SOLUTION SET

Chapter 2

WAVE NATURE OF LIGHT – THE INTERACTION OF LIGHT WITH MATERIALS

"LASER FUNDAMENTALS"

Second Edition

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1. Show how to go from the integral form to the differential form of Maxwell's four equations.

(a) $$\oint_S \bar{E}\cdot d\bar{s} \overset{\text{divergence Theorem}}{=} \int_V \bar{\nabla}\cdot \bar{E}\, dV = \int_V \bar{\nabla}\left(\frac{\bar{D}}{\epsilon_0}\right) dV$$

$$\Rightarrow \frac{1}{\epsilon_0}\int \bar{\nabla}\cdot\bar{D}\, dV = \frac{1}{\epsilon_0}\int_V \rho\, dV \Rightarrow \underline{\bar{\nabla}\cdot\bar{D} = \rho}$$

(b) $$\oint_S \bar{B}\cdot d\bar{s} = \int_V \bar{\nabla}\cdot\bar{B}\, dV = 0 \Rightarrow \underline{\bar{\nabla}\cdot\bar{B} = 0}$$

(c) $$\oint \bar{B}\cdot d\bar{l} \overset{\text{Stokes Theorem}}{=} \iint_A \bar{\nabla}\times\bar{B}\cdot d\bar{A} = \mu_0 I = \iint_A \mu_0\left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right)\cdot d\bar{A}$$

$$\Rightarrow \bar{\nabla}\times\bar{B} = \underline{\mu_0\left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right)}$$

(d) $$\oint \bar{E}\cdot d\bar{l} \overset{\text{Stokes Theorem}}{=} \iint_A \bar{\nabla}\times\bar{E}\cdot d\bar{A} = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\iint_A \bar{B}\cdot d\bar{A} = \iint_A -\frac{d\bar{B}}{dt}\cdot d\bar{A}$$

$$\Rightarrow \underline{\bar{\nabla}\times\bar{E} = -\frac{d\bar{B}}{dt}}$$

2. Calculate the electrical force of attraction between a positive and negative charge separated by a distance of 5.3×10^{-11} m (the average distance between an electron in its ground state and the nucleus). How does this force compare to the gravitational force an electron would experience at sea level?

The force is given by Coulomb's Law: $F = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$

The charge on the electron and proton is:
1.6×10^{-19} C

Hence $F = \frac{(1.6 \times 10^{-19})^2}{4\pi\,(8.85 \times 10^{-12}\,F/m)(5.3 \times 10^{-11}\,m)^2}$

$= \underline{8.21 \times 10^{-8}\ N/m}$

The gravitational force is:

$F = ma$ $\quad m = 9.1 \times 10^{-31}$ kg $\quad a = 9.8\ m/s^2$

$F = (9.1 \times 10^{-31}\,kg)(9.8\ m/s^2) = \underline{8.92 \times 10^{-30}\ N/m}$

$\therefore$ Gravitational force $\sim 10^{22}$ Time smaller

3. Show that the function $\mathbf{E} = \mathbf{E}_0 e^{-i(k_z z - \omega t)}$ is a solution to Maxwell's wave equation (eqn. 2.26), assuming that $c = 1/(\mu_0 \varepsilon_0)^{1/2}$.

$$\bar{E} = \bar{E}_0 e^{-i(k_z z - \omega t)}$$

$$\nabla^2 \bar{E} = \frac{d^2}{dz^2}\left(\bar{E}_0 e^{-i(k_z z - \omega t)}\right) = -k_z^2 \bar{E}$$

$$\mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}\left(\bar{E}_0 e^{-i(k_z z - \omega t)}\right) = -\mu_0 \epsilon_0 \omega^2 \bar{E}$$

$$= -\frac{\omega^2}{c^2} \bar{E} = -k_z^2 \bar{E}$$

Hence! $\underline{\nabla^2 \bar{E} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2}}$

4. Consider two waves, having slightly different angular frequencies ω_1 and ω_2 and wave numbers k_1 and k_2, of the form

$$E_1 = E_1^0 e^{-i(k_1 z - \omega_1 t)} \quad \text{and} \quad E_2 = E_2^0 e^{-i(k_2 z - \omega_2 t)},$$

where $E_1^0 = E_2^0$. Assume their "difference" wave numbers and frequencies can be written as

$$\Delta k = k_1 - k_2 \quad \text{and} \quad \Delta\omega = \omega_1 - \omega_2.$$

Show that the sum of these two waves leads to a slowly varying amplitude function associated with the difference frequencies and wave numbers multiplied by a term associated with the actual frequencies and wave numbers. *Hint:* Reduce the sum to the product of two cosine functions.

$$E_1 = E_1^0 e^{-i(k_1 z - \omega_1 t)} \qquad E_2 = E_2^0 e^{-i(k_2 z - \omega_2 t)}$$

$$k_1 = k + \frac{\Delta k}{2} \qquad k_2 = k - \frac{\Delta k}{2} \quad \text{where } k = \frac{k_1 + k_2}{2}$$

$$\omega_1 = \omega + \frac{\Delta\omega}{2} \qquad \omega_2 = \omega - \frac{\Delta\omega}{2} \quad \text{where } \omega = \frac{\omega_1 + \omega_2}{2}$$

$$\therefore \quad E_1 = E_1^0 e^{-i(kz - \omega t)} e^{-i\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)}$$

$$E_2 = E_2^0 e^{-i(kz - \omega t)} e^{+i\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)}$$

$$E_1^0 = E_2^0 = E_0$$

Hence;

$$E_1 + E_2 = E_0 e^{-i(kz - \omega t)} \left[e^{-\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)} + e^{+i\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)} \right]$$

$$= 2 E_0 e^{-i(kz - \omega t)} \underbrace{\cos\left[\tfrac{1}{2}(\Delta k z - \Delta\omega t)\right]}_{\text{slowly varying amplitude}}$$

5. Show that the group velocity and the phase velocity are equal when there is no dispersion and are different when dispersion is present. *Hint:* Begin with the definitions of group and phase velocity.

$$V_p = \frac{dz}{dt} = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{V_p} = \frac{\omega}{c/n} = \frac{n\omega}{c}$$

$$V_G = \frac{d\omega}{dk} = \frac{1}{dk/d\omega} = \frac{1}{\frac{n}{c} + \frac{\omega}{c}\frac{dn}{d\omega}}$$

If no dispersion Then n independent of ω

and $\frac{dn}{d\omega} = 0$

Then $V_G = \frac{c}{n} = V_p$

If dispersion Then $\frac{dn}{d\omega} \neq 0$

and $V_G \neq V_p$

6. Obtain (2.101) from (2.90) and (2.95).

$$K_z^2 = \frac{\omega^2}{c^2}\left(1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right]\right)$$

$$K_z = \frac{\omega}{c}\eta$$

substituting:

$$\left(\frac{\cancel{\omega}}{\cancel{c}}\eta\right)^2 = \frac{\cancel{\omega^2}}{\cancel{c^2}}\left(1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right]\right)$$

$$\underline{\eta^2 = 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right]}$$

7. Express (2.102) and (2.103) in terms of frequency (instead of angular frequency) and τ (instead of γ).

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi} \qquad \tau = \frac{1}{\gamma}$$

$$n^2 - \kappa^2 = 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right]$$

$$= 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{(2\pi)^2(\nu_0^2 - \nu^2)}{(2\pi)^4(\nu_0^2 - \nu^2)^2 + (2\pi)^2\frac{\gamma^2}{\tau^2}}\right]$$

$$= 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{(\nu_0^2 - \nu^2)}{(2\pi)^2(\nu_0^2 - \nu^2) + \frac{\gamma^2}{\tau^2}}\right]$$

$$2n\kappa = \frac{Ne^2}{m\epsilon_0}\left[\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right]$$

$$= \frac{Ne^2}{m\epsilon_0}\left[\frac{2\pi\nu/\tau}{(2\pi)^4(\nu_0^2 - \nu^2) + (2\pi)^2\nu^2/\tau^2}\right]$$

$$= \frac{Ne^2}{m\epsilon_0}\left[\frac{\nu/\tau}{(2\pi)^3(\nu_0^2 - \nu^2) + 2\pi\,\nu^2/\tau^2}\right]$$

$$= \frac{Ne^2}{2\pi m\epsilon_0}\left[\frac{\nu/\tau}{(2\pi)^2(\nu_0^2 - \nu^2) + \nu^2/\tau^2}\right]$$

8. For a medium (such as a gas) of low density and with a single resonance frequency, show that, if the imaginary part κ of the complex index of refraction is significantly smaller than the real part η, then for values of ω far from ω_0 the expressions for η and κ can be expressed as follows:

$$\eta = 1 + \frac{Ne^2}{2m\varepsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2}\right) \quad \text{and} \quad \kappa = \frac{Ne^2}{2m\varepsilon_0}\left(\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2}\right).$$

$$\eta^2 - \kappa^2 = 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right]$$

$$2\eta\kappa = \frac{Ne^2}{m\epsilon_0}\left[\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right]$$

For $\eta >> \kappa$ and $\omega^2 - \omega_0^2 \gg 0 \cong \omega^2$

and for gases, $\gamma << \omega$ at optical frequencies

Then

$$\eta^2 - \kappa^2 \cong \eta^2 \cong 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{1}{\omega_0^2 - \omega^2}\right]$$

$$\eta \cong \left[1 + \frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2}\right)\right]^{1/2}$$

but for gases, N is relatively low $\sim 10^{23} - 10^{24}/m^3$

$$\therefore \boxed{\eta \cong 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2}\right)}$$

Also $$2\eta\kappa \cong \frac{Ne^2}{m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2}$$

or $$\kappa \cong \frac{\frac{Ne^2}{m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2}}{2\left[1 + \underbrace{\cancel{\frac{1}{2}\frac{Ne^2}{m\epsilon_0(\omega_0^2 - \omega^2)}}}_{\text{small}}\right]}$$ using η from above

$$\therefore \boxed{\kappa \cong \frac{Ne^2}{2m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2}}$$

9. Assume that a dielectric material has a single resonance frequency at $\nu_0 = 3 \times 10^{14}$ Hz, the polarization decay time is $\tau = 2 \times 10^{-7}$ s, and the density of polarizable charges is 5×10^{26} m^{-3}. Determine the full width at half maximum (FWHM) of the absorption resonance in the material and determine the maximum numerical value of α.

$\alpha = \frac{2\omega K}{c}$ $\qquad$ α_{max} occurs at $\omega = \omega_0$

at $\omega = \omega_0$ $\quad n^2 - K^2 = 1$ and $2nK = \frac{C_1}{\gamma\omega}$ $\quad C_1 = \frac{Ne^2}{m\epsilon_0}$

$$4n^2K^2 = \frac{C_1^2}{\gamma^2\omega^2} \Rightarrow 4(1+K^2)K^2 = \frac{C_1^2}{\gamma^2\omega^2}$$

$$K^4 + K^2 = \frac{C_1^2}{4\gamma^2\omega^2} \quad \text{or} \quad K^4 + K^2 - \frac{C_1^2}{4\gamma^2\omega^2} = 0$$

$$K^2 = \frac{-1 \pm \sqrt{1 + \frac{4C_1^2}{4\gamma^2\omega^2}}}{2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{C_1^2}{\gamma^2\omega^2}}$$

$$C_1 = \frac{Ne^2}{m\epsilon_0} = \frac{5\times10^{26}(1.6\times10^{-19})^2}{9.1\times10^{-31}\ 8.85\times10^{-12}} = 1.59\times10^{30}$$

$$\gamma = \frac{1}{\tau} = \frac{1}{2\times10^{-7}} = 5\times10^6/s \qquad \omega_0 = 2\pi\nu = 2\pi\ 3\times10^{14} = 1.88\times10^{15}/s$$

$$K^2 = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{(1.59\times10^{30})^2}{(5\times10^6)^2(1.88\times10^{15})^2}}$$

$$K^2 = -\frac{1}{2} + 8.46\times10^7 \Rightarrow K = 9.20\times10^3$$

$$\alpha_{max} = \frac{2\omega K}{c} = \frac{2\ 1.88\times10^{15}\ 9.2\times10^3}{3\times10^8}$$

$$= \boxed{1.16\times10^{11}}$$

$$\Delta\omega_{FWHM} = \frac{\gamma}{2} \Rightarrow 2\Delta\omega = \gamma = 5\times10^6/s$$

10. A species of atomic weight 60 is doped into an Al_2O_3 crystal at a concentration of 0.1% by weight (cf. Section 5.3). The combined material is found to have an absorbing feature that peaks at 750 nm and a damping constant of $10^{13}\ s^{-1}$. Assume that each atom of the species contributes to the macroscopic polarization associated with that absorbing feature. Make a plot of η and κ versus wavelength in the wavelength region of the absorbing feature.

Al_2O_3 is 102 gm/mole

$\lambda = 750\ nm \quad \omega = \frac{2\pi c}{\lambda} = 2.51\times10^{15}/s$

$\gamma = 10^{13}/s$

species of atomic weight 60 is doped at 0.1% by weight or 0.001

$$60\ \frac{\cancel{gms}}{mole}\ \frac{1\ \cancel{cm^3}}{(0.001)(3.99\ \cancel{gms})}\ \frac{m^3}{10^6\ \cancel{cm^3}}\ \frac{mole}{6\times10^{23}\ atoms} = 2.51\times10^{-26}\ \frac{m^3}{atom}$$

or $3.99\times10^{25}\ atoms/m^3$

α_{max} at $\omega=\omega_0$ is $\alpha_{max} = \frac{2\omega K}{c}$

at α_{max}:

$$K^2 = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \frac{c_1^2}{\gamma^2\omega^2}} \qquad c_1 = \frac{Ne^2}{m\epsilon_0} = \frac{3.99\times10^{25}(1.6\times10^{-19})^2}{9.1\times10^{-31}\ 8.85\times10^{-12}}$$

$$= 1.27\times10^{29}$$

$$K^2 = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \left[\frac{1.27\times10^{29}}{10^{13}\ 2.51\times10^{15}}\right]^2}$$

$$= -\frac{1}{2} + \frac{1}{2}(5.16) = 2.08 \Rightarrow \boxed{K = 1.44}$$

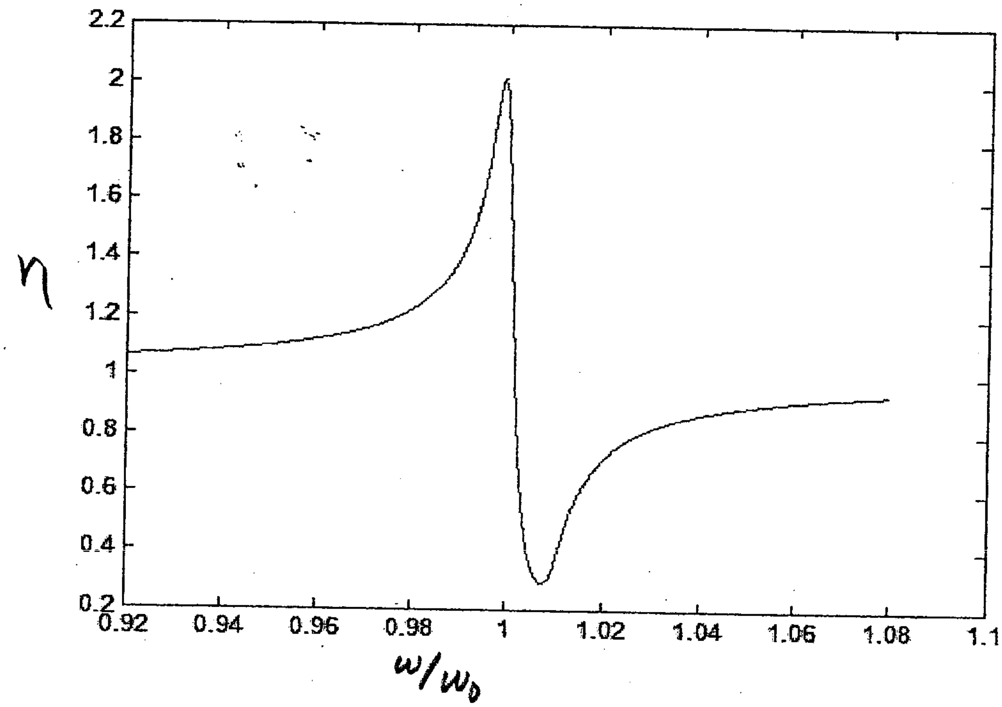
$$\alpha_{max} = \frac{2\omega K}{c} = \frac{2\ 2.51\times10^{15}\ 1.44}{3\times10^8} = \boxed{2.41\times10^7/m}$$

half max $\cong \gamma = 10^{13}/s$

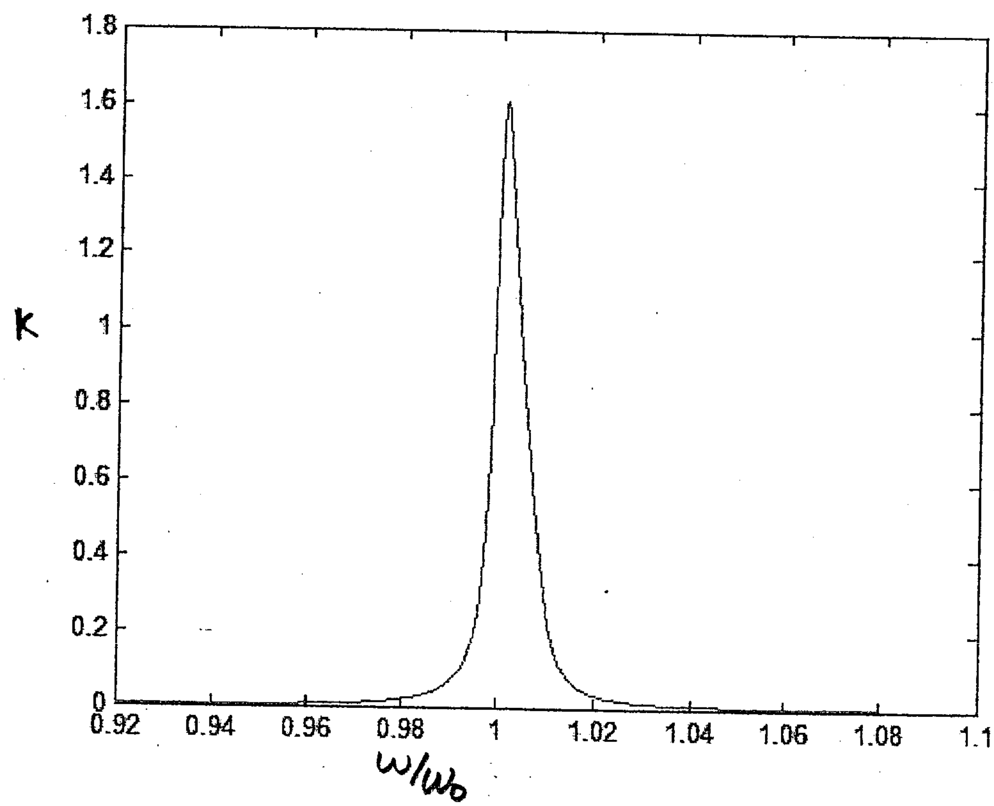
$$n\ (\omega=\omega_0) = \frac{c_1}{2\gamma\omega K} = \frac{1.27\times10^{29}}{2\ 10^{13}\ 2.51\times10^{15}\ 1.44}$$

$$\boxed{n = 1.76}$$

10 (cont)



2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
1.08
1.1
η
ω/ω0



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.92
0.94
0.96
0.98
1
1.02
1.04
1.06
1.08
1.1
k
ω/ω0

11. Show that, for a low-density medium such as a gas (as in Problem 8), the maximum and minimum values of η are located at frequencies that correspond to the half-maximum values of κ. *Hint:* Assume that in this frequency region $\gamma^2\omega^2 \gg (\omega_0^2 - \omega^2)^2$ near resonance.

$$(\eta + i\kappa)^2 = 1 + \frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$$

For a gas $N \sim 10^{23}/m^3 \Rightarrow \frac{Ne^2}{m\epsilon_0} \sim 3\times10^{26}$

and $\omega^2 \sim (2\pi\times10^{14})^2 \sim 4\times10^{29}$

Hence $1 \gg \frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$

using Taylor expansion $(1+x)^{1/2} \cong 1 + \frac{1}{2}x + \cdots$ for small x

$$\therefore \eta + i\kappa \cong 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$$

$$= 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left[\frac{1}{(\omega_0-\omega)(\omega_0+\omega) - i\gamma\omega}\right]$$

But near resonance $\omega_0 + \omega \cong 2\omega_0$ and $\gamma\omega \cong \gamma\omega_0$

$$\therefore \eta + i\kappa \cong 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\frac{1}{2\omega_0}\left[\frac{1}{(\omega_0-\omega) - i(\gamma/2)}\right]$$

$$= 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\left[\frac{\omega_0-\omega}{(\omega_0-\omega)^2 + (\frac{\gamma}{2})^2} + i\frac{\gamma/2}{(\omega_0-\omega)^2 + (\frac{\gamma}{2})^2}\right]$$

$$\therefore \eta = 1 + \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\left[\frac{\omega_0-\omega}{(\omega_0-\omega)^2 + (\gamma/2)^2}\right] \qquad \kappa = \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\left[\frac{\gamma/2}{(\omega_0-\omega)^2 + (\gamma/2)^2}\right]$$

at $\omega = \omega_0$ $\kappa_{max} = \frac{Ne^2}{2m\epsilon_0\omega_0\gamma}$

$$\frac{d\eta}{d\omega} = \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\left\{\frac{-(\omega-\omega_0)^2 - (\frac{\gamma}{2})^2 + (\omega_0-\omega)2(\omega_0-\omega)}{[(\omega_0-\omega)^2 + (\frac{\gamma}{2})^2]^2}\right\} = 0$$

$$= \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\frac{(\omega_0-\omega)^2 - (\gamma/2)^2}{(\omega_0-\omega)^2 + (\gamma/2)^2} = 0 \quad \text{when } \omega = \omega_0 \pm \gamma/2$$

at $\omega = \omega_0 \pm \gamma/2$
$$\kappa = \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\left(\frac{1}{2\omega_0}\right)\frac{\gamma/2}{2(\gamma/2)^2} = \frac{1}{2}\frac{Ne^2}{m\epsilon_0}\frac{1}{2\omega_0}\frac{1}{\gamma}$$

$$= \frac{1}{4}\frac{Ne^2}{m\epsilon_0\omega_0\gamma} = \underline{\frac{1}{2}\kappa_{max}}$$

12. In regions of the spectrum where the absorption is small, η can be expressed as

$$\eta^2 = 1 + \frac{Ne^2}{m\varepsilon_0}\left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right).$$

Convert this equation to wavelength λ instead of angular frequency ω. Show that it is of the form

$$(\eta^2 - 1)^{-1} = -C(\lambda^{-2} - \lambda_0^{-2}).$$

Use the following index data for crown glass in the expression just displayed to plot $(\eta^2 - 1)^{-1}$ versus λ^{-2} and obtain the values of C, N, and λ_0.

Wavelength (nm)	**Index of refraction**
728.135	1.5346
706.519	1.5352
667.815	1.53629
587.562	1.53954
504.774	1.54417
501.567	1.54473
492.193	1.54528
471.314	1.54624
447.148	1.54943
438.793	1.55026
414.376	1.55374
412.086	1.55402
401.619	1.55530
388.865	1.55767

$$\eta^2 = 1 + \frac{Ne^2}{m\epsilon_0}\left[\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right]$$

but in region where absorption is small

$(\omega_0^2 - \omega^2)^2 \gg \gamma^2\omega^2$ and assume $\omega > \omega_0$

Then $\eta^2 - 1 \cong -\dfrac{Ne^2}{m\epsilon_0}\left[\dfrac{1}{\omega_0^2 - \omega^2}\right]$

$$= -\frac{Ne^2}{m\epsilon_0}\left[\frac{1}{\left(\frac{2\pi c}{\lambda_0}\right)^2 - \left(\frac{2\pi c}{\lambda}\right)^2}\right]$$

$$= -\frac{Ne^2}{m\epsilon_0}\frac{1}{(2\pi c)^2}\left[\frac{1}{\lambda_0^{-2} - \lambda^{-2}}\right]$$

$$\frac{1}{\eta^2 - 1} = (\eta^2 - 1)^{-1} = -\frac{m\epsilon_0(2\pi c)^2}{Ne^2}(\lambda_0^{-2} - \lambda^{-2})$$

$$\therefore\ (\eta^2 - 1)^{-1} = -C(\lambda_0^{-2} - \lambda^{-2})$$

where $C = \dfrac{m\epsilon_0(2\pi c)^2}{Ne^2}$

Use data to obtain C, N, λ_0

$C = 7.76 \times 10^{-15}$ m

$\lambda_0 = 1.015 \times 10^{-7}$ m

$N = 1.44 \times 10^{29}$ atoms/m^3

13. Two electromagnetic waves in the blue spectral region (450 nm) are separated in frequency by 1 MHz and in wavelength by 1 pm. What is the group velocity of this wave combination?

$$V_G = \frac{\Delta\omega}{\Delta k}$$

$\omega = 2\pi\nu$ $\qquad \Delta\omega = 2\pi\Delta\nu$

$k = 2\pi/\lambda$ $\qquad \Delta k = \frac{2\pi}{\lambda^2}\Delta\lambda$

$\Delta\nu = 10^6/s$

$\Delta\lambda = 10^{-12} m$

$$\therefore V_G = \frac{2\pi\Delta\nu}{2\pi\,\Delta\lambda/\lambda^2} = \lambda^2\frac{\Delta\nu}{\Delta\lambda}$$

$$= \frac{(450\times10^{-9}m)^2\,10^6/s}{10^{-12}m} = \underline{2.03\times10^5\,m/s}$$

14. Two electromagnetic waves of nearly the same (600-nm) wavelength are measured on a spectrum analyzer to have a frequency difference of 1.0×10^8 Hz. While traveling through a dispersive medium, an interferometer is used to measure their wavelength difference of 0.0010 nm. What is the group velocity of the wave combination in that medium?

$\lambda = 600\ nm$

$\Delta\nu = 10^8\ Hz \qquad \Delta\lambda = 0.001\ nm$

$$V_G = \frac{\Delta\omega}{\Delta k} = \frac{2\pi\,\Delta\nu}{\Delta k} \qquad \Delta k = \frac{2\pi}{\lambda^2}\Delta\lambda$$

$$\therefore\ V_G = \lambda^2\frac{\Delta\nu}{\Delta\lambda} = \frac{(600\times10^{-9}m)^2\ 10^8/s}{0.001\times10^{-9}\,m}$$

$$= \boxed{3.6\times10^7\ m/s}$$

Their wavelength difference in vacuum would be

$$\Delta\lambda = \frac{\lambda^2\,\Delta\nu}{c} = \frac{(600\times10^{-9}m)^2\ 10^8/s}{3\times10^8\ m/s}$$

$$= 1.2\times10^{-13}\ m$$

15. Determine what emission frequency width would be required to have a temporal coherence length of 10 m at a source wavelength of 488 nm.

$\lambda = 488\ nm$ $\qquad$ $l_c = 10\ m$

$$l_c = \frac{\lambda^2}{\Delta\lambda}$$

$$\Delta\lambda = \frac{\lambda^2}{l_c} = \frac{(488 \times 10^{-9}\ m)^2}{10\ m} = 2.38 \times 10^{-14}\ m$$

$$\nu = \frac{c}{\lambda} \qquad \Delta\nu = \frac{c}{\lambda^2}\Delta\lambda$$

$$\Delta\nu = \frac{3 \times 10^{8}\ m/s}{(488 \times 10^{-9}\ m)^2}\ 2.38 \times 10^{-14}\ m$$

$$= 3.0 \times 10^{7}/s = \boxed{3.0 \times 10^{7}\ Hz}$$

16. If a photographic film has a minimum resolution of 10 μm, what minimum feature size could be observed at a distance of 2 m without observing coherent effects? (Assume the minimum feature size is equal to the minimum resolution.)

Spatial coherence length $l_T = \frac{r\lambda}{s} = \frac{\lambda}{\theta_s}$

assume $\lambda = 500\ \text{nm}$

$r = 2\ \text{m}$

$l_T = 10\ \mu\text{m} = 10^{-5}\ \text{m}$

Then $s = \frac{r\lambda}{l_T} = \frac{2\ \text{m}\ (500\times10^{-9}\ \text{m})}{10^{-5}\ \text{m}}$

$= 0.1\ \text{m} = \boxed{10\ \text{cm}}$