

Chapter 2

1. The speed (assumed constant) is $v = (90 \text{ km/h})(1000 \text{ m/km}) / (3600 \text{ s/h}) = 25 \text{ m/s}$. Thus, in 0.50 s , the car travels a distance $d = vt = (25 \text{ m/s})(0.50 \text{ s}) \approx 13 \text{ m}$.

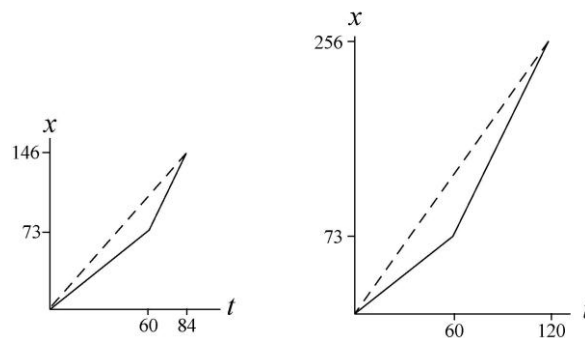
2. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m/s}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



3. **THINK** This one-dimensional kinematics problem consists of two parts, and we are asked to solve for the average velocity and average speed of the car.

EXPRESS Since the trip consists of two parts, let the displacements during first and second parts of the motion be Δx_1 and Δx_2 , and the corresponding time intervals be Δt_1 and Δt_2 , respectively. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is simply $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2-2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time Δt if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\Delta t$.

ANALYZE

(a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time taken is

$$t_1 = \frac{(40 \text{ km})}{(30 \text{ km/h})} = 1.33 \text{ h.}$$

Similarly, during the second part of the trip the displacement is $\Delta x_2 = 40$ km and the time interval is

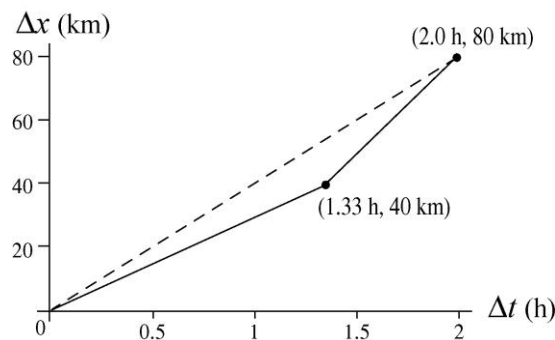
$$t_2 = \frac{(40 \text{ km})}{(60 \text{ km/h})} = 0.67 \text{ h.}$$

The total displacement is $\Delta x = \Delta x_1 + \Delta x_2 = 40 \text{ km} + 40 \text{ km} = 80 \text{ km}$, and the total time elapsed is $\Delta t = \Delta t_1 + \Delta t_2 = 2.00 \text{ h}$. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{(80 \text{ km})}{(2.0 \text{ h})} = 40 \text{ km/h.}$$

(b) In this case, the average speed is the same as the magnitude of the average velocity: $s_{\text{avg}} = 40 \text{ km/h}$.

(c) The graph of the entire trip, shown below, consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(\Delta t_1, \Delta x_1) = (1.33 \text{ h}, 40 \text{ km})$ and the second having a slope of 60 km/h and connecting $(\Delta t_1, \Delta x_1)$ to $(\Delta t, \Delta x) = (2.00 \text{ h}, 80 \text{ km})$.



From the graphical point of view, the slope of the dashed line drawn from the origin to $(\Delta t, \Delta x)$ represents the average velocity.

LEARN The average velocity is a vector quantity that depends only on the net displacement (also a vector) between the starting and ending points.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the

motion) we have speed = D/t . Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in $v_{\text{up}} = 40$ km/h and $v_{\text{down}} = 60$ km/h, yields 48 km/h for the average speed.

5. THINK In this one-dimensional kinematics problem, we're given the position function $x(t)$, and asked to calculate the position and velocity of the object at a later time.

EXPRESS The position function is given as $x(t) = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3$. The position of the object at some instant t_0 is simply given by $x(t_0)$. For the time interval $t_1 \leq t \leq t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2-2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

ANALYZE (a) Plugging in $t = 1$ s into $x(t)$ yields

$$x(1 \text{ s}) = (3 \text{ m/s})(1 \text{ s}) - (4 \text{ m/s}^2)(1 \text{ s})^2 + (1 \text{ m/s}^3)(1 \text{ s})^3 = 0.$$

(b) With $t = 2$ s we get $x(2 \text{ s}) = (3 \text{ m/s})(2 \text{ s}) - (4 \text{ m/s}^2)(2 \text{ s})^2 + (1 \text{ m/s}^3)(2 \text{ s})^3 = -2$ m.

(c) With $t = 3$ s we have $x(3 \text{ s}) = (3 \text{ m/s})(3 \text{ s}) - (4 \text{ m/s}^2)(3 \text{ s})^2 + (1 \text{ m/s}^3)(3 \text{ s})^3 = 0$ m.

(d) Similarly, plugging in $t = 4$ s gives

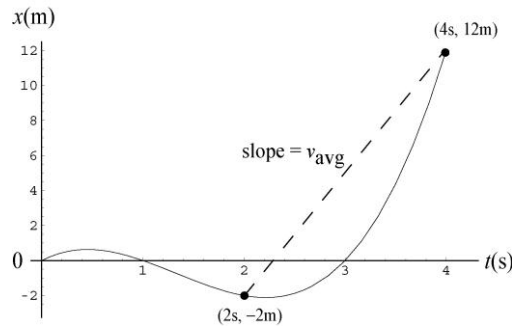
$$x(4 \text{ s}) = (3 \text{ m/s})(4 \text{ s}) - (4 \text{ m/s}^2)(4 \text{ s})^2 + (1 \text{ m/s}^3)(4 \text{ s})^3 = 12 \text{ m}.$$

(e) The position at $t = 0$ is $x = 0$. Thus, the displacement between $t = 0$ and $t = 4$ s is $\Delta x = x(4 \text{ s}) - x(0) = 12 \text{ m} - 0 = 12$ m.

(f) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement: $\Delta x = x(4 \text{ s}) - x(2 \text{ s}) = 12 \text{ m} - (-2 \text{ m}) = 14$ m. Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14 \text{ m}}{2 \text{ s}} = 7 \text{ m/s}.$$

(g) The position of the object for the interval $0 \leq t \leq 4$ is plotted below. The straight line drawn from the point at $(t, x) = (2 \text{ s}, -2 \text{ m})$ to $(4 \text{ s}, 12 \text{ m})$ would represent the average velocity, answer for part (f).



LEARN Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

6. Huber's speed is

$$v_0 = (200 \text{ m}) / (6.509 \text{ s}) = 30.72 \text{ m/s} = 110.6 \text{ km/h},$$

where we have used the conversion factor $1 \text{ m/s} = 3.6 \text{ km/h}$. Since Whittingham beat Huber by 19.0 km/h , his speed is $v_1 = (110.6 \text{ km/h} + 19.0 \text{ km/h}) = 129.6 \text{ km/h}$, or 36 m/s ($1 \text{ km/h} = 0.2778 \text{ m/s}$). Thus, using Eq. 2-2, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200 \text{ m}}{36 \text{ m/s}} = 5.554 \text{ s}.$$

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h , the total time that elapses before they crash is $t = (60 \text{ km}) / (60 \text{ km/h}) = 1.0 \text{ h}$. During this time, the bird travels a distance of $x = vt = (60 \text{ km/h})(1.0 \text{ h}) = 60 \text{ km}$.

8. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L / v_s$. With each additional person, the depth increases by one body depth d

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L / v_s} = \frac{dv_s}{L} = \frac{(0.25 \text{ m})(3.50 \text{ m/s})}{1.75 \text{ m}} = 0.50 \text{ m/s}$$

(b) The amount of time required to reach a depth of $D = 5.0 \text{ m}$ is

$$t = \frac{D}{R} = \frac{5.0 \text{ m}}{0.50 \text{ m/s}} = 10 \text{ s}$$

9. Converting to seconds, the running times are $t_1 = 147.95 \text{ s}$ and $t_2 = 148.15 \text{ s}$, respectively. If the runners were equally fast, then

$$s_{\text{avg1}} = s_{\text{avg2}} \Rightarrow \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1 \right) L_1 = \left(\frac{148.15}{147.95} - 1 \right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000 \text{ m}$ in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

10. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1 , the car moves in the same direction as the wind. Then the effective speed of the car is given by $v_{\text{eff},1} = v_c + v_w$, and the distance traveled is $d = v_{\text{eff},1} t_1 = (v_c + v_w) t_1$. On the other hand, for the return trip during time interval t_2 , the car moves in the opposite direction of the wind and the effective speed would be $v_{\text{eff},2} = v_c - v_w$. The distance traveled is $d = v_{\text{eff},2} t_2 = (v_c - v_w) t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2} \left(\frac{d}{t_1} + \frac{d}{t_2} \right)$. Thus, method 1 gives the car's speed v_c in windless situation.

(b) If method 2 is used, the result would be

$$v'_c = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{v_c + v_w} + \frac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c \left[1 - \left(\frac{v_w}{v_c} \right)^2 \right].$$

The fractional difference is

$$\frac{v_c - v'_c}{v_c} = \left(\frac{v_w}{v_c} \right)^2 = (0.0240)^2 = 5.76 \times 10^{-4}.$$

11. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed $v = (160 \text{ km}) / (1.25 \text{ h}) = 128 \text{ km/h}$ is needed.

12. (a) Let the fast and the slow cars be separated by a distance d at $t = 0$. If during the time interval $t = L / v_s = (12.0 \text{ m}) / (5.0 \text{ m/s}) = 2.40 \text{ s}$ in which the slow car has moved a distance of $L = 12.0 \text{ m}$, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition implies a separation of

$$d = vt - L = (25 \text{ m/s})(2.4 \text{ s}) - 12.0 \text{ m} = 48.0 \text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0 \text{ m}$. At a later time t , the slow and

the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d + x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00 \text{ m/s}}{25.0 \text{ m/s} - 5.00 \text{ m/s}} (96.0 \text{ m}) = 24.0 \text{ m},$$

which in turn gives $t = (24.0 \text{ m}) / (5.00 \text{ m/s}) = 4.80 \text{ s}$. Since the rear of the slow-car pack has moved a distance of $\Delta x = x - L = 24.0 \text{ m} - 12.0 \text{ m} = 12.0 \text{ m}$ downstream, the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0 \text{ m}}{4.80 \text{ s}} = 2.50 \text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as T and D , respectively, the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55 \text{ km/h})(T/2) + (90 \text{ km/h})(T/2)}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

which should be rounded to 68 km/h.

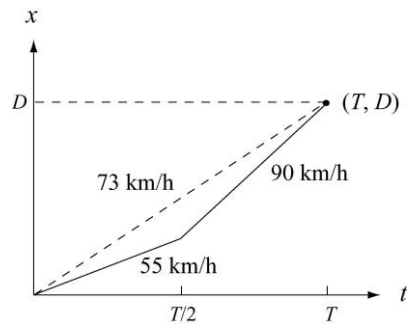
(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km/h}} + \frac{D}{68.3 \text{ km/h}}} = 70 \text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an atlas to look it up); the student can just as easily arbitrarily set T instead of D , as will be clear in the following discussion. We briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the

graphical point of view, is the slope of a line drawn from the origin to (T, D) . The graph (not drawn to scale) is depicted below:



14. Using the general property $\frac{d}{dx} \exp(bx) = b \exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt} \right) \cdot e^{-t} + (19t) \cdot \left(\frac{de^{-t}}{dt} \right).$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where $T = 1$ second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1 - t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when $t = 1$ s. Now that we know *when* it stops, we find out *where* it stops by plugging our result $t = 1$ into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find $x = 5.9$ m.

15. We use Eq. 2-4 to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt} (4 - 12t + 3t^2) = -12 + 6t.$$

Thus, at $t = 1$ s, the velocity is $v = (-12 + (6)(1)) = -6$ m/s.

(b) Since $v < 0$, it is moving in the $-x$ direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \rightarrow +\infty$, we have $v \rightarrow +\infty$). One can check that $v = 0$ when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

16. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

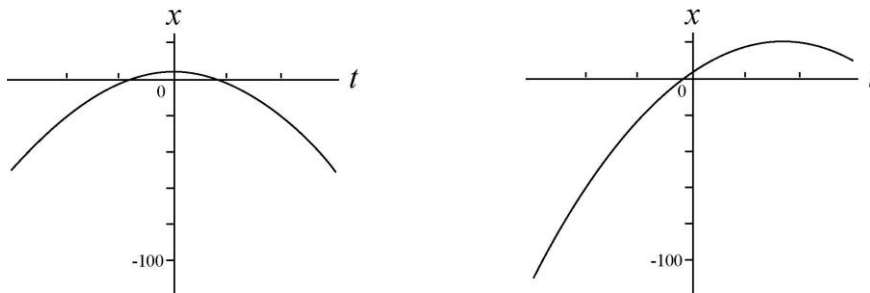
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m.

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the “shifted” graph that is relevant to part (f). In both cases, the time axis is given by $-3 \leq t \leq 3$ (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \leq t \leq 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25 \text{ cm} - 21.75 \text{ cm}}{3.00 \text{ s} - 2.00 \text{ s}}$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time $t = 2.00$ s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At $t = 3.00$ s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

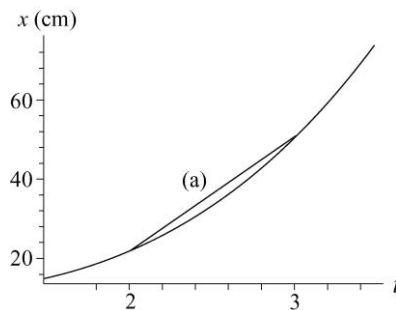
(d) At $t = 2.50$ s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \Rightarrow t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x -vs- t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \quad \text{and} \quad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value $t = 3$ yields $x(3) = 54$ m.

(b) Similarly, plugging in the value $t = 3$ yields $v(3) = 18$ m/s.

(c) For $t = 3$, $a(3) = -12$ m/s².

(d) At the maximum x , we must have $v = 0$; eliminating the $t = 0$ root, the velocity equation reveals $t = 24/6 = 4$ s for the time of maximum x . Plugging $t = 4$ into the equation for x leads to $x = 64$ m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at $t = 4.0$ s.

(f) A maximum v requires $a = 0$, which occurs when $t = 24/12 = 2.0$ s. This, inserted into the velocity equation, gives $v_{\max} = 24$ m/s.

(g) From (f), we see that the maximum of v occurs at $t = 24/12 = 2.0$ s.

(h) In part (e), the particle was (momentarily) motionless at $t = 4$ s. The acceleration at that time is readily found to be $24 - 12(4) = -24$ m/s².

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at $t = 0$ and $t = 3$ s are needed; these are, respectively, $x = 0$ and $x = 54$ m (found in part (a)). Thus,

$$v_{\text{avg}} = \frac{54 - 0}{3 - 0} = 18 \text{ m/s.}$$

19. **THINK** In this one-dimensional kinematics problem, we're given the speed of a particle at two instants and asked to calculate its average acceleration.

EXPRESS We represent the initial direction of motion as the $+x$ direction. The average acceleration over a time interval $t_1 \leq t \leq t_2$ is given by Eq. 2-7:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

ANALYZE Let $v_1 = +18$ m/s at $t_1 = 0$ and $v_2 = -30$ m/s at $t_2 = 2.4$ s. Using Eq. 2-7 we find

$$a_{\text{avg}} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30 \text{ m/s}) - (+18 \text{ m/s})}{2.4 \text{ s} - 0} = -20 \text{ m/s}^2.$$

LEARN The average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18 \text{ m/s}) - (20 \text{ m/s}^2)t.$$

20. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{dt} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

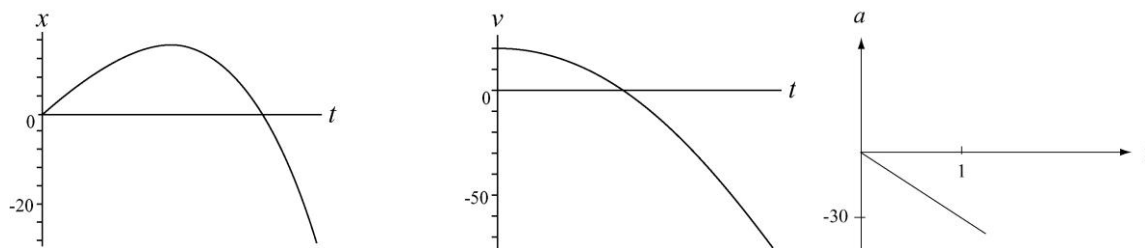
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$.

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



21. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during $5 \text{ min} \leq t \leq 10 \text{ min}$ is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) The entire interval considered is $\Delta t = 8 - 2 = 6 \text{ min}$, which is equivalent to 360 s , whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3 \text{ min} = 180 \text{ s}$. His position at $t = 2 \text{ min}$ is $x = 0$ and his position at $t = 8 \text{ min}$ is $x = v\Delta t' = (2.2)(180) = 396 \text{ m}$. Therefore,

$$v_{\text{avg}} = \frac{396 \text{ m} - 0}{360 \text{ s}} = 1.10 \text{ m/s}.$$

(b) The man is at rest at $t = 2 \text{ min}$ and has velocity $v = +2.2 \text{ m/s}$ at $t = 8 \text{ min}$. Thus, keeping the answer to 3 significant figures,

$$a_{\text{avg}} = \frac{2.2 \text{ m/s} - 0}{360 \text{ s}} = 0.00611 \text{ m/s}^2.$$

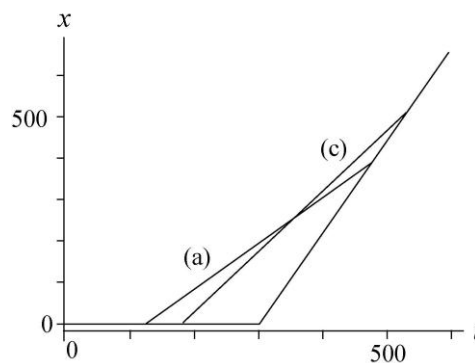
(c) Now, the entire interval considered is $\Delta t = 9 - 3 = 6 \text{ min}$ (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4 \text{ min} = 240 \text{ s}$. His position at $t = 3 \text{ min}$ is $x = 0$ and his position at $t = 9 \text{ min}$ is $x = v\Delta t' = (2.2)(240) = 528 \text{ m}$. Therefore,

$$v_{\text{avg}} = \frac{528 \text{ m} - 0}{360 \text{ s}} = 1.47 \text{ m/s}.$$

(d) The man is at rest at $t = 3 \text{ min}$ and has velocity $v = +2.2 \text{ m/s}$ at $t = 9 \text{ min}$. Consequently, $a_{\text{avg}} = 2.2/360 = 0.00611 \text{ m/s}^2$ just as in part (b).

(e) The horizontal line near the bottom of this x -vs- t graph represents the man standing at $x = 0$ for $0 \leq t < 300 \text{ s}$ and the linearly rising line for $300 \leq t \leq 600 \text{ s}$ represents his constant-velocity motion. The lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.

The graph of v -vs- t is not shown here, but would consist of two horizontal “steps” (one at $v = 0$ for $0 \leq t < 300 \text{ s}$ and the next at $v = 2.2 \text{ m/s}$ for $300 \leq$



$t \leq 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the “steps” at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

22. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t . The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time², or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time³, or m/s^3 .

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0 \text{ m/s}^2)}{3(2.0 \text{ m/s}^3)} = 1.0 \text{ s}.$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.0$ s, $x = 1.0 \text{ m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$ s).

(d) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4 \text{ s}) = (3.0 \text{ m/s}^2)(4.0 \text{ s})^2 - (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 = -80 \text{ m}.$$

The total path length it travels is $1.0 \text{ m} + 1.0 \text{ m} + 80 \text{ m} = 82 \text{ m}$.

(e) Its displacement is $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80 \text{ m}$. Thus, $\Delta x = -80 \text{ m}$.

The velocity is given by $v = 2ct - 3bt^2 = (6.0 \text{ m/s}^2)t - (6.0 \text{ m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain

$$v(1 \text{ s}) = (6.0 \text{ m/s}^2)(1.0 \text{ s}) - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2 = 0.$$

(g) Similarly, $v(2 \text{ s}) = (6.0 \text{ m/s}^2)(2.0 \text{ s}) - (6.0 \text{ m/s}^3)(2.0 \text{ s})^2 = -12 \text{ m/s}$.

(h) $v(3 \text{ s}) = (6.0 \text{ m/s}^2)(3.0 \text{ s}) - (6.0 \text{ m/s}^3)(3.0 \text{ s})^2 = -36 \text{ m/s}$.

(i) $v(4 \text{ s}) = (6.0 \text{ m/s}^2)(4.0 \text{ s}) - (6.0 \text{ m/s}^3)(4.0 \text{ s})^2 = -72 \text{ m/s}$.

The acceleration is given by $a = dv/dt = 2c - 6bt = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain $a(1 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(1.0 \text{ s}) = -6.0 \text{ m/s}^2$.

(k) $a(2 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(2.0 \text{ s}) = -18 \text{ m/s}^2$.

$$(l) \quad a(3 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(3.0 \text{ s}) = -30 \text{ m/s}^2.$$

$$(m) \quad a(4 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(4.0 \text{ s}) = -42 \text{ m/s}^2.$$

23. **THINK** The electron undergoes a constant acceleration. Given the final speed of the electron and the distance it has traveled, we can calculate its acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the electron can be readily analyzed using the equations given in Table 2-1:

$$v = v_0 + at \quad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2} at^2 \quad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad (2-16)$$

The acceleration can be found by solving Eq. 2-16.

ANALYZE With $v_0 = 1.50 \times 10^5 \text{ m/s}$, $v = 5.70 \times 10^6 \text{ m/s}$, $x_0 = 0$ and $x = 0.010 \text{ m}$, we find the average acceleration to be

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7 \times 10^6 \text{ m/s})^2 - (1.5 \times 10^5 \text{ m/s})^2}{2(0.010 \text{ m})} = 1.62 \times 10^{15} \text{ m/s}^2.$$

LEARN It is always a good idea to apply other equations in Table 2-1 not used for solving the problem as a consistency check. For example, since we now know the value of the acceleration, using Eq. 2-11, the time it takes for the electron to reach its final speed would be

$$t = \frac{v - v_0}{a} = \frac{5.70 \times 10^6 \text{ m/s} - 1.5 \times 10^5 \text{ m/s}}{1.62 \times 10^{15} \text{ m/s}^2} = 3.426 \times 10^{-9} \text{ s}$$

Substituting the value of t into Eq. 2-15, the distance the electron travels is

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2} at^2 = 0 + (1.5 \times 10^5 \text{ m/s})(3.426 \times 10^{-9} \text{ s}) + \frac{1}{2}(1.62 \times 10^{15} \text{ m/s}^2)(3.426 \times 10^{-9} \text{ s})^2 \\ &= 0.01 \text{ m} \end{aligned}$$

This is what was given in the problem statement. So we know the problem has been solved correctly.

24. In this problem we are given the initial and final speeds, and the displacement, and are asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2-16, $v^2 = v_0^2 + 2a(x - x_0)$.

(a) Given that $v_0 = 0$, $v = 1.6 \text{ m/s}$, and $\Delta x = 5.0 \mu\text{m}$, the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(1.6 \text{ m/s})^2}{2(5.0 \times 10^{-6} \text{ m})} = 2.56 \times 10^5 \text{ m/s}^2 = 2.6 \times 10^4 g$$

(b) During the speed-reduction stage, the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0 - (1.6 \text{ m/s})^2}{2(1.0 \times 10^{-3} \text{ m})} = -1.28 \times 10^3 \text{ m/s}^2 = -1.3 \times 10^2 g$$

The negative sign means that the spores are decelerating.

25. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20 \text{ m/s}$ and $a = 2.0 \text{ m/s}^2$. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20 \text{ m/s}$; $v = 0$ and $a = -1.0 \text{ m/s}^2$ (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10 \text{ s}$. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20 \text{ s}$, and the total is $t = t_1 + t_2 = 30 \text{ s}$.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20 \text{ m/s})^2 - (0)^2}{2(2.0 \text{ m/s}^2)} = 100 \text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100 \text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20 \text{ m/s})^2}{2(-1.0 \text{ m/s}^2)}.$$

Thus, the final position is $x = 300 \text{ m}$. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

26. The constant-acceleration condition permits the use of Table 2-1.

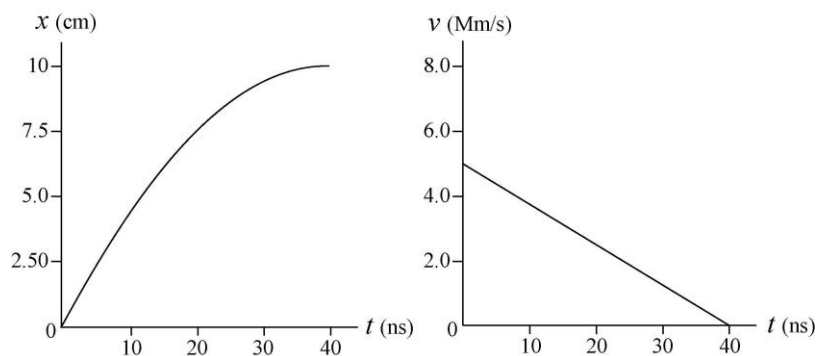
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2} \frac{v_0^2}{a} = -\frac{1}{2} \frac{(5.00 \times 10^6)^2}{-1.25 \times 10^{14}} = 0.100 \text{ m}.$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t , so that other equations from Table 2-1 (such as $v = v_0 + at$ and

$x = v_0t + \frac{1}{2}at^2$) are used in making these plots.



27. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals $+9.6$ m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2)(-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, $t = +2.5$ s and we find $v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2)(2.5 \text{ s}) = 18 \text{ m/s}$.

28. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s². We also take $x_0 = 0$.

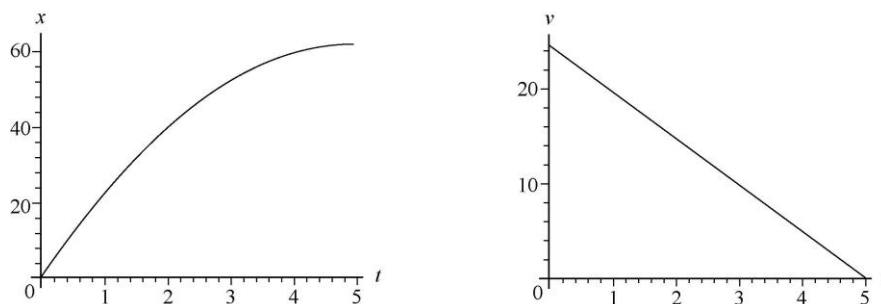
(a) The time to come to a halt is found using Eq. 2-11:

$$0 = v_0 + at \Rightarrow t = \frac{24.6 \text{ m/s}}{-4.92 \text{ m/s}^2} = 5.00 \text{ s}.$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \Rightarrow x = -\frac{(24.6 \text{ m/s})^2}{2(-4.92 \text{ m/s}^2)} = 61.5 \text{ m}.$$

(c) Using these results, we plot $v_0t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \leq t \leq 5$ s, with SI units understood.



29. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22 \text{ m/s}^2$ and $a_2 = -1.22 \text{ m/s}^2$. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08 \text{ m/s}$.

(a) We denote Δx as the distance moved during t_1 , and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \Rightarrow \Delta x = \frac{(5.08 \text{ m/s})^2}{2(1.22 \text{ m/s}^2)} = 10.59 \text{ m} \approx 10.6 \text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08 \text{ m/s}}{1.22 \text{ m/s}^2} = 4.17 \text{ s}.$$

The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33 \text{ s}$. The distances traveled during t_1 and t_2 are the same so that they total to $2(10.59 \text{ m}) = 21.18 \text{ m}$. This implies that for a distance of $190 \text{ m} - 21.18 \text{ m} = 168.82 \text{ m}$, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82 \text{ m}}{5.08 \text{ m/s}} = 33.21 \text{ s}.$$

Therefore, the total time is $8.33 \text{ s} + 33.21 \text{ s} \approx 41.5 \text{ s}$.

30. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

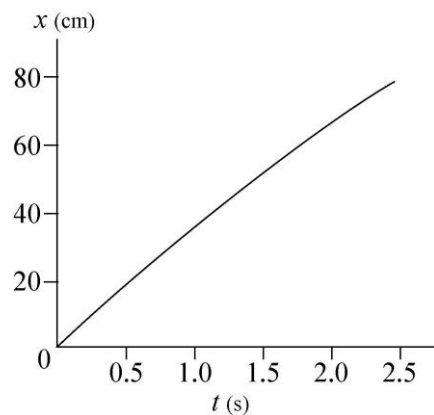
(a) Substituting $v_0 = 137 \text{ km/h} = 38.1 \text{ m/s}$, $v = 90 \text{ km/h} = 25 \text{ m/s}$, and $a = -5.2 \text{ m/s}^2$ into $v = v_0 + at$, we obtain

$$t = \frac{25 \text{ m/s} - 38 \text{ m/s}}{-5.2 \text{ m/s}^2} = 2.5 \text{ s}.$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = (38 \text{ m/s})t + \frac{1}{2}(-5.2 \text{ m/s}^2)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5 \text{ s}$ on the graph to the right. We have not shown the v -vs- t graph here; it is a descending straight line from v_0 to v .



31. **THINK** The rocket ship undergoes a constant acceleration from rest, and we want to know the time elapsed and the distance traveled when the rocket reaches a certain speed.

EXPRESS Since the problem involves constant acceleration, the motion of the rocket can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \quad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2} at^2 \quad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad (2-16)$$

ANALYZE (a) Given that $a = 9.8 \text{ m/s}^2$, $v_0 = 0$ and $v = 0.1c = 3.0 \times 10^7 \text{ m/s}$, we can solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{3.0 \times 10^7 \text{ m/s} - 0}{9.8 \text{ m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is about 1.2 months. So it takes 1.2 months for the rocket to reach a speed of $0.1c$ starting from rest with a constant acceleration of 9.8 m/s^2 .

(b) To calculate the distance traveled during this time interval, we evaluate $x = x_0 + v_0 t + \frac{1}{2} at^2$, with $x_0 = 0$ and $v_0 = 0$. The result is

$$x = \frac{1}{2} (9.8 \text{ m/s}^2) (3.1 \times 10^6 \text{ s})^2 = 4.6 \times 10^{13} \text{ m}.$$

LEARN In solving parts (a) and (b), we did not use Eq. (2-16): $v^2 = v_0^2 + 2a(x - x_0)$. This equation can be used to check our answers. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(9.8 \text{ m/s}^2)(4.6 \times 10^{13} \text{ m} - 0)} = 3.0 \times 10^7 \text{ m/s},$$

which is what was given in the problem statement. So we know the problems have been solved correctly.

32. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020 \text{ km/h}) \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right)}{1.4 \text{ s}} = 202.4 \text{ m/s}^2.$$

In terms of the gravitational acceleration g , this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right) g = 21g.$$

33. **THINK** The car undergoes a constant negative acceleration to avoid impacting a barrier. Given its initial speed, we want to know the distance it has traveled and the time elapsed prior to the impact.

EXPRESS Since the problem involves constant acceleration, the motion of the car can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \quad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2} at^2 \quad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad (2-16)$$

We take $x_0 = 0$ and $v_0 = 56.0 \text{ km/h} = 15.55 \text{ m/s}$ to be the initial position and speed of the car. Solving Eq. 2-15 with $t = 2.00 \text{ s}$ gives the acceleration a . Once a is known, the speed of the car upon impact can be found by using Eq. 2-11.

ANALYZE (a) Using Eq. 2-15, we find the acceleration to be

$$a = \frac{2(x - v_0 t)}{t^2} = \frac{2[(24.0 \text{ m}) - (15.55 \text{ m/s})(2.00 \text{ s})]}{(2.00 \text{ s})^2} = -3.56 \text{ m/s}^2,$$

or $|a| = 3.56 \text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car; the car is slowing down.

(b) The speed of the car at the instant of impact is

$$v = v_0 + at = 15.55 \text{ m/s} + (-3.56 \text{ m/s}^2)(2.00 \text{ s}) = 8.43 \text{ m/s}$$

which can also be converted to 30.3 km/h.

LEARN In solving parts (a) and (b), we did not use Eq. 1-16. This equation can be used as a consistency check. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{(15.55 \text{ m/s})^2 + 2(-3.56 \text{ m/s}^2)(24 \text{ m} - 0)} = 8.43 \text{ m/s},$$

which is what was calculated in (b). This indicates that the problems have been solved correctly.

34. Let d be the 220 m distance between the cars at $t = 0$, and v_1 be the $20 \text{ km/h} = 50/9 \text{ m/s}$ speed (corresponding to a passing point of $x_1 = 44.5 \text{ m}$) and v_2 be the $40 \text{ km/h} = 100/9 \text{ m/s}$ speed (corresponding to a passing point of $x_2 = 76.6 \text{ m}$) of the red car. We have two equations (based on Eq. 2-17):

$$d - x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 \quad \text{where } t_1 = x_1 / v_1$$

$$d - x_2 = v_0 t_2 + \frac{1}{2} a t_2^2 \quad \text{where } t_2 = x_2 / v_2$$

We simultaneously solve these equations and obtain the following results:

(a) The initial velocity of the green car is $v_o = -13.9$ m/s. or roughly -50 km/h (the negative sign means that it's along the $-x$ direction).

(b) The corresponding acceleration of the car is $a = -2.0$ m/s² (the negative sign means that it's along the $-x$ direction).

35. The positions of the cars as a function of time are given by

$$x_r(t) = x_{r0} + \frac{1}{2} a_r t^2 = (-35.0 \text{ m}) + \frac{1}{2} a_r t^2$$

$$x_g(t) = x_{g0} + v_g t = (270 \text{ m}) - (20 \text{ m/s})t$$

where we have substituted the velocity and not the speed for the green car. The two cars pass each other at $t = 12.0$ s when the graphed lines cross. This implies that

$$(270 \text{ m}) - (20 \text{ m/s})(12.0 \text{ s}) = 30 \text{ m} = (-35.0 \text{ m}) + \frac{1}{2} a_r (12.0 \text{ s})^2$$

which can be solved to give $a_r = 0.90$ m/s².

36. (a) Equation 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$$\Delta x_1 = v_{o1} t_1 + \frac{1}{2} a_1 t_1^2 \quad \text{where } a_1 = 2.25 \text{ m/s}^2 \text{ and } \Delta x_1 = \frac{900}{4} \text{ m}$$

$$\Delta x_2 = v_2 t_2 - \frac{1}{2} a_2 t_2^2 \quad \text{where } a_2 = -0.75 \text{ m/s}^2 \text{ and } \Delta x_2 = \frac{3(900)}{4} \text{ m}$$

In addition, $v_{o1} = v_2 = 0$. Solving these equations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s.

(b) Equation 2-16 is used for part 1 of the trip:

$$v^2 = (v_{o1})^2 + 2a_1 \Delta x_1 = 0 + 2(2.25) \left(\frac{900}{4} \right) = 1013 \text{ m}^2/\text{s}^2$$

which leads to $v = 31.8$ m/s for the maximum speed.

37. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a :

$$0.0 - (-2.0 \text{ m}) = v_0(1.0 \text{ s}) + \frac{1}{2}a(1.0 \text{ s})^2$$

$$6.0 \text{ m} - (-2.0 \text{ m}) = v_0(2.0 \text{ s}) + \frac{1}{2}a(2.0 \text{ s})^2.$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0 \text{ m/s}^2$.

(b) The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop ($v_2 = 0$) at the next station. The velocity at the midpoint is v_1 , which occurs at $x_1 = 806/2 = 403 \text{ m}$.

(a) Equation 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \Rightarrow v_1 = \sqrt{2(1.34 \text{ m/s}^2)(403 \text{ m})} = 32.9 \text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

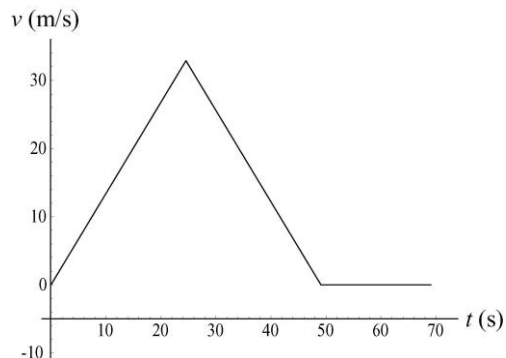
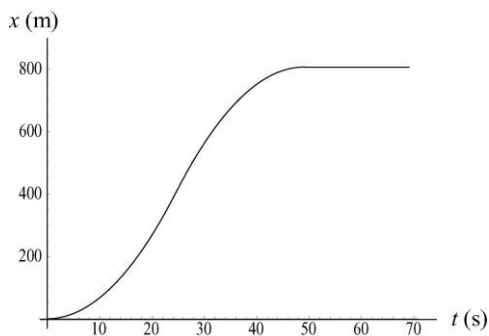
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \Rightarrow t_1 = \sqrt{\frac{2(403 \text{ m})}{1.34 \text{ m/s}^2}} = 24.53 \text{ s}.$$

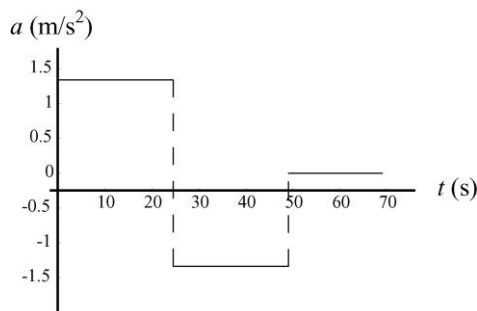
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain $t = 49.1 \text{ s}$ for the travel time between stations.

(c) With a “dead time” of 20 s, we have $T = t + 20 = 69.1 \text{ s}$ for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806 \text{ m}}{69.1 \text{ s}} = 11.7 \text{ m/s}.$$

(d) The graphs for x , v and a as a function of t are shown below. The third graph, $a(t)$, consists of three horizontal “steps” — one at 1.34 m/s^2 during $0 < t < 24.53 \text{ s}$, and the next at -1.34 m/s^2 during $24.53 \text{ s} < t < 49.1 \text{ s}$ and the last at zero during the “dead time” $49.1 \text{ s} < t < 69.1 \text{ s}$.





39. (a) We note that $v_A = 12/6 = 2$ m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at $x = 28$ m when $t = 4$ s. This must be the value of x for car B at that time; we use Eq. 2-15:

$$28 \text{ m} = (12 \text{ m/s})t + \frac{1}{2} a_B t^2 \quad \text{where } t = 4.0 \text{ s} .$$

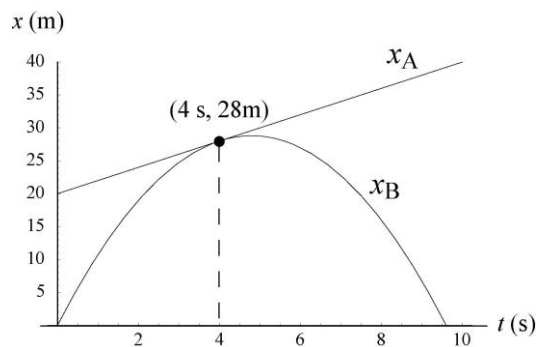
This yields $a_B = -2.5 \text{ m/s}^2$.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides $t = 4$ s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \frac{1}{2} a_B t^2$$

where $a_B = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_B)}$ is zero. In our case, it is zero – which means there is only one root. The cars are side by side only once at $t = 4$ s.

(c) A sketch is shown below. It consists of a straight line (x_A) tangent to a parabola (x_B) at $t = 4$.



(d) We only care about real roots, which means $10^2 - 2(-20)(a_B) \geq 0$. If $|a_B| > 5/2$ then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \Rightarrow$ two real roots. The cars are side by side at two different times.

40. We take the direction of motion as $+x$, so $a = -5.18 \text{ m/s}^2$, and we use SI units, so $v_0 = 55(1000/3600) = 15.28 \text{ m/s}$.

(a) The velocity is constant during the reaction time T , so the distance traveled during it is

$$d_r = v_0 T = (15.28 \text{ m/s})(0.75 \text{ s}) = 11.46 \text{ m}.$$

We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \Rightarrow d_b = -\frac{(15.28 \text{ m/s})^2}{2(-5.18 \text{ m/s}^2)}$$

which yields $d_b = 22.53 \text{ m}$. Thus, the total distance is $d_r + d_b = 34.0 \text{ m}$, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0 , the car would enter the intersection in $t = (40 \text{ m})/(15.28 \text{ m/s}) = 2.6 \text{ s}$, which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is $32/15.28 = 2.1 \text{ s}$, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

41. The displacement (Δx) for each train is the “area” in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is $1/2(\text{base}) \times (\text{height})$. Thus, the (absolute value of the) displacement for one train $(1/2)(40 \text{ m/s})(5 \text{ s}) = 100 \text{ m}$, and that of the other train is $(1/2)(30 \text{ m/s})(4 \text{ s}) = 60 \text{ m}$. The initial “gap” between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is $200 - 160 = 40 \text{ m}$.

42. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two-second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2-15) 51.11 m. In light of the fact that the initial “gap” between cars was 25 m, this means the gap has narrowed by 10.0 m – that is, to a distance of 15.0 m between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2-15) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it t_0) the gap between the cars is 10.6 m. The speed of the police car at t_0 is $30.56 - 5(2.4) = 18.56 \text{ m/s}$. Collision occurs at time t when $x_{\text{you}} = x_{\text{police}}$ (we choose coordinates such that your position is $x = 0$ and the police car’s position is $x = 10.6 \text{ m}$ at t_0). Eq. 2-15 becomes, for each car:

$$\begin{aligned} x_{\text{police}} - 10.6 &= 18.56(t - t_0) - \frac{1}{2}(5)(t - t_0)^2 \\ x_{\text{you}} &= 30.56(t - t_0) - \frac{1}{2}(5)(t - t_0)^2 \end{aligned} .$$

Subtracting equations, we find

$$10.6 = (30.56 - 18.56)(t - t_0) \Rightarrow 0.883 \text{ s} = t - t_0.$$

At that time your speed is $30.56 + a(t - t_0) = 30.56 - 5(0.883) \approx 26$ m/s (or 94 km/h).

43. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them, D , as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell.$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell \right) \left(\frac{v_\ell - v_t}{D} \right) = -\frac{1}{2D} (v_\ell - v_t)^2.$$

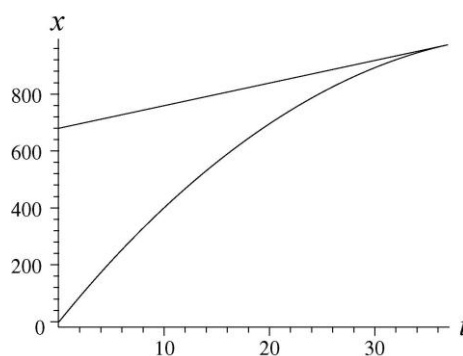
Hence,

$$a = -\frac{1}{2(0.676 \text{ km})} \left(29 \frac{\text{km}}{\text{h}} - 161 \frac{\text{km}}{\text{h}} \right)^2 = -12888 \text{ km/h}^2$$

which we convert as follows:

$$a = (-12888 \text{ km/h}^2) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is $|a| = 0.994 \text{ m/s}^2$. A graph is shown here for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

44. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2} g t^2$, with $y = 0.544 \text{ m}$ and $t = 0.200 \text{ s}$, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.544 \text{ m} + (9.8 \text{ m/s}^2)(0.200 \text{ s})^2/2}{0.200 \text{ s}} = 3.70 \text{ m/s}.$$

(b) The velocity at $y = 0.544 \text{ m}$ is

$$v = v_0 - gt = 3.70 \text{ m/s} - (9.8 \text{ m/s}^2)(0.200 \text{ s}) = 1.74 \text{ m/s}.$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{(3.7 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.698 \text{ m}.$$

Thus, the armadillo goes $0.698 - 0.544 = 0.154 \text{ m}$ higher.

45. **THINK** As the ball travels vertically upward, its motion is under the influence of gravitational acceleration. The kinematics is one-dimensional.

EXPRESS We neglect air resistance for the duration of the motion (between “launching” and “landing”), so $a = -g = -9.8 \text{ m/s}^2$ (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a constant motion:

$$v = v_0 - gt \quad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} gt^2 \quad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad (2-16)$$

We set $y_0 = 0$. Upon reaching the maximum height y , the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to y via the equation $0 = v^2 = v_0^2 - 2gy$. The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0/g$. Therefore, for the entire trip (from the time it leaves the ground until the time it returns to the ground), the total flight time is $T = 2t = 2v_0/g$.

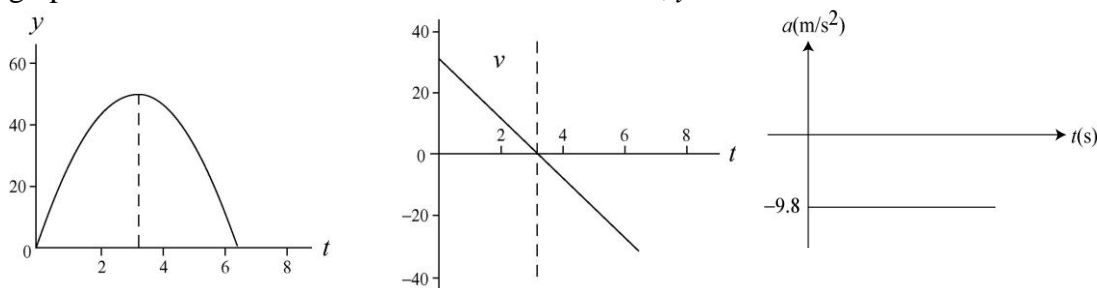
ANALYZE (a) At the highest point $v = 0$ and $v_0 = \sqrt{2gy}$. With $y = 50 \text{ m}$, we find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8 \text{ m/s}^2)(50 \text{ m})} = 31.3 \text{ m/s}.$$

(b) Using the result from (a) for v_0 , the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2(31.3 \text{ m/s})}{9.8 \text{ m/s}^2} = 6.39 \text{ s}$$

(c) The plots of y , v and a as a function of time are shown below. The acceleration graph is a horizontal line at -9.8 m/s^2 . At $t = 3.19 \text{ s}$, $y = 50 \text{ m}$.



LEARN In calculating the total flight time of the ball, we could have used Eq. 2-15. At $t = T > 0$, the ball returns to its original position ($y = 0$). Therefore,

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \Rightarrow T = \frac{2v_0}{g}$$

46. Neglect of air resistance justifies setting $a = -g = -9.8 \text{ m/s}^2$ (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8 \text{ m/s}^2)(-1700 \text{ m})} = -183 \text{ m/s}.$$

Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

47. **THINK** The wrench is in free fall with an acceleration $a = -g = -9.8 \text{ m/s}^2$.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \quad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} gt^2 \quad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad (2-16)$$

Since the wrench had an initial speed $v_0 = 0$, knowing its speed of impact allows us to apply Eq. 2-16 to calculate the height from which it was dropped.

ANALYZE (a) Using $v^2 = v_0^2 + 2a\Delta y$, we find the initial height to be

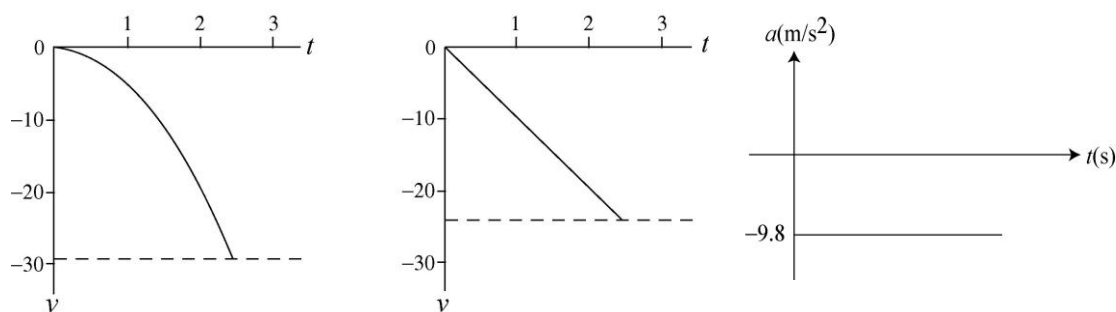
$$\Delta y = \frac{v_0^2 - v^2}{2a} = \frac{0 - (-24 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 29.4 \text{ m}.$$

So that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24 \text{ m/s})}{9.8 \text{ m/s}^2} = 2.45 \text{ s}.$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. The acceleration graph is a horizontal line at -9.8 m/s^2 .



LEARN As the wrench falls, with $a = -g < 0$, its speed increases but its velocity becomes more negative, as indicated by the second graph above.

48. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30 \text{ m}$, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t :

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12 \text{ m/s}$ since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12 \text{ m/s} + \sqrt{(-12 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(-30 \text{ m})}}{9.8 \text{ m/s}^2} = 1.54 \text{ s}.$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v ; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

49. **THINK** In this problem a package is dropped from a hot-air balloon which is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. This allows us to use Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \quad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \quad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad (2-16)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12 \text{ m/s}$ and that its initial coordinate is $y_0 = +80 \text{ m}$. The time it takes for the package to hit the ground can be found by solving Eq. 2-15 with $y = 0$.

ANALYZE (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for t):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 \text{ m/s} + \sqrt{(12 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(80 \text{ m})}}{9.8 \text{ m/s}^2} = 5.45 \text{ s}.$$

(b) The speed of the package when it hits the ground can be calculated using Eq. 2-11. The result is

$$v = v_0 - gt = 12 \text{ m/s} - (9.8 \text{ m/s}^2)(5.447 \text{ s}) = -41.38 \text{ m/s}.$$

Its final *speed* is 41.38 m/s.

LEARN Our answers can be readily verified by using Eq. 2-16 which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(0 - 80 \text{ m})} = -41.38 \text{ m/s}$$

which agrees with that calculated in (b).

50. The y coordinate of Apple 1 obeys $y - y_{01} = -\frac{1}{2} g t^2$ where $y = 0$ when $t = 2.0 \text{ s}$. This allows us to solve for y_{01} , and we find $y_{01} = 19.6 \text{ m}$.

The graph for the coordinate of Apple 2 (which is thrown apparently at $t = 1.0 \text{ s}$ with

velocity v_2) is

$$y - y_{o2} = v_2(t - 1.0) - \frac{1}{2} g (t - 1.0)^2$$

where $y_{o2} = y_{o1} = 19.6$ m and where $y = 0$ when $t = 2.25$ s. Thus, we obtain $|v_2| = 9.6$ m/s, approximately.

51. (a) With upward chosen as the $+y$ direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_o + at \quad \Rightarrow \quad 0 = v_o - (9.8 \text{ m/s}^2)(2.0 \text{ s})$$

which leads to $v_o = 19.6$ m/s. Now we use Eq. 2-15:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m} .$$

We note that the “2.0 s” in this second computation refers to the time interval $2 < t < 4$ in the graph (whereas the “2.0 s” in the first computation referred to the $0 < t < 2$ time interval shown in the graph).

(b) In our computation for part (b), the time interval (“6.0 s”) refers to the $2 < t < 8$ portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m} ,$$

or $|\Delta y| = 59$ m .

52. The full extent of the bolt’s fall is given by

$$y - y_0 = -\frac{1}{2} g t^2$$

where $y - y_0 = -90$ m (if upward is chosen as the positive y direction). Thus the time for the full fall is found to be $t = 4.29$ s. The first 80% of its free-fall distance is given by $-72 = -g \tau^2/2$, which requires time $\tau = 3.83$ s.

(a) Thus, the final 20% of its fall takes $t - \tau = 0.45$ s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v| = 38$ m/s, approximately.

(c) Similarly, $v_{final} = -g t \Rightarrow |v_{final}| = 42$ m/s.

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

EXPRESS The speed of the boat is constant, given by $v_b = d/t$, where d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling.

To calculate t , we take the time to be zero at the instant the key is dropped, we compute the time t when $y = 0$ using $y = y_0 + v_0 t - \frac{1}{2} g t^2$, with $y_0 = 45$ m. Once t is known, the speed of the boat can be readily calculated.

ANALYZE Since the initial velocity of the key is zero, the coordinate of the key is given by $y_0 = \frac{1}{2} g t^2$. Thus, the time it takes for the key to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45 \text{ m})}{9.8 \text{ m/s}^2}} = 3.03 \text{ s}.$$

Therefore, the speed of the boat is $v_b = \frac{12 \text{ m}}{3.03 \text{ s}} = 4.0 \text{ m/s}$.

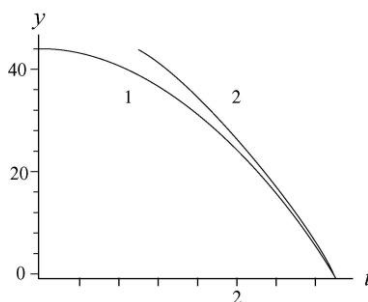
LEARN From the general expression $v_b = \frac{d}{t} = \frac{d}{\sqrt{2y_0/g}} = d \sqrt{\frac{g}{2y_0}}$, we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

54. (a) We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2} g t^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2} g (t-1)^2$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



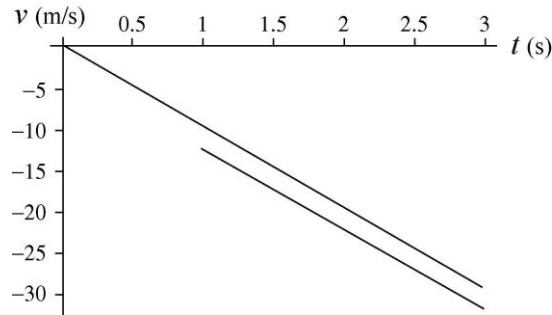
$$-43.9 \text{ m} = (-v_0)(1.99 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.99 \text{ s})^2$$

which leads to $v_0 = 12.3 \text{ m/s}$.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \quad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



55. **THINK** The free-falling moist-clay ball strikes the ground with a non-zero speed, and it undergoes deceleration before coming to rest.

EXPRESS During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t},$$

where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0 \times 10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground ($y = 0$).

ANALYZE (a) Now, to find the velocity just *before* contact, we take $t = 0$ to be when it is dropped. Using Eq. 2-16 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{0 - 2(9.8 \text{ m/s}^2)(0 - 15 \text{ m})} = -17.15 \text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0 - (-17.1 \text{ m/s})}{20.0 \times 10^{-3} \text{ s}} = 857 \text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward.

LEARN Since Δt is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the course of collision.

56. We use Eq. 2-16,

$$v_B^2 = v_A^2 + 2a(y_B - y_A),$$

with $a = -9.8 \text{ m/s}^2$, $y_B - y_A = 0.40$ m, and $v_B = \frac{1}{3} v_A$. It is then straightforward to solve: $v_A = 3.0 \text{ m/s}$, approximately.

57. The average acceleration during contact with the floor is $a_{\text{avg}} = (v_2 - v_1) / \Delta t$,

where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8 \text{ m/s}^2)(-4.00 \text{ m})} = -8.85 \text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8 \text{ m/s}^2)(-2.00 \text{ m} + 4.00 \text{ m})} = 6.26 \text{ m/s}.$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 \text{ m/s} - (-8.85 \text{ m/s})}{12.0 \times 10^{-3} \text{ s}} = 1.26 \times 10^3 \text{ m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h , so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$y' = \frac{1}{2}gt'^2 \Rightarrow t' = \sqrt{\frac{2y'}{g}}$$

$$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}}.$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50}.$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \Rightarrow t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields $t = 3.41$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find $h = 57$ m.

(c) In our approach, we did not use the quadratic formula, but we did “choose a root” when we assumed (in the last calculation in part (a)) that $\sqrt{0.50} = +0.707$ instead of -0.707 . If we had instead let $\sqrt{0.50} = -0.707$ then our answer for t would have been roughly 0.6 s, which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped), which certainly does not fit with the physical situation described in the problem.

59. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y -axis.

(a) The time drop 1 leaves the nozzle is taken as $t = 0$ and its time of landing on the floor t_1 can be computed from Eq. 2-15, with $v_0 = 0$ and $y_1 = -2.00$ m.

$$y_1 = -\frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00 \text{ m})}{9.8 \text{ m/s}^2}} = 0.639 \text{ s}.$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at $t = 0.639/3 = 0.213$ s and drop 3 leaves the nozzle at $t = 2(0.213 \text{ s}) = 0.426$ s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213 \text{ s} = 0.426$ s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.426 \text{ s})^2 = -0.889 \text{ m},$$

or about 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426 \text{ s} = 0.213$ s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.213 \text{ s})^2 = -0.222 \text{ m},$$

or about 22 cm below the nozzle.

60. To find the “launch” velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \Rightarrow 0 = v_0 - (9.8 \text{ m/s}^2)(2.5 \text{ s})$$

so that $v_0 = 24.5 \text{ m/s}$ (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking $y_0 = 0$ at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2} a t^2 \Rightarrow y - 0 = (24.5 \text{ m/s})(1.5 \text{ s}) - \frac{1}{2} (9.8 \text{ m/s}^2)(1.5 \text{ s})^2.$$

Thus, we obtain $y = 26 \text{ m}$.

61. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1 , and passes the bottom (which is at y_2) at time t_2 . We are told $y_2 - y_1 = 1.20 \text{ m}$ and $t_2 - t_1 = 0.125 \text{ s}$. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1 (t_2 - t_1) + \frac{1}{2} g (t_2 - t_1)^2$$

which immediately yields

$$v_1 = \frac{1.20 \text{ m} - \frac{1}{2} (9.8 \text{ m/s}^2)(0.125 \text{ s})^2}{0.125 \text{ s}} = 8.99 \text{ m/s}.$$

From this, Eq. 2-16 (with $v_0 = 0$) reveals the value of y_1 :

$$v_1^2 = 2gy_1 \Rightarrow y_1 = \frac{(8.99 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 4.12 \text{ m}.$$

It reaches the ground ($y_3 = H$) at t_3 . Because of the symmetry expressed in the problem (“upward flight is a reverse of the fall”) we know that $t_3 - t_2 = 2.00/2 = 1.00 \text{ s}$. And this means $t_3 - t_1 = 1.00 \text{ s} + 0.125 \text{ s} = 1.125 \text{ s}$. Now Eq. 2-15 produces

$$y_3 - y_1 = v_1 (t_3 - t_1) + \frac{1}{2} g (t_3 - t_1)^2$$

$$y_3 - 4.12 \text{ m} = (8.99 \text{ m/s})(1.125 \text{ s}) + \frac{1}{2} (9.8 \text{ m/s}^2)(1.125 \text{ s})^2$$

which yields $y_3 = H = 20.4 \text{ m}$.

62. The height reached by the player is $y = 0.76 \text{ m}$ (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8 \text{ m/s}^2)(0.76 \text{ m})} = 3.86 \text{ m/s}.$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1

= 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(0.61 \text{ m})} = 1.71 \text{ m/s} .$$

The time t_1 that the player spends *ascending* in the top $\Delta y_1 = 0.15$ m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2} (v_1 + v) t_1 \Rightarrow t_1 = \frac{2(0.15 \text{ m})}{1.71 \text{ m/s} + 0} = 0.175 \text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is $2(0.175 \text{ s}) = 0.35 \text{ s} = 350 \text{ ms}$.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15 \text{ m} = v_0 t_2 - \frac{1}{2} g t_2^2 = (3.86 \text{ m/s}) t_2 - \frac{1}{2} (9.8 \text{ m/s}^2) t_2^2 ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) $t_2 = 0.041 \text{ s} = 41 \text{ ms}$, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is $2(41 \text{ ms}) = 82 \text{ ms}$.

63. The time t the pot spends passing in front of the window of length $L = 2.0$ m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8 \text{ m/s}^2$ (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2} g t^2 \Rightarrow v = \frac{L}{t} - \frac{1}{2} g t .$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00 \text{ m}/0.25 \text{ s} - (9.80 \text{ m/s}^2)(0.25 \text{ s})/2)^2}{2(9.80 \text{ m/s}^2)} = 2.34 \text{ m} .$$

64. The graph shows $y = 25$ m to be the highest point (where the speed momentarily vanishes). The neglect of “air friction” (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with +y up)

$$y - y_0 = vt + \frac{1}{2} g_p t^2 \Rightarrow 25 \text{ m} - 0 = (0)(2.5 \text{ s}) + \frac{1}{2} g_p (2.5 \text{ s})^2$$

so that $g_p = 8.0 \text{ m/s}^2$.

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \Rightarrow 25 \text{ m} - 0 = \frac{1}{2} (v_0 + 0)(2.5 \text{ s})$$

Therefore, $v_0 = 20 \text{ m/s}$.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \left(\begin{array}{l} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right)$$

(a) From Fig. 2.15a, we see that the head begins to accelerate from rest ($v_0 = 0$) at $t_0 = 110 \text{ ms}$ and reaches a maximum value of 90 m/s^2 at $t_1 = 160 \text{ ms}$. The area of this region is

$$\text{area} = \frac{1}{2} (160 - 110) \times 10^{-3} \text{ s} \cdot (90 \text{ m/s}^2) = 2.25 \text{ m/s}$$

which is equal to v_1 , the speed at t_1 .

(b) To compute the speed of the torso at $t_1 = 160 \text{ ms}$, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_B = \frac{1}{2} (0.0600 \text{ s})(50.0 \text{ m/s}^2) = 1.50 \text{ m/s}.$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_C = (0.0200 \text{ s})(50.0 \text{ m/s}^2) = 1.00 \text{ m/s}.$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2} (0.0400 \text{ s})(50.0 + 20.0) \text{ m/s}^2 = 1.40 \text{ m/s}.$$

Substituting these values into Eq. 2-26, with $v_0 = 0$ then gives

$$v_1 - 0 = 0 + 1.50 \text{ m/s} + 1.00 \text{ m/s} + 1.40 \text{ m/s} = 3.90 \text{ m/s},$$

or $v_1 = 3.90 \text{ m/s}$.

66. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-30:

$$x_1 - x_0 = \left(\begin{array}{l} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right).$$

(a) To compute the position of the fist at $t = 50 \text{ ms}$, we divide the area in Fig. 2-37 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010 \text{ s})(2 \text{ m/s}) = 0.01 \text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040 \text{ s})(2 + 4) \text{ m/s} = 0.12 \text{ m}.$$

Substituting these values into Eq. 2-30 with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or $x_1 = 0.13 \text{ m}$.

(b) The speed of the fist reaches a maximum at $t_1 = 120 \text{ ms}$. From 50 to 90 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040 \text{ s})(4 + 5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030 \text{ s})(5 + 7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2-30, with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or $x_1 = 0.50 \text{ m}$.

67. The problem is solved using Eq. 2-31:

$$v_1 - v_0 = \left(\begin{array}{l} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{array} \right)$$

To compute the speed of the unhelmeted, bare head at $t_1 = 7.0 \text{ ms}$, we divide the area under the a vs. t graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s})(120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 ms to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s})(120 + 140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})(140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s})(200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with $v_0=0$ then gives

$$v_{\text{unhelmeted}} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0030 \text{ s})(40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_B = (0.0010 \text{ s})(40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.0020 \text{ s})(40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_D = \frac{1}{2}(0.0010 \text{ s})(80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with $v_0 = 0$ then gives

$$v_{\text{helmeted}} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_{\text{unhelmeted}} - v_{\text{helmeted}} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

68. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$\begin{aligned} v &= \text{area} = \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2 + 400 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(400 \text{ m/s}^2) \\ &= 5.0 \text{ m/s}. \end{aligned}$$

69. Since $v = dx/dt$ (Eq. 2-4), then $\Delta x = \int v dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base \times height) and triangular ($\frac{1}{2}$ base \times height) areas, we have

$$\begin{aligned} A &= A_{0 < t < 2} + A_{2 < t < 10} + A_{10 < t < 12} + A_{12 < t < 16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4) \right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}(6.00t^2 + 3.00t + 2.00) = 12.0t + 3.00.$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t) dt = 20.0 - 4.00t^2.$$

The condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 20.0 - 4.00t^2 \Rightarrow 4.00t^2 + 12.0t - 17.0 = 0$$

which can be solved to give (taking positive root) $t = (-3 + \sqrt{26})/2 = 1.05$ s. Thus, the velocity at this time is $v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6$ m/s.

71. (a) The derivative (with respect to time) of the given expression for x yields the “velocity” of the spot:

$$v(t) = 9 - \frac{9}{4} t^2$$

with 3 significant figures understood. It is easy to see that $v = 0$ when $t = 2.00$ s.

(b) At $t = 2$ s, $x = 9(2) - \frac{3}{4}(2)^3 = 12$. Thus, the location of the spot when $v = 0$ is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is $a = -\frac{9}{2} t$, which gives an acceleration of -9.00 cm/m² (negative sign indicating leftward) when the spot is 12 cm from the left edge of screen.

(d) Since $v > 0$ for times less than $t = 2$ s, then the spot had been moving rightward.

(e) As implied by our answer to part (c), it moves leftward for times immediately after $t = 2$ s. In fact, the expression found in part (a) guarantees that for all $t > 2$, $v < 0$ (that is, until the clock is "reset" by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some $t > 2$ s cannot be the right edge; it is the left edge ($x = 0$). Solving the expression given in the problem statement (with $x = 0$) for positive t yields the answer: the spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

72. We adopt the convention frequently used in the text: that "up" is the positive y direction.

(a) At the highest point in the trajectory $v = 0$. Thus, with $t = 1.60$ s, the equation $v = v_0 - gt$ yields $v_0 = 15.7$ m/s.

(b) One equation that is not dependent on our result from part (a) is $y - y_0 = vt + \frac{1}{2}gt^2$; this readily gives $y_{\max} - y_0 = 12.5$ m for the highest ("max") point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into $y - y_0 = v_0t + \frac{1}{2}gt^2$ with $t = 6.00$ s and $y = 0$ (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \frac{1}{2} (9.8 \text{ m/s}^2)(6.00 \text{ s})^2.$$

Therefore, y_0 (the height of the building) is equal to 82.3 m.

73. We denote the required time as t , assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v then

$$\Delta x = \left(\frac{1}{2} at^2 \right)_{\text{car}} = (vt)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5 \text{ m/s})}{2.2 \text{ m/s}^2} = 8.6 \text{ s} .$$

Therefore,

$$\Delta x = vt = (9.5 \text{ m/s})(8.6 \text{ s}) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = (2.2 \text{ m/s}^2)(8.6 \text{ s}) = 19 \text{ m/s} .$$

74. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3° , then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan \theta} = \frac{35 \text{ m}}{\tan 4.3^\circ} = 465.5 \text{ m} \approx 0.465 \text{ km}.$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{\text{avg}}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465 \text{ km}}{1300 \text{ km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

75. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + a t_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2} \frac{v_0^2}{a} = v_0 t_r - \frac{1}{2} \frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2} \frac{v_{01}^2}{a}, \quad x_2 = v_{02} t_r - \frac{1}{2} \frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2} \frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

(a) Substituting $x_1 = 56.7 \text{ m}$, $v_{01} = 80.5 \text{ km/h} = 22.4 \text{ m/s}$, $x_2 = 24.4 \text{ m}$ and $v_{02} = 48.3 \text{ km/h} = 13.4 \text{ m/s}$, we find

$$\begin{aligned} t_r &= \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})} = \frac{(13.4 \text{ m/s})^2 (56.7 \text{ m}) - (22.4 \text{ m/s})^2 (24.4 \text{ m})}{(22.4 \text{ m/s})(13.4 \text{ m/s})(13.4 \text{ m/s} - 22.4 \text{ m/s})} \\ &= 0.74 \text{ s}. \end{aligned}$$

(b) Similarly, substituting $x_1 = 56.7 \text{ m}$, $v_{01} = 80.5 \text{ km/h} = 22.4 \text{ m/s}$, $x_2 = 24.4 \text{ m}$, and

$v_{02} = 48.3 \text{ km/h} = 13.4 \text{ m/s}$ gives

$$a = -\frac{1}{2} \frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2} \frac{(13.4 \text{ m/s})(22.4 \text{ m/s})^2 - (22.4 \text{ m/s})(13.4 \text{ m/s})^2}{(13.4 \text{ m/s})(56.7 \text{ m}) - (22.4 \text{ m/s})(24.4 \text{ m})}$$

$$= -6.2 \text{ m/s}^2.$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2 . Although rounded-off values are displayed in the above substitutions, what we have input into our calculators are the “exact” values (such as $v_{02} = \frac{161}{12} \text{ m/s}$).

76. (a) A constant velocity is equal to the ratio of displacement to elapsed time. Thus, for the vehicle to be traveling at a constant speed v_p over a distance D_{23} , the time delay should be $t = D_{23}/v_p$.

(b) The time required for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p/a$. During this time interval, the distance traveled is $\Delta x_0 = at_0^2/2 = v_p^2/2a$. The car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2, and the time elapsed is $t_1 = (D_{12} - \Delta x_0 - d)/v_p$. Thus, the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - (v_p^2/2a) - d}{v_p}$$

$$= t_r + \frac{1}{2} \frac{v_p}{a} + \frac{D_{12} - d}{v_p}$$

77. **THINK** The speed of the rod changes due to a nonzero acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the rod can be readily analyzed using the equations given in Table 2-1. We take $+x$ to be in the direction of motion, so

$$v = (60 \text{ km/h}) \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = +16.7 \text{ m/s}$$

and $a > 0$. The location where the rod starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

ANALYZE (a) Using Eq. 2-7, we find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7 \text{ m/s} - 0}{5.4 \text{ s} - 0} = 3.09 \text{ m/s}^2.$$

(b) Assuming constant acceleration $a = a_{\text{avg}} = 3.09 \text{ m/s}^2$, the total distance traveled during the 5.4-s time interval is

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (3.09 \text{ m/s}^2)(5.4 \text{ s})^2 = 45 \text{ m}$$

(c) Using Eq. 2-15, the time required to travel a distance of $x = 250 \text{ m}$ is:

$$x = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250 \text{ m})}{3.1 \text{ m/s}^2}} = 12.73 \text{ s}$$

LEARN The displacement of the rod as a function of time can be written as

$x(t) = \frac{1}{2} (3.09 \text{ m/s}^2) t^2$. Note that we could have chosen Eq. 2-17 to solve for (b):

$$x = \frac{1}{2} (v_0 + v) t = \frac{1}{2} (16.7 \text{ m/s})(5.4 \text{ s}) = 45 \text{ m}.$$

78. We take the moment of applying brakes to be $t = 0$. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v'_0 = 72 \text{ km/h} = 20 \text{ m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when $t = 0$) and unprimed variables refer to the other (moving in the $-x$ direction and located at $x_0 = +950 \text{ m}$ when $t = 0$). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is $v_0 = -144 \text{ km/h} = -40 \text{ m/s}$. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v'_0)^2}{2a'} = \frac{0 - (20 \text{ m/s})^2}{-2 \text{ m/s}^2} = 200 \text{ m}.$$

The speed of the other train, when it reaches that location, is

$$\begin{aligned} v &= \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(-40 \text{ m/s})^2 + 2(1.0 \text{ m/s}^2)(200 \text{ m} - 950 \text{ m})} \\ &= 10 \text{ m/s} \end{aligned}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be -10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields $t = 20 \text{ s}$) and seeing where the unprimed train is at that moment (Eq. 2-18 yields $x = 350 \text{ m}$, still a good distance away from contact).

79. The y coordinate of Piton 1 obeys $y - y_{01} = -\frac{1}{2} g t^2$ where $y = 0$ when $t = 3.0 \text{ s}$.

This allows us to solve for y_{01} , and we find $y_{01} = 44.1 \text{ m}$. The graph for the coordinate of Piton 2 (which is thrown apparently at $t = 1.0 \text{ s}$ with velocity v_1) is

$$y - y_{02} = v_1(t-1.0) - \frac{1}{2} g (t-1.0)^2$$

where $y_{02} = y_{01} + 10 = 54.1$ m and where (again) $y = 0$ when $t = 3.0$ s. Thus we obtain $|v_1| = 17$ m/s, approximately.

80. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s, and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{(50 \text{ m/s})^2 - (30 \text{ m/s})^2}{2(160 \text{ m})} = 5.0 \text{ m/s}^2 .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160 \text{ m})}{30 \text{ m/s} + 50 \text{ m/s}} = 4.0 \text{ s} .$$

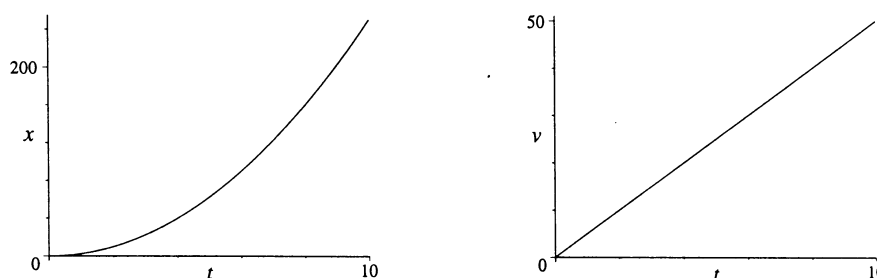
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

$$v_1 = v_0 + at_1 \Rightarrow t_1 = \frac{30 \text{ m/s}}{5.0 \text{ m/s}^2} = 6.0 \text{ s} .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1 . Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30 \text{ m/s})(6.0 \text{ s}) = 90 \text{ m} .$$

(e) The graphs are shown below, with SI units understood.



81. **THINK** The particle undergoes a *non-constant* acceleration along the $+x$ -axis. An integration is required to calculate velocity.

EXPRESS With a non-constant acceleration $a(t) = dv/dt$, the velocity of the

particle at time t_1 is given by Eq. 2-27: $v_1 = v_0 + \int_{t_0}^{t_1} a(t)dt$, where v_0 is the velocity at time t_0 . In our situation, we have $a = 5.0t$. In addition, we also know that $v_0 = 17 \text{ m/s}$ at $t_0 = 2.0 \text{ s}$.

ANALYZE Integrating (from $t = 2 \text{ s}$ to variable $t = 4 \text{ s}$) the acceleration to get the velocity and using the values given in the problem, leads to

$$v = v_0 + \int_{t_0}^t a dt = v_0 + \int_{t_0}^t (5.0t) dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(4^2 - 2^2) = 47 \text{ m/s}.$$

LEARN The velocity of the particle as a function of t is

$$v(t) = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(t^2 - 4) = 7 + 2.5t^2$$

in SI units (m/s). Since the acceleration is linear in t , we expect the velocity to be quadratic in t , and the displacement to be cubic in t .

82. The velocity v at $t = 6$ (SI units and two significant figures understood) is $v_{\text{given}} + \int_{-2}^6 a dt$. A quick way to implement this is to recall the area of a triangle ($\frac{1}{2}$ base \times height). The result is $v = 7 \text{ m/s} + 32 \text{ m/s} = 39 \text{ m/s}$.

83. The object, once it is dropped ($v_0 = 0$) is in free fall ($a = -g = -9.8 \text{ m/s}^2$ if we take down as the $-y$ direction), and we use Eq. 2-15 repeatedly.

(a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus, for $t_1 = 50.0 \text{ ms}$,

$$D_1 = \frac{(9.8 \text{ m/s}^2)(50.0 \times 10^{-3} \text{ s})^2}{2} = 0.0123 \text{ m} = 1.23 \text{ cm}.$$

$$(b) \text{ For } t_2 = 100 \text{ ms, } D_2 = \frac{(9.8 \text{ m/s}^2)(100 \times 10^{-3} \text{ s})^2}{2} = 0.049 \text{ m} = 4D_1.$$

$$(c) \text{ For } t_3 = 150 \text{ ms, } D_3 = \frac{(9.8 \text{ m/s}^2)(150 \times 10^{-3} \text{ s})^2}{2} = 0.11 \text{ m} = 9D_1.$$

$$(d) \text{ For } t_4 = 200 \text{ ms, } D_4 = \frac{(9.8 \text{ m/s}^2)(200 \times 10^{-3} \text{ s})^2}{2} = 0.196 \text{ m} = 16D_1.$$

$$(e) \text{ For } t_4 = 250 \text{ ms, } D_5 = \frac{(9.8 \text{ m/s}^2)(250 \times 10^{-3} \text{ s})^2}{2} = 0.306 \text{ m} = 25D_1.$$

84. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so $v = 1600(1000/3600) = 444$ m/s.

(a) Equation 2-11 gives $444 = a(1.8)$ or $a = 247$ m/s². We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right) g = 25g.$$

(b) Equation 2-17 readily yields

$$x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(444 \text{ m/s})(1.8 \text{ s}) = 400 \text{ m}.$$

85. Let D be the distance up the hill. Then

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time of travel}} = \frac{2D}{\frac{D}{20 \text{ km/h}} + \frac{D}{35 \text{ km/h}}} \approx 25 \text{ km/h}.$$

86. We obtain the velocity by integration of the acceleration:

$$v - v_0 = \int_0^t (6.1 - 1.2t') dt'.$$

Lengths are in meters and times are in seconds. The student is encouraged to look at the discussion in Section 2-7 to better understand the manipulations here.

(a) The result of the above calculation is $v = v_0 + 6.1t - 0.6t^2$, where the problem states that $v_0 = 2.7$ m/s. The maximum of this function is found by knowing when its derivative (the acceleration) is zero ($a = 0$ when $t = 6.1/1.2 = 5.1$ s) and plugging that value of t into the velocity equation above. Thus, we find $v = 18$ m/s.

(b) We integrate again to find x as a function of t :

$$x - x_0 = \int_0^t v dt' = \int_0^t (v_0 + 6.1t' - 0.6t'^2) dt' = v_0 t + 3.05t^2 - 0.2t^3.$$

With $x_0 = 7.3$ m, we obtain $x = 83$ m for $t = 6$. This is the correct answer, but one has the right to worry that it might not be; after all, the problem asks for the total distance traveled (and $x - x_0$ is just the *displacement*). If the cyclist backtracked, then his total distance would be greater than his displacement. Thus, we might ask, "did he backtrack?" To do so would require that his velocity be (momentarily) zero at some point (as he reversed his direction of motion). We could solve the above quadratic equation for velocity, for a positive value of t where $v = 0$; if we did, we would find that at $t = 10.6$ s, a reversal does indeed happen. However, in the time interval we are concerned with in our problem ($0 \leq t \leq 6$ s), there is no reversal and the displacement is the same as the total distance traveled.

87. **THINK** In this problem we're given two different speeds, and asked to find the difference in their travel times.

EXPRESS The time it takes to travel a distance d with a speed v_1 is $t_1 = d/v_1$. Similarly, with a speed v_2 the time would be $t_2 = d/v_2$. The two speeds in this problem are

$$v_1 = 55 \text{ mi/h} = (55 \text{ mi/h}) \frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 24.58 \text{ m/s}$$

$$v_2 = 65 \text{ mi/h} = (65 \text{ mi/h}) \frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 29.05 \text{ m/s}$$

ANALYZE With $d = 700 \text{ km} = 7.0 \times 10^5 \text{ m}$, the time difference between the two is

$$\begin{aligned} \Delta t = t_1 - t_2 &= d \left(\frac{1}{v_1} - \frac{1}{v_2} \right) = (7.0 \times 10^5 \text{ m}) \left(\frac{1}{24.58 \text{ m/s}} - \frac{1}{29.05 \text{ m/s}} \right) = 4383 \text{ s} \\ &= 73 \text{ min} \end{aligned}$$

or about 1.2 h.

LEARN The travel time was reduced from 7.9 h to 6.9 h. Driving at higher speed (within the legal limit) reduces travel time.

88. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17:

$$x = \frac{1}{2} (v + v_0) t = \frac{1}{2} (15.0 \text{ m/s} + v_0) (6.00 \text{ s}).$$

With $x = 60.0 \text{ m}$ (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: $v_0 = 5.00 \text{ m/s}$.

(b) Substituting $v = 15.0 \text{ m/s}$, $v_0 = 5.00 \text{ m/s}$, and $t = 6.00 \text{ s}$ into $a = (v - v_0)/t$ (Eq. 2-11), we find $a = 1.67 \text{ m/s}^2$.

(c) Substituting $v = 0$ in $v^2 = v_0^2 + 2ax$ and solving for x , we obtain

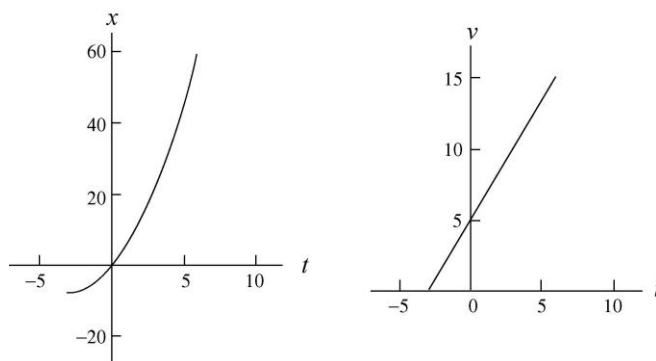
$$x = -\frac{v_0^2}{2a} = -\frac{(5.00 \text{ m/s})^2}{2(1.67 \text{ m/s}^2)} = -7.50 \text{ m},$$

or $|x| = 7.50 \text{ m}$.

(d) The graphs require computing the time when $v = 0$, in which case, we use $v = v_0 + at'$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5.00 \text{ m/s}}{1.67 \text{ m/s}^2} = -3.0 \text{ s}$$

indicates the moment the car was at rest. SI units are understood.



89. **THINK** In this problem we explore the connection between the maximum height an object reaches under the influence of gravity and the total amount of time it stays in air.

EXPRESS Neglecting air resistance and setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion, we analyze the motion of the ball using Table 2-1 (with Δy replacing Δx). We set $y_0 = 0$. Upon reaching the maximum height H , the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to H via the equation

$$0 = v^2 = v_0^2 - 2gH \Rightarrow v_0 = \sqrt{2gH}.$$

The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0 / g = \sqrt{2H/g}$.

ANALYZE If we want the ball to spend twice as much time in air as before, i.e., $t' = 2t$, then the new maximum height H' it must reach is such that $t' = \sqrt{2H'/g}$. Solving for H' we obtain

$$H' = \frac{1}{2}gt'^2 = \frac{1}{2}g(2t)^2 = 4\left(\frac{1}{2}gt^2\right) = 4H.$$

LEARN Since $H \sim t^2$, doubling t means that H must increase fourfold. Note also that for $t' = 2t$, the initial speed must be twice the original speed: $v'_0 = 2v_0$.

90. (a) Using the fact that the area of a triangle is $\frac{1}{2}$ (base) (height) (and the fact that the integral corresponds to the area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = dv/dt = \text{slope of the graph}$, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to -2.0 m/s^2 .

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 \text{ m} - 1 \text{ m}}{4 \text{ s}} = 3.5 \text{ m/s}.$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 \text{ m/s} - 2 \text{ m/s}}{4 \text{ s}} = 0.$$

91. Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + \frac{1}{2} g t^2$, which (with $v_0 = 0$) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, $y_1 = 50 \text{ m}$ so that $t_1 = \sqrt{\frac{2(50 \text{ m})}{9.8 \text{ m/s}^2}} = 3.2 \text{ s}$.

(b) For this next part of the motion, we note that the total displacement is $y_2 = 100 \text{ m}$. Therefore, the total time is

$$t_2 = \sqrt{\frac{2(100 \text{ m})}{9.8 \text{ m/s}^2}} = 4.5 \text{ s}.$$

The difference between this and the answer to part (a) is the time required to fall through that second 50 m distance: $\Delta t = t_2 - t_1 = 4.5 \text{ s} - 3.2 \text{ s} = 1.3 \text{ s}$.

92. Direction of $+x$ is implicit in the problem statement. The initial position (when the clock starts) is $x_0 = 0$ (where $v_0 = 0$), the end of the speeding-up motion occurs at $x_1 = 1100/2 = 550 \text{ m}$, and the subway train comes to a halt ($v_2 = 0$) at $x_2 = 1100 \text{ m}$.

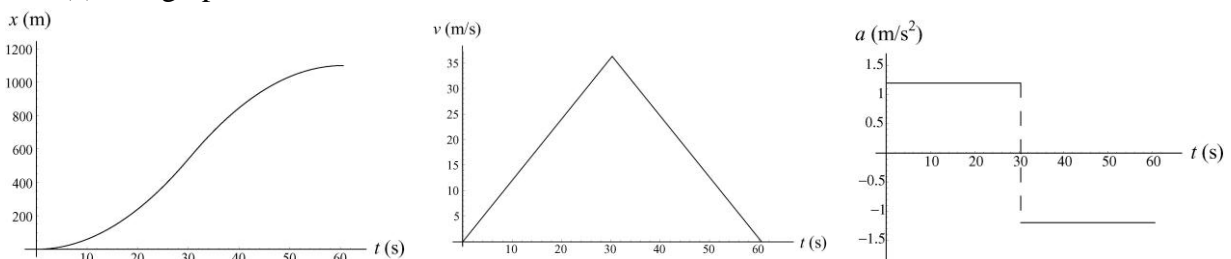
(a) Using Eq. 2-15, the subway train reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550 \text{ m})}{1.2 \text{ m/s}^2}} = 30.3 \text{ s}.$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is $t_2 = 2(30.3) = 60.6 \text{ s}$.

(b) Its maximum speed occurs at t_1 and equals $v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s}$.

(c) The graphs are shown below:



93. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_0 = 0$).

(a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$v_B^2 = v_0^2 - 2gy_B \Rightarrow \left(\frac{1}{2}v\right)^2 + 2g(y_A + 3) = v_0^2$$

$$v_A^2 = v_0^2 - 2gy_A \Rightarrow v^2 + 2gy_A = v_0^2$$

We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \Rightarrow 2g(3) = \frac{3}{4}v^2$$

which yields $v = \sqrt{2g(4)} = 8.85 \text{ m/s}$.

(b) An object moving upward at A with speed $v = 8.85 \text{ m/s}$ will reach a maximum height $y - y_A = v^2/2g = 4.00 \text{ m}$ above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B .

94. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y -axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$, and $y_0 = h = 60 \text{ m}$, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s}.$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3 \text{ s}$:

$$y - h = v_0(2.3 \text{ s}) - \frac{1}{2} g(2.3 \text{ s})^2 \Rightarrow y = 34 \text{ m}$$

where we again use the fact that $h = 60 \text{ m}$ and $v_0 = 0$.

95. **THINK** This problem involves analyzing a plot describing the position of an iceboat as function of time. The boat has a nonzero acceleration due to the wind.

EXPRESS Since we are told that the acceleration of the boat is constant, the equations of Table 2-1 can be applied. However, the challenge here is that v_0 , v , and a are not explicitly given. Our strategy to deduce these values is to apply the kinematic equation $x - x_0 = v_0 t + \frac{1}{2} a t^2$ to a variety of points on the graph and solve for the unknowns from the simultaneous equations.

ANALYZE (a) From the graph, we pick two points on the curve: $(t, x) = (2.0 \text{ s}, 16 \text{ m})$ and $(3.0 \text{ s}, 27 \text{ m})$. The corresponding simultaneous equations are

$$\begin{aligned} 16 \text{ m} - 0 &= v_0(2.0 \text{ s}) + \frac{1}{2} a(2.0 \text{ s})^2 \\ 27 \text{ m} - 0 &= v_0(3.0 \text{ s}) + \frac{1}{2} a(3.0 \text{ s})^2 \end{aligned}$$

Solving the equations lead to the values $v_0 = 6.0 \text{ m/s}$ and $a = 2.0 \text{ m/s}^2$.

(b) From Table 2-1,

$$x - x_0 = vt - \frac{1}{2} at^2 \Rightarrow 27 \text{ m} - 0 = v(3.0 \text{ s}) - \frac{1}{2} (2.0 \text{ m/s}^2)(3.0 \text{ s})^2$$

which leads to $v = 12 \text{ m/s}$.

(c) Assuming the wind continues during $3.0 \leq t \leq 6.0$, we apply $x - x_0 = v_0 t + \frac{1}{2} at^2$ to this interval (where $v_0 = 12.0 \text{ m/s}$ from part (b)) to obtain

$$\Delta x = (12.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2} (2.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 45 \text{ m}.$$

LEARN By using the results obtained in (a), the position and velocity of the iceboat as a function of time can be written as

$$x(t) = (6.0 \text{ m/s})t + \frac{1}{2} (2.0 \text{ m/s}^2)t^2 \quad \text{and} \quad v(t) = (6.0 \text{ m/s}) + (2.0 \text{ m/s}^2)t.$$

One can readily verify that the same answers are obtained for (b) and (c) using the above expressions for $x(t)$ and $v(t)$.

96. (a) Let the height of the diving board be h . We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D , and the total time for the ball to descend be T . The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}}.$$

Thus, $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80 \text{ s})\sqrt{(2)(9.80 \text{ m/s}^2)(5.20 \text{ m})} - 2(5.20 \text{ m}) = 38.1 \text{ m}.$$

(b) Using Eq. 2-2, the magnitude of the average velocity is

$$v_{\text{avg}} = \frac{D + h}{T} = \frac{38.1 \text{ m} + 5.20 \text{ m}}{4.80 \text{ s}} = 9.02 \text{ m/s}$$

(c) In our coordinate choices, a positive sign for v_{avg} means that the ball is going downward. If, however, upward had been chosen as the positive direction, then this answer in (b) would turn out negative-valued.

(d) We find v_0 from $\Delta y = v_0 t + \frac{1}{2} g t^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h + D}{T} - \frac{gT}{2} = \frac{5.20 \text{ m} + 38.1 \text{ m}}{4.80 \text{ s}} - \frac{(9.8 \text{ m/s}^2)(4.80 \text{ s})}{2} = 14.5 \text{ m/s}$$

(e) Here in our coordinate choices the negative sign means that the ball is being thrown upward.

97. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$, and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

(a) Equation 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8 \text{ m/s}^2)(120 \text{ m})} = 48.5 \text{ m/s}.$$

(b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120 \text{ m})}{9.8 \text{ m/s}^2}} = 4.95 \text{ s}.$$

(c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8 \text{ m/s}^2)(60 \text{ m})} = 34.3 \text{ m/s}.$$

(d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60 \text{ m})}{9.8 \text{ m/s}^2}} = 3.50 \text{ s}.$$

98. Taking +y to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\frac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2}g(t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10 \text{ m}$. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \Rightarrow t = (10/g) + 0.5 = 1.5 \text{ s}.$$

99. With +y upward, we have $y_0 = 36.6 \text{ m}$ and $y = 12.2 \text{ m}$. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \frac{1}{2}gt^2 \Rightarrow v = -22.0 \text{ m/s}$$

at $t = 2.00 \text{ s}$. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0 \text{ m/s}$.

100. During free fall, we ignore the air resistance and set $a = -g = -9.8 \text{ m/s}^2$ where we are choosing *down* to be the -y direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\frac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \frac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50 \text{ m} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.80 \text{ m/s}^2)t_1^2$$

which yields $t_1 = 3.2 \text{ s}$. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)(9.80 \text{ m/s}^2)(50 \text{ m})} = 31 \text{ m/s}.$$

If the final speed is v_2 , then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31 \text{ m/s} - 3.0 \text{ m/s}}{2 \text{ m/s}^2} = 14 \text{ s}.$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion — which makes it a deceleration). The total time of flight is therefore $t_1 + t_2 = 17 \text{ s}$.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\text{ m/s})^2 - (3.0\text{ m/s})^2}{(2)(2.0\text{ m/s}^2)} \approx 240\text{ m}.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by -1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of $h = 50 + d \approx 290\text{ m}$.

101. We neglect air resistance, which justifies setting $a = -g = -9.8\text{ m/s}^2$ (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g(y - y_0)} = \sqrt{v_0^2 + 2gh}.$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t , with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}.$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \Rightarrow t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$

102. We assume constant velocity motion and use Eq. 2-2 (with $v_{\text{avg}} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303 \frac{\text{km}}{\text{h}} \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) \right) (100 \times 10^{-3} \text{ s}) = 8.4 \text{ m}.$$

103. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is $\Delta x = v\Delta t$, where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have $160 \text{ km/h} = 44.4 \text{ m/s}$. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4 \text{ m}}{44.4 \text{ m/s}} = 0.414 \text{ s}.$$

The velocity-unit conversion implemented above can be figured “from basics” ($1000 \text{ m} = 1 \text{ km}$, $3600 \text{ s} = 1 \text{ h}$) or found in Appendix D.

104. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t . Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

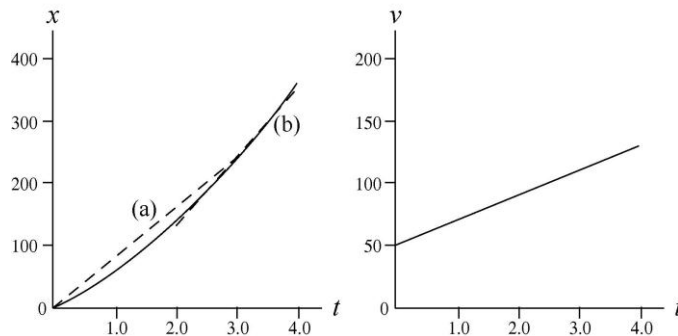
(a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80 \text{ m/s}.$$

(b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0 \text{ s}$, $v = 50 + (20)(3.0) = 110 \text{ m/s}$.

(c) The instantaneous acceleration at time t is given by $a = dv/dt = 20 \text{ m/s}^2$. It is constant, so the acceleration at any time is 20 m/s^2 .

(d) and (e) The graphs that follow show the coordinate x and velocity v as functions of time, with SI units understood. The dashed line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0 \text{ s}$, $x = 240 \text{ m}$. Its slope is the average velocity during the first 3 s of motion. The dashed line marked (b) is tangent to the x curve at $t = 3.0 \text{ s}$. Its slope is the instantaneous velocity at $t = 3.0 \text{ s}$.



105. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0$ m/s². The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -\frac{(30 \text{ m/s})^2}{2(-5 \text{ m/s}^2)} = 90 \text{ m}.$$

106. The problem consists of two constant-acceleration parts: part 1 with $v_0 = 0$, $v = 6.0$ m/s, $x = 1.8$ m, and $x_0 = 0$ (if we take its original position to be the coordinate origin); and, part 2 with $v_0 = 6.0$ m/s, $v = 0$, and $a_2 = -2.5$ m/s² (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-17 to find the time for the first part

$$x - x_0 = \frac{1}{2}(v_0 + v) t_1 \Rightarrow 1.8 \text{ m} - 0 = \frac{1}{2}(0 + 6.0 \text{ m/s}) t_1$$

so that $t_1 = 0.6$ s. And Eq. 2-11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \Rightarrow 0 = 6.0 \text{ m/s} + (-2.5 \text{ m/s}^2) t_2$$

from which $t_2 = 2.4$ s is computed. Thus, the total time is $t_1 + t_2 = 3.0$ s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but the one that makes no use of our part (a) results is Eq. 2-16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \Rightarrow 0 = (6.0 \text{ m/s})^2 + 2(-2.5 \text{ m/s}^2)\Delta x_2$$

which leads to $\Delta x_2 = 7.2$ m. Therefore, the total distance traveled by the shuffleboard disk is $(1.8 + 7.2) \text{ m} = 9.0$ m.

107. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100 \text{ km/h}) \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = 27.8 \text{ m/s}.$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

108. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a . Its minimum value is

$$a_{\min} = \frac{v_2 - v_0^2}{2\Delta x_{\max}} = \frac{(360 \text{ km/h})^2}{2(1.80 \text{ km})} = 36000 \text{ km/h}^2$$

which converts to 2.78 m/s².

109. (a) For the automobile $\Delta v = 55 - 25 = 30$ km/h, which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30 \text{ km/h}) \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right)}{(0.50 \text{ min})(60 \text{ s/min})} = 0.28 \text{ m/s}^2 .$$

(b) The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2 .

110. Converting to SI units, we have $v = 3400(1000/3600) = 944 \text{ m/s}$ (presumed constant) and $\Delta t = 0.10 \text{ s}$. Thus, $\Delta x = v\Delta t = 94 \text{ m}$.

111. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, $v = 11.0 \text{ m/s}$, $x = 12.0 \text{ m}$, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with $v = 11.0 \text{ m/s}$, $x_0 = 12.0$, and $x = 100.0 \text{ m}$.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}(v_0 + v)t_1 \Rightarrow 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that $t_1 = 2.2 \text{ s}$, and we find the time for part 2 simply from $88.0 = (11.0)t_2 \rightarrow t_2 = 8.0 \text{ s}$. Therefore, the total time is $t_1 + t_2 = 10.2 \text{ s}$.

(b) Here, the total time is required to be 10.0 s , and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$\begin{aligned} x_p - 0 &= \frac{1}{2}(0 + 11.0 \text{ m/s})t_1 \\ 100.0 \text{ m} - x_p &= (11.0 \text{ m/s})(10.0 \text{ s} - t_1) \end{aligned}$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that $t_1 = 1.8 \text{ s}$ and $x_p = 10.0 \text{ m}$.

112. The bullet starts at rest ($v_0 = 0$) and after traveling the length of the barrel ($\Delta x = 1.2 \text{ m}$) emerges with the given velocity ($v = 640 \text{ m/s}$), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use $\Delta x = \frac{1}{2}(v_0 + v)t$. Thus, we find $t = 0.00375 \text{ s}$ (or 3.75 ms).

113. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8 \text{ m/s}^2$ (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245 \text{ m/s}^2$.

(a) The time of fall is given by Eq. 2-15 with $v_0 = 0$ and $y = 0$. Thus,

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(145 \text{ m})}{9.8 \text{ m/s}^2}} = 5.44 \text{ s}.$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a))

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{2gy_0} = -53.3 \text{ m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is $|v| = 53.3 \text{ m/s}$.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity ($v_0 = -53.3 \text{ m/s}$) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{-(-53.3 \text{ m/s})^2}{2(245 \text{ m/s}^2)} = -5.80 \text{ m},$$

or $|\Delta y_2| = 5.80 \text{ m}$. The negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

114. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is $v_0 = 200(1000/3600) = 55.6 \text{ m/s}$. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with $\Delta x_b = 170 \text{ m}$) we find

$$v^2 = v_0^2 + 2a\Delta x_b \Rightarrow a = -\frac{v_0^2}{2\Delta x_b}$$

which yields $|a| = 9.08 \text{ m/s}^2$.

(b) We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{9.08 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right) g = 0.926g .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b = \frac{1}{2}(v_0 + v)T_b \Rightarrow T_b = \frac{2(170 \text{ m})}{55.6 \text{ m/s}} = 6.12 \text{ s} .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b = \left(\frac{6.12 \text{ s}}{400 \times 10^{-3} \text{ s}} \right) T_r = 15.3T_r .$$

(e) Since $T_b > T_r$, most of the full time required to stop is spent in braking.

(f) We are only asked what the *increase* in distance D is, due to $\Delta T_r = 0.100$ s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6 \text{ m/s})(0.100 \text{ s}) = 5.56 \text{ m}.$$

115. The total time elapsed is $\Delta t = 2 \text{ h } 41 \text{ min} = 161 \text{ min}$ and the center point is displaced by $\Delta x = 3.70 \text{ m} = 370 \text{ cm}$. Thus, the average velocity of the center point is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370 \text{ cm}}{161 \text{ min}} = 2.30 \text{ cm/min}.$$

116. Using Eq. 2-11, $v = v_0 + at$, we find the initial speed to be

$$v_0 = v - at = 0 - (-3400)(9.8 \text{ m/s}^2)(6.5 \times 10^{-3} \text{ s}) = 216.6 \text{ m/s}$$

117. The total number of days walked is (including the first and the last day, and leap year)

$$N = 340 + 365 + 365 + 366 + 365 + 365 + 261 = 2427$$

Thus, the average speed of the walk is

$$s_{\text{avg}} = \frac{d}{\Delta t} = \frac{3.06 \times 10^7 \text{ m}}{(2427 \text{ days})(86400 \text{ s/day})} = 0.146 \text{ m/s}.$$

118. (a) Let d be the distance traveled. The average speed with and without wings set as sails are $v_s = d/t_s$ and $v_{ns} = d/t_{ns}$, respectively. Thus, the ratio of the two speeds is

$$\frac{v_s}{v_{ns}} = \frac{d/t_s}{d/t_{ns}} = \frac{t_{ns}}{t_s} = \frac{25.0 \text{ s}}{7.1 \text{ s}} = 3.52$$

(b) The difference in time expressed in terms of v_s is

$$\Delta t = t_{ns} - t_s = \frac{d}{v_{ns}} - \frac{d}{v_s} = \frac{d}{(v_s/3.52)} - \frac{d}{v_s} = 2.52 \frac{d}{v_s} = 2.52 \frac{(2.0 \text{ m})}{v_s} = \frac{5.04 \text{ m}}{v_s}$$

119. (a) Differentiating $y(t) = (2.0 \text{ cm}) \sin(\pi t/4)$ with respect to t , we obtain

$$v_y(t) = \frac{dy}{dt} = \left(\frac{\pi}{2} \text{ cm/s} \right) \cos(\pi t/4)$$

The average velocity between $t = 0$ and $t = 2.0$ s is

$$\begin{aligned}
 v_{\text{avg}} &= \frac{1}{(2.0 \text{ s})} \int_0^2 v_y dt = \frac{1}{(2.0 \text{ s})} \left(\frac{\pi}{2} \text{ cm/s} \right) \int_0^2 \cos\left(\frac{\pi t}{4}\right) dt \\
 &= \frac{1}{(2.0 \text{ s})} (2 \text{ cm}) \int_0^{\pi/2} \cos x dx = 1.0 \text{ cm/s}
 \end{aligned}$$

(b) The instantaneous velocities of the particle at $t = 0$, 1.0 s, and 2.0 s are, respectively,

$$\begin{aligned}
 v_y(0) &= \left(\frac{\pi}{2} \text{ cm/s} \right) \cos(0) = \frac{\pi}{2} \text{ cm/s} \\
 v_y(1.0 \text{ s}) &= \left(\frac{\pi}{2} \text{ cm/s} \right) \cos(\pi/4) = \frac{\pi\sqrt{2}}{4} \text{ cm/s} \\
 v_y(2.0 \text{ s}) &= \left(\frac{\pi}{2} \text{ cm/s} \right) \cos(\pi/2) = 0
 \end{aligned}$$

(c) Differentiating $v_y(t)$ with respect to t , we obtain the following expression for acceleration:

$$a_y(t) = \frac{dv_y}{dt} = \left(-\frac{\pi^2}{8} \text{ cm/s}^2 \right) \sin(\pi t/4)$$

The average acceleration between $t = 0$ and $t = 2.0$ s is

$$\begin{aligned}
 a_{\text{avg}} &= \frac{1}{(2.0 \text{ s})} \int_0^2 a_y dt = \frac{1}{(2.0 \text{ s})} \left(-\frac{\pi^2}{8} \text{ cm/s}^2 \right) \int_0^2 \sin\left(\frac{\pi t}{4}\right) dt \\
 &= \frac{1}{(2.0 \text{ s})} \left(-\frac{\pi}{2} \text{ cm/s} \right) \int_0^{\pi/2} \sin x dx = \frac{1}{(2.0 \text{ s})} \left(-\frac{\pi}{2} \text{ cm/s} \right) = -\frac{\pi}{4} \text{ cm/s}^2
 \end{aligned}$$

(d) The instantaneous accelerations of the particle at $t = 0$, 1.0 s, and 2.0 s are, respectively,

$$\begin{aligned}
 a_y(0) &= \left(-\frac{\pi^2}{8} \text{ cm/s}^2 \right) \sin(0) = 0 \\
 a_y(1.0 \text{ s}) &= \left(-\frac{\pi^2}{8} \text{ cm/s}^2 \right) \sin(\pi/4) = -\frac{\pi^2\sqrt{2}}{16} \text{ cm/s}^2 \\
 a_y(2.0 \text{ s}) &= \left(-\frac{\pi^2}{8} \text{ cm/s}^2 \right) \sin(\pi/2) = -\frac{\pi^2}{8} \text{ cm/s}^2
 \end{aligned}$$