# Test Bank for Chapter 3

## Problem 3-1:

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

Formulate a linear programming model for this problem.

**Solution for Problem 3.1:**

The decision variables can be denoted and defined as follows:

$x_{P1L}$ = number of large units produced per day at Plant 1,
$x_{P1M}$ = number of medium units produced per day at Plant 1,
$x_{P1S}$ = number of small units produced per day at Plant 1,
$x_{P2L}$ = number of large units produced per day at Plant 2,
$x_{P2M}$ = number of medium units produced per day at Plant 2,
$x_{P2S}$ = number of small units produced per day at Plant 2,
$x_{P3L}$ = number of large units produced per day at Plant 3,
$x_{P3M}$ = number of medium units produced per day at Plant 3,
$x_{P3S}$ = number of small units produced per day at Plant 3.

Also letting P (or Z) denote the total net profit per day, the linear programming model for this problem is

$$\text{Maximize} \quad P = 420\,x_{P1L} + 360\,x_{P1M} + 300\,x_{P1S} + 420\,x_{P2L} + 360\,x_{P2M} + 300\,x_{P2S} + 420\,x_{P3L} + 360\,x_{P3M} + 300\,x_{P3S},$$

subject to

$$x_{P1L} + x_{P1M} + x_{P1S} \leq 750$$
$$x_{P2L} + x_{P2M} + x_{P2S} \leq 900$$
$$x_{P3L} + x_{P3M} + x_{P3S} \leq 450$$
$$20\ x_{P1L} + 15\ x_{P1M} + 12\ x_{P1S} \leq 13000$$
$$20\ x_{P2L} + 15\ x_{P2M} + 12\ x_{P2S} \leq 12000$$
$$20\ x_{P3L} + 15\ x_{P3M} + 12\ x_{P3S} \leq 5000$$
$$x_{P1L} + x_{P2L} + x_{P3L} \leq 900$$
$$x_{P1M} + x_{P2M} + x_{P3M} \leq 1200$$
$$x_{P1S} + x_{P2S} + x_{P3S} \leq 750$$
$$\frac{1}{750}(x_{P1L} + x_{P1M} + x_{P1S}) - \frac{1}{900}(x_{P2L} + x_{P2M} + x_{P2S}) = 0$$
$$\frac{1}{750}(x_{P1L} + x_{P1M} + x_{P1S}) - \frac{1}{450}(x_{P3L} + x_{P3M} + x_{P3S}) = 0$$

and

$$x_{P1L} \geq 0,\ x_{P1M} \geq 0,\ x_{P1S} \geq 0,\ x_{P2L} \geq 0,\ x_{P2M} \geq 0,\ x_{P2S} \geq 0,$$
$$x_{P3L} \geq 0,\ x_{P3M} \geq 0,\ x_{P3S} \geq 0.$$

The above set of equality constraints also can include the following constraint:

$$\frac{1}{900}\left(x_{P2L} + x_{P2M} + x_{P2S}\right) - \frac{1}{450}\left(x_{P3L} + x_{P3M} + x_{P3S}\right) = 0.$$

However, any one of the three equality constraints is redundant, so any one (say, this one) can be deleted.

## Problem 3-2:

Comfortable Hands is a company which features a product line of winter gloves for the entire family — men, women, and children. They are trying to decide what mix of these three types of gloves to produce.

Comfortable Hands' manufacturing labor force is unionized. Each full-time employee works a 40-hour week. In addition, by union contract, the number of full-time employees can never drop below 20. Nonunion, part-time workers can also be hired with the following union-imposed restrictions: (1) each part-time worker works 20 hours per week, and (2) there must be at least 2 full-time employees for each part-time employee.

All three types of gloves are made out of the same 100% genuine cowhide leather. Comfortable Hands has a long term contract with a supplier of the leather, and receives a 5,000 square feet shipment of the material each week. The material requirements and labor requirements, along with the *gross profit* per glove sold (not considering labor costs) is given in the following table.

| Glove | Material Required (square feet) | Labor Required (minutes) | Gross Profit (per pair) |
|---|---|---|---|
| Men's | 2 | 30 | \$8 |
| Women's | 1.5 | 45 | \$10 |
| Children's | 1 | 40 | \$6 |

Each full-time employee earns \$13 per hour, while each part-time employee earns \$10 per hour. Management wishes to know what mix of each of the three types of gloves to produce per week, as well as how many full-time and how many part-time workers to employ. They would like to maximize their *net profit* — their gross profit from sales minus their labor costs.

Formulate a linear programming model for this problem.

**Solution for Problem 3-2:**

The decision variables can be denoted and defined as follows:

M = number of men's gloves to produce per week,
W = number of women's gloves to produce per week,
C = number of children's gloves to produce per week,
F = number of full-time workers to employ,
PT = number of part-time workers to employ.

(Alternative notation for the decision variables is $x_M$, $x_W$, $x_C$, $x_F$, and $x_{PT}$, respectively.) Also letting P (or Z) denote the total net profit per week, the linear programming model for this problem is

$$\text{Maximize} \quad P = 8M + 10W + 6C - 13(40)F - 10(20)PT,$$

subject to

$$2M + 1.5W + C \le 5000$$
$$30M + 45W + 40C \le 40(60)F + 20(60)PT$$
$$F \ge 20$$
$$F \ge 2PT$$

and

$$M \ge 0,\ W \ge 0,\ C \ge 0,\quad F \ge 0,\quad PT \ge 0.$$

## Problem 3-3:

Slim-Down Manufacturing makes a line of nutritionally complete, weight-reduction beverages. One of their products is a strawberry shake which is designed to be a complete meal. The strawberry shake consists of several ingredients. Some information about each of these ingredients is given below.

| Ingredient | Calories from fat (per tbsp) | Total Calories (per tbsp) | Vitamin Content (mg/tbsp) | Thickeners (mg/tbsp) | Cost (¢/tbsp) |
|---|---|---|---|---|---|
| Strawberry flavoring | 1 | 50 | 20 | 3 | 10 |
| Cream | 75 | 100 | 0 | 8 | 8 |
| Vitamin supplement | 0 | 0 | 50 | 1 | 25 |
| Artificial sweetener | 0 | 120 | 0 | 2 | 15 |
| Thickening agent | 30 | 80 | 2 | 25 | 6 |

The nutritional requirements are as follows. The beverage must total between 380 and 420 calories (inclusive). No more than 20% of the total calories should come from fat. There must be at least 50 milligrams (mg) of vitamin content. For taste reasons, there must be at least two tablespoons (tbsp) of strawberry flavoring for each tbsp of artificial sweetener. Finally, to maintain proper thickness, there must be exactly 15 mg of thickeners in the beverage.

Management would like to select the quantity of each ingredient for the beverage which would minimize cost while meeting the above requirements.

Formulate a linear programming model for this problem.

**Solution for Problem 3-3:**

The decision variables can be denoted and defined as follows:

S = Tablespoons of strawberry flavoring,
CR = Tablespoons of cream,
V = Tablespoons of vitamin supplement,
A = Tablespoons of artificial sweetener,
T = Tablespoons of thickening agent.

(Alternative notation for the decision variables is $x_S$, $x_C$, $x_V$, $x_A$, and $x_T$, respectively.) Also letting C (or Z) denote cost, the linear programming model for this problem is

Minimize $C = 10\,S + 8\,CR + 25\,V + 15\,A + 6\,T$,

subject to

$$50\,S + 100\,CR + 120\,A + 80\,T \geq 380$$
$$50\,S + 100\,CR + 120\,A + 80\,T \leq 420$$
$$S + 75\,CR + 30\,T \leq 0.2\,(50\,S + 100\,CR + 120\,A + 80\,T)$$

$$20\ S + 50\ V + 2\ T \geq 50$$
$$S \geq 2A$$
$$3\ S + 8\ CR + V + 2\ A + 25\ T = 15$$

and

$$S \geq 0,\ \ CR \geq 0,\ \ V \geq 0,\ \ A \geq 0,\ \ T \geq 0.$$

## Problem 3-4:

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

**(a**) Formulate and solve a linear programming model for this problem on a spreadsheet.
**(b)** Formulate this same model algebraically.
**(c)** Use the graphical method by hand to solve this model.

**Solution for Problem 3-4:**

**(a)**

To build a spreadsheet model for this problem, start by entering the data. The data for this problem are the unit profit of each type of backpack, the resource requirements (square feet of nylon and labor hours required), the availability of each resource, 5400 square feet of nylon and (35 laborers)(40 hours/laborer) = 1400 labor hours, and the sales forecast for each type of backpack (1000 Collegiates and 1200 Minis). In order to keep the units consistent in row 8 (hours), the labor required for each backpack (in cells C8 and D8) are converted from minutes to hours (0.75 hours = 45 minutes, 0.667 hours = 40 minutes). The range names UnitProfit (C4:D4), Available (G7:G8), and SalesForecast (C13:D13) are added for these data.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | | | |
| 6 | | Resource Used per Unit Produced | | | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | | | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | | | 1400 |
| 9 | | | | | | |
| 10 | | | | | | |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | Sales Forecast | 1000 | 1200 | | | |

The decision to be made in this problem is how many of each type of backpack to make. Therefore, we add two changing cells with range name UnitsProduced (C11:D11). The values in CallsPlaced will eventually be determined by the Solver. For now, arbitrary values of 10 and 10 are entered.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | | | |
| 6 | | Resource Used per Unit Produced | | | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | | | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | | | 1400 |
| 9 | | | | | | |
| 10 | | | | | | |
| 11 | Units Produced | 10 | 10 | | | |
| 12 | | | | | | |
| 13 | Sales Forecast | 1000 | 1200 | | | |

The goal is to produce backpacks so as to achieve the highest total profit. Thus, the objective cell should calculate the total profit, where the objective will be to maximize this objective cell. In this case, the total profit will be

Total Profit = ($32)(# of Collegiates) + ($24)(# of Minis)

or

Total Cost = SUMPRODUCT(UnitProfit, UnitsProduced).

This formula is entered into cell G11 and given a range name of TotalProfit. With 10 Collegiates and 10 Minis produced, the total profit would be ($32)(10) + ($24)(10) = $560.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | | | |
| 6 | | Resource Used per Unit Produced | | | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | | | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | | | 1400 |
| 9 | | | | | | |
| 10 | | | | | | Total Profit |
| 11 | Units Produced | 10 | 10 | | | $560 |
| 12 | | | | | | |
| 13 | Sales Forecast | 1000 | 1200 | | | |

| | G |
|---|---|
| 10 | Total Profit |
| 11 | =SUMPRODUCT(UnitProfit,UnitsProduced) |

The first set of constraints in this problem involve the limited available resources (nylon and labor hours). Given the number of units produced (UnitsProduced in C11:D11), we calculate the total resources required. For nylon, this will be =SUMPRODUCT(C7:D7, UnitsProduced) in cell E7. By using a range name or an absolute reference for the units produced, this formula can be copied into cell E8 to calculate the labor hours required. The total resources used (TotalResources in E7:E8) must be <= Available (in cells G7:G8), as indicated by the <= in F7:F8.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | Total | | |
| 6 | | Resource Used per Unit Produced | | Required | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | 50 | <= | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | 14.166667 | <= | 1400 |
| 9 | | | | | | |
| 10 | | | | | | Total Profit |
| 11 | Units Produced | 10 | 10 | | | $560 |

| | E |
|---|---|
| 5 | Total |
| 6 | Required |
| 7 | =SUMPRODUCT(C7:D7,UnitsProduced) |
| 8 | =SUMPRODUCT(C8:D8,UnitsProduced) |

The final constraint is that it does not make sense to produce more backpacks than can be sold (as predicted by the sales forecast). Therefore UnitsProduced (C11:D11) should be less-than-or-equal-to the SalesForecast (C13:D13), as indicated by the <= in C12:D12

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | Total | | |
| 6 | | Resource Used per Unit Produced | | Required | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | 50 | <= | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | 14.16666667 | <= | 1400 |
| 9 | | | | | | |
| 10 | | | | | | Total Profit |
| 11 | Units Produced | 10 | 10 | | | $560 |
| 12 | | <= | <= | | | |
| 13 | Sales Forecast | 1000 | 1200 | | | |

The Solver information and solved spreadsheet are shown below.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Collegiate | Mini | | | |
| 4 | Unit Profit | $32 | $24 | | | |
| 5 | | | | Total | | |
| 6 | | Resource Used per Unit Produced | | Required | | Available |
| 7 | Nylon (sq. ft.) | 3 | 2 | 4950 | <= | 5400 |
| 8 | Labor (hours) | 0.75 | 0.667 | 1400 | <= | 1400 |
| 9 | | | | | | |
| 10 | | | | | | Total Profit |
| 11 | Units Produced | 1000 | 975 | | | $55,400 |
| 12 | | <= | <= | | | |
| 13 | Sales Forecast | 1000 | 1200 | | | |

Solver Parameters

**Set Objective Cell:** TotalProfit
**To:** Max
**By Changing Variable Cells:**
UnitsProduced
**Subject to the Constraints:**
TotalRequired <= Available
UnitsProduced <= SalesForecast

**Solver Options:**
Make Variables Nonnegative
Solving Method: Simplex LP

| Range Name | Cells |
|---|---|
| Available | G7:G8 |
| SalesForecast | C13:D13 |
| TotalProfit | G11 |
| TotalRequired | E7:E8 |
| UnitProfit | C4:D4 |
| UnitsProduced | C11:D11 |

| | E |
|---|---|
| 5 | Total |
| 6 | Required |
| 7 | =SUMPRODUCT(C7:D7,UnitsProduced) |
| 8 | =SUMPRODUCT(C8:D8,UnitsProduced) |

| | G |
|---|---|
| 10 | Total Profit |
| 11 | =SUMPRODUCT(UnitProfit,UnitsProduced) |

Thus, they should produce 1000 Collegiates and 975 Minis to achieve the maximum total profit of $55,400.

**(b)**

To build an algebraic model for this problem, start by defining the decision variables. In this case, the two decisions are how many Collegiates to produce and how many Minis to produce. These variables are defined below:

Let $C$ = Number of Collegiates to produce,
$M$ = Number of Minis to produce.

Next determine the goal of the problem. In this case, the goal is to produce the number of each type of backpack to achieve the highest possible total profit. Each Collegiate yields a unit profit of $32 while each Mini yields a unit profit of $24. The objective function is therefore

Maximize Total Profit = $\$32C + \$24M$.

The first set of constraints in this problem involve the limited resources (nylon and labor hours). Given the number of backpacks produced, $C$ and $M$, and the required nylon and labor hours for each, the total resources used can be calculated. These total resources used need to be less than or equal to the amount available. Since the labor available is in units of hours, the labor required for each backpack needs to be in units of hours (3/4 hour and 2/3 hour) rather than minutes (45 minutes and 40 minutes). These constraints are as follows:

Nylon: $3C + 2M \leq 5400$ square feet,
Labor Hours: $(3/4)C + (2/3)M \leq 1400$ hours.

The final constraint is that they should not produce more of each backpack than the sales forecast. Therefore,

Sales Forecast: $C \leq 1000$
$M \leq 1200$.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let $C$ = Number of Collegiates to produce,
$M$ = Number of Minis to produce.

Maximize Total Profit = $\$32C + \$24M$,

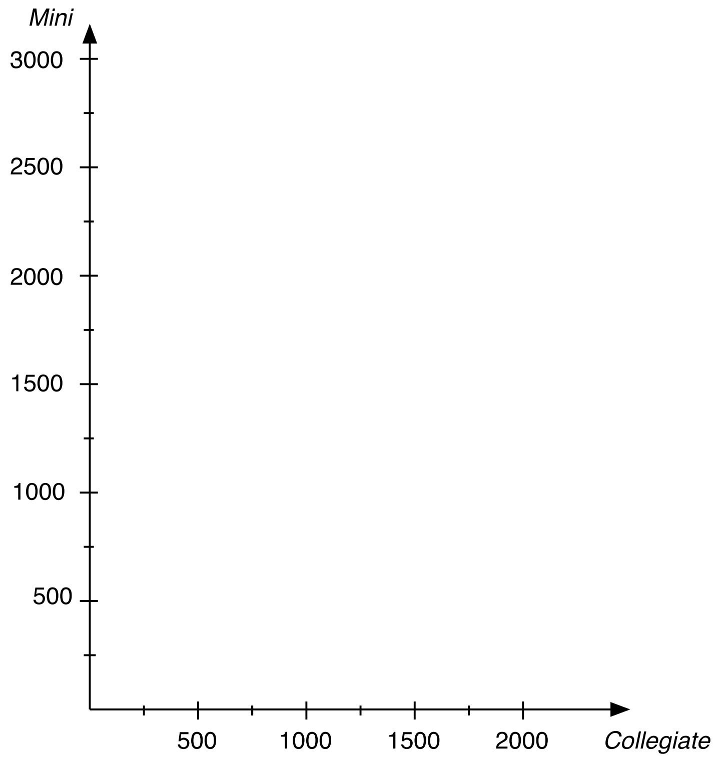
subject to

Nylon: $3C + 2M \leq 5400$ square feet,
Labor Hours: $(3/4)C + (2/3)M \leq 1400$ hours,
Sales Forecast: $C \leq 1000$
$M \leq 1200$.

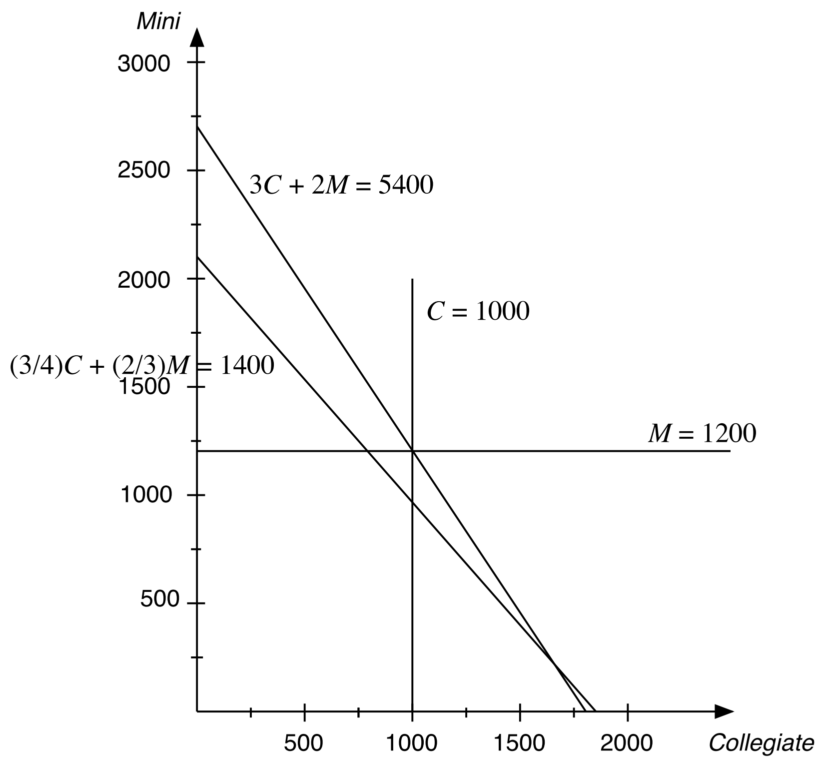
and $C \geq 0$, $M \geq 0$.

**(c)**

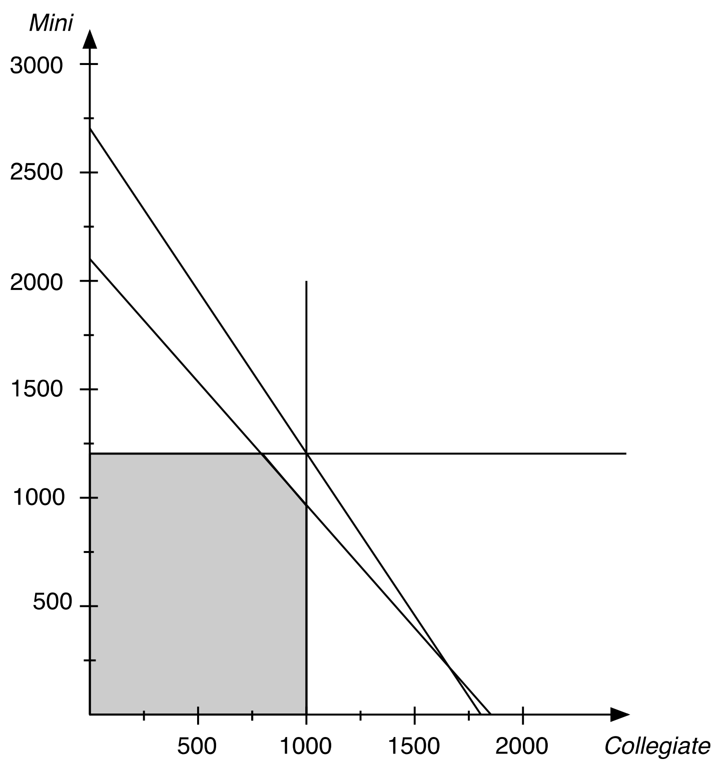
Start by plotting a graph with Collegiates (C) on the horizontal axis and Minis (M) on the vertical axis, as shown below.



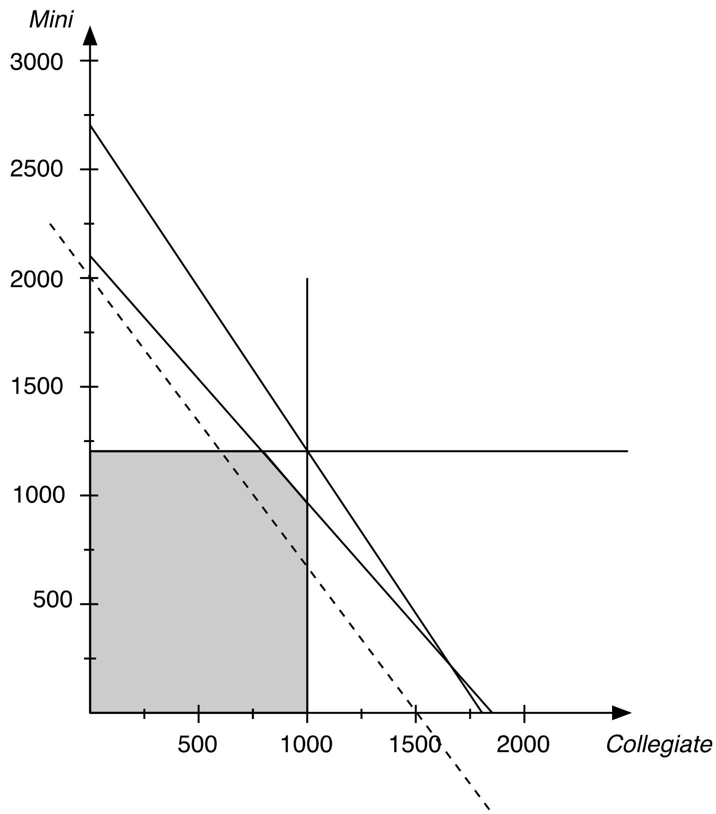

Next, the four constraint boundary lines (where the left-hand-side of the constraint exactly equals the right-hand-side) need to be plotted. The easiest way to do this is by determining where these lines intercepts the two axes. For the Nylon constraint boundary line ($3C + 2M = 5400$), setting $M = 0$ yields a C-intercept of 1800 while setting $C = 0$ yields an M-intercept of 2700. For the Labor constraint boundary line ($(3/4)C + (2/3)M = 1400$), setting $M = 0$ yields a C-intercept of 1866.67 while setting $C = 0$ yields an M-intercept of 2100. The sales forecast constraints are a horizontal line at $M = 1200$ and a vertical line at $C = 1000$. These constraint boundary lines are plotted below.




A feasible solution must be below and/or to the left of all four of these constraints while being above the Collegiate axis (since C ≥ 0) and to the right of the Mini axis (since M ≥ 0). This yields the feasible region shown below.

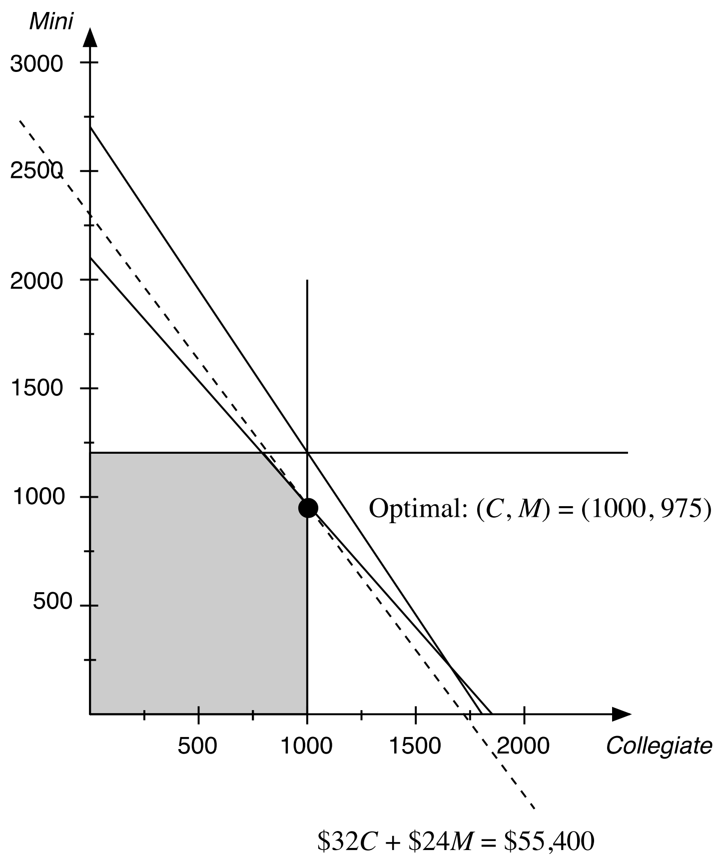


To find the optimal solution, an objective function line is plotted by setting the objective function equal to a value. For example, the objective function line when the value of the objective function is $48,000 is plotted as a dashed line below.




$\$32C + \$24M = \$48{,}000$

All objective function lines will be parallel to this one. To find the feasible solution that maximizes profit, slide this line out as far as possible while still touching the feasible region. This occurs when the profit is $55,400, and the objective function line intersect the feasible region at the single point with (C, M) = (1000, 975) as shown below.




Therefore, the optimal solution is to produce 1000 Collegiates and 975 Minis, yielding a total profit of $55,400.

## Problem 3-5:

The marketing group for a cell phone manufacturer plans to conduct a telephone survey to determine consumer attitudes toward a new cell phone that is currently under development. In order to have a sufficient sample size to conduct the analysis, they need to contact at least 100 young males (under age 40), 150 older males (over age 40), 120 young females (under age 40), and 200 older females (over age 40). It costs $1 to make a daytime phone call and $1.50 to make an evening phone call (due to higher labor costs). This cost is incurred whether or not anyone answers the phone. The table below shows the likelihood of a given customer type answering each phone call. Assume the survey is conducted with whoever first answers the phone. Also, because of limited evening staffing, at most one-third of phone calls placed can be evening phone calls. How should the marketing group conduct the telephone survey so as to meet the sample size requirements at the lowest possible cost?

| *Who Answers?* | *Daytime Calls* | *Evening Calls* |
|---|---|---|
| *Young Male* | *10%* | *20%* |
| *Older Male* | *15%* | *30%* |
| *Young Female* | *20%* | *20%* |
| *Older Female* | *35%* | *25%* |
| *No Answer* | *20%* | *5%* |

**(a**) Formulate and solve a linear programming model for this problem on a spreadsheet.
**(b)** Formulate this same model algebraically.

**Solution for Problem 3-5:**

**(a)**

To build a spreadsheet model for this problem, start by entering the data. The data for this problem are the cost of each type of phone call, the percentages of each customer type answering each type of phone call, and the total number of each customer type needed for the survey.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Daytime Call | Evening Call | | | |
| 4 | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | | | Responses |
| 6 | **Respondent** | | | | | Needed |
| 7 | Young Male | 10% | 20% | | | 100 |
| 8 | Older Male | 15% | 30% | | | 150 |
| 9 | Young Female | 20% | 20% | | | 120 |
| 10 | Older Female | 35% | 25% | | | 200 |

The decision to be made in this problem is how many of each type of phone call to make. Therefore, we add two changing cells with range name CallsPlaced (C13:D13). The values in CallsPlaced will eventually be determined by the Solver. For now, arbitrary values of 10 and 5 are entered.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Daytime Call | Evening Call | | | |
| 4 | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | | | Responses |
| 6 | **Respondent** | | | | | Needed |
| 7 | Young Male | 10% | 20% | | | 100 |
| 8 | Older Male | 15% | 30% | | | 150 |
| 9 | Young Female | 20% | 20% | | | 120 |
| 10 | Older Female | 35% | 25% | | | 200 |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | Calls Placed | 10 | 5 | | | |

The goal of the marketing group is to conduct the survey at the lowest possible cost. Thus, the objective cell should calculate the total cost, where the objective will be to minimize this objective cell. In this case, the total cost will be

Total Cost = ($1)(# of daytime calls) + ($1.50)(# of evening calls)

or

Total Cost = SUMPRODUCT(UnitCost, CallsPlaced).

This formula is entered into cell G13 and given a range name of TotalCost. With 10 daytime phone calls and 5 evening calls, the total cost would be ($1)(10) + ($1.50)(5) = $17.50.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Daytime Call | Evening Call | | | |
| 4 | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | | | Responses |
| 6 | **Respondent** | | | | | Needed |
| 7 | Young Male | 10% | 20% | | | 100 |
| 8 | Older Male | 15% | 30% | | | 150 |
| 9 | Young Female | 20% | 20% | | | 120 |
| 10 | Older Female | 35% | 25% | | | 200 |
| 11 | | | | | | |
| 12 | | | | | | Total Cost |
| 13 | Calls Placed | 10 | 5 | | | $17.50 |

| | G |
|---|---|
| 12 | Total Cost |
| 13 | =SUMPRODUCT(UnitCost,CallsPlaced) |

The first set of constraints in this problem involve the minimum responses required from each customer group. Given the number of calls placed (CallsPlaced in C13:D13), we calculate the total responses by each customer type. For young males, this will be =SUMPRODUCT(C7:D7, CallsPlaced). By using a range name or an absolute reference for the calls placed, this formula can be copied into cells E8-E10 to calculate the number of older males, young females, and older females reached. The total responses of each customer type (Total Responses in E7:E10) must be >= ResponsesNeeded (in cells G7:G10), as indicated by the >= in F7:F10.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Daytime Call | Evening Call | | | |
| 4 | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | Total | | Responses |
| 6 | **Respondent** | | | Responses | | Needed |
| 7 | Young Male | 10% | 20% | 2 | >= | 100 |
| 8 | Older Male | 15% | 30% | 3 | >= | 150 |
| 9 | Young Female | 20% | 20% | 3 | >= | 120 |
| 10 | Older Female | 35% | 25% | 4.75 | >= | 200 |
| 11 | | | | | | |
| 12 | | | | | | Total Cost |
| 13 | Calls Placed | 10 | 5 | | | $17.50 |

| | E |
|---|---|
| 5 | Total |
| 6 | Responses |
| 7 | =SUMPRODUCT(C7:D7,CallsPlaced) |
| 8 | =SUMPRODUCT(C8:D8,CallsPlaced) |
| 9 | =SUMPRODUCT(C9:D9,CallsPlaced) |
| 10 | =SUMPRODUCT(C10:D10,CallsPlaced) |

The final constraint is that at most one third of the total calls placed can be evening calls. In other words:

Evening Calls <= (1/3)(Total Calls Placed)

The two sides of this constraint (i.e., evening calls and 1/3 of total calls placed) are calculated in cells C15 and E15. Enter <= in D15 to show that C15 <= E15.

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 3 | | Daytime Call | Evening Call | | | |
| 4 | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | Total | | Responses |
| 6 | **Respondent** | | | Responses | | Needed |
| 7 | Young Male | 10% | 20% | 2 | >= | 100 |
| 8 | Older Male | 15% | 30% | 3 | >= | 150 |
| 9 | Young Female | 20% | 20% | 3 | >= | 120 |
| 10 | Older Female | 35% | 25% | 4.75 | >= | 200 |
| 11 | | | | | | |
| 12 | | | | | | Total Cost |
| 13 | Calls Placed | 10 | 5 | | | $17.50 |
| 14 | | | | | | |
| 15 | Evening Calls | 5 | <= | 5 | 33.33% | of Total Calls |

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 15 | Evening Calls | =D13 | <= | =F15*(C13+D13) | 0.333333: | of Total Calls |

The Solver information and solved spreadsheet are shown below.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **Conducting a Marketing Survey** | | | | | | |
| 2 | | | | | | | |
| 3 | | | Daytime Call | Evening Call | | | |
| 4 | | Unit Cost | $1.00 | $1.50 | | | |
| 5 | | | | | Total | | Responses |
| 6 | | **Respondent** | | | Responses | | Needed |
| 7 | | Young Male | 10% | 20% | 100 | >= | 100 |
| 8 | | Older Male | 15% | 30% | 150 | >= | 150 |
| 9 | | Young Female | 20% | 20% | 150 | >= | 120 |
| 10 | | Older Female | 35% | 25% | 237.5 | >= | 200 |
| 11 | | | | | | | |
| 12 | | | | | | | Total Cost |
| 13 | | Calls Placed | 500 | 250 | | | $875.00 |
| 14 | | | | | | | |
| 15 | | Evening Calls | 250 | <= | 250 | 33.33% | of Total Calls |

**Solver Parameters**

**Set Objective Cell:** TotalCost
**To:** Min
**By Changing Variable Cells:**
CallsPlaced
**Subject to the Constraints:**
EveningCalls <= E15
TotalResponses >= ResponsesNeeded
**Solver Options:**
Make Variables Nonnegative
Solving Method: Simplex LP

| Range Name | Cells |
|---|---|
| CallsPlaced | C13:D13 |
| EveningCalls | C15 |
| ResponsesNeeded | G7:G10 |
| TotalCost | G13 |
| TotalResponses | E7:E10 |
| UnitCost | C4:D4 |

| | E |
|---|---|
| 5 | Total |
| 6 | Responses |
| 7 | =SUMPRODUCT(C7:D7,CallsPlaced) |
| 8 | =SUMPRODUCT(C8:D8,CallsPlaced) |
| 9 | =SUMPRODUCT(C9:D9,CallsPlaced) |
| 10 | =SUMPRODUCT(C10:D10,CallsPlaced) |

| | G |
|---|---|
| 12 | Total Cost |
| 13 | =SUMPRODUCT(UnitCost,CallsPlaced) |

| | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| 15 | Evening Calls | =D13 | <= | =F15*(C13+D13) | 0.333333 | of Total Calls |

Thus, the marketing group should place 500 daytime calls and 250 evening calls at a total cost of $875.

**(b)**

To build an algebraic model for this problem, start by defining the decision variables. In this case, the two decisions are how many daytime calls and how many evening calls to place. These variables are defined below:

Let $D$ = Number of daytime calls to place
$E$ = Number of evening calls to place.

Next determine the goal of the problem. In this case, the goal is to conduct the marketing survey at the lowest possible cost. Each daytime call costs $1 while each evening call costs $1.50. The objective function is therefore

Minimize Total Cost = $\$1D + \$1.50E$.

The first set of constraints in this problem involve the minimum responses required from each customer group. Given the number of calls place, $D$ and $E$, and the percentage of calls answered by each customer group, the total responses for each customer group is calculated. These total responses need to be greater than or equal to the minimum responses required. These constraints are as follows:

| | |
|---|---|
| Young Males: | $(10\%)D + (20\%)E \geq 100$ |
| Older Males: | $(15\%)D + (30\%)E \geq 150$ |
| Young Females: | $(20\%)D + (20\%)E \geq 120$ |
| Older Females: | $(35\%)D + (25\%)E \geq 200$. |

The final constraint is that at most one third of the total calls placed can be evening calls. In other words:

Evening Calls <= (1/3)(Total Calls Placed)

Substituting $E$ for Evening Calls, and $D + E$ for Total Calls Placed yields the following constraint:

$E \leq (1/3)(D + E)$.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let $D$ = Number of daytime calls to place
$E$ = Number of evening calls to place.

Minimize Total Cost = $\$1D + \$1.50E$.

subject to

| | |
|---|---|
| Young Males: | $(10\%)D + (20\%)E \geq 100$ |
| Older Males: | $(15\%)D + (30\%)E \geq 150$ |
| Young Females: | $(20\%)D + (20\%)E \geq 120$ |
| Older Females: | $(35\%)D + (25\%)E \geq 200$ |
| Evening Call Ratio: | $E \leq (1/3)(D + E)$ |

and $D \geq 0$, $E \geq 0$.

## Problem 3-6

Dwight and Hattie have run the family farm for over thirty years. They are currently planning the mix of crops to plant on their 120-acre farm for the upcoming season. The table below gives the labor hours and fertilizer required per acre, as well as the total expected profit per acre for each of the potential crops under consideration. Dwight, Hattie, and their children can work at most 6,500 total hours during the upcoming season. They have 200 tons of fertilizer available. What mix of crops should be planted to maximize the family's total profit?

| Crop | Labor Required (hours per acre) | Fertilizer Required (tons per acre) | Expected Profit (per acre) |
|---|---|---|---|
| Oats | 50 | 1.5 | $500 |
| Wheat | 60 | 2 | $600 |
| Corn | 105 | 4 | $950 |

**(a)** Formulate and solve a linear programming model for this problem in a spreadsheet.
**(b)** Formulate this same model algebraically.

**Solution for Problem 3-6:**

**(a)**

This is a resource-allocation problem. The activities are the planting of the three crops and the limited resources are land, labor, and fertilizer. We will start to build a spreadsheet by entering the data. The data for this problem are the labor required, fertilizer required, and expected profit for each crop (per acre). The data in the spreadsheet would be entered as displayed below, where range names of ProfitPerAcre (C4:E4) and TotalAvailable (H7:H9) are assigned to the corresponding data cells.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Oats | Wheat | Corn | | | |
| 4 | Profit (per acre) | $500 | $600 | $950 | | | |
| 5 | | | | | | | Total |
| 6 | **Resources** | | Resources Used per Acre | | | | Available |
| 7 | Land (acres) | 1 | 1 | 1 | | | 120 |
| 8 | Labor (hours) | 50 | 60 | 105 | | | 6500 |
| 9 | Fertilizer (tons) | 1.5 | 2 | 4 | | | 200 |

The decisions to be made in this problem are how many acres of each crop to plant. Therefore, we add three changing cells in C12:E12 with range name AcresPlanted. The values in AcresPlanted (C12:E12) will eventually be determined by the Solver. For now, an arbitrary value of 1 is entered for each crop.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Oats | Wheat | Corn | | | |
| 4 | Profit (per acre) | $500 | $600 | $950 | | | |
| 5 | | | | | | | Total |
| 6 | **Resources** | | Resources Used per Acre | | | | Available |
| 7 | Land (acres) | 1 | 1 | 1 | | | 120 |
| 8 | Labor (hours) | 50 | 60 | 105 | | | 6500 |
| 9 | Fertilizer (tons) | 1.5 | 2 | 4 | | | 200 |
| 10 | | | | | | | |
| 11 | | | | | | | |
| 12 | Acres Planted | 1 | 1 | 1 | | | |

The goal is to maximize the family's total profit. Thus, the objective cell should calculate the total profit. In this case, the total profit will be

Total Profit = ($500)(acres of oats) + ($600)(acres of wheat) + ($950)(acres of corn)

or

Total Profit = SUMPRODUCT(ProfitPerAcre, AcresPlanted).

This formula is entered into cell H12. With 1 acre of each crop planted, the total cost would be ($500)(1) + ($600)(1) + ($950)(1) = $2,050.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Oats | Wheat | Corn | | | |
| 4 | Profit (per acre) | $500 | $600 | $950 | | | |
| 5 | | | | | | | Total |
| 6 | **Resources** | | Resources Used per Acre | | | | Available |
| 7 | Land (acres) | 1 | 1 | 1 | | | 120 |
| 8 | Labor (hours) | 50 | 60 | 105 | | | 6500 |
| 9 | Fertilizer (tons) | 1.5 | 2 | 4 | | | 200 |
| 10 | | | | | | | |
| 11 | | | | | | | Total Profit |
| 12 | Acres Planted | 1 | 1 | 1 | | | $2,050 |

| | H |
|---|---|
| 11 | Total Profit |
| 12 | =SUMPRODUCT(ProfitPerAcre,AcresPlanted) |

The functional constraints in this problem involve the limited resources of land, labor, and fertilizer. Given the AcresPlanted (the changing cells in C12:E12), we calculate the total resources used in TotalUsed (cells F7:F9). For land, this will be =SUMPRODUCT(C7:E7, AcresPlanted). Using a range name or an absolute reference for the acres planted, this formula can be copied into cells F8:F9 to calculate the amount of labor and fertilizer used. The total resources used must be <= TotalAvailable (H7:H9), as indicated by the <= in G7:G9.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Oats | Wheat | Corn | | | |
| 4 | Profit (per acre) | $500 | $600 | $950 | | | |
| 5 | | | | | Total | | Total |
| 6 | **Resources** | Resources Used per Acre | | | Used | | Available |
| 7 | Land (acres) | 1 | 1 | 1 | 3 | <= | 120 |
| 8 | Labor (hours) | 50 | 60 | 105 | 215 | <= | 6500 |
| 9 | Fertilizer (tons) | 1.5 | 2 | 4 | 8 | <= | 200 |
| 10 | | | | | | | |
| 11 | | | | | | | Total Profit |
| 12 | Acres Planted | 1 | 1 | 1 | | | $2,050 |

| | F |
|---|---|
| 5 | Total |
| 6 | Used |
| 7 | =SUMPRODUCT(C7:E7,AcresPlanted) |
| 8 | =SUMPRODUCT(C8:E8,AcresPlanted) |
| 9 | =SUMPRODUCT(C9:E9,AcresPlanted) |

The Solver information and solved spreadsheet are shown below.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **Farm Management** | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | Oats | Wheat | Corn | | | |
| 4 | | Profit (per acre) | $500 | $600 | $950 | | | |
| 5 | | | | | | Total | | Total |
| 6 | | **Resources** | Resources Used per Acre | | | Used | | Available |
| 7 | | Land (acres) | 1 | 1 | 1 | 120 | <= | 120 |
| 8 | | Labor (hours) | 50 | 60 | 105 | 6,400 | <= | 6500 |
| 9 | | Fertilizer (tons) | 1.5 | 2 | 4 | 200 | <= | 200 |
| 10 | | | | | | | | |
| 11 | | | | | | | | Total Profit |
| 12 | | Acres Planted | 80 | 40 | 0 | | | $64,000 |

| Solver Parameters |
|---|
| **Set Objective Cell:** TotalProfit |
| **To:** Max |
| **By Changing Variable Cells:** |
| AcresPlanted |
| **Subject to the Constraints:** |
| TotalUsed <= TotalAvailable |
| **Solver Options:** |
| Make Variables Nonnegative |
| Solving Method: Simplex LP |

| Range Name | Cells |
|---|---|
| AcresPlanted | C12:E12 |
| ProfitPerAcre | C4:E4 |
| TotalAvailable | H7:H9 |
| TotalProfit | H12 |
| TotalUsed | F7:F9 |

| | F |
|---|---|
| 5 | Total |
| 6 | Used |
| 7 | =SUMPRODUCT(C7:E7,AcresPlanted) |
| 8 | =SUMPRODUCT(C8:E8,AcresPlanted) |
| 9 | =SUMPRODUCT(C9:E9,AcresPlanted) |

| | H |
|---|---|
| 11 | Total Profit |
| 12 | =SUMPRODUCT(ProfitPerAcre,AcresPlanted) |

Thus, oats should be planted on 80 acres and wheat on 40 acres, while not planting any corn, with a resulting total profit of $64,000.

**(b)**

To build an algebraic model for this problem, start by defining the decision variables. In this case, the three decisions are how many acres of oats, wheat, and corn to plant. These variables are defined below:

Let $O$ = Acres of oats planted,
$W$ = Acres of wheat planted,
$C$ = Acres of corn planted.

Next determine the goal of the problem. In this case, the goal is to achieve the highest possible total profit. Each acre of oats yields a profit of \$500, each acre of wheat yields a profit of \$600, while each acre of corn yields a profit of \$950. The objective function is therefore

Maximize Total Profit = $\$500O + \$600W + \$950C$.

There are three limited resources in this problem: 120 acres of land, 6500 hours of labor, and 200 tons of fertilizer. Each acre of a given crop that is planted uses up one available acre. The data for labor hours used and fertilizer used per acre planted can be used to calculate the total resources used as a function of the decision variables. The total resources used need to be less than or equal to the amount available. These constraints are therefore as follows:

Land: $O + W + C \leq 120$ acres,
Labor: $50O + 60W + 105C \leq 6500$ hours,
Fertilizer: $1.5O + 2W + 4C \leq 200$ tons

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let $O$ = Acres of Oats planted,
$W$ = Acres of Wheat planted,
$C$ = Acres of Corn planted.

Maximize Total Profit = $\$500O + \$600W + \$950C$.

subject to

Land: $O + W + C \leq 120$ acres,
Labor: $50O + 60W + 105C \leq 6500$ hours,
Fertilizer: $1.5O + 2W + 4C \leq 200$ tons

and $O \geq 0$, $W \geq 0$, $C \geq 0$.

## Problem 3-7:

The kitchen manager for Sing Sing Prison is trying to decide what to feed its prisoners. She would like to offer some combination of milk, beans, and oranges. The goal is to minimize cost, subject to meeting the minimum nutritional requirements imposed by law. The cost and nutritional content of each food, along with the minimum nutritional requirements, are shown below. What diet should be fed to each prisoner?

| | Milk (gallons) | Navy Beans (cups) | Oranges (large Calif. Valencia) | Minimum Daily Requirement |
|---|---|---|---|---|
| Niacin (mg) | 3.2 | 4.9 | 0.8 | 13.0 |
| Thiamin (mg) | 1.12 | 1.3 | 0.19 | 1.5 |
| Vitamin C (mg) | 32.0 | 0.0 | 93.0 | 45.0 |
| Cost ($) | 2.00 | 0.20 | 0.25 | |

**(a)** Formulate and solve a linear programming model for this problem in a spreadsheet.
**(b)** Formulate this same model algebraically.

**Solution for Problem 3-7:**

**(a)**
This is a cost-benefit-trade-off problem. The activities are the quantities of food to feed each prisoner and the required benefits are the minimum nutritional requirements. We will start to build a spreadsheet by entering the data. The data for this problem are the nutrient content of each food, the minimum daily requirement for each nutrient, and the cost of each food. The data in the spreadsheet would be entered as displayed below, where range names of UnitCost (C5:E5), NutritionalContents (C9:E11), and MinimumRequirement (H9:H11) are assigned to the corresponding data cells.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Milk | Beans | | | | |
| 4 | | (gal.) | (cups) | Oranges | | | |
| 5 | Unit Cost | $2.00 | $0.20 | $0.25 | | | |
| 6 | | | | | | | |
| 7 | | | | | | | Minimum |
| 8 | | Nutritional Contents (mg) | | | | | Requirement |
| 9 | Niacin | 3.2 | 4.9 | 0.8 | | | 13 |
| 10 | Thiamin | 1.12 | 1.3 | 0.19 | | | 1.5 |
| 11 | Vitamin C | 32 | 0 | 93 | | | 45 |

The decisions to be made in this problem are how much of each food type should be fed to each prisoner. Therefore, we add three changing cells in C13:E13, with range name Quantity. The values in Quantity (C13:E13) will eventually be determined by Solver. For now, an arbitrary value of 1 is entered for each food type.

The goal is to minimize the total cost per prisoner. Thus, the objective cell should calculate this cost:

Total Cost = ($2)(gallons of milk) + ($0.20)(cups of beans) + ($0.25)(number of oranges)

or

Total Cost = SUMPRODUCT(UnitCost, Quantity).

This formula is entered into cell H14.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Milk | Beans | | | | |
| 4 | | (gal.) | (cups) | Oranges | | | |
| 5 | Unit Cost | $2.00 | $0.20 | $0.25 | | | |
| 6 | | | | | | | |
| 7 | | | | | | | Minimum |
| 8 | | Nutritional Contents (mg) | | | | | Requirement |
| 9 | Niacin | 3.2 | 4.9 | 0.8 | | | 13 |
| 10 | Thiamin | 1.12 | 1.3 | 0.19 | | | 1.5 |
| 11 | Vitamin C | 32 | 0 | 93 | | | 45 |
| 12 | | | | | | | |
| 13 | Quantity | 1 | 1 | 1 | | | Total Cost |
| 14 | (per prisoner) | | | | | | $2.45 |

| | H |
|---|---|
| 13 | Total Cost |
| 14 | =SUMPRODUCT(UnitCost,Quantity) |

The functional constraints in this problem involve the minimum daily requirement of each nutrient. Given the amount of food fed each prisoner (the changing cells in C13:E13), we calculate the total resources used in F9:F11. For niacin, this will be =SUMPRODUCT(C9:E9, $C$13:$E$13). Using an absolute reference for the acres planted, this formula can be copied into cells F10-F11 to calculate the thiamin and vitamin C. The benefit achieved (total of each nutrient) must be >= the minimum needed (H9:H11), as indicated by the >= in G9:G11.

| | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 3 | | Milk | Beans | | | | |
| 4 | | (gal.) | (cups) | Oranges | | | |
| 5 | Unit Cost | $2.00 | $0.20 | $0.25 | | | |
| 6 | | | | | | | |
| 7 | | | | | Total | | Minimum |
| 8 | | Nutritional Contents (mg) | | | Nutrients | | Requirement |
| 9 | Niacin | 3.2 | 4.9 | 0.8 | 8.9 | >= | 13 |
| 10 | Thiamin | 1.12 | 1.3 | 0.19 | 2.610 | >= | 1.5 |
| 11 | Vitamin C | 32 | 0 | 93 | 125 | >= | 45 |
| 12 | | | | | | | |
| 13 | Quantity | 1 | 1 | 1 | | | Total Cost |
| 14 | (per prisoner) | | | | | | $2.45 |

| | F |
|---|---|
| 7 | Total |
| 8 | Nutrients |
| 9 | =SUMPRODUCT(C9:E9,Quantity) |
| 10 | =SUMPRODUCT(C10:E10,Quantity) |
| 11 | =SUMPRODUCT(C11:E11,Quantity) |

The Solver information and solved spreadsheet are shown below.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **The Prison Diet Problem** | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | Milk | Beans | | | | |
| 4 | | | (gal.) | (cups) | Oranges | | | |
| 5 | | Unit Cost | $2.00 | $0.20 | $0.25 | | | |
| 6 | | | | | | | | |
| 7 | | | | | | Total | | Minimum |
| 8 | | | Nutritional Contents (mg) | | | Nutrients | | Requirement |
| 9 | | Niacin | 3.2 | 4.9 | 0.8 | 13 | >= | 13 |
| 10 | | Thiamin | 1.12 | 1.3 | 0.19 | 3.438 | >= | 1.5 |
| 11 | | Vitamin C | 32 | 0 | 93 | 45 | >= | 45 |
| 12 | | | | | | | | |
| 13 | | Quantity | 0 | 2.574 | 0.484 | | | Total Cost |
| 14 | | (per prisoner) | | | | | | $0.64 |

**Solver Parameters**

**Set Objective Cell:** TotalCost

**To:** Min

**By Changing Variable Cells:**

Quantity

**Subject to the Constraints:**

TotalNutrients >= MinimumRequirement

**Solver Options:**

Make Variables Nonnegative

Solving Method: Simplex LP

| Range Name | Cells |
|---|---|
| MinimumRequirement | H9:H11 |
| NutritionalContents | C9:E11 |
| Quantity | C13:E13 |
| TotalCost | H14 |
| TotalNutrients | F9:F11 |
| UnitCost | C5:E5 |

| | F |
|---|---|
| 7 | Total |
| 8 | Nutrients |
| 9 | =SUMPRODUCT(C9:E9,Quantity) |
| 10 | =SUMPRODUCT(C10:E10,Quantity) |
| 11 | =SUMPRODUCT(C11:E11,Quantity) |

| | H |
|---|---|
| 13 | Total Cost |
| 14 | =SUMPRODUCT(UnitCost,Quantity) |

Thus, each prisoner should be fed a daily average of 2.574 cups of beans and 0.484 oranges for a total cost of $0.64.

**(b)**
To build an algebraic model for this problem, start by defining the decision variables. In this case, the three decisions are how many gallons of milk, cups of beans, and how many oranges to feed each prisoner. These variables are defined below:

Let $M$ = gallons of milk fed to each prisoner,
$B$ = cups of beans fed to each prisoner,
$O$ = number of oranges fed to each prisoner.

Next determine the goal of the problem. In this case, the goal is to meet the nutritional requirements at the lowest possible cost. Each gallon of milk costs \$2, each cup of beans costs \$0.20, and each orange costs \$0.25. The objective function is therefore

Minimize Total Cost = $\$2.00M + \$0.20B + \$0.25O$.

The nutritional requirements include minimum requirements for niacin, thiamin, and vitamin C. The data for the nutritional contents of each type of food can be used to calculate the total level of each nutrient achieved as a function of the decision variables. The total nutrients need to be greater than or equal to the minimum requirement. These constraints are therefore as follows:

Niacin: $3.2M + 4.9B + 0.8O \geq 13$mg,
Thiamin: $1.12M + 1.3B + 0.19O \geq 1.5$mg,
Vitamin C: $32M + 93O \geq 45$mg.

After adding nonnegativity constraints, the complete algebraic formulation is given below:

Let $M$ = gallons of milk fed to each prisoner,
$B$ = cups of beans fed to each prisoner,
$O$ = number of oranges fed to each prisoner.

Minimize Total Cost = $\$2.00M + \$0.20B + \$0.25O$.

subject to

Niacin: $3.2M + 4.9B + 0.8O \geq 13$mg,
Thiamin: $1.12M + 1.3B + 0.19O \geq 1.5$mg,
Vitamin C: $32M + 93O \geq 45$mg.

and $M \geq 0$, $B \geq 0$, $O \geq 0$.

## Problem 3-8:

Surfs Up produces high-end surfboards. A challenge faced by Surfs Up is that their demand is highly seasonal. Demand exceeds production capacity during the warm summer months, but is very low in the winter months. To meet the high demand during the summer, Surfs Up typically produces more surfboards than are needed in the winter months and then carries inventory into the summer months. Their production facility can produce at most 50 boards per month using regular labor at a cost of \$125 each. Up to 10 additional boards can be produced by utilizing overtime labor at a cost of \$135 each. The boards are sold for $200. Because of storage cost and the opportunity cost of capital, each board held in inventory from one month to the next incurs a cost of $5 per board. Since demand is uncertain, Surfs Up would like to maintain an ending inventory (safety stock) of at least 10 boards during the warm months (May–September) and at least 5 boards during the other months (October–April). It is now the start of January and Surfs Up has 5 boards in inventory. The forecast of demand over the next 12 months is shown in the table below. Formulate and solve a linear programming model in a spreadsheet to determine how many surfboards should be produced each month to maximize total profit.

| Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 14 | 15 | 20 | 45 | 65 | 85 | 85 | 40 | 30 | 15 | 15 |

**Solution for Problem 3-8:**

This is a dynamic problem with 12 time periods (months). The activities are the production quantities in each of the 12 months using regular labor and the production quantities in each of the 12 months using overtime labor.

To get started, we sketch a spreadsheet model. Each of the 12 months will be a separate column in the spreadsheet. For each month, the regular production quantity (a changing cell) must be no more than the maximum regular production (50). Similarly, for each month the overtime production quantity (a changing cell) must be no more than the maximum overtime production (10). Each month will generate revenue, incur regular and overtime production costs, inventory holding costs, and achieve a resulting profit. The goal will be to maximize the total profit over all 12 months. This leads to the following sketch of a spreadsheet model.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Unit Cost (Reg) | | | | | | | | | | | | |
| Unit Cost (OT) | | | | | | | | | | | | |
| Selling Price | | | | | | | | | | | | |
| Holding Cost | | | | | | | | | | | | |
| Starting Inventory | | | | | | | | | | | | |
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| Regular Production | | | | | | | | | | | | |
| | | | | | | <= | | | | | | |
| Max Regular | | | | | | | | | | | | |
| OT Production | | | | | | | | | | | | |
| | | | | | | <= | | | | | | |
| Max OT | | | | | | | | | | | | |
| Forecasted Sales | | | | | | | | | | | | |
| Ending Inventory | | | | | | | | | | | | |
| | | | | | | >= | | | | | | |
| Safety Stock | | | | | | | | | | | | |
| Revenue | | | | | | | | | | | | |
| Regular Production Cost | | | | | | | | | | | | |
| Overtime Production Cost | | | | | | | | | | | | |
| Holding Cost | | | | | | | | | | | | |
| Profit | | | | | | | | | | | | |

The ending inventory each month will equal the starting inventory (the given starting inventory for January, or the previous month's ending inventory for future months) plus all production (regular and overtime) minus the forecasted sales. The ending inventory at the end of each month must be at least the minimum safety stock level. The revenue will equal the selling price times forecasted sales. The regular (or overtime) production cost will be the regular (or overtime) production quantity times the unit regular (or overtime) production cost. The holding cost will equal the ending inventory times the unit holding cost. The monthly profit will be revenue minus both production costs minus holding cost. Finally, the total profit will be the sum of the monthly profits. The final solved spreadsheet, formulas, and Solver information are shown below.

| | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **oduction and Inventory Planning at Surfs Up** | | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | | | |
| 3 | Unit Cost (Reg) | $125 | | | | | | | | | | | | |
| 4 | Unit Cost (OT) | $135 | | | | | | | | | | | | |
| 5 | Selling Price | $200 | | | | | | | | | | | | |
| 6 | Holding Cost | $5 | | | | | | | | | | | | |
| 7 | Starting Inventory | 5 | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | | | |
| 9 | | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | |
| 10 | Regular Production | 10 | 14 | 30 | 50 | 50 | 50 | 50 | 50 | 40 | 25 | 15 | 15 | |
| 11 | | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | |
| 12 | Max Regular | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 50 | |
| 13 | | | | | | | | | | | | | | |
| 14 | OT Production | 0 | 0 | 0 | 0 | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| 15 | | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | <= | |
| 16 | Max OT | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | |
| 17 | | | | | | | | | | | | | | |
| 18 | Forecasted Sales | 10 | 14 | 15 | 20 | 45 | 65 | 85 | 85 | 40 | 30 | 15 | 15 | |
| 19 | | | | | | | | | | | | | | |
| 20 | Ending Inventory | 5 | 5 | 20 | 50 | 65 | 60 | 35 | 10 | 10 | 5 | 5 | 5 | |
| 21 | | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | |
| 22 | Safety Stock | 5 | 5 | 5 | 5 | 10 | 10 | 10 | 10 | 10 | 5 | 5 | 5 | |
| 23 | | | | | | | | | | | | | | |
| 24 | | | | | | | | | | | | | | Total |
| 25 | Revenue | $2,000 | $2,800 | $3,000 | $4,000 | $9,000 | $13,000 | $17,000 | $17,000 | $8,000 | $6,000 | $3,000 | $3,000 | $87,800 |
| 26 | Regular Production Cost | $1,250 | $1,750 | $3,750 | $6,250 | $6,250 | $6,250 | $6,250 | $6,250 | $5,000 | $3,125 | $1,875 | $1,875 | $49,875 |
| 27 | Overtime Production Cost | $0 | $0 | $0 | $0 | $1,350 | $1,350 | $1,350 | $1,350 | $0 | $0 | $0 | $0 | $5,400 |
| 28 | Holding Cost | $25 | $25 | $100 | $250 | $325 | $300 | $175 | $50 | $50 | $25 | $25 | $25 | $1,375 |
| 29 | Profit | $725 | $1,025 | -$850 | -$2,500 | $1,075 | $5,100 | $9,225 | $9,350 | $2,950 | $2,850 | $1,100 | $1,100 | $31,150 |

**Solver Parameters**

**Set Objective Cell:** TotalProfit
**To:** Max
**By Changing Variable Cells:**
RegularProduction, OTProduction
**Subject to the Constraints:**
RegularProduction <= MaxRegular
OTProduction <= MaxOT
EndingInventory >= SafetyStock
**Solver Options:**
Make Variables Nonnegative
Solving Method: Simplex LP

| Range Name | Cells |
|---|---|
| EndingInventory | C20:N20 |
| ForecastedSales | C18:N18 |
| HoldingCost | C6 |
| MaxOT | C16:N16 |
| MaxRegular | C12:N12 |
| OTProduction | C14:N14 |
| RegularProduction | C10:N10 |
| SafetyStock | C22:N22 |
| SellingPrice | C5 |
| StartingInventory | C7 |
| TotalProfit | O29 |
| UnitCostOT | C4 |
| UnitCostReg | C3 |

| | B | C | D | E–N | O |
|---|---|---|---|---|---|
| 3 | Unit Cost (Reg) | 125 | | | |
| 4 | Unit Cost (OT) | 135 | | | |
| 5 | Selling Price | 200 | | | |
| 6 | Holding Cost | 5 | | | |
| 7 | Starting Inventory | 5 | | | |
| 8 | | | | | |
| 9 | | Jan | Feb | [illegible] | |
| 10 | Regular Production | 10 | 14 | [illegible] | |
| 11 | | <= | <= | [illegible] | |
| 12 | Max Regular | 50 | 50 | [illegible] | |
| 13 | | | | | |
| 14 | OT Production | 0 | 0 | [illegible] | |
| 15 | | <= | <= | [illegible] | |
| 16 | Max OT | 10 | 10 | [illegible] | |
| 17 | | | | | |
| 18 | Forecasted Sales | 10 | 14 | [illegible] | |
| 19 | | | | | |
| 20 | Ending Inventory | =StartingInventory+RegularProduction+OTProduction-ForecastedSales | =C20+RegularProduction+OTProduction-ForecastedSales | [illegible] | |
| 21 | | >= | >= | [illegible] | |
| 22 | Safety Stock | 5 | 5 | [illegible] | |
| 23 | | | | | |
| 24 | | | | | Total |
| 25 | Revenue | =SellingPrice*ForecastedSales | =SellingPrice*ForecastedSales | [illegible] | =SUM(C25:N25) |
| 26 | Regular Production Cost | =UnitCostReg*RegularProduction | =UnitCostReg*RegularProduction | [illegible] | =SUM(C26:N26) |
| 27 | Overtime Production Cost | =UnitCostOT*OTProduction | =UnitCostOT*OTProduction | [illegible] | =SUM(C27:N27) |
| 28 | Holding Cost | =HoldingCost*EndingInventory | =HoldingCost*EndingInventory | [illegible] | =SUM(C28:N28) |
| 29 | Profit | =C25-C26-C27-C28 | =D25-D26-D27-D28 | [illegible] | =SUM(C29:N29) |

The values in RegularProduction (C10:N10) and OTProduction (C14:N14) show how many surf boards Surfs Up should produce each month so as to achieve the maximum profit of $31,150.

## Problem 3-9:

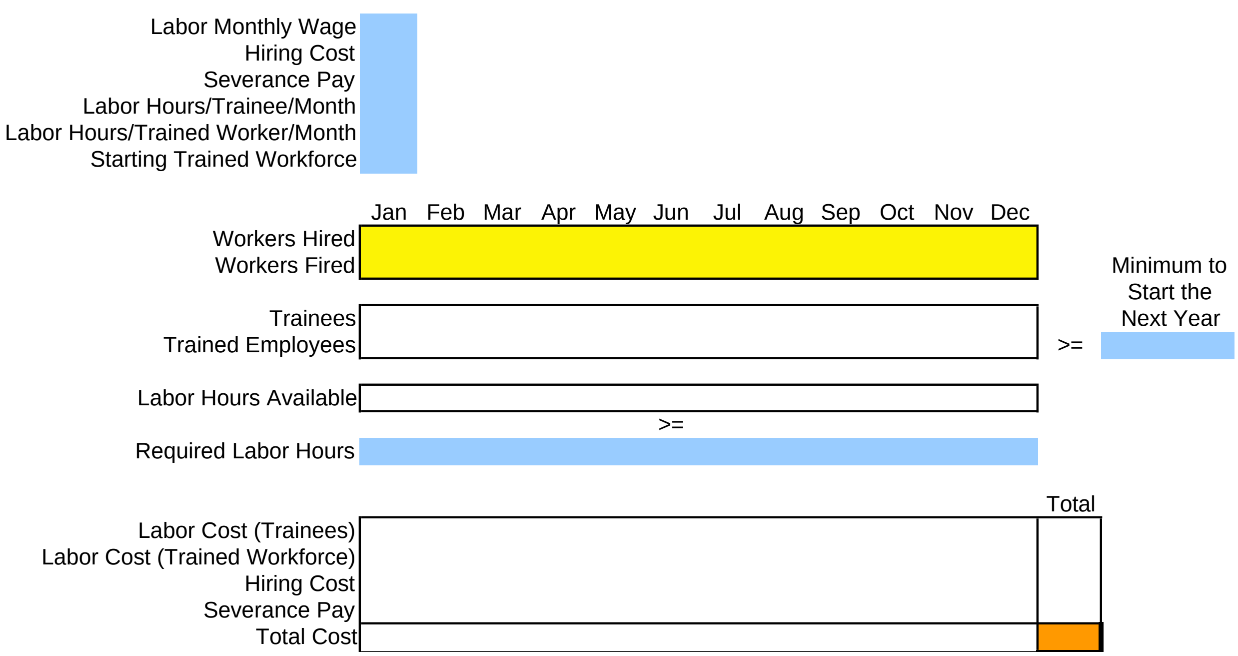
Cool Power produces air conditioning units for large commercial properties. Due to the low cost and efficiency of its products, the company has been growing from year to year. Also, due to seasonality in construction and weather conditions, production requirements vary from month to month. Cool Power currently has 10 fully trained employees working in manufacturing. Each trained employee can work 160 hours per month and is paid a monthly wage of $4000. New trainees can be hired at the beginning of any month. Due to their lack of initial skills and required training, a new trainee only provides 100 hours of useful labor in their first month, but are still paid a full monthly wage of $4000. Furthermore, because of required interviewing and training, there is a $2500 hiring cost for each employee hired. After one month, a trainee is considered fully trained. An employee can be fired at the beginning of any month, but must be paid two weeks of severance pay ($2000). Over the next 12 months, Cool Power forecasts the labor requirements shown in the table below. Since management anticipates higher requirements next year, Cool Power would like to end the year with at least 12 fully trained employees. How many trainees should be hired and/or workers fired in each month to meet the labor requirements at the minimum possible cost? Formulate and solve a linear programming spreadsheet model.

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1600 | 2000 | 2000 | 2000 | 2800 | 3200 | 3600 | 3200 | 1600 | 1200 | 800 | 800 |

**Solution for Problem 3-9:**

This is a dynamic problem with 12 time periods (months). The activities are the number of workers to hire and fire in each of the 12 months.

To get started, we sketch a spreadsheet model. Each of the 12 months will be a separate column in the spreadsheet. For each month, there are changing cells for both the number of workers hired and fired. Based on the values of these changing cells, we can determine the number of trainees and trained employees. The number of labor hours generated by the employees must be at least the required labor hours each month. Finally, labor costs (for trainees and the trained workforce), hiring cost, and severance pay leads to a total monthly cost. The goal will be to minimize the total cost over all 12 months. This leads to the following sketch of a spreadsheet model.



When an employee is first hired, he or she is a trainee for one month before becoming a fully-trained employee. Therefore, the number of trainees (row 14) is equal to the number of workers hired in that month, while the number of trained employees (row 15) is the number of trained employees and trainees from the previous month minus any employee that is fired. The labor hours available in each month equals the sumproduct of the labor hours provided by each type of worker (trained or trainees) with the number of each type of employee. The labor costs in each month are the monthly wage multiplied by the number of employees. The hiring cost is the unit hiring cost multiplied by the number of workers hired. The severance pay is the unit severance cost multiplied by the number of workers fired. Then, the total monthly cost is the sum of the labor costs, hiring cost, and severance pay. Finally, the total cost will be the sum of the monthly costs. For arbitrary values of workers hired and fired each month, this leads to the following spreadsheet.

| | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | Labor Monthly Wage | $4,000 | | | | | | | | | | | | | |
| 4 | Hiring Cost | $2,500 | | | | | | | | | | | | | |
| 5 | Severance Pay | $2,000 | | | | | | | | | | | | | |
| 6 | Labor Hours/Trainee/Month | 100 | | | | | | | | | | | | | |
| 7 | Labor Hours/Trained Worker/Month | 160 | | | | | | | | | | | | | |
| 8 | Starting Trained Workforce | 10 | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | | |
| 10 | | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | | |
| 11 | Workers Hired | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 2 | 2 | | |
| 12 | Workers Fired | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 3 | 3 | 0 | 0 | | Minimum to |
| 13 | | | | | | | | | | | | | | | Start the |
| 14 | Trainees | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 0 | 0 | 2 | 2 | | Next Year |
| 15 | Trained Employees | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 21 | 18 | 15 | 15 | 17 | >= | 12 |
| 16 | | | | | | | | | | | | | | | |
| 17 | Labor Hours Available | 1800 | 2120 | 2440 | 2760 | 3080 | 3400 | 3720 | 3360 | 2880 | 2400 | 2600 | 2920 | | |
| 18 | | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | | |
| 19 | Required Labor Hours | 1600 | 2000 | 2000 | 2000 | 2800 | 3200 | 3600 | 3200 | 1600 | 1200 | 800 | 800 | | |
| 20 | | | | | | | | | | | | | | | |
| 21 | | | | | | | | | | | | | | Total | |
| 22 | Labor Cost (Trainees) | $8,000 | $8,000 | $8,000 | $8,000 | $8,000 | $8,000 | $8,000 | $0 | $0 | $0 | $8,000 | $8,000 | $72,000 | |
| 23 | Labor Cost (Trained Workforce) | $40,000 | $48,000 | $56,000 | $64,000 | $72,000 | $80,000 | $88,000 | $84,000 | $72,000 | $60,000 | $60,000 | $68,000 | $792,000 | |
| 24 | Hiring Cost | $5,000 | $5,000 | $5,000 | $5,000 | $5,000 | $5,000 | $5,000 | $0 | $0 | $0 | $5,000 | $5,000 | $45,000 | |
| 25 | Severance Pay | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $6,000 | $6,000 | $6,000 | $0 | $0 | $18,000 | |
| 26 | Total Cost | $53,000 | $61,000 | $69,000 | $77,000 | $85,000 | $93,000 | $101,000 | $90,000 | $78,000 | $66,000 | $73,000 | $81,000 | $927,000 | |

| | B | C | D–N | O | P |
|---|---|---|---|---|---|
| 3 | Labor Monthly Wage | 4000 | | | |
| 4 | Hiring Cost | 2500 | | | |
| 5 | Severance Pay | 2000 | | | |
| 6 | Labor Hours/Trainee/Month | 100 | | | |
| 7 | Labor Hours/Trained Worker/Month | 160 | | | |
| 8 | Starting Trained Workforce | 10 | | | |
| 9 | | | | | |
| 10 | | Jan | [illegible] | | |
| 11 | Workers Hired | 2 | [illegible] | | |
| 12 | Workers Fired | 0 | [illegible] | | Minimum to |
| 13 | | | | | Start the |
| 14 | Trainees | =WorkersHired | [illegible] | | Next Year |
| 15 | Trained Employees | =StartingTrainedWorkforce-WorkersFired | [illegible] | >= | 12 |
| 16 | | | | | |
| 17 | Labor Hours Available | =SUMPRODUCT(LaborHoursPerTrainee:LaborHoursPerTrainedWorker,C14:C15) | [illegible] | | |
| 18 | | >= | [illegible] | | |
| 19 | Required Labor Hours | 1600 | [illegible] | | |
| 20 | | | | | |
| 21 | | | | Total | |
| 22 | Labor Cost (Trainees) | =LaborMonthlyWage*Trainees | [illegible] | =SUM(C22:N22) | |
| 23 | Labor Cost (Trained Workforce) | =LaborMonthlyWage*TrainedEmployees | [illegible] | =SUM(C23:N23) | |
| 24 | Hiring Cost | =HiringCost*WorkersHired | [illegible] | =SUM(C24:N24) | |
| 25 | Severance Pay | =SeverancePay*WorkersFired | [illegible] | =SUM(C25:N25) | |
| 26 | Total Cost | =SUM(C22:C25) | [illegible] | =SUM(C26:N26) | |

The Solver information is shown below, followed by the solved spreadsheet.

| | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | Labor Monthly Wage | $4,000 | | | | | | | | | | | | | |
| 4 | Hiring Cost | $2,500 | | | | | | | | | | | | | |
| 5 | Severance Pay | $2,000 | | | | | | | | | | | | | |
| 6 | Labor Hours/Trainee/Month | 100 | | | | | | | | | | | | | |
| 7 | Labor Hours/Trained Worker/Month | 160 | | | | | | | | | | | | | |
| 8 | Starting Trained Workforce | 10 | | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | | | | |
| 10 | | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec | | |
| 11 | Workers Hired | 2.1E-11 | 4 | 0 | 0 | 6 | 2 | 1 | 0 | 0 | 0 | 2 | 0 | | |
| 12 | Workers Fired | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 10 | 0 | 0 | 0 | | Minimum to |
| 13 | | | | | | | | | | | | | | | Start the |
| 14 | Trainees | 2.1E-11 | 4 | 0 | 0 | 6 | 2 | 1 | 0 | 0 | 0 | 2 | 0 | | Next Year |
| 15 | Trained Employees | 10 | 10 | 14 | 14 | 14 | 20 | 22 | 20 | 10 | 10 | 10 | 12 | >= | 12 |
| 16 | | | | | | | | | | | | | | | |
| 17 | Labor Hours Available | 1600 | 2000 | 2240 | 2240 | 2840 | 3400 | 3620 | 3200 | 1600 | 1600 | 1800 | 1920 | | |
| 18 | | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | >= | | |
| 19 | Required Labor Hours | 1600 | 2000 | 2000 | 2000 | 2800 | 3200 | 3600 | 3200 | 1600 | 1200 | 800 | 800 | | |
| 20 | | | | | | | | | | | | | | | |
| 21 | | | | | | | | | | | | | | Total | |
| 22 | Labor Cost (Trainees) | $0 | $16,000 | $0 | $0 | $24,000 | $8,000 | $4,000 | $0 | $0 | $0 | $8,000 | $0 | $60,000 | |
| 23 | Labor Cost (Trained Workforce) | $40,000 | $40,000 | $56,000 | $56,000 | $56,000 | $80,000 | $88,000 | $80,000 | $40,000 | $40,000 | $40,000 | $48,000 | $664,000 | |
| 24 | Hiring Cost | $0 | $10,000 | $0 | $0 | $15,000 | $5,000 | $2,500 | $0 | $0 | $0 | $5,000 | $0 | $37,500 | |
| 25 | Severance Pay | $0 | $0 | $0 | $0 | $0 | $0 | $0 | $6,000 | $20,000 | $0 | $0 | $0 | $26,000 | |
| 26 | Total Cost | $40,000 | $66,000 | $56,000 | $56,000 | $95,000 | $93,000 | $94,500 | $86,000 | $60,000 | $40,000 | $53,000 | $48,000 | $787,500 | |

**Solver Parameters**

**Set Objective Cell:** TotalCost
**To:** Min
**By Changing Variable Cells:**
WorkersHired, WorkersFired
**Subject to the Constraints:**
N15 >= MinimumToStartNewYear
LaborHoursAvailable >= RequiredLaborHours
WorkersHired = integer
WorkersFired = integer
**Solver Options:**
Make Variables Nonnegative
Solving Method: Simplex LP

| Range Name | Cells |
|---|---|
| HiringCost | C4 |
| LaborHoursAvailable | C17:N17 |
| LaborHoursPerTrainedWorker | C7 |
| LaborHoursPerTrainee | C6 |
| LaborMonthlyWage | C3 |
| MinimumToStartNextYear | P15 |
| RequiredLaborHours | C19:N19 |
| SeverancePay | C5 |
| StartingTrainedWorkforce | C8 |
| TotalCost | O26 |
| TrainedEmployees | C15:N15 |
| Trainees | C14:N14 |
| WorkersFired | C12:N12 |
| WorkersHired | C11:N11 |

| | B | C | D–N | O | P |
|---|---|---|---|---|---|
| 3 | Labor Monthly Wage | 4000 | | | |
| 4 | Hiring Cost | 2500 | | | |
| 5 | Severance Pay | 2000 | | | |
| 6 | Labor Hours/Trainee/Month | 100 | | | |
| 7 | Labor Hours/Trained Worker/Month | 160 | | | |
| 8 | Starting Trained Workforce | 10 | | | |
| 9 | | | | | |
| 10 | | Jan | [illegible] | | |
| 11 | Workers Hired | 2 | [illegible] | | |
| 12 | Workers Fired | 0 | [illegible] | | Minimum to |
| 13 | | | | | Start the |
| 14 | Trainees | =WorkersHired | [illegible] | | Next Year |
| 15 | Trained Employees | =StartingTrainedWorkforce-WorkersFired | [illegible] | >= | 12 |
| 16 | | | | | |
| 17 | Labor Hours Available | =SUMPRODUCT(LaborHoursPerTrainee:LaborHoursPerTrainedWorker,C14:C15) | [illegible] | | |
| 18 | | >= | [illegible] | | |
| 19 | Required Labor Hours | 1600 | [illegible] | | |
| 20 | | | | | |
| 21 | | | | Total | |
| 22 | Labor Cost (Trainees) | =LaborMonthlyWage*Trainees | [illegible] | =SUM(C22:N22) | |
| 23 | Labor Cost (Trained Workforce) | =LaborMonthlyWage*TrainedEmployees | [illegible] | =SUM(C23:N23) | |
| 24 | Hiring Cost | =HiringCost*WorkersHired | [illegible] | =SUM(C24:N24) | |
| 25 | Severance Pay | =SeverancePay*WorkersFired | [illegible] | =SUM(C25:N25) | |
| 26 | Total Cost | =SUM(C22:C25) | [illegible] | =SUM(C26:N26) | |

Thus, WorkersHired (C11:N11) shows the number of workers Cool Power should hire each month and WorkersFired (C12:N12) shows the number of workers Cool Power should fire each month so as to achieve the minimum TotalCost (O26) of $787,500.