Chapter Two: Linear Programming: Model Formulation and Graphical Solution

PROBLEM SUMMARY

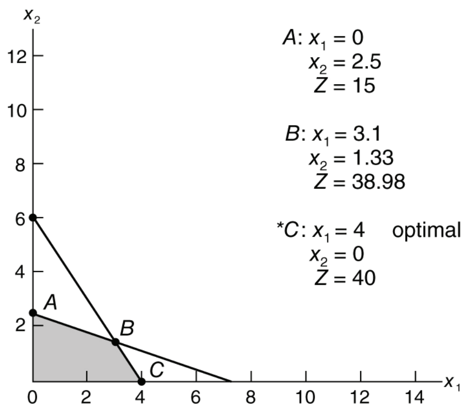
1. Maximization (1–28 continuation), graphical solution
2. Minimization, graphical solution
3. Sensitivity analysis (2–2)
4. Minimization, graphical solution
5. Maximization, graphical solution
6. Slack analysis (2–5), sensitivity analysis
7. Maximization, graphical solution
8. Slack analysis (2–7)
9. Maximization, graphical solution
10. Minimization, graphical solution
11. Maximization, graphical solution
12. Sensitivity analysis (2–11)
13. Sensitivity analysis (2–11)
14. Maximization, graphical solution
15. Sensitivity analysis (2–14)
16. Maximization, graphical solution
17. Sensitivity analysis (2–16)
18. Maximization, graphical solution
19. Standard form (2–18)
20. Maximization, graphical solution
21. Constraint analysis (2–20)
22. Minimization, graphical solution
23. Sensitivity analysis (2–22)
24. Sensitivity analysis (2–22)
25. Sensitivity analysis (2–22)
26. Minimization, graphical solution
27. Minimization, graphical solution
28. Sensitivity analysis (2–27)
29. Minimization, graphical solution
30. Maximization, graphical solution
31. Minimization, graphical solution
32. Maximization, graphical solution
33. Sensitivity analysis (2–32)
34. Minimization, graphical solution
35. Maximization, graphical solution
36. Maximization, graphical solution
37. Sensitivity analysis (2–36)
38. Maximization, graphical solution
39. Sensitivity analysis (2–38)
40. Maximization, graphical solution
41. Sensitivity analysis (2–40)
42. Minimization, graphical solution
43. Sensitivity analysis (2–42)
44. Maximization, graphical solution
45. Sensitivity analysis (2–44)
46. Maximization, graphical solution
47. Sensitivity analysis (2–46)
48. Maximization, graphical solution
49. Minimization, graphical solution
50. Sensitivity analysis (2–49)
51. Minimization, graphical solution
52. Sensitivity analysis (2–51)
53. Maximization, graphical solution
54. Minimization, graphical solution
55. Sensitivity analysis (2–54)
56. Maximization, graphical solution
57. Sensitivity analysis (2–56)
58. Maximization, graphical solution
59. Sensitivity analysis (2–58)
60. Multiple optimal solutions
61. Infeasible problem
62. Unbounded problem

PROBLEM SOLUTIONS

1. a) x_1 = # cakes
x_2 = # loaves of bread
maximize $Z = \$10x_1 + 6x_2$
subject to

$$3x_1 + 8x_2 \le 20 \text{ cups of flour}$$
$$45x_1 + 30x_2 \le 180 \text{ minutes}$$
$$x_1, x_2 \ge 0$$

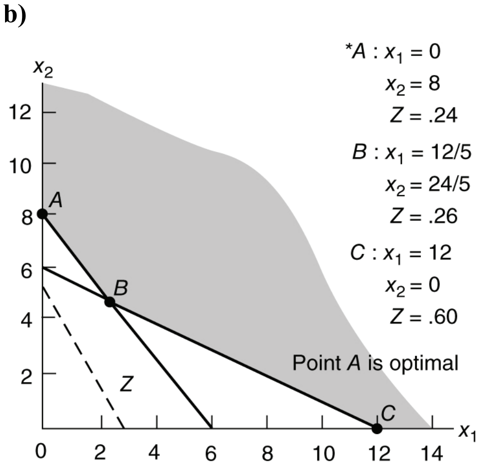
b)



2. a) Minimize $Z = .05x_1 + .03x_2$ (cost, \$)
subject to

$$8x_1 + 6x_2 \ge 48 \text{ (vitamin A, mg)}$$
$$x_1 + 2x_2 \ge 12 \text{ (vitamin B, mg)}$$
$$x_1, x_2 \ge 0$$

b)



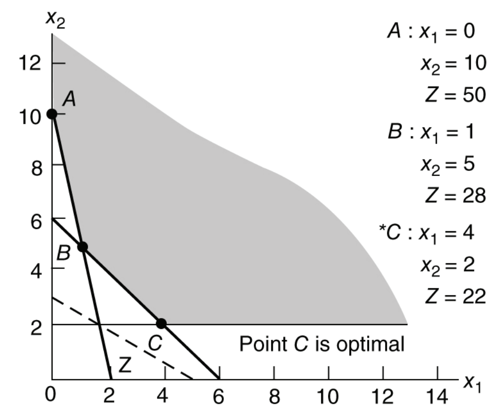
3. The optimal solution point would change from point A to point B, thus resulting in the optimal solution

$x_1 = 12/5 \quad x_2 = 24/5 \quad Z = .408$

4. a) Minimize $Z = 3x_1 + 5x_2$ (cost, \$)
subject to

$$10x_1 + 2x_2 \ge 20 \text{ (nitrogen, oz)}$$
$$6x_1 + 6x_2 \ge 36 \text{ (phosphate, oz)}$$
$$x_2 \ge 2 \text{ (potassium, oz)}$$
$$x_1, x_2 \ge 0$$

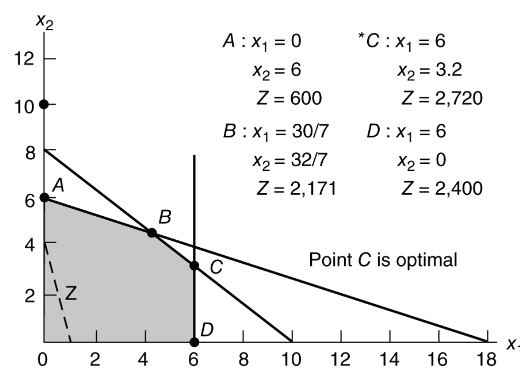
b)



5. a) Maximize $Z = 400x_1 + 100x_2$ (profit, \$)
subject to

$$8x_1 + 10x_2 \le 80 \text{ (labor, hr)}$$
$$2x_1 + 6x_2 \le 36 \text{ (wood)}$$
$$x_1 \le 6 \text{ (demand, chairs)}$$
$$x_1, x_2 \ge 0$$

b)



6. a) In order to solve this problem, you must substitute the optimal solution into the resource constraint for wood and the resource constraint for labor and determine how much of each resource is left over.

Labor

$$8x_1 + 10x_2 \le 80 \text{ hr}$$
$$8(6) + 10(3.2) \le 80$$
$$48 + 32 \le 80$$
$$80 \le 80$$

There is no labor left unused.

Wood

$$2x_1 + 6x_2 \le 36$$
$$2(6) + 6(3.2) \le 36$$
$$12 + 19.2 \le 36$$
$$31.2 \le 36$$
$$36 - 31.2 = 4.8$$

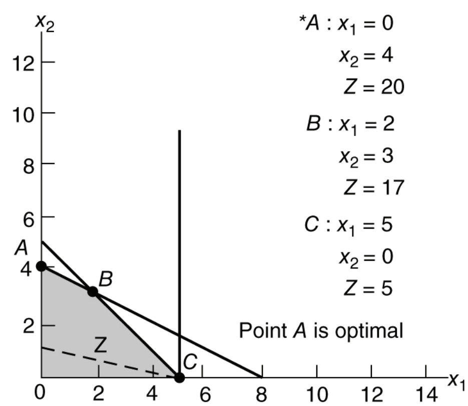
There is 4.8 lb of wood left unused.

b) The new objective function, $Z = 400x_1 + 500x_2$, is parallel to the constraint for labor, which results in multiple optimal solutions. Points B ($x_1 = 30/7$, $x_2 = 32/7$) and C ($x_1 = 6$, $x_2 = 3.2$) are the alternate optimal solutions, each with a profit of $4,000.

7. a) Maximize $Z = x_1 + 5x_2$ (profit, $)
subject to

$5x_1 + 5x_2 \le 25$ (flour, lb)
$2x_1 + 4x_2 \le 16$ (sugar, lb)
$x_1 \le 5$ (demand for cakes)
$x_1, x_2 \ge 0$

b)



8. In order to solve this problem, you must substitute the optimal solution into the resource constraints for flour and sugar and determine how much of each resource is left over.

Flour

$$5x_1 + 5x_2 \le 25 \text{ lb}$$
$$5(0) + 5(4) \le 25$$
$$20 \le 25$$
$$25 - 20 = 5$$

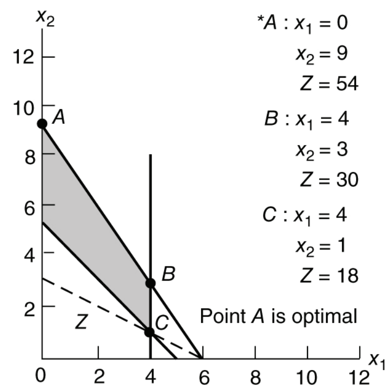
There are 5 lb of flour left unused.

Sugar

$$2x_1 + 4x_2 \le 16$$
$$2(0) + 4(4) \le 16$$
$$16 \le 16$$

There is no sugar left unused.

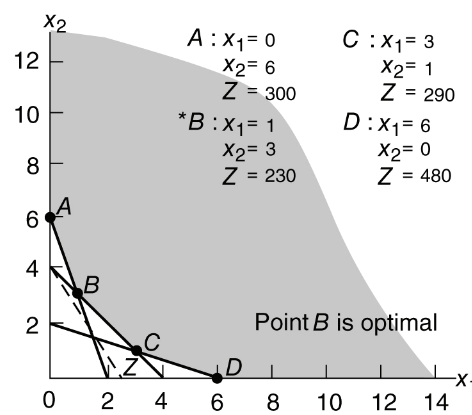
9.



10. a) Minimize $Z = 80x_1 + 50x_2$ (cost, $)
subject to

$3x_1 + x_2 \ge 6$ (antibiotic 1, units)
$x_1 + x_2 \ge 4$ (antibiotic 2, units)
$2x_1 + 6x_2 \ge 12$ (antibiotic 3, units)
$x_1, x_2 \ge 0$

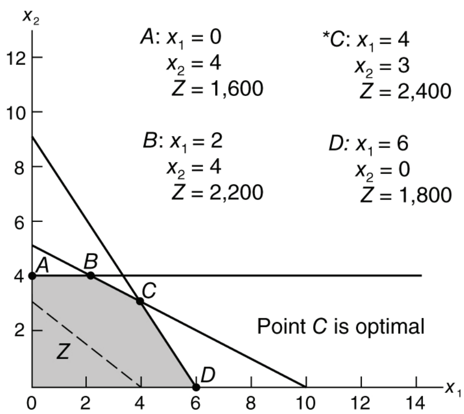
b)



11. a) Maximize $Z = 300x_1 + 400x_2$ (profit, $)
subject to

$3x_1 + 2x_2 \le 18$ (gold, oz)
$2x_1 + 4x_2 \le 20$ (platinum, oz)
$x_2 \le 4$ (demand, bracelets)
$x_1, x_2 \ge 0$

b)



12. The new objective function, $Z = 300x_1 + 600x_2$, is parallel to the constraint line for platinum, which results in multiple optimal solutions. Points B ($x_1 = 2$, $x_2 = 4$) and C ($x_1 = 4$, $x_2 = 3$) are the alternate optimal solutions, each with a profit of $3,000.

The feasible solution space will change. The new constraint line, $3x_1 + 4x_2 = 20$, is parallel to the existing objective function. Thus, multiple optimal solutions will also be present in this scenario. The alternate optimal solutions are at $x_1 = 1.33$, $x_2 = 4$ and $x_1 = 2.4$, $x_2 = 3.2$, each with a profit of $2,000.

13. a) Optimal solution: $x_1 = 4$ necklaces, $x_2 = 3$ bracelets. The maximum demand is not achieved by the amount of one bracelet.

b) The solution point on the graph which corresponds to no bracelets being produced must be on the x_1 axis where $x_2 = 0$. This is point D on the graph. In order for point D to be optimal, the objective function "slope" must change such that it is equal to or greater than the slope of the constraint line, $3x_1 + 2x_2 = 18$. Transforming this constraint into the form $y = a + bx$ enables us to compute the slope:

$$2x_2 = 18 - 3x_1$$
$$x_2 = 9 - 3/2x_1$$

From this equation the slope is −3/2. Thus, the slope of the objective function must be at least −3/2. Presently, the slope of the objective function is −3/4:

$$400x_2 = Z - 300x_1$$
$$x_2 = Z/400 - 3/4x_1$$

The profit for a necklace would have to increase to *$600* to result in a slope of −3/2:

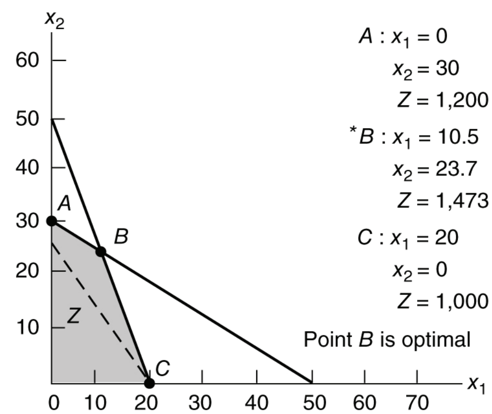
$$400x_2 = Z - 600x_1$$
$$x_2 = Z/400 - 3/2x_1$$

However, this creates a situation where both points C and D are optimal, ie., multiple optimal solutions, as are all
points on the line segment between
C and D.

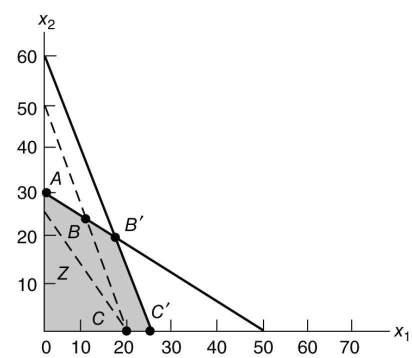
14. a) Maximize $Z = 50x_1 + 40x_2$ (profit, $) subject to

$$3x_1 + 5x_2 \leq 150 \text{ (wool, yd}^2\text{)}$$
$$10x_1 + 4x_2 \leq 200 \text{ (labor, hr)}$$
$$x_1, x_2 \geq 0$$

b)



15. The feasible solution space changes from the area *0ABC* to *0AB′C′*, as shown on the following graph.



The extreme points to evaluate are now *A*, *B′*, and *C′*.

A: $x_1 = 0$
$x_2 = 30$
$Z = 1,200$

**B′*: $x_1 = 15.8$
$x_2 = 20.5$
$Z = 1,610$

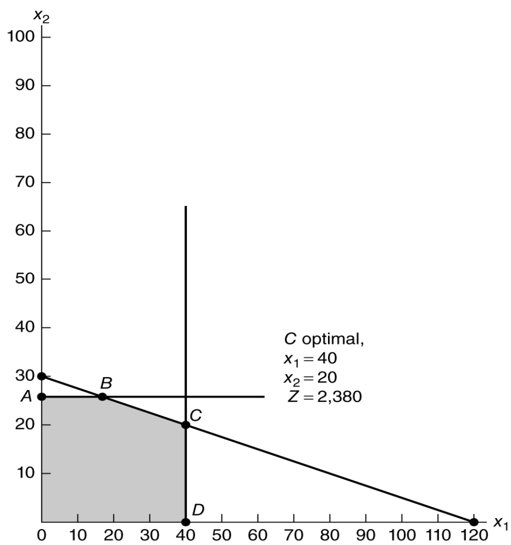
C': $x_1 = 24$
$x_2 = 0$
$Z = 1{,}200$

Point B' is optimal

16. a) Maximize $Z = 23x_1 + 73x_2$
subject to

$$x_1 \le 40$$
$$x_2 \le 25$$
$$x_1 + 4x_2 \le 120$$
$$x_1, x_2 \ge 0$$

b)



17. a) No, not this winter, but they might after they recover equipment costs, which should be after the 2nd winter.

b) $x_1 = 55$
$x_2 = 16.25$
$Z = 1{,}851$

No, profit will go down

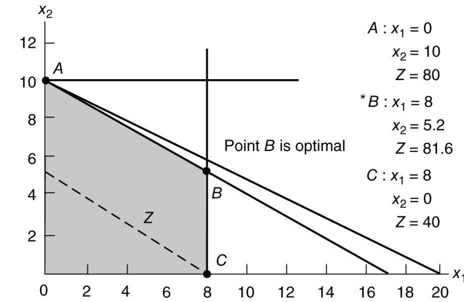
c) $x_1 = 40$
$x_2 = 25$
$Z = 2{,}435$

Profit will increase slightly

d) $x_1 = 55$
$x_2 = 27.72$
$Z = \$2{,}073$

Profit will go down from (c)

18.

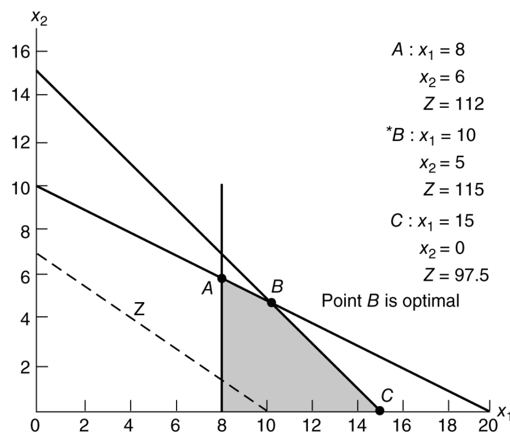


19. Maximize $Z = 5x_1 + 8x_2 + 0s_1 + 0s_3 + 0s_4$
subject to

$$3x_1 + 5x_2 + s_1 = 50$$
$$2x_1 + 4x_2 + s_2 = 40$$
$$x_1 + s_3 = 8$$
$$x_2 + s_4 = 10$$
$$x_1, x_2 \ge 0$$

A: $s_1 = 0$, $s_2 = 0$, $s_3 = 8$, $s_4 = 0$
B: $s_1 = 0$, $s_2 = 3.2$, $s_3 = 0$, $s_4 = 4.8$
C: $s_1 = 26$, $s_2 = 24$, $s_3 = 0$, $s_4 = 10$

20.

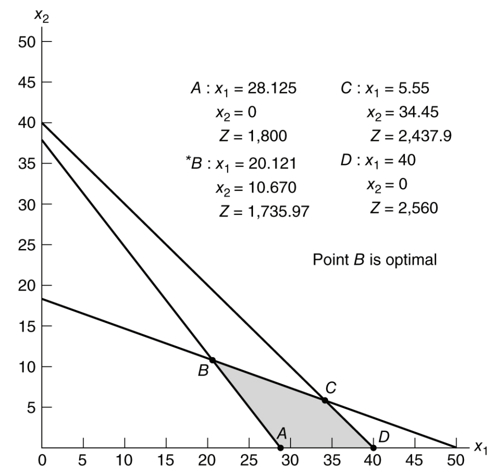


21. It changes the optimal solution to point A ($x_1 = 8$, $x_2 = 6$, $Z = 112$), and the constraint, $x_1 + x_2 \le 15$, is no longer part of the solution space boundary.

22. a) Minimize $Z = 64x_1 + 42x_2$ (labor cost, \$)
subject to

$$16x_1 + 12x_2 \ge 450 \text{ (claims)}$$
$$x_1 + x_2 \le 40 \text{ (workstations)}$$
$$0.5x_1 + 1.4x_2 \le 25 \text{ (defective claims)}$$
$$x_1, x_2 \ge 0$$

b)

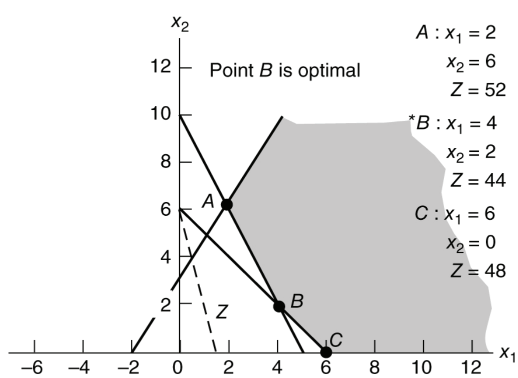


23. Changing the pay for a full-time claims solution to point A in the graphical solution where $x_1 = 28.125$ and $x_2 = 0$, i.e., there will be no part-time operators. Changing the pay for a part-time operator from \$42 to \$36 has no effect on the number of full-time and part-time operators hired, although the total cost will be reduced to \$1,671.95.

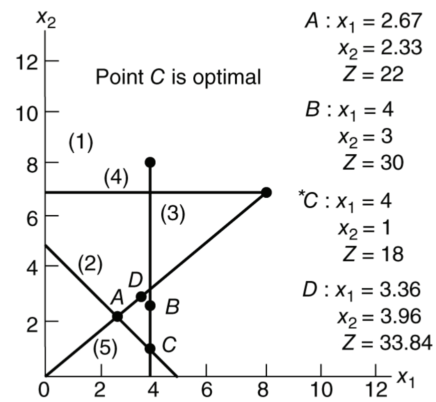
24. Eliminating the constraint for defective claims would result in a new solution, $x_1 = 0$ and $x_2 = 37.5$, where only part-time operators would be hired.

25. The solution becomes infeasible; there are not enough workstations to handle the increase in the volume of claims.

26.

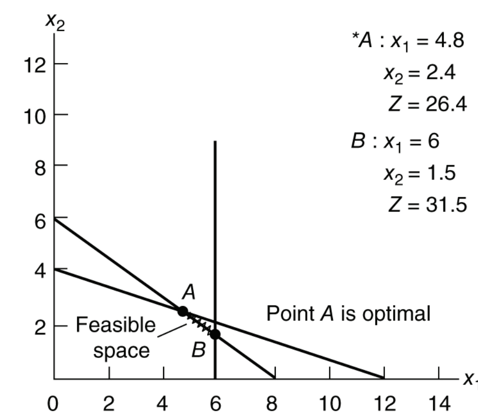


27.

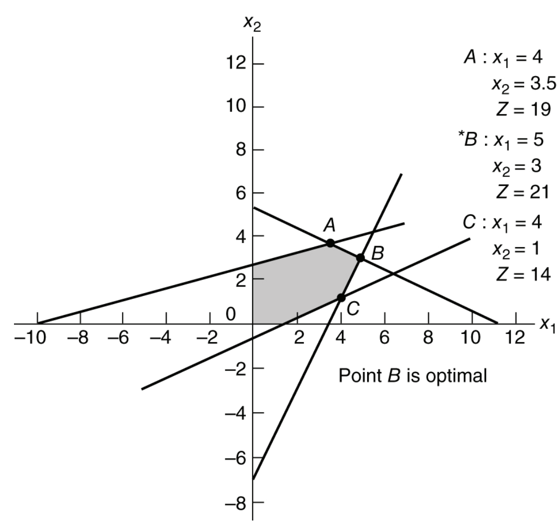


28. The problem becomes infeasible.

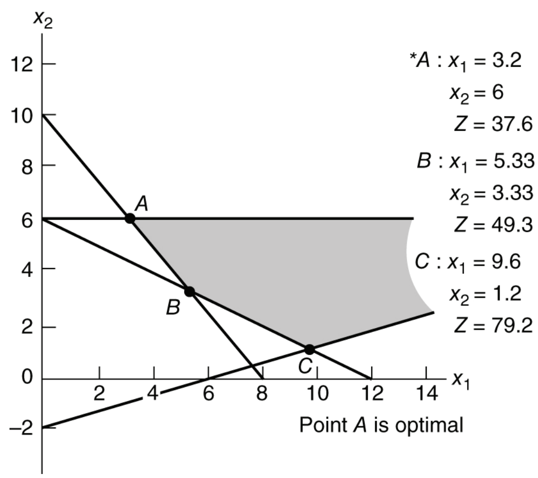
29.



30.



31.



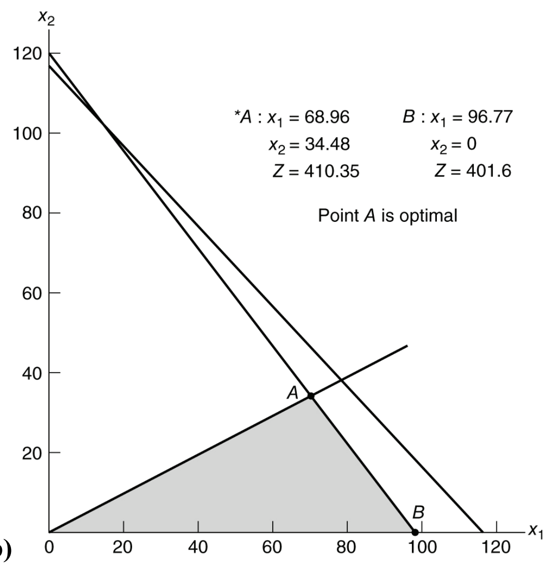
32. a) Maximize $Z = \$4.15x_1 + 3.60x_2$ (profit, \$) subject to

$x_1 + x_2 \leq 115$ (freezer space, gals.)

$0.93x_1 + 0.75x_2 \leq 90$ (budget, \$)

$\frac{x_1}{x_2} \geq \frac{2}{1}$ or $x_1 - 2x_2 \geq 0$ (demand)

$x_1, x_2 \geq 0$



b)

33. No additional profit, freezer space is not a binding constraint.

34. a) Minimize $Z = 200x_1 + 160x_2$ (cost, \$) subject to

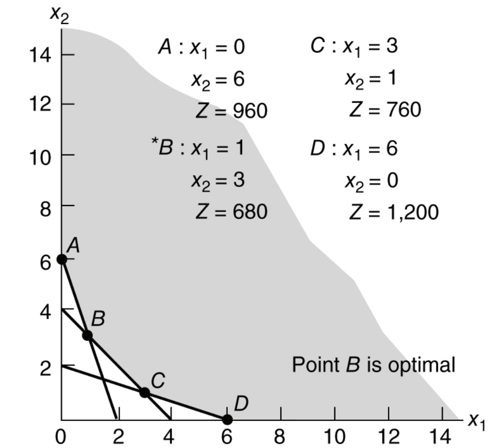
$6x_1 + 2x_2 \geq 12$ (high-grade ore, tons)

$2x_1 + 2x_2 \geq 8$ (medium-grade ore, tons)

$4x_1 + 12x_2 \geq 24$ (low-grade ore, tons)

$x_1, x_2 \geq 0$

b)



35. a) Maximize $Z = 800x_1 + 900x_2$ (profit, \$) subject to

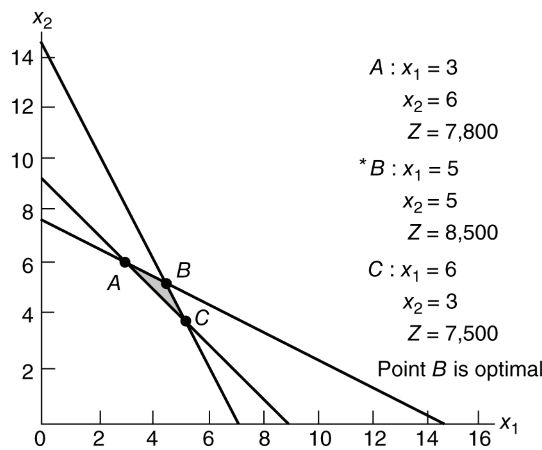
$2x_1 + 4x_2 \leq 30$ (stamping, days)

$4x_1 + 2x_2 \leq 30$ (coating, days)

$x_1 + x_2 \geq 9$ (lots)

$x_1, x_2 \geq 0$

b)



36. a) Maximize $Z = 30x_1 + 70x_2$ (profit, \$) subject to

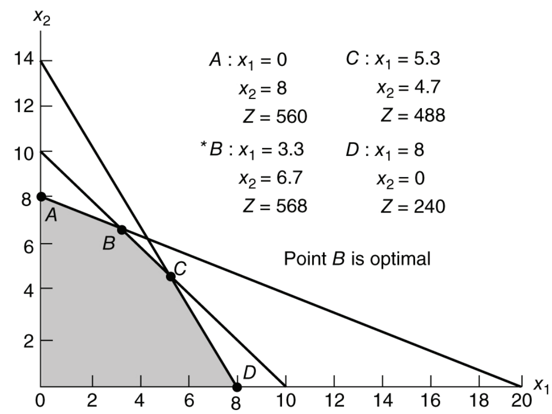
$4x_1 + 10x_2 \leq 80$ (assembly, hr)

$14x_1 + 8x_2 \leq 112$ (finishing, hr)

$x_1 + x_2 \leq 10$ (inventory, units)

$x_1, x_2 \geq 0$

b)



37. The slope of the original objective function is computed as follows:

$$Z = 30x_1 + 70x_2$$
$$70x_2 = Z - 30x_1$$
$$x_2 = Z/70 - 3/7x_1$$
$$\text{slope} = -3/7$$

The slope of the new objective function is computed as follows:

$$Z = 90x_1 + 70x_2$$
$$70x_2 = Z - 90x_1$$
$$x_2 = Z/70 - 9/7x_1$$
$$\text{slope} = -9/7$$

The change in the objective function not only changes the Z values but also results in a new solution point, C. The slope of the new objective function is steeper and thus changes the solution point.

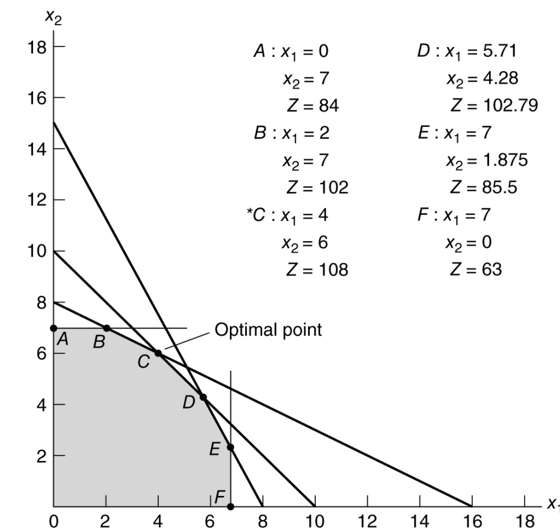
A: $x_1 = 0$, $x_2 = 8$, $Z = 560$ $\quad$ C: $x_1 = 5.3$, $x_2 = 4.7$, $Z = 806$

B: $x_1 = 3.3$, $x_2 = 6.7$, $Z = 766$ $\quad$ D: $x_1 = 8$, $x_2 = 0$, $Z = 720$

38. a) Maximize $Z = 9x_1 + 12x_2$ (profit, \$1,000s) subject to

$4x_1 + 8x_2 \leq 64$ (grapes, tons)
$5x_1 + 5x_2 \leq 50$ (storage space, yd^3)
$15x_1 + 8x_2 \leq 120$ (processing time, hr)
$x_1 \leq 7$ (demand, Nectar)
$x_2 \leq 7$ (demand, Red)
$x_1, x_2 \geq 0$

b)



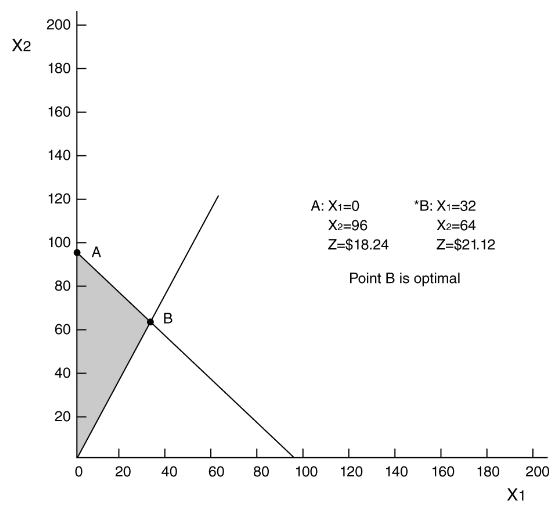
39. a) $15(4) + 8(6) \leq 120$ hr
$60 + 48 \leq 120$
$108 \leq 120$
$120 - 108 = 12$ hr left unused

b) Points C and D would be eliminated and a new optimal solution point at $x_1 = 5.09$, $x_2 = 5.45$, and $Z = 111.27$ would result.

40. a) Maximize $Z = .28x_1 + .19x_2$

$$x_1 + x_2 \leq 96 \text{ cans}$$
$$\frac{x_2}{x_1} \geq 2$$
$$x_1, x_2 \geq 0$$

b)



41. The model formulation would become,
maximize $Z = \$0.23x_1 + 0.19x_2$
subject to

$$x_1 + x_2 \le 96$$
$$-1.5x_1 + x_2 \ge 0$$
$$x_1, x_2 \ge 0$$

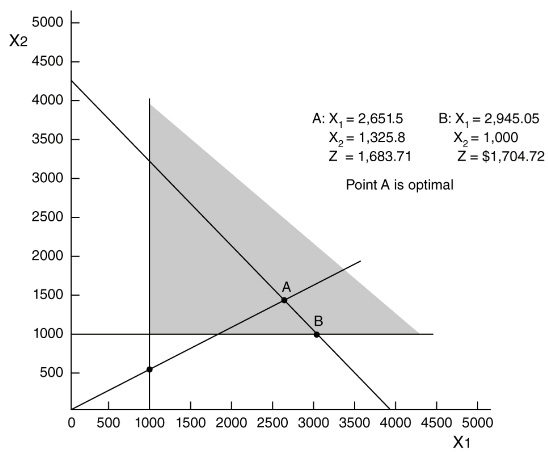
The solution is $x_1 = 38.4$, $x_2 = 57.6$, and $Z = \$19.78$

The discount would reduce profit.

42. a) Minimize $Z = \$0.46x_1 + 0.35x_2$
subject to

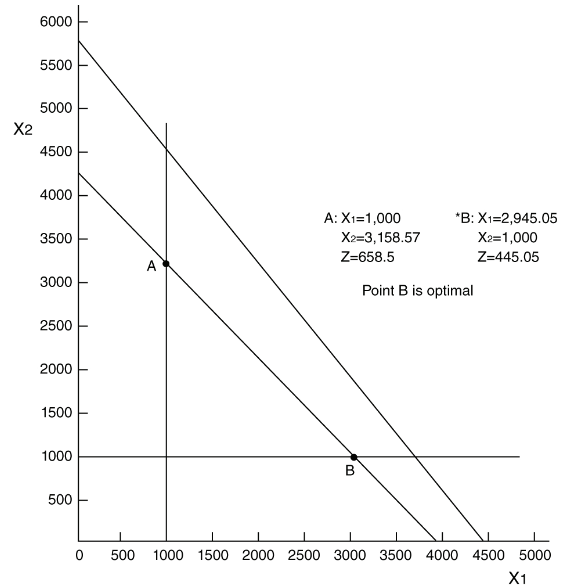
$$.91x_1 + .82x_2 = 3{,}500$$
$$x_1 \ge 1{,}000$$
$$x_2 \ge 1{,}000$$
$$.03x_1 - .06x_2 \ge 0$$
$$x_1, x_2 \ge 0$$

b)



43. a) Minimize $Z = .09x_1 + .18x_2$
subject to

$$.46x_1 + .35x_2 \le 2{,}000$$
$$x_1 \ge 1{,}000$$
$$x_2 \ge 1{,}000$$
$$.91x_1 - .82x_2 = 3{,}500$$
$$x_1, x_2 \ge 0$$

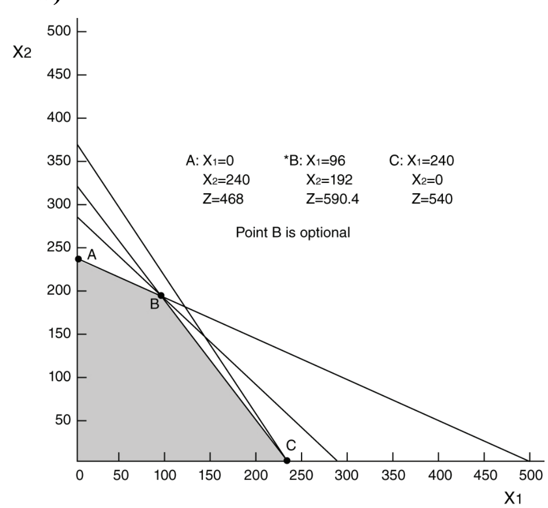


b) 477 − 445 = 32 fewer defective items

44. a) Maximize $Z = \$2.25x_1 + 1.95x_2$
subject to

$$8x_1 + 6x_2 \le 1{,}920$$
$$3x_1 + 6x_2 \le 1{,}440$$
$$3x_1 + 2x_2 \le 720$$
$$x_1 + x_2 \le 288$$
$$x_1, x_2 \ge 0$$

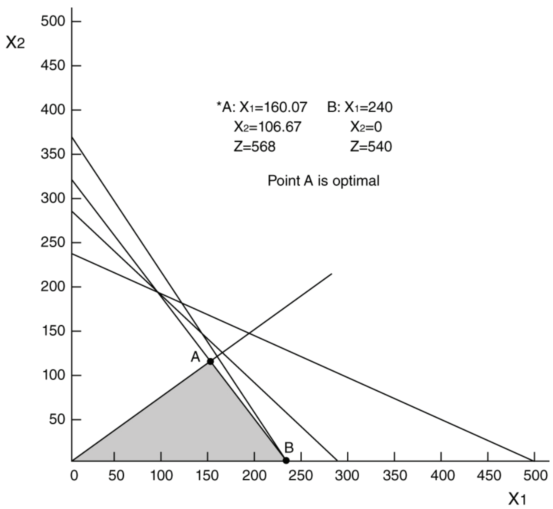
b)



45. A new constraint is added to the model in

$$\frac{x_1}{x_2} \geq 1.5$$

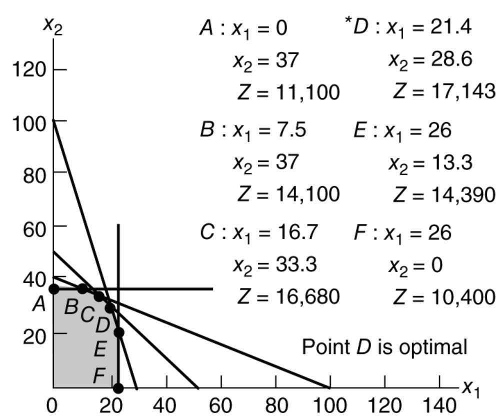
The solution is $x_1 = 160$, $x_2 = 106.67$, $Z = \$568$



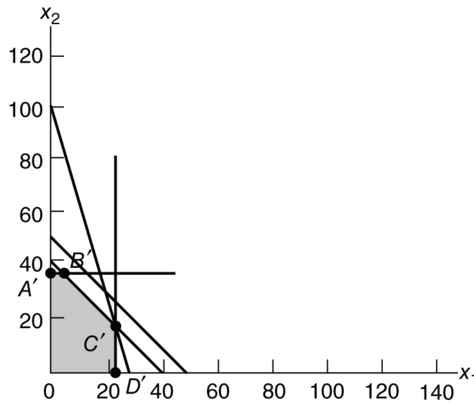
46. a) Maximize $Z = 400x_1 + 300x_2$ (profit, \$) subject to

$$x_1 + x_2 \leq 50 \text{ (available land, acres)}$$
$$10x_1 + 3x_2 \leq 300 \text{ (labor, hr)}$$
$$8x_1 + 20x_2 \leq 800 \text{ (fertilizer, tons)}$$
$$x_1 \leq 26 \text{ (shipping space, acres)}$$
$$x_2 \leq 37 \text{ (shipping space, acres)}$$
$$x_1, x_2 \geq 0$$

b)



47. The feasible solution space changes if the fertilizer constraint changes to $20x_1 + 20x_2 \leq 800$ tons. The new solution space is $A'B'C'D'$. Two of the constraints now have no effect.



The new optimal solution is point C':

A': $x_1 = 0$, $x_2 = 37$, $Z = 11{,}100$

$*C'$: $x_1 = 25.71$, $x_2 = 14.29$, $Z = 14{,}571$

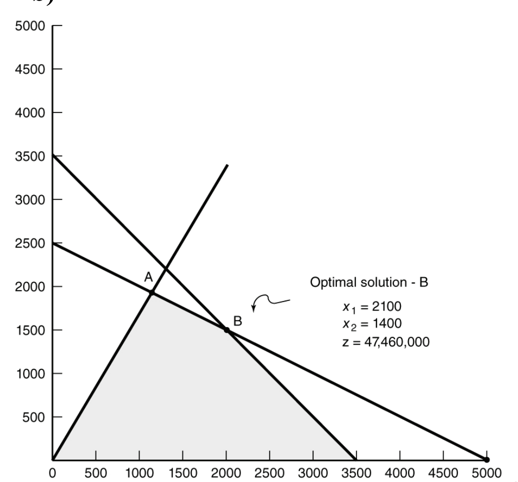
B': $x_1 = 3$, $x_2 = 37$, $Z = 12{,}300$

D': $x_1 = 26$, $x_2 = 0$, $Z = 10{,}400$

48. a) Maximize $Z = \$7{,}600x_1 + 22{,}500x_2$ subject to

$$x_1 + x_2 \leq 3{,}500$$
$$x_2/(x_1 + x_2) \leq .40$$
$$.12x_1 + .24x_2 \leq 600$$
$$x_1, x_2 \geq 0$$

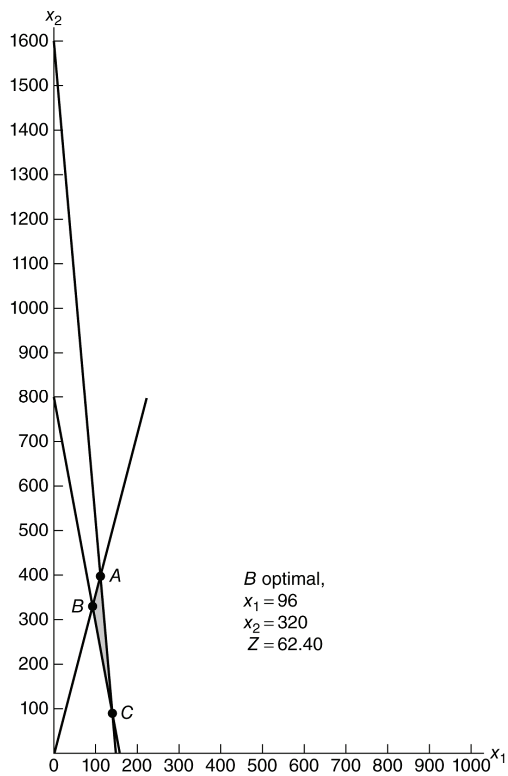
b)



49. a) Minimize $Z = \$(.05)(8)x_1 + (.10)(.75)x_2$
subject to

$$5x_1 + x_2 \geq 800$$
$$\frac{5x_1}{x_2} = 1.5$$
$$8x_1 + .75x_2 \leq 1{,}200$$
$$x_1, x_2 \geq 0$$
$$x_1 = 96$$
$$x_2 = 320$$
$$Z = \$62.40$$

b)



50. The new solution is

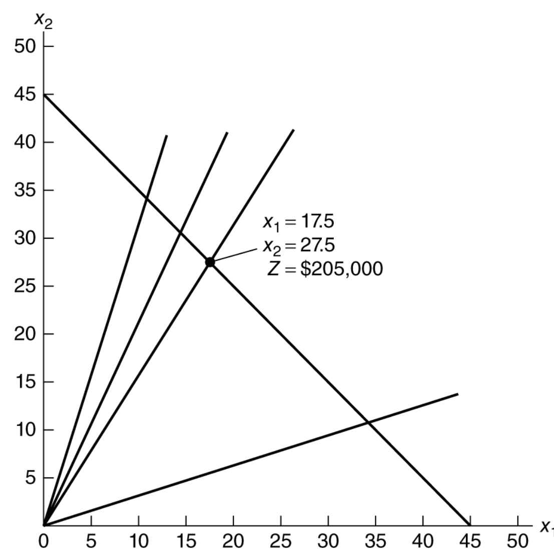
$$x_1 = 106.67$$
$$x_2 = 266.67$$
$$Z = \$62.67$$

If twice as many guests prefer wine to beer, then the Robinsons would be approximately 10 bottles of wine short and they would have approximately 53 more bottles of beer than they need. The waste is more difficult to compute. The model in problem 53 assumes that the Robinsons are ordering more wine and beer than they need, i.e., a buffer, and thus there logically would be some waste, i.e., 5% of the wine and 10% of the beer. However, if twice as many guests prefer wine, then there would logically be no waste for wine but only for beer. This amount "logically" would be the waste from 266.67 bottles, or \$20, and the amount from the additional 53 bottles, \$3.98, for a total of \$23.98.

51. a) Minimize $Z = 3700x_1 + 5100x_2$
subject to

$$x_1 + x_2 = 45$$
$$(32x_1 + 14x_2) / (x_1 + x_2) \leq 21$$
$$.10x_1 + .04x_2 \leq 6$$
$$\frac{x_1}{(x_1 + x_2)} \geq .25$$
$$\frac{x_2}{(x_1 + x_2)} \geq .25$$
$$x_1, x_2 \geq 0$$

b)



52. a) No, the solution would not change

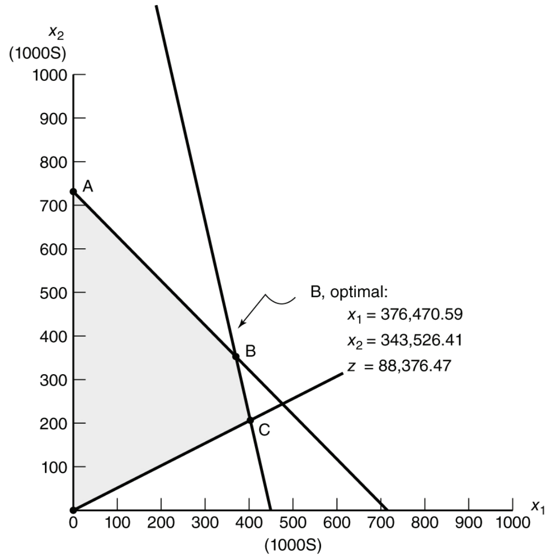
b) No, the solution would not change

c) Yes, the solution would change to China (x_1) = 22.5, Brazil (x_2) = 22.5, and $Z = \$198{,}000$.

53. a) x_1 = \$ invested in stocks
x_2 = \$ invested in bonds
maximize $Z = \$0.18x_1 + 0.06x_2$ (average annual return)
subject to

$x_1 + x_2 \leq \$720{,}000$ (available funds)
$x_1/(x_1 + x_2) \leq .65$ (% of stocks)
$.22x_1 + .05x_2 \leq 100{,}000$ (total possible loss)
$x_1, x_2 \geq 0$

b)



54. x_1 = exams assigned to Brad

x_2 = exams assigned to Sarah

minimize $Z = .10x_1 + .06x_2$

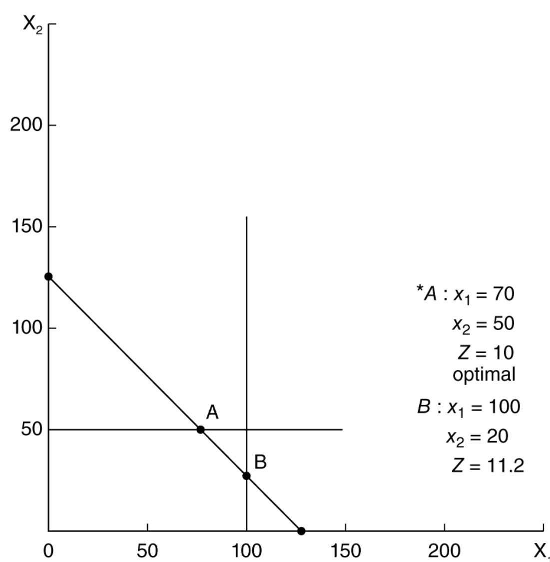
subject to

$x_1 + x_2 = 120$

$x_1 \le (720/7.2)$ or 100

$x_2 \le 50(600/12)$

$x_1, x_2 \ge 0$



55. If the constraint for Sarah's time became $x_2 \le 55$ with an additional hour then the solution point at A would move to $x_1 = 65$, $x_2 = 55$ and $Z = 9.8$. If the constraint for Brad's time became $x_1 \le 108.33$ with an additional hour then the solution point (A) would not change. All of Brad's time is not being used anyway so assigning him more time would not have an effect.

One more hour of Sarah's time would reduce the number of regraded exams from 10 to 9.8, whereas increasing Brad by one hour would have no effect on the solution. This is actually the marginal (or dual) value of one additional hour of labor, for Sarah, which is 0.20 fewer regraded exams, whereas the marginal value of Brad's is zero.

56. a) x_1 = # cups of Pomona

x_2 = # cups of Coastal

Maximize $Z = \$2.05x_1 + 1.85x_2$

subject to

$16x_1 + 16x_2 \le 3{,}840$ oz or (30 gal. × 128 oz)

$(.20)(.0625)x_1 + (.60)(.0625)x_2 \le 6$ lbs. Colombian

$(.35)(.0625)x_1 + (.10)(.0625)x_2 \le 6$ lbs. Kenyan

$(.45)(.0625)x_1 + (.30)(.0625)x_2 \le 6$ lbs. Indonesian

$x_2/x_1 = 3/2$

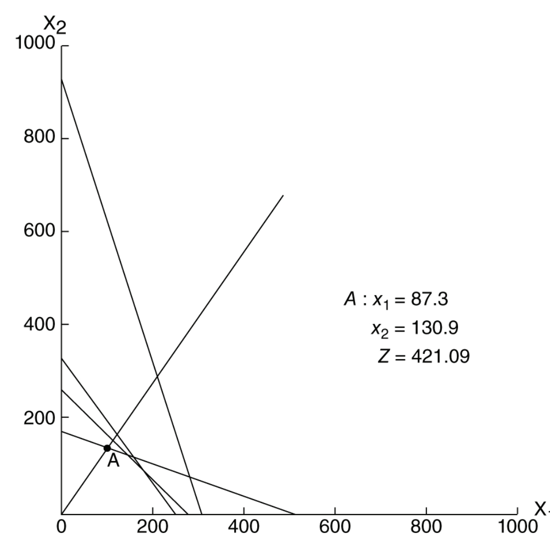
$x_1, x_2 \ge 0$

b) Solution:

$x_1 = 87.3$ cups

$x_2 = 130.9$ cups

$Z = \$421.09$



57. a) The only binding constraint is for Colombian; the constraints for Kenyan and Indonesian are nonbinding and there are already extra, or slack, pounds of these coffees available. Thus, only getting more Colombian would affect the solution.

One more pound of Colombian would increase sales from \$421.09 to \$463.20.

Increasing the brewing capacity to 40 gallons would have no effect since there is already unused brewing capacity with the optimal solution.

b) If the shop increased the demand ratio of Pomona to Coastal from 1.5 to 1 to 2 to 1 it would increase daily sales to \$460.00, so the shop should spend extra on advertising to achieve this result.

58. a) x_1 = 16 in. pizzas

x_2 = hot dogs

Maximize $Z = \$22x_1 + 2.35x_2$

Subject to

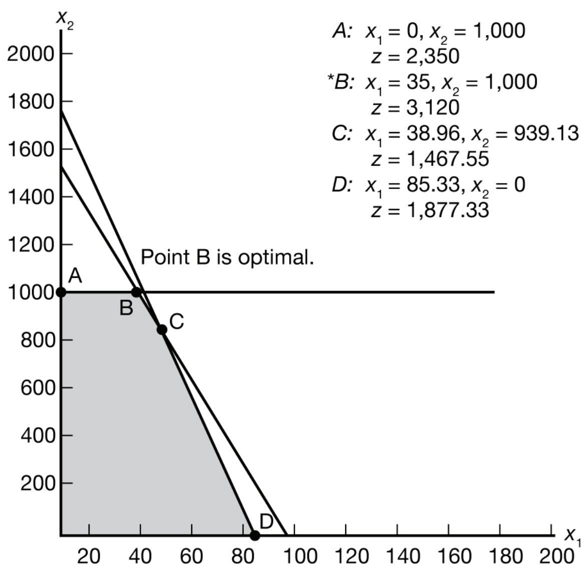
$$\$10x_1 + 0.65x_2 \leq \$1{,}000$$

$$324\text{ in}^2 x_1 + 16\text{ in}^2 x_2 \leq 27{,}648\text{ in}^2$$

$$x_2 \leq 1{,}000$$

$$x_1, x_2 \geq 0$$

b)



59. a) $x_1 = 35$, $x_2 = 1{,}000$, Z = \$3,120

Profit would remain the same (\$3,120) so the increase in the oven cost would decrease the season's profit from \$10,120 to \$8,120.

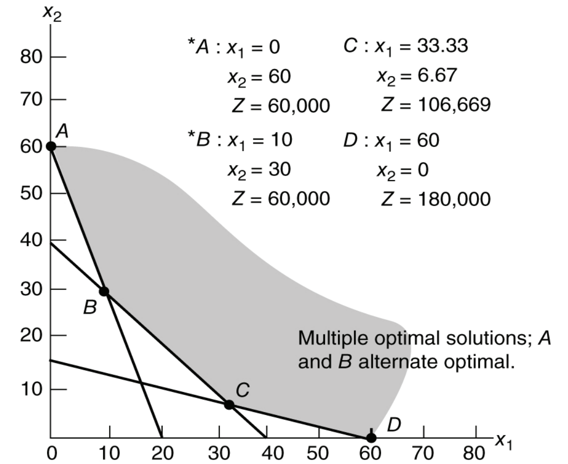
b) $x_1 = 35.95$, $x_2 = 1{,}000$, Z = \$3,140

Profit would increase slightly from \$10,120 to \$10, 245.46.

c) $x_1 = 55.7$, $x_2 = 600$, Z = \$3,235.48

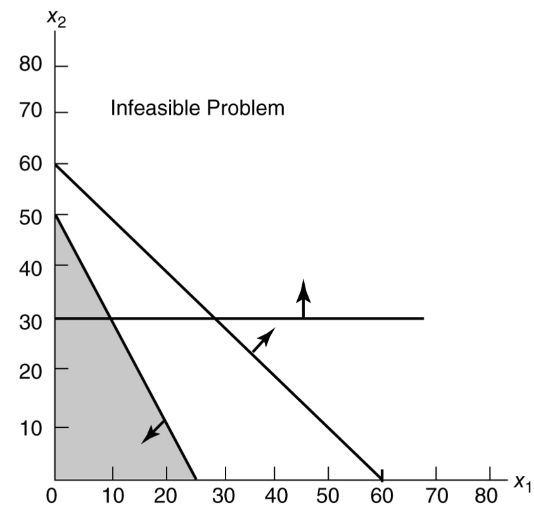
Profit per game would increase slightly.

60.

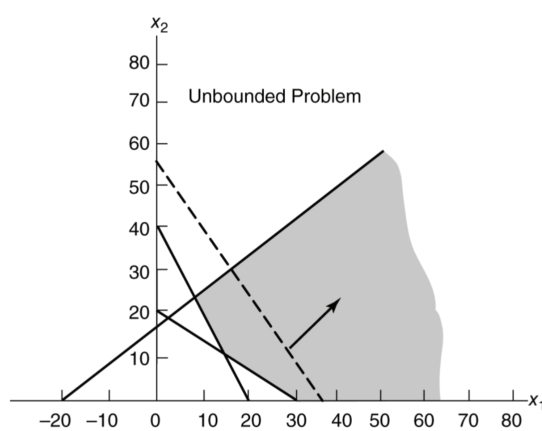


Multiple optimal solutions; *A* and *B* alternate optimal

61.



62.



CASE SOLUTION: METROPOLITAN POLICE PATROL

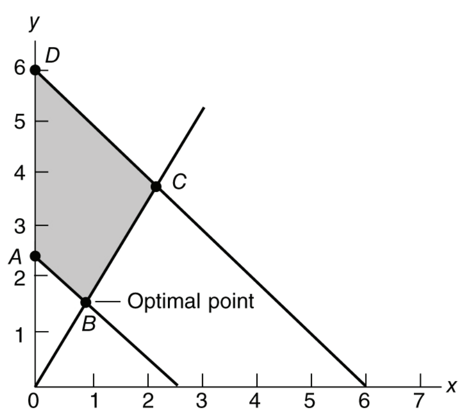
The linear programming model for this case problem is

Minimize $Z = x/60 + y/45$
subject to

$$2x + 2y \geq 5$$
$$2x + 2y \leq 12$$
$$y \geq 1.5x$$
$$x, y \geq 0$$

The objective function coefficients are determined by dividing the distance traveled, i.e., $x/3$, by the travel speed, i.e., 20 mph. Thus, the x coefficient is $x/3 \div 20$, or $x/60$. In the first two constraints,
$2x + 2y$ represents the formula for the perimeter of a rectangle.

The graphical solution is displayed as follows.



The optimal solution is $x = 1$, $y = 1.5$, and $Z = 0.05$. This means that a patrol sector is 1.5 miles by 1 mile and the response time is 0.05 hr, or 3 min.

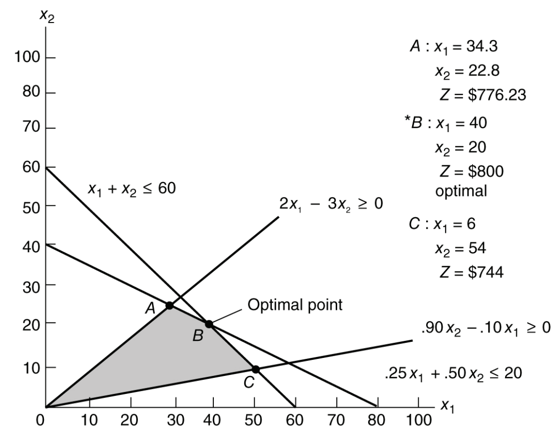
CASE SOLUTION: "THE POSSIBILITY" RESTAURANT

The linear programming model formulation is

Maximize = $Z = \$12x_1 + 16x_2$
subject to

$$x_1 + x_2 \leq 60$$
$$.25x_1 + .50x_2 \leq 20$$
$$x_1/x_2 \geq 3/2 \text{ or } 2x_1 - 3x_2 \geq 0$$
$$x_2/(x_1 + x_2) \geq .10 \text{ or } .90x_2 - .10x_1 \geq 0$$
$$x_1 x_2 \geq 0$$

The graphical solution is shown as follows.



Changing the objective function to $Z = \$16x_1 + 16x_2$ would result in multiple optimal solutions, the end points being B and C. The profit in each case would be \$960.

Changing the constraint from $.90x_2 - .10x_1 \geq 0$ to $.80x_2 - .20x_1 \geq 0$ has no effect on the solution.

CASE SOLUTION: ANNABELLE INVESTS IN THE MARKET

x_1 = no. of shares of index fund
x_2 = no. of shares of internet stock fund

Maximize $Z = (.17)(175)x_1 + (.28)(208)x_2$
$= 29.75x_1 + 58.24x_2$
subject to

$$175x_1 + 208x_2 = \$120,000$$
$$\frac{x_1}{x_2} \geq .33$$
$$\frac{x_2}{x_1} \leq 2$$
$$x_1, x_2 > 0$$

$x_1 = 203$
$x_2 = 406$
$Z = \$29,691.37$

Eliminating the constraint $\frac{x_2}{x_1} \geq .33$ will have no effect on the solution.

Eliminating the constraint $\frac{x_1}{x_2} \leq 2$ will change the solution to $x_1 = 149$, $x_2 = 451.55$, $Z = \$30,731.52$.

Increasing the amount available to invest (i.e., \$120,000 to \$120,001) will increase profit from Z = \$29,691.37 to Z = \$29,691.62 or approximately \$0.25. Increasing by another dollar will increase profit by another \$0.25, and increasing the amount available by one more dollar will again increase profit by \$0.25. This indicates that for each extra dollar invested a return of \$0.25 might be expected with this investment strategy.

Thus, the *marginal value* of an extra dollar to invest is \$0.25, which is also referred to as the "shadow" or "dual" price as described in Chapter 3.