CHAPTER 1

The Celestial Sphere

1.1 From Fig. 1.7, Earth makes $S/P_\oplus$ orbits about the Sun during the time required for another planet to make S/P orbits. If that other planet is a superior planet then Earth must make one extra trip around the Sun to overtake it, hence

$$\frac{S}{P_\oplus} = \frac{S}{P} + 1.$$

Similarly, for an inferior planet, that planet must make the extra trip, or

$$\frac{S}{P} = \frac{S}{P_\oplus} + 1.$$

Rearrangement gives Eq. (1.1).

1.2 For an inferior planet at greatest elongation, the positions of Earth (E), the planet (P), and the Sun (S) form a right triangle ($\angle EPS = 90°$). Thus $\cos(\angle PES) = \overline{EP}/\overline{ES}$.

From Fig. S1.1, the time required for a superior planet to go from opposition (point P_1) to quadrature (P_2) can be combined with its sidereal period (from Eq. 1.1) to find the angle $\angle P_1SP_2$. In the same time interval Earth will have moved through the angle $\angle E_1SE_2$. Since P_1, E_1, and S form a straight line, the angle $\angle P_2SE_2 = \angle E_1SE_2 - \angle P_1SP_2$. Now, using the right triangle at quadrature, $\overline{P_2S}/\overline{E_2S} = 1/\cos(\angle P_2SE_2)$.

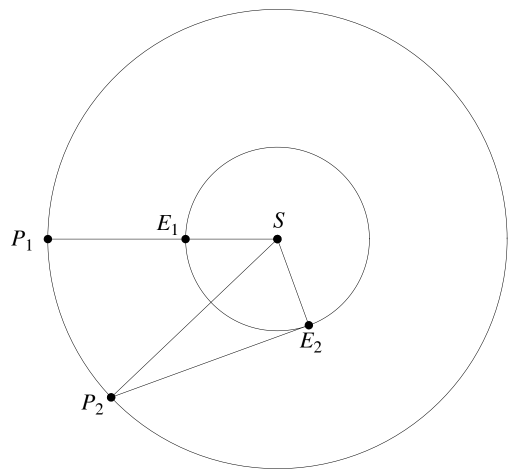

Figure S1.1: The relationship between synodic and sidereal periods for superior planets, as discussed in Problem 1.2.

1.3 (a) $P_{\text{Venus}} = 224.7$ d, $P_{\text{Mars}} = 687.0$ d

(b) Pluto. It travels the smallest fraction of its orbit before being "lapped" by Earth.

1.4 Vernal equinox: $\alpha = 0^{\text{h}}$, $\delta = 0°$
Summer solstice: $\alpha = 6^{\text{h}}$, $\delta = 23.5°$
Autumnal equinox: $\alpha = 12^{\text{h}}$, $\delta = 0°$
Winter solstice: $\alpha = 18^{\text{h}}$, $\delta = -23.5°$

1.5 (a) $(90° - 42°) + 23.5° = 71.5°$

(b) $(90° - 42°) - 23.5° = 24.5°$

1.6 (a) $90° - L < \delta < 90°$

(b) $L > 66.5°$

(c) Strictly speaking, only at $L = \pm 90°$. The Sun will move along the horizon at these latitudes.

1.7 (a) Both the year 2000 and the year 2004 were leap years, so each had 366 days. Therefore, the number of days between January 1, 2000 and January 1, 2006 is 2192 days. From January 1, 2006 to July 14, 2006 there are 194 days. Finally, from noon on July 14, 2006 to 16:15 UT is 4.25 hours, or 0.177 days. Thus, July 14, 2006 at 16:15 UT is JD 2453931.177.

(b) MJD 53930.677.

1.8 (a) $\Delta\alpha = 9^{\mathrm{m}}53.55^{\mathrm{s}} = 2.4731°$, $\Delta\delta = 2°9'16.2'' = 2.1545°$. From Eq. (1.8), $\Delta\theta = 2.435°$.

(b) $d = r\,\Delta\theta = 1.7 \times 10^{15}$ m = 11,400 AU.

1.9 (a) From Eqs. (1.2) and (1.3), $\Delta\alpha = 0.193628° = 0.774512^{\mathrm{m}}$ and $\Delta\delta = -0.044211° = -2.65266'$. This gives the 2010.0 precessed coordinates as $\alpha = 14^{\mathrm{h}}30^{\mathrm{m}}29.4^{\mathrm{s}}$, $\delta = -62°43'25.26''$.

(b) From Eqs. (1.6) and (1.7), $\Delta\alpha = -5.46^{\mathrm{s}}$ and $\Delta\delta = 7.984''$.

(c) Precession makes the largest contribution.

1.10 In January the Sun is at a right ascension of approximately 19^{h}. This implies that a right ascension of roughly 7^{h} is crossing the meridian at midnight. With about 14 hours of darkness this would imply observations of objects between right ascensions of 0 h and 14 h would be crossing the meridian during the course of the night (sunset to sunrise).

1.11 Using the identities, $\cos(90° - t) = \sin t$ and $\sin(90° - t) = \cos t$, together with the small-angle approximations $\cos\Delta\theta \approx 1$ and $\sin\Delta\theta \approx 1$, the expression immediately reduces to

$$\sin(\delta + \Delta\delta) = \sin\delta + \Delta\theta\cos\delta\cos\theta.$$

Using the identity $\sin(a + b) = \sin a\cos b + \cos a\sin b$, the expression now becomes

$$\sin\delta\cos\Delta\delta + \cos\delta\sin\Delta\delta = \sin\delta + \Delta\theta\cos\delta\cos\theta.$$

Assuming that $\cos\Delta\delta \approx 1$ and $\sin\Delta\delta \approx \Delta\delta$, Eq. (1.7) is obtained.

CHAPTER 2

Celestial Mechanics

2.1 From Fig. 2.4, note that

$$r^2 = (x - ae)^2 + y^2 \qquad \text{and} \qquad r'^2 = (x + ae)^2 + y^2.$$

Substituting Eq. (2.1) into the second expression gives

$$r = 2a - \sqrt{(x + ae)^2 + y^2}$$

which is now substituted into the first expression. After some rearrangement,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1.$$

Finally, from Eq. (2.2),

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

2.2 The area integral in Cartesian coordinates is given by

$$A = \int_{-a}^{a} \int_{-b\sqrt{1-x^2/a^2}}^{b\sqrt{1-x^2/a^2}} dy\,dx = \frac{2b}{a} \int_{-a}^{a} \sqrt{a^2 - x^2}\,dx = \pi ab.$$

2.3 (a) From Eq. (2.3) the radial velocity is given by

$$v_r = \frac{dr}{dt} = \frac{a(1 - e^2)}{(1 + e\cos\theta)^2}\, e \sin\theta \, \frac{d\theta}{dt}. \qquad \text{(S2.1)}$$

Using Eqs. (2.31) and (2.32)

$$\frac{d\theta}{dt} = \frac{2}{r^2}\frac{dA}{dt} = \frac{L}{\mu r^2}.$$

The angular momentum can be written in terms of the orbital period by integrating Kepler's second law. If we further substitute $A = \pi ab$ and $b = a(1 - e^2)^{1/2}$ then

$$L = 2\mu\pi a^2(1 - e^2)^{1/2}/P.$$

Substituting L and r into the expression for $d\theta/dt$ gives

$$\frac{d\theta}{dt} = \frac{2\pi(1 + e\cos\theta)^2}{P(1 - e^2)^{3/2}}.$$

This can now be used in Eq. (S2.1), which simplifies to

$$v_r = \frac{2\pi ae\sin\theta}{P(1 - e^2)^{1/2}}.$$

Similarly, for the transverse velocity

$$v_\theta = r\,\frac{d\theta}{dt} = \frac{2\pi a(1 + e\cos\theta)}{(1 - e^2)^{1/2} P}.$$

(b) Equation (2.36) follows directly from $v^2 = v_r^2 + v_\theta^2$, Eq. (2.37) (Kepler's third law), and Eq. (2.3).

2.4 The total energy of the orbiting bodies is given by

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G\frac{m_1 m_2}{r}$$

where $r = |\mathbf{r}_2 - \mathbf{r}_1|$. Now,

$$v_1 = \dot{r}_1 = -\frac{m_2}{m_1 + m_2}\dot{r} \qquad \text{and} \qquad v_2 = \dot{r}_2 = \frac{m_1}{m_1 + m_2}\dot{r}.$$

Finally, using $M = m_1 + m_2$, $\mu = m_1 m_2/(m_1 + m_2)$, and $m_1 m_2 = \mu M$, we obtain Eq. (2.25).

2.5 Following a procedure similar to Problem 2.4,

$$\begin{aligned}
\mathbf{L} &= m_1 \mathbf{r}_1 \times \mathbf{v}_1 + m_2 \mathbf{r}_2 \times \mathbf{v}_2 \\
&= m_1 \left[-\frac{m_2}{m_1 + m_2}\right]\mathbf{r} \times \left[-\frac{m_2}{m_1 + m_2}\right]\mathbf{v} \\
&\quad + m_2 \left[\frac{m_1}{m_1 + m_2}\right]\mathbf{r} \times \left[\frac{m_1}{m_1 + m_2}\right]\mathbf{v} \\
&= \mu \mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{p}
\end{aligned}$$

2.6 (a) The total orbital angular momentum of the Sun–Jupiter system is given by Eq. (2.30). Referring to the data in Appendicies A and C, $M_\odot = 1.989 \times 10^{30}$ kg, $M_J = 1.899 \times 10^{27}$ kg, $M = M_J + M_\odot = 1.991 \times 10^{30}$ kg, and $\mu = M_J M_\odot/(M_J + M_\odot) = 1.897 \times 10^{27}$ kg. Furthermore, $e = 0.0489$, $a = 5.2044$ AU $= 7.786 \times 10^{11}$ m. Substituting,

$$L_{\text{total orbit}} = \mu\sqrt{GMa\left(1 - e^2\right)} = 1.927 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

(b) The distance of the Sun from the center of mass is $a_\odot = \mu a/M_\odot = 7.426 \times 10^8$ m. The Sun's orbital speed is $v_\odot = 2\pi a_\odot/P_J = 12.46$ m s^{-1}, where $P_J = 3.743 \times 10^8$ s is the system's orbital period. Thus, for an assumed circular orbit,

$$L_{\text{Sun orbit}} = M_\odot a_\odot v_\odot = 1.840 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}.$$

(c) The distance of Jupiter from the center of mass is $a_J = \mu a/M_J = 7.778 \times 10^{11}$ m, and its orbital speed is $v_J = 2\pi a_J/P_J = 1.306 \times 10^4$ m s^{-1}. Again assuming a circular orbit,

$$L_{\text{Jupiter orbit}} = M_J a_J v_J = 1.929 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

This is in good agreement with

$$L_{\text{total orbit}} - L_{\text{Sun orbit}} = 1.925 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

(d) The moment of inertia of the Sun is approximately

$$I_\odot \sim \frac{2}{5}M_\odot R_\odot^2 \sim 3.85 \times 10^{47} \text{ kg m}^2$$

and the moment of inertia of Jupiter is approximately

$$I_J \sim \frac{2}{5}M_J R_J^2 \sim 3.62 \times 10^{42} \text{ kg m}^2.$$

(*Note:* Since the Sun and Jupiter are centrally condensed, these values are overestimates; see Section 23.2.) Using $\omega = 2\pi/P$,

$$L_{\text{Sun rotate}} = 1.078 \times 10^{42} \text{ kg m}^2 \text{ s}^{-1}$$

$$L_{\text{Jupiter rotate}} = 6.312 \times 10^{38} \text{ kg m}^2 \text{ s}^{-1}.$$

(e) Jupiter's orbital angular momentum.

2.7 (a) $v_{\text{esc}} = \sqrt{2GM_J/R_J} = 60.6 \text{ km s}^{-1}$

(b) $v_{\text{esc}} = \sqrt{2GM_\odot/1 \text{ AU}} = 42.1 \text{ km s}^{-1}$.

2.8 (a) From Kepler's third law (Eq. 2.37) with $a = R_\oplus + h = 6.99 \times 10^6$ m, $P = 5820$ s $= 96.9$ min.

(b) The orbital period of a geosynchronous satellite is the same as Earth's sidereal rotation period, or $P = 8.614 \times 10^4$ s. From Eq. (2.37), $a = 4.22 \times 10^7$ m, implying an altitude of $h = a - R_\oplus = 3.58 \times 10^7$ m $= 5.6\ \text{R}_\oplus$.

(c) A geosynchronous satellite must be "parked" over the equator and orbiting in the direction of Earth's rotation. This is because the center of the satellite's orbit is the center of mass of the Earth–satellite system (essentially Earth's center).

2.9 The integral average of the potential energy is given by

$$\langle U \rangle = \frac{1}{P}\int_0^P U(t)\,dt = -\frac{1}{P}\int_0^P \frac{GM\mu}{r(t)}\,dt.$$

Using Eqs. (2.31) and (2.32) to solve for dt in terms of $d\theta$, and making the appropriate changes in the limits of integration,

$$\langle U \rangle = -\frac{1}{P}\int_0^{2\pi} \frac{GM\mu^2 r}{L}\,d\theta.$$

Writing r in terms of θ via Eq. (2.3) leads to

$$\langle U \rangle = -\frac{GM\mu^2 a\left(1-e^2\right)}{PL}\int_0^{2\pi}\frac{d\theta}{1+e\cos\theta}$$

$$= -\frac{2\pi GM\mu^2 a\left(1-e^2\right)^{1/2}}{PL}.$$

Using Eq. (2.30) to eliminate the total orbital angular momentum L, and Kepler's third law (Eq. 2.37) to replace the orbital period P, we arrive at

$$\langle U \rangle = -G\frac{M\mu}{a}.$$

2.10 Using the integral average from Problem 2.9

$$\langle r \rangle = \frac{1}{P}\int_0^P r(t)\,dt.$$

Using substitutions similar to the solution of Problem 2.9 we eventually arrive at

$$\langle r \rangle = \frac{a}{2\pi}\left(1-e^2\right)^{5/2}\int_0^{2\pi}\frac{d\theta}{(1+e\cos\theta)^3}. \qquad \text{(S2.2)}$$

It is evident that for $e = 0$, $\langle r \rangle = a$, as expected for perfectly circular motion. However, $\langle r \rangle$ deviates from a for other values of e. This function is most easily evaluted numerically. Employing a simple trapezoid method with 10^6 intervals, gives the results shown in Table S2.1.

Table S2.1: Results of the numerical evaluation of Eq. (S2.2) for Problem 2.10.

e	$\langle r \rangle / a$
0.00000	1.000000
0.10000	1.005000
0.20000	1.020000
0.30000	1.045000
0.40000	1.080000
0.50000	1.125000
0.60000	1.180000
0.70000	1.245000
0.80000	1.320000
0.90000	1.405000
0.95000	1.451250
0.99000	1.490050
0.99900	1.499001
0.99990	1.499900
0.99999	1.499990
1.00000	0.000000

2.11 Since planetary orbits are very nearly circular (except Mercury and Pluto), the assumption of perfectly circular motion was a good approximation. Furthermore, since a geocentric model maintains circular motion, it was very difficult to make any observational distinction between geocentric and heliocentric universes. (Parallax effects are far too small to be noticeable with the naked eye.)

2.12 (a) The graph of $\log_{10} P$ vs. $\log_{10} a$ for the Galilean moons is given in Fig. S2.1.

(b) Using the data for Io and Callisto, we find a slope of 1.5.

(c) Assuming that the mass of Jupiter is much greater than the masses of any of the Galilean moons, Kepler's third law can be written as

$$\log M + 2\log P = \log\left(\frac{4\pi^2}{G}\right) + 3\log a,$$

or

$$\begin{aligned}\log P &= \frac{3}{2}\log a + \frac{1}{2}\log\left(\frac{4\pi^2}{G}\right) - \frac{1}{2}\log M.\\ &= m\log a + b\end{aligned}$$

where the y-intercept is

$$b = \frac{1}{2}\log\left(\frac{4\pi^2}{G}\right) - \frac{1}{2}\log M.$$

Solving for $\log M$ we have

$$\log M = \log\left(\frac{4\pi^2}{G}\right) - 2b.$$

Taking the slope as $m = 3/2$ and using the data for any of the Galilean moons we find $b = -7.753$ (in SI units). Solving gives $M = 1.900 \times 10^{27}$ kg, in good agreement with the value given in Appendix C and Problem 2.6.

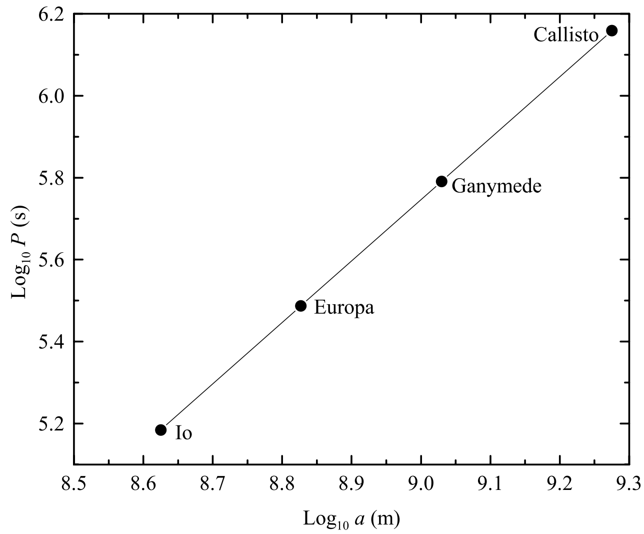

Figure S2.1: $\log_{10} P$ vs. $\log_{10} a$ for the Galilean moons.

2.13 (a) Since the velocity and position vectors are perpendicular at perihelion and aphelion, conservation of angular momentum leads to $r_p v_p = r_a v_a$. Thus

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{1+e}{1-e},$$

where the last relation is obtained from Eqs. (2.5) and (2.6).

(b) Conservation of energy at perihelion and aphelion gives

$$\frac{1}{2}\mu v_a^2 - G\frac{M\mu}{r_a} = \frac{1}{2}\mu v_p^2 - G\frac{M\mu}{r_p}.$$

Making use of Eqs. (2.5) and (2.6), and using the result of part (a) to replace v_p leads to

$$\frac{1}{2}v_a^2 - \frac{GM}{a(1+e)} = \frac{1}{2}v_a^2\left(\frac{1+e}{1-e}\right)^2 - \frac{GM}{1(1-e)}.$$

After some manipulation, we obtain Eq. (2.34); Eq. (2.33) follows immediately.

(c) The orbital angular momentum can now be obtained from

$$L = \mu r_p v_p = \mu a(1-e)\sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}.$$

Equation (2.30) follows directly.

2.14 (a) From Kepler's third law in the form $P^2 = a^3$ (P in years and a in AU), $a = 17.9$ AU.

(b) Since $m_{\text{comet}} \ll M_\odot$, Kepler's third law in the form of Eq. (2.37) gives

$$M_\odot \simeq \frac{4\pi^2 a^3}{GP^2} = 1.98 \times 10^{30} \text{ kg}.$$

(c) From Example 2.1.1, at perihelion $r_p = a(1-e) = 0.585$ AU and at aphelion $r_a = a(1+e) = 35.2$ AU.

(d) At perihelion, Eq. (2.33) gives $v_p = 55\ \mathrm{km\ s^{-1}}$, and at aphelion, Eq. (2.34) gives $v_a = 0.91\ \mathrm{km\ s^{-1}}$. When the comet is on the semiminor axis $r = a$, and Eq. (2.36) gives

$$v = \sqrt{\frac{GM_\odot}{a}} = 7.0\ \mathrm{km\ s^{-1}}.$$

(e) $K_p/K_a = \left(v_p/v_a\right)^2 = 3650$.

2.15 Using 50,000 time steps, $r = \sqrt{x^2 + y^2} \simeq 1$ AU when $t \simeq 0.105$ yr. (*Note:* `Orbit` can be downloaded from the companion web site at `http://www.aw-bc.com/astrophysics`.)

2.16 The data are plotted in Fig. S2.2. (*Note:* `Orbit` can be downloaded from the companion web site at `http://www.aw-bc.com/astrophysics`.)

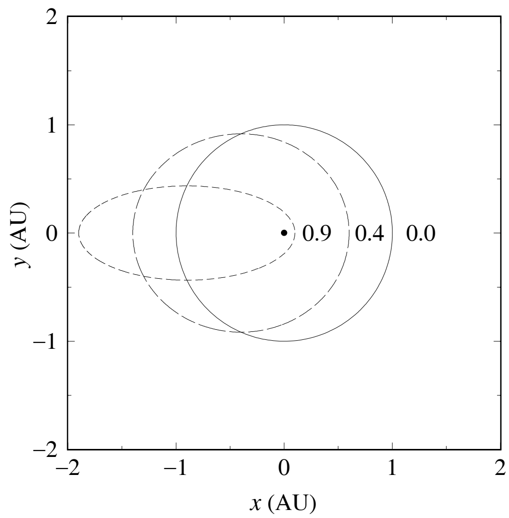

Figure S2.2: Results for Problem 2.16.

2.17 (*Note:* `Orbit` can be downloaded from the companion web site at `http://www.aw-bc.com/astrophysics`.)

(a) See Fig. S2.3.

(b) See Fig. S2.3.

(c) Figure S2.3 shows that the orbit of Mars is very close to a perfect circle, with the center of the circle slightly offset from the focal point of the ellipse. Kepler's early attempts at developing a model of the solar system based on perfect circles were not far off.

2.18 A modified Fortran 95 version of `Orbit` that works for this problem is given below.

(a) The orbits generated by the modified `Orbit` are shown in Fig. S2.4.

(b) The calculation indicates $S = 2.205$ yr.

(c) Eq. (1.1) yields a value of $S = 2.135$ yr. The results do not agree exactly because the derivation of Eq. (1.1) assumes constant speeds throughout the orbits.

(d) No, because the relative speeds during the partially-completed orbits are different.

(e) Since the orbits are not circular, Mars is at different distances from Earth during different oppositions. The closest opposition occurs when Earth is at aphelion and Mars is at perihelion, as in the start of this calculation.

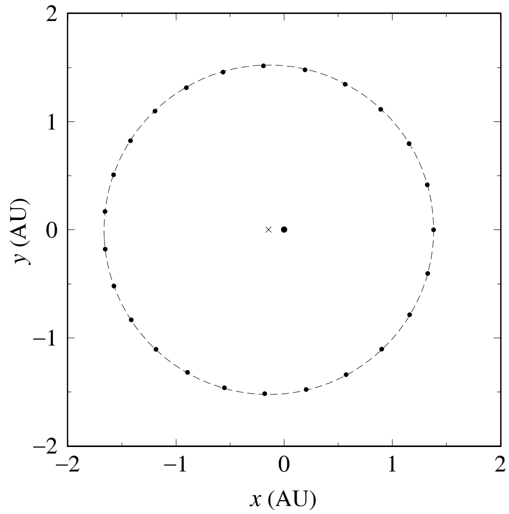

Figure S2.3: Results for Problem 2.17. The dots designate the elliptical orbit of Mars, and the principal focus of the ellipse is indicated by the circle at $(x, y) = (0, 0)$. The dashed line is for a perfect circle of radius $r = a = 1.5237$ AU centered at $x = -ae = -0.1423$ AU (marked by the $\times$).

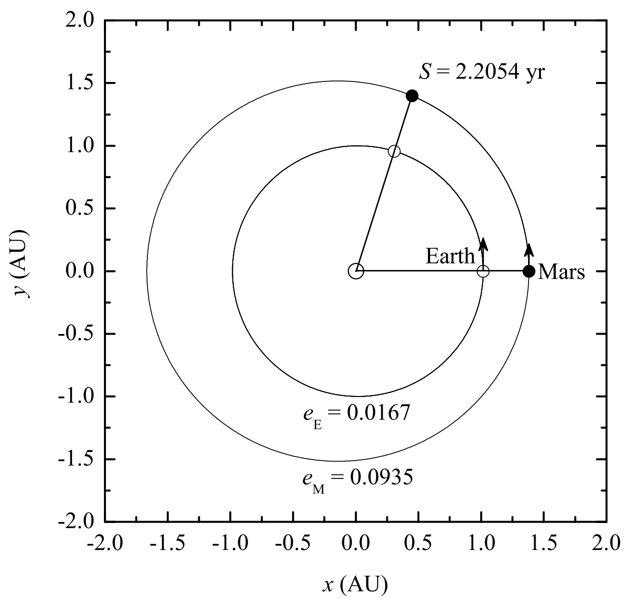

Figure S2.4: The orbits of Earth and Mars including correct eccentricities. The positions of two successive oppositions are shown. The first opposition occurs when Earth is at aphelion and Mars is at perihelion (the positions of closest approach).

```
PROGRAM Orbit
!
!   General Description:
!   ====================
!       Orbit computes the orbit of a small mass about a much larger mass,
!       or it can be considered as computing the motion of the reduced mass
!       about the center of mass.
!
!       "An Introduction to Modern Astrophysics", Appendix J
!       Bradley W. Carroll and Dale A. Ostlie
!       Second Edition, Addison Wesley, 2007
!
!       Weber State University
!       Ogden, UT
!       modastro@weber.edu
!-------------------------------------------------------------------

!       **************This version has been modified for Problem 2.17*****************

    USE Constants,  ONLY    :   i1, dp, G, AU, M_Sun, pi, two_pi, yr, &
                                radians_to_degrees, eps_dp

    IMPLICIT NONE
    REAL(dp)                ::  t, dt, LoM_E, LoM_M, P_E, P_M
    REAL(dp)                ::  Mstar, theta_E, dtheta_E, theta_M, dtheta_M, r_E, r_M
    INTEGER                 ::  n, k, kmax
    INTEGER(i1)             ::  ios         !I/O error flag
    REAL(dp)                ::  delta       !error range at end of period
    CHARACTER               ::  xpause

    REAL(dp), PARAMETER     ::  a_E = AU, a_M = 1.5236*AU, e_E = 0.0167, e_M = 0.0935

!   Open the output file
    OPEN (UNIT = 10, FILE = "Orbit.txt", STATUS = 'REPLACE', ACTION = 'WRITE', &
        IOSTAT = ios)
    IF (ios /= 0) THEN
        WRITE (*,'(" Unable to open Orbit.txt.  --- Terminating calculation")')
        STOP
    END IF


!   Convert entered values to conventional SI units
    Mstar = M_Sun

!   Calculate the orbital period of Earth in seconds using Kepler's Third Law (Eq. 2.37)
!   To be used to determine time steps
    P_E = SQRT(4*pi**2*a_E**3/(G*Mstar))

!   Enter the number of time steps and the time interval to be printed
    n = 100000
    n = n + 1   !increment to include t=0 (initial) point
    kmax = 100

!   Print header information for output file
    WRITE (10,'("t, theta_E, r_E, x_E, y_E, theta_M, r_M, x_M, y_M")')

!   Initialize print counter, angle, elapsed time, and time step.
    k = 1                   !printer counter
    theta_E = 0             !angle from direction to perihelion (radians)
    theta_M = 0

    t = 0                   !elapsed time (s)
    dt = P_E/(n-1)          !time step (s)
    delta = eps_dp          !allowable error at end of period

!   Start main time step loop
    DO
!       Calculate the distance from the principal focus using Eq. (2.3); Kepler's First Law.
        r_E = a_E*(1 - e_E**2)/(1 - e_E*COS(theta_E))       !Earth starts at aphelion
        r_M = a_M*(1 - e_M**2)/(1 + e_M*COS(theta_M))       !Mars starts at perihelion

!       If time to print, convert to cartesian coordinates.  Be sure to print last point also.
        IF (k == 1 .OR. (theta_E - theta_M)/two_pi > 1 + delta) &
            WRITE (10, '(9F10.4)') t/yr, theta_E*radians_to_degrees, r_E/AU, &
```

```
                                    r_E*COS(theta_E)/AU, r_E*SIN(theta_E)/AU, &
                                    theta_M*radians_to_degrees, r_M/AU, &
                                    r_M*COS(theta_M)/AU, r_M*SIN(theta_M)/AU

!       Exit the loop if Earth laps Mars.
        IF ((theta_E - theta_M)/two_pi > 1 + delta) EXIT

!       Prepare for the next time step:  Update the elapsed time.
        t = t + dt

!       Calculate the angular momentum per unit mass, L/m (Eq. 2.30).
        LoM_E = SQRT(G*Mstar*a_E*(1 - e_E**2))
        LoM_M = SQRT(G*Mstar*a_M*(1 - e_M**2))

!       Compute the next value for theta using the fixed time step by combining
!           Eq. (2.31) with Eq. (2.32), which is Kepler's Second Law.
        dtheta_E = LoM_E/r_E**2*dt
        theta_E = theta_E + dtheta_E

        dtheta_M = LOM_M/r_M**2*dt
        theta_M = theta_M + dtheta_M

!       Reset the print counter if necessary
        k = k + 1
        IF (k > kmax) k = 1

    END DO

    WRITE (*,'(/,"The calculation is finished and the data are in Orbit.txt")')

    WRITE (*,'(//,"Enter any character and press <enter> to exit:  ")', ADVANCE = 'NO')
    READ  (*,*) xpause
END PROGRAM Orbit
```