Chapter 2: Quadratic and Other Special Functions

Exercises 2.1

1. $2x^2+3=x^2-2x+4$
$x^2+2x-1=0$

2. $x^2-2x+5=2-2x^2$
$3x^2-2x+3=0$

3. $(y+1)(y+2)=4$
$y^2+3y+2=4$
$y^2+3y-2=0$

4. $(z-1)(z-3)=1$
$z^2-4z+2=0$

5. $x^2-4x=12$
$x^2-4x-12=0$
$x^2-6x+2x-12=0$
$x(x-6)+2(x-6)=0$
$(x-6)(x+2)=0$
$x-6=0$ or $x+2=0$
Solution: x = -2, 6

6. $x^2=11x-10$
$x^2-11x+10=0$
$x^2-10x-x+10=0$
$(x-10)(x-1)=0$
$x-10=0$ or $x-1=0$
Solution: x = 1, 10

7. $9-4x^2=0$
$(3+2x)(3-2x)=0$
$3+2x=0$ or $3-2x=0$
Solution: $x=-\frac{3}{2}, \frac{3}{2}$

8. $25x^2-16=0$
$(5x-4)(5x+4)=0$
$5x-4=0$ or $5x+4=0$
Solution: $x=\frac{4}{5}, -\frac{4}{5}$

9. $x=x^2$
$x^2-x=0$
$x(x-1)=0$
Solution: x = 0, 1
Never divide by a variable. A root is lost if you divide.

10. $t^2-4t=3t^2$
$0=2t^2+4t$
$0=2t(t+2)$
$2t=0$ or $t+2=0$
Solution: t = 0, –2

11. $4t^2-4t+1=0$
$(2t-1)(2t-1)=0$
$2t-1=0$
Solution: $t=\frac{1}{2}$

12. $49z^2+14z+1=0$
$(7z+1)(7z+1)=0$
$7z+1=0$
$7z=-1$
Solution: $z=-\frac{1}{7}$

13. a. $x^2-4x-4=0$
a = 1, b = –4, c = –4
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-4)}}{2(1)}$$
$$=\frac{4\pm\sqrt{32}}{2}=\frac{4\pm4\sqrt{2}}{2}=2\pm2\sqrt{2}$$

b. Since $\sqrt{2}\approx1.414$, the solutions are approximately 4.83, –0.83.

c. $x^2-6x+7=0$
a = 1, b = –6, c = 7
$$x=\frac{6\pm\sqrt{36-28}}{2}$$
$$=\frac{6\pm\sqrt{8}}{2}=\frac{6\pm2\sqrt{2}}{2}=3\pm\sqrt{2}$$
$\sqrt{2}\approx1.414$, the solutions are approximately 4.83, –0.83.

14. $x^2 - 6x + 7 = 0$
$a = 1$, $b = -6$, $c = 7$

$$x = \frac{6 \pm \sqrt{36-28}}{2}$$

$$= \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}$$

a. $3+\sqrt{2}, 3-\sqrt{2}$
b. 4.41, 1.59

15. $2w^2 + w + 1 = 0$
$a = 2$, $b = 1$, $c = 1$

$$w = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4}$$

There are no real solutions.

16. $z^2 + 2z + 4 = 0$
$a = 1$, $b = 2$, $c = 4$

$$z = \frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm \sqrt{-12}}{2}$$

No real solutions.

17. $y^2 = 7$

$$y = \pm\sqrt{7}$$

18. $z^2 = 12$

$$z = \pm\sqrt{12}$$

$$z = \pm 2\sqrt{3}$$

19. $5x^2 = 80$

$$x^2 = 16$$

$$x = \pm 4$$

20. $3x^2 = 75$

$$x^2 = 25$$

$$x = \pm 5$$

21. $(x+4)^2 = 25$

$$x + 4 = \pm 5$$

$$x = -4 \pm 5$$

Solution: $x = 1, -9$

22. $(x+1)^2 = 2$

$$x + 1 = \pm\sqrt{2}$$

$$x = -1 \pm \sqrt{2}$$

23. $x^2 + 5x = 21 + x$

$$x^2 + 4x - 21 = 0$$

$$(x+7)(x-3) = 0$$

Solution: $x = -7, 3$

24. $x^2 + 17x = 8x - 14$

$$x^2 + 9x + 14 = 0$$

$$(x+7)(x+2) = 0$$

Solution: $x = -7, -2$

25. $\frac{w^2}{8} - \frac{w}{2} - 4 = 0$

$$w^2 - 4w - 32 = 0$$

$$(w-8)(w+4) = 0$$

$$w - 8 = 0 \text{ or } w + 4 = 0$$

Solution: $w = 8, -4$

26. $\frac{y^2}{2} - \frac{11}{6}y + 1 = 0$

$$3y^2 - 11y + 6 = 0$$

$$(3y-2)(y-3) = 0$$

$$3y - 2 = 0 \text{ or } y - 3 = 0$$

Solution: $y = \frac{2}{3}, 3$

27. $16z^2 + 16z - 21 = 0$
$a = 16$, $b = 16$, $c = -21$

$$z = \frac{-16 \pm \sqrt{256+1344}}{32}$$

$$= \frac{-16 \pm 40}{32} = \frac{3}{4} \text{ or } -\frac{7}{4}$$

Solution: $z = -\frac{7}{4}, \frac{3}{4}$

28. $10y^2 - y - 65 = 0$

$a = 10,\ b = -1,\ c = -65$

$$y = \frac{1 \pm \sqrt{1-(-2600)}}{20}$$

$$= \frac{1 \pm \sqrt{2601}}{20} = \frac{1 \pm 51}{20} = -\frac{50}{20} \text{ or } \frac{52}{20}$$

Solution: $y = -\frac{5}{2}, \frac{13}{5}$

29. $(x-1)(x+5) = 7$

$x^2 + 4x - 5 = 7$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

Solution: $x = -6, 2$

30. $(x-3)(1-x) = 1$

$x - x^2 - 3 + 3x = 1$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$

Solution: $x = 2$

31. $5x^2 = 2x + 6$ or $5x^2 - 2x - 6 = 0$

$a = 5,\ b = -2,\ c = -6$

$$x = \frac{2 \pm \sqrt{4+120}}{10} = \frac{1 \pm \sqrt{31}}{5}$$

Solution: $x = \frac{1-\sqrt{31}}{5}, \frac{1+\sqrt{31}}{5}$

32. $3x^2 = -6x - 2$

$3x^2 + 6x + 2 = 0$

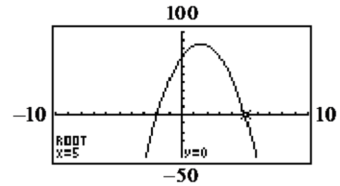
$a = 3,\ b = 6,\ c = 2$

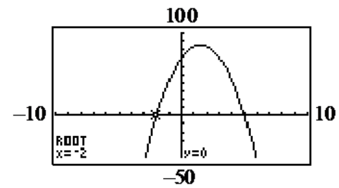
$$x = \frac{-6 \pm \sqrt{36-24}}{6} = \frac{-6 \pm \sqrt{12}}{6}$$

$$= \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

Solution: $x = \frac{-3-\sqrt{3}}{3}, \frac{-3+\sqrt{3}}{3}$

33. $21x + 70 - 7x^2 = 0$





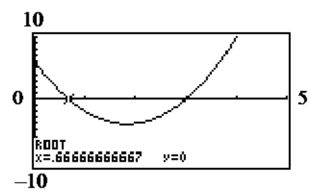
Divide by –7 and rearrange.

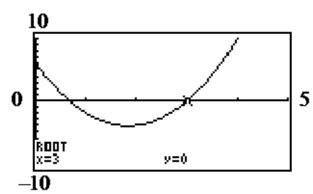
$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

Solution: $x = -2, 5$

34. $3x^2 - 11x + 6 = 0$

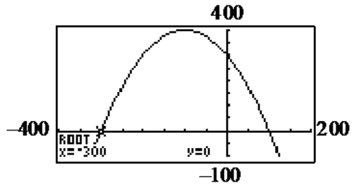


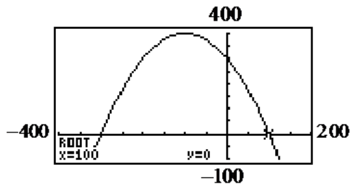


$(3x - 2)(x - 3) = 0$

Solution: $x = \frac{2}{3}, 3$

35. $300 - 2x - 0.01x^2 = 0$

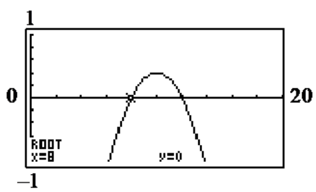


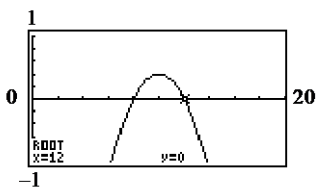


$a = -0.01$, $b = -2$, $c = 300$

$$x = \frac{2 \pm \sqrt{4+12}}{-0.02} = \frac{2 \pm 4}{-0.02}$$
$$= -300 \text{ or } 100$$

36. $-9.6 + 2x - 0.1x^2 = 0$

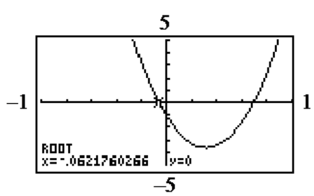


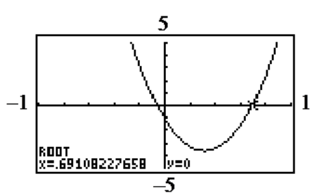


$$-9.6 + 2x - 0.1x^2 = 0$$
$$x^2 - 20x + 96 = 0$$
$$(x-12)(x-8) = 0$$

Solution; $x = 12, 8$

37. $25.6x^2 - 16.1x - 1.1 = 0$

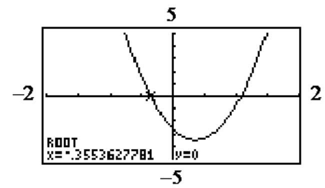


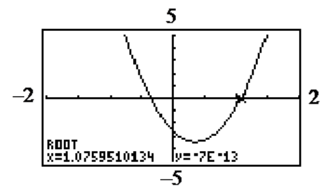


$a = 25.6$, $b = -16.1$, $c = -1.1$

$$x = \frac{16.1 \pm \sqrt{259.21 + 112.64}}{51.2}$$
$$= \frac{16.1 \pm \sqrt{371.85}}{51.2}$$
$$\approx 0.69 \text{ or } -0.06$$

38. $6.8z^2 - 4.9z - 2.6 = 0$





$$6.8z^2 - 4.9z - 2.6 = 0$$
$$z = \frac{4.9 \pm \sqrt{24.01 + 70.72}}{13.6}$$
$$= \frac{4.9 \pm \sqrt{94.73}}{13.6} = \frac{4.9 \pm 9.73}{13.6}$$
$$= 1.08 \text{ or } -0.36$$

39.
$$x + \frac{8}{x} = 9$$
$$x^2 + 8 = 9x$$
$$x^2 - 9x + 8 = 0$$
$$(x-8)(x-1) = 0$$

Solution: $x = 1, 8$

40.
$$\frac{x}{x-2} - 1 = \frac{3}{x+1}$$
$$x(x+1) - 1 \cdot (x-2)(x+1) = 3(x-2)$$
$$2x + 2 = 3x - 6$$
$$x = 8$$

Solution: $x = 8$

41. $\dfrac{x}{x-1} = 2x + \dfrac{1}{x-1}$

$x = (2x^2 - 2x) + 1$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

Solution: $x = \dfrac{1}{2}$

1 is not a root since division by zero is not defined.

42. $\dfrac{5}{z+4} - \dfrac{3}{z-2} = 4$

$5(z-2) - 3(z+4) = 4(z+4)(z-2)$

$2z - 22 = 4z^2 + 8z - 32$

$4z^2 + 6z - 10 = 0$

$2(2z+5)(z-1) = 0$

$2z + 5 = 0$ or $z - 1 = 0$

Solution: $z = -\dfrac{5}{2}, 1$

43. $(x+8)^2 + 3(x+8) + 2 = 0$

$[(x+8)+2][(x+8)+1] = 0$

$(x+8)+2 = 0$ or $(x+8)+1 = 0$

Solution: $x = -10, -9$

44. $(s-2)^2 - 5(s-2) - 24 = 0$

$[(s-2)-8][(s-2)+3] = 0$

$(s-2)-8 = 0$ or $(s-2)+3 = 0$

Solution: $s = 10, -1$

45. $P = -x^2 + 90x - 200$

$1200 = -x^2 + 90x - 200$

$0 = x^2 - 90x + 1400$

$0 = (x-20)(x-70)$

A profit of $1200 is earned at $x = 20$ units or $x = 70$ units of production.

46. $P = 16x - 0.1x^2 - 100$

When $P = 180$ we have

$180 = 16x - 0.1x^2 - 100$ or $0.1x^2 - 16x + 280 = 0$

$x = \dfrac{16 \pm \sqrt{256-112}}{0.2} = \dfrac{16 \pm \sqrt{144}}{0.2}$

$= \dfrac{16 \pm 12}{0.2} = 140$ or 20 units

47. a. $P = -18x^2 + 6400x - 400$

$61{,}800 = -18x^2 + 6400x - 400$

$18x^2 - 6400x + 62{,}200 = 0$

Factoring appears difficult, so let us apply the quadratic formula.

$x = \dfrac{6400 \pm \sqrt{6400^2 - 4(18)(62{,}200)}}{36}$

$= \dfrac{6400 \pm \sqrt{36{,}481{,}600}}{36}$

$= \dfrac{6400 \pm 6040}{36} = 10$ or 345.56

So, a profit of $61,800 is earned for 10 units or for 345.56 units.

b. Yes. Maximum profit occurs at vertex as seen using the graphing calculator.

48. a. $P = 50x - 300 - 0.01x^2$

When $P = 250$ we have

$250 = 50x - 300 - 0.01x^2$

or $0.01x^2 - 50x + 550 = 0.$

$x = \dfrac{50 \pm \sqrt{2500 - 22}}{0.02}$

$= \dfrac{50 \pm 49.78}{0.02} = 11$ or 4989 units

b. Yes. Try $P(4000)$ and $P(5000)$.

$P(4000) > \$250$.

49. $S = 100 + 96t - 16t^2$

$100 = 100 + 96t - 16t^2$

$0 = 96t - 16t^2 = 16t(6-t)$

The ball is 100 feet high 6 seconds later.

50. $D(t) = -16t^2 + 10t + 350$

$0 = -16t^2 + 10t + 350$

$-16t^2 + 10t + 350 = 0$

$8t^2 - 5t - 175 = 0$

$(8t + 35)(t - 5) = 0$

The answer $t = 5$ is the only one that makes sense in this case, so the ball hits the ground at 5 seconds.

51. $p = 25 - 0.01s^2$

a. $0 = 25 - 0.01s^2$

$= (5 + 0.1s)(5 - 0.1s)$

$p = 0$ if $5 - 0.1s = 0$ or $s = 50$.

b. $s \geq 0$. $p = 0$ means there is no particulate pollution.

52. $S = 100x - x^2$

a. $0 = x(100 - x)$

A dosage of 0 or 100 ml gives $S = 0$.

b. Dosage is effective if $0 < x < 100$.

53. $t = 0.001\left(0.732x^2 + 15.417x + 607.738\right)$

$8.99 = 0.001\left(0.732x^2 + 15.417x + 607.738\right)$

$8990 = 0.732x^2 + 15.417x + 607.738$

$0 = 0.732x^2 + 15.417x - 8382.262$

$$t = \frac{-15.417 \pm \sqrt{(15.417)^2 - 4(0.732)(-8382.262)}}{2(0.732)}$$

$t \approx 96.996$ or $t \approx -118.058$

The positive answer is the one that makes sense here, 97.0 mph.

54. $B = -0.0046t^2 - 0.033t + 6.05$

$-5 = -0.0046t^2 - 0.033t + 6.05$

$0.0046t^2 + 0.033t - 11.05 = 0$

Using the quadratic formula or a graphing utility gives the positive value $t \approx 45.6$. The fund is projected to be \$5 trillion in the red in the year 2046.

55. $p = 0.17t^2 - 2.61t + 52.64$

$55 = 0.17t^2 - 2.61t + 52.64$

$0.17t^2 - 2.61t - 2.36 = 0$

Using the quadratic formula or a graphing utility gives the positive value $t \approx 16.2$. In 2016 the percent of high school seniors who will have tried marijuana is predicted by the function to reach 55%.

56. a. $y = -0.0013x^2 + x + 10$

$0.0013x^2 - x - 10 = 0$

$x \approx -9.873$ or $x \approx 779.104$

b. $y = -\frac{x^2}{81} + \frac{4}{3}x + 10$

$\frac{x^2}{81} - \frac{4}{3}x - 10 = 0$

$x \approx 115.041$ or $x \approx -7.041$

Given that the distance x is not negative, the first projectile travels further (approximately 779 feet versus the second projectile's approximately 115 feet).

57. $P = \left(\frac{C}{100}\right) \cdot C$

We know that the selling price is \$144 and that the selling price equals the profit plus the cost C to the store.

$144 = \frac{C^2}{100} + C$

$14400 = C^2 + 100C$

$C^2 + 100C - 14400 = 0$

$C = -180$ or $C = 80$

The cost C of the necklace to the store is not negative, so C = \$80 is the amount the store paid for the necklace.

58. $y = 0.787x^2 - 11.0x + 290$

$1000 = 0.787x^2 - 11.0x + 290$

$0.787x^2 - 11x - 710 = 0$

Using the quadratic formula or a graphing utility gives the positive value $x \approx 38$.

Spending is projected to reach $1000 billion in the year 2028.

59. $E = 7.94x^2 + 33.2x + 2190$

$5000 = 7.94x^2 + 33.2x + 2190$

$7.94x^2 + 33.2x - 2810 = 0$

Using the quadratic formula or a graphing utility gives the positive value $x \approx 16.8$.

The model predict these expenditures will reach $5 trillion in 2022.

60. $v = k\left(R^2 - r^2\right)$

$v = 2\left(0.01 - r^2\right)$

In each case below only nonnegative values of r are reported.

a. $0.02 = 2\left(0.01 - r^2\right)$

$0.01 = 0.01 - r^2$

$r^2 = 0$

$r = 0$

b. $0.015 = 2\left(0.01 - r^2\right)$

$0.0075 = 0.01 - r^2$

$r^2 = 0.0025$

$r = 0.05$

c. $0 = 2\left(0.01 - r^2\right)$

$r^2 = 0.01$

$r = 0.1$

In this case the corpuscle is at the wall of the artery.

61. $K^2 = 16v + 4$

In each case below only positive values of K are reported.

a. $K^2 = 16(20) + 4 = 324$

$K = 18$

b. $K^2 = 16(60) + 4 = 964$

$K \approx 31$

c. Speed triples, but K changes only by a factor of 1.72.

62. Given that $s = 16t_1^2$ and $s = 1090t_2$,

$t_1 + t_2 = 3.9 \Rightarrow t_2 = 3.9 - t_1$

$16t_1^2 = 1090t_2$

$= 1090\left(3.9 - t_1\right)$

$= 4251 - 1090t_1$

$16t_1^2 + 1090t_1 - 4251 = 0$

Using the quadratic formula or a graphing utility gives the positive value $t_1 \approx 3.70$.

$s = 16t_1^2$

$\approx 16\left(3.70\right)^2$

≈ 219

The depth of the fissure is about 219 ft.

Chapter 2: Quadratic and Other Special Functions

Exercises 2.2

Chapter 2: Quadratic and Other Special Functions

1. $y = \frac{1}{2}x^2 + x$

a. $x = \frac{-b}{2a} = \frac{-1}{2(1/2)} = -1$

$y = \frac{1}{2}(-1)^2 + (-1) = -\frac{1}{2}$

Vertex is at $\left(-1, -\frac{1}{2}\right)$.

b. $a > 0$, so vertex is a minimum.

c. -1

d. $-\frac{1}{2}$

2. $y = x^2 - 2x$

a. $x = -\frac{b}{2a} = \frac{2}{2} = 1$

When $x = 1$, $y = -1$. The vertex is $(1, -1)$.

b. $a > 0$, so vertex is a minimum.

c. 1

d. -1

3. $y = 8 + 2x - x^2$

a. $x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$y = 8 + 2(1) - (1)^2 = 9$

Vertex is at $(1, 9)$.

b. $a < 0$, so vertex is a maximum.

c. 1

d. 9

4. $y = 6 - 4x - 2x^2$

a. $x = \frac{-b}{2a} = \frac{4}{-4} = -1$

When $x = -1$, $y = 8$. The vertex is $(-1, 8)$.

b. $a < 0$, so vertex is a maximum.

c. -1

d. 8

5. $f(x) = 6x - x^2$

a. $x = \frac{-b}{2a} = \frac{-6}{-2} = 3.$

$f(3) = 6(3) - (3)^2 = 9$

Vertex is at $(3, 9)$.

b. $a < 0$, so vertex is a maximum.

c. 3

d. 9

6. $f(x) = x^2 + 2x - 3$

a. $x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$f(-1) = (-1)^2 + 2(-1) - 3 = -4$

Vertex is at $(-1, -4)$.

b. $a > 0$, so vertex is a minimum.

c. -1

d. -4

7. $y = -\frac{1}{4}x^2 + x$

Vertex is a maximum point since $a < 0$.

V: $x = \frac{-b}{2a} = \frac{-1}{2(-1/4)} = 2$

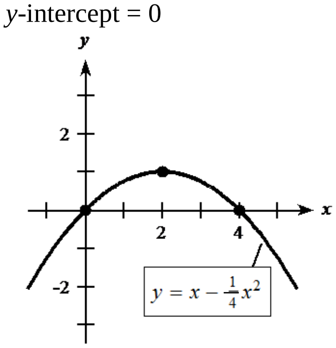
$y = -\frac{1}{4}(2)^2 + 2 = 1$

Zeros: $-\frac{1}{4}x^2 + x = 0$

$x\left(-\frac{1}{4}x + 1\right) = 0$

$x = 0, 4$

y-intercept = 0



8. $y = -2x^2 + 18x$

Vertex is a maximum since $a < 0$.

V: $x = \frac{-b}{2a} = \frac{-18}{-4} = \frac{9}{2}$

$y = -2\left(\frac{9}{2}\right)^2 + 18\left(\frac{9}{2}\right) = -\frac{81}{2} + \frac{162}{2} = \frac{81}{2}$

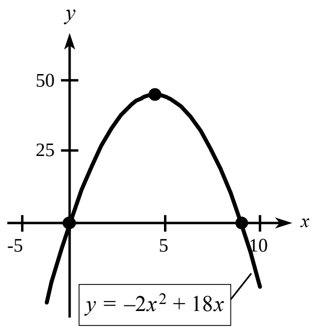
Zeros: $0 = -2x^2 + 18x$

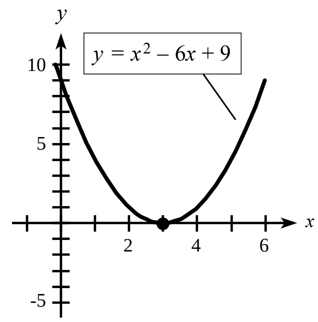
$0 = -2x(x - 9)$

$-2x = 0$ or $x - 9 = 0$

$x = 0 \quad x = 9$

y-intercept = 0





9. $y = x^2 + 4x + 4$

Vertex is a minimum point since $a > 0$.

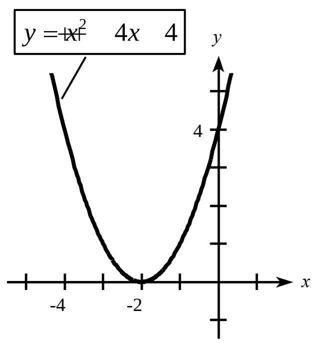
V: $x = \dfrac{-b}{2a} = \dfrac{-4}{2(1)} = -2$

$y = (-2)^2 + 4(-2) + 4 = 0$

Zeros: $x^2 + 4x + 4 = (x+2)(x+2) = 0$

$x = -2$

y-intercept = 4



10. $y = x^2 - 6x + 9$

Vertex is a minimum since $a > 0$.

V: $x = \dfrac{-b}{2a} = \dfrac{6}{2} = 3$

$y = 3^2 - 6(3) + 9 = 0$

Zeros: $0 = x^2 - 6x + 9$

$0 = (x-3)(x-3)$

$x - 3 = 0$

$x = 3$

y-intercept = 9

11. $y = \dfrac{1}{2}x^2 + x - 3$

Vertex is a minimum point since $a > 0$.

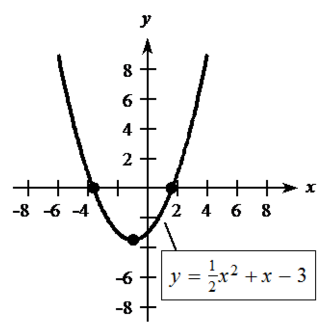
V: $x = \dfrac{-b}{2a} = \dfrac{-1}{2(1/2)} = -1$

$y = \dfrac{1}{2}(-1)^2 + (-1) - 3 = -\dfrac{7}{2}$

Zeros: $\dfrac{1}{2}x^2 + x - 3 = 0 \rightarrow x^2 + 2x - 6 = 0$

$x = \dfrac{-2 \pm \sqrt{4+24}}{2} = \dfrac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$

y-intercept $= -3$



12. $x^2 + x + 2y = 5$

$2y = -x^2 - x + 5$

$y = -\frac{1}{2}x^2 - \frac{1}{2}x + \frac{5}{2}$

Vertex is a maximum since $a < 0$.

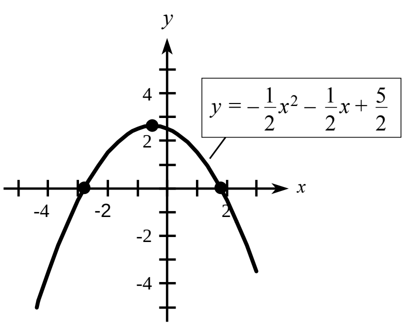
V: $x = \frac{-b}{2a} = -\frac{-\frac{1}{2}}{2\left(-\frac{1}{2}\right)} = -\frac{1}{2}$

$y = -\frac{1}{2}\left(-\frac{1}{2}\right)^2 - \frac{1}{2}\left(-\frac{1}{2}\right) + \frac{5}{2} = \frac{21}{8}$

Zeros: Using the quadratic formula,

$x = \frac{-1 \pm \sqrt{21}}{2}$.

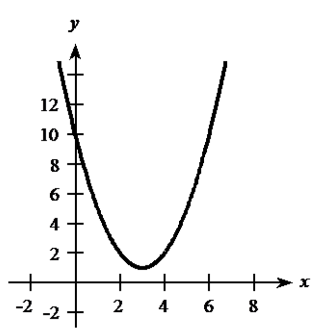
y-intercept $= \frac{5}{2}$



13. $y = (x-3)^2 + 1$

a. Graph is shifted 3 units to the right and 1 unit up.

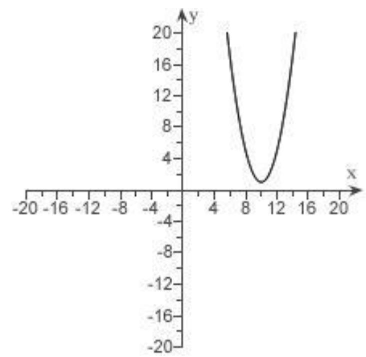
b.



14. $y = (x-10)^2 + 1$

a. Graph is shifted 10 units to the right and 1 units up.

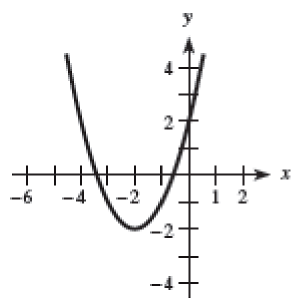
b.



15. $y = (x+2)^2 - 2$

a. Graph is shifted 2 units to the left and 2 units down.

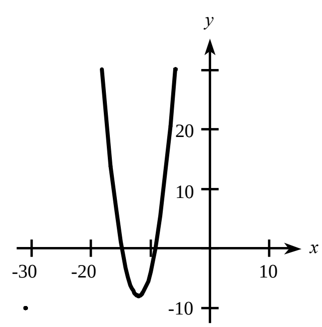
b.



16. $y = (x+12)^2 - 8$

a. Graph is shifted 12 units to the left and 8 units down.

b.



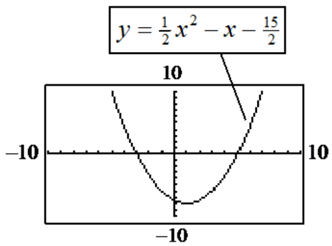
17. $y = \frac{1}{2}x^2 - x - \frac{15}{2}$

V: $x = \frac{-b}{2a} = \frac{-(-1)}{2(1/2)} = 1$

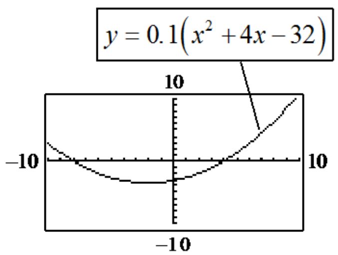
$y = \frac{1}{2}(1)^2 - 1 - \frac{15}{2} = -8$

Zeros: $x^2 - 2x - 15 = (x-5)(x+3) = 0$

$x = 5, -3$



18.



From the graph, the vertex is approximately $(-2, -3.5)$. The zeros are approximately –8 and 4. Algebraic check:

V: x-coordinate: $\frac{-b}{2a} = \frac{-4}{2} = -2$

y-coordinate: $0.1(4 - 8 - 32) = -3.6$

So, actual vertex is (–2, –3.6)

Zeros: $0 = x^2 + 4x - 32 = (x+8)(x-4)$

$x = -8, 4$

19. $y = \frac{1}{4}x^2 + 3x + 12$

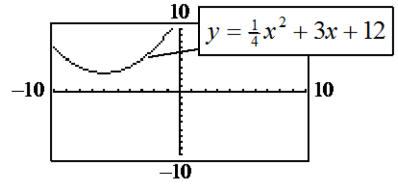
V: $x = \frac{-b}{2a} = \frac{-3}{2\left(\frac{1}{4}\right)} = -6$

$y = \frac{1}{4}(-6)^2 + 3(-6) + 12 = 3$

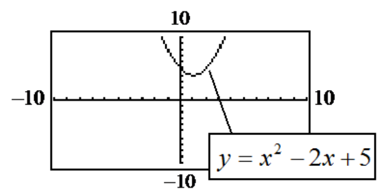
Zeros: $x^2 + 12x + 48 = 0$

$b^2 - 4ac = 144 - 192 < 0$

There are no zeros.



20. $y = x^2 - 2x + 5$



From the graph, the vertex is (1, 4).
There are no real zeros.
Algebraic check:

V: x-coordinate: $-\frac{b}{2a} = -\frac{-2}{2} = 1$

y-coordinate: $1^2 - 2(1) + 5 = 4$

The discriminant is negative, so no real zeros.

21. $f(x) = y = -5x - x^2$

Average Rate of Change $= \frac{f(1) - f(-1)}{1 - (-1)}$

$= \frac{-6 - 4}{2} = -\frac{10}{2} = -5$

22. $f(x) = y = 8 + 3x + 0.5x^2$

Average Rate of Change $= \frac{f(4) - f(2)}{4 - 2}$

$= \frac{28 - 16}{2} = \frac{12}{2} = 6$

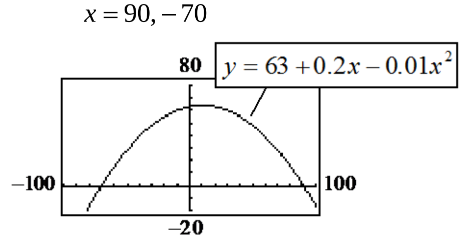
23. $y = 63 + 0.2x - 0.01x^2$

V: $x = \frac{-0.2}{-0.02} = 10$

$y = 63 + 2 - 1 = 64$

Zeros: $x^2 - 20x - 6300 = (x - 90)(x + 70) = 0$

$x = 90, -70$



24. $y = 0.2x^2 + 16x + 140$

V: x-coordinate: $\dfrac{-b}{2a} = \dfrac{-16}{2(0.2)} = -40$

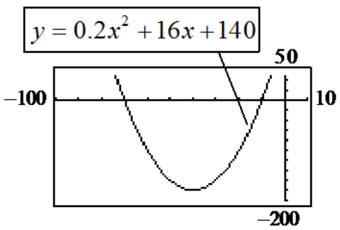
y-coordinate: $0.2(-40)^2 + 16(-40) + 140 = -180$

Zeros: $0 = 0.2(x^2 + 80x + 700)$

$= 0.2(x + 70)(x + 10)$

$x = -70, -10$

Graphing range: x-min = –100 y-min = –200
x-max = 0 y-max = 50



25. $y = 0.0001x^2 - 0.01$

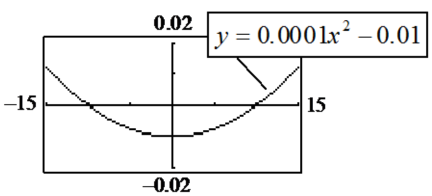
V: $x = \dfrac{-0}{2(0.0001)} = 0$

$y = 0 - 0.01 = -0.01$

Zeros: $0.0001x^2 - 0.01 = 0.01\left(0.01x^2 - 1\right) = 0$

$0.01(0.1x + 1)(0.1x - 1) = 0$

$x = -10, 10$



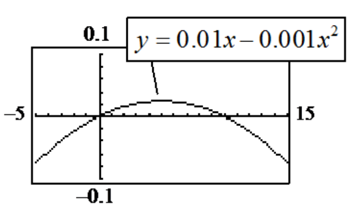
26. $y = 0.01x - 0.001x^2 = 0.001x(10 - x)$

Zeros: $x = 0$, $x = 10$

V: x-coordinate: $\dfrac{-b}{2a} = \dfrac{-0.01}{-0.002} = 5$

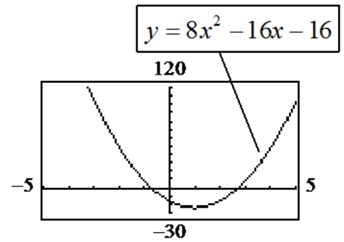
y-coordinate: $0.01(5) - 0.001(25) = 0.025$

Graphing range: x-min = –5 y-min = –0.1
x-max = 15 y-max = 0.1



27. $f(x) = 8x^2 - 16x - 16$

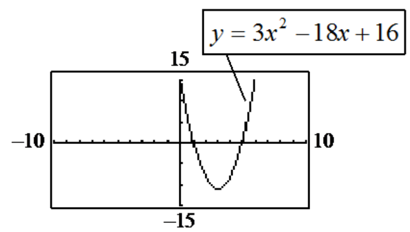
a. $x = \dfrac{-b}{2a} = \dfrac{16}{16} = 1$ and $f(1) = -24$



b. Graphical approximation gives
$x = -0.73,\ 2.73$

28. $f(x) = 3x^2 - 18x + 16$

a. $x = \dfrac{-b}{2a} = \dfrac{18}{6} = 3$ and $f(3) = -11$.



b. Graphical approximation gives
$x = 1.085,\ 4.915$

29. $f(x) = 3x^2 - 8x + 4$

a. The TRACE gives $x = 2$ as a solution.

b. $(x - 2)$ is a factor.

c. $3x^2 - 8x + 4 = (x - 2)(3x - 2)$

d. $(x - 2)(3x - 2) = 0$

$x - 2 = 0$ or $3x - 2 = 0$

Solution is $x = 2,\ 2/3$.

30. $f(x) = 5x^2 - 2x - 7$

a. The TRACE gives $x = -1$ as a solution.

b. $(x + 1)$ is the factor.

c. $5x^2 - 2x - 7 = (x + 1)(5x - 7)$

d. $(x + 1)(5x - 7) = 0$

$x + 1 = 0$ or $5x - 7 = 0$

Solution is $x = -1,\ 7/5$.

31. $P = -0.1x^2 + 16x - 100$
The vertex coordinates are the answers to the questions.

a. $a = -0.1$, $b = 16$

$$x = \frac{-b}{2a} = \frac{-16}{-0.2} = 80$$

Profit is maximized at a production level of 80 units.

b. $P(80) = -0.1(80)^2 + 16(80) - 100 = \$540$
is the maximum profit.

32. $P = 80x - 0.4x^2 - 200$

a. x-coordinate of vertex $= \dfrac{-b}{2a} = \dfrac{-80}{-0.8} = 100$

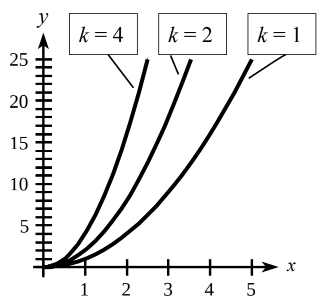
When $x = 100$, $P = 80(100) - 0.4(100)^2 - 200$
$= 8000 - 4000 - 200 = \$3800$

33. $Y = 800x - x^2$
Opens down so maximum Y is at vertex.

V: $x = \dfrac{-800}{-2} = 400$

Maximum yield occurs at $x = 400$ trees.

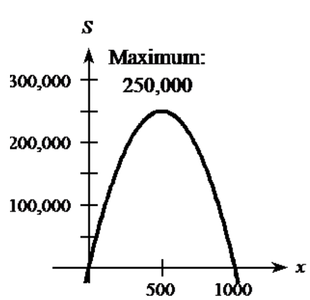
34. $y = kx^2$



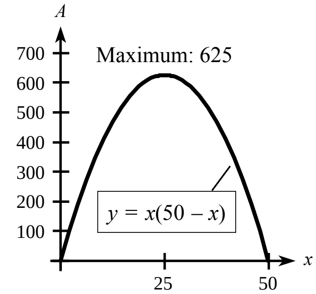
35. $S = 1000x - x^2$
Maximum sensitivity occurs at vertex.

V: $x = \dfrac{-1000}{-2} = 500$

The dosage for maximum sensitivity is 500.



36. $A = 50x - x^2$



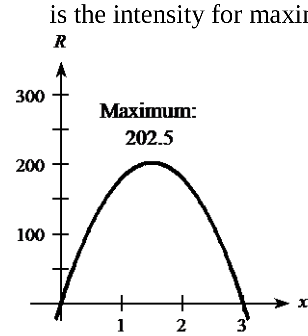
x-coordinate of the vertex $= \dfrac{-b}{2a} = \dfrac{-50}{-2} = 25$

A length of 25 feet and width of 25 feet gives a maximum area of 625 square feet.

37. $R = 270x - 90x^2$
Maximum rate occurs at vertex.

V: $x = \dfrac{-270}{2(-90)} = \dfrac{3}{2}$ (lumens)

is the intensity for maximum rate.



38. $s = 112t - 16t^2$
t-coordinate of the vertex

$= \dfrac{-b}{2a} = \dfrac{-112}{-32} = 3.5$ seconds

At $t = 3.5$, $s = 112(3.5) - 16(3.5)^2 = 196$ feet

39. a. $y = -0.0013x^2 + x + 10$

V: $x = \dfrac{-1}{-0.0026} = 384.62;$

$y = -0.0013(384.62)^2 + 384.62 + 10$
$= 202.31$

b. $y=-\frac{1}{81}x^2+\frac{4}{3}x+10$

V: $x=\frac{\frac{-4}{3}}{\frac{-2}{81}}=54$;

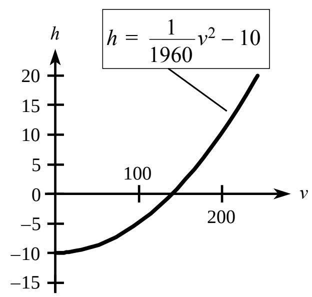
$y=-\frac{1}{81}(54)^2+\frac{4}{3}(54)+10=46$

Projectile **a.** goes 202.31 – 46 = 156.31 feet higher.

40. $v^2=1960(h+10)$

$\frac{v^2}{1960}=h+10$

$h=\frac{1}{1960}v^2-10$



41. a. From b to c. The average rate of change is the same as the slope of the segment. The segment from b to c is steeper.

b. Needs to satisfy $d>b$ to make the segment from a to d have a greater slope.

42. a. From b to c. The average rate of change is the same as the slope of the segment. The segment from b to c has a negative slope.

b. Needs to satisfy $d<b$ to make the segment from a to d have a greater slope.

43. a.

No. of Apts	Rent	Total Revenue
50	\$600	\$30,000
49	\$620	\$30,380
48	\$640	\$30,720

b. Revenue increases \$720

c. $R=(50-x)(600+20x)$

d. $R=-20x^2+400x+30,000$

R is maximized at $x=\frac{-400}{2(-20)}=10$.

Rent would be $\$600+\$200=\$800$.

44. a.

Price	No. of skaters	Total Revenue
12	50	\$600
11	60	\$660
10	70	\$700

b. The revenue increases.

c. $R(x)=(12-0.5x)(50+5x)$ where x is the number of each additional 5 skaters.

d. $R(x)=-2.5x^2+35x+600$. Maximum revenue is at $x=\frac{-35}{-5}=7$, or 85 skaters.

45. a. A quadratic function or parabola.

b. $a<0$ because the graph opens downward.

c. The vertex occurs after 2004 (or when $x>0$), so $-\frac{b}{2a}>0$. Hence with $a<0$ we must have $b>0$. The value $c=f(0)$ or the y-value during 2004 which is positive.

46. $y=ax^2+bx+c$

Zeros: (0, 0) and (40, 0)

Vertex: $\left(\frac{-b}{2a},40\right)$

(0, 0): $0=a(0)^2+b(0)+c$

$0=c$

So, $y=ax^2+bx$

(40, 0): $0=1600a+40b$

$b=-40a$

So, $y=ax^2-40ax$

So, x-coordinate of the vertex = $-\frac{-40a}{2a}=20$.

When x = 20, y = 40

$40=a(20)^2-40(a)(20)$

$40=400a-800a$

$-400a=40$

$a=-\frac{1}{10}$ and $b=4$

The equation is $y=-\frac{1}{10}x^2+4x$

47. $y = 20.61x^2 - 116.4x + 7406$

For 2010, $x = 10$ gives $y = 8303.$

For 2015, $x = 15$ gives $y = 10,297.25.$

For 2020, $x = 20$ gives $y = 13,322.$

Average rate of change from 2010 to 2015:

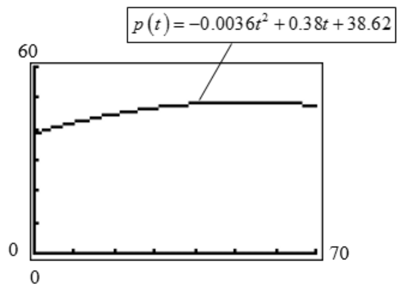
$$\frac{10,297.25 - 8303}{15 - 10} = 398.85$$

Average rate of change from 2015 to 2020:

$$\frac{13,322 - 10,297.25}{20 - 15} = 604.95$$

To the nearest dollar, the projected average rate of change of U.S. per capita health care costs from 2010 to 2015 will be \$399/year, and from 2015 to 2020 it will be \$605/year.

48. a.



b. Using the equation, we identify the maximum point by computing

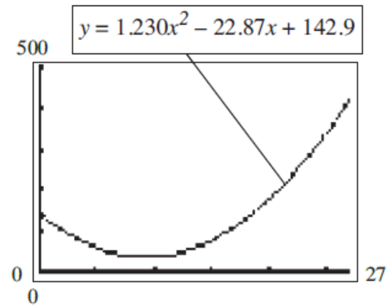
$$t = \frac{-b}{2a} = \frac{-0.38}{2(-0.0036)} \approx 52.78$$

$$p(52.78) \approx 48.65$$

The maximum point is (52.78, 48.65).

c. According to this model, the maximum percentage of women in the workforce occurs in the year $1970 + 53 = 2023$.

49.



50. The graphing calculator gives a minimum point at $(9.3, 36.6)$.

Exercises 2.3

1. $C(x) = x^2 + 40x + 2000$

$R(x) = 130x$

$x^2 + 40x + 2000 = 130x$

$x^2 - 90x + 2000 = 0$

$(x - 40)(x - 50) = 0$

$x = 40$ or $x = 50$

Break-even values are at $x = 40$ and 50 units.

2. At the break-even point, $R(x) = C(x)$.

$$3600 + 25x + \frac{1}{2}x^2 = 175x - \frac{1}{2}x^2$$

$x^2 - 150x + 3600 = 0$

$(x - 120)(x - 30) = 0$

$x = 120$ or $x = 30$ units

3. $C(x) = 15,000 + 35x + 0.1x^2$

$R(x) = 385x - 0.9x^2$

$15,000 + 35x + 0.1x^2 = 385x - 0.9x^2$

$x^2 - 350x + 15,000 = 0$

$(x - 300)(x - 50) = 0$

$x = 300$ or $x = 50$

4. At the break-even points, $R(x) = C(x)$.

$1600x - x^2 = 1600 + 1500x$

$0 = x^2 - 100x + 1600$

$0 = (x - 20)(x - 80)$

$x = 20$ or $x = 80$ units

5. $P(x) = -11.5x - 0.1x^2 - 150$

At the break-even points, $P(x) = 0$.

$$0 = 11.5x - 0.1x^2 - 150$$

$-0.1x^2 + 11.5x - 150 = 0$

$(x-15)(x-100) = 0$

Since production < 75 units, $x = 15$.

6. $P(x) = -1100 + 120x - x^2$

At the break-even points, $P(x) = 0$.

$$0 = -1100 + 120x - x^2$$

$x^2 - 120x + 1100 = 0$

$(x-110)(x-10) = 0$

Since production < 100 units, $x = 10$.

7. $R(x) = 385x - 0.9x^2$

$a = -0.9$, $b = 385$

Maximum revenue is at the vertex.

V: $x = \dfrac{-385}{-1.8} = 213.89$ or 214 total units

$R(214) = 385(214) - 0.9(214)^2 = \$41{,}173.60$

8. $R(x) = 1600x - x^2$

Maximum occurs at the vertex.

x-coordinate = $-\dfrac{1600}{-2} = 800$

$R(800) = 1600(800) - (800)^2 = \$640{,}000$

9. $R(x) = x(175 - 0.50x) = 175x - 0.5x^2$

$a = -0.50$, $b = 175$

Revenue is a maximum at $x = \dfrac{-175}{-1} = 175$.

Price that will maximize revenue is

$p = 175 - 87.50 = \$87.50$.

10. D: $p = 1600 - x \to x = 1600 - p$

Revenue: $R = px = p(1600 - p)$

$$R = 1600p - p^2$$

Max. revenue for $p = -\dfrac{1600}{-2} = \$800$.

11. $P(x) = -x^2 + 110x - 1000$

Maximum profit is at the vertex or when

$x = \dfrac{-110}{-2} = 55$.

$P(55) = \$2025$.

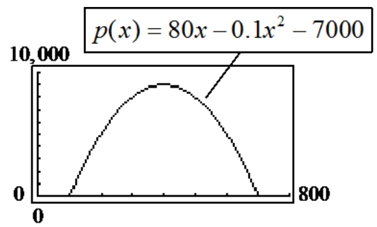
12. $P(x) = 88x - x^2 - 1200$

The x-coordinate giving the maximum profit is

$-\dfrac{b}{2a} = -\dfrac{88}{-2} = 44$.

$P(44) = 88(44) - (44)^2 - 1200 = \$736$

13. **a.**



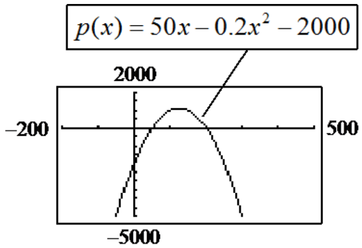
b. $(400, 9000)$ is the maximum

c. positive

d. negative

e. closer to 0

14. a.



b. $(125, 1125)$ is the maximum

c. positive

d. negative

e. closer to 0

15. $R(x) = 385x - 0.9x^2$

$C(x) = 15,000 + 35x + 0.1x^2$

a. $P(x) = 385x - 0.9x^2 - (15,000 + 35x + 0.1x^2)$

$= -x^2 + 350x - 15,000$

At the vertex we have $x = \frac{-350}{-2} = 175.$

So, $P(175)$ = \$15,625.

b. No. More units are required to maximize revenue.

c. The break-even values and zeros of $P(x)$ are the same.

16. a. $P(x) = R(x) - C(x)$

$= 1600x - x^2 - (1600 + 1500x)$

$= 100x - x^2 - 1600$

x-coordinate of max is $-\frac{100}{-2} = 50$

$P(50) = 100(50) - (50)^2 - 1600 = \$900$

b. No. More units are required to maximize revenue.

c. $0 = 100x - x^2 - 1600$

$x^2 - 100x + 1600 = 0$

$(x - 80)(x - 20) = 0$

The x-coordinates are the same.

17. a. $C(x) = 28,000 + \left(\frac{2}{5}x + 222\right)x$

$= \frac{2}{5}x^2 + 222x + 28,000$

$R(x) = \left(1250 - \frac{3}{5}x\right)x = 1250x - \frac{3}{5}x^2$

(The key is "per unit x.")

$R(x) = C(x)$

$1250x - \frac{3}{5}x^2 = \frac{2}{5}x^2 + 222x + 28,000$

$x^2 - 1028x + 28,000 = 0$

$(x - 1000)(x - 28) = 0$

Break-even values are at x = 28 and x = 1000.

b. Maximum revenue occurs at

$x = \frac{-1250}{-\frac{6}{5}} = 1042$ (rounded).

$R(1042) = \$651,041.60$ is the maximum revenue.

c. $P(x) = 1250x - \frac{3}{5}x^2 - \left(\frac{2}{5}x^2 + 222x + 28,000\right)$

$= -x^2 + 1028x - 28,000$

Maximum profit is at $x = \frac{-1028}{-2} = 514.$

$P(514) = \$236,196$ is the maximum profit.

d. Price that will maximize profit is

$p = 1250 - \frac{3}{5}(514) = \$941.60.$

18. a. $C(x) = 300 + \left(\frac{3}{4}x + 1460\right)x$

$= 300 + \frac{3}{4}x^2 + 1460x$

$R(x) = \left(1500 - \frac{1}{4}x\right)x = 1500x - \frac{1}{4}x^2$

At break-even points $C(x) = R(x)$.

$300 + \frac{3}{4}x^2 + 1460x = 1500x - \frac{1}{4}x^2$

$x^2 - 40x + 300 = 0$

$(x - 30)(x - 10) = 0$

$x = 30$ or $x = 10$

b. Maximum revenue:

x-coordinate: $-\frac{b}{2a} = -\frac{1500}{-\frac{1}{2}} = 3000$

$R(3000) = 1500(3000) - \frac{1}{4}(3000)^2$

$= \$2,250,000$

c. $P(x) = R(x) - C(x)$

$= 1500x - \frac{1}{4}x^2 - \left(300 + \frac{3}{4}x^2 + 1460x\right)$

$= 40x - x^2 - 300$

Maximum profit:

x-coordinate: $-\frac{b}{2a} = -\frac{40}{-2} = 20$

$P(20) = 40(20) - (20)^2 - 300 = \$100$

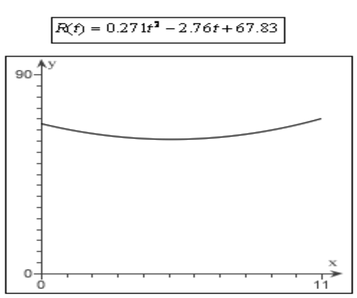
d. Selling price $= 1500 - \frac{1}{4}x$. When $x = 20$,

$p = 1500 - \frac{1}{4}(20) = \$1495$

19. a. $t \approx 5.1$, in 2012; $R \approx \$60.79$ billion

b. The data show a smaller revenue, R = $60.27 billion in 2011.

c.



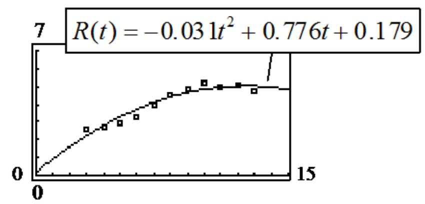
d. The model fits the data quite well.

20. $R(t) = -0.031t^2 + 0.776t + 0.179$

a. Maximum occurs at the vertex. The t-coordinate of the vertex is $-\frac{0.776}{-0.062} \approx 12.5$. Maximum revenue occurred during 2016. The maximum revenue predicted by the model is $R(12.5) \approx \$5.035$ million.

b. The entry in the table for 2016 is $4.7489 million, so the values are close. However, the 2013 revenues were greater than this.

c.

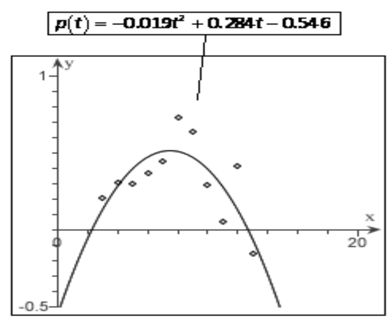


d. Although there are differences, the model appears to be a good quadratic fit for the data.

21. a. $p(t) = -0.019t^2 + 0.284t - 0.546$

b. 2011

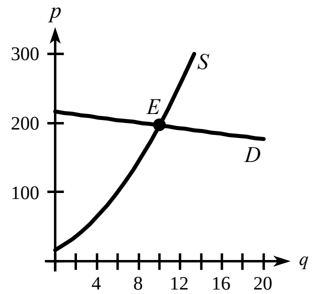
c.



d. The model projects decreasing profits, and, except for 2015, the data support this.

e. Management would be interested in increasing revenues or reducing costs (or both) to improve profit.

22. **a.** Supply: $p = q^2 + 8q + 16$ (see below)



Demand: $p = 216 - 2q$ (see below)

b. See E on the graph.

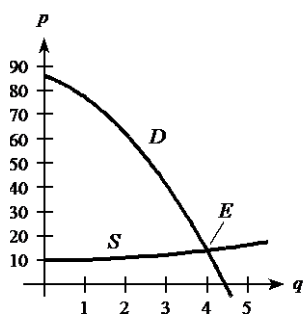
c. Supply = Demand

$$q^2 + 8q + 16 = 216 - 2q$$
$$q^2 + 10q - 200 = 0$$
$$(q - 10)(q + 20) = 0$$

q = 10 (only positive value)
p = 216 – 2(10) = 196
q = 10, p = \$196

23. **a.** Supply: $p = \frac{1}{4}q^2 + 10$ (see below)

Demand: $p = 86 - 6q - 3q^2$ (see below)



b. See E on graph.

c. $\frac{1}{4}q^2 + 10 = 86 - 6q - 3q^2$

$$q^2 + 40 = 344 - 24q - 12q^2$$
$$0 = 13q^2 + 24q - 304$$
$$0 = (q - 4)(13q + 76)$$

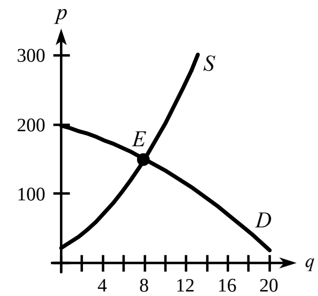
q = 4 must be positive.

$$p = \frac{1}{4}(4)^2 + 10 = 14$$

E: (4, 14)

24. **a.** Supply: $p = q^2 + 8q + 22$ (see below)

Demand: $p = 198 - 4q - \frac{1}{4}q^2$ (see below)



b. See E on the graph.

c. Supply = Demand

$$q^2 + 8q + 22 = 198 - 4q - \frac{1}{4}q^2$$
$$5q^2 + 48q - 704 = 0$$
$$(5q + 88)(q - 8) = 0$$

q = 8 (only positive value)

When q = 8, $p = (8)^2 + 8(8) + 22$
$p = 150$

So, E = (8, 150).

25. $p = q^2 + 8q + 16$

$$p = -3q^2 + 6q + 436$$
$$q^2 + 8q + 16 = -3q^2 + 6q + 436$$
$$4q^2 + 2q - 420 = 0$$
$$2q^2 + q - 210 = 0$$
$$(2q + 21)(q - 10) = 0$$
$$q = 10$$
$$p = 10^2 + 8(10) + 16 = 196$$

E: (10, 196)

26. S: $p = q^2 + 8q + 20$

D: $100 - 4q - q^2 = p$

$$q^2 + 8q + 20 = 100 - 4q - q^2$$
$$2q^2 + 12q - 80 = 0$$
$$2(q + 10)(q - 4) = 0$$

q = 4 (only positive value)

When q = 4, $p = 4^2 + 8(4) + 20 = \$68$

Equilibrium point: (4, 68)

27. $p^2 + 4q = 1600$

$300 - p^2 + 2q = 0$

$(300 + 2q) + 4q = 1600$

$6q = 1300$

$q = 216\frac{2}{3}$

$p^2 + 4\left(\frac{1300}{6}\right) = 1600$ or p^2

$= 733.33$ or $p = 27.08$

E: $\left(216\frac{2}{3}, 27.08\right)$

28. S: $4p - q = 42$ or $q = 4p - 42$

D: $(p+2)q = 2100$ or $q = \frac{2100}{p+2}$

$4p - 42 = \frac{2100}{p+2}$

$4p^2 - 34p - 84 = 2100$

$4p^2 - 34p - 2184 = 0$

$2(2p+39)(p-28) = 0$

$p = 28$ (only positive value)

When $p = \$28$, $q = 4(28) - 42 = 70$

Equilibrium point: (70, 28)

29. $p - q = 10$ or $q = p - 10$

$q(2p-10) = 2100$

$q = \frac{2100}{2p-10}$

$p - 10 = \frac{2100}{2p-10}$

$(p-10)(2p-10) = 2100$

$2p^2 - 30p + 100 = 2100$

$2p^2 - 30p - 2000 = 0$

$p^2 - 15p - 1000 = 0$

$(p-40)(p+25) = 0$

$p = 40$ or $p = -25$

(only the positive answer makes sense here)

$q = 40 - 10 = 30$

$E: (30, 40)$

30. S: $2p - q + 6 = 0$ or $q = 2p + 6$

D: $(p + q)(q + 10) = 3696$

Substitute $2p + 6$ for q in D and solve for p.

$(3p+6)(2p+16) = 3696$

$6p^2 + 60p - 3600 = 0$

$p^2 + 10p - 600 = 0$

$(p+30)(p-20) = 0$

$p = 20$ (only positive value)

When $p = 20$, $q = 2(20) + 6 = 46$.

Equilibrium point: (46, 20)

31. $2p - q - 10 = 0$

$(p + 10)(q + 30) = 7200$

So, $(p+10)(2p-10+30) = 7200$

$p^2 + 20p + 100 = 3600$

$p^2 + 20p - 3500 = 0$

$(p+70)(p-50) = 0$

$p = 50$

$q = 2(50) - 10 = 90$

E: $(q, p) = (90, 50)$

32. S: $2p - q = 50$ or $p = \frac{q+50}{2}$

D: $pq = 100 + 20q$ or $p = \frac{100+20q}{q}$

$\frac{q+50}{2} = \frac{100+20q}{q}$

$q^2 + 50q = 200 + 40q$

$q^2 + 10q - 200 = 0$

$(q+20)(q-10) = 0$

$q = 10$ (only positive value)

When $q = 10$, $p = 30$

Equilibrium point: (10, 30)

33. $p = \frac{1}{2}q + 5 + 22 = \frac{1}{2}q + 27$

So, $\left(\frac{1}{2}q + 27 + 10\right)(q + 30) = 7200$

$$(q+74)(q+30) = 14,400$$
$$q^2 + 104q - 12,180 = 0$$
$$(q+174)(q-70) = 0$$

$p = \frac{1}{2}(70) + 27 = 62$

E: (70, 62)

34. S: $p = \frac{q+50}{2} + 12.50$

D: $p = \frac{100+20q}{q}$

$$\frac{q+50}{2} + 12.50 = \frac{100+20q}{q}$$
$$q^2 + 75q = 200 + 40q$$
$$q^2 + 35q - 200 = 0$$
$$(q+40)(q-5) = 0$$

q = 5 (only positive value)

When q = 5, Equilibrium point: (5, 40)

Exercises 2.4

1. b

2. g

3. f

4. h

5. j

6. e

7. k

8. d

9. a

10. i

11. c

12. l

13. **a.** cubic
b. quartic

14. **a.** quartic
b. cubic

15. $y = x^3 - x = x(x+1)(x-1)$: e

16. $y = (x-3)^2(x+1)$: c

17. $y = 16x^2 - x^4 = x^2(4+x)(4-x)$: b

18. $y = x^4 - 3x^2 - 4 = (x^2 - 4)(x^2 + 1)$: h

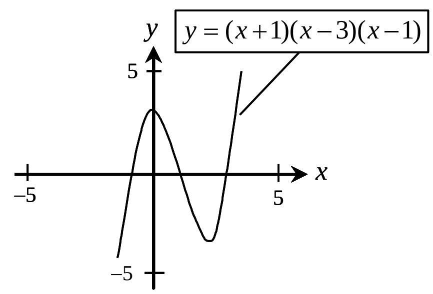
19. $y = x^2 + 7x = x(x+7)$: d

20. $y = 7x - x^2 = x(7-x)$: a

21. $y = \frac{x-3}{x+1}$: g

22. $y = \frac{1-3x}{2x+5}$: f

23.



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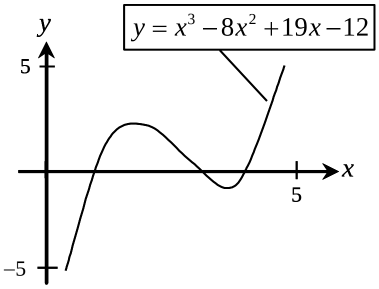
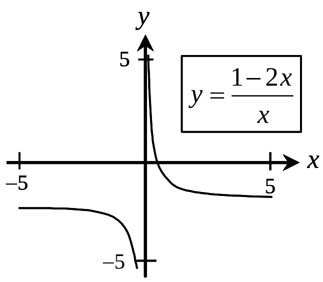
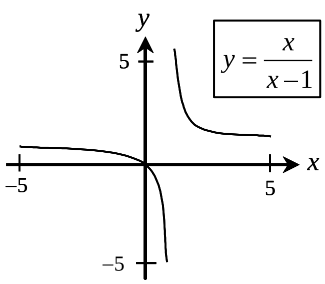


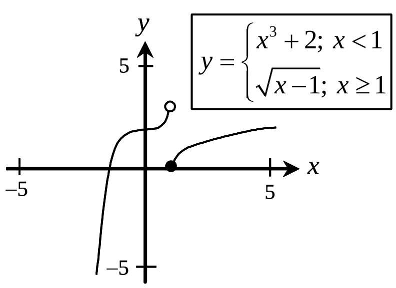
25.



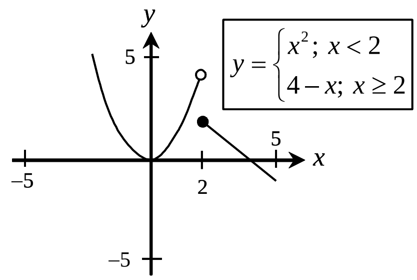
26.



27.



28.



29. $F(x) = \frac{x^2 - 1}{x}$

a. $F\left(-\frac{1}{3}\right) = \frac{\frac{1}{9} - 1}{-\frac{1}{3}} = \frac{8}{3}$

b. $F(10) = \frac{100 - 1}{10} = \frac{99}{10}$

$F(x) = \frac{x^2 - 1}{x}$

c. $F\left(-\frac{1}{3}\right) = \frac{\frac{1}{9} - 1}{-\frac{1}{3}} = \frac{8}{3}$

d. $F(10) = \frac{100 - 1}{10} = \frac{99}{10}$

e. $F(0.001) = \frac{0.000001 - 1}{0.001} = \frac{-0.999999}{0.001}$
$= -999.999$

f. $F(0)$ is not defined–division by zero.

30. $H(x) = |x - 1|$

a. $H(-1) = 2$
b. $H(1) = 0$
c. $H(0) = 1$
d. No

31. $f(x) = x^{3/2}$

a. $f(16) = (\sqrt{16})^3 = 64$

b. $f(1) = (\sqrt{1})^3 = 1$

c. $f(100) = (\sqrt{100})^3 = 1000$

d. $f(0.09) = (\sqrt{0.09})^3 = 0.027$

32. $k(x) = \begin{cases} 4-2x & \text{if } x<0 \\ |x-4| & \text{if } 0<x<4 \end{cases}$

a. $k(-0.1) = 4-2(-0.1) = 4.2$

b. $k(0.1) = |0.1-4| = |-3.9| = 3.9$

c. $k(3.9) = |3.9-4| = |-0.1| = 0.1$

d. $k(4.1)$ is undefined

33. $k(x) = \begin{cases} 2 & \text{if } x<0 \\ x+4 & \text{if } 0 \le x < 1 \\ 1-x & \text{if } x \ge 1 \end{cases}$

a. $k(-5) = 2$ since $x < 0$.

b. $k(0) = 0+4 = 4$

c. $k(1) = 1-1 = 0$

d. $k(-0.001) = 2$ since $x < 0$.

34. $g(x) = \begin{cases} 0.5x+4 & \text{if } x<0 \\ 4-x & \text{if } 0 \le x < 4 \\ 0 & \text{if } x > 4 \end{cases}$

a. $g(-4) = 0.5(-4)+4 = -2+4 = 2$

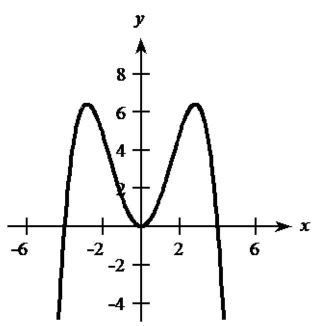
b. $g(1) = 4-1 = 3$

c. $g(7) = 0$

d. $g(3.9) = 4-3.9 = 0.1$

35. $y = 1.6x^2 - 0.1x^4$

a.



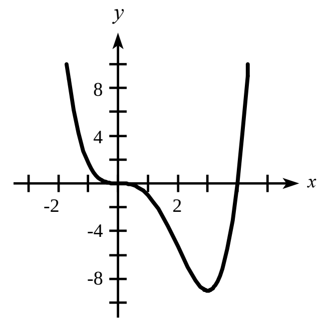
b. polynomial

c. no asymptotes

d. turning points at $x = 0$ and approximately $x = -2.8$ and $x = 2.8$

36. $f(x) = \dfrac{x^4 - 4x^3}{3}$

a.



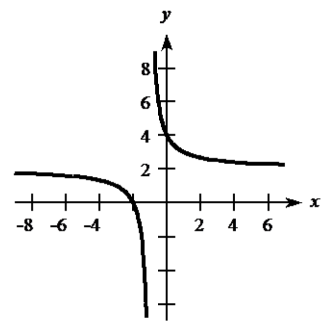
b. polynomial

c. no asymptotes

d. turning point at $x = 3$

37. $y = \dfrac{2x+4}{x+1}$

a.



b. rational

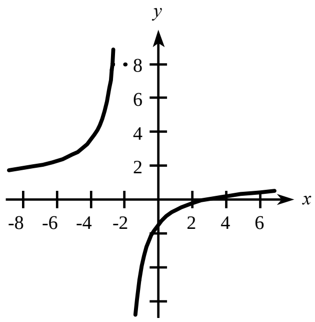
c. vertical: $x = -1$

horizontal: $y = 2$

d. no turning points

38. $f(x) = \dfrac{x-3}{x+2}$

a.



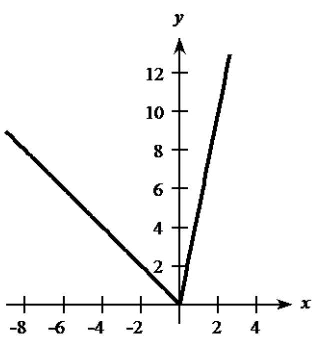
b. rational

c. vertical: $x = -2$
horizontal: $x = 1$

d. no turning points

39. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ 5x & \text{if } x \ge 0 \end{cases}$

a.



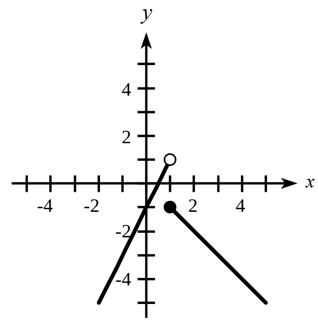
b. piecewise

c. no asymptotes

d. turning point at $x = 0$.

40. $f(x) = \begin{cases} 2x-1 & \text{if } x < 1 \\ -x & \text{if } x \ge 1 \end{cases}$

a.



b. piecewise

c. no asymptotes

d. no turning point (there is a jump at $x = 1$).

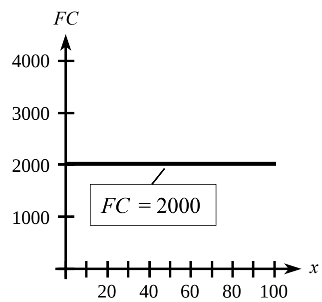
41. $V = V(x) = x^2(108-4x)$

a. $V(10) = 100(68) = 6800$ cubic inches
$V(20) = 400(28) = 11{,}200$ cubic inches

b. $108 - 4x > 0$

$$-4x > -108$$
$$0 < x < 27$$

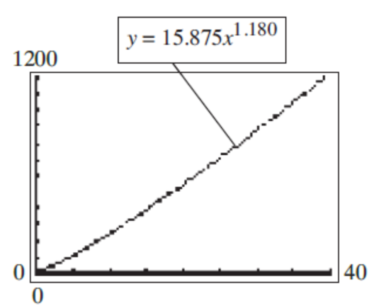
42.



43. $f(x) = 15.875x^{1.18}$

a. upward

b.

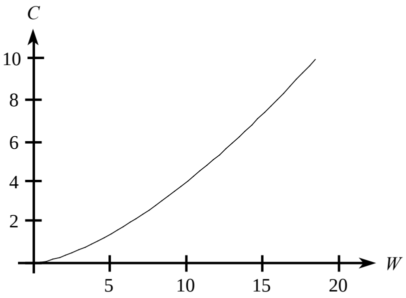


c. Intersecting the graphs of $y = 15.875x^{1.18}$ and $y = 1150$ gives $x \approx 37.7$. Global spending is expected to reach \$1,150,000,000,000 (\$1150 billion) in 1980 + 38 = 2018.

44. $C = 0.11W^{1.54}$

a. $W^{1.54}$ is "close" to W^2. The graph is turning up.

b.



c. $C(10) = 0.11(10)^{1.54} = 3.814$ grams

45. $C(p) = \dfrac{7300p}{100 - p}$

a. $0 \le p < 100$

b. $C(45) = \dfrac{7300 \cdot 45}{100 - 45} = \$5972.73$

c. $C(90) = \dfrac{7300 \cdot 90}{100 - 90} = \$65,700$

d. $C(99) = \dfrac{7300 \cdot 99}{100 - 99} = \$722,700$

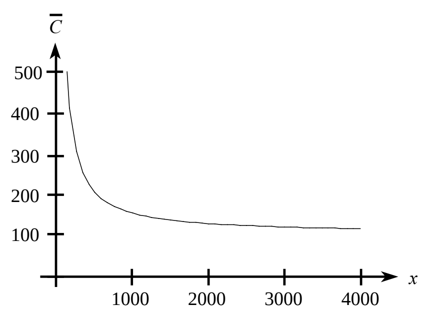
e. $C(99.6) = \dfrac{7300(99.6)}{100 - 99.6} = \$1,817,700$

f. To remove p% of the pollution would cost $C(p)$. Note how cost increases as p (the percent of pollution removed) increases.

46. $\overline{C} = \dfrac{50,000 + 105x}{x}$

a. $\overline{C}(3000) = \dfrac{50,000 + 105(3000)}{3000} = \$121.67$

b.



c. Yes. $\overline{C}(x) = \dfrac{50,000}{x} + 105$

47. $A = A(x) = x(50 - x)$

a. $A(2) = 2 \cdot 48 = 96$ square feet

$A(30) = 30 \cdot 20 = 600$ square feet

b. $0 < x < 50$ in order to have a rectangle.

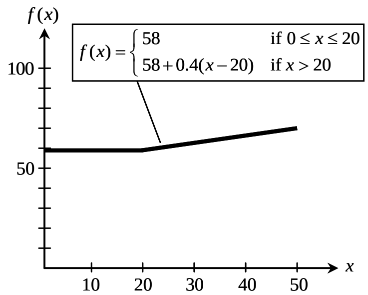
48. $f(x) = \begin{cases} 58 & \text{if } 0 \le x \le 20 \\ 58 + 0.4(x - 20) & \text{if } x > 20 \end{cases}$

a. $f(0.3) = \$58$

b. $f(30) = 58 + 0.4(30 - 20) = \$62$

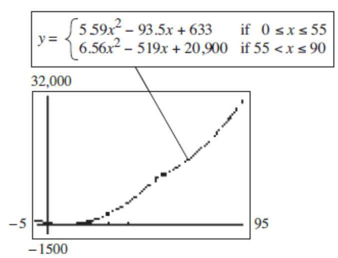
c. $f(40) = 58 + 0.4(40 - 20) = \$66$

d.



49. $y = \begin{cases} 5.59x^2 - 93.5x + 633 & \text{for } 0 \le x \le 55 \\ 6.56x^2 - 519x + 20{,}900 & \text{for } 55 < x \le 90 \end{cases}$

a.



b. $y(50) = 5.59(50)^2 - 93.5(50) + 633 = \$9933$ billion (\$9.933 trillion)

c. $y(75) = 6.56(75)^2 - 519(75) + 20{,}900 = \$18{,}875$ billion (\$18.875 trillion)

50. **a.** $C(5) = 7.52 + 0.1079(5) = \$8.06$

b. $C(6) = 19.22 + 0.1079(6) = \$19.87$

c. $C(3000) = 131.345 + 0.0321(3000) = \$227.65$

51. **a.** $P(x) = \begin{cases} 49 & \text{if } 0 < x \le 1 \\ 70 & \text{if } 1 < x \le 2 \\ 91 & \text{if } 2 < x \le 3 \\ 112 & \text{if } 3 < x \le 4 \end{cases}$

b. $P(1.2) = 70$; it costs 70 cents to mail a 1.2-oz letter.

c. Domain: $0 < x \le 4$; Range: $\{49, 70, 91, 112\}$

d. The postage for a 2-ounce letter is 70 cents; for a 201-ounce letter, it is 91 cents.

52. a. $T(x)=\begin{cases}0.10x & \text{if } 0\le x\le 16,750\\ 0.15(x-16,750)+1,675 & \text{if } 16,750<x\le 68,000\\ 0.25(x-68,000)+9,362.50 & \text{if } 68,000<x\le 137,300\end{cases}$

b. $T(70,000)=0.25(70,000-68,000)+9,362.50=\$9,862.50$

c. $T(50,000)=0.15(50,000-16,750)+1,675=\$6,662.50$

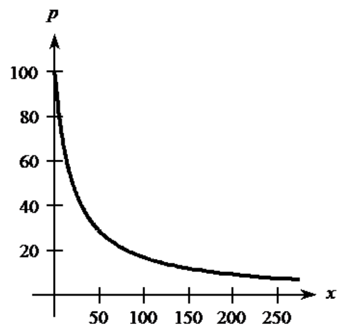
d. $T(68,000)=0.15(68,000-16,750)+1,675=\$9,362.50$

$T(68,001)=0.25(68,001-68,000)+9,362.50=\$9,362.75$

Jack's tax went up \$0.25 for the extra dollar earned. He is only charged 25% on the money he earns above \$68,000.

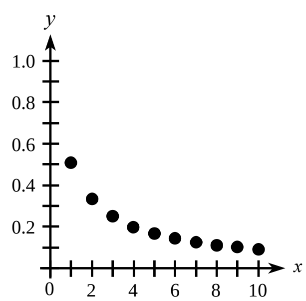
53. $p=\dfrac{200}{2+0.1x}$

a.



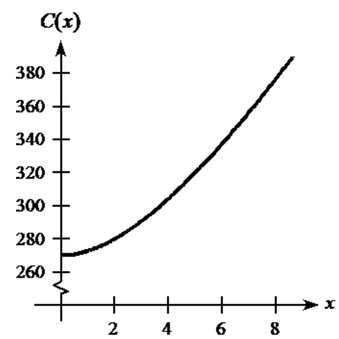
b. No

54. $y=\dfrac{1}{x+1}$, x positive integers



55. $C(x)=30(x-1)+\dfrac{3000}{x+10}$

a.



b. A turning point indicates a minimum or maximum cost.

c. This is the fixed cost of production.

Exercises 2.5

1. Linear: The points are in a straight line.

2. Power

3. Quadratic: The points appear to fit a parabola.

4. Linear

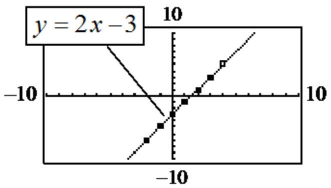
5. Quartic: The graph crosses the x-axis four times. Also there are three bends.

6. Cubic

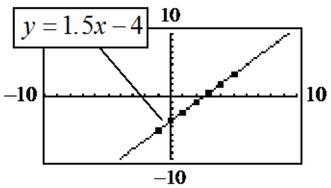
7. Quadratic: There is one bend. A parabola is the best fit.

8. Cubic

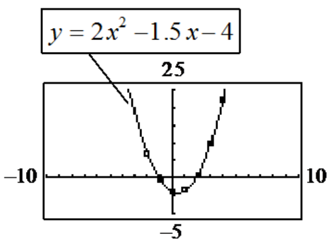
9. $y = 2x - 3$ is the best fit.



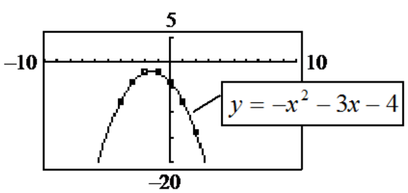
10. $y = 1.5x - 4$ is the best fit.



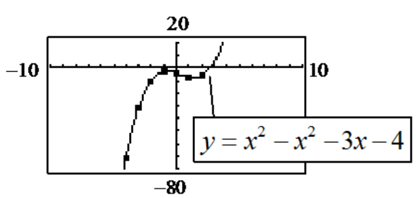
11. $y = 2x^2 - 1.5x - 4$ is the best fit.



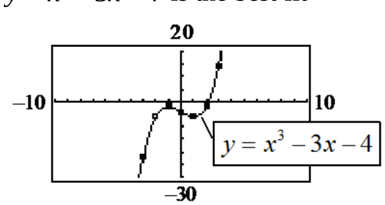
12. $y = -x^2 - 3x - 4$ is the best fit.



13. $y = x^3 - x^2 - 3x - 4$ is the best fit.

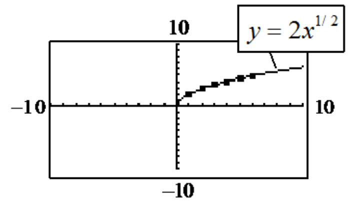


14. $y = x^3 - 3x - 4$ is the best fit

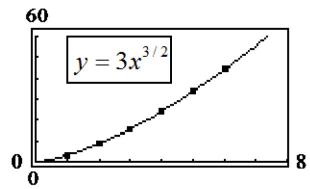


.

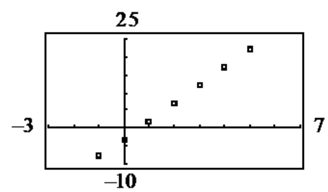
15. $y = 2x^{1/2}$ is the best fit.



16. $y = 3x^{3/2}$ is the best fit.



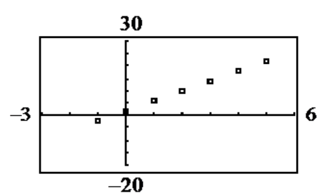
17. a.



b. linear

c. $y = 5x - 3$

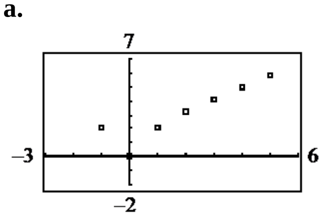
18. a.



b. linear

c. $y = 4x + 2$

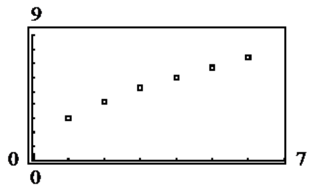
19. a.



b. quadratic

c. $y = 0.0959x^2 + 0.4656x + 1.4758$

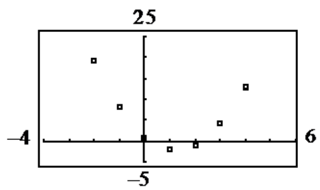
20. **a.**



b. power

c. $y = 3x^{1/2}$

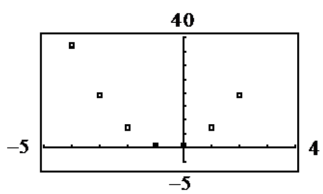
21. **a.**



b. quadratic

c. $y = 2x^2 - 5x + 1$

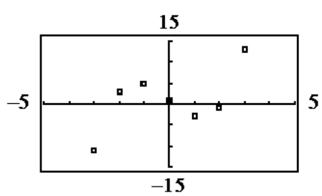
22. **a.**



b. quadratic

c. $3x^2 + 3x + 1$

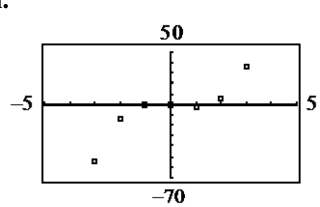
23. **a.**



b. cubic

c. $y = x^3 - 5x + 1$

24. **a.**



b. cubic

c. $y = 2x^3 - x^2 - 3x$

25. **a.** $y = 154.0x + 35{,}860$

b. $y(27) = 154.0(27) + 35{,}860 = 40{,}018$

The projected population of females under age 18 in 2037 is 40,018,000.

c. $45{,}000 = 154.0x + 35860 \Rightarrow x \approx 59.35$

This population will reach 45,000,000 in 2010 + 60 = 2070 according to this model.

26. **a.** $y = 18.96x + 321.5$

b. $y(14) = 18.96(14) + 321.5 \approx 586.9$ million metric tons

c. $m = 18.96;$ each year since 2010, carbon dioxide emissions in the U.S. are expected to change by 18.96 million metric tons.

27. **a.** A linear function is best; $y = 327.6x + 9591$

b. $y(17) = 327.6(17) + 9591 \approx \$15{,}160$ billion

c. $m = 327.6$ means the U.S. disposable income is increasing at the rate of about \$327.6 billion per year.

28. **a.** $y = 0.465x + 12.0$

b. $y(18) = 0.465(18) + 12.0 \approx 20.4\%$

c. $25 = 0.465x + 12.0 \Rightarrow x \approx 28$

This model predicts that the percent of U.S. adults with diabetes will each 25% in 2000 + 28 = 2028.

29. **a.** $y = 0.0052x^2 - 0.62x + 15$

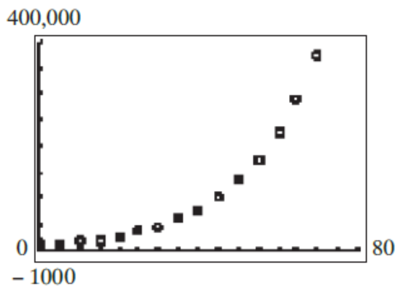
b. $x = \dfrac{-b}{2a} = \dfrac{0.62}{2(0.0052)} \approx 59.6$

c. No, it is unreasonable to feel warmer for winds greater than 60 mph.

30. **a.** $y = 0.0472x^2 + 2.64x + 12.1$

b. A maximum occurs at approximately $(28.0, 48.9)$. The model predicts that in the year 2000 + 28 = 2028, developing economies reach their maximum share, 48.9%, of the GDP.

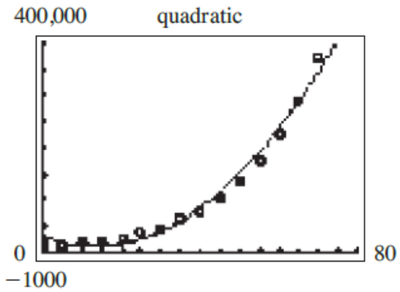
31. a.

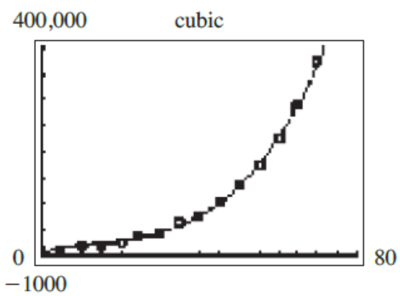


b. $y = 106x^2 - 2870x + 28,500$

c. $y = 1.70x^3 - 72.9x^2 + 1970x + 5270$

d.



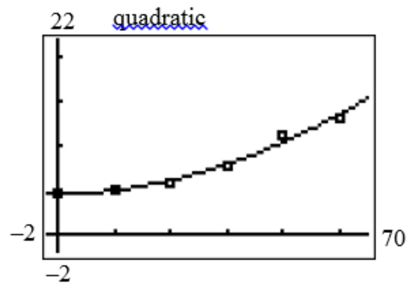


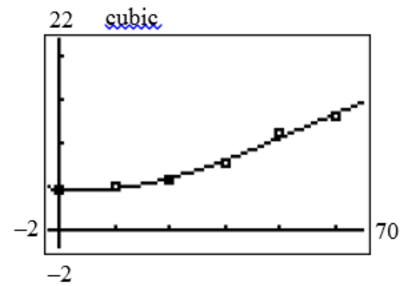
The cubic model fits better.

32. a. $y = 0.00336x^2 + 0.0127x + 4.47$

b. $y = -0.0000537x^3 + 0.00738x^2 - 0.0609x + 4.63$

c.



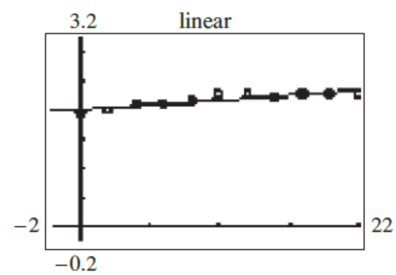


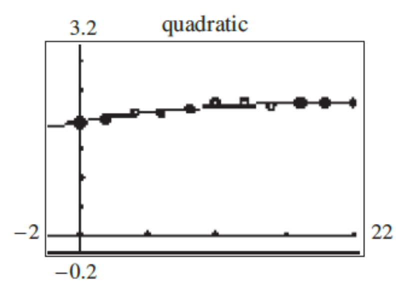
d. The fits look to be equally close.

33. a. $y = 0.0157x + 2.01$

b. $y = -0.00105x^2 + 0.367x + 1.94$

c.



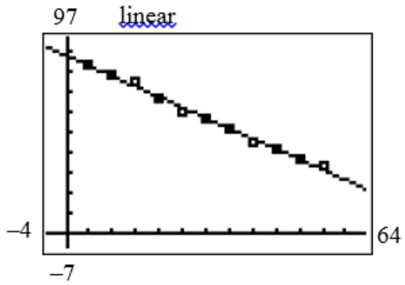


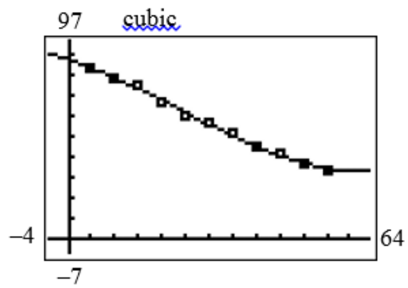
d. The quadratic model is a slightly better fit.

34. a. $y = -1.03x + 88.1$

b. $y = 0.000252x^3 - 0.0178x^2 - 0.0756x + 87.8$

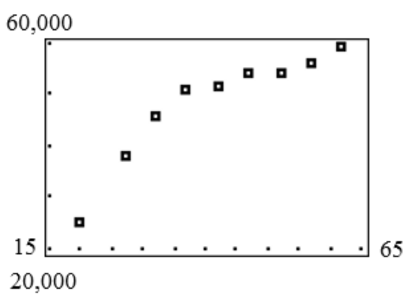
c.





d. The cubic model indicates that the percent of energy use may increase after 2035.

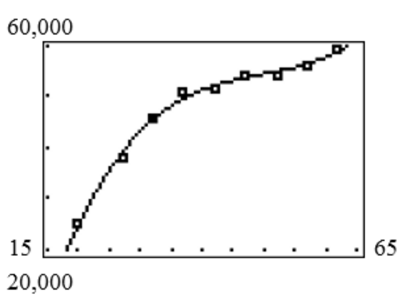
35. a.



A cubic model looks best because of the two bends.

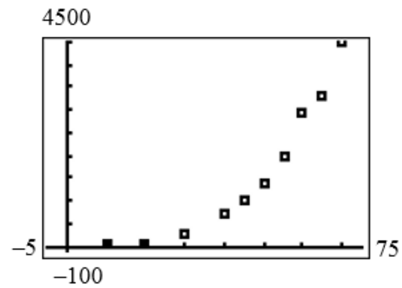
b. $y = 0.864x^3 - 128x^2 + 6610x - 62,600$

c.



d. Using the coefficient values reported by the calculator, the model estimates the median income to be $56,250 at age 57.

36. a.



It appears that both quadratic and power functions would make good models for these data.

b. power: $y = 0.0315x^{2.74}$

quadratic: $y = 1.76x^2 - 71.0x + 679$

c. power: $y(70) \approx \$3661$ billion

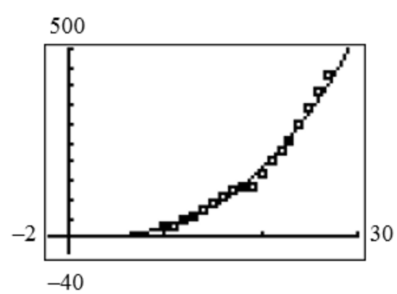
quadratic: $y(70) \approx \$4335$ billion

The quadratic model more accurately approximates the data point for 2020.

d. $y(75) \approx \$5257$ billion; $5257 billion is the national health-care expenditure predicted by the model for 2025.

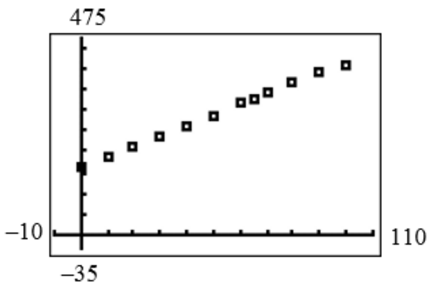
37. a. $y = 0.0514x^{2.73}$

b.



c. $y(30) \approx \$546$ billion

38. a.



b. Possible models are

linear: $y = 2.532x + 162.2$

quadratic: $y = -0.001020x^2 + 2.633x + 160.7$

cubic: $y = -0.00007456x^3 + 0.01030x^2 + 2.191x + 163.5$

c. linear: $y(90) \approx 390.08$

quadratic: $y(90) \approx 389.36$

cubic: $y(90) \approx 389.77$

The linear model most accurately approximates the data point for the year 2040.

d. Replacing y with 425 in the linear model gives $x \approx 103.8$. The U.S. population is predicted to reach 425 million in 1950 + 104 = 2054.

Chapter 2 Review Exercises

1. $3x^2 + 10x = 5x$

$3x^2 + 5x = 0$

$x(3x+5) = 0$

$x = 0 \text{ or } x = -\dfrac{5}{3}$

2. $4x - 3x^2 = 0$

$x(4-3x) = 0$

$x = 0 \text{ or } x = \dfrac{4}{3}$

3. $x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3$ or $x = -2$

4. $11 - 10x - 2x^2 = 0$

$a = -2$, $b = -10$, $c = 11$

$$x = \frac{10 \pm \sqrt{100+88}}{-4} = \frac{-5 \pm \sqrt{47}}{2}$$

5. $(x-1)(x+3) = -8$

$x^2 + 2x - 3 = -8$

$x^2 + 2x + 5 = 0$

$b^2 - 4ac < 0$

No real solution

6. $4x^2 = 3$

$x^2 = \dfrac{3}{4}$

$x = \pm\sqrt{\dfrac{3}{4}} = \pm\dfrac{\sqrt{3}}{2}$

7. $20x^2 + 3x = 20 - 15x^2$

$35x^2 + 3x - 20 = 0$

$(7x-5)(5x+4) = 0$

$x = \dfrac{5}{7} \text{ or } x = -\dfrac{4}{5}$

8. $8x^2 + 8x = 1 - 8x^2$

$16x^2 + 8x - 1 = 0$

$a = 16$, $b = 8$, $c = -1$

$$x = \frac{-8 \pm \sqrt{64+64}}{32} = \frac{-1 \pm \sqrt{2}}{4}$$

9. $7 = 2.07x - 0.02x^2$

$0.02x^2 - 2.07x + 7 = 0$

$a = 0.02$, $b = -2.07$, $c = 7$

$$x = \frac{2.07 \pm \sqrt{4.2849 - 0.56}}{0.04} = \frac{2.07 \pm 1.93}{0.04}$$

$= 100 \text{ or } 3.5$

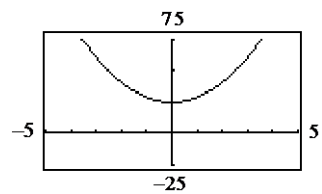
10. $46.3x - 117 - 0.5x^2 = 0$

$a = -0.5$, $b = 46.3$, $c = -117$

$$x = \frac{-46.3 \pm \sqrt{2143.69 + (-234)}}{-1} = \frac{-46.3 \pm 43.7}{-1}$$

$= 90 \text{ or } 2.6$

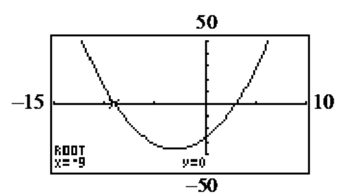
11. $4z^2 + 25 = 0$

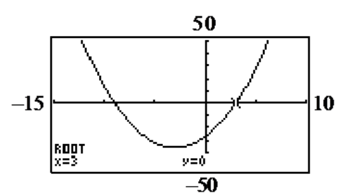


$4z^2 + 5^2 = 0$

The sum of 2 squares cannot be factored. There are no real solutions.

12. $f(z) = z^2 + 6z - 27$





From the graph, the zeros are –9 and 3.

Algebraic solution:

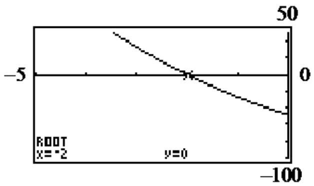
$z(z+6) = 27$

$z^2 + 6z - 27 = 0$

$(z+9)(z-3) = 0$

$z = -9$ or $z = 3$

13. $3x^2 - 18x - 48 = 0$



$3(x^2 - 6x - 16) = 0$

$3(x-8)(x+2) = 0$

$x = -2,\ x = 8$

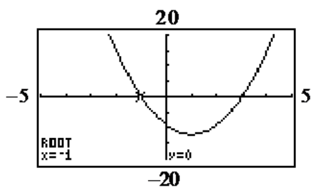
14. $f(x) = 3x^2 - 6x - 9$

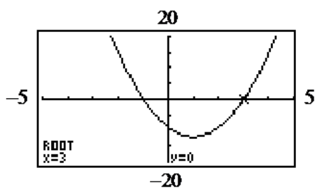
$3x^2 - 6x - 9 = 0$

$3\left(x^2 - 2x - 3\right) = 0$

$3\left(x-3\right)\left(x+1\right) = 0$

$x = 3,\quad x = -1$





15. $x^2 + ax + b = 0$

To apply the quadratic formula we have *"a"* = 1, *"b"* = *a*, and *"c"* = *b*.

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

16. $xr^2 - 4ar - x^2c = 0$

To solve for *r*, use the quadratic formula with "*a*" = *x*, "*b*" = –4*a*, and "*c*" = $-x^2c$.

$$r = \frac{4a \pm \sqrt{16a^2 + 4x(x^2c)}}{2x} = \frac{4a \pm \sqrt{16a^2 + 4x^3c}}{2x}$$

$$= \frac{4a \pm 2\sqrt{4a^2 + x^3c}}{2x} = \frac{2a \pm \sqrt{4a^2 + x^3c}}{x}$$

17. $-0.002x^2 - 14.1x + 23.1 = 0$

$$x = \frac{14.1 \pm \sqrt{198.81 + 0.1848}}{-0.004} = \frac{14.1 \pm 14.107}{-0.004}$$

$= -7051.64,\ 1.64,$ or 1.75 (using 14.107)

18. $1.03x^2 + 2.02x - 1.015 = 0$

a = 1.03, *b* = 2.02, *c* = –1.015

$$x = \frac{-2.02 \pm \sqrt{4.0804 + 4.1818}}{2.06} = \frac{-2.02 \pm 2.87}{2.06}$$

$= -2.38$ or 0.41

19. $y = \frac{1}{2}x^2 + 2x$

a > 0, thus vertex is a minimum.

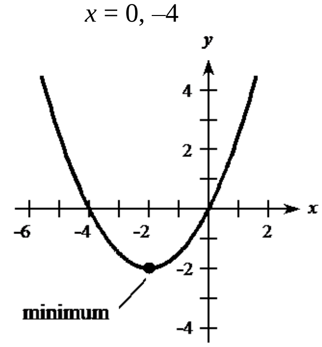
V: $x = \frac{-2}{2\left(\frac{1}{2}\right)} = -2$

$y = \frac{1}{2}(-2)^2 + 2(-2) = -2$

Zeros: $\frac{1}{2}x^2 + 2x = 0$

$x\left(\frac{1}{2}x + 2\right) = 0$

x = 0, –4



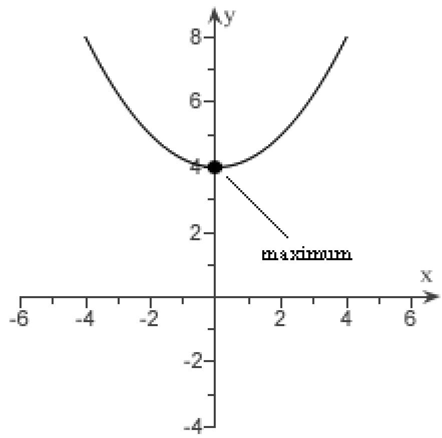
20. $y = 4 + \frac{1}{4}x^2$

V: *x*-coordinate = 0

y-coordinate = 4

(0, 4) is a maximum point

Zeros are $x = \pm 4$.



21. $y = 6 + x - x^2$

$a < 0$, thus vertex is a maximum.

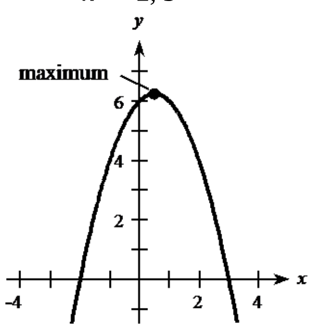
V: $x = \dfrac{-1}{2(-1)} = \dfrac{1}{2}$

$y = 6 + \dfrac{1}{2} - \left(\dfrac{1}{2}\right)^2 = \dfrac{25}{4}$

Zeros: $6 + x - x^2 = 0$

$(3 - x)(2 + x) = 0$

$x = -2, 3$



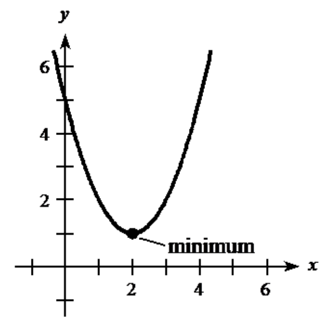
22. $y = x^2 - 4x + 5$

V: x-coordinate $= \dfrac{4}{2} = 2$

y-coordinate $= 2^2 - 4(2) + 5 = 1$

(2, 1) is a minimum point.

Zeros: Since the minimum point is above the x-axis, there are no zeros.



23. $y = x^2 + 6x + 9$

$a > 0$, thus vertex is a minimum.

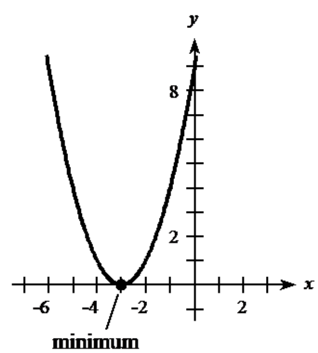
V: $x = \dfrac{-6}{2(1)} = -3$

$y = (-3)^2 + 6(-3) + 9 = 0$

Zeros: $x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$x = -3$



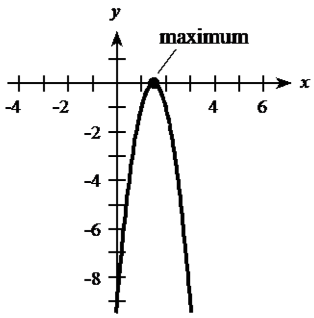
24. $y = 12x - 9 - 4x^2$

V: x-coordinate $= -\dfrac{12}{-8} = \dfrac{3}{2}$

y-coordinate $= 12\left(\dfrac{3}{2}\right) - 9 - 4\left(\dfrac{3}{2}\right)^2 = 0$

$\left(\frac{3}{2}, 0\right)$ is a maximum point.

Zeros: From the vertex we have that $x = \frac{3}{2}$ is the only zero.



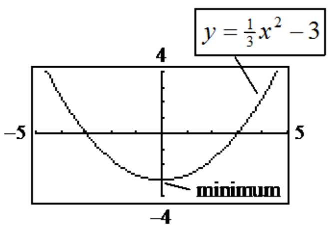
25. $y = \frac{1}{3}x^2 - 3$

V: (0, –3)

Zeros: $\frac{1}{3}x^2 - 3 = 0$

$x^2 = 9$

$x = \pm 3$



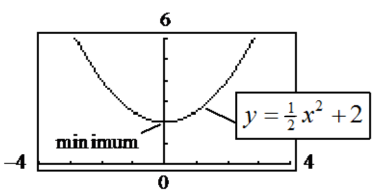
26. $y = \frac{1}{2}x^2 + 2$

Vertex: (0, 2) ← minimum

No zeros.

The graph using x-min = –4 y-min = 0
x-max = 4 y-max = 6

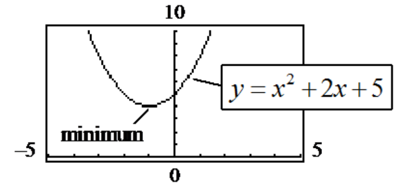
is shown below.



27. $y = x^2 + 2x + 5$

V: (–1, 4)

There are no real zeros.



28. $y = -10 + 7x - x^2$

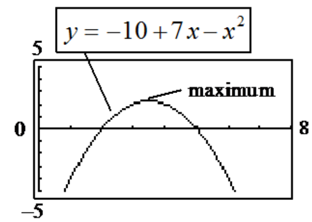
Vertex: $\left(\frac{7}{2}, \frac{9}{4}\right)$ ← maximum

Zeros: $x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5$ or $x = 2$

Graph using x-min = 0 y-min = –5
x-max = 8 y-max = 5



29. $y = 20x - 0.1x^2$

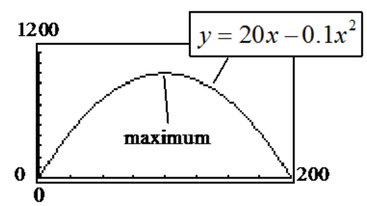
Zeros: $x(20 - 0.1x) = 0$

$x = 0, 200$

(This is an alternative method of getting the vertex.)

The x-coordinate of the vertex is halfway between the zeros.

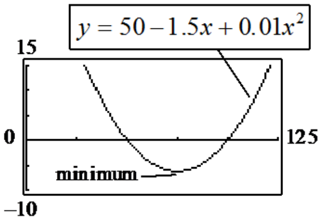
V: (100, 1000)



30. $y = 50 - 1.5x + 0.01x^2$
Vertex: (75, –6.25) ← minimum
Zeros: $0.01x^2 - 1.5x + 50 = 0$

$$0.01(x^2 - 150x + 5000) = 0$$
$$0.01(x - 50)(x - 100) = 0$$

x = 50 or x = 100
Graph using x-min = 0 y-min = –10
x-max = 125 y-max = 10



31. $\dfrac{f(50) - f(30)}{50 - 30} = \dfrac{2500 - 2100}{20} = \dfrac{400}{20} = 20$

32. $\dfrac{f(50) - f(10)}{50 - 10} = \dfrac{1022 + 178}{40} = \dfrac{1200}{40} = 30$

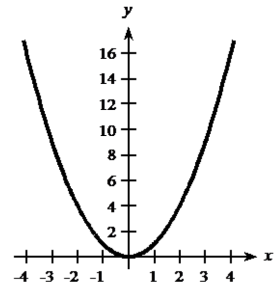
33. **a.** The vertex is halfway between the zeros. So, the vertex is $\left(1, -4\dfrac{1}{2}\right)$.
b. The zeros are where the graph crosses the x-axis. x = –2, 4.
c. The graph matches B.

34. From the graph,
a. Vertex is (0, 49)
b. Zeros are $x = \pm 7$.
c. Matches with D.

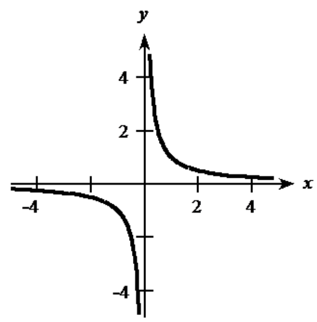
35. **a.** The vertex is halfway between the zeros. So, the vertex is (7, 24.5).
b. Zeros are x = 0, 14.
c. The graph matches A.

36. From the graph,
a. Vertex is (–1, 9).
b. Zeros are x = –4 and x = 2.
c. Matches with C.

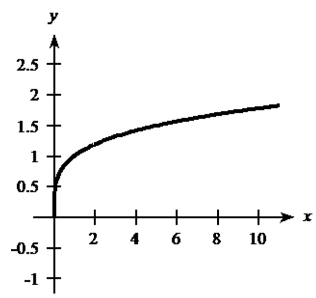
37. **a.** $f(x) = x^2$



b. $f(x) = \dfrac{1}{x}$



c. $f(x) = x^{1/4}$



38. $f(x) = \begin{cases} -x^2 & \text{if } x \le 0 \\ \dfrac{1}{x} & \text{if } x > 0 \end{cases}$

a. $f(0) = -(0^2) = 0$
b. $f(0.0001) = \dfrac{1}{0.0001} = 10{,}000$
c. $f(-5) = -(-5)^2 = -25$
d. $f(10) = \dfrac{1}{10} = 0.1$

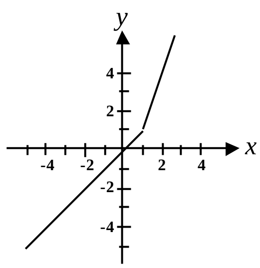
39. $f(x) = \begin{cases} x & \text{if } x \le 1 \\ 3x - 2 & \text{if } x > 1 \end{cases}$

a. $f(-2) = -2$
b. $f(0) = 0$

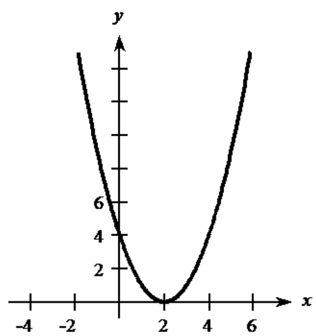
c. $f(1) = 1$

d. $f(2) = 3 \cdot 2 - 2 = 4$

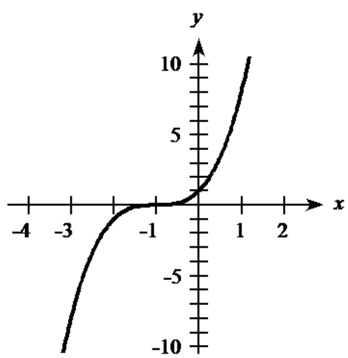
40. $f(x) = \begin{cases} x & \text{if } x \le 1 \\ 3x - 2 & \text{if } x > 1 \end{cases}$



41. a. $f(x) = (x-2)^2$

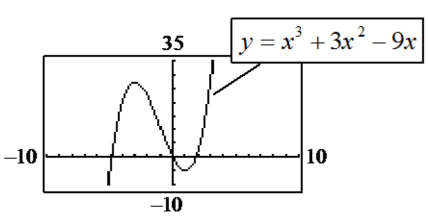


b. $f(x) = (x+1)^3$



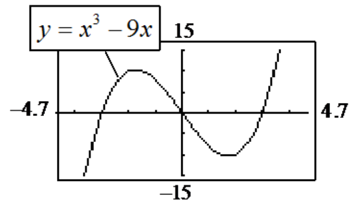
42. $y = x^3 + 3x^2 - 9x$

Using x-min = –10, x-max = 10, y-min = –10, y-max = 35, the turning points are at x = –3 and 1.



43. $y = x^3 - 9x$

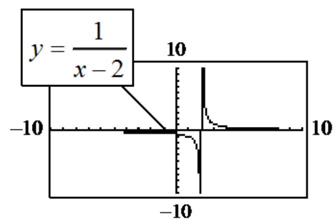
Using x-min = –4.7, x-max = 4.7, y-min = –15, y-max = 15, the turning points are at $x = \pm 1.732$.



Note: Your turning points in 42–43. may vary depending on your scale.

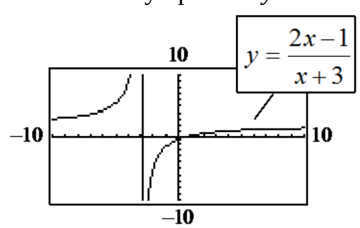
44. $y = \dfrac{1}{x-2}$

There is a vertical asymptote $x = 2$.
There is a horizontal asymptote $y = 0$.

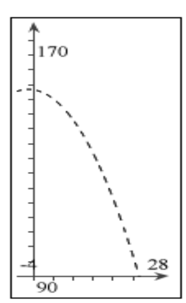


45. $y = \dfrac{2x-1}{x+3} = \dfrac{2 - \frac{1}{x}}{1 + \frac{3}{x}}$

Vertical asymptote is $x = -3$.
Horizontal asymptote is $y = 2$.



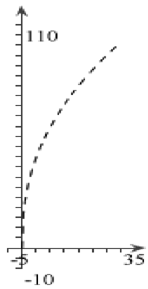
46. a.



b. $y = -2.1786x + 159.8571$ is a good fit to the data.

c. $y = -0.0818x^2 - 0.2143x + 153.3095$ is a slightly better fit.

47. **a.**



b. $y = 2.1413x + 34.3913$ is a good fit to the data.

c. $y = 22.2766x^{0.4259}$ is a slightly better fit.

48. $S = 96 + 32t - 16t^2$

a. $16(6 + 2t - t^2) = 0$

$$t = \frac{-2 \pm \sqrt{4+24}}{-2}$$

$t \approx -1.65$ or $t \approx 3.65$

b. $t \geq 0$ Use $t = 3.65$

c. After 3.65 seconds

49. $P(x) = -0.10x^2 + 82x - 1600$

$(-0.10x + 80)(x - 20) = 0$

Break-even at $x = 20, 800$

50. $E(t) = -0.0052t^2 + 0.080t + 12$

a. The employment is a maximum at

$$t = \frac{-b}{2a} = \frac{-0.080}{2(-0.0052)} \approx 7.69$$

$f(7.69) \approx 12.3$; the maximum employment in manufacturing in the U.S. is predicted to be 12.3 million in 2010 + 8 = 2018.

b. $11.5 = -0.0052t^2 + 0.080t + 12$

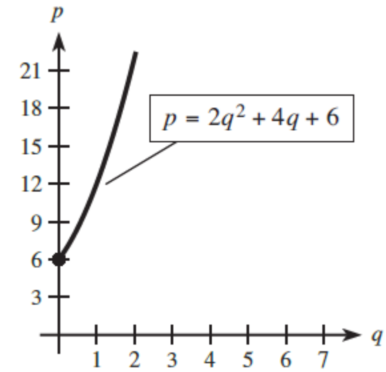
The quadratic formula gives $t \approx -4.8$ or $t \approx 20.2$. The employment in manufacturing in the U.S. will be 11.5 million in 2010 + 21 = 2031.

51. $A = -\frac{3}{4}x^2 + 300x$

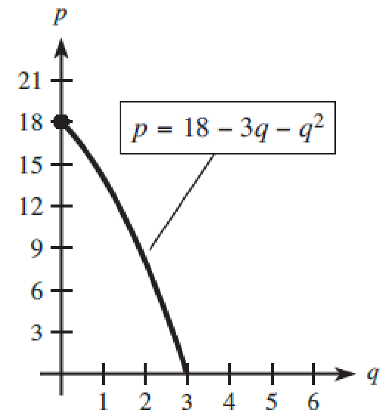
a. V: $x = \dfrac{-300}{-\frac{3}{2}} = 200$ ft

b. $A = -\frac{3}{4}(200)^2 + 300(200) = 30,000$ sq ft

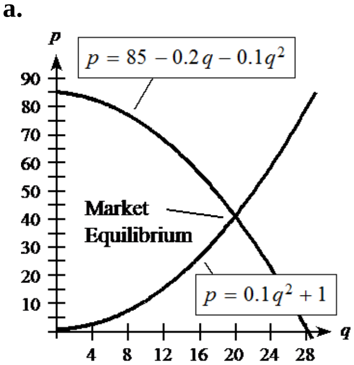
52.



53.



54. **a.**



b. $0.1q^2 + 1 = 85 - 0.2q - 0.1q^2$

$0.2q^2 + 0.2q - 84 = 0$

$0.2(q^2 + q - 420) = 0$

$0.2(q - 20)(q + 21) = 0$

$q = 20$ (only positive value)

$p = 0.1(20)^2 + 1 = 41$

55. $p = q^2 + 300$
$p = -q + 410$
$q^2 + 300 = -q + 410$
$q^2 + q - 110 = 0$
$(q + 11)(q - 10) = 0$
$q = 10$
$p = -10 + 410 = 400$
So, E: (10, 400).

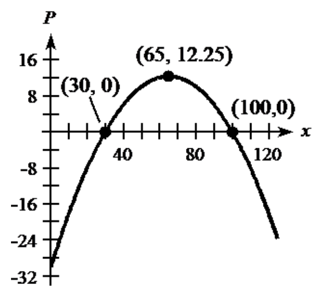
56. D: $p^2 + 5q = 200 \to p^2 = 200 - 5q$
S: $40 - p^2 + 3q = 0$
Substitute $200 - 5q$ for p^2 in the second equation and solve for q.
$40 - (200 - 5q) + 3q = 0$
$-160 = -8q$
$q = 20$
$p^2 = 200 - 5(20)$
$p^2 = 100$ or $p = 10$

57. $R(x) = 100x - 0.4x^2$
$C(x) = 1760 + 8x + 0.6x^2$
$100x - 0.4x^2 = 1760 + 8x + 0.6x^2$
$x^2 - 92x + 1760 = 0$
$$x = \frac{92 \pm \sqrt{1424}}{2} = 46 \pm 2\sqrt{89} \approx 64.87, 27.13$$
$(\sqrt{1424} = \sqrt{16 \cdot 89})$

58. $C(x) = 900 + 25x$
$R(x) = 100x - x^2$
$900 + 25x = 100x - x^2$
$x^2 - 75x + 900 = 0$
$(x - 60)(x - 15) = 0$
$x = 60$ or $x = 15$
$R(60) = 2400$; $R(15) = 1275$
(60, 2400) and (15, 1275)

59. $R(x) = 100x - x^2$
V: $x = \frac{-100}{-2} = 50$
$R(50) = 100(50) - 50^2$
$= \$2500$ max revenue
$P(x) = (100x - x^2) - (900 + 25x)$
$= -x^2 + 75x - 900$
V: $x = \frac{-75}{-2} = 37.5$
$P(37.5) = \$506.25$ max profit

60. $P(x) = 1.3x - 0.01x^2 - 30$
x-coordinate of the vertex $= \frac{1.3}{0.02} = 65$
$P(65) = 1.3(65) - 0.01(65)^2 - 30 = 12.25 \leftarrow$ max
Break-even points:
$0 = 1.3x - 0.01x^2 - 30$
$0 = -0.01(x^2 - 130x + 3000)$
$0 = -0.01(x - 30)(x - 100)$
$x = 30$ or $x = 100$



61. $P(x) = (50x - 0.2x^2) - (360 + 10x + 0.2x^2)$
$= -0.4x^2 + 40x - 360$
V: $x = \frac{-40}{-0.8} = 50$ units for maximum profit.
$P(50) = -0.4(50)^2 + 40(50) - 360$
$= \$640$ maximum profit.

62. **a.** $C(x) = 15,000 + (140 + 0.04x)x$
$= 15,000 + 140x + 0.04x^2$
$R(x) = (300 - 0.06x)x$
$= 300x - 0.06x^2$

b. $15,000 + 140x + 0.04x^2 = 300x - 0.06x^2$
$0.10x^2 - 160x + 15,000 = 0$
$0.1(x^2 - 1600x + 150,000) = 0$
$0.1(x - 100)(x - 1500) = 0$
$x = 100$ or $x = 1500$

c. Maximum revenue:

x-coordinate: $-\dfrac{300}{-0.12} = 2500$

d. $P(x) = R(x) - C(x)$

$= -0.10x^2 + 160x - 15{,}000$

x-coordinate of max = $-\dfrac{160}{-0.20} = 800$

e. $P(2500)$ = \$240,000 loss
$P(800)$ = \$49,000 profit

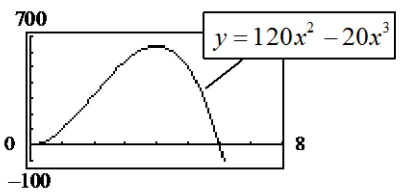
63. $D(t) = 4.95t^{0.495}$

a. power function

b. $D(20) \approx 21.8\%$

c. 24.4; in 2025 about 24.4% of U.S. adults are expected to have diabetes.

64. a.



b. $y = 20x^2(6 - x)$

Domain: $0 \le x \le 6$

65. $C(p) = \dfrac{4800p}{100 - p}$

a. rational function

b. Domain: $0 \le p < 100$

c. $C(0) = 0$ means that there is no cost if no pollution is removed.

d. $C(99) = \dfrac{4800(99)}{100 - 99} = \$475{,}200$

66. $C(x) = \begin{cases} 2.557x & 0 \le x \le 100 \\ 255.70 + 2.04(x - 100) & 100 < x \le 1000 \\ 2091.7 + 1.689(x - 1000) & x > 1000 \end{cases}$

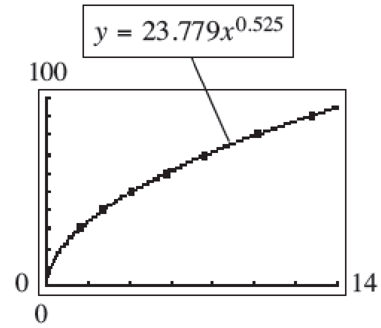
a. $C(12) = 2.557(12) = \$30.68$

b. $C(825) = 255.70 + 2.04(825 - 100) =$ \$1734.70

67. a. Linear, quadratic, cubic, and power functions are each reasonable.

b. $y = 23.779x^{0.525}$

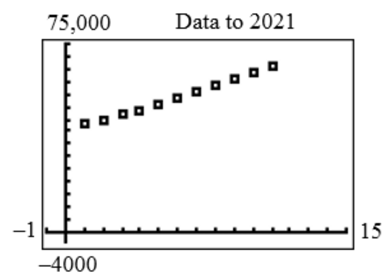
c.

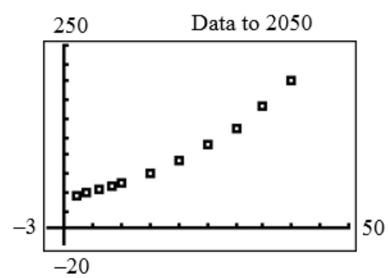


d. $f(5) = 23.779(5)^{0.525} \approx 55$ mph

e. Use the TRACE KEY. It will take 9.9 seconds.

68. a.





b. A quadratic model could be used.

$a(x) = 47.70x^2 + 1802x + 40{,}870;$

$A(x) = 0.07294x^2 + 0.9815x + 44.45$

c. 2020 data: \$63,676

$a(10) \approx \$63{,}664$–closer;

$A(10) \approx 61.564$ (\$61,564)

2050 data: 202.5 (\$202,500)

$a(40) \approx \$189{,}292;$

$A(40) \approx 200.413$ (\$200,413–closer)

d. $a = 150{,}000$ when $x \approx 32.5$, in 2043;

$A = 150{,}000$ when $x \approx 31.9$, in 2042

69. **a.** $O(x) = 0.743x + 6.97$

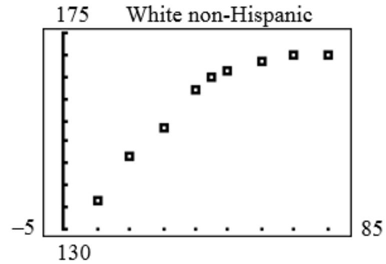
b. $S(x) = 0.264x - 2.57$

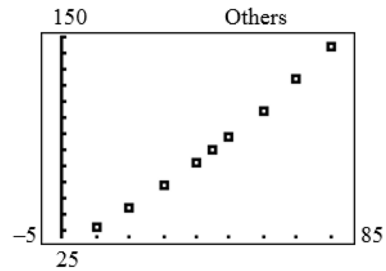
c. $F(x) = \dfrac{0.743x + 6.97}{0.264x - 2.57}$. This is called a rational function and measures the fraction of obese adults who are severely obese.

d. horizontal asymptote: $y \approx \dfrac{0.743}{0.264} \approx 0.355$.

This means that if this model remains valid far into the future, then the long-term projection is that about 0.355, or 35.5%, of obese adults will be severely obese.

70. **a.**





A quadratic function could be used to model each set of data.

$W(x) = -0.00903x^2 + 1.28x + 124$

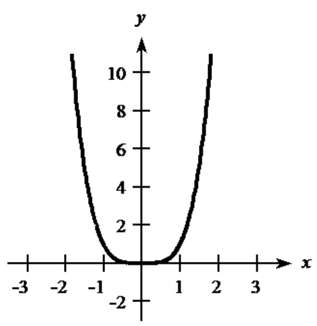
$O(x) = 0.00645x^2 + 1.02x + 20.0$

b. At $x \approx 91.1$, $W(x) = O(x) \approx 166.4$

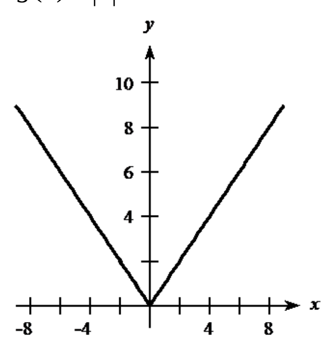
In 1970 + 92 = 2162, these population segments are predicted to be equal (at about 166.4 million each).

Chapter 2 Test

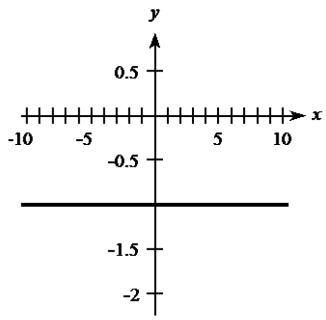
1. **a.** $f(x) = x^4$



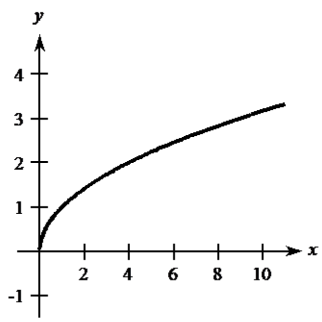
b. $g(x) = |x|$



c. $h(x) = -1$

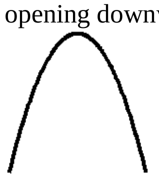


d. $k(x) = \sqrt{x}$

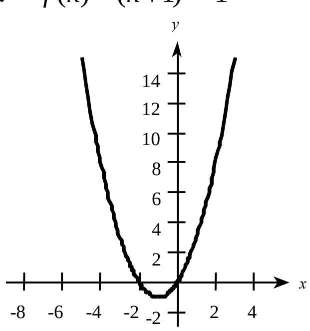


2. figure b is the graph for $b > 1$.
figure a is the graph for $0 < b < 1$.

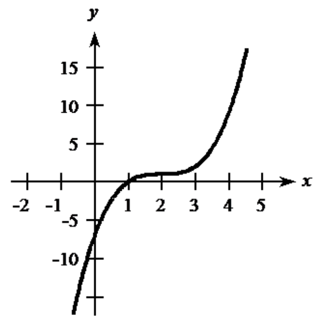
3. $f(x) = ax^2 + bx + c$ and a < 0 is a parabola opening downward.



4. **a.** $f(x) = (x+1)^2 - 1$



b. $f(x) = (x-2)^3 + 1$



5. $f(x) = x^3 - 4x^2 = x^2(x-4)$.
a. and **b.** are the cubic choices. $f(x) < 0$ if $0 < x < 4$. Answer: b

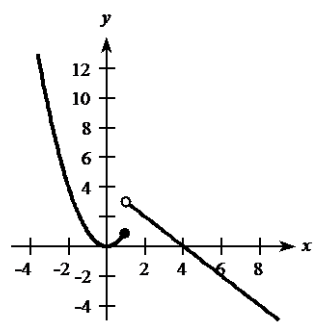
6. $f(x) = \begin{cases} 8x + \dfrac{1}{x} & \text{if } x < 0 \\ 4 & \text{if } 0 \le x \le 2 \\ 6 - x & \text{if } x > 2 \end{cases}$

a. $f(16) = 6 - 16 = -10$

b. $f(-2) = 8(-2) + \dfrac{1}{-2} = -16\dfrac{1}{2}$

c. $f(13) = 6 - 13 = -7$

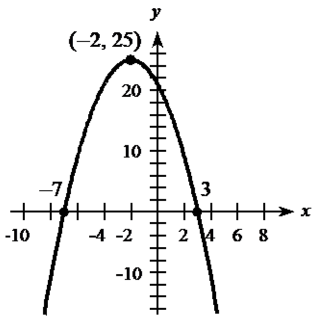
7. $g(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 4 - x & \text{if } x > 1 \end{cases}$



8. $f(x) = 21 - 4x - x^2 = (7+x)(3-x)$

Vertex: $x = \dfrac{-b}{2a} = \dfrac{-(-4)}{2(-1)} = -2$

Point: (–2, 25)
Zeros: $f(x) = 0$ at $x = -7$ or 3.



9. $$3x^2+2=7x$$
$$3x^2-7x+2=0$$
$$(3x-1)(x-2)=0$$
$3x - 1 = 0$ or $x - 2 = 0$
$$x=\frac{1}{3},2$$

10. $2x^2+6x-9=0$
$$x=\frac{-6\pm\sqrt{36+72}}{4}=\frac{-6\pm6\sqrt{3}}{4}=\frac{-3\pm3\sqrt{3}}{2}$$

11. $\left(\frac{1}{x}+2x=\frac{1}{3}+\frac{x+1}{x}\right)3x$
$$3+6x^2=x+3x+3$$
$$6x^2-4x=0$$
$$2x(3x-2)=0$$
$x=\frac{2}{3}$ is the only solution.

12. $g(x)=\frac{3(x-4)}{x+2}$

Vertical asymptote at $x = –2$.
$g(4) = 0$
Answer: c

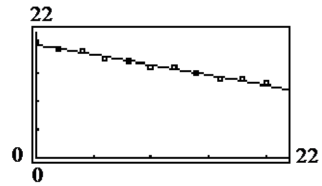
13. $f(x)=\frac{8}{2x-10}$

Horizontal: $y=0$
Vertical: $2x-10=0$
$$2x=10$$
$$x=5$$

14. $\frac{f(40)-f(10)}{40-10}=\frac{320-(-940)}{30}=\frac{1260}{30}=42$

15. **a.** quartic
b. cubic

16. **a.** $f(x) = –0.3577x + 19.9227$



b. $f(40) = 5.6$

c. $f(x) = 0$ if $x=\frac{19.9227}{0.3577}\approx 55.7$

17. S: $p=\frac{1}{6}q+30$

D: $p=\frac{30,000}{q}-20$
$$\left(\frac{1}{6}q+30=\frac{30,000}{q}-20\right)6q$$
$$q^2+180q=180,000-120q$$
$$q^2+300q-180,000=0$$
$$(q+600)(q-300)=0$$
$$\text{E}_q: q=300$$
$$\text{E}_p: p=50+30=80$$

18. $R(x)=285x-0.9x^2$
$C(x)=15,000+35x+0.1x^2$

a. $P(x)=285x-0.9x^2-(15,000+35x+0.1x^2)$
$$=-x^2+250x-15,000$$
$$=(100-x)(x-150)$$

b. Maximum profit is at vertex.
$$x=\frac{-250}{2(-1)}=125$$
Maximum profit = $P(125)$ = \$625

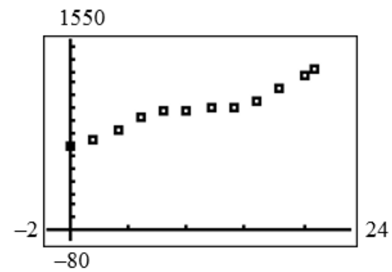
c. Break-even means $P(x) = 0$.
From **a.**, $x = 100, 150$.

19. **a.** Use middle rule for $s = 15$.
$f(15) = -19.5$ means that when the air temperature is 0ºF and the wind speed is 15 mph, then the air temperature feels like -19.5 ºF. In winter, the TV weather report usually gives the wind chill temperature.

b. $f(48) = -31.4$ºF

c. Break-even means $P(x) = 0$.
From **a.**, $x = 100, 150$.

20. **a.**



b. Linear: $y = 26.8x + 695$;

Cubic: $y = 0.175x^3 - 5.27x^2 + 65.7x + 654$

c. Linear: $y(21) \approx \$1258$;

Cubic: $y(21) \approx \$1326$

The cubic model is quite accurate, but both models are fairly close.

d. The linear model increases steadily, but the cubic model rises rapidly for years past 2021.

Chapter 2: Quadratic and Other Special Functions

Chapter 2 Extended Applications & Group Projects

I. Body Mass Index (Modeling)

1. Eight points in the table correspond to a BMI of 30. Converting heights to inches, we have:

Height (in.)	Weight (lb)
61	160
63	170
65	180
67	190
68	200
69	200
72	220
73	230

2. A linear model seems best as there appears to be roughly a constant rate of change of weight vs. height.
3. $y = 5.700x - 189.5$
4. We note that $y(61) \approx 158.1,$ lb close to the actual value of 160 lb, and $y(72) \approx 220.8,$ close to the actual value of 220 lb. The model seems to fit the data.
5. To test for obesity, substitute the person's height in inches for *x* in the model, computing *y*. If the person's weight is larger than *y*, then the person is considered obese. For a 5-foot-tall person, $y(60) \approx 152.4,$ so 152.4 lb is the obesity threshold for someone who is 5 feet tall. For a 6-feet-2-inches-tall person, $y(74) \approx 232.2,$ so 232.2 lb is the obesity threshold for someone who is 6 foot 2.
6. The Centers for Disease Control and Prevention (CDC) post the BMI formula
$$\text{BMI} = \frac{\text{weight (lb)}}{[\text{height (in)}]^2} \times 703$$
at their website (http://www.cdc.gov/healthyweight/assessing/bmi/adult_bmi/index.html, accessed September 15, 2014).
7. First solving the CDC formula for weight given a BMI of 30 gives $\text{weight} = \dfrac{30 \cdot [\text{height}]^2}{703}.$

Height (in.)	Weight (lb) from model	Weight (lb) from CDC definition	Weight (lb) from table
61	158	159	160
62	164	164	
63	170	169	170
64	175	175	
65	180	180	180
66	187	186	
67	192	192	190
68	198	197	200
69	204	203	200
70	209	209	
71	215	215	
72	221	221	220
73	227	227	230

Chapter 2: Quadratic and Other Special Functions

II. Operating Leverage and Business Risk

1. $R = xp$

2. **a.** $C = 100x + 10,000$

b. C is a linear function.

3. An equation that describes the break-even point is $xp = 100x + 10,000$

4. **a.** $xp = 100x + 10,000$

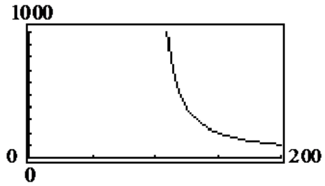
$$x = \frac{10,000}{p-100}$$

b. The solution is **4.a.** is a rational function.

c. The domain is all real numbers, $p \neq 100$.

d. The domain in the context of this problem is $p > 100$.

5. **a.**



b. The function decreases as p increases.

6. A price of $1100 would increase the revenue for each unit but demand would decrease.

7. A price of $101 per unit would increase demand but perhaps such a demand could not be met.

8. **a.** Increasing fixed costs gives a higher operating leverage. Using modern equipment would give the higher operating leverage.

b. To find the break-even point with current costs we have $200x = 100x + 10,000$

$$x = 100.$$

To find the break-even point with modern equipment we have $200x = 50x + 30,000$

$$x = 200.$$

The higher the break-even point the greater the business risk. The cost with the modern equipment creates a higher business risk.

c. In this case, higher operating leverage and higher business event together. This higher risk might give greater profits for increases in sales. It might also give a greater loss of sales fall.