Chapter 2 – Problem Solutions

2.2.1

$P = \gamma \cdot h$; where $\gamma = (1.03)(9810\ N/m^3) = 1.01x10^4\ N/m^3$

(using the specific weight of water at standard conditions since water gets very cold at great depths)

$P = \gamma \cdot h = (1.01x10^4\ N/m^3)(730\ m)$

$\mathbf{P = 7.37x10^6\ N/m^2 = 1{,}070\ psi}$

The pressure given is gage pressure. To get absolute pressure, atmospheric pressure must be added.

2.2.2

a) The force exerted on the tank bottom is equal to the weight of the water body.

$F = W = m \cdot g = [\rho \cdot (Vol)]\ (g)$

$F = [1.94\ slugs/ft^3\ (\pi \cdot (5\ ft)^2 \cdot 3\ ft)]\ (32.2\ ft/sec^2)$

$\mathbf{F = 1.47\ x\ 10^4\ lbs}$

(Note: $1\ slug = 1\ lb \cdot sec^2/ft$)

b) The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (62.3\ lb/ft^3)(3\ ft) = 187\ \ lb/ft^2$

$F = P \cdot A = (187\ \ lb/ft^2)(\ \pi \cdot (5\ ft)^2)$

$\mathbf{F = 1.47\ x\ 10^4\ lbs}$

2.2.3

γ_{water} at 30°C = $9.77\ kN/m^3$

P_{vapor} at 30°C = $4.24\ kN/m^2$

$P_{atm} = P_{column} + P_{vapor}$

$P_{atm} = (9.8\ m)(9.77\ kN/m^3) + (4.24\ kN/m^2)$

$\mathbf{P_{atm} = 95.7\ kN/m^2 + 4.24\ kN/m^2 = 99.9\ kN/m^2}$

2.2.3 (cont.)

The percentage error if the direct reading is used and the vapor pressure is ignored is:

$Error = (P_{atm} - P_{column})/(P_{atm})$

$Error = (99.9\ kN/m^2 – 95.7\ kN/m^2)/(99.9\ kN/m^2)$

Error = 0.0420 = **4.20%**

2.2.4

The atm. pressure found in problem 2.2.3 is $99.9\ kN/m^2$

$P_{atm} = (\gamma_{Hg})(h)$

$h = P_{atm}/\gamma_{Hg} = (99.9\ kN/m^2)\ /\ [(13.6)(\ 9.77\ kN/m^3)]$

h = 0.752 m = 75.2 cm = 2.47 ft

2.2.5

The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (9.79\ kN/m^3)(6\ m) = 58.7\ kN/m^2$

$F = P \cdot A = (58.7\ kN/m^2)(36\ m^2)$

F = 2,110 N

The force exerted on the sides of the tank may be found in like manner (pressure times the area). However, the pressure is not uniform on the tank sides since $P = \gamma \cdot h$. Therefore, the average pressure is required. Since the pressure is a linear relationship, the average pressure occurs at half the depth. Now,

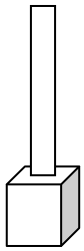
$P_{avg} = \gamma \cdot h_{avg} = (9.79\ kN/m^3)(3\ m) = 29.4\ kN/m^2$

$F = P_{avg} \cdot A = (29.4\ kN/m^2)(36\ m^2)$

F = 1,060 N

Obviously, the force on the bottom is greater than the force on the sides by a factor of two.

2.2.6



$W_{total} = (Vol_{container})(\gamma_{water}) + (Vol_{pipe})(\gamma_{water})$

$W_{total} = (3\text{ ft})^3(62.3\text{ lb/ft}^3)$
$\quad + [(\pi)(0.50\text{ ft})^2(30\text{ ft})](62.3\text{ lb/ft}^3)$

$\mathbf{W_{total}}$ = 1680 lb + 1470 lb = **3,150 lb**

$P_{bottom} = \gamma h = (62.3\text{ lb/ft}^3)(33\text{ ft}) = 2060\text{ lb/ft}^2$

$\mathbf{F_{bottom}} = (2060\text{ lb/ft}^2)(9\text{ ft}^2) =$ **18,500 lb**

Note: The weight of the water is not equal to the force on the bottom. Why? (Hint: Draw a free body diagram of the 3 ft x 3 ft x 3 ft water body labeling all forces (vertical) acting on it. Don't forget the pressure from the container top. Now, to determine the side force:

$P_{avg} = \gamma \cdot h_{avg} = (62.3\text{ lb/ft}^3)(31.5\text{ ft}) = 1960\text{ lb/ft}^2$

$F = P_{avg} \cdot A = (1960\text{ lb/ft}^2)(9\text{ ft}^2)$

F = 17,600 lb

2.2.7

$P_{bottom} = P_{gage} + (\gamma_{liquid})(1.4\text{ m})$; and

$\gamma_{liquid} = (SG)(\gamma_{water}) = (0.80)(9790\text{ N/m}^3) = 7830\text{ N/m}^3$

$\therefore P_{bottom} = 4.50\text{x}10^4\text{ N/m}^2 + (7830\text{ N/m}^3)(1.4\text{ m})$

$\mathbf{P_{bottom} = 5.60 \times 10^4\ N/m^2}$

The pressure at the bottom of the liquid column can be determined two different ways which must be equal. Hence,

$(h)(\gamma_{liquid}) = P_{gage} + (\gamma_{liquid})(1\text{ m})$

$h = (P_{gage})/(\gamma_{liquid}) + 1\text{m}$

h = $(4.50\text{x}10^4\text{ N/m}^2)/7830\text{ N/m}^3 + 1\text{ m} =$ **6.75 m**

2.2.8

$\gamma_{seawater} = (SG)(\gamma_{water}) = (1.03)(9790\text{ N/m}^3)$

$\gamma_{seawater} = 1.01 \times 10^4\text{ N/m}^3$

$P_{tank} = (\gamma_{water})(\Delta h) = (1.01 \times 10^4\text{ N/m}^3)(6\text{ m})$

$\mathbf{P_{tank} = 6.06 \times 10^4\ N/m^2}$ **(Pascals) = 8.79 psi**

2.2.9

$\gamma_{oil} = (SG)(\gamma_{water}) = (0.85)(\ 62.3\text{ lb/ft}^3) = 53.0\text{ lb/ft}^3$

$P_{10ft} = P_{air} + (\gamma_{oil})(10\text{ ft})$

$P_{air} = P_{10ft} - (\gamma_{oil})(10\text{ ft})$

$P_{air} = 23.7\text{ psi }(144\text{ in}^2/\text{ft}^2) - (53.0\text{ lb/ft}^3)(10\text{ ft})$

$\mathbf{P_{air} = 2.88 \times 10^4\ lb/ft^2}$ **(20.0 psi); Gage pressure**

$P_{abs} = P_{gage} + P_{atm} = 20.0\text{ psi} + 14.7\text{ psi}$

$\mathbf{P_{abs}}$ = **34.7 psi (5.00 x 10^4 lb/ft^2); Absolute pressure**

2.2.10

The mechanical advantage in the lever increases the input force delivered to the hydraulic jack. Thus,

$F_{input} = (9)(50\text{ N}) = 450\text{ N}$

The pressure developed in the system is:

$P_{input} = F/A = (450\text{ N})/(25\text{ cm}^2) = 18\text{ N/cm}^2$

$\mathbf{P_{input} = 180\ kN/m^2}$

From Pascal's law, the pressure at the input piston should equal the pressure at the two output pistons.

$\therefore$ The force exerted on each output piston is:

$P_{input} = P_{output}$ equates to: $18\text{ N/cm}^2 = F_{output}/250\text{ cm}^2$

$F_{output} = (18\text{ N/cm}^2)(250\text{ cm}^2)$

$\mathbf{F_{output} = 4.50\ kN}$

2.4.1

Since the line passing through points 7 and 8 represents an equal pressure surface;

$P_7 = P_8$ or $(h_{water})(\gamma_{water}) = (h_{oil})(\gamma_{oil})$

However; $(h_{oil})(\gamma_{oil}) = (h_{oil})(\gamma_{water})(SG_{oil})$, thus

$\mathbf{h_{oil}} = (h_{water})/(SG_{oil}) = (52.3 \text{ cm})/(0.85) = \mathbf{61.5\ cm}$

2.4.2

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$(3 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$h = (3 \text{ ft})(\gamma_{water}/\gamma_{Hg}) = (3 \text{ ft}) / (SG_{Hg}) = (3 \text{ ft}) / (13.6)$

h = 0.221 ft = 2.65 in.

2.4.3

The pressure at the bottom registered by the gage is equal to the pressure due to the liquid heights. Thus,

$(h_{Hg})(SG_{Hg})(\gamma_{water}) = (4h)(\gamma_{water}) + (h)(SG_{oil})(\gamma_{water})$

$h = (h_{Hg})(SG_{Hg})/(4 + SG_{oil})$

$\mathbf{h} = (26.3 \text{ cm})(13.6)/(4 + 0.82) = \mathbf{74.2\ cm}$

2.4.4

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_A + (y)(\gamma_{water}) = (h)(\gamma_{Hg})$

$P_A + (0.034 \text{ m})(\gamma_{water}) = (0.026 \text{ m})(\gamma_{Hg})$

$$P_A = (0.026 \text{ m})(13.6)(9{,}790 \text{ N/m}^3) - (0.034 \text{ m})(9{,}790 \text{ N/m}^3)$$

$\mathbf{P_A = 3{,}130\ N/m^2\ (Pascals) = 3.13\ kN/m^2}$

2.4.5

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (2 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$(16.8 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2) + (2 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$$(2.42 \times 10^3 \text{ lb/ft}^2) + (2 \text{ ft})(62.3 \text{ lb/ft}^3) = (h)(13.6)(62.3 \text{ lb/ft}^3)$$

h = 3.00 ft (manometer is correct)

2.4.6

Using the "swim through" technique, start at the end of the manometer which is open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipe. The computations are below:

$$0 - (0.66 \text{ m})(\gamma_{CT}) + [(0.66 + y + 0.58)\text{m}](\gamma_{air}) - (0.58 \text{ m})(\gamma_{oil}) = P_{pipe}$$

The specific weight of air is negligible when compared to fluids, so that term in the equation can be dropped.

$P_{pipe} = 0 - (0.66 \text{ m})(SG_{CT})(\gamma) - (0.58 \text{ m})(SG_{oil})(\gamma)$

$$P_{pipe} = 0 - (0.66 \text{ m})(1.60)(9790 \text{ N/m}^3) - (0.58 \text{ m})(0.82)(9790 \text{ N/m}^3)$$

$P_{pipe} = -15.0 \text{ kN/m}^2$ Pressure can be converted to height (head) of any liquid through $P = \gamma \cdot h$. Thus,

$\mathbf{h_{pipe}} = (-15{,}000 \text{ N/m}^2)/(9790 \text{ N/m}^3) = \mathbf{-1.53\ m\ of\ water}$

2.4.7

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P + (0.575 \text{ ft})(62.3 \text{ lb/ft}^3) = (2.00 \text{ ft})(13.6)(62.3 \text{ lb/ft}^3)$

$\mathbf{P = 1{,}660\ lb/ft^2 = 11.5\ psi}$

2.4.8

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P_{pipe} + (0.20\text{ m})(9790\text{ N/m}^3) = (0.67\text{ m})(13.6)(9790\text{ N/m}^3)$

$\mathbf{P_{pipe} = 8.72 \times 10^4\ N/m^2\ (Pascals) = 87.2\ KPa}$

When the manometer reading rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}{\cdot}h_1 = A_{tube}{\cdot}h_2$

$h_1 = h_2\,(A_{tube}/A_{res}) = h_2\,[(D_{tube})^2/(D_{res})^2]$

$\mathbf{h_1} = (10\text{ cm})[(0.5\text{ cm})^2/(5\text{ cm})^2] = \mathbf{0.1\ cm}$

2.4.9

Using the "swim through" technique, start at pipe *A* and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach pipe B. The computations are:

$P_A + (5.33\text{ ft})(\gamma) - (1.67\text{ft})(\gamma_{Hg}) - (1.0\text{ ft})(\gamma_{oil}) = P_B$

$P_A - P_B = (62.3\text{ lb/ft}^3)\,[(1.0\text{ ft})(0.82) - (5.33\text{ ft}) + (1.67\text{ ft})(13.6)]$

$\mathbf{P_A - P_B = 1{,}130\ lb/ft^2 = 7.85\ psi}$

2.4.10

Using the "swim through" technique, start at P_2 and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach P_1. The computations are:

$P_2 + (\Delta h)(\rho_1{\cdot}g) + (y)(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g) - (y)(\rho_1{\cdot}g) = P_1$

where y is the vertical elevation difference between the fluid surface in the left hand reservoir and the interface between the two fluids on the right side of the U-tube.

$P_1 - P_2 = (\Delta h)(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g)$

2.4.10 – cont.

When the manometer reading (h) rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}{\cdot}(\Delta h) = A_{tube}{\cdot}\ h$

$\Delta h = h\,(A_{tube}/A_{res}) = h\,[(d_2)^2/(d_1)^2]$; substituting yields

$\mathbf{P_1 - P_2 = h\,[(d_2)^2/(d_1)^2]\,(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g)}$

$\mathbf{P_1 - P_2 = h{\cdot}g\,[\rho_2 - \rho_1 + \rho_1\,\{(d_2)^2/(d_1)^2\}]}$

$\mathbf{P_1 - P_2 = h{\cdot}g\,[\rho_2 - \rho_1\,\{1 - (d_2)^2/(d_1)^2\}]}$

2.4.11

Using the "swim through" technique, start at both ends of the manometers which are open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipes in order to determine P_A and P_B. The computations are below:

$0 + (23)(13.6)(\gamma) - (44)(\gamma) = P_A;\quad P_A = 269{\cdot}\gamma$

$0 + (46)(0.8)(\gamma) + (20)(13.6)(\gamma) - (40)(\gamma) = P_B$

$P_B = 269{\cdot}\gamma$; Therefore, $P_A = P_B$ and $\mathbf{h = 0}$

2.4.12

Using the "swim through" technique, start at the sealed right tank where the pressure is known. Then "swim through" the tanks and pipes, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the left tank where the pressure is not known. The computations are as follows:

$20\text{ kN/m}^2 + (4.5\text{ m})(9.79\text{ kN/m}^3) - (2.5\text{ m})(1.6)(9.79\text{ kN/m}^3) - (5\text{ m})(0.8)(9.79\text{ kN/m}^3) = P_{left}$

$\mathbf{P_{left} = -14.3\ kN/m^2}$ **(or -14.3 kPa)**

$P_B = (-14.3\text{ kN/m}^2) / [(SG_{Hg})(\gamma)]$

$P_B = (-14.3\text{ kN/m}^2) / [(13.6)(9.79\text{ kN/m}^3)]$

$\mathbf{P_B = -0.107\ m = 10.7\ cm\ (Hg)}$

2.5.1

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)[(3m)/(3)]·[6 m^2]

F = 5.87 x 10^4 N = 58.7 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(4m)(3m)^3/36]}{[(4m)(3m)/2](1.00m)} + 1.00m$$

y_p = 1.50 m (depth to center of pressure)

2.5.2

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(30 ft)/2]·[(30 ft)(1 ft)]

F = 2.80 x 10^4 lbs per foot of length

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1ft)(30ft)^3/12]}{[(30ft)(1ft)](15ft)} + 15ft$$

y_p = 20.0 ft (depth to the center of pressure)

In summing moments about the toe of the dam ($\sum M_A$), the weight acts to stabilize the dam (called a righting moment) and the hydrostatic force tends to tip it over (overturning moment).

M = (Wt.)[(2/3)(10)] - F(10ft) =

[1/2 (10 ft)(30 ft)·(1 ft)](2.67)(62.3 lb/ft^3)·(6.67 ft) –

(2.80 x 10^4 lbs)·(10 ft) = -1.14 x 10^5 ft-lbs

M = 1.14 x 10^5 ft-lbs (overturning; dam is unsafe)

2.5.3

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(1 m)(π)(0.5 m)^2

F = 7.69 x 10^3 N = 7.69 kN ; $\bar{y} = \bar{h}/\sin 45°$;

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(1m)^4/64]}{[\pi(1m)^2/4](1.414m)} + 1.414m$$

y_p = 1.46 m (distance from water surface to the center of pressure along the incline).

2.5.4

$F_{square} = \gamma \cdot \bar{h} \cdot A$ = γ(L/2)(L^2) = (γ/2)·L^3

$F_{tri} = \gamma \cdot \bar{h} \cdot A$ = γ(L+H/3)(LH/2) = (γ/2)[L^{2}H + LH^2/3]

Setting the two forces equal: $F_{square} = Y_{tri}$;

(γ/2)·L^3 = (γ/2)[L^{2}H + LH^2/3]

L^2 - HL - H^2/3 = 0; divide by H^2 and solve quadratic

(L/H)^2 – (L/H) - 1/3 = 0; L/H = 1.26 or **H/L = 0.791**

2.5.5

$F_{left} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(0.5 m)[(1.41m)(3m)]

F_{left} = 20.7 kN (where A is "wet" surface area)

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3m)(1.41m)^3/12]}{[(3m)(1.41m)](0.705m)} + 0.705m$$

y_p = 0.940 m (inclined distance to center of pressure)

Location of this force from the hinge (moment arm):

Y' = 2 m – 1.41 m + 0.940 m = 1.53 m

$F_{right} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(h/2 m)[(h/cos45°)(3m)]

F_{right} = 20.8·h^2 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3)(1.41 \cdot h)^3/12]}{[(3)(1.41 \cdot h)](0.705 \cdot h)} + 0.705 \cdot h$$

y_p = (0.940·h)m; Moment arm of force from hinge:

Y" = 2 m – (h/sin 45)m + (0.940·h)m = 2m – (0.474·h)m

The force due to the gate weight: W = 20.0 kN
Moment arm of this force from hinge: X = 0.707 m

Summing moments about the hinge yields: $\sum M_{hinge}$= 0

(20.8·h^2)[2m – (0.474·h)] – 20.7(1.53) – 20(0.707) = 0

h = 1.25 m (gate opens when depth exceeds 1.25 m)

2.5.6

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(7 ft)]·[π(6 ft)^2/4]

F = 1.23 x 10^4 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(6ft)^4/64]}{[\pi(6ft)^2/4](7ft)} + 7ft$$

y_p = 7.32 ft (depth to the center of pressure)

Thus, summing moments: $\sum M_{hinge} = 0$

P(3 ft) – (1.23 x 10^4 lbs)(0.32 ft) = 0

P = 1.31 x 10^3 lbs

2.5.7

In order for the balance to be maintained at h = 4 feet, the center of pressure should be at the pivot point (i.e., the force at the bottom check block is zero). As the water rises above h = 4 feet, the center of pressure will rise above the pivot point and open the gate. Below h = 4 feet, the center of pressure will be lower than the pivot point and the gate will remain closed. Thus, for a unit width of gate, the center of pressure is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1ft)(10ft)^3/12]}{[(1ft)(10ft)](9ft)} + 9ft$$

y_p = 9.93 ft (vertical distance from water surface to the center of pressure)

Thus, the horizontal axis of rotation (0-0') should be 14 ft – 9.93 ft = 4.07 ft above the bottom of the gate.

2.5.8

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π){(1.5)^2–(0.5)^2}m^2]

F = 1.54 x 10^5 N = 154 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - \pi(1m)^4/64)]}{[\pi(3m)^2/4 - \pi(1m)^2/4](2.5m)} + 2.5m$$

y_p = 2.75 m (below the water surface)

2.5.9

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π)(1.5 m)^2–(1.0 m)^2]

F = 1.49 x 10^5 N = 149 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - (1m)(1m)^3/12)]}{[\pi(1.5m)^2 - (1m)^2](2.5m)} + 2.5m$$

y_p = 2.76 m (below the water surface)

2.5.10

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5m)[(5/cos 30°)(3 m)]

F = 4.24 x 10^5 N = 424 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3m)(5.77m)^3/12]}{[(3m)(5.77m)](2.89m)} + 2.89m$$

y_p = 3.85 m (inclined depth to center of pressure)

Summing moments about the base of the dam; $\sum M = 0$

(424 kN)(5.77 m – 3.85 m) – (F_{AB})(5.77m/2) = 0

F_{AB} = 282 kN

2.5.11

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(*d*/2)ft]·[{(*d* /cos30)ft}(8 ft)]

F = 288·d^2 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(8)(d/\cos 30°)^3/12]}{[(8)(d/\cos 30°)](d/2\cos 30°)} + (d/2\cos 30)$$

y_p = [(0.192·*d*) + 0.577·*d*)] ft = 0.769·*d* (inclined depth)

Thus, summing moments: $\sum M_{hinge} = 0$

(288·d^2)[(*d*/cos30)-0.769*d*] – (5,000)(15) = 0

d = 8.77 ft A depth greater than this will make the gate open, and anything less will make it close.

2.5.12

The force on the side of the gate can be found as:

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)[(h/2)m][(h m)(1 m)]
F = (4.90 x 10^3)h^2 N (per meter of gate width)

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1)(h)^3/12]}{[(1)(h)](h/2)} + h/2 = (2/3)h \text{ m}$$

The force on the bottom of the gate can be found as:

F = p·A = (9790 N/m^3)(h)[(1m)(1m)]
F = (9.79 x 10^3)h N (per meter of gate width)

This force is located 0.50 m from the hinge.
Summing moments about the hinge; $\sum M_h = 0$

[(4.90 x 10^3)h^2][h - (2/3)h] – [(9.79 x 10^3)h](0.5) = 0

h = 1.73 m

2.5.13

The total force from fluids A and B can be found as:

$F_A = \gamma \cdot \bar{h} \cdot A$ = (γ_A)(h_A)·[π(d)^2/4]
$F_B = \gamma \cdot \bar{h} \cdot A$ = (γ_B)(h_B)·[π(d)^2/4]

For equilibrium, forces must be equal, opposite, and collinear.

$F_A = F_B$; (γ_A)(h_A)·[π(d)^2/4] = (γ_B)(h_B)·[π(d)^2/4]

h_A = [(γ_B)/(γ_A)](h_B)

2.5.14

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(30 ft)]·[(10 ft)(6 ft)]

F = 1.12 x 10^5 lbs (Horizontal force on gate)

Thus, summing vertical forces: $\sum F_y = 0$
T_{up} – W – F($C_{friction}$) = 0

T = 3 tons (2000 lbs/1 ton) + (1.12 x 10^5 lbs)(0.2)

T = 2.84 x 10^4 lbs (lifting force required)

2.6.1

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(5m)[(10 m)(2 m)]

F_H = 9.79 x 10^5 N = 979 kN

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

$F_V = \gamma \cdot Vol$ = (9790 N/m^3)[(4m)(2m)+π/4(2m)^2](10 m)

F_V = 1.09 x 10^6 N = 1090 kN; The total force is

F = [(979 kN)^2 + (1090 kN)^2]$^{1/2}$ = **1470 kN**

θ = tan^{-1} (F_V/F_H) = **48.1°**

Since all hydrostatic pressures pass through point A (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point A.

2.6.2

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the viewing port.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (1.03·9790N/m^3)(4m)[π(1m)^2]=127 kN

Now obtain the net vertical component of the total hydrostatic pressure force by combining the weight of the water column above the top of the viewing port (which produces a downward force) and the upward force on the bottom of the viewing port (equivalent to the weight of the water above it). The difference in the two columns of water is the weight of water in a hemispherical volume (the viewing port) acting upwards.

$F_V = \gamma \cdot Vol$ =(1.03·9790 N/m^3)[(1/2)(4/3)π(1m)^3] = 21.1 kN

F = [(127 kN)^2 + (21.1 kN)^2]$^{1/2}$ = **129 kN**

θ = tan^{-1} (F_V/F_H) = **9.43°**

The resultant force will pass through the center of the hemisphere since all pressures pass through this point.

2.6.3

The vertical component of the total hydrostatic pressure force is equal to the weight of the water column above it to the free surface. In this case, it is the imaginary or displaced weight of water above the shell since the pressure is from below).

$F_V = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(1/2)(4/3)(π)(3.0 ft)^3] = 3,520 lb

The weight must be equal to this; thus **W = 3,520 lb**

2.6.4

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of curved surface AB (both sides).

$$F_H = (\gamma \cdot \bar{h} \cdot A)_{right} - (\gamma \cdot \bar{h} \cdot A)_{left} = (\gamma \cdot A)(\bar{h}_{right} - \bar{h}_{left})$$
$$= (9790\ \text{N/m}^3)\ [(1.75\ \text{m})(1\ \text{m})]\ (3.875\ \text{m} - 0.875\ \text{m})$$

$\mathbf{F_H}$ = 5.14 x 10^4 N **= 51.4 kN (towards the barge)**

Now obtain the resultant vertical component of the total hydrostatic pressure force subtracting the weight of the water column above the curved surface (the water in the barge) from the displaced weight for the case of the water on the outside of the barge.

$$F_V = (\gamma \cdot Vol)_{displaced} - (\gamma \cdot Vol)_{leaked}$$
$$= (9790\ \text{N/m}^3)[(1.75\ \text{m})(1\ \text{m})(3\ \text{m})]$$

$\mathbf{F_V}$ = 5.14 x 10^4 N **= 51.4 kN (upwards)**

2.6.5

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. The height of the vertical projection is (R)(sin 45˚) = 8.49 m. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(4.25m)[(10 m)(8.49 m)]

F_H = 3.53 x 10^6 N = 3,530 kN

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

2.6.5 (continued)

The volume of water above the gate is:

Vol = ($A_{rectangle}$ - $A_{triangle}$ - A_{arc})(length)
Vol =[(12m)(8.49m)-(1/2)(8.49m)(8.49m)-(π/8)(12m)^2](10 m)
Vol = 92.9 m^3

$F_V = \gamma \cdot Vol$ = (9790 N/m^3)(92.9 m^3)

F_V = 9.09 x 10^5 N = 909 kN; The total force is

F = [(3,530 kN)^2 + (909 kN)^2]$^{1/2}$ = **3650 kN**

θ = tan^{-1} (F_V/F_H) **= 14.4˚**

Since all hydrostatic pressures pass through point O (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point O.

2.6.6

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)(7.0 ft)[(8.0 ft)(1.0 ft)]

F_H = 3.49 x 10^3 lb = 3,490 lb (per unit length of gate)

Obtain the vertical component of the total hydrostatic pressure force by determining the imaginary (displaced) weight of the water column above the curved gate. The volume of displaced water above the gate is:

Vol = ($A_{rectangle}$ + A_{arc} - $A_{triangle}$)(length)

Vol = [(4 ft)(3 ft)+(53.1˚/360˚)π(10 ft)^2 - (1/2)(8 ft)(6 ft)](1 ft)

Vol = 34.3 ft^3

$F_V = \gamma \cdot Vol$ = (62.3 lb/ft^3)(34.3 ft^3)

F_V = 2.14 x 10^3 lb = 2,140 lb; The total force is

F = [(3,490 lb)^2 + (2,140 lb)^2]$^{1/2}$ = **4,090 lb**

θ = tan^{-1} (F_V/F_H) **= 31.5˚**

The resultant force will pass through the center of the gate radius since all pressures pass through this point.

2.6.7

The force on the end of the cylinder is:

$F = \gamma \cdot \bar{h} \cdot A$ = (0.9)(62.3 lb/ft^3)(10 ft)[π(2 ft)^2]

F = 7,050 lb

The force on the side of the cylinder is:

$F_H = \gamma \cdot \bar{h} \cdot A$ = (0.9)(62.3 lb/ft^3)(10.0 ft)[(10 ft)(4 ft)]

F_H = 22,400 lb

Based on the same theory as Example 2.6, the vertical force is downward and equal to the weight of the water in half of the tank. Thus,

$F_V = \gamma \cdot Vol$ = (0.9)(62.3 lb/ft^3)[π(2ft)^2/2](10 ft)

F_V = 3,520 lb (acting downward); The total force is

F = [(22,400 lb)^2 + (3,520 lb)^2]$^{1/2}$ = **22,700 lb**

θ = tan^{-1} (F_V/F_H) **= 8.93°** ↘

The resultant force will pass through the center of the tank since all pressures pass are normal to the tank wall and thus pass through this point.

2.6.8

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved surface ABC.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (γ)(R)[(2R)(1)] **= 2(γ)(R)^2**

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water above the curved surface ABC. The volume of water above the curved surface is:

Vol = ($A_{\text{quaarter circle}} + A_{\text{rectangle}} - A_{\text{quarter circle}}$)(unit length)

Vol = ($A_{\text{rectangle}}$)(unit length) = (2R)(R)(1) = 2(R)^2

$F_V = \gamma \cdot Vol$ = γ[2(R)^2] **= 2(γ)(R)^2**

2.6.9

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the projecting surface. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)(8.0 ft)[(12.0 ft)(1.0 ft)]

F_H = 5,980 lb (per unit length of surface)

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(1ft)(12ft)^3/12\right]}{\left[(1ft)(12ft)\right](8ft)} + 8ft \;\; \mathbf{= 9.50\ ft}$$

The vertical component of the total hydrostatic pressure force is equal to the weight of the water displaced by the quadrant and the triangle. In parts (and using Table 2.1 to locate the forces), we have

$F_{VTriangle} = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(1/2)(8 ft)(4 ft)](1 ft)

$F_{VTriangle}$ = 1000 lb upwards 1.33 ft from wall

$F_{Vquadrant} = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(π/4)(4 ft)^2](1 ft)

$F_{Vquadrant}$ = 780 lb upwards 1.70 ft from wall

2.6.10

There are four vertical forces at work on the cone plug. The cone unplugs when $\sum F_y = 0$.

The pressure force of fluid A on top of the plug (down):

$F_{ATop} = \gamma \cdot Vol$ = (9790 N/m^3)[π(0.15m)^2(0.3m)] = 208 N

The pressure force of fluid A on the cone sides (up):

$F_{ASides} = \gamma \cdot Vol$ = [π(0.15m)^2(0.3m) + (π/3) (0.15m)^2(0.3m) - (π/3) (0.05m)^2(0.1m)](9790 N/m^3) = 274 N

The pressure force of fluid B on the cone bottom (up):

$F_{Bbottom} = \gamma \cdot Vol$ = (0.8)(9790 N/m^3) [π(0.05m)^2(1.5m) + (π/3) (0.05m)^2(0.1m)] = 94.3 N

$\sum F_y$ = 208 – 274 – 94.3 + (γ_{cone})(π/3) (0.15m)^2(0.3m) = 0;

γ_{cone} = 22,700 N/m^3; S.G. = 2.32

2.6.11

Everything is the same as in problem 2.6.10 except:

The equivalent depth of oil based on the air pressure is:

$h = P/\gamma = (8{,}500\ \text{N/m}^2)/[(0.8)(9790\ \text{N/m}^3)] = 1.09\ \text{m}$

The pressure force of fluid B on the cone bottom (up):

$$F_{Bbottom} = \gamma \cdot Vol = (0.8)(9790\ \text{N/m}^3)\ [\pi(0.05\text{m})^2(1.09\text{m}) + (\pi/3)(0.05\text{m})^2(0.1\text{m})] = 69.1\ \text{N}$$

$\sum F_y = 208 - 274 - 69.1 + (\gamma_{cone})(\pi/3)(0.15\text{m})^2(0.3\text{m}) = 0;$

$\boldsymbol{\gamma_{cone} = 19{,}100\ \text{N/m}^3}$**; S.G. = 1.95**

2.6.12

The horizontal component of the hydrostatic pressure force due to fluids A and B are found as follows:

$F_{HALeft} = \gamma \cdot \bar{h} \cdot A = (0.8)(9790\ \text{N/m}^3)(6\ \text{m})[(1\ \text{m})(1.41\ \text{m})]$

$F_{HALeft} = 66.3\ \text{kN}$

$F_{HARight} = \gamma \cdot \bar{h} \cdot A = (0.8)(9790\ \text{N/m}^3)(5.65\ \text{m})[(1\text{m})(0.707\text{m})]$

$F_{HARight} = 31.3\ \text{kN}$

$F_{HBRight} = \gamma \cdot \bar{h} \cdot A = (1.5)(9790\ \text{N/m}^3)(5.35\ \text{m})[(1\ \text{m})(0.707\ \text{m})]$

$F_{HBRight} = 55.5\ \text{kN}$; Thus, $\mathbf{F_H = 20.5\ kN}$ $\leftarrow$

The vertical component of the total hydrostatic pressure force due to fluids A and B are found as follows:

$F_{VATop} = (0.8)(9790\text{N/m}^3)[(1.41\text{m})(1\text{m})(6\text{m}) - \pi/2(0.707\text{m})^2(1.0\text{m})]$

$F_{VATop} = 60.1\ \text{kN}$

$F_{VABottom} = (0.8)(9790\text{N/m}^3)[(0.707\text{m})(1\text{m})(6\text{m}) + \pi/4(0.707\text{m})^2(1.0\text{m})]$

$F_{VABottom} = 36.3\ \text{kN}$

$F_{VBBottom} = (1.5)(9790\text{N/m}^3)[(0.707\text{m})(1\text{m})(5\text{m}) + \pi/4(0.707\text{m})^2(1.0\text{m})]$

$F_{VBBottom} = 57.7\ \text{kN}$

$W_{Cylinder} = (2.0)(9790\text{N/m}^3)[\pi(0.707\text{m})^2(1.0\text{m})] = 30.7\ \text{kN}$

Thus, $\mathbf{F_V = 3.2\ kN}$ $\uparrow$

2.8.1

The buoyant force equals the weight reduction. Thus,

B = 301 N – 253 N = 48.0 N In addition,

B = wt. of water displaced = $\gamma \cdot Vol = (9790\ \text{N/m}^3)(Vol)$

Thus, Vol = $4.90 \times 10^{-3}\ \text{m}^3$ and

γ_{metal} = W/Vol = $\mathbf{6.14 \times 10^4\ N/m^3}$

S.G. = $(6.14 \times 10^4\ \text{N/m}^3)/9.79 \times 10^3\ \text{N/m}^3)$ **= 6.27**

2.8.2

For floating bodies, weight equals the buoyant force.

W = B; and using w & L for width & length of blocks

$\gamma_A(H)(w)(L) + \gamma_B(1.5 \cdot H)(w)(L) = \gamma\,(2 \cdot H)(w)(L)$

$\gamma_A + (1.5\gamma_A)(1.5) = \gamma\,(2);\quad \gamma_A\,(1 + 2.25) = \gamma\,(2);$

$\boldsymbol{\gamma_A = 0.615\gamma}$; and since $\boldsymbol{\gamma_B} = 1.5 \cdot \gamma_A = \mathbf{0.923\gamma}$

2.8.3

When the sphere is lifted off the bottom, equilibrium in the y-direction occurs with W = B. Therefore,

$W = \gamma_{sphere}\,[(4/3)\pi(0.15\text{m})^3] + \gamma_{buoy}[\pi(0.25\text{m})^2(2\text{m})]$

$W = (13.5\gamma)[0.0141\text{m}^3] + (0.45\gamma)[0.393\text{m}^3] = 0.367\ \gamma$

$B = \gamma_{sea}[(4/3)\pi(0.15\text{m})^3] + \gamma_{sea}[\pi(0.25\text{m})^2(0.30\text{m} + h)]$

$B = 0.0145\ \gamma + 0.0607\ \gamma + 0.202 \cdot h \cdot \gamma$

Equating; $\mathbf{h = 1.45\ m}$

2.8.4

Theoretically, the lake level will fall. When the anchor is in the boat, it is displacing a volume of water equal to its weight. When the anchor is thrown in the water, it is only displacing its volume. Since it has a specific gravity greater than 1.0, it will displace more water by weight than by volume.

2.8.5

When the anchor is lifted off the bottom, equilibrium in the y-direction occurs ($\sum F_y = 0$). Therefore,

T (sin60°) + B = W; where T = anchor line tension

B = buoyancy force, and W= anchor weight

B = (62.3 lb/ft^3)π(0.75 ft)^2(1.2 ft) = 132 lb

W = (2.7)(62.3 lb/ft^3)π(0.75 ft)^2(1.2 ft) = 357 lb

Substituting, **T = 260 lb**

2.8.6

Two forces act on the gate, the hydrostatic pressure and the buoy force. The hydrostatic pressure is

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(1.5m)[(1m/sin45°)^2]

F = 29.4 kN; acting normal to the gate surface.

The location of the force is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$y_P = \frac{\left[(1m/\sin 45°)(1m/\sin 45°)^3/12\right]}{\left[(1m/\sin 45°)(1m/\sin 45°)\right](1.5/\sin 45°)} + (1.5/\sin 45°)$$

y_p = 2.20 m; This is the distance down the incline from the water surface.

The distance up the incline from the hinge is

y' = (2m/sin45°) – 2.20 m = 0.628 m

The buoyant force (on half the sphere) is

B =γ·Vol=(9790 N/m^3)(1/2)(4/3)π(R)^3 = 20.5(R)^3 kN

$\sum M_{hinge}$ = 0, ignoring the weights (gate and buoy)

(29.4 kN)(0.628 m) – [20.5(R)^3 kN](1 m) = 0

R = 0.966 m

2.8.7

Three forces act on the rod; the weight, buoyant force, and the hinge force. The buoyant force is

B = (62.3 lb/ft^3)(0.5 ft)(0.5 ft)(7 ft/sin θ) = 109 lb/sin θ;

B = 109 lb/sin θ; ↑ The buoyant force acts at the center of the submerged portion. W = 150 lb

$\sum M_{hinge}$ = 0, and assuming the rod is homogeneous,

(109 lb/sin θ)(3.5 ft/tan θ) – (150 lb)[(6 ft)(cos θ)] = 0

Noting that tan θ = (sin θ/cos θ) and dividing by cos θ

(sin θ)^2 = 0.424; sin θ = 0.651; **θ = 40.6°**

2.8.8

The center of gravity (G) is given as 1 m up from the bottom of the barge. The center of buoyancy (B) is 0.75 m up from the bottom since the draft is 1.5 m. Therefore GB = 0.25 m, and GM is found using

$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{\text{Vol}} \pm \overline{GB}$; where Io is the waterline moment of inertial about the tilting axis. Chopping off the barge at the waterline and looking down we have a rectangle which is 14 m by 6 m. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{\left[(14m)(6m)^3/12\right]}{(14m)(6m)(1.5m)} - 0.25m = \mathbf{1.75\ m};$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

$M = W \cdot \overline{GM} \cdot \sin\theta$

M = [(1.03)(9790 N/m^3)(14 m)(6m)(1.5m)](1.75m)(sin 4°)

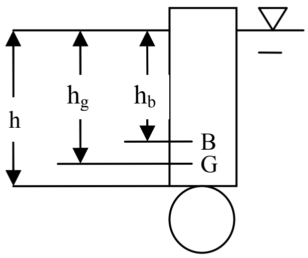
M = 155 kN·m (for a heel angle of 4°)

M = 309 kN·m (for a heel angle of 8°)

M = 462 kN·m (for a heel angle of 12°)

2.8.9

First determine how much the wooden pole is in the water. Summing forces in the y-direction, W = B



$(Vol_s)(SG_s)(\gamma) + (Vol_p)(SG_p)(\gamma) = (Vol_s)(\gamma) + (Vol_{p'})(\gamma)$

$[(4/3)\pi(0.25m)^3](1.4)+[\pi(0.125m)^2(2m)](0.62) =$

$[(4/3)\pi(0.25m)^3]+[\pi(0.125m)^2(h)]$;

$0.0916m^3 + 0.0609m^3 = 0.0654m^3 + (0.0491m^2)h$

h = 1.77 m; Find "B" using the principle of moments.

$[(Vol_s)(\gamma)+(Vol_{p'})(\gamma)](h_b)=(Vol_s)(\gamma)(h+0.25m)+(Vol_{p'})(\gamma)(h/2)$

$[(0.0654m^3 + (0.0491m^3)(1.77m)](h_b) =$
$(0.0654m^3)(1.77m + 0.25m) + [(0.0491m^3)(1.77m)](1.77m/2)$

h_b = 1.37 m; Find "G" using the principle of moments.

$(W)(h_g+0.23m) = (W_s)(2.0\text{ m} + 0.25\text{ m}) + (W_p)(2.0m/2)$

$[(0.0916m^3 + 0.0609m^3)](h_g +0.23m) =$
$(0.0916m^3)(2.25m) + (0.0609m^3)(1.00m)$

h_g = 1.52 m; GB = $h_g - h_b$ = 0.15 m

MB = I_o/Vol

MB = $[(1/64)\pi(0.25m)^4] / [(0.0654m^3 + (0.0491m^3)(1.77m)]$

MB = 1.26 x 10^{-3} m

GM = MB + GB = 1.26 x 10^{-3} m + 0.15 m **= 0.151 m**

2.8.10

If the metacenter is at the same position as the center of gravity, then GM = 0 and the righting moment is $M = W \cdot \overline{GM} \cdot \sin\theta$ = 0. **With no righting moment, the block will not be stable.**

2.8.11

The center of gravity (G) is estimated as 17 ft up from the bottom of the tube based on the depth of the water inside it. The center of buoyancy (B) is 21 feet from the bottom since 42 feet is in the water.
Therefore GB = 4.0 ft, and GM is found using

$\overline{GM} = \overline{MB} \pm \overline{GB} = \dfrac{I_0}{\text{Vol}} \pm \overline{GB}$; where Io is the waterline moment of inertial about the tilting axis. Chopping off the tube at the waterline and looking down we have a circle with a 36 ft diameter. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{[\pi(36ft)^4/64]}{(\pi/4)(36ft)^2(42ft)} + 4.0ft = \mathbf{5.93\ ft;}$$

Note: Vol is the submerged volume and a positive sign is used since G is located below the center of buoyancy.

$M = W \cdot \overline{GM} \cdot \sin\theta$

M = $[(1.02)(62.3\text{ lb/ft}^3)(\pi/4)(36\text{ ft})^2(42\text{ ft})](5.93\text{ ft})(\sin 4°)$

M = 1.12 x 10^6 ft·lb (for a heel angle of 4°)

2.8.12

The center of gravity (G) is roughly 1.7 m up from the bottom if the load is equally distributed. The center of buoyancy (B) is 1.4 m from the bottom since the draft is 2.0 m. Therefore GB = 0.3 m, and GM is found using

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{[(12m)(4.8m)^3/12]}{(12m)(4.8m)(2m)} - 0.3m = 0.660\text{ m;}$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

$M = W \cdot \overline{GM} \cdot \sin\theta$

M = $[(1.03)(9790\text{ N/m}^3)(12m)(4.8m)(2m)](0.660m)(\sin 15°)$

M = 198 kN·m; The distance G can be moved is

d = GM (sin θ) = (0.66 m)(sin 15°) **= 0.171 m**