CHAPTER 1

The Study of Motion

CHAPTER OUTLINE

CHAPTER OVERVIEW

This chapter introduces the measurement of motion. We start with systems of units and unit conversions. Metric and English units for length, area, volume, time, frequency and mass are presented. The concepts of speed, velocity, and the ideas of vectors and scalars follow, and are used to develop the idea of acceleration. Centripetal acceleration, plus examples of motion with constant velocity and constant acceleration are used to show how distance, speed, and time are related to each other in simple types of motion, and to show how relationships can be expressed in different ways—using words, equations, tables, and graphs. The chapter concludes with a section on Aristotle and Galileo. As with the Profiles in Physics sections in the other chapters, the material here can be skipped, treated lightly, stressed, or elaborated on.

LEARNING OBJECTIVES

A student who has mastered this material should be able to:

1. Explain what is meant by the term *fundamental physical quantity*, giving examples.
2. Remember the factors of ten associated with the common metric prefixes milli-, centi-, kilo-, mega-, and giga-.
3. Convert distances and speeds from metric to English units and vice versa.
4. Explain what area and volume measure.
5. Define period and frequency and explain their relationship to each other.
6. Remember what hertz (Hz) measures.
7. Try to explain the physical meaning of mass.
8. Make the distinction between average speed and instantaneous speed.
9. Calculate average speeds from position vs. time data.
10. Distinguish between speed and velocity.
11. Explain how positive and negative velocities can represent movement in opposite directions.
12. Do vector addition graphically as well as resolve a vector into its components.
13. Explain carefully the concept of acceleration and distinguish the physics usage of the term from its everyday meaning.
14. Compute accelerations from velocity versus time data.
15. Understand centripetal acceleration and do computations using the expression v^2/r.
16. Analyze simple types of motion (e.g., uniform motion, free fall, other motion with constant acceleration) using tables, graphs, and equations relating position, velocity and acceleration as functions of time.
17. Explain the physical meaning of the slope of distance versus time $(d\text{-}t)$ and velocity versus time $(v\text{-}t)$ graphs.
18. Relate a bit of the early history of physics from the work of Aristotle and Galileo.

TEACHING SUGGESTIONS AND LECTURE HINTS

The metric prefixes in Table P.2 are the standard way of presenting physical quantities with their units of measure, and the conversion of units should be stressed for future use. In connection with Table 1.1 you may want to go over scientific notation or point out the Math Review (Appendix B). The method of unit conversion presented is not the most common one and you may not want to use it. Have the students attempt a unit conversion during class as soon as you've introduced the material. Spending just a few minutes looking over their shoulders as they work will give you plenty to fix.

Measure your height with a one-foot ruler and with a meter stick.

To stress the concept of learning metric units, you might ask them to choose between jumping off a 10-meter high diving board or having a mass of 40 kilograms placed in their laps. Also, the story of the embarrassing loss of the Mars Climate Orbiter spacecraft due to contractors' confusion over units should be told.

Bring a pendulum for discussion of period, frequency, and timekeeping. Set it swinging and have the students count the number of oscillations between different start and stop times. Students counting the swings of a pendulum can suffer from the notorious "off by one" error (they start from *one* instead of from *zero*), and from doubling the count because of the symmetry of "back" and "forth". You might try to trip them up by picking a start or stop time in the middle of a swing.

The "Time Out" Commercial Application should help your students understand why new definitions of standards come to replace old ones as improved technology allows (and demands) greater precision. But I learned recently that my own students still wondered how such complicated numbers as 299,792,458 m/s or 9,192,631,770 Hz arose. One of them said these numbers looked "almost random." They're not. They result from "matching" the old definition to the new one as closely as possible. The idea in switching to a new definition isn't to change the size of the unit, but to pin it down to more decimal places. The previously established value, to the precision it was known, has to stay.

Have a dynamics cart, a ball, or other object to demonstrate weight (by lifting it) and mass (by accelerating it).

The distinction between speed and velocity has to be stressed, and I point out that changes in direction of motion are equally as important as changes in speed as far as mechanics is concerned. Vector addition and the resolution of a vector into components occur only a few times in the text and are presented geometrically.

You can "add" vertical and horizontal velocities by drawing a vertical line on the blackboard while standing still and then doing the same thing as you walk parallel to the board. The direction of the resultant velocity of the chalk is given by the line on the board. By varying the two speeds you can show that the direction changes. With a stopwatch and meter-stick you can measure the horizontal, vertical, and resultant speeds.

The precise concept of acceleration is new to most students and it deserves a lot of attention because of its importance in Chapter 2. The fact that a change in direction of motion is an acceleration is foreign to students and has to be explained. If you are in a colder climate, you might describe how a perfectly slick sheet of ice keeps a car from speeding up, slowing down, or going around a curve. I talk about having sun glasses in a case on the dashboard of my car and exiting an interstate at a constant speed of 40 mph. If the situation is right, the glasses case will slide across the dashboard, which is a demonstration of centripetal acceleration. All three would be accelerations.

You might bring up the 'cornering acceleration' of cars in g's as presented in automotive magazines, or the use of 'g-suits' by pilots of higher performance aircraft.

When discussing acceleration and free fall, carry a rubber ball around and drop it several times for visual effect. Roll it down a ramp to illustrate smaller acceleration. You may want to show a strobe photo of a falling ball (look at Figure 1.27), perhaps even take measurements on it and show how the speed increases. Swing a rubber stopper attached to a strong fishing line overhead when talking about centripetal acceleration.

The two main thrusts of Section 1.4 are to show how distance, speed, and time are related to each other in simple types of motion, and to show how relationships can be expressed in different ways—using words, equations, tables, and graphs. Try constructing motion graphs by moving around (back and forth along a line) in front of the class and having the students try to draw the d-t or v-t graph corresponding to your movement. Also try acting out graphs dreamed up by the students. (Sometimes this requires infinite quickness or the ability to split in two!) You may not want to stress the graphing, particularly the calculation of slope, as much as I have unless you use it in labs as I do.

I usually point out, but do not dwell on, the fact that air resistance eventually affects the acceleration of a falling body. This is treated in Section 2.5.

Use an air track to show constant acceleration by raising one end slightly. By measuring the tilt angle beforehand you can predict the acceleration. Predict the distance a glider will travel in 1 s (for example) and use photogates or perhaps a student with a stopwatch to time it.

A Demonstration Handbook for Physics (DHP), page M-6, item Mb-12 (a long string with balls tied along it with appropriate spacing so that when the whole thing is hung vertically and released the balls hit at equal time intervals) is an interesting way to show how distance traveled increases during free fall.

Do the standard acceleration of gravity lab that uses a spark timer and a strip of paper. I have my students graph the values of v, taken every 1/60th of a second, versus time and compute the slope of a 'best' straight line drawn through the data. They also have the option of using a linear regression program (I don't call it that) on a computer.

The graphs in Figures 1.29 (car acceleration) and 1.31 (karate chop) can lead to some interesting discussions. Encourage (make) the students go to the library and find the whole story on the karate blow in the September 1983 *American Journal of Physics*. You might jump ahead a bit and mention that the force on the fist during the deceleration is on the order of 500 lbs. (See Problem 13 in Chapter 2.) At about 25 m/s the position of the fist drops below zero, and the velocity of the fist goes more negative (downward faster), so the concrete block must have cracked and given way.

Use a long evacuated glass tube with a feather and a marble inside to show that all objects fall in the same way if there is no air resistance. You might use the idea of a deflated basketball to explain that a lighter object can fall faster than a heavier one if the air resistance is smaller.

Galileo's work with inclined planes can be presented with an air track or just a ball and a board. While the air track works very well for demonstrating Figure 1.35 (small accelerations on slight slopes, zero acceleration when horizontal), it is limited for use with Figure 1.36 (showing increase of acceleration with steeper slopes).

Aristotle and Galileo provide very fertile ground for discussion topics, particularly if you have an interest in philosophy and/or the history of physics. Some are: the role of philosophy in physics, the almost unquestioned acceptance of Aristotle's physics for centuries, Galileo's difficulty with the Catholic Church, and Galileo's scientific method.

The following websites contain valuable simulations you can use for physics exercises:

http://phet.colorado.edu/en/simulations/category/physics
http://www.physicslessons.com/iphysics.htm
http://jersey.uoregon.edu/vlab/

COMMON MISCONCEPTIONS

My students often express worries like, "Are we going to have to memorize all the conversion factors?" I reassure them that the answer is certainly no. The only conversion I think worth remembering is 1 inch = *exactly* 2.54 cm. Others simply stick with you if you end up using them a lot, but it is not worth trying to memorize them. Just look them up. Practicing *how to do* unit conversions is the important thing.

As mentioned in the Prologue, for over ten years I have been collecting student *reading memos* (see Edwin F. Taylor's Guest Comment in the March 1992 *American Journal of Physics*). The following questions are some examples.

Can kilograms be directly converted into pounds? Can't they be considered the same?

If we restrict ourselves to objects near the surface of the Earth, mass and weight are always in direct proportion, with weight equaling mass times 9.8 m/s^2 (Section 2.1a)—it is easy to think of them as nearly equivalent. This relationship disappears when we take a larger view, thinking of ourselves possibly on the surface of a different planet or in a spaceship in orbit.

When skating, for example, isn't turning increasing velocity?

Since velocity is a vector, the idea of an increase in velocity doesn't really make sense—you can speak of increases or decreases in *speed* without difficulty, but since velocity incorporates direction as well as speed there is no way to characterize a direction change as an increase or a decrease—it is simply a *change* in velocity. Of course you do encounter phrases like "increasing velocity" frequently, but in this case the word velocity is being used in the everyday fashion as a synonym for speed, not in the rigorous physical sense we are trying to learn.

This same conflict between the rigorous physical definition and the usage in everyday speech occurs with the concept of acceleration. Using the term "acceleration" to encompass both speeding up and slowing down as well as changes in direction can be mystifying, as shown by comments like these—

I still do not understand the concept of acceleration. To me, acceleration means "speed up in the motion."

I don't grasp why acceleration includes slowing down as well as speeding up. I can understand that slowing down refers to any change in speed or direction, but I guess when I think of accelerating, I think of moving forward at a faster rate, not a slower rate.

You could lose velocity but according to the book's definition still be accelerating.

Explain that the motivation behind the broader definition is the unity of bringing all these things together under one roof, recognizing the vector nature of velocity and acceleration. The very reason for introducing the concept of a vector is that it can be used in very compact notation. Stop to think about how much is hidden in the standard vector notation $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ (The text does not use this notation.[1]) It takes quite a bit of experience to decode this abbreviation correctly in a variety of different coordinate systems, yet it is possible to do many manipulations in this compact form and avoid a lot of clumsiness. The trouble is that unless you are thoroughly grounded in the underlying details and understand exactly what the notation means, you may find yourself moving symbols around but not really knowing what you are doing. Thus there is little point in trying to introduce this notation to students at this level, but do try to get across that *there is a reason* for learning about vectors and defining velocity and acceleration in the new ways.

The concept of a negative acceleration can cause confusion because it is very tempting to think that a negative acceleration is a deceleration. Example 1.4 points out that the negative sign represents an *opposite direction, not a decrease in magnitude*. This is a very important point, worth emphasizing. Plus and minus are used in vector component notation to represent opposite

[1] Using boldface type for vectors vs. plain italic for the magnitudes of those vectors is the traditional way to distinguish the two quantities, but it is such a subtle thing many students miss it. I recall seeing a new edition of a text where the choice of a different font made boldface and plain text look almost identical—even I couldn't see at first which symbols were supposed to be vectors and which were not! So I am willing to accept some notational abuse (using the same symbols for both vectors and scalars) as long as the context makes clear which we are referring to.

directions, not increasing vs. decreasing. This is the source of much trouble in more advanced courses (and in textbooks!). See Challenge 6.

Students are confused by the idea that only a change in direction and not speed is still an acceleration. Centripetal acceleration needs emphasis because it is indeed the motivation behind the introduction of the vector velocity.

Care should be taken to explore centripetal acceleration in cases other than motion along a circular path at constant speed. Show examples where there are acceleration components both along the path and centripetally (e.g., Challenge 8). Consider a curve other than a circle. It is not obvious right away what "the center of the curve" means in this case—explain the concept of center of curvature.

If you move toward the outside of a circle when going around it, why does the acceleration move toward the center?

It says that a ball on a string above your head is accelerating to the middle of the circle, why then, if let go would it go away from the middle?

These questions can best be answered by a move forward into Chapter 2. The idea is that something is *causing* the centripetal acceleration—something that vanishes when you let go of the string, something that is a bit weaker than necessary when you slide outward as you try to go around a curve in a car, say.

Is acceleration always a constant?

Students may get the wrong impression if their attention is devoted exclusively to constant acceleration problems. (I suspect the writer who gave Spock the line, "The acceleration is no longer a constant," in the movie *Star Trek IV* was a victim of this.) Perhaps have the students extend Concept Map 1.2 to show how they would include a few "Not So Simple" types of motion.

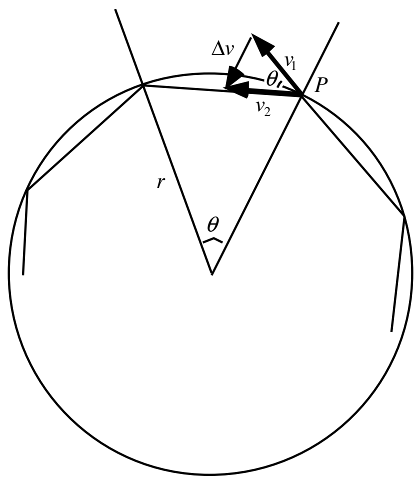
CONSIDER THIS—

With the advent of cheap GPS receivers (see the Time Out! on p. 16) and camcorders a host of "highway kinematics" experiments could be tried. I have tried videotaping the GPS display while driving (it's best for taping to be done by passengers, not the driver, obviously, though I admit I tried it solo with things taped to a 2×4), thus collecting real time data on position and velocity to analyze later. (My first effort uncovered a problem—the camcorder created interference causing the GPS receiver to get "lost"! A student later showed me how to hook up the GPS output port to a portable PC with a homemade cable and capture the data stream to a file—that worked great!) It would be a simple matter to measure the car's acceleration on a straight (and empty) section of highway.

But the GPS receiver displays both the speed and compass heading—having these recorded as functions of time would allow even more interesting experiments. From the change in compass heading and distance traveled, one can compute the radius of curvature of a section of highway, and then use it to calculate the centripetal acceleration of the car. It would be most fun to compute this after having actually experienced it while riding in the car.

The text does not attempt to derive the formula for centripetal acceleration. Here's a neat way to do it due to Newton (adapted from Arnold B. Arons' presentation in his wonderful book, *Teaching Introductory Physics* [ISBN 0-471-13707-3], p. 162).

Imagine a ball bouncing around the inside of a circle as shown below.



At each impact it undergoes a change in velocity that can be found using the technique shown in Figure 1.18. During the impact at point *P* the ball's velocity changes from v_1 to v_2 (while its speed v remains unchanged). Using a little triangle geometry one gets

$$\Delta v = 2v\sin\left(\frac{\theta}{2}\right)$$

Call the time between impacts Δt. The distance the ball goes between impacts, $v\Delta t$, is found by applying exactly the same argument to the triangle having its tip at the center of the circle:

$$v\Delta t = 2r\sin\left(\frac{\theta}{2}\right)$$

The acceleration is then

$$a = \frac{\Delta v}{\Delta t} = \frac{2v\sin\left(\frac{\theta}{2}\right)}{\left(\frac{2r\sin\left(\frac{\theta}{2}\right)}{v}\right)} = \frac{v^2}{r}$$

This is of course only the *average* acceleration during the time between impacts, but if one imagines the ball taking a progression of paths around the circle, each with more impacts per circuit, and extending this to imagine infinitely many impacts—true circular motion—one can see that the above formula is indeed the instantaneous acceleration in that case.

A look back at the above diagram to check the direction of the acceleration may cause a moment's puzzlement—shouldn't the Δv vector be pointed toward the center of the circle? It seems to be aimed too far left. What's wrong? One must realize that the change in velocity happens *at point P* on the diagram—redrawing the Δv vector *starting from point P* will confirm that it indeed points directly toward the circle's center.

This is an important issue in all drawings of vectors. Even though arrows of equal length pointing in the same direction represent the same vector wherever they might be placed in a drawing, the physical context ties the vector to a particular *point* in space. And because arrows have both a tail and a tip, there is a choice to be made—do we attach the tail of the vector to that point, or draw the arrowhead there? Sometimes the choice is easy, as with displacement vectors. Putting the tail of the vector at the starting point and the tip of the vector at the ending point is the obvious thing to do, because both ends of a displacement vector have physical meaning. But velocity and acceleration vectors are different—they describe motion at a single point in space, and there's no way to fit an arrow into just a point. Careful attention must be paid to handling this dilemma.

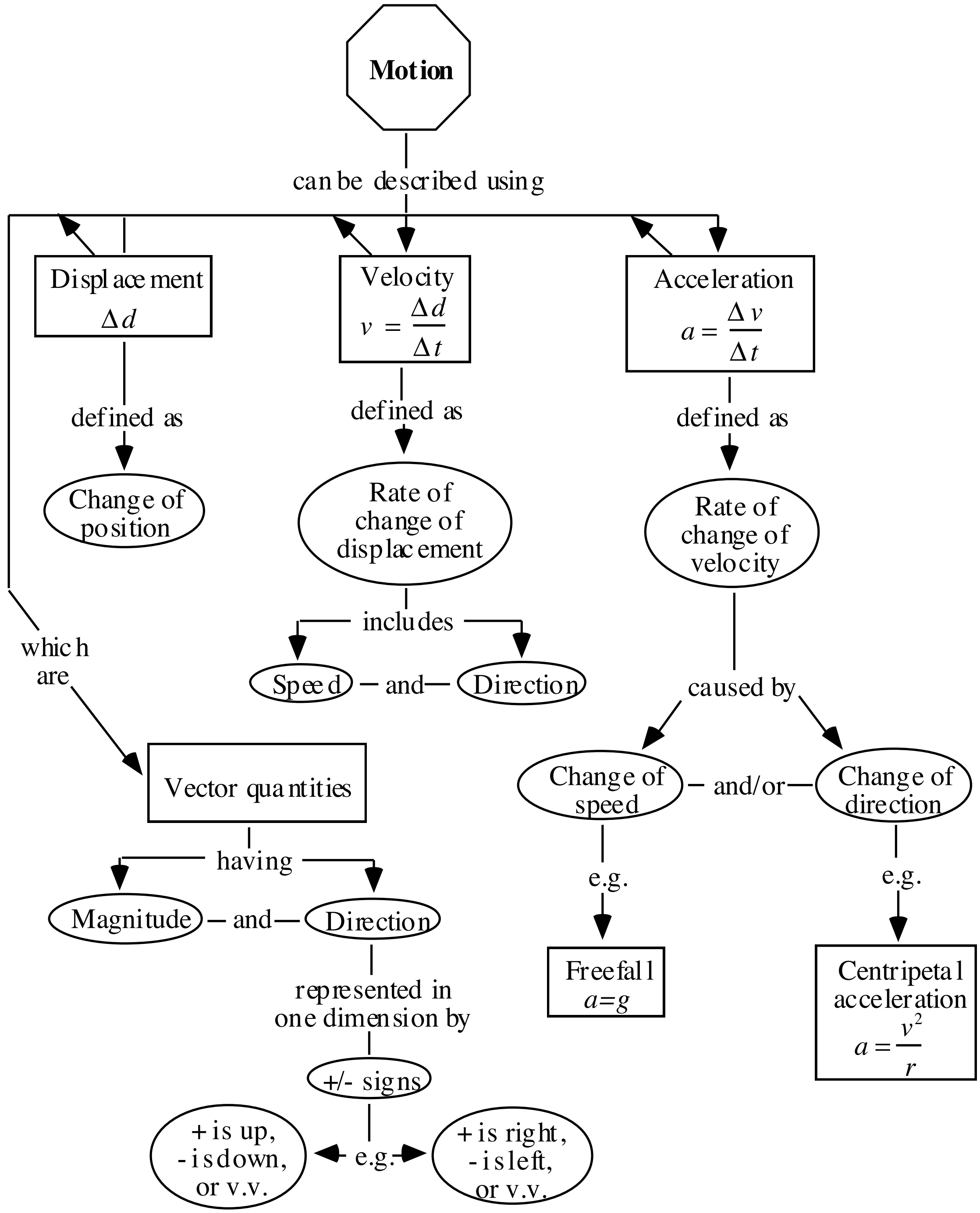
The vectors shown in the text's figures are drawn offset from the bodies whose motion they describe. For simple cases like a car traveling down a straight road (Figure 1.17) it is clear that the velocity vector pictured above the car describes the motion of every part of the car. On the other hand, for an object turning a corner (like the ball on a string in Figure 1.19), different parts of the object have slightly different velocities (e.g., the part of the ball outside the dotted line is moving slightly faster than the point on the inside where the string is tied, and points on the leading and trailing surfaces are going in slightly different directions), and one has to realize that the velocity vector drawn only applies to the motion of one point on the body (intended here to be the ball's center, I assume).

Find out more about the work of the National Institute of Standards and Technology (NIST). Research the history of refinements of the standards for mass, length, and time. Look into atomic clocks and coordinated universal time.

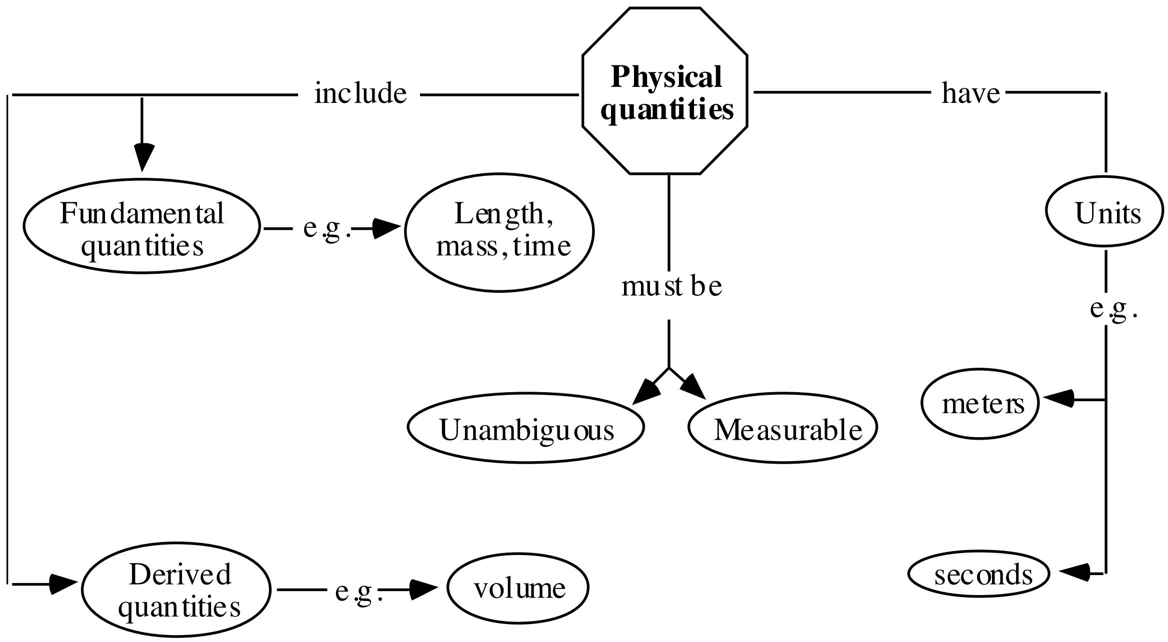
ANSWERS TO MAPPING IT OUT!

1. Some additional concepts that could be added about speed are that it is a relative quantity as well as a scalar, a distinction must be made between instantaneous and average speed, and that *c*, the speed of light, is the fastest speed possible. Velocity is a vector; plus and minus signs are used to represent the directions of the components of a velocity vector; vector addition must be used to find resultant velocities. Increasing speed, decreasing speed, and changing directions are all accelerations. A good example of acceleration due to speed increase is free fall. A good example of acceleration due to changing direction is uniform circular motion.

   A concept map incorporating some of the concepts discussed above appears on the next page.


Motion
can be described using
Displacement
Δd
Velocity
$v = \frac{\Delta d}{\Delta t}$
Acceleration
$a = \frac{\Delta v}{\Delta t}$
defined as
Change of position
defined as
Rate of change of displacement
defined as
Rate of change of velocity
includes
Speed
and
Direction
which are
Vector quantities
having
Magnitude
and
Direction
represented in one dimension by
+/- signs
e.g.
+ is up, - is down, or v.v.
+ is right, - is left, or v.v.
caused by
Change of speed
and/or
Change of direction
e.g.
Freefall
$a=g$
e.g.
Centripetal acceleration
$a = \frac{v^2}{r}$

2.

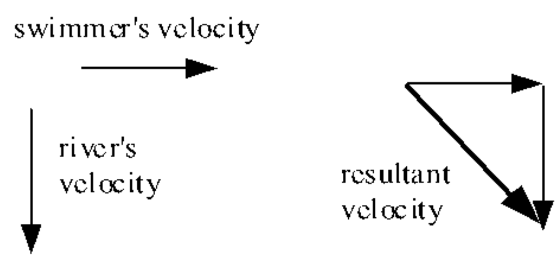


ANSWERS TO QUESTIONS

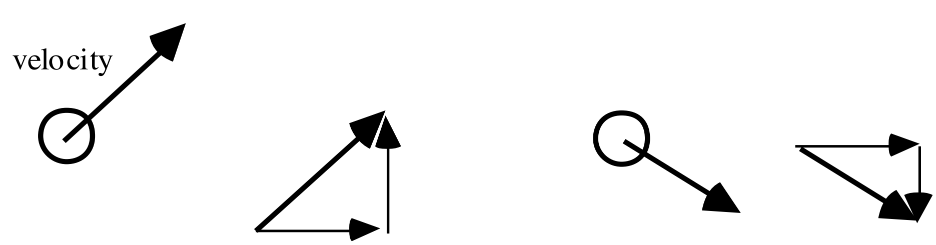
1. If the two rugs are "exactly the same shape," then if one is twice as long as the other it must be twice as wide also. Two factors of two arise, one in the 'length' and one in the 'width', so the area of the larger rug is *four* times that of the smaller one.
2. Derived units are combinations of fundamental units, such as those for area (m^2), volume (m^3), speed (m/s), and acceleration (m/s^2). Many others exist.
3. Smaller period means the frequency is larger. The clock will run *fast* since the pendulum will complete the number of cycles needed to register one minute in a shorter time.
4. Distance, time, and mass are the fundamental physical quantities because other physical quantities are based on these three. For example, speed is based on distance and time.
5. When walking towards the north, the person experiences a greater relative wind speed than when not walking. When walking towards the south, the person experiences lower relative wind speed when walking at a speed faster than the wind, and experiences a "tail wind" when walking slower than the wind speed.
6. The person should run as fast as possible in the direction opposite to that of the train's motion. This will reduce the relative speed between the person and the ground.
7. The physical quantities identified in this chapter are distance, area, volume, time, mass, period, frequency, speed, velocity, displacement, and acceleration. Displacement, area, and volume are derived from distance; period and frequency are derived from time; and speed,

velocity, and acceleration are derived from distance and time. Velocity, displacement, and acceleration are vectors and the others are scalars.

8. Velocity is speed (rate of motion, or distance traveled per unit time) with direction specified. An object moving with constant speed around a circular path, like a satellite in a circular orbit, has nonconstant velocity.
9. Yes, the velocity of the ball changes because there is a change in direction, even though the speed remains the same.
10. Vector addition is the addition of two physical quantities that have both magnitude and direction. It is done by placing the two arrows representing the two vectors together so that the tip of one vector touches the tail of the other vector. Then the vector that is the sum of the two vectors is represented by the arrow drawn from the tail of the first vector to the tip of the second. (See Figures 1.11, 1.12, and 1.13.)
11. The resultant velocity of two velocities has zero magnitude whenever the two velocities have equal magnitudes and are in opposite directions. Example: a person swimming 1 m/s upstream in a river that is flowing with speed 1 m/s has a resultant velocity of zero magnitude.
12. The following sketch shows the swimmer's velocity, the velocity of the river, and the swimmer's resultant velocity at the opposite bank of a river.



13. The basketball's velocity and its components just after the ball leaves the player's hands are shown below at the left. The basketball's velocity and the components of its velocity at the instant just before the ball reaches the basket are shown to the right.



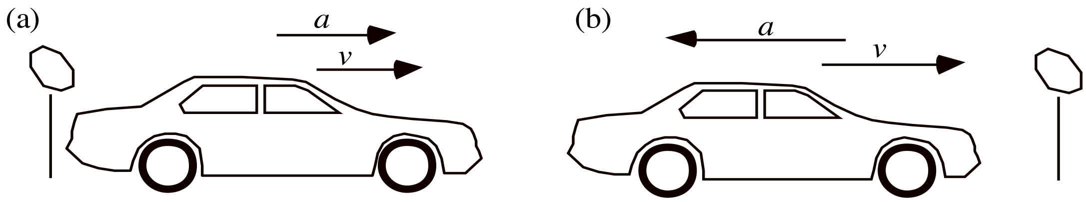
The horizontal component is the same but the vertical component is downward instead of up and smaller in magnitude than the original upward vertical component because the basket is higher than the shooter's hands at the instant of release and the ball reaches the basket before regaining its original vertical speed.

14. Acceleration is the rate of change of velocity. An object is undergoing acceleration whenever its velocity is changing—speed increasing, speed decreasing, or when the direction of motion is changing.
15. If the velocity of the car is zero, then the car is not moving compared to the given frame of reference. If the car is not moving, then there is no change in speed or direction, which means the acceleration must also be zero in the same frame of reference.
16. The speed of a freely falling body increases at a constant rate—9.8 m/s each second, 22 mph each second, or 32 ft/s each second. For a body starting from rest, the distance it has fallen (in m) is 4.9 times the square of the time (in s) elapsed. The acceleration is constant, equal to 9.8 m/s^2.
17. The acceleration (downward) of a falling person is constant (= g). The longer a person falls, the greater the speed ($v = g\ t$). The shorter the distance a person falls, the shorter the time spent accelerating ($d = -\ ½\ g\ t^2$). The person reaches only a slow speed after falling the height of a chair, and comes to a stop (with small upward acceleration) when hitting the floor. Falling from the height of a tall building would give the person a high speed with a huge upward acceleration upon hitting the ground, resulting in severe injury or death.
18. Centripetal acceleration is the acceleration that an object has due to motion along a circular path. It is always directed toward the center of the circular path, perpendicular to the object's velocity. Therefore, the direction of the centripetal acceleration of a car going around a curve is directed toward the center of the curve, perpendicular to the velocity of the car.
19. The runner on the inside track will have a larger centripetal acceleration than the runner on the outside track because the radius of the curve of the inside track is smaller. Because the speeds are the same, v^2 is the same for both runners, but v^2/r is larger for the smaller r.
20. The direction of the car's centripetal acceleration is toward the center of the curve and the direction of the acceleration due to increasing speed is forward (parallel to the velocity).
21. (a) The car's direction stayed constant but its speed decreased by 10 mph in 5 s, so it had an acceleration of 2 mph/s opposite the direction of the car's motion. The acceleration would be aimed at a heading of 250 degrees.

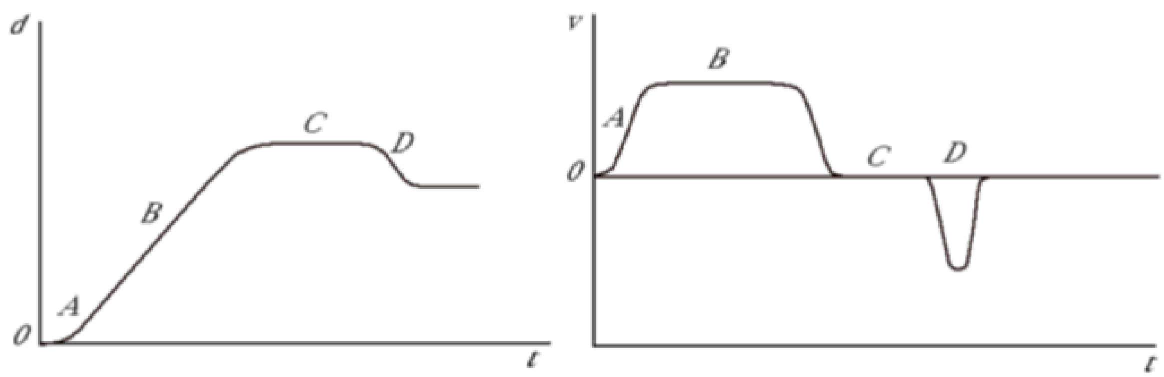
    (b) The car's direction stayed constant but its speed increased by 20 mph in 5 s, so it had an acceleration of 4 mph/s in the direction of the car's motion (at a heading of 70 degrees).

    (c) The car's speed stays constant, but because its direction is changing, there is a centripetal acceleration toward the center of the curve. If the curve is in the shape of a circular arc, then at the start of the 5-second interval the acceleration is initially aimed at a heading of 160 degrees, and as the car steers to heading 90 degrees, the acceleration moves to a heading of 180 degrees. Using the formula from the footnote on p. 19, the magnitude of the centripetal acceleration is 4.2 mph/s.

22. Figure (a) shows the direction of velocity and acceleration for a car speeding up from a stop sign. Figure (b) shows the direction of velocity and acceleration for the car slowing down as it approaches a stop sign.



23. Assuming there is no air resistance, the acceleration of a ball moving upward when thrown straight up into the air is the same as its acceleration when it reaches its highest point and stops in an instant. The ball's acceleration is *always* 9.8 m/s^2 downwards in both cases.
24. Velocity is the quantity physically represented by the slope of a distance-versus-time graph.
25. The sketches below graph the velocity versus time for the motion of the car illustrated in Figure 1.24.



At each point in time in Figure 1.24 the slope of the graph equals the speed. So the velocity is some positive number at *A*, some larger number at *B*, zero at *C*, and some negative number at *D*.

The acceleration is indicated by the slope of the velocity versus time graph. It is some positive number at *A* , zero at *B* and *C*, negative just to the left of *D*, positive just to the right of *D*, and zero at the bottom of the dip at *D*.

26. We establish ***east*** being reckoned as positive **(+)** and ***west*** as negative (-), with the observer standing beside the track watching the train and the occupant pass by. We are asked to rank situations on the resultant velocity of the walker/runner with respect to the observer.

Situation	Person [east(+) and west (-)]	Train [east(+) and west (-)]	Resultant Velocity	Rank
(a)	-2 m/s	+20 m/s	+18 m/s	4
(b)	+6 m/s	-30 m/s	-24 m/s	3

(c)	-4 m/s	-30 m/s	-34 m/s	1
(d)	-10 m/s	+10 m/s	0 m/s	5
(e)	+2 m/s	-20 m/s	-18 m/s	4
(f)	+4m/s	+25 m/s	+29 m/s	2

27. For this question, we must derive an equation for the distance (height) each arrow will travel based on having an initial velocity v_o and knowing the arrow is affected by the acceleration due to gravity (-g in this case). We know the velocity v at any time t is given by:
$v = v_0 + at$ where $a = -g \Rightarrow v = v_0 - gt$.
We also can establish that at the maximum distance (height) the velocity of the arrow will be zero. Therefore, at maximum height: $0 = v_0 - gt \Rightarrow v_0 = gt \Rightarrow \frac{v_0}{g} = t$. We also know that the distance traveled when there is an initial velocity is given by: $d = v_0 t + \frac{1}{2}at^2$. Thus we have:

$$d = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2 \text{ where } t = \frac{v_0}{g}$$

$$d = v_0\left(\frac{v_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0}{g}\right)^2$$

$$d = \frac{v_0^2}{g} - \frac{1}{2}g\frac{v_0^2}{g^2} = \left(\frac{v_0^2}{g}\right) - \frac{1}{2}\left(\frac{v_0^2}{g}\right) = \frac{1}{2}\frac{v_0^2}{g} = \frac{v_0^2}{2g}$$

The distance (height) traveled is independent of the mass of the arrow. This relationship will also be established in Chapter 3 based on the conservation of energy.

Arrow	*M*: Mass (kg)	*V*: Initial Velocity (m/s)	Distance (height) in meters	Rank
A	0.075	16	13.06	2
B	0.180	12	7.35	3
C	0.100	18	16.53	1
D	0.075	12	7.35	3
E	0.120	10	5.10	4
F	0.090	16	13.06	2
G	0.180	10	5.10	4

ANSWERS TO EVEN NUMBERED PROBLEMS

2. Two examples of height expressed in (a) meters and (b) centimeters.

   h = 6 ft 1 in. = 6.083 ft = 1.85 m = 185 cm

   h = 5 ft 7 in. = 5.583 ft = 1.70 m = 170 cm

4. Express 1,609 m (1 mile) in kilometers and centimeters.

   1 mile = 1,609 m = 1.609 km = 160,900 cm

6. The period T of a quartz crystal vibrating with frequency 32,768 Hz is

   $T = 1/f = 1/(32{,}768\ \text{Hz}) = 0.000030518\ \text{s}$

   $= 3.0518\times10^{-5}\ \text{s} = 0.030518\ \text{ms}.$

8. U.S. speed skater Apolo Ohno took the gold medal for the 500-m sprint by completing the course in 41.935 s. His average speed for the event was

   $v_{\text{average}} = 500\ \text{m}/41.935\ \text{s}$

   $= 11.923\ \text{m/s}.$

   See Example 1.2.

10. How long does it take a laser beam to go from Earth to the Moon and back at the speed of light, $3.0\times10^{8}\,\text{m/s}$? The Moon is about $3.8\times10^{8}\,\text{m}$ from Earth.

    $2\left(3.8\times10^{8}\,\text{m/s}\right)/\left(3.0\times10^{8}\,\text{m}\right) = 2.53\ \text{s}$

12. What is the velocity (speed and direction) of the air relative to a runner who jogs west at 4 m/s on a day when the wind is blowing toward the south at 3 m/s?

    Adding the wind velocity, magnitude 3 m/s, direction south, to the runner's velocity, magnitude 4 m/s, direction west, and solving for the resultant vector's magnitude using the Pythagorean theorem, the resultant vector has magnitude 5 m/s, pointing toward the southwest.

14. How far does a long-distance runner with an average speed of 4 m/s travel in 20 min?

    $d = v_{\text{average}}t = (4\ \text{m/s})(20\ \text{min})(60\ \text{s}/1\ \text{min}) = 4{,}800\ \text{m}$

16. Compute the elevator's velocity from the graph at the times marked a, b, and c. (See Figure 1.38.)

    at time a: v = 2 m/s

    at time b: v = 0 m/s

    at time c: v = –2 m/s

18. As a baseball is thrown, it goes from 0 to 40 m/s in 0.15 s.

    (a) The acceleration of the baseball is $a = 267\ \text{m/s}^2$,

    (b) or $a = \left(2.67\ \text{m/s}^2\right)/\left(9.8\ \text{m/s}^2\right) = 27.2g.$

20 A child's speed sitting on the edge of a spinning merry-go-round of radius 1.5 m is 2 m/s. The child's acceleration is

$$a=\frac{v^2}{r}=\frac{(2\text{ m/s})^2}{1.5\text{ m}}=2.67\text{ m/s}^2\ .$$ See Example 1.5.

22\. A car goes 50 m/s around a curved track of radius 250 m. The car's acceleration is

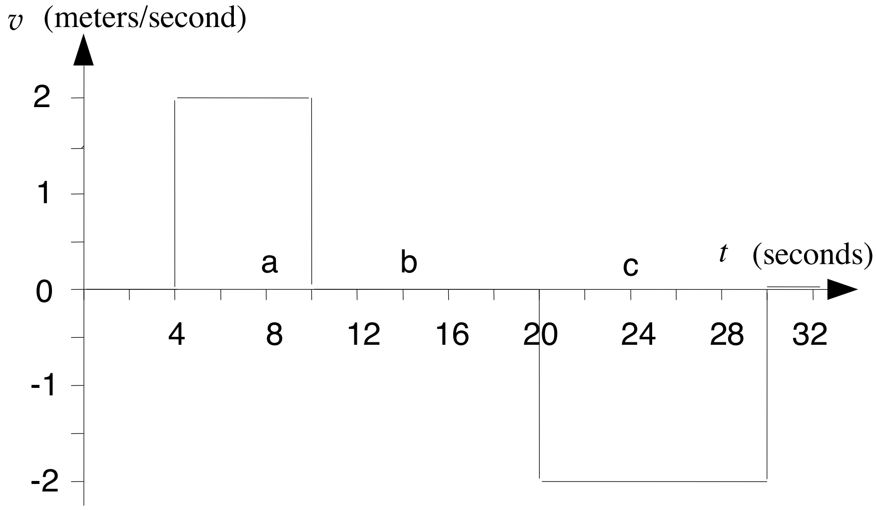
$$a=\frac{v^2}{r}=\frac{(50\text{ m/s})^2}{250\text{ m}}=10\text{ m/s}^2$$ toward the center of curvature.

24\. From rest, a train has a constant acceleration of 0.5 m/s^2.

(a) After 15 s, the train has speed $v=at=(0.5\text{ m/s}^2)(15\text{ s})=7.5\text{ m/s}.$

(b) The time for the train to reach a speed of 25 m/s is $t=v/a=(25\text{ m/s})/(0.5\text{ m/s}^2)=50\text{ s}.$

26\. The following is a graph of velocity versus time for the elevator in Problem 16.



28\. A rock is dropped off the side of a bridge and hits the water below 2 s later.

(a) The rock's velocity when it hits the water is $v=gt=(9.8\text{ m/s}^2)(2\text{ s})=19.6\text{ m/s}.$

(b) The rock's average velocity as it falls is $v_{\text{average}}=(19.6\text{ m/s})/(2\text{ s})=9.8\text{ m/s}.$

(c) The height of the bridge above the water is $h=\frac{1}{2}gt^2=\frac{1}{2}(9.8\text{ m/s}^2)(2\text{ s})^2=19.6\text{ m}.$

30\. During takeoff, an airplane goes from 0 to 50 m/s in 8 s.

(a) Its acceleration is $a=\Delta v/\Delta t=(50\text{ m/s}-0\text{ m/s})/(8\text{ s})=6.25\text{ m/s}^2.$

(b) After 5 s, its speed is $v=at=(6.25\text{ m/s}^2)(5\text{ s})=31.25\text{ m/s}.$

(c) At 50 m/s, it has traveled a distance $d=\frac{1}{2}at^2=\frac{1}{2}(6.25\text{ m/s}^2)(8\text{ s})^2=200\text{ m}.$

32. A bungee jumper falls for 1.3 s before the cord begins to stretch. Until the jumper has bounced back up to this height, the elastic cord causes the jumper to have an average acceleration upward of 4 m/s^2.

(a) The jumper has speed $v = gt = (9.8 \text{ m/s}^2)(1.3 \text{ s}) = 12.74 \text{ m/s}$ when the cord begins to stretch.

(b) The jumper is distance $d = \frac{1}{2}gt^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(1.3 \text{ s})^2 = 8.28 \text{ m}$ below the diving platform at that moment.

(c) The problem says the jumper has an average acceleration of 4 m/s^2 upward from the point where the bungee cord first starts to stretch till the time the jumper has bounced back up to that same point. Assuming the downward and upward halves of the bounce are symmetric, the average acceleration for just the downward part of the trip also will be 4 m/s^2. We solve for the time when the speed has slowed to zero.

$$v = v_0 - at \Rightarrow 0 = v_0 - at \Rightarrow \frac{v_0}{a} = t$$

$$t = \frac{12.74 \text{ m/s}}{4 \text{ m/s}^2} = 3.185 \text{ s}$$

(d) Using the formula on page 33, the distance fallen while the bungee cord is stretching is

$$d = v_0 t - \tfrac{1}{2}at^2 = (12.74 \text{ m/s})(3.185 \text{ s}) - \tfrac{1}{2}(4 \text{ m/s}^2)(3.185 \text{ s})^2 = 20.29 \text{ m.}$$

But the problem asks for the total distance below the platform, so the extra distance fallen from part (b) must be added, giving a total distance of $8.28 \text{ m} + 20.29 \text{ m} = 28.57 \text{ m}$.

ANSWERS TO CHALLENGES

1. A car accelerates and decelerates because of the force of friction between the tires and the road. Most cars have only two-wheel drive so the largest possible forward force is about one-half that when braking, in which all four wheels are used. Also, when accelerating, the force is supplied by the engine which has a limited power output. At higher speeds, the force is smaller, usually because of lower gear ratio. Race cars often have such powerful engines that this latter factor doesn't apply. By using adhesion between special tires and the road, and downward aerodynamic forces, many race cars can accelerate at more than 1 g.
2. (a) The Moon's orbital speed is determined by the following.

$$v = \frac{d}{t}; \quad d = \text{circumference} = 2\pi r$$

$$d = 2\pi\left(3.84\times10^{8}\text{ m}\right)$$

$$= 2.41\times10^{9}\text{m}$$

$$t = 27.3\text{ days} = 27.3\text{ days}\times24\text{ h}\times60\text{ min}\times60\text{ s}$$

$$= 2.36\times10^{6}\text{ s}$$

$$v = \frac{d}{t} = \frac{2.41\times10^{9}\text{m}}{2.36\times10^{6}\text{ s}}$$

$$v = 1{,}021\text{ m/s}$$

(b) The Moon's acceleration is determined by the following.

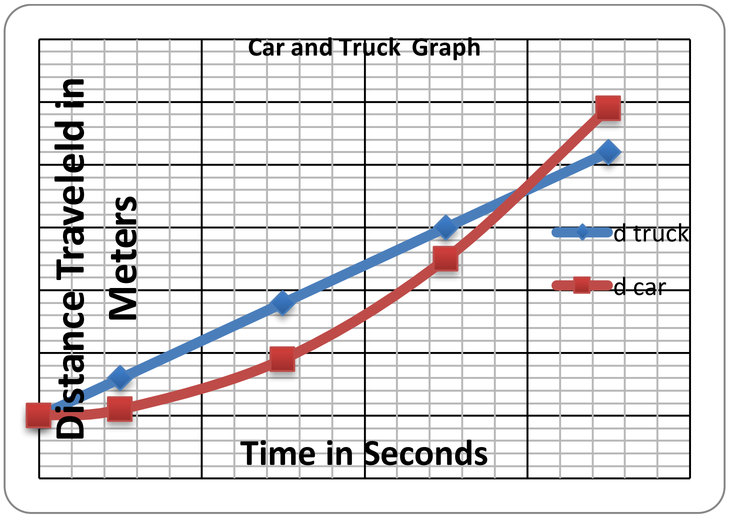
$$a = \frac{v^2}{r} = \frac{\left(1{,}021\text{ m/s}\right)^2}{3.84\times10^{8}\text{m}}$$

$$a = 0.00271\text{ m/s}^2$$

3. (a) For the car, a = 4.0 m/s^2 and for the truck, v = 12 m/s. The distance for the car is d_{car} = ½ at^2 and for the truck $d_{truck} = v\ t$.

d_{car} (m)	d_{truck} (m)	t (s)
0	0	0
2	12	1.0
18	36	3.0
50	60	5.0
98	84	7.0

(b)



At some time (t) the distance traveled by the car and truck are equal.

$d_{\text{car}} = \frac{1}{2}at^2$ and $d_{\text{truck}} = vt$; and at time t; $d_{\text{car}} = d_{\text{truck}}$

$$\frac{1}{2}at^2 = vt$$

$$\frac{at^2}{at} = \frac{2vt}{at}$$

$$t = \frac{2v}{a} = \frac{2 \times 12 \text{ m/s}}{4.0 \text{ m/s}^2} = 6.0 \text{ s}$$

4. (a) The maximum speed that the car can go around a curve of radius $r = 100$ m with acceleration $0.85\ g$ is calculated by the following.

$$a = (v^2)\ |\ r$$

$$v^2 = a\,r;\quad a = 0.85\ g = 0.85 \cdot 9.8 \text{ m/s}^2$$

$$a = 8.33 \text{ m/s}^2$$

$$v^2 = 8.33 \text{ m/s}^2 \cdot 100 \text{ m} = 833 \text{ m}^2/\text{s}^2$$

$$v = 28.9 \text{ m/s}$$

(b) The maximum speed for a 50-m radius curve is

$$v^2 = 8.33 \text{ m/s}^2 \cdot 50 \text{ m} = 416.5 \text{ m}^2/\text{s}^2$$

$$v = 20.4 \text{ m/s}.$$

(c) If the maximum cornering acceleration is $0.6g$, we recalculate the answers to part (a) where $r = 100$ m and to part (b) where $r = 50$ m.

$a = 0.6\ g\text{'s} = 0.6 \cdot 9.8\ \text{m/s}^2 = 5.88\ \text{m/s}^2$

For $r = 100$ m:

$v^2 = 5.88\ \text{m/s}^2 \cdot 100\ \text{m} = 588\ \text{m}^2/\text{s}^2$

$v = 24.2$ m/s

For $r = 50$ m:

$v^2 = 5.88\ \text{m/s}^2 \cdot 50\ \text{m} = 294\ \text{m}^2/\text{s}^2$

$v = 17.1$ m/s

5. Let a denote the acceleration on the newly discovered planet.

$$d = \tfrac{1}{2}at^2 \rightarrow a = \frac{2d}{t^2}$$

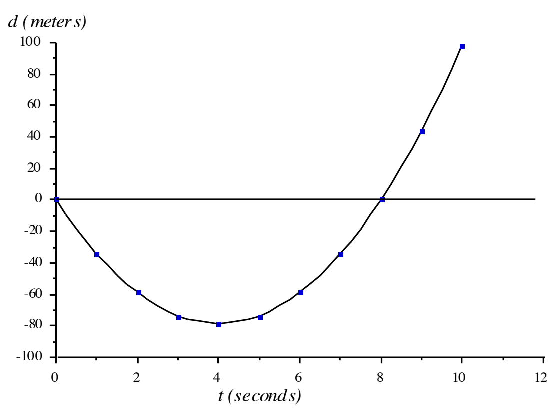
$$a = \frac{2(2.0\ \text{m})}{(0.71\ \text{s})^2} = \frac{4.0\ \text{m}}{0.504\ \text{s}^2}$$

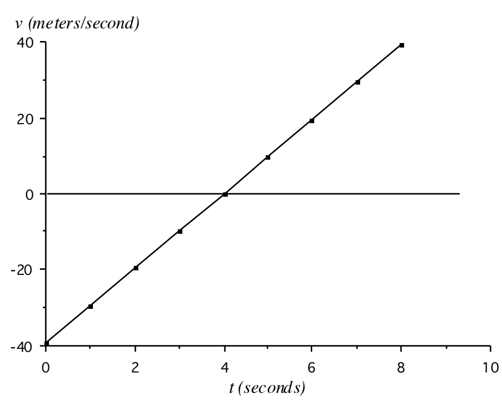
$$a = 7.9\ \text{m/s}^2$$

6. (a) The free-fall results on pages 31 through 33 were developed using positive velocities to represent *downward* movement. Upward movement can be represented using *negative* velocities (see page 31). The graph in Figure 1.26 and the corresponding table can be extended as shown below in part (b). Because the motion does not start from rest, the footnote on page 25 applies as well. We can use the equation

$$d = v_{\text{initial}} + \tfrac{1}{2}at^2$$

with v_{initial} a negative quantity in order to get the position as a function of time. Consistency with our choice of positive velocity indicating downward movement demands that positive values of d indicate positions *below* the starting point and negative values of d indicate positions *above* the starting point.





Time (s)	Velocity (m/s)
0	–39.2
1.0	–29.4
2.0	–19.6
3.0	–9.8
4.0	0
5.0	9.8
6.0	19.6
7.0	29.4
8.0	39.2

(b) The graphs above represent the speed and distance of the baseball thrown vertically upward at a s peed of 39.2 m/s (88 mph).

The highest point is reached at the time the baseball switches from traveling upward to traveling downward—at this instant its velocity is zero. The time can be found by inspecting the graphs above and the table to the right, or by using the following equation.

$$v = v_{\text{initial}} + at$$

$$0 = -39.2\text{ m/s} + 9.8\text{ m/s}^2 \cdot t$$

$$t = \frac{39.2\text{ m/s}}{9.8\text{ m/s}^2} = 4.0\text{ s}$$

The height at time $t = 4.0$ s is given by

$$d = v_{\text{initial}}t + \tfrac{1}{2}at^2 = (-39.2\text{ m/s})(4.0\text{ s}) + \tfrac{1}{2}(9.8\text{ m/s}^2)(4.0\text{ s})^2 = -78.4\text{ m}$$

(Remember the negative sign means the distance is *above* the starting point, as counterintuitive as that may seem.)

7. We derive the equation relating speed to distance for an object starting at rest with constant acceleration.

$$v = at; \quad t = v \div a$$

$$d = \tfrac{1}{2}a\left(\frac{v}{a}\right)^2 = \frac{av^2}{2a^2}$$

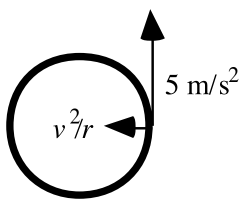
$$d = \frac{v^2}{2a}$$

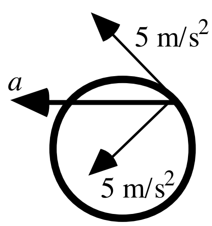
$$v^2 = 2ad$$

When the object is dropped from a height of 4.9 m, we have

$$v = \sqrt{2ad}$$
$$= \sqrt{2gd} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 4.9 \text{ m}} = \sqrt{96.04 \text{ m}^2/\text{s}^2}$$
$$= 9.8 \text{ m/s.}$$

8. The race car's acceleration has a "straight ahead" component and a centripetal component. The straight ahead component is the rate of increase of the car's speed, 5 m/s^2.





The centripetal component has magnitude v^2/r with r =100 m. Because the track is circular, the two components are always aimed at right angles to each other. The car's vector acceleration will be aimed 45° away from straight ahead when the two components are equal in magnitude, i.e., when $v^2/100 \text{ m} = 5 \text{ m/s}^2$, which makes $v = \sqrt{(100 \text{ m})(5 \text{ m/s}^2)} = 22.36 \text{ m/s.}$ The car's speed has been increasing from rest at a constant rate of 5 m/s^2, therefore we can use the equation $v = at$ with a = 5 m/s^2 and v = 22.36 m/s to solve for the time t required.

$$v = at$$
$$22.36 \text{ m/s} = 5 \text{ m/s}^2 \cdot t$$
$$t = \frac{22.36 \text{ m/s}}{5 \text{ m/s}^2} = 4.47 \text{ s}$$

The magnitude of the car's vector acceleration at this time is

$$a = \sqrt{a^2_{\text{straight ahead}} + a^2_{\text{centripetal}}} = \sqrt{\left(5 \text{ m/s}^2\right)^2 + \left(5 \text{ m/s}^2\right)^2} = 7.07 \text{ m/s}^2.$$

9. Col. Stapp reached a velocity of 282.4 m/s after which the sled came to a stop in 1.4 s.

   (a) The average (negative) acceleration he experienced as the sled came to a stop is

$$a = \frac{\Delta v}{\Delta t} = \frac{0\text{ m/s} - 282.4\text{ m/s}}{1.4\text{ s}} = -201.7\text{ m/s}^2.$$

The acceleration in g's $= \dfrac{a}{9.8\text{ m/s}^2} = -\dfrac{201.7\text{ m/s}^2}{9.8\text{ m/s}^2} = -20.6g.$

(b) The distance traveled during the time it took to stop the sled is

$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2}\left(201.7\text{ m/s}^2\right)\left(1.4\text{ s}\right)^2 = 197.7\text{ m}.$$