To Measure the Sky

Hints and solutions to the end-of-chapter exercises

Chapter 1

2. $\lambda = \frac{\text{hc}}{\text{E}} = \frac{1239.85 \text{ eV nm}}{\text{E}}$

a) $\lambda = \frac{1240}{13.6} = 91.2 \text{ nm (Lyman limit)}$

b)

3. f_λ has units of brightness/wavelength = $\left[\frac{\text{W}}{\text{m}^3}\right]$

4. $dF = f_\lambda \text{d}\lambda = f_\upsilon \text{d}\upsilon$ for any amount of flux, so

$$\frac{f_\lambda}{f_\upsilon} = \frac{\text{d}\upsilon}{\text{d}\lambda} = -\frac{\upsilon^2}{c} = -\frac{\text{c}}{\lambda^2}$$

5. The observed flux from either star alone is proportional to its surface area times its surface brightness, so, if we treat both stars as blackbodies, the brightness maximum will be:

$$F_{\text{max}} = \frac{\sigma R^2}{d^2}\left[T_1^4 + a^2 T_2^4\right]$$

When star 1 completely occults star 2, the relative brightness of the system is

$$\frac{F_1}{F_{\text{max}}} = \frac{T_1^4}{\left[T_1^4 + a^2 T_2^4\right]} = \frac{1}{1 + a^2 (T_2^4 / T_1^4)} = \frac{1}{1+b}$$

When star 2 completely overlaps star 1, then the relative brightness is:

$$\frac{F_2}{F_{\text{max}}} = \frac{T_1^4 + a^2 T_2^4 - a^2 T_1^4}{\left[T_1^4 + a^2 T_2^4\right]} = \frac{1 + a^2\ T_2^4 / T_1^4 - a^2}{1 + a^2 (T_2^4 / T_1^4)} = 1 - \frac{a^2}{1+b}$$

6. The energy of 1 photon at frequency 10^6 Hz is $h\nu$=6.63 x 10^{-34} x 10^6=6.63 x 10^{-28} J

$$\text{Flux is} \int_{1{,}000{,}000}^{1{,}000{,}001} \text{f}_\upsilon\, \text{d}\upsilon \cong 10^{-26}\,\text{Wm}^{-2}\text{Hz}^{-1} \cdot 1\text{Hz} = 10^{-26}\,\text{Wm}^{-2} = 10^{-26}\,\text{Js}^{-1}\text{m}^{-2}$$

$$\text{N (photons): } \frac{10^{-26}\,\text{Js}^{-1}\text{m}^{-2}}{6.63 \times 10^{-28}\,\text{J/photon}} = 15 \text{ photons m}^{-2}\text{s}^{-1}$$

7. A magnitude of zero means: $-2.5\log(2.65\times10^{-8})+\mathrm{K}=0 \Rightarrow \mathrm{K}=-18.94$.

8. Assume that the band-pass magnitude is directly proportional to the monochromatic flux at the center of the band, and use the magnitude difference formula to compute the monochromatic flux at the center of the B band in star X:

$$f_x = f_1 \cdot 10^{-0.4(\mathrm{m_x}-\mathrm{m_1})} = 375 \cdot 10^{-0.4(17.79)} = 2.87 \text{ x } 10^{-5} \text{ Jy}$$

(b) The total energy collected is
E = (monochromatic flux) x (bandwidth) x (area) x (exposure time)

Or, converting units:

E = ([2.87 x 10^{-5} Jy] x [10^{-26} J s^{-1} m^{-2} Hz^{-1}/1 Jy) x (2.5 x 10^{14} Hz) x 100 s x 5 m^2
= 3.59 x 10^{-14} J

The corresponding number of photons is

$$N = \frac{E\lambda}{hc} = \frac{(3.59\times10^{-14})(4.4\times10^{-7})}{(6.63\times10^{-34})(3\times10^{8})} = 7942$$

9. The difference between the magnitude of two stars together and the magnitude of one star alone is:

$$\mathrm{m}_{both} - \mathrm{m}_{one} = -2.5\log\left(\frac{\mathrm{F}+\mathrm{F}}{\mathrm{F}}\right) = -2.5\log 2 = -0.753\text{, so}$$

$$\mathrm{m}_{both} = \mathrm{m}_{one} - 0.753 = 7.59$$

10. (a) Apply the reasoning of the previous problem: The magnitude difference between the total nebula and a one arcsec square is:

$$\Delta\mathrm{m} = -2.5\log 144 = -5.40$$
$$\mathrm{m}_{144} = \mathrm{m}_1 - 5.4 = 17.77 - 5.40 = 12.37$$

(b) If distance to the nebula doubles, the surface brightness stays the same, the angular area decreases by a factor of 4, and the total apparent magnitude increases by 2.5 log(4) = 1.51 and becomes 13.88.

11. As in the previous problem, if distance to the nebula doubles, the *intrinsic* surface brightness stays the same, the angular area decreases by a factor of 10^4, and the total apparent magnitude increases by 2.5 log(10^4) = 10 and becomes 22.37. However, in this case one cannot observe the intrinsic surface area or surface brightness because of the limited telescopic resolution. Note that the intrinsic diameter of the nebula would be

$$D_{\text{intrinsic}} = \left[\frac{4}{\pi} A_{\text{intrinsic}}\right]^{\frac{1}{2}} = \left[\frac{4}{\pi}\frac{144}{10^4}\right]^{\frac{1}{2}} = 0.141 \text{ arcsec}$$

This is considerably smaller that the actual observed image diameter of 1.2 arcsec, so the observed image will be spread over an area of

$$A_{obs} = \frac{\pi}{4}(1.2)^2 = 1.13 \text{ arcsec}^2$$

The observed surface brightness will therefore be reduced by a factor of

$$\frac{A_{\text{intrinsic}}}{A_{obs}} = \frac{0.0144}{1.13} = 1.27 \times 10^{-2}$$

over the intrinsic value. Or in magnitudes, the surface brightness is

$$S_{obs} = S_{\text{intrinsic}} - 2.5 \log(1.27 \times 10^{-2}) = 17.77 + 4.74 = 22.51 \text{ mag arcsec}^{-2}$$

12. (a) The distance modulus formula, $m - M = 5 \log(r) - 5$, gives:

$$13.25 - (-19.6) + 5 = 37.8 = 5\log(r)$$
$$r = 3.72 \times 10^7 \text{ pc} = 37.2 \text{ Mpc}$$

(b) To correct for the absorption, repeat the computation above but with the apparent magnitude brightened by 1.5 magnitudes:

$$13.25 - (-19.6) - 1.5 + 5 = 36.3 = 5\log(r)$$
$$r = 1.82 \times 10^7 \text{ pc} = 18.2 \text{ Mpc}$$

13. Begin with the definition of flux: $F = \dfrac{L}{4\pi r^2}$, then let

F=flux observed from an arbitrary distance , r
m = apparent magnitude observed from a distance r
F_{10} = Flux observed from a distance of 10 pc
M = apparent magnitude observed from a distance 10 pc (the absolute magnitude)

Applying the formula for magnitude difference:

$$\Delta m = m - M = -2.5\log\frac{F}{F_{10}} = -2.5\log\frac{10^2}{r^2} = 5\log r - 5$$

14. Note that the definition of logarithms means $F = e^{\ln F}$ and therefore

$$\log_{10} F = \ln F \cdot \log_{10} e$$

So, how much does the magnitude change if the flux changes by amount ΔF? Take the derivative of the magnitude equation: $m = -2.5\log_{10} F + K$, that is:

$$dm = -2.5d(\ln F \cdot \log_{10} e) = -2.5\frac{dF}{F}\log_{10} e = 1.086\frac{dF}{F}, \text{ so}$$

$$\Delta m \simeq \frac{\Delta F}{F}$$

15. It is important that the aperture used for each star be identical in size; i.e. use the same total number of pixels for each star image. One of many reasonable measurements might go like this:

34	16	26	33	37	22	25	25	29	19	28	25
22	20	44	34	22	26	14	30	30	20	19	17
31	70	98	66	37	25	35	36	39	39	23	20
34	99	229	107	38	28	46	102	159	93	37	22
33	67	103	67	36	32	69	240	393	248	69	30
22	33	34	29	36	24	65	241	363	244	68	24
28	22	17	16	32	24	46	85	157	84	42	22
18	25	27	26	17	18	30	29	35	24	30	27
32	23	16	29	25	24	30	28	20	35	22	23
28	28	28	24	26	26	17	19	30	35	30	26

Background: the 18 pixels 3 X 6 box in the lower left corner have a mean value of 24.11, a median of 25, and a mode 25. Either 24.11 or 25 is a reasonable choice for the background. Assume B=24.11.

Brighter star: This star is very symmetric around a point midway between the pixels with values 393 and 363. Add the values of the 16 shaded pixels: Sum of star and background = 2680, so the total number of counts for the brighter star alone is 2680 – (16 x 24.11) = 2294.2

Fainter star: Also very symmetric, but around the center of the pixel with value 229. Take the 21 pixels indicated, but give only 13 pixels full weight (dark shading) and give the 8 outer pixels a weight of 3/8. Thus the effective number of pixels in both star apertures are the same. For the fainter star alone the total counts are:

$$1056 + ((3/8) \times 253) - (16 \times 24.11) = 765.1$$

Magnitude: If the magnitude of the brighter star is 9.000, the magnitude of the fainter is

$$m_f = -2.5\log(765.1/2294.2) + 9.0 = 10.192$$