# Chapter 2
# Modeling with Linear Functions

**Homework 2.1**

**2. a.**



Years since 2000
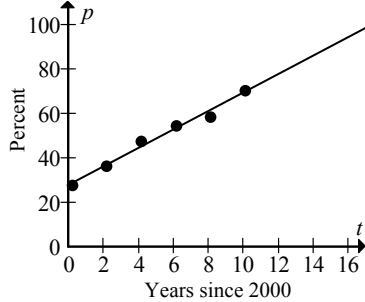
**b.** Draw a line through the points. See the graph in part (a).

**c.** When half of Americans send in their tax returns, $p$ equals 50. When $p$ equals 50, $t$ is approximately 5. Therefore, when half of Americans sent in their tax returns, the year was approximately 2005.

**d.** When $p$ equals 80, $t$ equals approximately 13, which represents the year 2013. Therefore, it is estimated that the 80% tax return percentage was achieved in the year 2013. The goal was not achieved.

**e.** The goal was not reached by 2011, according to the model.

**4.** Responses for this exercise are the same for parts (c) and (d) of Exercise 3. Responses for this exercise are different for parts (e) and (f) of Exercise 3.
Answers may vary.

**6. a.**



Elevation (in thousands of meters)
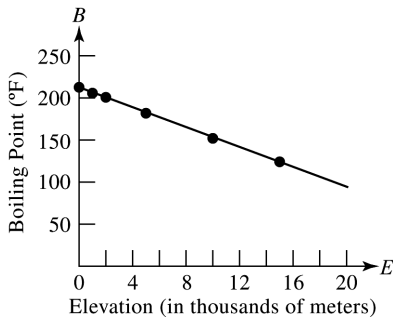
**b.** Draw a line through the points. See the graph in part (a).

**c.** The elevation 8850 meters is represented by $E = 8.85$. Locating this on the linear model, note that $B$ is approximately 160. Therefore, the boiling point of water is 160°F at the peak of Mount Everest. Interpolation was used, owing to our examination of predicted data in between points.

**d.** According to the linear model, when $B = 98.6$, the elevation is approximately 19 thousand meters. Therefore, at an elevation of 19,000 meters, boiling water feels lukewarm. Extrapolation was used, owing to the examination of predicted data outside of the data points.

**e.** Answers may vary. Example:
Keep in mind that the higher the boiling point (lower elevation), the less time it takes for an egg to cook. Likewise, the lower the boiling point (higher elevation), the more time it takes for an egg to cook. The linear model should be increasing.

**8. a.**



Age (in years)
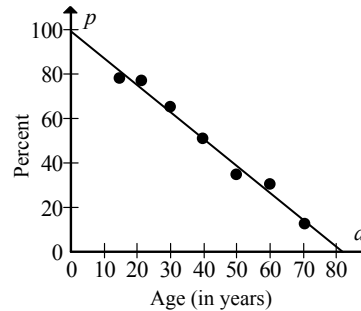
**b.** Draw a line through the points. See the graph in part (a).

**c.** When $p$ equals 70, $a$ is approximately 24. Therefore, 70% of Americans have a profile page at age 24.

**d.** When $a$ is 0, $p$ is approximately 100. This means that at birth, all Americans have a personal profile page. Model breakdown has likely occurred.

**e.** When $p$ is 0, $a$ is approximately 82. This means by age 82, Americans do not have a personal profile page. Model breakdown has likely occurred.
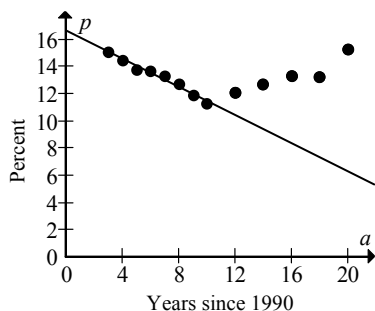
**10. a.**
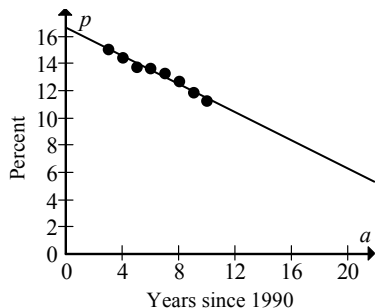


**b.** Draw a line through the points. See the graph in part (a).

**c.** According to the model, the estimated percentage of people living below the poverty level in 2010 ($t = 20$) was approximately 6.5%.

**d.**



**e.** The estimated value from part (c) was 6.5%. The actual value for 2010 was 15.3%. The error in prediction is $6.5\% - 15.3\% = -8.8\%$ . The linear model breaks down after the year 2000, so the predicted value is much too low.

**12. a.** To find the $t$-intercept, estimate what the temperature will be when $w = 0$. This estimate is approximately (13, 0). This means that if the wind speed is 10 mph and the temperature is 13 degrees Fahrenheit, it will feel like it is 0 degrees Fahrenheit.

**b.** To find the $w$-intercept, estimate what the windchill will be when $t = 0$. This estimate is approximately (0, −16). This means that if the wind speed is 10 mph and the temperature is 0 degrees Fahrenheit, it will feel like it is 16 degrees Fahrenheit below 0.

**14.** We usually have more faith in a result obtained by interpolation.
Answers may vary.

**16.** Answers may vary. Example:
All points will probably not describe the situation exactly, but they are likely to describe the situation quite well.

**18.** Answers may vary.

**Homework 2.2**

**2.** Use (90, 11.1) and (111, 109.8) to write the equation of the line. First, find the slope.
$$m = \frac{109.8 - 11.1}{111 - 90} = \frac{98.7}{21} = 4.7$$
So, $n = 4.7t + b$. Substitute 90 for $t$ and 11.1 for $n$ since the line contains (90, 11.1) and then solve for $b$.
$$n = mt + b$$
$$11.1 = 4.7(90) + b$$
$$11.1 = 423 + b$$
$$-411.9 = b$$
The equation of the line is $n = 4.7t - 411.9$.

**4.** Use (7, 2.6) and (11, 2.1) to write the equation of the line. First, find the slope.
$$m = \frac{2.1 - 2.6}{11 - 7} = \frac{-0.5}{4} = -0.125$$
So, $n = -0.125t + b$. Substitute 7 for $t$ and 2.6 for $n$ since the line contains (7, 2.6) and then solve for $b$.
$$n = mt + b$$
$$2.6 = -0.125(7) + b$$
$$2.6 = -0.875 + b$$
$$3.475 = b$$
The equation of the line is $n = -0.125t + 3.475$.

**6.** Use (7, 176) and (11, 131) to write the equation of the line. First, find the slope.
$$m = \frac{131 - 176}{11 - 7} = \frac{-45}{4} = -11.25$$
So, $n = -11.25t + b$. Substitute 7 for $t$ and 176 for $n$ since the line contains (7, 176) and then solve for $b$.
$$n = mt + b$$
$$176 = -11.25(7) + b$$
$$176 = -78.75 + b$$
$$254.75 = b$$
The equation of the line is $n = -11.25t + 254.75$.

**8.** Use (4, 0.10) and (10, 0.19) to write the equation of the line. First, find the slope.

$$m = \frac{0.19 - 0.10}{10 - 4} = \frac{0.09}{6} = 0.015$$

So, $c = 0.015w + b$. Substitute 4 for $t$ and 0.10 for $c$ since the line contains (4, 0.10) and then solve for $b$.

$$c = mw + b$$
$$0.10 = 0.015(4) + b$$
$$0.10 = 0.06 + b$$
$$0.04 = b$$

The equation of the line is $c = 0.015w + 0.04$.

**10.** Use (1, 18) and (10, 5) to write the equation of the line. First, find the slope.

$$m = \frac{5 - 18}{10 - 1} = \frac{-13}{9} \approx -1.44$$

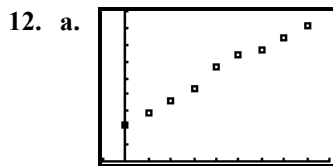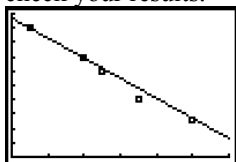So, $y = -1.44x + b$. Substitute 1 for $x$ and 18 for $y$ since the line contains (1, 18) and then solve for $b$.

$$y = mx + b$$
$$18 = -1.44(1) + b$$
$$18 = -1.44 + b$$
$$19.44 = b$$

The equation of the line is $y = -1.44x + 19.44$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.) Use the graphing calculator to check your results.



**12. a.**



**b.** Use the points (0, 10.7) and (40, 40.8) to write the equation of the line. First find the slope.

$$m = \frac{40.8 - 10.7}{40 - 0} \approx 0.75$$

So, $p = 0.75t + b$. Substitute 0 for $t$ and 10.7 for $p$, since the line contains (0, 10.7) and then solve for $b$.
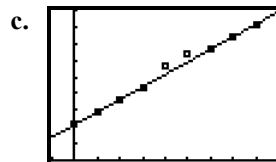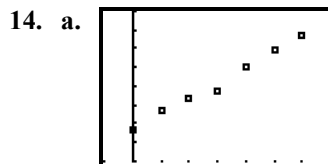
$$p = mt + b$$
$$10.7 = 0.75(0) + b$$
$$10.7 = 0 + b$$
$$10.7 = b$$

The equation of the line is $p = 0.75t + 10.7$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**c.**



Notice that the points (0, 10.7) and (40, 40.8) are filled in, showing that the line goes through these points.

The slopes are the same but the $p$-intercepts are different. Though the data is the same (hence the same slope), using a different number for the year shifts the graph, and therefore the value of $p$.

**14. a.**



**b.** Use the points (0, 73.7) and (20, 77.0) to find the equation of the line.

$$m = \frac{77.0 - 73.7}{20 - 0} = \frac{3.3}{20} = 0.165$$

So, $L = 0.165t + b$. Substitute 0 for $t$ and 73.7 for $L$ since the line contains (0, 73.7) and then solve for $b$.
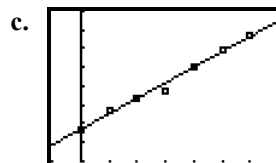
$$L = mt + b$$
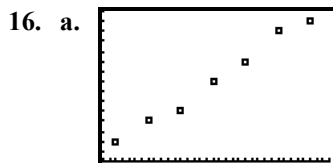$$73.7 = 0.165(0) + b$$
$$73.7 = 0 + b$$
$$73.7 = b$$

The linear equation that describes the data is $L = 0.165t + 73.7$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**c.**

**16. a.**



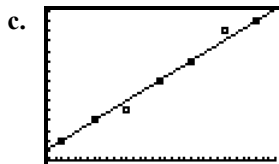**b.** Use the points (37, 2) and (67, 14) to find the equation of the line.
$$m = \frac{14-2}{67-37} = \frac{12}{30} = 0.40$$
So, $p = 0.40a + b$. Substitute 37 for $a$ and 2 for $p$ since the line contains (37, 2) and then solve for $b$.
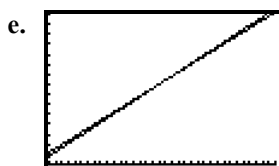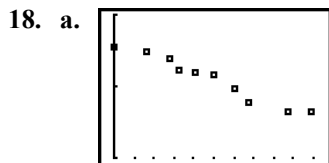$$p = ma + b$$
$$2 = 0.40(37) + b$$
$$2 = 14.8 + b$$
$$-12.8 = b$$
The equation of the line
is $p = 0.40a - 12.8$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**c.**



**d.** The equation of the line is
$p = 0.42a - 13.91$.

**e.**



The two graphs are very similar.

**18. a.**



**b.** Use the points (16, 47.4) and (88, 43.29) to find the equation of the line.
$$m = \frac{43.29 - 47.4}{88 - 16} = \frac{-4.11}{72} \approx -0.057$$
So, $r = -0.057t + b$. Substitute 16 for $t$ and 47.4 for $r$ since the line contains (16, 47.4) and then solve for $b$.
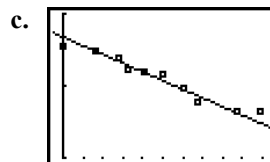
$$r = mt + b$$
$$47.4 = -0.057(16) + b$$
$$47.4 \approx -0.91 + b$$
$$48.31 \approx b$$
The equation is $r = -0.057t + 48.31$.
(Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**c.**



The line comes close to the data points.

**20. a.** The equation of the line is
$r = 0.080s + 1.766$.

**b.** The equation of the line is
$r = 0.079s + 1.657$.

**c.** The women's equation has the larger $r$-coordinate (1.766). This tells you that the women's regression line is above the men's line when $s = 0$.

**d.** The women's model has a slightly larger slope. This indicates the line rises more quickly in comparison to the men's line.

**e.** The women's line begins above the men's line and also rises faster than the men's line, so the two lines can never intersect in quadrant I. Women, in general, have faster stride rates, regardless of speed.
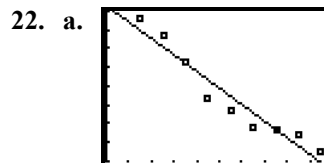
**22. a.**



A linear model works well, since a line can be drawn that comes close to the data points. The regression equation is $A = -11.02t + 457.74$. (Your equation may be slightly different if you used data points to form a model.)

**b.**
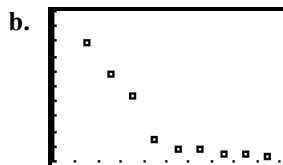


No, the first three data points fall on one line, and the last six data points fall on a different, less steep line.
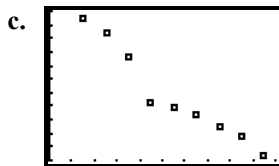
**c.**



A linear model works well, since a line can be drawn that comes close to the data points. The regression equation is $A = -124.3t + 5734$. (Your equation may be slightly different if you used data points to form a model.)

**d.**  **i.** Aggravated assaults continued to decline even though the economy did poorly, which supports the theory that crime and the economy are not related.

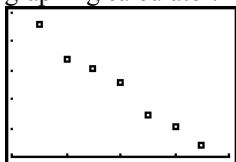**ii.** Burglaries almost leveled off after 2001, indicating that the changes in the economy may not have had much effect on them.

**iii.** All types of crime declined linearly, in spite of the ups and downs of the economy.

**24.** Check the scatter plot obtained from the graphing calculator.



Student C made the best choice of points. The lines obtained by students A and B will have a slope that is steeper than the actual slope.

**26.**
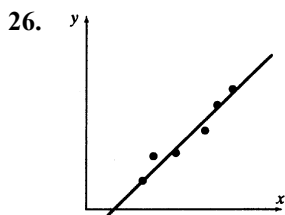


In this model, $m$ needs to be greater (steeper slope) than it was in the original model. The value of $b$ should decrease.

**28.** Answers may vary.

**Homework 2.3**

**2.** $f(-2) = 6(-2) - 4 = -12 - 4 = -16$

**4.** $f\left(\dfrac{5}{2}\right) = 6\left(\dfrac{5}{2}\right) - 4 = \dfrac{30}{2} - 4 = 15 - 4 = 11$

**6.** $f(a-3) = 6(a-3) - 4 = 6a - 18 - 4 = 6a - 22$

**8.** $g(3) = 2(3^2) - 5(3) = 2(9) - 5(3) = 18 - 15 = 3$

**10.** $g(-2) = 2((-2)^2) - 5(-2) = 2(4) - 5(-2)$
$= 8 - (-10) = 18$

**12.** $h(-4) = \dfrac{3(-4) - 4}{5(-4) + 2} = \dfrac{-12 - 4}{-20 + 2} = \dfrac{-16}{-18} = \dfrac{8}{9}$

**14.** $h(3a) = \dfrac{3(3a) - 4}{5(3a) + 2} = \dfrac{9a - 4}{15a + 2}$

**16.** $g(-1) = -3(-1)^2 + 2(-1) = -3 - 2 = -5$

**18.** $f(-4) = -2(-4) + 7 = 8 + 7 = 15$

**20.** $h(-9) = -4$

**22.** $f(-3a) = -4(-3a) - 7 = 12a - 7$

**24.** $f\left(\dfrac{3a}{2}\right) = -4\left(\dfrac{3a}{2}\right) - 7 = -\dfrac{12a}{2} - 7 = -6a - 7$

**26.** $f(a-4) = -4(a-4) - 7$
$= -4a + 16 - 7$
$= -4a + 9$

**28.** Substituting $a + h$ wherever there is an $x$ in $f(x)$:
$f(a+h) = -4(a+h) - 7 = -4a - 4h - 7$

**30.** To find $x$ when $f(x) = 0$, substitute 0 for $f(x)$ and solve for $x$:
$0 = -3x + 7$
$3x = 7$
$x = \dfrac{7}{3}$

**32.** To find $x$ when $f(x) - \dfrac{4}{3}$, substitute $-\dfrac{4}{3}$ for $f(x)$ and solve for $x$:

$$-\dfrac{4}{3} = -3x + 7$$
$$-4 = -9x + 21$$
$$9x - 4 = 21$$
$$9x = 25$$
$$x = \dfrac{25}{9}$$

**34.** To find $x$ when $f(x) = a + 2$, substitute $a + 2$ for $f(x)$ and solve for $x$:

$$a + 2 = -3x + 7$$
$$a - 5 = -3x$$
$$\dfrac{a - 5}{-3} = x$$
$$\dfrac{5 - a}{3} = x$$

**36.** $f(17.28) = -5.95(17.28) + 183.22$
$$= -102.816 + 183.22 = 80.404$$
$$\approx 80.40$$

**38.** To find $x$ when $f(x) = 72.06$, substitute $72.06$ for $f(x)$ and solve for $x$:

$$72.06 = -5.95x + 183.22$$
$$0 = -5.95x + 183.22 - 72.06$$
$$5.95x = 111.16$$
$$x \approx 18.68$$

**40.** $f(4) = 0$

**42.** When $f(x) = 4$, $x$ is 2.

**44.** Since the line contains the point (0, 2), $f(0) = 2$.

**46.** Since the line contains the point $\left(-\dfrac{11}{2}, \dfrac{23}{6}\right)$,
$$f\left(-\dfrac{11}{2}\right) = \dfrac{23}{6}.$$

**48.** Since the line contains the point (3, 1), $x = 3$ when $f(x)$ or $y = 1$.

**50.** Since the line contains the point (–4.5, 3.5), $x = -4.5$ when $f(x)$ or $y = 3.5$.

**52.** Since the line contains the point $\left(-\dfrac{3}{2}, \dfrac{5}{2}\right)$,
$$x = -\dfrac{3}{2} \text{ when } f(x) \text{ or } y = \dfrac{5}{2}.$$

**54.** The range is $-\infty < f(x) < \infty$.

**56.** Since the function contains the point (2, 3), $x = 2$ when $g(x)$ or $y = 3$.

**58.** $-2 \le g(x) \le 4$

**60.** Since the function contains the point $(-2, -1)$, $x = -2$ when $h(x)$ or $y = -1$.

**62.** $-4 \le h(x) \le 1$

**64.** $f(x)$ or $y = 4x + 2$
To find the $y$-intercept, set $x = 0$ and solve for $y$.
$$y = 4(0) + 2 = 0 + 2 = 2$$
The $y$-intercept is (0, 2).
To find the $x$-intercept, set $y = 0$ and solve for $x$.
$$0 = 4x + 2$$
$$-2 = 4x$$
$$x = -\dfrac{2}{4} = -\dfrac{1}{2}$$
The $x$-intercept is $\left(-\dfrac{1}{2}, 0\right)$.

**66.** $f(x)$ or $y = -7x$
To find the $y$-intercept, set $x = 0$ and solve for $y$.
$$y = -7(0) = 0$$
The $y$-intercept is (0, 0).
To find the $x$-intercept, set $y = 0$ and solve for $x$.
$$0 = -7x$$
$$0 = x$$
The $x$-intercept is (0, 0).

**68.** $f(x)$ or $y = -2$ is the equation of a horizontal line. The $y$-intercept of the line will be (0, –2). There is no $x$-intercept.

**70.** $f(x)$ or $y = -\dfrac{3}{4}x + \dfrac{1}{2}$
To find the $y$-intercept, set $x = 0$ and solve for $y$.
$$y = -\dfrac{3}{4}(0) + \dfrac{1}{2} = 0 + \dfrac{1}{2} = \dfrac{1}{2}$$
The $y$-intercept is $\left(0, \dfrac{1}{2}\right)$.
To find the $x$-intercept, set $y = 0$ and solve for $x$.

$$0 = -\frac{3}{4}x + \frac{1}{2}$$

$$-\frac{1}{2} = -\frac{3}{4}x$$

$$x = -\frac{1}{2}\left(-\frac{4}{3}\right) = \frac{2}{3}$$

The *x*-intercept is $\left(\frac{2}{3}, 0\right)$.

**72.** $f(x)$ or $y = -4.29x + 37.58$

To find the *y*-intercept, set $x = 0$ and solve for *y*.

$y = -4.29(0) + 37.58 = 37.58$

The *y*-intercept is (0, 37.58).

To find the *x*-intercept, set $y = 0$ and solve for *x*.

$$0 = -4.29x + 37.58$$

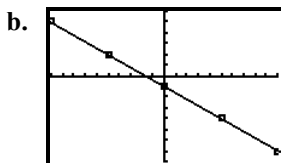$$4.29x = 37.58$$

$$x \approx 8.76$$

The *x*-intercept is (8.76, 0).

**74.  a**.  Your table may be different if you chose different *x* values.

| *x* | *f(x)* |
|-----|--------|
| −10 | 5 |
| −5 | 2 |
| 0 | −1 |
| 5 | −4 |
| 10 | −7 |

**b.**



**c.**  For each input-output pair, the output is 1 less than $-\frac{3}{5}$ times the input.

**76.  a.**  $f(t) = 0.76t + 11.09$

**b.**  $f(48) = 0.76(48) + 11.09$
$= 36.48 + 11.09$
$= 47.57$
When $t = 48, f = 47.57$. This means that in the year 1970 + 48 = 2018, the percentage of births outside marriage will be about 47.6%. This is the same as the result obtained in Exercise 75.

**c.**     $47 = 0.76t + 11.09$
$35.91 = 0.76t$
$t = 47.25$
When $f = 47, t = 47.25$. This means that the percentage of births outside marriage will be 47% in the year 1970 + 47 = 2017. This is the same as the result obtained in Exercise 75.

**d.**     $100 = 0.76t + 11.09$
$88.91 = 0.76t$
$t \approx 117$
In the year 1970 + 117 = 2087, all births will be outside marriage. This is the same as the result obtained in Exercise 75.

**e.**  Find *f* when $t = 1997 - 1970 = 27$.
$f(27) = 0.76(27) + 11.09$
$= 20.52 + 11.09$
$= 31.61$
This means that in the year 1997, the percentage of births outside marriage was 31.6%. Since the actual percentage was 32.4%, the error is 31.6% − 32.4% = −0.8%. This is nearly the same as the result obtained in Exercise 75.

**78.  a.**
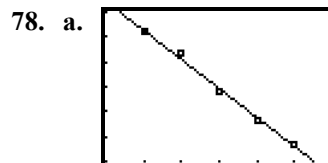


The regression equation is
$f(t) = -5.83t + 137.09$.
(Your equation may be slightly different if you used two data points to form a model.)
The model fits the data well.

**b.**  $f(18) = -5.83(18) + 137.09$
$= -104.94 + 137.09$
$= 32.15$
According to the model, in the year $2000 + 18 = 2018$, first-class mail volume will be about 32 billion pieces.

**c.**  To find *t* when $f(t)$ is 18, substitute 18 for $f(t)$ and solve for *t*:
$18 = -5.83t + 137.09$
$-119.09 = -5.83t$
$20.43 \approx t$
According to the model, first-class mail volume will be about 18 billion pieces in the year $2000 + 20.43 = 2020$.

**d.** To find the *v*-intercept, set *t* equal to 0 and solve for *v*.

$$v = -5.83(0) + 137.09 = 137.09$$

The *v*-intercept is (0, 137.09). The model estimates that in the year 2000, first-class mail volume was about 137 billion pieces.

**e.** To find the *t*-intercept, set *v* equal to 0 and solve for *t*.

$$0 = -5.83t + 137.09$$
$$-137.09 = -5.83t$$
$$23.51 \approx t$$

The *t*-intercept is (23.51, 0). The model predicts that in the year $2000 + 23.51 \approx 2024$, first-class mail volume will be 0.

**f.** Answers may vary. Example:
The use of the Internet for receiving and paying bills, getting news, reading magazines, and reaching people with advertising greatly increased in that time period.

**80. a.** Since *t* is in years since 1980, we need $t = 2017 - 1980 = 37$.

$$L = 0.16(37) + 73.71 = 5.92 + 73.71 = 79.63$$

Life expectancy for people born in the year 2017 will be approximately 79.6 years.

**b.** To find the birth year at which life expectancy is 80 years, we must find *t* when $L = 80$. To find this, substitute 80 for *L* and solve for *t*.

$$80 = 0.16t + 73.71$$
$$6.29 = 0.16t$$
$$39.31 \approx t$$

Since *t* is the number of years from 1980, $39.31 + 1980 \approx 2019$ is the birth year when the life expectancy will be 80.

**c.** Answers may vary. Example:
For a person born in 1980, $t = 0$. Find *L*.

$$L = 0.16(0) + 73.71 = 73.71$$

Therefore, the estimated life expectancy of an American born in the year 1980 is approximately 73.7 years. Generally, a person's current life expectancy tends to be greater than it was at the time of birth.

**d.** To find the *t*-intercept, set $L = 0$ and solve for *t*.

$$0 = 0.16t + 73.71$$
$$-73.71 = 0.16t$$
$$-461 \approx t$$

This means that in the year $1980 - 461 = 1519$, life expectancy at birth was zero. Model breakdown has occurred.

**e.** Answers may vary. The graph should show that human life expectancy has increased more rapidly in the recent past than earlier in human history.

**82. a.**
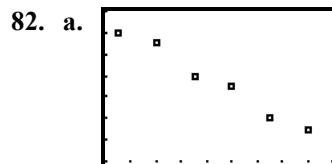


**b.** The regression equation is $P = -0.638t + 31.48$, where the population is in millions. (Your equation may be slightly different if you used two data points to form a model.)
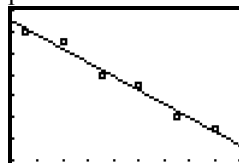


Graphing the line with the scattergram, we see that the model fits the data fairly well.

**c.** To find the *t*-intercept, set *A* equal to 0 and solve for *t*.

$$0 = -0.638t + 31.48$$
$$-31.48 = -0.638t$$
$$49.34 \approx t$$

The *t*-intercept is (49.34, 0). The model predicts that in the year $1990 + 49.34 \approx 2039$, the average gasoline tax will be 0.

**d.** Answers may vary. Example:
Inflation would cause the average gasoline tax per 1000 miles driven to increase, since tax is a percent of the amount paid. This increase is offset by the increased gas mileage of cars. Since cars go further on a gallon of gas, less gas is needed to go 1000 miles, and less tax is collected.

**e.** **i.** To find the tax paid for 1000 miles driven in 2016, set *t* equal to 26 and solve for *A*.

$$A = -0.638(26) + 31.48$$
$$= -16.588 + 31.48$$
$$\approx 14.89$$

Total tax paid: $7.624 \cdot 14.89 \approx 113.52$

To find the tax paid for 1000 miles driven in 2017, set $t$ equal to 27 and solve for $A$.

$A = -0.638(27) + 31.48$
$\quad = -17.226 + 31.48$
$\quad \approx 14.25$

Total tax paid: $7.624 \cdot 14.25 \approx 108.64$

To find the tax paid for 1000 miles driven in 2018, set $t$ equal to 28 and solve for $A$.

$A = -0.638(28) + 31.48$
$\quad = -17.864 + 31.48$
$\quad \approx 13.62$

Total tax paid: $7.624 \cdot 13.62 \approx 103.84$

In 2016-2018, a teenage will pay $113.52 + 108.64 + 103.84 = 326$ dollars in gasoline taxes.

**ii.** First find the tax that would have been paid for 3 years driving 7624 miles per year at the rate of \$28 per 1000 miles driven.

$3 \cdot 7.624 \cdot 28 \approx 640.42$

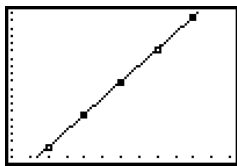The additional tax a teenager would pay is $640.42 - 326 = 314.42$ dollars.

**84. a.** Two of the data points are (50, 43) and (90, 215). To find the equation for $g$, find the slope using these points.

$m = \dfrac{215 - 43}{90 - 50} = \dfrac{172}{40} = 4.3$

So, $g(F) = 4.3F + b$. To solve for $b$, substitute 50 for $F$ and 43 for $g$ since the line contains (50, 43).

$43 = 4.3(50) + b$
$43 = 215 + b$
$-172 = b$

The equation for $g$ is $g(F) = 4.3F - 172$.



**b.** $g(73) = 4.3(73) - 172 = 313.9 - 172 = 141.9$

When the temperature is 73°F, crickets will chirp approximately 142 times per minute.

**c.** $100 = 4.3F - 172$
$\quad 272 = 4.3F$
$\quad 63.26 \approx F$

When the temperature is 63°F, crickets will chirp approximately 100 times per minute.

**d.** When crickets are not chirping, $g(F) = 0$.

We need to find $F$, when $g(F) = 0$.

$\quad 0 = 4.3F - 172$
$172 = 4.3F$
$\quad 40 = F$

When the temperature is 40°F or below, crickets will not chirp.

**86. a.** $f(a) = 0.42a - 13.91$

**b.** $f(40) = 0.42(40) - 13.91$
$\qquad\quad = 16.8 - 13.91$
$\qquad\quad = 2.89$

or about 2.9%

**c.** To find $a$ when $f(a)$ is 7%, substitute 7 for $f(a)$ and solve for $a$:

$\quad 7 = 0.42a - 13.91$
$20.91 = 0.42a$
$49.79 \approx a$

or about age 50

**d.** To find the $a$-intercept, substitute 0 for $f(a)$ and solve for $a$.

$\quad 0 = 0.42a - 13.91$
$13.91 = 0.42a$
$33.12 \approx a$

The model states that at approximately age 33, the percentage of Americans diagnosed with diabetes is 0. Model breakdown has occurred.

**e.** Answers may vary. Example: Perhaps people who reach the age of 70 are less likely to have had diabetes.

**88.** If we let $t$ be the number of years after 2000 and let $n$ be the number of words in the federal tax code (in millions), we can make a linear model of the given information. To start, we can use the two points (5, 3.4) and (12, 3.8) to find the slope of the model:

$m = \dfrac{3.8 - 3.4}{12 - 5} = \dfrac{0.4}{7} \approx 0.057$

So $n = 0.057t + b$. To solve for $b$, substitute 5 for $t$ and 3.4 for $n$, since the line contains (5, 3.4).

$3.4 = 0.057(5) + b$

$3.4 = 0.285 + b$

$3.115 = b$

The equation for $n$ is $n = 0.057t + 3.115$.

To find the number of words in 2018, we substitute 18 for $t$ and solve for $n$.

$n = 0.057(18) + 3.115 = 1.026 + 3.115 = 4.141$

In 2018 there will be about 4.1 million words in the federal tax code.

**90.** If we let $t$ be the number of years after 1970 and let $p$ be the percentage of female workers who prefer a female boss over a male boss, we can make a linear model of the given information. To start, we can use the two points (5, 10) and (41, 27) to find the slope of the model:

$m = \dfrac{27 - 10}{41 - 5} = \dfrac{17}{36} \approx 0.47$

So $p = 0.47t + b$. To solve for $b$, substitute 5 for $t$ and 10 for $p$, since the line contains (5, 10).

$10 = 0.47(5) + b$

$10 = 2.35 + b$

$7.65 = b$

The equation for $n$ is $p = 0.47t + 7.65$.

To find when 30% of female workers will prefer a female boss, we substitute 30 for $p$ and solve for $t$.

$30 = 0.47t + 7.65$

$22.35 = 0.47t$

$47.55 \approx t$

By $1970 + 47.55 \approx 2018$, 30% of female workers will prefer a female boss.

**92. a.** If we let $t$ be the number of years after 2000 and let $a$ be the average math score on the SAT, we can make a linear model of the given information. To start, we can use the two points (6, 518) and (12, 514) to find the slope of the model:

$m = \dfrac{514 - 518}{12 - 6} = \dfrac{-4}{6} \approx -0.67$

So $a = -0.67t + b$. To solve for $b$, substitute 6 for $t$ and 518 for $a$, since the line contains (6, 518).

$518 = -0.67(6) + b$

$518 = -4.02 + b$

$522.02 = b$

The equation for $a$ is $a = -0.67t + 522.02$.

To find the average math score on the SAT in 2017, or when $t = 17$, substitute 17 for $t$ and solve for $a$.

$a = -0.67(17) + 522.02$

$= -11.39 + 522.02$

$= 510.63$

According to the model, in 2017 the average math score on the SAT will be about 511.

**b.** To find when the average math score on the SAT will be 509, or when $a = 509$, substitute 509 for $a$ and solve for $t$.

$509 = -0.67t + 522.02$

$-13.02 = -.67t$

$19.43 \approx t$

According to the model, the average math score on the SAT will be 509 in $2000 + 19.43 \approx 2019$.

**94.** We are given two data points, (4, 3067) and (6, 4114). We can now calculate the slope of the linear model.

$m = \dfrac{4114 - 3067}{6 - 4} = \dfrac{1047}{2} = 523.5$, which

yields $f(n) = 523.5n + b$, where $n$ is the number of people in the family. We can now substitute 4 for $n$ and 3067 for $f(n)$ to get $b$.

$3067 = 523.5(4) + b$

$3067 = 2094 + b$

$973 = b$

$f(n) = 523.5n + 973$

To find the monthly income limit for a family of 7, substitute 7 for $n$ and solve for $f(n)$.

$f(7) = 523.5(7) + 973$

$f(7) = 3664.5 + 973$

$f(7) = 4637.5$

**96. a.** From the given information, we have the data points (0, 12) and (5, 0). Use these points to find the slope of the equation for $f$.
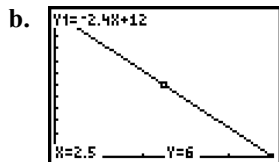
$m = \dfrac{0 - 12}{5 - 0} = \dfrac{-12}{5} = -2.4$

So, $f(t) = -2.4t + b$. Since the line contains (0, 12), substitute 0 for $t$ and 12 for $f(t)$ and solve for $b$.

$12 = -2.4(0) + b$

$12 = b$

The linear model is $f(t) = -2.4t + 12$.

**b.** 

**c.** Since it takes 5 minutes to eat the ice cream, the domain is $0 \le t \le 5$. Since there are 12 ounces of ice cream, the range is $0 \le f(t) \le 12$.

**98.** Instead of taking the opposite of 5 squared, the student should find –5 squared, since the function should be evaluated at –5. The correct procedure is:
$$g(-5) = (-5)^2 = 25$$

**100. a.** $f(5) = 3(5) + 2 = 15 + 2 = 17$
$f(4) = 3(4) + 2 = 12 + 2 = 14$
$f(5) - f(4) = 17 - 14 = 3$
This value and the slope are the same.

**b.** $f(7) = 2(7) + 5 = 14 + 5 = 19$
$f(6) = 2(6) + 5 = 12 + 5 = 17$
$f(7) - f(6) = 19 - 17 = 2$
This value and the slope are the same.

**c.** $f(3) = 4(3) + 1 = 12 + 1 = 13$
$f(2) = 4(2) + 1 = 8 + 1 = 9$
$f(3) - f(2) = 13 - 9 = 4$
This value and the slope are the same.

**d.** $f(a+1) = m(a+1) + b = ma + m + b$
$f(a) = m(a) + b = ma + b$
$f(a+1) - f(a) = ma + m + b - ma - b = m$
This value and the slope are the same. This indicates that for every increment of one in a linear function, the next value will have a $y$ value increased by the slope value.

**102.** No. Answers may vary. Example:
$f(4)$ means that you should evaluate the function $f$ at 4, that is, substitute 4 for the independent variable and solve.

**104.** Answers may vary. Example:
The four steps are: **1.** Create a scattergram of the data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find the equation of a linear model. **2.** Find an equation of your model. **3.** Verify your

equation by checking that the graph of your model contains the two chosen points and comes close to all of the data points. **4.** Use the equation of your model to make estimates, make predictions, and draw conclusions.

**Homework 2.4**

**2.** The average rate of change of the percentage of Americans who have trust in newspapers is given by dividing the change in the percent of Americans who have trust in newspapers by the change in years.
$$m = \frac{\text{change in percentage}}{\text{change in years}}$$
$$= \frac{28 - 22}{2011 - 2007}$$
$$= \frac{6}{4}$$
$$= 1.5$$
This means that the average rate of change of the percentage of Americans who have trust in newspapers is 1.5 percent per year, or an increase of 1.5 percent per year.

**4.** The average rate of change of the population of Pittsburgh is given by dividing the change in population by the change in years.
$$m = \frac{\text{change in population}}{\text{change in years}}$$
$$= \frac{306 - 370}{2010 - 1990}$$
$$= \frac{-64}{20}$$
$$= -3.2$$
The average rate of change is about –3.2 thousand people per year.

**6.** The average rate of change of the number of laws enacted by Congress is given by dividing the change in the number of laws enacted by the change in years.
$$m = \frac{\text{change in number of laws enacted}}{\text{change in years}}$$
$$= \frac{383 - 498}{2010 - 2004}$$
$$= \frac{-115}{6}$$
$$\approx -19.2$$
The average rate of change is about –19.2 laws enacted per year.

**8.** The average rate of change of the average price with respect to the number of bedrooms. is given by dividing the change in the average price by the change in the number of bedrooms.

$$m = \frac{\text{change in price}}{\text{change in number of bedrooms}}$$
$$= \frac{6,315,496 - 1,694,547}{4 - 2}$$
$$= \frac{4,620,949}{2}$$
$$= 2,310,474.5$$

The average rate of change is $2,310,474.50 per bedroom.

**10. a.**

| $t$ (hr) | $d$ (mi) |
|---|---|
| 1 | 40 |
| 2 | 80 |
| 3 | 120 |
| 4 | 160 |
| 5 | 200 |
| 6 | 240 |
| 7 | 280 |
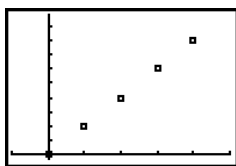


**b.** There is a linear relationship because the train is moving at a constant speed. The slope is 40 miles per hour, so the distance increases 40 miles for each hour of travel.

**12. a.** Yes, there is an approximate linear relationship between $s$ and $t$. The slope is $2.6 million per year, which is the annual increase in sales of men's skin care products.

**b.** The $s$-intercept of the model is $81.7 million. This is the sales of men's skin care products in 2011, the base year.

**c.** $s(t) = 2.6t + 81.7$

**d.** $\$\text{million} = \dfrac{\$\text{million}}{\text{year}} \cdot \text{year} + \$\text{million}$
$\$\text{million} = \$\text{million} + \$\text{million}$
$\$\text{million} = \$\text{million}$
This unit analysis shows that this model uses the correct units.

**e.** To predict this value, substitute 100 for $s(t)$ and solve for $t$.
$$100 = 2.6t + 81.7$$
$$18.3 = 2.6t$$
$$7.04 \approx t$$
Retail sales of men's skin care products will be approximately $100 million in $2011 + 7.04 \approx 2018$.

**14. a.** The slope is −2.4. The percentage of middle-income families who say it's possible to save for a secure retirement is decreasing by 2.4% per year.

**b.** $f(t) = -2.4t + 28$

**c.** To find this quantity, we substitute 14 for $f(t)$ and solve for $t$.
$$14 = -2.4t + 28$$
$$-14 = -2.4t$$
$$5.8 \approx t$$
The model predicts that 14% of middle-income families will say it's possible to save for a secure retirement in $2011 + 5.8 \approx 2017$.

**d.** To find the $t$-intercept of the model, set $f(t)$ to 0 and solve for $t$.
$$0 = -2.4t + 28$$
$$-28 = -2.4t$$
$$11.67 \approx t$$
The model predicts that no middle-income families will say it's possible to save for a secure retirement in $2011 + 11.67 \approx 2023$.

**e.** 2018 is 7 years away from 2011. Thus $t = 7$. We substitute 7 for $t$ and solve for $f(t)$.
$$f(7) = -2.4(7) + 28 = -16.8 + 28 = 11.2$$
Then we substitute 7 for $t$ and solve for $g(t)$.
$$g(7) = 0.035(7) + 15.87$$
$$= 0.245 + 15.87$$
$$= 16.115$$
So, 11.2% of 16.115 million families is
$$0.112 \cdot 16.115 \approx 1.8$$
The models predict that 1.8 million middle-income families who will say it's possible to save for a secure retirement in 2018.

**16. a.** The slope of the graph of $h$ is 96 dollars per credit. Each additional credit costs $96 per semester.

**b.** $h(c) = 96c + 17$

**c.** $\text{dollars} = \dfrac{\text{dollars}}{\text{credit hour}} \text{credit hour} + \text{dollars}$
$\text{dollars} = \text{dollars} + \text{dollars}$
$\text{dollars} = \text{dollars}$
This unit analysis shows that this model uses the correct units.

**d.** To find $h(9)$, substitute 9 for $c$ in the equation we found and compute.
$h(9) = 96(9) + 17 = 864 + 17 = 881$
This means that for 9 credit hours, the student's total bill will be $881.

**e.** To find $c$ when $h(c)$ is 593, substitute 593 for $h(c)$ and solve for $c$.
$593 = 96c + 17$
$576 = 96c$
$6 = c$
This means that if the student pays $593 for one semester, the student is taking 6 credits.

**18. a.** The slope is 0.0303 atm/ft. This means that for every foot you descend, the pressure increases by 0.0303 atm.

**b.** $f(d) = 0.0303d + 1$

**c.** $\text{atm} = \dfrac{\text{atm}}{\text{ft}}(\text{ft}) + \text{atm}$
$\text{atm} = \text{atm} + \text{atm}$
$\text{atm} = \text{atm}$
This unit analysis shows that this model uses the correct units.

**d.** Twice the pressure at sea level would be 2 atm. We must find the depth where this pressure occurs. To do this, substitute 2 for $f(d)$ and solve for $d$.
$2 = 0.0303d + 1$
$1 = 0.0303d$
$33.0 \approx d$
The depth at which the pressure is 2 atm is 33 ft.

**e.** To find the pressure at 1943 feet, substitute 1943 for $d$ and compute.
$f(1943) = 0.0303(1943) + 1$
$f(1943) \approx 58.87 + 1$
$f(1943) \approx 59.87$
The pressure at Crater Lake's greatest depth is about 60 atm.

**20. a.** Let $n$ be the number of U.S. brewery openings in the year that is $t$ years since 2010.
$n = 12.4t + 154$

**b.** $\text{no. of openings} = \dfrac{\text{no. of openings}}{\text{year}} \text{years}$
$\qquad\qquad\qquad + \text{no. of openings}$
$\text{no. of openings} = \text{no. of openings}$
$\qquad\qquad\qquad + \text{no. of openings}$
$\text{no. of openings} = \text{no. of openings}$
This unit analysis shows that this model uses the correct units.

**c.** The slope of the model is 12.4 openings per year. Each additional year has about 12.4 additional openings.

**d.** To find $c$ when $n$ is $5 \cdot 50 = 250$, substitute 250 for $n$ and solve for $t$.
$250 = 12.4t + 154$
$96 = 12.4t$
$7.74 \approx t$
This means that in $2010 + 7.74 \approx 2018$, there will be an average of 5 brewery openings per state.

**22. a.** Let $c$ be the total cost of $n$ teenagers traveling to Disneyland Resort in a bus.
$c = 92n + 26$

**b.** To find $n$ when $c$ is 2142, substitute 2142 for $c$ and solve for $n$.
$2142 = 92n + 26$
$2116 = 92n$
$23 = n$
This means there are 23 teenagers traveling to Disneyland Resort in a bus.

**24.** Let $p$ be the percentage of Americans who have ever listened to an audio podcast in the year that is $t$ years since 2010.
$p = 3t + 23$
To find this quantity, we substitute 50 for $p$ and solve for $t$.

$50 = 3t + 23$

$27 = 3t$

$9 = t$

In $2010 + 9 = 2019$, 50% of Americans will have listened to an audio podcast

**26.** The customer paid for $n$ chocolate bars and 1 half-gallon of milk, with a total of $14.75 spent. Since we know the prices for these items, we can set up an equation to find $n$.

$14.75 = 0.95n + 2.40$

$12.35 = 0.95n$

$13 = n$

The person bought 13 chocolate bars.

**28.** The slope of this model is $-5.83$, indicating that first-class mail volume decreases by $-5.83$ billion pieces per year.

**30.** The slope of this model is $-0.48$, indicating that the percentage of adult Americans who smoke decreases by 0.48 per year.

**32.** The slope of this model is 4.3, indicating that for every degree Fahrenheit increase in temperature, the number of cricket chirps increases by 4.3 per minute.
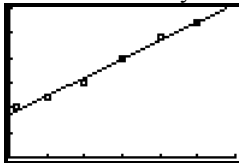
**34. a.** Create a scattergram using the data in the table.
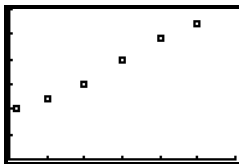


The regression equation is $f(t) = 0.074t + 0.86$.

(Your equation may differ slightly if you used two data points to form a model.) Graphing the equation on the same graph as the scattergram, we see that the equation fits the data fairly well.



**b.** The slope of the model is 0.074, indicating that the population of Nevada increases by 0.074 million per year.

**c.** To predict the population in 2018, we substitute $t = 2018 - 1985 = 33$ into the model, and calculate.

$f(33) = 0.074(33) + 0.86$

$= 2.442 + 0.86$

$\approx 3.3$

In 2018, Nevada's population will be about 3.3 million.

**d.** To find the $t$-intercept of the model, set $f(t)$ to 0 and solve for $t$.

$0 = 0.074t + 0.86$

$-0.86 = 0.074t$

$-11.6 \approx t$

The model indicates that Nevada's population was zero in $1985 - 11.6 \approx 1973$. This is false. Model breakdown has occurred.

**e.** To find this quantity, substitute 3.6 for $f(t)$ and solve for $t$.

$3.6 = 0.074t + 0.86$

$2.74 = 0.074t$

$37.0 \approx t$

Nevada's population will reach 3.6 million in about $1985 + 37 = 2022$.

**36. a.** Create a scattergram using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $A(t) = -0.007t + 4.153$. (Your equation may differ slightly if you used two data points to form a model.) Graphing the equation on the same graph as the scattergram, we see that the equation fits the data fairly well.
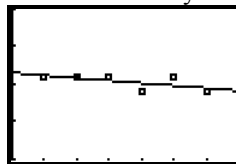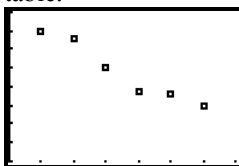


**b.** Use the data points (5, 4.1) and (20, 3.9) to find the slope.

$m = \dfrac{3.9 - 4.1}{20 - 5} = \dfrac{-0.2}{15} \approx -0.013$

This rate of change is more negative than the one we found in part (a) using linear regression.

**c.** Create a scattergram using the data in the table.



The data is approximately linear, so it can be modeled using a linear model. The regression equation that models this data is $H(t) = -0.086t + 7.93$. (Your equation may differ slightly if you chose two data points to form a model.)

**d.** The rate of change of the percentage of disposable personal income that Americans spent on food eaten at home was about $-0.09\%$ per year.

**e.** To find the year, substitute 4.7 for $H(t)$ and solve for $t$.

$$4.7 = -0.086t + 7.93$$
$$-3.23 = -0.086t$$
$$37.6 \approx t$$

The model predicts that in $1980 + 37.6 \approx 2018$, Americans will spend about 4.7% of their disposable personal income on food eaten at home.

**38. a.** The regression equation for the data is $f(p) = 0.11p + 69.48$. Your answer may differ slightly if you used two data points to model the data.

**b.** The slope of the model is 0.11. The slope tells us that for every increase of one in the relative humidity there will be a 0.11 degree increase in the heat index.

**c.** To find the average rate of change of the heat index with respect to relative humidity, divide the differences in the heat index by the differences in the humidity.

**i.** $m = \dfrac{80 - 69}{100 - 0} = \dfrac{11}{100} = 0.11$

This is the same rate of change that we found in part (b).

**ii.** $m = \dfrac{78 - 72}{80 - 20} = \dfrac{6}{60} = 0.1$

This is a slightly shallower rate of change than we found in part (b).

**iii.** $m = \dfrac{76 - 74}{60 - 40} = \dfrac{2}{20} = 0.1$

This is a slightly shallower rate of change than we found in part (b).

**d.** To find the relative humidity, substitute 77.6 for $f(p)$ and solve for $p$.
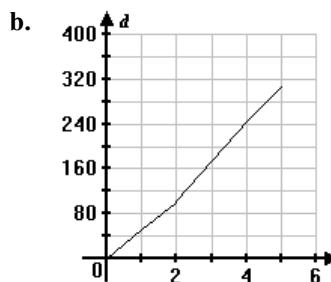
$$77.6 = 0.11p + 69.48$$
$$8.12 = 0.11p$$
$$73.82 \approx p$$

The relative humidity is approximately 74%.

**40. a.**

**Accelerating from 50 mph to 70 mph in One Minute**

| Time | $t$ (hours) | $d$ (in miles) |
|------|------|------|
| 12:00 | 0 | $50(0) = 0$ |
| 1:00 | 1 | $50(1) = 50$ |
| 2:00 | 2 | $50(2) = 100$ |
| 2:01 | 2.017 | $100 + 1 = 101$ |
| 3:01 | 3.017 | $101 + 70(1) = 171$ |
| 4:01 | 4.017 | $101 + 70(2) = 241$ |
| 5:01 | 5.017 | $101 + 70(3) = 311$ |

**b.**



**42.** The points (5, 40) and (25, 30) are on the line and can be used to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 40}{25 - 5} = \frac{-10}{20} = -0.5$$
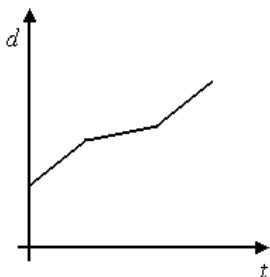
The population is decreasing by about 0.5 thousand or 500 people per year.

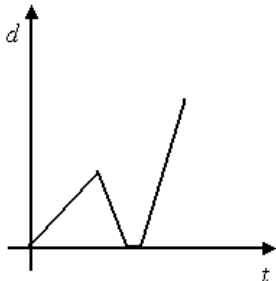**44.** The points (1, 60) and (5, 240) are on the line and can be used to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{240 - 60}{5 - 1} = \frac{180}{4} = 45$$

The total cost increases $45 for each additional person.

**46.** Answers may vary. Example:



**48.** Answers may vary. Example:



**50.** Answers may vary. Example:
The rate of change is the quotient of the change in the dependent variable and the change in the independent variable. If one change is negative and the other is positive, the quotient is negative.

**52.** Answers may vary.

**Chapter 2 Review**

**1.**    $f(3) = 3(3)^2 - 7 = 3(9) - 7 = 27 - 7 = 20$

**2.**  $f(-3) = 3(-3)^2 - 7 = 3(9) - 7 = 27 - 7 = 20$

**3.**  $g(2) = \dfrac{2(2)+5}{3(2)+6} = \dfrac{4+5}{6+6} = \dfrac{9}{12} = \dfrac{3}{4}$

**4.**  $h\left(\dfrac{3}{5}\right) = -10\left(\dfrac{3}{5}\right) - 3 = -2(3) - 3 = -6 - 3 = -9$

**5.**  $h(a+3) = -10(a+3) - 3$
$= -10a - 30 - 3$
$= -10a - 33$

**6.**    $-6 = 2x + 3$
$-6 - 3 = 2x + 3 - 3$
$-9 = 2x$
$\dfrac{-9}{2} = x$

**7.**    $\dfrac{2}{3} = 2x + 3$
$\dfrac{2}{3} - 3 = 2x + 3 - 3$
$\dfrac{-7}{3} = 2x$
$\dfrac{1}{2}\left(\dfrac{-7}{3}\right) = \dfrac{1}{2}(2x)$
$\dfrac{-7}{6} = x$

**8.**  $a + 7 = 2x + 3$
$a + 7 - 3 = 2x + 3 - 3$
$a + 4 = 2x$
$\dfrac{a+4}{2} = \dfrac{2x}{2}$
$\dfrac{a}{2} + 2 = x$

**9.** Since the graph includes the point (2, 0), $f(2) = 0$.

**10.** Since the graph includes the point (0, 1), $f(0) = 1$.

**11.** Since the graph includes a point close to $(-3, 3.5), f(-3) \approx 3.5$.

**12.** Since the graph includes the point (–2, 3), $x = -2$ when $f(x) = 3$.

**13.** Since the graph includes the point (2, 0), $x = 2$ when $f(x) = 0$.

**14.** Since the graph includes the point (4, –1), $x = 4$ when $f(x) = -1$.

**15.** The domain of $f$ is $-5 \le x \le 6$.

**16.** The range of $f$ is $-2 \le y \le 4$.

**17.** Since $x$ is 0 when $f(x) = 1, f(0) = 1$.

**18.** Since $x$ is 2 when $f(x) = 4, f(2) = 4$.

**19.** When $f(x) = 0, x = 4$.

**20.** When $f(x) = 2, x = 1$.

**21.** $f(x)$ or $y = -7x + 3$

To find the $x$-intercept, set $y = 0$ and solve for $x$.

$0 = -7x + 3$

$-3 = -7x$

$x = \dfrac{-3}{-7} = \dfrac{3}{7}$

The $x$-intercept is $\left(\dfrac{3}{7}, 0\right)$.

To find the $y$-intercept, set $x = 0$ and solve for $y$.

$y = -7(0) + 3 = 0 + 3 = 3$

The $y$-intercept is (0, 3).

**22.** $f(x)$ or $y = 4$ is the equation of a horizontal line. The $y$-intercept of the line is (0, 4). There is no $x$-intercept.

**23.** $f(x)$ or $y = -\dfrac{4}{7}x + 2$

To find the $y$-intercept, set $x = 0$ and solve for $y$.

$y = -\dfrac{4}{7}(0) + 2 = 0 + 2 = 2$

The $y$-intercept is (0, 2).

To find the $x$-intercept, set $y = 0$ and solve for $x$.

$0 = -\dfrac{4}{7}x + 2$

$-2 = -\dfrac{4}{7}x$

$-2\left(-\dfrac{7}{4}\right) = x$

$x = \dfrac{7}{2}$

The $x$-intercept is $\left(\dfrac{7}{2}, 0\right)$.

**24.** To find the $x$-intercept, set $y$ equal to zero and solve for $x$;
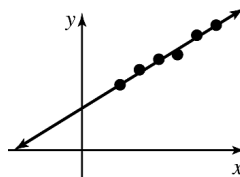
$2.56x - 9.41(0) = 78.25$

$2.56x = 78.25$

$x \approx 30.57$

The $x$-intercept is (30.57, 0).

To find the $y$-intercept, set $x$ equal to zero and solve for $y$;

$2.56(0) - 9.41y = 78.25$

$-9.41y = 78.25$

$y \approx -8.32$

The $y$-intercept is (0, –8.32).

**25.**



The new line has an increased slope since it is steeper than the model. The $y$-intercept is lower in the new line as it intersects the $y$-axis at a lower point than the model.

**26. a.** The student's car has 13 gallons of gas in the tank when no hours have been driven, so when $t = 0$, $b = 13$. Since the car uses 1.8 gallons of gas per hour, the slope is $-1.8$. An equation for $f$ is

$f(t) = -1.8t + 13$.

**b.** The slope of $f$ is $-1.8$. This means that the amount of gasoline decreases at a constant rate of 1.8 gallons per hour of driving.

**c.** To find the $A$-intercept of $f$, let $t = 0$.

$A = f(t) = -1.8(0) + 13 = 0 + 13 = 13$

The $A$-intercept is (0, 13). This represents the amount of gas in a full tank before any time has been spent driving ($t = 0$). At that time, the car has 13 gallons of gas in the tank.

**d.** gallons = gallons $- \dfrac{\text{gallons}}{\text{hour}}$ hour

= gallons – gallons

= gallons

This unit analysis shows that this model uses the correct units.

**e.** To find the $t$-intercept of $f$, let $f(t) = 0$.

$0 = -1.8t + 13$

$0 + 1.8t = -1.8t + 13 + 1.8t$

$1.8t = 13$

$t = \dfrac{13}{1.8}$

$t \approx 7.22$

The $t$-intercept is (7.22, 0). This means that the student can drive for 7.22 hours before running out of gas.

**f.** Since the car can be driven for 7.22 hours before running out of gas, the domain is $0 \le t \le 7.22$. Since the gas tank has between 0 and 13 gallons of gas, the range is $0 \le A \le 13$.

**27. a.** The slope of *C* is 37. The median annual compensation is increasing by $37 thousand per year.

**b.** In 2010, *t* = 0 and the median annual compensation was $870 thousand. Therefore, *b* = 870. Since the slope is 37, the equation for *C* is *C*(*t*) = 37*t* + 870.

**c.** To find *C*(8), substitute 8 for *t*.
$$C(8) = 37(8) + 870 = 296 + 870 = 1166$$
The median annual compensation will be $1166 thousand ($1.166 million) in 2018.

**d.** Substitute 1000 for *C*(*t*) and solve for *t*.
$$1000 = 37t + 870$$
$$130 = 37t$$
$$3.51 \approx t$$
The median annual compensation will be $1000 thousand ($1 million) in 2014.

**28.** The rate of change of annual sales of new light-duty vehicles is given by the difference in the sales divided by the difference in years.
$$m = \frac{\text{change in sales}}{\text{change in years}}$$
$$= \frac{5.65 - 7.78}{2010 - 2006}$$
$$= \frac{-2.13}{4}$$
$$\approx -0.53$$
The rate of change of annual sales is −0.53 million vehicles per year (−530 thousand vehicles per year).

**29.** If we let *t* be the number of years after 1990 and *p* be the average U.S. personal income, we can make a linear model of the given information. To start, we can use the two points (0, 14,420) and (18, 27,589) to find the slope of the model:
$$m = \frac{27,589 - 14,420}{18 - 0} = \frac{13,169}{18} \approx 731.6$$
So *p* = 731.6*t* + *b* . To solve for *b* substitute 0 for *t* and 14,420 for *p* since the line contains (0, 14,420).
$$14,420 = 488(0) + b$$
$$14,420 = b$$
The equation for *p* is *p* = 731.6*t* + 14,420 .

To find the average U.S. personal income in 2010, we can substitute 20 for *t* and solve for *p*.
$$p = 731.6(20) + 14,420$$
$$= 14,632 + 14,420$$
$$= 29,052$$
According to the model, in 2010, the average U.S. personal income was $29,052. The actual average personal income for 2010 was $26,059, so the result from the model is an overestimate. Answers may vary. Example:
The average personal income in the years 2008–2010 may not have followed the linear model due to the poor economy.

**30. a.**
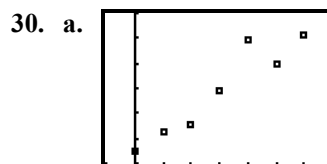


From the given information, we can select the data points (0, 32.5) and (12, 55.5). Use these points to find the slope of the equation for *f*.
$$m = \frac{55.5 - 32.5}{12 - 0} = \frac{23}{12} \approx 1.92$$
So, $f(t) = 1.92t + b$ . Since the line contains (0, 32.5), substitute 0 for *t* and 32.5 for $f(t)$ and solve for *b*.
$$32.5 = 1.92(0) + b$$
$$32.5 = b$$
The linear model is $f(t) = 1.92t + 32.5$ .

(Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**b.** The slope of the model is 1.92. This means that for every year between 2000 and 2012, the standard mileage rate increases by 1.92 cents per mile.

**c.** The *M*-intercept is given by letting *t* equal zero and solving for *M*.
$$M = 1.92(0) + 32.5 = 32.5$$
This means that during 2000 the standard mileage rate was 32.5 cents per mile.

**d.** To predict when the standard mileage rate will be 69 cents per mile, substitute 69 for $M$ and solve for $t$.

$$69 = 1.92t + 32.5$$
$$36.5 = 1.92t$$
$$19.0 \approx t$$

The model predicts that the standard mileage rate will be about 69 cents per mile in $2000 + 19 = 2019$.

**e.** 2017 corresponds to $t = 17$.

$$M = 1.92(17) + 32.5 = 32.64 + 32.5 = 65.14$$

The model predicts that the standard mileage rate in 2017 will be 65 cents per mile, or $0.65 per mile.
If someone will drive 12,500 miles in 2017, their deduction will be given by the standard mileage rate times the distance they traveled.
$$12,500(0.65) = 8125$$
This person will be able to deduct $8125.

**f.** The estimate of 54.5 is the average of 50.5 and 58.5.

**31. a.**
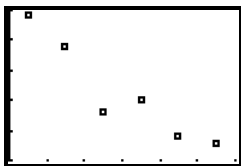


From the given information, we can select the data points (3, 34) and (9, 19). Use these points to find the slope of the equation for $f$.

$$m = \frac{19 - 34}{9 - 3} = \frac{-15}{6} = -2.5$$

So, $f(t) = -2.5t + b$. To solve for $b$, substitute 3 for $t$ and 34 for $f(t)$ since the line contains (3, 34).

$$34 = -2.5(3) + b$$
$$34 = -7.5 + b$$
$$41.5 = b$$

The equation for $f$ is $f(t) = -2.5t + 41.5$.

(Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**b.** The slope is –2.5. This means that the percentage of Americans who think the First Amendment goes too far in the rights it guarantees decreases by 2.5 percentage points per year.

**c.** Set $f(t)$ equal to 3 and solve for $t$.

$$3 = -2.5t + 41.5$$
$$-38.5 = -2.5t$$
$$15.4 = t$$

According to the model, in $2000 + 15.4 \approx 2015$, 3% of Americans will think the First Amendment goes too far in the rights it guarantees.

**d.** $f(8) = -2.5(8) + 41.5 = -20 + 41.5 = 21.5$

The model predicts that in $2000 + 8 = 2008$, 21.5 % of Americans thought the First Amendment went too far in the rights it guarantees.

**e.**

$$8 = -2.5t + 41.5$$
$$-33.5 = -2.5t$$
$$13.4 = t$$

The model predicts that in $2000 + 13.4 \approx 2013$, 8% of Americans thought the First Amendment went too far in the rights it guarantees.

**f.** To find the $t$-intercept, set $f(t)$ equal to 0 and solve for $t$.

$$0 = -2.5t + 41.5$$
$$-41.5 = -2.5t$$
$$16.6 = t$$

According to the model, in $2000 + 16.6 \approx 2017$, there will be no Americans (0%) who think the First Amendment went too far in the rights it guarantees. Model breakdown has occurred.

**Chapter 2 Test**

**1.** Since the line includes the point (−3, −2), $f(-3) = -2$.

**2.** Since the line includes the point (3, 0), $f(3) = 0$.

**3.** Since the line includes the point (0, −1), $f(0) = -1$.

**4.** Since the line includes the point $\left(-5, \frac{-8}{3}\right)$, $f(-5) = -\frac{8}{3}$ or approximately −2.7.

**5.** Since the line includes the point (−6, −3), $x = -6$ when $f(x) = -3$.

**6.** Since the line includes the point $(-3, -2)$, $x = -3$ when $f(x) = -2$.

**7.** Since the line includes the point $(3, 0)$, $x = 3$ when $f(x) = 0$.

**8.** Since the line includes the point $(4.5, 0.5)$, $x = 4.5$ when $f(x) = 0.5$.

**9.** The domain of $f$ is all the $x$-coordinates of the points in the graph. In this case $f$ has a domain of $-6 \le x \le 6$.

**10.** The range of $f$ is all the $y$-coordinates of the points in the graph. In this case $f$ has a range of $-3 \le y \le 1$.

**11.** To find $f(-3)$, substitute $-3$ for $x$.
$$f(-3) = -4(-3) + 7 = 12 + 7 = 19$$

**12.** To find $f(a-5)$, substitute $a-5$ for $x$.
$$\begin{aligned} f(a-5) &= -4(a-5) + 7 \\ &= -4a + 20 + 7 \\ &= -4a + 27 \end{aligned}$$

**13.** To find $x$ when $f(x) = 2$, substitute $a$ for $f(x)$.
$$\begin{aligned} 2 &= -4x + 7 \\ -5 &= -4x \\ \frac{5}{4} &= x \end{aligned}$$

**14.** To find $x$ when $f(x) = a$, substitute 2 for $f(x)$.
$$\begin{aligned} a &= -4x + 7 \\ a - 7 &= -4x \\ \frac{a-7}{-4} &= x \quad \text{or} \quad x = \frac{-a+7}{4} \end{aligned}$$

**15.** To find the $x$-intercept, let $f(x) = 0$ and solve for $x$.
$$\begin{aligned} 0 &= 3x - 7 \\ 7 &= 3x \\ \frac{7}{3} &= x \end{aligned}$$
The $x$-intercept is $\left( \frac{7}{3}, 0 \right)$.

To find the $y$-intercept, let $x = 0$ and solve for $y$ or $f(x)$ since $y = f(x)$.
$$y = 3(0) - 7 = -7$$
The $y$-intercept is $(0, -7)$.

**16.** To find the $x$-intercept, let $g(x) = 0$ and solve for $x$.
$$\begin{aligned} 0 &= -2x \\ 0 &= x \end{aligned}$$
The $x$-intercept is $(0, 0)$.
To find the $y$-intercept, let $x = 0$ and solve for $y$ or $g(x)$ since $y = g(x)$.
$$y = -2(0) = 0$$
The $y$-intercept is $(0, 0)$.

**17.** To find the $x$-intercept, let $k(x) = 0$ and solve for $x$.
$$\begin{aligned} 0 &= \frac{1}{3}x - 8 \\ 8 &= \frac{1}{3}x \\ 24 &= x \end{aligned}$$
The $x$-intercept is $(24, 0)$.
To find the $y$-intercept, let $x = 0$ and solve for $y$ or $k(x)$ since $y = k(x)$.
$$y = \frac{1}{3}(0) - 8 = -8$$
The $y$-intercept is $(0, -8)$.

**18. a.** Use the first three steps of the modeling process to find the equation. Begin by creating a scattergram using the data in the table.
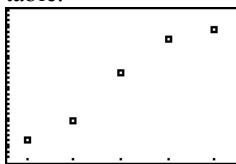


A line close to the data points passes through (16, 56.8) and (20, 81.8). Use those points to find the slope of the line.
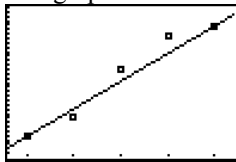
$$m = \frac{56.8 - 81.8}{16 - 20} = 6.25$$

So $f(a) = 6.25a + b$. To solve for $b$, substitute 16 for $a$ and 56.8 for $f(a)$ since the line contains (16, 56.8).

$$56.8 = 6.25(16) + b$$
$$56.8 = 100 + b$$
$$-43.2 = b$$

The equation for $f$ is $f(a) = 6.25a - 43.2$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)
The graph of the model fits the data well.



**b.** The slope of the model is 6.25. This means that the percentage of teenagers who have driver's licenses increases by 6.25 percentage points per year of age.

**c.**   $0 = 6.25a - 43.2$
$$43.2 = 6.25a$$
$$7 \approx a$$
According to the model, 0% of 7 year olds have a driver's license. While this might be true, it implies that some percentage of 8–15 year olds do have a driver's license, something that probably doesn't happen.

**d.** Let $a = 21$ and solve for $f(a)$.
$$f(21) = 6.25(21) - 43.2$$
$$= 131.25 - 43.2$$
$$= 88.05$$
This means that 88.05 percent of 21-year-old adults have a driver's license.

**e.** Let $f(a) = 100$ and solve for $a$.
$$100 = 6.25a - 43.2$$
$$143.2 = 6.25a$$
$$23 \approx a$$
This means that 100 percent of 23-year-old adults have a driver's license. Model breakdown has likely occurred.

**19. a.** Use the first three steps of the modeling process to find the equation. Begin by creating a scattergram using the data in the table.
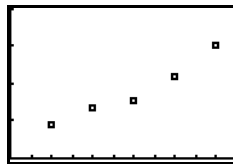


A line close to the data points passes through (8, 2.7) and (16, 4.4). Use those points to find the slope of the line.

$$m = \frac{4.4 - 2.7}{16 - 8} = \frac{1.7}{8} \approx 0.21$$

So $f(t) = 0.21t + b$. To solve for $b$, substitute 8 for $t$ and 2.7 for $f(t)$ since the line contains (8, 2.7).

$$2.7 = 0.21(8) + b$$
$$2.7 = 1.68 + b$$
$$1.02 = b$$

The equation for $f$ is $f(t) = 0.21t + 1.02$. (Your equation may be slightly different if you chose different points, or if you used the linear regression feature of your graphing calculator.)

**b.** The slope of the model is 0.21. This means the number of farmers markets is increasing by 0.21 thousand (210) markets per year.

**c.** $f(8) = 0.21(8) + 1.02 = 1.68 + 1.02 = 2.70$

This means in 1998, there were about 2.7 thousand farmers markets.

**d.** Let $f(t) = 8$ and solve for $t$.
$$8 = 0.21t + 1.02$$
$$6.98 = 0.21t$$
$$33.24 \approx t$$
This means in $1990 + 33.24 \approx 2023$, there will be 8 thousand farmers markets.

**e.** To find the $t$-intercept, set $f(t)$ equal to 0 and solve for $t$.

$$0 = 0.21t + 1.02$$
$$-1.02 = 0.21t$$
$$-4.86 \approx t$$

According to the model, in $1990 - 4.86 \approx 1985$, there were no farmers markets. Model breakdown has occurred.

**20.** Use the points (8, 239.1) and (20, 613.8) to find the slope of the line.

$$m = \frac{613.8 - 239.1}{20 - 8} = \frac{374.7}{12} \approx 31.23$$

Let $t$ be the number of years after 1990. So $f(t) = 31.23t + b$. To solve for $b$, substitute 8 for $t$ and 239.1 for $f(t)$ since the line contains (8, 239.1).

$$239.1 = 31.23(8) + b$$
$$239.1 = 249.84 + b$$
$$-10.74 = b$$

The equation for $f$ is $f(t) = 31.23t - 10.74$. Predicting for 2017 using this equation, let $t = 27$ for $f(t)$.

$$f(27) = 31.23(27) - 10.74$$
$$= 843.21 - 10.74$$
$$= 832.47$$

This means that in 2017, total ad spending for the NCAA basketball tournament can be predicted to be about $832.5 million.

**21. a.** Yes, the rate of increase has remained constant, showing a linear relationship. The slope in this case is 1.7, or an increase of 1.7 states per year.

**b.** The $n$-intercept is the $n$ value when $t = 0$, or during 2012. In 2012, $n = 29$, meaning there were 29 states with ethanol plants in 2012.

**c.** Based on the first two parts, the equation of the model is $n(t) = 1.7t + 29$.

**d.** Since $2018 - 2012 = 6$, $t = 6$ corresponds to 2018.

$$f(6) = 1.7(6) + 29 = 10.2 + 29 = 39.2$$

The model predicts that in 2012, 39 states will have ethanol plants.

**e.** To find when all states have ethanol plants, substitute 50 for $n$, and solve for $t$.

$$50 = 1.7t + 29$$
$$21 = 1.7t$$
$$12.35 \approx t$$

The model predicts that in $2012 + 12.35 \approx 2024$, all states will have ethanol plants.

**22.** Answers may vary. Example: