Chapter 2

ECONOMIC OPTIMIZATION

QUESTIONS & ANSWERS

Q2.1 *In 2007, Chrysler Group said it would cut 13,000 jobs, close a major assembly plant and reduce production at other plants as part of a restructuring effort designed to restore profitability at the auto maker by 2008. Its German parent, DaimlerChrysler said it is looking into further strategic options with partners to optimize and accelerate the plan as it seeks the best solutions for its struggling U.S. unit. Does this decision reflect an application of the global or partial optimization concept? Explain.*

Q2.1 *ANSWER*

Chrysler's decision to scale back employment at four assembly plants is a reflection of partial optimization because its decision alternatives were constrained by past decisions. The complexity of a completely integrated decision analysis approach--or global optimization--sometimes confines its use to major planning decisions. For many day-to-day operating decisions, managers often employ much less complicated partial optimization techniques. Partial optimization abstracts from the complexity of a completely integrated decision process by concentrating on more limited objectives within the firm's various operating departments. For example, the marketing department is usually required to determine the price and advertising policy that will achieve some sales goal given the firm's current product line and marketing budget. Alternatively, a production department might be expected to minimize the cost of a specified quantity of output at a stated quality level. In both instances, the fundamentals of economic analysis provide the basis for optimal managerial decisions.

Q2.2 *"The personal computer is a calculating device and a communicating device. Spreadsheets incorporate the best of both characteristics by allowing managers to determine and communicate the optimal course of action." Discuss this statement and explain why computer spreadsheets are a popular means for expressing and analyzing economic relations.*

Q2.2 *ANSWER*

When tables of economic data are displayed electronically in the format of an accounting income statement or balance sheet, such tables are often referred to as spreadsheets. *Microsoft Excel* and other spreadsheet software programs are popular means for expressing economic relations because they incorporate methods for manipulating and analyzing economic data. When the underlying relation between economic data is very simple, tables and spreadsheets by themselves may be sufficient for analytical purposes. In other instances, a simple graph or visual representation of the data can provide valuable insight. With spreadsheet software, creating graphs is quick and easy. When the complex nature of economic relations requires that more sophisticated methods of expression be employed, spreadsheet formulas can be used to generate equations, or analytical expressions of functional relationships, that offer a very useful means for characterizing the connection among economic variables. Equations are frequently used to express both simple and complex economic relations. When the underlying relation among economic variables is uncomplicated, equations offer a useful compact means for data description. When underlying relations are complex, equations are helpful because they permit the powerful tools of mathematical and statistical analysis to be employed.

Q2.3 *For those 50 or older, membership in AARP, formerly known as the American Association of Retired Persons, brings numerous discounts for health insurance, hotels, auto rentals, shopping, travel planning, etc. Use the marginal profit concept to explain why vendors seek out bargain-priced business with AARP members.*

Q2.3 **ANSWER**

The rise (or fall) in total profit associated with a one-unit increase in output is marginal profit. The marginal profit concept is critical in managerial economics because the optimization process requires an analysis of change in one or more important economic variables. A total profit function, for example, will be maximized when marginal profits equal zero, $M\pi = 0$, so long as total profit is falling as output expands beyond that point.

Vendors seek out bargain-priced business with AARP members so long as marginal revenue exceeds marginal cost, and marginal profit is positive. Particularly when fixed costs are high and marginal costs are very low, as they are in hotel lodging, for example, discounted or bargain-priced business with AARP members can bring a large marginal profit contribution and be very appealing to vendors.

Q2.4 *If a baseball player hits .285 during a given season, the player's lifetime batting average of .278 will rise. Use this observation to explain why the marginal cost curve always intersects the related average cost curve at either a maximum or a minimum point.*

Q2.4 **ANSWER**

The marginal observation can increase, decrease, or have no effect on the average. If the marginal is greater than the average, the average will rise. If the marginal is less than the average, the average will fall. At points where the average reaches an extreme (maxima or minima) and is neither rising nor falling, the marginal will equal the average. In terms of cost, average cost (AC) is rising if marginal cost (MC) is higher than average cost. Average cost is falling if MC < AC. AC reaches a minimum if MC = AC, and AC is rising with an increase in production. AC reaches a maximum if MC = AC, and AC is falling with an increase in production.

Q2.5 *Southwest Airlines is known for offering cut-rate promotional fares to build customer awareness, grow market share, and boost revenues in new markets. Would you expect total revenue to be maximized at an output level that is typically greater than or less than the short-run profit-maximizing output level? Is such an approach incompatible with long-run profit maximization?*

Q2.5 **ANSWER**

Marginal revenue equals zero at the output level where total revenue is maximized. On the other hand, marginal revenue equals marginal cost at the output level where total profit is maximized. Given a typically downward sloping demand curve and positive marginal costs, it is reasonable to expect that the revenue-maximizing output level where MR = 0 will be greater than the short-run profit maximizing output level where MR = MC > 0. However, it is important to recognize that there is no incompatibility between short-run revenue maximization and long-run profit maximization. Many successful firms achieve long-run profit maximization through a measured approach to building consumer awareness, market share, revenues, and long-term profits.

Total revenue is typically maximized at an output level that is typically greater than the short-run profit-maximizing output level, but such an approach can be and typically is fully compatible with long-run profit maximization.

Q2.6 *Intel Corp. designs, develops, manufactures and sells integrated circuit solutions for wireless data and personal computer (PC) applications. The company is expanding rapidly to achieve hoped-for reductions in average costs as output expands. Does the point of minimum long-run average costs always represent the optimal activity level?*

Q2.6 **ANSWER**

No, the point of minimum long-run average costs, where MC = AC, simply indicates the point of lowest average production and/or distribution costs. Determination of the optimal activity level requires that both revenue (demand) and cost (supply) conditions be considered. It would be inefficient to produce an average-cost minimizing level of output if such output could only be sold at such low prices that MR < MC. Similarly, production beyond the average-cost minimizing level of output can be justified so long as MR > MC.

Q2.7 *McDonald's restaurants do the bulk of their business at lunchtime, but have found that promotionally-priced meals at breakfast and dinner make a significant profit contribution. Does the success of McDonald's restaurants in this regard reflect an effective application of the marginal profit concept or the incremental profit concept? Explain.*

Q2.7 **ANSWER**

The success of McDonald's restaurants in offering promotionally-priced breakfast and dinner items reflects an effective application of the incremental profit concept. Marginal profit refers to the increase in total profit following a single-unit increase in output. On the other hand, incremental profit refers to the increase in total profit due to a relevant managerial decision that may involve a multiple-unit expansion in output. Either profit concept can be relevant for pricing purposes, depending on output production and demand relations. McDonald's does the bulk of its business at lunchtime, and these peak revenues must be sufficient to cover relevant costs, including fixed costs related to capital expenditures. During off-peak periods, even promotionally-priced breakfast and dinner items make a big profit contribution and represent a big part of McDonald's amazing success.

Q2.8 *Economists have long argued that if you want to tax away excess profits without affecting allocative efficiency, you should use a lump-sum tax instead of an excise or sales tax. Use the concepts developed in the chapter to support this position.*

Q2.8 **ANSWER**

Lump-sum taxes only affect total fixed costs. They are invariant with respect to the activity level of the firm. Thus, lump-sum taxes will not appear in the marginal revenue, marginal cost, or marginal profit functions of the firm. They cannot affect the determination of the optimal output level in the short run, as would a sales tax, an excise tax, or any such tax tied to the level of production. Of course, to the extent that lump-sum taxes reduce profits below a risk-adjusted normal rate of return, they too have the potential to affect the level of output. In the extreme, if a lump-sum tax

reduced profits below the minimum required level, the firm's very existence could be imperiled in the long run.

Q2.9   *"It is often impossible to obtain precise information about the pattern of future revenues, costs, and interest rates. Therefore, the process of economic optimization is futile." Discuss this statement.*

Q2.9   ANSWER

A view of the process of economic optimization as futile, given the obvious uncertainty regarding the future pattern of economic activity, is plainly incorrect. Economic decisions concerning investment projects, for example, are made on the basis of expected rather than actual values. Decision making based upon expectations is necessary because it is impossible to learn future values before the fact. Importantly, the costs of information gathering, a key element in the process of forming accurate expectations, is explicitly incorporated into the optimization process through the impact of search on marginal costs. Far from futile, the process of economic optimization provides a practical guide for understanding the basis for managerial decision making.

Q2.10   *In estimating regulatory benefits, the Environmental Protection Agency (EPA) and other government agencies typically assign a value of approximately $6 million to each life saved. What factors might the EPA consider in arriving at such a valuation? How would you respond to criticism directed at the EPA that life is precious and cannot be valued in dollar terms?*

Q2.10   ANSWER

From an economic standpoint, the effectiveness of regulatory policy can and should be measured in terms of resulting costs and benefits. When clean-air standards result in a reduction of smog and other pollutants, important benefits are experienced in terms of improved public health and safety. When sickness is avoided, social benefits are measured in terms of reduced health care expenses, cutbacks in the number of sick days for affected workers, and so on. Placing appropriate values on the social benefits enjoyed when the general public simply feels better is much harder to accomplish, of course.

When deaths rates fall following an improvement in clean-air standards, for example, important economic and personal benefits are realized. In light of current interest rates and employment opportunities, an EPA estimate of $6.1 million per each life saved represents the agency's present-value estimate of the dollar value derived from a typical person's gainful economic activity. In other words, a life saved is "worth" $6.1 million in terms of preserved economic activity.

To be sure, adoption of such an approach is not to deny the sanctity of human life. It is merely a practical means to ensure that government, business, and the public consider both large and small social benefits when judging the cost effectiveness of health and safety regulation.

SELF-TEST PROBLEMS AND SOLUTIONS

ST2.1 **Profit versus Revenue Maximization.** *Presto Products, Inc., recently introduced an innovative new frozen dessert maker with the following revenue and cost relations:*

$$P = \$60 - \$0.005Q \qquad\qquad TC = \$88,000 + \$5Q + \$0.0005Q^2$$

$$MR = \partial TR/\partial Q = \$60 - \$0.01Q \qquad MC = \partial TC/\partial Q = \$5 + \$0.001Q$$

   A. *Set up a spreadsheet for output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), total profit (π), and marginal profit (Mπ). Establish a range for Q from 0 to 10,000 in increments of 1,000 (i.e., 0, 1,000, 2,000, ..., 10,000).*

   B. *Use the spreadsheet to, create a graph with TR, TC, and π as dependent variables, and units of output (Q) as the independent variable. At what price/output combination is total profit maximized? At what price/output combination is total revenue maximized?*

   C. *Determine these profit-maximizing and revenue-maximizing price/output combinations analytically. In other words, use the profit and revenue equations to confirm your answers to part B.*

   D. *Compare the profit-maximizing and revenue-maximizing price/output combinations, and discuss any differences. When will short-run revenue maximization lead to long-run profit maximization?*

ST2.1 **SOLUTION**

A. A table or spreadsheet for Presto output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), total profit (π), and marginal profit (Mπ) appears as follows:

Presto Products

Units	Price	Total Revenue	Marginal Revenue	Total Cost	Marginal Cost	Total Profit	Marginal Profit
0	$60	$0	$60	$88,000	$5	-$88,000	$55

1,000	55	55,000	50	93,500	6	-38,500	44
2,000	50	100,000	40	100,000	7	0	33
3,000	45	135,000	30	107,500	8	27,500	22
4,000	40	160,000	20	116,000	9	44,000	11
5,000	35	175,000	10	125,500	10	49,500	0
6,000	30	180,000	0	136,000	11	44,000	-11
7,000	25	175,000	-10	147,500	12	27,500	-22
8,000	20	160,000	-20	160,000	13	0	-33
9,000	15	135,000	-30	173,500	14	-38,500	-44
10,000	10	100,000	-40	188,000	15	-88,000	-55

B.   The price/output combination at which total profit is maximized is P = $35 and Q = 5,000 units. At that point, MR = MC and total profit is maximized at $49,500. The price/output combination at which total revenue is maximized is P = $30 and Q = 6,000 units.   At that point, MR = 0 and total revenue is maximized at $180,000. Using the Presto table or spreadsheet, a graph with TR, TC, and π as dependent variables, and units of output (Q) as the independent variable appears as follows:

Presto Products, Inc.
Profit vs. Revenue Maximization
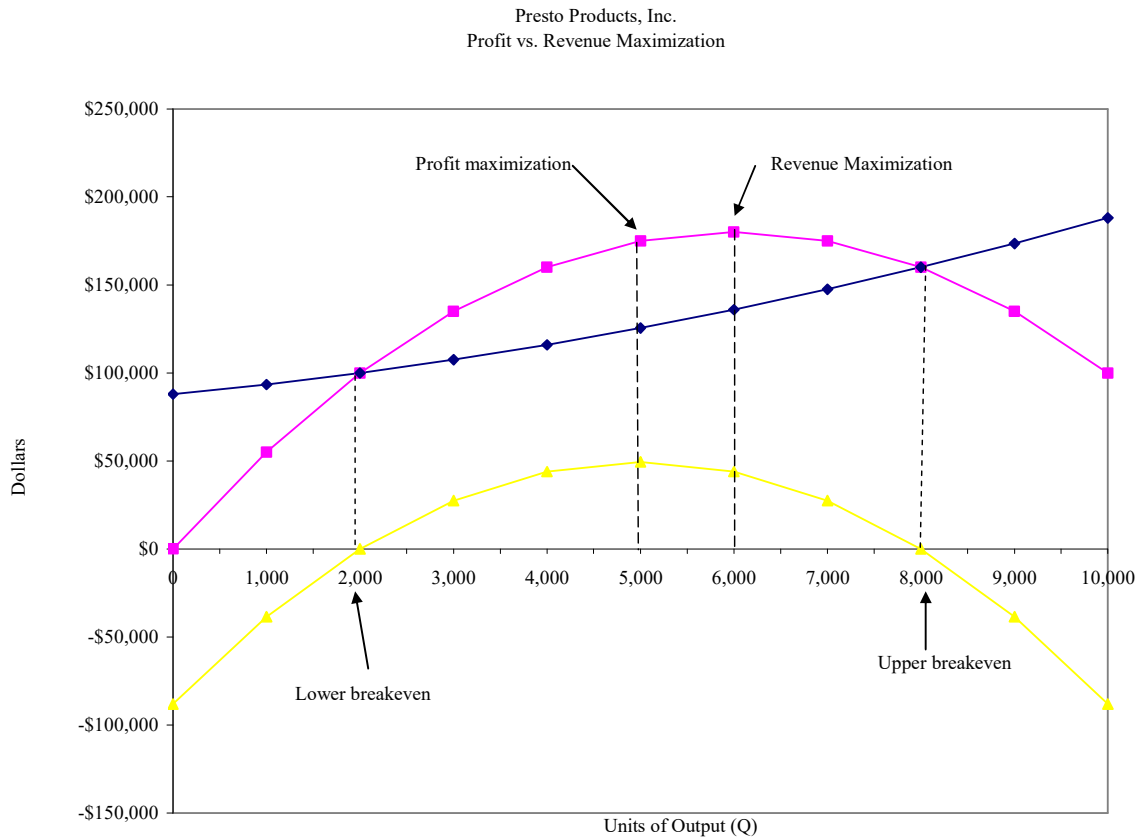
C.   To find the profit-maximizing output level analytically, set $M\pi = MR - MC = 0$ or MR = MC, and solve for Q. Because

$$MR = MC$$

$$\$60 - \$0.01Q = \$5 + \$0.001Q$$

$$0.011Q = 55$$

$$Q = 5,000$$

At Q = 5,000,

$$P = \$60 - \$0.005(5,000)$$

$$= \$35$$

$$\pi = -\$188,000 + \$55(5,000) - \$0.0055(5,000^2)$$

$$= \$49,500$$

This is a profit maximum because total profit is falling for Q > 5,000. To find the revenue-maximizing output level, set MR = 0, and solve for Q. Thus,

$$MR = \$60 - \$0.01Q = 0$$

$$0.01Q = 60$$

$$Q = 6,000$$

At Q = 6,000,

$$P = \$60 - \$0.005(6,000)$$

$$= \$30$$

$$\pi = TR - TC$$

$$= (\$60 - \$0.005Q)Q - \$88,000 - \$5Q - \$0.0005Q^2$$

$$= -\$88,000 + \$55Q - \$0.0055Q^2$$

$$= \quad \text{-}\$88,000 + \$55(6,000) - \$0.0055(6,000^2)$$

$$= \quad \$44,000$$

This is a revenue maximum because total revenue is decreasing for output beyond Q > 6,000.

D.   Given downward sloping demand and marginal revenue curves and positive marginal costs, the profit-maximizing price/output combination is *always* at a higher price and lower production level than the revenue-maximizing price-output combination. This stems from the fact that profit is maximized when MR = MC, whereas revenue is maximized when MR = 0. It follows that profits and revenue are only maximized at the same price/output combination in the unlikely event that MC = 0.

     In pursuing a short-run revenue rather than profit-maximizing strategy, Presto can expect to gain a number of important advantages, including enhanced product awareness among consumers, increased customer loyalty, potential economies of scale in marketing and promotion, and possible limitations in competitor entry and growth. To be consistent with long-run profit maximization, these advantages of short-run revenue maximization must be at least worth Presto's short-run sacrifice of $5,500 (= $49,500 - $44,000) in monthly profits.

ST2.2   ***Average Cost-Minimization.*** *Pharmed Caplets is an antibiotic product with monthly revenues and costs of:*

$$TR = \$900Q - \$0.1Q^2 \qquad\qquad TC = \$36,000 + \$200Q + \$0.4Q^2$$

$$MR = \partial TR/\partial Q = \$900 - \$0.2Q \qquad MC = \partial TC/\partial Q = \$200 + \$0.8Q$$

A.   *Set up a spreadsheet for output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), average cost (AC), total profit (π), and marginal profit (Mπ). Establish a range for Q from 0 to 1,000 in increments of 100 (i.e., 0, 100, 200, ..., 1,000).*

B.   *Using the spreadsheet to, create a graph with MR, MC, and AC as dependent variables and units of output (Q) as the independent variable.   At what price/output combination is total profit maximized?   Why?   At what price/output combination is average cost minimized?   Why?*

C.   *Determine these profit-maximizing and average-cost minimizing price/output combinations analytically.   In other words, use revenue and cost equations to confirm your answers to part B.*

> **D.** *Compare the profit-maximizing and average-cost minimizing price/output combinations, and discuss any differences.   When will average-cost minimization lead to long-run profit maximization?*

ST2.2 SOLUTION

A. A table or spreadsheet for output (Q), price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), average cost (AC), total profit (π), and marginal profit (Mπ) appears as follows:

Units	Price	Total Revenue	Marginal Revenue	Total Cost	Marginal Cost	Average Cost	Total Profit	Marginal Profit
0	$900	$0	$900	$36,000	$200	---	($36,000)	$700
100	$890	89,000	$880	$60,000	$280	600.00	29,000	600
200	$880	176,000	$860	$92,000	$360	460.00	84,000	500
300	$870	261,000	$840	$132,000	$440	440.00	129,000	400
400	$860	344,000	$820	$180,000	$520	450.00	164,000	300
500	$850	425,000	$800	$236,000	$600	472.00	189,000	200
600	$840	504,000	$780	$300,000	$680	500.00	204,000	100
700	$830	581,000	$760	$372,000	$760	531.43	209,000	0
800	$820	656,000	$740	$452,000	$840	565.00	204,000	(100)
900	$810	729,000	$720	$540,000	$920	600.00	189,000	(200)
1,000	$800	800,000	$700	$636,000	$1,000	636.00	164,000	(300)

B. The price/output combination at which total profit is maximized is P = $830 and Q = 700 units. At that point, MR = MC and total profit is maximized at $209,000. The price/output combination at which average cost is minimized is P = $870 and Q = 300 units. At that point, MC = AC = $440.

Using the spreadsheet, a graph with AC, and MC as dependent variables and units of output (Q) as the independent variable appears as follows:

Pharmed Caplets
Average Cost Minimization
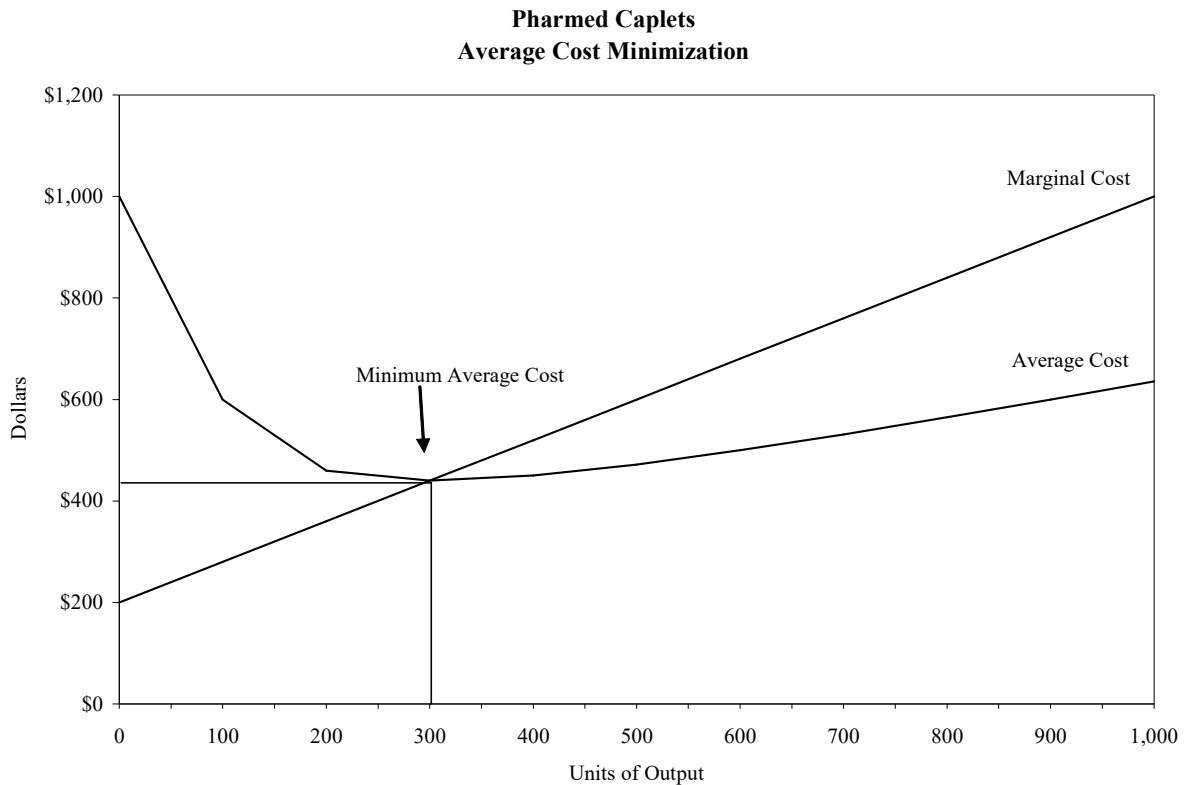
C.   To find the profit-maximizing output level analytically, set $M\pi = MR - MC = 0$ or MR = MC, and solve for Q:

$$MR = MC$$

$$\$900 - \$0.2Q = \$200 + \$0.8Q$$

$$Q = 700$$

At Q = 700,

$$P = TR/Q$$

$$= (\$900Q - \$0.1Q^2)/Q$$

$$= \$900 - \$0.1(700)$$

$$= \$830$$

$$\pi = TR - TC$$

$$= \$900Q - \$0.1Q^2 - \$36,000 - \$200Q - \$0.4Q^2$$

$$= -\$36,000 + \$700(700) - \$0.5(700^2)$$

$$= \$209,000$$

This is a profit maximum because profits are falling for $Q > 700$.

To find the average-cost minimizing output level, set MC = AC, and solve for Q:

$$AC = TC/Q$$

$$= (\$36,000 + \$200Q + \$0.4Q^2)/Q$$

$$= \$36,000Q^{-1} + \$200 + \$0.4Q,$$

It follows that:

$$MC = AC$$

$$\$200 + \$0.8Q = \$36,000Q^{-1} + \$200 + \$0.4Q$$

$$0.4Q = 36,000Q^{-1}$$

$$0.4Q^2 = 36,000$$

$$Q^2 = 36,000/0.4$$

$$Q^2 = 90,000$$

$$Q = 300$$

At Q = 300,

$$P = \$900 - \$0.1(300)$$

$$= \quad \$870$$

$$\pi \quad = \quad \text{-}\$36,\!000 + \$700(300) \text{ - } \$0.5(300^2)$$

$$= \quad \$129,\!000$$

This is an average-cost minimum because average cost is rising for Q > 300.

D.   Given downward sloping demand and marginal revenue curves and a U-shaped (or quadratic) AC function, the profit-maximizing price/output combination will *often* be at a different price and production level than the average-cost minimizing price-output combination. This stems from the fact that profit is maximized when MR = MC, whereas average cost is minimized when MC = AC. Profits are maximized at the same price/output combination as where average costs are minimized in the unlikely event that MR = MC *and* MC = AC and, therefore, MR = MC = AC.

It is often true that the profit-maximizing output level differs from the average cost-minimizing activity level. In this instance, expansion beyond Q = 300, the average cost-minimizing activity level, can be justified because the added gain in revenue more than compensates for the added costs. Note that total costs rise by $240,000, from $132,000 to $372,000 as output expands from Q = 300 to Q = 700, as average cost rises from $440 to $531.43. Nevertheless, profits rise by $80,000, from $129,000 to $209,000, because total revenue rises by $320,000, from $261,000 to $581,000. The profit-maximizing activity level can be less than, greater than, or equal to the average-cost minimizing activity level depending on the shape of relevant demand and cost relations.

PROBLEMS & SOLUTIONS

P2.1   Graphic Analysis
A.   Given the output (Q) and price (P) data in the following table, calculate total revenue (TR) and marginal revenue (MR):

Quantity	*Price*	*Total Revenue* $TR = P \times Q$	*Marginal Revenue* $MR = \partial TR / \partial Q$
0	*$10*		
1	*9*		
2	*8*		
3	*7*		
4	*6*		
5	*5*		
6	*4*		
7	*3*		
8	*2*		

9	*1*
10	*0*

B.   Graph these data using "dollars" on the vertical axis and "quantity" on the horizontal axis. At what output level is revenue maximized?

C.   Why is marginal revenue less than average revenue at each price level?

P2.1   SOLUTION

A.

Quantity	Price	Total Revenue TR=P×Q	Marginal Revenue MR=∂TR/∂Q
0	$10	0	--
1	9	$9	$9
2	8	16	7
3	7	21	5
4	6	24	3
5	5	25	1
6	4	24	-1
7	3	21	-3
8	2	16	-5
9	1	9	-7
10	0	0	-9

B.   If production of partial units is not possible, revenue is maximized at an output level of Q = 5 because MR switches from positive to negative at Q = 6. Because output is lumpy, there is no single point at which MR = 0.

Price, Total Revenue and Marginal Revenue
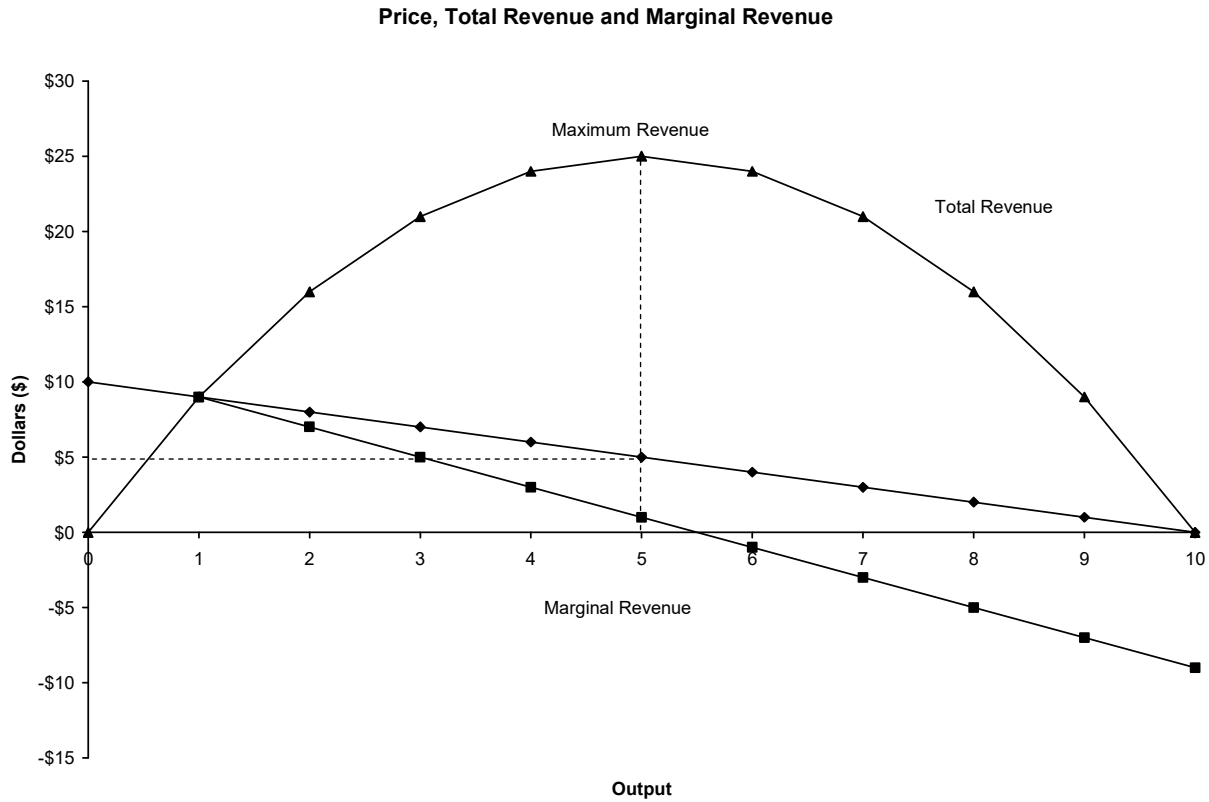
C. At every price level, price must be cut by $1 in order to increase sales by an additional unit. This means that the "benefit" of added sales from new customers is only gained at the "cost" of some loss in revenue from current customers. The net increase in revenue from added sales is always less than the change in gross revenue, and marginal revenue is always less than average revenue (or price) when the demand curve is downward sloping.

P2.2 *A. Fill in the missing data for price (P), total revenue (TR), marginal revenue (MR), total cost (TC), marginal cost (MC), profit (π), and marginal profit (Mπ) in the following table:*

Q	*P*	*TR=P×Q*	*MR=∂TR/∂Q*	*TC*	*MC=∂TC/∂Q*	*π*	*Mπ=∂π/∂Q*
0	*$160*	*$0*	*--*	*$0*	*0*	*$0*	*--*
1	*150*	*150*	*150*	*25*	*25*	*125*	*125*
2	*140*			*55*	*30*		*100*
3		*390*			*35*	*300*	*75*
4			*90*	*130*		*350*	
5	*110*	*550*		*175*			
6		*600*	*50*		*55*	*370*	
7		*630*		*290*	*60*		*-30*

8	80	640	355	285	
9				75	-85
10		600	525		

B. At what output level is profit maximized?

C. At what output level is revenue maximized?

D. Discuss any differences in your answers to parts B and C.

P2.2 **SOLUTION**

A.

Q	P	TR=P×Q	MR=∂TR/∂Q	TC	MC=∂TC/∂Q	π	Mπ=∂π/∂Q
0	$160	$0	--	$0	0	$0	--
1	150	150	150	25	25	125	125
2	140	280	130	55	30	225	100
3	130	390	110	90	35	300	75
4	120	480	90	130	40	350	50
5	110	550	70	175	45	375	25
6	100	600	50	230	55	370	-5
7	90	630	30	290	60	340	-30
8	80	640	10	355	65	285	-55
9	70	630	-10	430	75	200	-85
10	60	600	-30	525	95	75	-125

B. Profit increases as long as MR > MC and Mπ > 0. In this problem, profit is maximized at Q = 5 where π = $375 and TR = $550.

C. Total Revenue increases so long as MR > 0. In this problem, revenue is maximized at Q = 8 where TR = $640 and π = $285.

D. Given a downward sloping demand curve and MC > 0, as is typically the case, profits will be maximized at an output level that is less than the revenue-maximizing level. Revenue maximization requires lower prices and greater output than would be true with profit maximization. The potential long-run advantage of a revenue-maximizing strategy is that it might generate rapid market expansion and long-run benefits in terms of customer loyalty and future unit-cost reductions. The cost is, of course, measured in terms of lost profits in the short-run (here the loss is $90 in profits).

P2.3 *Marginal Analysis. Characterize each of the following statements as true or false, and explain your answer.*

A.   *If marginal revenue is less than average revenue, the demand curve will be downward sloping.*

B.   *Profits will be maximized when total revenue equals total cost.*

C.   *Given a downward-sloping demand curve and positive marginal costs, profit-maximizing firms will always sell less output at higher prices than will revenue-maximizing firms.*

D.   *Marginal cost must be falling for average cost to decline as output expands.*

E.   *Marginal profit is the difference between marginal revenue and marginal cost and will always equal zero at the profit-maximizing activity level.*

P2.3   SOLUTION

A.   True. The demand curve is the average-revenue curve. Since average revenue is falling along a downward sloping demand curve, marginal revenue is less than average revenue.

B.   False. Profits are maximized when marginal revenue equals marginal cost. Profits equal zero at the breakeven point where total revenue equals total cost.

C.   True. Profit maximization involves setting marginal revenue equal to marginal cost. Revenue maximization involves setting marginal revenue equal to zero. Given a downward-sloping demand curve and positive marginal costs, revenue-maximizing firms will charge lower prices and offer greater quantities of output than will firms that seek to maximize profits.

D.   False. Average cost will fall as output expands so long as marginal cost is simply less than average cost. If this condition is met, average cost will decline whether marginal costs are falling, rising, or constant.

E.   True. Marginal profit equals marginal revenue minus marginal cost and will equal zero at the profit-maximizing activity level.

P2.4   ***Marginal Analysis: Tables***. *Meredith Grey is a regional sales representative for Dental Laboratories, Inc., a company that sells alloys created from gold, silver, platinum, and other precious metals to several dental laboratories in Washington, Oregon, and Idaho. Grey's goal is to maximize total monthly commission income, which is figured at 8 percent of gross sales. In reviewing monthly experience over the past year, Grey found the following relations between days spent in each state and monthly sales generated:*

	Washington		Oregon		Idaho
Days	*Gross Sales*	*Days*	*Gross Sales*	*Days*	*Gross Sales*
0	*$10,000*	*0*	*$0*	*0*	*$6,250*
1	*25,000*	*1*	*8,750*	*1*	*12,500*
2	*37,500*	*2*	*16,250*	*2*	*17,500*
3	*47,500*	*3*	*22,500*	*3*	*21,250*
4	*55,000*	*4*	*26,250*	*4*	*23,750*
5	*60,000*	*5*	*28,750*	*5*	*25,000*
6	*62,500*	*6*	*30,000*	*6*	*25,000*
7	*62,500*	*7*	*31,250*	*7*	*25,000*

A.   *Construct a table showing Grey's marginal sales per day in each state.*

B.   *If administrative duties limit Grey to only ten selling days per month, how should they be spent to maximize commission income?*

C.   *Calculate Grey's maximum monthly commission income.*

P2.4   SOLUTION

A.

	Washington		Oregon		Idaho
Days	Marginal Sales	Days	Marginal Sales	Days	Marginal Sales
0	---	0	---	0	---
1	$15,000	1	$8,750	1	$6,250
2	12,500	2	7,500	2	5,000
3	10,000	3	6,250	3	3,750
4	7,500	4	3,750	4	2,500
5	5,000	5	2,500	5	1,250
6	2,500	6	1,250	6	0
7	0	7	1,250	7	0

B.   The maximum commission income is earned by allocating selling days on the basis of obtaining the largest marginal sales for each additional day of selling activity. Using the data in part A, we see that five days should be spent in Washington, three days in Oregon, and two days should be spent in Idaho.

C.   Given this time allocation, Grey's maximum commission income is

State	Sales
Washington (5 days)	$60,000
Oregon (3 days)	22,500
Idaho (2 days)	17,500
Total	$100,000
× Commission rate	× 0.08
	$ 8,000 per month

P2.5   ***Marginal Analysis: Tables***. *Climate Control Devices, Inc., estimates that sales of defective thermostats cost the firm $50 each for replacement or repair. Boone Carlyle, an independent engineering consultant, has recommended hiring quality control inspectors so that defective thermostats can be identified and corrected before shipping. The following schedule shows the expected relation between the number of quality control inspectors and the thermostat failure rate, defined in terms of the percentage of total shipments that prove to be defective.*

Number of Quality Control Inspectors	Thermostat Failure Rate (percent)
0	*5.0*
1	*4.0*
2	*3.2*
3	*2.6*
4	*2.2*
5	*2.0*

The firm expects to ship 250,000 thermostats during the coming year, and quality control inspectors each command a salary of $60,000 per year.

A.   *Construct a table showing the marginal failure reduction (in units) and the dollar value of these reductions for each inspector hired.*

B.   *How many inspectors should the firm hire?*

C. How many inspectors should be hired if additional indirect costs (lost customer goodwill and so on) were to average 30 percent of direct replacement or repair costs?

P2.5 **SOLUTION**

A.

Number of Quality Control Inspectors (col. 1)	Thermostat Failure rate (percent) (col. 2)	Number of Failures (= 250,000 × (col. 2 ÷ 100) (col. 3)	Marginal Failure Reduction (col. 4)	Marginal Value of Failure Reduction (= $50 × (col. 4) (col. 5)
0	5.0	12,500	---	---
1	4.0	10,000	2,500	$125,000
2	3.2	8,000	2,000	100,000
3	2.6	6,500	1,500	75,000
4	2.2	5,500	1,000	50,000
5	2.0	5,000	500	25,000

B. I = 3. With a $60,000 inspector salary, the firm will enjoy a net marginal return of $15,000 (= $75,000 - $60,000) from hiring a third inspector. Hiring a fourth inspector would result in a marginal loss of $10,000 (= $50,000 - $60,000).

C. I = 4. If additional indirect costs total 30 percent of direct replacement costs, the marginal value of inspectors (column 5) would rise by 30 percent. Under these circumstances, the marginal value of a fourth inspector would rise from $50,000 to $65,000 (= 1.3 × 50,000), and hiring four inspectors could be justified since doing so would increase profits by $5,000 (= $65,000 - $60,000).

P2.6 ***Price and Total Revenue.*** *The Portland Sea Dogs, the AA affiliate of the Boston Red Sox major league baseball team, have enjoyed a surge in popularity. During a recent home stand, suppose the club offered $5 off the $12 regular price of reserved seats, and sales spurted from 3,200 to 5,200 tickets per game.*

A. *Derive the function that describes the price/output relation with price expressed as a function of quantity (tickets sold). Also express tickets sold as a function of price.*

B. *Use the information derived in part A to calculate total revenues at prices in $1 increments from $5 to $15 per ticket. What is the revenue-maximizing ticket*

price? If variable costs are negligible, is this amount also the profit-maximizing ticket price?

P2.6   **SOLUTION**

A.   When a linear demand curve is written as:

$$P = a + bQ$$

a is the intercept and *b* is the slope coefficient. Because 3,200 seats were sold at a regular price of $12 per game, and 5,200 seats were sold at the discount price of $7, two points on the firm's linear demand curve are identified. Given this information, it is possible to identify the linear demand curve by solving the system of two equations with two unknowns, *a* and *b*:

$$
\begin{array}{rcl}
12 &=& a + b(3,200) \\
\text{minus } 7 &=& a + b(5,200) \\
5 &=& -2,000\,b
\end{array}
$$

$$b = -0.0025$$

By substitution, if *b* = -0.0025, then:

$$12 = a + b(3,200)$$

$$12 = a - 0.0025(3,200)$$

$$12 = a - 8$$

$$a = 20$$

With price expressed as a function of quantity, the reserved seat demand curve can be written:

$$P = \$20 - \$0.0025Q$$

Similarly, the number of tickets sold (quantity) can be expressed as a function of price:

$$P = \$20 - \$0.0025Q$$

$$0.0025Q = \$20 - P$$

$$Q = 8,000 - 400P$$

This simple linear characterization of the firm's demand curve can be used to profitably guide production, pricing and promotion decisions.

B. The Portland Sea Dogs could use the estimated linear market demand curve to estimate the quantity demanded during the same marketing period for ticket prices in the range from $5 to $15 per ticket, using $1 increments:

Price	Quantity	TR=P×Q
$5	6,000	30,000
6	5,600	33,600
7	5,200	36,400
8	4,800	38,400
9	4,400	39,600
10	4,000	40,000
11	3,600	39,600
12	3,200	38,400
13	2,800	36,400
14	2,400	33,600
15	2,000	30,000

From the table, the revenue-maximizing ticket price is $10. This is also the profit-maximizing ticket price if variable costs and, hence, marginal costs are negligible. The pricing promotion resulted in declining revenues, and the $7 price results in an activity level that is above the revenue-maximizing output. Because the marginal cost of fan attendance cannot be less than zero, the profit-maximizing price cannot be less than the revenue-maximizing price of $10.

P2.7 **Profit Maximization: Equations.** *21st Century Insurance offers mail-order automobile insurance to preferred-risk drivers in the Los Angeles area. The company is the low-cost provider of insurance in this market but doesn't believe its annual premium of $1,500 can be raised for competitive reasons. Rates are expected to remain stable during coming periods; hence, P = MR = $1,500. Total and marginal cost relations for the company are as follows:*

$$TC = \$41,000,000 + \$500Q + \$0.005Q^2$$

$$MC = \partial TC/\partial Q = \$500 + \$0.01Q$$

A.   Calculate the profit-maximizing activity level.

B.   Calculate the company's optimal profit, and optimal profit as a percentage of sales revenue (profit margin).

P2.7   **SOLUTION**

A.   Set MR = MC and solve for Q to find the profit-maximizing activity level:

$$MR = MC$$

$$\$1,500 = \$500 + \$0.01Q$$

$$0.01Q = \$1,000$$

$$Q = 100,000$$

This is a profit maximum because profits are decreasing for Q > 100,000.

B.   The total revenue function for 21st Century Insurance is:

$$TR = P \times Q = \$1,500Q$$

Then, total profit is

$$
\begin{aligned}
\pi &= TR - TC \\
&= \$1,500Q - \$41,000,000 - \$500Q - \$0.005Q^2 \\
&= 1,500(100,000) - 41,000,000 - 500(100,000) - 0.005(100,000^2) \\
&= \$9,000,000 \\
TR &= \$1,500(100,000) \\
&= \$150,000,000 \text{ or } \$150 \text{ million}
\end{aligned}
$$

Profit Margin   = π/TR

$$= \$9{,}000{,}000/\$150{,}000{,}000$$

$$= 0.06 \text{ or } 6 \text{ percent}$$

P2.8   ***Not-for-Profit Analysis****. The Denver Athlete's Club (DAC) is a private, not-for-profit athletic club located in Denver, Colorado. DAC currently has 3,500 members but is planning on a membership drive to increase this number significantly. An important issue facing John Blutarsky, DAC's administrative director, is the determination of an appropriate membership level. In order to efficiently employ scarce DAC resources, the board of directors has instructed Blutarsky to maximize DAC's operating surplus, defined as revenues minus operating costs. They have also asked Blutarsky to determine the effects of a proposed agreement between DAC and a neighboring club with outdoor recreation and swimming pool facilities. Plan A involves paying the neighboring club $100 per DAC member. Plan B involves payment of a fixed fee of $400,000 per year. Finally, the board has determined that the basic membership fee for the coming year will remain constant at $2,500 per member irrespective of the number of new members added and whether plan A or plan B is adopted.*
      In the calculations for determining an optimal membership level, Blutarsky regards price as fixed; therefore, P = MR = $2,500. Before considering the effects of any agreement with the neighboring club, Blutarsky projects total and marginal cost relations during the coming year to be as follows:

$$TC = \$3{,}500{,}000 + \$500Q + \$0.25Q^2$$

$$MC = \partial TC/\partial Q = \$500 + \$0.5Q$$

where Q is the number of DAC members.

A.   *Before considering the effects of the proposed agreement with the neighboring club, calculate DAC's optimal membership and operating surplus levels.*

B.   *Calculate these levels under plan A.*

C.   *Calculate these levels under plan B.*

P2.8   **SOLUTION**

A.   Set MR = MC and solve for Q to find the operating surplus (profit)-maximizing activity level:

$$\begin{aligned}
MR &= MC \\
\$2{,}500 &= \$500 + \$0.5Q \\
0.5Q &= 2{,}000 \\
Q &= 4{,}000 \\
Surplus &= P \times Q - TC \\
&= \$2{,}500(4{,}000) - \$3{,}500{,}000 - \$500(4{,}000) - \$0.25(4{,}000^2) \\
&= \$500{,}000
\end{aligned}$$

This is a profit maximum because surplus is decreasing for $Q > 4{,}000$.

B.   When operating costs increase by \$100 per member, the marginal cost function and optimal activity level are both affected. Under plan A set MR = MC + \$100, and solve for Q to find the new operating surplus (profit)-maximizing activity level.

$$\begin{aligned}
MR &= MC + \$100 \\
\$2{,}500 &= \$500 + \$0.5Q + \$100 \\
0.5Q &= 1{,}900 \\
Q &= 3{,}800 \\
Surplus &= P \times Q - TC - \text{Plan A cost} \\
&= \$2{,}500(3{,}800) - \$3{,}500{,}000 - \$500(3{,}800) \\
&\quad - \$0.25(3{,}800^2) - \$100(3{,}800) \\
&= \$110{,}000
\end{aligned}$$

C.   When operating costs increase by a flat \$400,000 per year, the marginal cost function and operating surplus (profit)-maximizing activity level are unaffected. As in part A, $Q = 4{,}000$.
    The new operating surplus (profit) level is:

$$Surplus = PQ - TC - \text{Plan B cost}$$

$$= \$500,000 - \$400,000$$

$$= \$100,000$$

Here, the DAC would be slightly better off under plan A. In general, a fixed-sum increase in costs will decrease the operating surplus (profit) by a like amount, but have no influence on price and activity levels in the short-run. In the long run, however, both price and activity levels will be affected if cost increases depress the operating surplus (profit) below a normal (or required) rate of return.

P2.9 **Average Cost Minimization.** *Giant Screen TV, Inc., is a Miami-based importer and distributor of 60-inch screen HDTVs for residential and commercial customers. Revenue and cost relations are as follows:*

$$TR \;=\; \$1,800Q - \$0.006Q^2$$

$$MR \;=\; \partial TR/\partial Q = \$1,800 - \$0.012Q$$

$$TC \;=\; \$12,100,000 + \$800Q + \$0.004Q^2$$

$$MC \;=\; \partial TC/\partial Q = \$800 + \$0.008Q$$

A. *Calculate output, marginal cost, average cost, price, and profit at the average cost-minimizing activity level.*

B. *Calculate these values at the profit-maximizing activity level.*

C. *Compare and discuss your answers to parts A and B.*

P2.9 **SOLUTION**

A. To find the average cost-minimizing level of output, set MC = AC and solve for Q. Because,

$$AC \;=\; TC/Q$$

$$=\; (\$12,100,000 + \$800Q + \$0.004Q^2)/Q$$

$$=\; \$12,100,000/Q + \$800 + \$0.004Q$$

Therefore,

$$MC \quad = \quad AC$$

$$\$800 + \$0.008Q \quad = \quad \$12,100,000/Q + \$800 + \$0.004Q$$

$$0.004Q \quad = \quad 12,100,000/Q$$

$$Q^2 \quad = \quad 12,100,000/0.004$$

$$Q \quad = \quad (12,100,000/0.004)^{1/2}$$

$$= \quad 55,000$$

And,

$$MC \quad = \quad \$800 + \$0.008(55,000)$$

$$= \quad \$1,240$$

$$AC \quad = \quad \$12,100,000/(55,000) + \$800 + \$0.004(55,000)$$

$$= \quad \$1,240$$

$$P \quad = \quad TR/Q$$

$$= \quad (\$1,800Q - \$0.006Q^2)/Q$$

$$= \quad \$1,800 - \$0.006Q$$

$$= \quad \$1,800 - \$0.006(55,000)$$

$$= \quad \$1,470$$

$$\pi \quad = \quad P \times Q - TC$$

$$= \quad \$1,470(55,000) - \$12,100,000 - \$800(55,000) - \$0.004(55,000^2)$$

$$= \quad \$12,650,000$$

This is an average-cost minimum because average cost is rising for $Q > 55,000$.

B.   To find the profit-maximizing level of output, set MR = MC and solve for Q (this is also where $M\pi = 0$):

$$MR = MC$$

$$\$1,800 - \$0.012Q = \$800 + \$0.008Q$$

$$0.02Q = 1,000$$

$$Q = 50,000$$

And

$$MC = \$800 + \$0.008(50,000)$$

$$= \$1,200$$

$$AC = \$12,100,000/(50,000) + \$800 + \$0.004(50,000)$$

$$= \$1,242$$

$$P = \$1,800 - \$0.006(50,000)$$

$$= \$1,500$$

$$\pi = TR - TC$$

$$= \$1,800Q - \$0.006Q^2 - \$12,100,000 - \$800Q - \$0.004Q^2$$

$$= -\$0.01Q^2 + \$1,000Q - \$12,100,000$$

$$= -\$0.01(50,000^2) + \$1,000(50,000) - \$12,100,000$$

$$= \$12,900,000$$

This is a profit maximum because profit is falling for Q > 50,000.

C. Average cost is minimized when MC = AC = $1,240. Given P = $1,470, a $230 profit per unit of output is earned when Q = 55,000. Total profit π = $12.65 million.
  Profit is maximized when Q = 50,000 since MR = MC = $1,200 at that activity level. Since MC = $1,200 < AC = $1,242, average cost is falling. Given P = $1,500 and AC = $1,242, a $258 profit per unit of output is earned when Q = 50,000. Total profit π = $12.9 million.

Total profit is higher at the Q = 50,000 activity level because the modest $2 (= $1,242 - $1,240) decline in average cost is more than offset by the $30 (= $1,500 - $1,470) price cut necessary to expand sales from Q = 50,000 to Q = 55,000 units.

P2.10   ***Incremental Analysis.***   *Founded in 1985, Starbucks Corporation offers brewed coffees, espresso beverages, cold blended beverages, various complementary food items, and related products at over 12,000 retail outlets in the United States Canada, the United Kingdom, Thailand, Australia, Germany, China, Singapore, Puerto Rico, Chile, and Ireland. Over 100 outlets are featured in the Greater Chicago Land area alone. For a new unit in Chicago's O'Hare Airport, suppose beverage customers spend an average $4 on beverages with an 80 percent gross margin, and food customers spend an average $5 on sandwiches and salads with a 50 percent gross margin. In both cases, gross margin is simply price minus input cost and does not reflect variable labor and related expenses. Customer traffic throughout the day is as follows:*

Hour of day	Beverage Customers	Food Customers	Profit Contribution
6:00	150	50	$605.00
7:00	250	100	1,050.00
8:00	200	75	827.50
9:00	175	50	685.00
10:00	100	25	382.50
11:00	200	75	827.50
12:00	200	175	1,077.50
13:00	125	150	775.00
14:00	75	75	427.50
15:00	50	50	285.00
16:00	100	25	382.50
17:00	75	50	365.00
18:00	50	75	347.50
19:00	50	25	222.50
20:00	25	25	142.50
21:00	25	10	105.00
22:00	25	10	105.00
Totals	1,875	1,045	$8,612.50

A.   *Assume labor, electricity, and other incremental costs are $175 per hour of operation; calculate the profit-maximizing hours of operation per day.*

B.   *Assume the store is open 365 days per year, and that incremental rental costs are $2 million per year.   Calculate optimal incremental profits. Should Starbucks close this site?*

P2.10   SOLUTION

A.   Incremental profit is the profit gain or loss associated with a given managerial decision. Total profit increases so long as incremental profit is positive. When incremental profit is negative, total profit declines. Similarly, incremental profit is positive (and total profit increases) if the incremental revenue associated with a decision exceeds the incremental cost. Because incremental costs are $175 per hour, the optimal hours of operation for this Chicago O'Hare Starbucks outlet are from 6:00-19:00 hours per day (or from 6:00 am to 7:00 pm). Up until 7:00 pm, incremental revenues per hour exceed incremental costs and incremental profits are positive. After 7:00 pm, incremental revenues per hour fall short of incremental costs and incremental profits are negative.

B.   Because the optimal hours of operation for this Chicago O'Hare Starbucks outlet are from 6:00-19:00 hours per day (or from 6:00 am to 7:00 pm), the calculation of optimal incremental profits is based on the following data:

Hour of day	Beverage Customers	Food Customers	Gross Margin	Incremental Costs/hour	Incremental Profits/hour
6:00	150	50	$605.00	175.00	$430.00
7:00	250	100	1,050.00	175.00	875.00
8:00	200	75	827.50	175.00	652.50
9:00	175	50	685.00	175.00	510.00
10:00	100	25	382.50	175.00	207.50
11:00	200	75	827.50	175.00	652.50
12:00	200	175	1,077.50	175.00	902.50
13:00	125	150	775.00	175.00	600.00
14:00	75	75	427.50	175.00	252.50
15:00	50	50	285.00	175.00	110.00
16:00	100	25	382.50	175.00	207.50
17:00	75	50	365.00	175.00	190.00
18:00	50	75	347.50	175.00	172.50
19:00	50	25	222.50	175.00	47.50
Totals	1,800	1,000	$8,260.00	$2,450.00	$5,810.00

Gross margin is $8,260 after ingredient costs but before incremental labor and related expenses of $2,450 per day. This means that incremental profits are $5,810 per day. With 365 days per year, incremental profits before rental expenses are $2,120,650 (= $5,810 × 365) per year. After rental expenses, optimal incremental profits for the unit are $120,650 (= $2,120,650 - $2,000,000). The site has the potential to be profitable with optimal operation between the hours of 6:00 am to 7:00 pm and should not be closed.

CASE STUDY FOR CHAPTER 2

Spreadsheet Analysis of the EOQ at the Neighborhood Pharmacy, Inc.

A spreadsheet is a table of data organized in a logical framework similar to an accounting income statement or balance sheet. At first, this marriage of computers and accounting information might seem like a minor innovation. However, it is not. For example, with computerized spreadsheets it becomes possible to easily reflect the effects on revenue, cost, and profit of a slight change in demand conditions. Similarly, the effects on the profit-maximizing or breakeven activity levels can be easily determined. Various "what if?" scenarios can also be tested to determine the optimal or profit-maximizing activity level under a wide variety of operating conditions. Thus, it becomes easy to quantify in dollar terms the pluses and minuses (revenues and costs) of alternate decisions. Each operating and planning decision can be easily evaluated in light of available alternatives. Through the use of spreadsheet formulas and so-called "macros," managers are able to locate maximum or minimum values for any objective function based on the relevant marginal relations. Therefore, spreadsheets are a very useful tool that can be employed to analyze a variety of typical optimization problems.

To illustrate the use of spreadsheets in economic analysis, consider the hypothetical case of The Neighborhood Pharmacy, Inc. (NPI), a small but rapidly growing operator of a number of large-scale discount pharmacies in the greater Boston, Massachusetts, metropolitan area. A key contributor to the overall success of the company is a system of tight controls over inventory acquisition and carrying costs. The company's total annual costs for acquisition and inventory of pharmaceutical items are composed of the purchase cost of individual products supplied by wholesalers (purchase costs); the clerical, transportation, and other costs associated with placing each individual order (order costs); and the interest, insurance, and other expenses involved with carrying inventory (carrying costs). The company's total inventory-related costs are given by the expression:

$$TC = P \times X + \Theta \times X/Q + C \times Q/2$$

where TC is inventory-related total costs during the planning period, P is the purchase price of the inventory item, X is the total quantity of the inventory item that is to be ordered (used) during the planning period (use requirement), Θ is the cost of placing an individual order for the inventory item (order cost), C is inventory carrying costs expressed on a per unit of inventory basis (carrying cost), and Q is the quantity of inventory ordered at any one point in time (order quantity). Here Q is NPI's decision variable, whereas each other variable contained in the total cost function is beyond control of the firm (exogenous). In analyzing this total cost relation, NPI is concerned with picking the order quantity that will minimize total inventory-related costs. The optimal or total cost minimizing order quantity is typically referred to as the "economic order quantity."

During the relevant planning period, the per unit purchase cost for an important prescribed (ethical) drug is P = \$4, the total estimated use for the planning period is X = 5,000, the cost of placing an order is Θ = \$50; and the per unit carrying cost is C = \$0.50, calculated as the current interest rate of 12.5 percent multiplied by the per unit purchase cost of the item.

A. Set up a table or spreadsheet for NPI's order quantity (Q), inventory-related total cost (TC), purchase price (P), use requirement (X), order cost (Θ), and carrying cost (C). Establish a range for Q from 0 to 2,000 in increments of 100 (i.e., 0, 100, 200, ..., 2,000).

B. Based on the NPI table or spreadsheet, determine the order quantity that will minimize the company's inventory-related total costs during the planning period.

C. Placing inventory-related total costs, TC, on the vertical or y-axis and the order quantity, Q, on the horizontal or x-axis, plot the relation between inventory-related total costs and the order quantity.

CASE STUDY SOLUTION

A. The table or spreadsheet for NPI's order quantity (Q), inventory-related total cost (TC), purchase price (P), use requirement (X), order cost (Θ), and carrying cost (C) appears as follows:

Quantity (Q)	Total Cost (TC)	Price (P)	Use Requirement (X)	Order Cost θ	Carrying Cost (C)
0		4	5,000	$50	$0.50
100	$22,525	4	5,000	50	0.50
200	21,300	4	5,000	50	0.50
300	20,908	4	5,000	50	0.50
400	20,725	4	5,000	50	0.50
500	20,625	4	5,000	50	0.50
600	20,567	4	5,000	50	0.50
700	20,532	4	5,000	50	0.50
800	20,513	4	5,000	50	0.50
900	20,503	4	5,000	50	0.50
1,000	20,500	4	5,000	50	0.50
1,100	20,502	4	5,000	50	0.50
1,200	20,508	4	5,000	50	0.50
1,300	20,517	4	5,000	50	0.50
1,400	20,529	4	5,000	50	0.50
1,500	20,542	4	5,000	50	0.50
1,600	20,556	4	5,000	50	0.50
1,700	20,572	4	5,000	50	0.50
1,800	20,589	4	5,000	50	0.50
1,900	20,607	4	5,000	50	0.50
2,000	20,625	4	5,000	50	0.50

B.   Based on the NPI spreadsheet, the order quantity that will minimize the company's inventory-related order costs during the planning period is Q = 1,000, the total cost-minimizing order level.

C.   Using inventory-related total costs, TC, on the vertical *Y* axis, and the order quantity, Q, on the horizontal *X* axis, a plot of the relation between inventory-related total costs and the order quantity appears as follows:

Neighborhood Pharmacy EOQ
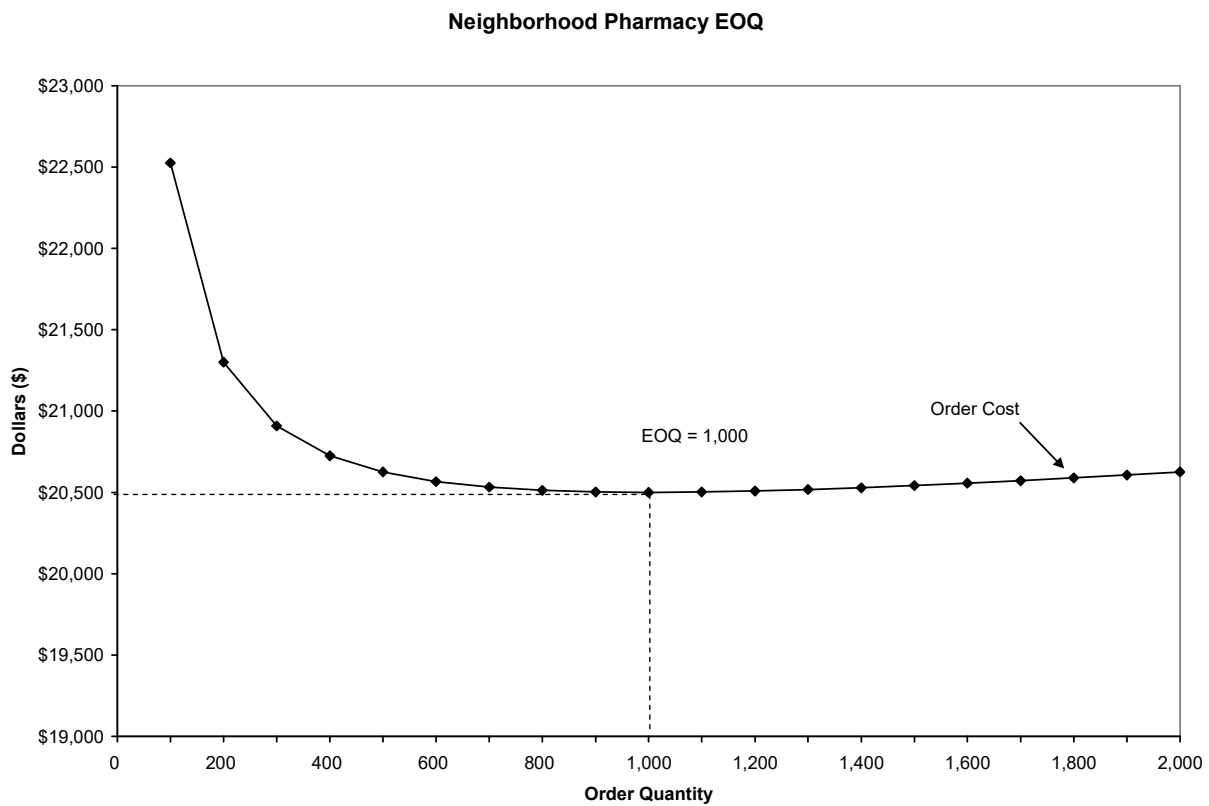
Appendix 2B

MULTIVARIATE OPTIMIZATION AND THE LAGRANGIAN TECHNIQUE

PROBLEM AND SOLUTION

2B.1   ***Lagrangian Multipliers.*** *Amos Jones and Andrew Brown own and operate Amos & Andy, Inc., a Minneapolis-based installer of conversion packages for vans manufactured by the major auto companies. Amos & Andy has fixed capital and labor expenses of $1.2 million per year, and variable materials expenses average $2,000 per van conversion. Recent operating experience suggests the following annual demand relation for Amos & Andy products:*

$$Q = 1,000 - 0.1P$$

where Q is the number of van conversions (output) and P is price.

A.   *Calculate Amos & Andy's profit-maximizing output, price, and profit levels.*

B.   *Using the Lagrangian multiplier method, calculate profit-maximizing output, price, and profit levels in light of a parts shortage that limits Amos & Andy's output to 300 conversions during the coming year.*

C.   *Calculate and interpret λ, the Lagrangian multiplier.*

D.   *Calculate the value to Amos & Andy of having the parts shortage eliminated.*

P2B.1   SOLUTION

A.   Since Q = 1,000 - 0.1P,

$$P = \$10,000 - \$10Q$$

$$TR = PQ$$

$$= \$10,000Q - \$10Q^2$$

Furthermore, given fixed expenses of $1,200,000 per year and average variable costs of $2,000 per unit, the relevant total cost function for the coming year is:

$$TC = \$1,200,000 + \$2,000Q$$

The Amos & Andy profit function is

$$\pi \quad = \quad TR - TC$$

$$= \quad \$10,000Q - \$10Q^2 - \$1,200,000 - \$2,000Q$$

$$= \quad -\$10Q^2 + \$8,000Q - \$1,200,000$$

$$d\pi/dQ \quad = \quad -20Q + 8,000 = 0$$

$$20Q \quad = \quad 8,000$$

$$Q \quad = \quad 400$$

$$P \quad = \quad \$10,000 - \$10Q$$

$$= \quad \$10,000 - \$10(400)$$

$$= \quad \$6,000$$

$$\pi \quad = \quad -\$10(400^2) + \$8,000(400) - \$1,200,000$$

$$= \quad \$400,000$$

(*Note*: $d^2\pi/dQ^2 = -20 < 0$, and $Q = 400$ is a profit maximum.)

B.   With Amos & Andy output limited to $Q = 300$, the constraint $0 = 300 - Q$ becomes active. Amos & Andys' constrained optimization problem can then be written

$$L_\pi \quad = \quad -\$10Q^2 + \$8,000Q - \$1,200,000 + \lambda(300 - Q)$$

where

(1) $\partial L_\pi / \partial Q = -20Q + 8,000 - \lambda = 0$

(2) $\partial L_\pi / \partial \lambda = 300 - Q = 0$

Multiplying (2) by 20 and subtracting from (1) provides:

(1)	$-20Q + 8,000 - \lambda$	$= 0$
minus 20 x (2)	$\underline{-20Q + 6,000}$	$\underline{= 0}$
	$2,000 - \lambda$	$= 0$

$$\lambda \ = \ 2{,}000$$

Then substituting $\lambda = 2{,}000$ into (1) yields:

$$-20Q + 8{,}000 - 2{,}000 \ = \ 0$$

$$20Q \ = \ 6{,}000$$

$$Q \ = \ 300$$

$$P \ = \ \$10{,}000 - \$10Q$$

$$= \ \$10{,}000 - \$10(300)$$

$$= \ \$7{,}000$$

$$\pi \ = \ -\$10Q^2 + \$8{,}000Q - \$1{,}200{,}000$$

$$= \ -\$10(300^2) + \$8{,}000(300) - \$1{,}200{,}000$$

$$= \ \$300{,}000$$

C.   From part B, note that $\lambda = \partial\pi/\partial Q = 2{,}000$, which simply means that profits would increase by \$2,000 if output were to expand by one unit.

D.   With no vehicle shortage (part A) Amos & Andy earned \$400,000, but only \$300,000 with a shortage (part B). Thus, having the vehicle shortage eliminated has a maximum value of \$100,000 (= \$400,000 - \$300,000) to Amos & Andy.

Case 2   EOQ Estimation for the Neighborhood Pharmacy

Quantity (Q)	Total Cost (TC)	Price (P)	Use Requirement	Order Cost θ	Carrying Cost (C)
0		4	5,000	$50	$0.50
100	$22,525	4	5,000	50	0.50
200	21,300	4	5,000	50	0.50
300	20,908	4	5,000	50	0.50
400	20,725	4	5,000	50	0.50
500	20,625	4	5,000	50	0.50
600	20,567	4	5,000	50	0.50
700	20,532	4	5,000	50	0.50
800	20,513	4	5,000	50	0.50
900	20,503	4	5,000	50	0.50
1,000	20,500	4	5,000	50	0.50
1,100	20,502	4	5,000	50	0.50
1,200	20,508	4	5,000	50	0.50
1,300	20,517	4	5,000	50	0.50
1,400	20,529	4	5,000	50	0.50
1,500	20,542	4	5,000	50	0.50
1,600	20,556	4	5,000	50	0.50
1,700	20,572	4	5,000	50	0.50
1,800	20,589	4	5,000	50	0.50
1,900	20,607	4	5,000	50	0.50
2,000	20,625	4	5,000	50	0.50

Neighborhood Pharmacy EOQ

EOQ = 1,000

Order Cost