2 Sets

Exercise Set 2-1

1. A *set* is a well-defined collection of objects.

2. (a) Roster method; (b) Descriptive method; (c) Set-builder notation

3. Equal sets have exactly the *same elements*. Equivalent sets have exactly the *same number* of elements.

4. In a *finite* set, the number of elements is either 0 or some natural number. In an *infinite* set, the number of elements exceeds any natural number.

5. Each element of one set can be associated (paired) with exactly one element of the other set, and no element in either set is left alone.

6. The empty set admits no elements. Two examples:

(a) {5-leg horses} = ∅

(b) $\left\{\text{integers between } \frac{1}{10} \text{ and } \frac{9}{10}\right\} = \varnothing$

7. S = {s, t, r, e}.

8. A = {A, L, B, M}

9. Natural numbers are the counting numbers so:

P = {51, 52, 53, 54, 55, 56, 57, 58, 59}

10. R = {12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38}

11. Q = {1, 3, 5, 7, 9, 11, 13}

12. M = {2, 4, 6}

13. G = {11, 12, 13, … }

14. B = {101, 102, 103, … }

15. Y = {2,001, 2,002, 2,003, … , 2,999}

16. Z = {501, 502, 503, … , 5,999}

17. W = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

18. C = {red, white, blue}

19. D = {hearts, diamonds, spades, clubs}

20. F = {jack, queen, king}

21. This is the set of even natural numbers.

22. This is the set of odd natural numbers.

23. This is the set of the first four multiples of 9.

24. This is the set of the first four multiples of 5.

25. This is the set of letters in Mary.

26. This is the set of letters in Thomas.

27. This is the set of natural numbers from 100 to 199.

28. This is the set of natural numbers from 21 to 30.

29. $\{x \mid x$ is a multiple of 10$\}$

30. $\{x \mid x$ is a multiple of 5 between 50 and 90$\}$

31. $X = \{x \mid x \in N \text{ and } x > 20\}$

32. $Z = \{x \mid x \in E \text{ and } x < 12\}$

33. $\{x \mid x$ is an odd natural number less than 10$\}$

34. $\{x \mid x$ is a multiple of 3 between 15 and 33$\}$

35. There are no natural numbers less than zero so $H = \varnothing$.

36. {71, 72, 73, 74, 75, 76, 77, 78, 79}

37. {7, 14, 21, 28, 35, 42, 49, 56, 63}

38. {5, 12, 19, 26, 33, 40}

39. {102, 104, 106, 108, 110, 112, 114, 116, 118}

40. {91, 93, 95, 97, 99}

41. The collection is well-defined since the elements can be objectively determined to be in the collection or not.

42. The collection is well-defined since the elements can be objectively determined to be in the collection or not.

43. The collection is not well-defined since "excellent" is subjective.

44. The collection is well-defined since the elements can be objectively determined to be in the collection or not.

45. The collection is well-defined since the elements can be objectively determined to be in the collection or not.

46. The collection is not well-defined since "good" is subjective.

47. The collection is not well-defined since no pattern can be established to determine the numbers in the collection.

48. The collection is well-defined since the elements can be objectively determined to be in the collection or not.

49. This is true since 3 is in the set *B*.

50. This is false since a is not in the set *C*.

51. This is true since Wednesday is not in the set *A*.

52. This is true since 7 is not in the set *B*.

53. This is true since r is in the set *C*.

54. This is false since q is not in the set *B*.

55. Infinite: There is not a fixed number of even numbers

56. Finite: There are a fixed number of elements, 1000.

57. There are 26 letters in the English alphabet, therefore *K* is finite.

58. Finite: The set of past presidents in the United States has a fixed number of elements, so the set of years in which they were born has a fixed number of elements as well.

59. Infinite: The … indicates that the set continues indefinitely.

60. Finite: There are a fixed number of elements, zero.

61. Finite: There are a fixed number of television programs that are currently airing.

62. Infinite: A fraction is the quotient of two integers and there is not a fixed number of integers therefore there isn't a fixed number of fractions.

63. Equal and equivalent

64. Equivalent

65. Neither

66. Neither

67. Equivalent

68. Equal and equivalent

69. Neither

70. Equal and equivalent

71.
$$\begin{array}{cccc} \{10, & 20, & 30, & 40\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{40, & 10, & 20, & 30\} \end{array}$$

72.
$$\begin{array}{cccc} \{w, & x, & y, & z\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{1, & 2, & 3, & 4\} \end{array}$$

73.
$$\begin{array}{cccccc} \{1, & 2, & 3, & \ldots, & 25, & 26\} \\ \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \updownarrow \\ \{a, & b, & c, & \ldots, & y, & z\} \end{array}$$

74.
$$\begin{array}{ccccc} \{1, & 3, & 5, & 7, & 9\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{2, & 4, & 6, & 8, & 10\} \end{array}$$

75. $n(A) = 5$ (since there are 5 elements in the set)

76. $n(B) = 37$ (since there are 37 elements in the set)

77. $n(C) = 7$ (since there are seven days in a week)

78. $n(D) = 12$ (since there are 12 months in a year)

79. $n(E) = 1$ (the word "three" is the only element in the set)

80. $n(F) = 4$ (though there appears to be five elements in the set, the letter e is repeated so it is not counted twice)

81. $n(G) = 0$ (there are no negative natural numbers therefore G is the set with no elements, that is, the empty set)

82. $n(H) = 0$ (since there are no elements in the empty set)

83. True, the two sets have exactly the same elements and are therefore equal.

84. True, though the two sets have the same number of elements they do not have exactly the same elements and are therefore not equal.

85. True, for two sets to be equal the must have exactly the same elements which means they will also have the same cardinal number so they will be equivalent.

86. False, The sets in exercise 83 are equal sets that are also equivalent.

87. False, the first set has no elements, the second has the element ∅.

88. False, the two sets do not have exactly the same elements. The number 12 is in the first but not the second.

89. False, there is one element in the set {∅}, namely ∅.

90. False, the set continues on to include all even natural numbers and does not have a fixed number of elements.

91. a) {California, New York, Florida}

b) {Massachusetts, Georgia, Virginia, Maryland}

c) {California, New York, Florida, Texas, New Jersey}

d) {Texas, New Jersey, Illinois}

92. a) {25-30, 35-44, 45-54} People between 25 and 54 have a higher percentage in ISPs, web search portals and data processing companies, than the younger or older age groups.

b) {16-19, 55-64, 65 and older} Older people and high-school aged people are less likely to be working in these categories.

c) {10.1, 26.8, 31.8}

d) {35-44}

e) {19.6, 7.8}

f) ∅

93. a) {Drunk driving, Injury, Assault}

b) {Injury, Health problems}

c) {Injury, Assault, Drunk driving}

d) {97,000, 1,700, 150,000}

e) Answers vary

94. a){Psychology, Computers, Philosophy}

b) {Education, Health professions, Engineering, Physical Sciences}

c) {Education, Psychology, Health professions, Engineering}

d) {Psychology, Health professions, Computers, Physical sciences, Philosophy}

e) {Education, Psychology, Engineering, Physical sciences, Mathematics, Philosophy}

f) {Business, Communications, Computers, Philosophy}

95. a) {Employment fraud, Bank fraud}

b) {18-29, 30-39, 40-49}

c) {Utilities/phone fraud, Credit card fraud, Other}

d) {20, 13, 9}

e) {Employment fraud, Bank fraud, Utilities/phone fraud}

96. a) {16-24, 25-34}

b) {35-44, 45-54, 55+}

c) {16-24, 25-34}

d) {28, 15}

97. a) {2000, 2001, 2002}

b) {2002, 2003, 2004, 2005}

c) {2002, 2003, 2004, 2005, 2006}

d) {2000, 2001, 2002}

98. a) {2005, 2006, 2007, 2008}

b) {2004}

c) {2004, 2005, 2006}

d) {2007, 2008}

99. Yes; $A \cong B$ means A and B have the same number of elements. $A \cong C$ means A and C have the same number of elements. Then B and C have the same number of elements, so $B \cong C$.

100. No; $\emptyset$ contains no elements, but {0} contains one element: zero. So $\emptyset$ and {0} do not have the same number of elements.

101. Answers vary, one possible answer is: The set of people who are currently enrolled in at least one college class.

102. Answers vary, one possible answer is: The set of male students in your math class. The set of students in your math class that belong to the Republican Party. The set of history majors in your math class.

103. Answers vary

104. Answers vary

Exercise Set 2-2

1. If every element of set A is also in set B, then A is a *subset* of B.

2. A *subset* of (say) set M can equal M, but a *proper subset* of M cannot equal M.

3. A subset is a set in its own right, hence a well-defined *collection* of objects. An element of a set is just an *individual member* of the set.

4. $\emptyset \subseteq \emptyset$ because the requirement for being a subset is trivially true, there being no elements to be tested for inclusion. But $\emptyset \not\subset \emptyset$ because $\emptyset = \emptyset$ always.

5. The union of sets A and B consists of all elements that are in *at least one* of A and B. The intersection of A and B consists of all elements that are *in both* A and B.

6. When they have no elements in common, two sets are said to be disjoint.

7. The set of all elements used in a particular problem or situation is called a universal set.

8. The *complement* of a set (say) A is the set of elements that belong to the universal set but not to A.

9. Answers vary, for instance, the set of math students in your class who are male or over the age of 23. It represents a union because to be in this set you are in at least one of the categories "male" or "over 23."

10. Answers vary, for instance, the set of students in your math class who are liberal arts majors but not freshman. This is a difference because it's the subset of liberal arts majors with those who are freshman taken out.

11. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in A:
U = {2, 3, ~~5, 7, 11, 13~~, 17, 19}

A' = {2, 3, 17, 19}

12. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in B:
U = {~~2~~, 3, 5, 7, 11, 13, 17, 19}

B' = {3, 5, 7, 11, 13, 17, 19}

13. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in C:
U = {2, 3, 5, 7, 11, ~~13, 17, 19~~}

C' = {2, 3, 5, 7, 11}

14. U = {2, 3, 5, 7, 11, 13, 17, 19}

Cross off those elements in D:
U = {~~2, 3, 5~~, 7, 11, 13, 17, 19}

$D' = \{7, 11, 13, 17, 19\}$

15. ∅; {r}; {s}; {t}; {r, s}; {r, t}; {s, t}; {r, s, t}

16. ∅; {2}; {5}; {7}, {2, 5}; {2, 7}; {5, 7}; {2, 5, 7}

17. ∅; {1}; {3}; {1, 3}

18. ∅; {p}; {q}; {p, q}

19. { } or ∅

20. { } or ∅

21 . ∅; {5}; {12}; {13}; {14}; {5, 12}; {5, 13}; {5, 14}; {12, 13}; {12, 14}; {13, 14}, {5, 12, 13}; {5, 12, 14}; {5, 13, 14}; {12, 13, 14}; {5, 12, 13, 14}

22. ∅; {m}; {o}; {r}; {e}; {m, o} {m, r}; {m, e}; {o, r}; {o, e}; {r, e}; {m, o, r}; {m, o, e}; {o, r, e}; {m, r, e}; {m, o, r, e}

23. ∅; {1}; {10}; {20}; {1, 10}; {1, 20}; {10, 20}

24. ∅; {March}; {April}; {May}; {March, April}; {March, May}; {April, May}

25. ∅

26. ∅

27. None

28. None

29. True

30. False, since a proper subset cannot equal the original set.

31. False; the second set contains one element, 123.

32. False; a proper subset cannot be equal to itself.

33. False; {   } has no elements.

34. False; the sun is not a planet.

35. False; {3} is not an element of the second set, $\subset$ or $\subseteq$ should be used with subsets.

36. True

37. True

38. False; ∅ is a subset or proper subset of any set. It is not an element of {r, s, t, u}.

39. $2^3 = 8$

40. $2^{10} = 1024$

41. $2^0 = 1$

42. $2^1 = 2$

43. $2^2 = 4$

44. $2^5 = 32$

45. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

46. $A = \{2, 3, 5, 9\}$

47. $B = \{5, 6, 7, 8, 9\}$

48. $A \cap B = \{5, 9\}$

49. $A \cup B = \{2, 3, 5, 6, 7, 8, 9\}$

50. $A' = \{1, 4, 6, 7, 8\}$

51. $B' = \{1, 2, 3, 4\}$

52. $(A \cup B)' = \{1, 4\}$

53. $(A \cap B)' = \{1, 2, 3, 4, 6, 7, 8\}$

54. $A \cap B' = \{2, 3\}$

55. $A \cup C = \{10, 30, 40, 50, 60, 70, 90\}$

56. $A \cap B = \varnothing$

57. $A' = \{20, 40, 60, 80, 100\}$

58. $A \cap B = \varnothing$
$(A \cap B) \cup C = C = \{30, 40, 50, 60\}$

59. $B \cup C = \{20, 30, 40, 50, 60, 80, 100\}$
$A' = \{20, 40, 60, 80, 100\}$
$A' \cap (B \cup C) = \{20, 40, 60, 80, 100\}$

60. $A \cap B = \varnothing$
$(A \cap B) \cap C = \varnothing$

61. $A \cup B = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

$(A \cup B)' = \varnothing$

$(A \cup B)' \cap C = \varnothing$

62. $B' = \{10, 30, 50, 70, 90\}$
$A \cap B' = A = \{10, 30, 50, 70, 90\}$

63. $B \cup C = \{20, 30, 40, 50, 60, 80, 100\}$
$A' = \{20, 40, 60, 80, 100\}$
$(B \cup C) \cap A' = \{20, 40, 60, 80, 100\}$

64. $A' = \{20, 40, 60, 80, 100\}$
$B' = \{10, 30, 50, 70, 90\}$
$A' \cup B' = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

$C' = \{10, 20, 70, 80, 90, 100\}$
$(A' \cup B') \cup C'$
$= \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

65. $P \cap Q = \{b, d\}$

66. $Q \cup R = \{a, b, c, d, e, f, g\}$

67. $P' = \{a, c, e, h\}$

68. $Q' = \{e, f, g, h\}$

69. $R' = \{a, b, c, d, h\}$
$P' = \{a, c, e, h\}$
$R' \cap P' = \{a, c, h\}$

70. $Q \cap R = \varnothing$
$P \cup (Q \cap R) = P = \{b, d, f, g\}$

71. $Q \cup P = \{a, b, c, d, f, g\}$
$(Q \cup P)' = \{e, h\}$
$(Q \cup P)' \cap R = \{e\}$

72. $Q \cap R = \varnothing$
$P \cap (Q \cap R) = \varnothing$

73. $P \cup Q = \{a, b, c, d, f, g\}$
$P \cup R = \{b, d, e, f, g\}$
$(P \cup Q) \cap (P \cup R) = \{b, d, f, g\}$

74. $Q' = \{e, f, g, h\}$
$R' = \{a, b, c, d, h\}$
$Q' \cup R' = \{a, b, c, d, e, f, g, h\} = U$, the universal set

75. $W \cap Y = \{2, 4, 6\}$

76. $X \cup Z = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12\}$, which is all of U except 4

77. $W \cup X$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$= U$, the universal set

78. $X \cap Y = \{1, 3, 5\}$
$(X \cap Y) \cap Z = \{5\}$

79. $W \cap X = \varnothing$

80. $Y \cup Z = \{1, 2, 3, 4, 5, 6, 8, 10, 11, 12\}$
$(Y \cup Z)' = \{7, 9\}$

81. $X \cup Y = \{1, 2, 3, 4, 5, 6, 7, 9, 11\}$
$(X \cup Y) \cap Z = \{2, 5, 6, 11\}$

82. $Z \cap Y = \{2, 5, 6\}$
$(Z \cap Y) \cup W = \{2, 4, 5, 6, 8, 10, 12\}$

83. $W' = \{1, 3, 5, 7, 9, 11\}$
$X' = \{2, 4, 6, 8, 10, 12\}$
$W' \cap X' = \varnothing$

84. $Z \cup X = \{1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12\}$
$(Z \cup X)' = \{4\}$
$(Z \cup X)' \cap Y = \{4\}$

85. $A \cap B = B$ since $B \subset A$.

86. $A' = \{\textit{natural numbers that are not multiples of 3}\}$
$A' \cap C = \{\textit{all even natural numbers that are not multiples of 3}\}$
$= \{2, 4, 8, 10, 14, \ldots\}$

87. $C' = \{\textit{all odd natural numbers}\}$
$B \cup C' = \{\textit{odd natural numbers or multiples of 9}\}$

$A \cap (B \cup C') = \{x \mid x \textit{ is an odd multiple of 3 or an even multiple of 9}\}$
$= \{3, 9, 15, 18, 21, 27, 33, 36, 39, \ldots\}$

88. $A \cup B = A$ since $B \subset A$.

89. List the members of A: {20, 60, 100, 110}

Cross off those that are in B: {20, ~~60, 100~~, 110}

$A - B = \{20, 110\}$

90. List the members of A: {20, 60, 100, 110}

Cross off those that are in C: {20, 60, ~~100~~, ~~110~~}

$A - C = \{20, 60\}$

91. List the members of B: {60, 80, 100}

Cross off those that are in C: {60, ~~80, 100~~}

$B - C = \{60\}$

92. List the members of B: {60, 80, 100}

Cross off those that are in A: {~~60~~, 80, ~~100~~}
$B - A = \{80\}$

Note: In general, $A \cap B' = \{x \mid x \in A \text{ and } x \notin B\}$ which is the same way $A - B$ is defined so the two are used interchangeably in some of the following problems.

93. $C \cap B' = C - B$ so, list the members of C: {80, 100, 110}

Cross off those in B: {~~80, 100~~, 110}

$C \cap B' = \{110\}$

94. $A \cap C' = A - C$ so, list the members of A: {20, 60, 100, 110}

Cross off those in C: {20, 60, ~~100, 110~~}

$A \cap C' = \{20, 60\}$

95. List the members of C: {p, r, t, v}

Cross off those that are in B: {p~~, r, t, v~~}

$C - B = \{p\}$

96. List the elements in A: {p, q, r, s, t}

Cross off those in C: {~~p~~, q, ~~r~~, s, ~~t~~}

$A - C = \{q, s\}$

97. List the elements in B: {r, s, t, u, v}

Cross of those in C: {~~r~~, s, ~~t,~~ u, ~~v~~}

$B - C = \{s, u\}$

98. List the elements in B: {r, s, t, u, v}

Cross off those in A: {~~r, s, t~~, u, v}

$B - A = \{u, v\}$

99. $B \cap C' = B - C$ so, list the elements in B: {r, s, t, u, v}

Cross of those in C: {~~r~~, s, ~~t,~~ u, ~~v~~}

$B \cap C' = \{s, u\}$

100. $C \cap A' = C - A$ so, List the elements in C: {p, r, t, v}

Cross off those in A: {~~p, r, t~~, v}

$C \cap A' = \{v\}$

101. $A \times B = \{(9, 1), (9, 2), (9, 3), (12, 1), (12,2), (12, 3), (18, 1), (18, 2), (18, 3)\}$

102. $B \times A = \{(1, 9), (1, 12), (1, 18), (2, 9), (2, 12), (2, 18), (3, 9), (3, 12), (3, 18)\}$

103. $A \times A = \{(9, 9), (9, 12), (9, 18), (12, 9), (12, 12), (12, 18), (18, 9), (18, 12), (18, 18)\}$

104. $B \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

105. $A \times B = \{(1, 1), (1, 3), (2, 1), (2, 3), (4, 1), (4, 3), (8, 1), (8, 3)\}$

106. $B \times A = \{(1, 1), (1, 2), (1, 4), (1, 8), (3, 1), (3, 2), (3, 4), (3, 8)\}$

107. $B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

108. $A \times A = \{(1, 1), (1, 2), (1, 4), (1, 8), (2, 1), (2, 2), (2, 4), (2, 8), (4, 1), (4, 2), (4, 4), (4, 8), (8, 1), (8, 2), (8, 4), (8, 8)\}$

109. {cell phone, laptop, iPod}, {cell phone, laptop}, {cell phone, iPod}, {laptop, iPod}, {cell phone}, {laptop}, {iPod}, $\varnothing$

110. $2^5 = 32$

111. $2^7 - 1 = 127$

112. $2^6 = 64$

113. $2^4 = 16$

114. {treadmill, cycle, stair stepper}; {treadmill, cycle}; {treadmill, stair stepper}; {cycle, stair stepper}; {treadmill}; {cycle}; {stair stepper}

115. Answers vary

116. Answers vary

117. Answers vary

118. Answers vary

Exercise Set 2-3

1. Answers vary

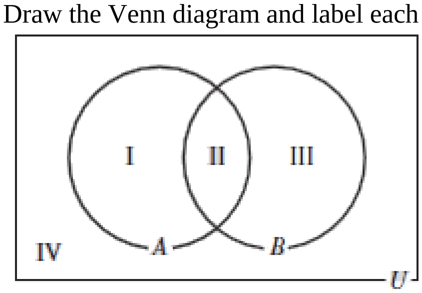
2. Answers vary

3. The complement of the union is the intersection of the complements and the complement of the intersection is the union of the complements.

4. Add the cardinal numbers of the two sets then subtract the cardinal number of their intersection.

For Exercises 5-10, all solutions have the same first two steps, given here.

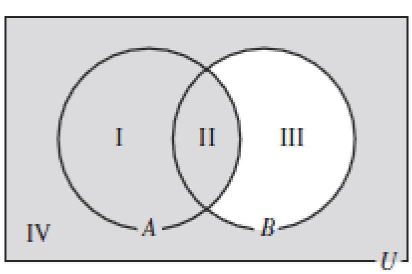
Step 1 Draw the Venn diagram and label each area.



Step 2 From the diagram, list the regions in each set. U = {I, II, III, IV} A = {I, II} B = {II, III}

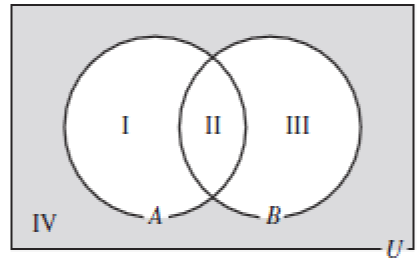
5. **Step 3** B' = {I, IV}, so $A \cup B'$ = {I, II, IV}

Step 4 Shade regions I, II, and IV



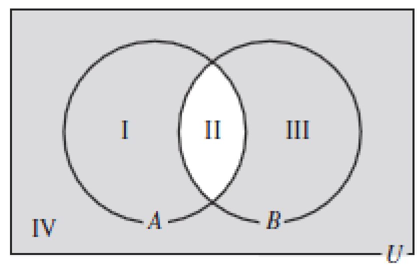
6. **Step 3** $A \cup B$ = {I, II, III}, so $(A \cup B)'$ = {IV}

Step 4 Shade region IV



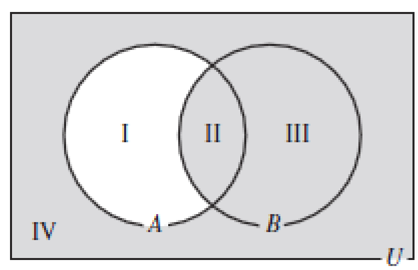
7. **Step 3** A' = {III, IV} and B' = {I, IV}, so $A' \cup B'$ = {I, III, IV}

Step 4 Shade regions I, III, and IV



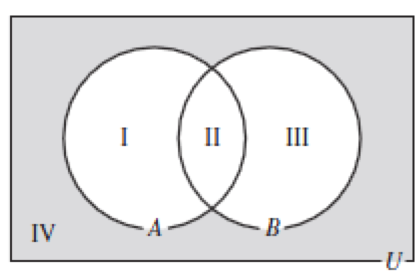
8. **Step 3** A' = {III, IV} so $A' \cup B$ = {II, III, IV}

Step 4 Shade regions II, III, and IV



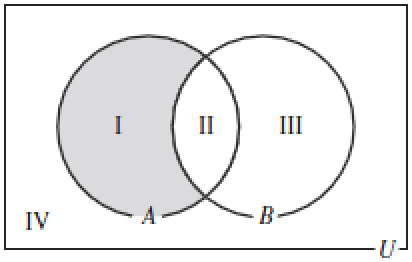
9. **Step 3** A' = {III, IV} and B' = {I, IV}, so $A' \cap B'$ = {IV}

Step 4 Shade region IV



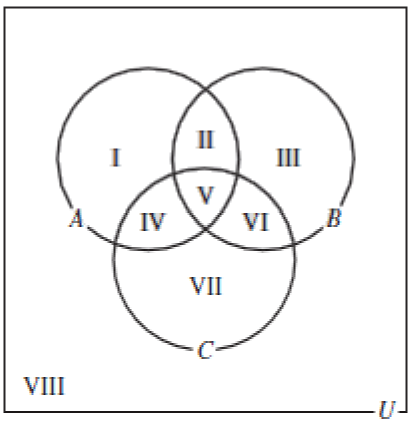
10. **Step** B' = {I, IV}, so $A \cap B'$ = {I}

Step 4 Shade region I



For Exercises 11-28, all solutions have the same first two steps, given here.

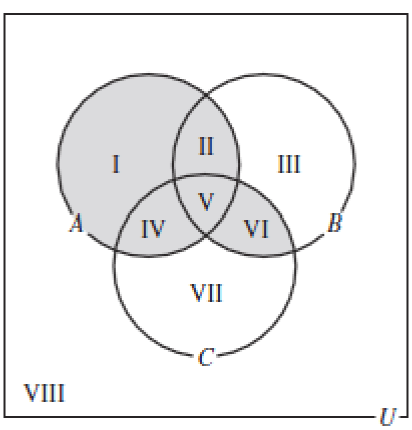
Step 1 Draw and label the diagram as shown



Step 2 From the diagram, list the regions in each set.
U = {I, II, III, IV, V, VI, VII, VIII}
A = {I, II, IV, V}
B = {II, III, V, VI}
C = {IV, V, VI, VII}

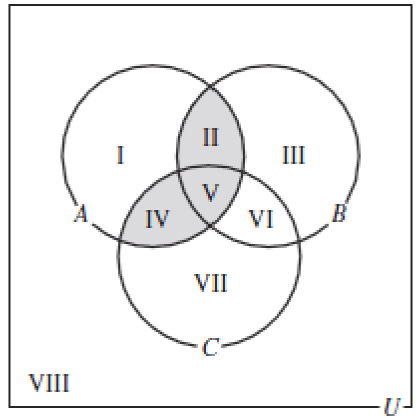
11. Step 3 $B \cap C$ = {V, VI}, So $A \cup (B \cap C)$ = {I, II, IV, V, VI}.

Step 4 Shade regions I, II, IV, V, and VI



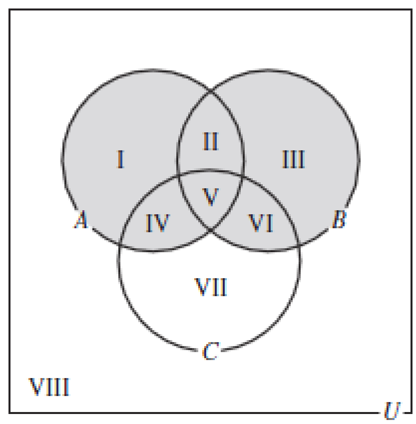
12. Step 3 $B \cup C$ = {II, III, IV, V, VI, VII}, so $A \cap (B \cup C)$ = {II, IV, V}.

Step 4 Shade regions II, IV, and V



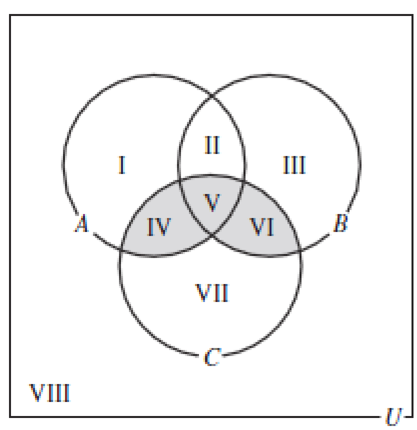
13. Step 3 $A \cup B$ = {I, II, III, IV, V, VI} and $A \cap C$ = {IV, V}, so $(A \cup B) \cup (A \cap C)$ = {I, II, III, IV, V, VI }.

Step 4 Shade regions I, II, III, IV, V, and VI



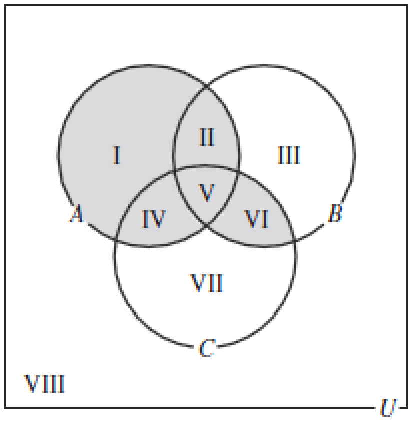
14. Step 3 $A \cup B$ = {I, II, III, IV, V, VI} , so $(A \cup B) \cap C$ = { IV, V, VI }.

Step 4 Shade regions IV, V, and VI



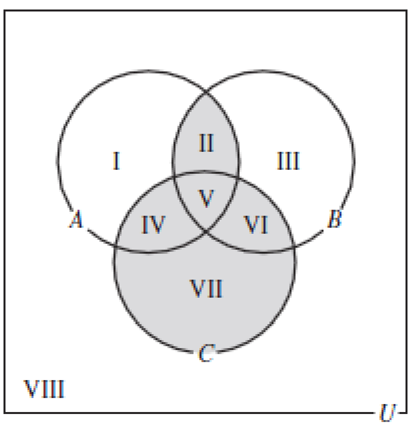
15. Step 3 $A \cup B$ = {I, II, III, IV, V, VI} and $A \cup C$ = {I, II, IV, V, VI, VII}, so $(A \cup B) \cap (A \cup C)$ = { I, II, III, IV, V, VI}.

Step 4 Shade regions I, II, IV, V, and VI



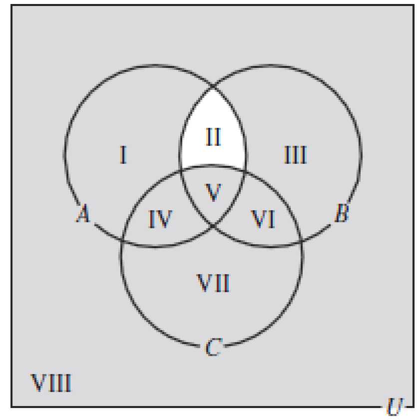
16. Step 3 $A \cap B$ ={II, V}, so $(A \cap B) \cup C$ = {II, IV, V, VI, VII}.

Step 4 Shade regions II, IV, V, VI, and VII



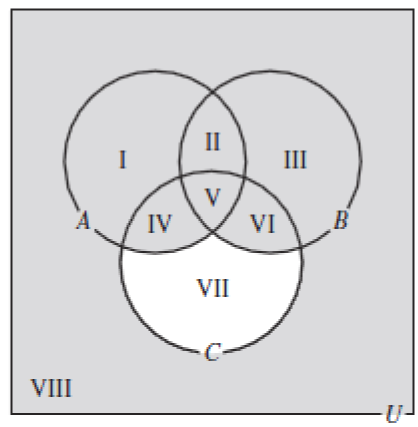
17. Step 3 $A \cap B$ = {II, V}, so $(A \cap B)'$ = {I, III, IV, VI, VII, VIII} and $(A \cap B)' \cup C$ = { I, III, IV, V, VI, VII, VIII }.

Step 4 Shade regions I, III, IV, V, VI, VII, and VIII



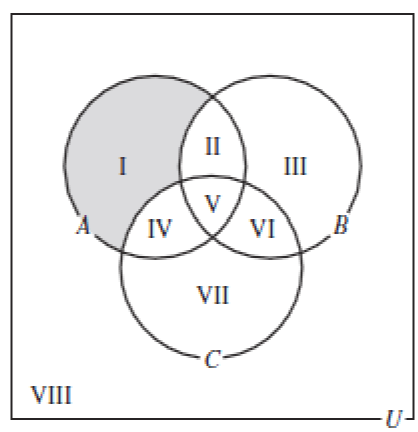
18. Step 3 $A \cup B$ = {I, II, III, IV, V, VI} and C' = {I, II, III, VIII}, so $(A \cup B) \cup C'$ = {I, II, III, IV, V, VI, VIII}

Step 4 Shade regions I, II, III, IV, V, VI, and VIII



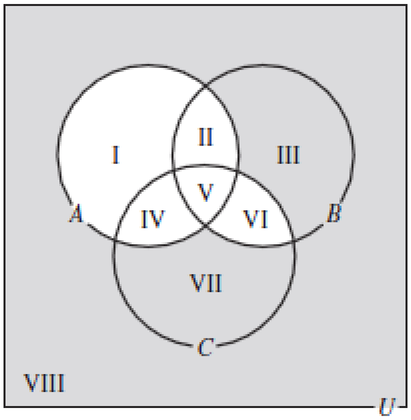
19. Step 3 B ∪ C = {II, III, IV, V, VI, VII}, so (B ∪ C)′ = {I, VIII} and A ∩ (B ∪ C)′ = {I}.

Step 4 Shade region I



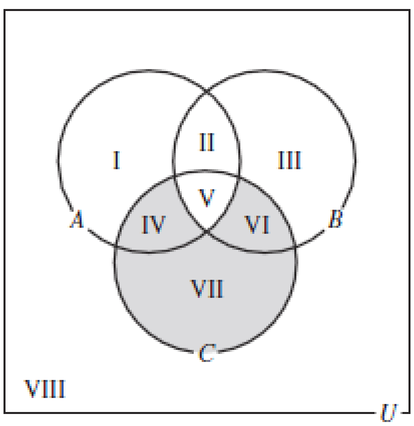
20. Step 3 $B' \cup C' = (B \cap C)'$ (by DeMorgan's law) = {I, II, III, IV, VII, VIII} and A' = {III, VI, VII, VIII}, So $A' \cap (B' \cup C')$ = {III, VII, VIII}

Step 4 Shade regions III, VII, and VIII



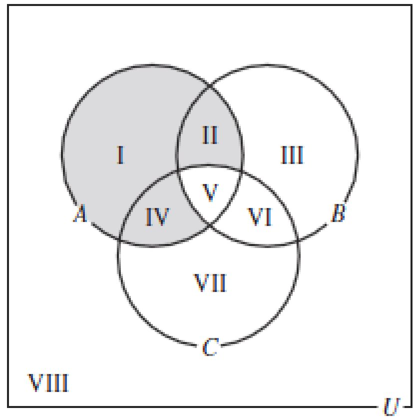
21. Step 3 $A' \cup B' = (A \cap B)'$ (by DeMorgan's law) = {I, III, IV, VI, VII, VIII} , So $(A' \cup B') \cap C$ = {IV, VI, VII}

Step 4 Shade regions IV, VI, and VII



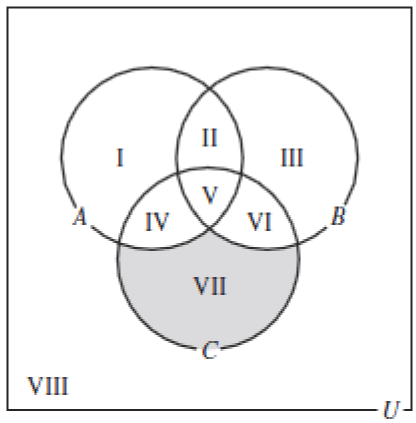
22. Step 3 $(B \cap C)'$ = {I, II, III, IV, VII, VIII}, so $A \cap (B \cap C)'$ = {I, II, IV}

Step 4 Shade regions I, II, and IV



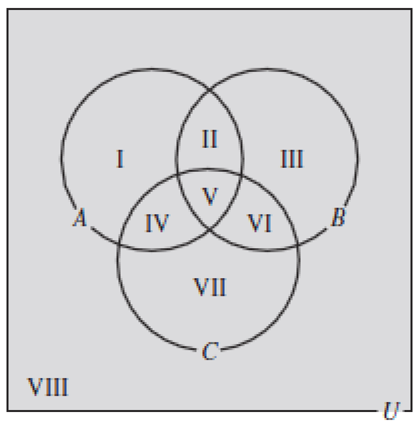
23. Step 3 $(A \cup B)'$ = {VII, VIII} and $A \cup C$ = {I, II, IV, V, VI, VII}, so $(A \cup B)' \cap (A \cup C)$ = {VII}

Step 4 Shade region VII



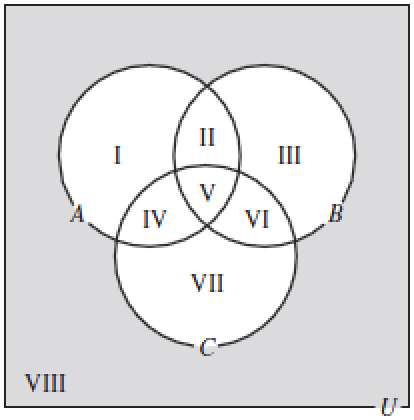
24. Step 3 $B \cup C$ = {II, III, IV, V, VI, VII} and C'={I, II, III, VIII}, So $(B \cup C) \cup C'$ = {I, II, III, IV, V, VI, , VII, VIII} = U

Step 4 Shade all regions



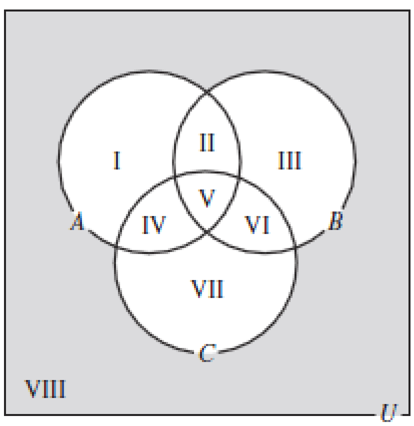
25. Step 3 $B' \cap C' = (B \cup C)'$ ={I, VIII} and A' = {III, VI, VII, VIII}, so $A' \cap (B' \cap C')$={VIII}

Step 4 Shade region VIII



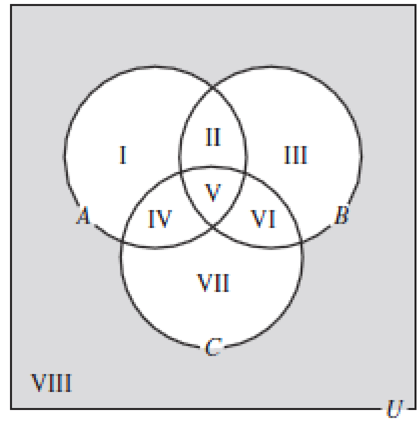
26. Step 3 $(A \cup B)'$ = {VII, VIII} and C' = {I, II, III, VIII}, so $(A \cup B)' \cap C'$ = {VIII}

Step 4 Shade region VIII



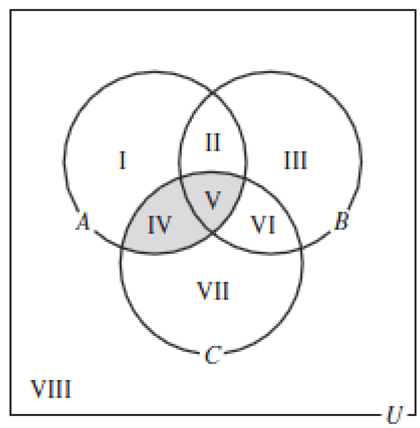
27. Step 3 $(B \cup C)'$ ={I, VIII} and A' = {III, VI, VII, VIII}, so $A' \cap (B \cup C)'$={VIII}

Step 4 Shade region VIII



28. Step 3 $A \cup B$ = {I, II, III, IV, V, VI} and $A \cap C$ = {IV, V}, so $(A \cup B) \cap (A \cap C)$ = {IV, V}

Step 4 Shade regions IV, and V



For Exercises 29-36, use the labeled Venn diagram from Step 1 of the solutions for Exercises 11-28.

29. $A \cap B$ = {II, V}
$(A \cap B)'$ = {I, III, IV, VI, VII, VIII}
A' = {III, VI, VII, VIII}
B' = {I, IV, VII, VIII}
$A' \cup B'$ = {I, III, IV, VI, VII, VIII}
Yes, $(A \cap B)'$ is equal to $A' \cup B'$.

30 . $A \cup B$ = {I, II, III, IV, V, VI}
$(A \cup B)'$ = {VII, VIII}
A' = {III, VI, VII, VIII}
B' = {I, IV, VII, VIII}
$A' \cup B'$ = {I, III, IV, VI, VII, VIII}
No, $(A \cup B)'$ is not equal to $A' \cup B'$.

31. $(A \cup B) \cup C = \{I, II, III, IV, V, VI, VII\}$
$A \cup (B \cup C) = \{I, II, III, IV, V, VI, VIII\}$
Yes, $(A \cup B) \cup C$ is equal to $A \cup (B \cup C)$.

32. $B \cup C = \{II, III, IV, V, VI, VII\}$
$A \cap (B \cup C) = \{II, IV, V\}$
$A \cap B = \{II, V\}$
$A \cap C = \{VI, V\}$
$(A \cap B) \cup (A \cap C) = \{II, VI, V\}$
Yes, $A \cap (B \cup C)$ is equal to $(A \cap B) \cup (A \cap C)$.

33. $C' = \{I, II, III, VIII\}$
$B \cap C' = \{II, III\}$
$A' = \{III, VI, VII, VIII\}$
$A' \cup (B \cap C') = \{II, III, VI, VII, VIII\}$
$A' \cup B = \{II, III, V, VI, VII, VIII\}$
$(A' \cup B) \cap C' = \{II, III, VIII\}$
No, $A' \cup (B \cap C')$ is not equal to $(A' \cup B) \cap C'$

34. $A \cap B = \{II, V\}$
$C' = \{I, II, III, VIII\}$
$(A \cap B) \cup C' = \{I, II, III, V, VIII\}$
$B \cap C' = \{II, III\}$
$(A \cap B) \cup (B \cap C') = \{II, III, V\}$
No, $(A \cap B) \cup C'$ is not equal to $(A \cap B) \cup (B \cap C')$

35. $A \cap B = \{II, V\}$
$(A \cap B)' = \{I, III, IV, VI, VII, VIII\}$
$(A \cap B)' \cup C = \{I, III, IV, V, VI, VII, VIII\}$
$A' = \{III, VI, VII, VIII\}$
$B' = \{I, IV, VII, VIII\}$
$A' \cup B' = \{I, III, IV, VI, VII, VIII\}$
$(A' \cup B') \cap C = \{IV, VI, VII\}$
No, $(A \cap B)' \cup C$ is not equal to $(A' \cup B') \cap C$

36. $A' = \{III, VI, VII, VIII\}$
$B' = \{I, IV, VII, VIII\}$
$C = \{IV, V, VI, VII\}$
$(A' \cup B') \cup C = \{I, III, IV, V, VI, VII, VIII\}$
$A \cap B = \{II, V\}$
$(A \cap B)' = \{I, III, IV, VI, VII, VIII\}$
$C' = \{I, II, III, VIII\}$
$(A \cap B)' \cap C' = \{I, III, VIII\}$
No, $(A' \cup B') \cup C$ is not equal to $(A \cap B)' \cap C'$

37. $n(A) = 6$

38. $n(B) = 5$

39. $n(A \cap B) = 2$

40. $n(A \cup B) = 9$

41. $n(A') = 6$

42. $n(B') = 7$

43. $n(A' \cap B') = 3$

44. $n(A' \cup B') = 10$

45. $n(A - B) = 4$

46. $n(B - A) = 3$

47. $A = \{1, 3, 5, 7, 9, 11\}$
$n(A) = 6$

48. $B = \{2, 3, 5, 7, 11\}$
$n(B) = 5$

49. $A \cap B = \{3, 5, 7, 11\}$
$n(A \cap B) = 4$

50. $A \cup B = \{1, 2, 3, 5, 7, 9, 11\}$
$n(A \cup B) = 7$

51. $A \cap B' = \{1\}$
$n(A \cap B') = 1$

52. $A' \cup B = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12\}$
$n(A' \cup B) = 10$

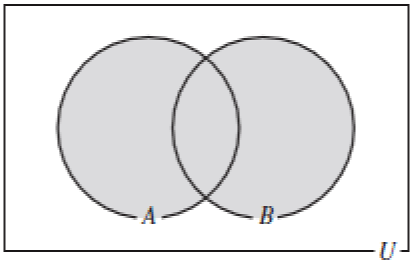
53. $A' = \{2, 4, 6, 8, 10, 12\}$
$n(A') = 6$

54. $B' = \{1, 4, 6, 8, 9, 10, 12\}$
$n(B') = 7$

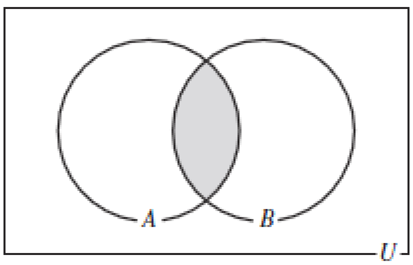
55. $A - B = \{1, 9\}$
$n(A - B) = 2$

56. $B' - A = \{4, 6, 8, 10, 12\}$
$n(B' - A) = 5$

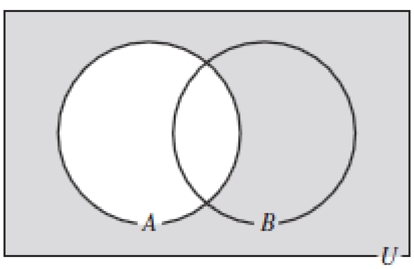
57. People that drive an SUV or a hybrid vehicle.



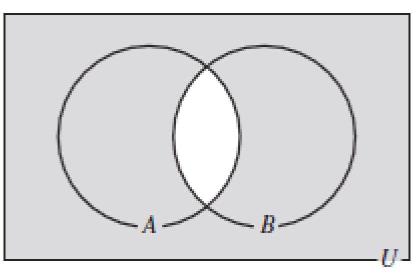
58. People who drive a hybrid SUV.



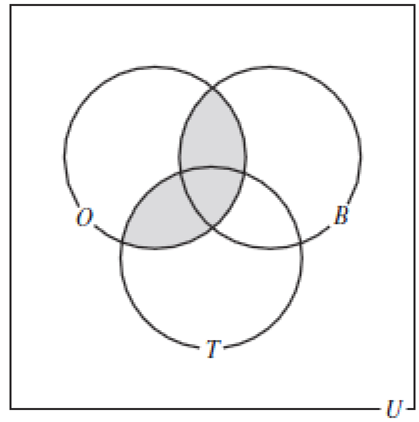
59. People who do not drive an SUV.



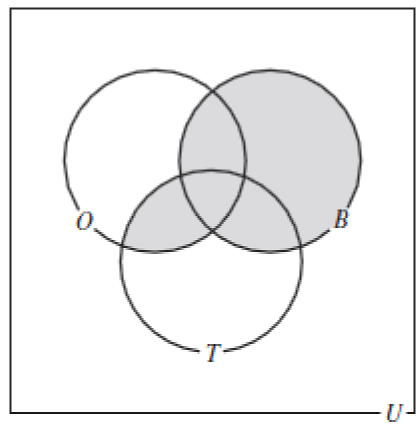
60. People who do not drive a hybrid SUV.



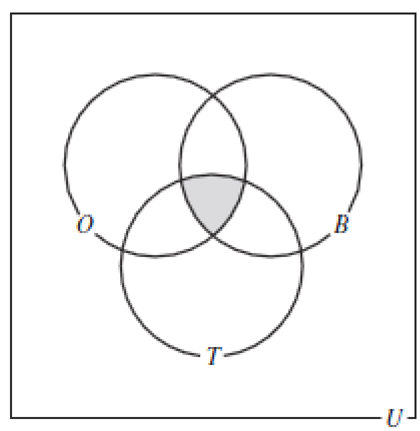
61. Students in online courses and blended or traditional courses.



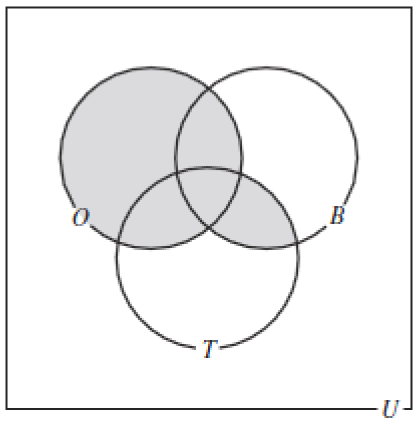
62. Students who are in blended courseS or online and traditional courses.



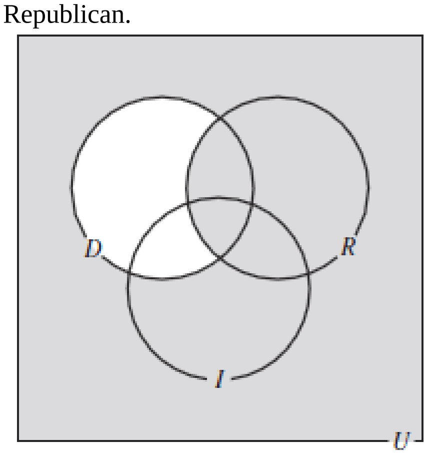
63. Students who are in blended, online, and traditional courses.



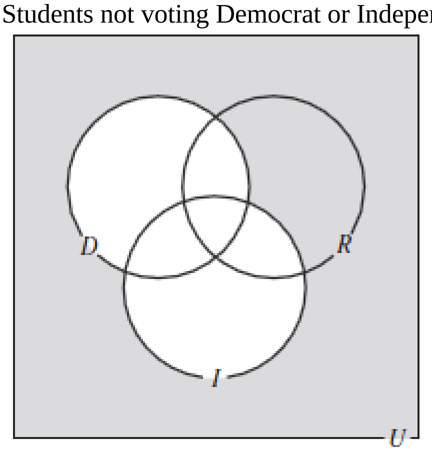
64. Students in blended and traditional or online courses.



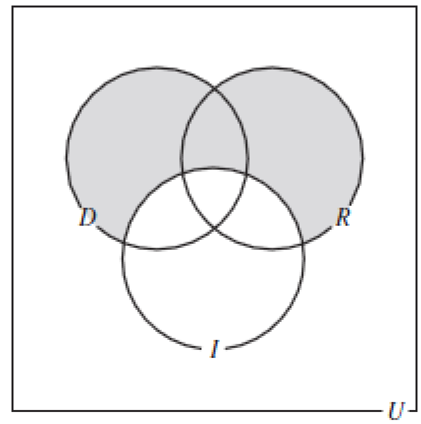
65. Students not voting Democrat or voting Republican.



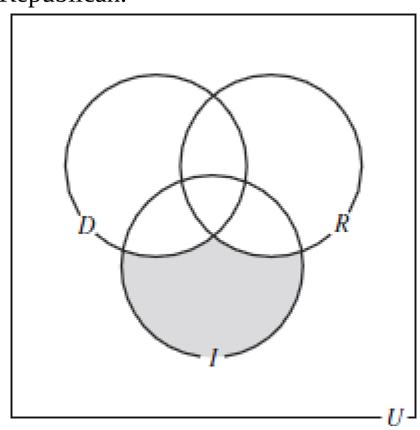
66. Students not voting Democrat or Independent.



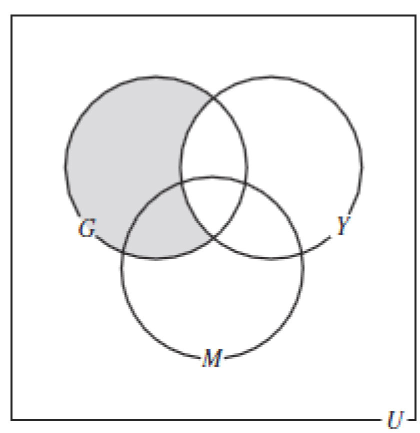
67. Students voting Democrat or Republican but not Independent.



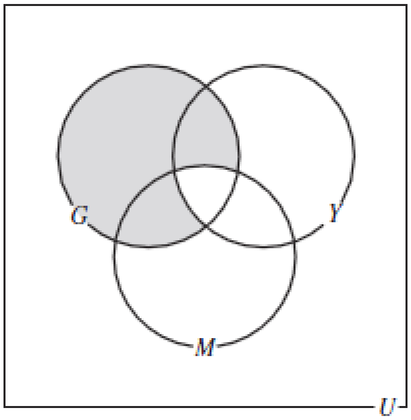
68. Students voting Independent but not Democrat or Republican.



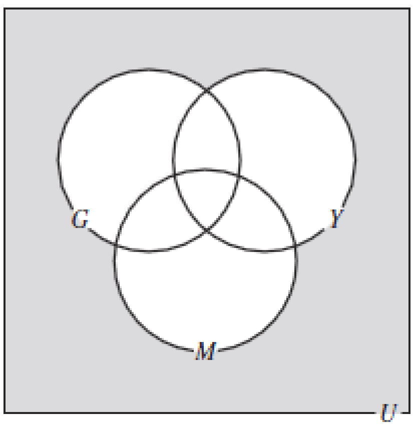
69. People who regularly use Google but not Yahoo!.



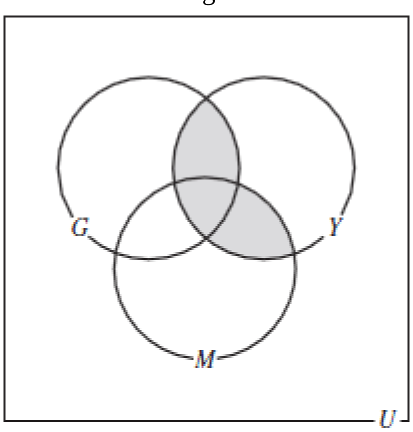
70. People who regularly use Google and either Yahoo! or MSN Live but not all three.



71. People who do not regularly use Google, Yahoo!, or MSN Live.



72. People who regularly use Yahoo! and MSN Live or Yahoo! and Google.



73. The Boston Red Sox were in the playoffs in 2005 and 2007 so they are in *A* and *C* but not *B* which is region IV.

74. The Los Angeles Angels were in the playoffs in 2005 and 2007 so they are in *A* and *C* but not *B* which is region IV.

75. The Cleveland Indians were in the playoffs in 2007 only, so they are in region VII.

76. The Minnesota Twins were in the playoffs in 2006 only, so they are in region III.

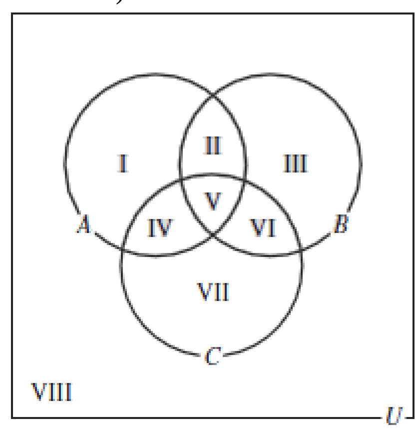
77. The New York Yankees were in the playoffs all three years, so they are in region V.

78. The Oakland A's were in the playoffs in 2006 only, so they are in region III.

79. No; $n(A - B) = n(A) - n(A \cap B)$

80. There is no formula for $n(A \cap B)$ in terms of $n(A)$ and $n(B)$ only. Answers vary for possible formulas, one example is $n(A \cap B) = n(A \cup B) - n(A - B) - n(B - A)$.

81. $(A \cup B \cup C)' = A' \cap B' \cap C'$



$A \cup B \cup C = \{\text{I, II, III, IV, V, VI, VII}\}$ so $(A \cup B \cup C)' = \{\text{VIII}\}$.

$A' = \{\text{III, VI, VII, VIII}\}$
$B' = \{\text{I, IV, VII, VIII}\}$
$C' = \{\text{I, II, III, VIII}\}$ so $A' \cap B' \cap C' = \{\text{VIII}\}$.

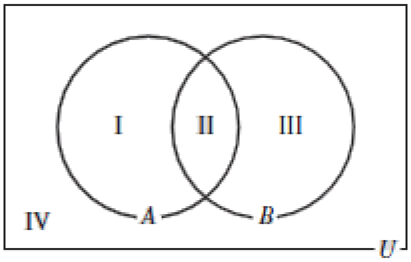
Therefore $(A \cup B \cup C)' = A' \cap B' \cap C'$

82. $(A \cap B \cap C)' = A' \cup B' \cup C'$
(Using the Venn diagram in the previous solution)
$A \cap B \cap C = \{\text{V}\}$ so $(A \cap B \cap C)' = \{\text{I, II, III, IV, VI, VII, VIII}\}$

$A' = \{\text{III, VI, VII, VIII}\}$
$B' = \{\text{I, IV, VII, VIII}\}$
$C' = \{\text{I, II, III, VIII}\}$ so $A' \cup B' \cup C' = \{\text{I, II, III, IV, VI, VII, VIII}\}$
Therefore $(A \cap B \cap C)' = A' \cup B' \cup C'$

Exercise Set 2-4

The solutions to exercises 1-4 refer to the regions labeled in the following Venn Diagram.



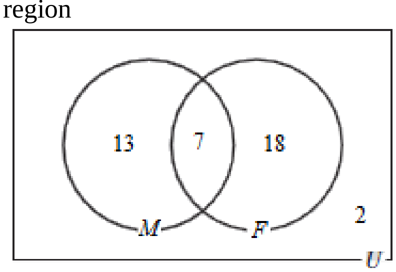
1. **Step 1** Draw a Venn diagram, where
U = universal set
M = people who use myspace.com
F = people who use facebook.com

Step 2 Since 7 people use both facebook.com and myspace.com, place 7 in region II.

Step 3 Since 20 people use myspace.com and 7 people use both, subtract 20 – 7 = 13 to get the number of people who use myspace.com only. Put this number in region I.

By subtracting 25 – 7 = 18 we can find the number of people who use facebook.com only. Put this number in region III.

Step 4 Find the number of people who used neither myspace.com nor facebook.com by adding, 13 + 7 + 18 = 38, and subtracting that number from the total number of students, 40; 40 – 38 = 2. Place 2 in region



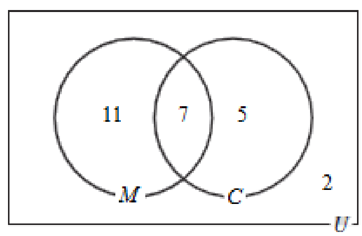
(a) The number of students that use myspace.com only is 13.
(b) The number of students that use Facebook.com only is 18.
(c) The number of students that use neither is 2.

2. **Step 1** Draw a Venn diagram, where
U = universal set
M = mathematics majors
C = computer science majors

Step 2 Since 7 students were dual majors in mathematics and computer science, place 7 in region II.

Step 3 Since 18 students were mathematics majors and 7 students were dual majors, subtract 18 – 7 = 11 to get the number of students who were majoring in mathematics only. Place this number in region I.
By subtracting, find the number of students majoring in computer science only; 12 – 7 = 5.
Place this number in region III.

Step 4 Find the number of students who were not mathematics or computer science majors by adding, 7 + 11 + 5 = 23, and subtracting that number from the total number of students, 25; 25 – 23 = 2. Place 2 in region IV.



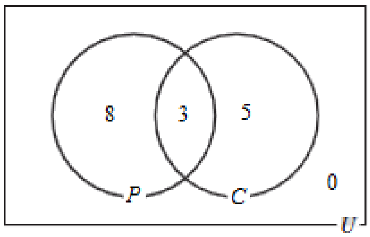
(a) The number of students majoring in mathematics only is 11.
(b) The number of students not majoring in computer science is 11+ 2 = 13.
(c) The number of students who were not mathematics or computer science majors is 2.

3. **Step 1** Draw a Venn diagram, where
U = universal set
P = bins that contained paper
C = bins that contained plastic.

Step 2 Since 3 bins contained both paper and plastic, place 3 in region II.

Step 3 Since 8 bins contained **only** paper, this number goes in region I.
Since 5 bins contained **only** plastic, this number goes in region III.

Step 4 By subtracting 8 + 5 + 3 = 16 from 16 we see there is no bin that contained neither paper nor plastic, so put 0 in region IV.



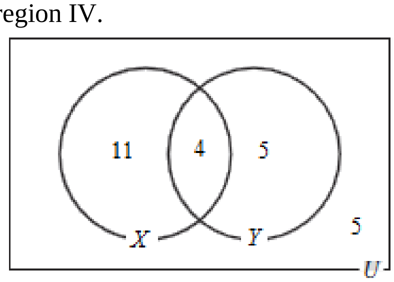
(a) The number of bins that contained only paper is 8.
(b) The number of bins that contained only plastic is 5.
(c) The number of bins that contained neither paper nor plastic is 0.

4. **Step 1** Draw a Venn diagram, where
U = universal set
X = students enrolled in psychology
Y = students enrolled in physics

Step 2 Since 4 students were enrolled in both psychology and physics, place 4 region II.

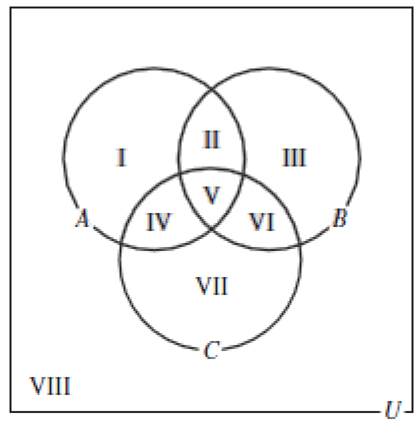
Step 3 Since 15 students were enrolled in psychology and 4 were enrolled in both psychology and physics, subtract 15 – 4 = 11 to get the number of students enrolled in psychology only. Place this number in region I.
By subtracting, find the number of students enrolled in physics only; 9 – 4 = 5. Place this number in region III.

Step 4 Find the number of students not enrolled in psychology or physics by adding,
4 + 11 + 5 = 20, and subtracting that number from the total number of students, 25; 25 – 20 = 5. Place 5 in region IV.



(a) The number of students enrolled in psychology only is 11.
(b) The number of students not enrolled in physics is 11 + 5 = 16.
(c) The number of students enrolled in at least one of these courses is
11 + 4 + 5 = 20.

The solutions to exercises 5-18 refer to the regions labeled in the following Venn diagram.



5. **Step 1** Draw a Venn diagram.

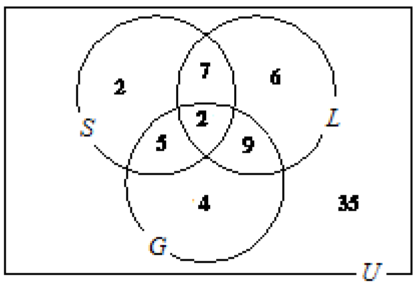
Step 2 Since 2 students receive all three types of aid, place 2 in region V.

Step 3 Find the number of students who had scholarships and loans but did not receive a grant. Subtract the number of students who had all three types of aid (2) from the number of students who had scholarships and grants (9), 9 – 2 = 7. Place 7 in region II.
Find the number of students that had loans and grants but did not have scholarships by subtracting 11 – 2 = 9. Place 9 in region VI.
Find the number of students who had scholarships and grants but did have student loans;
7 – 2 = 5. Place 5 in region IV.

Step 4 By subtracting, find the number of students that had only scholarships;
16 – (7 + 5 + 2) = 2. Place 2 in region I.
By subtracting, find the number of that had only student loans;
24 – (7 + 9 + 2) = 6. Place 6 in region III.
By subtracting, find the number of

students that had only grants;
20 – (9 + 5 + 2) = 4. Place 4 in region VII.

Step 5 Find the number of students who did not receive any financial aid by adding all the numbers, 2 + 7 + 6 + 5 + 2 + 9 + 4 = 35, and subtracting that number from the total number of students, 70; 70 – 35 = 35. Place 35 in region VIII.



(a) The number of students who had only scholarships is 2.
(b) The number of students who received grants and loans but not scholarships is 9.
(c) The number of students who did not receive any of these types of aid is 35.

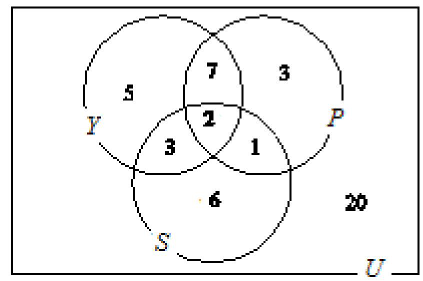
6. **Step 1** Draw a Venn diagram.

Step 2 Since 2 students were interested in all three, place 2 in region V.

Step 3 Find the number of students that were interested in yoga and Pilates but not spinning. Subtract the number of students interested in all three (2) from the number of students that were interested in yoga and Pilates (9), 9 – 2 = 7. Place 7 in region II.
Find the number of students that were interested in Pilates and spinning but not yoga by subtracting 3 – 2 = 1. Place 1 in region VI.
Find the number of students that were interested in yoga and spinning but not Pilates by subtracting 5 – 2 = 3. Place 3 in region IV.

Step 4 By subtracting, find the number of students interested in yoga only;
17 – (2 + 7 + 3) = 5. Place 5 in region I.
By subtracting, find the number of students interested in Pilates only;
13 – (2 + 7 + 1) = 3. Place 3 in region III.
By subtracting, find the number of students interested in spinning only;
12 – (2 + 1 + 3) = 6. Place 6 in region VII.

Step 5 Find the number of students that had interest in none of the classes by adding all the numbers, 5 + 7 + 3 + 3 + 2 + 1 + 6 = 27, and subtracting that number from the total number of students, 47; 47 – 27 = 20. Place 20 in region VIII.



(a) The number of students who are interested in yoga or spinning but not Pilates is 5 + 3 + 6 = 14.
(b) The number of students who are interested in exactly two of the classes is 7 + 3 + 1 = 11.
(c) The number of students who are interested in yoga but not Pilates is 5 + 3 = 8.

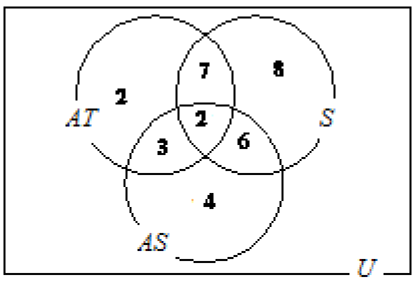
7. **Step 1** Draw a Venn diagram.

Step 2 Since 2 students failed because of all three reasons, place 2 in region V.

Find the number of students who failed because of poor attendance and not studying but not because of not turning in assignments. Subtract the number of students that failed because of all three conditions (2) from the number of students that failed because of poor attendance and not studying (9), 9 – 2 = 7. Place 7 in region II.
Find the number of students who failed because of not studying and not turning in assignments but not because of poor attendance by subtracting 8 – 2 = 6. Place 6 in region VI.
Find the number of students who failed because poor attendance and not turning in assignments but not

because of not studying by subtracting 5 – 2 = 3. Place 3 in region IV.

Step 4 By subtracting, find the number of students that failed because of poor attendance only; 14 – (2 + 7 + 3) = 2. Place 2 in region I.
By subtracting, find the number of students that failed because of not studying only; 23 – (2 + 7 + 6) = 8. Place 8 in region III.
By subtracting, find the number of students that failed because of not turning in assignments only; 15 – (2 + 6 + 3) = 4. Place 4 in region VII.



(a) The number of students who failed for exactly two of the three reasons is 7 + 3 + 6 = 16.
(b) The number of students who failed because of poor attendance and not studying but not because of not turning in assignments is 7.
(c) The number of students who failed because of exactly one of the three reasons is 2 + 8 + 4 = 14.
(d) The number of students who failed because of poor attendance and not turning in assignments but not because of not studying is 2 + 3 + 4 = 9.

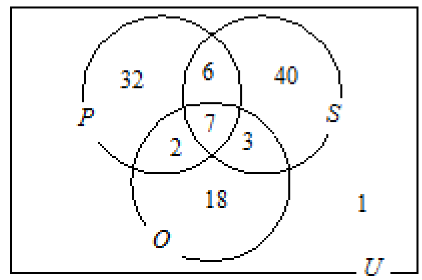
8. Step 1 Draw a Venn diagram.

Step 2 Since 7 customers ordered all three this number goes in region V.

Step 3 By subtracting 13 – 7 = 6 we find the number of customers who ordered pepperoni and sausage but not onions. Place 6 in region II.
By subtracting 10 – 7 = 3 we find the number of customers who ordered sausage and onions but not pepperoni. Place 3 in region VI.
By subtracting 9 – 7 = 2 we find the number of customers who ordered pepperoni and onions but not sausage. Place 2 in region IV.

Step 4 Since 32 customers ordered just pepperoni, place this number in region I.
Since 40 customers ordered just sausage, place this number in region III.
Since 18 customers ordered just onions, place this in region VII.

Step 5 Add all the numbers 32 + 6 + 40 + 2 + 7 + 3 + 18 = 108 subtract from 109 to get 1 customer in region VIII.



(a) There were 32 + 6 + 40 + 2 + 7 + 3 = 90 customers who ordered their pizzas with pepperoni or sausage.
(b) There were 6 + 40 + 2 + 7 + 3 + 18 = 76 customers who ordered sausage or onions.
(c) There was one customer that ordered their pizza without sausage, pepperoni, or onions.

9. Step 1 Draw a Venn diagram.

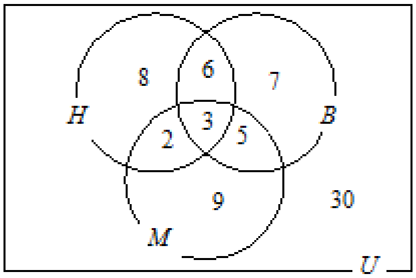
Step 2 Since 3 students were selling back all three types of books, place this number in region V.

Step 3 By subtracting 9 – 3 = 6 we get the number of students who sold back history and business books but not math. Place 6 in region II.
By subtracting 8 – 3 = 5 we get the number of students who sold back business and math books but not history. Place 5 in region VI.
By subtracting 5 – 3 = 2 we get the number of students who sold back history and math books but not business. Place 2 in region IV.

Step 4 Subtract 19 – (6 + 3 + 2) = 8 to get the number of students who sold back only history books. Place 8 in region I.

Subtract 21 – (6 + 3 + 5) = 7 to get the number of students who sold back only business books. Place 7 in region III.
Subtract 19 – (2 + 3 + 5) = 9 to get the number of students who sold back only math books. Place 9 in region VII.

Step 5 Subtract 70 – (8 + 6 + 7 + 2 + 3 + 5 + 9) = 30 to get the number of students who sold back none of the three types of books. Place 30 in region VIII.



(a) At most two means one or two so add 8 + 6 + 7 + 2 + 5 + 9 = 37 students who sold back at most two types of books.
(b) There were 2 students who sold back history and math books but not business books.
(c) There were 7 + 30 = 37 who were selling back neither history nor math books.

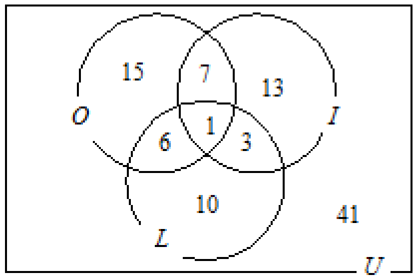
10. **Step 1** Draw a Venn diagram.

Step 2 Since 1 student read all three news sources, place 1 in region V.

Step 3 By subtracting 8 – 1 = 7 we get the number of students who read the *Campus Observer* and the Internet news, but not the local paper. Place 7 in region II.
By subtracting 4 – 1 = 3 we get the number of students who read the Internet news and the local paper, but not the *Campus Observer*. Place 3 in region VI.
By subtracting 7 – 1 = 6 we get the number of students who read the *Campus Observer* and the local paper, but not the Internet news. Place 6 in region IV.

Step 4 By subtracting 29 – (6+ 7 + 1) = 15 we get the number of students who read only the *Campus Observer*. Place 15 in region I.
By subtracting 24 – (7 + 3 + 1) = 13 we get the number of students who read only the Internet news. Place 13 in region III.
By subtracting 20 – (6 + 3 + 1) = 10 we get the number of students who read only the local paper. Place 10 in region VII.

Step 5 By subtracting 96 – (15 + 7 + 13 + 6 + 1 + 3 + 10) = 41 we get the number of students who read none of the three sources. Place 41 in region VIII.



(a) 7 + 13 + 6 + 10 = 36 students read the Internet news or local paper but not both.
(b) There were 3 students who read the Internet news and local paper but not the *Campus Observer*.
(c) 15 + 7 + 13 + 6 + 1 + 3 = 45 students read the *Campus Observer* or the Internet news.

11. **Step 1** Draw a Venn diagram

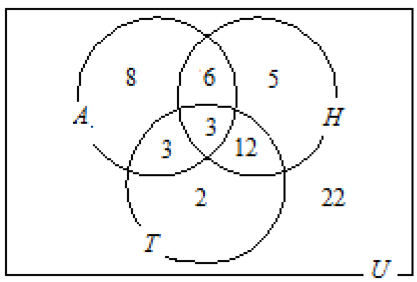
Step 2 Since 3 students liked all three types of music, place this number in region V.

Step 3 By subtracting 9 – 3 = 6 we get the number of students who liked alternative and hip-hop, but not techno. Place 6 in region II.
By subtracting 15 – 3 = 12 we get the number of students who liked hip-hop and techno, but not alternative. Place 12 in region VI.
By subtracting 6 – 3 = 3 we get the number of students who liked alternative and techno, but not hip-hop. Place 3 in region IV.

Step 4 By subtracting 20 – (6 + 3 + 3) = 8 we get the number of students who

liked alternative only. Place 8 in region I.
By subtracting 26 – (6 + 3 + 12) = 5 we get the number of students who liked only hip-hop. Place 5 in region III.
By subtracting 20 – (3 + 3 + 12) = 2 we get the number of students who liked only techno. Place 2 in region VII.

Step 5 By subtracting 61 – (8 + 6 + 5 + 3 + 3 + 1 2 + 2) = 22 we get the number of students who didn't like any of the three types of music. Place 22 in region VIII.



(a) There were 22 students who liked none of the three types of music.
(b) There were 3 students who liked exactly three of these types of music.
(c) There were 8 + 5 + 2 = 15 students who liked exactly one of the three types of music.

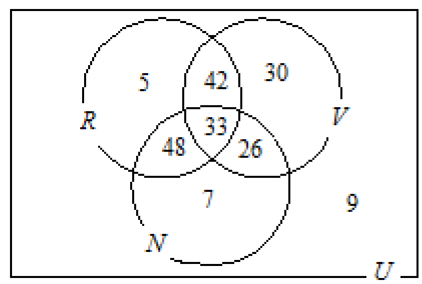
12. **Step 1** Draw a Venn diagram

Step 2 Since 33 students watched all three types of shows, place this number in region V.

Step 3 By subtracting 75 – 33 = 42 we get the number of students who watched a reality show and an MTV video but not a CNN news show. Place 42 in region II.
By subtracting 59 – 33 = 26 we get the number of students who watched an MTV video and CNN news show, but not a reality show. Place 26 in region VI.
By subtracting 81 – 33 = 48 we get the number of students who watched a reality show and a CNN news show, but not an MTV video. Place 48 in region IV.

Step 4 By subtracting 128 – (48 + 33 + 42) = 5 we get then number of students who watched only a reality show. Place 5 in region I.
By subtracting 131 – (42 + 33 + 26) = 30 we get the number of students who watched an MTV video only. Place 30 in region III.
By subtracting 114 – (48 + 33 + 26) = 7 we get the number of students who watched a CNN news show only. Place 7 in region VII.

Step 5 By subtracting 200 – (5 + 42 + 30 + 48 + 33 + 26 + 7) = 9 we get the number of students who watched none of the three types of shows. Place 9 in region VIII.



(a) Add 48 + 42 + 26 = 116 students watched exactly two types of shows.
(b) Add 5 + 30 + 7 = 42 students watched only one of the types of shows.
(c) Add 30 + 26 + 7 + 9 = 72 students did not watch a reality show.

13. **Step 1** Draw a Venn diagram

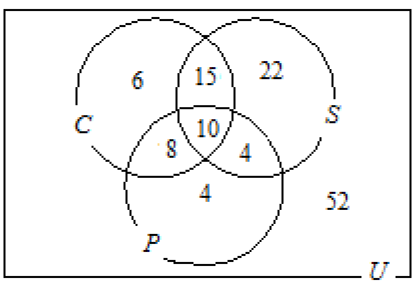
Step 2 Since 10 students were involved in all three place this number in region V.

Step 3 By subtracting 25 – 10 = 15 we get the number of students who belonged to a club and attended sporting events but did not belong to a professional organization. Place 15 in region II.
By subtracting 14 – 10 = 4 we get the number of students who attended sporting events and were in professional organizations but did not belong to a club. Place 4 in region VI.
By subtracting 18 – 10 = 8 we get the number of students who belonged to clubs and professional organizations

but did not attend sporting events. Place 8 in region IV.

Step 4 By subtracting 39 – (8 + 10 + 15) = 6 we get the number of students who belonged to clubs only. Place 6 in region I.
By subtracting 51 – (15 + 10 + 4) = 22 we get the number of students who attended sporting events only. Place 22 in region III.
By subtracting 26 – (8 + 10 + 4) = 4 we get the number of students who belonged only to professional organizations. Place 4 in region VII.

Step 5 By subtracting 121 – (6 + 15 + 22 + 8 + 10 + 4 + 4) = 52 we get the number of students who were not involved in any of the three activities. Place 52 in region VIII.



(a) Add 6 + 22 + 4 = 32 students who did exactly one of the activities.
(b) Add 15 + 22 = 37 students attended sports events but did not belong to a professional organization.
(c) Add 6 + 52 = 58 students who neither attended sporting events nor belonged to professional organizations.

14. Twenty students took the exam, 2 of whom answered neither bonus question, so $n(F \cup S) = 18$. We know that $n(F) = 15$ and $n(S) = 13$. We also know that $n(F \cup S) = n(F) + n(S) - n(F \cap S)$ (cardinal number formula), or equivalently, $n(F \cap S) = n(F) + n(S) - n(F \cup S)$. So the number of students who answered both bonus questions was 15 + 13 – 18 = 10.

15. There were 34 people waiting, 2 of which didn't order a latte or cappuccino, so $n(L \cup C) = 32$. We know that $n(L) = 20$ and $n(C) = 18$. We know also that $n(L \cup C) = n(L) + n(C) - n(L \cap C)$ (cardinal number formula) or equivalently $n(L \cap C) = n(L) + n(C) - n(L \cup C)$. So the number of people who wanted both latte and cappuccino is 18 + 20 – 32 = 6.

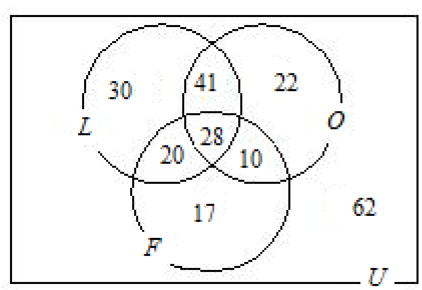
16. Step 1 Draw a Venn diagram

Step 2 Since there were 28 students who watch all three, place this number in region V.

Step 3 By subtracting 69 – 28 = 41 we get the number of students who watch Letterman and O'Brien but not Fallon. Place 41 in region II.
By subtracting 38 – 28 = 10 we get the number of students who watch O'Brien and Fallon but not Letterman. Place 10 in region VI.
By subtracting 48 – 28 = 20 we get the number of students who watch Letterman and Fallon but not O'Brien. Place 20 in region IV.

Step 4 By subtracting 119 – (20 + 28 + 41) = 30 we get the number of students who watched only Letterman. Place in 30 region I.
By subtracting 101 – (41 + 28 + 10) = 22 we get the number of students who watched O'Brien only. Place 22 in region III.
By subtracting 75 – (20 + 28 + 10) = 17 we get the number of students who watched Fallon only. Place 17 in region VII.

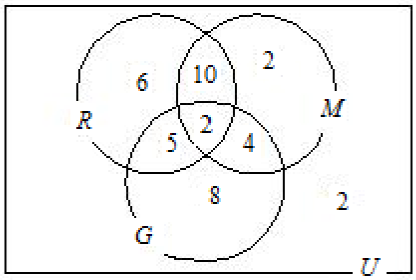
Step 5 By subtracting 230 – (30 + 41 + 22 + 20 + 28 + 10 + 17) = 62 we get the number of students who watched none of the three. Place 62 in region VIII.



(a) There were 62 students that watch none of the three shows.

(b) 30 + 22 + 17 = 69 students watch only one of the three shows.
(c) 20 + 41 + 10 = 71 students watch exactly two of the three shows.

17. The Venn diagram that summarizes the researcher's results would be as follows.



The total of the eight regions is 39 but the researcher surveyed 40 people.

18. Answers vary; one possible answer is: of the 40 email recipients, only 3 looked at all three advertisements.

Exercise Set 2-5

1. An *infinite set* is one that does not have a fixed number of elements. Cantor's definition: A set is infinite if it can be put into a one-to-one correspondence with a subset of itself.

2. A general term of an infinite set is written in terms of *n*, such that when 1 is substituted for *n*, one gets the first term of the set. When 2 is substituted for *n*, one gets the second term of the set, etc.

3. A countable set is one that can be put into one-to-one correspondence with a subset of the natural numbers.

4. Natural numbers {1, 2, 3, … } can be put into one-to-one correspondence with the even numbers {0, 2, –2, 4, –4, 6, –6, …} as follows. For every odd number n in N let n correspond to $n - 1$. For every even number n in N let n correspond to $-n$. Since the two sets can be put into a one-to-one correspondence with each other they have the same cardinality.

5. Use inductive reasoning.
7(1) = 7
7(2) = 14
7(3) = 21
7(4) = 28
7(5) = 35
etc.
A general term is $7n$.

6. Use inductive reasoning.
$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
etc.
A general term is n^3.

7. Use inductive reasoning.
$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1024$
etc.
A general term is 4^n.

8. Use inductive reasoning.
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
etc.
A general term is n^2.

9. Use inductive reasoning.
–3(1) = –3
–3(2) = –6
–3(3) = –9
–3(4) = –12
–3(5) = –15
etc.
A general term is $-3n$.

10. Use inductive reasoning.
22(1) = 22
22(2) = 44
22(3) = 66
22(4) = 88
22(5) = 110
etc.
A general term is $22n$.

11. Use inductive reasoning.
$\frac{1}{1+1}=\frac{1}{2}$
$\frac{1}{2+1}=\frac{1}{3}$
$\frac{1}{3+1}=\frac{1}{4}$
$\frac{1}{4+1}=\frac{1}{5}$
$\frac{1}{5+1}=\frac{1}{6}$
etc.
A general term is $\frac{1}{n+1}$.

12. Use inductive reasoning. A general term is $\frac{n}{3}$.

13. Use inductive reasoning.
4(1) – 2 = 2
4(2) – 2 = 6
4(3) – 2 = 10
4(4) – 2 = 14
4(5) – 2 = 18
etc.
A general term is $4n - 2$.

14. Use inductive reasoning.
3(1) – 2 = 1
3(2) – 2 = 4
3(3) – 2 = 7
3(4) – 2 = 10
3(5) – 2 = 13
etc.
A general term is $3n - 2$.

15. Use inductive reasoning

$\frac{2}{3}=\frac{1+1}{1+2}$
$\frac{3}{4}=\frac{2+1}{2+2}$
$\frac{4}{5}=\frac{3+1}{3+2}$
$\frac{5}{6}=\frac{4+1}{4+2}$
$\frac{6}{7}=\frac{5+1}{5+2}$
etc.

A general term is $\frac{n+1}{n+2}$.

16. Use inductive reasoning

$\frac{1}{1}=\frac{1}{1^2}$
$\frac{1}{4}=\frac{1}{2^2}$
$\frac{1}{9}=\frac{1}{3^2}$
$\frac{1}{16}=\frac{1}{4^2}$
$\frac{1}{25}=\frac{1}{5^2}$
etc.

A general term is $\frac{1}{n^2}$.

17. Use inductive reasoning

100 = 100(1)
200 = 100 (2)
300 = 100 (3)
400 = 100 (4)
500 = 100 (5)
etc.
A general term is $100n$.

18. Use inductive reasoning
50 = 50(1)
100 = 50 (2)
150 = 50 (3)
200 = 50 (4)
250 = 50 (5)
etc.
A general term is $50n$.

19. Use inductive reasoning
–4 = –3(1) – 1
–7 = –3(2) – 1
–10 = –3(3) – 1
–13 = –3(4) – 1
–16 = –3(5) – 1
etc.
A general term is $-3n - 1$.

20. Use inductive reasoning
–3 = –2(1) – 1
–5 = –2(2) – 1
–7 = –2(3) – 1
–9 = –2(4) – 1
–11 = –2(5) – 1
A general term is $-2n - 1$.

For 21 through 30 we will show each set is infinite by putting it into a one-to-one correspondence with a proper subset of itself.

21. $\{3, 6, 9, 12, 15, \ldots, 3n, \ldots\}$
$\updownarrow$
$\{6, 12, 18, 24, 30, \ldots, 6n, \ldots\}$

22. $\{10, 15, 20, 25, 30, \ldots, 5n+5, \ldots\}$
$\updownarrow$
$\{15, 25, 35, 45, 55, \ldots, 10n+5, \ldots\}$

23. $\{9, 18, 27, 36, 45, \ldots, 9n, \ldots\}$
$\updownarrow$
$\{18, 36, 54, 72, 90, \ldots, 18n, \ldots\}$

24. $\{4, 10, 16, 22, 28, \ldots, 6n-2, \ldots\}$
$\updownarrow$
$\{10, 22, 34, 46, 58, \ldots, 12n-2, \ldots\}$

25. $\{2, 5, 8, 11, \ldots, 3n-1, \ldots\}$
$\updownarrow$
$\{5, 11, 17, 23, \ldots, 6n-1, \ldots\}$

26. $\{20, 24, 28, \ldots, 16+4n, \ldots\}$
$\updownarrow$
$\{24, 28, 32, \ldots, 20+4n, \ldots\}$

27. $\{10, 100, \ldots, 10^n, \ldots\}$
$\updownarrow$
$\{100, 10{,}000, \ldots, 10^{2n}, \ldots\}$

28. $\{100, 200, 300, 400, \ldots, 100n, \ldots\}$
$\updownarrow$
$\{200, 400, 600, 800, \ldots, 200n, \ldots\}$

29. $\left\{\frac{5}{1}, \frac{5}{2}, \frac{5}{3}, \ldots, \frac{5}{n}, \ldots\right\}$
$\updownarrow$
$\left\{\frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots, \frac{5}{n+1}, \ldots\right\}$

30. $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots\right\}$
$\updownarrow$
$\left\{\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots, \frac{1}{2^{n+1}}, \ldots\right\}$

31. $\aleph_0 + 1 = \aleph_0$ and $\aleph_0 + \aleph_0 = \aleph_0$

32. $\{1, 2, 3, 4, 5, \ldots, 2n+1, 2n, \ldots\}$
$\updownarrow$
$\{0, -1, 1, -2, 2, \ldots, n, -n, \ldots\}$

33. Answers vary: Though it may seem that there are more rational numbers than natural numbers there are not. The rational numbers can be put into a one-to-one correspondence with the natural numbers.

34. Answers vary: Although the idea of infinity extends beyond what is tangible.

Review Exercises

1. $D = \{52, 54, 56, 58\}$

2. $F = \{5, 7, 9, ..., 39\}$

3. $L = \{$l, e, t, r$\}$

4. $A = \{$a, r, k, n, s$\}$

5. $B = \{501, 502, 503, ...\}$

6. $C = \{6, 7, 8, 9, 10, 11\}$

7. $M = \varnothing$

8. $G = \varnothing$

9. $\{x \mid x$ is even and $16 < x < 26\}$

10. $\{x \mid x$ is a multiple of 5 between 0 and 25$\}$

11. $\{x \mid x$ is an odd natural number greater than 100$\}$

12. $\{x \mid x$ is a positive multiple of 8 less than 73$\}$

13. Infinite

14. Infinite

15. Finite

16. Finite

17. Finite

18. Finite

19. False since 100 is in the first set but not the second.

20. True since every element of {6} is also in {6, 12, 18} and the sets are not equal.

21. False since 6 is in the first set but not the second.

22. False because proper subsets cannot be equal.

23. $\varnothing$; {r}; {s}; {t}; {r, s}; {r, t}; {s, t}; {r, s, t}

24. $\varnothing$; {m}; {n}; {o}; {m, n}; {m, o}; {n, o}; {m, n, o}

25. $2^5 = 32$ subsets; 31 proper subsets

26. $2^6 = 64$ subsets; 63 proper subsets

27. $A \cap B$ = {t, u, v}

28. $B \cup C$ = {s, t, u, v, w, x, y, z}

29. $A \cap B$ = {t, u, v}
$(A \cap B) \cap C = \varnothing$

30. B' = {p, q, r, s, w, z}

31. List the elements in A = {p, r, t, u, v}
Cross off those elements of A that are in B: {p, r, ~~t, u, v~~} so $A - B$ = {p, r}.

32. List the elements in B = {t, u, v, x, y}.
Cross off those in B which are also in A: {~~t, u, v~~, x, y} so $B - A$ = {x, y}

33. $A \cup B$ = {p, r, t, u, v, x, y}
$(A \cup B)'$ = {q, s, w, z}
$(A \cup B)' \cap C$ = {s, w, z}

34. B' = {p, q, r, s, w, z}
C' = {p, q, r, t, u, v, x, y}
$B' \cap C'$ = {p, q, r}

35. $B \cup C$ = {s, t, u, v, w, x, y, z}
A' = {q, s, w, x, y, z}
$(B \cup C) \cap A'$ = {s, w, x, y, z}

36. $A \cup B$ = {p, r, t, u, v, x, y}
C' = {p, q, r, t, u, v, x, y}
$(A \cup B) \cap C'$ = {p, r, t, u, v, x, y}

37. B' = {p, q, r, s, w, z}
C' = {p, q, r, t, u, v, x, y}
$B' \cap C'$ = {p, q, r}
A' = {q, s, w, x, y, z}
$(B' \cap C') \cup A'$ = {p, q, r, s, w, x, y, z}

38. A' = {q, s, w, x, y, z}
$A' \cap B$ = {x, y}
$(A' \cap B) \cup C$ = {s, w, x, y, z}

39. $M \times N$ = {(s, v), (s, w), (s, x), (t, v), (t, w), (t, x), (u, v), (u, w), (u, x)}

40. $N \times M$ = {(v, s), (v, t), (v, u), (w, s), (w, t), (w, u), (x, s), (x, t), (x, u)}

41. $M \times M$ = {(s, s), (s, t), (s, u), (t, s), (t, t), (t, u), (u, s), (u, t), (u, u)}

42. $N \times N$ = {(v, v), (v, w), (v, x), (w, v), (w, w), (w, x), (x, v), (x, w), (x, x)}

43. Region I contains the elements in A which are not also in B: $A - B$

44. Region II contains the elements that are in both A and B: $A \cap B$

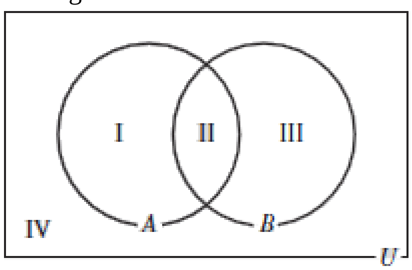
45. Region III contains the elements in B which are not also in A: $B - A$

46. Region IV contains the elements which are neither in A nor B: $(A \cup B)'$

47. Regions I and III contain the elements which are in A or B but not both A and B:
$(A \cup B) - (A \cap B)$

48. Regions I and IV contain the elements which are not in B: B'

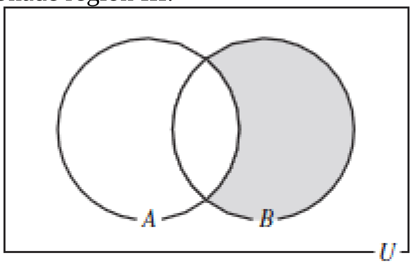
49. **Step 1** Draw the Venn diagram and label each region



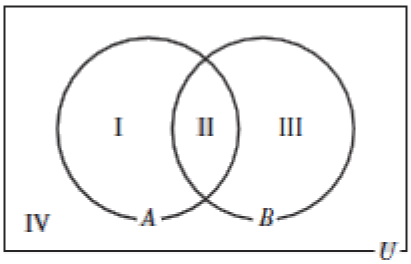
Step 2 From the diagram, list the regions elements in each set.
U = {I, II, III, IV}
A = {I, II}
B = {II, III}

Step 3 A' = {III, IV}
$A' \cap B$ = {III}

Step 4 Shade region III.



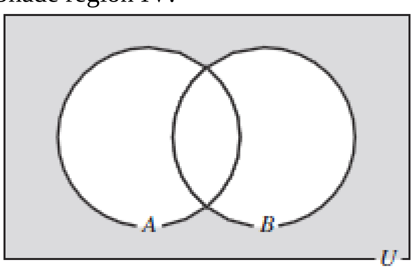
50. Step 1 Draw the Venn diagram and label each region.



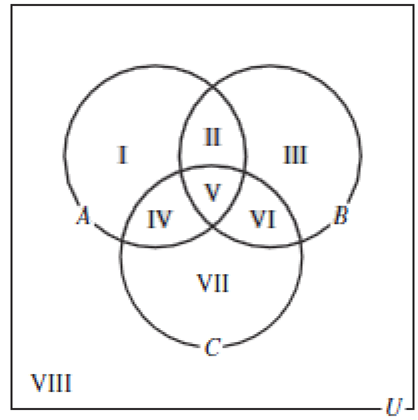
Step 2 From the Venn diagram, list the regions in each set.
U = {I, II, III, IV}
A = {I, II}
B = {II, III}

Step 3 $A \cup B$ = {I, II, III}
$(A \cup B)'$ = {IV}

Step 4 Shade region IV.



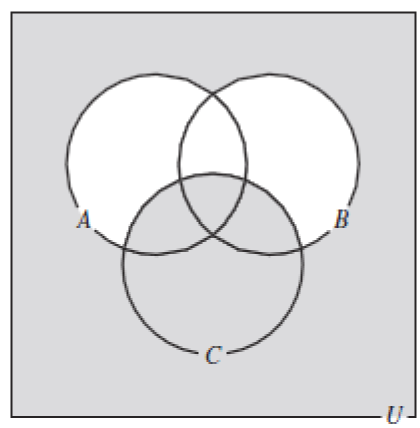
51. Step 1 Draw and label the diagram as shown.



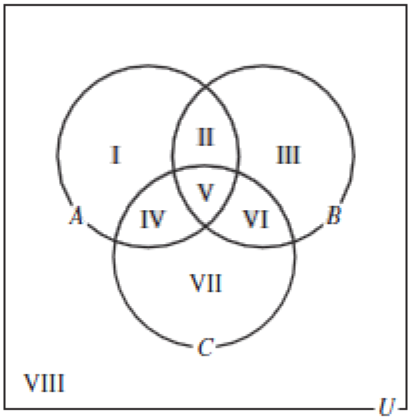
Step 2 From the diagram, list the regions in each set.
U = {I, II, III, IV, V, VI, VII, VIII}
A = {I, II, IV, V}
B = {II, III, V, VI}
C = {IV, V, VI, VII}

Step 3 Find the solution to $(A' \cap B') \cup C$
A' = {III, VI, VII, VIII}
B' = {I, IV, VII, VIII
$A' \cap B'$ = {VII, VIII}
$(A' \cap B') \cup C$ = {IV, V, VI, VII, VIII}

Step 4 Shade regions IV, V, VI, VII, and VIII.



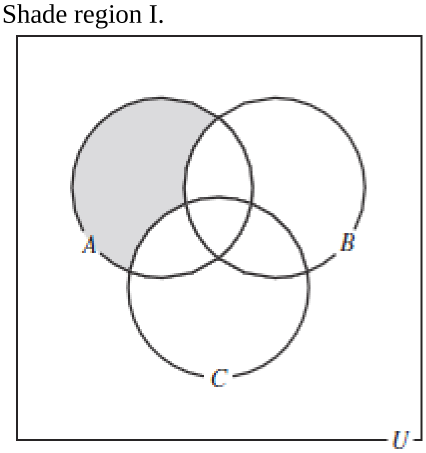
52. **Step 1** Draw and label the diagram as shown.



Step 2 From the diagram, list the regions in each set.
U = {I, II, III, IV, V, VI, VII, VIII}
A = {I, II, IV, V}
B = {II, III, V, VI}
C = {IV, V, VI, VII}

Step 3 $B \cup C$ = {II, III, IV, V, VI, VII}
$(B \cup C)'$ = {I, VIII}
$A \cap (B \cup C)'$ = {I}

Step 4 Shade region I.



53. The cardinal number formula says $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so $n(A \cup B) = 15 + 9 - 4 = 20$.

54. $n(A \cup B) = 24 + 20 - 14 = 30$.

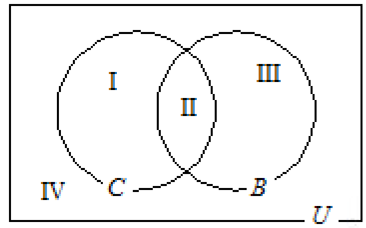
55. Since Florida appears in all three years it would be in region V.

56. Since Louisiana appears in 2006 only, it would be in region VII.

57. Since New Mexico appears in 2004 and 2005 but not 2006, it would be in region II.

58. Since Nevada appears in 2006 only it would be in region VII.

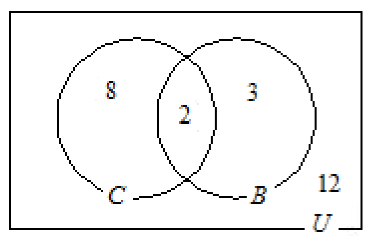
59. **Step 1** Draw a Venn diagram with regions I, II, III, and IV as follows:



Step 2 Since 2 students used a chat room and posted a new blog, put 2 in region II.

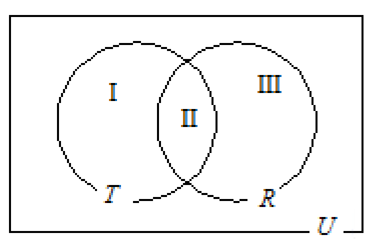
Step 3 By subtracting 10 – 2 = 8 we get the number of students that used a chat room but did not post a new blog. Place 8 in region I.
By subtracting 5 – 2 = 3 we get the number of students that posted a new blog but did not use a chat room. Place 3 in region III.

Step 4 By subtracting 25 – (2 + 8 + 3) = 12 we get the number of students who did neither. Place 12 in region IV.



(a) There were 12 students that didn't use a chat room or post a new blog.
(b) There were 3 students who posted a new blog only.

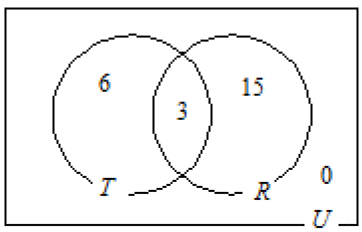
60. **Step 1** Draw a Venn diagram with regions I, II, III, and IV indicated as follows:



Step 2 Since 3 students tailgated and went to a rave, put 3 in region II.

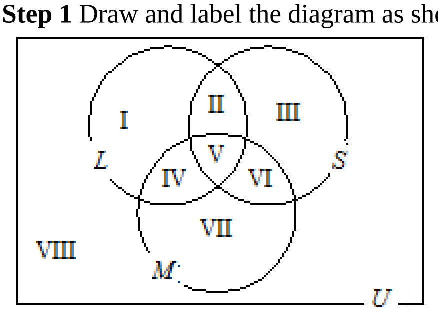
Step 3 By subtracting 9 – 3 = 6 we get the number of students who tailgated only. Place 6 in region I.
By subtracting 18 – 3 = 15 we get the number of students who only went to a rave. Place 15 in region III.

Step 4 By subtracting 24 – (6 + 15 + 3) = 0 we get the number of students who did neither. Place 0 in region IV.



(a) There were 15 students who went to a rave only.
(b) There were 6 + 0 = 6 students who did not go to a rave.

61. Step 1 Draw and label the diagram as shown

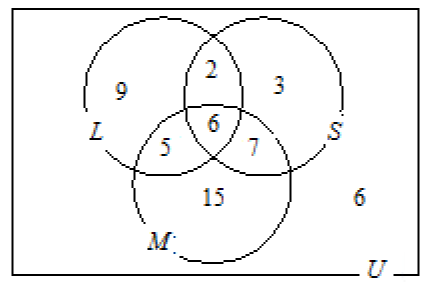


Step 2 Since there are 6 callers who listen to all three, place 6 in region V.

Step 3 By subtracting 8 – 6 = 2 we get the number of callers that listen to local radio and satellite radio but not MP3 players. Place 2 in region II.
By subtracting 13 – 6 = 7 we get the number of callers who listen to satellite and MP3 but not local radio. Place 7 in region VI.
By subtracting 11 – 6 = 5 we get the number of callers that listen to local radio and MP3 but not satellite. Place 5 in region IV.

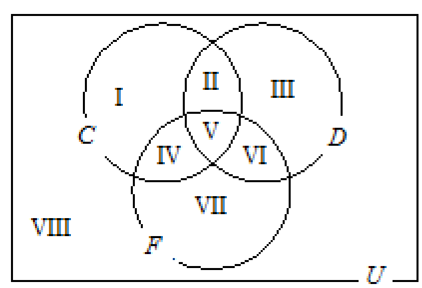
Step 4 By subtracting 22 – (5 + 6 + 2) = 9 we get the number of callers who listened to local radio only. Place 9 in region I.
By subtracting 18 – (6 + 2 + 7) = 3 we get the number of callers who listened to satellite radio only. Place 3 in region III.
By subtracting 33 – (5 + 6 + 7) = 15 we get the number of callers who listened to MP3 players only. Place 15 in region VII.

Step 5 By subtracting 53 – (9 + 2 + 3 + 5 + 6 + 7 + 15) = 6 we get the number of callers who listened to none of the three. Place 6 in region VIII.



(a) There were 3 callers that listened to satellite radio only.
(b) There were 5 callers that listened to local radio and MP3 but not satellite.
(c) There were six callers that listened to none of the three.

62. Step 1 Draw and label the Venn diagram as shown



Step 2 Since there is only one student who used all three, place 1 in region V.

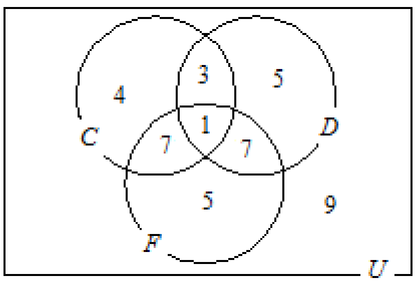
Step 3 By subtracting 4 – 1 = 3 we get the number of students who used cash and debit but not financial aid. Place 3 in region II.

By subtracting 8 – 1 = 7 we get the number of students that used debit and financial aid but not cash. Place 7 in region VI.

By subtracting 8 – 1 = 7 we get the number of students that used cash and financial aid but not debit. Place 7 in region IV.

Step 4 By subtracting 15 – (3 + 1 + 7) = 4 we get the number of students that used cash only. Place 4 in region I. By subtracting 16 – (3 + 1 + 7) = 5 we get the number of students that used debit only. Place 5 in region III. By subtracting 20 – (7 + 1 + 7) = 5 we get the number of students who used financial aid vouchers only. Place5 in region VII.

Step 5 By subtracting 41 – (4 + 3 + 5 + 7 + 1 + 7 + 5) = 9 we get the number of students who used none of the three types of payments. Place 9 in region VIII.



(a) There were 9 students that used none of the three types of payments.
(b) There were 5 students who used only debit cards.
(c) There were 7 students that used cash and financial aid but not debit.

63. Use inductive reasoning.
–3 – 2(1) = –5
–3 – 2(2) = –7
–3 – 2(3) = –9
–3 – 2(4) = –11
–3 – 2(5) = –13
etc.
A general term is $-3 - 2n$.

64. We will show the set is infinite by putting it into a one-to-one correspondence with a subset of itself: $\{12, 24, 36, \ldots, 12n, \ldots\}$
$\updownarrow \quad \updownarrow \quad \updownarrow \qquad \updownarrow$
$\{24, 48, 72, \ldots, 24n, \ldots\}$

Chapter Test

1. $P = \{92, 94, 96, 98\}$

2. $J = \{41, 43, 45, 47, 49\}$

3. K = {e, n, v, l, o, p}

4. W = {w, a, s, h, i, n, g, t, o}

5. $X = \{1, 2, 3, 4, \ldots, 79\}$

6. $Y = \{17, 18, 19, 20, 21, 22, 23, 24\}$

7. J = {January, June, July}

8. $L = \{\ \}$ or $\varnothing$

9. $\{x \mid x \in E \text{ and } 10 < x < 20\}$

10. {x | x is a multiple of 5 between 25 and 50}

11. {x | x is an odd natural number greater than 200}

12. $\{x \mid x = 2^{n+1}$ when n is a natural number less than 7$\}$

13. Infinite

14. Infinite

15. Finite

16. Finite

17. Finite

18. ∅; {d}; {e}; {f}; {d, e}; {d, f}; {e, f}; {d, e, f}

19. ∅; {p}; {q}; {r}; {p, q}; {p, r}; {q, r}; {p, q, r}

20. $2^5 = 32$ subsets; 31 proper subsets

21. $A \cap B$ = {a}

22. $B \cup C$ = {a, e, g, h, i, j, k}

23. $B' = \{b, c, d, e, f, h\}$

24. $A \cup B = \{a, b, d, e, f, g, i, j, k\}$
$(A \cup B)' = \{c, h\}$

25. $B' = \{b, c, d, e, f, h\}$
$A \cap B' = \{b, d, e, f\}$
$C' = \{a, b, c, d, f, g, i, k\}$
$(A \cap B') \cup C' = \{a, b, c, d, e, f, g, i, k\}$

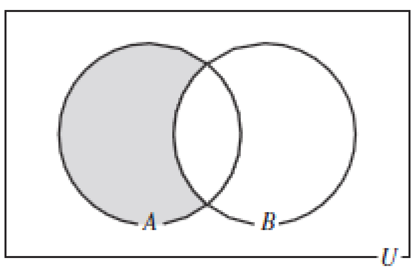
26. List elements in *A* and cross off those which are also in *B*: {~~a~~, b, d, e, f}, so,
$A - B = \{b, d, e, f\}$

27. List the elements in *B* and cross off those which are also in *C*: {a, g, i, ~~j~~, k},
so $B - C = \{a, g, i, k\}$

28. From exercise 26, $A - B = \{b, d, e, f\}$, cross off those also in *C*: {b, d, ~~e~~, f}, so
$(A - B) - C = \{b, d, f\}$

29. List elements in *A* and cross off those which are also in *C*: {a, b, d, ~~e~~, f}, so,
$A - C = \{a, b, d, f\}$

30.



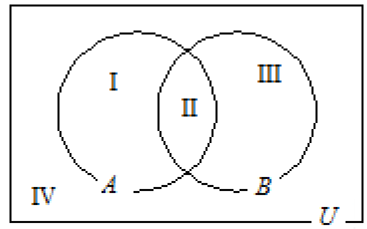
31. $A \times B = \{(@, \pi), (@, \#), (!, \pi), (!, \#), (\alpha, \pi), (\alpha, \#)\}$

32. $B \times A = \{(\pi, @), (\pi, !), (\pi, \alpha), (\#, @), (\#, !), (\#, \alpha)\}$

33. $B \times B = \{(\pi, \pi), (\pi, \#), (\#, \pi), (\#, \#)\}$

34. $A \times A = \{(@, @), (@, !), (@, \alpha), (!, @), (!, !), (!, \alpha), (\alpha, @), (\alpha, !), (\alpha, \alpha)\}$

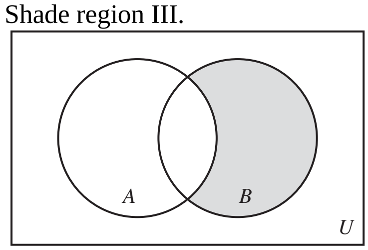
35. **Step 1** Draw the Venn diagram and label each region.



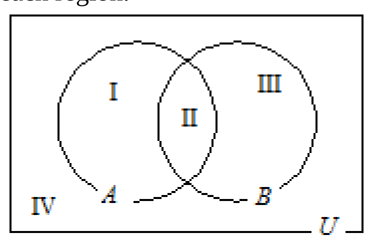
Step 2 From the diagram, list the regions in each set.
$U = \{I, II, III, IV\}$
$A = \{I, II\}$
$B = \{II, III\}$

Step 3 $A' = \{III, IV\}$
$A' \cap B = \{III\}$

Step 4 Shade region III.



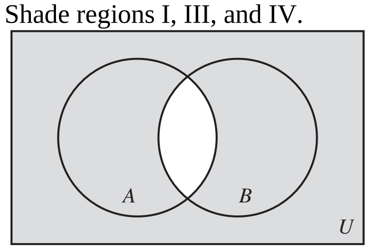
36. **Step 1** Draw the Venn diagram and label each region.



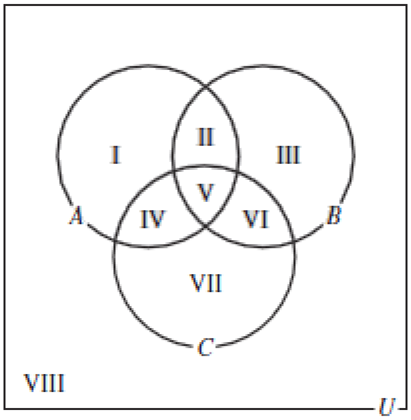
Step 2 From the diagram, list the regions in each set.
$U = \{I, II, III, IV\}$
$A = \{I, II\}$
$B = \{II, III\}$

Step 3 $A \cap B = \{II\}$
$(A \cap B)' = \{I, III, IV\}$

Step 4 Shade regions I, III, and IV.



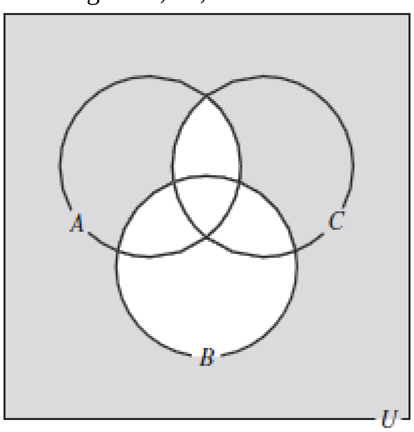
37. Step 1 Draw and label the Venn diagram as shown.



Step 2 From the diagram, list the regions in each set.
U = {I, II, III, IV, V, VI, VII, VIII}
A = {I, II, IV, V}
B = {II, III, V, VI}
C = {IV, V, VI, VII}

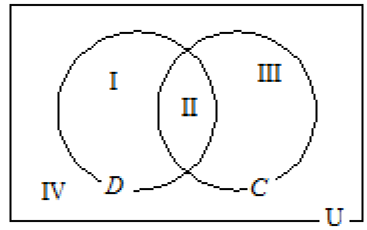
Step 3 $A' = \{\text{III, VI, VII, VIII}\}$
$B' = \{\text{I, IV, VII, VIII}\}$
$A' \cup B' = \{\text{I, III, IV, VI, VII, VIII}\}$
$C' = \{\text{I, II, III, VIII}\}$
$(A' \cup B') \cap C' = \{\text{I, III, VIII}\}$

Step 4 Shade regions I, III, and VIII.



38. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 1{,}500 + 1{,}150 - 350 = 2{,}300$

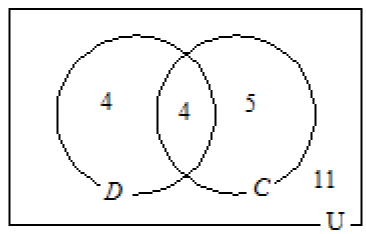
39. Step 1 Draw a Venn diagram with regions I, II, III, and IV as follows:



Step 2 Since 4 students used both, place 4 in region II.

Step 3 By subtracting 8 – 4 = 4 we get the number of students who used a digital camera only. Place 4 in region I. By subtracting 9 – 4 = 5 we get the number of students who used their cell phone cameras only. Place 5 in region III.

Step 4 By subtracting 24 – (4 + 4 + 5) = 11 we get the number of students who used neither. Place 11 in region IV.



(a) There were 11 students who used neither.
(b) There were 4 students who used a digital camera only.

40. Use inductive reasoning.
15(1) = 15
15(2) = 30
15(3) = 45
15(4) = 60
15(5) = 75
A general term is 15*n*.

41. Place the set into a one-to-one correspondence with a subset of itself:

$$\begin{array}{cccccccccc} \{1, & -1, & 2, & -2, & 3, & -3, & \ldots, & n, & -n\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & & \updownarrow & \updownarrow \\ \{1, & 2, & 3, & 4, & 5, & 6, & \ldots, & 2n-1, & 2n\} \end{array}$$

42. False; the elements of the sets are not the same.

43. True; the cardinality is the same.

44. True

45. True

46. True

47. True

48. False; y is an element of the first set but not the second set.

49. False; $12 \in \{12, 24, 36, ...\}$ but $\{12\} \notin \{12, 24, 36, ...\}$.

51. True

52. False; $\varnothing$ has no elements.

53. False; for any set $A \cap \varnothing = \varnothing$.

54. True