Chapter 2

Section 2.1 Practice

1. a. Use the perimeter formula $P = 2l + 2w$.

$$\begin{aligned} P &= 2(7) + 2(5) \\ &= 14 + 10 \\ &= 24 \end{aligned}$$

The perimeter of the picture is 24 inches.

b. Find the length and width of the frame.

$$\begin{aligned} \text{Length} &= 1 + 7 + 1 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{Width} &= 1 + 5 + 1 \\ &= 7 \end{aligned}$$

Find the perimeter of the outside edge of the frame.

$$\begin{aligned} P &= 2(9) + 2(7) \\ &= 18 + 14 \\ &= 32 \end{aligned}$$

The perimeter of the outside edge of the frame is 32 inches.

2. a. Use the formula $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(24) \\ &= 48\pi \end{aligned}$$

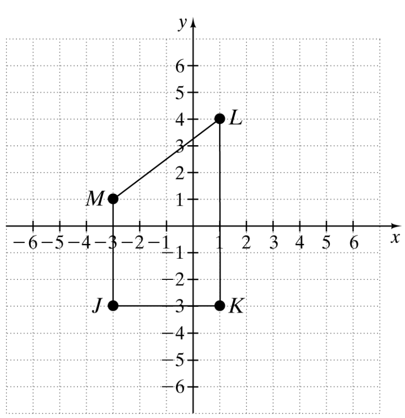
The exact circumference is 48π meters.

b. Use the formula $C = \pi d$.

$$\begin{aligned} C &= \pi d \\ &= \pi(3) \\ &= 3\pi \\ &\approx 9.4 \end{aligned}$$

The circumference is about 9.4 meters.

3.



Find the length of each segment in the quadrilateral using the Distance Formula or the Ruler Postulate.

$$JK = |-3 - 1| = 4$$

$$KL = |-3 - 4| = 7$$

$$\begin{aligned} LM &= \sqrt{[1 - (-3)]^2 + (4 - 1)^2} \\ &= \sqrt{(4)^2 + (3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$MJ = |1 - (-3)| = 4$$

Then add the side lengths to find the perimeter.

$$\begin{aligned} JK + KL + LM + MJ &= 4 + 7 + 5 + 4 \\ &= 20 \end{aligned}$$

The perimeter of quadrilateral *JKLM* is 20 units.

4. First, convert 3 yd into feet.

$$3\text{ yd} \cdot \frac{3\text{ ft}}{1\text{ yd}} = 9\text{ ft}$$

Then find the area of the poster.

$$\begin{aligned} A &= lw \\ &= 9 \cdot 8 \\ &= 72 \end{aligned}$$

The area of the poster is 72 square feet. You would need 72 ft^2 of paper.

5. a. First, find the radius of the circle.

$$r = \frac{d}{2} = \frac{14}{2} = 7$$

The radius of the circle is 7 feet.

Find the area using the formula $A = \pi r^2$.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(7)^2 \\ &= 49\pi \end{aligned}$$

The exact area in terms of π is 49π square feet.

b. Use 3.14 for π.

$$\begin{aligned} 49\pi &= 49(3.14) \\ &= 153.86 \end{aligned}$$

The area using 3.14 for π is 153.86 square feet.

Vocabulary & Readiness Check 2.1

1. The perimeter of a geometric figure is the distance around the figure.

2. The distance around a circle is called the circumference.

3. The exact ratio of circumference to diameter is π.

4. The diameter of a circle is double its radius.

5. Both $\frac{22}{7}$ and 3.14 are approximations for π.

6. The area of a geometric figure is the number of square units it encloses.

Exercise Set 2.1

2. $$\begin{aligned} P &= 2(14)+2(5) \\ &= 28+10 \\ &= 38 \text{ m} \end{aligned}$$

4. $$\begin{aligned} P &= 5+11+10 \\ &= 26 \text{ units} \end{aligned}$$

6. $$\begin{aligned} P &= 4(9) \\ &= 36 \text{ cm} \end{aligned}$$

8. $$\begin{aligned} P &= 4(50) \\ &= 200 \text{ m} \end{aligned}$$

10. $$\begin{aligned} P &= 6(15) \\ &= 90 \text{ yd} \end{aligned}$$

12. $$\begin{aligned} C &= \pi d \\ &= \pi(26) \\ &= 26\pi \text{ m} \\ &\approx 81.64 \text{ m} \end{aligned}$$

14. $$\begin{aligned} C &= 2\pi r \\ &= 2\pi(10) \\ &= 20\pi \text{ yd} \\ &\approx 62.8 \text{ yd} \end{aligned}$$

16. $$\begin{aligned} C &= 2\pi r \\ &= 2\pi(2.5) \\ &= 5\pi \text{ in.} \\ &\approx 15.7 \text{ in.} \end{aligned}$$

18. $$\begin{aligned} A &= lw \\ &= 7 \cdot 1.2 \\ &= 8.4 \text{ ft}^2 \end{aligned}$$

20. $$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}\left(4\frac{1}{2}\right)(5) \\ &= 11\frac{1}{4} \text{ ft}^2 \end{aligned}$$

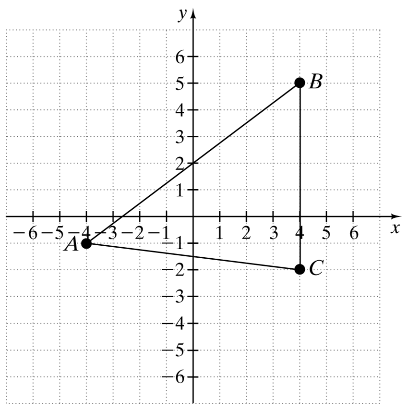
22. $$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(5)(7) \\ &= 17\frac{1}{2} \text{ ft}^2 \end{aligned}$$

24. $$\begin{aligned} A &= \pi r^2 \\ &= \pi(5)^2 \\ &= 25\pi \text{ cm}^2 \approx 78.5 \text{ cm}^2 \end{aligned}$$

26. $$\begin{aligned} A &= s^2 \\ &= 12^2 \\ &= 144 \text{ yd}^2 \end{aligned}$$

28. $$r = \frac{d}{2} = \frac{20}{2} = 10$$
$$\begin{aligned} A &= \pi r^2 \\ &= \pi(10)^2 \\ &= 100\pi \text{ m}^2 \approx 314 \text{ m}^2 \end{aligned}$$

30.



Find the length of each segment in the triangle using the Distance Formula or the Ruler Postulate.

$$\begin{aligned} AB &= \sqrt{\left[4-(-4)\right]^2+\left[5-(-1)\right]^2} \\ &= \sqrt{(8)^2+(6)^2} \\ &= \sqrt{64+36} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

$$BC = |-2-5| = 7$$

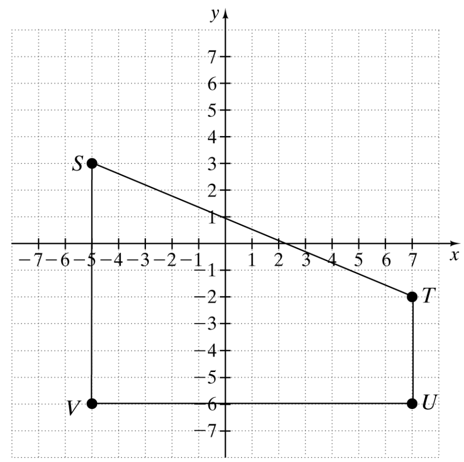
$$\begin{aligned} AC &= \sqrt{\left[4-(-4)\right]^2+\left[-2-(-1)\right]^2} \\ &= \sqrt{(8)^2+(-1)^2} \\ &= \sqrt{64+1} \\ &= \sqrt{65} \end{aligned}$$

Then add the side lengths to find the perimeter.

$$\begin{aligned} AB+BC+AC &= 10+7+\sqrt{65} \\ &= 17+\sqrt{65} \end{aligned}$$

The perimeter of triangle ABC is $17+\sqrt{65}$ units, or approximately 25.1 units.

32.



Find the length of each segment in the quadrilateral using the Distance Formula or the Ruler Postulate.

$$\begin{aligned} ST &= \sqrt{\left[7-(-5)\right]^2+(-2-3)^2} \\ &= \sqrt{(12)^2+(-5)^2} \\ &= \sqrt{144+25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$TU = |-6-(-2)| = 4$$

$$UV = |-5-7| = 12$$

$$SV = |-6-3| = 9$$

Then add the side lengths to find the perimeter.

$$\begin{aligned} ST+TU+UV+SV &= 13+4+12+9 \\ &= 38 \end{aligned}$$

The perimeter of quadrilateral $STUV$ is 38 units.

34. Find the area of one panel.

$$\begin{aligned} A &= lw \\ &= 6\cdot 7 \\ &= 42\text{ ft}^2 \end{aligned}$$

Now find the area of four panels.

$$4\cdot 42\text{ ft}^2 = 168\text{ ft}^2$$

36.

$$\begin{aligned} A &= lw \\ &= 197\cdot 66 \\ &= 13{,}002\text{ ft}^2 \end{aligned}$$

38. Find the length and width of the matting.

$$\text{Length} = 3+6+3 = 12$$

$$\text{Width} = 3+4+3 = 10$$

Find the perimeter of the outside of the matting.

$$P = 2(12)+2(10) = 24+20 = 44$$

The perimeter of the outside of the matting is 44 inches.

40. $$C = \pi d = \pi(150) = 150\pi \text{ ft} \approx 471.2 \text{ ft}$$

42. The gutters go along the sides of the roof, so we are concerned with perimeter.

44. The baseboards go around the edges of the room, so we are concerned with perimeter.

46. Fertilizer needs to cover the entire yard, so we are concerned with area.

48. The grass seed needs to cover the entire yard, so we are concerned with area.

50. Find the area of the picture and the frame together.

$$A = lw = 7\cdot 5 = 35 \text{ cm}^2$$

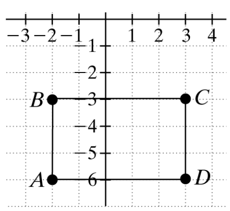
Find the area of the picture.

$$A = lw = 2\cdot 4 = 8 \text{ cm}^2$$

Subtract the area of the picture to find the area of the frame.

$$35 \text{ cm}^2 - 8 \text{ cm}^2 = 27 \text{ cm}^2$$

52.



Find the lengths of two perpendicular segments in the rectangle using the Distance Formula or the Ruler Postulate.

$$AB = |-3-(-6)| = 3$$

$$BC = |3-(-2)| = 5$$

Find the perimeter.

$$P = 2(3)+2(5) = 6+10 = 16$$

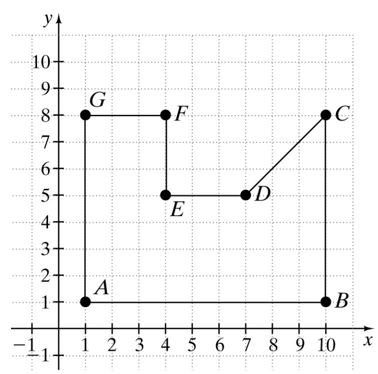
The perimeter is 16 units.

Find the area.

$$A = lw = 5\cdot 3 = 15$$

The area is 15 square units.

54.



Find the length of each of the segments using the Distance Formula or the Ruler Postulate.

$$AB = |10-1| = 9$$

$$BC = |8-1| = 7$$

$$CD = \sqrt{(10-7)^2+(8-5)^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$DE = |4-7| = 3$$

$$EF = |5-8| = 3$$

$$FG = |4-1| = 3$$

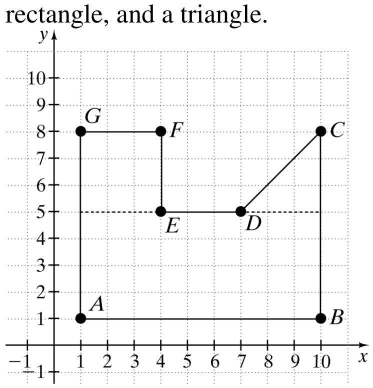
$$AG = |1-8| = 7$$

Add the lengths of the sides to find the perimeter.

$$P = AB + BC + CD + DE + EF + FG + AG$$
$$= 9 + 7 + \sqrt{18} + 3 + 3 + 3 + 7$$
$$= 32 + \sqrt{18}$$
$$\approx 36.2$$

The perimeter is $32 + \sqrt{18}$ units, or approximately 36.2 units.

Find the area. Split the figure into smaller figures and find the area of each figure. There are many methods to do this, but one is to split the figure on a line through points D and E. This breaks the figure into a square, a rectangle, and a triangle.



The square has side lengths of 3 units.

$$A = s^2$$
$$= (3)^2$$
$$= 9$$

The rectangle has side lengths 9 units and 4 units.

$$A = lw$$
$$= 9 \cdot 4$$
$$= 36$$

The triangle has a base of 3 units and a height of 3 units.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(3)(3)$$
$$= 4.5$$

Add the areas to find the area of the whole figure.

$$A = 9 + 36 + 4.5$$
$$= 49.5$$

The area is 49.5 square units.

56. Find the areas of the six surfaces of the figure. Notice that each surface has the same area as its opposing surface, so the areas of only three surfaces need to be calculated.

$$A = lw$$
$$= 6 \cdot 4$$
$$= 24$$
$$A = lw$$
$$= 8 \cdot 6$$
$$= 48$$
$$A = lw$$
$$= 8 \cdot 4$$
$$= 32$$

Add the areas of the six surfaces.

$$\text{SA} = 2(24) + 2(48) + 2(32)$$
$$= 48 + 96 + 64$$
$$= 208 \text{ in.}^2$$

58. **a.** No, because every rectangle does not have four equal sides.

b. Solve the perimeter formula for s and then substitute it into the area formula.

$$P = 4s$$
$$s = \frac{P}{4}$$
$$A = s^2$$
$$= \left(\frac{P}{4}\right)^2$$
$$= \frac{P^2}{16}$$

60. Answers will vary. Possible answer: Building a fence around a yard would need the perimeter. Planting grass seed in the yard would need the area.

62. $7^2 = 49$

64. $20^2 = 400$

66. $5^2 + 3^2 = 25 + 9 = 34$

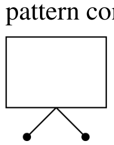
68. $1^2 + 6^2 = 1 + 36 = 37$

Section 2.2 Practice

1. **a.** The pattern rotates between multiplying the previous number by 2, 3, and 4. If the pattern continues the next number will be $2 \cdot 1728$, or 3456.
   b. The pattern rotates between adding 8, adding 8, then subtracting 14. If the pattern continues the next number will be $21 - 14$, or 7.

2. **a.** Each new number is the sum of the two previous numbers. If the pattern continues the next number will be $29 + 47$, or 76.
   b. Each new number is the sum of the previous number and an increasing multiple of 2, starting with 1 for the first number and 2 for the second. If the pattern continues the next number will be 257.

3. There are two separate patterns occurring here. The first pattern is alternating between a square and a triangle. The next shape is a rectangle. The second pattern is the legs on the bottom rotating between 1 leg, 2 legs, and 3 legs. The next figure will have 2 legs. If the pattern continues, the next figure is shown.



4. **a.** The colors of the shapes are rotating between purple, red and green. Since there are 3 different colors, every multiple of 3 will be in green. If the number is higher than a multiple of 3 by 1, or lower by 2, then the shape will be purple. If the number is higher than a multiple of 3 by 2, or lower by 1, then the shape will be red. Since 11 is 1 lower than the closest multiple of 3, which is 12, the 11th figure will be red.
   b. Each circle has twice as many regions as the circle before it, so if n represents the number of the circle then each circle will have 2^n regions. Thus, the 11th circle will have 2^{11}, or 2048, regions.
   c. The appearance of the 11th circle is red with 2048 regions formed by the diameters.
   d. The 30th circle is green. The 30th circle has 2^{30}, or 1,073,741,824, regions formed by the diameters.

5. Since we predict that the dealership will sell 66 cars in October, use that to help predict the number of cars that will be sold in November, and then December. Using inductive reasoning, and the estimated increase of 4 cars per year, we predict that the dealership sells 70 hybrid cars in November, since $66 + 4 = 70$. Now using inductive reasoning, we predict that the dealership sells 74 hybrid cars in December.

6. **a.** Answers may vary.
   Counterexample: $4 \cdot \frac{1}{2} = 2$. Notice that $2 \not> 4$. Thus, the conjecture is false because it is not always true.
   b. Answers may vary.
   Counterexample**:** Let the length and width of a square be 3. The area would then be equal to $3 \cdot 3$ or 9. Notice that 9 is not an even number. Thus, the conjecture is false.

Vocabulary & Readiness Check 2.2

1. Another word for an educated guess is a <u>conjecture</u>.

2. A <u>counterexample</u> is an example that shows that a conjecture is false.

3. The process of arriving at a general conclusion based on observing patterns or specific examples is called <u>inductive reasoning</u>.

Exercise Set 2.2

2. Each number is 4 times the previous number. If the pattern continues, then the next number will be $320 \cdot 4$, or 1280.

4. Each number is 2 less than the previous number. If the pattern continues, then the next number will be $14 - 2$, or 12.

6. Every third number from the first number is decreasing by one, starting at 5. Every third number from the second number is increasing by 1, starting at −5. Every third number from the third number is 0. If the pattern continues, then the next number will be $-4 + 1$, or -3.

8. Each number is the sum of the previous number and an even number, which increases with every sum. If the pattern continues, then the next number will be $21+10$, or 31.

10. The pattern rotates between multiplying the previous number by 3 and 2. If the pattern continues, then the next number will be $72 \cdot 3$, or 216.

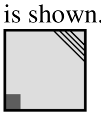
12. In the tens place, each number is rotating between 7 and 6. In the ones place, each number is decreasing by 1. If the pattern continues, then the next number will be 75.

14. The pattern rotates between multiplying the previous number by 2 and 3. If the pattern continues, then the next number will be $72 \cdot 2$, or 144.

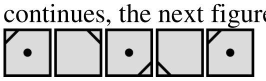
16. A 1 is added to the end of each previous number. If the pattern continues, then the next number will be 0.1111.

18. The numerator is a sum of the numerator and denominator of the previous number and the denominator is the previous denominator multiplied by 3. If the pattern continues, then the next number will be $\frac{40+81}{81 \cdot 3}$, or $\frac{121}{243}$.

20. There are two things occurring in this pattern. The first pattern is that both the smaller shaded box and the lines are rotating clockwise around to each corner. The second pattern is that there is 1 line being added each time. If the pattern continues, the next figure is shown.



22. In this pattern, the previous figure is the beginning of the new figure in the pattern and a box is added on with odd ones having a dot in them with the line in the corner and the even ones just having the line. The line in the corner is rotating clockwise around to the next corner each time. If the pattern continues, the next figure is shown.



24. The colors of the shapes are rotating between purple, red and green. Since there are 3 different colors, every multiple of 3 will be in green. If the number is higher than a multiple of 3 by 1, or lower by 2, then the shape will be purple. If the number is higher than a multiple of 3 by 2, or lower by 1, then the shape will be red. Since 40 is 1 higher than the closest multiple of 3, which is 39, the 40th figure will be purple.

26. The shapes are rotating between a circle, triangle, square and star. Since there are 4 different shapes, every multiple of 4 will be in a star. If the number is higher than a multiple of 4 by 1, or lower by 3, then the shape will be a circle. If the number is higher than a multiple of 4 by 2, or lower by 2, then the shape will be triangle. If the number is higher than a multiple of 4 by 3, or lower by 1, then the shape will be a square. Since 50 is 2 higher than the closest multiple of 4, which is 48, the 50th figure will be a triangle.

28. **a.** The colors of the shapes are rotating between purple, red and green. Since there are 3 different colors, every multiple of 3 will be in green. If the number is higher than a multiple of 3 by 1, or lower by 2, then the shape will be purple. If the number is higher than a multiple of 3 by 2, or lower by 1, then the shape will be red. Since 20 is 1 lower than the closest multiple of 3, which is 21, the 20th figure will be red.

b. The shapes are rotating between a circle, triangle, square and star. Since there are 4 different shapes, every multiple of 4 will be in a star. If the number is higher than a multiple of 4 by 1, or lower by 3, then the shape will be a circle. If the number is higher than a multiple of 4 by 2, or lower by 2, then the shape will be triangle. If the number is higher than a multiple of 4 by 3, or lower by 1, then the shape will be a square. Since 20 is a multiple of 4, the 20th figure will be a star.

c. The appearance of the 20th figure will be a red star.

30. Answers may vary. Sample Answer: The sum of an even and an odd number is odd.

32. Answers may vary. Sample Answer: The product of two odd numbers is odd.

34. Answers may vary. Sample Answer: The product of a number and its reciprocal is one.

36. **a.** The color of the shapes is rotating between orange and purple. The odd numbered half-circles are orange and the even numbered half-circles are purple. Since 13 is odd, the 13th figure will be orange.

b. The number of regions in each pattern is increasing by 1 each time. Each figure has the number of regions that it is numbered in the list. Therefore, the 13th figure will have 13 regions

c. The 13th figure will have an appearance of an orange half-circle with 13 regions

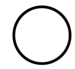
d. Following the patterns from parts a and b, since 29 is an odd number, the 29th figure will be orange and will have 29 regions. Therefore, the appearance will be an orange half-circle with 29 regions

38. **a.** The colors of the shapes are rotating between green, green, red, and blue. Since there are 4 different colors, every multiple of 4 will be blue. If the number is higher than a multiple of 4 by 1, or lower by 3, then the shape will be green. If the number is higher than a multiple of 4 by 2, or lower by 2, then the shape will also be green. If the number is higher than a multiple of 4 by 3, or lower by 1, then the shape will be red. Since 12 is a multiple of 4, the 12th figure will be blue.

b. The shapes are rotating between a triangle, a circle, and a square. Since there are 3 different shapes, every multiple of 3 will be a square. If the number is higher than a multiple of 3 by 1, or lower by 2, then the shape will be a triangle. If the number is higher than a multiple of 3 by 2, or lower by 1, then the shape will be a circle. Since 12 is a multiple of 3, the 12th figure will be a square.

c. Since 13 is 1 higher than the closest multiple of 4, which is 12, the 13th figure will be green. Since 13 is 1 higher than the closest multiple of 3, which is 12, the shape will be a triangle. The 13th figure will be a green triangle.

d. Since 26 is 2 higher than the closest multiple of 4, which is 24, the 26th figure will be green. Since 26 is 1 lower than the closest multiple of 3, which is 27, the shape will be a circle. The 13th figure will be a green circle.



40. Each time the number of chirps per 14 seconds increases by 5, the temperature increases by 10. Thus, if you hear a cricket chirp 20 times in 14 seconds then the temperature will be $65+10,$ or $75°.$

42. Answers may vary. Sample Answer:
$\angle A = 60°, \angle B = 90°, \angle C = 30°$
Thus, the conjecture is false because it is not always true.

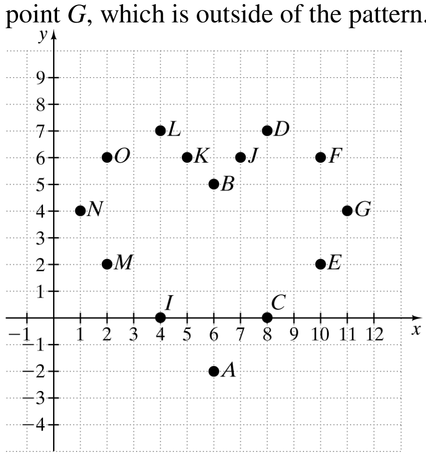
44. Answers may vary. Sample Answer:
$-1 \cdot -2 = 3.$ Notice that $3 \nless -1$ and $3 \nless -2.$
Thus, the conjecture is false because it is not always true.

46. Answers may vary. Sample Answer: Let the length of a rectangle be 4 and the width be $\frac{1}{2}.$ $A = \frac{1}{2} \cdot 4 = 2.$ Notice that $2 \ngtr 4.$
Thus, the conjecture is false because it is not always true.

48. The prediction is 555555555. This is because the pattern is increasing by 111111111 each time.
$12345679 \cdot 45 = 555555555$

50. There are sixty-four 1×1 squares, forty-nine 2×2 squares, thirty-six 3×3 squares, twenty-five 4×4 squares, sixteen 5×5 squares, nine 6×6 squares, four $7\times7,$ and one 8×8 square. Notice that number of squares from 1×1 to 8×8 is decreasing to the next perfect square number, starting from $8^2 = 64$ all the way to $1^2 = 1.$
So the total number of squares is 204.

52. The majority of the points together create a heart. Notice that point H does not fit the pattern because it is located to the right of point G, which is outside of the pattern.



54. Answers may vary. Sample Answer: The last six times I went to the beach, the traffic was light on Wednesdays and heavy on Sundays. My conclusion is that weekdays have lighter traffic than weekends.

56. Each year, the number of bird species is increasing by about 5. Since $2019-2012=7$ and $7\cdot 5=35$, 35 will be the approximate increase from 2012 to 2019. So, $90+35=125$, which means in 2019, there will be about 125 bird species.

58. Clay is conjecturing that each number is increasing by 2. Ott is conjecturing that each number is being doubled. Stacie is conjecturing that each number is going to increase by 1 more than the previous number. There is not enough information to decide who is correct.

60. a. $\dfrac{100(100+1)}{2}=5,050$

b. $\dfrac{n(n+1)}{2}$

62. $12(2)+8=32$

64. True. Explanations may vary. Sample: If three odd numbers are represented by $2a+1$, $2b+1$, $2c+1$, then

$$\begin{aligned}2a+1+2b+1+2c+1 &= (2a+2b+2c)+3\\ &= 2(a+b+c)+3,\end{aligned}$$

which is an odd number.

Section 2.3 Practice

1. a. Hypothesis (p): an animal is a pig
Conclusion (q): the animal has 44 teeth

b. Hypothesis (p): $x=7$
Conclusion (q): $x^2=49$

2. a. If an angle measures 36°, then the angle is an acute angle.

b. If you live in Texas, then you live in the continental U.S.

3. If a number is a fraction, then it is a real number.

4. a. False; complementary angles must have their measures add up to 90°, but they do not need to be adjacent.

b. True; the only months that start with M are March and May, and both have 31 days.

5. a. If the figure has three sides, then the figure is a triangle.

b. If the figure is not a triangle, then the figure does not have three sides.

c. If the figure does not have three sides, then the figure is not a triangle.

Vocabulary & Readiness Check 2.3

1. In symbols, an “if-then” statement can be written as $\underline{p \to q}$.

2. In symbols, the negation of p is written as $\underline{\sim p}$.

3. For an “if-then” statement, the part following “then” is called the <u>conclusion</u>.

4. For an “if-then” statement, the part following “if” is called the <u>hypothesis</u>.

5. The statement name for $\sim p \to \sim q$ is <u>inverse</u>.

6. The statement name for $\sim q \to \sim p$ is <u>contrapositive</u>.

7. The statement name for $q \to p$ is <u>converse</u>.

8. The statement name for $\sim p$ is <u>negation</u>.

9. Two statements that are both always true or both always false are called equivalent statements.

10. Conditional and contrapositive statements are equivalent/conditional statements.

Exercise Set 2.3

2. Hypothesis (p): you want to be healthy
Conclusion (q): you should eat vegetables

4. Hypothesis (p): the figure is a pentagon.
Conclusion (q): the figure has five sides.

6. If the animal is a frog, then the animal is an amphibian.

8. If $x = 9$, then $\sqrt{x} = 3$.

10. If an angle is a right angle, then the angle measures 90º.

12. If an animal is a whale, then the animal is a mammal.

14. If $2x + 3 = 11$, then $2x = 8$.

16. If the side of a square measures 6 feet, then the side of the square measures 2 yards.

18. False; there are other sports that are played with a bat and a ball, like cricket for example, so it is not guaranteed that the sport is baseball.

20. True; a polygon with eight sides is called an octagon by definition.

22. c; a rectangle does not have three sides.

24. a; the avocado is not soft.

26. Converse $(q \rightarrow p)$: If you play soccer, then you are a goalkeeper.
Inverse $(\sim p \rightarrow \sim q)$: If you are not a goalkeeper, then you do not play soccer.
Contrapositive $(\sim q \rightarrow \sim p)$: If you do not play soccer, then you are not a goalkeeper.

28. Converse $(q \rightarrow p)$: If $x = 7$, then $5x - 3 = 32$.
Inverse $(\sim p \rightarrow \sim q)$: If $5x - 3 \neq 32$, then $x \neq 7$.
Contrapositive $(\sim q \rightarrow \sim p)$: If $x \neq 7$, then $5x - 3 \neq 32$.

30. The statement is neither the converse, the inverse, nor the contrapositive. It shows $p \rightarrow \sim q$, which does not have a name.

32. Converse; the statement shows $q \rightarrow p$.

34. Inverse; the statement shows $\sim p \rightarrow \sim q$.

36. Contrapositive; the statement shows $\sim q \rightarrow \sim p$.

38. If a number is not divisible by 2, then it is not even. This is true because even numbers are divisible by 2 by definition, so if a number is not divisible by 2, it follows that it is not even.

40. If I do not live in South America, then I do not live in Brazil. This is true because Brazil is in South America, so it is impossible to live in Brazil if you do not live in South America.

42. If $\angle B$ is not acute, then $m\angle B \neq 15°$. This is true because if an angle is not acute, its measure must be greater than or equal to 90º.

44. If there are two points, then there is exactly one line through them.

46. If-then form $(p \rightarrow q)$ If two lines lie in the same plane, then the lines are coplanar. This statement is true because the word coplanar means that the lines are in the same plane by definition.
Converse $(q \rightarrow p)$: If two lines are coplanar, then the lines lie in the same plane. This statement is true by the definition of coplanar.
Inverse $(\sim p \rightarrow \sim q)$: If two lines do not lie in the same plane, then the lines are not coplanar. This statement is true by the definition of coplanar.
Contrapositive $(\sim q \rightarrow \sim p)$: If two lines are not coplanar, then the lines do not lie in the same plane. This statement is true by the definition of coplanar.

48. Converse $(q \to p)$: If $-y$ is positive, then y is negative. This is true because $-y$ means "the opposite of y," so it must have the opposite sign of y.

50. If $x^2 > 0$, then $x < 0$. This statement is false. Squaring any value greater than 0 results in a value that is also greater than 0. For example, $5^2 > 0$ but 5 is not less than 0.

52. If an event has a probability of 0, then the event is certain not to occur.

54. If someone has never made a mistake, then he or she has never tried anything new.

56. Answers may vary. Sample answer:
True conditional with a true converse: If $x = 2$, then $2 = x$. The converse is "if $2 = x$, then $x = 2$," which is also true by the Symmetric Property of Equality.
True conditional with a false converse: If it is nighttime, then the sun is not shining. The converse is "if the sun is not shining, then it is nighttime," which is not true because it could be cloudy during the day.

58. No; the square of 2 is guaranteed to be 4, and if $x^2 \neq 4$, then x is guaranteed not to be 2.

60. If you are eating at Subway, then you are eating fresh.

62. If you are eating at Burger King, then you are having it your way.

64. $r \to t$ is true because multiplying any whole number by 2 results in an even number.
$t \to r$ is false because an odd number is even when multiplied by 2. For example, 2(11) is even, but 11 is odd.
$r \to u$ is false because any even value of a cannot be multiplied by 2 to get a product that is odd. For example, $4 \cdot 2 = 8$, which is even.
$u \to r$ is false because any odd value for $2a$ cannot be divided by 2 to get an even number. For example, $13 \div 2 = 6.5$, which is not even.
$t \to u$ is false because a number cannot be both even and odd. For example, $2 \cdot 11 = 22$, which is even but not odd.
$u \to t$ is false because a number cannot be both even and odd. For example, $2 \cdot 11 = 22$, which is even but not odd.

66. The square of a number between 0 and 1 is less than the number. For example,
$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

68. Use the formula for the perimeter of a rectangle.

$$\begin{aligned} P &= 2l + 2w \\ &= 2(3.5\text{ cm}) + 2(7\text{ cm}) \\ &= 7\text{ cm} + 14\text{ cm} \\ &= 21\text{ cm} \end{aligned}$$

70. Use the formula for the perimeter of a rectangle. Make sure that the units are equivalent.

$$\begin{aligned} P &= 2l + 2w \\ &= 2(11\text{ yd}) + 2\left(60\text{ ft} \cdot \frac{1\text{ yd}}{3\text{ ft}}\right) \\ &= 22\text{ yd} + 2(20\text{ yd}) \\ &= 22\text{ yd} + 40\text{ yd} \\ &= 62\text{ yd} \end{aligned}$$

Section 2.4 Practice

1. **a.** Conditional Statement: If two angles are complimentary, then the sum of their measures is 90°.
b. Converse Statement: If the sum of the measures of two angles is 90°, then the two angles are complimentary.

2. **a.** Converse Statement: If $AB + BC = AC$, then point B lies between points A and C.
b. The converse statement is true.
c. Biconditional Statement: Point B is between points A and C if and only if $AB + BC = AC$.

3. **a.** Converse Statement: If $|x| = 7$, then $x = 7$.
b. The converse statement is false.
c. For a counterexample, notice that if $|x| = 7$, then x may also be -7, since $|-7| = 7$.

4. Choice a is not reversible. A rectangle has right angles, but it may not be a square.
Choice b is not reversible. June is a month that begins with the letter J, but it is not the month of January.
Choice c is a good definition. It is reversible, and all terms in the definition are clearly defined.
Choice d is not a good definition. "29 days" is not precise. February can have 28 days as well as 29 days.

5. Conditional Statement: If the length of an object is a yard, then the object measures 36 inches. TRUE
Converse Statement: If the length of an object measures 36 inches, then the object is one yard long. TRUE
Biconditional Statement: An object is a yard long if and only if it measures 36 inches. This means that the original definition is a good definition and can be used both "forward" and "backward."

Vocabulary & Readiness Check 2.4

1. A biconditional statement is written using a conditional statement and its converse.

2. A biconditional statement may be written using the phrase if and only if.

3. The converse of statement $p \to q$ is $\underline{q \to p}$.

4. The symbol for if and only if is $\underline{\leftrightarrow}$.

5. A good definition may be written as a biconditional (or if and only if) statement.

6. A true biconditional statement $p \leftrightarrow q$ may be used "forward" and "backward." In symbols, forward and backward mean, respectively $\underline{p \to q}$ and $\underline{q \to p}$.

Exercise Set 2.4

2. Conditional Statement: If a ray bisects a segment, then the ray intersects the segment only at the midpoint.
Converse Statement: If a segment is intersected by a ray at its midpoint, then the ray bisects the segment.

4. Conditional Statement: If an integer is divisible by 5, then its last digit is 0 or 5.
Converse Statement: If an integer's last digit is 0 or 5, then the integer is divisible by 5.

6. Conditional Statement: If you live in Baton Rouge, Louisiana, then you live in the capital of the state of Louisiana.
Converse Statement: If you live in the capital of the state of Louisiana, then you live in Baton Rouge, Louisiana.

8. Conditional Statement: If $|x| = 13,$ then $x = 13$ or $x = -13.$
Converse Statement: If $x = 13$ or $x = -13,$ then $|x| = 13.$

10. **a.** Converse Statement: If two angles are congruent, then they have the same degree measure.
b. The converse statement is true.
c. Two angles have the same degree measure if and only if they are congruent.

12. **a.** Converse Statement: If $8x + 14 = 14,$ then $x = 0.$
b. The converse statement is true.
c. $x = 0$ if and only if $8x + 14 = 14.$

14. **a.** Converse Statement: If a number is a multiple of 3, then it is divisible by 6.
b. The converse statement is false.
c. For a counterexample, 3 is a multiple of 3, but 3 is not divisible by 6.

16. **a.** Converse Statement: If it is Thanksgiving in the United States, then it is the fourth Thursday in November.
b. The converse statement is true.
c. In the United States, it is the fourth Thursday in November if and only if it is Thanksgiving.

18. **a.** Converse Statement: If $|x| = 5,$ then $x = -5.$
b. The converse statement is false.
c. For a counterexample, x could be 5, since $|5| = 5.$

20. **a.** Conditional Statement: If two angles are vertical angles, then they are congruent.
b. The conditional statement is true.
c. Converse Statement: If two angles are congruent, then they are vertical angles.
d. The converse statement is false.
e. For a counterexample, notice that two angles of an isosceles triangle are congruent, but they are not vertical angles.

22. **a.** Conditional Statement: If angle B is obtuse, then $90° < m\angle B < 180°$.
b. The conditional statement is true.
c. Converse Statement: If $90° < m\angle B < 180°$, then angle B is obtuse.
d. The converse statement is true.
e. Biconditional Statement: An angle B is obtuse if and only if $90° < m\angle B < 180°$.

24. This statement is not a good definition. The converse is false. For a counterexample, a chicken is a mammal, but a chicken is not a dolphin.

26. This statement is not a good definition. The converse is false. For a counterexample, a ruler is a geometric tool, but a ruler is not a compass.

28. This statement is a good definition because the conditional and converse statements are true and can be combined to form the following true biconditional statement: Two intersecting lines are perpendicular if and only if they intersect to form right angles.

30. Statement c and its converse form a true biconditional statement. $x = 10$ if and only if $5x + 5 = 55$.

32. A point is in quadrant I if and only if it has two positive coordinates.

34. An object is a triangle if and only if it is a three-sided polygon.

36. The last digit of an integer is even if and only if it is divisible by two.

38. The statement is a good definition
a. If an angle is obtuse, then its measure is greater than 90°. If the measure of an angle is greater than 90°, then it is obtuse.
b. The two conditional statements are converses of each other.

40. The statement is not a good definition. "Gigantic animal" is vague. For example, a whale is a gigantic animal, but a whale is not an elephant. The converse is false.

42. "A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays" is a better definition of a linear pair because both the conditional statement and the converse statement are true and can be combined to form the following true biconditional statement: A pair of angles is a linear pair if and only if they are adjacent angles with noncommon sides that are opposite rays.

44. If $\angle A$ and $\angle B$ are a linear pair, then $\angle A$ and $\angle B$ are adjacent.

46. $\angle A$ and $\angle B$ are a linear pair if and only if $\angle A$ and $\angle B$ are adjacent and supplementary angles.

48. The description of the letter K is not a good one. For a counterexample, the letter V is also formed by making a V with the two fingers beside the thumb.

50. The description of the letter B is a good one because there is no other letter that can be formed the same way.

52. If you have a good voice, then you are in the school chorus.

54. If a conditional statement is true, its converse is <u>sometimes true and sometimes false.</u>

56. Given the list 2500, 500, 100, 20, …, notice that each number in the list is $\frac{1}{5}$ of the previous number. Therefore, the next two terms in the list are 4 and $\frac{4}{5}$.

58. Given the list 1, 3, 2, 4, 3, …, notice that the pattern alternates between adding 2 and subtracting 1. Therefore, the next two terms in the list are 5 and 4.

Section 2.5 Practice

1. a. If a number is a real number, then it is an integer. $(q \to p)$

b. If a number is not an integer, then it is not a real number. $(\sim p \to \sim q)$

c. If a number is not a real number, then it is not an integer. $(\sim q \to \sim p)$

2. Conclusion: You have a license plate on the front and the back of your car.

3. a. In symbols, the first sentence is a true conditional $(p \to q)$. The second sentence is a true hypothesis (p); thus q, the third sentence, is true.
This argument is valid.

b. In symbols, the first sentence is a true conditional $(p \to q)$. The first part of the second sentence is a true conclusion (q), not a true hypothesis (p).
This argument is not valid.

4. The first conditional statement $p \to q$ is true, where "a natural number ends in 0" is a true hypothesis (p) and "it is divisible by 10" is a true conclusion (q).
The second conditional statement $q \to r$ is true, where "a natural number is divisible by 10" is a true hypothesis (q) and "it is divisible by 5" is a true conclusion (r).
Conclusion: If a natural number ends in 0, then it is divisible by 5. The conditional statement $p \to r$ is true, where "a natural number ends in 0" is a true hypothesis (p) and "it is divisible by 5" is a true conclusion (r).

5. a. The given statements are in the form
$p \to q$
p
Conclusion: The figure has three sides. (q is then true.) The Law of Detachment is used.

b. The given statements are in the form
$p \to q$
$q \to r$
Conclusion: All squares have diagonals that bisect each other. ($p \to r$ is then true.) The Law of Syllogism is used.

Vocabulary & Readiness Check 2.5

1. The <u>Law of Detachment</u> says that if $p \to q$ is true and p is true, then q is true.

2. The <u>Law of Syllogism</u> says that if $p \to q$ is true and $q \to r$ is true, then $p \to r$ is true.

3. <u>Deductive</u> reasoning is used to prove theorems.

4. With <u>inductive</u> reasoning, your conclusion may or may not be true.

Exercise Set 2.5

2. $\angle A$ is not congruent to $\angle B$.

4. If $\angle A$ is congruent to $\angle B$, then $m\angle A = m\angle B$.

6. If $m\angle A = m\angle B$, then $\angle A$ is not congruent to $\angle B$.

8. If $\angle A$ is not congruent to $\angle B$, then $m\angle A \neq m\angle B$.

10. Converse statement

12. Contrapositive statement

14. $q \to p$ is equivalent to $\sim p \to \sim q$.

16. If a figure has seven sides, then it is a heptagon.

18. If a figure is not a heptagon, then it does not have seven sides.

20. The given statements are in the form
$p \to q$
p
Conclusion: Rashid must study hard.

22. The given statements are in the form
$p \to q$
$\sim p$
It is not possible to make a conclusion. The Law of Detachment says that if $p \to q$ is true and p is true, then q is true. In this case, $\sim p$ is given as true, not p.

24. The given statements are in the form
$p \to q$
p
Conclusion: Points X, Y, and Z are coplanar.

26. The given statements are in the form
$p \to q$
$q \to r$
Conclusion: If a line intersects a segment at its midpoint, then it divides the segments into two congruent segments.

28. The given statements are in the form
$p \to q$
$r \to p$
Since the order of the statements does not matter, it is possible to make a conclusion.
Conclusion: If you read often, then you will improve your score on a standardized test.

30. **a.** Conclusion: You are exercising.
b. Law of Detachment

32. **a.** Conclusion: If an Alaskan mountain is more than 20,300 ft high, then it is the highest in the United States.
b. Law of Syllogism

34. **a.** Answers may vary.
Sample table:

Choose an integer.	-12	0	2	90
Multiply the integer by 3.	-36	0	6	270
Add 6 to the product.	-30	6	12	276
Divide the sum by 3.	-10	2	4	92

The result is the original number plus 2.

b. n
$3n$
$3n+6$
$\frac{3n+6}{3}=n+2$

36. If a figure is a square, then it is a rectangle.
Conclusion: $ABCD$ is a rectangle.

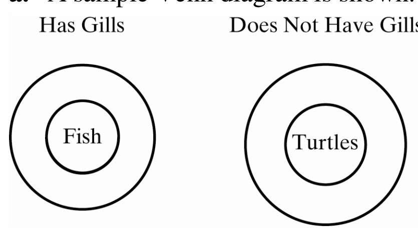
38. If a person is a high school student, then he or she likes art.
It is not possible to make a conclusion. The Law of Detachment says that if $p \to q$ is true and p is true, then q is true. In this case, q is given as true, not p.

40. Must be true; since e is true, a must be true. Since breakfast time is a mealtime, Curtis is drinking water by c.

42. Is not true; since e is true, a must be true. Since breakfast time is a mealtime, Curtis is drinking water by c, not juice.

44. Is not true; since e is true, a must be true. Since it is breakfast time, Julie is drinking juice by d, not milk.

46. **a.** A sample Venn diagram is shown.



b. The circle representing turtles does not overlap with the circle representing fish at any point, so a turtle cannot be a fish.

48. $\angle BOC$ and $\angle COB$

50. By the Angle Addition Postulate,
$m\angle AOC = m\angle 1 + m\angle 2.$

$$\begin{aligned} m\angle AOC &= m\angle 1 + m\angle 2 \\ &= 43^\circ + 43^\circ \\ &= 86^\circ \end{aligned}$$

Since $86^\circ < 90^\circ$, $\angle AOC$ is acute.

Section 2.6 Practice

1.

Statements	Reasons
$9x-45=45$	Given
$9x=90$	Addition Property of Equality
$x=10$	Division Property of Equality

2.

Statements	Reasons
$20x-8(3+2x)=28x$	Given
$20x-24-16x=28x$	Distributive Property
$4x-24=28x$	Simplify
$-24=24x$	Addition Property of Equality
$-1=x$	Division Property of Equality

3. a.

Statements	Reasons
$m\angle B=m\angle E$	Given
$m\angle E=m\angle B$	Symmetric Property of Equality

b.

Statements	Reasons
$AD=NF$	Given
$NF=PG$	Given
$AD=PG$	Transitive Property of Equality

c.

Statements	Reasons
$m\angle 2+m\angle 4+m\angle 6=75°$	Given
$m\angle 4+m\angle 6=60°$	Given
$m\angle 2+60°=75°$	Substitution Property

4.

Statements	Reasons
1. $AB=CD$	**1.** Given
2. $BC=BC$	**2.** Reflexive Property of Equality
3. $AB+BC=CD+BC$	**3.** Addition Property of Equality
4. $AB+BC=AC$	**4.** Segment Addition Postulate
5. $CD+BC=BD$	**5.** Segment Addition Postulate
6. $AC=BD$	**6.** Substitution Property

5.

Statements	Reasons
1. $m\angle A=32°$	**1.** Given
2. $m\angle B=32°$	**2.** Given
3. $32°=m\angle B$	**3.** Symmetric Property of Equality
4. $m\angle A=m\angle B$	**4.** Transitive Property of Equality or Substitution Property
5. $\angle A\cong\angle B$	**5.** Definition of congruent angles

Vocabulary & Readiness Check 2.6

1. The statement "If $m\angle 1=m\angle 2,$ then $m\angle 2=m\angle 1$" is an example of the Symmetric Property.

2. The statement "$AB=AB$" is an example of the Reflexive Property.

3. The statement "If $m\angle C=m\angle G$ and $m\angle G=m\angle Q,$ then $m\angle C=m\angle Q$" is an example of the Transitive Property.

4. A proof is a logical argument used to establish the truth of a statement.

Exercise Set 2.6

2.

Statements	Reasons
$11x+14=80$	Given
$11x=66$	**a.** Subtraction Property of Equality
$x=6$	**b.** Division Property of Equality

4.

Statements	Reasons
$3x+6=-2x+31$	Given
$5x+6=31$	**a.** Addition Property of Equality
$5x=25$	**b.** Subtraction Property of Equality
$x=5$	**c.** Division Property of Equality

6.

Statements	Reasons
$5(x+3)=-4$	Given
$5x+15=-4$	**a.** Multiply or Distributive Property
$5x=-19$	**b.** Subtraction Property of Equality
$x=-\frac{19}{5}$	**c.** Division Property of Equality

8.

Statements	Reasons
$-20+2(11-2x)=26$	Given
$-20+22-4x=26$	**a.** Multiply or Distributive Property
$2-4x=26$	**b.** Simplify or Combine like terms
$-4x=24$	**c.** Subtraction Property of Equality
$x=-6$	**d.** Division Property of Equality

10.

Statements	Reasons
$XZ+ZY=XY$	**a.** Segment Addition Postulate
$3(n+4)+3n=42$	**b.** Substitution Property
$3n+12+3n=42$	**c.** Multiply or Distributive Property
$6n+12=42$	**d.** Simplify or Combine like terms
$6n=30$	**e.** Subtraction Property of Equality
$n=5$	**f.** Division Property of Equality

12. Each side of the first statement, $5x=20$, is divided by 5 to get the second statement, $x=4$. The Division Property of Equality justifies this operation.

14. BC is added to each side of the first statement, $AB-BC=12$, to get the second statement, $AB=12+BC$. The Addition Property of Equality justifies this operation.

16. The Transitive Property of Equality states that if $a=b$ and $b=c$, then $a=c$. So, D is correct.

18. The Addition Property states that if $a=b$, then $a+c=b+c$. Adding 12 to both sides of $3x-12=15$ results in $3x=27$. So, C is correct.

20. The Distributive Property states that $a(b+c)=ab+ac$. Distributing -2 to each term of $x-7$ results in $-2x+14$ on the left side of the equation. So, E is correct.

22. The Symmetric Property of Equality states that if $a=b$, then $b=a$. So, if $AB=YU$, then $\underline{YU=AB}$.

24. The Reflexive Property of Congruence states that $\angle A \cong \angle A$. So, $\angle POR \cong \underline{\angle POR}$.

26. The Substitution Property states that if $a=b$, then b can replace a in any expression. So, if $LM=7$ and $EF+LM=NP$, then $\underline{EF+7}=NP$.

28.

Statements	Reasons
1. $\overrightarrow{AC}$ is the bisector of $\angle DAG$.	**1. a.** Given
2. $\angle DAC \cong \angle CAG$	**2. b.** Definition of an angle bisector
3. $m\angle DAC = m\angle CAG$	**3.** $\cong$ angles have equal measure.
4. $9x=6x+9$	**4. c.** Substitution Property
5. d. $3x=9$	**5.** Subtraction Property of Equality
6. $x=3$	**6. e.** Division Property of Equality

30.

Statements	Reasons
1. $KM=KL+LM$	**1.** Segment Addition Postulate
2. $KM=35$	**2.** Given
3. $KL+LM=35$	**3.** Transitive Property of Equality or Substitution Property
4. $2x-5+2x=35$	**4.** Substitution Property
5. $4x-5=35$	**5.** Simplify or Combine like terms
6. $4x=40$	**6.** Addition Property of Equality
7. $x=10$	**7.** Division Property of Equality
8. $KL=2x-5$	**8.** Given
9. $KL=2(10)-5$	**9.** Substitution Property
10. $KL=20-5$	**10.** Multiply
11. $KL=15$	**11.** Simplify or Combine like terms

32. The Transitive Property of Falling Dominoes: If Domino A causes Domino B to fall, and Domino B causes Domino C to fall, then Domino A causes Domino C to fall.
This follows from the Transitive Property of Equality, which states that if $a = b$ and $b = c$, then $a = c$.

34. Choose a point on $\overrightarrow{AB}$ to the right of point B and call it C. So, the four angles formed by $\overline{AC}$ and $\overline{DE}$ are $\angle 1$, $\angle 2$, $\angle ABD$, and $\angle DBC$.
$\angle 1$ and $\angle 2$ form a linear pair and are supplementary. Thus, $m\angle 1 + m\angle 2 = 180°$.
$\angle ABD$ and $\angle DBC$ form a linear pair and are supplementary. Thus,
$m\angle ABD + m\angle DBC = 180°$.
$\angle 1$ and $\angle ABD$ form a linear pair and are supplementary. Thus,
$m\angle 1 + m\angle ABD = 180°$.
$\angle 2$ and $\angle DBC$ form a linear pair and are supplementary. Thus,
$m\angle 2 + m\angle DBC = 180°$.
Since $m\angle 1 + m\angle 2 = 180°$,
$m\angle 1 = 180° - m\angle 2$, or $m\angle 2 = 180° - m\angle 1$.
Substitute $180° - m\angle 2$ for $m\angle 1$ into the equation $m\angle 1 + m\angle ABD = 180°$.

$$\begin{aligned} m\angle 1 + m\angle ABD &= 180° \\ (180° - m\angle 2) + m\angle ABD &= 180° \\ -m\angle 2 + m\angle ABD &= 0 \\ m\angle ABD &= m\angle 2 \end{aligned}$$

So, $m\angle ABD = m\angle 2$.
Now, substitute $180° - m\angle 1$ for $m\angle 2$ into the equation $m\angle 2 + m\angle DBC = 180°$.

$$\begin{aligned} m\angle 2 + m\angle DBC &= 180° \\ (180° - m\angle 1) + m\angle DBC &= 180° \\ -m\angle 1 + m\angle DBC &= 0 \\ m\angle DBC &= m\angle 1 \end{aligned}$$

So, $m\angle DBC = m\angle 1$.

36. The relationship is not reflexive because the following statement cannot be true: A lives in a different state than A.
The relationship is symmetric because the following statement is true: If A lives in a different state than B, then B lives in a different state than A.
The relationship is not transitive because the following statement may not necessarily be true: If A lives in a different state than B and B lives in a different states than C, then A lives in a different state than C.

38. The relationship is not reflexive because the following statement cannot be true: A is shorter than A.
The relationship is not symmetric because the following statement cannot be true: If A is shorter than B, then B is shorter than A.
The relationship is transitive because the following statement is true: If A is shorter than B and B is shorter than C, then A is shorter than C.

40. The error is in the fifth step of the proof. The Division Property of Equality states that if $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. Since $x = -3$ is given, use substitution to evaluate $3 + x$.
$3 + x = 3 + (-3) = 0$
Since the value of $3 + x$ is 0 at $x = -3$, and division by 0 is not defined, the Division Property of Equality cannot be used to divide both sides of the equation by $3 + x$.

42. By the Angle Addition Postulate,
$m\angle DOB = m\angle COB + m\angle COD$. So,
$m\angle DOB = 20° + 45° = 65°$.

44. By the Angle Addition Postulate,
$m\angle BOE = m\angle BOC + m\angle COD + m\angle DOE$.
So, $m\angle BOE = 20° + 45° + 25° = 90°$.

46. By the Angle Addition Postulate, the angles that measure $55°$ and $y°$ form an angle with measure $(55 + y)°$. This angle and the right angle form a linear pair, so they are supplementary and the sum of their measures is $180°$. The measure of the right angle is $90°$ by definition.

$$\begin{aligned} (55 + y)° + 90° &= 180° \\ y + 145 &= 180 \\ y &= 35 \end{aligned}$$

Section 2.7 Practice

1. Equal Supplements Theorem: Supplements of the same angle (or of equal angles) are equal in measure.
If two angles are supplementary to the same angle (or to equal angles), then they are equal in measure.

2. Linear Pair Theorem: If two angles form a linear pair, then the angles are supplementary.
Angles that form a linear pair are supplementary.

3. Vertical Angles Theorem: Vertical angles are congruent.
If two angles are vertical angles, then they are congruent.

4.

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles.	**1.** Given
2. $m\angle 1 = 90°$; $m\angle 2 = 90°$	**2.** Definition of right angles
3. $m\angle 1 = m\angle 2$	**3.** Transitive Property
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

5. The angles are vertical angles and are thus equal in measure.

$$9x = 7x + 11$$
$$2x = 11$$
$$x = \frac{11}{2} \text{ or } 5.5$$

Vocabulary & Readiness Check 2.7

1. The new format of proof shown in this section is the <u>paragraph</u> proof.

2. The type of proof used most often thus far is the <u>two column</u> proof.

Exercise Set 2.7

2. $m\angle 2 = 115°$ by the Vertical Angles Theorem.

4. $m\angle 4 = 90° - 30° = 60°$ by the definition of a right angle.

6. $m\angle 6 = 90°$ by the Equal Supplementary Angles Theorem.

8. By the Vertical Angles Theorem, $m\angle 1 = 90°$, $m\angle 2 = 75°$, and $m\angle 3 = 15°$.

10. $\angle LQP$ and $\angle MQN$ are vertical angles and are thus equal in measure.

$$m\angle MQN = m\angle LQP$$
$$4x° = (x + 90)°$$
$$3x = 90$$
$$x = 30$$

12. $\angle RZS$ and $\angle WZT$ are vertical angles and are thus equal in measure.

$$m\angle RZS = m\angle WZT$$
$$3x° = 87°$$
$$x = 29$$

$\angle WZT$ and $\angle TZS$ form a linear pair, so they are supplementary and the sum of their angle measures is 180°.

$$m\angle WZT + m\angle TZS = 180°$$
$$m\angle TZS = 180° - m\angle WZT$$
$$y° = 180° - 87°$$
$$= 93$$

14. Use the value of x to find $m\angle LQP$ and $m\angle NQM$.

$$\begin{aligned} m\angle LQP &= (x+90)° \\ &= (30+90)° \\ &= 120° \\ m\angle NQM &= (4x)° \\ &= (4 \cdot 30)° \\ &= 120° \end{aligned}$$

Use the Linear Pair Theorem to find $m\angle LQM$.

$$m\angle LQP + m\angle LQM = 180°$$
$$m\angle LQM = 180° - m\angle LQP$$
$$y = 180° - 120°$$
$$= 60°$$

Use the Vertical Angles Theorem to find $m\angle NQP$.

$$m\angle NQP = m\angle LQM = 60°$$

16. $m\angle WZT = 87°$ is given.
Use the Vertical Angles Theorem to find $m\angle RZS$.
$m\angle WZT = m\angle RZS = 87°$
Use the value of y found in Exercise 12 to find $m\angle TZS$.
$m\angle TZS = y = 93°$
Use the Vertical Angles Theorem to find $m\angle RZW$.
$m\angle RZW = m\angle TZS = 93°$

18.

Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 1 \cong \angle 3$; $\angle 2 \cong \angle 4$	**2.** Vertical Angles Theorem
3. $\angle 3 = \angle 4$	**3.** Transitive Property

20. $\angle AOD \cong \angle BOC$ and $\angle AOB \cong \angle DOC$ by the Vertical Angles Theorem.

22. $\angle KPL \cong \angle MPL$ and $\angle KPJ \cong \angle MPJ$ by the Equal Supplements Theorem.

24. This is true by the Equal Supplementary Angles Theorem, which states that supplements of the same angle (or of equal angles) are equal in measure.

26. $m\angle 1 = m\angle 2$ and $m\angle 1 + m\angle 2 = 180°$ are given. $m\angle 1$ can be found by substitution.

$$m\angle 1 + m\angle 1 = 180°$$
$$2(m\angle 1) = 180°$$
$$m\angle 1 = 90°$$

Since $m\angle 1 = m\angle 2$ and $m\angle 1 = 90°$, by the Transitive Property, $m\angle 2 = 90°$. By the definition of right angles, $\angle 1$ and $\angle 2$ are both right angles.

28. Since the angle is formed by the path of the ball and the line perpendicular to the wall at the point of contact, $40° + x = 90°$.

$$40° + x = 90°$$
$$x = 50°$$

The ball bounces off the wall at the same angle it hit the wall, so $y = x$.

$$y = x$$
$$y = 50°$$

30. Use the Vertical Angles Theorem to find the value of x.

$$m\angle CHG = m\angle PHN$$
$$(3x+8)° = (5x-20)°$$
$$-2x = -28$$
$$x = 14$$

Use the value of x to find $m\angle CHG$.

$$m\angle CHG = (3x+8)°$$
$$= (3\cdot 14+8)°$$
$$= (42+8)°$$
$$= 50°$$

Use the Vertical Angles Theorem to find $m\angle PHN$.
$m\angle PHN = m\angle CHG = 50°$
Use the Linear Pair Theorem and the value of x to find y.

$$m\angle PHN + m\angle GHN = 180°$$
$$(5x-20)° + (5x+4y)° = 180°$$
$$5\cdot 14 - 20 + 5\cdot 14 + 4y = 180$$
$$70 - 20 + 70 + 4y = 180$$
$$120 + 4y = 180$$
$$4y = 60$$
$$y = 15$$

Use the value of y to find $m\angle GHN$.

$$m\angle GHN = (5\cdot 14 + 4\cdot 15)°$$
$$= (70+60)°$$
$$= 130°$$

Use the Vertical Angles Theorem to find $m\angle PHC$.
$m\angle PHC = m\angle GHN = 130°$

32. $m\angle A$ and $m\angle B$ are complements, so $m\angle A + m\angle B = 90°$.
It is given that $\angle A$ is half as large as $\angle B$, so

$$m\angle A = \frac{1}{2} m\angle B.$$

Use substitution to solve for $m\angle B$.

$$\frac{1}{2} m\angle B + m\angle B = 90°$$
$$\frac{3}{2}(m\angle B) = 90°$$
$$m\angle B = 60°$$

Solve the first equation for $m\angle A$.

$$m\angle A + m\angle B = 90°$$
$$m\angle A + 60° = 90°$$
$$m\angle A = 30°$$

34. $m\angle A$ and $m\angle B$ are supplements, so
$m\angle A + m\angle B = 180°$
It is given that $\angle A$ is half as large as twice of $\angle B$, so $m\angle A = \frac{1}{2}2(m\angle B) = m\angle B.$
Use substitution to solve for $m\angle B.$
$m\angle B + m\angle B = 180°$
$2(m\angle B) = 180°$
$m\angle B = 90°$
Solve the first equation for $m\angle A.$
$m\angle A + m\angle B = 180°$
$m\angle A + 90° = 180°$
$m\angle A = 90°$

36. Use the Linear Pair Theorem.
$(x-y)° + (x+y)° = 180°$
$x = 90$
Use the Vertical Angles Theorem and the value of *x*.
$2y° = (x-y)°$
$2y = 90 - y$
$3y = 90$
$y = 30$
The angle measures are as follows:
$(x-y)° = 2y° = 60°$
$(x+y)° = 120°$
The angle vertical to $(x+y)°$ is $120°.$

38. Use the Linear Pair Theorem and solve for *y*.
$2x° + (x+y+10)° = 180°$
$3x + y = 170$
$y = 170 - 3x$
Use substitution and the Linear Pair Theorem to solve for *x*.
$4y° + 2x° = 180°$
$4(170-3x) + 2x = 180$
$x = 50$
Use the value of *x* to solve for the value of *y*.
$y = 170 - 3x$
$y = 170 - 3(50)$
$y = 170 - 150$
$y = 20$
The angle measures are as follows:
$4y° = (x+y+10)° = 100°$
$2x° = 80°$
The angle vertical to $2x°$ is $80°.$

40. All angles that form a right angle are congruent and they all have a measure of $90°.$

42. Only the points along the line $y = -\frac{1}{3}x$ for $x > 0$ form an angle that is adjacent and complementary to the given angle. Example: $(3, -1)$

44. This is justified by the Division Property of Equality, which allows you to divide both sides of the equation by the same nonzero value.

46. Symmetric Property: If $a = b$, then $b = a.$

48. No, point *B* is not on the same line as points *A* and *G*, so they are not collinear.

50. *E* lies on line *r*, which can also be labeled as $\overleftrightarrow{EH}$, $\overleftrightarrow{HE}$, $\overleftrightarrow{GH}$, $\overleftrightarrow{HG}$, $\overleftrightarrow{HC}$, $\overleftrightarrow{CH}$, $\overleftrightarrow{EC}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{CG}$ or $\overleftrightarrow{GC}$.

52. Lines *t* and *r* intersect at point *H*.

Chapter 2 Vocabulary Check

1. The distance around a circle is given by a special name, called <u>circumference</u>.

2. A <u>counterexample</u> is an example that shows that a conjecture is false.

3. The process of arriving at a general conclusion based on observing specific examples is called <u>inductive</u> reasoning.

4. The distance around a geometric figure is called <u>perimeter</u>.

5. The number of square units a geometric figure encloses is called <u>area</u>.

6. The statement "If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$" is an example of the <u>Symmetric</u> Property.

7. The statement "$AB = AB$" is an example of the <u>Reflexive</u> Property.

8. The statement "if $m\angle C = m\angle G$ and $m\angle G = m\angle Q$, then $m\angle C = m\angle Q$" is an example of <u>Transitive</u> Property.

9. For an "if-then" statement, the part following "then" is called the conclusion.

10. For an "if-then" statement, the part following "if" is called the hypothesis.

11. Two statements that are both always true or both always false are called equivalent statements.

12. In symbols, the negation of p is written as $\underline{\sim p}$.

13. In symbols, an "if p then q" statement can be written as $\underline{p \to q}$.

14. The converse of the statement $p \to q$ is $\underline{q \to p}$.

15. The symbol for "if and only if" is $\underline{\leftrightarrow}$.

16. A biconditional statement is written using a conditional statement and its converse.

17. A biconditional statement may be written using the phrase if and only if.

18. A good definition may be rewritten as biconditional statement.

19. A true biconditional statement $p \leftrightarrow q$ may be used "forward" and "backward." In symbols, forward and backward mean $\underline{p \to q}$ and $\underline{q \to p}$, respectively.

20. The Law of Detachment says that if $p \to q$ is true and p is true, then q is true.

21. The Law of Syllogism says that if $p \to q$ is true and $q \to r$ is true, then $p \to r$ is true.

22. Deductive reasoning is used to prove theorems.

23. With inductive reasoning, your conclusion may or may not be true.

24. $p \to q$ is a conditional statement.

25. $\sim p \to \sim q$ is an inverse.

26. $\sim q \to \sim p$ is a contrapositive.

27. $q \to p$ is a converse.

28. $\sim p$ is a negation.

Chapter 2 Review

1. The figure is a square because all four sides are the same length and all four angles are right angles. Use the formula for the perimeter of a square.
$P = 4s = 4(8 \text{ cm}) = 32 \text{ cm}$
Use the formula for the area of a square.
$A = s^2 = (8 \text{ cm})^2 = 64 \text{ cm}^2$

2. Use the formula for the perimeter of a rectangle.
$$\begin{aligned} P &= 2l + 2w \\ &= 2(12 \text{ m}) + 2(8 \text{ m}) \\ &= 24 \text{ m} + 16 \text{ m} \\ &= 40 \text{ m} \end{aligned}$$
Use the formula for the area of a rectangle.
$$\begin{aligned} A &= lw \\ &= (12 \text{ m})(8 \text{ m}) \\ &= 96 \text{ m}^2 \end{aligned}$$

3. Use the formula for the circumference of a circle.
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3 \text{ in.}) \\ &= 6\pi \text{ in.} \end{aligned}$$
Use the formula for the area of a circle.
$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3 \text{ in.})^2 \\ &= 9\pi \text{ in.}^2 \end{aligned}$$

4. Divide the diameter by 2 to find the radius. The radius is 7.5 m. Use the formula for the circumference of a circle.
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(7.5 \text{ m}) \\ &= 15\pi \text{ m} \end{aligned}$$
Use the formula for the area of a circle.
$$\begin{aligned} A &= \pi r^2 \\ &= \pi(7.5 \text{ m})^2 \\ &= 56.25\pi \text{ m}^2 \end{aligned}$$

5. Divide the previous term by 10. The next two terms are 1 and $\frac{1}{10}$.

6. Multiply the previous term by -1. The next two terms are 5 and -5.

7. Subtract 7 from the previous term. The next two terms are 6 and -1.

8. Multiply the previous term by 4. The next two terms are 1536 and 6144.

9. The conjecture is false for any negative integer. For example:
$-1 \cdot 2 = -2$
$-2 < 2$

10. There is a city of Portland in Maine, so the city of Portland is not necessarily in Oregon.

11. If a person is a motorcyclist, then that person wears a helmet.

12. If two lines are nonparallel, then the lines will intersect at one point.

13. If two angles form a linear pair, then they are supplementary.

14. If school is closed, then it is a certain holiday.

15. Converse $(q \to p)$: If the measure of an angle is greater than 90º and less than 180º, then the angle is obtuse.
Inverse $(\sim p \to \sim q)$: If an angle is not obtuse, then it is not true that its measure is greater than 90º and less than 180º.
Contrapositive $(\sim q \to \sim p)$: If it is not true that the measure of an angle is greater than 90º and less than 180º, then the angle is not obtuse.
The original conditional statement, the converse, the inverse, and the contrapositive are all true.

16. Converse $(q \to p)$: If a figure has four sides, then the figure is a square.
Inverse $(\sim p \to \sim q)$: If a figure is not a square, then the figure does not have four sides.
Contrapositive $(\sim q \to \sim p)$: If a figure does not have four sides, then the figure is not a square.
The original conditional statement and contrapositive are true. The converse and inverse are false.

17. Converse $(q \to p)$: If you play an instrument, then you play the tuba.
Inverse $(\sim p \to \sim q)$: If you do not play the tuba, then you do not play an instrument.
Contrapositive $(\sim q \to \sim p)$: If you do not play an instrument, then you do not play the tuba.
The original conditional statement and contrapositive are true. The converse and inverse are false.

18. Converse $(q \to p)$: If you are busy on Saturday night, then you are the manager of a restaurant. False.
Inverse $(\sim p \to \sim q)$: If you are not the manager of a restaurant, then you are not busy on Saturday night. False.
Contrapositive $(\sim q \to \sim p)$: If you are not busy on Saturday night, then you are not the manager of a restaurant. False.
The original conditional statement, converse, inverse, and contrapositive are all false.

19. This is a good definition because it satisfies a biconditional statement. An animal is a bird if and only if it has feathers.

20. This is not a good definition because the converse of the definition is not true. Magazines or websites can also have articles that you read.

21. This is a good definition because it satisfies a biconditional statement. A pair of adjacent angles is a linear pair if and only if the angles' noncommon sides are opposite rays.

22. This is not a good definition because its converse is not true. A geometric figure does not have to be an angle.

23. A phrase is an oxymoron if and only if it contains contradictory terms.

24. If two angles are complementary, then the sum of their measures is 90º.
If the sum of the measures of two angles is 90º, then they are complementary.

25. Colin will become a better player.

26. The sum of the measures of $\angle 1$ and $\angle 2$ is 180º.

27. If two angles are vertical, then their measures are equal.

28. If your father buys new gardening gloves, then he will plant tomatoes.

29–34.

Statements	Reasons
1. $QS = 42$	**1.** <u>Given</u> (Exercise 29)
2. $QR + RS = QS$	**2.** <u>Segment Addition Postulate</u> (Exercise 30)
3. $(x+3)+2x = 42$	**3.** <u>Substitution</u> (Exercise 31)
4. $3x+3=42$	**4.** <u>Simplify</u> (Exercise 32)
5. $3x = 39$	**5.** <u>Subtraction Property of Equality</u> (Exercise 33)
6. $x = 13$	**6.** Division Property of Equality (Exercise 34)

35. If $2(AX) = 2(BY)$, then $\underline{AX = BY}$.

36. $3p - 6q = 3(\underline{p-2q})$

37. Use the vertical angles theorem. The angles are congruent, so they have the same measure.

$$\begin{aligned} 3y+20 &= 5y-16 \\ -2y+20 &= -16 \\ -2y &= -36 \\ y &= 18 \end{aligned}$$

38. Substitute the value of y found in Exercise 37 into the angle measurement.
$3y+20 = 3(18)+20 = 54+20 = 74°$

39. Substitute the value of y found in Exercise 37 into the angle measurement.
$5y-16 = 5(18)-16 = 90-16 = 74°$

40. $\angle AEB$ forms a linear pair with $\angle AEC$, so the angles are supplementary.
$m\angle AEB = 180 - 74 = 106°$

41.

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complementary; $\angle 3$ and $\angle 4$ are complementary	**1.** Given
2. $\angle 2 \cong \angle 4$	**2.** Given
3. $m\angle 1 + m\angle 2 = 90°$; $m\angle 3 + m\angle 4 = 90°$	**3.** Definition of complementary angles
4. $m\angle 2 = m\angle 4$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**5.** Transitive Property
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	**6.** Substitution
7. $m\angle 1 = m\angle 3$	**7.** Subtraction Property of Equality
8. $\angle 1 \cong \angle 3$	**8.** Definition of congruent angles

42. Multiply the previous term by -3. The next two terms are 81 and -243.

43. The converse is "if you are younger than 20, then you are a teenager," and it is false because people aged 12 or under are not considered teenagers.

44. Transitive Property of Congruence

45.

Statements	Reasons
1. $\angle 1 \cong \angle 4$	**1.** Given
2. $\angle 1$ and $\angle 2$ are vertical angles; $\angle 3$ and $\angle 4$ are vertical angles	**2.** Definition of vertical angles
3. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	**3.** Vertical Angles Theorem
4. $\angle 1 \cong \angle 3$	**4.** Transitive Property of Congruence
5. $\angle 2 \cong \angle 3$	**5.** Substitution

46. The definition is not reversible because four-sided figures do not have to be rectangles.

47. Use the Law of Detachment.
If you play hockey, then you are a varsity athlete.

Chapter 2 Test

1. The figure is a square because all four sides are the same length and all four angles are right angles. Use the formula for the perimeter of a square.
$P = 4s = 4(9 \text{ cm}) = 36 \text{ cm}$
Use the formula for the area of a square.
$A = s^2 = (9 \text{ cm})^2 = 81 \text{ cm}^2$

2. Extend the length and width of the garden by 2 ft to get the figure that contains both the walkway and the garden. Use the formula for the perimeter of a rectangle.
$$\begin{aligned} P &= 2l + 2w \\ &= 2(6 \text{ ft} + 2 \text{ ft}) + 2(5 \text{ ft} + 2 \text{ ft}) \\ &= 2(8 \text{ ft}) + 2(7 \text{ ft}) \\ &= 16 \text{ ft} + 14 \text{ ft} \\ &= 30 \text{ ft} \end{aligned}$$

3. Use the formula for the circumference of a circle. The diameter is given, so divide the diameter by 2 to get the radius.
$$C = 2\pi r = 2\pi\left(\frac{9}{2} \text{ cm}\right) = 9\pi \text{ cm}$$
Use the formula for the area of a circle.
$$A = \pi r^2 = \pi\left(\frac{9}{2} \text{ cm}\right)^2 = \frac{81}{4}\pi \text{ cm}^2$$

4. Use the formula for the circumference of a circle.
$C = 2\pi r = 2\pi(5 \text{ ft}) = 10\pi \text{ ft}$
$C \approx 10(3.14) \text{ ft} = 31.4 \text{ ft}$
Use the formula for the area of a circle.
$A = \pi r^2 = \pi(5 \text{ ft})^2 = 25\pi \text{ ft}^2$
$A \approx 25(3.14) \text{ ft} = 78.5 \text{ ft}^2$

5. Divide the previous term by -2. The next two terms are -1 and $\frac{1}{2}$.

6. These are the squares of the whole numbers in order. The next two terms are 36 and 49.

7. Answers may vary. Sample answer: Garter snakes are not poisonous.

8. Two 45º angles are complementary and congruent.

9. Hypothesis (p): $x + 9 = 11$
Conclusion (q): $x = 2$

10. If a polygon is a quadrilateral, then the polygon has four sides.

11. c; an obtuse angle does not measure 79º.

12. Converse $(q \to p)$: If a figure has at least two right angles, then the figure is a square.
Inverse $(\sim p \to \sim q)$: If a figure is not a square, then the figure does not have at least two right angles.
Contrapositive $(\sim q \to \sim p)$: If a figure does not have at least two right angles, then the figure is not a square.

13. Converse $(q \to p)$: If a square's perimeter is 12 meters, then its side length is 3 meters.
Inverse $(\sim p \to \sim q)$: If a square does not have side length 3 meters, then its perimeter is not 12 meters.
Contrapositive $(\sim q \to \sim p)$: If a square's perimeter is not 12 meters, then its side length is not 3 meters.

14. If a fish is a bluegill, then it is a bluish, fresh-water sunfish.
If a fish is a bluish, fresh-water sunfish, then it is a bluegill.

15. **a.** If A, B, and C lie on the same line, then they are collinear.
b. True; this is the definition of collinear.
c. A, B, and C are collinear if and only if they lie on the same line.

16. d; A yard is a unit of measure exactly 3 feet long. The converse of this definition is true, and it confirms that nothing else can be mistaken for a yard.

17. Supplementary angles do not have to be a linear pair. As a counterexample, any two angles of a rectangle can be supplementary, but in no cases do they form a straight line.

18. Transitive Property of Equality

19. Subtraction Property of Equality

20. Reflexive Property of Congruence

21. Symmetric Property of Congruence

22. $\angle LNP \cong \angle VNM$ and $\angle LNV \cong \angle PNM$, by the Vertical Angles Theorem.

23. $\angle BCE \cong \angle DCF$ is given. $\angle BCF \cong \angle DCE$ by the Equal Supplements Theorem.

24. $\angle BZR \cong \angle NZP$ by the Vertical Angles Theorem. This means their measures are equal by definition of congruence.

$$2x = 5x - 63$$
$$-3x = -63$$
$$x = 21$$
$$m\angle BZR = 2x^\circ = 2(21)^\circ = 42^\circ$$
$$m\angle NZP = m\angle BZR = 42^\circ$$
$$m\angle RZN = 180^\circ - 42^\circ = 138^\circ$$
$$m\angle BZP = \angle RZN = 138^\circ$$

25. Not possible; no conclusion can be made using the Law of Detachment because the conclusion is given instead of the hypothesis.

26. Use the Law of Syllogism.
If the traffic light is red, then you must apply your brakes.

27.

Statements	Reasons
1. $\underline{B \text{ is the midpoint of } \overline{AC}}$ (**a.**)	**1.** Given
2. $AB = BC$	**2.** Definition of midpoint (**b.**)
3. $AB + BC = AC$	**3.** Segment Addition Postulate (**c.**)
4. $AB + AB = AC$	**4.** Substitution (**d.**)
5. $\underline{2AB = AC}$ (**e.**)	**5.** Simplify.
6. $AB = \dfrac{AC}{2}$	**6.** Division Property of Equality (**f.**)

28. $\angle FED$ and $\angle DEW$ are complementary because it is given. By definition of complementary angles, $\underline{m\angle FED + m\angle DEW = 90^\circ}$. $m\angle FED + m\angle DEW = m\angle FEW$ by the Angle Addition Postulate. $90^\circ = m\angle FEW$ by the Substitution or Transitive Property of Equality. Then $\angle FEW$ is a right angle by the definition of a right angle.

