SECTION 2.1

1. a. "The set of all black automobiles" is well-defined because an automobile is either black or not black.
   b. "The set of all inexpensive automobiles" is not well-defined because we need a level of cost (e.g., under $8,000) specified to determine whether an automobile is expensive or inexpensive.
   c. "The set of all prime numbers" is well-defined as it can be determined whether or not a number is prime or not prime.
   d. "The set of all large numbers" is not well-defined because different people will have different definitions for large.

2. a. $n(A) = 7$
   b. True
   c. False
   d. True

3. Proper: { }, {Lennon}, {McCartney};
   Improper: {Lennon, McCartney}

4. Proper: { }; Improper: {0}

5. Proper: { }, {yes}, {no}, {undecided}, {yes, no}, {yes, undecided}, {no, undecided}
   Improper: {yes, no, undecided}

6. Proper: { }, {classical}, {country}, {jazz}, {rock}, {classical, country}, {classical, jazz}, {classical, rock}, {country, jazz}, {country, rock}, {jazz, rock}, {classical, country, jazz}, {classical, country, rock}, {classical, jazz, rock}, {country, jazz, rock}
   Improper:{classical, country, jazz, rock}

7. a. {4, 5}
   b. {1, 2, 3, 4, 5, 6, 7, 8}
   c. {0, 6, 7, 8, 9}
   d. {0, 1, 2, 3, 9}

8. a. {2, 7}
   b. {2, 3, 4, 5, 6, 7}
   c. {0, 1, 4, 6, 8, 9}
   d. {0, 1, 3, 5, 8, 9}

9. a. { }
   b. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
   c. {0, 2, 4, 6, 8}
   d. {1, 3, 5, 7, 9}

10. a. { }
    b. {3, 4, 6, 8, 9}
    c. {0, 1, 2, 4, 5, 7, 8}
    d. {0, 1, 2, 3, 5, 6, 7, 9}

11. {Friday}

12. {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

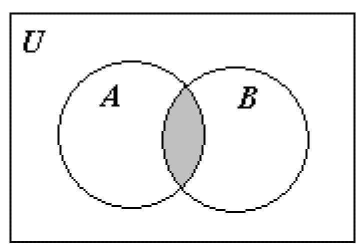
13. {Monday, Tuesday, Wednesday, Thursday}

14. {Saturday, Sunday}

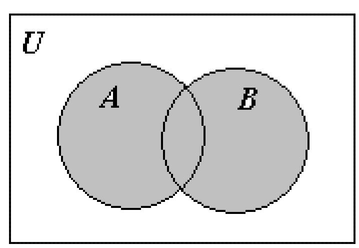
15. {Friday, Saturday, Sunday}

16. {Monday, Tuesday, Wednesday, Thursday}

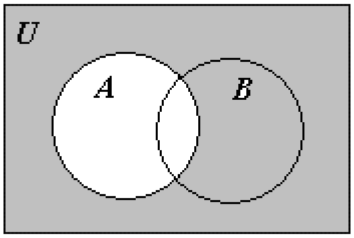
17.



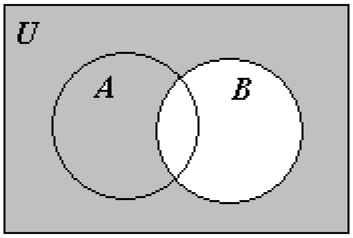
18.



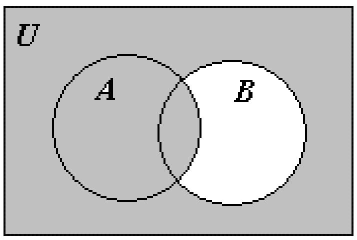
19.



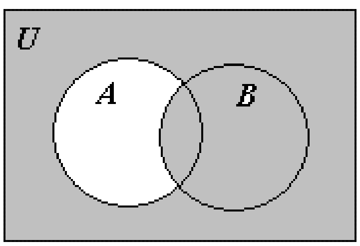
20.



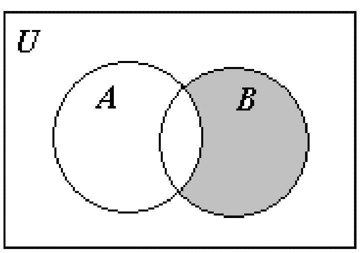
21.



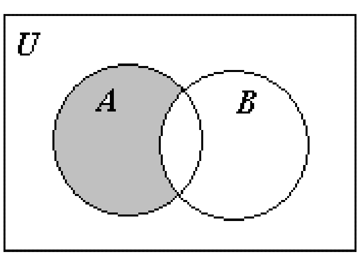
22.



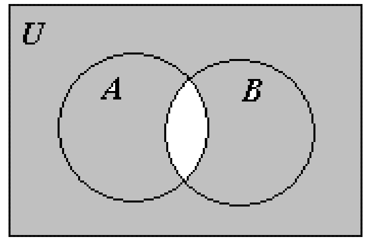
23.



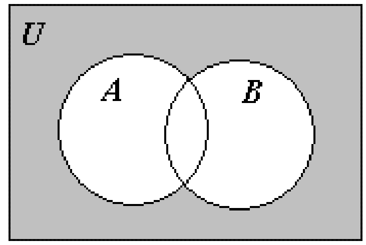
24.



25.



26.

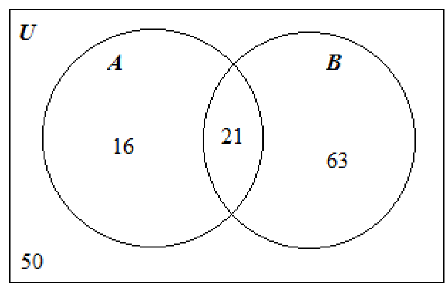


27.a. Rearranging the Cardinal Number Formula for the Union of Sets:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(A \cap B) = 37 + 84 - 100$$

$$n(A \cap B) = 21$$

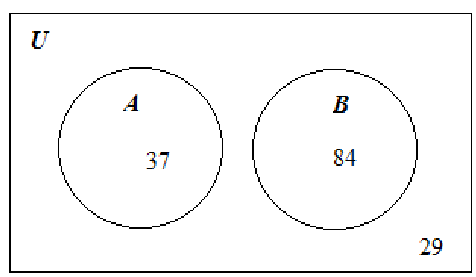


b. Using the formula from part a:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(A \cap B) = 37 + 84 - 121$$

$$n(A \cap B) = 0$$



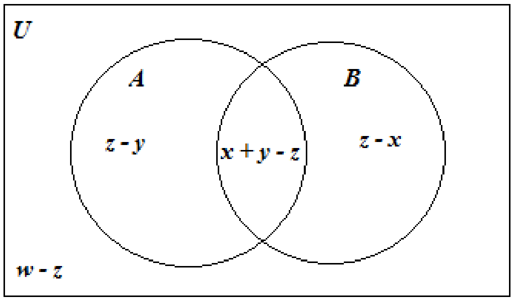
28. a. $A \subseteq (A \cup B)$. Thus, $n(A) \le n(A \cup B)$. Hence, $x \le z$.

b. $w \ge z$, $w > y$, $y \le z$, $x < w$

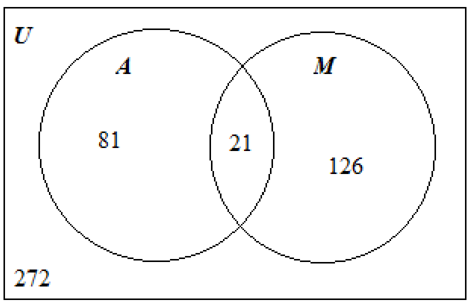
c. Rearranging the Cardinal Number Formula for the Union of Sets:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$n(A \cap B) = x + y - z$$

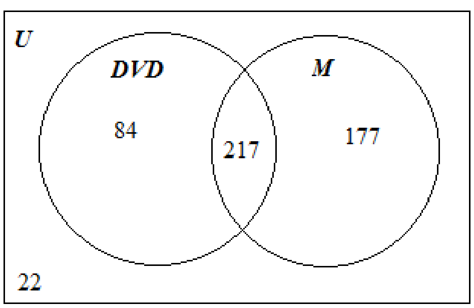


29 a.



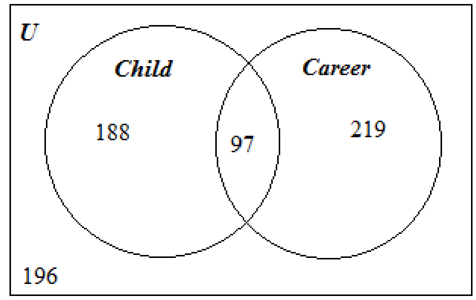
b. $\dfrac{81+21+126}{500} = \dfrac{228}{500} = 0.456 = 45.6\%$

30. a.



b. $\dfrac{84+217+177}{500} = \dfrac{478}{500} = 0.956 = 95.6\%$

31. a. Rearranging the Cardinal Number Formula for the Union of Sets:



$$n(child \cap career) = n(child) + n(career) - n(child \cup career)$$

$$n(child \cap career) = 285 + 316 - (700 - 196)$$

$$n(child \cap career) = 97$$

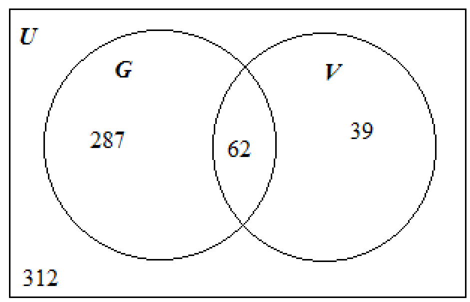
b. $\dfrac{97}{700} \approx 0.13857 = 13.857\%$

32. a. Rearranging the Cardinal Number Formula for the Union of Sets:

$$n(G \cap V) = n(G) + n(V) - n(G \cup V)$$

$$n(G \cap V) = 349 + 101 - (700 - 312)$$

$$n(G \cap V) = 62$$



b. $\dfrac{62}{700} \approx 0.08857 = 8.857\%$

For questions 33 – 36, the following facts are useful:

$n(U) = 50$

$n(A) = 4$, since A = {Alabama, Alaska, Arizona, Arkansas}

$n(I) = 4$, since I = {Idaho, Illinois, Indiana, Iowa}

$n(M) = 8$, since M = {Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana}

$n(N) = 8$, since N = {Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, North Dakota}

$n(O) = 3$, since O = {Ohio, Oklahoma, Oregon}

33. $n(M') = n(U) - n(M) = 50 - 8 = 42$

34. $n(A \cup N) = n(A) + n(N) - n(A \cap N)$
$= 4 + 8 - 0 = 12$

35. Rearranging the Cardinal Number Formula for the Union of Sets:
$n(I' \cap O') = n(I') + n(O') - n(I' \cup O')$
But, $n(I') = n(U) - n(I) = 50 - 4 = 46$
and $n(O') = n(U) - n(O) = 50 - 3 = 47$.
Thus, $n(I' \cap O') = 46 + 47 - 50 = 43$

36. $n(M \cap I) = 0$, since no state can start with both M and I.

For questions 37 – 40, the following facts are useful:

$n(U) = 12$

$n(J) = 3$, since J = {January, June, July}

$n(Y) = 4$, since Y = {January, February, May, July}

$n(V) = 3$, since V = {April, August, October}

$n(R) = 4$, since R = {September, October, November, December}

37. $n(R') = n(U) - n(R) = 12 - 4 = 8$

38. $n(J \cap V) = 0$, since no month starts with J and begins with a vowel.

39. $n(J \cup Y) = n(J) + n(Y) - n(J \cap Y)$
$= 3 + 4 - 2 = 5$

40. $n(V \cap R) = 1$, since October is the only month that starts with a vowel and ends with the letter R.

41. $n(S \cup A) = n(S) + n(A) - n(S \cap A)$
$= 13 + 4 - 1 = 16$
Note: $n(S \cap A) = 1$, since there is only one ace which is also a spade.

42. $n(C \cup T) = n(C) + n(T) - n(C \cap T)$
$= 13 + 4 - 1 = 16$
Note: $n(C \cap T) = 1$, since there is only one two which is also a club.

43. $n(F \cup B) = n(F) + n(B) - n(F \cap B)$
$= 12 + 26 - 6 = 32$
Note: $n(F \cap B) = 6$, since there are six face cards that are also black.

44. $n(F \cup D) = n(F) + n(D) - n(F \cap D)$
$= 12 + 13 - 3 = 22$

45. $n(F \cap B) = 6$, since there are 6 black face cards.

46. $n(F \cap D) = 3$, since there are 3 face cards which are diamonds.

47. $n(A \cup E) = n(A) + n(E) - n(A \cap E)$
$= 4 + 4 - 0 = 8$
Note: $n(A \cap E) = 0$, since there are no a aces which are also eights.

48. $n(T \cup S) = n(T) + n(S) - n(T \cap S)$
$= 4 + 4 - 0 = 8$
Note: $n(T \cap S) = 0$, since there are no threes which are also sixes.

49. $n(A \cap E) = 0$, since there are no aces which are also eights.

50. $n(T \cap S) = 0$, since there are no threes which are also sixes.

51. a. {1, 2, 3}
 b. {1, 2, 3, 4, 5, 6}
 c. $E \subseteq F$
 d. $E \subseteq F$

52. a. *A*, Example: *A* = {1, 2}, *B* = {1, 2, 3}
 b. *B*, Example: *A* = {1, 2}, *B* = {1, 2, 3}

53. a. { }, {*a*} 2 subsets
 b. { }, {*a*}, {*b*}, {*a*, *b*} 4 subsets
 c. { }, {*a*}, {*b*}, {*c*}, {*a*, *b*}, {*a*, *c*}, {*b*, *c*}, {*a*, *b*, *c*} 8 subsets
 d. { }, {*a*}, {*b*}, {*c*}, {*d*}, {*a*, *b*}, {*a*, *c*}, {*a*, *d*}, {*b*, *c*}, {*b*, *d*}, {*c*, *d*}, {*a*, *b*, *c*}, {*a*, *b*, *d*}, {*a*, *c*, *d*}, {*b*, *c*, *d*}, {*a*, *b*, *c*, *d*} 16 subsets
 e. Yes! The number of subsets of $A = 2^{n(A)}$
 f. Since $n(A) = 6$, the number of subsets is $2^6 = 64$.

54. Prove: $n(U) = n(A) + n(A')$
 Proof: Note that $A \cup A' = U$
 $n(U) = n(A \cup A')$
 $= n(A) + n(A') - n(A \cap A')$
 Thus, $n(U) = n(A) + n(A')$ since $A \cap A' = \{\ \}$

55. The two sets are disjoint.

56. *A* = *B* = { }

57. The set {0} contains one member, namely 0. The other set, $\varnothing$, contains no elements.

58. 0 is a number, not a set. {0} is a set containing one element, namely 0.

59. Only if *A* = { }

60. If *A* is a proper subset of *B*, there must be at least one element in *B* that is not in *A*.

61. The roster method is advantageous if the number of elements is small. Set-builder notation is advantageous when the number of elements is large.

62. a. *x* is an element in the intersection of *A* and *B* if and only if *x* is an element in *A* and *x* is an element in *B*.
 b. *x* is an element in the complement of *A* if and only if *x* is not an element in *A*.
 c. *A* is a subset of *B* if and only if the following is true: If *x* is an element of *A*, then *x* is an element of *B*.

63. Venn was a professor of moral sciences at Cambridge.

64. Venn utilized diagrams to analyze logical arguments. Venn's contriubtion was to make these diagrams uniform.

65. a. doesn't conform because M & O are in the same group
 b. doesn't conform because J & P are in the same group
 c. doesn't conform because N is in John's group, but P is not in Juneko's group.
 d. conforms
 e. doesn't conform because M & O are in the same group

66. (a) J; If N is in John's group, then P must be in Juneko's group. But, if P is in Juneko's group, then J must be in John's group.

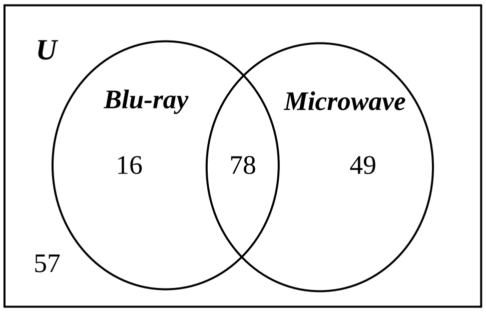
67. (c) L, M, P; Juneko's group must contain both M & P and Juneko's group can't contain O or J.

68. (b) If J is in Juneko's group, P can't be in it. But, if N was in John's group, then P would have to be in Juneko's group.

69. e. If K is in John's group, then M must be in Juneko's group. But, M & O cannot be in the same group, so O cannot be in Juneko's group. O could be in John's group, but it could also be the poster that is not used. Nothing is known about J, L, or N which means answer (e) is the only one that can be true.

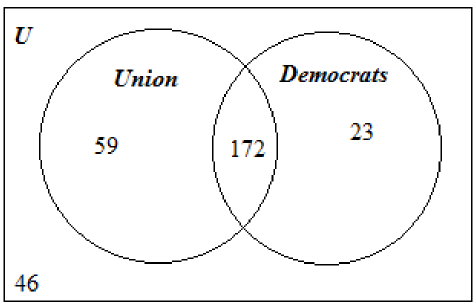
SECTION 2.2

1.



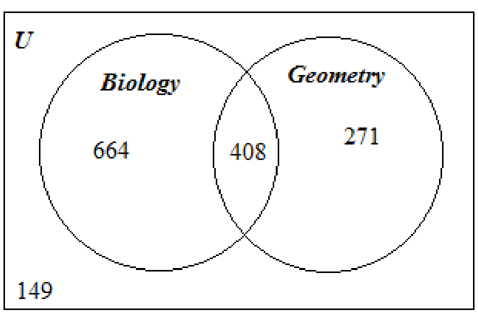
a. 16 + 78 + 49 = 143
b. 16
c. 49
d. 57

2.



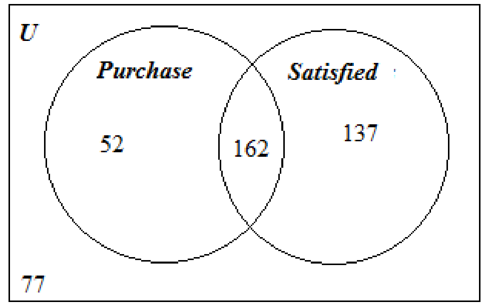
a. 59 + 172 + 23 = 254
b. 59
c. 23
d. 46

3.



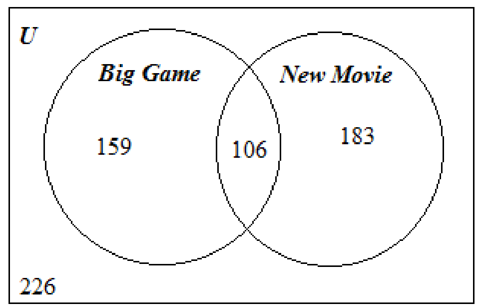
a. 408
b. 664 + 408 + 217 = 1,343
c. 664
d. 149

4.



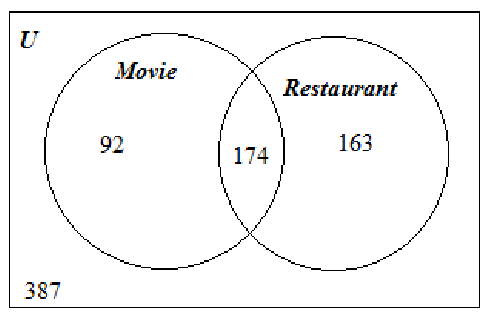
a. 162
b. 351
c. 137
d. 77

5.



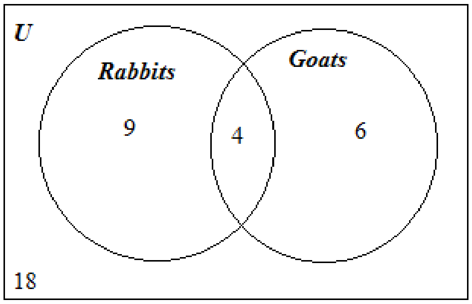
a. 106
b. 448
c. 256
d. 159

6.



a. 174
b. 429
c. 337
d. 163

7.

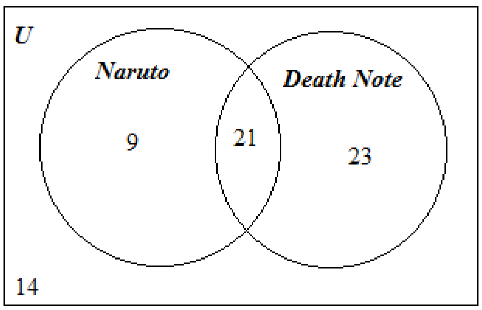


a. $\frac{19}{37} \approx 0.51351 = 51.351\%$

b. $\frac{9}{37} \approx 0.24324 = 24.324\%$

c. $\frac{6}{37} \approx 0.162162 = 16.216\%$

8.

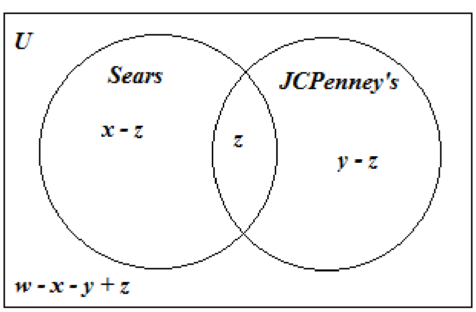


a. $\frac{53}{67} \approx 0.79104 = 79.104\%$

b. $\frac{9}{67} \approx 0.13433 = 13.433\%$

c. $\frac{23}{67} \approx 0.34328 = 34.328\%$

9.



$w-(x-z)-z-(y-z)=w-x-y+z$

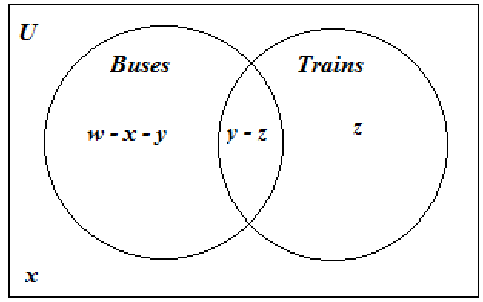
a. $(x-z)+z+(y-z)=x+y-z$

b. $x-z$

c. $y-z$

d. $w-x-y+z$

10.



$w-(y-z)-z-x=w-x-y$

a. $y-z$

b. $w-x-y$

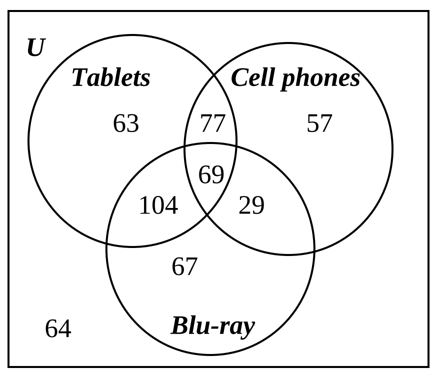
c. $(w-x-y)+(y-z)=w-x-z$

d. $(w-x-y)+(y-z)+z=w-x$

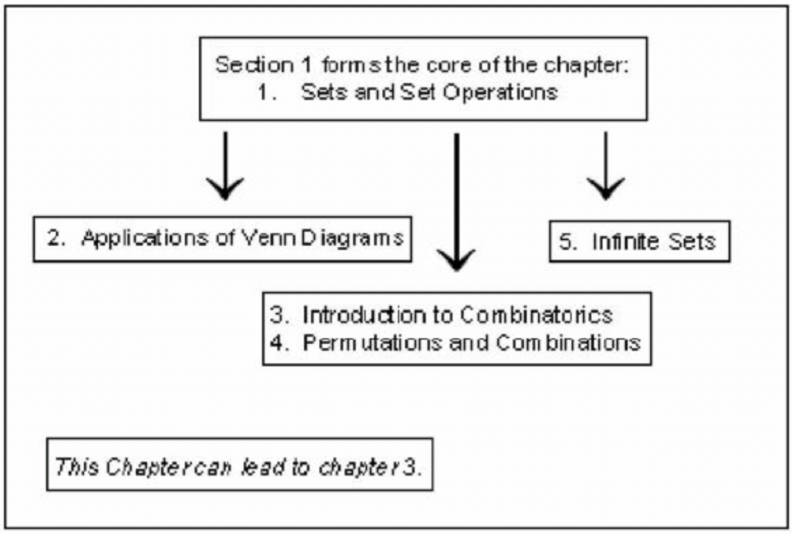
11. Find the total number of people surveyed:
63+77+69+104+57+29+67+64 = 530

a. $\frac{n(\text{cell phones})}{n(\text{surveyed})} = \frac{57+77+69+29}{530}$
$\approx 0.438 = 43.8\%$

b. $\frac{n(\text{only a cell phone})}{n(\text{surveyed})} = \frac{57}{530}$
$\approx 0.108 = 10.8\%$



Chapter 2
Sets and Counting



This chapter focuses on using sets and counting techniques to sort information regarding surveys and sample spaces. Several sections contain exercises pertaining to questions similar to those typically encountered on the Graduate Record Examination (GRE) or the Law School Admission Test (LSAT); this feature is called "The Next Level."

Section 2.1 forms the core of the chapter. Sections 2.2 through 2.5 are optional. Section 2.2 utilizes

Venn diagrams in the analysis of surveys. Section 2.3 and 2.4 provide an investigation of combinatorics: tree diagrams and the multiplying principle, permutations, and combinations.

Section 2.5 provides a brief study of countable versus uncountable infinite sets. Most of the chapter is used to service Chapter 3, Probability. Specifically, section 2.1 is prerequisite for section 3.2, and sections 2.3 and 2.4 are prerequisites for sections 3.4 and 3.5.

2.1
Sets and Set Operations

Topics

set notation
Venn diagrams
Cardinal number of a set
Subset (the distinction of proper versus improper is required for section 2.5.)
Set operations:
    intersection (page 77)
    union (page 78)
    complementation (page 80)

cardinal number rules:
for the union/intersection of sets (page 79)
for the complement of a set (page 82)
comparison of the set operations and the logical connectives from Chapter 1. (charts on page 83)
Historical Note on John Venn, page 81

Exercises

1: determining whether or not sets are well-defined
2: true/false questions regarding set notation
3–6: listing all possible subsets
7–16: listing the elements of specified sets
17–26: shading the appropriate regions of Venn diagrams
27–28: use of the Cardinal Number Rule for the Union/Intersection of Sets
29–32: applications of the Cardinal Number Rule for the Union/Intersection of Sets
33–40: interpreting set-builder notation, set operations, and cardinal numbers
41–50: the Cardinal Number is applied to a standard deck of playing cards
51–52: interpretation of set operations
53: development of the relationship between n(A) and the number of subsets of A
54: proof of Cardinal Number Formula
55–62: Concept Questions
63–64: History Questions
65–69: The Next Level

2.2
Applications of Venn Diagrams

Topics

using Venn diagrams to sort the information obtained from a survey
De Morgan’s Laws
Historical Note on Augustus D Morgan, page 81
Topic X: Blood Types/Set Theory in the Real World, page 82

Exercises

1–10: analyzing surveys that consist of two categories
11–20: analyzing surveys that consist of three categories
21–22: analyzing surveys that consist of four categories
23–26: using De Morgan’s Laws to simplify set operations
27–32: shading the appropriate regions of Venn diagrams
33–34: using Venn diagrams to prove set operation identities
35–43: exercises regarding Set Theory in the Real World: Blood Types
44–45: History Questions
46–52: 82
53: Project

2.3 Introduction to Combinatorics

Topics

tree diagrams
The Fundamental Principle of counting
factorials

Exercises

1–4: constructing tree diagrams to list all possible outcomes
5–22: applications of the Fundamental Principle of Counting
23–46: factorial arithmetic
47–48: Concept Question
49: History Questions
50–54: The Next Level

2.4 Permutations and Combinations

Topics

with versus without replacement
permutations (formula on page 108)
combinations (formula on page 111)
Pascal's triangle
Historical Note on Chu Shih-chich, page 116
conbinatoric flowchart, page 115
permutations of identical items (formula on page 118)

Exercises

1–12: evaluating expressions of the form ${}_nP_r$ and ${}_nC_r$
13–16: listing all possible permutations and combinations of given elements
17–26: applications of combinatorics
27–32: using combinations to determine the number of possible poker hands
33–44: various types of lotteries are examined
45–48: analyzing patterns in Pascal's Triangle
49–60: permutations of identical items
61–62: Concept Questions
63–67: The Next Level
68: Project

2.5
Infinite Sets

Topics

Historical Note on Georg Cantor, page 124
one-to-one correspondence and equivalent sets
countable sets and $\aleph_0$ uncountable sets and points on a line exercises

Exercises

1–10: one-to-one correspondences between finite sets
11–16: investigating countable infinite sets and their correspondences
17–23: draw geometric figures to obtain one-to-one correspondence between uncountable infinite sets
24: Concept Question
25–27: History Questions
28: Project