

Instructor's Manual

Mathematics for Economics and Business

Eighth edition

Ian Jacques

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SOLUTIONS TO PROBLEMS

CHAPTER 1

Linear Equations

Section 1.1 Introduction to algebra

Practice Problems

- (a) -30 (b) 2 (c) -5
(d) 5 (e) 36 (f) -1
- (a) -1 (b) -7 (c) 5
(d) 0 (e) -91 (f) -5
- (a) 19 (b) 1500 (c) 32
(d) 35
- (a) $x + 9y$ (b) $2y + 4z$
(c) not possible
(d) $8r^2 + s + rs - 3s^2$ (e) $-4f$
(f) not possible (g) 0
- (a) $5z - 2z^2$
(b) $6x - 6y + 3y - 6x = -3y$
(c) $x - y + z - x^2 - x + y = z - x^2$
- (a) $7(d + 3)$ (b) $4(4w - 5q)$
(c) $3(3x - y + 3z)$ (d) $5Q(1 - 2Q)$
- (a) $x^2 - 2x + 3x - 6 = x^2 + x - 6$
(b) $x^2 - xy + yx - y^2 = x^2 - y^2$
(c) $x^2 + xy + yx + y^2 = x^2 + 2xy + y^2$

$$\begin{aligned} \text{(d)} \quad & 5x^2 - 5xy + 5x + 2yx - 2y^2 + 2y \\ & = 5x^2 - 3xy + 5x - 2y^2 + 2y \end{aligned}$$

8. (a) $(x + 8)(x - 8)$
 (b) $(2x + 9)(2x - 9)$

Exercise 1.1 (p. 18)

1. (a) -20 (b) 3 (c) -4 (d) 1
 (e) -12 (f) 5 (g) -5 (h) 3
 (i) 30 (j) 4.
2. (a) -1 (b) -3 (c) -11 (d) 16
 (e) -1 (f) -13 (g) 11 (h) 0
 (i) -31 (j) -2
3. (a) -3 (b) 2 (c) 18 (d) -15
 (e) -41 (f) -3 (g) 18 (h) -6
 (i) -25 (j) -6.
4. (a) $2PQ$ (b) $8I$ (c) $3xy$ (d) $4qwz$
 (e) b^2 (f) $3k^2$
5. (a) $19w$ (b) $4x - 7y$ (c) $9a + 2b - 2c$
 (d) $x^2 + 2x$ (e) $4c - 3cd$ (f) $2st + s^2 + t^2 + 9$.
6. (a) 10 (b) 18 (c) 2000
 (d) 96 (e) 70
7. (a) 1 (b) 5 (c) -6 (d) -16
 (e) -30 (f) 44.
8. (a) 16.

(b) Presented with the calculation, -4^2 , your calculator uses BIDMAS, so squares first to get 16 and then subtracts from zero to give a final answer, -16. To obtain the correct answer you need to use brackets:

$$\boxed{(-)} \quad \boxed{-} \quad \boxed{4} \quad \boxed{)} \quad \boxed{x^2} \quad \boxed{=}$$

9. (a) 9 (b) 21 no.
10. (a) 43.96 (b) 1.13 (c) 10.34 (d) 0.17
 (e) 27.38 (f) 3.72 (g) 62.70 (h) 2.39
11. (a) $7x - 7y$ (b) $15x - 6y$ (c) $4x + 12$ (d) $21x - 7$
 (e) $3x + 3y + 3z$ (f) $3x^2 - 4x$ (g) $y + 2z - 2x - 6y + 2z = -2x - 5y + 4z$
12. (a) $5(5c + 6)$ (b) $9(x - 2)$ (c) $x(x + 2)$ (d) $4(4x - 3y)$
 (e) $2x(2x - 3y)$ (f) $5(2d - 3e + 10)$
13. (a) $x^2 + 7x + 10$ (b) $a^2 + 3a - 4$ (c) $d^2 - 5d - 24$
 (d) $6s^2 + 23s + 21$ (e) $2y^2 + 5y + 3$ (f) $10t^2 - 11t - 14$
 (g) $9n^2 - 4$ (h) $a^2 - 2ab + b^2$
14. (a) $6x + 2y$ (b) $11x^2 - 3x - 3$ (c) $14xy + 2x$
 (d) $6xyz + 2xy$ (e) $10a - 2b$ (f) $17x + 22y$
 (g) $11 - 3p$ (h) $x^2 + 10x$
15. (a) $(x + 2)(x - 2)$ (b) $(Q + 7)(Q - 7)$ (c) $(x + y)(x - y)$
 (d) $(3x + 10y)(3x - 10y)$
16. (a) $4x^2 + 8x - 2$ (b) $3x^2 + 2x - 3x^2 - 15x = -13x$
17. $S = 1.2N + 3000E + 1000(A - 21)$; \$204,000
18. (a) $C = 80 + 60L + K$ (b) $C = 10 + 1.25x$ (c) $H = 5a + 10b$ (d) $X = Cd + cm$

Section 1.2 Further algebra

Practice Problems

1. (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2y}$ (d) $\frac{1}{2 + 3x}$ (e) $\frac{1}{x - 4}$
2. (a) $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$

$$(b) \cancel{17} \times \frac{1}{\cancel{14}_2} = \frac{1}{2}$$

$$(c) \frac{2}{3} \div \frac{8}{9} = \frac{\cancel{2}}{1\cancel{3}} \times \frac{\cancel{9}^3}{\cancel{8}_4} = \frac{3}{4}$$

$$(d) \frac{8}{9} \div 16 = \frac{\cancel{8}}{9} \times \frac{1}{\cancel{16}_2} = \frac{1}{18}$$

3. (a) $\frac{3}{7} - \frac{1}{7} = \frac{2}{7}$

(b) $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$

(c) $\frac{7}{18} - \frac{1}{4} = \frac{14}{36} - \frac{9}{36} = \frac{5}{36}$

4. (a) $\frac{\cancel{5}}{\cancel{x-1}} \times \frac{\cancel{x-1}}{x+2} = \frac{5}{x+2}$

(b) $\frac{x^2}{x+10} \div \frac{x}{x+1} = \frac{x^2}{x+10} \times \frac{x+1}{x} = \frac{x(x+1)}{x+10}$

(c) $\frac{4}{x+1} + \frac{1}{x+1} = \frac{4+1}{x+1} = \frac{5}{x+1}$

(d) $\frac{2}{x+1} - \frac{1}{x+2}$
 $= \frac{2(x+2)}{(x+1)(x+2)} - \frac{(1)(x+1)}{(x+1)(x+2)}$
 $= \frac{(2x+4) - (x+1)}{(x+1)(x+2)} = \frac{(x+3)}{(x+1)(x+2)}$

5. (a) $4x + 1 = 25$

$4x = 24$ (subtract 1 from both sides)

$x = 6$ (divide both sides by 4)

(b) $4x + 5 = 5x - 7$

$5 = x - 7$ (subtract $4x$ from both sides)

$12 = x$ (add 7 to both sides)

(c) $3(3 - 2x) + 2(x - 1) = 10$

$$9 - 6x + 2x - 2 = 10 \quad (\text{multiply out brackets})$$

$$7 - 4x = 10 \quad (\text{collect like terms})$$

$$-4x = 3 \quad (\text{subtract 7 from both sides})$$

$$x = -\frac{3}{4} \quad (\text{divide both sides by } -4)$$

(d) $\frac{4}{x-1} = 5$

$$4 = 5(x - 1) \quad (\text{multiply both sides by } x - 1)$$

$$4 = 5x - 5 \quad (\text{multiply out brackets})$$

$$9 = 5x \quad (\text{add 5 to both sides})$$

$$\frac{9}{5} = x \quad (\text{divide both sides by 5})$$

(e) $\frac{3}{x} = \frac{5}{x-1}$

$$3(x - 1) = 5x \quad (\text{cross-multiplication})$$

$$3x - 3 = 5x \quad (\text{multiply out brackets})$$

$$-3 = 2x \quad (\text{subtract } 3x \text{ from both sides})$$

$$-\frac{3}{2} = x \quad (\text{divide both side by 2})$$

6. (a) $12 > 9$ (true) (b) $12 > 6$ (true)

(c) $3 > 0$ (true) (d) same as (c)

(e) $2 > 1$ (true) (f) $-24 > -12$ (false)

(g) $-6 > -3$ (false) (h) $-2 > -1$ (false)

(i) $-4 > -7$ (true).

7. (a) $2x < 3x + 7$

$$-x < 7 \quad (\text{subtract } 3x \text{ from both sides})$$

$$x > -7 \quad (\text{divide both sides by } -1 \text{ changing sense because } -1 < 0)$$

(b) $21x - 19 \geq 4x + 15$

$17x - 19 \geq 15$ (subtract $4x$ from both sides)

$17x \geq 34$ (add 19 to both sides)

$x \geq 2$ (divide both sides by 17, leaving inequality unchanged because $17 > 0$)

Exercise 1.2 (p. 36)

1. (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{1}{3}$ (e) $\frac{4}{3} = 1\frac{1}{3}$

2. (a) $\frac{35}{100} = \frac{7}{20}$; (b) $\frac{56}{35} = 1\frac{3}{5}$

3. (a) $\frac{2x}{3}$ (b) $\frac{1}{2x}$ (c) $\frac{1}{ac}$ (d) $\frac{2}{3xy}$ (e) $\frac{3a}{4b}$

4. (a) $\frac{2p}{2(2q+3r)} = \frac{p}{2q+3r}$ (b) $\frac{x}{x(x-4)} = \frac{1}{x-4}$ (c) $\frac{3ab}{3a(2a+1)} = \frac{b}{2a+1}$

(d) $\frac{14d}{7d(3-e)} = \frac{2}{3-e}$ (e) $\frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2}$ (using the difference of two squares for the denominator)

5. $\frac{x-1}{2x-2} = \frac{x-1}{2(x-1)} = \frac{1}{2}$; other two have no common factors on top and bottom.

6. (a) $\frac{3}{7}$ (b) $-\frac{1}{3}$ (c) $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

(d) $\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$ (e) $\frac{3}{18} + \frac{4}{18} = \frac{7}{18}$ (f) $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

(g) $\frac{5}{2} \times \frac{3^1}{4} = \frac{5}{8}$ (h) $\frac{2^4}{5} \times \frac{3^1}{2} = \frac{2}{5}$ (i) $\frac{7}{4} \times \frac{2^1}{3} = \frac{7}{12}$

(j) $\frac{1^2}{15} \times \frac{5^1}{4} = \frac{1}{30}$ (k) $\frac{2}{9} \times \frac{1}{3} = \frac{2}{27}$ (l) $\frac{3}{1} \times \frac{7}{2} = \frac{21}{2} = 10\frac{1}{2}$

7. $47\frac{1}{2} \div 1\frac{1}{4} = \frac{95}{2} \div \frac{5}{4} = \frac{95}{2} \times \frac{4}{5} = 38$

8. (a) $\frac{2}{3x} + \frac{1}{3x} = \frac{3}{3x} = \frac{1}{x}$ (b) $\frac{2}{1^x} \times \frac{x^1}{5} = \frac{2}{5}$ (c) $\frac{3}{x} - \frac{2}{x^2} = \frac{3x}{x^2} - \frac{2}{x^2} = \frac{3x-2}{x^2}$

$$(d) \frac{7}{x} + \frac{2}{y} = \frac{7y}{xy} + \frac{2x}{xy} = \frac{7y+2x}{xy}$$

$$(e) \frac{a}{2} \div \frac{a}{6} = \frac{1a}{2} \times \frac{6^1}{a_1} = 3$$

$$(f) \frac{5c}{12} + \frac{5d}{18} = \frac{15c}{36} + \frac{10d}{36} = \frac{15c+10d}{36}$$

$$(g) \frac{x+2}{\cancel{1}y-5} \times \frac{\cancel{y-5}^1}{x+3} = \frac{x+2}{x+3}$$

$$(h) \frac{4gh}{7} \div \frac{2g}{9h} = \frac{24gh}{7} \times \frac{9h}{2g} = \frac{18h^2}{7}$$

$$(i) \frac{t}{4} \div 5 = \frac{t}{4} \times \frac{1}{5} = \frac{t}{20}$$

$$(j) \frac{1P}{1Q} \times \frac{Q^1}{P_1} = 1$$

9. (a) $x + 2 = 7$

$$x = 5 \quad (\text{subtract 2 from both sides})$$

(b) $3x = 18$

$$x = 6 \quad (\text{divide both sides by 3})$$

(c) $\frac{x}{9} = 2$

$$x = 18 \quad (\text{multiply both sides by 9})$$

(d) $x - 4 = -2$

$$x = 2 \quad (\text{add 4 to both sides})$$

(e) $2x - 3 = 17$

$$2x = 20 \quad (\text{add 3 to both sides})$$

$$x = 10 \quad (\text{divide both sides by 2})$$

(f) $3x + 4 = 1$

$$3x = -3 \quad (\text{subtract 4 from both sides})$$

$$x = -1 \quad (\text{divide both sides by 3})$$

(g) $\frac{x}{6} - 7 = 3$

$$\frac{x}{6} = 10 \quad (\text{add 7 to both sides})$$

$$x = 60 \quad (\text{multiply both sides by 6})$$

(h) $3(x - 1) = 2$

$$3x - 3 = 2 \quad (\text{multiply out brackets})$$

$$3x = 5 \quad (\text{add 3 to both sides})$$

$$x = \frac{5}{3} = 1\frac{2}{3} \quad (\text{divide both sides by 3})$$

(i) $4 - x = 9$
 $-x = 5$ (subtract 4 from both sides)
 $x = -5$ (divide both sides by -1)

(j) $6x + 2 = 5x - 1$
 $x + 2 = -1$ (subtract $5x$ from both sides)
 $x = -3$ (subtract 2 from both sides)

(k) $5(3x + 8) = 10$
 $15x + 40 = 10$ (multiply out brackets)
 $15x = -30$ (subtract 40 from both sides)
 $x = -2$ (divide both sides by 15)

(l) $2(x - 3) = 5(x + 1)$
 $2x - 6 = 5x + 5$ (multiply out brackets)
 $-3x - 6 = 5$ (subtract $5x$ from both sides)
 $-3x = 11$ (add 6 to both sides)
 $x = \frac{-11}{3} = -3\frac{2}{3}$ (divide both sides by -3)

(m) $\frac{4x - 7}{3} = 2$
 $4x - 7 = 6$ (multiply both sides by 3)
 $4x = 13$ (add 7 to both sides)
 $x = \frac{13}{4} = 3\frac{1}{4}$ (divide both sides by 4)

(n) $\frac{4}{x+1} = 1$
 $4 = x + 1$ (multiply both sides by $x + 1$)
 $3 = x$ (subtract 1 from both sides)

(o) $5 - \frac{1}{x} = 1$
 $5 = 1 + \frac{1}{x}$ (add $\frac{1}{x}$ to both sides)
 $4 = \frac{1}{x}$ (subtract 1 from both sides)
 $4x = 1$ (multiply both sides by x)
 $x = \frac{1}{4}$ (divide both sides by 4)

10. (a), (d), (e), (f).

11. (a) $2x > x + 1$

$$x > 1 \quad (\text{subtract } x \text{ from both sides})$$

(b) $7x + 3 \leq 9 + 5x$

$$2x + 3 \leq 9 \quad (\text{subtract } 5x \text{ from both sides})$$

$$2x \leq 6 \quad (\text{subtract } 3 \text{ from both sides})$$

$$x \leq 3 \quad (\text{divide both sides by } 2)$$

(c) $x - 5 > 4x + 4$

$$-3x - 5 > 4 \quad (\text{subtract } 4x \text{ from both sides})$$

$$-3x > 9 \quad (\text{add } 5 \text{ to both sides})$$

$$x < -3 \quad (\text{divide both sides by } -3)$$

(d) $x - 1 < 2x - 3$

$$-x - 1 < -3 \quad (\text{subtract } 2x \text{ from both sides})$$

$$-x < -2 \quad (\text{add } 1 \text{ to both sides})$$

$$x > 2 \quad (\text{divide both sides by } -1)$$

$$12. \frac{4}{x^2 y} \div \frac{2x}{y} = \frac{4}{x^2 y} \times \frac{y}{2x} = \frac{2}{x^3}$$

13. (a) $6(2 + x) = 5(1 - 4x)$

$$12 + 6x = 5 - 20x \quad (\text{multiply out brackets})$$

$$12 + 26x = 5 \quad (\text{add } 20x \text{ to both sides})$$

$$26x = -7 \quad (\text{subtract } 12 \text{ from both sides})$$

$$x = -\frac{7}{26} \quad (\text{divide both sides by } 26)$$

(b) $3x + 6 \geq 5x - 14$

$$-2x + 6 \geq -14 \quad (\text{subtract } 5x \text{ from both sides})$$

$$-2x \geq -20 \quad (\text{subtract } 6 \text{ from both sides})$$

$$x \leq 10 \quad (\text{divide both sides by } -2)$$

Section 1.3 Graphs of linear equations

Practice Problems

1. From Figure S1.1 note that all five points lie on a straight line.

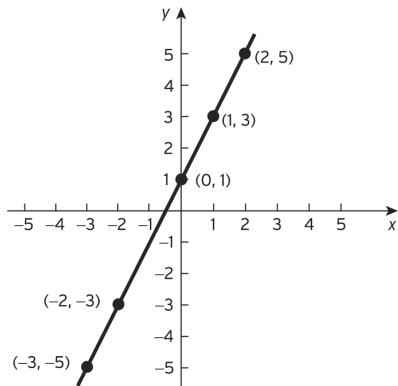


Figure S1.1

- 2.

Point	Check	
$(-1, 2)$	$2(-1) + 3(2) = -2 + 6 = 4$	✓
$(-4, 4)$	$2(-4) + 3(4) = -8 + 12 = 4$	✓
$(5, -2)$	$2(5) + 3(-2) = 10 - 6 = 4$	✓
$(2, 0)$	$2(2) + 3(0) = 4 + 0 = 4$	✓

The graph is sketched in Figure S1.2.

The graph shows that $(3, -1)$ does not lie on the line. This can be verified algebraically:

$$2(3) + 3(-1) = 6 - 3 = 3 \neq 4$$

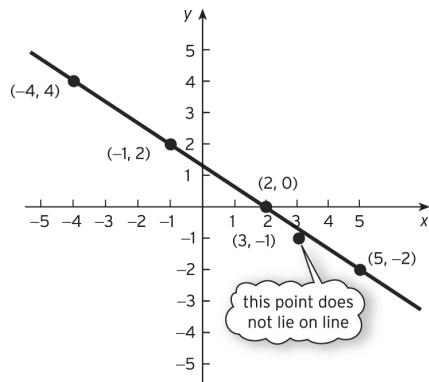


Figure S1.2

$$\begin{aligned}
 3. \quad & 3x - 2y = 4 \\
 & 3(2) - 2y = 4 \quad (\text{substitute } x = 2) \\
 & 6 - 2y = 4 \\
 & -2y = -2 \quad (\text{subtract 6 from both sides}) \\
 & y = 1 \quad (\text{divide both sides by } -2)
 \end{aligned}$$

Hence (2,1) lies on the line.

$$\begin{aligned}
 & 3x - 2y = 4 \\
 & 3(-2) - 2y = 4 \\
 & -6 - 2y = 4 \quad (\text{substitute } x = -2) \\
 & -2y = 10 \quad (\text{add 6 to both sides}) \\
 & y = -5 \quad (\text{divide both sides by } -2)
 \end{aligned}$$

Hence (-2, -5) lies on the line.

The line is sketched in Figure S1.3.

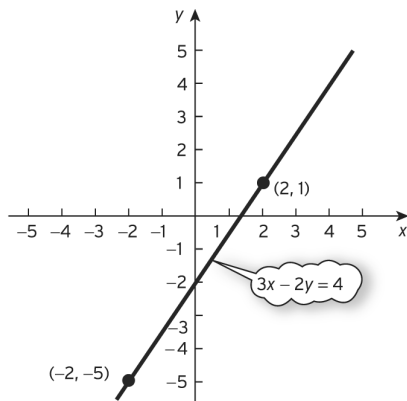


Figure S1.3

$$\begin{aligned}
 4. \quad & x - 2y = 2 \\
 & 0 - 2y = 2 \quad (\text{substitute } x = 0) \\
 & -2y = 2 \\
 & y = -1 \quad (\text{divide both sides by } -2)
 \end{aligned}$$

Hence (0, -1) lies on the line.

$$\begin{aligned}
 & x - 2y = 2 \\
 & x - 2(0) = 2 \quad (\text{substitute } y = 0) \\
 & x - 0 = 2 \\
 & x = 2
 \end{aligned}$$

Hence (2,0) lies on the line.

The graph is sketched in Figure S1.4.

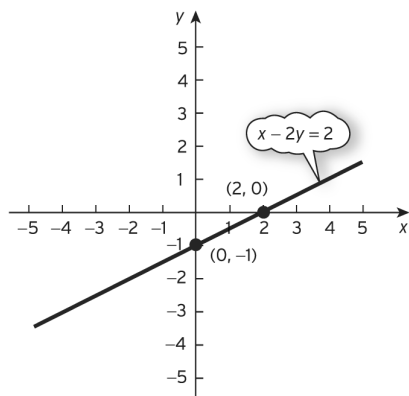


Figure S1.4

5. From Figure S1.5 the point of intersection is $(1, -1/2)$.

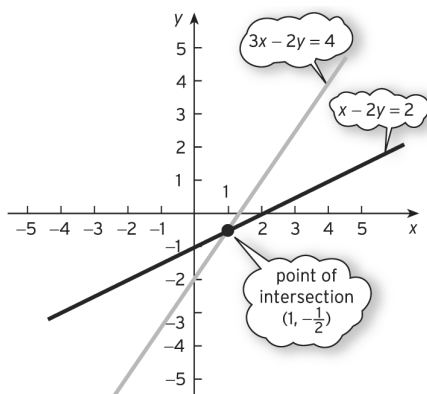


Figure S1.5

6. (a) $a = 1, b = 2$. The graph is sketched in Figure S1.7.

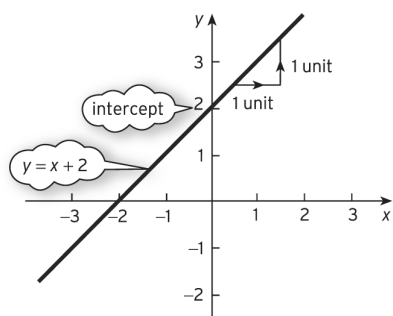


Figure S1.7