus the inertia force of the weight W_2.

Since the inertia force varies with distance from point C, we now must consider an element of length $d\xi$ at distance ξ, where ξ varies from x to L.

Mass of element $d = \dfrac{d}{L}\left(\dfrac{W_1}{g}\right)$

Acceleration of element $= \xi\omega^2$

Centrifugal force produced by element

$$= (\text{mass})(\text{ acceleration}) = \frac{W_1\omega^2}{gL} d$$

Centrifugal force produced by weight W_2

$$= \left(\frac{W_2}{g}\right)(L\omega^2)$$

Axial force $F(x)$

$$F(x) = \int_{=x}^{=L} \frac{W_1\omega^2}{gL} d + \frac{W_2 L\omega^2}{g}$$

$$= \frac{W_1\omega^2}{2gL}(L^2 - x^2) + \frac{W_2 L\omega^2}{g}$$

Elongation of bar BC

$$\delta = \int_0^L \frac{F(x)\,dx}{EA}$$

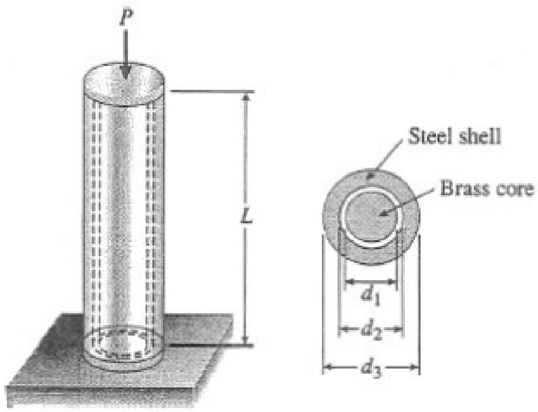
$$= \int_0^L \frac{W_1\omega^2}{2gL}(L^2 - x^2)dx + \int_0^L \frac{W_2 L\omega^2 dx}{gEA}$$

$$= \frac{W1L\omega^2}{2gLEA}\left[\int_0^L L^2\,dx - \int_0^L x^2\,dx\right] + \frac{W_2 L\omega^2 dx}{gEA}\int_0^L dx$$

$$= \frac{W_1 L^2\omega^2}{3gEA} + \frac{W_2 L^2\omega^2}{gEA}$$

$$= \frac{L^2\omega^2}{3gEA} + (W_1 + 3W_2) \quad \leftarrow$$

Problem 2.4-1



EQUATION OF EQULIBRIUM

$$\Sigma F_{vert} = 0,\ P_b + P_s - P = 0 \tag{1}$$

EQUATION OF COMPATIBILITY

$$\delta_s = \delta_b \tag{2}$$

FORCE DISPLACEMENT RELATIONS

$$\delta = \frac{P_s L}{E_s A_s},\quad \delta = \frac{P_b L}{E_b A_b} \tag{3}$$

SUBSTITUTE INTO Eq. (2):

$$\frac{P_s L}{E_s A_s} = \frac{P_b L}{E_b A_b} \tag{4}$$

SOLUTION OF EQUATIONS

Solve simultaneously Eqs. (1) and (4):

$$P_s = \frac{E_s A_s P}{E_s A_s + E_b A_b},\ P_b = \frac{E_b A_b P}{E_s A_s + E_b A_b} \tag{5}$$

Substitute into Eq. (3):

$$\delta = \delta_s = \delta_b = \frac{PL}{E_s A_s + E_b A_b} \tag{6}$$

STRESSES

$$\sigma = \frac{P_s}{A_s} = \frac{E_s P}{E_s A_s + E_b A_b},\ \sigma = \frac{P_b}{A_b} = \frac{E_b P}{E_s A_s + E_b A_b} \tag{7}$$

NUMERICAL VALUES

Steel: $A_s = \left(\frac{\pi}{4}\right)[(9\text{ mm})^2 - (7\text{ mm})^2] = 25.13\text{ mm}^2$

$E_s = 200\text{ GPa},\ E_s A_s = 5.027 \times 10^6\text{ N}$

Brass: $A_b = \left(\frac{\pi}{4}\right)(6.0\text{ mm})^2 = 28.27\text{ mm}^2$

$E_b = 100\text{ GPa},\ E_b A_b = 2.827 \times 10^6\text{ N}$

$E_b A_b + E_s A_s = 7.854 \times 10^6\text{ N},\ L = 85\text{ mm}$

(a) DECREASE IN LENGTH

$$\delta = \frac{PL}{E_s A_s + E_b A_b} = 0.1\text{ mm},\ P = 9.24\text{ kN}$$

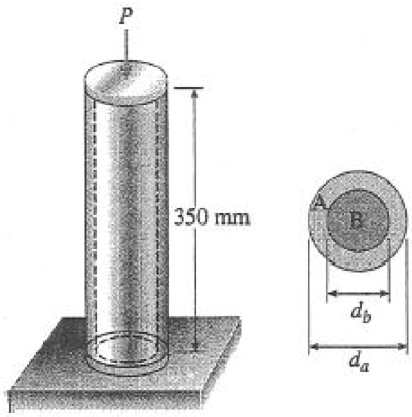
(b) ALLOWABLE LOAD

$$\sigma = 180\text{ MPa},\ P_s = \sigma_s \frac{E_b A_b + E_s A_s}{E_s} = 7.07\text{ kN}$$

$$\sigma = 250\text{ MPa},\ P_b = \sigma_b \frac{E_b A_b + E_s A_s}{E_b} = 11.00\text{ kN}$$

Steel governs. $P_{\text{allow}} = 7.07\text{ kN}$ ←

Problem 2.4-2



A = aluminum

B = brass

$L = 350$ mm

$d_a = 40$ mm

$d_b = 25$ mm

$$A_a = \frac{\pi}{4}(d_a^2 - d_b^2)$$

$$= 765.8 \text{ mm}^2$$

$E_a = 72$ GPa $E_b = 100$ GPa $A_b = \frac{\pi}{4}d_b^2$

$= 490.9 \text{ mm}^2$

(a) Decrease in length

($\delta = 0.1\%$ of $L = 0.350$ mm)

Use Eq. (2-18) of Example 2-6.

$$\delta = \frac{PL}{E_aA_a + E_bA_b} \quad \text{or}$$

$$P = (E_aA_a + E_bA_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$\begin{aligned} E_aA_a + E_bA_b &= (72 \text{ GPa})(765.8 \text{ mm}^2) \\ &\quad +(100 \text{ GPa})(490.9 \text{ mm}^2) \\ &= 55.135 \text{ MN} + 49.090 \text{ MN} \\ &= 104.23 \text{ MN} \end{aligned}$$

$$P = (104.23 \text{ MN})\left(\frac{0.350 \text{ mm}}{350 \text{ mm}}\right)$$

$$= 104.2 \text{ kN} \quad \longleftarrow$$

(b) Allowable load

$\sigma_a = 80$ MPa $\sigma_b = 120$ MPa

Use Eqs. (2-17a and b) of Example 2-6.

For aluminum:

$$\sigma_a = \frac{PE_a}{E_aA_a + E_bA_b} \quad P_a = (E_aA_a + E_bA_b)\left(\frac{\sigma_a}{E_a}\right)$$

$$P_a = (104.23 \text{ MN})\left(\frac{80 \text{ MPa}}{72 \text{ GPa}}\right) = 115.8 \text{ kN}$$

For brass:

$$\sigma_b = \frac{PE_b}{E_aA_a + E_bA_b} \quad P_b = (E_aA_a + E_bA_b)\left(\frac{\sigma_b}{E_b}\right)$$

$$P_b = (104.23 \text{ MN})\left(\frac{120 \text{ MPa}}{100 \text{ GPa}}\right) = 125.1 \text{ kN}$$

Aluminum governs. $P_{max} = 116$ kN $\longleftarrow$

Problem 2.4-3

$E = 200GPa \qquad A = 5100mm^2$

Use superposition - select A_y as the redundant

Released structure with actual load P at C $\qquad \delta_{A1} = \frac{10kN \cdot (2m)}{E \cdot A} = 0.02 \cdot mm \qquad$ upward

Released structure with redundant A_y applied at A $\qquad \delta_{A2} = A_y \cdot \left(\frac{1m + 2m}{E \cdot A}\right)$

$$\frac{1m + 2m}{E \cdot A} = 2.941 \times 10^{-3} \cdot \frac{mm}{kN}$$

Compatibility equation $\qquad \delta_{A1} + \delta_{A2} = 0 \qquad$ solve for redundant $A_y \qquad A_y = \frac{-\delta_{A1}}{\frac{1m+2m}{E \cdot A}} = -6.667 \cdot kN$

Statics $\qquad B_y = -(A_y + 10kN) = -3.333 \cdot kN$

Axial displacement at C $\qquad \frac{-B_y \cdot (2m)}{E \cdot A} = 6.536 \times 10^{-3} \cdot mm$ upward ... or

$$\frac{-A_y \cdot (1m)}{E \cdot A} = 6.536 \times 10^{-3} \cdot mm$$

use either extension of segment BC or compression of AC to find upward displ. δ_C

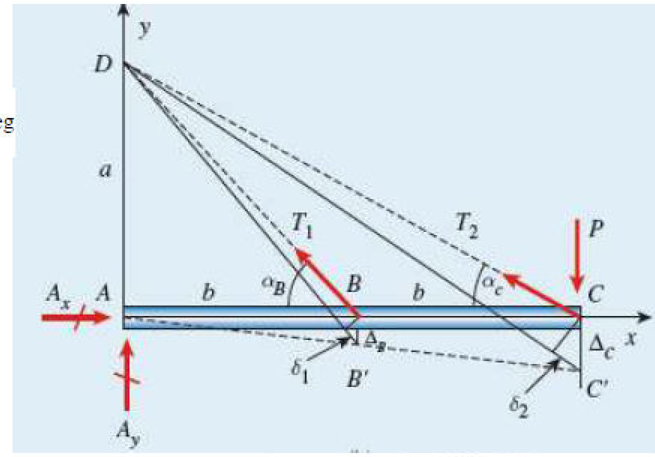
Problem 2.4-4

$P = 10kN \qquad E = 200GPa \qquad \sigma_Y = 400MPa \qquad FS_Y = 2 \qquad \sigma_a = \frac{\sigma_Y}{FS_Y} = 200\cdot MPa$

Static equilibrium - cut through cables, use lower FBD (see fig.)

$a = 1.5m \qquad b = 1.5m \qquad \alpha_B = \operatorname{atan}\left(\frac{a}{b}\right) = 45\cdot deg$

$\alpha_C = \operatorname{atan}\left(\frac{a}{2\cdot b}\right) = 26.565\cdot deg$



$\Sigma M_A = 0$

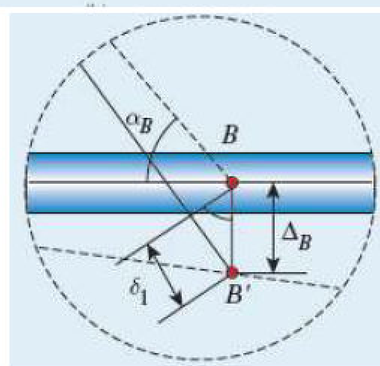
$T_1\cdot\sin(\alpha_B) + 2\cdot T_2\cdot\sin(\alpha_C) = P\cdot(2)$

Compatibility - from figure, see that $\quad \Delta_C = 2\cdot\Delta_B$

Cable elongations

$\delta_1 = \Delta_B\cdot\sin(\alpha_B) \qquad \delta_2 = \Delta_C\cdot\sin(\alpha_C)$

so $\quad \delta_2 = 2\cdot\left(\frac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\delta_1 \qquad 2\cdot\left(\frac{\sin(\alpha_C)}{\sin(\alpha_B)}\right) = 1.26491$



Force-displacement relations for cables

$L_1 = \sqrt{a^2 + b^2} = 2.121m \qquad L_2 = \sqrt{a^2 + (2\cdot b)^2} = 3.354m$

$\delta_1 = T_1\cdot f_1 \qquad f_1 = \frac{L_1}{E\cdot A_1} \qquad \delta_2 = T_2\cdot f_2 \qquad f_2 = \frac{L_2}{E\cdot A_2}$

where $\quad T_2\cdot f_2 = 2\cdot\left(\frac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot T_1\cdot f_1 \quad$ or $\quad T_2 = 2\cdot\left(\frac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\left(\frac{f_1}{f_2}\right)\cdot T_1 \quad$ and $A_1 = A_2$ so $\quad \frac{f_1}{f_2} = \frac{L_1}{L_2}$

Substitute T_2 expression into equilibrium equation and solve for T_1 then solve for T_2

$$T_1 = \left(\frac{2\cdot f_2\cdot\sin(\alpha_B)}{f_2\cdot\sin(\alpha_B)^2 + 4\cdot f_1\cdot\sin(\alpha_C)^2}\right)\cdot P \quad \text{or} \quad T_1 = \left[\frac{2\cdot\sin(\alpha_B)}{\sin(\alpha_B)^2 + 4\cdot\left(\frac{L_1}{L_2}\right)\cdot\sin(\alpha_C)^2}\right]\cdot P = 14.058\cdot kN$$

and

$$T_2 = 2\cdot\left(\frac{\sin(\alpha_C)}{\sin(\alpha_B)}\right)\cdot\left(\frac{L_1}{L_2}\right)\cdot T_1 = 11.247\cdot kN$$

Use allowable stress σ_a to find minimum required cross sectional area of each cable

$A_1 = \frac{T_1}{\sigma_a} = 70.291\cdot mm^2 \qquad A_2 = \frac{T_2}{\sigma_a} = 56.233\cdot mm^2 \qquad$ so $\qquad A_{reqd} = 70.3mm^2$

Problem 2.4-5

$\sigma_{ys} = 340\text{MPa}$ $\sigma_{yA} = 410\text{MPa}$ $A_s = 7700\text{mm}^2$ $A_A = 3800\text{mm}^2$ $L = 500\text{mm}$

$E_s = 200\text{GPa}$ $E_A = 73\text{GPa}$

Axial stiffnesses of cylinder and tube - treat as springs in parallel

$$k_s = \frac{E_s \cdot A_s}{L} = 3.08 \times 10^6 \cdot \frac{\text{kN}}{\text{m}} \qquad k_A = \frac{E_A \cdot A_A}{L} = 5.548 \times 10^5 \cdot \frac{\text{kN}}{\text{m}}$$

$$k_T = k_s + k_A = 3.635 \times 10^6 \cdot \frac{\text{kN}}{\text{m}}$$

Each "spring" carries a force in proportion to its stiffness

$$P_s(P) = \frac{k_s}{k_T} \cdot P \text{ float, 5} \rightarrow 0.84736 \cdot P \qquad P_A(P) = \frac{k_A}{k_T} \cdot P \text{ float, 5} \rightarrow 0.15264 \cdot P$$

Maximum force in each component is governed by its yield stress

$$\sigma_{ys}(P) = \frac{P_s(P)}{A_s} \text{ float, 5} \rightarrow \frac{0.00011005 \cdot P}{\text{mm}^2}$$

$$\sigma_{ys}(P) - 340\text{MPa} \;\Big|\; \begin{matrix}\text{solve, P} \\ \text{float, 5}\end{matrix} \rightarrow 3.0895\text{e}6 \cdot \text{MPa} \cdot \text{mm}^2 = 3089.5 \cdot \text{kN}$$

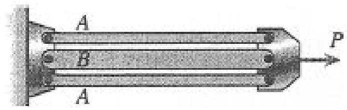
$$\sigma_{yA}(P) = \frac{P_A(P)}{A_A} \text{ float, 5} \rightarrow \frac{0.000040168 \cdot P}{\text{mm}^2}$$

$$\sigma_{yA}(P) - 410\text{MPa} \;\Big|\; \begin{matrix}\text{solve, P} \\ \text{float, 5}\end{matrix} \rightarrow 1.0207\text{e}7 \cdot \text{MPa} \cdot \text{mm}^2 = 10207 \cdot \text{kN}$$

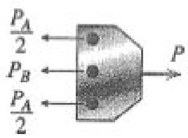
So the allowable load P is limited by yield stress in steel cylinder

$$P_{all} = 3090\text{kN}$$

Problem 2.4-6



FREE-BODY DIAGRAM OF END PLATE



EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{horiz}} = 0 \quad P_A + P_B - P = 0 \tag{1}$$

EQUATION OF COMPATIBILITY

$$\delta_A = \delta_B \tag{2}$$

FORCE-DISPLACEMENT RELATIONS

A_A = total area of both outer bars

$$\delta_A = \frac{P_A L}{E_A A_k} \quad \delta_B = \frac{P_B L}{E_B A_B} \tag{3}$$

Substitute into Eq. (2):

$$\frac{P_A L}{E_A A_A} = \frac{P_B L}{E_B A_B} \tag{4}$$

SOLUTION OF THE EQUATIONS

Solve simultaneously Eqs. (1) and (4):

$$P_A = \frac{E_A A_A P}{E_A A_A + E_B A_B} \quad P_B = \frac{E_B A_B P}{E_A A_A + E_B A_B} \tag{5}$$

Substitute into Eq. (3):

$$\delta = \delta_A = \delta_B = \frac{PL}{E_A A_A + E_B A_B} \tag{6}$$

STRESSES:

$$\sigma_A = \frac{P_A}{A_A} = \frac{E_A P}{E_A A_A + E_B A_B}$$

$$\sigma_B = \frac{P_B}{A_B} = \frac{E_B P}{E_A A_A + E_B A_B} \tag{7}$$

(a) **LOAD IN MIDDLE BAR**

$$\frac{P_B}{P} = \frac{E_B A_B}{E_A A_A + E_B A_B} = \frac{1}{\frac{E_A A_A}{E_B A_B} + 1}$$

$$\text{Given: } \frac{E_A}{E_B} = 2 \quad \frac{A_A}{A_B} = \frac{1+1}{1.5} = \frac{4}{3}$$

$$\therefore \frac{P_B}{P} = \frac{1}{\left(\frac{E_A}{E_B}\right)\left(\frac{A_A}{A_B}\right) + 1} = \frac{1}{\frac{8}{3} + 1} = \frac{3}{11} \quad \leftarrow$$

(b) **RATIO OF STRESSES**

$$\frac{\sigma_B}{\sigma_A} = \frac{E_B}{E_A} = \frac{1}{2} \quad \leftarrow$$

(c) **RATIO OF STRAINS**

All bars have the same strain

Ratio = 1 $\leftarrow$

Problem 2.4-7

(a) REACTIONS AT A AND B DUE TO LOAD P AT $L/2$

$$A_{AC} = \frac{\pi}{4}\left[d^2 - \left(\frac{d}{2}\right)^2\right] \qquad A_{AC} = \frac{3}{16}\pi d^2$$

$$A_{CB} = \frac{\pi}{4}d^2$$

Select R_B as the redundant; use superposition and a compatibility equation at B:

if $x \leq L/2$ $$\delta_{B1a} = \frac{Px}{EA_{AC}} + \frac{P\left(\frac{L}{2} - x\right)}{EA_{CB}} \qquad \delta_{B1a} = \frac{P}{E}\left(\frac{x}{\frac{3}{16}\pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4}d^2}\right)$$

$$\delta_{B1a} = \frac{2}{3}P\frac{2x + 3L}{E\pi d^2}$$

if $x \geq L/2$ $$\delta_{B1b} = \frac{P\frac{L}{2}}{EA_{AC}} \qquad \delta_{B1b} = \frac{P\frac{L}{2}}{E\left(\frac{3}{16}\pi d^2\right)} \qquad \delta_{B1b} = \frac{8}{3}\frac{PL}{E\pi d^2}$$

The following expression for δ_{B2} is good for all x:

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L - x}{A_{CB}}\right) \qquad \delta_{B2} = \frac{R_B}{E}\left(\frac{x}{\frac{3}{16}\pi d^2} + \frac{L - x}{\frac{\pi}{4}d^2}\right)$$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L - x}{\pi d^2}\right)$$

Solve for R_B and R_A assuming that $x \leq L/2$:

Compatibility: $\delta_{B1a} + \delta_{B2} = 0$ $$R_{Ba} = \frac{-\left(\frac{2}{3}P\frac{2x + 3L}{\pi d^2}\right)}{\left(\frac{16}{3}\frac{x}{\pi d^2} + 4\frac{L - x}{\pi d^2}\right)} \qquad R_{Ba} = \frac{-1}{2}P\frac{2x + 3L}{x + 3L} \quad \leftarrow$$

^ check—if $x = 0$, $R_B = -P/2$

Statics: $R_{Aa} = -P - R_{Ba}$ $$R_{Aa} = -P - \frac{-1}{2}P\frac{2x + 3L}{x + 3L} \qquad R_{Aa} = \frac{-3}{2}P\frac{L}{x + 3L} \quad \leftarrow$$

^ check—if $x = 0$, $R_{A_a} = -P/2$

Solve for R_B and R_A assuming that $x \geq L/2$:

Compatibility: $\delta_{B1b} + \delta_{B2} = 0$ $\quad R_{Bb} = \dfrac{\dfrac{-8}{3}\dfrac{PL}{\pi d^2}}{\left(\dfrac{16}{3}\dfrac{x}{\pi d^2} + 4\dfrac{L-x}{\pi d^2}\right)}$ $\quad R_{Bb} = \dfrac{-2PL}{x + 3L}$ $\leftarrow$

^ check—if $x = L$, $R_B = -P/2$

Statics: $R_{Ab} = -P - R_{Bb}$ $\quad R_{Ab} = -P - \left(\dfrac{-2PL}{x + 3L}\right)$ $\quad R_{Ab} = -P\dfrac{x + L}{x + 3L}$ $\leftarrow$

(b) Find δ at point of load application; axial force for segment 0 to $L/2 = -R_A$ and δ = elongation of this segment

Assume that $x \leq L/2$:

$$\delta_a = \frac{-R_{Aa}}{E}\left(\frac{x}{A_{AC}} + \frac{\frac{L}{2} - x}{A_{CB}}\right) \qquad \delta_a = \frac{-\left(\frac{-3}{2}P\frac{L}{x+3L}\right)}{E}\left(\frac{x}{\frac{3}{16}\pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4}d^2}\right)$$

$$\delta_a = PL\frac{2x + 3L}{(x + 3L)E\pi d^2}$$

For $x = L/2$, $\quad \delta_a = \dfrac{8}{7}L\dfrac{P}{E\pi d^2}$ $\leftarrow$

Assume that $x \geq L/2$:

$$\delta_b = \frac{(-R_{Ab})\frac{L}{2}}{EA_{AC}} \qquad \delta_b = \frac{\left(P\frac{x+L}{x+3L}\right)\frac{L}{2}}{E\left(\frac{3}{16}\pi d^2\right)} \qquad \delta_b = \frac{8}{3}P\left(\frac{x+L}{x+3L}\right)\frac{L}{E\pi d^2} \leftarrow$$

for $x = L/2$ $\quad \delta_b = \dfrac{8}{7}P\dfrac{L}{E\pi d^2}$ $\quad <$ same as δ_a above (OK)

(c) For what value of x is $R_B = (6/5)\,R_A$?
Guess that $x < L/2$ here and use R_{Ba} expression above to find x:

$$\frac{-1}{2}P\frac{2x + 3L}{x + 3L} - \frac{6}{5}\left(\frac{-3}{2}P\frac{L}{x + 3L}\right) = 0 \qquad \frac{-1}{10}P\frac{10x - 3L}{x + 3L} = 0 \qquad x = \frac{3L}{10} \leftarrow$$

Now try $R_{Bb} = (6/5)R_{Ab}$, assuming that $x > L/2$

$$\frac{-2PL}{x + 3L} - \frac{6}{5}\left(-P\frac{x + L}{x + 3L}\right) = 0 \qquad \frac{2}{5}P\frac{-2L + 3x}{x + 3L} = 0 \qquad x = \frac{2}{3}L \leftarrow$$

So, there are two solutions for x.

(d) FIND REACTIONS IF THE BAR IS NOW ROTATED TO A VERTICAL POSITION, LOAD P IS REMOVED, AND THE BAR IS HANGING UNDER ITS OWN WEIGHT (ASSUME MASS DENSITY = ρ). ASSUME THAT $x = L/2$.

$$A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4}d^2$$

Select R_B as the redundant; use superposition and a compatibility equation at B

from (a) above. compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$\delta_{B2} = \frac{R_B}{E}\left(\frac{x}{A_{AC}} + \frac{L-x}{A_{CB}}\right) \qquad \text{For } x = L/2,\ \delta_{B2} = \frac{R_B}{E}\left(\frac{14}{3}\frac{L}{\pi d^2}\right)$$

$$\delta_{B1} = \int_0^{\frac{L}{2}} \frac{N_{AC}}{EA_{AC}}d\zeta + \int_{\frac{L}{2}}^{L} \frac{N_{CB}}{EA_{CB}}d\zeta$$

Where axial forces in bar due to self weight are $W_{AC} = \rho g A_{AC}\frac{L}{2}$ $\quad W_{CB} = \rho g A_{CB}\frac{L}{2}$
(assume ζ is measured upward from A):

$$N_{AC} = -\left[\rho g A_{CB}\frac{L}{2} + \rho g A_{AC}\left(\frac{L}{2} - \zeta\right)\right] \qquad A_{AC} = \frac{3}{16}\pi d^2 \qquad A_{CB} = \frac{\pi}{4}d^2$$

$$N_{CB} = -[\rho\, g A_{CB}(L-\zeta)]$$

$$N_{AC} = \frac{-1}{8}\rho g\pi\, d^2\, L - \frac{3}{16}\rho g\pi\, d^2\left(\frac{1}{2}L - \zeta\right) \qquad N_{CB} = -\left[\frac{1}{4}\rho g\pi\, d^2(L-\zeta)\right]$$

$$\delta_{B1} = \int_0^{\frac{L}{2}} \frac{\frac{-1}{8}\rho g\pi d^2 L - \frac{3}{16}\rho g\pi d^2\left(\frac{1}{2}L - \zeta\right)}{E\left(\frac{3}{16}\pi d^2\right)}d\zeta + \int_{\frac{L}{2}}^{L} \frac{-\left[\frac{1}{4}\rho g\pi d^2(L-\zeta)\right]}{E\left(\frac{\pi}{4}d^2\right)}d\zeta$$

$$\delta_{B1} = \left(\frac{-11}{24}\rho g\frac{L^2}{E} + \frac{-1}{8}\rho g\frac{L^2}{E}\right) \qquad \delta_{B1} = \frac{-7}{12}\rho\, g\frac{L^2}{E} \qquad \frac{7}{12} = 0.583$$

Compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{-\left(\frac{-7}{12}\rho g\frac{L^2}{E}\right)}{\left(\frac{14}{3}\frac{L}{E\pi d^2}\right)} \qquad R_B = \frac{1}{8}\rho g\pi d^2 L \quad \leftarrow$$

Statics: $\quad R_A = (W_{AC} + W_{CB}) - R_B$

$$R_A = \left[\left[\rho g\left(\frac{3}{16}\pi d^2\right)\frac{L}{2} + \rho g\left(\frac{\pi}{4}d^2\right)\frac{L}{2}\right] - \frac{1}{8}\rho g\pi d^2 L\right]$$

$$R_A = \frac{3}{32}\rho g\pi\, d^2\, L \quad \leftarrow$$

Problem 2.4-8

$P = 200kN \quad L = 2m \quad t = 20mm \quad b_1 = 100mm \quad b_2 = 115mm \quad E = 96GPa$

Select reaction R_C as the redundant; use superposition

axial displacement at C due to actual load P at B

$$\delta_{C1} = \frac{P \cdot \left(\frac{3 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 1.165 \cdot mm$$

axial displacement at C due to redundant R_C

$$\delta_{C2} = R_C \cdot \left[\frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right)\right]$$

$$\frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 9.706 \times 10^{-3} \cdot \frac{mm}{kN}$$

Compatibility equation $\quad \delta_{C1} + \delta_{C2} = 0 \quad$ solve for R_C

$$R_C = \frac{-\delta_{C1}}{\left[\frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right)\right]} = -120 \cdot kN$$

Statics $\quad \Sigma F = 0 \quad R_A = -(P + R_C) = -80 \cdot kN$

Negative reactions so both act to left

Compute extension of AB or compression of BC to find displ. δ_B (to the right)

$$-R_A \cdot \left[\frac{\left(\frac{3 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right)\right] = 0.466 \cdot mm \qquad \text{or} \qquad -R_C \cdot \left[\frac{\left(\frac{2 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right)\right] = 0.466 \cdot mm$$

Problem 2.4-9

$P = 90\text{kN}$ $\quad L = 1\text{m}$ $\quad t = 6\text{mm}$ $\quad b_1 = 50\text{mm}$ $\quad b_2 = 60\text{mm}$ $\quad E = 72\text{GPa}$

$A_{BC} = b_1 \cdot t = 300 \cdot \text{mm}^2 \qquad b_{ave} = \dfrac{b_1 + b_2}{2} = 55 \cdot \text{mm}$

Select reaction R_C as the redundant; use superposition

axial displacement at C due to actual load P at middle of AB

$$\delta_{C1} = \frac{P \cdot \left(\frac{L}{4}\right)}{E \cdot t \cdot (b_2 - b_{ave})} \cdot \ln\left(\frac{b_2}{b_{ave}}\right) = 0.906 \cdot \text{mm}$$

axial displacement at C due to redundant R_C

$$\delta_{C2} = R_C \cdot \left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot A_{BC}}\right]$$

flexibility constant for bar

$$\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot A_{BC}} = 0.044 \cdot \frac{\text{mm}}{\text{kN}}$$

Compatibility equation $\quad \delta_{C1} + \delta_{C2} = 0 \quad$ solve for R_C

$$R_C = \frac{-\delta_{C1}}{\left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot A_{BC}}\right]} = -20.483 \cdot \text{kN}$$

Statics $\qquad \Sigma F = 0 \qquad R_A = -(P + R_C) = -69.517 \cdot \text{kN}$

Negative reactions so both act to left

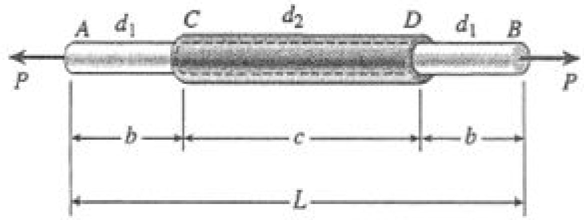
Compute deformations of AB (two terms, more difficult) or deformation of BC (easier) to find displ. δ_B (to the right)

$$-R_A \cdot \left[\frac{\left(\frac{L}{4}\right)}{E \cdot t \cdot (b_2 - b_{ave})} \cdot \ln\left(\frac{b_2}{b_{ave}}\right)\right] + \frac{(-R_A - P) \cdot \frac{L}{4}}{E \cdot t \cdot (b_{ave} - b_1)} \cdot \ln\left(\frac{b_{ave}}{b_1}\right) = 4.741 \times 10^{-1} \cdot \text{mm}$$

or

$$-R_C \cdot \left(\frac{\frac{L}{2}}{E \cdot A_{BC}}\right) = 0.474 \cdot \text{mm}$$

Problem 2.4-10



$P = 12$ kN $\quad d_1 = 30$ mm $\quad b = 100$ mm

$L = 500$ mm $\quad d_2 = 45$ mm $\quad c = 300$ mm

Rod: $E_1 = 3.1$ GPa

Sleeve: $E_2 = 2.5$ GPa

$$\text{Rod: } A_1 = \frac{\pi d_1^2}{4} = 706.86 \text{ mm}^2$$

$$\text{Sleeve: } A_2 = \frac{\pi}{4}(d_2^2 - d_1^2) = 883.57 \text{ mm}^2$$

$$E_1A_1 + E_2A_2 = 4.400 \text{ MN}$$

(a) Elongation of rod

$$\text{Part } AC\text{: } \delta_{AC} = \frac{Pb}{E_1A_1} = 0.5476 \text{ mm}$$

$$\text{Part } CD\text{: } \delta_{CD} = \frac{Pc}{E_1A_1 + E_2A_2}$$

$$= 0.81815 \text{ mm}$$

(From Eq. 2-16 of Example 2-8)

$$\delta = 2\delta_{AC} + \delta_{CD} = 1.91 \text{ mm} \quad \leftarrow$$

(b) Sleeve at full length

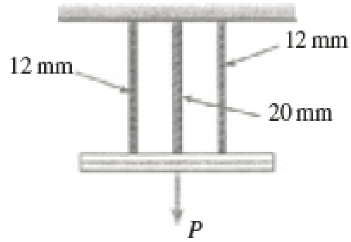
$$\delta = \delta_{CD}\left(\frac{L}{c}\right) = (0.81815 \text{ mm})\left(\frac{500 \text{ mm}}{300 \text{ mm}}\right)$$

$$= 1.36 \text{ mm} \quad \leftarrow$$

(c) Sleeve removed

$$\delta = \frac{PL}{E_1A_1} = 2.74 \text{ mm} \quad \leftarrow$$

Problem 2.4-11



AREAS OF CABLES (from Table 2-1)

Middle cable: $A_M = 173 \text{ mm}^2$

Outer cables: $A_O = 77 \text{ mm}^2$

(for each cable)

FIRST LOADING

$$P_1 = 60 \text{ kN}\left(\text{Each cable carries } \frac{P_1}{3} \text{ or } 20 \text{ kN}\right)$$

SECOND LOADING

$P_2 = 40$ kN (additional load)

EQUATION OF EQUILIBRIUM

$$\Sigma F_{vert} = 0 \qquad 2P_O + P_M - P_2 = 0 \qquad (1)$$

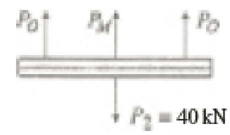
EQUATION OF COMPATIBILITY

$$\delta_M = \delta_O \qquad (2)$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_M = \frac{P_M L}{EA_M} \quad \delta_O = \frac{P_o L}{EA_o}$$

SUBSTITUTE INTO COMPATIBILITY EQUATION:



$$\frac{P_M L}{EA_M} = \frac{P_O L}{EA_O} \quad \frac{P_M}{A_M} = \frac{P_O}{A_O} \qquad (3)$$

SOLVE SIMULTANEOUSLY EQS. (1) AND (3):

$$P_M = P_2\left(\frac{A_M}{A_M + 2A_O}\right) = 21 \text{ kN}$$

$$P_o = P_2\left(\frac{A_o}{A_M + 2A_O}\right) = 9.418 \text{ kN}$$

FORCES IN CABLES

Middle cable: Force = 20 kN + 21 kN = 41 kN

Outer cables: Force = 20 kN + 9.418 kN = 29.4 kN

(a) PERCENT OF TOTAL LOAD CARRIED BY MIDDLE CABLE

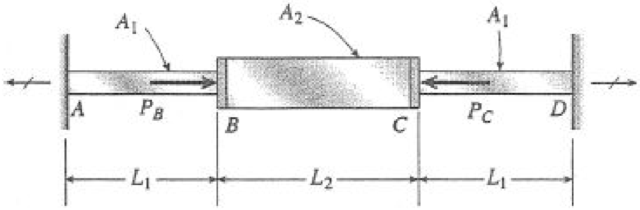
$$\text{Percent} = \frac{41 \text{ KN}}{100 \text{ KN}}(100\%) = 41.2\% \quad \leftarrow$$

(b) STRESSES IN CABLES ($\sigma = P/A$)

$$\text{Middle cable: } \sigma_M = \frac{41 \text{ kN}}{173 \text{ mm}^2} = 238 \text{ MPa} \quad \leftarrow$$

$$\text{Outer cables: } \sigma_O = \frac{29.41 \text{ kN}}{77 \text{ mm}^2} = 383 \text{ MPa} \quad \leftarrow$$

Problem 2.4-12



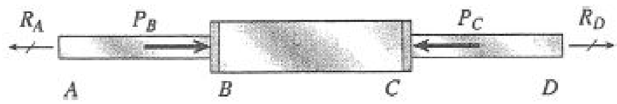
$P_B = 25.5$ kN $\quad P_C = 17.0$ kN

$L_1 = 200$ mm $\quad L_2 = 250$ mm

$A_1 = 840\ \text{mm}^2 \quad A_2 = 1260\ \text{mm}^2$

m = meter

FREE-BODY DIAGRAM



EQUATION OF EQUILIBRIUM

$\Sigma F_{\text{horiz}} = 0 \quad \overset{+}{\rightarrow} \quad \leftarrow$

$P_B + R_D - P_C - R_A = 0$ or

$R_A - R_D = P_B - P_C = 8.5$ kN (Eq. 1)

EQUATION OF COMPATIBILITY

δ_{AD} = elongation of entire bar

$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 0$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AB} = \frac{R_A L_1}{EA_1} = \frac{R_A}{E}\left(238.05\frac{1}{\text{m}}\right) \quad \text{(Eq. 3)}$$

$$\delta_{BC} = \frac{(R_A - P_B)L_2}{EA_2}$$

$$= \frac{R_A}{E}\left(198.413\frac{1}{\text{m}}\right) - \frac{P_B}{E}\left(198.413\frac{1}{\text{m}}\right) \quad \text{(Eq. 4)}$$

$$\delta_{CD} = \frac{R_D L_1}{EA_1} = \frac{R_D}{E}\left(238.095\frac{1}{\text{m}}\right) \quad \text{(Eq. 5)}$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3), (4), and (5) into Eq. (2):

$$\frac{R_A}{E}\left(238.095\frac{1}{\text{m}}\right) + \frac{R_A}{E}\left(198.413\frac{1}{\text{m}}\right)$$

$$-\frac{P_B}{E}\left(198.413\frac{1}{\text{m}}\right) + \frac{R_D}{E}\left(238.095\frac{1}{\text{m}}\right) = 0$$

Simplify and substitute $P_B = 25.5$ kN:

$$R_A\left(436.508\frac{1}{\text{m}}\right) + R_D\left(238.095\frac{1}{\text{m}}\right)$$

$$= 5{,}059.53\ \text{kN/m} \quad \text{(Eq. 6)}$$

(a) REACTIONS R_A AND R_D

Solve simultaneously Eqs. (1) and (6).

From (1): $R_D = R_A - 8.5$ kN

Substitute into (6) and solve for R_A:

$$R_A\left(674.603\frac{1}{\text{m}}\right) = 7083.34\ \text{kN/m}$$

$R_A = 10.5$ kN $\quad \leftarrow$

$R_D = R_A - 8.5\ \text{kN} = 2.0$ kN $\quad \leftarrow$

(b) COMPRESSIVE AXIAL FORCE F_{BC}

$F_{BC} = P_B - R_A = P_C - R_D = 15.0$ kN $\quad \leftarrow$

Problem 2.4-13

NUMERICAL DATA

$n = 6 \qquad d_b = 12.5 \text{ mm} \qquad \sigma_a = 96 \text{ MPa} \qquad A_b = \frac{\pi}{4} d_b^{\,2} = 122.718 \text{ mm}^2$

(a) FORMULAS FOR REACTIONS F

Segment ABC flexibility: $f_1 = \dfrac{2\left(\frac{L}{4}\right)}{EA} = \dfrac{L}{2EA}$

Segment CDE flexibility: $f_2 = \dfrac{2\left(\frac{L}{4}\right)}{\frac{1}{2}EA} = \dfrac{L}{EA}$

Loads at points B and D:

$P_B = -2P \qquad P_D = 3P$

(1) Select R_E as the redundant; find axial displacement $\delta_1 =$ displ. at E due to loads P_B and P_D:

$$\delta_1 = \frac{(P_B + P_D)\frac{L}{4}}{EA} + \frac{P_D\frac{L}{4}}{EA} + \frac{P_D\frac{L}{4}}{\frac{1}{2}EA} = \frac{5LP}{2EA}$$

(2) Next apply redundant R_E and find axial displ. $\delta_2 =$ displ. at E due to redundant R_E:

$$\delta_2 = R_E(f_1 + f_2) = \frac{3LR_E}{2EA}$$

(3) Use compatibility equation to find redundant R_E, then use statics to find R_A:

$\delta_1 + \delta_2 = 0$ solving for $R_E \qquad R_E = \dfrac{-5}{3}P$

$R_A = -R_E - P_B - P_D \qquad R_A = \dfrac{2P}{3} \qquad \boxed{R_A = \dfrac{2P}{3}} \qquad \boxed{R_E = \dfrac{-5P}{3}}$

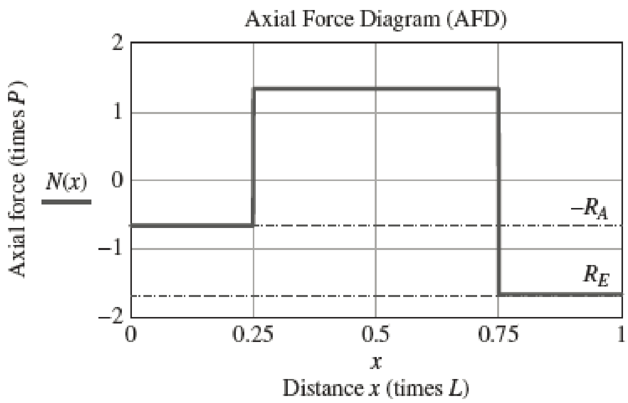
(b) DETERMINE THE AXIAL DISPLACEMENTS δ_B, δ_C, AND δ_D AT POINTS B, C, AND D, RESPECTIVELY.

$$\delta_B \frac{\left(\frac{-2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = -\frac{LP}{6EA} \quad \text{leftward}$$

$$\delta_C = \delta_B + \frac{\left(2P - \frac{2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = \frac{LP}{6EA} \quad \text{to the right}$$

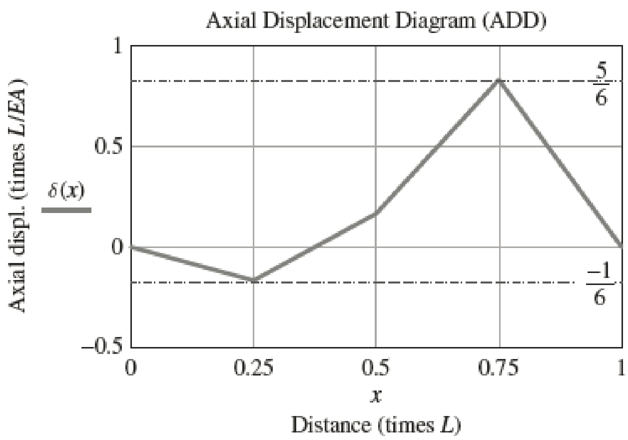
$$\delta_D = \frac{\left(\frac{5P}{3}\right)\left(\frac{L}{4}\right)}{\frac{EA}{2}} = \frac{5LP}{6EA} \quad \text{to the right}$$

(c) DRAW AN AXIAL-DISPLACEMENT DIAGRAM (ADD) IN WHICH THE ABSCISSA IS THE DISTANCE x FROM SUPPORT A TO ANY POINT ON THE BAR AND THE ORDINATE IS THE HORIZONTAL DISPLACEMENT δ AT THAT POINT.



AFD for use below in Part (d)

AFD is composed of 4 constant segments, so ADD is linear with zero displacements at supports A and E.



Plot displacements δ_B, δ_C, and δ_D from part (b) above, then connect points using straight lines showing linear variation of axial displacement between points.

$\delta_{max} = \delta_D \qquad \delta_{max} = \dfrac{5LP}{6EA}$ to the right

Boundary conditions at supports:

$\delta_A = \delta_E = 0$

(d) MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE P BASED ON ALLOWABLE NORMAL STRESS IN FLANGE BOLTS

FROM AFD, FORCE AT $L/2$:

$$F_{max} = \frac{4}{3}P \qquad \text{and} \qquad F_{max} = n\sigma_a A_b = 70.686 \text{ kN}$$

$$P_{max} = \frac{3}{4}F_{max} = 53.01 \text{ kN} \qquad \boxed{P_{max} = 53 \text{ kN}}$$

Problem 2.4-14

(a) STRESSES AND REACTIONS: SELECT R_1 AS REDUNDANT AND DO SUPERPOSITION ANALYSIS (HERE $q = 0$; DEFLECTION POSITIVE UPWARD)

$d_1 = 50 \text{ mm}$ $\quad d_2 = 60 \text{ mm}$ $\quad d_3 = 57 \text{ mm}$ $\quad d_4 = 64 \text{ mm}$ $\quad A_1 = \frac{\pi}{4}(d_2^2 - d_1^2) = 863.938 \text{ mm}^2$

$E = 110 \text{ MPa}$

$A_2 = \frac{\pi}{4}(d_4^2 - d_3^2) = 665.232 \text{ mm}^2$

SEGMENT FLEXIBILITIES $\quad L_1 = 2 \text{ m}$ $\quad L_2 = 3 \text{ m}$

$f_1 = \frac{L_1}{EA_1} = 0.02105 \text{ mm/N}$ $\quad f_2 = \frac{L_2}{EA_2} = 0.041 \text{ mm/N}$ $\quad \frac{f_1}{f_2} = 0.513$

TENSILE stress (σ_1) is known in upper segment so $R_1 = \sigma_1 \times A_1$ $\quad \sigma_1 = 10.5 \text{ MPa}$ $\quad R_1 = \sigma_1 A_1 = 9.07 \text{ kN}$

$\delta_{1a} = -Pf_2$ $\quad \delta_{1b} = R_1(f_1 + f_2)$ $\quad$ Compatibility: $\delta_{1a} + \delta_{1b} = 0$

Solve for P: $\quad P = R_1\left(\frac{f_1 + f_2}{f_2}\right) = 13.73 \text{ kN}$

Finally, use statics to find R_2: $\quad R_2 = P - R_1 = 4.66 \text{ kN}$ $\quad \sigma_2 = \frac{R_2}{A_2} = 7 \text{ MPa}$ $\quad$ < **compressive** since R_2 is positive (upward)

$\boxed{P = 13.73 \text{ kN}}$ $\quad \boxed{R_1 = 9.07 \text{ kN}}$ $\quad \boxed{R_2 = 4.66 \text{ kN}}$ $\quad \boxed{\sigma_2 = 7 \text{ MPa}}$

(b) DISPLACEMENT AT CAP PLATE

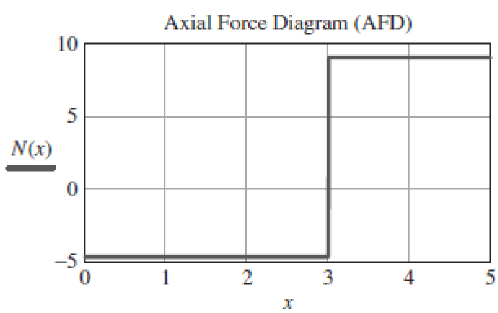
$\delta_c = R_1 f_1 = 190.909 \text{ mm}$ $\quad$ < downward $\quad$ OR $\quad \delta_c = (R_2)f_2 = 190.909 \text{ mm}$ $\quad$ < downward (neg. x-direction)

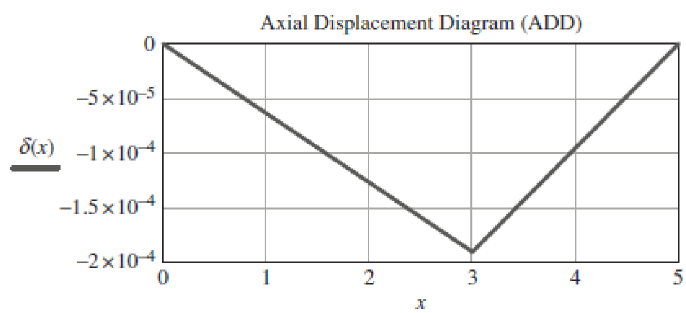
$\delta_{cap} = \delta_c = 0.191 \text{ m}$ $\quad \boxed{\delta_{cap} = 190.9 \text{ mm}}$

AFD and ADD: $\quad R_1 = 9.071$ $\quad R_2 = 4.657$ $\quad L_1 = 2$ $\quad A_1 = 863.938$ $\quad A_2 = 665.232$ $\quad E = 110$

$L_2 = 3$

NOTE: x is measured up from lower support.





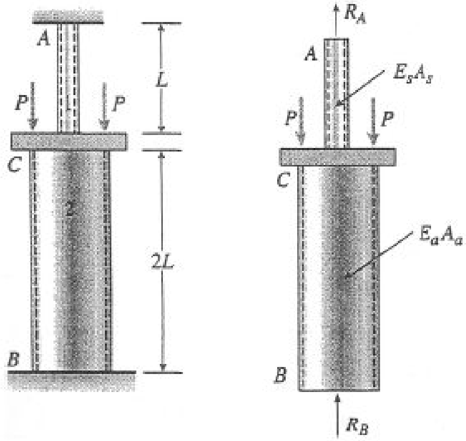
(c) UNIFORM LOAD Q ON SEGMENT 2 SUCH THAT $R_2 = 0$

$P = 13.728 \text{ kN}$ $\quad R_1 = \sigma_1 A_1 = 9.071 \text{ kN}$ $\quad L_2 = 3 \text{ m}$

Equilibrium: $\quad R_1 + R_2 = P - qL_2$ < set $R_2 = 0$, solve for req'd q $\quad q = \frac{P - R_1}{L_2} = 1.552 \text{ kN/m}$

$\boxed{q = 1.552 \text{ kN/m}}$

Problem 2.4-15



Pipe 1 is steel.
Pipe 2 is aluminum.

EQUATION OF EQUILIBRIUM

$\Sigma F_{vert} = 0, \qquad R_A + R_B = 2P$ (Eq. 1)

EQUATION OF COMPATIBILITY

$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0$ (Eq. 2)

(A positive value of δ means elongation.)

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AC} = \frac{R_A L}{E_s A_s} \quad \delta_{BC} = -\frac{R_B(2L)}{E_a A_a} \qquad \text{(Eqs. 3)}$$

SOLUTION OF EQUATIONS

Substitute Eq. (3) into Eq. (2):

$$\frac{R_A L}{E_s A_s} - \frac{R_B(2L)}{E_a A_a} = 0 \qquad \text{(Eq. 4)}$$

Solve simultaneously Eqs. (1) and (4):

$$R_A = \frac{4E_s A_s P}{E_a A_a + 2E_s A_s} \quad R_B = \frac{2E_a A_a P}{E_a A_a + 2E_s A_s} \qquad \text{(Eqs. 5)}$$

(a) AXIAL STRESSES

$$\text{Steel: } \sigma_s = \frac{R_A}{A_s} = \frac{4E_s P}{E_a A_a + 2E_s A_s} \quad \leftarrow \qquad \text{(Eq. 6)}$$

(tension)

$$\text{Aluminum: } \sigma_a = \frac{R_B}{A_a} = \frac{2E_a P}{E_a A_a + 2E_s A_s} \quad \leftarrow$$

(compression) (Eq. 7)

(b) NUMERICAL RESULTS

$P = 50\text{ kN} \qquad A_a = 6000\text{ mm}^2 \qquad A_s = 600\text{ mm}^2$

$E_a = 70\text{ GPa} \qquad E_s = 200\text{ GPa}$

$E_a A_a + 2E_s A_s = 660 \times 10^3\text{ kN}$

From Eq. (6): $\sigma_s = 60.6$ MPa (tension) $\leftarrow$

From Eq. (7): $\sigma_a = 10.6$ MPa (compression) $\leftarrow$

Problem 2.4-16

Numerical data:

$W = 800\text{ N}$ $\quad L = 150\text{ mm}$

$a = 50\text{ mm}$ $\quad d_S = 2\text{ mm}$

$d_A = 4\text{ mm}$ $\quad E_S = 210\text{ GPa}$

$E_A = 70\text{ GPa}$

$\sigma_{Sa} = 220\text{ MPa}$ $\quad \sigma_{Aa} = 80\text{ MPa}$

$$A_A = \frac{\pi}{4} d_A^2 \qquad A_S = \frac{\pi}{4} d_S^2$$

$$A_A = 13\text{ mm}^2 \qquad A_S = 3\text{ mm}^2$$

(a) P_{allow} AT CENTER OF BAR

One-degree statically indeterminate - use reaction (R_A) at top of aluminum bar as the redundant

compatibility: $\delta_1 - \delta_2 = 0$ Statics: $2R_S + R_A = P + W$

$$\delta_1 = \frac{P + W}{2}\left(\frac{L}{E_S A_S}\right)$$

< downward displacement due to elongation of each steel wire under $P + W$ if aluminum wire is cut at top

$$\delta_2 = R_A\left(\frac{L}{2E_S A_S} + \frac{L}{E_A A_A}\right)$$

< upward displ. due to shortening of steel wires and elongation of aluminum wire under redundant R_A

Enforce compatibility and then solve for R_A:

$$\delta_1 = \delta_2 \quad \text{so} \quad R_A = \frac{\frac{P+W}{2}\left(\frac{L}{E_S A_S}\right)}{\frac{L}{2E_S A_S} + \frac{L}{E_A A_A}} \qquad R_A = (P + W)\frac{E_A A_A}{E_A A_A + 2E_S A_S} \quad \text{and} \quad \sigma_{Aa} = \frac{R_A}{A_A}$$

Now use statics to find R_S:

$$R_S = \frac{P + W - R_A}{2} \qquad R_S = \frac{P + W - (P + W)\frac{E_A A_A}{E_A A_A + 2E_S A_S}}{2} \qquad R_S = (P + W)\frac{E_S A_S}{E_A A_A + 2E_S A_S}$$

$$\text{and} \quad \sigma_{Sa} = \frac{R_S}{A_S}$$

Compute stresses and apply allowable stress values:

$$\sigma_{Aa} = (P + W)\frac{E_A}{E_A A_A + 2E_S A_S} \qquad \sigma_{Sa} = (P + W)\frac{E_S}{E_A A_A + 2E_S A_S}$$

Solve for allowable load P:

$$P_{Aa} = \sigma_{Aa}\left(\frac{E_A A_A + 2E_S A_S}{E_A}\right) - W \qquad P_{Sa} = \sigma_{Sa}\left(\frac{E_A A_A + 2E_S A_S}{E_S}\right) - W$$

(lower value of P controls)

$P_{Aa} = 1713\text{ N}$ $\qquad P_{Sa} = 1504\text{ N}$ $\leftarrow$ P_{allow} is controlled by steel wires

(b) P_{allow} IF LOAD P AT $x = a/2$

Again, cut aluminum wire at top, then compute elongations of left and right steel wires:

$$\delta_{1L} = \left(\frac{3P}{4} + \frac{W}{2}\right)\left(\frac{L}{E_S A_S}\right) \quad \delta_{1R} = \left(\frac{P}{4} + \frac{W}{2}\right)\left(\frac{L}{E_S A_S}\right)$$

$$\delta_1 = \frac{\delta_{1L} + \delta_{1R}}{2} \qquad \delta_1 = \frac{P + W}{2}\left(\frac{L}{E_S A_S}\right)$$ where δ_1 = displacement at $x = a$

Use δ_2 from part (a):

$$\delta_2 = R_A\left(\frac{L}{2E_S A_S} + \frac{L}{E_A A_A}\right)$$

So equating δ_1 and δ_2, solve for R_A: $R_A = (P + W)\dfrac{E_A A_A}{E_A A_A + 2E_S A_S}$

^ same as in part (a)

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{R_A}{2}$$ < stress in left steel wire exceeds that in right steel wire

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{(P + W)\dfrac{E_A A_A}{E_A A_A + 2E_S A_S}}{2}$$

$$R_{SL} = \frac{PE_A A_A + 6PE_S A_S + 4WE_S A_S}{4E_A A_A + 8E_S A_S} \qquad \sigma_{Sa} = \frac{PE_A A_A + 6PE_S A_S + 4WE_S A_S}{4E_A A_A + 8E_S A_S}\left(\frac{1}{A_S}\right)$$

Solve for P_{allow} based on allowable stresses in steel and aluminum:

$$P_{Sa} = \frac{\sigma_{Sa}(4A_S E_A A_A + 8E_S A_S^2) - (4WE_S A_S)}{E_A A_A + 6E_S A_S} \qquad P_{Aa} = 1713 \text{ N}$$ < same as in part(a)

$P_{Sa} = 820$ N ← steel controls

(c) P_{allow} IF WIRES ARE SWITCHED AS SHOWN AND $x = a/2$

Select R_A as the redundant; statics on the two released structures:

(1) Cut aluminum wire—apply P and W, compute forces in left and right steel wires, then compute displacements at each steel wire:

$$R_{SL} = \frac{P}{2} \qquad R_{SR} = \frac{P}{2} + W$$

$$\delta_{1L} = \frac{P}{2}\left(\frac{L}{E_S A_S}\right) \quad \delta_{1R} = \left(\frac{P}{2} + W\right)\left(\frac{L}{E_S A_S}\right)$$

By geometry, δ at aluminum wire location at far right is $\quad \delta_1 = \left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_S A_S}\right)$

(2) Next apply redundant R_A at right wire, compute wire force and displacement at aluminum wire:

$$R_{SL} = -R_A \qquad R_{SR} = 2R_A \qquad \delta_2 = R_A\left(\frac{5L}{E_S A_S} + \frac{L}{E_A A_A}\right)$$

(3) Compatibility equate δ_1, δ_2 and solve for R_A, then P_{allow} for aluminum wire:

$$R_A = \frac{\left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_S A_S}\right)}{\frac{5L}{E_S A_S} + \frac{L}{E_A A_A}} \qquad R_A = \frac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S} \qquad \sigma_{Aa} = \frac{R_A}{A_A}$$

$$\sigma_{Aa} = \frac{E_A P + 4E_A W}{10E_A A_A + 2E_S A_S}$$

$$P_{Aa} = \frac{\sigma_{Aa}(10E_A A_A + 2E_S A_S) - 4E_A W}{E_A} \qquad P_{Aa} = 1713 \text{ N}$$

(4) Statics or superposition—find forces in steel wires, then P_{allow} for steel wires:

$$R_{SL} = \frac{P}{2} + R_A \qquad R_{SL} = \frac{P}{2} + \frac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S}$$

$$R_{SL} = \frac{6E_A A_A P + PE_S A_S + 4E_A A_A W}{10E_A A_A + 2E_S A_S}$$ < larger than R_{SR}, so use in allowable stress calculations

$$R_{SR} = \frac{P}{2} + W - 2R_A \qquad R_{SR} = \frac{P}{2} + W - \frac{E_A A_A P + 4E_A A_A W}{5E_A A_A + E_S A_S}$$

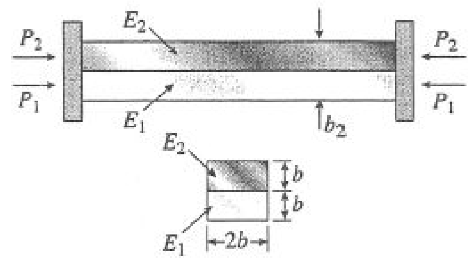
$$R_{SR} = \frac{3E_A A_A P + PE_S A_S + 2E_A A_A W + 2WE_S A_S}{10E_A A_A + 2E_S A_S}$$

$$\sigma_{Sa} = \frac{R_{SL}}{A_S} \qquad P_{Sa} = \sigma_{Sa} A_S\left(\frac{10E_A A_A + 2E_S A_S}{6E_A A_A + E_S A_S}\right) - \frac{4E_A A_A W}{6E_A A_A + E_S A_S}$$

$$P_{Sa} = \frac{10\sigma_{Sa} A_S E_A A_A + 2\sigma_{Sa} A_S^2 E_S - 4E_A A_A W}{6E_A A_A + E_S A_S} \qquad P_{Sa} = 703 \text{ N} \quad \leftarrow$$

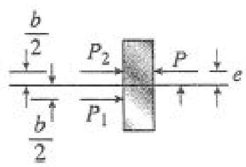
^ steel controls

Problem 2.4-17



FREE-BODY DIAGRAM

(Plate at right-hand end)



EQUATIONS OF EQUILIBRIUM

$\Sigma F = 0 \quad P_1 + P_2 = P$ (Eq. 1)

$\Sigma M = 0 \; \circlearrowleft \quad Pe + P_1\left(\frac{b}{2}\right) - P_2\left(\frac{b}{2}\right) = 0$ (Eq. 2)

EQUATION OF COMPATIBILITY

$\delta_2 = \delta_1$

$\frac{P_2 L}{E_2 A} = \frac{P_1 L}{E_1 A} \quad \text{or} \quad \frac{P_2}{E_2} = \frac{P_1}{E_1}$ (Eq. 3)

(a) AXIAL FORCES

Solve simultaneously Eqs. (1) and (3):

$$P_1 = \frac{PE_1}{E_1 + E_2} \quad P_2 = \frac{PE_2}{E_1 + E_2} \quad \longleftarrow$$

(b ECCENTRICITY OF LOAD P

Substitute P_1 and P_2 into Eq. (2) and solve for e:

$$e = \frac{b(E_2 - E_1)}{2(E_2 + E_1)} \quad \longleftarrow$$

(c) RATIO OF STRESSES

$$\sigma_1 = \frac{P_1}{A} \quad \sigma_2 = \frac{P_2}{A} \quad \frac{\sigma_1}{\sigma_2} = \frac{P_1}{P_2} = \frac{E_1}{E_2} \quad \longleftarrow$$

Problem 2.4-18

NUMERICAL DATA

$L = 2.5\ \text{m}$ $\quad b = 0.71$ $\quad L = 1.775\ \text{m}$ $\quad E = 210\ \text{GPa}$ $\quad A = 3500\ \text{mm}^2$ $\quad P = 185\ \text{kN}$ $\quad \theta_A = 60°$

$\sigma_a = 150\ \text{MPa}$

FIND MISSING DIMENSIONS AND ANGLES IN PLANE TRUSS FIGURE

$x_c = b\cos(\theta_A) = 0.8875\ \text{m} \qquad y_c = b\sin(\theta_A) = 1.5372\ \text{m}$

$$\frac{b}{\sin(\theta_B)} = \frac{L}{\sin(\theta_A)} \qquad \text{so} \qquad \theta_B = a\sin\left(\frac{b\sin(\theta_A)}{L}\right) = 37.94306°$$

$$\theta_C = 180° - (\theta_A + \theta_B) = 82.05694°$$

$$c = \frac{L}{\sin(\theta_A)}\sin(\theta_C) = 2.85906\ \text{m} \qquad \text{or} \qquad c = \sqrt{b^2 + L^2 - 2bL\cos(\theta_C)} = 2.85906\ \text{m}$$

(a) SELECT B_x AS THE REDUNDANT; PERFORM SUPERPOSITION ANALYSIS TO FIND B_x THEN USE STATICS TO FIND REMAINING REACTIONS. FINALLY USE METHOD OF JOINTS TO FIND MEMBER FORCES (SEE EXAMPLE 1-1)

δ_{Bx1} = displacement in x-direction in released structure acted upon by loads P and $2P$ at joint C:

$\delta_{Bx1} = 1.2789911\ \text{mm}$ < this displacement equals force in AB divided by flexibility of AB

δ_{Bx2} = displacement in x-direction in released structure acted upon by redundant B_x: $\quad \delta_{BX2} = B_x \dfrac{c}{EA}$

COMPATIBILITY EQUATION: $\quad \delta_{BX1} + \delta_{BX2} = 0 \quad$ so $\quad B_X = \dfrac{-EA}{c}\delta_{BX1} = -328.8\ \text{kN}$

STATICS: $\quad \Sigma F_X = 0 \qquad A_X = -B_X - 2P = -41.2\ \text{kN}$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{c}[2P(b\sin(\theta_A)) + P(b\cos(\theta_A))] = 256.361\ \text{kN}$$

$$\Sigma F_y = 0 \qquad A_y = P - B_y = -71.361\ \text{kN}$$

REACTIONS:

$\boxed{A_x = -41.2\ \text{kN}} \quad \boxed{A_y = -71.4\ \text{kN}} \quad \boxed{B_x = -329\ \text{kN}} \quad \boxed{B_y = 256\ \text{kN}}$

(b) FIND MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE P IF ALLOWABLE NORMAL STRESS IS 150 MPA

(1) Use reactions and Method of Joints to find member forces in each member for above loading.

Results: $F_{AB} = 0 \qquad F_{BC} = -416.929\ \text{kN} \qquad F_{AC} = 82.40\ \text{kN}$

(2) Compute member stresses:

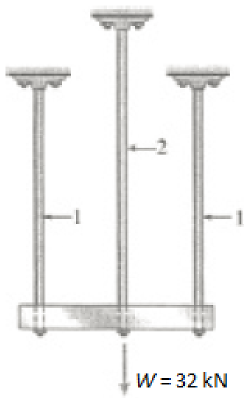
$$\sigma_{AB} = 0 \qquad \sigma_{BC} = \frac{-416.93\ \text{kN}}{A} = -119.123\ \text{MPa} \qquad \sigma_{AC} = \frac{82.4\ \text{kN}}{A} = 23.543\ \text{MPa}$$

(3) Maximum stress occurs in member BC. For linear analysis, the stress is proportional to the load so

$$\boxed{P_{max} = \frac{\sigma_a}{\sigma_{BC}} P = 233\ \text{kN}}$$

So when downward load $P = 233$ kN is applied at C and horizontal load $2P = 466$ kN is applied to the right at C, the stress in BC is 150 MPa

Problem 2.4-19



BAR 1 ALUMINUM

$E_1 = 70$ GPa

$d_1 = 10$ mm

$L_1 = 1$ m

$\sigma_1 = 165$ MPa

BAR 2 MAGNESIUM

$E_2 = 42$ GPa

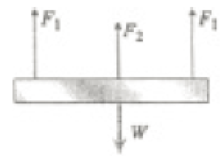
$d_2 = ?$ $\quad L_2 = ?$

$\sigma_2 = 90$ MPa

$W = 30$ kN

FREE-BODY DIAGRAM OF RIGID BAR

EQUATION OF EQUILIBRIUM



$\Sigma F_{vert} = 0$

$2F_1 + F_2 - W = 0$ (Eq. 1)

FULLY STRESSED RODS

$F_1 = \sigma_1 A_1 \qquad F_2 = \sigma_2 A_2$

$$A_1 = \frac{\pi d_1^2}{4} \qquad A_2 = \frac{\pi d_2^2}{4}$$

Substitute into Eq. (1):

$$2\sigma_1\left(\frac{\pi d_1^2}{4}\right) + \sigma_2\left(\frac{\pi d_2^2}{4}\right) = W$$

Diameter d_1 is known; solve for d_2:

$$d_2 = \sqrt{\frac{4W}{\pi\sigma_2} - \frac{2\sigma_1 d_1^2}{\sigma_2}} \quad \leftarrow \qquad \text{(Eq. 2)}$$

SUBSTITUTE NUMERICAL VALUES:

$d_2 = 9.28$ mm $\leftarrow$

EQUATION OF COMPATIBILITY

$\delta_1 = \delta_2$ (Eq. 3)

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} = \sigma_1\left(\frac{L_1}{E_1}\right) \qquad \text{(Eq. 4)}$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} = \sigma_2\left(\frac{L_2}{E_2}\right) \qquad \text{(Eq. 5)}$$

Substitute Eqs. (4) and (5) into Eq. (3):

$$\sigma_1\left(\frac{L_1}{E_1}\right) = \sigma_2\left(\frac{L_2}{E_2}\right)$$

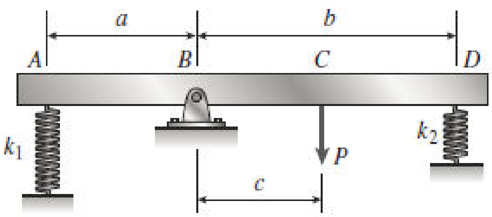
Length L_1 is known; solve for L_2:

$$L_2 = L_1\left(\frac{\sigma_1 E_2}{\sigma_2 E_1}\right) \quad \leftarrow \qquad \text{(Eq. 6)}$$

SUBSTITUTE NUMERICAL VALUES:

$L_2 = 1.10$ m

Problem 2.4-20



NUMERICAL DATA

$a = 250$ mm

$b = 500$ mm

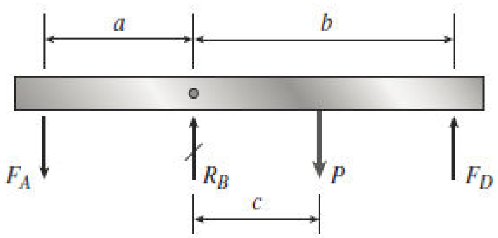
$c = 200$ mm

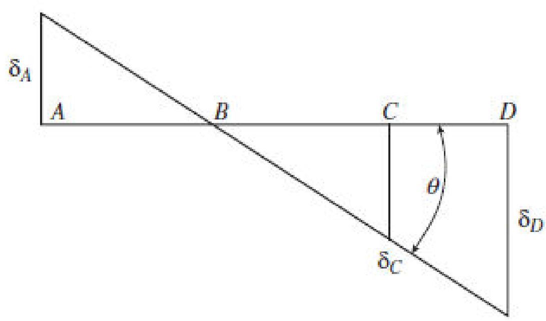
$k_1 = 10$ kN/m

$k_2 = 25$ kN/m

$\theta_{max} = 3° = \frac{\pi}{60}$ rad

FREE-BODY DIAGRAM AND DISPLACEMENT DIAGRAM





EQUATION OF EQUILIBRIUM

$$\Sigma M_B = 0 + - F_A(a) - P(c) + F_D(b) = 0 \qquad \text{(Eq. 1)}$$

EQUATION OF COMPATIBILITY

$$\frac{\delta_A}{a} = \frac{\delta_D}{b} \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_A = \frac{F_A}{k_1} \quad \delta_D = \frac{F_D}{k_2} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{F_A}{ak_1} = \frac{F_D}{bk_2} \qquad \text{(Eq. 5)}$$

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$$F_A = \frac{ack_1P}{a^2k_1 + b^2k_2} \qquad F_D = \frac{bck_2P}{a^2k_1 + b^2k_2}$$

ANGLE OF ROTATION

$$\delta_D = \frac{F_D}{k_2} = \frac{bcP}{a^2k_1 + b^2k_2} \qquad \theta = \frac{\delta_D}{b} = \frac{cP}{a^2k_1 + b^2k_2}$$

MAXIMUM LOAD

$$P = \frac{\theta}{c}(a^2k_1 + b^2k_2)$$

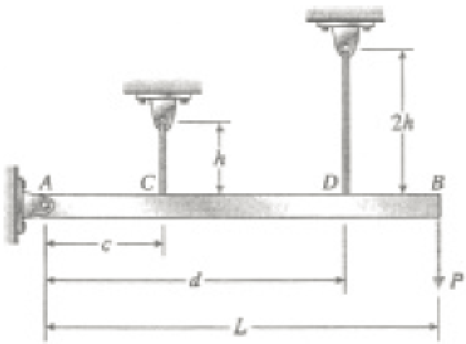
$$P_{max} = \frac{\theta_{max}}{c}(a^2k_1 + b^2k_2) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$P_{max} = \frac{\pi/60 \text{ rad}}{200 \text{ mm}}[(250 \text{ mm})^2(10 \text{ kN/m}) + (500 \text{ mm})^2(25 \text{ kN/m})]$$

$$= 1800 \text{ N} \quad \leftarrow$$

Problem 2.4-21



$h = 0.4$ m

$2h = 0.8$ m

$c = 0.5$ m

$d = 1.2$ m

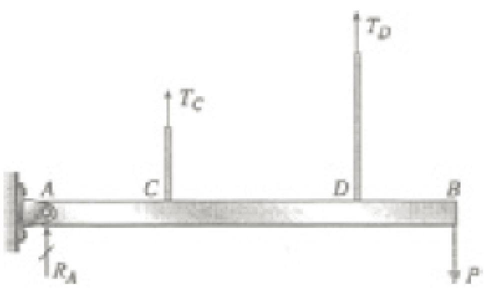
$L = 1600$ mm

$E = 200$ GPa

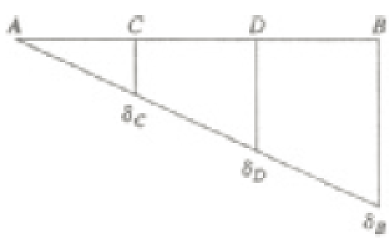
$A = 16\ \text{mm}^2$

$P = 970$ N

Free-body diagram



Displacement diagram



Equation of equilibrium

$$\Sigma M_A = 0 \curvearrowleft\!+ \curvearrowleft \quad T_C(c) + T_D(d) = PL \qquad \text{(Eq. 1)}$$

Equation of compatibility

$$\frac{\delta_C}{c} = \frac{\delta_D}{d} \qquad \text{(Eq. 2)}$$

Force-displacement relations

$$\delta_C = \frac{T_C h}{EA} \quad \delta_D = \frac{T_D(2h)}{EA} \qquad \text{(Eqs. 3, 4)}$$

Solution of equations

Substitute Eqs. (3) and (4) into Eq. (2):

$$\frac{T_C h}{cEA} = \frac{T_D(2h)}{dEA} \quad \text{or} \quad \frac{T_C}{c} = \frac{2T_D}{d} \qquad \text{(Eq. 5)}$$

Tensile forces in the wires

Solve simultaneously Eqs. (1) and (5):

$$T_C = \frac{2cPL}{2c^2 + d^2} \quad T_D = \frac{dPL}{2c^2 + d^2}$$

Tensile stresses in the wires

$$\sigma_C = \frac{T_C}{A} = \frac{2cPL}{A(2c^2 + d^2)}$$

$$\sigma_D = \frac{T_D}{A} = \frac{dPL}{A(2c^2 + d^2)}$$

Substitute numerical values

$$A(2c^2 + d^2) = (60\ \text{mm})^2[2(500\ \text{mm})^2 + (1200\ \text{mm})^2]$$
$$= 31.04 \times 10^6\ \text{mm}^4$$

$$\sigma_C = \frac{2(500\ \text{mm})(970\ \text{N})(1600\ \text{mm})}{31.04 \times 10^6\ \text{mm}^4} = 50.0\ \text{MPa} \quad \leftarrow$$

$$\sigma_D = \frac{(1200\ \text{mm})(970\ \text{N})(1600\ \text{mm})}{31.04 \times 10^6\ \text{mm}^4} = 60.0\ \text{MPa} \quad \leftarrow$$

Displacement at end of bar

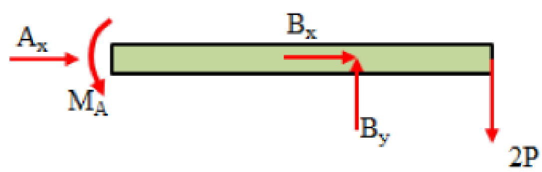
$$\delta_B = \delta_D\left(\frac{L}{d}\right) = \frac{2hT_D}{EA}\left(\frac{L}{d}\right) = \frac{2hPL^2}{EA(2c^2 + d^2)}$$

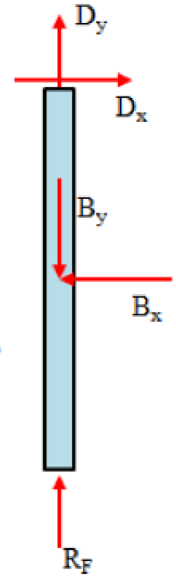
Substitute numerical values

$$\delta_B = \frac{2(400\ \text{mm})(970\ \text{N})(1600\ \text{mm})^2}{(200\ \text{GPa})(31.04 \times 10^6\ \text{mm}^4)}$$
$$= 0.320\ \text{mm} \quad \leftarrow$$

Problem 2.4-22

Remove pin at B; draw separate FBD's of beam and column. Find selected forces using statics





From FBD of column DBF

$\Sigma M_B = D_x \cdot \frac{L}{2} = 0 \qquad D_x = 0$

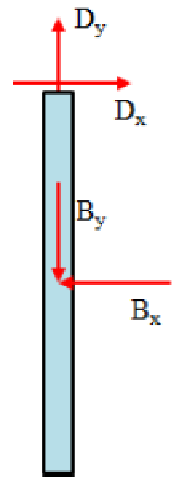
$\Sigma F_x = D_x - B_x = 0 \qquad B_x = D_x$

From FBD of beam ABC

$\Sigma F_x = A_x + B_x = 0 \qquad A_x = 0$

$\Sigma M_B = M_A - 2P \cdot \frac{L}{3} = 0 \qquad M_A = 2P \cdot \frac{L}{3}$

$\Sigma F_y = B_y - 2P = 0 \qquad B_y = 2P$

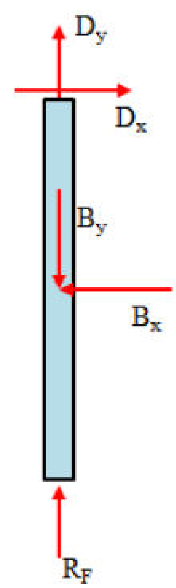


Remove reaction R_F to create the release structure; find vertical displacement at F due to actual load 2P at C

$$\delta_{F1} = \frac{B_y \cdot \frac{L}{2}}{EA} \qquad \delta_{F1} = \frac{P \cdot L}{EA}$$

Apply redundant R_F to released structure; find vertical displacement at F

$$B_y = 0 \qquad \delta'_{F2} = \frac{-R_F \cdot \frac{L}{2}}{2EA} - \frac{R_F \cdot \frac{L}{2}}{EA} \qquad \delta'_{F2} = -R_F \cdot \left(\frac{L}{4 \cdot EA} + \frac{L}{2EA}\right) \qquad \delta'_{F2} = -R_F \cdot \frac{3L}{4 \cdot EA}$$



Compatibility equation - solve for R_F

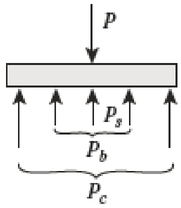
$$\delta_{F1} + \delta'_{F2} = 0 \qquad R_F = \frac{\frac{P \cdot L}{EA}}{\left(\frac{3L}{4 \cdot EA}\right)} \qquad R_F = \frac{4}{3}P$$

Finally solve for reaction D_y using FBD of DBF

$$\Sigma F_y = 0 \qquad D_y = B_y - R_F \qquad D_y = 2P - \frac{4}{3}P \qquad D_y = \frac{2}{3}P$$

Problem 2.4-23

Free-body diagram of rigid end plate



Equation of equilibrium

$\Sigma F_{vert} = 0 \qquad P_s + P_b + P_c = P \qquad$ (Eq. 1)

Equations of compatibility

$\delta_s = \delta_b \qquad \delta_c = \delta_s \qquad$ (Eqs. 2)

Force-displacement relations

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_b = \frac{P_b L}{E_b A_b} \quad \delta_c = \frac{P_c L}{E_c A_c}$$

Solution of equations

Substitute into Eqs. (2):

$$P_b = P_s \frac{E_b A_b}{E_s A_s} \qquad \text{(Eqs. 3, 4)}$$

$$P_c = P_s \frac{E_c A_c}{E_s A_s}$$

Solve simultaneously Eqs. (1), (3), and (4):

$$P_s = P \frac{E_s A_s}{E_s A_s + E_b A_b + E_c A_c}$$

$$P_b = P \frac{E_b A_b}{E_s A_s + E_b A_b + E_c A_c}$$

$$P_c = P \frac{E_c A_c}{E_s A_s + E_b A_b + E_c A_c} \qquad \text{(Eq. 5)}$$

Compressive stresses

Let $\Sigma EA = E_s A_s + E_b A_b + E_c A_c$

$$\sigma_s = \frac{P_s}{A_s} = \frac{PE_s}{\Sigma EA} \qquad \sigma_b = \frac{P_b}{A_b} = \frac{PE_b}{\Sigma EA}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{PE_c}{\Sigma EA}$$

Substitute numerical values:

$E_s = 210$ GPa, $\quad E_b = 100$ GPa, $\quad E_c = 120$ GPa

$d_c = 20$ mm, $\quad d_b = 15$ mm, $\quad d_s = 10$ mm

$$A_s = \frac{\pi}{4} d_s^2 = \frac{\pi}{4}(10 \text{ mm})^2 = 78.54 \text{ mm}^2$$

$$A_b = \frac{\pi}{4}(d_b^2 - d_s^2) = \frac{\pi}{4}[(15 \text{ mm})^2 - (10 \text{ mm})^2]$$

$$= 98.17 \text{ mm}^2$$

$$A_c = \frac{\pi}{4}(d_c^2 - d_b^2) = \frac{\pi}{4}[(20 \text{ mm})^2 - (15 \text{ mm})^2]$$

$$= 137.44 \text{ mm}^2$$

$P = 12$ kN, $\quad \Sigma EA = 42.80 \times 10^6$ N

$$\sigma_s = \frac{PE_s}{\Sigma EA} = 58.9 \text{ MPa}$$

$$\sigma_b = \frac{PE_b}{\Sigma EA} = 28.0 \text{ MPa} \quad \leftarrow$$

$$\sigma_c = \frac{PE_c}{\Sigma EA} = 33.6 \text{ MPa}$$

Problem 2.4-24

Remove R_F to create released structure; use superposition to find redundant R_F = y-dir reaction at F

Released structure under actual load; use FBD of ABC to find pin force B_y

$\Sigma M_A = 0 \qquad B_y = \frac{1}{\frac{L}{3}} \cdot (3 \cdot P \cdot L) \qquad B_y \rightarrow 9 \cdot P$ acts upward on ABC so acts downward on DBF

Find vert. displ. of F in released structure under actual loads $\delta_{F1} = \frac{-B_y \cdot L}{2 \cdot EA} \qquad \delta_{F1} \rightarrow -\frac{9 \cdot L \cdot P}{2 \cdot EA}$ downward

Apply redundant R_F and find vertical displ. at F in released structure $\delta_{F2} = R_F \cdot \left(\frac{\frac{L}{3}}{EA} + \frac{L}{2 \cdot EA} \right) \qquad \delta_{F2} \rightarrow \frac{5 \cdot L \cdot R_F}{6 \cdot EA}$ upward

Compatibility equ. $\delta_{F1} + \delta_{F2} = 0 \qquad R_F = \frac{-\delta_{F1}}{\frac{5}{6} \cdot \frac{L}{EA}} \qquad R_F \rightarrow \frac{27 \cdot P}{5}$

Now use statics to find all remaining reactions FBD of DBF $\Sigma M_B = 0$ so $D_x = 0$

Entire structure $\Sigma F_x = 0 \quad A_x = 0 \qquad \Sigma M_B = 0 \quad A_y = \frac{1}{\frac{L}{3}} \left[-3 \cdot P \cdot \left(\frac{2 \cdot L}{3} \right) \right] \qquad A_y \rightarrow -6 \cdot P$

$\Sigma F_y = 0 \qquad D_y = -R_F + 3 \cdot P - A_y \qquad D_y \rightarrow \frac{18 \cdot P}{5}$

Problem 2.5-1

The rails are prevented from expanding because of their great length and lack of expansion joints.

Therefore, the rail is in the same condition as a bar with fixed ends (see Example 2-9).

The compressive stress in the rails may be calculated as follows:

$$\Delta T = 52°C - 10°C = 42°C$$

$$\sigma = E\alpha(\Delta T)$$

$$= (200 \text{ GPa})(12 \times 10^{-6}/°C)(42°C)$$

$$= 100.8 \text{ MPa} \quad \leftarrow \quad \text{(compression)}$$

Problem 2.5-2

INITIAL CONDITIONS

$L_a = 60$ m $\quad T_0 = 10°C$

$L_s = 60.005$ m $\quad T_0 = 10°C$

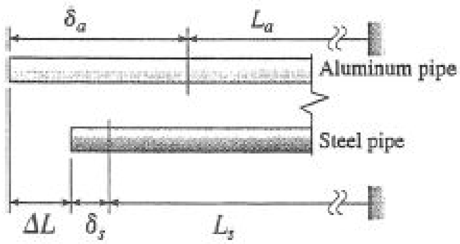
$\alpha_a = 23 \times 10^{-6}/°C \quad \alpha_s = 12 \times 10^{-6}/°C$

FINAL CONDITIONS

Aluminum pipe is longer than the steel pipe by the amount $\Delta L = 15$ mm.

ΔT = increase in temperature

$\delta_a = \alpha_a(\Delta T)L_a \qquad \delta_s = \alpha_s(\Delta T)L_s$



From the figure above:

$\delta_a + L_a = \Delta L + \delta_s + L_s$

or, $\alpha_a(\Delta T)L_a + L_a = \Delta L + \alpha_s(\Delta T)L_s + L_s$

Solve for ΔT:

$$\Delta T = \frac{\Delta L + (L_s - L_a)}{\alpha_a L_a - \alpha_s L_s} \quad \leftarrow$$

Substitute numerical values:

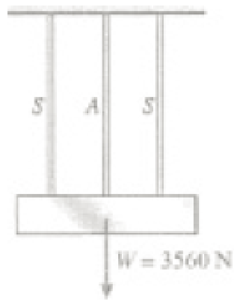
$\alpha_a L_a - \alpha_s L_s = 659.9 \times 10^{-6}$ m/°C

$$\Delta T = \frac{15 \text{ mm} + 5 \text{ mm}}{659.9 \times 10^{-6} \text{ m/°C}} = 30.31°C$$

$T = T_0 + \Delta T = 10°C + 30.31°C$

$= 40.3°C \quad \leftarrow$

Problem 2.5-3



S = steel $\quad$ A = aluminum

$W = 3560\text{ N}$

$d = 3.2\text{ mm}$

$$A_s = \frac{\pi d^2}{4} = 8.042\text{ mm}^2$$

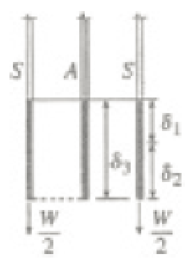
$E_s = 205\text{ GPa}$

$E_s A_s = 1{,}648{,}706\text{ N}$

$\alpha_s = 12 \times 10^{-6}/°\text{C}$

$\alpha_a = 24 \times 10^{-6}/°\text{C}$

L = Initial length of wires



δ_1 = increase in length of a steel wire due to temperature increase ΔT

$= \alpha_s(\Delta T)L$

δ_2 = increase in length of a steel wire due to load $W/2$

$$= \frac{WL}{2E_s A_s}$$

δ_3 = increase in length of aluminum wire due to temperature increase ΔT

$= \alpha_a(\Delta T)L$

For no load in the aluminum wire:

$$\delta_1 + \delta_2 = \delta_3$$

$$\alpha_s(\Delta T)L + \frac{WL}{2E_s A_s} = \alpha_a(\Delta T)L$$

or

$$\Delta T = \frac{W}{2E_s A_s(\alpha_a - \alpha_s)} \quad \leftarrow$$

Substitute numerical values:

$$\Delta T = \frac{3560\text{ N}}{(2)(1{,}648{,}706\text{ N})(12 \times 10^{-6}/°\text{C})}$$

$$= 90°\text{C} \quad \leftarrow$$

NOTE: If the temperature increase is larger than ΔT, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than ΔT, the aluminum wire will be in tension and carry part of the load.

Problem 2.5-4

NUMERICAL PROPERTIES

$$d_r = 15 \text{ mm} \quad d_b = 12 \text{ mm} \quad d_w = 20 \text{ mm} \quad t_c = 10 \text{ mm} \quad t_{wall} = 18 \text{ mm}$$

$$\tau_b = 45 \text{ MPa} \quad \alpha = 12(10^{-6}) \quad E = 200 \text{ GPa}$$

(a) TEMPERATURE DROP RESULTING IN BOLT SHEAR STRESS $\quad \varepsilon = \alpha\Delta T \quad \sigma = E\alpha\Delta T$

Rod force $= P = (E\alpha\Delta T)\frac{\pi}{4}d_r^2$ and bolt in double shear with shear stress $\tau = \dfrac{\frac{P}{2}}{A_s} \quad \tau = \dfrac{P}{2\frac{\pi}{4}d_b^2}$

$$\tau_b = \frac{2}{\pi d_b^2}\left[(E\alpha\Delta T)\frac{\pi}{4}d_r^2\right] \qquad \tau_b = \frac{E\alpha\Delta T}{2}\left(\frac{d_r}{d_b}\right)^2$$

$\tau_b = 45 \text{ MPa}$

$$\Delta T = \frac{2\tau_b}{E(1000)\alpha}\left(\frac{d_b}{d_r}\right)^2 \qquad \Delta T = 24°\text{C} \qquad P = (E\alpha\Delta T)\frac{\pi}{4}d_r^2 \qquad P = 10 \text{ kN}$$

$$\sigma_{rod} = \frac{P\,1000}{\frac{\pi}{4}d_r^2} \qquad \boxed{\sigma_{rod} = 57.6 \text{ MPa}}$$

(b) BEARING STRESSES

BOLT AND CLEVIS $\quad \sigma_{bc} = \dfrac{\frac{P}{2}}{d_b t_c} \qquad \boxed{\sigma_{bc} = 42.4 \text{ MPa}}$

WASHER AT WALL $\quad \sigma_{bw} = \dfrac{P}{\frac{\pi}{4}(d_w^2 - d_r^2)} \qquad \boxed{\sigma_{bw} = 74.1 \text{ MPa}}$

(c) If the connection to the wall at B is changed to an end plate with two bolts (see Fig. b), what is the required diameter d_b of each bolt if temperature drop $\Delta T = 38°\text{C}$ and the allowable bolt stress is 90 MPa?

Find force in rod due to temperature drop.

$\Delta T = 38°\text{C} \qquad P = (E\alpha\Delta T)\frac{\pi}{4}d_r^2$

$$P = 200\ GPa\frac{\pi}{4}(15 \text{ mm})^2\left[12\left(10^{-6}\right)\right](38) = 16116 \text{ N} \qquad P = 16.12 \text{ kN}$$

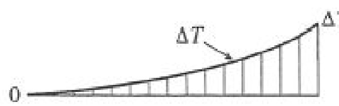
Each bolt carries one half of the force P:

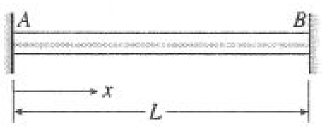
$$d_b = \sqrt{\frac{\frac{16.12 \text{ kN}}{2}}{\frac{\pi}{4}(90 \text{ MPa})}} = 10.68 \text{ mm}) \qquad \boxed{d_b = 10.68 \text{ mm}}$$

Problem 2.5-5

(a) One degree statically indeterminate—use superposition select reaction R_B as the redundant; follow procedure

Bar with nonuniform temperature change.

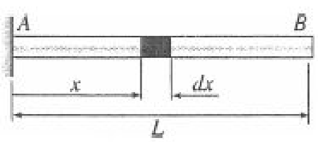




At distance x:

$$\Delta T = \Delta T_B \left(\frac{x^3}{L^3}\right)$$

Remove the support at the end B of the bar:



Consider an element dx at a distance x from end A.

$d\delta$ = Elongation of element dx

$$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx$$

$d\delta$ = elongation of bar

$$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_B)\left(\frac{x^3}{L^3}\right)dx = \frac{1}{4}\alpha(\Delta T_B)L$$

Compressive force P required to shorten the bar by the amount δ

$$P = \frac{EA\delta}{L} = \frac{1}{4}EA\alpha(\Delta T_B)$$

Compressive stress in the bar

$$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_B)}{4} \quad \leftarrow$$

(b) One degree statically indeterminate—use superposition.

Select reaction R_B as the redundant then compute bar elongations due to ΔT and due to R_B

$$\delta_{B1} = \alpha \Delta T_B \frac{L}{4} \quad \text{due to temperature from above}$$

$$\delta_{B2} = R_B\left(\frac{1}{k} + \frac{L}{EA}\right)$$

Compatibility: solve for R_B: $\quad \delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{-\left(\alpha \Delta T_B \frac{L}{4}\right)}{\left(\frac{1}{k} + \frac{L}{EA}\right)}$$

$$R_B = -\alpha \Delta T_B \left[\frac{EA}{4\left(\frac{EA}{kL} + 1\right)}\right]$$

So compressive stress in bar is

$$\sigma_c = \frac{R_B}{A} \qquad \sigma_c = \frac{E\alpha(\Delta T_B)}{4\left(\frac{EA}{kL} + 1\right)} \quad \leftarrow$$

NOTE: σ_c in part (b) is the same as in part (a) if spring constant k goes to infinity.

Problem 2.5-6

$A = 2 \cdot (1913 mm^2) = 3826 \cdot mm^2 \quad k = 1750 \frac{kN}{m} \quad \Delta T = 45 \quad \alpha = 12 \cdot (10^{-6}) \quad L = 3m \quad E = 205GPa$

Assume that beam and spring are stress free at the start, then apply temperature increase ΔT.
Select R_C as the redundant to remove to create the released structure

Apply ΔT to beam in released structure $\quad \delta_{C1} = \alpha \cdot \Delta T \cdot L = 1.62 \cdot mm$

Apply redundant R_C $\quad \delta_{C2} = R_C \cdot \left(\frac{L}{E \cdot A} + \frac{1}{k} \right) \quad \frac{L}{E \cdot A} + \frac{1}{k} = 0.575 \cdot \frac{mm}{kN}$

Compatibility equation and solution for redundant $\quad \delta_{C1} + \delta_{C2} = 0 \quad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left(\frac{L}{E \cdot A} + \frac{1}{k} \right)} = -2.816 \cdot kN$

Axial normal compressive stress in beam $\quad \sigma_T = \frac{R_C}{A} = -0.736 \cdot MPa$

Displacement at B using superposition $\quad \delta_B = \frac{R_C \cdot L}{E \cdot A} + \alpha \cdot \Delta T \cdot L = 1.609 \cdot mm \quad \frac{R_C}{k} = -1.609 \cdot mm$

elongation of beam is equal to shortening of spring

Problem 2.5-7

$E = 200GPa \qquad \alpha = 12 \cdot 10^{-6} \qquad \Delta T = 10 \qquad A = 33.4cm^2 \qquad L = 3m$

Select reaction R_B as the redundant; remove R_B to create released structure. Use superposition - apply ΔT to released structure, then apply redundant. Solve compatibility equation to find R_B then use statics to get R_A

$$\delta_{B1} = \alpha \cdot \Delta T \cdot L = 0.014 \cdot in \qquad \delta_{B2} = R_B \cdot \frac{L}{EA}$$

Compatibility $\delta_{B1} + \delta_{B2} = 0$ solve for R_B

$$R_B = \frac{-E \cdot A}{L} \cdot (\alpha \cdot \Delta T \cdot L) = -80.16 \cdot kN$$ negative so R_B acts to left

Statics $R_A + R_B = 0$ so $R_A = -R_B = 80.16 \cdot kN$

Beam is in uniform axial compression due to temperature change; compressive normal stress is

$$\sigma_T = \frac{R_B}{A} = -24 \cdot MPa$$

Problem 2.5-8

NUMERICAL DATA

$d_1 = 50$ mm $d_2 = 75$ mm

$L_1 = 225$ mm $L_2 = 300$ mm

$E = 6.0$ GPa $\alpha = 100 \times 10^{-6}/°C$

$\Delta T = 30°C$ $k = 50$ MN/m

(a) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PT. C

$$A_1 = \frac{\pi}{4}d_1^{\,2} \quad A_2 = \frac{\pi}{4}d_2^{\,2}$$

One-degree statically indeterminate—use R_B as redundant

$$\delta_{B1} = \alpha\Delta T(L_1 + L_2)$$

$$\delta_{B2} = R_B\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right)$$

Compatibility: $\delta_{B1} = \delta_{B2}$, solve for R_B

$$R_B = \frac{\alpha\Delta T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2}} \quad N = R_B$$

$N = 51.8$ kN $\leftarrow$

Maximum compressive stress in AC since it has the smaller area ($A_1 < A_2$):

$$\sigma_{cmax} = \frac{N}{A_1} \quad \sigma_{cmax} = 26.4 \text{ MPa}$$

Displacement δ_C of point C = superposition of displacements in two released structures at C:

$$\delta_C = \alpha\Delta T(L_1) - R_B\frac{L_1}{EA_1}$$

$\delta_C = -0.314$ mm $\leftarrow$ (−) sign means joint C moves left

(b) COMPRESSIVE FORCE N, MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PART C FOR ELASTIC SUPPORT CASE

Use R_B as redundant as in part (a):

$$\delta_{B1} = \alpha\Delta T(L_1 + L_2)$$

$$\delta_{B_2} = R_B\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k}\right)$$

Now add effect of elastic support; equate δ_{B1} and δ_{B2} then solve for R_B:

$$R_B = \frac{\alpha\Delta T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k}} \quad N = R_B$$

$N = 31.2$ kN $\leftarrow$

$$\sigma_{cmax} = \frac{N}{A_1} \quad \sigma_{cmax} = 15.91 \text{ MPa} \leftarrow$$

Superposition:

$$\delta_C = \alpha\Delta T(L_1) - R_B\left(\frac{L_1}{EA_1} + \frac{1}{k}\right)$$

$\delta_C = -0.546$ mm $\leftarrow$ (−) sign means joint C moves left

Problem 2.5-9

$\Delta T = 10 \quad \alpha = 23 \cdot \left(10^{-6}\right) \quad E = 72GPa \quad L = 1m \quad b_1 = 50mm \quad b_2 = 60mm \quad t = 6mm$

Select reaction R_C as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha \cdot \Delta T \cdot L = 0.23 \cdot mm \qquad \delta_{C2} = R_C \cdot \left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)}\right]$$

$$\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} = 0.044 \cdot \frac{mm}{kN}$$

Write compatibility equation then solve for R_C

$$\delta_{C1} + \delta_{C2} = 0 \qquad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)}\right]} = -5.198 \cdot kN$$

Statics $\quad R_A + R_C = 0 \qquad R_A = -R_C = 5.198 \cdot kN$

Displacement at B using superposition

$$\delta_B = -R_A \cdot \left[\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right)\right] + \alpha \cdot \Delta T \cdot \frac{L}{2} = 5.318 \times 10^{-3} \cdot mm$$

joint B moves to right

OR

$$\frac{R_C \cdot \frac{L}{2}}{E \cdot (b_1 \cdot t)} + \alpha \cdot \Delta T \cdot \frac{L}{2} = -5.32 \times 10^{-3} \cdot mm$$

shortening of BC

Problem 2.5-10

$\Delta T = 30 \quad \alpha = 19\cdot(10^{-6}) \quad L = 2m \quad t = 20mm \quad b_1 = 100mm \quad b_2 = 115mm \quad E = 96GPa$

Select reaction R_c as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha\cdot\Delta T\cdot L = 1.14\cdot mm \qquad \delta_{C2} = R_C\cdot\left[\frac{\frac{3\cdot L}{5}+\frac{2\cdot L}{5}}{E\cdot t\cdot(b_2-b_1)}\cdot \ln\left(\frac{b_2}{b_1}\right)\right] \qquad \frac{L}{E\cdot t\cdot(b_2-b_1)}\cdot\ln\left(\frac{b_2}{b_1}\right) = 9.706\times10^{-3}\cdot\frac{mm}{kN}$$

Write compatibility equation then solve for R_c

$$\delta_{C1}+\delta_{C2} = 0 \qquad R_C = \frac{-(\alpha\cdot\Delta T\cdot L)}{\left[\frac{L}{E\cdot t\cdot(b_2-b_1)}\cdot\ln\left(\frac{b_2}{b_1}\right)\right]} = -117.457\cdot kN$$

Statics $\quad R_A + R_C = 0 \qquad R_A = -R_C = 117.457\cdot kN$

Displacement at B using superposition

$$\delta_B = -R_A\cdot\left[\frac{\frac{3\cdot L}{5}}{E\cdot t\cdot(b_2-b_1)}\cdot\ln\left(\frac{b_2}{b_1}\right)\right] + \alpha\cdot\Delta T\cdot\frac{3\cdot L}{5} = 0\cdot mm$$

no elongation of AB

OR

$$R_C\cdot\left[\frac{\frac{2\cdot L}{5}}{E\cdot t\cdot(b_2-b_1)}\cdot\ln\left(\frac{b_2}{b_1}\right)\right] + \alpha\cdot\Delta T\cdot\frac{2L}{5} = 0\cdot mm$$

no shortening of BC

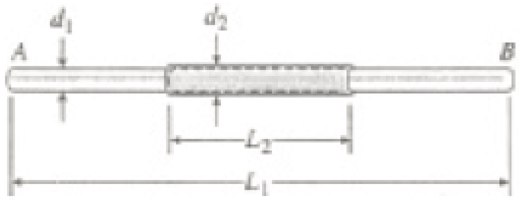
Extra - find displ. at x = 2L/5 $\quad b_{2L5} = b_2 - \frac{2}{3}\cdot(b_2-b_1) \qquad b_{2L5} \rightarrow 105\cdot mm$

$$\delta_{2L5} = -R_A\cdot\left[\frac{\frac{2\cdot L}{5}}{E\cdot t\cdot(b_2-b_{2L5})}\cdot\ln\left(\frac{b_2}{b_{2L5}}\right)\right] + \alpha\cdot\Delta T\cdot\frac{2\cdot L}{5} = 0.011\cdot mm$$

OR

$$R_C\cdot\left[\frac{\frac{2\cdot L}{5}}{E\cdot t\cdot(b_2-b_1)}\cdot\ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{5}}{E\cdot t\cdot(b_{2L5}-b_1)}\cdot\ln\left(\frac{b_{2L5}}{b_1}\right)\right] + \alpha\cdot\Delta T\cdot\frac{3L}{5} = -0.011\cdot mm$$

Problem 2.5-11



ELONGATION OF THE TWO OUTER PARTS OF THE BAR

$$\delta_1 = \alpha_s(\Delta T)(L_1 - L_2)$$
$$= 2.940 \text{ mm}$$

ELONGATION OF THE MIDDLE PART OF THE BAR

The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-10. Thus, we can calculate the elongation from Eq. (2-21):

$$\delta_2 = \frac{(\alpha_s E_s A_s + \alpha_b E_b A_b)(\Delta T)L_2}{E_s A_s + E_b A_b}$$

SUBSTITUTE NUMERICAL VALUES

$\alpha_s = 12 \times 10^{-6}/°C \quad \alpha_b = 20 \times 10^{-6}/°C$

$E_s = 210 \text{ GPa} \qquad E_b = 110 \text{ GPa}$

$$A_s = \frac{\pi}{4} d_1^2 = 176.7 \text{ mm}^2$$

$$A_b = \frac{\pi}{4}(d_2^2 - d_1^2) = 169.6 \text{ mm}^2$$

$\Delta T = 350°C \quad L_2 = 400 \text{ mm}$

$\delta_2 = 2.055 \text{ mm}$

TOTAL ELONGATION

$\delta = \delta_1 + \delta_2 = 5.0 \text{ mm} \quad \leftarrow$

Problem 2.5-12

$\Delta T = 15$ $\quad \alpha_T = 23\cdot(10^{-6})$ $\quad L = 1.8m$ $\quad r = 36mm$ $\quad E = 72GPa$ $\quad a = \frac{r}{8} = 4.5\cdot mm$

$A_1 = \pi\cdot r^2 = 4071.504\cdot mm^2$ Use formulas in ***Appendix E***, ***Case 15*** for area of slotted segment

$\alpha = acos\left(\frac{a}{r}\right) = 1.445$ $\quad b = \sqrt{r^2 - a^2} = 35.718\cdot mm$ $\quad A_2 = 2\cdot r^2\cdot\left(\alpha - \frac{a\cdot b}{r^2}\right) = 3425.196\cdot mm^2$ $\quad \frac{A_2}{A_1} = 0.841$

Select reaction R_c as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$\delta_{C1} = \alpha_T\cdot\Delta T\cdot L = 0.621\cdot mm$ $\quad \delta_{C2} = R_C\cdot\left(\frac{2\cdot\frac{L}{4}}{E\cdot A_1} + \frac{\frac{L}{2}}{E\cdot A_2}\right)$ $\quad \frac{2\cdot\frac{L}{4}}{E\cdot A_1} + \frac{\frac{L}{2}}{E\cdot A_2} = 6.72\times 10^{-3}\cdot\frac{mm}{kN}$

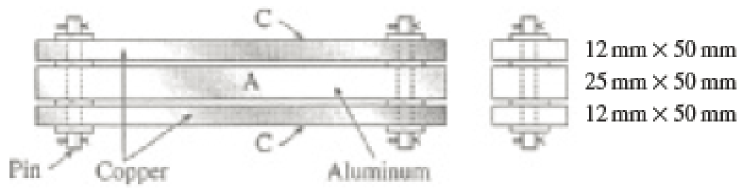
Write compatibility equation then solve for R_c $\quad \delta_{C1} + \delta_{C2} = 0$ $\quad R_C = \frac{-(\alpha_T\cdot\Delta T\cdot L)}{\left(\frac{2\cdot\frac{L}{4}}{E\cdot A_1} + \frac{\frac{L}{2}}{E\cdot A_2}\right)} = -92.417\cdot kN$

Statics $\quad R_A + R_C = 0$ $\quad R_A = -R_C = 92.417\cdot kN$

Thermal compressive stress in solid bar segments $\quad \sigma_{T1} = \frac{R_C}{A_1} = -22.698\cdot MPa$

and in slotted middle segment $\quad \sigma_{T2} = \frac{R_C}{A_2} = -26.982\cdot MPa$

Problem 2.5-13



Diameter of pin: $d_P = 111$ mm

Area of pin: $A_P = \frac{\pi}{4} d_P^2 = 95 \text{ mm}^2$

Area of two copper bars: $A_c = 1200 \text{ mm}^2$

Area of aluminum bar: $A_a = 1250 \text{ mm}^2$

$\Delta T = 40°\text{C}$

Copper: $E_c = 124$ GPa $\quad \alpha_c = 20 \times 10^{-6}/°\text{C}$

Aluminum: $E_a = 69$ GPa

$\alpha_a = 26 \times 10^{-6}/°\text{C}$

Use the results of Example 2-10.

Find the forces P_a and P_c in the aluminum bar and copper bar, respectively, from Eq. (2-19).

Replace the subscript "S" in that equation by "a" (for aluminum) and replace the subscript "B" by "c" (for copper):

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that P_a is the compressive force in the aluminum bar and P_c is the combined tensile force in the two copper bars.

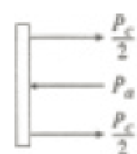
Substitute numerical values:

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_c A_c}{1 + \dfrac{E_c A_c}{E_a A_a}}$$

$$P_a = P_c = \frac{(6 \times 10^{-6}/°\text{C})(40°\text{C})(124 \text{ GPa})(1200 \text{ mm}^2)}{1 + \dfrac{124}{69}\left(\dfrac{1200}{1250}\right)}$$

$= 12.861$ kN

Free-body diagram of pin at the left end



V = shear force in pin

$= P_c/2$

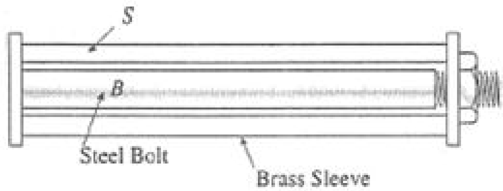
$= 6430.5$ N

τ = average shear stress on cross section of pin

$$\tau = \frac{V}{A_P} = \frac{6430.5 \text{ N}}{95 \text{ mm}^2}$$

$\tau = 67.7$ MPa ←

Problem 2.5-14



Subscript S means "sleeve".

Subscript B means "bolt".

Use the results of Example 2-10.

σ_S = compressive force in sleeve

EQUATION (2-20a):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B} \text{(Compression)}$$

SOLVE FOR ΔT:

$$\Delta T = \frac{\sigma_S(E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B)E_S E_B A_B}$$

or

$$\Delta T = \frac{\sigma_S}{E_S(\alpha_S - \alpha_B)}\left(1 + \frac{E_S A_S}{E_B A_B}\right) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_S = 25$ MPa

$d_2 = 36$ mm $\quad d_1 = 26$ mm $\quad d_B = 25$ mm

$E_S = 100$ GPa $\quad E_B = 200$ GPa

$\alpha_S = 21 \times 10^{-6}/°C \quad \alpha_B = 10 \times 10^{-6}/°C$

$$A_S = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(620 \text{ mm}^2)$$

$$A_B = \frac{\pi}{4}(d_B)^2 = \frac{\pi}{4}(625 \text{ mm}^2) \; 1 + \frac{E_S A_S}{E_B A_B} = 1.496$$

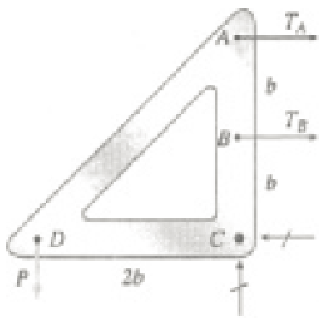
$$\Delta T = \frac{25 \text{ MPa } (1.496)}{(100 \text{ GPa})(11 \times 10^{-6}/°C)}$$

$\Delta T = 34°C \quad \leftarrow$

(Increase in temperature)

Problem 2.5-15

Free-body diagram of frame

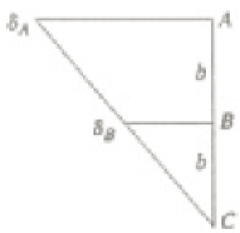


Equation of equilibrium

$\Sigma M_C = 0$ ↻↺

$P(2b) - T_A(2b) - T_B(b) = 0$ or $2T_A + T_B = 2P$ (Eq. 1)

Displacement diagram



Equation of compatibility

$\delta_A = 2\delta_B$ (Eq. 2)

(a) Load P only

Force-displacement relations:

$$\delta_A = \frac{T_A L}{EA} \quad \delta_B = \frac{T_B L}{EA} \qquad \text{(Eq. 3)}$$

(L = length of wires at A and B.)

Substitute Eq. (3) into Eq. (2):

$$\frac{T_A L}{EA} = \frac{2T_B L}{EA}$$

or $T_A = 2T_B$ (Eq. 4)

Solve simultaneously Eqs. (1) and (4):

$$T_A = \frac{4P}{5} \quad T_B = \frac{2P}{5} \qquad \text{(Eqs. 5)}$$

For $P = 2.2$ kN, we obtain

$T_A = 1760$ N $\quad T_B = 880$ N ←

(b) Load P and temperature increase ($\Delta T = 100°C$)

Force-displacement and temperature-displacement relations:

$$\delta_A = \frac{T_A L}{EA} + \alpha(\Delta T)L \qquad \text{(Eq. 6)}$$

$$\delta_B = \frac{T_B L}{EA} + \alpha(\Delta T)L$$

Substitute Eq. (6) into Eq. (2):

$$\frac{T_A L}{EA} + \alpha(\Delta T)L = \frac{2T_B L}{EA} + 2\alpha(\Delta T)L$$

or $\quad T_A - 2T_B = EA\alpha(\Delta T)$ (Eq. 7)

Solve simultaneously Eqs. (1) and (7):

$$T_A = \frac{1}{5}[4P + EA\alpha(\Delta T)] \qquad \text{(Eq. 8)}$$

$$T_B = \frac{2}{5}[P - EA\alpha(\Delta T)] \qquad \text{(Eq. 9)}$$

Substitute numerical values:

$P = 2.2$ kN = 2200 N $\quad EA = 540$ kN = 540,000 N

$\Delta T = 100°C$

$\alpha = 23 \times 10^{-6}/°C$

$T_A = 1760$ N + 248 N) = 2008 N ←

$T_B = 880$ N − 497 N = 383 N ←

(c) Wire B becomes slack

Set $T_B = 0$ in Eq. (9):

$$P = EA\alpha(\Delta T)$$

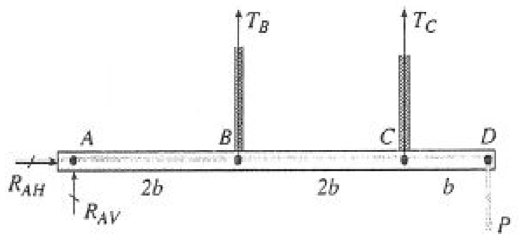
or

$$\Delta T = \frac{P}{EA\alpha} = \frac{2200 \text{ N}}{(540 \text{ kN})(23 \times 10^{-6}/°C)}$$

$$= 177°C$$

Problem 2.5-16

FREE-BODY DIAGRAM OF BAR $ABCD$



T_B = force in cable B $\quad T_C$ = force in cable C

$d_B = 12$ mm $\quad d_C = 20$ mm

From Table 2-1:

$A_B = 76.7\ \text{mm}^2 \quad E = 140$ GPa

$\Delta T = 60°\text{C} \quad A_C = 173\ \text{mm}^2$

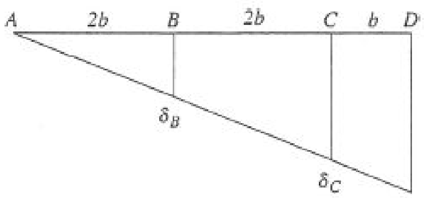
$\alpha = 12 \times 10^{-6}/°\text{C}$

EQUATION OF EQUILIBRIUM

$\Sigma M_A = 0 \quad T_B(2b) + T_C(4b) - P(5b) = 0$

or $2T_B + 4T_C = 5P$ (Eq. 1)

DISPLACEMENT DIAGRAM



COMPATIBILITY:

$\delta_C = 2\delta_B$ (Eq. 2)

FORCE-DISPLACEMENT AND TEMPERATURE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{T_B L}{EA_B} + \alpha(\Delta T)L \quad \text{(Eq. 3)}$$

$$\delta_C = \frac{T_C L}{EA_C} + \alpha(\Delta T)L \quad \text{(Eq. 4)}$$

SUBSTITUTE EQS. (3) AND (4) INTO EQ. (2):

$$\frac{T_C L}{EA_C} + \alpha(\Delta T)L = \frac{2T_B L}{EA_B} + 2\alpha(\Delta T)L$$

or

$$2T_B A_C - T_C A_B = -E\alpha(\Delta T)A_B A_C \quad \text{(Eq. 5)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (5):

$T_B(346) - T_C(76.7) = -1{,}338{,}000$ (Eq. 6)

in which T_B and T_C have units of newtons.

SOLVE SIMULTANEOUSLY EQS. (1) AND (6):

$T_B = 0.2494\ P - 3{,}480$ (Eq. 7)

$T_C = 1.1253\ P + 1{,}740$ (Eq. 8)

in which P has units of newtons.

SOLVE EQS. (7) AND (8) FOR THE LOAD P:

$P_B = 4.0096\ T_B + 13{,}953$ (Eq. 9)

$P_C = 0.8887\ T_C - 1{,}546$ (Eq. 10)

ALLOWABLE LOADS

From Table 2-1:

$(T_B)_{\text{ULT}} = 102{,}000$ N $\quad (T_C)_{\text{ULT}} = 231{,}000$ N

Factor of safety = 5

$(T_B)_{\text{allow}} = 20{,}400$ N $\quad (T_C)_{\text{allow}} = 46{,}200$ N

From Eq. (9): $P_B = (4.0096)(20{,}400\ \text{N}) + 13{,}953\ \text{N} = 95{,}700$ N

From Eq. (10): $P_C = (0.8887)(46{,}200\ \text{N}) - 1546\ \text{N} = 39{,}500$ N

Cable C governs.

$P_{\text{allow}} = 39.5$ kN $\quad \leftarrow$

Problem 2.5-17

NUMERICAL DATA

$L = 0.635 \text{ m} \quad d = 0.050 \text{ m} \quad \delta = 2(10^{-4}) \text{ m}$

$k = 210(10^6) \text{ N/m} \quad E = 110(10^9) \text{ Pa}$

$\alpha = 17.5(10^{-6}) \quad \Delta T = 27°\text{C}$

$A = \frac{\pi}{4}d^2 \quad A = 1.9635\ 10^{-3} \text{ m}^2$

(a) ONE-DEGREE STATICALLY INDETERMINATE IF GAP CLOSES

$\Delta = \alpha \Delta T L \quad \Delta = 3.00037\ 10^{-4} \text{ m}$ <exceeds gap

Select R_A as redundant and do superposition analysis:

$$\delta_{A1} = \Delta \quad \delta_{A2} = R_A\left(\frac{L}{EA} + \frac{1}{k}\right)$$

Compatibility: $\delta_{A1} + \delta_{A2} = \delta \quad \delta_{A2} = \delta - \delta_{A1}$

$$\frac{\Delta}{\delta} = 1.50019$$

$$R_A = \frac{\delta - \Delta}{\frac{L}{EA} + \frac{1}{k}} \quad R_A = -1.29886 \times 10^4 \text{ N}$$

Compressive stress in bar:

$$\sigma = \frac{R_A}{A} \quad \sigma = -6.62 \text{ MPa}$$

(b) FORCE IN SPRING $F_k = R_C$

STATICS $R_A + R_C = 0$

$R_C = -R_A$

$R_C = -1.29886 \times 10^4 \text{ N} \quad \leftarrow$

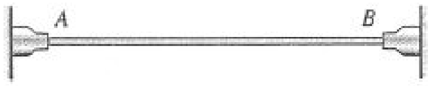
$F_k = 12.99 \text{ kN(C)}$

(c) FIND COMPRESSIVE STRESS IN BAR IF k GOES TO INFINITY. FROM EXPRESSION FOR R_A ABOVE, $1/k$ GOES TO ZERO, SO

$$R_A = \frac{\delta - \Delta}{\frac{L}{EA}} \quad R_A = -3.40261 \times 10^4 \text{ N}$$

$$\sigma = \frac{R_A}{A} \qquad \sigma = -17.33 \text{ MPa} \quad \leftarrow$$

Problem 2.5-18



Initial prestress: $\sigma_1 = 42$ MPa

Initial temperature: $T_1 = 20°C$

$E = 200$ GPa

$\alpha = 14 \times 10^{-6}/°C$

(a) Stress σ when temperature drops to 0°C

$T_2 = 0°C \quad \Delta T = 20°C$

NOTE: *Positive* ΔT means a *decrease* in temperature and an *increase* in the stress in the wire.

Negative ΔT means an *increase* in temperature and a *decrease* in the stress.

Stress σ equals the initial stress σ_1 plus the additional stress σ_2 due to the temperature drop.

$\sigma_2 = E\alpha(\Delta T)$

$\sigma = \sigma_1 + \sigma_2 = \sigma_1 + E\alpha(\Delta T)$

$= 42 \text{ MPa} + (200 \text{ GPa})(14 \times 10^{-6}/°C)(20°C)$

$= 42 \text{ MPa} + 56 \text{ MPa} = 98 \text{ MPa} \quad \leftarrow$

(b) Temperature when stress equals zero

$\sigma = \sigma_1 + \sigma_2 = 0 \quad \sigma_1 + E\alpha(\Delta T) = 0$

$$\Delta T = -\frac{\sigma_1}{E\alpha}$$

(Negative means increase in temp.)

$$\Delta T = -\frac{42 \text{ MPa}}{(200 \text{ GPa})(14 \times 10^{-6}/°C} = -15°C$$

$T = 20°C + 15°C = 35°C \quad \leftarrow$

Problem 2.5-19

$n = 1.5 \qquad p = 1.6mm \qquad A_s = 550mm^2 \qquad A_A = 2900mm^2 \qquad d = \sqrt{\frac{4}{\pi} \cdot A_s} = 26.463 \cdot mm$

$L = 500mm \qquad E_s = 200GPa \qquad E_A = 73GPa$

Select force in tube as the redundant. Cut through aluminum tube at right end to expose internal force F_A to create released structure. Apply n turns of turnbuckles to released structure to find relative displacement between ends of cut tube

$\delta_1 = 2 \cdot n \cdot p = 4.8 \cdot mm$ Note that n turns of a turnbuckle moves ends together by factor of two

Now apply pair of internal forces F_T to ends of tube then again find relative displacement. Force F_A shortens both cables and elongates the tube.

$$\delta_2 = F_A \cdot \left(\frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} \right) \qquad \frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} = 4.635 \times 10^{-3} \cdot \frac{mm}{kN}$$

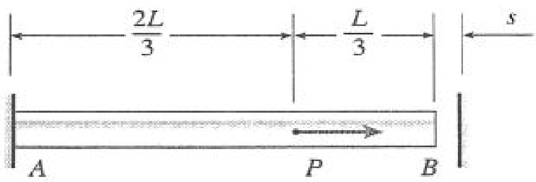
Compatibility equation $\quad \delta_1 + \delta_2 = 0 \quad$ solve for F_A

$$F_A = \frac{-2 \cdot n \cdot p}{\frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s}} = -1.036 \times 10^3 \cdot kN$$

Statics - force in each cable = F_s $\quad 2 \cdot F_s + F_A = 0 \qquad F_s = \frac{-F_A}{2} = 517.849 \cdot kN$

Shortening of aluminum tube $\quad \delta_A = \frac{F_A \cdot L}{E_A \cdot A_A} = -2.4461 \cdot mm$

Problem 2.5-20



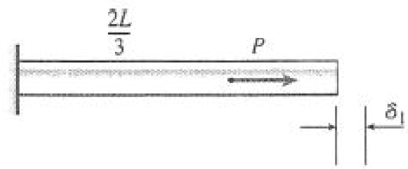
L = length of bar

s = size of gap

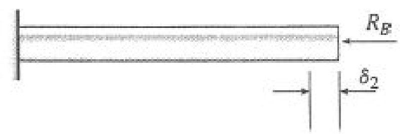
EA = axial rigidity

Reactions must be equal; find s.

FORCE-DISPLACEMENT RELATIONS



$$\delta_1 = \frac{P\left(\frac{2L}{3}\right)}{EA}$$



$$\delta_2 = \frac{R_B L}{EA}$$

COMPATIBILITY EQUATION

$\delta_1 - \delta_2 = s$ or

$$\frac{2PL}{3EA} - \frac{R_B L}{EA} = s \qquad \text{(Eq. 1)}$$

EQUILIBRIUM EQUATION

R_A = reaction at end A (to the left)

R_B = reaction at end B (to the left)

$P = R_A + R_B$

Reactions must be equal.

$\therefore R_A = R_B \quad P = 2R_B \quad R_B = \dfrac{P}{2}$

Substitute for R_B in Eq. (1):

$$\frac{2PL}{3EA} - \frac{PL}{2EA} = s \quad \text{or} \quad s = \frac{PL}{6EA} \longleftarrow$$

NOTE: The gap closes when the load reaches the value $P/4$. When the load reaches the value P, equal to $6EAs/L$, the reactions are equal ($R_A = R_B = P/2$). When the load is between $P/4$ and P, R_A is greater than R_B. If the load exceeds P, R_B is greater than R_A.

Problem 2.5-21

NUMERICAL PROPERTIES (N, m)

$E_1 = 210(10^9) \quad E_2 = 96(10^9)$

$L_1 = 1.4 \quad L_2 = 0.9 \quad s = 1.25(10^{-3})$

$d_1 = 0.152 \quad t_1 = 0.0125$

$d_2 = 0.127 \quad t_2 = 0.0065$

$\alpha_1 = 12(10^{-6}) \quad \alpha_2 = 21(10^{-6})$

$A_1 = \frac{\pi}{4}[d_1^2 - (d_1 - 2t_1)^2] \quad A_1 = 5.478 \times 10^{-3}$

$A_2 = \frac{\pi}{4}[d_2^2 - (d_2 - 2t_2)^2]$

$A_2 = 2.461 \times 10^{-3}$

(a) Find reactions at A and B for applied force P_1. First compute P_1 required to close gap:

$$P_1 = \frac{E_1 A_1}{L_1} s \quad P_1 = 1027 \text{ kN} \quad \leftarrow$$

Stat-indet. analysis with R_B as the redundant:

$$\delta_{B1} = -s \quad \delta_{B2} = R_B\left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}\right)$$

Compatibility: $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{s}{\left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}\right)} \quad R_B = 249 \text{ kN} \quad \leftarrow$$

$R_A = -R_B$

(b) Find reactions at A and B for applied force P_2:

$$P_2 = \frac{E_2 A_2}{\frac{L_2}{2}} s \quad P_2 = 656 \text{ kN} \quad \leftarrow$$

Stat-indet. analysis after removing P_2 is same as in part (a).

(c) Max. shear stress in pipe 1 or 2 when either P_1 OR P_2 is applied:

$$\tau_{maxa} = \frac{\frac{P_1}{A_1}}{2} \quad \tau_{maxa} = 93.8 \text{ MPa} \quad \leftarrow$$

$$\tau_{maxb} = \frac{\frac{P_2}{A_2}}{2} \quad \tau_{maxb} = 133.3 \text{ MPa} \quad \leftarrow$$

(d) Required ΔT and reactions at A and B

$\Delta T_{reqd} = 35°\text{C}$

If pin is inserted but temperature remains at ΔT above ambient temperature, reactions are zero.

(e) If temp. returns to original ambient temperature, find reactions at A and B
Stat-indet analysis with R_B as the redundant:
Compatibility: $\delta_{B1} + \delta_{B2} = 0$
Analysis is the same as in parts (a) and (b) above since gap s is the same, so reactions are the same as above.

Problem 2.5-22

With gap s closed due to ΔT, structure is one-degree statically-indeterminate; select internal force (Q) at juncture of bar and spring as the redundant. Use superposition of two released structures in the solution.

δ_{rel1} = relative displacement between end of bar at C and end of spring due to ΔT

$\delta_{rel1} = \alpha\Delta T(L_1 + L_2)$

δ_{rel1} is greater than gap length s

δ_{rel2} = relative displacement between ends of bar and spring due to pair of forces Q, one on end of bar at C and the other on end of spring

$$\delta_{rel2} = Q\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right) + \frac{Q}{k_3}$$

$$\delta_{rel2} = Q\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}\right)$$

Compatibility: $\delta_{rel1} + \delta_{rel2} = s \quad \delta_{rel2} = s - \delta_{rel1}$

$$\delta_{rel2} = s - \alpha\Delta T(L_1 + L_2)$$

$$Q = \frac{s - \alpha\Delta T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}$$

$$Q = \frac{EA_1A_2k_3}{L_1A_2k_3 + L_2A_1k_3 + EA_1A_2}$$
$$[s - \alpha\Delta T(L_1 + L_2)]$$

(a) Reactions at A and D

Statics: $R_A = -Q \quad R_D = Q$

$$R_A = \frac{-s + \alpha\Delta T(L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \quad \leftarrow$$

$$R_D = -R_A \quad \leftarrow$$

(b) Displacements at B and C

Use superposition of displacements in the two released structures:

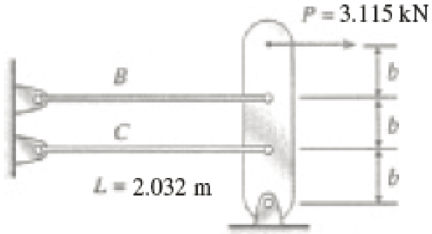
$$\delta_B = \alpha\Delta T(L_1) - R_A\left(\frac{L_1}{EA_1}\right) \quad \leftarrow$$

$$\delta_B = \alpha\Delta T(L_1) - \frac{[-s + \alpha\Delta T(L_1 + L_2)]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}\left(\frac{L_1}{EA_1}\right)$$

$$\delta_C = \alpha\Delta T(L_1 + L_2) - R_A\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right) \quad \leftarrow$$

$$\delta_C = \alpha\Delta T(L_1 + L_2) - \frac{[-s + \alpha\Delta T(L_1 + L_2)]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}\left(\frac{L_1}{EA_1} + \frac{L_2}{EA_2}\right)$$

Problem 2.5-23



$P = 3.115$ kN

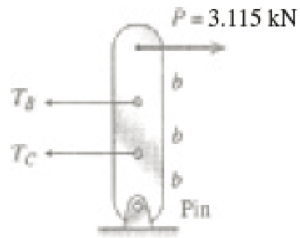
$A = 19.3\ \text{mm}^2$

$E = 210$ GPa

$L_B = 2.031$ m

$L_C = 2.030$ m

EQUILIBRIUM EQUATION



$\Sigma M_{\text{pin}} = 0 \quad \leftarrow$

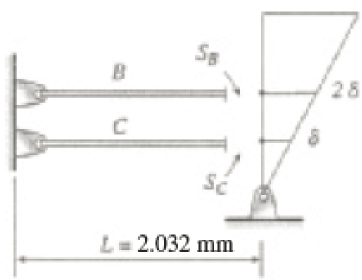
$T_C(b) + T_B(2b) = P(3b)$

$2T_B + T_C = 3P \qquad \text{(Eq. 1)}$

DISPLACEMENT DIAGRAM

$S_B = 2.032\ \text{m} - L_B = 1\ \text{mm}$

$S_C = 2.032\ \text{m} - L_C = 2\ \text{mm}$



Elongation of wires:

$\delta_B = S_B + 2\delta \qquad \text{(Eq. 2)}$

$\delta_C = S_C + \delta \qquad \text{(Eq. 3)}$

FORCE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{T_B L}{EA} \quad \delta_C = \frac{T_C L}{EA} \qquad \text{(Eqs. 4, 5)}$$

SOLUTION OF EQUATIONS

Combine Eqs. (2) and (4):

$$\frac{T_B L}{EA} = S_B + 2\delta \qquad \text{(Eq. 6)}$$

Combine Eqs. (3) and (5):

$$\frac{T_C L}{EA} = S_C + \delta \qquad \text{(Eq. 7)}$$

Eliminate δ between Eqs. (6) and (7):

$$T_B - 2T_C = \frac{EAS_B}{L} - \frac{2EAS_C}{L} \qquad \text{(Eq. 8)}$$

Solve simultaneously Eqs. (1) and (8):

$$T_B = \frac{6P}{5} + \frac{EAS_B}{5L} - \frac{2EAS_C}{5L} \quad \leftarrow$$

$$T_C = \frac{3P}{5} - \frac{2EAS_B}{5L} + \frac{4EAS_C}{5L} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

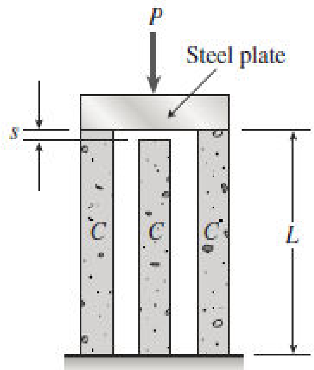
$$\frac{EA}{5L} = 398.917\ \text{kN/m.}$$

$T_B = 3738 + 398.911\ \text{N} - 1595.668 = 2541\ \text{N} \quad \leftarrow$

$T_C = 1869 - 797.834 + 3191.336 = 4263\ \text{N} \quad \leftarrow$

(Both forces are positive, which means tension, as required for wires.)

Problem 2.5-24



s = size of gap = 1.0 mm

L = length of posts = 2.0 m

A = 40,000 mm^2

σ_{allow} = 20 MPa

E = 30 GPa

C = concrete post

DOES THE GAP CLOSE?

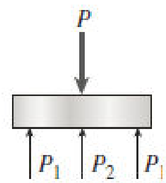
Stress in the two outer posts when the gap is just closed:

$$\sigma = E\varepsilon = E\left(\frac{s}{L}\right) = (30\text{ GPa})\left(\frac{1.0\text{ mm}}{2.0\text{ m}}\right)$$

$$= 15\text{ MPa}$$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

EQUILIBRIUM EQUATION



$$2P_1 + P_2 = P \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

δ_1 = shortening of outer posts

δ_2 = shortening of inner post

$$\delta_1 = \delta_2 + s \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{P_1 L}{EA} \quad \delta_2 = \frac{P_2 L}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} = \frac{P_2 L}{EA} + s \quad \text{or} \quad P_1 - P_2 = \frac{EAs}{L} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P = 3P_1 - \frac{EAs}{L}$$

By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $\sigma_{allow} A$.

$$P_{allow} = 3\sigma_{allow} A - \frac{EAs}{L}$$

$$= 2400\text{ kN} - 600\text{ kN} = 1800\text{ kN}$$

$$= 1.8\text{ MN} \quad \leftarrow$$

Problem 2.5-25

The figure shows a section through the pipe, cap and rod.

NUMERICAL PROPERTIES

$L_{ci} = 1.6 \text{ m}$ $E_s = 210 \text{ GPA}$ $E_b = 96 \text{ GPa}$

$E_c = 83 \text{ GPa}$ $t_c = 25 \text{ mm}$ $p = 1.3 \text{ mm}$ $n = \dfrac{1}{4}$

$d_w = 19 \text{ mm}$ $d_r = 12 \text{ mm}$ $d_o = 150 \text{ mm}$

$d_i = 143 \text{ mm}$

(a) FORCES AND STRESSES IN PIPE AND ROD

One degree stat-indet. - cut rod at cap and use force in rod (Q) as the redundant.

δ_{rel1} = relative displ. between cut ends of rod due to 1/4 turn of nut

$\delta_{rel1} = -np$ < ends of rod move apart, not together, so this is $(-)$

δ_{rel2} = relative displ. between cut ends of rod due to pair of forces Q

$$\delta_{rel2} = Q\left(\frac{L + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}}\right)$$

$A_{rod} = \dfrac{\pi}{4} d_r^2$ $A_{pipe} = \dfrac{\pi}{4}(d_o^2 - d_i^2)$

$A_{rod} = 1.131 \times 10^{-4} \text{ m}^2$ $A_{pipe} = 1.611 \times 10^{-3} \text{ m}^2$

Compatibility equation: $\delta_{rel1} + \delta_{rel2} = 0$

$$Q = \frac{np}{\dfrac{L_{ci} + 2t_c}{E_b A_{rod}} + \dfrac{L_{ci}}{E_c A_{pipe}}}$$

$Q = 1.982 \times 10^3 \text{ N}$

Statics: $F_{rod} = Q$ $F_{pipe} = -Q$

Stresses: $\sigma_p = \dfrac{F_{pipe}}{A_{pipe}}$ $\sigma_p = -1.231 \text{ MPa}$ ←

$\sigma_r = \dfrac{F_{rod}}{A_{rod}}$ $\sigma_r = 17.53 \text{ MPa}$ ←

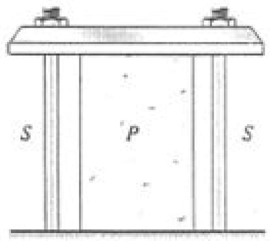
(b) BEARING AND SHEAR STRESSES IN STEEL CAP

$d_w = 0.019 \text{ m}$ $d_r = 0.012 \text{ m}$ $t_c = 0.025 \text{ m}$

$\sigma_b = \dfrac{F_{rod}}{\dfrac{\pi}{4}(d_w^2 - d_r^2)}$ $\sigma_b = 11.63 \text{ MPa}$ ←

$\tau_c = \dfrac{F_{rod}}{\pi d_w t_c}$ $\tau_c = 1.328 \text{ MPa}$ ←

Problem 2.5-26



$L = 200$ mm

$P = 1.0$ mm

$E_s = 200$ GPa

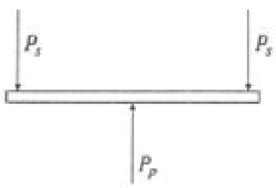
$A_s = 36.0\ \text{mm}^2$ (for one bolt)

$E_p = 7.5$ GPa

$A_p = 960\ \text{mm}^2$

$n = 1$ (See Eq. 2-22)

EQUILIBRIUM EQUATION

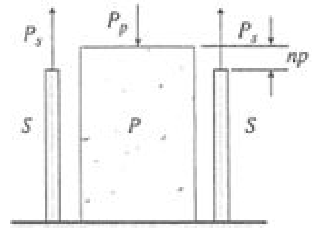


P_s = tensile force in one steel bolt

P_p = compressive force in plastic cylinder

$P_p = 2P_s$ (Eq. 1)

COMPATIBILITY EQUATION



δ_s = elongation of steel bolt

δ_p = shortening of plastic cylinder

$\delta_s + \delta_p = np$ (Eq. 2)

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2npE_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2np\, E_s\, A_s\, E_p}{L(E_p A_p + 2E_s A_s)}$$

SUBSTITUTE NUMERICAL VALUES:

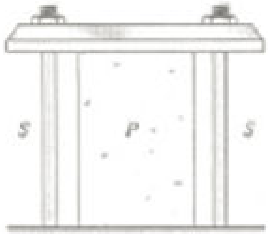
$N = E_s\, A_s\, E_p = 54.0 \times 10^{15}\ \text{N}^2/\text{m}^2$

$D = E_p A_p + 2E_s A_s = 21.6 \times 10^6$ N

$$\sigma_p = \frac{2np}{L}\left(\frac{N}{D}\right) = \frac{2(1)(1.0\text{ mm})}{200\text{ mm}}\left(\frac{N}{D}\right)$$

$= 25.0$ MPa ←

Problem 2.5-27



$L = 300$ mm
$p = 1.5$ mm
$E_s = 210$ GPa

$A_s = 50 \text{ mm}^2$ (for one bolt)
$E_p = 3.5$ GPa
$A_p = 1000 \text{ mm}^2$
$n = 1$ (see Eq. 2-22)

EQUILIBRIUM EQUATION

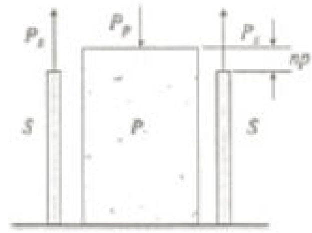
P_s = tensile force in one steel bolt
P_p = compressive force in plastic cylinder

$$P_p = 2P_s \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

δ_s = elongation of steel bolt
δ_p = shortening of plastic cylinder

$$\delta_s + \delta_p = np \qquad \text{(Eq. 2)}$$



FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p} \qquad \text{(Eq. 3, Eq. 4)}$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2\,np\,E_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2\,np\,E_s A_s E_p}{L(E_p A_p + 2E_s A_s)} \quad \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$N = E_s A_s E_p$$
$$D = E_p A_p + 2E_s A_s$$

$$\sigma_p = \frac{2np}{L}\left(\frac{N}{D}\right)$$
$$= 15.0 \text{ MPa} \quad \longleftarrow$$

Problem 2.5-28

The figure shows a section through the sleeve, cap, and bolt.

NUMERICAL PROPERTIES

$n = \frac{1}{2}$ $\quad p = 1.0$ mm $\quad \Delta T = 30°C$

$E_c = 120$ GPa $\quad \alpha_c = 17 \times (10^{-6})/°C$

$E_s = 200$ GPa $\quad \alpha_s = 12 \times (10^{-6})/°C$

$\tau_{aj} = 18.5$ MPa $\quad s = 26$ mm $\quad d_b = 5$ mm

$L_1 = 40$ mm $\quad t_1 = 4$ mm $\quad L_2 = 50$ mm $\quad t_2 = 3$ mm

$d_1 = 25$ mm $\quad d_1 - 2t_1 = 17$ mm $\quad d_2 = 17$ mm

$$A_b = \frac{\pi}{4} d_b^2 \qquad A_1 = \frac{\pi}{4}[d_1^2 - (d_1 - 2t_1)^2]$$

$A_b = 19.635\ \text{mm}^2 \qquad A_1 = 263.894\ \text{mm}^2$

$$A_2 = \frac{\pi}{4}[d_2^2 - (d_2 - 2t_2)^2] \qquad A_2 = 131.947\ \text{mm}^2$$

(a) FORCES IN SLEEVE AND BOLT

One-degree statically indeterminate—cut bolt and use force in bolt (P_B) as redundant (see sketches):

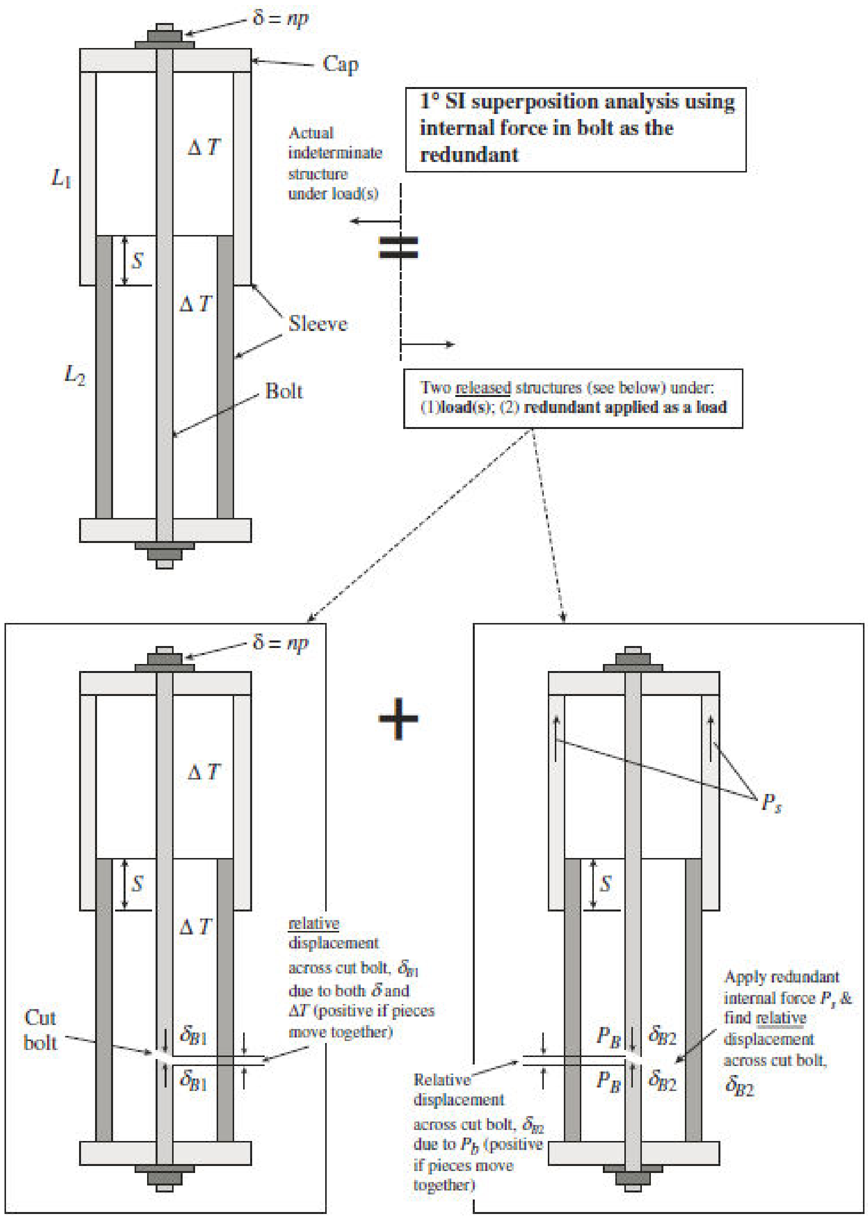
$$\delta_{B1} = -np + \alpha_s \Delta T (L_1 + L_2 - s)$$

$$\delta_{B2} = P_B\left[\frac{L_1 + L_2 - s}{E_s A_b} + \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c(A_1 + A_2)}\right]$$

Compatibility: $\quad \delta_{B1} + \delta_{B2} = 0$

$$P_B = \frac{-[-np + \alpha_s \Delta T(L_1 + L_2 - s)]}{\left[\frac{L_1 + L_2 - s}{E_s A_b} + \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c(A_1 + A_2)}\right]} \qquad P_B = 25.4\ \text{kN} \quad \longleftarrow \qquad P_s = -P_B \quad \longleftarrow$$

Sketches illustrating superposition procedure for statically-indeterminate analysis



(b) Required length of solder joint≈

$$\tau = \frac{P}{A_s} \qquad A_s = \pi d_2 s$$

$$s_{\text{reqd}} = \frac{P_B}{\pi d_2 \tau_{aj}} \qquad s_{\text{reqd}} = 25.7 \text{ mm}$$

(c) Final elongation

δ_f = net of elongation of bolt (δ_b) and shortening of sleeve (δ_s)

$$\delta_b = P_B\left(\frac{L_1 + L_2 - s}{E_s A_b}\right) \qquad \delta_b = 0.413 \text{ mm}$$

$$\delta_s = P_s\left[\frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c(A_1 + A_2)}\right]$$

$$\delta_s = -0.064 \text{ mm}$$

$$\delta_f = \delta_b + \delta_s \qquad \delta_f = 0.35 \text{ mm} \quad \leftarrow$$

Problem 2.5-29

PROPERTIES & DIMENSIONS (N, m)

$d_o = 0.150 \qquad t = 0.003 \qquad E_t = 0.7 \times 10^9$

$A_t = \frac{\pi}{4}[d_o^{\,2} - (d_o - 2t)^2] \qquad A_t = 1.385 \times 10^{-3}$

MUST REDEFINE L AND L_1 FROM ABOVE

$L_1 = 0.308 > L = 0.305 \qquad k = 262.5\,(10^3)$

Spring 3 mm longer than tube $\qquad \delta = L_1 - L \qquad \delta = 3 \times 10^{-3}$

$\alpha_k = 12(10^{-6}) < \alpha_t = 140(10^{-6})$

$\Delta T = 0$ $\qquad$ < note that Q result below is for zero temp.

(a) FORCE IN SPRING F_k = REDUNDANT Q

Flexibilities: $f = \frac{1}{k} \quad f_t = \frac{L}{E_t A_t} \quad f_t = 3.145 \times 10^{-7}$

$$Q = \frac{-\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t}$$

$Q = -727\text{ N}$ $\qquad$ < COMPRESSIVE FORCE in spring (F_k)

(b) F_t = FORCE IN TUBE = $-Q$
^ also tensile force in tube

NOTE: if tube is rigid, $F_k = -k\delta = -787.5$

(c) FINAL LENGTH OF TUBE AND SPRING

$L_f = L + \delta_{c1} + \delta_{c2}$ $\qquad$ < i.e., add displacements in cap for the two released structures to initial tube length L

$L_f = L - Qf_t + \alpha_t(\Delta T)L \qquad L_f = 305.2\text{ mm}$

$k(L_f - L) = -727.446 \qquad \frac{Q}{k} = -2.771 \times 10^{-3}$

$Qf = -2.771 \times 10^{-3}$

STRESS IN POLYETHYLENE TUBE

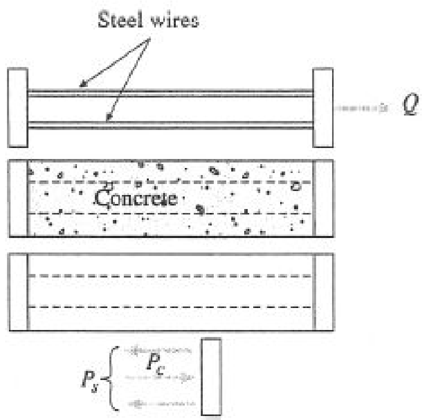
$\sigma_t = \frac{Q}{A_t} \qquad \sigma_t = -5.251 \times 10^5\text{ Pa}$

(d) SET $Q = 0$ TO FIND ΔT REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{reqd} = \frac{\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$\Delta T_{reqd} = 76.9\ ^\circ\text{C}$ $\qquad$ < since $\alpha_t > \alpha_k$, a temp. increase is req'd to expand tube so that spring force goes to zero

Problem 2.5-30



L = length

σ_0 = initial stress in wires

$= \dfrac{Q}{A_s} = 620 \text{ MPa}$

A_s = total area of steel wires

A_c = area of concrete

$= 50\,A_s$

$E_s = 12\,E_c$

P_s = final tensile force in steel wires

P_c = final compressive force in concrete

EQUILIBRIUM EQUATION

$P_s = P_c$ (Eq. 1)

COMPATIBILITY EQUATION AND FORCE-DISPLACEMENT RELATIONS

δ_1 = initial elongation of steel wires

$= \dfrac{QL}{E_sA_s} = \dfrac{\sigma_0 L}{E_s}$

δ_2 = final elongation of steel wires

$= \dfrac{P_sL}{E_sA_s}$

δ_3 = shortening of concrete

$= \dfrac{P_cL}{E_cA_c}$

$\delta_1 - \delta_2 = \delta_3$ or

$$\frac{\sigma_0 L}{E_s} - \frac{P_sL}{E_sA_s} = \frac{P_cL}{E_cA_c} \qquad \text{(Eq. 2, Eq. 3)}$$

Solve simultaneously Eqs. (1) and (3):

$$P_s = P_c = \frac{\sigma_0 A_s}{1 + \dfrac{E_sA_s}{E_cA_c}}$$

STRESSES

$$\sigma_s = \frac{P_s}{A_s} = \frac{\sigma_0}{1 + \dfrac{E_sA_s}{E_cA_c}} \quad \leftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{\sigma_0}{\dfrac{A_c}{A_s} + \dfrac{E_s}{E_c}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_0 = 620 \text{ MPa} \qquad \frac{E_s}{E_c} = 12 \quad \frac{A_s}{A_c} = \frac{1}{50}$$

$$\sigma_s = \frac{620 \text{ MPa}}{1 + \dfrac{12}{50}} = 500 \text{ MPa (Tension)} \quad \leftarrow$$

$$\sigma_c = \frac{620 \text{ MPa}}{50 + 12} = 10 \text{ MPa (Compression)} \quad \leftarrow$$

Problem 2.5-31

The figure shows a section through the tube, cap and spring.

PROPERTIES AND DIMENSIONS (N, m)

$d_0 = 0.150$ mm $\quad t = 0.003$ mm $\quad E_t = 0.7(10^9)$

$L = 0.305$ mm $> L_1 = 0.302$ mm $\quad k = 262.5(10^3)\,\dfrac{\text{N}}{\text{m}}$

$$\alpha_k = 12(10^{-6}) < \alpha_t = 140(10^{-6})$$

$$A_t = \frac{\pi}{4}[\,d_0^2 - (\,d_0 - 2t)^2]$$

$A_t = 1.385 \times 10^{-3}$ spring is 3 mm shorter than tube

PRETENSION AND TEMPERATURE

$\delta = L - L_1 \quad \delta = 3 \times 10^{-3} \quad \Delta T = 0$

< note that Q result below is for ZERO TEMP (until part (d))

FLEXIBILITIES $\quad f = \dfrac{1}{k} \quad f_t = \dfrac{L}{E_t A_t} \quad f_t = 3.145 \times 10^{-7}$

(a) FORCE IN SPRING (F_k) = REDUNDANT (Q)

$$Q = \frac{\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t} = F_k$$

$Q = 727$ N

$F_k = 727$ N $\quad \leftarrow$ also the compressive force in the tube

(b) FORCE IN TUBE $F_t = -Q \quad \leftarrow$

(c) FINAL LENGTH OF TUBE AND SPRING $\quad L_f = L + \delta_{c1} + \delta_{c2}$

$L_f = L - Qf_t + \alpha_t(\Delta T)L \quad L_f = 304.8$ mm $\quad \leftarrow$

$k(L_f - L_t) = 727.446$

$\dfrac{Q}{k} = 2.771 \times 10^{-3}$

$Qf = 2.771 \times 10^{-3}$ same as $Q = F_k$

STRESS IN POLYETHYLENE TUBE

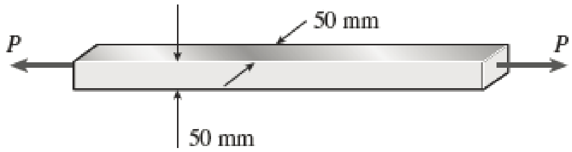
$\sigma_t = \dfrac{Q}{A_t} \quad \sigma_t = 5.251 \times 10^5$

(d) SET $Q = 0$ TO FIND ΔT REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{reqd} = \frac{-\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$\Delta T_{reqd} = -76.8°\text{C}$ $\quad$ since $\alpha_t > \alpha_k$, a temp. drop is req'd to shrink tube so that spring force goes to zero

Problem 2.6-1



NUMERICAL DATA

$A = 2.5 \times 10^{-3}\text{m}^2$

$\sigma_a = 125\ \text{MPa}$

$\tau_a = 76\ \text{MPa}$

MAXIMUM LOAD–tension

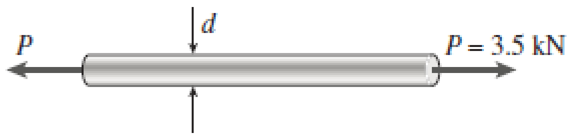
$P_{max\sigma} = \sigma_a A \qquad P_{max\sigma} = 312\ \text{kN}$

MAXIMUM LOAD–shear

$P_{max\tau} = 2\tau_a A \qquad P_{max\tau} = 380\ \text{kN}$

Because τ_{allow} is more than one-half of σ_{allow}, the normal stress governs.

Problem 2.6-2



NUMERICAL DATA $P = 3.5\ \text{kN}$ $\sigma_a = 118\ \text{MPa}$

$\tau_a = 48\ \text{MPa}$

Find P_{max} then rod diameter.

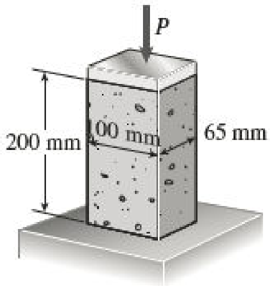
since τ_a is less than 1/2 of σ_a, shear governs.

$$P_{max} = 2\tau_a\left(\frac{\pi}{4}d_{min}^2\right)$$

$$d_{min} = \sqrt{\frac{2}{\pi\tau_a}P}$$

$$d_{min} = 6.81\ \text{mm} \quad \leftarrow$$

Problem 2.6-3



$A = 65\text{ mm} \times 100\text{ mm} = 6500\text{ mm}^2$
Maximum normal stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

$\sigma_{ult} = 26\text{ MPa} \qquad \tau_{ult} = 8\text{ MPa}$

Because τ_{ult} is less than one-half of σ_{ult}, the shear stress governs.

$$\tau_{max} = \frac{P}{2A} \quad \text{or} \quad P_{max} = 2A\tau_{ult}$$

$$P_{max} = 2(6500\text{ mm}^2)(8\text{ MPa}) = 104\text{ kN} \leftarrow$$

Problem 2.6-4

NUMERICAL DATA

$d = 2.42$ mm $\quad T = 98$ N
$\alpha = 19.5\,(10^{-6})/°C \quad E = 110$ GPa

(a) ΔT_{max} (DROP IN TEMPERATURE)

$$\sigma = \frac{T}{A} - (E\alpha\,\Delta T) \qquad \tau_{max} = \frac{\sigma}{2}$$

$$\tau_a = \frac{T}{2A} - \frac{E\alpha\,\Delta T}{2}$$

$$\tau_a = 60 \text{ MPa} \qquad A = \frac{\pi}{4}d^2$$

$$\Delta T_{max} = \frac{\frac{T}{A} - 2\,\tau_a}{E\,\alpha}$$

$$\Delta T_{max} = -46°\text{C (drop)}$$

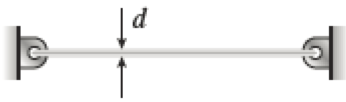
(b) ΔT AT WHICH WIRE GOES SLACK

Increase ΔT until $\sigma = 0$:

$$\Delta T = \frac{T}{E\,\alpha A}$$

$$\Delta T = 9.93°\text{C (increase)}$$

Problem 2.6-5



NUMERICAL DATA (N, m)

$d = 0.0016$ m $T = 200$ N $\alpha = 21.2(10^{-6})$

$E = 110(10^9)$ Pa $\Delta T = -30°C$

$A = \frac{\pi}{4}d^2$

(a) τ_{max} (DUE TO DROP IN TEMPERATURE)

$$\tau_{max} = \frac{\sigma_x}{2} \qquad \tau_{max} = \frac{\frac{T}{A} - (E\alpha\Delta T)}{2}$$

$\tau_{max} = 84.7$ MPa ←

(b) ΔT_{max} FOR ALLOW. SHEAR STRESS

$\tau_a = 70(10^6)$ Pa

$$\Delta T_{max} = \frac{\frac{T}{A} - 2\tau_a}{E\alpha}$$

$\Delta T_{max} = -17.38°C$ ←

(c) ΔT AT WHICH WIRE GOES SLACK

Increase ΔT until $\sigma = 0$

$$\Delta T = \frac{T}{E\alpha A}$$

$\Delta T = 42.7°C$ (increase) ←

Problem 2.6-6

(a) $d = 12\text{ mm}$ $\quad P = 9.5\text{ kN}$ $\quad A = \frac{\pi}{4}d^2 = 1.131 \times 10^{-4}\text{ m}^2$

$$\boxed{\sigma_x = \frac{P}{A} = 84\text{ MPa}}$$

(b) $\boxed{\tau_{max} = \frac{\sigma_x}{2} = 42\text{ MPa}}$ On plane stress element rotated 45°

(c) Rotated stress element (45°) has normal tensile stress $\sigma_x/2$ on all faces, $-T_{max}$ (CW) on $+x$-face, and $+T_{max}$ (CCW) on $+y$-face

$\tau_{x1y1} = \tau_{max}$ $\quad \sigma_{x1} = \frac{\sigma_x}{2}$ $\quad \sigma_{y1} = \sigma_{x1}$

On rotated x-face: $\boxed{\sigma_{x1} = 42\text{ MPa}}$ $\quad \boxed{\tau_{x1y1} = 42\text{ MPa}}$

On rotated y-face: $\boxed{\sigma_{y1} = 42\text{ MPa}}$

(d) $\theta = 22.5°$ $\quad$ < CCW rotation of element

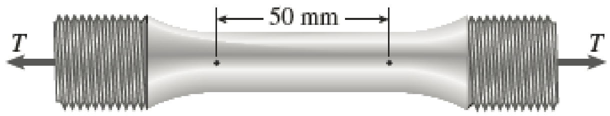
$\sigma_\theta = \sigma_x \cos(\theta)^2 = 71.7\text{ MPa}$ $\quad$ < on rotated x face $\quad \sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 = 12.3\text{ MPa}$ $\quad$ < on rotated y face

Eq. 2-29b $\quad \tau_\theta = \frac{-\sigma_x}{2}\sin(2\theta) = -29.7\text{ MPa}$ $\quad$ < CW on rotated x-face

On rotated x-face: $\boxed{\sigma_{x1} = 71.7\text{ MPa}}$ $\quad \boxed{\tau_{x1y1} = -29.7\text{ MPa}}$

On rotated y-face: $\boxed{\sigma_{y1} = 12.3\text{ MPa}}$

Problem 2.6-7



Elongation: $\delta = 0.004$ mm

Strain: $\varepsilon = \frac{\delta}{L} = \frac{0.004\,\text{mm}}{50\,\text{mm}} = 0.00008$

Hooke's law: $\sigma_x = E\varepsilon = (210\text{ GPa})(0.00008)$

$= 16.8\text{ MPa}$

(a) Maximum normal stress

σ_x is the maximum normal stress.

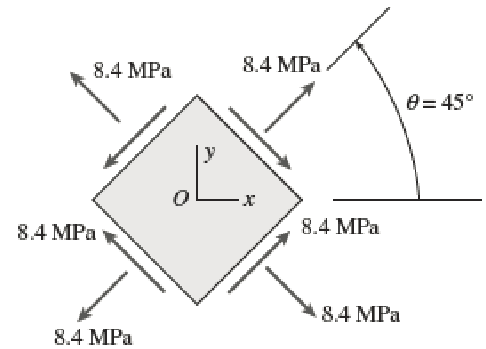
$\sigma_{max} = 16.8\text{ MPa}$ ←

(b) Maximum shear stress

The maximum shear stress is on a 45° plane and equals $\sigma_x/2$.

$\tau_{max} = \frac{\sigma_x}{2} = 8.4\text{ MPa}$ ←

(c) Stress element at $\theta = 45°$



Problem 2.6-8

(a) $\alpha = 17.5\left(10^{-6}\right)$ $\Delta T = 50$ $E = 120\text{ GPa}$

$\sigma_x = -E\alpha\,\Delta T = -105\text{ MPa}$ $\tau_{max} = \dfrac{\sigma_x}{2} = -52.5\text{ MPa}$ < at $\theta = 45°$

(compression)

Element A: $\sigma_x = 105\text{ MPa}$ (compression);
Element B: $\tau_{max} = 52.5\text{ MPa}$

(b) $\tau_\theta = 48\text{ MPa}$

Eq. 2-29b $\tau_\theta = \dfrac{-\sigma_x}{2}\sin(2\theta)$

so $\theta = \dfrac{1}{2}\operatorname{asin}\left(\dfrac{2\tau_\theta}{-\sigma_x}\right) = 33.1°$ < CCW rotation of element $\boxed{\theta = 33.1°}$

$\sigma_\theta = \sigma_x\cos(\theta)^2 = -73.8\text{ MPa}$ < on rotated x face

$\sigma_y = \sigma_x\cos\left(\theta + \dfrac{\pi}{2}\right)^2 = -31.2\text{ MPa}$ < on rotated y face

Problem 2.6-9

$P = 45kN \qquad L = 1m \qquad A = 5200mm^2 \qquad \theta = 35deg$

Normal compressive stress $\qquad \sigma_x = \frac{-P}{A} = -8.654 \cdot MPa$

Plane stress transformations

$\sigma_\theta(\theta) = \sigma_x \cdot \cos(\theta)^2 \qquad \sigma_\theta(\theta) = -5.807 \cdot MPa \qquad \sigma_\theta\left(\theta + \frac{\pi}{2}\right) = -2.847 \cdot MPa$

$\tau_\theta(\theta) = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta) \qquad \tau_\theta(\theta) = 4.066 \cdot MPa \qquad \tau_\theta\left(\theta + \frac{\pi}{2}\right) = -4.066 \cdot MPa$

Problem 2.6-10

$L = 1m \quad A = 1200mm^2 \quad \Delta T = 25 \quad \alpha = 12 \cdot (10^{-6}) \quad \theta = 45deg \quad E = 200GPa$

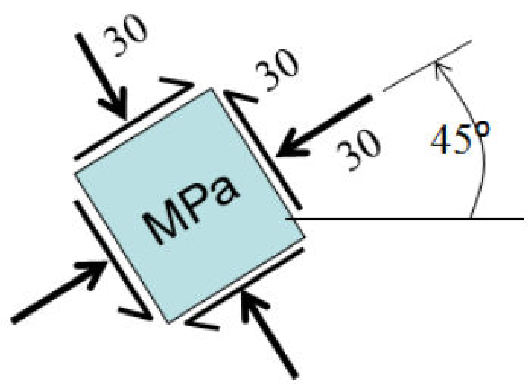
Compressive thermal stress $\sigma_T = E \cdot \alpha \cdot \Delta T = 60 \cdot MPa$

Support reactions $R_A = \sigma_T \cdot A = 72 \cdot kN \qquad R_B = -R_A$

Plane stress transformations $\sigma_x = \dfrac{R_B}{A} = -60 \cdot MPa$

$\sigma_\theta = \sigma_x \cdot \cos(\theta)^2 = -30 \cdot MPa \qquad \sigma_x \cdot \cos\left(\theta + \dfrac{\pi}{2}\right)^2 = -30 \cdot MPa \qquad \tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta) = 30 \cdot MPa$

Rotated stress element



Problem 2.6-11

NUMERICAL DATA

$L = 3\ \text{m}\quad b = 0.71\,L\quad P = 220\ \text{kN}\quad \sigma_a = 96\ \text{MPa}\quad \tau_a = 52\ \text{MPa}\quad A = 37.4\ \text{cm}^2\quad <\text{UPN }220\quad b = 2.13\ \text{m}$

(a) FIND REACTIONS, THEN MEMBER FORCES (SEE SOLU. APPROACH IN EG. 1-1)

$$\theta_A = 60°\quad \theta_B = \arcsin\left(\frac{b}{L}\sin(\theta_A)\right) = 37.943°\quad \theta_C = 180° - (\theta_A + \theta_B) = 82.057°$$

$$c = L\left(\frac{\sin(\theta_C)}{\sin(\theta_A)}\right) = 3.431\ \text{m}\quad B_y = \frac{Pb\cos(\theta_A) + 2Pb\sin(\theta_A)}{c} = 304.861\ \text{kN}\quad A_y = P - B_y = -84.861\ \text{kN}$$

$$A_x = -2P = -440\ \text{kN}\quad F_{AC} = \frac{-A_y}{\sin(\theta_A)} = 97.99\ \text{kN}\quad F_{AB} = -A_x - F_{AC}\cos(\theta_A) = 391.005\ \text{kN}$$

$$F_{BC} = \frac{-B_y}{\sin(\theta_B)} = 495.808\ \text{kN}$$

Normal stresses in each member: $\sigma_{AC} = \dfrac{F_{AC}}{A} = 26.2\ \text{MPa}\quad \sigma_{AB} = \dfrac{F_{AB}}{A} = 104.547\ \text{MPa}$

$$\sigma_{BC} = \frac{F_{BC}}{A} = -132.569\ \text{MPa}$$

From Eq. 2-31:

$$\boxed{\tau_{maxAC} = \frac{\sigma_{AC}}{2} = 13.1\ \text{MPa}}\quad \boxed{\tau_{maxAB} = \frac{\sigma_{AB}}{2} = 52.3\ \text{MPa}}\quad \boxed{\tau_{maxBC} = \frac{\sigma_{BC}}{2} = -66.3\ \text{MPa}}$$

(b) $\sigma_a < 2\tau_a$ so normal stress will control; lowest value governs here.

MEMBER AC: $P_{max\sigma} = \dfrac{P}{F_{AC}}(\sigma_a A) = 806.094\ \text{kN}\qquad P_{max\tau} = \dfrac{P}{F_{AC}}(2\tau_a A) = 873.268\ \text{kN}$

MEMBER AB: $P_{max\sigma} = \dfrac{P}{F_{AB}}(\sigma_a A) = 202.015\ \text{kN}\qquad P_{max\tau} = \dfrac{P}{F_{AB}}(2\tau_a A) = 218.849\ \text{kN}$

MEMBER BC: $\boxed{P_{max\sigma} = \left|\dfrac{P}{F_{BC}}\right|(\sigma_a A) = 159.3\ \text{kN}}\qquad P_{max\tau} = \left|\dfrac{P}{F_{BC}}\right|(2\tau_a A) = 172.589\ \text{kN}$

Problem 2.6-12

Numerical data

$d = 32\text{ mm}$ $\quad A = \frac{\pi}{4}d^2$

$P = 190\text{ N}$ $\quad A = 804.25\text{ mm}^2$

$a = 100\text{ mm}$

$b = 300\text{ mm}$

(a) Statics—Find compressive force F and stresses in plastic bar

$F = \frac{P(a+b)}{a}$ $\quad F = 760\text{ N}$

$\sigma_x = \frac{F}{A}$ $\quad \sigma_x = 0.945\text{ MPa}$ or $\sigma_x = 945\text{ kPa}$

From (1), (2), and (3) below:

$\sigma_{max} = \sigma_x$ $\quad \sigma_{max} = -945\text{ kPa}$

$\tau_{max} = 472\text{ kPa}$ $\quad \frac{\sigma_x}{2} = -472\text{ kPa}$

(1) $\theta = 0°$ $\quad \sigma_x = -945\text{ kPa}$ ←

(2) $\theta = 22.50°$

On $+x$-face:

$\sigma_\theta = \sigma_x\cos(\theta)^2$

$\sigma_\theta = -807\text{ kPa}$ ←

$\tau_\theta = -\sigma_x\sin(\theta)\cos(\theta)$

$\tau_\theta = 334\text{ kPa}$ ←

On +y-face: $\quad \theta = \theta + \frac{\pi}{2}$

$\sigma_\theta = \sigma_x\cos(\theta)^2$

$\sigma_\theta = -138.39\text{ kPa}$

$\tau_\theta = -\sigma_x\sin(\theta)\cos(\theta)$

$\tau_\theta = -334.1\text{ kPa}$

(3) $\theta = 45°$

On $+x$-face:

$\sigma_\theta = \sigma_x\cos(\theta)^2$

$\sigma_\theta = -472\text{ kPa}$ ←

$\tau_\theta = -\sigma_x\sin(\theta)\cos(\theta)$

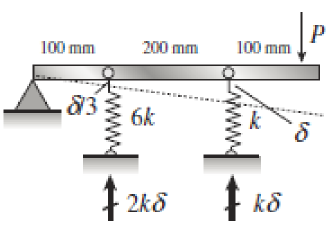
$\tau_\theta = 472\text{ kPa}$ ←

On +y-face: $\quad \theta = \theta + \frac{\pi}{2}$

$\sigma_\theta = \sigma_x\cos(\theta)^2$ $\quad \sigma_\theta = -472.49\text{ kPa}$

$\tau_\theta = -\sigma_x\sin(\theta)\cos(\theta)$ $\quad \tau_\theta = -472.49\text{ kPa}$

(b) Add spring—Find maximum normal and shear stresses in plastic bar



$\sum M_{pin} = 0$

$P(400) = [2k\delta(100) + k\delta(300)]$

$\delta = \frac{4}{5}\frac{P}{k}$

Force in plastic bar: $\quad F = (2k)\left(\frac{4}{5}\frac{P}{k}\right)$

$F = \frac{8}{5}P$ $\quad F = 304\text{ N}$

Normal and shear stresses in plastic bar:

$\sigma_x = \frac{F}{A}$ $\quad \sigma_x = 0.38$

$\sigma_{max} = -378\text{ kPa}$ ←

$\tau_{max} = \frac{\sigma_x}{2}$ $\quad \tau_{max} = -189\text{ kPa}$ ←

Problem 2.6-13

NUMERICAL DATA (N, m)

$b = 0.038 \quad h = 0.075 \quad A = bh \quad A = 2.85 \times 10^{-3}\,\text{m}^2$

$\Delta T = 70 - 20 \quad \Delta T = 50°\text{C}$

$\sigma_{pq} = -8.7(10^6)$

$\alpha = 95\,(10^{-6})/°\text{C}$

$E = 2.4\,(10^9)\,\text{Pa}$

(a) SHEAR STRESS ON PLANE PQ

STAT-INDET. ANALYSIS GIVES FOR REACTION AT RIGHT SUPPORT:

$R = -EA\alpha\Delta T \qquad R = -32.49\text{ kN}$

$\sigma_x = \dfrac{R}{A} \qquad \sigma_x = -11.4\text{ MPa}$

Using $\sigma_\theta = \sigma_x\cos(\theta)^2$: $\quad \cos(\theta)^2 = \dfrac{\sigma_{pq}}{\sigma_x}$

$\sigma_x = -11.4\text{ MPa} \quad \sigma_{pq} = -8.7\text{ MPa} \qquad \sqrt{\dfrac{\sigma_{pq}}{\sigma_x}} = 0.87$

$\theta = \arccos\left(\sqrt{\dfrac{\sigma_{pq}}{\sigma_x}}\right) \qquad \theta = 0.87°$

Now with θ, we can find shear stress on plane pq:

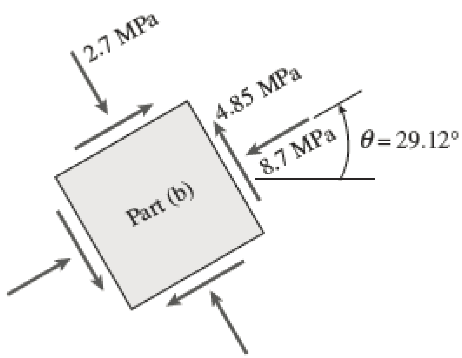
$\tau_{pq} = -\sigma_x\sin(\theta)\cos(\theta) \qquad \tau_{pq} = 4.85\text{ MPa} \qquad \leftarrow$

$\sigma_{pq} = \sigma_x\cos(\theta)^2 \qquad \sigma_{pq} = -8.7\text{ MPa}$

Stresses at $\theta + \pi/2$ (y-face):

$\sigma_y = \sigma_x cos\left(\theta + \dfrac{\pi}{2}\right)^2 \qquad \sigma_y = -2.7\text{ MPa}$

(b) STRESS ELEMENT FOR PLANE PQ



(c) MAX. LOAD AT QUARTER POINT $\quad \sigma_a = 23(10^6)\text{ Pa}$

$\tau_a = 11.3(10^6) \quad 2\tau_a = 22.6\text{ MPa}$ $\quad <$ less than σ_a, so shear controls

Stat-indet. analysis for P at $L/4$ gives for reactions:

$R_{R2} = \dfrac{-P}{4} \qquad R_{L2} = \dfrac{-3}{4}P$

(tension for 0 to $L/4$ and compression for rest of bar)

From part (a) (for temperature increase ΔT):

$R_{R1} = -EA\alpha\Delta T \qquad R_{L1} = -EA\alpha\Delta T$

Stresses in bar (0 to $L/4$):

$\sigma_x = -E\alpha\Delta T + \dfrac{3P}{4A} \qquad \tau_{max} = \dfrac{\sigma_x}{2}$

Set $\tau_{max} = \tau_a$ and solve for P_{max1}:

$\tau_a = \dfrac{-E\alpha\Delta T}{2} + \dfrac{3P}{8A} \qquad \tau_a = 11.3\text{ MPa}$

$P_{max1} = \dfrac{4A}{3}(2\tau_a + E\alpha\Delta T)$

$P_{max1} = 129{,}200\text{ N}$

$\tau_{max} = \dfrac{-E\alpha\Delta T}{2} + \dfrac{3P_{max1}}{8A}$

$\tau_{max} = 11.3\text{ MPa} \qquad <$ check

$\sigma_x = -E\alpha\Delta T + \dfrac{3P_{max1}}{4A}$

$\sigma_x = 22.6\text{ MPa} \qquad <$ less than σ_a

Stresses in bar ($L/4$ to L):

$\sigma_x = -E\alpha\Delta T - \dfrac{P}{4A} \qquad \tau_{max} = \dfrac{\sigma_x}{2}$

Set $\tau_{max} = \tau_a$ and solve for P_{max2}:

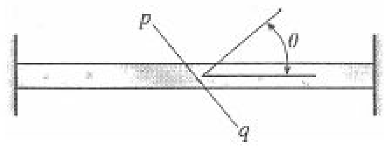
$P_{max2} = -4A(-2\tau_a + E\alpha\Delta T)$

$P_{max2} = 127.7\text{ kN} \qquad \leftarrow$ shear in segment ($L/4$ to L) controls

$\tau_{max} = \dfrac{-E\alpha\Delta T}{2} - \dfrac{P_{max2}}{8A} \qquad \tau_{max} = -11.3\text{ MPa}$

$\sigma_x = -E\alpha\Delta T - \dfrac{P_{max2}}{4A} \qquad \sigma_x = -22.6\text{ MPa}$

Problem 2.6-14



NUMERICAL DATA

$$\theta = 55\left(\frac{\pi}{180}\right) \text{ rad}$$

$b = 18$ mm $\quad h = 40$ mm

$A = bh \quad A = 720\ \text{mm}^2$

$\sigma_{pqa} = 60$ MPa $\quad \tau_{pqa} = 30$ MPa

$\alpha = 17 \times (10^{-6})/°C \quad E = 120$ GPa

$\Delta T = 20°C \quad P = 15$ kN

(a) FIND ΔT_{max} BASED ON ALLOWABLE NORMAL AND SHEAR STRESS VALUES ON PLANE pq

$$\sigma_x = -E\alpha\Delta T_{max} \qquad \Delta T_{max} = \frac{-\sigma_x}{E\alpha}$$

$$\sigma_{pq} = \sigma_x\cos(\theta)^2 \qquad \tau_{pq} = -\sigma_x\sin(\theta)\cos(\theta)$$

Set each equal to corresponding allowable and solve for σ_x:

$$\sigma_{x1} = \frac{\sigma_{pqa}}{\cos(\theta)^2} \qquad \sigma_{x1} = 182.38 \text{ MPa}$$

$$\sigma_{x2} = \frac{\tau_{pqa}}{-\sin(\theta)\cos(\theta)} \qquad \sigma_{x2} = -63.85 \text{ MPa}$$

Lesser value controls, so allowable shear stress governs.

$$\Delta T_{max} = \frac{-\sigma_{x2}}{E\alpha} \qquad \Delta T_{max} = 31.3°C \quad \leftarrow$$

(b) STRESSES ON PLANE PQ FOR MAXIMUM TEMPERATURE

$$\sigma_x = -E\alpha\Delta T_{max} \qquad \sigma_x = -63.85 \text{ MPa}$$

$$\sigma_{pq} = \sigma_x\cos(\theta)^2 \qquad \sigma_{pq} = -21.0 \text{ MPa} \quad \leftarrow$$

$$\tau_{pq} = -\sigma_x\sin(\theta)\cos(\theta) \qquad \tau_{pq} = 30 \text{ MPa} \quad \leftarrow$$

(c) ADD LOAD P IN $+x$-DIRECTION TO TEMPERATURE CHANGE AND FIND LOCATION OF LOAD

$\Delta T = 28°C$

$P = 15$ kN from one-degree statically indeterminate analysis, reactions R_A and R_B due to load P:

$$R_A = -(1-\beta)P \qquad R_B = \beta P$$

Now add normal stresses due to P to thermal stresses due to ΔT (tension in segment 0 to βL, compression in segment βL to L).

Stresses in bar (0 to βL):

$$\sigma_x = -E\alpha\Delta T + \frac{R_A}{A} \qquad \tau_{max} = \frac{\sigma_x}{2}$$

Shear controls so set $\tau_{max} = \tau_a$ and solve for β:

$$2\tau_a = -E\alpha\Delta T + \frac{(1-\beta)P}{A}$$

$$\beta = 1 - \frac{A}{P}[2\tau_a + E\alpha\Delta T]$$

$\beta = -5.1$

Impossible so evaluate segment (βL to L):

Stresses in bar (βL to L):

$$\sigma_x = -E\alpha\Delta T - \frac{R_B}{A} \qquad \tau_{max} = \frac{\sigma_x}{2}$$

set $\tau_{max} = \tau_a$ and solve for P_{max2}

$$2\tau_a = -E\alpha\Delta T - \frac{\beta P}{A}$$

$$\beta = \frac{-A}{P}[-2\tau_a + E\alpha\Delta T]$$

$\beta = 0.62 \quad \leftarrow$

Problem 2.6-15

NUMERICAL DATA

$P = 30 \text{ kN} \qquad \alpha = 36° \qquad \sigma_a = 90 \text{ MPa}$

$\tau_a = 48 \text{ MPa}$ <in the brass bars

$\theta = \frac{\pi}{2} - \alpha \quad \theta = 54°$

$\sigma_{ja} = 40 \text{ MPa}$

$\tau_{ja} = 20 \text{ MPa}$ <on brazed joint

tensile force N_{AC} Method of Joints at C

$$N_{AC} = \frac{P}{\sin(60°)} \qquad \text{(tension)}$$

$N_{AC} = 34.6 \text{ kN} \quad \leftarrow$

min. required diameter of bar AC

(1) Check tension and shear in bars; $\tau_a > \sigma_a/2$ so normal stress controls

$$\sigma_a = \frac{N_{AC}}{A} \qquad \sigma_x = \sigma_a$$

$$A_{reqd} = \frac{N_{AC}}{\sigma_a} \qquad A_{reqd} = 384.9 \text{ mm}^2$$

$$d_{min} = \sqrt{\frac{4}{\pi} A_{reqd}} \qquad d_{min} = 22.14 \text{ mm}$$

(2) Check tension and shear on brazed joint:

$$\sigma_x = \frac{N_{AC}}{A} \qquad \sigma_x = \frac{N_{AC}}{\frac{\pi}{4} d^2} \qquad d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}}$$

Tension on brazed joint:

$\sigma_\theta = \sigma_x \cos(\theta)^2$ set equal to σ_{ja} and solve for σ_x, then d_{reqd}

$$\sigma_x = \frac{\sigma_{ja}}{\cos(\theta)^2} \qquad \sigma_x = 115.78 \text{ MPa}$$

$$d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \qquad d_{reqd} = 19.52 \text{ mm}$$

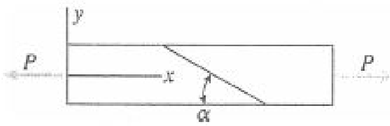
Shear on brazed joint:

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta)$$

$$\sigma_x = \left| \frac{N_{AC}}{-(\sin(\theta)\cos(\theta))} \right| \qquad \sigma_x = 42.06 \text{ MPa}$$

$$d_{reqd} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \qquad d_{reqd} = 32.4 \text{ mm} \qquad \leftarrow \text{governs}$$

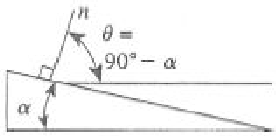
Problem 2.6-16



$10° \leq \alpha \leq 40°$

Due to load P: $\sigma_x = 4.9$ MPa

(a) STRESSES ON JOINT WHEN $\alpha = 20°$



$$\theta = 90° - \alpha = 70°$$

$$\sigma_\theta = \sigma_x \cos^2\theta = (4.9 \text{ MPa})(\cos 70°)^2$$
$$= 0.57 \text{ MPa} \quad \leftarrow$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$
$$= (-4.9 \text{ MPa})(\sin 70°)(\cos 70°)$$
$$= -1.58 \text{ MPa} \quad \leftarrow$$

(b) LARGEST ANGLE α IF $\tau_{allow} = 2.25$ MPa

$$\tau_{allow} = -\sigma_x \sin\theta \cos\theta$$

The shear stress on the joint has a negative sign. Its numerical value cannot exceed $\tau_{allow} = 2.25$ MPa. Therefore,

$-2.25 \text{ MPa} = -(4.9 \text{ MPa})(\sin\theta)(\cos\theta)$ or $\sin\theta\cos\theta = 0.4592$

From trigonometry: $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

Therefore: $\sin 2\theta = 2(0.4592) = 0.9184$

Solving: $2\theta = 66.69°$ or $113.31°$

$\theta = 33.34°$ or $56.66°$

$\alpha = 90° - \theta \qquad \therefore \alpha = 56.66°$ or $33.34°$

Since α must be between $10°$ and $40°$, we select

$\alpha = 33.3° \quad \leftarrow$

NOTE: If α is between $10°$ and $33.3°$,

$|\tau_\theta| < 2.25$ MPa.

If α is between $33.3°$ and $40°$,

$|\tau_\theta| > 2.25$ MPa.

(c) WHAT IS α if $\tau_\theta = 2\sigma_\theta$?

Numerical values only:

$|\tau_\theta| = \sigma_x \sin\theta\cos\theta \qquad |\sigma_\theta| = \sigma_x \cos^2\theta$

$$\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2$$

$$\sigma_x \sin\theta\cos\theta = 2\sigma_x\cos^2\theta$$

$\sin\theta = 2\cos\theta$ or $\tan\theta = 2$

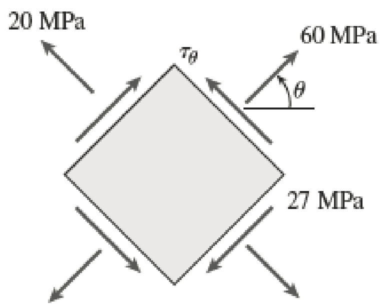
$\theta = 63.43° \qquad \alpha = 90° - \theta$

$\alpha = 26.6° \quad \leftarrow$

NOTE: For $\alpha = 26.6°$ and $\theta = 63.4°$, we find $\sigma_\theta = 0.98$ MPa and $\tau_\theta = -1.96$ MPa.

Thus, $\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2$ as required.

Problem 2.6-17



(a) ANGLE θ AND SHEAR STRESS τ_θ

PLANE AT ANGLE θ

$\sigma_\theta = \sigma_x \cos^2\theta$

$\sigma_\theta = 60$ MPa

$$\sigma_x = \frac{\sigma_\theta}{\cos^2\theta} = \frac{60\text{ MPa}}{\cos^2\theta} \quad (1)$$

PLANE AT ANGLE $\theta + 90°$

$$\sigma_{\theta+90°} = \sigma_x[\cos(\theta + 90°)]^2 = \sigma_x[-\sin\theta]^2 = \sigma_x \sin^2\theta$$

$\sigma_{\theta+90°} = 20$ MPa

$$\sigma_x = \frac{\sigma_{\theta+90°}}{\sin^2\theta} = \frac{20\text{ MPa}}{\sin^2\theta} \quad (2)$$

Equate (1) and (2):

$$\frac{60\text{ MPa}}{\cos^2\theta} = \frac{20\text{ MPa}}{\sin^2\theta}$$

$$\tan^2\theta = \frac{1}{3} \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \theta = 30° \quad \longleftarrow$$

From Eq. (1) or (2):

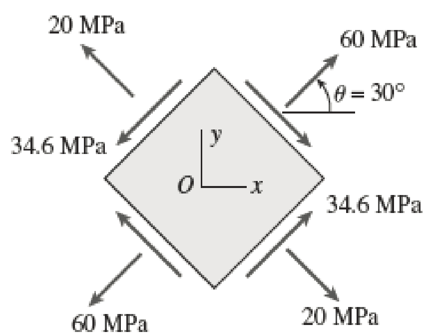
$\sigma_x = 80$ MPa (tension)

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= (-80\text{ MPa})(\sin 30°)(\cos 30°)$

$= -34.6$ MPa $\longleftarrow$

Minus sign means that τ_θ acts clockwise on the plane for which $\theta = 30°$.

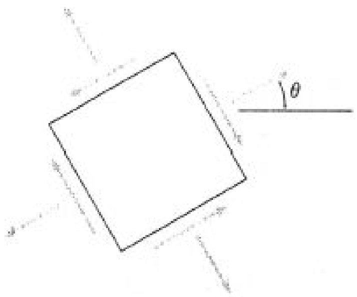


(b) MAXIMUM NORMAL AND SHEAR STRESSES

$\sigma_{max} = \sigma_x = 80$ MPa $\longleftarrow$

$$\tau_{max} = \frac{\sigma_x}{2} = 40\text{ MPa} \quad \longleftarrow$$

Problem 2.6-18



Find θ and σ_x for stress state shown in figure.

$\sigma_\theta = \sigma_x \cos(\theta)^2 \qquad \cos(\theta) = \sqrt{\frac{\sigma_\theta}{\sigma_x}}$

so $\qquad \sin(\theta) = \sqrt{1 - \frac{\sigma_\theta}{\sigma_x}}$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta)$

$$\frac{\tau_\theta}{\sigma_x} = -\sqrt{1 - \frac{\sigma_\theta}{\sigma_x}}\sqrt{\frac{\sigma_\theta}{\sigma_x}}$$

$$\left(\frac{\tau_\theta}{\sigma_x}\right)^2 = \frac{\sigma_\theta}{\sigma_x} - \left(\frac{\sigma_\theta}{\sigma_x}\right)$$

$$\left(\frac{23}{\sigma_x}\right)^2 = \frac{65}{\sigma_x} - \left(\frac{65}{\sigma_x}\right)^2$$

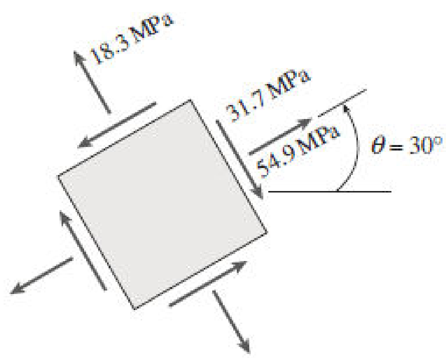
$$\left(\frac{65}{\sigma_x}\right)^2 - \left(\frac{65}{\sigma_x}\right) + \left(\frac{23}{\sigma_x}\right)^2 = 0$$

$$\frac{-(-4754 + 65\sigma_x)}{\sigma_x^2} = 0$$

$$\sigma_x = \frac{4754}{65}$$

$\sigma_x = 73.1 \text{ MPa} \qquad \sigma_\theta = 65 \text{ MPa}$

$\theta = acos\left(\sqrt{\frac{\sigma_\theta}{\sigma_x}}\right) \qquad \theta = 19.5°$



Now find σ_θ and τ_θ for $\theta = 30°$:

$\sigma_{\theta 1} = \sigma_x \cos(\theta)^2 \qquad \sigma_{\theta 1} = 54.9 \text{ MPa} \quad \leftarrow$

$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) \qquad \tau_\theta = -31.7 \text{ MPa} \quad \leftarrow$

$\sigma_{\theta 2} = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 \qquad \sigma_{\theta 2} = 18.3 \text{ MPa} \quad \leftarrow$

Problem 2.6-19

Eq. (2-29a)

$\sigma_\theta = \sigma_x \cos^2\theta$

PLANE pq:

$\sigma_1 = 57\,\text{MPa}$

$\theta = \theta_1, \sigma_1 = \sigma_x \cos^2\theta_1$ (1)

PLANE rs: $\sigma_2 = 23\,\text{MPa}$, $\theta = \theta_1 + \beta = \theta_1 + 30°$

$\sigma = \sigma_x \cos^2(\theta_1 + 30°)$ (2)

From (1) and (2): $\sigma_x = \dfrac{\sigma_1}{\cos^2\theta_1} = \dfrac{\sigma_2}{\cos^2(\theta_1 + 30°)}$ (3)

From (3): $\left[\dfrac{\cos\theta_1}{\cos(\theta_1 + 30°)}\right] = \dfrac{\sigma_1}{\sigma_2}$ or

$$\frac{\cos\theta_1}{\cos(\theta_1 + 30°)} = \sqrt{\frac{\sigma_1}{\sigma_2}} = 1.5742$$

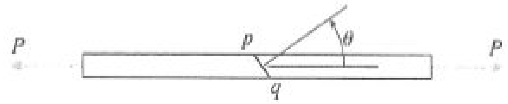
Solve by iteration or use a computer program:

$\theta_1 = 25°$

From (1) and (2): $\sigma_{max} = \sigma_x = 64.4\,\text{MPa}$

$\tau_{max} = \dfrac{\sigma_x}{2} = 34.7\,\text{MPa}$ ←

Problem 2.6-20



$25° < \theta < 45°$

$A = 225\ \text{mm}^2$

On glued joint: $\sigma_{allow} = 5.0$ MPa

$\tau_{allow} = 3.0$ MPa

ALLOWABLE STRESS σ_x IN TENSION

$$\sigma_\theta = \sigma_x \cos^2\theta \qquad \sigma_x = \frac{\sigma_\theta}{\cos^2\theta} = \frac{5.0\text{ MPa}}{\cos^2\theta} \qquad (1)$$

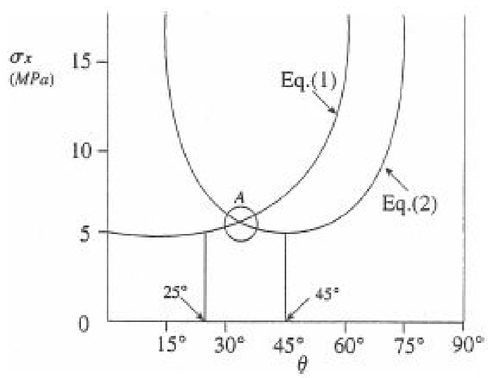
$$\tau_\theta = -\sigma_x \sin\theta\cos\theta$$

Since the direction of τ_θ is immaterial, we can write:
$\tau_\theta| = \sigma_x \sin\theta\cos\theta$

or

$$\sigma_x = \frac{|\tau_\theta|}{\sin\theta\cos\theta} = \frac{3.0\text{ MPa}}{\sin\theta\cos\theta} \qquad (2)$$

GRAPH OF EQS. (1) AND (2)



(a) DETERMINE ANGLE Θ FOR LARGEST LOAD

Point A gives the largest value of σ_x and hence the largest load. To determine the angle θ corresponding to point A, we equate Eqs. (1) and (2).

$$\frac{5.0\text{ MPa}}{\cos^2\theta} = \frac{3.0\text{ MPa}}{\sin\theta\cos\theta}$$

$$\tan\theta = \frac{3.0}{5.0} \qquad \theta = 30.96° \quad \leftarrow$$

(b) DETERMINE THE MAXIMUM LOAD

From Eq. (1) or Eq. (2):

$$\sigma_x = \frac{5.0\text{ MPa}}{\cos^2\theta} = \frac{3.0\text{ MPa}}{\sin\theta\cos\theta} = 6.80\text{ MPa}$$

$$P_{max} = \sigma_x A = (6.80\text{ MPa})(225\text{ mm}^2)$$
$$= 1.53\text{ kN} \quad \leftarrow$$

Problem 2.6-21

NUMERICAL DATA

$\alpha = 95(10^{-6})/°C \quad E = 2.8\text{ GPa} \quad L = 0.6\text{ m} \quad \Delta T = 48°C \quad k = 3150\text{ kN/m} \quad f = \dfrac{1}{k} = 3.175 \times 10^{-4}\text{ kN/m}$

$b = 19\text{ mm} \quad h = 38\text{ mm} \quad A = bh \quad L_\theta = 0.46\text{ m} \quad \sigma_a = -6.9\text{ MPa} \quad \tau_a = -3.9\text{ MPa} \quad \sigma_\theta = -5.3\text{ MPa}$

(a) FIND θ AND T_θ

$$R_2 = \text{redundant} \qquad R_2 = \frac{-\alpha\,\Delta TL}{\left(\frac{L}{EA}\right) + f} = -4.454\text{ kN} \qquad \sigma_x = \frac{R_2}{A} = -6.169\text{ MPa} \qquad \sqrt{\frac{\sigma_\theta}{\sigma_x}} = 0.927$$

$$\theta = \arccos\left(\sqrt{\frac{\sigma_\theta}{\sigma_x}}\right) = 0.385 \qquad \cos(2\theta) = 0.718 \quad \theta = 22.047°$$

$$\sigma_x \cos(\theta)^2 = -5.3\text{ MPa} \quad \text{OR} \quad \frac{\sigma_x}{2}(1 + \cos(2\theta)) = -5.3\text{ MPa} \quad \sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 = -0.869\text{ MPa}$$

$$\theta = 0.385\text{ radians} \quad \theta = 22.047° \quad \sigma_x = -6.169\text{ MPa} \quad 2\theta = 0.77\text{ radians}$$

$$\tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) = 2.146\text{ MPa} \quad \text{OR} \quad \tau_\theta = \frac{-\sigma_x}{2}\sin(2\theta) = 2.146\text{ MPa}$$

$\boxed{\tau_\theta = 2.15\text{MPa}}$ $\boxed{\theta = 22°}$

(b) FIND σ_{x1} AND σ_{y1}

$$\sigma_{x1} = \sigma_x \cos(\theta)^2 \quad \sigma_{y1} = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2$$

$\boxed{\sigma_{x1} = -5.3\text{ MPa}}$ $\boxed{\sigma_{y1} = -0.869\text{ MPa}}$

(c) GIVEN $L = 0.6$ m, FIND k_{max}

$$k_{max1} = \frac{\sigma_a A}{-\alpha\Delta TL - \sigma_a A\left(\frac{L}{EA}\right)} = 3961.895\text{ kN/m} \quad < \text{controls (based on } \sigma_{allow})$$

$$\text{OR} \quad k_{max2} = \frac{2\tau_a A}{-\alpha\Delta TL - 2\tau_a A\left(\frac{L}{EA}\right)} = 5290.016\text{ kN/m} \quad < \text{based on allowable shear stress}$$

$\boxed{k_{max} = 3962\text{ kN/m}}$

(d) GIVEN ALLOWABLE NORMAL AND SHEAR STRESSES, FIND L_{max}

$k = 3150$ kN/m

$$\sigma_x = \frac{R_2}{A} \quad \sigma_a A = \frac{-\alpha\Delta TL}{\left(\frac{L}{EA}\right) + f} \quad L_{max1} = \frac{\sigma_a A(f)}{-\left(\alpha\Delta T + \frac{\sigma_a}{E}\right)} = 0.755\text{ m} \quad < \text{controls based on } \sigma_{allow}$$

$$\text{OR} \quad L_{max2} = \frac{2\tau_a A(f)}{-\left(\alpha\Delta T + \frac{2\tau_a}{E}\right)} = 1.088\text{ m} \quad < \text{based on } \tau_{allow}$$

$\boxed{L_{max} = 0.755\text{ m}}$

(e) Find ΔT_{max} given L, k, and allowable stresses $k = 3150\ \text{kN/m}$ $L = 0.6\ \text{m}$ $\sigma_a = -6.9\ \text{MPa}$

$\tau_a = -3.9\ \text{MPa}$

$$\Delta T_{max1} = \frac{\left(\frac{L}{EA} + f\right)\sigma_a A}{-\alpha L} = 53.686\,°\text{C} \quad < \text{based on } \sigma_{allow}$$

$$\Delta T_{max2} = \frac{\left(\frac{L}{EA} + f\right)2\,\tau_a A}{-\alpha L} = 60.688°\text{C} \quad < \text{based on } \tau_{allow}$$

$$\boxed{\Delta T_{max} = 53.7°\text{C}}$$

Problem 2.6-22

$b = 50mm \qquad \alpha = 35deg$

$\sigma_a = 11.5MPa \qquad \tau_a = 4.5MPa$

$\sigma_{ga} = 3.5MPa \qquad \tau_{ga} = 1.25MPa$

Rotate stress element CW by angle θ to align with glue joint (see fig.)

$$\theta = \alpha - 90deg = -55 \cdot deg$$

Plane stress transformations $\qquad \sigma_x = \frac{P}{A} \qquad A = b^2 = 2500 \cdot mm^2$

$$\sigma_\theta = \sigma_x \cdot \cos(\theta)^2 \qquad \tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta)$$

Equate σ_θ and τ_θ to allowable values and solve for P - min. P controls

$$\sigma_{max} = \sigma_x \qquad P_{max1} = \sigma_a \cdot A = 28.75 \cdot kN$$

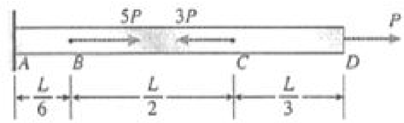
$$\tau_{max} = -\left(\frac{P}{A}\right) \cdot \sin(45deg) \cdot \cos(45deg) \qquad P_{max2} = \frac{\tau_a}{2} \cdot A = 5.625 \cdot kN$$

< shear in wood controls

$$P_{max3} = \frac{\sigma_{ga} \cdot A}{\cos(\theta)^2} = 26.597 \cdot kN$$

$$P_{max4} = \frac{\tau_{ga} \cdot A}{-\sin(\theta) \cdot \cos(\theta)} = 6.651 \cdot kN$$

Problem 2.7-1



$P = 27$ kN

$L = 130$ cm

$E = 72$ GPa

$A = 18$ cm^2

Internal axial forces

$N_{AB} = 3P \qquad N_{BC} = -2P \qquad N_{CD} = P$

Lengths

$L_{AB} = \frac{L}{6} \qquad L_{BC} = \frac{L}{2} \qquad L_{CD} = \frac{L}{3}$

(a) Strain energy of the bar (Eq. 2-40)

$$U = \Sigma \frac{N_i^2 L_i}{2E_i A_i}$$

$$= \frac{1}{2EA}\left[(3P)^2\left(\frac{L}{6}\right) + (-2P)^2\left(\frac{L}{2}\right) + (P)^2\left(\frac{L}{3}\right)\right]$$

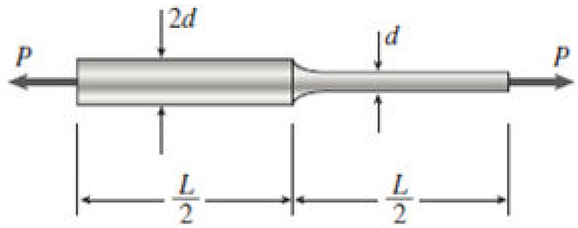
$$= \frac{P^2 L}{2EA}\left(\frac{23}{6}\right) = \frac{23P^2L}{12EA} \quad \leftarrow$$

(b) Substitute numerical values:

$$U = \frac{23P^2L}{12EA}$$

$$= 14.02 \text{ N}\cdot\text{m} \quad \leftarrow$$

Problem 2.7-2



(a) STRAIN ENERGY OF THE BAR

Add the strain energies of the two segments of the bar (see Eq. 2-40).

$$U = \sum_{i=1}^{2} \frac{N_i^2 L_i}{2\, E_i A_i} = \frac{P^2(L/2)}{2E}\left[\frac{1}{\frac{\pi}{4}(2d)^2} + \frac{1}{\frac{\pi}{4}(d^2)}\right]$$

$$= \frac{P^2 L}{\pi E}\left(\frac{1}{4d^2} + \frac{1}{d^2}\right) = \frac{5P^2 L}{4\pi E d^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

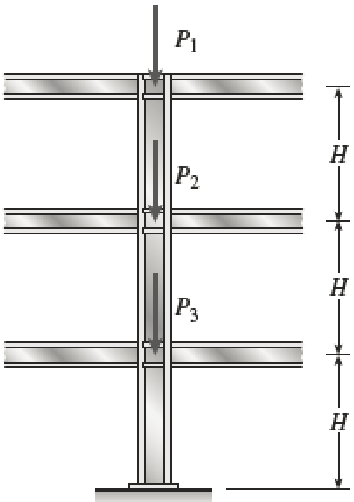
$P = 27$ kN $\quad L = 600$ mm

$d = 40$ mm $\quad E = 105$ GPa

$$U = \frac{5(27\text{ kN}^2)(600\text{ mm})}{4\pi(105\text{ GPa})(40\text{ mm})^2}$$

$$= 1.036\text{ N}\cdot\text{m} = 1.036\text{ J} \quad \leftarrow$$

Problem 2.7-3



Add the strain energies of the three segments (see Eq. 2-40).

Upper segment: $N_1 = -P_1$

Middle segment: $N_2 = -(P_1 + P_2)$

Lower segment: $N_3 = -(P_1 + P_2 + P_3)$

STRAIN ENERGY

$$U = \Sigma \frac{N_i^2 L_i}{2E_i A_i} = \left[\frac{H}{(2EA)}\right] \sum_{i=1}^{3} N_i^2$$

$$= \frac{H}{2EA}[P_1^2 + (P_1 + P_2)^2 + (P_1 + P_2 + P_3)^2]$$

Substitute numerical values:

$H = 3.0 \text{ m} \qquad E = 200 \text{ GPa}$

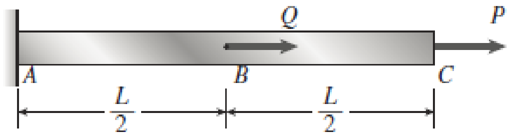
$A = 7500 \text{ mm}^2 \qquad P_1 = 150 \text{ kN}$

$P_2 = P_3 = 300 \text{ kN}$

$$U = \frac{(3.0 \text{ m})}{2(200 \text{ GPa})(7500 \text{ mm}^2)}[(150 \text{ kN})^2 + (450 \text{ kN})^2 + (750 \text{ kN})^2]$$

$= 788 \text{ J} \quad \leftarrow$

Problem 2.7-4



(a) Force P acts alone ($Q = 0$)

$$U_1 = \frac{P^2L}{2EA} \quad \leftarrow$$

(b) Force Q acts alone ($P = 0$)

$$U_2 = \frac{Q^2(L/2)}{2EA} = \frac{Q^2L}{4EA} \quad \leftarrow$$

(c) Forces P and Q act simultaneously

$$\text{Segment } BC\text{: } U_{BC} = \frac{P^2(L/2)}{2EA} = \frac{P^2L}{4EA}$$

$$\text{Segment } AB\text{: } U_{AB} = \frac{(P+Q)^2(L/2)}{2EA}$$

$$= \frac{P^2L}{4EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA}$$

$$U_3 = U_{BC} + U_{AB} = \frac{P^2L}{2EA} + \frac{PQL}{2EA} + \frac{Q^2L}{4EA} \quad \leftarrow$$

(Note that U_3 is *not* equal to $U_1 + U_2$. In this case, $U_3 > U_1 + U_2$. However, if Q is reversed in direction, $U_3 < U_1 + U_2$. Thus, U_3 may be larger or smaller than $U_1 + U_2$.)

Problem 2.7-5

DATA

Material	Weight density (kN/m^3)	Modulus of elasticity (GPa)	Proportional limit (MPa)
Mild steel	77.1	207	248
Tool steel	77.1	207	827
Aluminum	26.7	72	345
Rubber (soft)	11.0	2	1.38

STRAIN ENERGY PER UNIT VOLUME

$$U = \frac{P^2 L}{2EA} \qquad \text{Volume } V = AL$$

$$u = \frac{U}{V} = \frac{\sigma^2}{2E}$$

At the proportional limit:

$u = u_R =$ modulus of resistance

$$u_R = \frac{\sigma_{PL}^2}{2E} \qquad \text{(Eq. 1)}$$

STRAIN ENERGY PER UNIT WEIGHT

$$U = \frac{P^2 L}{2EA} \qquad \text{Weight } W = \gamma AL$$

$\gamma =$ weight density

$$u_W = \frac{U}{W} = \frac{\sigma^2}{2\gamma E}$$

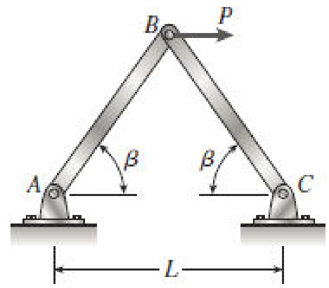
At the proportional limit:

$$u_W = \frac{\sigma_{PL}^2}{2\gamma E} \qquad \text{(Eq. 2)}$$

RESULTS

	u_R (kPa)	u_w (m)
Mild steel	149	1.9
Tool steel	1652	21
Aluminum	826	31
Rubber (soft)	476	143

Problem 2.7-6



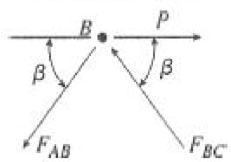
$\beta = 60°$

$L_{AB} = L_{BC} = L$

$\sin\beta = \sqrt{3}/2$

$\cos\beta = 1/2$

FREE-BODY DIAGRAM OF JOINT B



$\Sigma F_{vert} = 0 \quad \uparrow_+ \quad \downarrow^-$

$-F_{AB}\sin\beta + F_{BC}\sin\beta = 0$

$F_{AB} = F_{BC}$ (Eq. 1)

$\Sigma F_{horiz} = 0 \rightarrow \leftarrow$

$-F_{AB}\cos\beta - F_{BC}\cos\beta + P = 0$

$F_{AB} = F_{BC} = \dfrac{P}{2\cos\beta} = \dfrac{P}{2(1/2)} = P$ (Eq. 2)

Axial forces: $N_{AB} = P$ (tension)

$N_{BC} = -P$ (compression)

(a) STRAIN ENERGY OF TRUSS (EQ. 2-40)

$$U = \Sigma\frac{N_i^2 L_i}{2E_i A_i} = \frac{(N_{AB})^2 L}{2EA} + \frac{(N_{BC})^2 L}{2EA} = \frac{P^2 L}{EA} \quad \leftarrow$$

(b) HORIZONTAL DISPLACEMENT OF JOINT B (EQ. 2-42)

$$\delta_B = \frac{2U}{P} = \frac{2}{P}\left(\frac{P^2 L}{EA}\right) = \frac{2PL}{EA} \quad \leftarrow$$

Problem 2.7-7

$L_{BC} = 1.5\text{ m} \qquad \theta = 30°$

$P_1 = 1.3\text{ kN} \qquad P_2 = 4\text{ kN}$

$L_{AB} = \dfrac{L_{BC}}{\cos(\theta)} \qquad L_{AB} = 1.732\text{ m}$

$E = 200\text{ GPa} \qquad A = 1500\text{ mm}^2$

(a) Load P_1 acts alone

$F_{BC} = P_1 \qquad F_{AB} = 0$

$$U_1 = \frac{F_{BC}^2 L_{BC}}{2EA} \qquad U_1 = 0.00422\text{ J}$$

(b) Load P_2 acts alone

$$F_{AB} = \frac{P_2}{\sin(\theta)} \qquad F_{AB} = 8\text{ kN}$$

$$F_{BC} = -F_{AB}\cos(\theta) \qquad F_{BC} = -6928.203\text{ N}$$

$$U_2 = \frac{F_{AB}^2 L_{AB}}{2EA} + \frac{F_{BC}^2 L_{BC}}{2EA} \qquad U_2 = 0.305\text{ J}$$

(c) Loads P_1 and P_2 act simultaneously

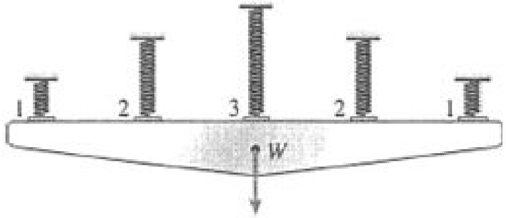
$$F_{AB} = \frac{P_2}{\sin(\theta)} = 8\text{ kN}$$

$$F_{BC} = P_1 + F_{BC} = -5.628\text{ kN}$$

$$U_3 = \frac{F_{AB}^2 L_{AB}}{2EA} + \frac{F_{BC}^2 L_{BC}}{2EA} \qquad U_3 = 0.264\text{ J}$$

NOTE: The strain energy U_3 is *not* equal to $U_1 + U_2$.

Problem 2.7-8



$k_1 = 3k$

$k_2 = 1.5k$

$k_3 = k$

δ = downward displacement of rigid bar

For a spring: $U = \dfrac{k\delta^2}{2}$ Eq. (2-38b)

(a) STRAIN ENERGY U OF ALL SPRINGS

$$U = 2\left(\frac{3k\delta^2}{2}\right) + 2\left(\frac{1.5k\delta^2}{2}\right) + \frac{k\delta^2}{2} = 5k\delta^2 \quad \leftarrow$$

(b) DISPLACEMENT δ

Work done by the weight W equals $\dfrac{W\delta}{2}$

Strain energy of the springs equals $5k\delta^2$

$$\therefore \frac{W\delta}{2} = 5k\delta^2 \quad \text{and} \quad \delta = \frac{W}{10k} \quad \leftarrow$$

(c) FORCES IN THE SPRINGS

$$F_1 = 3k\delta = \frac{3W}{10} \quad F_2 = 1.5k\delta = \frac{3W}{20} \quad \leftarrow$$

$$F_3 = k\delta = \frac{W}{10} \quad \leftarrow$$

(d) NUMERICAL VALUES

$W = 600$ N $\quad k = 7.5$ N/mm $= 7500$ N/mm

$$U = 5k\delta^2 = 5k\left(\frac{W}{10k}\right)^2 = \frac{W^2}{20k}$$

$$= 2.4 \text{ N}\cdot\text{m} = 2.4 \text{ J} \quad \leftarrow$$

$$\delta = \frac{W}{10k} = 8.0 \text{ mm} \quad \leftarrow$$

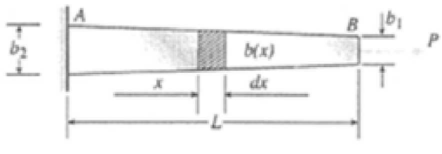
$$F_1 = \frac{3W}{10} = 180 \text{ N} \quad \leftarrow$$

$$F_2 = \frac{3W}{20} = 90 \text{ N} \quad \leftarrow$$

$$F_3 = \frac{W}{10} = 60 \text{ N} \quad \leftarrow$$

NOTE: $W = 2F_1 + 2F_2 + F_3 = 600$ N (Check)

Problem 2.7-9



$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = tb(x)$$

$$= t\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]$$

(a) Strain energy of the bar

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad \text{(Eq. 2-41)}$$

$$= \int_0^L \frac{P^2 dx}{2Etb(x)} = \frac{P^2}{2Et}\int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \qquad (1)$$

From Appendix C: $\displaystyle\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et}\left[\frac{1}{-(b_2 - b_1)\left(\frac{1}{L}\right)}\ln\left[b_2 - \frac{(b_2 - b_1)x}{L}\right]\right]_0^L$$

$$= \frac{P^2}{2Et}\left[\frac{-L}{(b_2 - b_1)}\ln b_1 - \frac{-L}{(b_2 - b_1)}\ln b_2\right]$$

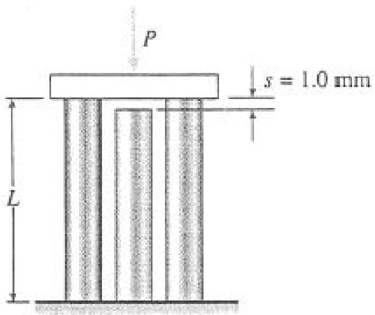
$$U = \frac{P^2 L}{2Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad \leftarrow$$

(b) Elongation of the bar (Eq. 2-42)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)}\ln\frac{b_2}{b_1} \quad \leftarrow$$

NOTE: This result agrees with the formula derived in Prob. 2.3-17.

Problem 2.7-10



$s = 1.0$ mm

$L = 1.0$ m

For each bar:

$A = 3000\ \text{mm}^2$

$E = 45$ GPa

$\frac{EA}{L} = 135 \times 10^6$ N/m

(a) Load P_1 required to close the gap

In general, $\delta = \frac{PL}{EA}$ and $P = \frac{EA\delta}{L}$

For two bars, we obtain:

$$P_1 = 2\left(\frac{EAs}{L}\right) = 2(135 \times 10^6\ \text{N/m})(1.0\ \text{mm})$$

$P_1 = 270$ kN ←

(b) Displacement δ for $P = 400$ kN

Since $P > P_1$, all three bars are compressed. The force P equals P_1 plus the additional force required to compress all three bars by the amount $\delta - s$.

$$P = P_1 + 3\left(\frac{EA}{L}\right)(\delta - s)$$

or $400\ \text{kN} = 270\ \text{kN} + 3(135 \times 10^6\ \text{N/m})(\delta - 0.001\ \text{m})$

Solving, we get $\delta = 1.321$ mm ←

(c) Strain energy U for $P = 400$ kN

$$U = \Sigma \frac{EA\delta^2}{2L}$$

Outer bars: $\delta = 1.321$ mm

Middle bar: $\delta = 1.321\ \text{mm} - s = 0.321$ mm

$$U = \frac{EA}{2L}[2(1.321\ \text{mm})^2 + (0.321\ \text{mm})^2]$$

$$= \frac{1}{2}(135 \times 10^6\ \text{N/m})(3.593\ \text{mm}^2)$$

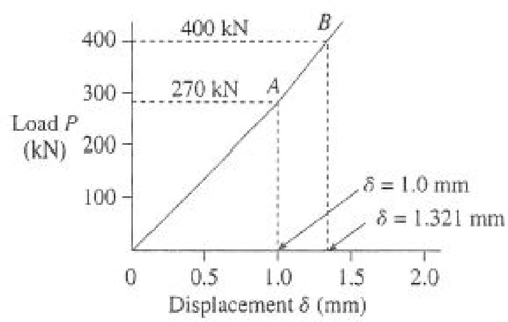
$$= 243\ \text{N·m} = 243\ \text{J}$$ ←

(d) Load-displacement diagram

$U = 243\ \text{J} = 243\ \text{N·m}$

$$\frac{P\delta}{2} = \frac{1}{2}(400\ \text{kN})(1.321\ \text{mm}) = 264\ \text{N·m}$$

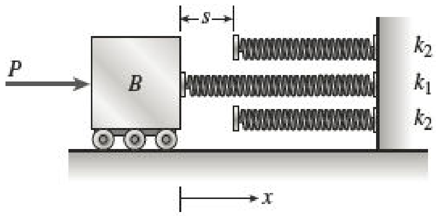
The strain energy U is *not* equal to $\frac{P\delta}{2}$ = because the load-displacement relation is not linear.



U = area under line OAB.

$\frac{P\delta}{2}$ = area under a straight line from O to B, which is larger than U.

Problem 2.7-11



Force P_0 required to close the gap:

$$P_0 = k_1 s \qquad (1)$$

FORCE-DISPLACEMENT RELATION BEFORE GAP IS CLOSED

$$P = k_1 x \qquad (0 \le x \le s)(0 \le P \le P_0) \qquad (2)$$

FORCE-DISPLACEMENT RELATION AFTER GAP IS CLOSED

All three springs are compressed. Total stiffness equals $k_1 + 2k_2$. Additional displacement equals $x - s$. Force P equals P_0 plus the force required to compress all three springs by the amount $x - s$.

$$P = P_0 + (k_1 + 2k_2)(x - s)$$

$$= k_1 s + (k_1 + 2k_2)x - k_1 s - 2k_2 s$$

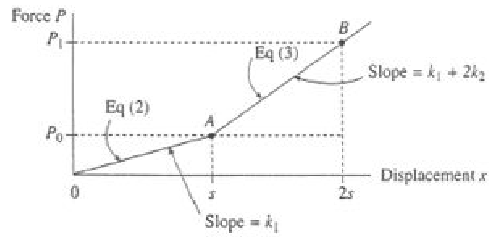
$$P = (k_1 + 2k_2)x - 2k_2 s \qquad (x \ge s); (P \ge P_0) \qquad (3)$$

P_1 = force P when $x = 2s$

Substitute $x = 2s$ into Eq. (3):

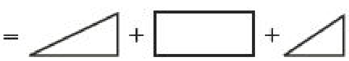
$$P_1 = 2(k_1 + k_2)s \qquad (4)$$

(a) FORCE-DISPLACEMENT DIAGRAM



(b) STRAIN ENERGY U_1 WHEN $x = 2s$

U_1 = Area below force-displacement curve

= 

$$= \frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 - P_0)s = P_0 s + \frac{1}{2}P_1 s$$

$$= k_1 s^2 + (k_1 + k_2)s^2$$

$$U_1 = (2k_1 + k_2)s^2 \quad \leftarrow \qquad (5)$$

(c) STRAIN ENERGY U_1 IS NOT EQUAL TO $\frac{P\delta}{2}$

For $\delta = 2s$: $\frac{P\delta}{2} = \frac{1}{2}P_1(2s) = P_1 s = 2(k_1 + k_2)s^2$

(This quantity is greater than U_1.)

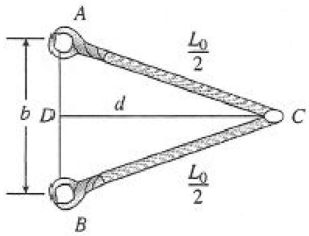
U_1 = area under line OAB.

$\frac{P\delta}{2}$ = area under a straight line from O to B, which is larger than U_1.

Thus, $\frac{P\delta}{2}$ is *not* equal to the strain energy because the force-displacement relation is not linear.

Problem 2.7-12

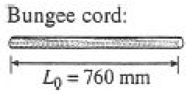
DIMENSIONS BEFORE THE LOAD P IS APPLIED



$L_0 = 760$ mm $\qquad \frac{L_0}{2} = 380$ mm

$b = 380$ mm

Bungee cord:

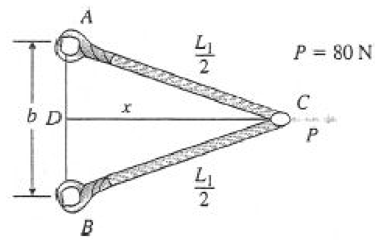


$k = 140$ N/m

From triangle ACD:

$$d = \frac{1}{2}\sqrt{L_0^2 - b^2} = 329.09 \text{ mm} \qquad (1)$$

DIMENSIONS AFTER THE LOAD P IS APPLIED



Let x = distance CD

Let L_1 = stretched length of bungee cord

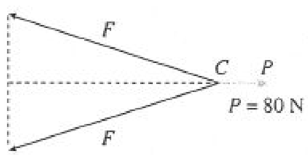
From triangle ACD:

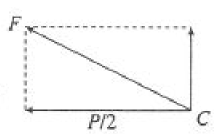
$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \qquad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \qquad (3)$$

EQUILIBRIUM AT POINT C

Let F = tensile force in bungee cord





$$\frac{F}{P/2} = \frac{L_1/2}{x} \qquad F = \left(\frac{P}{2}\right)\left(\frac{L_1}{2}\right)\left(\frac{1}{x}\right)$$

$$= \frac{P}{2}\sqrt{1 + \left(\frac{b}{2x}\right)^2} \qquad (4)$$

ELONGATION OF BUNGEE CORD

Let δ = elongation of the entire bungee cord

$$\delta = \frac{F}{k} = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (5)$$

Final length of bungee cord = original length + δ

$$L_1 = L_0 + \delta = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} \qquad (6)$$

SOLUTION OF EQUATIONS

Combine Eqs. (6) and (3):

$$L_1 = L_0 + \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

$$\text{or} \quad L_1 = L_0 + \frac{P}{4kx}\sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$$

$$L_0 = \left(1 - \frac{P}{4kx}\right)\sqrt{b^2 + 4x^2} \qquad (7)$$

This equation can be solved for x.

SUBSTITUTE NUMERICAL VALUES INTO EQ. (7):

$$760 \text{ mm} = \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})x}\right]$$

$$\times \sqrt{(380 \text{ mm})^2 + 4x^2} \qquad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right)\sqrt{144{,}400 + 4x^2} \qquad (9)$$

Units: x is in millimeters

Solve for x (Use trial-and-error or a computer program):

$x = 497.88$ mm

(a) STRAIN ENERGY U OF THE BUNGEE CORD

$$U = \frac{k\delta^2}{2} \qquad k = 140 \text{ N/m} \qquad P = 80 \text{ N}$$

From Eq. (5):

$$\delta = \frac{P}{2k}\sqrt{1 + \frac{b^2}{4x^2}} = 305.81 \text{ mm}$$

$$U = \frac{1}{2}(140 \text{ N/m})(305.81 \text{ mm})^2 = 6.55 \text{ N·m}$$

$U = 6.55$ J $\leftarrow$

(b) DISPLACEMENT δ_C OF POINT C

$$\delta_C = x - d = 497.88 \text{ mm} - 329.09 \text{ mm}$$

$$= 168.8 \text{ mm} \quad \leftarrow$$

(c) Comparison of strain energy U with the quantity $P\delta_C/2$

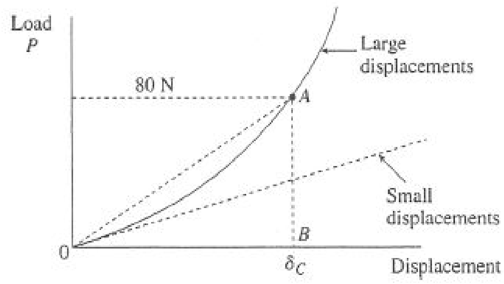
$U = 6.55$ J

$$\frac{P\delta_C}{2} = \frac{1}{2}(80\text{ N})(168.8\text{ mm}) = 6.75\text{ J}$$

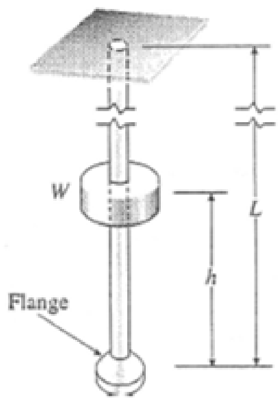
The two quantities are not the same. The work done by the load P is *not* equal to $P\delta_C/2$ because the load-displacement relation (see below) is non-linear when the displacements are large. (The *work* done by the load P is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)

U = area OAB under the curve OA.

$\frac{P\delta_C}{2}$ = area of triangle OAB, which is greater than U.



Problem 2.8-1



$W = 650\text{ N}$

$h = 50\text{ mm}$ $L = 1.2\text{ m}$

$E = 210\text{ GPa}$ $A = 5\text{ cm}^2$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA}$$

Eq. (2-53):

$$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.869\text{ mm} \quad \leftarrow$$

(b) Maximum tensile stress (Eq. 2-55)

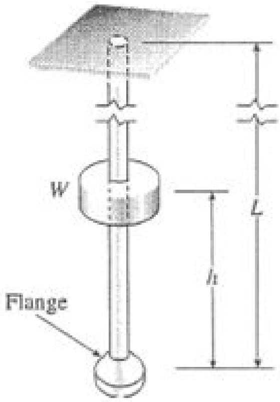
$$\sigma_{max} = \frac{E\delta_{max}}{L} = 152.1\text{ MPa} \quad \leftarrow$$

(c) Impact factor (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}}$$

$$= 117 \quad \leftarrow$$

Problem 2.8-2



$M = 80$ kg

$W = Mg = (80 \text{ kg})(9.81 \text{ m/s}^2)$

$= 784.8$ N

$h = 0.5$ m $\quad L = 3.0$ m

$E = 170$ GPa $\quad A = 350 \text{ mm}^2$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA} = 0.03957 \text{ mm}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

$= 6.33$ mm ←

(b) Maximum tensile stress (Eq. 2-55)

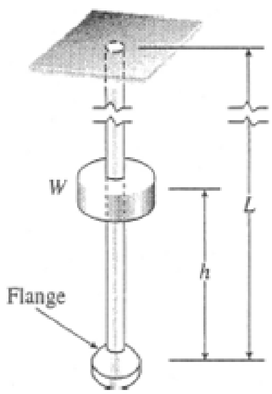
$$\sigma_{max} = \frac{E\delta_{max}}{L} = 359 \text{ MPa} \quad \leftarrow$$

(c) Impact factor (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{6.33 \text{ mm}}{0.03957 \text{ mm}}$$

$= 160$ ←

Problem 2.8-3



$W = 200$ N $\qquad h = 50$ mm

$L = 0.9$ m

$E = 210$ GPa $\qquad A = 15\ \text{cm}^2$

(a) Downward displacement of flange

$$\delta_{st} = \frac{WL}{EA}$$

Eq. (2-53): $$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$= 0.762 \text{ mm} \quad \leftarrow$$

$$\frac{\delta_{max}}{L} = 8.463 \times 10^{-4}$$

(b) Maximum tensile stress (Eq. 2-55)

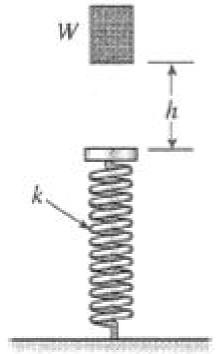
$$\sigma_{max} = \frac{E\delta_{max}}{L} = 177.7 \text{ MPa} \quad \leftarrow$$

(c) Impact factor (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}}$$

$$= 133 \quad \leftarrow$$

Problem 2.8-4



$W = 5.0\text{ N} \qquad h = 200\text{ mm} \qquad k = 90\text{ N/m}$

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k} = \frac{5.0\text{ N}}{90\text{ N/m}} = 55.56\text{ mm}$$

Eq. (2-53): $\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

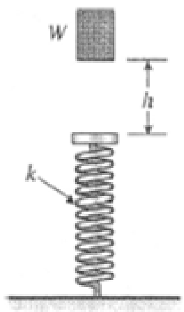
$= 215\text{ mm} \quad \leftarrow$

(b) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{max}}{\delta_{st}} = \frac{215\text{ mm}}{55.56\text{ mm}}$$

$= 3.9 \quad \leftarrow$

Problem 2.8-5



$W = 8\text{ N} \qquad h = 300\text{ mm} \qquad k = 125\text{ N/m}$

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k}$$

Eq. (2-53): $\delta_{\max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$

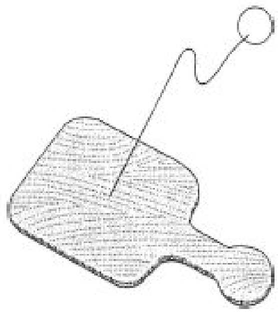
$= 270\text{ mm} \quad \leftarrow$

(b) IMPACT FACTOR (EQ. 2-61)

$$\text{Impact factor} = \frac{\delta_{\max}}{\delta_{st}}$$

$= 4.2 \quad \leftarrow$

Problem 2.8-6



$g = 9.81 \text{ m/s}^2 \qquad E = 2.0 \text{ MPa}$

$A = 1.6 \text{ mm}^2 \qquad L_0 = 200 \text{ mm}$

$L_1 = 900 \text{ mm} \qquad W = 450 \text{ mN}$

When the ball leaves the paddle

$$KE = \frac{Wv^2}{2g}$$

When the rubber cord is fully stretched:

$$U = \frac{EA\delta^2}{2L_0} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

Conservation of energy

$$KE = U \quad \frac{Wv^2}{2g} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

$$v^2 = \frac{gEA}{WL_0}(L_1 - L_0)^2$$

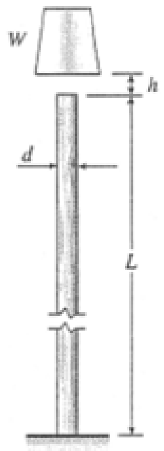
$$v = (L_1 - L_0)\sqrt{\frac{gEA}{WL_0}} \quad \leftarrow$$

Substitute numerical values:

$$v = (700 \text{ mm})\sqrt{\frac{(9.81 \text{ m/s}^2)(2.0 \text{ MPa})(1.6 \text{ mm}^2)}{(450 \text{ mN})(200 \text{ mm})}}$$

$$= 13.1 \text{ m/s} \quad \leftarrow$$

Problem 2.8-7



$W = 20 \text{ kN} \qquad d = 300 \text{ mm}$

$L = 5.5 \text{ m}$

$$A = \frac{\pi d^2}{4} = 0.0707 \text{ m}^2$$

$E = 10 \text{ GPa}$

$\sigma_{allow} = 17 \text{ MPa}$

Find h_{max}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{20 \text{ kN}}{0.0707 \text{ m}^2} = 283 \text{ KPa}$$

MAXIMUM HEIGHT h_{max}

$$\text{Eq. (2-59):} \quad \sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

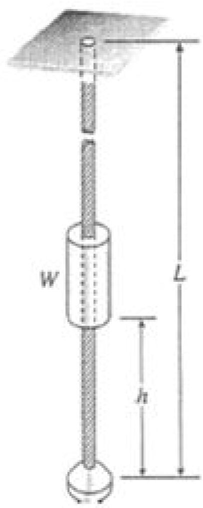
$$h = h_{max} = \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} - 2\right) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_{max} = \sigma_{allow} = 17 \text{ MPa}$

$h_{max} = 0.27 \text{ m} \quad \leftarrow$

Problem 2.8-8



$W = Mg = (35 \text{ kg})(9.81 \text{ m/s}^2) = 343.4 \text{ N}$

$A = 40 \text{ mm}^2 \qquad E = 130 \text{ GPa}$

$h = 1.0 \text{ m} \qquad \sigma_{allow} = \sigma_{max} = 500 \text{ MPa}$

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{343.4 \text{ N}}{40 \text{ mm}^2} = 8.585 \text{ MPa}$$

MINIMUM LENGTH L_{min}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

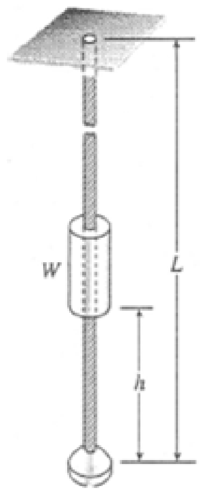
$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2(130 \text{ GPa})(1.0 \text{ m})(8.585 \text{ MPa})}{(500 \text{ MPa})[500 \text{ MPa} - 2(8.585 \text{ MPa})]}$$

$$= 9.25 \text{ m} \quad \leftarrow$$

Problem 2.8-9



$W = 145$ N

$A = 0.5 \text{ cm}^2 \qquad E = 150$ GPa

$h = 120$ cm $\qquad \sigma_a = 480$ MPa

Find minimum length L_{min}.

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = 2.9 \text{ MPa}$$

MINIMUM LENGTH L_{min}

Eq. (2-59): $\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for L:

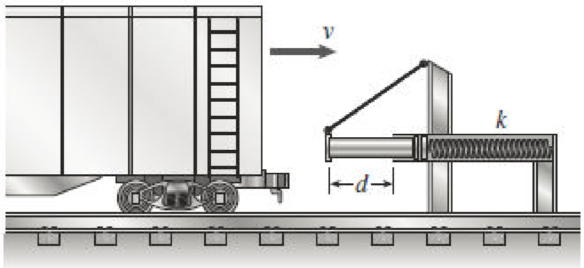
$$L = L_{min} = \frac{2Eh\sigma_{st}}{\sigma_{max}(\sigma_{max} - 2\sigma_{st})} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$L_{min} = \frac{2Eh\sigma_{st}}{\sigma_a(\sigma_a - 2\sigma_{st})}$$

$$= 4.59 \text{ m} \quad \leftarrow$$

Problem 2.8-10



$k = 8.0$ MN/m $\qquad W = 545$ kN

d = maximum displacement of spring

$d = \delta_{max} = 450$ mm

Find v_{max}.

KINETIC ENERGY BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS COMPRESSED TO THE MAXIMUM ALLOWABLE AMOUNT

$$U = \frac{k\delta_{max}^2}{2} = \frac{kd^2}{2}$$

CONSERVATION OF ENERGY

$$KE = U \quad \frac{Wv^2}{2g} = \frac{kd^2}{2} \quad v^2 = \frac{kd^2}{W/g}$$

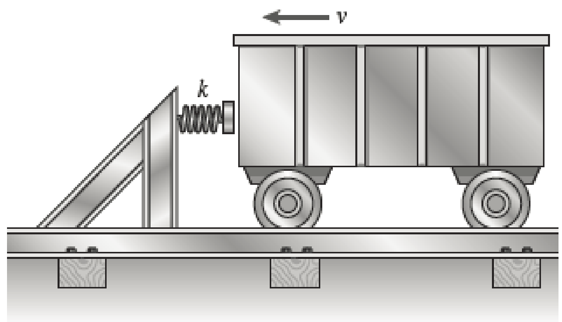
$$v = v_{max} = d\sqrt{\frac{k}{W/g}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$v_{max} = (450\text{ mm})\sqrt{\frac{8.0\text{ MN/m}}{(545\text{ kN})/(9.81\text{ m/s}^2)}}$$

$$= 5400\text{ mm/s} = 5.4\text{ m/s} \quad \leftarrow$$

Problem 2.8-11



$k = 176$ kN/m $\quad W = 14$ kN

$v = 8$ km/hr

$g = 9.81$ m/s^2

SHORTENING OF THE SPRING

Conservation of energy:

KE before import = strain energy when spring is fully compressed

$$\frac{Mv^2}{2} = \frac{k\delta_{max}^2}{2}$$

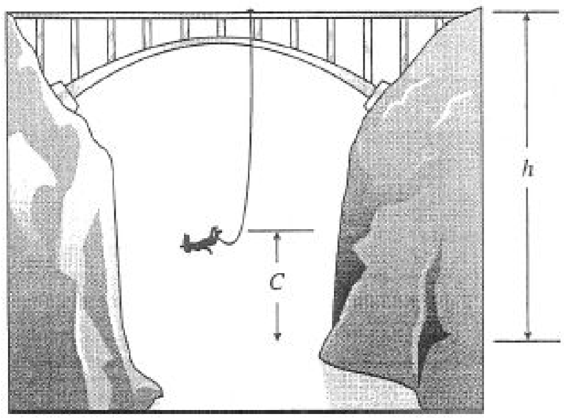
Solve for δ_{max}: $\delta_{max} = \sqrt{\frac{Mv^2}{k}} = \sqrt{\frac{Wv^2}{gk}} \quad \leftarrow$

SUBSTITUTE NUMERICAL VALUES:

$$\delta_{max} = \sqrt{\frac{W \cdot v^2}{g \cdot k}} = \sqrt{\frac{14000\text{N}\left(\frac{8000\text{m}}{3600\text{s}}\right)^2}{9.81\frac{\text{m}}{\text{s}^2}\left(176000\,\frac{\text{N}}{\text{m}}\right)}} = 200\,\text{mm}$$

$\delta_{max} = 0.2\,\text{m} = 200\,\text{mm} \quad \leftarrow$

Problem 2.8-12



$W = Mg = (55 \text{ kg})(9.81 \text{ m/s}^2)$

$= 539.55 \text{ N}$

$EA = 2.3 \text{ kN}$

Height: $h = 60 \text{ m}$

Clearance: $C = 10 \text{ m}$

Find length L of the bungee cord.

$P.E.$ = Potential energy of the jumper at the top of bridge (with respect to lowest position)

$= W(L + \delta_{max})$

U = strain energy of cord at lowest position

$$= \frac{EA\delta_{max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \qquad W(L + \delta_{max}) = \frac{EA\delta_{max}^2}{2L}$$

$$\text{or} \qquad \delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

$$= \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

VERTICAL HEIGHT

$$h = C + L + \delta_{max}$$

$$h - C = L + \frac{WL}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]$$

SOLVE FOR L:

$$L = \frac{h - C}{1 + \frac{W}{EA}\left[1 + \left(1 + \frac{2EA}{W}\right)^{1/2}\right]} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{W}{EA} = \frac{539.55 \text{ N}}{2.3 \text{ kN}} = 0.234587$$

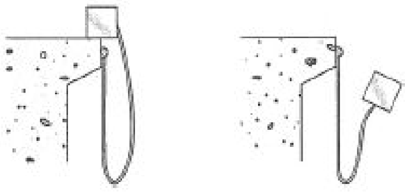
Numerator $= h - C = 60 \text{ m} - 10 \text{ m} = 50 \text{ m}$

$$\text{Denominator} = 1 + (0.234587) \times \left[1 + \left(1 + \frac{2}{0.234587}\right)^{1/2}\right]$$

$$= 1.9586$$

$$L = \frac{50 \text{ m}}{1.9586} = 25.5 \text{ m} \quad \leftarrow$$

Problem 2.8-13



W = Weight

Properties of elastic cord:

E = modulus of elasticity

A = cross-sectional area

L = original length

δ_{max} = elongation of elastic cord

$P.E.$ = potential energy of weight before fall (with respect to lowest position)

$P.E. = W(L + \delta_{max})$

Let U = strain energy of cord at lowest position.

$$U = \frac{EA\delta_{max}^2}{2L}$$

Conservation of energy

$$P.E. = U \qquad W(L + \delta_{max}) = \frac{EA\delta_{max}^2}{2L}$$

or $$\delta_{max}^2 - \frac{2WL}{EA}\delta_{max} - \frac{2WL^2}{EA} = 0$$

Solve quadratic equation for δ_{max}:

$$\delta_{max} = \frac{WL}{EA} + \left[\left(\frac{WL}{EA}\right)^2 + 2L\left(\frac{WL}{EA}\right)\right]^{1/2}$$

Static elongation

$$\delta_{st} = \frac{WL}{EA}$$

Impact factor

$$\frac{\delta_{max}}{\delta_{st}} = 1 + \left[1 + \frac{2EA}{W}\right]^{1/2} \quad \leftarrow$$

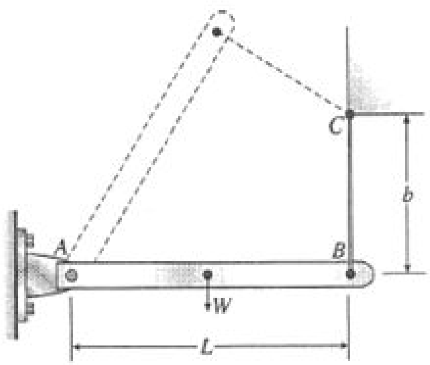
Numerical values

$$\delta_{st} = (2.5\%)(L) = 0.025L$$

$$\delta_{st} = \frac{WL}{EA} \qquad \frac{W}{EA} = 0.025 \qquad \frac{EA}{W} = 40$$

$$\text{Impact factor} = 1 + [1 + 2(40)]^{1/2} = 10 \quad \leftarrow$$

Problem 2.8-14



RIGID BAR:

$W = Mg = (1.0 \text{ kg})(9.81 \text{ m/s}^2)$

$= 9.81 \text{ N}$

$L = 0.5 \text{ m}$

NYLON CORD:

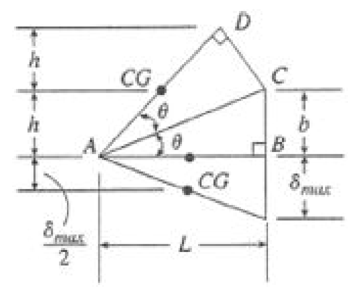
$A = 30 \text{ mm}^2$

$b = 0.25 \text{ m}$

$E = 2.1 \text{ GPa}$

Find maximum stress σ_{max} in cord BC.

GEOMETRY OF BAR AB AND CORD BC



$\overline{CD} = \overline{CB} = b$

$\overline{AD} = \overline{AB} = L$

h = height of center of gravity of raised bar AD

δ_{max} = elongation of cord

From triangle ABC: $\sin\theta = \dfrac{b}{\sqrt{b^2 + L^2}}$

$\cos\theta = \dfrac{L}{\sqrt{b^2 + L^2}}$

From line AD: $\sin 2\theta = \dfrac{2h}{AD} = \dfrac{2h}{L}$

From Appendix C: $\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \frac{2h}{L} = 2\left(\frac{b}{\sqrt{b^2 + L^2}}\right)\left(\frac{L}{\sqrt{b^2 + L^2}}\right) = \frac{2bL}{b^2 + L^2}$$

and $h = \dfrac{bL^2}{b^2 + L^2}$ (Eq. 1)

CONSERVATION OF ENERGY

$P.E.$ = potential energy of raised bar AD

$$= W\left(h + \frac{\delta_{max}}{2}\right)$$

U = strain energy of stretched cord $= \dfrac{EA\delta_{max}^2}{2b}$

$$P.E. = U \quad W\left(h + \frac{\delta_{max}}{2}\right) = \frac{EA\delta_{max}^2}{2b} \qquad \text{(Eq. 2)}$$

For the cord: $\delta_{max} = \dfrac{\sigma_{max} b}{E}$

Substitute into Eq. (2) and rearrange:

$$\sigma_{max}^2 - \frac{W}{A}\sigma_{max} - \frac{2WhE}{bA} = 0 \qquad \text{(Eq. 3)}$$

Substitute from Eq. (1) into Eq. (3):

$$\sigma_{max}^2 - \frac{W}{A}\sigma_{max} - \frac{2WL^2E}{A(b^2 + L^2)} = 0 \qquad \text{(Eq. 4)}$$

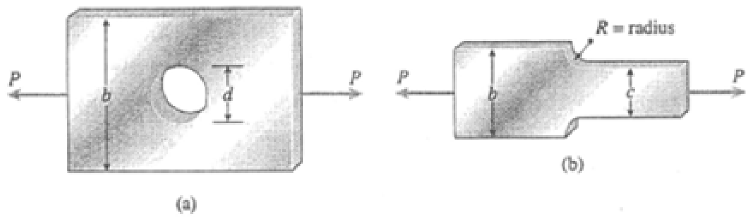
SOLVE FOR σ_{max}:

$$\sigma_{max} = \frac{W}{2A}\left[1 + \sqrt{1 + \frac{8L^2EA}{W(b^2 + L^2)}}\right] \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$\sigma_{max} = 33.3 \text{ MPa}$ ←

Problem 2.10-1



$P = 13$ kN $\quad t = 6$ mm

(a) Bar with circular hole ($b = 150$ mm)

Obtain K from Fig. 2-84

For $d = 25$ mm: $\quad c = b - d = 125$ mm

$$\sigma_{nom} = \frac{P}{ct} = 17.33 \text{ MPa}$$

$d/b = 0.167 \quad K \approx 2.60$

$\sigma_{max} = K\sigma_{nom} \approx 45$ MPa $\leftarrow$

For $d = 50$ mm: $c = b - d = 100$ mm

$$\sigma_{nom} = \frac{P}{ct} = 21.67 \text{ MPa}$$

$d/b = 0.33 \quad K \approx 2.31$

$\sigma_{max} = K\sigma_{nom} \approx 50$ MPa $\leftarrow$

(b) Stepped bar with shoulder fillets

$b = 100$ mm $\quad c = 64$ mm; obtain K from Fig. 2-86.

$$\sigma_{nom} = \frac{P}{ct} = 33.85 \text{ MPa}$$

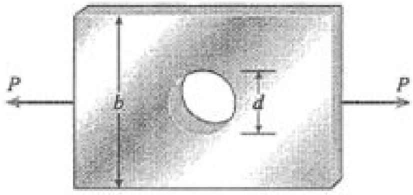
For $R = 6$ mm: $R/c = 0.1 \quad b/c = 1.5$

$K \approx 2.25$, $\sigma_{max} = K\sigma_{nom} \approx 76$ MPa $\leftarrow$

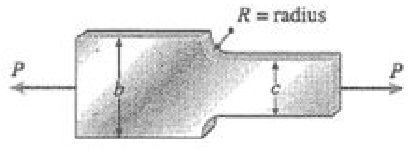
For $R = 12$ mm: $R/c = 0.19, \quad b/c = 1.5$

$K \approx 1.87 \quad \sigma_{max} = K\sigma_{nom} \approx 61$ MPa $\leftarrow$

Problem 2.10-2



(a)



(b)

$P = 2.5\text{ kN} \qquad t = 5.0\text{ mm}$

(a) Bar with circular hole ($b = 60$ mm)

Obtain K from Fig. 2-84

For $d = 12$ mm: $c = b - d = 48$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(48\text{ mm})(5\text{ mm})} = 10.42\text{ MPa}$$

$$d/b = \frac{1}{5} \quad K \approx 2.51$$

$\sigma_{max} = K\sigma_{nom} \approx 26$ MPa ←

For $d = 20$ mm: $c = b - d = 40$ mm

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(40\text{ mm})(5\text{ mm})} = 12.50\text{ MPa}$$

$$d/b = \frac{1}{3} \quad K \approx 2.31$$

$\sigma_{max} = K\sigma_{nom} \approx 29$ MPa ←

(b) Stepped bar with shoulder fillets

$b = 60$ mm $\qquad c = 40$ mm;

Obtain K from Fig 2-86

$$\sigma_{nom} = \frac{P}{ct} = \frac{2.5\text{ kN}}{(40\text{ mm})(5\text{ mm})} = 12.50\text{ MPa}$$

For $R = 6$ mm: $R/c = 0.15 \qquad b/c = 1.5$

$K \approx 2.00 \qquad \sigma_{max} = K\sigma_{nom} \approx 25$ MPa ←

For $R = 10$ mm: $R/c = 0.25 \qquad b/c = 1.5$

$K \approx 1.75 \qquad \sigma_{max} = K\sigma_{nom} \approx 22$ MPa ←

Problem 2.10-3

t = thickness

σ_t = allowable tensile stress

Find P_{max}

Find K from Fig. 2-84

$$P_{max} = \sigma_{nom} ct = \frac{\sigma_{max}}{K} ct = \frac{\sigma_t}{K}(b - d)t$$

$$= \frac{\sigma_t}{K} bt\left(1 - \frac{d}{b}\right)$$

Because σ_t, b, and t are constants, we write:

$$P^* = \frac{P_{max}}{\sigma_t bt} = \frac{1}{K}\left(1 - \frac{d}{b}\right)$$

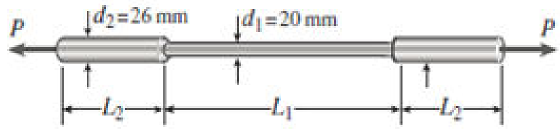
$\frac{d}{b}$	K	P^*
0	3.00	0.333
0.1	2.73	0.330
0.2	2.50	0.320
0.3	2.35	0.298
0.4	2.24	0.268

We observe that P_{max} decreases as d/b increases. Therefore, the maximum load occurs when the hole becomes very small.

$$\left(\frac{d}{b} \to 0 \quad \text{and} \quad K \to 3\right)$$

$$P_{max} = \frac{\sigma_t bt}{3} \quad \leftarrow$$

Problem 2.10-4



$E = 100$ GPa

$\delta = 0.12$ mm

$L_2 = 0.1$ m

$L_1 = 0.3$ m

$$R = \text{radius of fillets} = \frac{26\text{ mm} - 20\text{ mm}}{2} = 3\text{ mm}$$

$$\delta = 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $\quad P = \dfrac{\delta E A_1 A_2}{2L_2A_1 + L_1A_2}$

Use Fig. 2-87 for the stress-concentration factor:

$$\sigma_{nom} = \frac{P}{A_1} = \frac{\delta E A_2}{2L_2A_1 + L_1A_2} = \frac{\delta E}{2L_2\left(\frac{A_1}{A_2}\right) + L_1}$$

$$= \frac{\delta E}{2L_2\left(\frac{d_1}{d_2}\right)^2 + L_1}$$

Substitute numerical values:

$$\sigma_{nom} = \frac{(0.12\text{ mm})(100\text{ GPa})}{2(0.1\text{ m})\left(\frac{20}{26}\right)^2 + 0.3\text{ m}} = 28.68\text{ MPa}$$

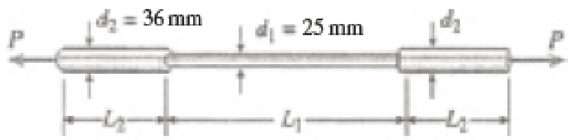
$$\frac{R}{D_1} = \frac{3\text{ mm}}{20\text{ mm}} = 0.15$$

Use the dashed curve in Fig. 2-87. $K \approx 1.6$

$$\sigma_{max} = K\sigma_{nom} \approx (1.6)(28.68\text{ MPa})$$

$$\approx 46\text{ MPa} \quad \leftarrow$$

Problem 2.10-5



$E = 170$ GPa

$\delta = 0.1$ mm

$L_1 = 500$ mm

$L_2 = 125$ mm

R = radius of fillets $\quad R = \dfrac{36 \text{ mm} - 25 \text{ mm}}{2}$

$= 5.5$ mm

$$\delta = 2\left(\frac{PL_2}{EA_2}\right) + \frac{PL_1}{EA_1}$$

Solve for P: $P = \dfrac{\delta E A_1 A_2}{2L_2 A_1 + L_1 A_2}$

Use Fig. 2-87 for the stress-concentration factor.

$$\sigma_{nom} = \frac{P}{A_1} = \frac{\delta E A_2}{2L_2 A_1 + L_1 A_2} = \frac{\delta E}{2L_2\left(\frac{A_1}{A_2}\right) + L_1}$$

$$= \frac{\delta E}{2L_2\left(\frac{d_1}{d_2}\right)^2 + L_1}$$

SUBSTITUTE NUMERICAL VALUES:

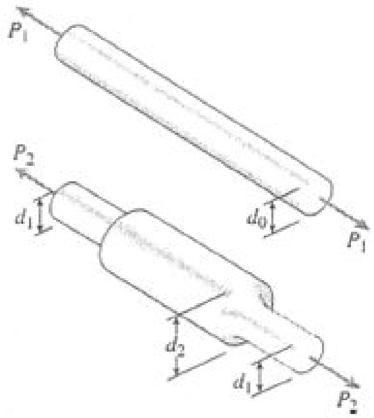
$$\sigma_{nom} = \frac{(0.1)(170)}{\left[2(125)\left(\frac{25}{36}\right)^2 + 500\right]} = 27.39 \text{ MPa}$$

$$\frac{R}{D_1} = \frac{5.5 \text{ mm}}{25 \text{ mm}} = 0.22$$

Use the dashed curve in Fig. 2-87. $K \approx 1.53$

$$\sigma_{max} = K\sigma_{nom} \approx (1.53)(27.39 \text{ MPa})$$
$$\approx 41.9 \text{ MPa} \quad \leftarrow$$

Problem 2.10-6



$d_0 = 20$ mm
$d_1 = 20$ mm
$d_2 = 25$ mm

Fillet radius: $R = 2$ mm

Allowable stress: $\sigma_t = 80$ MPa

(a) Comparison of bars

Prismatic bar: $P_1 = \sigma_t A_0 = \sigma_t\left(\frac{\pi d_0^2}{4}\right)$

$= (80 \text{ MPa})\left(\frac{\pi}{4}\right)(20\text{mm})^2 = 25.1 \text{ kN} \quad \leftarrow$

Stepped bar: See Fig. 2-87 for the stress-concentration factor.

$R = 2.0$ mm $\quad D_1 = 20$ mm $\quad D_2 = 25$ mm

$R/D_1 = 0.10 \quad D_2/D_1 = 1.25 \quad K \approx 1.75$

$$\sigma_{nom} = \frac{P_2}{\frac{\pi}{4}d_1^2} = \frac{P_2}{A_1} \quad \sigma_{nom} = \frac{\sigma_{max}}{K}$$

$$P_2 = \sigma_{nom} A_1 = \frac{\sigma_{max}}{K}A_1 = \frac{\sigma_t}{K}A_1$$

$$= \left(\frac{80 \text{ MPa}}{1.75}\right)\left(\frac{\pi}{4}\right)(20 \text{ mm})^2$$

$$\approx 14.4 \text{ kN} \quad \leftarrow$$

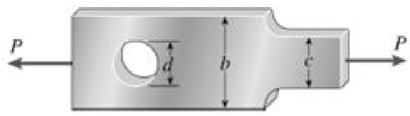
Enlarging the bar makes it *weaker*, not stronger. The ratio of loads is $P_1/P_2 = K = 1.75$

(b) Diameter of prismatic bar for the same allowable load

$$P_1 = P_2 \quad \sigma_t\left(\frac{\pi d_0^2}{4}\right) = \frac{\sigma_t}{K}\left(\frac{\pi d_1^2}{4}\right) \quad d_0^2 = \frac{d_1^2}{K}$$

$$d_0 = \frac{d_1}{\sqrt{K}} \approx \frac{20 \text{ mm}}{\sqrt{1.75}} \approx 15.1 \text{ mm} \quad \leftarrow$$

Problem 2.10-7



$b = 60$ mm

$c = 40$ mm

Fillet radius: $R = 5$ mm

Find d_{max}.

Based upon fillets (Use Fig. 2-86.)

$b = 60$ mm $\quad c = 40$ mm $\quad R = 5$ mm

$R/c = 0.125 \quad b/c = 1.5 \quad K \approx 2.10$

$$P_{max} = \sigma_{nom} ct = \frac{\sigma_{max}}{K} ct = \frac{\sigma_{max}}{K}\left(\frac{c}{b}\right)(bt)$$

$$\approx 0.317 bt\, \sigma_{max}$$

Based upon hole (Use Fig. 2-84)

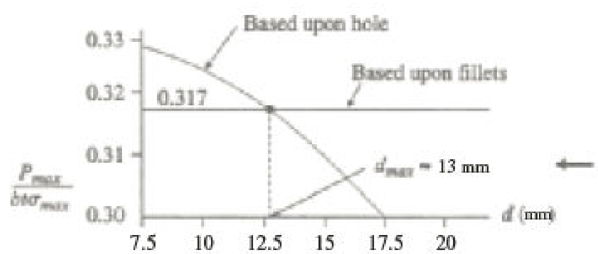
$b = 60$ mm $\quad d$ = diameter of the hole (mm)

$c_1 = b - d$

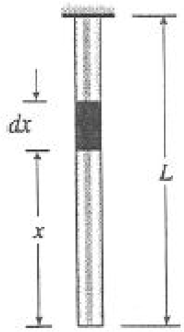
$$P_{max} = \sigma_{nom}\, c_1 t = \frac{\sigma_{max}}{K}(b - d)t$$

$$= \frac{1}{K}\left(1 - \frac{d}{b}\right) bt\sigma_{max}$$

d(mm)	d/b	K	$P_{max}/bt\sigma_{max}$
12	0.20	2.5	0.32
13	0.22	2.45	0.318
14	0.23	2.4	0.321
15	0.25	2.35	0.319



Problem 2.11-1



Let A = cross-sectional area

Let N = axial force at distance x

$N = \gamma A x$

$\sigma = \frac{N}{A} = \gamma x$

STRAIN AT DISTANCE x

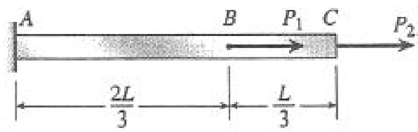
$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma_0\alpha}{E}\left(\frac{\sigma}{\sigma_0}\right)^m = \frac{\gamma x}{E} + \frac{\sigma_0}{\alpha E}\left(\frac{\gamma x}{\sigma_0}\right)^m$$

ELONGATION OF BAR

$$\delta = \int_0^L \varepsilon dx = \int_0^L \frac{\gamma x}{E}dx + \frac{\sigma_0\alpha}{E}\int_0^L \left(\frac{\gamma x}{\sigma_0}\right)^m dx$$

$$= \frac{\gamma L^2}{2E} + \frac{\sigma_0\alpha L}{(m+1)E}\left(\frac{\gamma L}{\sigma_0}\right)^m \qquad \text{Q.E.D.} \quad \leftarrow$$

Problem 2.11-2



$L = 1.8$ m $\qquad A = 480$ mm^2

$P_1 = 30$ kN $\qquad P_2 = 60$ kN

Ramberg–Osgood equation:

$$\varepsilon = \frac{\sigma}{45{,}000} + \frac{1}{618}\left(\frac{\sigma}{170}\right)^{10} \quad (\sigma = \text{MPa})$$

Find displacement at end of bar.

(a) P_1 ACTS ALONE

$$AB\text{: } \sigma = \frac{P_1}{A} = \frac{30 \text{ kN}}{480 \text{ mm}^2} = 62.5 \text{ MPa}$$

$$\varepsilon = 0.001389$$

$$\delta_c = \varepsilon\left(\frac{2L}{3}\right) = 1.67 \text{ mm} \quad \leftarrow$$

(b) P_2 ACTS ALONE

$$ABC\text{:}\sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

$$\varepsilon = 0.002853$$

$$\delta_c = \varepsilon L = 5.13 \text{ mm} \quad \leftarrow$$

(c) BOTH P_1 AND P_2 ARE ACTING

$$AB\text{:}\sigma = \frac{P_1 + P_2}{A} = \frac{90 \text{ kN}}{480 \text{ mm}^2} = 187.5 \text{ MPa}$$

$$\varepsilon = 0.008477$$

$$\delta_{AB} = \varepsilon\left(\frac{2L}{3}\right) = 10.17 \text{ mm}$$

$$BC\text{:}\sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

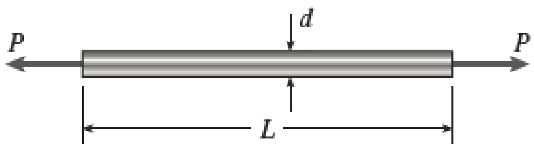
$$\varepsilon = 0.002853$$

$$\delta_{BC} = \varepsilon\left(\frac{L}{3}\right) = 1.71 \text{ mm}$$

$$\delta_C = \delta_{AB} + \delta_{BC} = 11.88 \text{ mm} \quad \leftarrow$$

(Note that the displacement when both loads act simultaneously is *not* equal to the sum of the displacements when the loads act separately.)

Problem 2.11-3

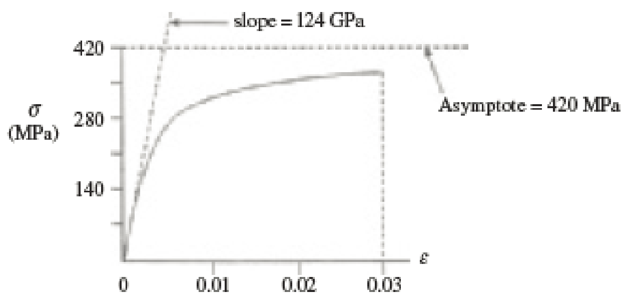


$L = 810$ mm $\quad d = 19$ mm

$$A = \frac{\pi d^2}{4} \quad A = \frac{\pi 19^2}{4} = 283.529$$

(a) Stress-strain diagram

$$\sigma = \left(\frac{18{,}000\varepsilon}{1 + 300\varepsilon}\right)(6.809) \quad 0 \le \varepsilon \le 0.03 \quad (\sigma = \text{MPa})$$



(b) Allowable load P

Max. elongation $\delta_{max} = 6$ mm

Max. stress $\sigma_{max} = 275$ MPa

Based upon elongation:

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{6}{810} = 7.407 \times 10^{-3}$$

$$\sigma_{max} = 6.809\left(\frac{18{,}000\varepsilon_{max}}{1 + 300\varepsilon_{max}}\right) = 281.752$$

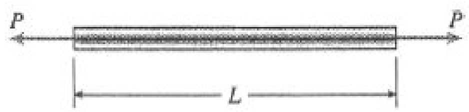
so max. stress controls.

Based upon stress:

$P_a = \sigma_{max}$ A

$P_a = 79.9$ kN $\leftarrow$

Problem 2.11-4



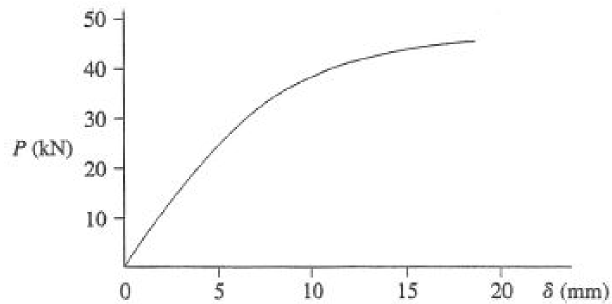
$L = 2.0$ m

$A = 249$ mm^2

STRESS-STRAIN DIAGRAM
(See the problem statement for the diagram)

LOAD-DISPLACEMENT DIAGRAM

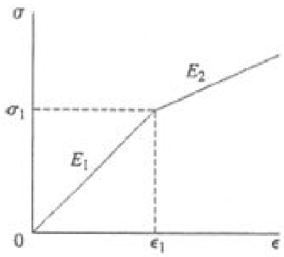
P (kN)	$\sigma = P/A$ (MPa)	ε (from diagram)	$\delta = \varepsilon L$ (mm)
10	40	0.0009	1.8
20	80	0.0018	3.6
30	120	0.0031	6.2
40	161	0.0060	12.0
45	181	0.0081	16.2



NOTE: The load-displacement curve has the same shape as the stress-strain curve.

Problem 2.11-5

STRESS-STRAIN DIAGRAM



$$E_1 = 69 \text{ GPa}$$
$$E_2 = 16.5 \text{ GPa}$$
$$\sigma_1 = 83 \text{ MPa}$$
$$\varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{83 \text{ MPa}}{69 \text{ GPa}}$$
$$= 0.0012$$

For $0 \leq \sigma \leq \sigma_1$:

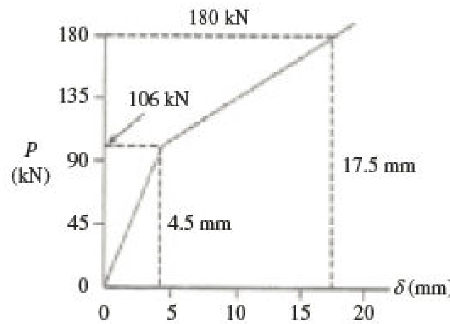
$$\varepsilon = \frac{\sigma}{E_1} = \frac{\sigma}{69 \text{ GPa}} \quad (\sigma = P/A) \qquad \text{(Eq. (1))}$$

For $\sigma \geq \sigma_1$:

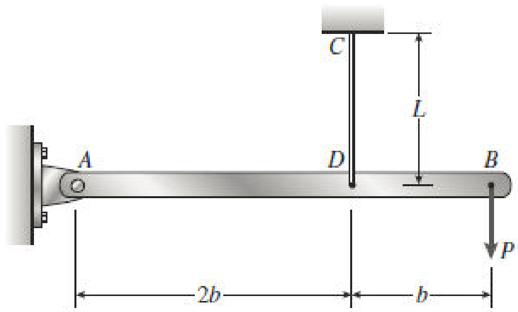
$$\varepsilon = \varepsilon_1 + \frac{\sigma - \sigma_1}{E_2} = 0.0012 + \frac{\sigma - 83 \text{ MPa}}{16.5 \text{ GPa}}$$
$$= \frac{\sigma}{16.5 \text{ GPa}} - 0.0038 \quad (\sigma = P/A) \qquad \text{(Eq. (2))}$$

LOAD-DISPLACEMENT DIAGRAM

P (kN)	$\sigma = P/A$ (MPa)	ε (from Eq. 1 or Eq. 2)	$d = \varepsilon L$ (mm)
35	27	0.00039	1.5
70	54	0.00078	3.0
106	82	0.00121	4.5
140	108	0.00275	10.5
180	139	0.00462	17.5



Problem 2.11-6



Wire: $E = 210$ GPa

$\sigma_Y = 820$ MPa

$L = 1.0$ m

$d = 3$ mm

$A = \dfrac{\pi d^2}{4} = 7.0686\ \text{mm}^2$

STRESS-STRAIN DIAGRAM

$$\sigma = E\varepsilon \qquad (0 \le \sigma \le \sigma_Y) \qquad (1)$$

$$\sigma = \sigma_Y\left(\frac{E\varepsilon}{\sigma_Y}\right)^n \qquad (\sigma \ge \sigma_Y) \qquad (n = 0.2) \qquad (2)$$

(a) DISPLACEMENT δ_B AT END OF BAR

$$\delta = \text{elongation of wire } \delta_B = \frac{3}{2}\delta = \frac{3}{2}\varepsilon L \qquad (3)$$

Obtain ε from stress-strain equations:

$$\text{From Eq. (1): } \varepsilon = \frac{\sigma E}{(0 \le \sigma \le \sigma_Y)} \qquad (4)$$

$$\text{From Eq. (2): } \varepsilon = \frac{\sigma_Y}{E}\left(\frac{\sigma}{\sigma_Y}\right)^{1/n} \qquad (5)$$

Axial force in wire: $F = \dfrac{3P}{2}$

$$\text{Stress in wire: } \sigma = \frac{F}{A} = \frac{3P}{2A} \qquad (6)$$

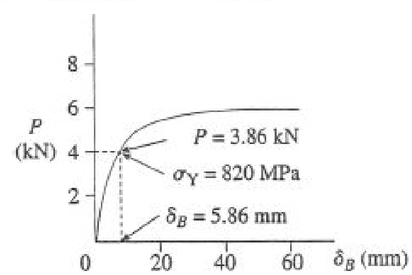
PROCEDURE: Assume a value of P
Calculate σ from Eq. (6)
Calculate ε from Eq. (4) or (5)
Calculate δ_B from Eq. (3)

P (kN)	σ (MPa) Eq. (6)	ε Eq. (4) or (5)	δ_B (mm) Eq. (3)
2.4	509.3	0.002425	3.64
3.2	679.1	0.003234	4.85
4.0	848.8	0.004640	6.96
4.8	1018.6	0.01155	17.3
5.6	1188.4	0.02497	37.5

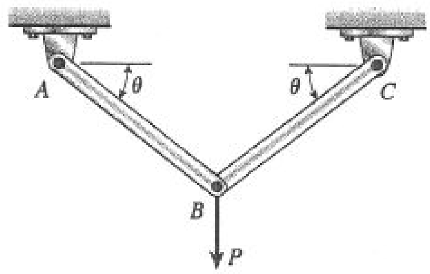
For $\sigma = \sigma_Y = 820$ MPa:

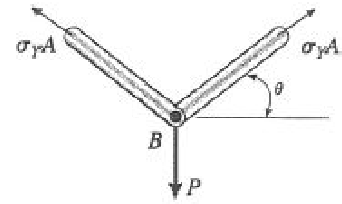
$\varepsilon = 0.0039048 \qquad P = 3.864$ kN $\qquad \delta_B = 5.86$ mm

(b) LOAD-DISPLACEMENT DIAGRAM



Problem 2.12-1





Structure is statically determinate. The yield load P_Y and the plastic lead P_P occur at the same time, namely, when both bars reach the yield stress.

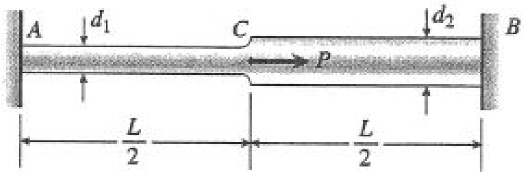
JOINT B

$\Sigma F_{\text{vert}} = 0$

$(2\sigma_Y A) \sin \theta = P$

$P_Y = P_P = 2\sigma_Y A \sin \theta \quad \leftarrow$

Problem 2.12-2

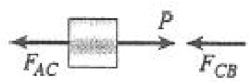


$d_1 = 20$ mm $\quad d_2 = 25$ mm $\quad \sigma_Y = 250$ MPa

DETERMINE THE PLASTIC LOAD P_P:

At the plastic load, all parts of the bar are stressed to the yield stress.

Point C:



$$F_{AC} = \sigma_Y A_1 \qquad F_{CB} = \sigma_Y A_2$$

$$P = F_{AC} + F_{CB}$$

$$P_P = \sigma_Y A_1 + \sigma_Y A_2 = \sigma_Y (A_1 + A_2) \quad \leftarrow$$

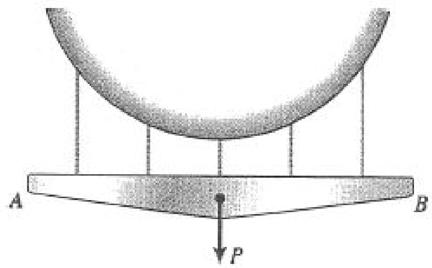
SUBSTITUTE NUMERICAL VALUES:

$$P_P = (250 \text{ MPa})\left(\frac{\pi}{4}\right)(d_1^2 + d_2^2)$$

$$= (250 \text{ MPa})\left(\frac{\pi}{4}\right)[(20 \text{ mm})^2 + (25 \text{ mm})^2]$$

$$= 201 \text{ kN} \quad \leftarrow$$

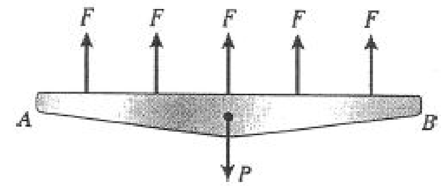
Problem 2.12-3



(a) PLASTIC LOAD P_P

At the plastic load, each wire is stressed to the yield stress. $\therefore P_P = 5\sigma_Y A$ ←

$F = \sigma_Y A$



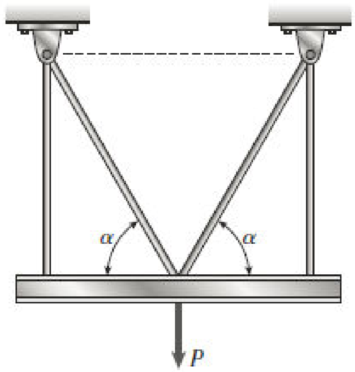
(b) BAR AB IS FLEXIBLE

At the plastic load, each wire is stressed to the yield stress, so the plastic load is not changed. ←

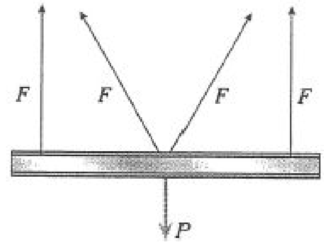
(c) RADIUS R IS INCREASED

Again, the forces in the wires are not changed, so the plastic load is not changed. ←

Problem 2.12-4



At the plastic load, all four rods are stressed to the yield stress.



$F = \sigma_Y A$

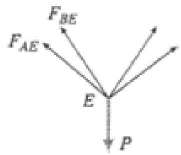
Sum forces in the vertical direction and solve for the load:

$$P_P = 2F + 2F \sin \alpha$$

$$P_P = 2\sigma_Y A\,(1 + \sin \alpha) \quad \leftarrow$$

Problem 2.12-5

Joint E



Equilibrium:

$$2F_{AE}\left(\frac{3}{5}\right) + 2F_{BE}\left(\frac{4}{5}\right) = P$$

or

$$P = \frac{6}{5}F_{AE} + \frac{8}{5}F_{BE}$$

Plastic load P_P

At the plastic load, all bars are stressed to the yield stress.

$$F_{AE} = \sigma_Y A_{AE} \qquad F_{BE} = \sigma_Y A_{BE}$$

$$P_P = \frac{6}{5}\sigma_Y A_{AE} + \frac{8}{5}\sigma_Y A_{BE} \quad \longleftarrow$$

Substitute numerical values:

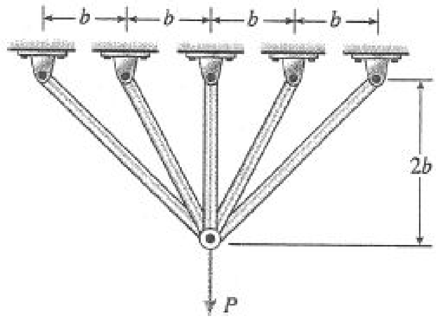
$$A_{AE} = 200 \text{ mm}^2 \quad A_{BE} = 400 \text{ mm}^2$$

$$\sigma_Y = 250 \text{ MPa}$$

$$P_P = \frac{6}{5}(250 \text{ MPa})(200 \text{ mm}^2) + \frac{8}{5}(250 \text{ MPa})(400 \text{ mm}^2)$$

$$= 60 \text{ kN} + 160 \text{ kN} = 220 \text{ kN} \quad \longleftarrow$$

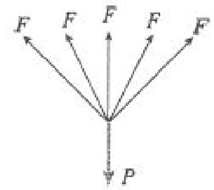
Problem 2.12-6



$d = 10$ mm

$A = \frac{\pi d^2}{4} = 78.54\ \text{mm}^2$

$\sigma_Y = 250$ MPa



At the plastic load, all five bars are stressed to the yield stress

$F = \sigma_Y A$

Sum forces in the vertical direction and solve for the load:

$$P_P = 2F\left(\frac{1}{\sqrt{2}}\right) + 2F\left(\frac{2}{\sqrt{5}}\right) + F$$

$$= \frac{\sigma_Y A}{5}(5\sqrt{2} + 4\sqrt{5} + 5)$$

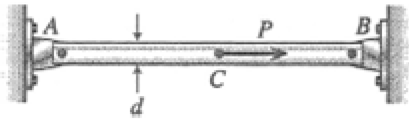
$$= 4.2031\sigma_Y A \quad \leftarrow$$

Substitute numerical values:

$$P_P = (4.2031)(250\ \text{MPa})(78.54\ \text{mm}^2)$$

$$= 82.5\ \text{kN} \quad \leftarrow$$

Problem 2.12-7



$d = 15$ mm

$\sigma_Y = 290$ MPa

Tensile stress = 60 MPa

(a) PLASTIC LOAD P_P

The presence of the initial tensile stress does not affect the plastic load. Both parts of the bar must yield in order to reach the plastic load.

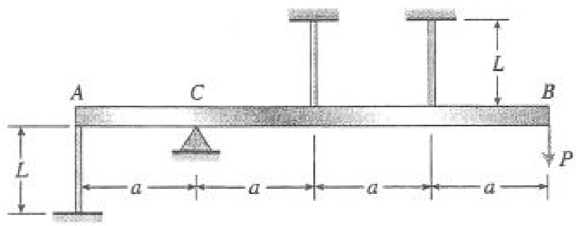
$$\therefore\ P_p = 2\sigma_Y A \quad \leftarrow$$

$$P_P = (2)(290\text{ MPa})\left(\frac{\pi}{4}\right)(15\text{ mm})^2$$

$$= 102\text{ kN} \quad \leftarrow$$

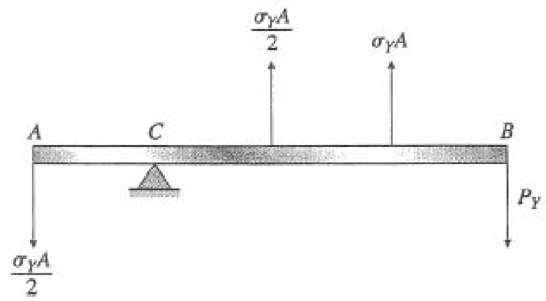
(b) P_P is not changed.

Problem 2.12-8



(a) Yield load P_Y

Yielding occurs when the most highly stressed wire reaches the yield stress σ_Y



$\Sigma M_C = 0$

$P_Y = \sigma_Y A$ ←

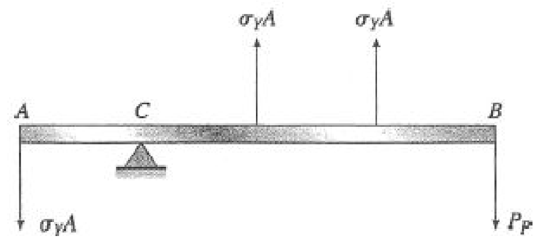
At point A:

$$\delta_A = \left(\frac{\sigma_Y A}{2}\right)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{2E}$$

At point B:

$$\delta_B = 3\delta_A = \delta_Y = \frac{3\sigma_Y L}{2E} \quad \leftarrow$$

(b) Plastic load P_P



At the plastic load, all wires reach the yield stress.

$\Sigma M_C = 0$

$$P_P = \frac{4\sigma_Y A}{3} \quad \leftarrow$$

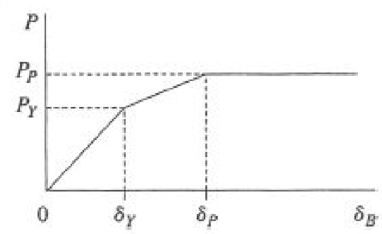
At point A:

$$\delta_A = (\sigma_Y A)\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{E}$$

At point B:

$$\delta_B = 3\delta_A = \delta_P = \frac{3\sigma_Y L}{E} \quad \leftarrow$$

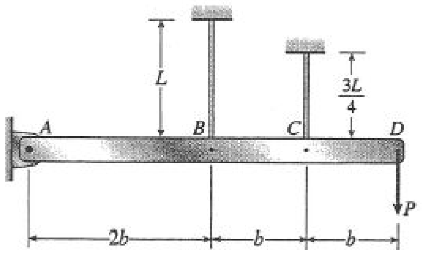
(c) Load-displacement diagram



$$P_P = \frac{4}{3}P_Y$$

$$\delta_P = 2\delta_Y$$

Problem 2.12-9

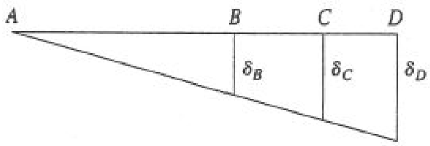


A = cross-sectional area

σ_Y = yield stress

E = modulus of elasticity

DISPLACEMENT DIAGRAM

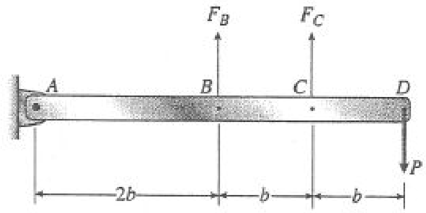


COMPATIBILITY:

$$\delta_C = \frac{3}{2}\delta_B \qquad (1)$$

$$\delta_D = 2\delta_B \qquad (2)$$

FREE-BODY DIAGRAM



EQUILIBRIUM:

$$\Sigma M_A = 0 \quad F_B(2b) + F_C(3b) = P(4b)$$

$$2F_B + 3F_C = 4P \qquad (3)$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{F_B L}{EA} \quad \delta_C = \frac{F_C\left(\frac{3}{4}L\right)}{EA} \qquad (4, 5)$$

Substitute into Eq. (1):

$$\frac{3F_C L}{4EA} = \frac{3F_B L}{2EA}$$

$$F_C = 2F_B \qquad (6)$$

STRESSES

$$\sigma_B = \frac{F_B}{A} \quad \sigma_C = \frac{F_C}{A} \quad \sigma_C = 2\sigma_B \qquad (7)$$

Wire C has the larger stress. Therefore, it will yield first.

(a) YIELD LOAD

$$\sigma_C = \sigma_Y \qquad \sigma_B = \frac{\sigma_C}{2} = \frac{\sigma_Y}{2} \qquad \text{(From Eq. 7)}$$

$$F_C = \sigma_Y A \qquad F_B = \frac{1}{2}\sigma_Y A$$

From Eq. (3):

$$2\left(\frac{1}{2}\sigma_Y A\right) + 3(\sigma_Y A) = 4P$$

$$P = P_Y = \sigma_Y A \quad \leftarrow$$

From Eq. (4):

$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{2E}$$

From Eq. (2):

$$\delta_D = \delta_Y = 2\delta_B = \frac{\sigma_Y L}{E} \quad \leftarrow$$

(b) PLASTIC LOAD

At the plastic load, both wires yield.

$$\sigma_B = \sigma_Y = \sigma_C \qquad F_B = F_C = \sigma_Y A$$

From Eq. (3):

$$2(\sigma_Y A) + 3(\sigma_Y A) = 4P$$

$$P = P_P = \frac{5}{4}\sigma_Y A \quad \leftarrow$$

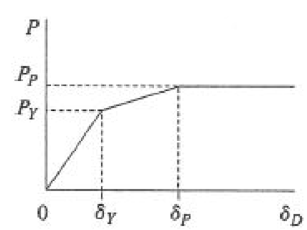
From Eq. (4):

$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{E}$$

From Eq. (2):

$$\delta_D = \delta_P = 2\delta_B = \frac{2\sigma_Y L}{E} \quad \leftarrow$$

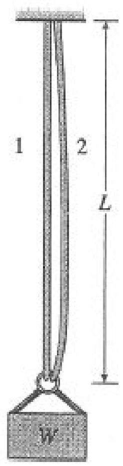
(c) LOAD-DISPLACEMENT DIAGRAM



$$P_P = \frac{5}{4}P_Y$$

$$\delta_P = 2\delta_Y$$

Problem 2.12-10



$L = 40$ m $\quad A = 48.0$ mm^2

$E = 160$ GPa

d = difference in length = 100 mm

$\sigma_Y = 500$ MPa

INITIAL STRETCHING OF CABLE 1

Initially, cable 1 supports all of the load. Let W_1 = load required to stretch cable 1 to the same length as cable 2

$$W_1 = \frac{EA}{L}d = 19.2 \text{ kN}$$

$\delta_1 = 100$ mm (elongation of cable 1)

$$\sigma_1 = \frac{W_1}{A} = \frac{Ed}{L} = 400 \text{ MPa } (\sigma_1 < \sigma_Y \therefore > \text{OK})$$

(a) YIELD LOAD W_Y

Cable 1 yields first. $F_1 = \sigma_Y A = 24$ kN

δ_{1Y} = total elongation of cable 1

δ_{1Y} = total elongation of cable 1

$$\delta_{1Y} = \frac{F_1 L}{EA} = \frac{\sigma_Y L}{E} = 0.125 \text{ m} = 125 \text{ mm}$$

$\delta_Y = \delta_{1Y} = 125$ mm $\leftarrow$

δ_{2Y} = elongation of cable 2

$= \delta_{1Y} - d = 25$ mm

$$F_2 = \frac{EA}{L}\delta_{2Y} = 4.8 \text{ kN}$$

$W_Y = F_1 + F_2 = 24$ kN $+ 4.8$ kN

$= 28.8$ kN $\leftarrow$

(b) PLASTIC LOAD W_P

$F_1 = \sigma_Y A \qquad F_2 = \sigma_Y A$

$W_P = 2\sigma_Y A = 48$ kN $\leftarrow$

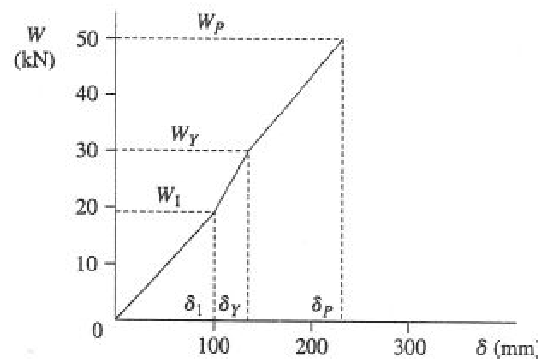
δ_{2P} = elongation of cable 2

$$= F_2\left(\frac{L}{EA}\right) = \frac{\sigma_Y L}{E} = 0.125 \text{ mm} = 125 \text{ mm}$$

$\delta_{1P} = \delta_{2P} + d = 225$ mm

$\delta_P = \delta_{1P} = 225$ mm $\leftarrow$

(c) LOAD-DISPLACEMENT DIAGRAM



$$\frac{W_Y}{W_1} = 1.5 \qquad \frac{\delta_Y}{\delta_1} = 1.25$$

$$\frac{W_P}{W_Y} = 1.667 \qquad \frac{\delta_P}{\delta_Y} = 1.8$$

$0 < W < W_1$: slope = 192,000 N/m

$W_1 < W < W_Y$: slope = 384,000 N/m

$W_Y < W < W_P$: slope = 192,000 N/m

Problem 2.12-11

$L = 380$ mm

$c = 0.26$ mm

$E = 200$ GPa

$\sigma_Y = 250$ MPa

TUBE:

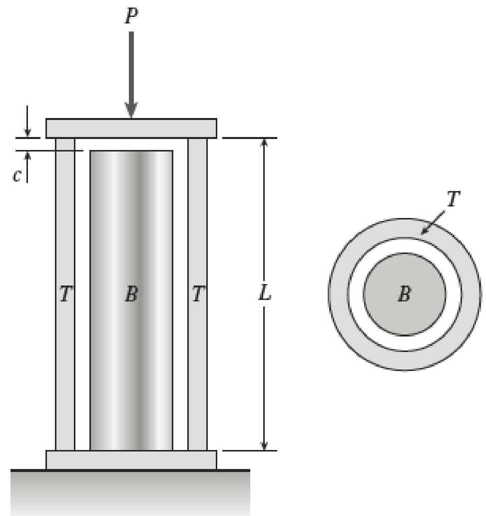
$d_2 = 76$ mm

$d_1 = 70$ mm

$d_b = 38$ mm

$$A_T = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$A_B = \frac{\pi}{4}d_b^2$$



INITIAL SHORTENING OF TUBE T

Initially, the tube supports all of the load.

Let P_1 = load required to close the clearance

$$P_1 = \frac{EA_T}{L}c = 94.1 \text{ kN}$$

Let δ_1 = shortening of tube $\qquad \delta_1 = c = 0.26$ mm

$$\sigma_1 = \frac{P_1}{A_T} = 136.8 \text{ MPa} \qquad (\sigma_1 < \sigma_Y \;\therefore\; \text{OK})$$

(a) YIELD LOAD AND SHORTENING OF TUBE

Because the tube and bar are made of the same material, and because the strain in the tube is larger than the strain in the bar, the tube will yield first.

$F_T = \sigma_Y A_T = 172$ kN

δ_{TY} = shortening of tube at the yield stress

$$\sigma_{TY} = \frac{\sigma_Y L}{E} = 0.475 \text{ mm}$$

$\delta_Y = \delta_{TY} \quad \leftarrow$

δ_{BY} = shortening of bar

$= \delta_{TY} - c = 0.215$ mm

$$F_B = \frac{EA_B}{L}\delta_{BY} = 128.3 \text{ kN}$$

$P_Y = F_T + F_B$

$= 300$ kN

$P_Y = 300$ kN $\quad \leftarrow$

(b) PLASTIC LOAD P_P

$F_T = \sigma_Y A_T \qquad F_B = \sigma_Y A_B$

$P_P = F_T + F_B = \sigma_Y(A_T + A_B)$

$= 456$ kN $\quad \leftarrow$

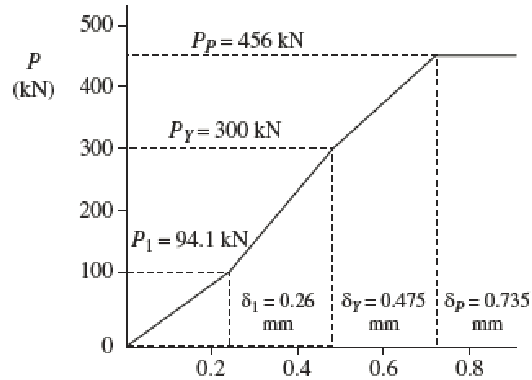
δ_{BP} = shortening of bar

$$= \frac{\sigma_Y L}{E} = 0.475 \text{ mm}$$

$\delta_{TP} = \delta_{BP} + c = 0.735$ mm

$\delta_P = \delta_{TP} \quad \leftarrow$

(c) LOAD-DISPLACEMENT DIAGRAM



$$\frac{P_Y}{P_1} = 3.19 \qquad \frac{\delta_Y}{\delta_1} = 1.827$$

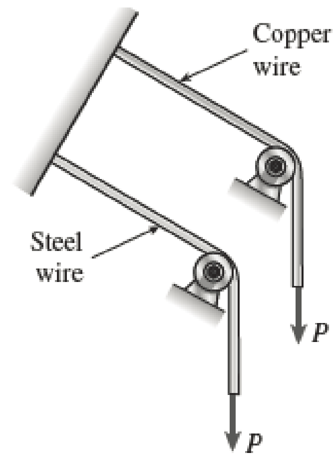
$$\frac{P_P}{P_Y} = 1.517 \qquad \frac{\delta_P}{\delta_Y} = 1.547$$

$$0 < P < P_1\text{: slope} = \frac{P_1}{\delta_1} \qquad \text{slope} = 362 \text{ kN/mm}$$

$$P_1 < P < P_Y\text{: slope} = \frac{P_Y - P_1}{\delta_Y - \delta_1} \qquad \text{slope} = 959 \text{ kN/mm}$$

$$P_1 < P < P_P\text{: slope} = \frac{P_P - P_Y}{\delta_P - \delta_Y} \qquad \text{slope} = 597 \text{ kN/mm}$$

Problem R-2.1



$E_s = 210 \text{ GPa} \qquad E_c = 120 \text{ GPa}$

Displacements are equal: $\delta_s = \delta_c$

or $\dfrac{PL}{E_s A_s} = \dfrac{PL}{E_c A_c}$

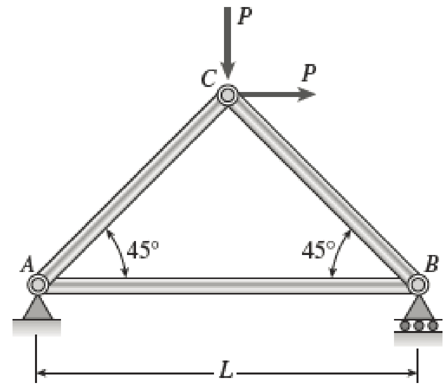
so $E_s A_s = E_c A_c$

and $\dfrac{A_c}{A_s} = \dfrac{E_s}{E_c}$

Express areas in terms of wire diameters, then find ratio:

$$\frac{\dfrac{\pi d_c^2}{4}}{\left(\dfrac{\pi d_s^2}{4}\right)} = \frac{E_s}{E_c} \qquad \text{so} \qquad \frac{d_c}{d_s} = \sqrt{\frac{E_s}{E_c}} = 1.323 \qquad \leftarrow$$

Problem R-2.2



$L = 4.5 \text{ m} \qquad E = 170 \text{ GPa}$

$A = 4500 \text{ mm}^2 \qquad \delta_{max} = 2.7 \text{ mm}$

Statics: sum moments about A to find reaction at B

$$R_B = \frac{P\frac{L}{2} + P\frac{L}{2}}{L} \qquad R_B = P$$

Method of Joints at B:

$F_{AB} = P$ (tension)

Force-displ. relation:

$$P_{max} = \frac{EA}{L}\delta_{max} = 459 \text{ kN} \qquad \leftarrow$$

Check normal stress in bar AB: $\sigma = \frac{P_{max}}{A} = 102.0 \text{ MPa}$ < well below yield stress of 290 MPa in tension

Problem R-2.3

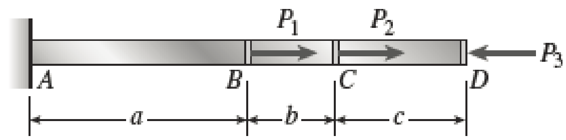
$E = 110 \text{ GPa} \qquad A = 250 \text{ mm}^2$

$a = 2 \text{ m} \qquad b = 0.75 \text{ m}$

$c = 1.2 \text{ m}$

$P_1 = 15 \text{ kN} \qquad P_2 = 10 \text{ kN}$

$P_3 = 8 \text{ kN}$



Segment forces (tension is positive): $N_{AB} = P_1 + P_2 - P_3 = 17.00 \text{ kN}$

$N_{BC} = P_2 - P_3 = 2.00 \text{ kN}$

$N_{CD} = -P_3 = -8.00 \text{ kN}$

Change in length:

$$\delta_D = \frac{1}{EA}(N_{AB}a + N_{BC}b + N_{CD}c) = 0.942 \text{ mm} \qquad \leftarrow$$

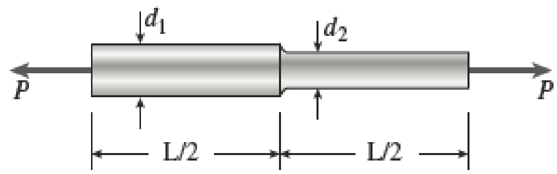
$$\frac{\delta_D}{a + b + c} = 2.384 \times 10^{-4}$$

^positive so elongation

Check max. stress:

$$\frac{N_{AB}}{A} = 68.0 \text{ MPa} \qquad < \text{ well below yield stress for brass, so OK}$$

Problem R-2.4



$L = 2.5\text{ m} \qquad P = 25\text{ kN}$

$d_1 = 18\text{ mm} \qquad d_2 = 12\text{ mm}$

$E = 110\text{ GPa}$

$$A_1 = \frac{\pi}{4}d_1^{\,2} = 254.469\text{ mm}^2$$

$$A_2 = \frac{\pi}{4}d_2^{\,2} = 113.097\text{ mm}^2$$

Volume of nonprismatic bar:

$$\text{Vol}_{\text{nonprismatic}} = (A_1 + A_2)\frac{L}{2} = 459{,}458\text{ mm}^3$$

Diameter of prismatic bar of same volume: $d = \sqrt{\dfrac{\text{Vol}_{nonprismatic}}{\frac{\pi}{4}L}} = 15.30\text{ mm}$

$$A_{\text{prismatic}} = \frac{\pi}{4}d^2 = 184\text{ mm}^2$$

$$V_{\text{prismatic}} = A_{\text{prismatic}}L = 459{,}458\text{ mm}^3$$

Elongation of prismatic bar:

$$\delta = \frac{PL}{EA_{\text{prismatic}}} = 3.09\text{ mm} \quad \leftarrow \quad < \text{less than } \delta \text{ for nonprismatic bar}$$

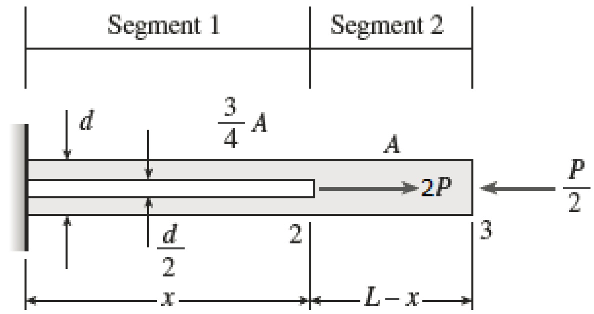
Elongation of nonprismatic bar shown in figure above:

$$\Delta = \frac{PL}{2E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right) = 3.63\text{ mm}$$

Problem R-2.5

Forces in Segments 1 and 2:

$$N_1 = \frac{3P}{2} \qquad N_2 = \frac{-P}{2}$$



Displacement at free end:

$$\delta_3 = \frac{N_1 x}{E\left(\frac{3}{4}A\right)} + \frac{N_2(L - x)}{EA}$$

$$\delta_3 = \frac{\frac{3P}{2}x}{E\left(\frac{3}{4}A\right)} + \frac{\frac{-P}{2}(L - x)}{EA} = -\frac{P(L - 5x)}{2AE}$$

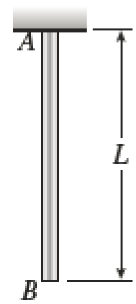
Set δ_3 equal to PL/EA and solve for x:

$$-\frac{P(L - 5x)}{2AE} = \frac{PL}{EA} \quad \text{or}$$

$$-\frac{P(L - 5x)}{2AE} - \frac{PL}{EA} = 0 \text{ simplify then solve for } x\text{: } \rightarrow -\frac{P(3L - 5x)}{2AE} = 0$$

So $x = 3L/5$ $\leftarrow$

Problem R-2.6



$E = 2.1\ \text{GPa} \qquad L = 4.5\ \text{m} \qquad d = 12\ \text{mm}$

$$A = \frac{\pi d^2}{4} = 113.097\ \text{mm}^2$$

$$\gamma = 11\,\frac{\text{kN}}{\text{m}^3}$$

$$W = \gamma L A = 5.598\ \text{N}$$

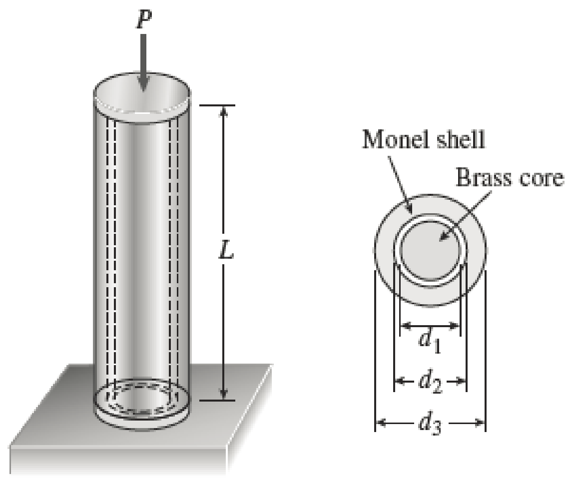
$$\delta_B = \frac{WL}{2EA} \qquad \text{or} \qquad \delta_B = \frac{(\gamma L A)L}{2EA}$$

$$\text{so} \qquad \delta_B = \frac{\gamma L^2}{2E} = 0.053\ \text{mm} \qquad \leftarrow$$

Check max. normal stress at top of bar $\sigma_{max} = \dfrac{W}{A} = 0.050\ \text{MPa}$

$<$ ok - well below ult. stress for nylon

Problem R-2.7



$E_m = 170$ GPa $\quad E_b = 96$ GPa

$d_1 = 6$ mm $\quad d_2 = 8$ mm

$d_3 = 12$ mm $\quad L = 100$ mm

$$A_m = \frac{\pi}{4}(d_3^2 - d_2^2) = 62.832 \text{ mm}^2$$

$$A_b = \frac{\pi}{4}d_1^2 = 28.274 \text{ mm}^2$$

Compatibility: $\quad \delta_m = \delta_b$

$$\frac{P_m L}{E_m A_m} = \frac{P_b L}{E_b A_b} \qquad P_m = \frac{E_m A_m}{E_b A_b} P_b$$

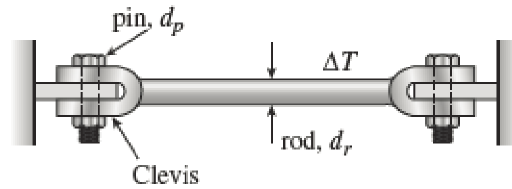
Statics: $\quad P_m + P_b = P \quad$ so $\quad P_b = \dfrac{P}{\left(1 + \dfrac{E_m A_m}{E_b A_b}\right)}$

Set δ_b equal to 0.10 mm and solve for load P:

$$\delta_b = \frac{P_b L}{E_b A_b} \quad \text{so} \quad P_b = \frac{E_b A_b}{L}\delta_b \quad \text{with} \quad \delta_b = 0.10 \text{ mm}$$

and then $\quad P = \dfrac{E_b A_b}{L}\delta_b\left(1 + \dfrac{E_m A_m}{E_b A_b}\right) = 13.40 \text{ kN} \quad \leftarrow$

Problem R-2.8



$E_s = 210$ GPa

$d_r = 12$ mm $\qquad d_p = 15$ mm

$A_r = \frac{\pi}{4} d_r^2 = 113.097 \text{ mm}^2$

$A_p = \frac{\pi}{4} d_p^2 = 176.715 \text{ mm}^2$

$\alpha_s = 12(10^{-6})/°C$

$\tau_a = 45$ MPa $\qquad \sigma_a = 70$ MPa

Force in rod due to temperature drop ΔT: And normal stress in rod:

$$F_r = E_s A_r (\alpha_s) \Delta T \qquad \sigma_r = \frac{F_r}{A_r}$$

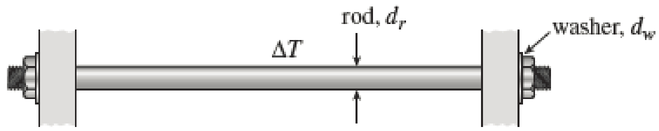
So ΔT_{max} associated with normal stress in **rod**:

$$\Delta T_{maxrod} = \frac{\sigma_a}{E_s \alpha_s} = 27.8 \quad \leftarrow \quad \text{degrees Celsius (decrease) < controls}$$

Now check ΔT based on shear stress in **pin** (in double shear): $\tau_{pin} = \frac{F_r}{2A_p}$

$$\Delta T_{maxpin} = \frac{\tau_a (2A_p)}{E_s A_r \alpha_s} = 55.8$$

Problem R-2.9



$E_s = 210\text{ GPa} \qquad d_r = 15\text{ mm} \qquad d_w = 22\text{ mm}$

$A_r = \dfrac{\pi}{4}d_r^2 = 176.7\text{ mm}^2$

$A_w = \dfrac{\pi}{4}(d_w^2 - d_r^2) = 203.4\text{ mm}^2$

$\alpha_s = 12(10^{-6})/°\text{C}$

$\sigma_{ba} = 55\text{ MPa} \qquad \sigma_a = 90\text{ MPa}$

Force in rod due to temperature drop ΔT: And normal stress in rod:

$F_r = E_s A_r(\alpha_s)\Delta T \qquad \sigma_r = \dfrac{F_r}{A_r}$

So ΔT_{max} associated with normal stress in **rod**:

$\Delta T_{maxrod} = \dfrac{\sigma_a}{E_s \alpha_s} = 35.7$ degrees Celsius (decrease)

Now check ΔT based on bearing stress beneath **washer**: $\sigma_b = \dfrac{F_r}{A_w}$

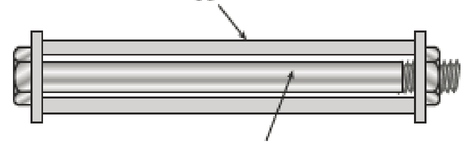
$\Delta T_{maxwasher} = \dfrac{\sigma_{ba}(A_w)}{E_s A_r \alpha_s} = 25.1 \quad \leftarrow$ degrees Celsius (decrease) $<$ controls

Problem R-2.10

$E_s = 210$ GPa $\quad E_c = 110$ GPa $\quad L = 0.5$ m

$A_c = 400\ \text{mm}^2 \quad A_s = 130\ \text{mm}^2$

$n = 0.25 \quad p = 1.25$ mm



Compatibility: shortening of tube and elongation of bolt = applied displacement of $n \times p$

$$\frac{P_c L}{E_c A_c} + \frac{P_s L}{E_s A_s} = np$$

Statics: $\quad P_c = P_s$

Solve for P_s:

$$\frac{P_s L}{E_c A_c} + \frac{P_s L}{E_s A_s} = np \quad \text{or} \quad P_s = \frac{np}{L\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)} = 10.529\ \text{kN}$$

Stress in steel bolt:

$$\sigma_s = \frac{P_s}{A_s} = 81.0\ \text{MPa} \quad \leftarrow \quad < \text{tension}$$

Stress in copper tube:

$$\sigma_c = \frac{P_s}{A_c} = 26.3\ \text{MPa} \quad < \text{compression}$$

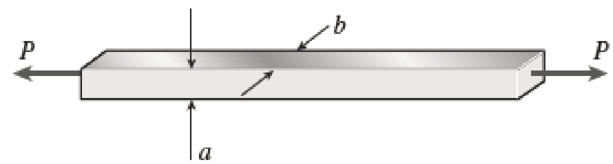
Problem R-2.11

$a = 38$ mm $b = 50$ mm

$A = ab = 1900\ \text{mm}^2$

$\sigma_a = 50$ MPa

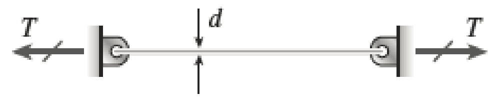
$\tau_a = 24$ MPa



Bar is in uniaxial tension so $\tau_{max} = \sigma_{max}/2$; since $2\tau_a < \sigma_a$, shear stress governs.

$P_{max} = 2\tau_a A = 91.2$ kN   ←

Problem R-2.12



$E = 110 \text{ GPa} \qquad d = 2.0 \text{ mm}$

$\alpha_b = 19.5(10^{-6})/°C \qquad T = 85 \text{ N}$

$A = \frac{\pi}{4} d^2 = 3.14 \text{ mm}^2$

Normal tensile stress in wire due to pretension T and temperature increase ΔT:

$$\sigma = \frac{T}{A} - E\alpha_b\,\Delta T$$

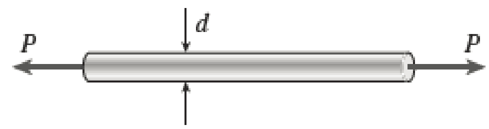
Wire goes slack when normal stress goes to zero; solve for ΔT:

$$\Delta T = \frac{\frac{T}{A}}{E\alpha_b} = +12.61 \qquad \leftarrow \quad \text{degrees Celsius (increase in temperature)}$$

Problem R-2.13

$E = 110 \text{ GPa} \qquad d = 10 \text{ mm}$

$A = \frac{\pi}{4} d^2 = 78.54 \text{ mm}^2$



$P = 11.5 \text{ kN}$

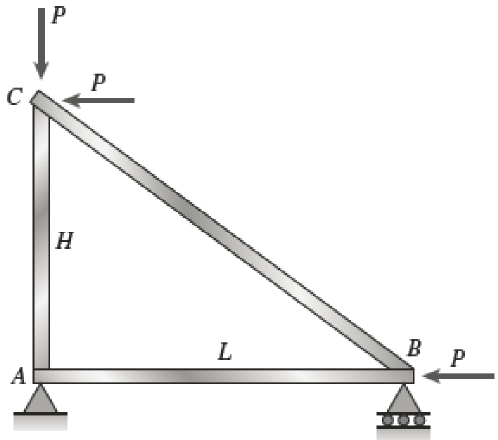
Normal stress in bar:

$$\sigma = \frac{p}{A} = 146.4 \text{ MPa}$$

For bar in uniaxial stress, max. shear stress is on a plane at 45° to axis of bar and equals 1/2 of normal stress:

$$\tau_{max} = \frac{\sigma}{2} = 73.2 \text{ MPa} \qquad \leftarrow$$

Problem R-2.14



$P = 200\text{ kN} \qquad A = 3970\text{ mm}^2 \qquad H = 3\text{ m} \qquad L = 4\text{ m}$

Statics: sum moments about A to find vertical reaction at B

$$B_{vert} = \frac{-PH}{L} = -150.000\text{ kN}$$

(downward)

Method of Joints at B:

$$CB_{vert} = -B_{vert} \qquad CB_{horiz} = \frac{L}{H}CB_{vert} = 200.0\text{ kN}$$

So bar force in AB is: $\quad AB = P + CB_{horiz}$

$$= 400.0\text{ kN} \quad \text{(compression)}$$

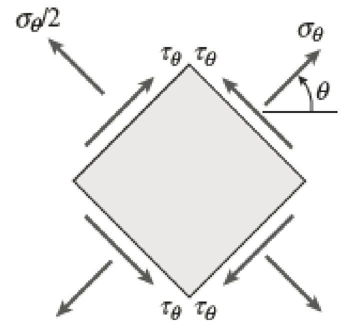
Max. normal stress in AB: $\quad \sigma_{AB} = \dfrac{AB}{A} = 100.8\text{ MPa}$

Max. shear stress is 1/2 of max. normal stress for bar in uniaxial stress and is on plane at 45° to axis of bar:

$$\tau_{max} = \frac{\sigma_{AB}}{2} = 50.4\text{ MPa} \qquad \leftarrow$$

Problem R-2.15

$\sigma_\theta = 78\text{ MPa}$



Plane stress transformation formulas for uniaxial stress:

$$\sigma_x = \frac{\sigma_\theta}{\cos(\theta)^2} \qquad \text{and} \qquad \sigma_x = \frac{\frac{\sigma_\theta}{2}}{\sin(\theta)^2}$$

$\wedge$ on element face at angle θ $\qquad$ $\wedge$ on element face at angle $\theta + 90$

Equate above formulas and solve for σ_x

$$\tan(\theta)^2 = \frac{1}{2}$$

$$\text{so} \quad \theta = \arctan\left(\frac{1}{\sqrt{2}}\right) = 35.264°$$

$$\sigma_x = \frac{\sigma_\theta}{\cos(\theta)^2} = 117.0\text{ MPa} \qquad \text{also} \quad \tau_\theta = -\sigma_x \sin(\theta)\cos(\theta) = -55.154\text{ MPa}$$

Max. shear stress is 1/2 of max. normal stress for bar in uniaxial stress and is on plane at 45° to axis of bar:

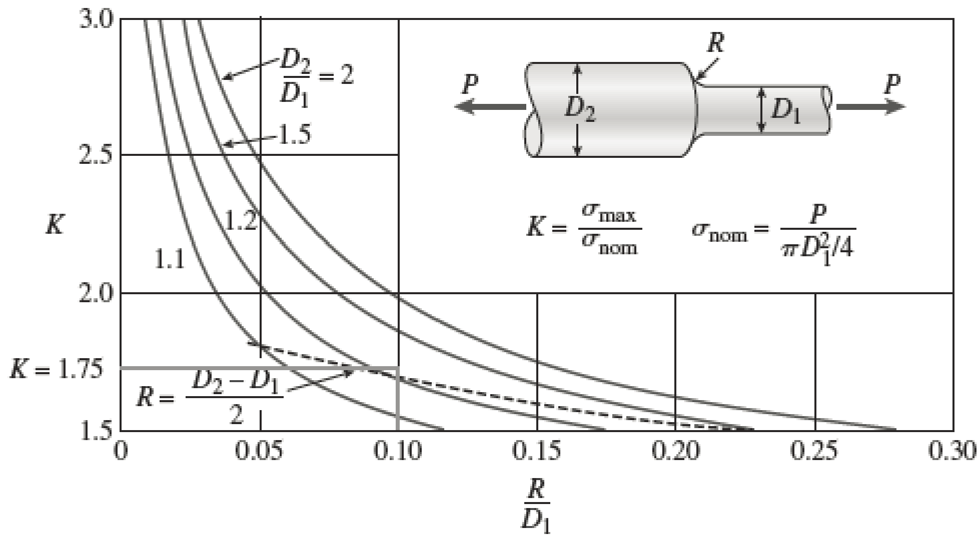
$$\tau_{max} = \frac{\sigma_x}{2} = 58.5\text{ MPa} \quad \leftarrow$$

Problem R-2.16

Prismatic bar $P_{1\,max} = \sigma_{allow}\left(\frac{\pi d_0^{\,2}}{4}\right) = (75\text{ MPa})\left[\frac{\pi(18\text{ mm})^2}{4}\right] = 19.1\text{ kN}$

Stepped bar $\frac{R}{d_1} = \frac{2\text{ mm}}{20\text{ mm}} = 0.100 \quad \frac{d_2}{d_1} = \frac{25\text{ mm}}{20\text{ mm}} = 1.250 \quad \text{so} \quad K = 1.75$

From stress conc. Fig. 2-66:



$$P_{2\,max} = \frac{\sigma_{allow}}{K}\left(\frac{\pi d_1^{\,2}}{4}\right) = \left(\frac{75\text{ MPa}}{K}\right)\left[\frac{\pi(20\text{ mm})^2}{4}\right] = 13.5\text{ kN}$$

$$\frac{P_{1\,max}}{P_{2\,max}} = \frac{19.1\text{ kN}}{13.5\text{ kN}} = 1.41 \quad \leftarrow$$