

Chapter Three

ISENTROPIC FLOW OF A PERFECT GAS

Problem 1. – Air flows at Mach 0.25 through a circular duct with a diameter of 60 cm. The stagnation pressure of the flow is 500 kPa; the stagnation temperature is 175°C. Calculate the mass flow rate through the channel, assuming $\gamma = 1.4$ and that the air behaves as a perfect gas with constant specific heats.

$$p = \left(\frac{p}{p_o} \right) 500 \text{ kPa} = 0.9575(500) = 478.7500 \text{ kPa}$$

$$T = \left(\frac{T}{T_o} \right) (175 + 273) = 0.9877(448) = 442.4896 \text{ K}$$

$$\rho = \frac{p}{RT} = \frac{(478.75 \text{ kN/m}^2)}{(0.287 \text{ kN} \cdot \text{m/kg} \cdot \text{K})(442.4896 \text{ K})} = 3.7698 \text{ kg/m}^3$$

$$A = \frac{\pi}{4} (0.6)^2 = 0.2827 \text{ m}^2$$

$$V = M \sqrt{\gamma RT} = 0.25 \sqrt{1.4(287 \text{ N} \cdot \text{m/kg} \cdot \text{K})(442.4896 \text{ K})} = 105.4136 \text{ m/s}$$

$$\dot{m} = \rho AV = 112.3603 \text{ kg/s}$$

Problem 2. – Helium flows at Mach 0.50 in a channel with cross-sectional area of 0.16 m². The stagnation pressure of the flow is 1 MPa, and stagnation temperature is 1000 K. Calculate the mass flow rate through the channel, with $\gamma = 5/3$.

$$p = \left(\frac{p}{p_o} \right) 1 \text{ MPa} = 0.8186(1000 \text{ kPa}) = 818.6 \text{ kPa}$$

$$T = \left(\frac{T}{T_o} \right) (1000 \text{ K}) = 0.9231(1000) = 923.1 \text{ K}$$

$$R = 2.077 \text{ kJ/kg} \cdot \text{K}$$

$$\rho = \frac{p}{RT} = \frac{818.6}{(2.077)(923.1)} = 0.4270 \text{ kg/m}^3$$

$$V = M\sqrt{\gamma RT} = 0.50\sqrt{(5/3)(2077 \text{ N} \cdot \text{m/kg} \cdot \text{K})(923.1 \text{ K})} = 893.7931 \text{ m/s}$$

$$\dot{m} = \rho AV = (0.4270 \text{ kg/m}^3)(0.16 \text{ m}^2)(893.7931 \text{ m/s}) = 61.0639 \text{ kg/s}$$

Problem 3. – In Problem 2, the cross-sectional area is reduced to 0.12 m^2 . Calculate the Mach number and flow velocity at the reduced area. What percent of further reduction in area would be required to reach Mach 1 in the channel?

$$\frac{A_2}{A^*} = \frac{A_2}{A_1} \frac{A_1}{A^*} = \left(\frac{0.12}{0.16}\right) 1.3203 = 0.9902$$

So, $A_2 < A^*$ for $M_1 = 0.5$. Therefore, $M_2 = 1$ and M_1 will be reduced below 0.5. Since the exit Mach number is 1, then $A_2 = A^*$,

$$\frac{A_1}{A^*} = \frac{A_1}{A_2} \frac{A_2}{A^*} = \left(\frac{0.16}{0.12}\right) 1 = 1.3333$$

Using this area ratio we find: $M_1 = 0.4930$. Now $M_2 = 1$ so

$$T_2 = \left(\frac{T_2}{T_0}\right) T_0 = (0.7500) 1000 = 750.0 \text{ K}$$

$$V_2 = M_2 \sqrt{\gamma RT} = 1.0 \sqrt{(5/3)(2077) 750} = 1611.2883 \text{ m/s}$$

Problem 4. – (a) For small Mach numbers, determine an expression for the density ratio ρ/ρ_0 . (b) Using Eqs. (3.15) and (3.17), prove that

$$\left(\frac{p_0}{p}\right) \left(\frac{\rho}{\rho_0}\right) = \left(\frac{T_0}{T}\right) = \left(\frac{a_0}{a}\right)^2$$

(a)

$$\frac{\rho}{\rho_o} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{\gamma-1}} \cong 1 + \left(\frac{\gamma-1}{2}\right) \left(-\frac{1}{\gamma-1}\right) M^2 + \dots = 1 - \frac{M^2}{2} + \dots$$

(b)

$$\begin{aligned} \left(\frac{p_o}{p}\right) \left(\frac{\rho}{\rho_o}\right) &= \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} M^2\right)^1 \\ &= \frac{T_o}{T} = \frac{a_o^2/\gamma R}{a^2/\gamma R} = \left(\frac{a_o}{a}\right)^2 \end{aligned}$$

Problem 5. – An airflow at Mach 0.6 passes through a channel with a cross-sectional area of 50 cm². The static pressure in the airstream is 50 kPa; static temperature is 298 K.

(a) Calculate the mass flow rate through the channel.

(b) What percent of reduction in area would be necessary to increase the flow Mach number to 0.8? to 1.0?

(c) What would happen if the area were reduced more than necessary to reach Mach 1?

(a)
$$\rho = \frac{p}{RT} = \frac{50 \text{ kPa}}{(0.287 \text{ kN} \cdot \text{m/kg} \cdot \text{K}) 298 \text{ K}} = 0.5846 \text{ kg/m}^3$$

$$V = M\sqrt{\gamma RT} = 0.6\sqrt{1.4(287)298} = 207.6177 \text{ m/s}$$

$$\dot{m} = \rho AV = (0.5846)(0.0050 \text{ m}^2)(207.6177 \text{ m/s}) = 0.6069 \text{ kg/s}$$

(b) For M = 0.8, A/A* = 1.0382

For M = 0.6, A/A* = 1.1882

$$(\% \text{ reduction in area to reach Mach 0.8}) = \frac{1.1882 - 1.0382}{1.1882} 100 = 12.62\%$$

$$(\% \text{ reduction in area to reach Mach 1.0}) = \frac{1.1882 - 1}{1.1882} 100 = 15.84\%$$

(c) Flow would be reduced.

Problem 6. – A converging nozzle with an exit area of 1.0 cm² is supplied from an oxygen reservoir in which the pressure is 500 kPa and the temperature is 1200 K. Calculate the mass flow rate of oxygen for back pressures of 0, 100, 200, 300, and 400 kPa. Assume that $\gamma = 1.3$.

For $\gamma = 1.3$, the critical pressure ratio is: $\frac{p^*}{p_o} = 0.5457$. So, the back pressure is

$$p_b = \left(\frac{p^*}{p_o}\right)p_o = 0.5457(500) = 272.8500 \text{ kPa},$$

Thus, the nozzle is choked for back-pressures below 272.85 kPa, i.e., for 0, 100, and 200 kPa. For these back pressures, $p_e = 272.8$ kPa and

$$T_e = \left(\frac{T_e}{T_o}\right)T_o = 0.8696(1200 \text{ K}) = 1043.5200 \text{ K}$$

$$V_e = M_e \sqrt{\gamma RT_e} = \sqrt{1.3(259.8)(1043.52)} = 593.6653 \text{ m/s}$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{272.85 \text{ kN/m}^2}{(0.2598 \text{ kJ/kg} \cdot \text{K})(1043.52 \text{ K})} = 1.0064 \text{ kg/m}^3$$

$$\dot{m} = \rho_e A_e V_e = (1.0064)(1 \times 10^{-4})(593.6653) = 0.05975 \text{ kg/s}$$

For $p_b = p_e = 300$ kPa; thus, $\frac{p_e}{p_o} = \frac{300}{500} = 0.6$, from which we find $M_e = 0.9133$

$$T_e = \left(\frac{T_e}{T_o}\right)T_o = 0.8888(1200 \text{ K}) = 1066.5600 \text{ K}$$

$$V_e = M_e \sqrt{\gamma RT_e} = 0.9133 \sqrt{1.3(259.8)(1066.56)} = 548.1474 \text{ m/s}$$

$$\dot{m} = \left(\frac{p_e}{RT_e}\right)(1 \times 10^{-4} \text{ m}^2)(548.1474 \text{ m/s}) = \frac{300}{0.2598(1066.56)} \frac{\text{kg}}{\text{m}^3} (1 \times 10^{-4} \text{ m}^2)(548.1474 \text{ m/s})$$

$$= 0.05935 \text{ kg/s}$$

For $p_b = p_e = 400$ kPa, $\frac{p_e}{p_o} = 0.8$, $M_e = 0.5935$

$$T_e = 0.9498(1200) = 1139.7600 \text{ K}$$

$$\dot{m} = \left(\frac{400}{0.2598 \times 1139.76} \right) (10^{-4}) \left[0.5935 \sqrt{1.3(259.8)1139.76} \right] = 0.04974 \text{ kg/s}$$

Problem 7. – Compressed air is discharged through the converging nozzle as shown in Figure P3.7. The tank pressure is 500 kPa, and local atmospheric pressure is 101 kPa. The inlet area of the nozzle is 100 cm²; the exit area is 34 cm². Find the force of the air on the nozzle, assuming the air to behave as a perfect gas with constant $\gamma = 1.4$. Take the temperature in the tank to be 300 K.

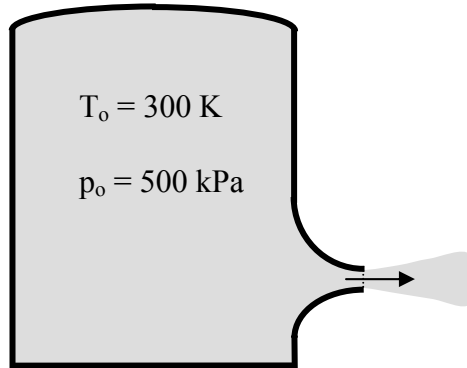


Figure P3.7

Assume the nozzle is choked. Accordingly, $p_e = 0.5283 (500 \text{ kPa}) = 264.15 \text{ kPa}$. Since this pressure exceeds the back pressure, the assumption is valid.

$$M_e = 1.0$$

$$T_e = 0.8333(300) = 249.9900 \text{ K}$$

$$V_e = M_e \sqrt{\gamma R T_e} = \sqrt{1.4(287)249.99} = 316.9321 \text{ m/s}$$

At the nozzle inlet, $\frac{A_i}{A^*} = \frac{100}{34} = 2.9412$, from which we find $M_i = 0.2038$

$$\frac{T_i}{T_o} = 0.9938, \text{ so } T_i = 0.9938(300) = 298.1400 \text{ K}$$

$$\frac{p_i}{p_o} = 0.9735, \text{ } p_i = 0.9735(500) = 486.7500 \text{ kPa}$$

$$V_i = 0.2038 \sqrt{1.4(287)298.14} = 70.5374 \text{ m/s}$$

$$\dot{m}_e = \frac{264.15}{0.287(249.99)}(0.0034)(316.9321) = 3.9673 \text{ kg/s}$$

$$p_i A_i + F_T - p_e A_e - p_{\text{atm}}(A_i - A_e) = \dot{m}(V_e - V_i)$$

$$\therefore F_T = p_e A_e - p_i A_i + p_{\text{atm}}(A_i - A_e) + \dot{m}(V_e - V_i)$$

$$\begin{aligned} F_T &= (264.15 \text{ kN/m}^2)(34 \times 10^{-4} \text{ m}^2) - (486.75 \text{ kN/m}^2)(100 \times 10^{-4} \text{ m}^2) \\ &\quad + (101.0 \text{ kN/m}^2)(100 - 34)10^{-4} \\ &\quad + \frac{(3.9673 \text{ kg/s})(316.9321 - 70.5374 \text{ m/s})}{1000 \text{ N/kN}} \end{aligned}$$

$$F_T = 0.8981 - 4.8675 + 0.6666 + 0.9775 = -2.3253 \text{ kN}$$

The force of the fluid on the nozzle (equal but opposite) is 2.3253 kN to the right.

Problem 8. – A converging nozzle has an exit area of 56 cm. Nitrogen stored in a reservoir is to be discharged through the nozzle to an ambient pressure of 100 kPa. Determine the flow rate through the nozzle for reservoir pressures of 120 kPa, 140 kPa, 200 kPa, and 1 MPa. Assume isentropic nozzle flow. In each case, determine the increase in mass flow to be gained by reducing the back pressure from 100 to 0 kPa. Reservoir temperature is 298 K.

For N_2 , $\gamma = 1.40$. The nozzle is choked for

$$p_o = \frac{p_b}{(p^*/p_o)} = \frac{100}{0.5283} = 189.2864 \text{ kPa}$$

Case 1. $p_o = 120 \text{ kPa}$ and $p_b = 100 \text{ kPa}$

$$\frac{p_e}{p_o} = \frac{100}{120} = 0.8333, \quad M_e = 0.5171, \quad \frac{T_e}{T_o} = 0.9492$$

$$T_e = 0.9492(298) = 282.8616 \text{ K}$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{100 \text{ kN/m}^2}{(0.2968 \text{ kJ/kg} \cdot \text{k})282.8616 \text{ K}} = 1.1911 \text{ kg/m}^3$$

$$V_e = M_e \sqrt{\gamma RT_e} = 0.5171 \sqrt{1.4(296.8)282.8616} = 177.2791 \text{ m/s}$$

$$\dot{m} = \rho_e A_e V_e = (1.1911) \left(56 \times 10^{-4} \right) 177.2791 = 1.1825 \text{ kg/s}$$

Case 2. $p_o = 140 \text{ kPa}$ and $p_b = 100 \text{ kPa}$

$$\frac{p_e}{p_o} = \frac{100}{140} = 0.7143, \quad M_e = 0.7103, \quad T_e = 0.9083(298) = 270.6734 \text{ K}$$

$$\rho_e = \frac{100}{0.2968(270.6734)} = 1.2448 \text{ kg/m}^3$$

$$V_e = 0.7103 \sqrt{1.4(296.8)270.6734} = 238.2103 \text{ m/s}$$

$$\dot{m} = 1.2448 \left(56 \times 10^{-4} \right) 238.2103 = 1.6605 \text{ kg/s}$$

Case 3. $p_o = 200 \text{ kPa}$ and $p_b = 100 \text{ kPa}$

Since p_o is above the critical reservoir pressure the nozzle is choked, therefore $M_e = 1.0$

$$p_e = 0.5283(200) = 105.6600 \text{ kPa}$$

$$T_e = 0.8333(298) = 248.3234 \text{ K}$$

$$\rho_e = \frac{105.66}{0.2968(248.3234)} = 1.4336 \text{ kg/m}^3$$

$$V_e = 1.0 \sqrt{1.4(296.8)248.3234} = 321.2216 \text{ m/s}$$

$$\dot{m} = (1.4336) \left(56 \times 10^{-4} \right) (321.2216) = 2.5788 \text{ kg/s}$$

Case 4. $p_o = 1 \text{ MPa} = 1000 \text{ kPa}$ and $p_b = 100 \text{ kPa}$

$$\dot{m} = 2.5788 \left(\frac{1000}{200} \right) = 12.8941 \text{ kg/s}$$

Case 5. $p_o = 120 \text{ kPa}$ and $p_b = 0 \text{ kPa}$

For **Case 1**, lowering back pressure to 0 kPa will change the flow and the nozzle will now be choked. Therefore,

$$M_e = 1.0,$$

$$V_e = 321.2216 \text{ m/s}$$

$$\rho_e = \frac{0.5283(120)}{0.2968(248.3234)} = 0.8602 \text{ kg/m}^3$$

$$\dot{m} = (0.8602)(56 \times 10^{-4})(321.2216) = 1.5473 \text{ kg/s}$$

Case 6. $p_o = 140 \text{ kPa}$ and $p_b = 0 \text{ kPa}$

The nozzle is choked, so $M_e = 1$

$$\dot{m} = \frac{140}{120}(1.5473) = 1.8052 \text{ kg/s}$$

Case 7.

$$\dot{m} = \frac{200}{120}(1.5473) = 2.5788 \text{ kg/s}$$

Case 8.

$$\dot{m} = \frac{1000}{120}(1.5473) = 12.8941 \text{ kg/s}$$

Problem 9. – Pressurized liquid water flows from a large reservoir through a converging nozzle. Assuming isentropic nozzle flow with a negligible inlet velocity and a back pressure of 101 kPa, calculate the reservoir pressure necessary to choke the nozzle. Assume that the isothermal compressibility of water is constant at $5 \times 10^{-7} \text{ (kPa)}^{-1}$ and equal to the isentropic compressibility. Exit density of the water is 1000 kg/m^3 .

$$\int \frac{dp}{\rho} + \frac{V^2}{2} = c$$

$$k_T \approx k_s = \frac{1}{\rho} \frac{\partial \rho}{\partial p} = \frac{1}{\rho} \frac{d\rho}{dp}$$

$$\frac{1}{k_T} \int_1^2 \frac{d\rho}{\rho^2} + \frac{V_2^2}{2} = 0$$

$$\frac{1}{k_T} \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) + \frac{V_2^2}{2} = 0$$

$$V_2 = a_2 = \sqrt{\frac{1}{\rho_2 k_s}} = \sqrt{\frac{1}{(1000 \text{ kg/m}^3) 5 \times 10^{-7} (\text{kPa})^{-1}}} = 1414.2136 \text{ m/s}$$

$$\frac{1}{\rho_1} - \frac{1}{\rho_2} = \left(-\frac{1414.2136^2 \text{ m}^2}{2 \text{ s}^2} \right) (5 \times 10^{-7}) \text{ kPa}^{-1} = -0.0005 \text{ m}^3/\text{kg}$$

$$\frac{1}{\rho_1} = \frac{1}{1000 \text{ kg/m}^3} - 0.0005 \text{ m}^3/\text{kg}$$

$$\rho_1 = 2000.0 \text{ kg/m}^3$$

$$\int_1^2 dp = \frac{1}{k_T} \int_1^2 \frac{d\rho}{\rho}$$

$$p_2 - p_1 = \frac{1}{k_T} \ln\left(\frac{\rho_2}{\rho_1}\right) = \frac{1}{5 \times 10^{-7} (\text{kPa})^{-1}} \ln\left(\frac{1000}{2000}\right) = -1.3863 \times 10^6 \text{ kPa}$$

or $p_r = 101 + 1.3863 \times 10^6 = 1.3864 \times 10^6 \text{ kPa}$

Problem 10. – Calculate the stagnation temperature in an airstream traveling at Mach 5 with a static temperature of 273 K (see Figure P3.10). An insulated flat plate is inserted into this flow, aligned parallel with the flow direction, with a boundary layer building up along the plate. Since the absolute velocity at the plate surface is zero, would you expect the plate temperature to reach the free stream stagnation temperature? Explain.

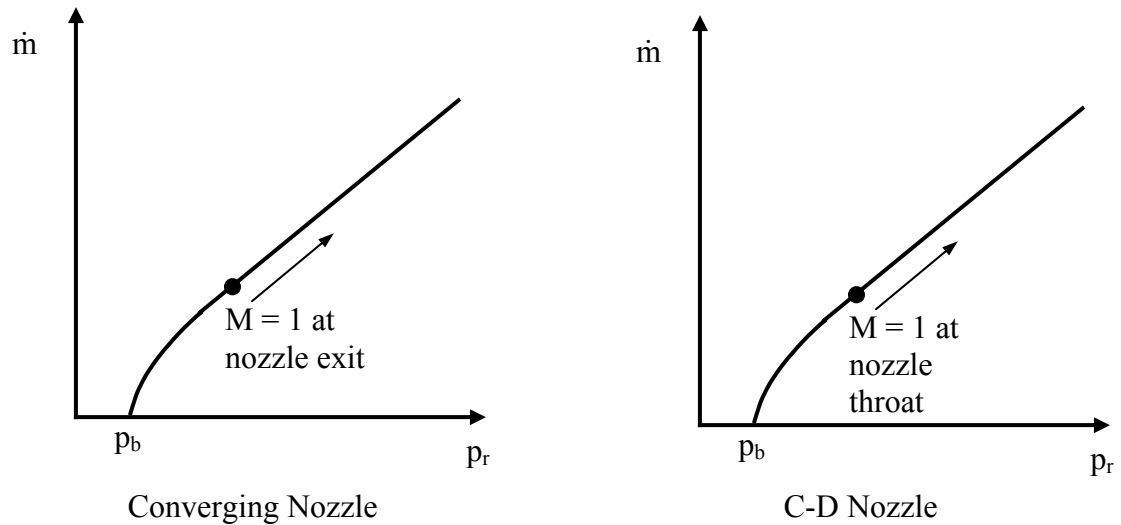


Figure P3.10

$$T_0 = \frac{273}{0.1667} = 1637.7 \text{ K}$$

No. In general the reduction to zero speed is not an adiabatic process. However, it could be if viscous heating counteracts heat conduction back through the boundary layer.

Problem 11. – A gas stored in a large reservoir is discharged through a converging nozzle. For a constant back pressure, sketch a plot of mass flow rate versus reservoir pressure. Repeat for a converging-diverging nozzle.



Problem 12. – A converging-diverging nozzle is designed to operate isentropically with air at an exit Mach number of 1.75. For a constant chamber pressure and temperature of 5 MPa and 200°C, respectively, calculate the following:

- (a) Maximum back pressure to choke nozzle
- (b) Flow rate in kilograms per second for a back pressure of 101 kPa
- (c) Flow rate for a back pressure of 1 MPa Nozzle exit area is 0.12 m².

(a) For $M = 1.75$, $\frac{A}{A^*} = 1.3865$

For $\frac{A}{A^*} = 1.3865$, $M = 0.4770$, $\frac{p}{p_0} = 0.8558$

Maximum back pressure to choke nozzle = $5(0.8558) = 4.2790$ MPa

(b) $p_b = 101$ kPa, nozzle choked

$$A_{\text{throat}} = \frac{0.12 \text{ m}^2}{1.3865} = 0.086549 \text{ m}^2$$

$$p_{\text{throat}} = 5 \text{ MPa}(0.5283) = 2.6415 \text{ MPa}$$

$$T_{\text{throat}} = (200 + 273)0.8333 = 394.1509 \text{ K}$$

$$V_{\text{throat}} = \sqrt{1.4(287)394.1509} = 397.9571 \text{ m/s}$$

$$\rho_{\text{throat}} = \frac{2641.5 \text{ kPa}}{0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (394.1509 \text{ K})} = 23.3510 \text{ kg/m}^3$$

$$\dot{m} = \rho_{\text{throat}} A_{\text{throat}} V_{\text{throat}} = (23.3510)(0.08655)(397.9571) = 804.2829 \text{ kg/s}$$

(c) $\dot{m} = 804.2829 \text{ kg/s}$

Problem 13. – A supersonic flow is allowed to expand indefinitely in a diverging channel. Does the flow velocity approach a finite limit, or does it continue to increase indefinitely? Assume a perfect gas with constant specific heats.

For adiabatic flow, $c_p T_o = c_p T + \frac{V^2}{2}$. However, T cannot be less than 0 K (second law)

So,

$$V_{\text{max}} = \sqrt{2c_p T_o} \quad \text{and} \quad V_{\text{max}} \text{ is finite}$$

Problem 14. – A converging-diverging frictionless nozzle is used to accelerate an airstream emanating from a large chamber. The nozzle has an exit area of 30 cm² and a throat area of 15 cm². If the ambient pressure surrounding the nozzle is 101 kPa and the chamber temperature is 500 K, calculate the following:

- (a) Minimum chamber pressure to choke the nozzle
- (b) Mass flow rate for a chamber pressure of 400 kPa
- (c) Mass flow rate for a chamber pressure of 200 kPa

(a) $\frac{A_{\text{exit}}}{A_{\text{throat}}} = 2.0$

For $\frac{A}{A^*} = 2.0$, $M = 0.3059$, $\frac{P}{P_o} = 0.9372$

Minimum chamber pressure to choke = $\frac{101}{0.9372} = 107.7678 \text{ kPa}$

(b) Nozzle choked for $p_c = p_o = 400 \text{ kPa}$

$p_{\text{throat}} = 0.5283(400) = 211.3200 \text{ kPa}$

$$T_{\text{throat}} = 0.8333(500) = 416.6500 \text{ K}$$

$$\dot{m} = \rho AV = \frac{211.32 \text{ kPa}}{(0.287) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (416.65 \text{ K})} \left(15 \times 10^{-4} \text{ m}^2\right) \sqrt{1.4(287)(416.65)}$$

$$= (1.7672 \text{ kg/m}^3) \left(15 \times 10^{-4} \text{ m}^2\right) 409.1576 \text{ m/s}$$

$$= 1.0846 \text{ kg/s}$$

(c) $\dot{m} = 1.0846 \left(\frac{200}{400}\right) = 0.5423 \text{ kg/s}$

Problem 15. – Sketch p versus x for the case shown in Figure P3.15.

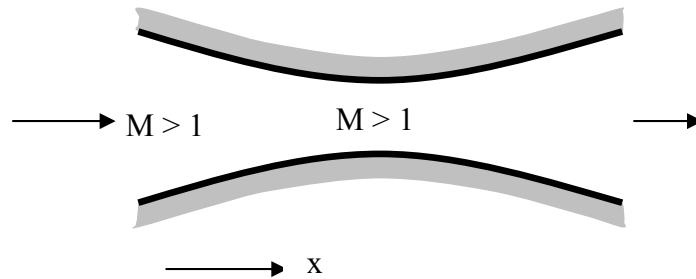
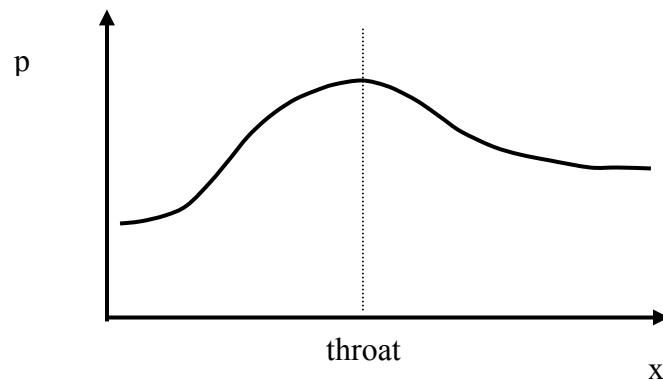


Figure P3.15



Problem 16. – Steam is to be expanded to Mach 2.0 in a converging-diverging nozzle from an inlet velocity of 100 m/s. The inlet area is 50 cm²; inlet static temperature is 500 K. Assuming isentropic flow, determine the throat and exit areas required. Assume the steam to behave as a perfect gas with constant $\gamma = 1.3$.

$$M_i = \frac{100}{\sqrt{1.3(461.5)500}} = \frac{100}{547.6997} = 0.1826$$

$$\frac{A}{A^*} = 3.2669, \quad A_{\text{throat}} = \frac{50}{3.2669} = 15.3050 \text{ cm}^2$$

$$\text{For } M = 2.0, \quad \frac{A}{A^*} = \frac{A_e}{A_t} = 1.7732 \quad \text{so } A_{\text{exit}} = 27.1389 \text{ cm}^2$$

Problem 17. – Write a computer program that will yield values of T/T_o , p/p_o , and A/A^* for isentropic flow of a perfect gas with constant $\gamma = 1.27$. Use Mach number increments of 0.05 over the range $M = 0$ to $M = 2.0$.

$$b = \frac{\gamma - 1}{\gamma + 1}, \quad \frac{T}{T_o} = \frac{1 - b}{1 + b(M^2 - 1)}, \quad \frac{p}{p_o} = \left(\frac{T}{T_o} \right)^{\frac{b+1}{2b}}, \quad \frac{A}{A^*} = \frac{[1 + b(M^2 - 1)]^{\frac{1}{2b}}}{M}$$

| M | T/T _o | p/p _o | A/A* |
|------|------------------|------------------|----------|
| 0.00 | 1.00000 | 1.00000 | infinite |
| 0.05 | 0.99966 | 0.99841 | 11.76142 |
| 0.10 | 0.99865 | 0.99367 | 5.90577 |
| 0.15 | 0.99697 | 0.98584 | 3.96515 |
| 0.20 | 0.99463 | 0.97499 | 3.00342 |
| 0.25 | 0.99163 | 0.96125 | 2.43340 |
| 0.30 | 0.98800 | 0.94478 | 2.05940 |
| 0.35 | 0.98373 | 0.92575 | 1.79759 |
| 0.40 | 0.97886 | 0.90437 | 1.60608 |
| 0.45 | 0.97339 | 0.88086 | 1.46164 |
| 0.50 | 0.96735 | 0.85545 | 1.35034 |
| 0.55 | 0.96076 | 0.82839 | 1.26335 |
| 0.60 | 0.95365 | 0.79994 | 1.19481 |
| 0.65 | 0.94604 | 0.77035 | 1.14069 |
| 0.70 | 0.93795 | 0.73986 | 1.09813 |
| 0.75 | 0.92942 | 0.70873 | 1.06506 |
| 0.80 | 0.92047 | 0.67720 | 1.03995 |
| 0.85 | 0.91113 | 0.64547 | 1.02166 |
| 0.90 | 0.90143 | 0.61378 | 1.00931 |

| | | | |
|----------|------------------------|------------------------|-------------|
| 0.95 | 0.89139 | 0.58230 | 1.00226 |
| 1.00 | 0.88106 | 0.55121 | 1.00000 |
| 1.05 | 0.87045 | 0.52067 | 1.00215 |
| 1.10 | 0.85959 | 0.49081 | 1.00844 |
| 1.15 | 0.84851 | 0.46177 | 1.01864 |
| 1.20 | 0.83724 | 0.43362 | 1.03264 |
| 1.25 | 0.82581 | 0.40646 | 1.05032 |
| 1.30 | 0.81423 | 0.38035 | 1.07166 |
| 1.35 | 0.80254 | 0.35534 | 1.09664 |
| 1.40 | 0.79076 | 0.33147 | 1.12530 |
| 1.45 | 0.77891 | 0.30875 | 1.15768 |
| 1.50 | 0.76702 | 0.28718 | 1.19389 |
| 1.55 | 0.75509 | 0.26678 | 1.23404 |
| 1.60 | 0.74316 | 0.24752 | 1.27826 |
| 1.65 | 0.73124 | 0.22939 | 1.32672 |
| 1.70 | 0.71935 | 0.21236 | 1.37960 |
| 1.75 | 0.70750 | 0.19640 | 1.43712 |
| 1.80 | 0.69570 | 0.18147 | 1.49952 |
| 1.85 | 0.68398 | 0.16753 | 1.56703 |
| 1.90 | 0.67234 | 0.15453 | 1.63996 |
| 1.95 | 0.66079 | 0.14244 | 1.71860 |
| 2.00 | 0.64935 | 0.13121 | 1.80329 |
| M | T/T₀ | p/p₀ | A/A* |

Problem 18. – A gas is known to have a molecular mass of 18, with $c_p = 2.0 \text{ kJ/kg} \cdot \text{K}$. The gas is expanded from negligible initial velocity through a converging-diverging nozzle with an area ratio of 5.0. Assuming an isentropic expansion in the nozzle with initial stagnation pressure and temperature 1 MPa and 1000 K, respectively, determine the exit nozzle velocity.

$$R = \frac{8314.3 \text{ J/kg} \cdot \text{mole} \cdot \text{K}}{18 \text{ kg/kg} \cdot \text{mole}} = 461.9056 \text{ J/kg} \cdot \text{K}$$

$$c_p = \frac{R\gamma}{\gamma - 1} = 2.0 \text{ kJ/kg} \cdot \text{K}$$

$$\frac{\gamma}{\gamma - 1} = \frac{2.0}{0.4619056} = 4.3299$$

$$\therefore \gamma = 1.300$$

$$\frac{A}{A^*} = 5.0, \quad M_e = 2.9723, \quad T_e = 1000(0.4301) = 430.1000 \text{ K}$$

$$V_e = 2.9723\sqrt{1.3(461.9056)430.1} = 1510.5171 \text{ m/s}$$

Problem 19. – A jet plane is flying at 10 km with a cabin pressure of 101 kPa and a cabin temperature of 20°C. Suddenly a bullet is fired inside the cabin and pierces the fuselage; the resultant hole is 2 cm in diameter. Assume that the temperature within the cabin remains constant and that the flow through the hole behaves as that through a converging nozzle with an exit diameter of 2.0 cm. Take the cabin volume to be 100 m³. Calculate the time for the cabin pressure to decrease to one-half the initial value. At 10 km, p = 26.5 kPa and T = 223.3 K.

Because the back pressure to cabin pressure is 26.5/101 = 0.2624, which is less than 0.5283 the critical pressure ratio at $\gamma = 1.4$, the flow is choked and $Me = 1$. Hence, the mass flow rate is

$$\begin{aligned} \dot{m} &= \rho AV = \frac{p}{RT} AM\sqrt{\gamma RT} = \frac{(0.5283)p_c}{287(0.8333)(293)} \left(\frac{\pi}{4} 0.02^2 \right) (1) \sqrt{1.4(287)(0.8333)(293)} \\ &= 7.4186 \times 10^{-7} p_c \end{aligned}$$

In the cabin,

$$p_c \nabla = mRT$$

$$\therefore \frac{dp_c}{dt} = \frac{RT}{\nabla} \frac{dm}{dt} = -\frac{RT}{\nabla} \dot{m}$$

$$\frac{dp_c}{p_c} = -\frac{RT}{\nabla} (7.4186 \times 10^{-7}) dt$$

Integration produces,

$$\ln \frac{p_{c \text{ final}}}{p_{c \text{ initial}}} = -\frac{RT}{\nabla} (7.4186 \times 10^{-7}) t$$

$$\ln 2 = \frac{(287)(293)}{100} (7.4186 \times 10^{-7}) t$$

$$t = 1111.1096 \text{ s} = 0.3086 \text{ h}$$

Problem 20. – A rocket nozzle is designed to operate isentropically at 20 km with a chamber pressure of 2.0 MPa and chamber temperature of 3000 K. If the products of combustion are assumed to behave as a perfect gas with constant specific heats ($\gamma = 1.3$ and $MM = 20$), determine the design thrust for a nozzle throat area of 0.25 m^2 .

At 20 km, $p = 5.53 \text{ kPa}$

$$\frac{p_b}{p_r} = \frac{5.53}{2000} = 0.002765 = \frac{p_e}{p_o}$$

$$M_e = 4.3923, \quad T_e = \left(\frac{T}{T_o}\right)_e T_o = 0.2568(3000) = 770.4000 \text{ K}$$

At design

$$\text{Thrust} = mV_e + (p_e)A_e$$

$$V_e = 4.3923 \sqrt{1.3 \left(\frac{8314.3}{20}\right) 770.4} = 2834.1293 \text{ m/s}$$

Now at the throat $M_t = 1$, so $(p/p_o)_t = 0.5457$ and $(T/T_o)_t = 0.8696$.

$$\dot{m}_t = \left[\frac{(0.5457)2000 \text{ kN/m}^2}{\left(\frac{8.3143 \text{ kNm}}{20 \text{ kg} \cdot \text{K}}\right)(0.8696)(3000)\text{K}} \right] (0.25\text{m}^2) (1) \sqrt{1.3 \left(\frac{8314.3}{20}\right) (0.8696)(3000)} \text{ m/s}$$

$$= (1.0063 \text{ kg/m}^3) (0.25\text{m}^2) (1187.3805 \text{ m/s})$$

$$= 298.7290 \text{ kg/s}$$

$$\begin{aligned} \text{Thrust} &= [(298.7290)(2834.1293) + (5530)(0.25)] / (1,000,000) \\ &= 0.8466 + 0.0014 \\ &= 0.8480\text{MN} \end{aligned}$$

Problem 21. – A converging nozzle has a rectangular cross section of a constant width of 10 cm. For ease of manufacture, the sidewalls of the nozzle are straight, making an angle of 10° with the horizontal, as shown in Figure P3.21. Determine and plot the variation of M , T , and p with x , taking $M_1 = 0.4$, $P_{o1} = 200 \text{ kPa}$, and $T_{o1} = 350 \text{ K}$. Assume the

working fluid to be air, which behaves as a perfect gas with constant specific heats ($\gamma = 1.4$), and that the flow is isentropic.

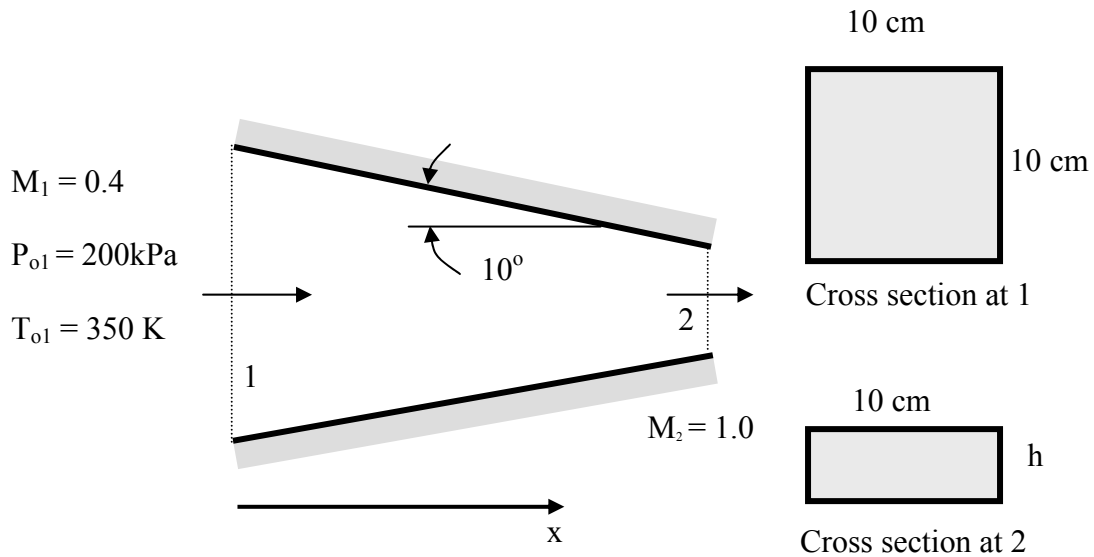


Figure P3.21

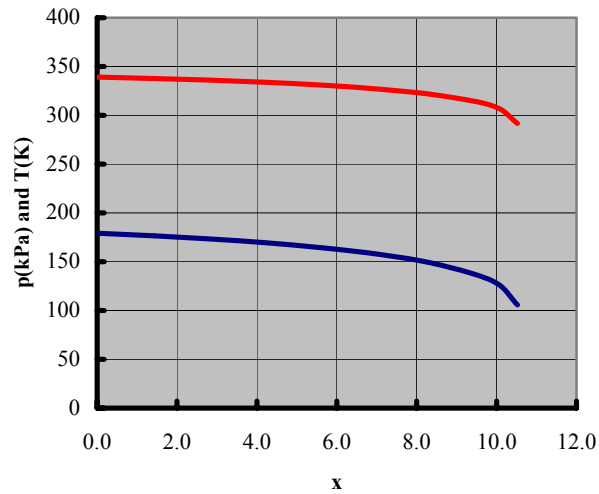
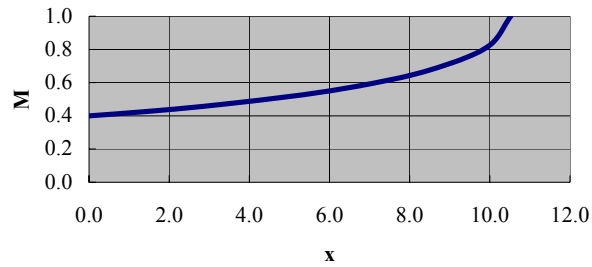
$$A_2 = A^*$$

$$A_1 = 100 \text{ cm}^2, M_1 = 0.4, \frac{A_1}{A^*} = 1.5901$$

$$A_2 = \frac{100}{1.5901} = 62.89 \text{ cm}^2, \text{ so } h = 6.289 \text{ cm}$$

$$x = \frac{h_1 - h}{2 \tan(10)}$$

| h (cm) | x (cm) | A/A* | M | p (kPa) | T (K) |
|--------|--------|--------|-------|---------|-------|
| 10.00 | 0 | 1.5901 | 0.400 | 179.1 | 339.2 |
| 9.50 | 1.418 | 1.5106 | 0.426 | 176.5 | 337.7 |
| 9.00 | 2.836 | 1.4311 | 0.457 | 173.3 | 336.0 |
| 8.50 | 4.253 | 1.3516 | 0.494 | 169.3 | 333.7 |
| 8.00 | 5.671 | 1.2721 | 0.539 | 164.1 | 330.8 |
| 7.50 | 7.089 | 1.1926 | 0.596 | 157.3 | 326.8 |
| 7.00 | 8.507 | 1.1131 | 0.676 | 147.3 | 320.7 |
| 6.50 | 9.925 | 1.0336 | 0.812 | 129.7 | 309.3 |
| 6.29 | 10.523 | 1.0000 | 1.000 | 105.7 | 291.7 |



Problem 22. – A spherical tank contains compressed air at 500 kPa; the volume of the tank is 20 m³. A 5-cm burst diaphragm in the side of the tank ruptures, causing air to escape from the tank. Find the time required for the tank pressure to drop to 200 kPa. Assume the temperature of the air in the tank remains constant at 280 K, the ambient pressure is 101 kPa and that the airflow through the opening can be treated as isentropic flow through a converging nozzle with a 5-cm exit diameter.

$$\text{For } p_{\text{tank}} = 200 \text{ kPa, } \frac{p_b}{p_o} = \frac{101}{200} = 0.505 (< 0.5283 \text{ so choked})$$

$$p_e = 0.5283 p_o, \quad T_e = 0.8333(280) = 233.3240 \text{ K}$$

$$V_e = \sqrt{\gamma RT_e} = \sqrt{1.4(287.0)233.3240} = 306.1855 \text{ m/s}$$

$$\begin{aligned} \dot{m} &= \frac{0.5283 p_o}{0.287(233.3240)} \left(\frac{\pi}{4} 0.05^2 \right) 306.1855 \\ &= 0.004743 p_o \text{ kg/s with } p_o \text{ in kPa} \end{aligned}$$

In the tank,

$$p_o \nabla = mRT$$

$$\frac{dp_o}{dt} = \frac{RT}{\nabla} \frac{dm}{dt} = -\frac{RT}{\nabla} \dot{m} = -\frac{RT}{\nabla} (0.004743 p_o)$$

$$\frac{dp_o}{p_o} = \frac{-0.287(270)}{20} (0.004743) dt$$

$$\ln \frac{200}{500} = -0.01838 t$$

$$t = \frac{\ln(0.4)}{-0.01838} = 49.8526 \text{ s}$$

Problem 23. – A converging-diverging nozzle has an area ratio of 3.3 to 1. The nozzle is supplied from a tank containing a gas at 100 kPa and 270 K (see Figure P3.23). Determine the maximum mass flow possible through the nozzle and the range of back pressures over which the mass flow can be attained assuming the gas is (a) helium ($\gamma = 1.67$, $R = 2.077 \text{ kJ/kg}\cdot\text{K}$) and (b) hydrogen ($\gamma = 1.4$, $R = 4.124 \text{ kJ/kg}\cdot\text{K}$).

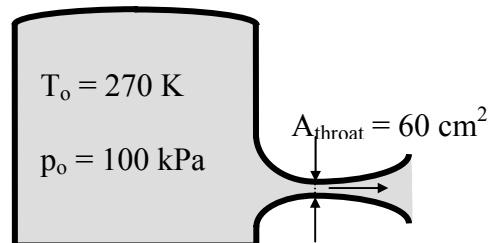


Figure P3.23

(a) Helium: $\gamma = 1.67$, $\frac{A_e}{A^*} = 3.3$

$$M_e = 0.1739, 3.1494$$

$$\text{Maximum } p_b \text{ to choke nozzle: at } M_e = 0.1739, \left(\frac{p}{p_o} \right)_e = 0.9752$$

$$\text{Maximum } p_b \text{ to choke nozzle} = 97.52 \text{ kPa}$$

$$\text{Nozzle choked for } p_b \leq 97.52 \text{ kPa}$$

$$\begin{aligned}\dot{m}_{\max} &= \frac{P_{\text{throat}}}{RT_{\text{throat}}} AM_{\text{throat}} \sqrt{\gamma RT_{\text{throat}}} \\ &= \frac{(0.4867)100}{2.077(0.7491)270} (60 \times 10^{-4})(1) \sqrt{1.67(2077)(0.7491)270}\end{aligned}$$

$$\dot{m}_{\max} = 0.5822 \text{ kg/s}$$

(b) Hydrogen: $\gamma = 1.40$, $\frac{A_e}{A^*} = 3.3$

$$M_e = 0.1787, \quad \left(\frac{p}{p_o} \right)_e = 0.9780$$

Nozzle choked for all $p_b \leq 97.8 \text{ kPa}$

$$\begin{aligned}\dot{m}_{\max} &= \frac{(0.5283)100}{4.124(0.8333)270} (60 \times 10^{-4})(1) \sqrt{1.4(4124)(0.8333)270} \\ &= 0.3894 \text{ kg/s}\end{aligned}$$

Problem 24. – Superheated steam is stored in a large tank at 6 MPa and 800°C. The steam is exhausted isentropically through a converging-diverging nozzle. Determine the velocity of the steam flow when the steam starts to condense, assuming the steam to behave as a perfect gas with $\gamma = 1.3$.

Solution Using Steam Table Data

At 6 MPa, 800°C: $s_1 = 7.6554 \text{ kJ/kg} \cdot \text{K}$

$$\begin{aligned}h_1 &= u_1 + p_1 v_1 \\ &= 3641.2 \frac{\text{kJ}}{\text{kg}} + (6000)(0.08159) \frac{\text{kJ}}{\text{kg}} \\ &= 4130.7 \text{ kJ/kg}\end{aligned}$$

Steam will just condense for $s_2 = s_g = s_1$

At 45 kPa, $s_g = 7.6307$; at 40 kPa, $s_g = 7.6709$

Interpolation gives

$$p_2 = 42 \text{ kPa}, \quad T_2 = 77^\circ\text{C}, \quad h_2 = 2638.8 \frac{\text{kJ}}{\text{kg}}$$

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(4130.7 - 2638.8)1000} = 1727 \text{ m/s}$$

Solution Assuming Steam is a Perfect Gas

$$\frac{p}{p_o} = \frac{42}{6000} = 0.007, \quad M_e = 3.7794, \quad T_e = 273 + 600 = 873 \text{ K}$$

$$T_2 = 873 (0.3182) = 277.7886 \text{ K}$$

$$V_2 = 3.7794 \sqrt{1.3(461.5)277.7886} = 1542.8994 \text{ m/s}$$

Because the second answer assumes that the steam is a perfect gas with constant specific heats, the first answer is more accurate.

Problem 25. – Air is stored in a tank 0.037661 m^3 in volume at an initial pressure of 5,760.6 kPa and a temperature of 321.4K. The gas is discharged through a converging nozzle with an exit area of $3.167 \times 10^{-5} \text{ m}^2$. For a back-pressure of 101 kPa, assuming a spatially lumped polytropic process in the tank, i.e., $p v^n = \text{constant}$, and isentropic flow in the nozzle, i.e., $p v^\gamma = \text{constant}$, compare predicted tank pressures to the measured values contained the following table. Try various values of the polytropic exponent, n , from 1.0 (isothermal) to 1.4 (isentropic). Perform only a Stage I analysis, i.e., the nozzle is choked.

| | | | | | | | | | | | | | | |
|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| time, sec | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 | 9.0 | 11.0 | 13.0 | 15.0 | 17.0 | 19.0 |
| p_o/p_{o1} | 1.000 | 0.717 | 0.551 | 0.448 | 0.358 | 0.281 | 0.241 | 0.199 | 0.142 | 0.104 | 0.078 | 0.059 | 0.044 | 0.033 |

Now from the continuity equation

$$\frac{dm}{dt} = -\dot{m}_e = -\rho_e A_e V_e$$

For polytropic expansion within the tank

$$\frac{p_o}{\rho_o^n} = \frac{p_{o1}}{\rho_{o1}^n}$$

So

$$\rho_o = \rho_{o1} \left(\frac{p_o}{p_{o1}} \right)^{\frac{1}{n}}$$

And for isentropic expansion in the nozzle

$$\frac{p_o}{\rho_o^\gamma} = \frac{p_e}{\rho_e^\gamma}$$

So

$$\rho_e = \rho_o \left(\frac{p_o}{p_{o1}} \right)^{\frac{1}{\gamma}}$$

For a choked flow: $M_e = 1$, $V_e = a_e = \sqrt{\gamma p_e / \rho_e}$ and

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2 \right)^{\gamma/(\gamma-1)} = \left(\frac{\gamma+1}{2} \right)^{\gamma/(\gamma-1)}$$

So,

$$p_e = p_o \left(\frac{\gamma+1}{2} \right)^{\gamma/(1-\gamma)}$$

Therefore,

$$\rho_e = \rho_{o1} \left(\frac{\gamma+1}{2} \right)^{\frac{1}{1-\gamma}} \left(\frac{p_o}{p_{o1}} \right)^{\frac{1}{n}}$$

Now,

$$\dot{m}_e = \rho_e A_e V_e = \rho_e A_e a_e = \rho_e A_e \sqrt{\gamma \frac{p_e}{\rho_e}} = A_e \sqrt{\gamma p_e \rho_e}$$

$$= \sqrt{\gamma} A_e p_o^{1/2} \rho_{o1} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{2(1-\gamma)}} \rho_{o1}^{1/2} \left(\frac{\gamma+1}{2} \right)^{\frac{1}{2(1-\gamma)}} \left(\frac{p_o}{p_{o1}} \right)^{\frac{1}{2n}}$$

Using $a_{ol} = \sqrt{\gamma p_{ol}/\rho_{ol}}$ the mass flow rate at the exit can be written as

$$\dot{m}_e = A_e a_{ol} \frac{\rho_{ol}}{p_{ol}^{1/2}} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} \left(\frac{1}{p_{ol}} \right)^{\frac{1}{2n}} p_o^{\frac{n+1}{2n}}$$

Now the time rate of change of the mass within the tank is given by

$$\frac{dm}{dt} = \forall \frac{dp_o}{dt} = \left[\frac{\rho_{ol} \forall}{n p_{ol}^{1/n}} \right] p_o^{\frac{(1-n)}{n}} \frac{dp_o}{dt}$$

Equating this to the exiting flow rate gives

$$\left[\frac{\rho_{ol} \forall}{n p_{ol}^{1/n}} \right] p_o^{\frac{(1-n)}{n}} \frac{dp_o}{dt} = -\dot{m}_e = -A_e a_{ol} \frac{\rho_{ol}}{p_{ol}^{1/2}} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} \left(\frac{1}{p_{ol}} \right)^{\frac{1}{2n}} p_o^{\frac{n+1}{2n}}$$

or

$$\begin{aligned} \frac{p_o^{\frac{1-n}{n}}}{p_o^{\frac{n+1}{2n}}} \frac{dp_o}{dt} &= p_o^{\left(\frac{1-n}{n} \right) - \left(\frac{n+1}{2n} \right)} \frac{dp_o}{dt} = p_o^{\frac{1-3n}{2n}} \frac{dp_o}{dt} = \frac{-n A_e a_{ol}}{\forall} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} p_{ol}^{1/n-1/2-1/2n} \\ &= \frac{-n A_e a_{ol}}{\forall} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} p_{ol}^{\frac{1-n}{2n}} \end{aligned}$$

Integration yields, (note: $\frac{1-3n}{2n} + 1 = \frac{1-n}{2n}$)

$$p_o^{\frac{1-n}{2n}} - p_{ol}^{\frac{1-n}{2n}} = \left(\frac{n-1}{2} \right) \left(\frac{A_e a_{ol}}{\forall} \right) \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} p_{ol}^{\frac{1-n}{2n}} t$$

Rearrangement brings

$$\frac{p_o}{p_{o1}} = \left[1 + \left(\frac{n-1}{2} \right) \left(\frac{A_e a_{o1}}{\nabla} \right) \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} t \right]^{\frac{2n}{1-n}}$$

Note this is not valid for $n=1$, the isothermal case which must be treated separately. For $n=1$

$$\dot{m}_e = A_e a_{o1} \frac{\rho_{o1}}{p_{o1}} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} p_o$$

$$\rho_o = \rho_{o1} \left(\frac{p_o}{p_{o1}} \right)$$

$$\frac{dm}{dt} = \nabla \frac{d\rho_o}{dt} = \nabla \frac{\rho_{o1}}{p_{o1}} \frac{dp_o}{dt} = -\dot{m}_e = -A_e a_{o1} \frac{\rho_{o1}}{p_{o1}} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} p_o$$

Canceling, separating variables, integrating and rearranging yields,

$$\frac{p_o}{p_{o1}} = e^{-\frac{A_e a_{o1}}{\nabla} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{2(1-\gamma)}} t}$$

A spreadsheet program was written and run for various n . A table of the results is as follows

| n = | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | |
|------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|---|
| t | p_o/p_{o1} | p_o/p_{o1} | p_o/p_{o1} | p_o/p_{o1} | p_o/p_{o1} | p_o/p_{o1} (exp) |
| 0.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.000 |
| 1.0 | 0.8396 | 0.8257 | 0.8122 | 0.7990 | 0.7862 | 0.717 |
| 2.0 | 0.7049 | 0.6830 | 0.6620 | 0.6421 | 0.6230 | 0.551 |
| 3.0 | 0.5918 | 0.5658 | 0.5415 | 0.5187 | 0.4974 | 0.448 |
| 4.0 | 0.4969 | 0.4695 | 0.4443 | 0.4212 | 0.3999 | 0.358 |
| 5.0 | 0.4172 | 0.3902 | 0.3658 | 0.3437 | 0.3237 | 0.281 |
| 6.0 | 0.3503 | 0.3247 | 0.3021 | 0.2818 | 0.2636 | 0.241 |
| 7.0 | 0.2941 | 0.2707 | 0.2502 | 0.2321 | 0.2159 | 0.199 |
| 9.0 | 0.2073 | 0.1890 | 0.1731 | 0.1594 | 0.1473 | 0.142 |
| 11.0 | 0.1461 | 0.1327 | 0.1211 | 0.1112 | 0.1025 | 0.104 |
| 13.0 | 0.1030 | 0.0937 | 0.0856 | 0.0787 | 0.0726 | 0.078 |
| 15.0 | 0.0726 | 0.0665 | 0.0611 | 0.0564 | 0.0522 | 0.059 |
| 17.0 | 0.0512 | 0.0474 | 0.0440 | 0.0410 | 0.0382 | 0.044 |
| 19.0 | 0.0361 | 0.0340 | 0.0320 | 0.0301 | 0.0283 | 0.033 |