Chapter 2: Analysis of Graphs and Functions

2.1: Graphs of Basic Functions and Relations; Symmetry

1. $(-\infty, \infty)$.
2. $(-\infty, \infty)$; $[0, \infty)$
3. $(0, 0)$
4. $[0, \infty)$; $[0, \infty)$
5. increases
6. $(-\infty, 0]$; $[0, \infty)$
7. x-axis
8. even
9. odd
10. y-axis; origin
11. The domain can be all real numbers; therefore, the function is continuous for the interval $(-\infty, \infty)$.
12. The domain can be all real numbers; therefore, the function is continuous for the interval $(-\infty, \infty)$.
13. The domain can only be values where $x \geq 0$; therefore, the function is continuous for the interval $[0, \infty)$.
14. The domain can only be values where $x \leq 0$; therefore, the function is continuous for the interval $(-\infty, 0]$.
15. The domain can be all real numbers except -3; therefore, the function is continuous for the interval $(-\infty, -3) \cup (-3, \infty)$.
16. The domain can be all real numbers except 1; therefore, the function is continuous for the interval $(-\infty, 1) \cup (1, \infty)$.
17. (a) The function is increasing for the interval $(3, \infty)$
    (b) The function is decreasing for the interval $(-\infty, 3)$
    (c) The function is never constant; therefore, none.
    (d) The domain can be all real numbers; therefore, the interval $(-\infty, \infty)$.
    (e) The range can only be values where $y \geq 0$; therefore, the interval $[0, \infty)$.
18. (a) The function is increasing for the interval $(4, \infty)$
    (b) The function is decreasing for the interval $(-\infty, -1)$
    (c) The function is constant for the interval $(-1, 4)$
    (d) The domain can be all real numbers; therefore, the interval $(-\infty, \infty)$.
    (e) The range can only be values where $y \geq 3$; therefore, the interval $[3, \infty)$.
19. (a) The function is increasing for the interval $(-\infty, 1)$
    (b) The function is decreasing for the interval $(4, \infty)$

(c) The function is constant for the interval $(1,4)$

(d) The domain can be all real numbers; therefore, the interval $(-\infty,\infty)$.

(e) The range can only be values where $y \le 3$; therefore, the interval $(-\infty,3]$.

20. (a) The function is never increasing; therefore, none.

(b) The function is always decreasing; therefore, the interval $(-\infty,\infty)$.

(c) The function is never constant; therefore, none.

(d) The domain can be all real numbers; therefore, the interval $(-\infty,\infty)$.

(e) The range can be all real numbers; therefore, the interval $(-\infty,\infty)$.

21. (a) The function is never increasing; therefore, none

(b) The function is decreasing for the intervals $(-\infty,-2)$ and $(3,\infty)$

(c) The function is constant for the interval $(-2,3)$.

(d) The domain can be all real numbers; therefore, the interval $(-\infty,\infty)$.

(e) The range can only be values where $y \le 1.5$ or $y \ge 2$; therefore, the interval $(-\infty,1.5]\cup[2,\infty)$.

22. (a) The function is increasing for the interval $(3,\infty)$.

(b) The function is decreasing for the interval $(-\infty,-3)$.

(c) The function is constant for the interval $(-3,3)$

(d) The domain can be all real numbers except -3; therefore, the interval $(-\infty,-3)\cup(-3,\infty)$.

(e) The range can only be values where $y > 1$; therefore, the interval $(1,\infty)$.

23. Graph $f(x)=x^5$. See Figure 23. As x increases for the interval $(-\infty,\infty)$, y increases; therefore, the function is increasing.

24. Graph $f(x)=-x^3$. See Figure 24. As x increases for the interval $(-\infty,\infty)$, y decreases; therefore, the function is decreasing.

25. Graph $f(x)=x^4$. See Figure 25. As x increases for the interval $(-\infty,0)$ y decreases; therefore, the function is decreasing on $(-\infty,0)$

26. Graph $f(x)=x^4$. See Figure 26. As x increases for the interval $(0,\infty)$, y increases; therefore, the function is increasing on $(0,\infty)$

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

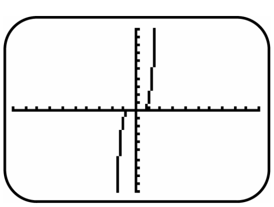

Figure 23

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

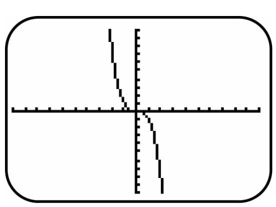

Figure 24

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

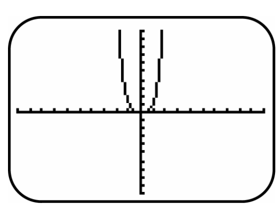

Figure 25

[-10,10] by [-10,10]
Xscl = 1 Yscl= 1

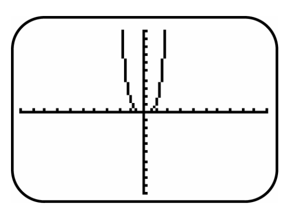

Figure 26

27. Graph $f(x) = -|x|$. See Figure 27. As x increases for the interval $(-\infty, 0)$, y increases; therefore, the function is increasing on $(-\infty, 0)$.

28. Graph $f(x) = -|x|$. See Figure 28. As x increases for the interval $(0, \infty)$, y decreases; therefore, the function is decreasing on $(0, \infty)$.

29. Graph $f(x) = -\sqrt[3]{x}$. See Figure 29. As x increases for the interval $(-\infty, \infty)$, y decreases; therefore, the function is decreasing.

30. Graph $f(x) = -\sqrt{x}$. See Figure 30. As x increases for the interval $(0, \infty)$ y decreases; therefore, the function is decreasing.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

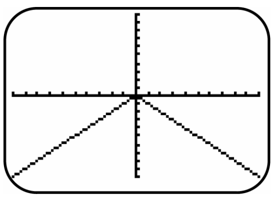

Figure 27

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

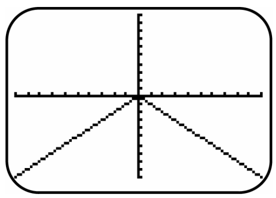

Figure 28

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

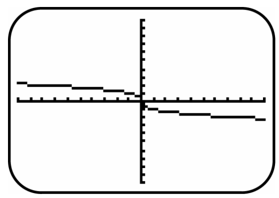

Figure 29

[-10,10] by [-10,10]
Xscl = 1 Yscl= 1

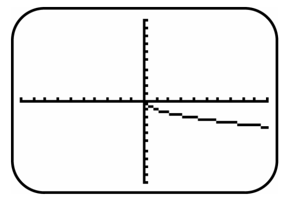

Figure 30

31. Graph $f(x) = 1 - x^3$. See Figure 31. As x increases for the interval $(-\infty, \infty)$, y decreases; therefore, the function is decreasing.

32. Graph $f(x) = x^2 - 2x$. See Figure 32. As x increases for the interval $(1, \infty)$ y increases; therefore, the function is increasing on $(1, \infty)$.

33. Graph $f(x) = 2 - x^2$. See Figure 33. As x increases for the interval $(-\infty, 0)$ y increases; therefore, the function is increasing on $(-\infty, 0)$.

34. Graph $f(x) = |x+1|$. See Figure 34. As x increases for the interval $(-\infty, -1)$ y decreases; therefore, the function is decreasing on $(-\infty, -1)$.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

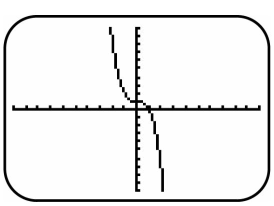

Figure 31

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

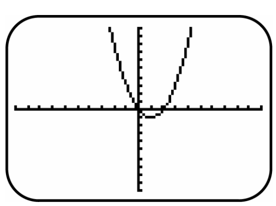

Figure 32

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

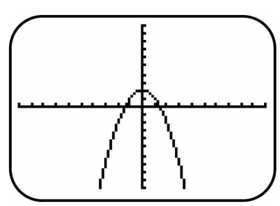

Figure 33

[-10,10] by [-10,10]
Xscl = 1 Yscl= 1

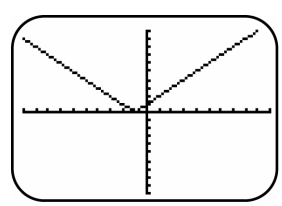

Figure 34

35. (a) No (b) Yes (c) No
36. (a) Yes (b) No (c) No
37. (a) Yes (b) No (c) No
38. (a) No (b) No (c) Yes
39. (a) Yes (b) Yes (c) Yes
40. (a) Yes (b) Yes (c) Yes
41. (a) No (b) No (c) Yes
42. (a) No (b) Yes (c) No
43. (a) Since $f(-x) = f(x)$, this is an even function and is symmetric with respect to the y-axis. See Figure 43a.
(b) Since $f(-x) = -f(x)$, this is an odd function and is symmetric with respect to the origin. See Figure 43b.

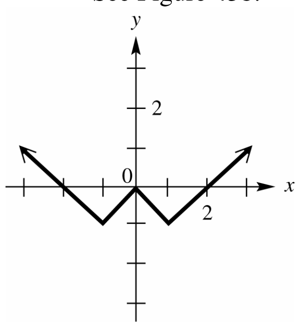

Figure 43a

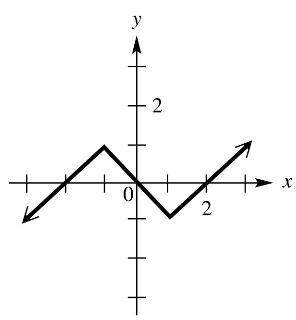

Figure 43b

44. (a) Since this is an odd function and is symmetric with respect to the origin. See Figure 44a.
(b) Since this is an even function and is symmetric with respect to the y-axis. See Figure 44b

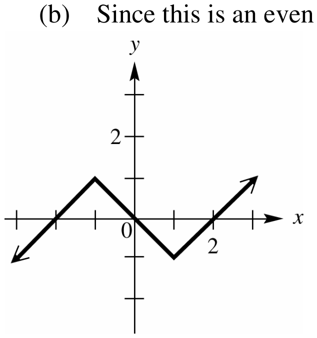

Figure 44a

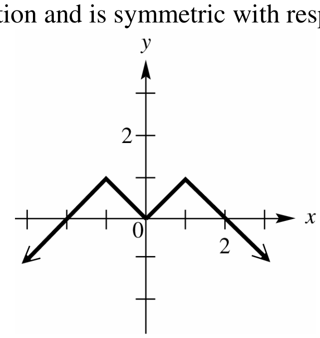

Figure 44b

45. If f is an even function then $f(-x) = f(x)$ or opposite domains have the same range. See Figure 45
46. If g is an odd function then $g(-x) = -g(x)$ or opposite domains have the opposite range. See Figure 46

x	$f(x)$
–3	21
–2	–12
–1	–25
1	–25
2	–12
3	21

Figure 45

x	$g(x)$
–5	13
–3	1
–2	–5
0	0
2	5
3	–1
5	–13

Figure 46

47. This is an even function since opposite domains have the same range.

48. This is an even function since opposite domains have the same range.

49. This is an odd function since opposite domains have the opposite range.

50. This is an odd function since opposite domains have the opposite range.

51. This is neither even nor odd since the opposite domains are neither the opposite or same range.

52. This is neither even nor odd since the opposite domains are neither the opposite or same range.

53. If $f(x) = x^4 - 7x^2 + 6,$ then $f(-x) = (-x)^4 - 7(-x)^2 + 6 \Rightarrow f(-x) = x^4 - 7x^2 + 6.$ Since $f(-x) = f(x),$ the function is even.

54. If $f(x) = -2x^6 - 8x^2,$ then $f(-x) = -2(-x)^6 - 8(-x)^2 \Rightarrow f(-x) = -2x^6 - 8x^2.$ Since $f(-x) = f(x),$ the function is even.

55. If $f(x) = 3x^3 - x,$ then $f(-x) = 3(-x)^3 - (-x) \Rightarrow f(-x) = -3x^3 + x$ and $-f(x) = -(3x^3 - x) \Rightarrow -f(x) = -3x^3 + x.$ Since $f(-x) = -f(x),$ the function is odd.

56. If $f(x) = -x^5 + 2x^3 - 3x,$ then $f(-x) = -(-x)^5 + 2(-x)^3 - 3(-x) \Rightarrow f(-x) = x^5 - 2x^3 + 3x$ and $-f(x) = -(-x^5 + 2x^3 - 3x) \Rightarrow -f(x) = x^5 - 2x^3 + 3x.$ Since $f(-x) = -f(x),$ the function is odd.

57. If $f(x) = x^6 - 4x^4 + 5$ then $f(-x) = (-x)^6 - 4(-x)^4 + 5 \Rightarrow f(-x) = x^6 - 4x^4 + 5.$ Since $f(-x) = f(x),$ the function is even.

58. If $f(x) = 8,$ then $f(-x) = 8.$ Since $f(-x) = f(x),$ the function is even.

59. If $f(x) = 3x^5 - x^3 + 7x,$ then $f(-x) = 3(-x)^5 - (-x)^3 + 7(-x) \Rightarrow f(-x) = -3x^5 + x^3 - 7x$ and $-f(x) = -(3x^5 - x^3 + 7x) \Rightarrow -f(x) = -3x^5 + x^3 - 7x.$ Since $f(-x) = -f(x),$ the function is odd.

60. If $f(x) = x^3 - 4x,$ then $f(-x) = (-x)^3 - 4(-x) \Rightarrow f(-x) = -x^3 + 4x$ and $-f(x) = -(x^3 - 4x) \Rightarrow -f(x) = -x^3 + 4x.$ Since $f(-x) = -f(x),$ the function is odd.

61. If $f(x) = |5x|,$ then $f(-x) = |5(-x)| \Rightarrow f(-x) = |5x|.$ Since $f(-x) = f(x),$ the function is even.

62. If $f(x) = \sqrt{x^2 + 1},$ then $f(-x) = \sqrt{(-x)^2 + 1} \Rightarrow f(-x) = \sqrt{x^2 + 1}.$ Since $f(-x) = f(x),$ the function is even.

63. If $(-3,11)$ and $(2,9)$ then $f(-x) = \dfrac{1}{2(-x)} \Rightarrow f(-x) = -\dfrac{1}{2x}$ and $-f(x) = -\left(\dfrac{1}{2x}\right) \Rightarrow -f(x) = -\dfrac{1}{2x}$ Since $f(-x) = -f(x),$ the function is odd.

64. If $f(x) = 4x - \dfrac{1}{x},$ then $f(-x) = 4(-x) - \dfrac{1}{(-x)} \Rightarrow f(-x) = -4x + \dfrac{1}{x}$ and $-f(x) = -\left(4x - \dfrac{1}{x}\right) \Rightarrow -f(x) = -4x + \dfrac{1}{x}.$ Since $f(-x) = -f(x),$ the function is odd.

65. If $f(x) = -x^3 + 2x,$ then $f(-x) = -(-x)^3 + 2(-x) \Rightarrow f(-x) = x^3 - 2x$ and

$-f(x)=-(-x^3+2x)\Rightarrow -f(x)=x^3-2x.$ Since $f(-x)=-f(x)$, the function is symmetric with respect to the origin. Graph $f(x)=-x^3+2x;$ the graph supports symmetry with respect to the origin.

66. If $f(x)=x^5-2x^3$, then $f(-x)=(-x)^5-2(-x)^3\Rightarrow f(-x)=-x^5+2x^3$ and $-f(x)=-(x^5-2x^3)\Rightarrow -f(x)=-x^5+2x^3$. Since $f(-x)=-f(x)$, the function is symmetric with respect to the origin. Graph $f(x)=x^5-2x^3$; the graph supports symmetry with respect to the origin.

67. If $f(x)=0.5x^4-2x^2+1$, then $f(-x)=0.5(-x)^4-2(-x)^2+1\Rightarrow f(-x)=0.5x^4-2x^2+1$. Since $f(-x)=f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x)=0.5x^4-2x^2+1$; the graph supports symmetry with respect to the y-axis.

68. If $f(x)=0.75x^2+|x|+1$, then $f(-x)=0.75(-x)^2+|(-x)|+1\Rightarrow f(-x)=0.75x^2+|x|+1$. Since $f(-x)=f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x)=.75x^4+|x|+1$; the graph supports symmetry with respect to the y-axis.

69. If $f(x)=x^3-x+3$, then $f(-x)=(-x)^3-(-x)+3\Rightarrow f(-x)=-x^3+x+3$ and $-f(x)=-(x^3-x+3)\Rightarrow -f(x)=-x^3+x-3$. Since $f(x)\neq f(-x)\neq -f(x)$, the function is not symmetric with respect to the y-axis or the origin.

70. If $f(x)=x^4-5x+2$, then $f(-x)=(-x)^4-5(-x)+2\Rightarrow f(-x)=x^4+5x+2$ and $-f(x)=-(x^4-5x+2)\Rightarrow -f(x)=-x^4+5x-2$. Since $f(x)\neq f(-x)\neq -f(x)$, the function is not symmetric with respect to the y-axis or the origin. Graph $f(x)=x^4-5x+2$; the graph supports no symmetry with respect to the y-axis or the origin.

71. If $f(x)=x^6-4x^3$, then $f(-x)=(-x)^6-4(-x)^3\Rightarrow f(-x)=x^6+4x^3$ and $-f(x)=-(x^6-4x^3)\Rightarrow -f(x)=-x^6+4x^3$. Since $f(x)\neq f(-x)\neq -f(x)$, the function is not symmetric with respect to the y-axis or the origin. Graph $f(x)=x^6-4x^3$; the graph supports no symmetry with respect to the y-axis or the origin.

72. If $f(x)=x^3-3x$, then $f(-x)=(-x)^3-3(-x)\Rightarrow f(-x)=-x^3+3x$ and $-f(x)=-(x^3-3x)\Rightarrow -f(x)=-x^3+3x$. Since $f(-x)=-f(x)$, the function is symmetric with respect to the origin. Graph $f(x)=x^3-3x$; the graph supports symmetry with respect to the origin.

73. If $f(x)=-6$, then $f(-x)=-6$, Since $f(-x)=f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x)=-6$; the graph supports symmetry with respect to the y-axis.

74. If $f(x)=|x|$, then $f(-x)=|(-x)|\Rightarrow f(-x)=|x|$. Since $f(-x)=f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x)=|x|$; the graph supports symmetry with respect to the y-axis.

75. If $f(x)=\frac{1}{4x^3}$, then $f(-x)=\frac{1}{4(-x)^3}\Rightarrow f(-x)=-\frac{1}{4x^3}$ and $-f(x)=-\left(\frac{1}{4x^3}\right)\Rightarrow -f(x)=-\frac{1}{4x^3}$. Since $f(-x)=-f(x)$, the function is symmetric with respect to the origin. Graph $f(x)=\frac{1}{4x^3}$; the graph supports symmetry with respect to the origin.

76. If $f(x)=\sqrt{x^2}\Rightarrow f(x)=|x|$, then $f(-x)=\sqrt{(-x)^2}\Rightarrow f(-x)=\sqrt{x^2}\Rightarrow f(-x)=|x|$. Since $f(-x)=f(x)$, the function is symmetric with respect to the y-axis. Graph $f(x)=\sqrt{x^2}$; the graph

2.2: Vertical and Horizontal Shifts of Graphs

1. The equation $y=x^2$ shifted 3 units upward is $y=x^2+3$.
2. The equation $y=x^3$ shifted 2 units downward is $y=x^3-2$.
3. The equation $y=\sqrt{x}$ shifted 4 units downward is $y=\sqrt{x}-4$.
4. The equation $y=\sqrt[3]{x}$ shifted 6 units upward is $y=\sqrt[3]{x}+6$.
5. The equation $y=|x|$ shifted 4 units to the right is $y=|x-4|$.
6. The equation $y=|x|$ shifted 3 units to the left is $y=|x+3|$.
7. The equation $y=x^3$ shifted 7 units to the left is $y=(x+7)^3$.
8. The equation $y=\sqrt{x}$ shifted 9 units to the right is $y=\sqrt{x-9}$.
9. The equation $y=x^2$ shifted 2 units downward and 3 units right is $y=(x-3)^2-2$.
10. The equation $y=x^2$ shifted 4 units upward and 1 unit left is $y=(x+1)^2+4$.
11. The equation $y=\sqrt{x}$ shifted 3 units upward and 6 units to the left is $y=\sqrt{x+6}+3$.
12. The equation $y=|x|$ shifted 1 unit downward and 5 units to the right is $y=|x-5|-1$.
13. The equation $y=x^2$ shifted 500 units upward and 2000 units right is $y=(x-2000)^2+500$.
14. The equation $y=x^2$ shifted 255 units downward and 1000 units left is $y=(x+1000)^2-255$.
15. Shift the graph of f 4 units upward to obtain the graph of g.
16. Shift the graph of f 4 units to the left to obtain the graph of g.
17. The equation $y=x^2-3$ is $y=x^2$ shifted 3 units downward; therefore, graph B.
18. The equation $y=(x-3)^2$ is $y=x^2$ shifted 3 units to the right; therefore, graph C.
19. The equation $y=(x+3)^2$ is $y=x^2$ shifted 3 units to the left; therefore, graph A.
20. The equation $y=|x|+4$ is $y=|x|$ shifted 4 units upward; therefore; graph A.

21. The equation $y = |x+4| - 3$ is $y = |x|$ shifted 4 units to the left and 3 units downward; therefore, graph B.

22. The equation $y = |x-4| - 3$ is $y = f(x)$ shifted 4 units to the right and 3 units downward; therefore, graph C.

23. The equation $y = (x-3)^3$ is $y = x^3$ shifted 3 units to the right; therefore, graph C.

24. The equation $y = (x-2)^3 - 4$ is $y = x^3$ shifted 2 units to the right and 4 units downward; therefore, graph A.

25. The equation $y = (x+2)^3 - 4$ is $[-a, -b]$. shifted 2 units to the left and 4 units downward; therefore, graph B.

26. Using $Y_2 = Y_1 + k$ and $x = 0.$ we get $19 = 15 + k \Rightarrow k = 4.$

27. Using $Y_2 = Y_1 + k$ and $x = 0,$ we get $-5 = -3 + k \Rightarrow k = -2.$

28. Using $Y_2 = Y_1 + k$ and $x = 0$, we get $5.5 = 4 + 1.5 \Rightarrow k = 1.5.$

29. From the graphs, $(6, 2)$ is a point on Y_1 and $(6, -1)$ a point on Y_2. Using $Y_2 = Y_1 + k$ and $x = 6,$ we get $-1 = 2 + k \Rightarrow k = -3.$

30. From the graphs, $(-4, 3)$ is a point on Y_1 and $(-4, 8)$ a point on Y_2. Using $Y_2 = Y_1 + k$ and $x = -4,$ we get $8 = 3 + k \Rightarrow k = 5.$

31. For the equation $y = x^2$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$.

32. For the equation $y = x^2$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 3 units to the right gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[0, \infty)$.

33. For the equation $y = |x|$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 4 units to the left and 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$.

34. For the equation $y = |x|$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 4 units to the right and 3 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-3, \infty)$.

35. For the equation $y = x^3$, the Domain is $(-\infty, \infty)$ and the Range is $(-\infty, \infty)$. Shifting this 3 units to the right gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$

36. For the equation $y = x^3$, the Domain is $(-\infty, \infty)$ and the Range is $(-\infty, \infty)$. Shifting this 2 units to the right and 4 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$

37. For the equation $y = x^2$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 1 unit to the right and 5 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[-5, \infty)$.

38. For the equation $y = x^2$, the Domain is $(-\infty, \infty)$ and the Range is $[0, \infty)$. Shifting this 8 units to the left and 3 units upward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $[3, \infty)$.

39. For the equation $y = \sqrt{x}$, the Domain is $[0, \infty)$. and the Range is $[0, \infty)$. Shifting this 4 units to the right gives us: (a) Domain: $[4, \infty)$. (b) Range: $[0, \infty)$.

40. For the equation $y = \sqrt{x}$, the Domain is $[0, \infty)$. and the Range is $[0, \infty)$. Shifting this 1 units to the left and 10 units downward gives us: (a) Domain: $[-1, \infty)$. (b) Range: $[-10, \infty)$.

41. For the equation $y = x^3$, the Domain is $(-\infty, \infty)$ and the Range is $(-\infty, \infty)$. Shifting this 1 unit to the right and 4 units upward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$

42. For the equation $y = \sqrt[3]{x}$, the Domain is $(-\infty, \infty)$ and the Range is $(-\infty, \infty)$. Shifting this 7 units to the left and 10 units downward gives us: (a) Domain: $(-\infty, \infty)$ (b) Range: $(-\infty, \infty)$

43. The graph of $y = f(x)$ is the graph of the equation $y = x^2$ shifted 1 unit to the right. See Figure 43.

44. The graph of $y = \sqrt{x+2}$ is the graph of the equation $y = \sqrt{x}$ shifted 2 units to the left. See Figure 44.

45. The graph of $y = x^3 + 1$ is the graph of the equation $y = x^3$ shifted 1 unit upward. See Figure 45.

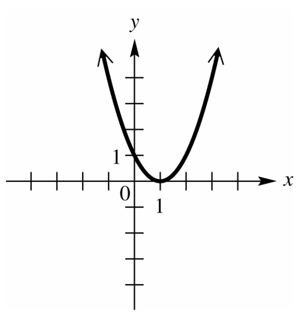

Figure 43

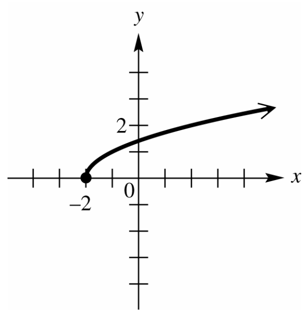

Figure 44

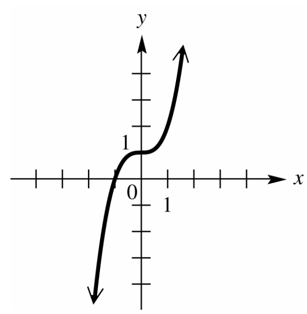

Figure 45

46. The graph of $y = |x+2|$ is the graph of the equation $y = |x|$ shifted 2 units to the left. See Figure 46.

47. The graph of $y = (x-1)^3$ is the graph of the equation $y = x^3$ shifted 1 unit to the right. See Figure 47.

48. The graph of $y = |x| - 3$ is the graph of the equation $y = |x|$ shifted 3 units downward. See Figure 48.

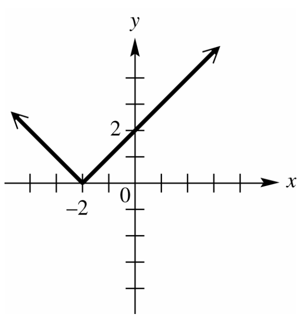

Figure 46

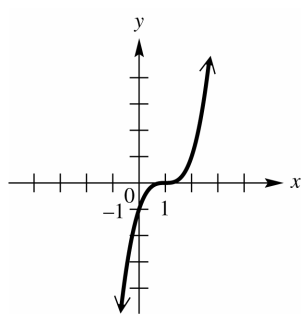

Figure 47

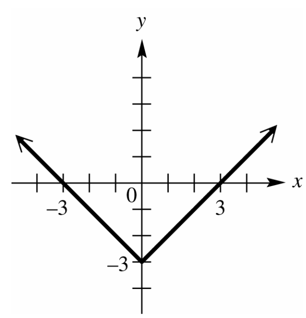

Figure 48

49. The graph of $y = \sqrt{x-2} - 1$ is the graph of the equation $y = \sqrt{x}$ shifted 2 units to the right and 1 unit downward. See Figure 49.

50. The graph of $y = \sqrt{x+3} - 4$ is the graph of the equation $y = \sqrt{x}$ shifted 3 units to the left and 4 units downward. See Figure 50.

51. The graph of $f(x)$ is the graph of the equation $y = x^2$ shifted 2 units to the left and 3 units upward. See Figure 51.

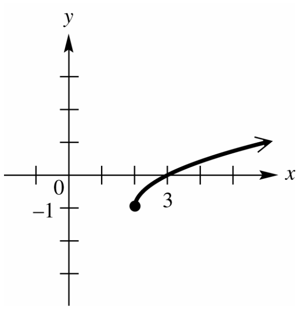

Figure 49

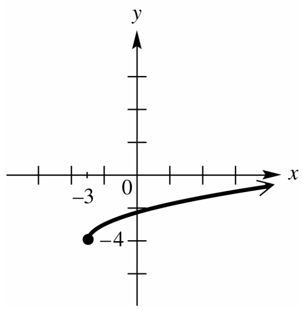

Figure 50

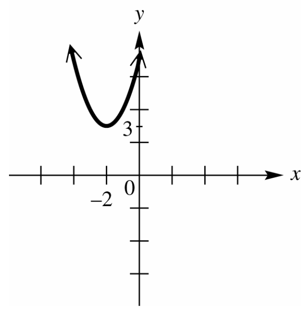

Figure 51

52. The graph of $y=(x-4)^2-4$ is the graph of the equation $y=x^2$ shifted 4 units to the right and 4 units downward. See Figure 52.

53. The graph of $y=|x+4|-2$ is the graph of the equation $y=|x|$ shifted 4 units to the left and 2 units downward. See Figure 53.

54. The graph of $y=(x+3)^3-1$ is the graph of the equation $y=x^3$ shifted 3 units to the left and 1 unit downward. See Figure 54.

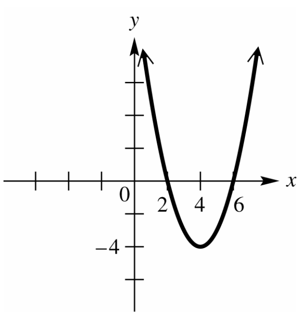

Figure 52

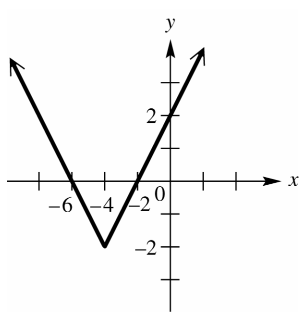

Figure 53

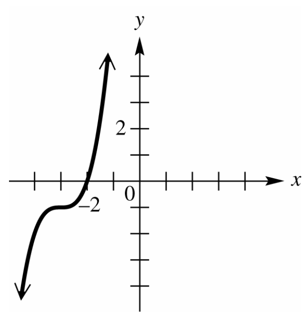

Figure 54

55. Since *h* and *k* are positive, the equation is $y=x^2$ shifted to the right and down; therefore, B.

56. Since *h* and *k* are positive, the equation is $y=x^2$ shifted to the left and down; therefore, D.

57. Since *h* and *k* are positive, the equation is $y=x^2$ shifted to the left and up; therefore, A.

58. Since *h* and *k* are positive, the equation is $y=x^2$ shifted to the right and up; therefore, C.

59. The equation $y=f(x)+2$ is $y=f(x)$ shifted up 2 units or add 2 to the *y*-coordinate of each point as follows: $(-3,2)\Rightarrow(-3,0);(-1,4)\Rightarrow(-1,6);(5,0)\Rightarrow(5,2)$. See Figure 59.

60. The equation $y=f(x)-2$ is $y=f(x)$ shifted down 2 units or subtract 2 from the *y*-coordinate of each point as follows: $(-3,-2)\Rightarrow(-3,-4);(-1,4)\Rightarrow(-1,2);(5,0)\Rightarrow(5,-2)$. See Figure 60.

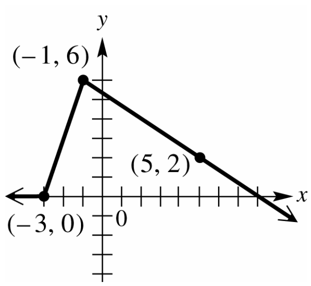

Figure 59

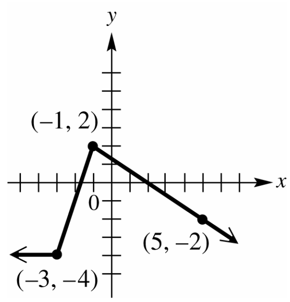

Figure 60

61. The equation $y = f(x+2)$ is $y = f(x)$ shifted left 2 units or subtract 2 from the x-coordinate of each point as follows: $(-3,-2) \Rightarrow (-5,-2); (-1,4) \Rightarrow (-3,4); (5,0) \Rightarrow (3,0)$. See Figure 61.

62. The equation $y = f(x-2)$ is $y = f(x)$ shifted right 2 units or add 2 to the x-coordinate of each point as follows: $(-3,-2) \Rightarrow (-1,-2); (-1,4) \Rightarrow (1,4); (5,0) \Rightarrow (7,0)$. See Figure 62.

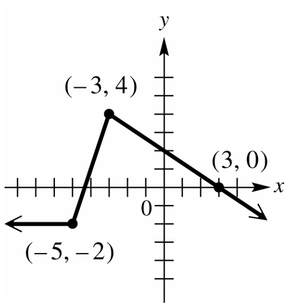

Figure 61

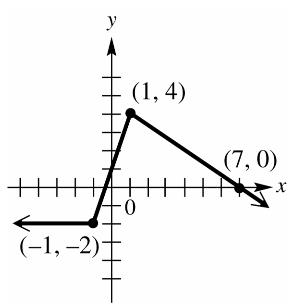

Figure 62

63. The graph is the basic function $y = x^2$ translated 4 units to the left and 3 units up; therefore, the new equation is $y = (x+4)^2 + 3$. The equation is now increasing for the interval: (a) $(-4, \infty)$ and decreasing for the interval: (b) $(-\infty, -4)$.

64. The graph is the basic function $y = \sqrt{x}$ translated 5 units to the left; therefore, the new equation is $y = \sqrt{x+5}$. The equation is now increasing for the interval: (a) $(-5, \infty)$ and does not decrease; therefore: (b) none.

65. The graph is the basic function $y = x^3$ translated 5 units down; therefore, the new equation is $y = x^3 - 5$. The equation is now increasing for the interval: (a) $(-\infty, \infty)$ and does not decrease; therefore: (b) none.

66. The graph is the basic function $y = |x|$ translated 10 units to the left; therefore, the new equation is $y = |x + 10|$. The equation is now increasing for the interval: (a) $(-10, \infty)$ and decreasing for the interval: (b) $(-\infty, -10)$

67. The graph is the basic function $y = \sqrt{x}$ translated 2 units to the right and 1 unit up; therefore, the new equation is $y = \sqrt{x-2} + 1$. The equation is now increasing for the interval: (a) $(2, \infty)$ and does not decrease; therefore: (b) none.

68. The graph is the basic function $y = x^2$ translated 2 units to the right and 3 units down; therefore, the new equation is $y = (x-2)^2 - 3$. The equation is now increasing for the interval: (a) $(2,\infty)$ and decreasing for the interval: (b) $(-\infty,2)$.

69. (a) $f(x) = 0$: $\{3,4\}$

(b) $f(x) > 0$: for the intervals $(-\infty,3)\cup(4,\infty)$.

(c) $f(x) < 0$: for the interval $(3,4)$.

70. (a) $f(x) = 0$: $\{\sqrt{2}\}$

(b) $f(x) > 0$: for the interval $(\sqrt{2},\infty)$.

(c) $f(x) < 0$: for the interval $(-\infty,\sqrt{2})$.

71. (a) $f(x) = 0$: $\{-4,5\}$

(b) $f(x) \ge 0$: for the intervals $(-\infty,-4]\cup[5,\infty)$

(c) $f(x) \le 0$: for the interval $[-4,5]$.

72. (a) $f(x) = 0$: never; therefore: $\varnothing$.

(b) $f(x) \ge 0$: for the interval $[1,\infty)$.

(c) $f(x) \le 0$: never; therefore: $\varnothing$.

73. The translation is 3 units to the left and 1 unit up; therefore, the new equation is $y = |x+3| + 1$. The form $y = |x-h| + k$ will equal $y = |x+3| + 1$ when: $h = -3$ and $k = 1$.

74. The equation $y = x^2$ has a Domain: $(-\infty,\infty)$ and a Range: $[0,\infty)$. After the translation the Domain is still: $(-\infty,\infty)$ but now the Range is $(38,\infty)$, a positive or upward shift of 38 units. Therefore, the horizontal shift can be any number of units, but the vertical shift is up 38. This makes h any real number and $k = 38$.

75. (a) $B(4) = 66.25(4) + 160 = 425$; In 2010, 425,000 bankruptcies were filed.

(b) We will use the point (2006, 160) and the slope of 66.25 in the point slope form for the equation of a line. $y - y_1 = m(x - x_1) \Rightarrow y - 160 = 66.25(x-2006) \Rightarrow y = 66.25(x-2006) + 160$

(c) $y = 66.25(2010-2006) + 160 = 66.25(4) + 160 = 425$, In 2010, 425,000 bankruptcies were filed.

(d) $293 = 66.25(x-2006) + 160 \Rightarrow 133 = 66.25(x-2006) \Rightarrow \dfrac{133}{66.25} = x - 2006 \Rightarrow x = 2006 + \dfrac{133}{66.25}$.

There will be 293 thousand bankruptcies in 2008.

76. (a) $S(14) = -\dfrac{3}{7}(14) + 15 = 9$; In 2013, sales were \$9 billion.

(b) We will use the point (1999, 15) and the slope of $-\frac{3}{7}$ in the point slope form for the equation of a line. $y - y_1 = m(x - x_1) \Rightarrow y - 15 = -\frac{3}{7}(x - 1999) \Rightarrow y = -\frac{3}{7}(x - 1999) + 15$

(c) $y = -\frac{3}{7}(2013 - 1999) + 15 = -\frac{3}{7}(14) + 15 = 9$; In 2013, sales were \$9 billion.

(d) $12 = -\frac{3}{7}(x - 1999) + 15 \Rightarrow -3 = -\frac{3}{7}(x - 1999) \Rightarrow 7 = x - 1999 \Rightarrow x = 2006$

77. $U(2011) = 13(2011 - 2006)^2 + 115 = 13(25) + 115 = 440$; The average U.S. household spent \$440 on Apple products in 2011.

78. The formula for $W(x)$ can be found by shifting $U(x) = 13(x - 2006)^2 + 115$ to the right 4 units.

$W(x) = 13(x - 2010)^2 + 115$; $W(x) = 13(2015 - 2010)^2 + 115 = 13(25) + 115 = 440$

In 2015, the average worldwide household spending on Apple products was \$440, which equaled U.S. spending 4 years earlier.

79. (a) Enter the year in L_1 and enter tuition and fees in L_2. The year 2000 corresponds to $x = 0$ and so on. The regression equation is $y \approx 402.5x + 3460$.

(b) Since $x = 0$ corresponds to 2000, the equation when the exact year is entered is $y = 402.5(x - 2000) + 3460$

(c) $y \approx 402.5(2009 - 2000) + 3460 \Rightarrow y \approx \$7100$

80. (a) Enter the year in L_1 and enter the percent of women in the workforce in L_2. The year 1970 corresponds to $x = 0$ and so on. The regression equation is $y \approx 0.40167x + 46.36$.

(b) Since $x = 0$ corresponds to 1970, the equation when the exact year is entered is $y \approx 0.40167(x - 1970) + 46.36$.

(c) $y \approx 0.40167(2015 - 1970) + 46.36 \Rightarrow y \approx 64.4$

81. See Figure 81.

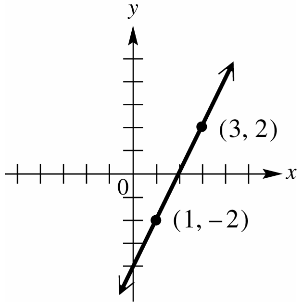

Figure 81

82. $m = \frac{2-(-2)}{3-1} \Rightarrow m = \frac{4}{2} = 2$

83. Using slope-intercept form yields: $y_1 - 2 = 2(x-3) \Rightarrow y_1 - 2 = 2x - 6 \Rightarrow y_1 = 2x - 4$

84. $(1, -2+6)$ and $(3, 2+6) \Rightarrow (1,4)$ and $(3,8)$

85. $m = \frac{8-4}{3-1} \Rightarrow m = \frac{4}{2} = 2$

86. Using slope-intercept form yields: $y_2 - 4 = 2(x-1) \Rightarrow y_2 - 4 = 2x - 2 \Rightarrow y_2 = 2x + 2$.

87. Graph $y_1 = 2x - 4$ and $y_2 = 2x + 2$ See Figure 87. The graph y_2 can be obtained by shifting the graph of y_1 upward 6 units. The constant 6, comes from the 6 we added to each y-value in Exercise 84.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

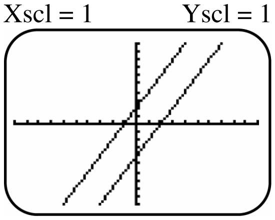

Figure 87

88. c; c; the same as; c; upward (or positive vertical)

2.3: Stretching, Shrinking, and Reflecting Graphs

1. The function $y = x^2$ vertically stretched by a factor of 2 is $y = 2x^2$.
2. The function $y = x^3$ vertically shrunk by a factor of $\frac{1}{2}$ is $y = \frac{1}{2}x^3$.
3. The function $y = \sqrt{x}$ reflected across the y-axis is $y = \sqrt{-x}$.
4. The function $y = \sqrt[3]{x}$ reflected across the x-axis is $y = -\sqrt[3]{x}$.
5. The function $y = |x|$ vertically stretched by a factor of 3 and reflected across the x-axis is $y = -3|x|$.
6. The function $y = |x|$ vertically shrunk by a factor of $\frac{1}{3}$ and reflected across the y-axis is $y = \frac{1}{3}|-x|$.
7. The function $y = x^3$ vertically shrunk by a factor of 0.25 and reflected across the y-axis is $y = 0.25(-x^3)$ or $y = -0.25x^3$.
8. The function $y = \sqrt{x}$ vertically shrunk by a factor of 0.2 and reflected across the x-axis is $y = -0.2\sqrt{x}$.
9. Graph $y_1 = x$, $y_2 = x + 3$ (y_1 shifted up 3 units), and $y_3 = x - 3$ (y_1 shifted down 3 units). See Figure 9.
10. Graph $y_1 = x^3$, $y_2 = x^3 + 4$ (y_1 shifted up 4 units), and $y_3 = x^3 - 4$ (y_1 shifted down units). See Figure 10.
11. Graph $y_1 = |x|$, $y_2 = |x-3|$ (y_1 shifted right 3 units), and $y_3 = |x+3|$ (y_1 shifted left 3 units). See Figure 11.

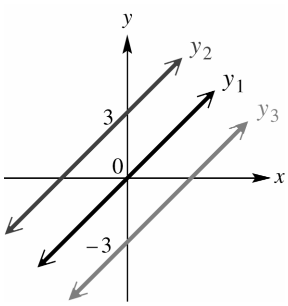

Figure 9

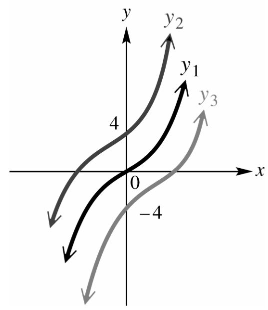

Figure 10

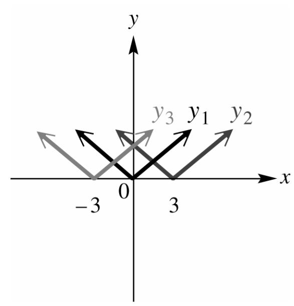

Figure 11

12. Graph $y_1 = |x|$, $y_2 = |x| - 3$ (y_1 shifted down 3 units), and $y_3 = |x| + 3$ (y_1 shifted up 3 units). See Figure 12.

13. Graph $y_1 = \sqrt{x}$, $y_2 = \sqrt{x+6}$ (y_1 shifted left 6 units), and $y_3 = \sqrt{x-6}$ (y_1 shifted right 6 units). See Figure 13.

14. Graph $y_1 = |x|$, $y_2 = 2|x|$ (y_1 stretched vertically by a factor of 2), and $y_3 = 2.5|x|$ (y_1 stretched vertically by a factor of 2.5). See Figure 14

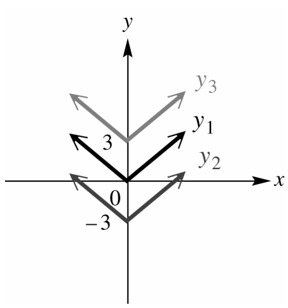

Figure 12

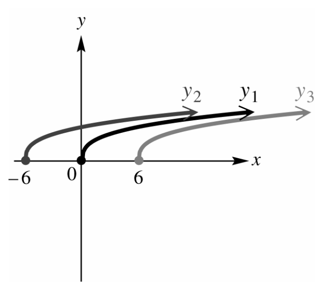

Figure 13

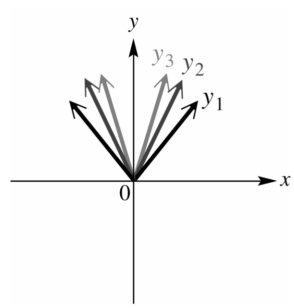

Figure 14

15. Graph $y_1 = \sqrt[3]{x}$, $y_2 = -\sqrt[3]{x}$ (y_1 reflected across the x-axis), and $y_3 = -2\sqrt[3]{x}$ (y_1 reflected across the x-axis and stretched vertically by a factor of 2). See Figure 15.

16. Graph $y_1 = x^2$, $y_2 = (x-2)^2 + 1$ (y_1 shifted right 2 units and up 1 unit), and $y_3 = -(x+2)^2$ (y_1 shifted left 2 units and reflected across the x-axis). See Figure 16

17. Graph $y_1 = |x|$, $y_2 = -2|x-1| + 1$ (y_1 reflected across the x-axis, stretched vertically by a factor of 2, shifted right 1 unit, and shifted up 1 unit), and $y_3 = -\frac{1}{2}|x| - 4$ (y_1 reflected across the x-axis, shrunk by factor of $\frac{1}{2}$, and shifted down 4 units). See Figure 17

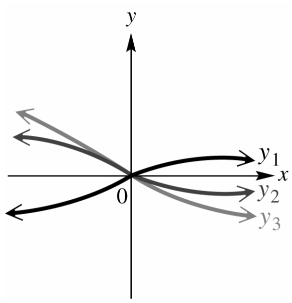

Figure 15

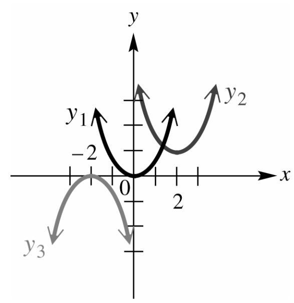

Figure 16

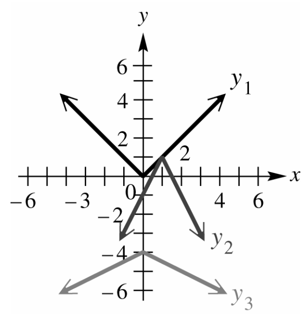

Figure 17

18. Graph $y_1 = \sqrt{x}$, $y_2 = -\sqrt{x}$ (y_1 reflected across the x-axis), and $y_3 = \sqrt{-x}$ (y_1 reflected across the y-axis). See Figure 18.

19. Graph $y_1 = x^2 - 1$ (which is $y = x^2$ shifted down 1 unit), $y_2 = \left(\frac{1}{2}x\right)^2 - 1$ (y_1 shrunk vertically by A factor of $\frac{1}{2}$), and $y_3 = (2x)^2 - 1$ (y_1 stretched vertically by a factor of 2^2 or 4). See Figure 19.

20. Graph $y_1 = 3 - |x|$ (which is $y = |x|$ reflected across the x-axis and shifted up 3 units), $y_2 = 3 - |3x|$ stretched vertically by a factor of 3), and $y_3 = 3 - \left|\frac{1}{3}x\right|$ (y_1 shrunk vertically by a factor of $\frac{1}{3}$). See Figure 20

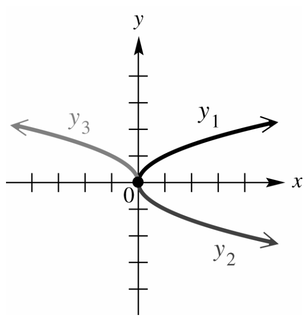

Figure 18

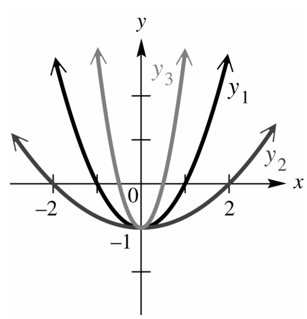

Figure 19

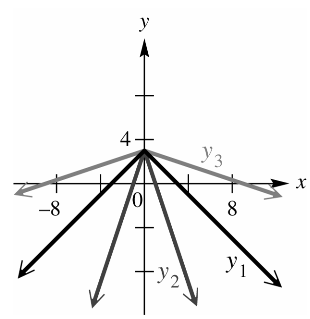

Figure 20

21. Graph $y_1 = \sqrt[3]{x}$, $y_2 = \sqrt[3]{-x}$ (y_1 reflected across the y-axis), and $y_3 = \sqrt[3]{-(x-1)}$ (y_1 reflected across the y-axis and shifted right 1 unit). See Figure 21.

22. Graph $y_1 = \sqrt[3]{x}$, $y_2 = 2 - \sqrt[3]{x}$ (y_1 reflected across the x-axis and shifted up 2 units), and $y_3 = 1 + \sqrt[3]{x}$ (y_1 shifted up 1 unit). See Figure 22.

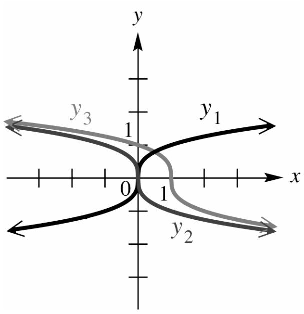

Figure 21

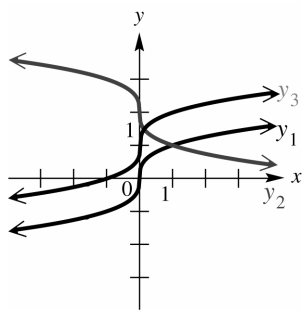

Figure 22

23. The graph $y = f(x) = x^2$ has been reflected across the *x*-axis, shifted 5 units to the right, and shifted 2 units downward; therefore, the equation of $g(x)$ is $g(x) = -(x-5)^2 - 2$.

24. The graph $y = f(x) = x^3$ has been shifted 4 units to the right and shifted 3 units upward; therefore, the equation of $g(x)$ is $g(x) = (x-4)^3 + 3$.

25. The graph $y = f(x) = \sqrt{x}$ has been reflected across the *y*-axis and shifted 1 unit upward; therefore, the equation of $g(x)$ is $g(x) = \sqrt{-x} + 1$.

26. The graph $y = f(x) = \sqrt[3]{x}$ has been reflected across the *x*-axis and shifted 2 units to the right; therefore, the equation of $g(x)$ is $g(x) = -\sqrt[3]{x-2}$.

27. 4; *x*

28. 6; *x*

29. 2; left; $\frac{1}{4}$; *x*; 3; downward (or negative)

30. *y*; $\frac{2}{5}$; *x*; 6 upward (or positive)

31. 3; right; 6

32. 2; left; 0.5

33. The function $y = x^2$ is vertically shrunk by a factor of $\frac{1}{2}$ and shifted 7 units down; therefore, $y = \frac{1}{2}x^2 - 7$.

34. The function $y = x^3$ is vertically stretched by a factor of 3, reflected across the *x*-axis, and shifted 8 units upward; therefore, $y = 3x + 8$.

35. The function $y = \sqrt{x}$ is shifted 3 units right, vertically stretched by a factor of 4.5, and shifted 6 units down; therefore, $y = 4.5\sqrt{x-3} - 6$.

36. The function $y = \sqrt[3]{x}$ is shifted 2 units left, vertically stretched by a factor of 1.5, and shifted 8 units upward; therefore, $y = 1.5\sqrt[3]{x+2} + 8$.

37. The function $f(x) = \sqrt{x-3} + 2$ is $f(x) = \sqrt{x}$ shifted 3 units right and 2 units upward. See Figure 37.

38. The function $f(x)=|x+2|-3$ is $f(x)=|x|$ shifted 2 units left and 3 units downward. See Figure 38.

39. The function $f(x)=\sqrt{2x}=\sqrt{2}\sqrt{x}$ is $f(x)=\sqrt{x}$ stretched vertically by a factor of $\sqrt{2}$. See Figure 39.

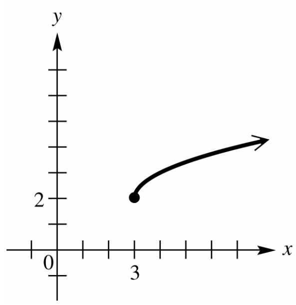

Figure 37

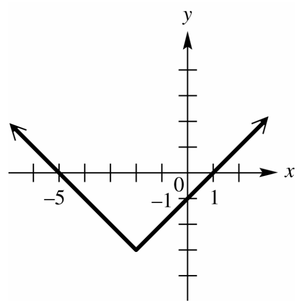

Figure 38

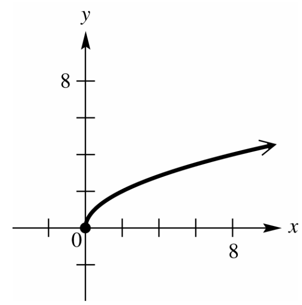

Figure 39

40. The function $f(x)=\frac{1}{2}(x+2)^2$ is $f(x)=x^2$ shifted 2 units left and shrunk vertically by a factor of $\frac{1}{2}$.

See Figure 40.

41. The function $f(x)=|2x|=2|x|$ is $f(x)=|x|$ stretched vertically by a factor of 2. See Figure 41.

42. The function $f(x)=\frac{1}{2}|x|$ is $f(x)=|x|$ shrunk vertically by a factor of $\frac{1}{2}$. See Figure 42

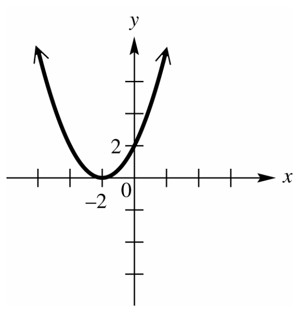

Figure 40

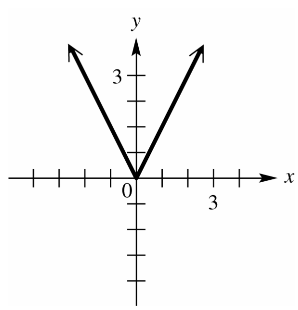

Figure 41

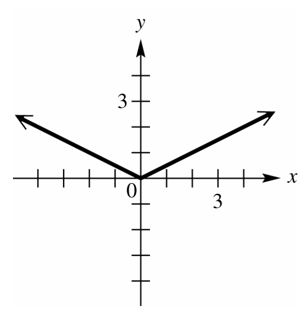

Figure 42

43. The function $f(x)=1-\sqrt{x}$ is $f(x)=\sqrt{x}$ reflected across the *x*-axis and shifted 1 unit upward. See Figure 43.

44. The function $f(x)=2\sqrt{x-2}-1$ is $f(x)=\sqrt{x}$ shifted 2 units right, stretched vertically by a factor of 2, and shifted 1 unit downward. See Figure 44.

45. The function $f(x)=-\sqrt{1-x}=-\sqrt{-(x-1)}$ is $f(x)=\sqrt{x}$ reflected across both the *x*-axis, the *y*-axis and shifted 1 unit right. See Figure 45.

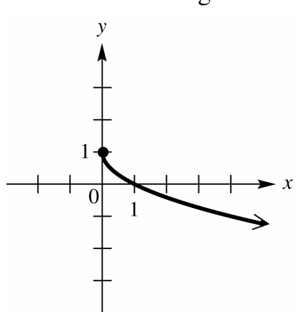

Figure 43

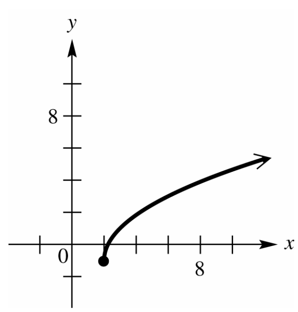

Figure 44

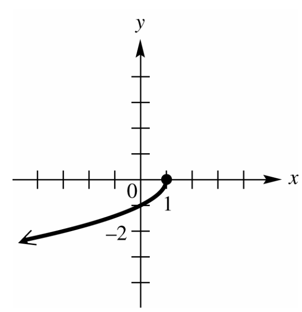

Figure 45

46. The function $f(x)=\sqrt{-x}-1$ is $f(x)=\sqrt{x}$ reflected across the y-axis and shifted 1 unit downward. See Figure 46.

47. The function $f(x)=\sqrt{-(x+1)}$ is $f(x)=\sqrt{x}$ reflected across the y-axis and shifted 1 unit left. See Figure 47.

48. The function $f(x)=2+\sqrt{-(x-3)}$ is $f(x)=\sqrt{x}$ reflected across the y-axis, shifted 3 units right, and shifted 2 units upward. See Figure 48.

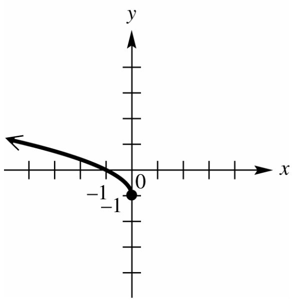

Figure 46

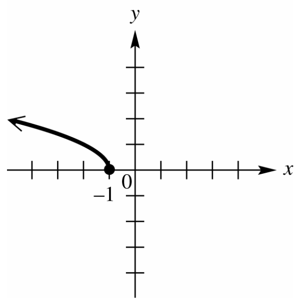

Figure 47

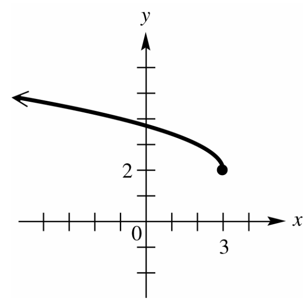

Figure 48

49. The function $f(x)=(x-1)^3$ is $f(x)=x^3$ shifted 1 unit right. See Figure 49.

50. The function $f(x)=(x+2)^3$ is $f(x)=x^3$ shifted 2 unit left. See Figure 50.

51. The function $f(x)=-x^3$ is $f(x)=x^3$ reflected across the x-axis. See Figure 51.

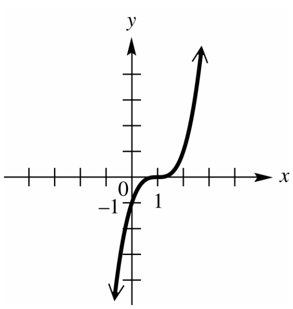

Figure 49

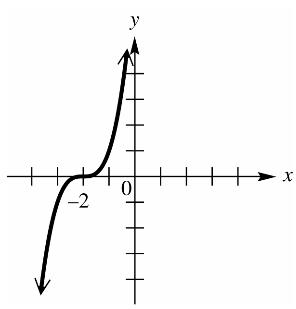

Figure 50

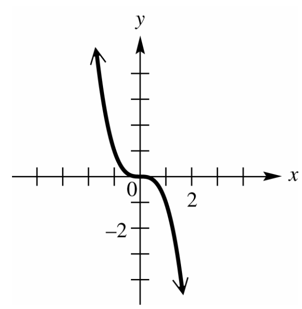

Figure 51

52. The function $f(x)=(-x)^3+1$ is $f(x)=x^3$ reflected across the y-axis and shifted 1 unit upward. See Figure 52.

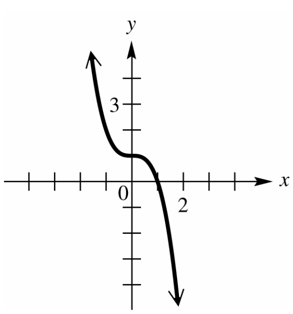

Figure 52

53. (a) The equation $y=-f(x)$ is $y=f(x)$ reflected across the x-axis. See Figure 53a.

(b) The equation $y=f(-x)$ is $y=f(x)$ reflected across the y-axis. See Figure 53b.

(c) The equation $y=2f(x)$ is $y=f(x)$ stretched vertically by a factor of 2. See Figure 53c.

(d) From the graph $f(0)=1$.

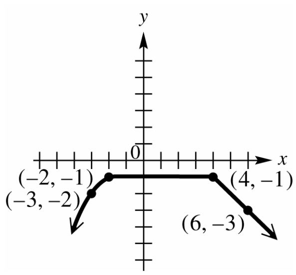

Figure 53a

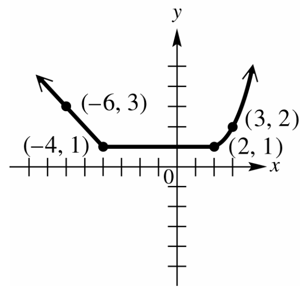

Figure 53b

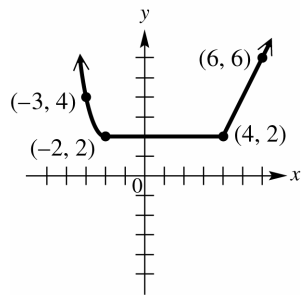

Figure 53c

54. (a) The equation $y=-f(x)$ is $y=f(x)$ reflected across the *x*-axis. See Figure 54a.

 (b) The equation $y=f(-x)$ is $y=f(x)$ reflected across the *y*-axis. See Figure 54b.

 (c) The equation $y=3f(x)$ is $y=f(x)$ stretched vertically by a factor of 3. See Figure 54c.

 (d) From the graph $f(4)=1$

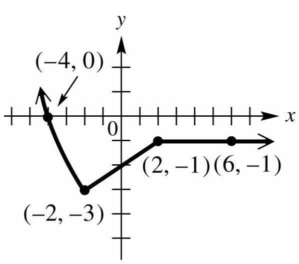

Figure 54a

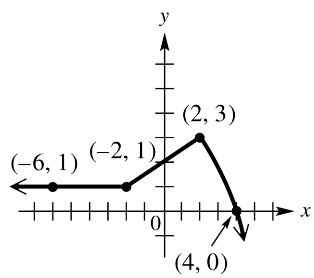

Figure 54b

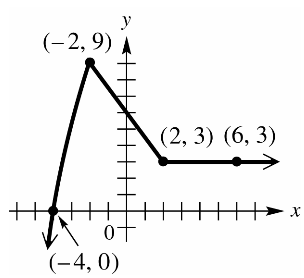

Figure 54c

55. (a) The equation $f(x)$ is $y=f(x)$ reflected across the *x*-axis. See Figure 55a.

 (b) The equation $y=f(-x)$ is $y=f(x)$ reflected across the *y*-axis. See Figure 55b.

 (c) The equation $y=f(x+1)$ is $y=f(x)$ shifted 1 unit to the left. See Figure 55c.

 (d) From the graph, there are two *x*-intercepts, $(-1,0)$ and $(4,0)$.

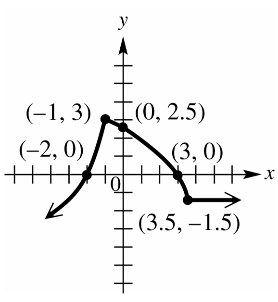

Figure 55a

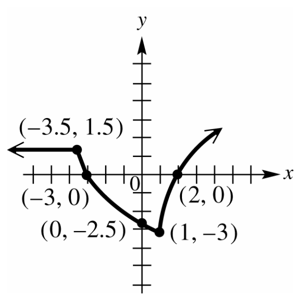

Figure 55b

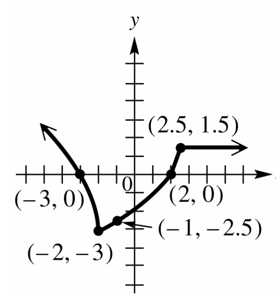

Figure 55c

56. (a) The equation $y=-f(x)$ is $y=f(x)$ reflected across the *x*-axis. See Figure 56a.

 (b) The equation $y=f(-x)$ is $y=f(x)$ reflected across the *y*-axis. See Figure 56b.

 (c) The equation $y=\frac{1}{2}f(x)$ is $(20,5)$. shrunk vertically by a factor of $\frac{1}{2}$. See Figure 56c.

 (d) From the graph $f(x)<0$ for the interval: $(-\infty,0)$.

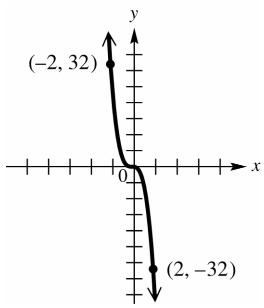

Figure 56a

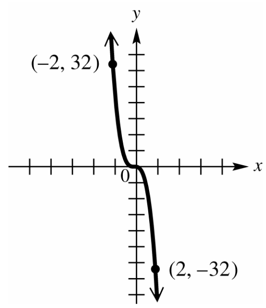

Figure 56b

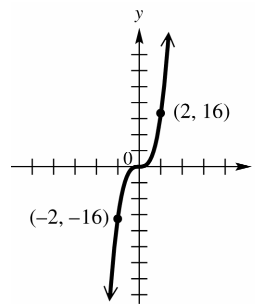

Figure 56c

57. (a) The equation $y = -f(x)$ is $y = f(x)$ reflected across the x-axis. See Figure 57a.

    (b) The equation $y = f\left(\frac{1}{3}x\right)$ is $y = f(x)$ stretched horizontally by a factor of 3. See Figure 57b.

    (c) The equation $y = 0.5f(x)$ is $y = f(x)$ shrunk vertically by a factor of 0.5. See Figure 57c.

    (d) From the graph, symmetry with respect to the origin.

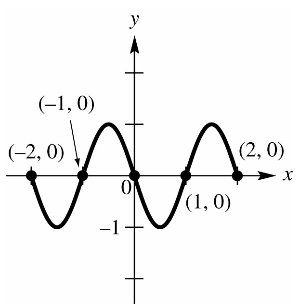

Figure 57a

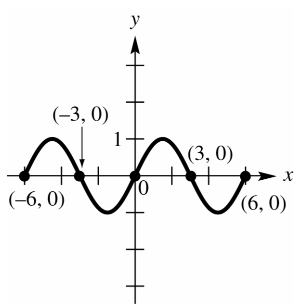

Figure 57b

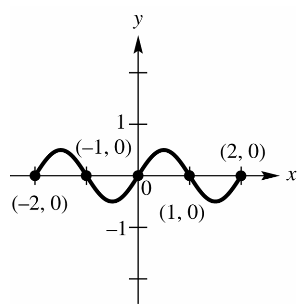

Figure 57c

58. (a) The equation $y = f(2x)$ is $y = f(x)$ stretched horizontally by a factor of $\frac{1}{2}$. See Figure 58a.

    (b) The equation $y = f(-x)$ is $y = f(x)$ reflected across the y-axis. See Figure 58b.

    (c) The equation $y = 3f(x)$ is $y = f(x)$ stretched vertically by a factor of 3. See Figure 58c.

    (d) From the graph, symmetry with respect to the y-axis.

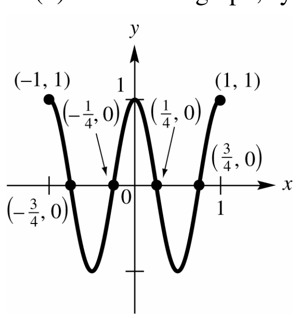

Figure 58a

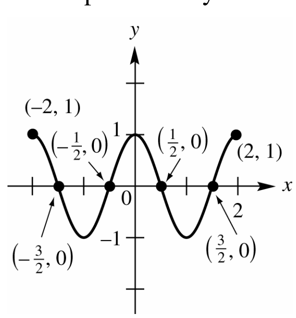

Figure 58b

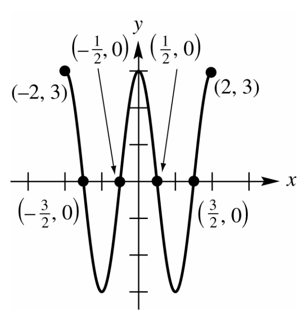

Figure 58c

59. (a) The equation $y = f(x)+1$ is $y = f(x)$ shifted 1 unit upward. See Figure 59a.

    (b) The equation -30° is $y = f(x)$ reflected across the x-axis and shifted 1 unit down. See Figure 59b.

(c) The equation $y = 2f\left(\frac{1}{2}x\right)$ is $y = f(x)$ stretched vertically by a factor of 2 and horizontally by a factor of 2. See Figure 59c.

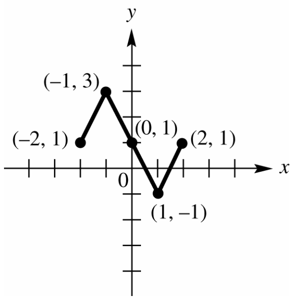

Figure 59a

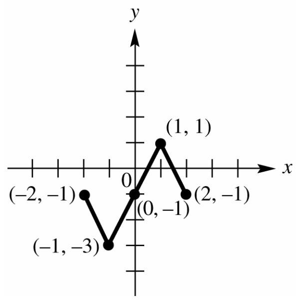

Figure 59b

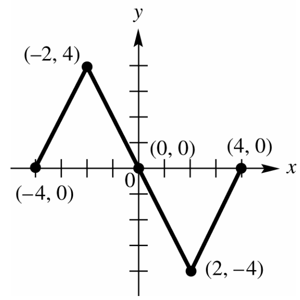

Figure 59c

60. (a) The equation $y = f(x) - 2$ is $y = f(x)$ shifted 2 units downward. See Figure 60a.

(b) The equation $y = f(x-1) + 2$ is $y = f(x)$ shifted 1 unit right and 2 units upward. See Figure 60b.

(c) The equation $y = 2f(x)$ is $y = f(x)$ stretched vertically by a factor of 2. See Figure 60c.

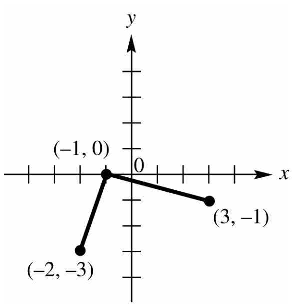

Figure 60a

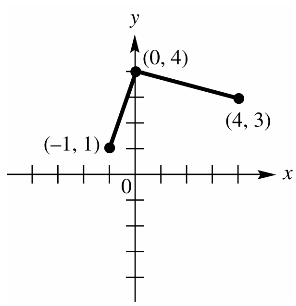

Figure 60b

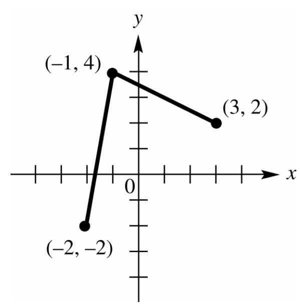

Figure 60c

61. (a) The equation $y = f(2x) + 1$ is x shrunk horizontally by a factor of $\{2\}$ and shifted 1 unit upward. See Figure 61a.

(b) The equation $y = 2f\left(\frac{1}{2}x\right) + 1$ is $y = f(x)$ stretched vertically by a factor of 2, stretched horizontally by a factor of 2, and shifted 1 unit upward. See Figure 61b.

(c) The equation $y = \frac{1}{2}f(x-2)$ is $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$ and shifted 2 units to the right. See Figure 61c.

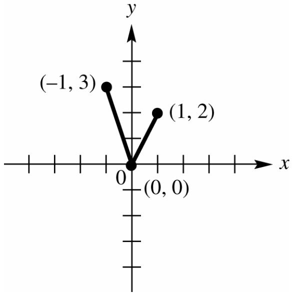

Figure 61a

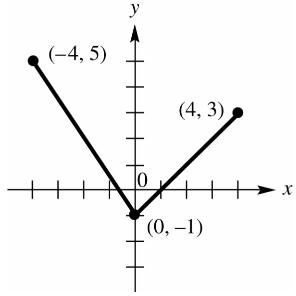

Figure 61b

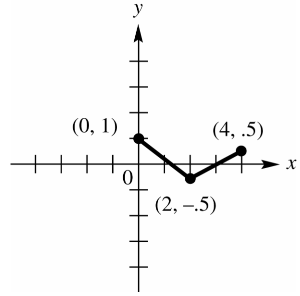

Figure 61c

62. (a) The equation $y = f(2x)$ is $y = f(x)$ shrunk horizontally by a factor of $\frac{1}{2}$. See Figure 62a.

(b) The equation $y = f\left(\frac{1}{2}x\right) - 1$ is $y = f(x)$ stretched horizontally by a factor of 2, and shifted 1 unit downward. See Figure 62b.

(c) The equation $y = 2f(x) - 1$ is $y = f(x)$ stretched vertically by a factor of 2 and shifted 1 unit downward. See Figure 62c.

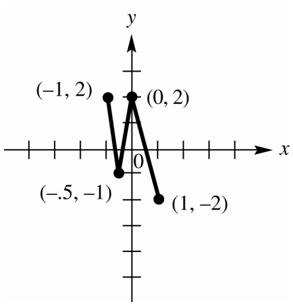

Figure 62a

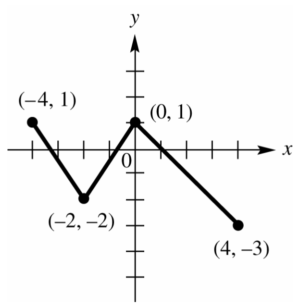

Figure 62b

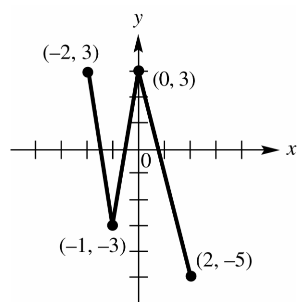

Figure 62c

63. (a) If $(r, 0)$ is the x-intercept of $y = f(x)$ and $y = -f(x)$ is $y = f(x)$ reflected across the x-axis, then $(r, 0)$ is also the x-intercept of $y = -f(x)$.

(b) If $(r, 0)$ is the x-intercept of $y = f(x)$ and $y = f(-x)$ is $f(x)$ reflected across the y-axis, then $(-r, 0)$ is the x-intercept of $y = -f(x)$

(c) If $(r, 0)$ is the x-intercept of $y = f(x)$ and $y = -f(-x)$ is $y = f(x)$ reflected across both the x-axis and y-axis, then $(-r, 0)$, is the x-intercept of $y = -f(-x)$.

64. (a) If $(0, b)$ is the y-intercept of $y = f(x)$ and $y = -f(x)$ is $y = f(x)$ reflected across the x-axis, then $(0, -b)$ is the y-intercept of $y = -f(x)$.

(b) If $(0, b)$ is the y-intercept of $y = f(x)$ and $y = f(-x)$ is $y = f(x)$ reflected across the y-axis, then $(0, b)$ is also the y-intercept of $y = f(-x)$.

(c) If $(0, b)$ is the y-intercept of $y = f(x)$ and $y = 5f(x)$ is $y = f(x)$ stretched vertically by a factor of 5, then $(0, 5b)$ is the y-intercept of $y = 5f(x)$.

(d) If $(0, b)$ is the y-intercept of $y = f(x)$ and $y = -3f(x)$ is $y = f(x)$ reflected across the x-axis and stretched vertically by a factor of 3, then $(0, -3b)$ is the y-intercept of $y = -3f(x)$.

65. Since $y = f(x-2)$ is $y = f(x)$ shifted 2 units to the right, the domain of $f(x-2)$ is $[-1+2, 2+2]$ or $[1, 4]$, and the range is the same: $[0, 3]$.

66. Since $5f(x+1)$ is $f(x)$ shifted 1 unit to the left, the domain of $5f(x+1)$ is $[-1-1, 2-1]$ or $[-2, 1]$ and, stretched vertically by a factor of 5, the range is $[5(0), 5(3)]$ or $[0, 15]$.

67. Since $-f(x)$ is $f(x)$ reflected across the x-axis, the domain of $-f(x)$ is the same: $[-1,2]$, and the range is $[-3,0]$. .

68. Since $f(x-3)+1$ is $f(x)$ shifted 3 unit to the right, $f(x-3)+1$ is $[-1+3,2+3]$ or $[2,5]$, an shifted 1 unit upward, the range is $[0+1,3+1]$ or $[1,4]$

69. Since $f(2x)$ is $f(x)$ shrunk horizontally by a factor of $\frac{1}{2}$, the domain of $f(2x)$ is $\left[\frac{1}{2}(-1),\frac{1}{2}(2)\right]$ or $\left[-\frac{1}{2},1\right]$, and the range is the same: $[0,3]$.

70. Since $2f(x-1)$ is $f(x)$ shifted 1 unit to the right, the domain of $2f(x-1)$ is $[-1+1,2+1]$ or $[0,3]$, and stretched vertically by a factor of 3, the range is $[2(-2),2(3)]$ or $[0,6]$.

71. Since $3f\left(\frac{1}{4}x\right)$ is $f(x)$ stretched horizontally by a factor of 4, the domain of $3f\left(\frac{1}{4}x\right)$ is $[4(-1),4(2)]$ or $[-4,8]$, and stretched vertically by a factor of 3, the range is $[3(0),3(3)]$ or $[0,9]$.

72. Since $-2f(4x)$ is $f(x)$ shrunk horizontally by a factor of $\frac{1}{4}$, the domain of $2f(4x)$ is $\left[\frac{1}{4}(-1),\frac{1}{4}(2)\right]$ or $\left[-\frac{1}{4},\frac{1}{2}\right]$; and reflected across the x-axis while being stretched vertically by a factor of 2, the range is $[-2(0),-2(3)]=[0,-6]$ or $[-6,0]$.

73. Since $f(-x)$ is $f(x)$ reflected across the y-axis, the domain of $f(-x)$ is $[-(-1),-(2)]=[1,-2]$ or $[-2,1]$; and the range is the same: $[0,3]$.

74. Since $-2f(-x)$ is $f(x)$ reflected across the y-axis, the domain of $-2f(-x)$ is $[-(-1),-(2)]=[1,-2]$ or $[-2,1]$, and reflected across the x-axis while being stretched vertically by a factor of 2, the range is $[-2(0),-2(3)]=[0,-6]$ or $[-6,0]$.

75. Since $f(-3x)$ is $f(x)$ reflected across the y-axis and shrunk horizontally by a factor of $\frac{1}{3}$, the domain of $f(-3x)$ is $\left[-\frac{1}{3}(-1),-\frac{1}{3}(2)\right]=\left[\frac{1}{3},-\frac{2}{3}\right]$ or $\left[-\frac{2}{3},\frac{1}{3}\right]$, and the range is the same: $[0,3]$.

76. Since $\frac{1}{3}f(x-3)$ is $f(x)$ shifted 3 units to the right, the domain of $\frac{1}{3}f(x-3)$ is $[-1+3,2+3]$ or $[2,5]$, and shrunk vertically by a factor of $\frac{1}{3}$, the range is $\left[\frac{1}{3}(0),\frac{1}{3}(3)\right]$ or $[0,1]$.

77. Since $y=\sqrt{x}$ has an endpoint of (0, 0), and the graph of $y=10\sqrt{x-20}+5$ is the graph of $y=\sqrt{x}$ shifted 20 units right, stretched vertically by a factor of 10, and shifted 5 units upward, the endpoint of $y=10\sqrt{x-20}+5$ is $(0+20,10(0)+5)$ or $(20,5)$. Therefore, the domain is $[20,\infty)$, and the range is $[5,\infty)$.

78. Since $y=\sqrt{x}$ has an endpoint of (0, 0), and the graph of $y=-2\sqrt{x+15}-18$ is the graph of $y=\sqrt{x}$ shifted 15 units left, reflected across the *x*-axis, stretched vertically by a factor of 2, and shifted 18 units downward, the endpoint of $y=-2\sqrt{x+15}-18$ is $(0-15,-2(0)-18)$ or $(-15,-18)$. Therefore, the domain is $[-15,\infty)$, and the range, because of the reflection across the *x*-axis, is $(-\infty,-18]$.

79. Since $y=\sqrt{x}$ has an endpoint of (0, 0), and the graph of $y=-.5\sqrt{x+10}+5$ is the graph of $y=\sqrt{x}$ shifted 10 units left, reflected across the *x*-axis, shrunk vertically by a factor of .5, and shifted 5 units upward, the endpoint of $y=-.5\sqrt{x+10}+5$ is $(0-10,-.5(0)+5)$ or $(-10,5)$. Therefore, the domain is $[-10,\infty)$, and the range, because of the reflection across the *x*-axis, is $(-\infty,5]$.

80. Using ex. 77, the domain is $[h,\infty)$, and the range is $[k,\infty)$.

81. The graph of $y=-f(x)$ is $y=f(x)$ reflected across the *x*-axis; therefore, $y=-f(x)$ is decreasing for the interval (a,b).

82. The graph of $y=f(-x)$ is $y=f(x)$ reflected across the *y*-axis; therefore, $y=f(-x)$ is decreasing for the interval $(-b,-a)$.

83. The graph of $y=-f(-x)$ is $y=f(x)$ reflected across both the *x*-axis and *y*-axis; therefore, $y=-f(-x)$ is increasing for the interval $(-b,-a)$

84. The graph of $y=-c\cdot f(x)$ is $y=f(x)$ reflected across the *x*-axis; therefore, $y=-c\cdot f(x)$ is decreasing for the interval (a,b).

85. (a) the function is increasing for the interval: $(-1,2)$.

    (b) the function is decreasing for the interval: $(-\infty,-1)$.

    (c) the function is constant for the interval: $(2,\infty)$.

86. (a) the function is increasing for the interval: $(-\infty,-1)$.

    (b) the function is decreasing for the interval: $(-1,2)$.

    (c) the function is constant for the interval: $(2,\infty)$.

87. (a) the function is increasing for the interval: $(1,\infty)$.

    (b) the function is decreasing for the interval: $(-2,1)$.

(c) the function is constant for the interval: $(-\infty,-2)$.

88. (a) the function is increasing for the interval: $(-\infty,-3)$.

(b) the function is decreasing for the interval: $(-3,\infty)$.

(c) the function is not constant for any interval.

89. From the graph, the point on y_2 is approximately $(8,10)$.

90. From the graph, the point on y_2 is approximately $(-27,-15)$.

91. Use two points on the graph to find the slope. Two points are $(-2,-1)$ and $(-1,1)$; therefore, the slope is $m=\dfrac{1-(-1)}{-1-(-2)}=\dfrac{2}{1}\Rightarrow m=2.$ The stretch factor is 2 and the graph has been shifted 2 units to the left and 1 unit down; therefore, the equation is $y=2|x+2|-1.$

92. Use two points on the graph to find the slope. Two points are $(1,2)$ and $(5,0)$; therefore, the slope is $m=\dfrac{0-2}{5-1}=\dfrac{-2}{4}\Rightarrow m=-\dfrac{1}{2}.$ The shrinking factor is $\dfrac{1}{2},$ the graph has been reflected across the *x*-axis, shifted 1 unit to the right, and shifted 2 units upward; therefore, the equation is $y=-\dfrac{1}{2}|x-1|+2.$

93. Use two points on the graph to find the slope. Two points are $(0,2)$ and $(1,-1)$; therefore, the slope is $m=\dfrac{-1-2}{1-0}=\dfrac{-3}{1}\Rightarrow m=-3.$ The stretch factor is 3, the graph has been reflected across the *x*-axis, and shifted 2 units upward; therefore, the equation is $y=-3|x|+2.$

94. Use two points on the graph to find the slope. Two points are $(-1,-2)$ and $(0,1)$; therefore, the slope is $m=\dfrac{1-(-2)}{0-(-1)}=-\dfrac{3}{1}\Rightarrow m=3.$ The stretch factor is 3 and the graph has been shifted 1 unit to the left and 2 units down; therefore, the equation is $y=3|x+1|-2.$

95. Use two points on the graph to find the slope. Two points are (0,-4) and (3,0); therefore, the slope is $m=\dfrac{-4-0}{0-3}=\dfrac{4}{3}\Rightarrow m=\dfrac{4}{3}.$ The stretch factor is $\dfrac{4}{3}$ and the graph has been shifted 4 units down.; therefore, the equation is $y=\dfrac{4}{3}|x|-4$.

96. Use two points on the graph to find the slope. Two points are (0,4) and (-3,5); therefore, the slope is $m=\dfrac{5-4}{-3-0}=-\dfrac{1}{3}\Rightarrow m=-\dfrac{1}{3}.$ The stretch factor is $-\dfrac{1}{3}$ and the graph has been shifted 3 units to the left and 5 units up.; therefore, the equation is $y=-\dfrac{1}{3}|x+3|+5$.

97. Since $y = f(x)$ is symmetric with respect to the y-axis, for every (x, y) on the graph, $(-x, y)$ is also on the graph. Reflection across the y-axis reflect onto itself and will not change the graph. It will be the same.

Reviewing Basic Concepts (Sections 2.1—2.3)

1. (a) The function $f(x) = |x|$ shifted up one unit yields the function $f(x) = |x| + 1$. Therefore, this function has a domain of $(-\infty, \infty)$ and a range of of $[1, \infty)$. The function is increasing from $(0, \infty)$ and decreasing from $(-\infty, 0)$.

   (b) The function $f(x) = x^2$ shifted to the right 2 units yields the function $f(x) = (x-2)^2$. Therefore, this function has a domain of $(-\infty, \infty)$ and a range of of $[0, \infty)$. The function is increasing from $(2, \infty)$ and decreasing from $(-\infty, 2)$.

   (c) The function $f(x) = \sqrt{x}$ reflected over the x-axis yields the function $f(x) = -\sqrt{x}$. Therefore, this function has a domain of $[0, \infty)$ and a range of of $(-\infty, 0]$. The function is never increasing and decreasing from $(0, \infty)$.

2. (a) If $y = f(x)$ is symmetric with respect to the origin, then another function value is $f(-3) = -6$.

   (b) If $y = f(x)$ is symmetric with respect to the y-axis, then another function value is $f(-3) = 6$.

   (c) If $f(-x) = -f(x), y = f(x)$ is symmetric with respect to both the x-axis and y-axis, then another function value is $f(-3) = -6$.

   (d) If $y = f(-x),\ \ y = f(x)$ is symmetric with respect to the y-axis, then another function value is $f(-3) = 6$.

3. (a) The equation $y = (x-7)^2$ is $y = x^2$ shifted 7 units to the right: B.

   (b) The equation $y = x^2 - 7$ is $y = x^2$ shifted 7 units downward: D.

   (c) The equation $y = 7x^2$ is $y = x^2$ stretches vertically by a factor of 7: E.

   (d) The equation $y = (x+7)^2$ is $y = x^2$ shifted 7 units to the left: A.

   (e) The equation $y = \left(\frac{1}{3}x\right)^2$ is $y = x^2$ stretches horizontally by a factor of 3: C.

4. (a) The equation $y = x^2 + 2$ is $y = x^2$ shifted 2 units upward: B.

   (b) The equation $y = x^2 - 2$ is $y = x^2$ shifted 2 units downward: A.

   (c) The equation $y = (x+2)^2$ is $y = x^2$ shifted 2 units to the left: G.

   (d) The equation $y = (x-2)^2$ is $y = x^2$ shifted 2 units to the right: C.

   (e) The equation $y = 2x^2$ is $y = x^2$ stretched vertically by a factor of 2: F.

   (f) The equation $y = -x^2$ is $y = x^2$ reflected across the x-axis D.

   (g) The equation $y = (x-2)^2 + 1$ is $y = x^2$ shifted 2 units to the right and 1 unit upward: H.

   (h) The equation $y = (x+2)^2 + 1$ is $y = x^2$ shifted 2 units to the left and 1 unit upward: E.

5. (a) The equation $y = |x| + 4$ is $y = |x|$ shifted 4 units upward. See Figure 5a.

   (b) The equation $y = |x + 4|$ is $y = |x|$ shifted 4 units to the left. See Figure 5b.

   (c) The equation $y = |x - 4|$ is $y = |x|$ shifted 4 units to the right. See Figure 5c.

   (d) The equation $y = |x + 2| - 4$ is $y = |x|$ shifted 2 units to the left and 4 units down. See Figure 5d.

   (e) The equation $y = -|x - 2| + 4$ is $y = |x|$ reflected across the *x*-axis, shifted 2 units to the right, and 4 units upward. See Figure 5e.

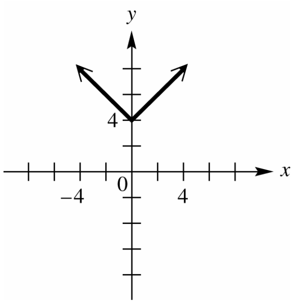

Figure 5a

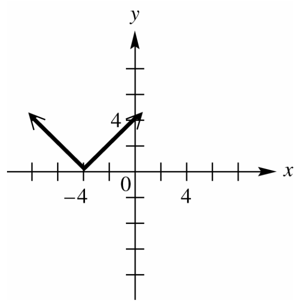

Figure 5b

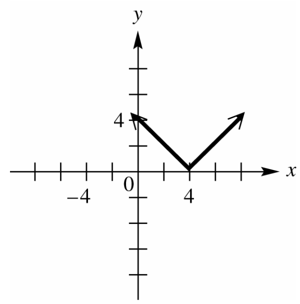

Figure 5c

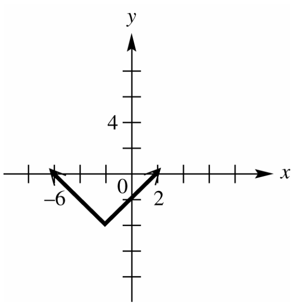

Figure 5d

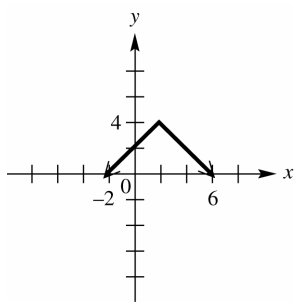

Figure 5e

6. (a) The graph is the function $f(x) = |x|$ reflected across the *x*-axis, shifted 1 unit left and 3 units upward Therefore, the equation is $y = -|x + 1| + 3.$

   (b) The graph is the function $g(x) = \sqrt{x}$ reflected across the *x*-axis, shifted 4 units left and 2 units upward. Therefore, the equation is $y = -\sqrt{x + 4} + 2.$

   (c) The graph is the function $g(x) = \sqrt{x}$ stretches vertically by a factor of 2, shifted 4 units left and 4 units downward. Therefore, the equation is $y = 2\sqrt{x + 4} - 4.$

   (d) The graph is the function $f(x) = |x|$ shrunk vertically by a factor of $\frac{1}{2}$, shifted 2 units right and 1 unit downward. Therefore, the equation is $y = \frac{1}{2}|x - 2| - 1.$

7. (a) The graph of $g(x)$ is the graph $f(x)$ shifted 2 units upward. Therefore, $c = 2.$

   (b) The graph of $g(x)$ is the graph $f(x)$ shifted 4 units to the left. Therefore, $c = 4.$

8. The graph of $y = F(x + h)$ is a horizontal translation of the graph of $y = F(x)$. The graph of $y = F(x) + h$ is not the same as the graph of $y = F(x + h)$, because the graph of $y = F(x) + h$ is a vertical translation of the graph of $y = F(x)$.

9. (a) If f is even, then $f(x) = f(-x)$. See Figure 9a.

   (b) If f is odd, then $f(-x) = -f(-x)$. See Figure 9b.

x	$f(x)$
-3	4
-2	-6
-1	5
1	5
2	-6
3	4

Figure 9a

x	$f(x)$
-3	4
-2	-6
-1	5
1	-5
2	6
3	-4

Figure 9b

10. (a) $R(x) = 5(7) + 2 = 37$, In 2011, Google's ad revenues were \$37 billion.

    (b) Using the point (2004, 2) and the slope of 5 with the point slope formula we will have $y - 2 = 5(x - 2004) \Rightarrow y = 5(x - 2004) + 2$.

    (c) $y = 5(2011 - 2004) + 2 = 5(7) + 2 = 37$, In 2011, Google's ad revenues were \$37 billion.

    (d) $27 = 5(x - 2004) + 2 \Rightarrow 25 = 5(x - 2004) \Rightarrow 5 = x - 2004 \Rightarrow x = 2009$

<u>2.4: Absolute Value Functions</u>

1. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 1.
2. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 2.
3. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 3.

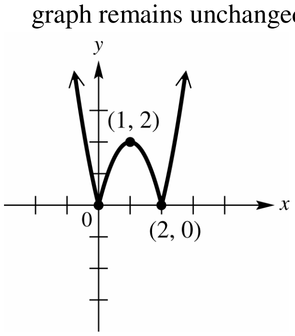

Figure 1

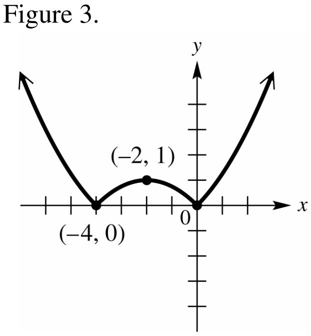

Figure 2

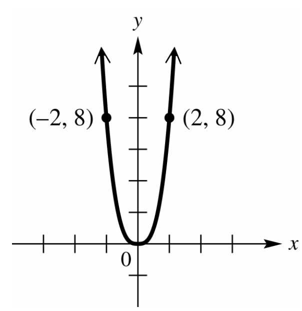

Figure 3

4. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 4.
5. Since for all y, $y \geq 0$, the graph remains unchanged. That is, $y = |f(x)|$ has the same graph as $y = f(x)$.
6. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 6.

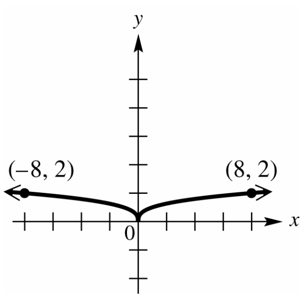

Figure 4

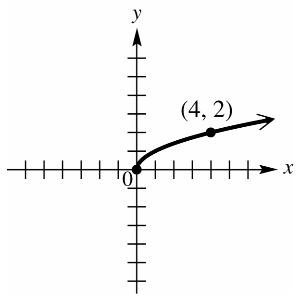

Figure 6

7. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 7.
8. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 8.
9. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 9.

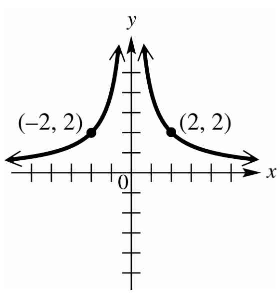

Figure 7

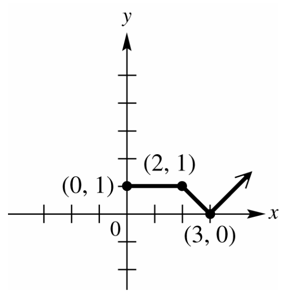

Figure 8

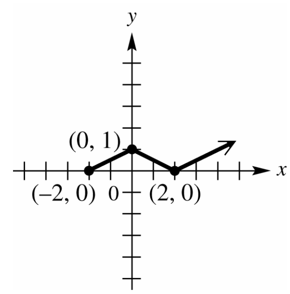

Figure 9

10. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged.
11. If $f(a) = -5$, then $|f(a)| = |-5| = 5$.
12. Since $f(x) = x^2$ is an even function, $f(x) = x^2$ and $f(x) = |x^2|$ are the same graph.
13. If $f(x) = -x^2$, then $y = |f(x)| \Rightarrow y = |-x^2| \Rightarrow y = x^2$. Therefore, the range of $y = |f(x)|$ is $[0, \infty)$.
14. If the range of $y = f(x)$ is $[-2, \infty)$, the range of $y = |f(x)|$ is $[0, \infty)$ since all negative values of y are reflected across the x-axis.
15. If the range of $y = f(x)$ is $(-\infty, -2]$, the range of $y = |f(x)|$ is $[2, \infty)$ since all negative values of y are reflected across the x-axis.
16. $|f(x)|$ is greater than or equal to 0 for any value of x. Since -1 is less than 0, -1 cannot be in the range of f

17. From the graph of $y=(x+1)^2-2$ the domain of $f(x)$ is $(-\infty,\infty)$, and the range is $[-2,\infty)$.

From the graph of $y=\left|(x+1)^2-2\right|$ the domain of $|f(x)|$ is $(-\infty,\infty)$, and the range is $[0,\infty)$.

18. From the graph of $y=2-\frac{1}{2}x$ the domain of $f(x)$ is $(-\infty,\infty)$, and the range is $(-\infty,\infty)$. From the graph of $y=\left|2-\frac{1}{2}x\right|$ the domain of $|f(x)|$ is $(-\infty,\infty)$, and the range is $[0,\infty)$.

19. From the graph of $y=-1-(x-2)^2$ the domain of $f(x)$ is $(-\infty,\infty)$, and the range is $(-\infty,-1]$. From the graph of $y=\left|-1-(x-2)^2\right|$ the domain of $|f(x)|$ is $(-\infty,\infty)$, and the range is $[1,\infty)$.

20. From the graph of $y=-|x+2|-2$ the domain of $f(x)$ is $(-\infty,\infty)$, and the range is $(-\infty,-2]$. From the graph of $y=\left|(-|x+2|-2)\right|$ the domain of $|f(x)|$ is $(-\infty,\infty)$, and the range is $[2,\infty)$.

21. From the graph, the domain of $f(x)$ is $[-2,3]$, and the range is $[-2,3]$. For the function $y=|f(x)|$, we reflect the graph of $y=f(x)$ across the x-axis for all points for which $y<0$, and, where $y\ge 0$, the graph remains unchanged. Therefore, the domain of $y=|f(x)|$ is [–2, 3], and the range is [0, 3].

22. From the graph, the domain of $f(x)$ is [–3, 2], and the range is [–2, 2]. For the function $y=|f(x)|$, we reflect the graph of $y=f(x)$ across the x-axis for all points for which $y<0$ and, where $y\ge 0$, the graph remains unchanged. Therefore, the domain of $y=|f(x)|$ is [–3, 2], and the range is [0, 2].

23. From the graph, the domain of $f(x)$ is [–2, 3], and the range is [–3, 1]. For the function $y=|f(x)|$, we reflect the graph of $y=f(x)$ across the x-axis for all points for which $y<0$, and, where $y\ge 0$, the graph remains unchanged. Therefore, the domain of $y=|f(x)|$ is [–2, 3], and the range is [0, 3].

24. From the graph, the domain of $y=f(x)$ is [–3, 3], and the range is [–3, –1]. For the function $y=|f(x)|$, we reflect the graph of $y=f(x)$ across the x-axis for all points for which $y<0$, and, where $y\ge 0$, the graph remains unchanged. Therefore, the domain of $y=|f(x)|$ is [–3, 3], and the range is [1, 3].

25. (a) The function $y=f(-x)$ is the function $y=f(x)$ reflected across the y-axis. See Figure 25a.

(b) The function $y=-f(-x)$ is the function $y=f(x)$ reflected across both the x-axis and y-axis. See Figure 25b.

(c) For the function $y=|-f(-x)|$ we reflect the graph of $y=-f(-x)$ (ex. b) across the x-axis for all points for which $y<0$, and where $y\ge 0$, the graph remains unchanged. See Figure 25c.

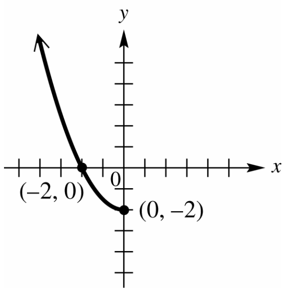

Figure 25a

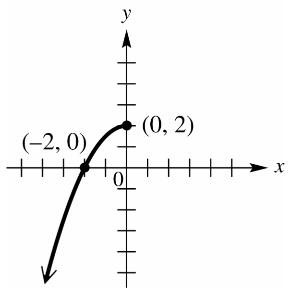

Figure 25b

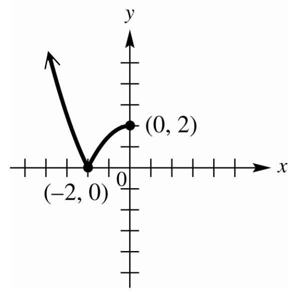

Figure 25c

26. (a) The function $y = f(-x)$ is the function $y = f(x)$ reflected across the y-axis. See Figure 26a.

(b) The function $y = -f(-x)$ is the function $y = f(x)$ reflected across both the x-axis and y-axis. See Figure 26b

(c) For the function $y = |-f(-x)|$ we reflect the graph of $y = -f(-x)$ (ex. b) across the x-axis for all points for which $y < 0$, and, where $y \geq 0$, the graph remains unchanged. See Figure 26c

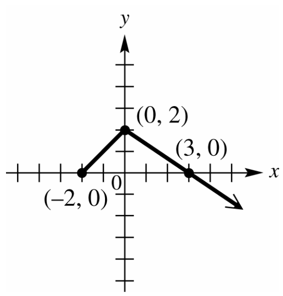

Figure 26a

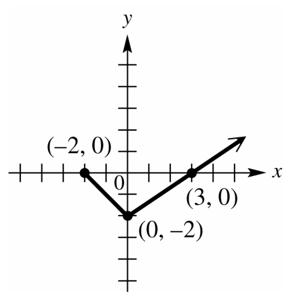

Figure 26b

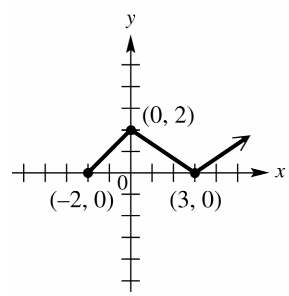

Figure 26c

27. The graph of $y = |f(x)|$ can not be below the x-axis; therefore, Figure A shows the graph of $y = f(x)$, while Figure B shows the graph of $y = |f(x)|$.

28. The graph of $y = |f(x)|$ can not be below the x-axis; therefore, Figure B shows the graph of $y = f(x)$, while Figure A shows the graph of $y = |f(x)|$.

29. (a) From the graph, $y_1 = y_2$ at the coordinates $(-1,5)$ and $(6,5)$; therefore, the solution set is $\{-1,6\}$.

(b) From the graph, $y_1 < y_2$ for the interval $(-1,6)$.

(c) From the graph, $y_1 > y_2$ for the intervals $(-\infty,-1) \cup (6,\infty)$.

30. (a) From the graph, $y_1 = y_2$ at the coordinates $(0,-2)$ and $(8,-2)$; therefore, the solution set is $\{0,8\}$.

(b) From the graph, $y_1 < y_2$ for the intervals $(-\infty,0) \cup (8,\infty)$.

(c) From the graph, $y_1 > y_2$ for the interval $(0,8)$.

31 (a) From the graph, $y_1 = y_2$ at the coordinate $(4,1)$; therefore, the solution set is $\{4\}$.

(b) From the graph, $y_1 < y_2$ never occurs; therefore, the solution set is $\varnothing$.

(c) From the graph, $y_1 > y_2$ for all values for x except 4; therefore, the solution set is the intervals $(-\infty,4)\cup(4,\infty)$.

32. (a) From the graph, $y_1 = y_2$ never occurs; therefore, the solution set is $\varnothing$.

(b) From the graph, $y_1 < y_2$ for all values for x; therefore, the solution set is $(-\infty,\infty)$.

(c). From the graph, $y_1 > y_2$ never occurs; therefore, the solution set is $\varnothing$.

33. The V-shaped graph is that of $f(x)=|.5x+6|$, since this is typical of the graphs of absolute value functions of the form $f(x)=|ax+b|$

34. The straight line graph is that of $g(x)=3x-14$ which is a linear function.

35. The graphs intersect at $(8,10)$, so the solution set is $\{8\}$.

36. From the graph, $f(x)>g(x)$ for the interval $(-\infty,8)$.

37. From the graph, $f(x)<g(x)$ for the interval $(8,\infty)$.

38. If $|.5x+6|-(3x-14)=0$ then $|.5x+6|=3x-14$. Therefore, the solution is the intersection of the graphs, or $\{8\}$.

39. (a) $|x+4|=9 \Rightarrow x+4=9$ or $x+4=-9 \Rightarrow x=5$ or $x=-13$. The solution set is $\{-13,5\}$, which is supported by the graphs of $y_1=|x+4|$ and $y_2=9$.

(b) $|x+4|>9 \Rightarrow x+4>9$ or $x+4<-9 \Rightarrow x>5$ or $x<-13$. The solution is $(-\infty,-13) \cup (5,\infty)$, which is supported by the graphs of $y_1=|x+4|$ and $y_2=9$.

(c) $|x+4|<9 \Rightarrow -9<x+4<9 \Rightarrow -13<x<5$. The solution is $(-13,5)$, which is supported by the graphs of $|x+4|$ and $y_2=9$.

40. (a) $|x-3|=5 \Rightarrow x-3=5$ or $x-3=-5 \Rightarrow x=8$ or $x=-2$. The solution set is $\{-2,8\}$, which supported by the graphs of $y_1=|x-3|$ and $y_2=5$.

(b) $|x-3|>5 \Rightarrow x-3>5$ or $x-3<-5 \Rightarrow x>8$ or $x<-2$. The solution is $(-\infty,-2)\cup(8,\infty)$ which is supported by the graphs of $y_1=|x-3|$ and $y_2=5$.

(c) $|x-3|<5 \Rightarrow -5<x-3<5 \Rightarrow -2<x<8$. The solution is $(-2,8)$, which is supported by the graphs of $y_1=|x-3|$ and $y_2=5$.

41. (a) $|7-2x|=3 \Rightarrow 7-2x=3$ or $7-2x=-3 \Rightarrow -2x=-4$ or $-2x=-10 \Rightarrow x=2$ or $x=5$. The solution set is $\{2,5\}$, which is supported by the graphs of $y_1=|7-2x|$ and $y_2=3$.

(b) $|7-2x|\ge 3 \Rightarrow 7-2x\ge 3$ or $7-2x\le -3 \Rightarrow -2x\ge -4$ or $-2x\le -10 \Rightarrow x\le 2$ or $x\ge 5$. The solution set is $(-\infty,2]\cup[5,\infty)$, which is supported by the graphs of $y_1=|7-2x|$ and $y_2=3$.

(c) $|7-2x| \le 3 \Rightarrow -3 \le 7-2x \le 3 \Rightarrow -10 \le -2x \le -4 \Rightarrow 5 \ge x \ge 2$ or $2 \le x \le 5$. The solution is $[2,5]$, which is supported by the graphs of $y_1 = |7-2x|$ and $y_2 = 3$.

42. (a) $|-9-3x| = 6 \Rightarrow -9-3x = 6$ or $-9-3x = -6 \Rightarrow -3x = 15$ or $-3x = 3 \Rightarrow x = -5$ or x = -1. The solution set is $\{-5,-1\}$, which is supported by the graphs of $y_1 = |-9-3x|$ and $y_2 = 6$.

(b) $|-9-3x| \ge 6 \Rightarrow -9-3x \ge 6$ or $-9-3x \le -6 \Rightarrow -3x \ge 15$ or $-3x \le 3 \Rightarrow x \le -5$ or $x \ge -1$. The solution is $(-\infty,-5] \cup [-1,\infty)$, which is supported by the graphs of $y_1 = |-9-3x|$ and $y_2 = 6$.

(c) $|-9-3x| \le 6 \Rightarrow -6 \le -9-3x \le 6 \Rightarrow 3 \le -3x \le 15 \Rightarrow -1 \ge x \ge -5$ or $-5 \le x \le -1$. The solution is $[-5,-1]$, which is supported by the graphs of $y_1 = |-9-3x|$ and $y_2 = 6$.

43. (a) $|2x+1|+3 = 5 \Rightarrow 2x+1 = 2$ or $2x+1 = -2 \Rightarrow 2x = 1$ or $2x = -3 \Rightarrow x = \frac{1}{2}$ or $x = -\frac{3}{2}$. The solution set is $\left\{-\frac{3}{2}, \frac{1}{2}\right\}$, which is supported by the graphs of $y_1 = |2x+1|+3$ and $y_2 = 5$.

(b) $|2x+1|+3 \le 5 \Rightarrow -2 \le 2x+1 \le 2 \Rightarrow -3 \le 2x \le 1 \Rightarrow -\frac{3}{2} \le x \le \frac{1}{2}$. The solution is $\left[-\frac{3}{2}, \frac{1}{2}\right]$, which is supported by the graphs of $y_1 = |2x+1|+3$ and $y_2 = 5$.

(c) $|2x+1|+3 \ge 5 \Rightarrow 2x+1 \ge 2$ or $2x+1 \le -2 \Rightarrow 2x \ge 1$ or $2x \le -3 \Rightarrow x \ge \frac{1}{2}$ or $x \le -\frac{3}{2}$. The solution is $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$, which is supported by the graphs of $y_1 = |2x+1|+3$ and $y_2 = 5$.

44. (a) $|4x+7|+4 = 4 \Rightarrow 4x+7 = 0 \Rightarrow 4x = -7 \Rightarrow x = -\frac{7}{4}$. The solution set is $\left\{-\frac{7}{4}\right\}$, which is supported by the graphs of $y_1 = |4x+7|+4$ and $y_2 = 4$.

(b) $|4x+7|+4 > 4 \Rightarrow 4x+7 > 0$ or $4x+7 < 0 \Rightarrow 4x > -7 \quad 4x < -7 \Rightarrow x > -\frac{7}{4}$ or $x < -\frac{7}{4}$. The solution is $\left(-\infty, -\frac{7}{4}\right) \cup \left(-\frac{7}{4}, \infty\right)$, which is supported by the graphs of $y_1 = |4x+7|+4$ and $y_2 = 4$.

(c) $|4x+7|+4 < 4 \Rightarrow 0 < 4x+7 < 0 \Rightarrow -7 < 4x < -7 \Rightarrow -\frac{7}{4} < x < -\frac{7}{4} \Rightarrow$ the solution set is $\varnothing$, which is supported by the graphs of $y_1 = |4x+7|+4$ and $y_2 = 4$.

45. (a) $|5-7x| = 0 \Rightarrow 5-7x = 0 \Rightarrow 7x = 5 \Rightarrow x = \frac{5}{7}$. The solution set is $\left\{\frac{5}{7}\right\}$, which is supported by the graphs of $y_1 = |5-7x|$ and $y_2 = 0$.

(b) $|5-7x| \geq 0 \Rightarrow 5-7x \geq 0$ or $5-7x \leq 0 \Rightarrow 7x \geq 5$ or $7x \leq 5 \Rightarrow x \geq \frac{5}{7}$ or $x \leq \frac{5}{7}$. The solution is $(-\infty, \infty)$, which is supported by the graphs of $y_1 = |5-7x|$ and $y_2 = 0$.

(c) $|5-7x| \leq 0 \Rightarrow 0 \leq 5-7x \leq 0 \Rightarrow 5 \geq 7x \geq 5 \Rightarrow \frac{5}{7} \geq x \geq \frac{5}{7}$. The solution set is $\left\{\frac{5}{7}\right\}$, which is supported by the graphs of $y_1 = |5-7x|$ and $y_2 = 0$.

46. (a) Absolute value is always positive; therefore, the solution set is $\varnothing$, which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.

(b) Absolute value is always positive, and so cannot be less than -4; therefore, the solution set is $\varnothing$, which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.

(c) Absolute value is always positive, and so is always greater than -4; therefore, the solution set is $(-\infty, \infty)$, which is supported by the graphs of $y_1 = |\pi x + 8|$ and $y_2 = -4$.

47. (a) Absolute value is always positive; therefore, the solution set is $\varnothing$, which is supported by the graphs of $y_1 = \left|\sqrt{2}x - 3.6\right|$ and $y_2 = -1$.

(b) Absolute value is always positive, and so cannot be less than or equal to -1; therefore, the solution set is $\varnothing$, which is supported by the graphs of $y_1 = \left|\sqrt{2}x - 3.6\right|$ and $y_2 = -1$.

(c) Absolute value is always positive, and so is always greater than -1; therefore, the solution is $(-\infty, \infty)$, which is supported by the graphs of $y_1 = \left|\sqrt{2}x - 3.6\right|$ and $y_2 = -1$.

48. $|2x+4|+2=10 \Rightarrow |2x+4|=8 \Rightarrow 2x+4=8$ or $2x+4=-8 \Rightarrow 2x=4$ or $2x=-12 \Rightarrow x=2$ and $x=-6$. Therefore, the solution set is $\{-6, 2\}$.

49. $3|4-3x|-4=8 \Rightarrow 3|4-3x|=12 \Rightarrow |4-3x|=4 \Rightarrow 4-3x=4$ or $4-3x=-4 \Rightarrow -3x=0$ or $-3x=-8 \Rightarrow x=0$ or $x=\frac{8}{3}$. Therefore, the solution set is $\left\{0, \frac{8}{3}\right\}$.

50. $5|x+3|-2=18 \Rightarrow 5|x+3|=20 \Rightarrow |x+3|=4 \Rightarrow x+3=4$ or $x+3=-4 \Rightarrow x=1$ or $x=-7$. Therefore, the solution set is $\{-7, 1\}$.

51. $\frac{1}{2}\left|-2x+\frac{1}{2}\right|=\frac{3}{4} \Rightarrow \left|-2x+\frac{1}{2}\right|=\frac{3}{2} \Rightarrow -2x+\frac{1}{2}=\frac{3}{2}$ or $-2x+\frac{1}{2}=-\frac{3}{2} \Rightarrow -2x=1$ or $-2x=-2 \Rightarrow x=-\frac{1}{2}$ or $x=1$. Therefore, the solution set is $\left\{-\frac{1}{2}, 1\right\}$.

52. $|3(x-5)+2|+3=9 \Rightarrow |3(x-5)+2|=6 \Rightarrow 3(x-5)+2=6$ or $3(x-5)+2=-6 \Rightarrow 3(x-5)=4$ or $3(x-5)=-8 \Rightarrow x-5=\frac{4}{3}$ or $x-5=-\frac{8}{3} \Rightarrow x=\frac{19}{3}$ or $x=\frac{7}{3}$. Therefore, the solution set is $\left\{\frac{7}{3}, \frac{19}{3}\right\}$.

53. $4.2|.5-x|+1=3.1 \Rightarrow 4.2|.5-x|=2.1 \Rightarrow |.5-x|=.5 \Rightarrow .5-x=.5$ or $.5-x=-.5 \Rightarrow -x=0$ or $-x=-1 \Rightarrow x=0$ or $x=1$. Therefore, the solution set is $\{0,1\}$.

54. $|3x-1|<8 \Rightarrow -8<3x-1<8 \Rightarrow -7<3x<9 \Rightarrow -\frac{7}{3}<x<3$. Therefore, the solution is $\left(-\frac{7}{3},3\right)$.

55. $|15-x|<7 \Rightarrow -7<15-x<7 \Rightarrow 22>x>8$ or $8<x<22$. Therefore, the solution is $(8,22)$.

56. $|7-4x|\le 11 \Rightarrow -11\le 7-4x\le 11 \Rightarrow -18\le -4x\le 4 \Rightarrow -1\le x\le \frac{9}{2}$. Therefore, the solution is $\left[-1,\frac{9}{2}\right]$.

57. $|2x-3|>1 \Rightarrow 2x-3>1$ or $2x-3<-1 \Rightarrow 2x>4$ or $\left(\frac{f}{g}\right)(-3)=\frac{-3(-3)-4}{(-3)^2}=\frac{5}{9}$. or $x<1$. Therefore, the solution is $(-\infty,1)\cup(2,\infty)$.

58. $|4-3x|>1 \Rightarrow 4-3x>1$ or $4-3x<-1 \Rightarrow -3x>-3$ or $-3x<-5 \Rightarrow x<1$ or $x>\frac{5}{3}$. Therefore, the solution is $(-\infty,1)\cup\left(\frac{5}{3},\infty\right)$.

59. $|-3x+8|\ge 3 \Rightarrow -3x+8\ge 3$ or $-3x+8\le -3 \Rightarrow -3x\ge -5$ or $-3x\le -11 \Rightarrow x\le \frac{5}{3}$ or $x\ge \frac{11}{3}$.

Therefore, the solution is $\left(-\infty,\frac{5}{3}\right]\cup\left[\frac{11}{3},\infty\right)$.

60. Absolute value is always positive, and so is always greater than -1; therefore, the solution is $(-\infty,\infty)$.

61. $\left|6-\frac{1}{3}x\right|>0 \Rightarrow 6-\frac{1}{3}x>0$ or $6-\frac{1}{3}x<0 \Rightarrow -\frac{1}{3}x>-6$ or $-\frac{1}{3}x<-6 \Rightarrow x<18$ or $x>18$. Therefore, the solution is every real number except 18: $(-\infty,18)\cup(18,\infty)$.

62. Absolute value is always positive, and cannot be less than 0; therefore, the solution set is $\varnothing$.

63. Absolute value is always positive, and so cannot be less than or equal to -6; therefore, the solution set is $\varnothing$.

64. Absolute value is always positive, and so cannot be less than -4; therefore, the solution set is $\varnothing$.

65. Absolute value is always positive, and so is always greater than -5; therefore, the solution is $(-\infty,\infty)$.

66. To solve such an equation, we must solve the compound equation $ax+b=cx+d$ or $ax+b=cx+d$. The solution set consists of the union of the two individual solution sets.

67. (a) $3x+1=2x-7 \Rightarrow x+1=-7 \Rightarrow x=-8$ $3x+1=-(2x-7) \Rightarrow 3x+1=-2x+7 \Rightarrow 5x=6 \Rightarrow x=\frac{6}{5}$.

Therefore, the solution set is $\left\{-8,\frac{6}{5}\right\}$.

(b) Graph $y_1=|3x+1|$ and $y_2=|2x-7|$. See Figure 67. From the graph, $|f(x)|>|g(x)|$ when $y_1>y_2$ which is for the interval $(-\infty,-8)\cup\left(\frac{6}{5},\infty\right)$.

(c) Graph $y_1 = |3x+1|$ and $y_2 = |2x-7|$ See Figure 67. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval $\left(-8, \frac{6}{5}\right)$.

68. (a) $x - 4 = 7x + 12 \Rightarrow -6x - 4 = 12 \Rightarrow -6x = 16 \Rightarrow x = -\frac{8}{3}$ or $x - 4 = -(7x+12) \Rightarrow$

$x - 4 = -7x - 12 \Rightarrow 8x - 4 = -12 \Rightarrow 8x = -8 \Rightarrow x = -1$. Therefore, the solution set is $\left\{-\frac{8}{3}, -1\right\}$.

(b) Graph $y_1 = |x-4|$ and $y_2 = |7x+12|$. See Figure 68. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$ which is for the interval $\left(-\frac{8}{3}, -1\right)$.

(c) Graph $y_1 = |x-4|$ and $y_2 = |7x+12|$. See Figure 68. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$ which is for the interval $\left(-\infty, -\frac{8}{3}\right) \cup (-1, \infty)$.

[-20,20] by [-10,50]
Xscl = 2 Yscl = 5

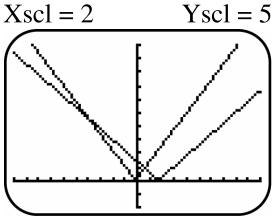

Figure 67

[-10,10] by [-4,16]
Xscl = 1 Yscl = 1

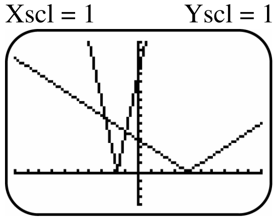

Figure 68

69. (a) $-2x + 5 = x + 3 \Rightarrow -3x = -2 \Rightarrow x = \frac{2}{3}$ or $-2x + 5 = -(x+3) \Rightarrow -2x + 5 = -x - 3 \Rightarrow$

$-x = -8 \Rightarrow x = 8$. Therefore, the solution set is $\left\{\frac{2}{3}, 8\right\}$.

(b) Graph $y_1 = |-2x+5|$ and $y_2 = |x+3|$. See Figure 69. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$, which it is for the interval $\left(-\infty, \frac{2}{3}\right) \cup (8, \infty)$.

(c) Graph $y_1 = |-2x+5|$ and $y_2 = |x+3|$. See Figure 69. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(\frac{2}{3}, 8\right)$.

70. (a) $-5x + 1 = 3x - 4 \Rightarrow -8x = -5 \Rightarrow x = \frac{5}{8}$ or $-5x + 1 = -(3x - 4) \Rightarrow -5x + 1 = -3x + 4 \Rightarrow -2x = 3 \Rightarrow x = -\frac{3}{2}$.

Therefore, the solution set is $\left\{-\frac{3}{2}, \frac{5}{8}\right\}$.

(b) Graph $y_1 = |-5x+1|$ and $y_2 = |3x-4|$. See Figure 70. From the graph, $|f(x)| > |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{5}{8}, \infty\right)$.

(c) Graph $y_1 = |-5x+1|$ and $y_2 = |3x-4|$. See Figure 70. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\frac{3}{2}, \frac{5}{8}\right)$.

[-10,10] by [-4,16]
Xscl = 1 Yscl = 1

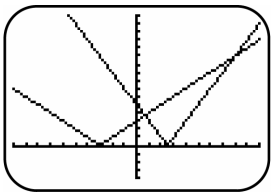

Figure 69

[-3,3] by [-4,16]
Xscl = 1 Yscl = 2

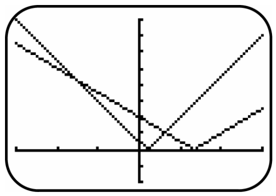

Figure 70

[-6,6] by [-2,10]
Xscl= 1 Yscl = 1

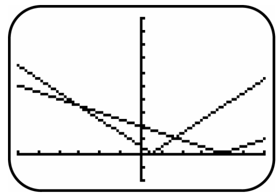

Figure 71

71. (a) $x - \frac{1}{2} = \frac{1}{2}x - 2 \Rightarrow \frac{1}{2}x = -\frac{3}{2} \Rightarrow x = -3$ or $x - \frac{1}{2} = -\left(\frac{1}{2}x - 2\right) \Rightarrow x - \frac{1}{2} = -\frac{1}{2}x + 2 \Rightarrow$ $\frac{3}{2}x = \frac{5}{2} \Rightarrow x = \frac{5}{3}$. Therefore, the solution set is $\left\{-3, \frac{5}{3}\right\}$.

(b) Graph $y_1 = \left|x - \frac{1}{2}\right|$ and $y_2 = \left|\frac{1}{2}x - 2\right|$. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$, which it is for the interval $(-\infty, -3) \cup \left(\frac{5}{3}, \infty\right)$.

(c) Graph $y_1 = \left|x - \frac{1}{2}\right|$ and $y_2 = \left|\frac{1}{2}x - 2\right|$ .See Figure 71. From the graph $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-3, \frac{5}{3}\right)$.

72. (a) $x + 3 = \frac{1}{3}x + 8 \Rightarrow \frac{2}{3}x = 5 \Rightarrow x = \frac{15}{2}$ or $x + 3 = -(\frac{1}{3}x + 8) \Rightarrow x + 3 = -\frac{1}{3}x - 8 \Rightarrow$ $\frac{4}{3}x = -11 \Rightarrow x = -\frac{33}{4}$. Therefore, the solution set is $\left\{-\frac{33}{4}, \frac{15}{2}\right\}$.

(b) Graph $y_1 = |x+3|$ and $y_2 = \left|\frac{1}{3}x + 8\right|$. See Figure 72. From the graph, $|f(x)| > |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\infty, -\frac{33}{4}\right) \cup \left(\frac{15}{2}, \infty\right)$.

(c) Graph $y_1 = |x+3|$ and $y_2 = \left|\frac{1}{3}x + 8\right|$. See Figure 72. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\frac{33}{4}, \frac{15}{2}\right)$.

73. (a) $4x + 1 = 4x + 6 \Rightarrow 1 = 6 \Rightarrow \varnothing$ or $4x + 1 = -(4x + 6) \Rightarrow 4x = 1 = -4x - 6 \Rightarrow 8x = -7 \Rightarrow -\frac{7}{8}$.

Therefore, the solution set is $\left\{-\frac{7}{8}\right\}$.

(b). Graph $y_1 = |4x+1|$ or $y_2 = |4x+6|$. See Figure 73. From the graph $|f(x)| > |g(x)|$ when $y_1 > y_2$, which it is for the interval $\left(-\infty, \frac{7}{8}\right)$.

(c) Graph $y_1 = |4x+1|$ or $y_2 = |4x+6|$. See Figure 73. From the graph, $|f(x)| < |g(x)|$ when $y_1 > y_2$, which is for the interval $\left(-\frac{7}{8}, \infty\right)$.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

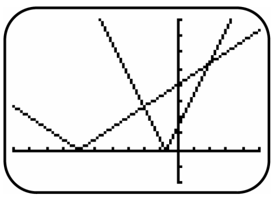

Figure 72

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

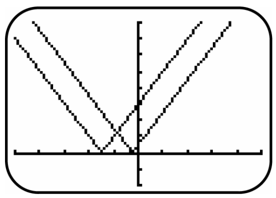

Figure 73

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

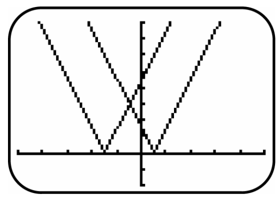

Figure 74

74. (a) $6x+9 = 6x-3 \Rightarrow 9 = -3 \Rightarrow \varnothing$ or $6x+9 = -(6x+9) \Rightarrow 6x+9 = -6x+3 \Rightarrow 12x = -6 \Rightarrow$ $-6 \Rightarrow x = -\frac{6}{12} = -\frac{1}{2}$. Therefore, the solution set is $\left\{-\frac{1}{2}\right\}$.

(b) Graph $y_1 = |6x+9|$ and $y_2 = |6x-3|$. See Figure 74. From the graph, $|f(x)| > |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\frac{1}{2}, \infty\right)$.

(c) Graph $y_1 = |6x+9|$ and $y_2 = |6x-3|$. See Figure 74. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $\left(-\infty, -\frac{1}{2}\right)$.

75. (a) $0.25x+1 = 0.75x-3 \Rightarrow -0.50x = -4 \Rightarrow x = 8$ or $0.25x+1 = -(0.75x-3) \Rightarrow 0.25x+1 = -0.75x+3 \Rightarrow$ $x = 2$. Therefore, the solution set is $\{2, 8\}$.

(b) Graph $y_1 = |.25x+1|$ and $y_2 = |.75x-3|$. See Figure 75. From the graph, $|f(x)| > |g(x)|$ when $y_1 < y_2$, which it is for the interval $(2, 8)$.

(c) Graph $y_1 = |.25x+1|$ and $y_2 = |.75x-3|$. See Figure 75. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $(-\infty, 2) \cup (8, \infty)$.

76. (a) $.40x+2 = .60x-5 \Rightarrow -20x = -7 \Rightarrow x = 35$ or $.40x+2 = -(.60x-5) \Rightarrow$ $.40x+2 = -.60x+5 \Rightarrow x = 3$. Therefore, the solution set is $\{3, 35\}$.

(b) Graph $y_1 = |.40x+2|$ and $y_2 = |.60x-5|$. See Figure 76. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$, which it is for the interval $(3, 35)$.

(c) Graph $y_1 = |.40x + 2|$ and $y_2 = |.60x - 5|$. See Figure 76. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, which it is for the interval $(-\infty, 3) \cup (35, \infty)$.

[-20,20] by [-4,16]
Xscl = 2 Yscl = 1

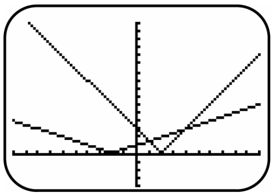

Figure 75

[-30,50] by [-5,30]
Xscl = 5 Yscl = 5

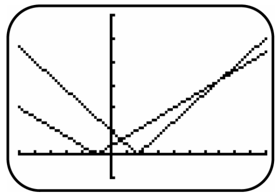

Figure 76

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

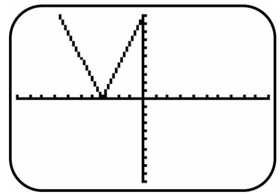

Figure 77

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

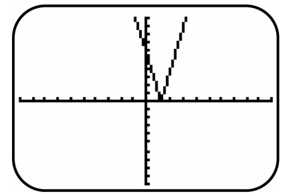

Figure 78

77. (a) $3x + 10 = -(-3x - 10) \Rightarrow 3x + 10 = 3x + 10 \Rightarrow$ there are an infinite number of solutions. Therefore, the solution set is $(-\infty, \infty)$.

(b) Graph $y_1 = |3x + 10|$ and $y_2 = |-3x - 10|$. See Figure 77. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$, for which there is no solution.

(c) Graph $y_1 = |3x + 10|$ and $y_2 = |-3x - 10|$. See Figure 77. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, for which there is no solution.

78. (a) $5x - 6 = -(-5x + 6) \Rightarrow 5x - 6 = 5x - 6 \Rightarrow$ there are an infinite number of solutions. Therefore, the solution set is $(-\infty, \infty)$.

(b) Graph $y_1 = |5x - 6|$ and $y_2 = |-5x + 6|$. See Figure 78. From the graph, $|f(x)| > |g(x)|$ when $y_1 > y_2$, for which there is no solution.

(c) Graph $y_1 = |5x - 6|$ and $y_2 = |-5x + 6|$. See Figure 78. From the graph, $|f(x)| < |g(x)|$ when $y_1 < y_2$, for which there is no solution.

79. Graph $y_1 = |x + 1| + |x - 6|$ and $y_2 = 11$. See Figure 79. From the graph, the lines intersect at $(-3, 11)$ and (2,9). Therefore, the solution set is $\{-3, 8\}$.

80. Graph $y_1 = |2x + 2| + |x + 1|$ and $y_2 = 9$. See Figure 80. From the graph, the lines intersect at $(-4, 9)$ and (2,9). Therefore, the solution set is $\{-4, 2\}$.

81. Graph $y_1 = |x| + |x - 4|$ and $y_2 = 8$. See Figure 81. From the graph, the lines intersect at $(-2, 8)$ and (6,8). Therefore, the solution set is $\{-2, 6\}$.

82. Graph $y_1 = |\ .5x + 2| + |.25x + 4|$ and $y_2 = 9$. See Figure 82. From the graph, the lines intersect at $(-20, 9)$ and (4,9). Therefore, the solution set is $\{-20, 4\}$.

[-10,10] by [-4,16]

Xscl = 1 Yscl = 1

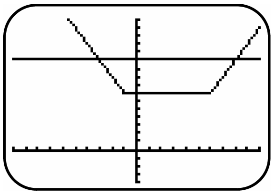

Figure 79

[-10,10] by [-4,16]

Xscl = 1 Yscl = 1

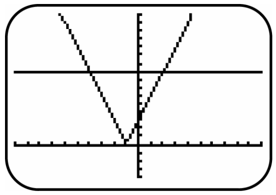

Figure 80

[-10,10] by [-4,16]

Xscl= 1 Yscl = 1

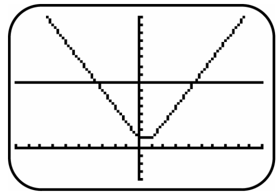

Figure 81

[-30,10] by [-4,16]

Xscl = 5 Yscl= 1

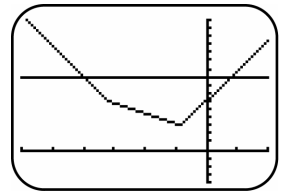

Figure 82

83. (a) $|T-50| \le 22 \Rightarrow -22 \le T-50 \le 22 \Rightarrow 28 \le T \le 72.$

(b) The average monthly temperatures in Boston vary between a low of 28° F and a high of 72° F. The monthly averages are always within 22° of 50° F.

84. (a) $|T-10| \le 36 \Rightarrow -36 \le T-10 \le 36 \Rightarrow -26 \le T \le 46..$

(b) The average monthly temperatures in Chesterfield vary between a low of -26° F and a high of 46° F. The monthly averages are always within 36° of 10° F.

85. (a) $|T-61.5| \le 12.5 \Rightarrow -12.5 \le T-61.5 \le 12.5 \Rightarrow 49 \le T \le 74..$

(b) The average monthly temperatures in Buenos Aires vary between a low of 49° F (possibly in July) and a high of 74° F (possibly in January). The monthly averages are always within 12.5° of 61.5° F.

86. (a) $|T-43.5| \le 8.5 \Rightarrow -8.5 \le T-43.5 \le 8.5 \Rightarrow 35 \le T \le 52.$

(b) The average monthly temperatures in Punta Arenas vary between a low of 35° F and a high of 52° F. The monthly averages are always within 8.5° of 43.5° F.

87. $|x-8.0| \le 1.5 \Rightarrow -1.5 \le x-8.0 \le 1.5 \Rightarrow 6.5 \le x \le 9.5;$ therefore, the range is the interval $[6.5, 9.5]$.

88. If $\dfrac{680+780}{2} = 730$ is the midpoint, then $680 \le F \le 780 \Rightarrow 680-730 \le F-730 \le 780-730 \Rightarrow$ $-50 \le F-730 \le 50 \Rightarrow |F-730| \le 50$ (or $|730-F| \le 50$).

89. (a) $P_d = |116-125| \Rightarrow P_d = |-9| = 9$.

(b) $17 = |P-130| \Rightarrow P-130 = 17 \text{ or } P-130 = -17 \Rightarrow P = 147 \text{ or } \text{P}=113$.

90. If $\dfrac{98+148}{2} = 123$ is the midpoint, then $98 \le x \le 148 \Rightarrow 98-123 \le x-123 \le x-123 \le 148-123 \Rightarrow$ $-25 \le x-123 \le 25 \Rightarrow |x-123| \le 25$ (or $|123-x| \le 25$); and if $\dfrac{16+26}{2} = 21$ is the midpoint, then $16 \le x \le 26 \Rightarrow 16-21 \le x-21 \le 26-21 \Rightarrow -5 \le x-21 \le 5 \Rightarrow |x-21| \le 5$ (or $|21-x| \le 5$).

91. If the difference between y and 1 is less than .1, then $|y-1| < .1 \Rightarrow |2x+1-1| < .1 \Rightarrow$ $|2x| < .1 \Rightarrow -.1 < 2x < .1 \Rightarrow -.05 < x < .05.$ The open interval of x is $(-.05, .05)$.

92. If the difference between y and 2 is less than .01, then $|y-2|<.01 \Rightarrow |3x-6-2|<.01 \Rightarrow |3x-8|<.01 \Rightarrow$ $-.01<3x-8<.01 \Rightarrow 7.99<3x<8.01 \Rightarrow 2.663<x<2.67$. The open interval of x is $\left(2.66\bar{3}, 2.67\right)$.

93. If the difference between y and 3 is less than .001, then $|y-3|<.001 \Rightarrow |4x-8-3|<.001 \Rightarrow$ $|4x-11|<.001 \Rightarrow -.001<4x-11<.001 \Rightarrow 10.999<4x<11.001 \Rightarrow 2.74975<x<2.75025$. The open interval of x is $(2.74975, 2.75025)$.

94. If the difference between y and 4 is less than .0001, then $|y-4|<.0001 \Rightarrow |5x+12-4|<.0001 \Rightarrow$ $|5x+8|<.0001 \Rightarrow -.0001<5x+8<.0001 \Rightarrow -8.0001<5x<-7.9999 \Rightarrow -1.60002<x<-1.59998$. The open interval of x is $(-1.60002, -1.59998)$.

95. If $|2x+7|=6x-1$ then $|2x+7|-(6x-1)=0$. Graph $y_1=|2x+7|-(6x-1)$, See Figure 95. The x-intercept is 2; therefore, the solution set is $\{2\}$.

96. If $-|3x-12| \ge -x-1$ then $-|3x-12|-(-x-1) \ge 0$. Graph $y_1=-|3x-12|-(-x-1)$, See Figure 96. The equation is ≥ 0, or the graph intersects or is above the x-axis, for the interval: $[2.75, 6.5]$.

97. If $|x-4|>.5x-6$ then $|x-4|-(.5x-6)>0$. Graph $y_1=|x-4|-(.5x-6)$, See Figure 97. The equation is >0, or the graph is above the x-axis, for the interval: $(-\infty, \infty)$.

98. If $2x+8>-|3x+4|$ then $2x+8-(-|3x+4|)>0$. Graph $y_1=2x+8-(-|3x+4|)$, See Figure 98. The equation is >0, or the graph is above the x-axis, for the interval: $(-\infty, \infty)$.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

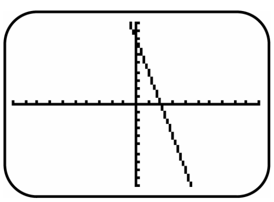

Figure 95

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

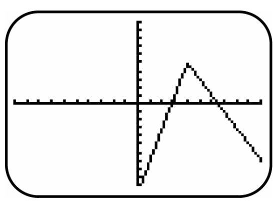

Figure 96

[-10,10] by [-10,10]
Xscl= 1 Yscl = 1

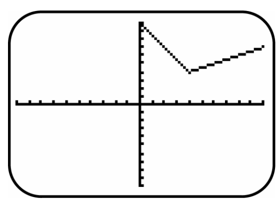

Figure 97

[-10,10] by [-4,16]
Xscl = 1 Yscl= 1

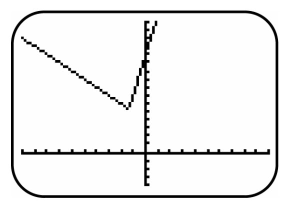

Figure 98

99. If $|3x+4|<-3x-14$ then $|3x+4|-(-3x-14)<0$. Graph $y_1=|3x+4|-(-3x-14)$, See Figure 99. The equation is <0, or the graph is below the x-axis, never or for the solution set: $\varnothing$.

100. If $\left|x-\sqrt{13}\right|+\sqrt{6} \le -x-\sqrt{10}$ then $\left|x-\sqrt{13}\right|+\sqrt{6}-\left(-x-\sqrt{10}\right) \le 0$. Graph $y_1=\left|x-\sqrt{13}\right|+\sqrt{6}-\left(-x-\sqrt{10}\right)$ See Figure 100. The equation is ≤ 0, or the graph intersects or is below the x-axis, never or for the solution set: $\varnothing$.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

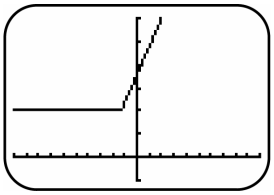

Figure 99

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

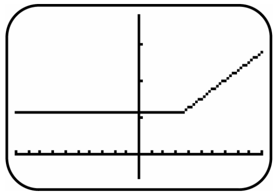

Figure 100

2.5: Piecewise-Defined Functions

1. (a) From the graph, the speed limit is 40 mph.
   (b) From the graph, the speed limit is 30 mph for 6 miles.
   (c) From the graph, $f(5)=40$ mph; $f(13)=30$ mph; and $f(19)=55$ mph.
   (d) From the graph, the graph is discontinuous at $x=4,6,8,12,\text{and}16$. The speed limit changes at each discontinuity.
2. (a) From the graph, the Initial amount was: $1,000; and the final amount was: $600.
   (b) From the graph, $f(10)=\$900; f(50)=\$600$. The function f is not continuous.
   (c) From the graph, the discontinuity shows 3 drops or withdrawals.
   (d) From the graph, the largest drop or largest withdrawal of $300 occurred after 15 minutes.
   (e) From the graph, the one increase or deposit was $200.
3. (a) From the graph, the Initial amount was: 50,000 gal.; and the final amount was: 30,000 gal.
   (b) From the graph, during the first and fourth days.
   (c) From the graph, $f(2)=45{,}000$ gal; $f(4)=40{,}000$ gal.
   (d) From the graph, between days 1 and 3 the water dropped: $\frac{50{,}000-40{,}000}{2}=\frac{10{,}000}{2}=5{,}000$ gal./day.
4. (a) From the graph, when x = 1 the tank had 20 gallons of gas.
   (b) Since x = 0 represents 3:15 pm and 3 hours later the tank was filled to 20 gallons, the time was 6:15 pm
   (c) When the graph is horizontal, the engine is not running; when the graph is decreasing, the engine is burning gasoline; and when the graph is increasing, gasoline is being put into the tank.
   (d) When the graph is decreasing, gasoline was burned the fastest between 1 and 2.9 hours
5. (a) $f(-5)=2(-5)=-10$ (b) $f(-1)=2(-1)=-2$
   (c) $f(0)=0-1=-1$ (d) $f(3)=3-1=2$
6. (a) $f(-5)=-5-2=-7$ (b) $f(-1)=-1-2=-3$
   (c) $f(0)=0-2=-2$ (d) $f(3)=5-3=2$
7. (a) $f(-5)=2+(-5)=-3$ (b) $f(-1)=-(-1)=1$
   (c) $f(0)=-(0)=0$ (d) $f(3)=3(3)=9$
8. (a) $f(-5)=-2(-5)=10$ (b) $f(-1)=3(-1)-1=-4$
   (c) $f(0)=3(0)-1=-1$ (d) $f(3)=-4(3)=-12$

9. Yes, continuous. See Figure 9.
10. Yes, continuous. See Figure 10.
11. Not continuous. See Figure 11

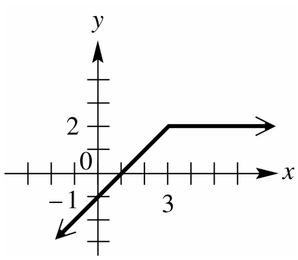

Figure 9

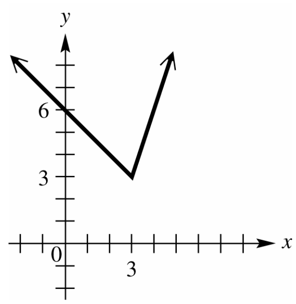

Figure 10

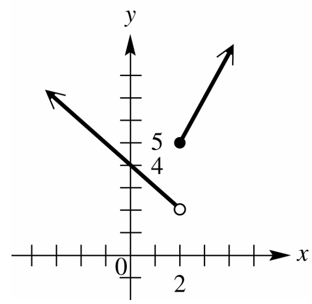

Figure 11

12. Not continuous. See Figure 12.
13. Not continuous. See Figure 13.
14. Not continuous. See Figure 14.

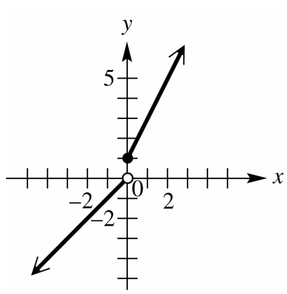

Figure 12

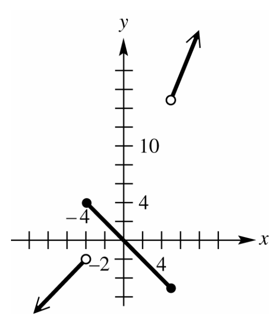

Figure 13

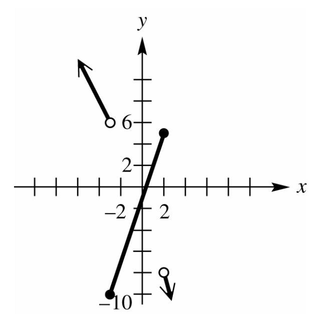

Figure 14

15. Not continuous. See Figure 15.
16. Not continuous. See Figure 16.
17. Yes, continuous. See Figure 17.

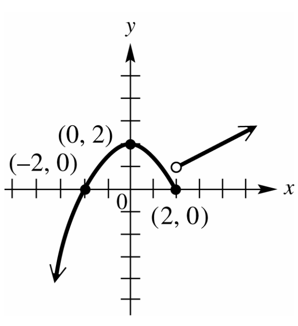

Figure 15

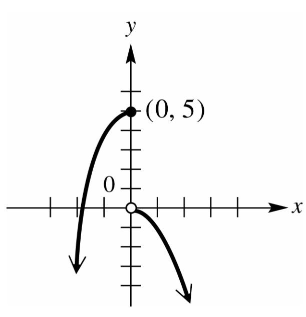

Figure 16

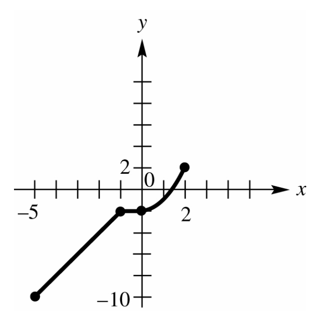

Figure 17

18. Not continuous. See Figure 18.
19. Yes, continuous. See Figure 19.
20. Yes, continuous. See Figure 20.

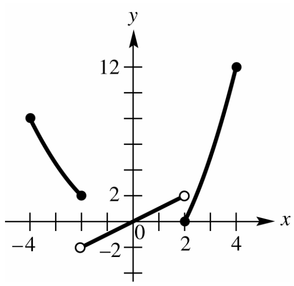

Figure 18

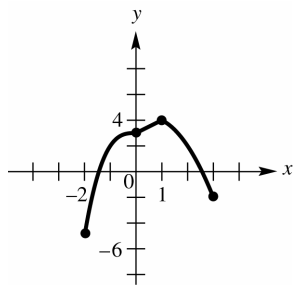

Figure 19

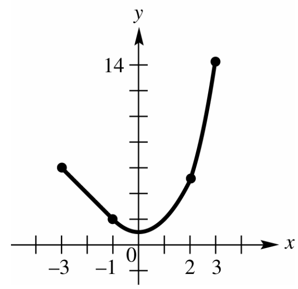

Figure 20

21. Look for a $y = x^2$ graph if $x \ge 0$; and a linear graph if $x < 0$. Therefore: B.

22. Look for a $y = |x|$ graph if $x \ge -1$; and a reflected $y = x^2$ graph if $x < -1$. Therefore: A.

23. Look for a horizontal graph above the x-axis if $x \ge 0$; and a horizontal graph below the x-axis if $x < 0$. Therefore: D.

24. Look for a $y = \sqrt{x}$ graph if $x \ge 0$; and a reflected $y = x^2$ graph if $x < 0$. Therefore: C.

25. Graph $y_1 = (x-1)(x \le 3) + (2)(x > 3)$, See Figure 25.

26. Graph $y_1 = (6-x)(x \le 3) + (3x-6)(x > 3)$, See Figure 26.

27. Graph $y_1 = (4-x)(x < 2) + (1+2x)(x \ge 2)$, See Figure 27.

28. Graph $y_1 = (2x+1)(x \ge 0) + (x)(x < 0)$, See Figure 28.

[-4,6] by [-2,4]
Xscl = 1 Yscl = 1

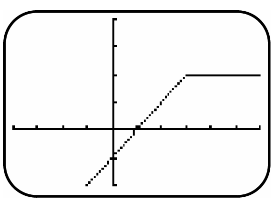
Figure 25

[-2,8] by [-2,10]
Xscl = 1 Yscl = 1

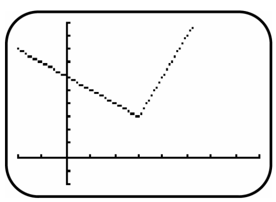
Figure 26

[-4,6] by [-2,8]
Xscl= 1 Yscl = 1

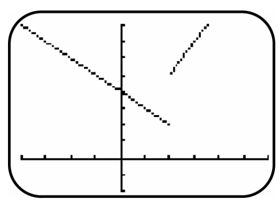
Figure 27

[-5,4] by [-3,8]
Xscl = 1 Yscl= 1

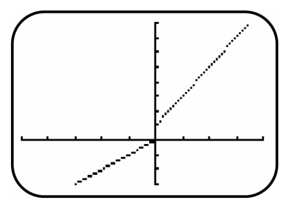
Figure 28

29. Graph $y_1 = (2+x)(x < -4) + (-x)(-4 \le x \text{ and } x \le 5) + (3x)(x > 5)$, See Figure 29.

30. Graph $y_1 = (-2x)(x < -3) + (3x-1)(-3 \le x \text{ and } x \le 2) + (-4x)(x > 2)$, See Figure 30.

31. Graph $y_1 = \left(-\frac{1}{2}x + 2\right)(x \le 2) + \left(\frac{1}{2}x\right)(x > 2)$, See Figure 31.

32. Graph $y_1 = (x^3 + 5)(x \le 0) + (-x^2)(x > 0)$, See Figure 32.

[-12,12] by [-6,20]
Xscl = 2 Yscl = 2

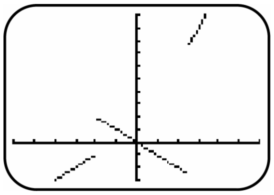

Figure 29

[-6,6] by [-10,8]
Xscl = 1 Yscl = 1

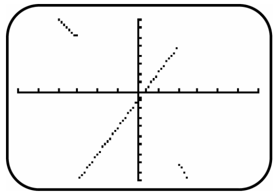

Figure 30

[-5,6] by [-2,4]
Xscl= 1 Yscl = 1

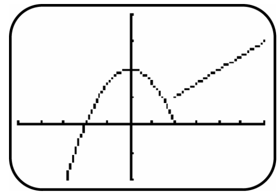

Figure 31

[-3,4] by [-3,6]
Xscl = 1 Yscl= 1

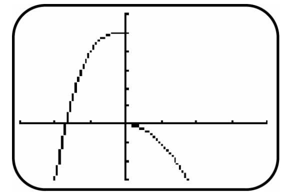

Figure 32

33. From the graph, the function is $f(x)=\begin{cases}2 & \text{if } x\le 0\\ -1 & \text{if } x>1\end{cases}$; domain : $(-\infty,0]\cup(1,\infty)$; range: $\{-1,2\}$.

34. From the graph, the function is $f(x)=\begin{cases}1 & \text{if } x\le -1\\ -1 & \text{if } x>2\end{cases}$; domain : $(-\infty,-1]\cup(2,\infty)$; range: $\{-1,1\}$.

35. From the graph, the function is $f(x)=\begin{cases}x & \text{if } x\le 0\\ 2 & \text{if } x>0\end{cases}$; domain : $(-\infty,\infty)$; range: $(-\infty,0]\cup\{2\}$.

36. From the graph, the function is $f(x)=\begin{cases}-3 & \text{if } x<0\\ \sqrt{x} & \text{if } x\ge 0\end{cases}$; domain : $(-\infty,\infty]$; range: $\{-3\}\cup[0,\infty)$.

37. From the graph, the function is $f(x)=\begin{cases}\sqrt[3]{x} & \text{if } x<1\\ x+1 & \text{if } x\ge 1\end{cases}$; domain : $(-\infty,\infty)$; range : $(-\infty,1)\cup[2,\infty)$.

38. From the graph, the function is $f(x)=\begin{cases}3 & \text{if } x=2\\ 2x-3 & \text{if } x\ne 2\end{cases}$; domain : $(-\infty,\infty)$; range : $(-\infty,1)\cup(1,\infty)$.

39. There is an overlap of intervals since the number 4 satisfies both conditions. To be a function, every *x*-value is used only once.

40. The value $f(4)$ cannot be found since using the first formula, $f(4)=11$, and using the second formula, $f(4)=16$. To have two different values violates the definition of function.

41. The graph of $y=[\![x]\!]$ is shifted 1.5 units downward.

42. The graph of $y=[\![x]\!]$ is reflected across the *y*-axis.

43. The graph of $y=[\![x]\!]$ is reflected across the *x*-axis.

44. The graph of $y=[\![x]\!]$ is shifted 2 units to the left.

45. Graph $y=[\![x]\!]-1.5$, See Figure 45.

46. Graph $y=[\![-x]\!]$, See Figure 46.

47. Graph $y=-[\![x]\!]$, See Figure 47.

48. Graph $y=[\![x+2]\!]$, See Figure 48.

[-5,5] by [-3,3]

Xscl = 1 Yscl = 1

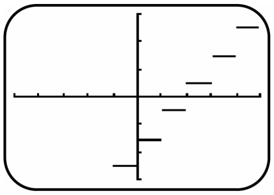

Figure 45

[-5,5] by [-3,3]

Xscl = 1 Yscl = 1

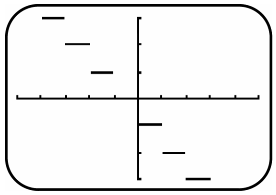

Figure 46

[-5,5] by [-3,3]

Xscl= 1 Yscl = 1

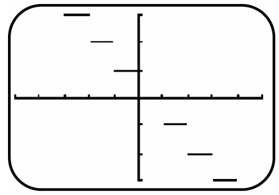

Figure 47

[-5,5] by [-3,3]

Xscl = 1 Yscl= 1

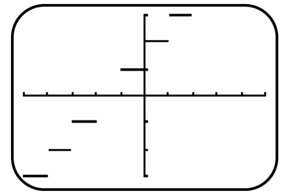

Figure 48

49. When $0 \le x \le 3$ the slope is 5, which means the inlet pipe is open and the outlet pipe is closed; when $3 < x \le 5$ the slope is 2, which means both pipes are open; when $5 < x \le 8$ the slope is 0, which means both pipes are closed; when $8 < x \le 10$ the slope is -3, which means the inlet pipe is closed and the outlet pipe is open.

50. (a) Since for each year given the shoe size is 1 size smaller than his age, the formula is $y = x - 1$

(b) From the table and question, graph the function. See figure 50.

51. (a) $f(1.5) = 1.12, f(3) = 1.32$, It costs \$1.12 to mail 1.5 oz and \$1.32 to mail 3 oz.

(b) Domain: $(0,5]$; Range: $\{0.92, 1.12, 1.32, 1.52, 1.72\}$. See figure 51.

52. (a) $f(1967) = 0.0475(1967) - 93.3 = 0.1325$, In 1967, the average Super Bowl ad cost \$0.1325 million.

(b) $f(1998) = 0.475(1998) - 93.3 = 1.605$, In 1998, the average Super Bowl ad cost \$1.605 million.

(c) $f(2013) = 0.1333(2013) - 264.7284 = 3.6045$, In 2013, the average Super Bowl ad cost \$3.6045 million.

(d) The function $f(x)$ is continuous on its domain since there are no breaks in the graph of $f(x)$.

(e) See Figure 52.

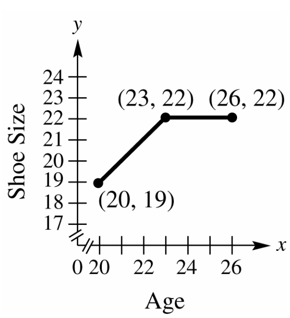

Figure 50

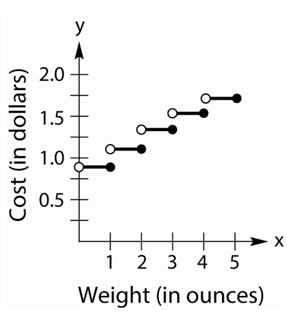

Figure 51

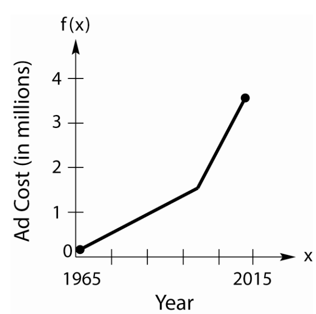

Figure 52

53. (a) From the table, graph the piecewise function. See Figure 53.

(b) The likelihood of being a victim peaks from age 16 up to age 20, then decreases.

54. (a) From the table, graph the piecewise function. See Figure 54.

(b) Housing starts increased, decreased, and increased, with the maximum occurring during the 1960's.

55. (a) From the graph, the highest speed is 55 mph and the lowest speed is 30 mph.

(b) There are approximately 12 miles of highway with a speed of 55 mph.

(c) Fromt the graph, $f(4) = 40; f(12) = 30; f(18) = 55.$

56. From the table, graph the piecewise function. See Figure 56.

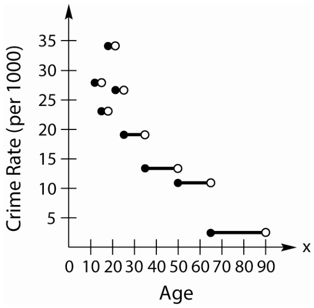

Figure 53

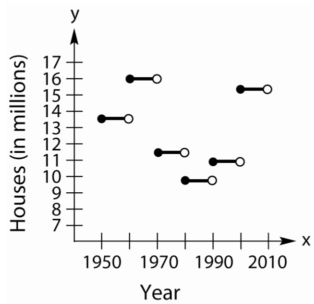

Figure 54

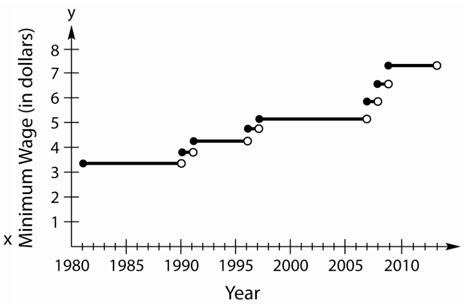

Figure 56

57. (a) A 3.5 minute call would round up to 4 minutes. A 4 minute call would cost: $.50 + 3(.25) = \$1.25.$

(b) We use a piecewise defined function where the cost increases after each whole number as follows:

$$f(x) = \begin{cases} .50 & \text{if } 0 < x \le 1 \\ .75 & \text{if } 1 < x \le 2 \\ 1.00 & \text{if } 2 < x \le 3 \\ 1.25 & \text{if } 3 < x \le 4 \\ 1.50 & \text{if } 4 < x \le 5 \end{cases}.$$ Another possibility is $$f(x) = \begin{cases} .50 & \text{if } 0 < x \le 1 \\ .50 - .25[\![1 - x]\!] & \text{if } 1 < x \le 5 \end{cases}.$$

58. (a) Since cost is rounded down to the nearest 2 foot interval, we can use the greatest integer function. The function is $f(x) = .80\left[\!\!\left[\frac{x}{2}\right]\!\!\right]$ from $6 \le x \le 18.$

(b) Graph $f(x) = .80\left[\!\!\left[\frac{x}{2}\right]\!\!\right]$ from $6 \le x \le 18.$ See Figure 58.

(c) $f(8.5) = .80\left[\!\!\left[\frac{8.5}{2}\right]\!\!\right] = .80[\![4.25]\!] = .80(4) \Rightarrow f(8.5) = \$3.20$

$f(15.2) = .80\left[\!\!\left[\frac{15.2}{2}\right]\!\!\right] = .80[\![7.6]\!] = .80(7) \Rightarrow f(15.2) = \$5.60$

[6,18] by [0,8]

Xscl = 1 Yscl = 1

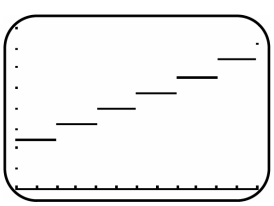

Figure 58

59. For x in the interval $(0,2]$, $y = 25$. For x in the interval $(2,3]$, $y = 25 + 3 = 28$. For x in the interval $(3,4]$, $y = 28 + 3 = 31$ and so on. The graph is a step function. In this case, the first step has a different width. See Figure 59.

60. Sketch a piecewise function that measures miles over minutes. The first piece increases at a rate of 40 mph or $\frac{40}{60} = \frac{2}{3}$ miles per minute, for 30 minutes (the time it takes to travel 20 miles at that rate). The second piece stays at a constant distance of 20 miles for the 2 hour period at the park. The third piece decreases (returns home) at a rate of 20 mph or $\frac{20}{60} = \frac{1}{3}$ miles per minute, for 60 minutes (the time it takes to travel 20 miles at that rate). See Figure 60.

61. Sketch a piecewise function that fills a tank at a rate of 5 gallons a minute for the first 20 minutes (the time it takes to fill the 100 gallon tank) and then drains the tank at a rate of 2 gallons per minute for 50 minutes (the time it takes to drain the 100 gallon tank). See Figure 61.

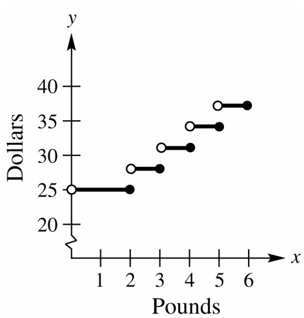

Figure 59

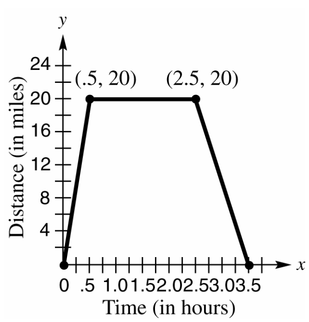

Figure 60

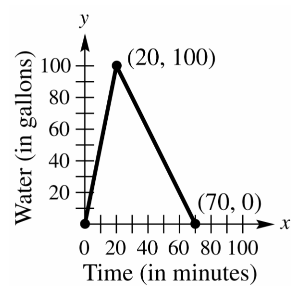

Figure 61

62. (a) Use the points (1990, 1.3) and (2007, 2.7) to find the slope. $m = \frac{2.7 - 1.3}{2007 - 1990} = \frac{1.4}{17} \approx 0.082$

$y - y_1 = m(x - x_1) \Rightarrow y - 1.3 = 0.082(x - 1990) \Rightarrow y = 0.082x + 161.88$ Use the points (2007, 2.7) and (2012, 1.75) to find the slope. $m = \frac{1.75 - 2.7}{2012 - 2007} = -\frac{0.95}{5} = -0.19$ $y - y_1 = m(x - x_1) \Rightarrow$

$y - 1.75 = -0.19(x - 2012) \Rightarrow y = -0.19x + 384.03$.

(b) Use the two functions above to create $f(x) = \begin{cases} 0.082x + 161.88 & 1990 \le x \le 2007 \\ -0.19x + 384.03 & 2007 < x \le 2012 \end{cases}$

<u>2.6: Operations and Composition</u>

1. $x^2 + (2x - 5) = x^2 + 2x - 5 \Rightarrow E.$
2. $x^2 - (2x - 5) = x^2 - 2x + 5 \Rightarrow B.$
3. $x^2(2x - 5) = 2x^3 - 5x^2 \Rightarrow F.$
4. $\frac{x^2}{2x - 5} \Rightarrow D.$

5. $(2x-5)^2 = 4x^2 - 20x + 25 \Rightarrow A$.

6. $(2(x^2)-5) = 2x^2 - 5 \Rightarrow C$.

7. $(f \circ g)(3) = f(g(3)) = f(2(3)-1) = f(5) = 5^2 + 3(5) = 40$

8. $(g \circ f)(-2) = g(f(-2)) = g((-2)^2 + 3(-2)) = g(-2) = 2(-2) - 1 = -5$

9. $(f \circ g)(x) = f(g(x)) = f(2x-1) = (2x-1)^2 + 3(2x-1) = 4x^2 - 4x + 1 + 6x - 3 = 4x^2 + 2x - 2$

10. $(g \circ f)(x) = g(f(x)) = g(x^2 + 3x) = 2(x^2 + 3x) - 1 = 2x^2 + 6x - 1$

11. $(f+g)(3) = f(3) + g(3) = ((3)^2 + 3(3)) + (2(3) - 1) = 23$

12. $(f+g)(-5) = f(-5) + g(-5) = \left((-5)^2 + 3(-5)\right) + (2(-5) - 1) = -1$

13. $(f \cdot g)(4) = f(4) \cdot g(4) = \left((4)^2 + 3(4)\right) \cdot (2(4) - 1) = 196$

14. $(f \cdot g)(-3) = f(-3) \cdot g(-3) = \left((-3)^2 + 3(-3)\right) \cdot \left(2(-3) - 1\right) = 0$

15. $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{(-1)^2 + 3(-1)}{2(-1) - 1} = \frac{2}{3}$

16. $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{(4)^2 + 3(4)}{2(4) - 1} = \frac{28}{7} = 4$

17. $(f-g)(2) = f(2) - g(2) = \left((2)^2 + 3(2)\right) - \left(2(2) - 1\right) = 7$

18. $(f-g)(-2) = f(-2) - g(-2) = \left((-2)^2 + 3(-2)\right) - \left(2(-2) - 1\right) = 3$

19. $(g-f)(-2) = g(-2) - f(-2) = \left(2(-2) - 1\right) - \left((-2)^2 + 3(-2)\right) = -3$

20. $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right) = \frac{f\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)} = \frac{\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)}{2\left(\frac{1}{2}\right) - 1} = \frac{\frac{7}{4}}{0} \Rightarrow$ undefined.

21. $\left(\frac{g}{f}\right)(0) = \frac{f(0)}{g(0)} = \frac{2(0) - 1}{(0)^2 + 3(0)} = \frac{-1}{0} \Rightarrow$ undefined.

22. Because the two compositions are opposites of each other.

23. (a) $(f+g)(x) = (4x-1) + (6x+3) = 10x + 2$, $(f-g)(x) = (4x-1) - (6x+3) = -2x - 4$

$(fg)(x) = (4x-1)(6x+3) = 24x^2 + 12x - 6x - 3 = 24x^2 + 6x - 3$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{4x-1}{6x+3}$; all values can replace x, except $-\frac{1}{2}$; therefore, the domain is $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

(d) $(f \circ g)(x) = f\left[g(x)\right] = 4(6x+3) - 1 = 24x + 12 - 1 = 24x + 11$; all values can be input for x; therefore, the domain is $(-\infty, \infty)$

(e) $(g \circ f)(x) = g\left[f(x)\right] = 6(4x-1)+3 = 24x-6+3 = 24x-3;$ all values can replace x; therefore, the domain is $(-\infty, \infty)$.

24. (a) $(f+g)(x) = (9-2x)+(-5x+2) = -7x+11$, $(f-g)(x) = (9-2x)-(-5x+2) = 3x+7$

$(fg)(x) = (9-2x)(-5x+2) = -45x+18+10x^2-4x = 10x^2-49x+18$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{9-2x}{-5x+2}$; all values can replace x, except $\frac{2}{5}$; therefore, the domain is $\left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$.

(d) $(f \circ g)(x) = f\left[g(x)\right] = 9-2(-5x+2) = 9+10x-4 = 10x+5$; all values can replace x; therefore, the domain is $(-\infty, \infty)$

(e) $(g \circ f)(x) = g\left[f(x)\right] = -5(9-2x)+2 = -45+10x+2 = 10x-43;$ all values can replace x; therefore, the domain is $(-\infty, \infty)$

25. (a) $(f+g)(x) = |x+3|+2x$, $(f-g)(x) = |x+3|-2x$, $(fg)(x) = |x+3|(2x)$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{|x+3|}{2x}$; all values can replace x, except 0; therefore, the domain is $(-\infty, 0) \cup (0, \infty)$

(d) $(f \circ g)(x) = f\left[g(x)\right] = |(2x)+3| = |2x+3|$; all values can replace x; therefore, the domain is $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g\left[f(x)\right] = 2\left(|x+3|\right) = 2|x+3|$; all values can replace x; therefore, the domain is $(-\infty, \infty)$

26. (a) $(f+g)(x) = |2x-4|+(x+1) = |2x-4|+x+1$, $(f-g)(x) = |2x-4|-(x+1) = |2x-4|-x-1$

$(fg)(x) = |2x-4|(x+1) = (x+1)|2x-4|$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{|2x-4|}{x+1}$ all values can replace x, except -1; therefore, the domain is $(-\infty, -1) \cup (-1, \infty)$.

(d) $(f \circ g)(x) = f\left[g(x)\right] = |2(x+1)-4| = |2x-2|$; all values can replace x, so the domain is $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g\left[f(x)\right] = |2x-4|+1$; all values can replace x, so the domain is $(-\infty, \infty)$.

27. (a) $(f+g)(x) = \sqrt[3]{x+4}+(x^3+5) = \sqrt[3]{x+4}+x^3+5$, $(f-g)(x) = \sqrt[3]{x+4}-(x^3+5) = \sqrt[3]{x+4}-x^3-5$

$(fg)(x) = \left(\sqrt[3]{x+4}\right)(x^3+5)$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty, \infty)$ in all cases.

(c) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt[3]{x+4}}{(x^3+5)}$ all values can replace x, except $\sqrt[3]{-5}$, so the domain is $(-\infty, \sqrt[3]{-5}) \cup (\sqrt[3]{-5}, \infty)$.

(d) $(f \circ g)(x) = f\left[g(x)\right] = \sqrt[3]{(x^3+5)+4} = \sqrt[3]{x^3+9}$; all values can replace x, so the domain is $(-\infty, \infty)$.

(e) $(g\circ f)(x)=g\left[f(x)\right]=\left(\sqrt[3]{x+4}\right)^3+5=x+4+5=x+9;$ all values can replace x, so the domain is $(-\infty,\infty)$.

28. (a) $(f+g)(x)=\sqrt[3]{6-3x}+(2x^3+1)\ =\sqrt[3]{6-3x}+2x^3+1$

$(f-g)(x)=\sqrt[3]{6-3x}-(2x^3+1)\ =\sqrt[3]{6-3x}-2x^3-1$

$(fg)(x)=\left(\sqrt[3]{6-3x}\right)(2x^3+1)$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty,\infty)$ in all cases.

(c) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt[3]{6-3x}}{(2x^3+1)};$ all values can replace x, except $\sqrt[3]{-\dfrac{1}{2}}$, so the domain is $\left(-\infty,\sqrt[3]{-\dfrac{1}{2}}\right)\cup\left(\sqrt[3]{-\dfrac{1}{2}},\infty\right)$

(d) $(f\circ g)(x)=f\left[g(x)\right]=\sqrt[3]{6-3(2x^3+1)}=\sqrt[3]{-6x^3+3};$ all values can replace x, so the domain is $(-\infty,\infty)$.

(e) $(g\circ f)(x)=g\left[f(x)\right]=2\left(\sqrt[3]{6-3x}\right)^3+1=2(6-3x)+1=-6x+13;$ all values can replace x; therefore, the domain is $(-\infty,\infty)$.

29. (a) $(f+g)(x)=\sqrt{x^2+3}+(x+1)=\sqrt{x^2+3}+x+1$, $(f-g)(x)=\sqrt{x^2+3}-(x+1)=\sqrt{x^2+3}-x-1$

$(fg)(x)=\left(\sqrt{x^2+3}\right)(x+1)$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty,\infty)$ in all cases.

(c) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x^2+3}}{(x+1)};$ all values can replace x, except $-1;$ therefore, the domain is $(-\infty,-1)\cup(-1,\infty)$.

(d) $(f\circ g)(x)=f\left[g(x)\right]=\sqrt{(x+1)^2+3}=\sqrt{x^2+2x+1+3}=\sqrt{x^2+2x+4};$ all values can replace x; therefore, the domain is $(-\infty,\infty)$.

(e) $(g\circ f)(x)=g\left[f(x)\right]=\left(\sqrt{x^2+3}\right)+1=\sqrt{x^2+3}+1;$ all values can replace x; therefore, the domain is, $(-\infty,\infty)$.

30. (a) $(f+g)(x)=\sqrt{2+4x^2}+(x)=\sqrt{2+4x^2}+x$, $(f-g)(x)=\sqrt{2+4x^2}-(x)=\sqrt{2+4x^2}-x$

$(fg)(x)=\left(\sqrt{2+4x^2}\right)(x)=x\sqrt{2+4x^2}$

(b) All values can replace x in all three equations; therefore, the domain is $(-\infty,\infty)$ in all cases.

(c) $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{2+4x^2}}{x};$ all values can replace x, except 0; therefore, the domain is $(-\infty,0)\cup(0,\infty)$.

(d) $(f \circ g)(x) = f[g(x)] = \sqrt{2+4(x)^2} = \sqrt{2+4x^2}$; all values can replace x; therefore, the domain is $(-\infty, \infty)$.

(e) $(g \circ f)(x) = g[f(x)] = \left(\sqrt{2+4x^2}\right) = \sqrt{2+4x^2}$; all values can replace x; therefore, the domain is $(-\infty, \infty)$.

31. (a) From the graph, $4+(-2)=2$.
    (b) From the graph, $1-(-3)=4$.
    (c) From the graph, $(0)(-4)=0$.
    (d) From the graph, $\frac{1}{-3} = -\frac{1}{3}$.

32. (a) From the graph, $0+2=2$.
    (b) From the graph, $-2-1=-3$.
    (c) From the graph, $(2)(1)=2$.
    (d) From the graph, $\frac{4}{-2} = -2$.

33. (a) From the graph, $0+3=3$.
    (b) From the graph, $-1-4=-5$.
    (c) From the graph, $(2)(1)=2$.
    (d) From the graph, $\frac{3}{0} \Rightarrow$ undefined.

34. (a) From the graph, $-3+1=-2$.
    (b) From the graph, $-2-0=-2$.
    (c) From the graph, $(-3)(-1)=3$.
    (d) From the graph, $\frac{-3}{1} = -3$.

35. (a) From the table, $7+(-2)=5$.
    (b) From the table, $10-5=5$.
    (c) From the table, $(0)(6)=0$.
    (d) From the table, $\frac{5}{0} =$ undefined.

36. (a) From the table, $5+4=9$.
    (b) From the table, $0-0=0$.
    (c) From the table, $(-4)(2)=-8$.
    (d) From the table, $\frac{8}{-1} = -8$.

37. See Table 37.

38. See Table 38.

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	6	-6	0	0
0	5	5	0	undefined
2	5	9	-14	-3.5
4	15	5	50	2

Figure 37

x	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$
-2	-2	-6	-8	-2
0	7	9	-8	-8
2	9	1	20	1.25
4	0	0	0	undefined

Figure 38

39. $M(2004) \approx 260$ and $F(2004) \approx 400 \Rightarrow T(2004) = M(2004) + F(2004) = 260 + 400 = 660$

40. $M(2008) \approx 290$ and $F(2008) \approx 470 \Rightarrow T(2008) = M(2008) + F(2008) = 290 + 470 = 760$

41. The slopes of the line segments for the period 2000-2004 are much steeper than the slopes of the corresponding line segments for the period 2004-2008. Thus, the number of associate's degrees increased more rapidly during the period 2000-2004.

42. If $2000 \le k \le 2008, T(k) = r,$ and $F(k) = s,$ then $M(k) = r - s$.

43. $(T-S)(2000) = T(2000) - S(2000) = 19 - 13 = 6$, This represents the billions of dollars spent for general science in 2000.

44. $(T-G)(2010) = T(2010) - G(2010) = 29 - 11 = 18$, This represents the billions of dollars spent for space and other technologies in 2010.

45. In space and other technologies spending was almost static in the years 1995-2000.

46. In space and other technologies spending increased the most during the years 2005-2010.

47. (a) $(f \circ g)(4) = f\left[g(4)\right]$, so from the graph find $g(4) = 0$. Now find $f(0) = -4$; therefore, $(f \circ g)(4) = -4$.

(b) $(g \circ f)(3) = g\left[f(3)\right]$, so from the graph find $f(3) = 2$. Now find $g(2) = 2$; therefore, $(g \circ f)(3) = 2$.

(c) $(f \circ f)(2) = f\left[f(2)\right]$, so from the graph find $f(2) = 0$. Now find $f(0) = -4$; therefore, $(f \circ f)(2) = -4$.

48. (a) $(f \circ g)(2) = f\left[g(2)\right]$, so from the graph find $g(2) = -2$. Now find $f(-2) = -4$; therefore, $(f \circ g)(2) = -4$.

(b) $(g \circ g)(0) = g\left[g(0)\right]$, so from the graph find $g(0) = 2$. Now find $g(2) = -2$; therefore, $(g \circ g)(0) = -2$.

(c) $(g \circ f)(4) = g\left[f(4)\right]$, so from the graph find $f(4) = 2$. Now find $g(2) = -2$; therefore, $(g \circ f)(4) = -2$.

49. (a) $(f \circ g)(1) = f\left[g(1)\right]$, so from the graph find $g(1) = 2$. Now find $f(2) = -3$; therefore, $(f \circ g)(1) = -3$.

(b) $(g \circ f)(-2) = g\left[f(-2)\right]$, so from the graph find $f(-2) = -3$. Now find $g(-3) = -2$; therefore, $(g \circ f)(-2) = -2$.

(c) $(g \circ g)(-2) = g\left[g(-2)\right]$, so from the graph find $g(-2) = -1$. Now find $g(-1) = 0$; therefore, $(g \circ g)(-1) = 0$.

50. (a) $(f \circ g)(-2) = f\left[g(-2)\right]$, so from the graph find $g(-2) = 4$. Now find $f(4) = 2$; therefore, $(f \circ g)(-2) = 2$.

(b) $(g \circ f)(1) = g\left[f(1)\right]$, so from the graph find $f(1) = 1$. Now find $g(1) = 1$; therefore, $(g \circ f)(1) = 1$.

(c) $(f \circ f)(0) = f[f(0)]$, so from the graph find $f(0) = 0$. Now find $f(0) = 0$; therefore, $(f \circ f)(0) = 0$.

51. (a) $(g \circ f)(1) = g[f(1)]$, so from the table find $f(1) = 4$. Now find $g(4) = 5$; therefore, $(g \circ f)(1) = 5$.

(b) $(f \circ g)(4) = f[g(4)]$, so from the table find $g(4) = 5$. $f(5)$ is undefined; therefore, $(f \circ g)(4)$ is undefined.

(c) $(f \circ f)(3) = f[f(3)]$, so from the table find $f(3) = 1$. Now find $f(1) = 4$; therefore, $(f \circ f)(3) = 4$.

52. (a) $(g \circ f)(1) = g[f(1)]$, so from the table find $f(1) = 2$. Now find $g(2) = 4$; therefore, $(g \circ f)(1) = 4$.

(b) $(f \circ g)(4) = f[g(4)]$, so from the table we find $g(4)$ is undefined; therefore, $(f \circ g)(4)$ is undefined.

(c) $(f \circ f)(3) = f[f(3)]$, so from the table find $f(3) = 6$. Now find $f(6) = 7$; therefore, $(f \circ f)(3) = 7$.

53. From the table, $g(3) = 4$ and $f(4) = 2$.

54. From the table, $f(6) = 7$ and $g(7) = 0$.

55. Since $Y_3 = Y_1 \circ Y_2$ and $X = -1$, we solve $Y_1[Y_2(-1)]$. First solve $Y_2 = (-1)^2 = 1$, now solve $Y_1 = 2(1) - 5 = -3$; therefore, $Y_3 = -3$.

56. Since $Y_3 = Y_1 \circ Y_2$ and $X = -2$, we solve $Y_1[Y_2(-2)]$. First solve $Y_2 = (-2)^2 = 4$, now solve $Y_1 = 2(4) - 5 = 3$; therefore, $Y_3 = 3$.

57. Since $Y_3 = Y_1 \circ Y_2$ and $X = 7$, we solve $Y_1[Y_2(7)]$. First solve $Y_2 = (7)^2 = 49$, now solve $Y_1 = 2(49) - 5 = 93$; therefore, $Y_3 = 93$.

58. Since $Y_3 = Y_1 \circ Y_2$ and $X = 8$, we solve $Y_1 = [Y_2(8)]$. First solve $Y_2 = (8)^2 = 64$, now solve $Y_1 = 2(64) - 5 = 123$; therefore, $Y_3 = 123$.

59. (a) $(f \circ g)(x) = f[g(x)] = (x^2 + 3x - 1)^3$; all values can be input for *x*; therefore, the domain is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = (x^3)^2 + 3(x^3) - 1 = x^6 + 3x^3 - 1$; all values can be input for *x*; therefore, the domain is $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = (x^3)^3 = x^9$; all values can be input for *x*; therefore, the domain is $(-\infty, \infty)$.

60. (a) $(f \circ g)(x) = f[g(x)] = 2 - \left(\frac{1}{x^2}\right) = 2 - \frac{1}{x^2}$; all values can be input for *x*, except 0; therefore, the domain is $(-\infty, 0) \cup (0, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = \frac{1}{(2-x)^2}$; all values can be input for *x*, except 2; therefore, the domain is $(-\infty, 2) \cup (2, \infty)$.

(c) $(f \circ f)(x) = f[f(x)] = 2 - (2 - x) = x$; all values can be input for *x*; therefore, the domain is $(-\infty, \infty)$.

61. (a) $(f \circ g)(x) = f[g(x)] = \left(\sqrt{1-x}\right)^2 = 1 - x$; all values can be input for *x*; therefore, the domain is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g\left[f(x)\right] = \left(\sqrt{1-(x^2)}\right) = \sqrt{1-x^2}$; only values where $x^2 \le 1$ can be input for x; therefore, the domain is $\left[-1,1\right]$.

(c) $(f \circ f)(x) = f\left[f(x)\right] = (x^2)^2 = x^4$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

62. (a) $(f \circ g)(x) = f\left[g(x)\right] = (x^4 + x^2 - 3x - 4) + 2 = x^4 + x^2 - 3x - 2$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

(b) $(g \circ f)(x) = g\left[f(x)\right] = (x+2)^4 + (x+2)^2 - 3(x+2) - 4$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

(c) $(f \circ f)(x) = f\left[f(x)\right] = (x+2) + 2 = x + 4$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

63. (a) $(f \circ g)(x) = f[g(x)] = \frac{1}{(5x)+1} = \frac{1}{5x+1}$; all values can be input for x, except $-\frac{1}{5}$; therefore, the domain is $\left(-\infty,-\frac{1}{5}\right) \cup \left(-\frac{1}{5},\infty\right)$.

(b) $(g \circ f)(x) = g[f(x)] = 5\left(\frac{1}{x+1}\right) = \frac{5}{x+1}$; all values can be input for x, except, -1; therefore, the domain is $(-\infty.-1) \cup (-1,\infty)$.

(c) $(f \circ f)(x) = f[f(x)] = \frac{1}{\left(\frac{1}{x+1}\right)+1} = \frac{1}{\frac{1}{x+1} + \frac{x+1}{x+1}} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$; all values can be input for x, except those that make $\frac{x+1}{x+2} = 0$ or undefined. That would be -1 and -2; therefore, the domain is $(-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$.

64. (a) $(f \circ g)(x) = f[g(x)] = (\sqrt{4-x^2}) + 4 = \sqrt{4-x^2} + 4$; only values where $x^2 \le 4$ can be input for x; therefore, the domain is $[-2, 2]$.

(b) $(g \circ f)(x) = g[f(x)] = \sqrt{4-(x+4)^2}$; only values where $(x+4)^2 \le 4$ can be input for x; therefore, the domain is $[-6, -2]$.

(c) $(f \circ f)(x) = f[f(x)] = (x+4) + 4 = x + 8$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

65. (a) $(f \circ g)(x) = f[g(x)] = 2(4x^3 - 5x^2) + 1 = 8x^3 - 10x^2 + 1$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

(b) $(g \circ f)(x) = g[f(x)] = 4(2x+1)^3 - 5(2x+1)^2 = 4(8x^3 + 12x^2 + 6x + 1) - 5(4x^2 + 4x + 1) =$ $32x^3 + 48x^2 + 24x + 4 - (20x^2 + 20x + 5) = 32x^3 + 28x^2 + 4x - 1$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

(c) $(f \circ f)(x) = f[f(x)] = 2(2x+1) + 1 = 4x + 3$; all values can be input for x; therefore, the domain is $(-\infty,\infty)$.

66. (a) $(f \circ g)(x) = f[g(x)] = \dfrac{(2x+3)-3}{2} = \dfrac{2x}{3} = x$; all values can be input for x; therefore, the domain is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g[f(x)] = 2\left(\dfrac{x-3}{2}\right) + 3 = (x-3)+3 = x$; all values can be input for x; therefore, the domain is $(-\infty, \infty)$.

(c) $(f \circ f)(x) f[f(x)] = \dfrac{\left(\dfrac{x-3}{2}\right)-3}{2} = \dfrac{\left(\dfrac{x-3}{2} - \dfrac{6}{2}\right)}{2} = \dfrac{\left(\dfrac{x-9}{2}\right)}{2} = \dfrac{x-9}{4}$; all values can be input for x; therefore, the domain is $(-\infty, \infty)$.

67. (a) $(f \circ g)(x) = f(g(x)) = f(5) = 5$, all values can be input for x, thefore the domain is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(5) = 5$, all values can be input for x, therefore the domain is $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(5) = 5$, all values can be input for x, therefore the domain is $(-\infty, \infty)$.

68. Let $f(x) = c$, then $(f \circ f)(x) = f(f(x)) = f(c) = c$. Therefore, there is only one value in the range of $(f \circ f)(x)$.

69. $(f \circ g)(x) = f[g(x)] = 4\left(\dfrac{1}{4}(x-2)\right) + 2 = x - 2 + 2 = x$, $(g \circ f)(x) = g[f(x)] = \dfrac{1}{4}((4x+2)-2) = \dfrac{1}{4}(4x) = x$

70. $(f \circ g)(x) = f[g(x)] = -3\left(-\dfrac{1}{3x}\right) = x$, $(g \circ f)(x) = g[f(x)] = -\dfrac{1}{3}(-3x) = x$

71. $(f \circ g)(x) = f\left[g(x)\right] = \sqrt[3]{5\left(\dfrac{1}{5}x^3 - \dfrac{4}{5}\right) + 4} = \sqrt[3]{(x^3-4)+4} = \sqrt[3]{x^3} = x$

$(g \circ f)(x) = g\left[f(x)\right] = \dfrac{1}{5}\left(\sqrt[3]{5x+4}\right)^3 - \dfrac{4}{5} = \dfrac{1}{5}(5x+4) - \dfrac{4}{5} = x + \dfrac{4}{5} - \dfrac{4}{5} = x$

72. $(f \circ g)(x) = f\left[g(x)\right] = \sqrt[3]{(x^3-1)+1} = \sqrt[3]{x^3} = x$, $(g \circ f)(x) = g\left[f(x)\right] = \left(\sqrt[3]{x+1}\right)^3 - 1 = x + 1 - 1 = x$

73. Graph $y_1 = \sqrt[3]{x-6}$, $y_2 = x^3 + 6$, and $y_3 = x$ in the same viewing window. See Figures 73. The graph of y_2 can be obtained by *reflecting* the graph of y_1 across the line $y_3 = x$.

74. Graph $y_1 = 5x - 3$, $y_2 = \dfrac{1}{5}(x+3)$, and $y_3 = x$ in the same viewing window. See Figures 74. The graph of y_2 can be obtained by *reflecting* the graph of y_1 across the line $y_3 = x$.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

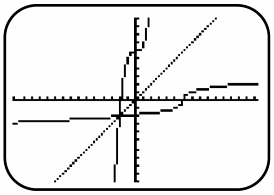

Figure 73

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

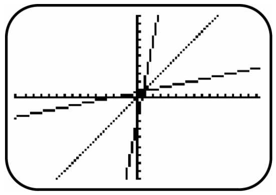

Figure 74

75. $f(x)=x^2-4 \Rightarrow f(x+h)=(x+h)^2-4=x^2+2xh+h^2-4;$
$f(x)+f(h)=\left(x^2-4\right)+\left(h^2-4\right)=x^2+h^2-8$

76. $f(x)=5x^2+x \Rightarrow f(x+h)=5(x+h)^2+(x+h)=5(x^2+2xh+h^2)+(x+h)=5x^2+10xh+5h^2+x+h;$
$f(x)+f(h)=\left(5x^2+x\right)+\left(5h^2+h\right)=5x^2+x+5h^2+h$

77. $f(x)=3x-x^2 \Rightarrow f(x+h)=3(x+h)-(x+h)^2=3x+3h-\left(x^2+2xh+h^2\right)=-x^2-2xh+3x-h^2+3h\,;$
$f(x)+f(h)=\left(3x-x^2\right)+\left(3h-h^2\right)=-x^2+3x-h^2+3h$

78. $f(x)=x^3 \Rightarrow f(x+h)=(x+h)^3=\left(x^2+2xh+h^2\right)\left(x+h\right)=x^3+3hx^2+3h^2x+h^3$
$f(x)+f(h)=x^3+h^3$

79. Using $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$ gives: $\dfrac{4\left(x+h\right)+3-\left(4x+3\right)}{h}=\dfrac{4x+4h+3-4x-3}{h}=\dfrac{4h}{h}=4.$

80. Using $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$ gives: $\dfrac{5\left(x+h\right)-6-\left(5x-6\right)}{h}=\dfrac{5x+5h-6-5x+6}{h}=\dfrac{5h}{h}=5.$

81. Using $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$ gives: $\dfrac{-6\left(x+h\right)^2-\left(x+h\right)+4-\left(-6x^2-x+4\right)}{h}$

$=\dfrac{-6\left(x^2+2xh+h^2\right)-x-h+4+6x^2+x-4}{h}=\dfrac{-6x^2-12xh-6h^2-x-h+4+6x^2+x-4}{h}$

$=\dfrac{-12xh-6h^2-h}{h}=-12x-6h-1.$

82. Using $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$ gives: $\dfrac{\frac{1}{2}\left(x+h\right)^2+4\left(x+h\right)-\left(\frac{1}{2}x^2+4x\right)}{h}$

$=\dfrac{\frac{1}{2}\left(x^2+2xh+h^2\right)+4x+4h-\frac{1}{2}x^2-4x}{h}=\dfrac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2+4x+4h-\frac{1}{2}x^2-4x}{h}$

$=\dfrac{xh+\frac{1}{2}h^2+4h}{h}=x+\dfrac{1}{2}h+4.$

83. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{(x+h)^3-x^3}{h}=\dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=\dfrac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2.$

84. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{-2(x+h)^3-(-2x^3)}{h}=\dfrac{-2(x^3+3x^2h+3xh^2+h^3)+2x^3}{h}=$

$\dfrac{-2x^3-6x^2h-6xh^2-2h^3+2x^3}{h}=\dfrac{-6x^2h-6xh^2-2h^3}{h}=-6x^2-6xh-2h^2.$

85. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{1-(x+h)^2-(1-x^2)}{h}=\dfrac{1-\left(x^2+2xh+h^2\right)-1+x^2}{h}=$

$\dfrac{1-x^2-2xh-h^2-1+x^2}{h}=\dfrac{-2xh-h^2}{h}=-2x-h$

86. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{(x+h)^2+2(x+h)-(x^2+2x)}{h}=\dfrac{\left(x^2+2xh+h^2\right)+2x+2h-x^2-2x}{h}=$

$\dfrac{2xh+h^2+2h}{h}=2x+h+2$

87. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{3(x+h)^2-(3x^2)}{h}=\dfrac{3\left(x^2+2xh+h^2\right)-3x^2}{h}=$

$\dfrac{3x^2+6xh+3h^2-3x^2}{h}=\dfrac{6xh+3h^2}{h}=6x+3h$

88. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}=\dfrac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\dfrac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=$

$\dfrac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

89. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}=\dfrac{\frac{2x-(2x+2h)}{2x(2x+2h)}}{h}=\dfrac{\frac{-2h}{2x(2x+2h)}}{h}=\dfrac{-1}{2x(x+h)}$

90. Using $\dfrac{f(x+h)-f(x)}{h}$ gives: $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\dfrac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}=\dfrac{\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}}{h}=\dfrac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h}=$

$\dfrac{\frac{h(-2x-h)}{x^2(x+h)^2}}{h}=\dfrac{-2x-h}{x^2(x+h)^2}$

91. One possible solution is $f(x)=x^2$ and $g(x)=6x-2.$ Then $(f\circ g)(x)=f[g(x)]=(6x-2)^2.$

92. One possible solution is $f(x)=x^2$ and $g(x)=11x^2+12x.$ Then $(f\circ g)(x)=f[g(x)]=(11x^2+12x)^2.$

93. One possible solution is $f(x)=\sqrt{x}$ and $g(x)=x^2-1.$ Then $(f\circ g)(x)=f[g(x)]=\sqrt{x^2-1}.$

94. One possible solution is $f(x)=x^3$ and $g(x)=2x-3.$ Then $(f\circ g)(x)=f[g(x)]=(2x-3)^3.$

95. One possible solution is $f(x)=\sqrt{x}+12$ and $g(x)=6x.$ Then $(f\circ g)(x)=f[g(x)]=\sqrt{6x}+12.$

96. One possible solution is $f(x)=\sqrt[3]{x}-4$ and $g(x)=2x+3.$ Then $(f\circ g)(x)=f[g(x)]=\sqrt[3]{2x+3}-4.$

97. (a) With a cost of \$10 to produce each item and a fixed cost of \$500, the cost function is $C(x) = 10x + 500$.

(b) With a selling price of \$35 for each item, the revenue function is $R(x) = 35x$.

(c) The profit function is $P(x) = R(x) - C(x) \Rightarrow P(x) = 35x - (10x + 500) \Rightarrow P(x) = 25x - 500$.

(d) A profit is shown when $P(x) > 0 \Rightarrow 25x - 500 > 0 \Rightarrow 25x > 500 \Rightarrow x > 20$. Therefore, 21 items must be produced and sold to realize a profit.

(e) Graph $y_1 = 25x - 500$. The smallest whole number for which $P(x) > 0$ is 21. Use a window of $[0, 30]$ by $[-1000, 500]$, for example.

98. (a) With a cost of \$11 to produce each item and a fixed cost of \$180, the cost function is $C(x) = 11x + 180$.

(b) With a selling price of \$20 for each item, the revenue function is $R(x) = 20x$.

(c) The profit function is $P(x) = R(x) - C(x) \Rightarrow P(x) = 20x - (11x + 180) \Rightarrow P(x) = 9x - 180$.

(d) A profit is shown when $P(x) > 0 \Rightarrow 9x - 180 > 0 \Rightarrow 9x > 180 \Rightarrow x > 20$. Therefore, 21 items must be produced and sold to realize a profit.

(e) Graph $y_1 = 9x - 180$. The smallest whole number for which $P(x) > 0$ is 21. Use a window of $[-5, 30]$ by $[-200, 200]$, for example.

99. (a) With a cost of \$100 to produce each item and a fixed cost of \$2700, the cost function is $C(x) = 100x + 2700$.

(b) With a selling price of \$280 for each item, the revenue function is $R(x) = 280x$.

(c) The profit function is $P(x) = R(x) - C(x) \Rightarrow P(x) = 280x - (100x + 2700) \Rightarrow P(x) = 180x - 2700$.

(d) A profit is shown when $P(x) > 0 \Rightarrow 180x - 2700 > 0 \Rightarrow 180x > 2700 \Rightarrow x > 15$. Therefore, 16 items must be produced and sold to realize a profit.

(e) Graph $y_1 = 180x - 2700$, the smallest whole number for which $P(x) > 0$ is 16. Use a window of $[0, 30]$ by $[-3000, 500]$, for example.

100. (a) With a cost of \$200 to produce each item and a fixed cost of \$1000, the cost function is $C(x) = 200x + 1000$.

(b) With a selling price of \$240 for each item, the revenue function is $R(x) = 240x$.

(c) The profit function is $P(x) = R(x) - C(x) \Rightarrow P(x) = 240x - (200x + 1000) \Rightarrow P(x) = 40x - 1000$.

(d) A profit is shown when $P(x) > 0 \Rightarrow 40x - 1000 > 0 \Rightarrow 40x > 1000 \Rightarrow x > 25$. Therefore, 26 items must be produced and sold to realize a profit.

(e) Graph $y_1 = 40x - 1000$, the smallest whole number for which $P(x) > 0$ is 26. Use a window of $[-5, 40]$ by $[-1200, 600]$, for example.

101. (a) If $V(r) = \frac{4}{3}\pi r^3$, then a 3 inch increase would be $V(r) = \frac{4}{3}\pi (r+3)^3$, and the volume gained would be $V(r) = \frac{4}{3}\pi (r+3)^3 - \frac{4}{3}\pi r^3$.

(b) Graph $y_1 = \frac{4}{3}\pi (x+3)^3 - \frac{4}{3}\pi x^3$ in the window $[0, 10]$ by $[0, 1500]$. See Figure 101. Although this appears to be a portion of a parabola, it is actually a cubic function.

(c) From the graph in exercise 91b, an input value of $x = 4$ results in a gain of $y \approx 1168.67$.

(d) $V(4)=\frac{4}{3}\pi(4+3)^3-\frac{4}{3}\pi(4)^3=\frac{4}{3}\pi(343)-\frac{4}{3}\pi(64)=\frac{1372}{3}\pi-\frac{256}{3}\pi=\frac{1116}{3}\pi=372\pi\approx 1168.67.$

102. If $S(r)=4\pi r^2$, then doubling the radius would give us a surface area gained function of $S(r)=4\pi(2r)^2-4\pi r^2=16\pi r^2-4\pi r^2=12\pi r^2$.

103. (a) If $x=$ width, then $2x=$ length. Since the perimeter formula is $P=2W+2L$ our perimeter function is $P(x)=2(x)+2(2x)=2x+4x\Rightarrow P(x)=6x$. This is a linear function.

(b) Graph $P(x)=6x$ in the window $[0,10]$ by $[1,100]$. See Figure 103b. From the graph when $x=4,\ y=24$. The 4 represents the width of a rectangle and 24 represents the perimeter.

(c) If $x=4$ is the width of a rectangle then $2x=8$ is the length. See Figure 103c. Using the standard perimeter formula yields $P=2(4)+2(8)=24$. This compares favorably with the graph result in part b.

(d) (Answers may vary.) If the perimeter *y* of a rectangle satisfying the given conditions is 36, then the width *x* is 6. See Figure 103d.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

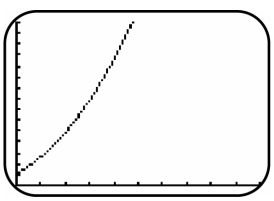

Figure 101

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

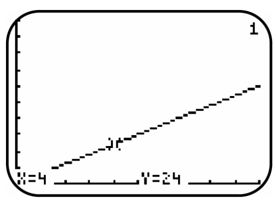

Figure 103b

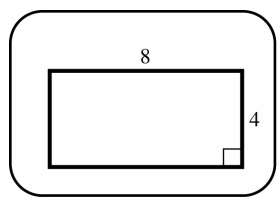

Figure 103c

[-10,10] by [-10,10]
Xscl = 1 Yscl= 1

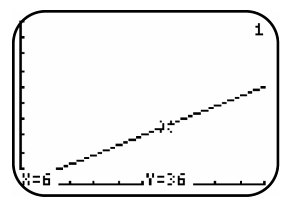

Figure 103d

104. (a) If $x=4s$ then $s=\frac{x}{4}$.

(b) If $y=s^2$ and $s=\frac{x}{4}$, then $y(x)=\left(\frac{x}{4}\right)^2\Rightarrow y(x)=\frac{x^2}{16}$.

(c) Sine *x* is the perimeter and $x=6,\ y(6)=\frac{(6)^2}{16}=\frac{36}{16}=\frac{9}{4}=2.25$.

(d) Show that the point $(6,2.25)$ is on the graph $y=\frac{x^2}{16}$. A square with perimeter 6 will have area 2.25 square u nits.

105. (a) $A(2x)=\frac{\sqrt{3}}{4}(2x)^2=\frac{\sqrt{3}}{4}(4x^2)\Rightarrow A(2x)=\sqrt{3}x^2$

(b) $A(x)=\frac{\sqrt{3}}{4}(16)^2=\frac{\sqrt{3}}{4}(256)\Rightarrow A(x)=64\sqrt{3}$ square units.

(c) On the graph of $y=\frac{\sqrt{3}}{4}x^2$, locate the point where $x=16$ to find $y\approx 110.85$, an approximation for $64\sqrt{3}$.

106. (a) If $A(r)=\pi r^2$ and $r(t)=2t$ then $(A\circ r)(t)=A[r(t)]=A[2t]=\pi(2t)^2=4\pi t^2$.

(b) $(A \circ r)(t)$ is a composite function that expresses the area of the circular region covered by the pollutants as a function of time *t* (in hours).

(c) Since $t = 0$ is 8 A.M., noon would be $t = 4$. $(A \circ r)(4) = 4\pi(4)^2 = 64\pi \text{ mi}^2$.

(d) Graph $y_1 = 4\pi x^2$ and show that for $x = 4$, $y \approx 201$ (an approximation for 64π).

107. (a) $A(2100) = 42,$; The average age of a person in 2100 is projected to be 42 years. $T(2100) = 430$, In 2100, the living world's population will have a combined life experience of 430 billion years.

(b) $\dfrac{T(2100)}{A(2100)} = \dfrac{430}{42} \approx 10.2$, The world population will be about 10.2 billion in 2100.

(c) *P(x)* gives the world's population during year *x*.

108. (a) The domain is the years. See table below. Therefore, D= { 2009, 2010, 2011, 2012 }.

x	2009	2010	2011	2012
h(x)	77.5	78.1	75.3	76.1

(b) The function *h* computes the number of acres of soybeans harvested in the U.S. in millions of acres.

109. (a) $(f + g)(2010) = 13.0 + 74.3 = 87.3$

(b) The function $(f + g)(x)$ computes the total SO_2 and Carbon Monoxide during year *x*.

(c) Add functions *f* and *g*.

x	1970	1980	1990	2000	2010
(f+g)(x)	235.2	211.3	177.3	130.8	87.3

110. (a) The function *h* is the addition of functions *f* and *g*.

x	1990	2000	2010	2020	2030
h(x)	32	35.5	39	42.5	46

(b) The function *h* is the addition of functions *f* and *g*. Therefore, $h(x) = f(x) + g(x)$.

111. (a) The function *h* is the subtraction of function *f* from *g*. Therefore, $h(x) = g(x) - f(x)$.

(b) $h(1996) = g(1996) - f(1996) = 841 - 694 = 147$, $h(2006) = g(2006) = f(2006) = 1165 - 1012 = 153$

(c) Using the points (1996, 147) and (2006, 153) from part b, the slope is $m = \dfrac{153 - 147}{2006 - 1996} = \dfrac{6}{10} = .6$. Now using point slope form: $y - 147 = .6(x - 1996) \Rightarrow y = .6(x - 1996) + 147$.

112. (a) Graph $h(x) = \dfrac{1900(x - 1982)^2 + 619}{3200(x - 1982)^2 + 1586}$, in the window [1982, 1994] by [0, 1] See Figure 112a.

Approximately 59% of the people who contracted AIDS during this time period died.

(b) Divide number of deaths by number of cases for each year. The results compare favorably with the graph. See Figure 112b.

[-10,10] by [-10,10]

Xscl = 1 Yscl = 1

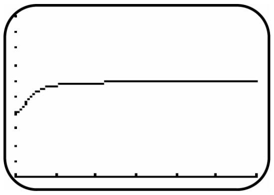

Figure 112a

Year	1982	1984	1986	1988	1990	1992	1994
Ratio	.39	.51	.59	.58	.61	.60	.61

Figure 112b

Reviewing Basic Concepts (Sections 2.4—2.6)

1. (a) $\left|\frac{1}{2}x+2\right|=4 \Rightarrow \frac{1}{2}x+2=4 \Rightarrow \frac{1}{2}x=2 \Rightarrow x=4$ or $\frac{1}{2}x+2=-4 \Rightarrow \frac{1}{2}x=-6 \Rightarrow x=-12.$

   Therefore, the solution set is $\{-12, 4\}$.

   (b) $\left|\frac{1}{2}x+2\right|>4 \Rightarrow \frac{1}{2}x+2>4 \Rightarrow \frac{1}{2}x>2 \Rightarrow x>4$ or $\frac{1}{2}x+2<-4 \Rightarrow \frac{1}{2}x<-6 \Rightarrow x<-12.$ Therefore, the solution interval is $(-\infty,-12)\cup(4,\infty)$.

   (c) $\left|\frac{1}{2}x+2\right|\le 4 \Rightarrow -4\le \frac{1}{2}x+2\le 4 \Rightarrow -6\le \frac{1}{2}x\le 2 \Rightarrow -12\le x\le 4.$

   Therefore, the solution interval is [-12,4].

2. For the graph of $y=|f(x)|$, we reflect the graph of $y=f(x)$ across the *x*-axis for all points for which $y<0$. Where $y\ge 0$, the graph remains unchanged. See Figure 2.

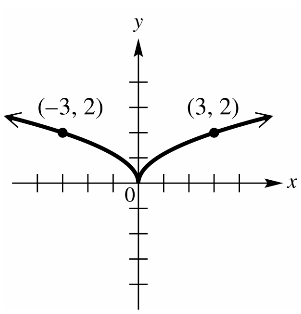

Figure 2

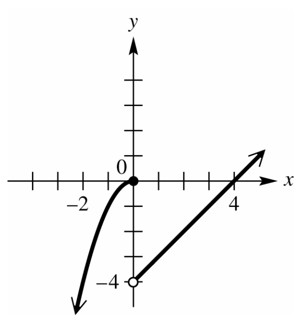

Figure 5a

3. $|2x+4|=|1-3x| \Rightarrow 2x+4=1-3x \Rightarrow 5x=-3 \Rightarrow x=-\frac{3}{5}$ or $2x+4=-(1-3x) \Rightarrow 2x+4=3x-1 \Rightarrow 5=x$

   The solution set is $\left\{-\frac{3}{5}, 5\right\}$.

4. (a) $f(-3)=2(-3)+3=-3$ (b) $f(0)=(0)^2+4=4$ (c) $f(2)=(2)^2+4=8$

5. (a) See Figure 5a.

   (b) Graph $y_1=(-x^2)*(x\le 0)+(x-4)*(x>0)$ in the window [-10,10] by [-10,10]. See Figure 5b.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

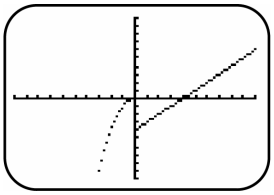

Figure 5b

6. (a) $(f+g)(x)=(-3x-4)+(x^2)=x^2-3x-4.$ Therefore, $(f+g)(1)=(1)^2-3(1)-4=-6.$

   (b) $(f-g)(x)=(-3x-4)-(x^2)=-x^2-3x-4.$ Therefore, $(f-g)(3)=-(3)^2-3(3)-4=-22.$

   (c) $(fg)(x)=(-3x-4)(x^2)=-3x^3-4x^2.$ Therefore, $(fg)(1)=-3(-2)^3-4(-2)^2=24-16=8.$

   (d) $\left(\frac{f}{g}\right)(x)=\frac{-3x-4}{x^2}.$ Therefore, $\left(\frac{f}{g}\right)(-3)=\frac{-3(-3)-4}{(-3)^2}=\frac{5}{9}.$

   (e) $(f\circ g)(x)=f[g(x)]=-3(x)^2-4\Rightarrow(f\circ g)(x)=-3x^2-4$

   (f) $(g\circ f)(x)=g[f(x)]=(-3x-4)^2\Rightarrow(g\circ f)(x)=9x^2+24x+16$

7. One of many possible solutions for $(f\circ g)(x)=h(x)$ is $f(x)=x^4$ and $g(x)=x+2.$ Then $(f\circ g)(x)=f[g(x)]=(x+2)^4.$

8. $\frac{-2(x+h)^2+3(x+h)-5-(-2x^2+3x-5)}{h}=\frac{-2(x^2+2xh+h^2)+3x+3h-5+2x^2-3x+5}{h}=$

$\frac{-2x^2-4xh-2h^2+3x+3h-5+2x^2-3x+5}{h}=\frac{-4xh-2h^2+3h}{h}=-4x-2h+3.$

9. (a) At 4% simple interest the equation for interest earned is $y_1=.04x.$

   (b) If he invested *x* dollars in the first account, then he invested *x*+500 in the second account. The equation for the amount of interest earned on this account is $y_2=.025(x+500)\Rightarrow y_2=.025x+12.5.$

   (c) It represents the total interest earned in both accounts for 1 year.

   (d) Graph $y_1+y_2=.04x+(.025x+12.5)\Rightarrow y_1+y_2=.04x+.025x+12.5$ in the window [0,1000] by [0,100]. See Figure 9. An input value of $x=250,$ results in \$28.75 earned interest.

   (e) At $x=250, y_1+y_2=.04(250)+.025(250)+12.5=10+6.25+12.5=\$28.75.$

[0,1000] by [-20,100]
Xscl = 100 Yscl = 10

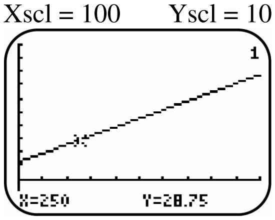

Figure 9

10. If the radius is r, then the height is $2r$ and the equation is

$$S = \pi r\sqrt{r^2 + (2r)^2} = \pi r\sqrt{r^2 + 4r^2} = \pi r\sqrt{5r^2} \Rightarrow S = \pi r^2\sqrt{5}.$$

Chapter 2 Review Exercises

The graphs for exercises 1–10 can be found in the "Function Capsule" boxes located in section 2.1 in the text.

1. True Both $f(x) = x^2$ and $f(x) = |x|$ have the interval: $[0, \infty)$ as the range.
2. True Both $f(x) = x^2$ and $f(x) = |x|$ increase on the interval: $[0, \infty)$.
3. False The function $f(x) = \sqrt{x}$ has the domain: $[0, \infty)$ and $f(x) = \sqrt[3]{x}$ the domain: $(-\infty, \infty)$.
4. False The function $f(x) = \sqrt[3]{x}$ increases on its entire domain.
5. True The function $f(x) = x$ has a domain and range of: $(-\infty, \infty)$
6. False The function $f(x) = \sqrt{x}$ is not defined on $(-\infty, 0)$, so certainly cannot be continuous.
7. True All of the functions show increases on the interval: $[0, \infty)$
8. True Both $f(x) = x$ and $f(x) = x^3$ have graphs that are symmetric with respect to the origin.
9. True Both $f(x) = x^2$ and $f(x) = |x|$ have graphs that are symmetric with respect to the y-axis.
10. True No graphs are symmetric with respect to the x-axis.
11. Only values where $x \geq 0$ can be input for x, therefore the domain of $f(x) = \sqrt{x}$ is: $[0, \infty)$
12. Only positive solution are possible in absolute value functions, therefore the range of $f(x) = \sqrt{x}$ is: $[0, \infty)$
13. All solution are possible in cube root functions, therefore the range of $f(x) = \sqrt[3]{x}$ is: $(-\infty, \infty)$.
14. All values can be input for x, therefore the domain of $f(x) = x^2$ is: $(-\infty, \infty)$.
15. The function $f(x) = \sqrt[3]{x}$ increases for all inputs for x, therefore the interval is: $(-\infty, \infty)$.
16. The function $f(x) = |x|$ increases for all inputs where $x \geq 0$, therefore the interval is: $[0, \infty)$
17. The equation is the equation $y = \sqrt{x}$. Only values where $x \geq 0$ can be input for x, therefore the domain of $y = \sqrt{x}$ is: $[0, \infty)$
18. The equation $y^2 = x$ is the equation $y = \sqrt{x}$ Square root functions have both positive and negative solutions and all solution are possible, therefore the range of $y = \sqrt{x}$ is: $(-\infty, \infty)$.
19. The graph of $f(x) = (x+3) - 1$ is the graph $y = x$ shifted 3 units to the left and 1 unit downward. See Figure 19.
20. The graph of $f(x) = -\frac{1}{2}x + 1$ is the graph reflected $y = x$ across the x-axis, vertically shrunk by a factor of $\frac{1}{2}$, and shifted 1 unit upward. See Figure 20.
21. The graph of $f(x) = (x+1)^2 - 2$ is the graph $y = x^2$ shifted 1 unit to the left and 2 units downward. See Figure 21.

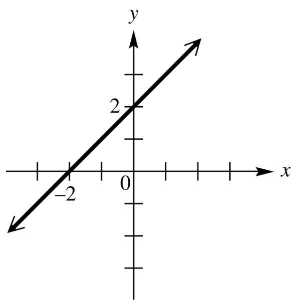

Figure 19

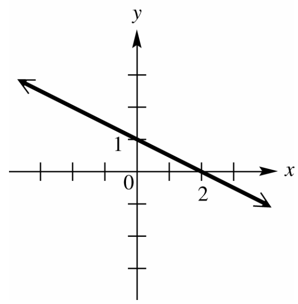

Figure 20

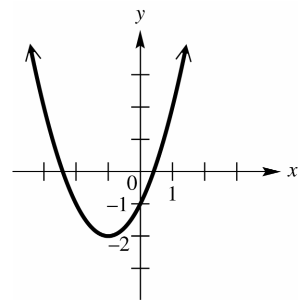

Figure 21

22. The graph of $f(x) = -2x^2 + 3$ is the graph $y = x^2$ reflected across the *x*-axis, vertically stretched by a factor of , and shifted 3 units upward. See Figure 22.

23. The graph of $f(x) = -x^3 + 2$ is the graph $y = x^3$ reflected across the *x*-axis and shifted 2 units upward. See Figure 23.

24. The graph of $f(x) = (x-3)^3$ is the graph $y = x^3$ shifted 3 units to the right. See Figure 24.

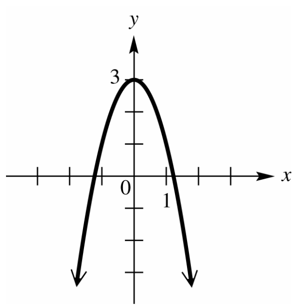

Figure 22

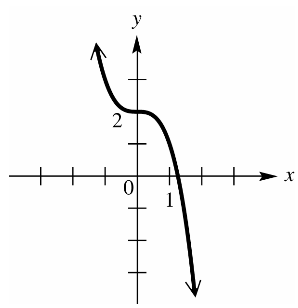

Figure 23

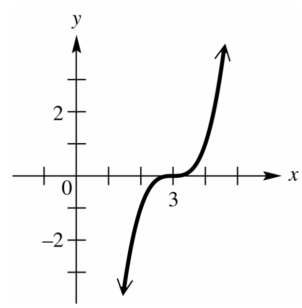

Figure 24

25. The graph of $f(x) = \sqrt{\frac{1}{2}x}$ is the graph $y = \sqrt{x}$ horizontally stretched by a factor of 2. See Figure 25.

26. The graph of $f(x) = \sqrt{x-2} + 1$ is the graph $y = \sqrt{x}$ shifted 2 units to the right and 1 unit upward. See Figure 26.

27. The graph of $f(x) = 2\sqrt[3]{x}$ is the graph $y = \sqrt[3]{x}$ vertically stretched by a factor of . See Figure 27.

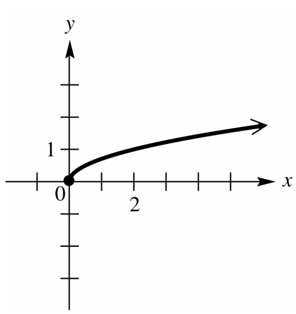

Figure 25

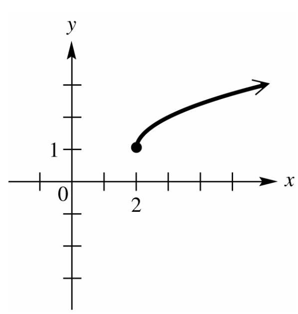

Figure 26

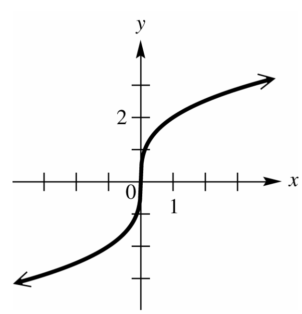

Figure 27

28. The graph of $f(x) = \sqrt[3]{x} - 2$ is the graph $y = \sqrt[3]{x}$ shifted 2 units downward. See Figure 28.

29. The graph of $f(x)=|x-2|+1$ is the graph $y=|x|$ shifted 2 units right and 1 unit upward. See Figure 29.

30. The graph of $f(x)=|-2x+3|$ is the graph $y=|x|$ horizontally shrunk by a factor of $\frac{1}{2}$, shifted $\left(\frac{1}{2}\right)(3)$ or $\frac{3}{2}$ units to the left, and reflected across the y-axis. See Figure 30.

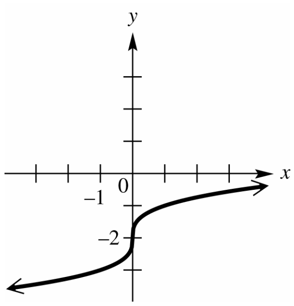

Figure 28

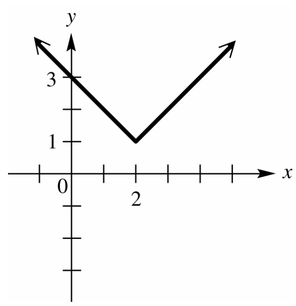

Figure 29

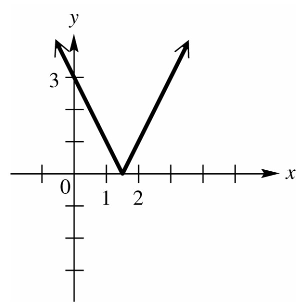

Figure 30

31. (a) From the graph, the function is continuous for the intervals: $(-\infty,-2),[-2,1]$ and $(1,\infty)$

(b) From the graph, the function is increasing for the interval: $[-2,1]$

(c) From the graph, the function is decreasing for the interval: $(-\infty,-2)$

(d) From the graph, the function is constant for the interval: $(1,\infty)$

(e) From the graph, all values can be input for x, therefore the domain is: $(-\infty,\infty)$

(f) From the graph, the possible values of y or the range is: $\{-2\}\cup[-1,1]\cup(2,\infty)$

32. $x=y^2-4\Rightarrow y^2=x+4\Rightarrow y=\sqrt{x+4}$ and $y=-\sqrt{x+4}$.

33. From the graph, the relation is symmetric with respect to the x-axis, y-axis, and origin. The relation is not a function since some inputs x have two outputs y.

34. If $F(x)=x^3-6$,then $F(-x)=(-x)^3-6\Rightarrow F(-x)=-x^{-3}-6$ and $-F(x)=-(x^3-6)\Rightarrow -F(x)=-x^3+6$.

Since $F(x)\neq F(-x)\neq -F(x)$ the function has no symmetry and is neither an ever nor an odd function.

35. If $f(x)=|x|+4$,then $f(-x)=|(-x)|+4\Rightarrow f(-x)=|x|+4$ and $-f(x)=-|x|-4$ Since $f(-x)=f(x)$ the function is symmetric with respect to the y-axis and is an even function.

36. If $f(x)=\sqrt{x-5}$ then $f(-x)=\sqrt{(-x)-5}$ and $-f(x)=-\sqrt{x-5}$ .Since $f(x)\neq f(-x)\neq -f(x)$, the function has no symmetry and is neither an even nor an odd function.

37. If $y^2=x-5$ then $y=\pm\sqrt{x-5}$. Since $f(x)=-\sqrt{x-5}$ is the reflection of $f(x)=-\sqrt{x-5}$ across the x-axis, the relation has symmetry with respect to the x-axis. Also, one x inputs can produce two y outputs the relation is not a function.

38. If $f(x)=3x^4+2x^2+1$ then $f(-x)=3(-x)^4+2(-x)^2+1 \Rightarrow f(-x)=3x^4+2x^2+1$ and $-f(x)=-3x^4-2x^2-1$. Since $f(-x)=f(x)$ the function is symmetric with respect to the *y*-axis and is an even function.

39. True, a graph that is symmetrical with respect to the *x*-axis means that for every (x, y) there is also $(x,-y)$.

40. True, since an even function and one that is symmetric with respect to the *y*-axis both contain the points (x, y) and $(-x, y)$.

41. True, since an odd function and one that is symmetric with respect to the origin both contain the points (x, y) and $(-x,-y)$.

42. False, for an even function, if (a,b) is on the graph, then $(-a,b)$ is on the graph and not $(a,-b)$. For example, $f(x)=x^2$ is even, and (2, 4) is on the graph, but (2, −4) is not.

43. False, for an odd function, if (a,b) is on the graph, then $(-a,-b)$ is on the graph and not $(-a,b)$ For example, $f(x)=x^3$ is odd, and $(2,8)$ is on the graph, but $(-2,8)$ is not.

44. True, if $(x,0)$ is on the graph of $f(x)=0$ then $(-x,0)$ is on the graph.

45. The graph of $y=-3(x+4)^2-8$ is the graph of $y=x^2$ shifted 4 units to the left, vertically stretched by a factor of 3, reflected across the *x*-axis, and shifted 8 units downward.

46. The equation $y=\sqrt{x}$ reflected across the *y*-axis is: $f(x)=\sqrt{-x}$ then reflected across the *x*-axis is: $y=-\sqrt{-x}$ now vertically shrunk by a factor of $\frac{2}{3}$ is: $y=-\frac{2}{3}\sqrt{-x}$, and finally shifted 4 units upward is: $y=-\frac{2}{3}\sqrt{-x}+4$.

47. Shift the function *f* upward 3 units. See Figure 47.

48. Shift the function *f* to the right 2 units. See Figure 48.

49. Shift the function *f* to the left 3 units and downward 2 units. See Figure 49.

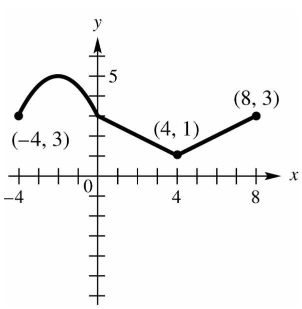

Figure 47

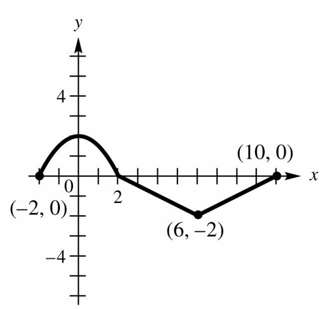

Figure 48

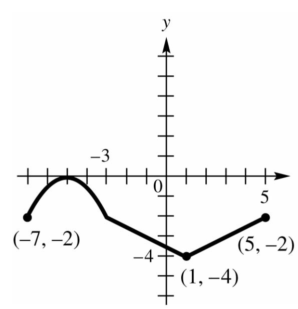

Figure 49

50. For values where $f(x) > 0$ the graph remains the same. For values where $f(x) < 0$ reflect the graph across the x-axis. See Figure 50.

51. Horizontally shrink the function f by a factor of $\frac{1}{4}$. See Figure 51.

52. Horizontally stretch the function f by a factor of 2. See Figure 52.

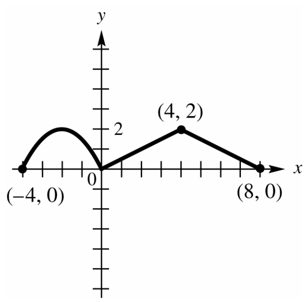

Figure 50

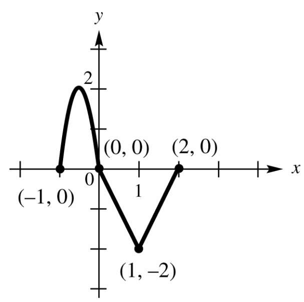

Figure 51

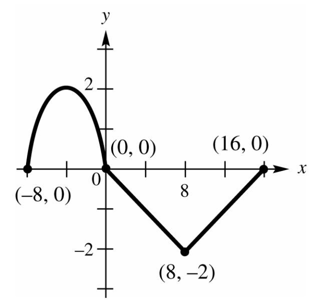

Figure 52

53. The function is shifted upward 4 units, therefore the domain remains the same: $[-3,4]$ and the range is increased by 4 and is: $[2,9]$.

54. The function is shifted left 10 units, therefore the domain is decreased by 10 and is $[-13,-6]$; and the function is stretched vertically by a factor of 5, therefore the range is multiplied by 5 and is: $[-10,25]$.

55. The function is horizontally shrunk by a factor of $\frac{1}{2}$, therefore the domain is divided by 2 and is: $\left[-\frac{3}{2},2\right]$; and the function is reflected across the x-axis, therefore the range is opposites of the original and is: $[-5,2]$.

56. The function is shifted right 1 unit, therefore the domain is increased by 1 and is: $[-2,5]$; and the function is also shifted upward 3 units, therefore the range is increased by 3 and is: $[1,8]$.

57. We reflect the graph of $y = f(x)$ across the x-axis for all points for which $y < 0$. Where $y \geq 0$, the graph remains unchanged. See Figure 57.

58. We reflect the graph of $y = f(x)$ across the x-axis for all points for which < 0. Where $y \geq 0$, the graph remains unchanged. See Figure 58.

59. Since the range is $\{2\}, y \geq 0$, so the graph remains unchanged.

60. Since the range is $\{-2\}, y < 0$, so we reflect the graph across the x-axis. See Figure 60.

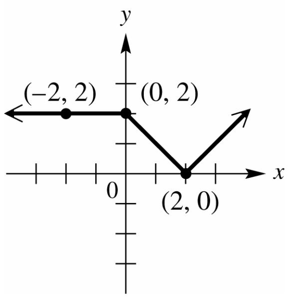

Figure 57

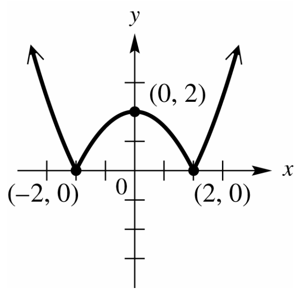

Figure 58

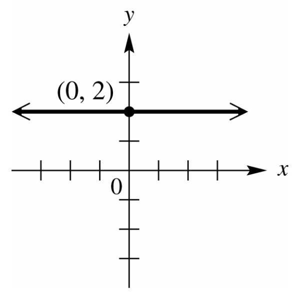

Figure 60

61. $|4x+3|=12 \Rightarrow 4x+3=12 \Rightarrow 4x=9 \Rightarrow x=\frac{9}{4}$ or $4x+3=-12 \Rightarrow 4x=-15 \Rightarrow x=-\frac{15}{4}$, therefore the solution set is: $\left\{-\frac{15}{4},\frac{9}{4}\right\}$.

62. $|-2x-6|+4=1 \Rightarrow |-2x-6|=-3$. Since an absolute value equation can not have a solution less than zero, the solution set is: $\varnothing$

63. $|5x+3|=|x+11| \Rightarrow 5x+3=x+11 \Rightarrow 4x=8 \Rightarrow x=2$ or $5x+3=-(x+11) \Rightarrow 6x=-14 \Rightarrow x=-\frac{14}{6}=-\frac{7}{3}$, therefore the solution set is: $\left\{-\frac{7}{3},2\right\}$.

64. $|2x+5|=7 \Rightarrow 2x+5=7 \Rightarrow 2x=2 \Rightarrow x=1$ or $2x+5=-7 \Rightarrow 2x=-12 \Rightarrow x=-6$, therefore the solution set is: $\{-6,1\}$.

65. $|2x+5|\le 7 \Rightarrow -7\le 2x+5\le 7 \Rightarrow -12\le 2x\le 2 \Rightarrow -6\le x\le 1$, therefore the interval is: $\{-6,1\}$.

66. $|2x+5|\ge 7 \Rightarrow 2x+5\ge 7 \Rightarrow 2x\ge 2 \Rightarrow x\ge 1$ or $2x+5\le -7 \Rightarrow 2x\le -12 \Rightarrow x\le -6$, therefore the solution is the interval: $(-\infty,-6]\cup[1,\infty)$.

67. $|5x-12|0 \Rightarrow 5x-12>0 \Rightarrow 5x>12 \Rightarrow x>\frac{12}{5}$ or $5x-12<0 \Rightarrow 5x=12 \Rightarrow x<\frac{12}{5}$, therefore the solution is the interval: $\left(-\infty,\frac{12}{5}\right)\cup\left(\frac{12}{5},\infty\right)$ or $\left\{x \mid x\ne\frac{12}{5}\right\}$.

68. Since an absolute value equation can not have a solution less than zero, the solution set is: $\varnothing$

69. $2|3x-1|+1=21 \Rightarrow 2|3x-1|=20 \Rightarrow |3x-1|=10 \Rightarrow 3x-1=10 \Rightarrow 3x=11 \Rightarrow x=\frac{11}{3}$ or $3x-1=-10 \Rightarrow 3x=-9 \Rightarrow x=-3$, therefore the solution set is: $\left\{-3,\frac{11}{3}\right\}$.

70. $|2x+1|=|-3x+1| \Rightarrow 2x+1=-3x+1 \Rightarrow 5x=0 \Rightarrow x=0$ or $2x+1=-(-3x+1) \Rightarrow -x=-2 \Rightarrow x=2$, therefore the solution set is: $\{0,2\}$.

71. The x-coordinates of the points of intersection of the graphs are -6 and 1.Thus, $\{-6,1\}$ is the solution set of $y_1 = y_2$ The graph of y_1 lies on or below the graph of y_2 between -6 and 1, so the solution set of $y_1 \le y_2$ is $[-6,1]$ The graph of y_1 lies above the graph of y_2 everywhere else, so the solution set of $y_1 \ge y_2$ is $(-\infty,-6]\cup[1,\infty)$.

72. Graph $y_1 = |x+1|+|x-3|$ and $\{-3,-1\}$ See Figure 72. The intersections are $x=-3$ and $x=5,$ therefore the solution set is: $\{-3,5\}$. **Check:** $|(-3)+1|+|(-3)-3| = 8 \Rightarrow |-2|+|-6| = 8 \Rightarrow 2+6=8 \Rightarrow 8=8$ and $|(5)+1|+|(5)-3| = 8 \Rightarrow |6|+|2| = 8 \Rightarrow 6+2=8 \Rightarrow 8=8$

[-10,10] by [-4,16]

Xscl = 1 Yscl = 1

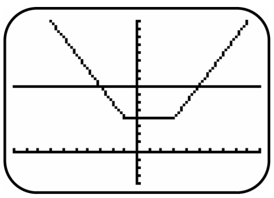

Figure 72

73. Initially, the car is at home. After traveling 30 mph for 1 hr, the car is 30 mi away from home. During the second hour the car travels 20 mph until it is 50 mi away. During the third hour the car travels toward home at 30 mph until it is 20 mi away. During the fourth hour the car travels away from home at 40 mph until it is 60 mi away from home. During the last hour, the car travels 60 mi at 60 mph until it arrives home.

74. See Figure 74

75. See Figure 75

76. See Figure 76

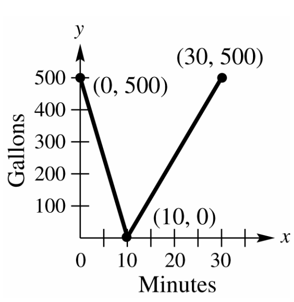

Figure 74

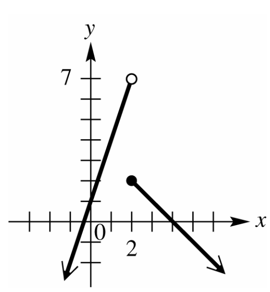

Figure 75

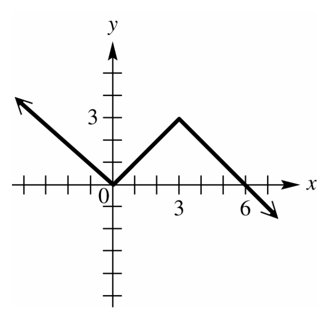

Figure 76

77. Graph $y_1 = (3x+1)*(x<2)+(-x+4)*(x\ge 2)$ in the window $[-10,10]$ by

$|4x+8| > 4 \Rightarrow 4x+8>4 \Rightarrow 4x>-4 \Rightarrow -1$ See Figure 77.

78. See Figure 78.

[-10,10] by [-10,10]
Xscl = 1 Yscl = 1

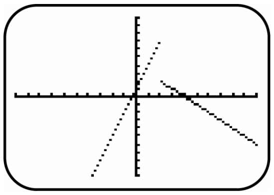

Figure 77

[-5,5] by [-5,5]
Xscl = 1 Yscl = 1

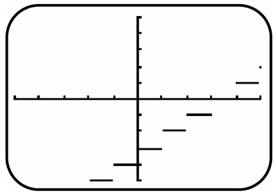

Figure 78

79. From the graphs $(f+g)(1)=2+3=5$

80. From the graphs $(f-g)(0)=1-4=-3$

81. From the graphs $(fg)(-1)=(0)(3)=0$

82. From the graphs $\left(\frac{f}{g}\right)(2)=\frac{3}{2}$

83. From the graphs $(f\circ g)(2)=f[g(2)]=f(2)=3$

84. From the graphs $(g\circ f)(2)=g[f(2)]=g(3)=2$

85. From the graphs $(g\circ f)(-4)=g[f(-4)]=g(2)=2$

86. From the graphs $(f\circ g)(-2)=f[g(-2)]=f(2)=3$

87. From the table $(f+g)(1)=7+1=8$

88. From the table $(f-g)(3)=9-1=0$

89. From the table $(fg)(-1)=(3)(-2)=-6$

90. From the table $\left(\frac{f}{g}\right)(0)=\frac{5}{0}$, which is undefined.

91. From the tables $(g\circ f)(-2)=g[f(-2)]=g(1)=2$

92. From the graphs $(f\circ g)(3)=f[g(3)]=f(-2)=1$

93. $\frac{2(x+h)+9-(2x+9)}{h}=\frac{2x+2h+9-2x-9}{h}=\frac{2h}{h}=2$

94. $\frac{(x+h)^2-5(x+h)+3-(x^2-5x+3)}{h}=\frac{x^2+2xh+h^2-5x-5h+3-x^2+5x-3}{h}=\frac{2xh+h^2-5h}{h}=2x+h-5$

95. One of many possible solutions for $(f\circ g)(x)=h(x)$ is: $f(x)=x^2$ and $g(x)=x^3-3x$ Then $(f\circ g)(x)=\left(x^3-3x\right)^2$.

96. One of many possible solutions for $(f\circ g)(x)=h(x)$ is $f(x)=\frac{1}{x}$ and $g(x)=x-5$ Then

$$(f\circ g)(x)=f[g(x)]=\frac{1}{x-5}$$

97. If $V(r)=\frac{4}{3}\pi r^3$, then a 4 inch increase would be: $V(r)=\frac{4}{3}\pi (r+4)^3$, and the volume gained would be: $V(r)=\frac{4}{3}\pi (r+4)^3-\frac{4}{3}\pi r^3$.

98. (a) Since $h=d, r=\frac{d}{2}$ and the formula for the volume of a can is: $V=\pi r^2 h$, the function is:

$$V(d)=\pi\left(\frac{d}{2}\right)^2 d \Rightarrow V(d)=\frac{\pi d^3}{4}$$

(b) Since $h=d, r=\frac{d}{2}, c=2\pi r$ and the formula for the surface area of a can is: $A=2\pi rh+2\pi r^2$, the function is: $S(d)=2\pi\left(\frac{d}{2}\right)d+2\pi\left(\frac{d}{2}\right)^2 \Rightarrow S(d)=\pi d^2+\frac{\pi d^2}{2} \Rightarrow S(d)=\frac{3\pi d^2}{2}$

99. The function for changing yards to inches is: $f(x)=36x$ and the function for changing miles to yards is: $g(x)=1760x$ The composition of this which would change miles into inches is: $f[g(x)]=36[1760(x)] \Rightarrow (f\circ g)(x)=63,360x$.

100. If $x=$ width, then length $=2x$ A formula for Perimeter can now be written as: $P=x+2x+x+2x$ and the function is: $P(x)=6x$ This is a linear function.

Chapter 2 Test

1. (a) D, only values where $x\geq 0$ can be input into a square root function.
   (b) D, only values where $y\geq 0$ can be the solution to a square root function.
   (c) C, all values can be input for x in a squaring function.
   (d) B, only values where $y\geq 3$ can be the solution to $f(x)=x^2+3$
   (e) C, all values can be input for x in a cube root function.
   (f) C, all values can be a solution in a cube root function.
   (g) C, all values can be input for x in an absolute value function.
   (h) D, only values where $y\geq 0$ can be the solution to an absolute value function.
   (i) D, if $x=y^2$ then $y=\sqrt{x}$ and only values where $x\geq 0$ can be input into a square root function.
   (j) C, all values can be a solution in this function.
2. (a) This is $f(x)$ shifted 2 units upward. See Figure 2a.
   (b) This is $f(x)$ shifted 2 units to the left. See Figure 2b.
   (c) This is $f(x)$ reflected across the x-axis. See Figure 2c.
   (d) This is $f(x)$ reflected across the y-axis. See Figure 2d.
   (e) This is $f(x)$ vertically stretched by a factor of 2. See Figure 2e.
   (f) We reflect the graph of $y=f(x)$ across the x-axis for all points for which \$2.75 Where $y\geq 0$ the graph remains unchanged. See Figure 2f.

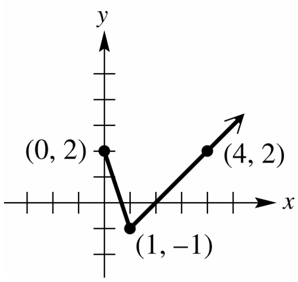

Figure 2a

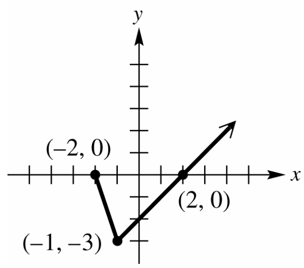

Figure 2b

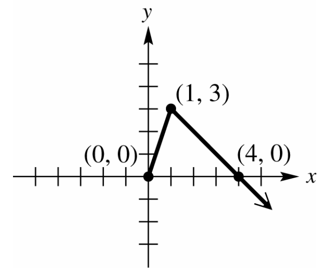

Figure 2c

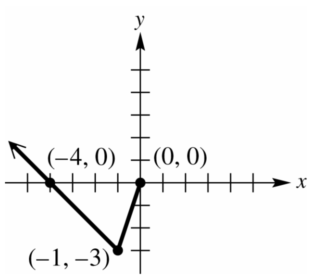

Figure 2d

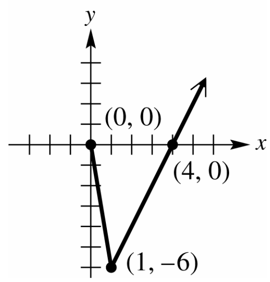

Figure 2e

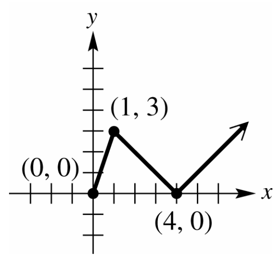

Figure 2f

3. (a) Since $y = f(2x)$ is $y = f(x)$ horizontally shrunk by a factor of $\frac{1}{2}$, the point $(-2,4)$ on $y = f(x)$ becomes the point $(-1,4)$ on the graph of $y = f(2x)$.

(b) Since $y = f\left(\frac{1}{2}x\right)$ is $y = f(x)$ horizontally stretched by a factor of 2, the point $(-2,4)$ on $y = f(x)$ becomes the point $(-4,4)$ on the graph of $y = f\left(\frac{1}{2}x\right)$.

4. (a) The graph of $f(x) = -x(x-2)^2 + 4$ is the basic graph $f(x) = x^2$ reflected across the x-axis, shifted 2 units to the right, and shifted 4 units upward. See Figure 4a.

(b) The graph of $f(x) = -2\sqrt{-x}$ is the basic graph $f(x) = \sqrt{x}$ reflected across the y-axis and vertically stretched by a factor of 2. See Figure 4b.

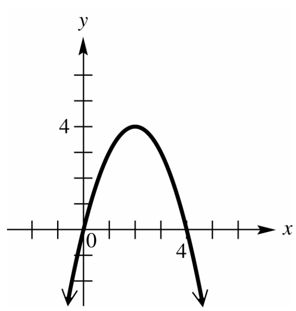

Figure 4a

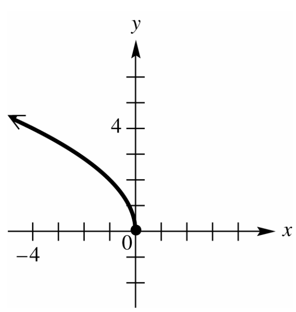

Figure 4b

5. (a) If the graph is symmetric with respect to the y-axis, then $(x, y) \Rightarrow (-x, y)$ therefore $(3,6) \Rightarrow (-3,6)$.

(b) If the graph is symmetric with respect to the x-axis, then $(x, y) \Rightarrow (-x, -y)$ therefore $(3,6) \Rightarrow (-3,-6)$.

(c) See Figure 5. We give an actual screen here. The drawing should resemble it.

[-4,4] by [0,]
Xscl = 1 Yscl = 1

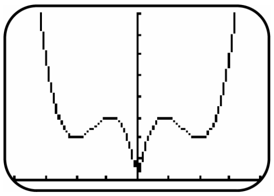

Figure 5

6. (a) Shift the graph of $y=\sqrt[3]{x}$ to the left 2 units, vertically stretch by a factor of 4, and shift 5 units downward.

(b) Graph $y=|x|$ reflected across the x-axis, vertically shrunk by a factor of $\frac{1}{2}$ shifted 3 units to the right, and shifted up 2 units. See Figure 6. From the graph the domain is: $(-\infty,\infty)$; and the range is: $(-\infty,2]$.

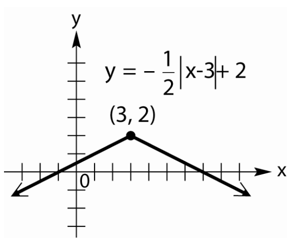

Figure 6

7. (a) From the graph, the function is increasing for the interval: $(-\infty,-3)$

(b) From the graph, the function is decreasing for the interval: $(4,\infty)$

(c) From the graph, the function is constant for the interval: $(-3,4)$

(d) From the graph, the function is continuous for the intervals: $(-\infty,-3),(-3,4),(4,\infty)$.

(e) From the graph, the domain is: $(-\infty,\infty)$

(f) From the graph, the range is: $(-\infty,2)$

8. (a) $|4x+8|=4 \Rightarrow 4x+8=4 \Rightarrow 4x=-4 \Rightarrow x=-1$ or $4x+8=-4 \Rightarrow 4x=-12 \Rightarrow x=-3$, therefore the solution set is: $\{-3,-1\}$

(b) $|4x+8|<4 \Rightarrow -4<4x+8<4 \Rightarrow -12<4x<-4 \Rightarrow -3<x<-1$, therefore the solution is: $(-3,-1)$.

(c) $|4x+8|>4 \Rightarrow 4x+8>4 \Rightarrow 4x>-4 \Rightarrow x>-1$ or $4x+8<-4 \Rightarrow 4x<-12 \Rightarrow x<-3$ therefore the solution is: $(-\infty,-3)\cup(-1,\infty)$.

9. (a) $(f-g)(x)=2x^2-3x+2-(-2x+1) \Rightarrow (f-g)(x)=2x^2-x+1$

(b) $\left(\frac{f}{g}\right)(x)=\frac{2x^2-3x+2}{-2x+1}$

(c) The domain can be all values for x, except any that make $g(x)=0$. Therefore

$-2x+1\neq 0 \Rightarrow -2x\neq -1 \Rightarrow x\neq \frac{1}{2}$ or the interval: $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

(d) $(f\circ g)(x)=f\left[g\left(x\right)\right]=2(-2x+1)^2-3(-2x+1)+2=2(4x^2-4x+1)+6x-3+2=$

$8x^2-8x+2+6x-3+2=8x^2-2x+1$

(e) $(g\circ f)(x)=g\left[f\left(x\right)\right]=-2(2x^2-3x+2)+1=-4x^2+6x-4+1=-4x^2+6x-3$

(f) $\dfrac{2(x+h)^2-3(x+h)+2-(2x^2-3x+2)}{h}=\dfrac{2(x^2+2xh+h^2)-3x-3h+2-2x^2+3x-2}{h}=$

$\dfrac{2x^2+4xh+2h^2-3x-3h+2-2x^2+3x-2}{h}=\dfrac{4xh+2h^2-3h}{h}=4x+2h-3$

10. (a) See Figure 10a.

(b) Graph $y_1=(-x^2+3)*(x\leq 1)+\left(\sqrt[3]{x}+2\right)+\left(\sqrt[3]{x}+2\right)*(x>1)$ in the window $[-4.7,4.7]$ by $[-5.1,5.1]$

See Figure 10b.

(c) The graph is not connected at $x=1$ and thus f is not continuous when $x=1$.

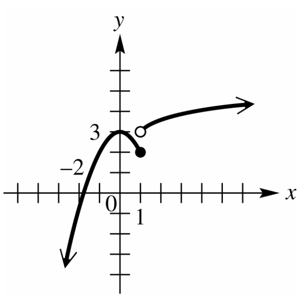

Figure 10a

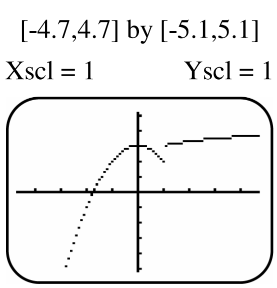

Figure 10b

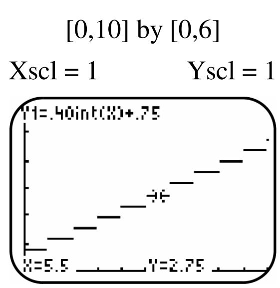

Figure 11

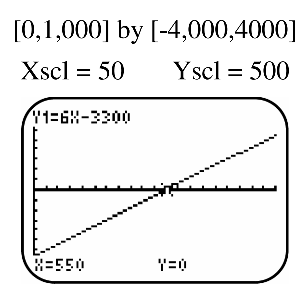

Figure 12

11. (a) See Figure 11.

(b) Set $x=5.5$ then \$2.75 is the cost of a 5.5 minute call. See the display at the bottom of the screen.

12. (a) With an initial set-up cost of \$3300 and a production cost of \$4.50 the function is: $C(x)=3300+4.50x$

(b) With a selling price of \$10.50 the revenue function is: $R(x)=10.50x$

(c) $P(x)=R(x)-C(x)\Rightarrow P(x)=10.50x-(3300+4.50x)\Rightarrow P(x)=6x-3300$

(d) To make a profit $P(x) > 0,$ therefore $6x - 3300 > 0 \Rightarrow 6x > 3300 \Rightarrow x > 550$

Tyler needs to sell 551 before he earns a profit.

(e) Graph $y_1 = 6x - 3300$, See Figure 12. The first integer x-value for which $P(x) > 0$ is 551