

Chapter 5: Thermodynamic Processes and Cycles

5-6) This problem examines the Rankine heat engine introduced in Figure 5-5. Saturated steam at $T = 250^\circ\text{C}$ enters the turbine and the condenser operates at $T = 40^\circ\text{C}$.

- Assuming the turbine is reversible, give your best estimate of the efficiency of the cycle, and indicate the quality of the stream leaving the turbine.
- Assuming the turbine has an efficiency of 75%, give your best estimate of the efficiency of the cycle, and indicate the quality of the stream leaving the turbine.
- Find the flow rate of circulating water needed to produce a net power of 1 MW from the cycle with the turbine efficiency of 75%.

Solution:

A)

$$\eta_{\text{H.E.}} = \frac{|W_{\text{net}}|}{|Q_{\text{added}}|}$$

Set an entropy balance around the adiabatic reversible turbine

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{\text{in}}\hat{S}_{\text{in}} - \dot{m}_{\text{out}}\hat{S}_{\text{out}} + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Cancelling terms

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

Find \hat{S}_{out}

$$(\hat{S}_{\text{in}}) \rightarrow \text{Saturated Water vapor at } 250^\circ\text{C} \rightarrow 6.072 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{S}_{\text{out}} = 6.072 \frac{\text{kJ}}{\text{kg K}}$$

Find quality of mixture leaving reversible turbine:

$$\hat{S}_{\text{out}} = 6.072 \frac{\text{kJ}}{\text{kg K}} = (1 - q)\hat{S}_L + q\hat{S}_V$$

At 40°C :

$$\hat{S}_L = 0.572 \frac{\text{kJ}}{\text{kg K}}; \hat{S}_V = 8.256 \frac{\text{kJ}}{\text{kg K}}$$

$$q = 0.716$$

Set an energy balance around the reversible turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{W}_S$$

Find enthalpies of the inlet and outlet stream using the steam tables

$$\hat{H}_{in} \rightarrow \text{Saturated water at } 250^\circ\text{C} \rightarrow 2800.9 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{out} \rightarrow \text{Water at } 40^\circ\text{C and } q = 0.716 \rightarrow 1889.7 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_{S,turbine}}{\dot{m}} = 1889.7 \frac{\text{kJ}}{\text{kg}} - 2800.9 \frac{\text{kJ}}{\text{kg}} = -911.2 \frac{\text{kJ}}{\text{kg}}$$

Find Q_C using energy balance around condenser

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} \\ + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{Q}$$

$$\hat{H}_{in} \rightarrow \text{VLE mixture at } 40^\circ\text{C with } q=0.716 \rightarrow 1889.7 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{out} \rightarrow \text{Saturated liquid at } 40^\circ\text{C} \rightarrow 167.5 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}_C}{\dot{m}} = 167.5 \frac{\text{kJ}}{\text{kg}} - 1889.7 \frac{\text{kJ}}{\text{kg}} = -1722.2 \frac{\text{kJ}}{\text{kg}}$$

Calculate pump work using equation 3.77:

$$\frac{\dot{W}_{s,pump}}{\dot{m}} \approx \hat{V}(P_{out} - P_{in})$$

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Inlet and outlet pressure are saturation pressures at 40 and 250°C:

$$\frac{\dot{W}_{s,pump}}{\dot{m}} \approx \left(0.001008 \frac{\text{m}^3}{\text{kg}}\right) (39.76 - 0.0738 \text{ bar}) \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}}\right) \left(\frac{1 \text{ J}}{\text{Nm}}\right) \left(\frac{1 \frac{\text{N}}{\text{m}^2}}{\text{Pa}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) = 4.0 \frac{\text{kJ}}{\text{kg}}$$

Energy balance around whole engine:

$$0 = \frac{\dot{Q}_C}{\dot{m}} + \frac{\dot{Q}_H}{\dot{m}} + \frac{\dot{W}_{s,pump}}{\dot{m}} + \frac{\dot{W}_{s,turbine}}{\dot{m}}$$

$$\frac{\dot{Q}_H}{\dot{m}} = 2629.4 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{H.E.}} = \frac{\left| \left(-911.2 \frac{\text{kJ}}{\text{kg}}\right) + \left(4.0 \frac{\text{kJ}}{\text{kg}}\right) \right|}{\left(2629.4 \frac{\text{kJ}}{\text{kg}}\right)} = \mathbf{0.345}$$

B) Now given a turbine efficiency of 0.75. We calculated this entire problem based on the assumption that the turbine was reversible, in other words, an efficiency of 1. The boiler and pump conditions are unaffected by this change. The lower turbine efficiency means less work is removed in the turbine and more heat is subsequently removed in the condenser.

$$\eta_{\text{turbine}} = \frac{W_{s,\text{actual}}}{W_{s,\text{reversible}}}$$

$$W_{s,\text{reversible}} \times 0.75 = W_{s,\text{actual}}$$

$$-911.2 \frac{\text{kJ}}{\text{kg}} \times 0.75 = -683.4 \frac{\text{kJ}}{\text{kg}}$$

Find efficiency of the cycle

$$\eta_{\text{H.E.}} = \frac{\left| \left(-683.4 \frac{\text{kJ}}{\text{kg}}\right) + \left(4.0 \frac{\text{kJ}}{\text{kg}}\right) \right|}{\left(2629.4 \frac{\text{kJ}}{\text{kg}}\right)} = \mathbf{0.258}$$

Set an energy balance around the actual turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\hat{H}_{\text{out}} = \hat{H}_{\text{in}} + \frac{\dot{W}_S}{\dot{m}}$$

$$\hat{H}_{\text{out}} = 2800.9 \frac{\text{kJ}}{\text{kg}} - 683.4 \frac{\text{kJ}}{\text{kg}} = 2117.5 \frac{\text{kJ}}{\text{kg}}$$

Water at 40°C and $2117.5 \frac{\text{kJ}}{\text{kg}} \rightarrow \mathbf{q=0.810}$

C)

$$1\text{MW} \left(\frac{1 \text{ kg}}{679.4 \text{ kJ}} \right) \left(\frac{\text{MJ}}{\text{sec}} \right) \left(\frac{1 \times 10^3 \text{ kJ}}{1 \text{ MJ}} \right) = \mathbf{1.47 \frac{\text{kg}}{\text{sec}}}$$

5-7) A refrigerator runs on the vapor-compression cycle. The boiler operates at $T=265 \text{ K}$ and the condenser operates at 305 K . The compressor has an efficiency of 85%. Thermodynamic data for two different refrigerants is located in Appendix F.

- A) What is the maximum attainable coefficient of performance for Freon 22?
- B) What is the maximum attainable coefficient of performance for refrigerant R-422A?
- C) Besides coefficient of performance, two considerations in choosing a refrigerant are price and safety. Do a little research on Freon 22 and R-422A and comment on their suitability as refrigerants.

Solution

A) Set an entropy balance on the reversible compressor

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{\text{in}}\hat{S}_{\text{in}} - \dot{m}_{\text{out}}\hat{S}_{\text{out}} + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Cancelling terms

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

The condenser operates at 305 K , or 32°C . From the figure, the vapor pressure corresponding to this temperature is $\sim 13 \text{ bar}$.

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The inlet to the compressor is saturated vapor at 265 K, or -8°C . Follow the line of constant entropy from this point up to the pressure 13 bar. This reveals that $\hat{H}_{\text{out}} = 435 \frac{\text{kJ}}{\text{kg}}$, for a reversible compressor. Now we will perform an energy balance on the reversible compressor.

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find \hat{H}_{in}

$$\hat{H}_{\text{in}} \rightarrow \text{Saturated Freon 22 vapor at } 265\text{K} \rightarrow 402 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 435 \frac{\text{kJ}}{\text{kg}} - 402 \frac{\text{kJ}}{\text{kg}} = 35 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{compressor}} = \frac{W_{s,\text{reversible}}}{W_{s,\text{actual}}}$$

$$W_{s,\text{actual}} = \frac{\left(35 \frac{\text{kJ}}{\text{kg}} \right)}{(0.85)} = 41.2 \frac{\text{kJ}}{\text{kg}}$$

Set an energy balance on the boiler, remembering that enthalpy is unchanged in the valve, so the enthalpy entering the boiler is the same as the enthalpy leaving the condenser.

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{Q}$$

$$\frac{\dot{Q}}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$$\hat{H}_{in} \rightarrow \text{Freon 22 saturated liquid at 305 K} \rightarrow 240 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{out} \rightarrow \text{Freon 22 at 265K} \rightarrow 402 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}_C}{\dot{m}} = 402 \frac{\text{kJ}}{\text{kg}} - 240 \frac{\text{kJ}}{\text{kg}} = 162 \frac{\text{kJ}}{\text{kg}}$$

Use the definition of the C.O.P

$$\text{C. O. P.} = \frac{\dot{Q}_C}{\dot{W}_S}$$

$$\text{C. O. P.} = \frac{162 \frac{\text{kJ}}{\text{kg}}}{41.7 \frac{\text{kJ}}{\text{kg}}} = \mathbf{3.88}$$

B) Set an entropy balance on the reversible compressor

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{in}\hat{S}_{in} - \dot{m}_{out}\hat{S}_{out} + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Cancelling terms

$$\hat{S}_{in} = \hat{S}_{out}$$

Again the outlet from the reversible compressor is found by following the line of constant entropy from the inlet condition (saturated vapor at 265 K ~ 17°F, $\hat{H} = 104 \frac{\text{BTU}}{\text{lb}}$) to the outlet condition. For this refrigerant T=305 K = 89°F corresponds to a vapor pressure of ~120 psia. Again following the line of constant entropy from the compressor inlet condition to 120 psia gives $\hat{H}_{out} = 118 \frac{\text{BTU}}{\text{lb}}$. For the reversible compressor:

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{out} - \hat{H}_{in} = 118 - 104 \frac{\text{BTU}}{\text{lb}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 14 \frac{\text{BTU}}{\text{lb}}$$

$$\eta_{\text{compressor}} = \frac{W_{S,\text{reversible}}}{W_{S,\text{actual}}}$$

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$$W_{s,\text{actual}} = \frac{\left(14 \frac{\text{BTU}}{\text{lb}}\right)}{(0.85)} = 16.5 \frac{\text{BTU}}{\text{lb}}$$

And for the boiler

$$\frac{\dot{Q}}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$$\hat{H}_{\text{in}} \rightarrow \text{same enthalpy as R-422A saturated liquid at } T=89^\circ\text{F} \rightarrow 42 \frac{\text{BTU}}{\text{lb}}$$

$$\hat{H}_{\text{out}} \rightarrow \text{same as stream entering compressor} \rightarrow \hat{H} = 104 \frac{\text{BTU}}{\text{lb}}$$

$$\frac{\dot{Q}}{\dot{m}} = 104 \frac{\text{BTU}}{\text{lb}} - 42 \frac{\text{BTU}}{\text{lb}} = 62 \frac{\text{BTU}}{\text{lb}}$$

Use the definition of the C.O.P

$$\text{C. O. P.} = \frac{\dot{Q}_c}{\dot{W}_s}$$

$$\text{C. O. P.} = \frac{62 \frac{\text{BTU}}{\text{lb}}}{16.5 \frac{\text{BTU}}{\text{lb}}} = 3.77$$

The answers to parts A and B are very close and, considering the level of accuracy of our readings from the figures, should probably be considered indistinguishable.

C) Freon 22 is a powerful greenhouse gas with a global warming potential 1810 times greater than CO_2 . Freon 22 has mostly been phased out in new equipment in the United States and replaced by other refrigerants. HFC134a is a halualkaline refrigerant with a lower ozone depletion potential than Freon. HFC134a has an LD_{50} (lethal concentration for 50% of subjects) in rats of $1,500 \text{ g/m}^3$ making it relatively non-toxic.

5-8) The engine on a steam ship runs on the Rankine cycle. The steam leaves the boiler at 20 bar and 350°C. The turbine has an efficiency of 75% and an outlet pressure of 1 bar. The pressure changes in the boiler and condenser can be considered negligible and the liquid leaving the condenser is saturated.

A) Determine the operating temperature of the condenser, and compute the efficiency of a Carnot cycle operating between the boiler and condenser temperatures.

B) Determine the actual efficiency of this Rankine cycle and compare it to the Carnot efficiency.

C) When the *Titanic* was sinking, the *Carpathia* received the S.O.S. and immediately set course to attempt a rescue. The captain of the *Carpathia* ordered the hot water turned off in the passengers' cabins (Lord, 1955). What effect do you expect this action had?

Solution:

A) The condenser operates at 1 bar, so $T=99.6^\circ\text{C}$ according to saturated steam table. Boiler outlet temperature is 350°C (though the phase change occurs at 212.4°C , the boiling temperature at 20 bar). The Carnot efficiency corresponding to these temperatures is:

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{99.6 + 273.15}{350 + 273.15} = 0.402$$

B) For steam at 350°C and 20 bar:

$$\hat{H} = 3137.7 \frac{\text{kJ}}{\text{kg}} \quad \hat{S} = 6.9583 \frac{\text{kJ}}{\text{kg K}}$$

For saturated liquid and vapor at 1 bar:

$$\hat{H}^V = 2674.9 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^V = 7.3588 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{H}^L = 417.5 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^L = 1.3028 \frac{\text{kJ}}{\text{kg K}}$$

The entropy balance for a reversible turbine (steady state, adiabatic and reversible) is:

$$\hat{S}_{out,rev} = \hat{S}_{in}$$

Thus for the reversible turbine in this case:

$$\hat{S}_{out,rev} = 6.9583 \frac{\text{kJ}}{\text{kg K}} = q_{rev} \hat{S}^V + (1 - q_{rev}) \hat{S}^L$$

$$q_{rev} = 0.934$$

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This is used to find the enthalpy leaving the reversible turbine:

$$\hat{H}_{out,rev} = q_{rev}\hat{H}^V + (1 - q_{rev})\hat{H}^L = 2525.6 \frac{kJ}{kg}$$

Reversible work for the turbine is:

$$\frac{\dot{W}_{s,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 2525.6 - 3137.7 \frac{kJ}{kg} = -612.1 \frac{kJ}{kg}$$

For an efficiency of 75%:

$$\frac{\dot{W}_{s,act}}{\dot{M}} = \eta \left(\frac{\dot{W}_{s,rev}}{\dot{M}} \right) = (0.75) \left(-612.1 \frac{kJ}{kg} \right) = -459.1 \frac{kJ}{kg}$$

The actual enthalpy of the stream leaving the turbine can be determined by closing the energy balance:

$$\begin{aligned} \frac{\dot{W}_{s,act}}{\dot{M}} &= \hat{H}_{out,act} - \hat{H}_{in} \\ -459.1 \frac{kJ}{kg} &= \hat{H}_{out,act} - 3137.7 \frac{kJ}{kg} \\ \hat{H}_{out,act} &= 2678.6 \frac{kJ}{kg} \end{aligned}$$

The energy balance for the condenser is:

$$\frac{\dot{Q}}{\dot{M}} = \hat{H}_{out} - \hat{H}_{in}$$

The stream exiting the turbine is the same as the stream entering the condenser, and the stream exiting the condenser is saturated liquid at 1 bar:

$$\frac{\dot{Q}_C}{\dot{M}} = 417.5 - 2678.6 \frac{kJ}{kg} = -2261.1 \frac{kJ}{kg}$$

In the pump, the pressure is increased from 1 bar to 20 bar. Using equation 3.77 (an integrated form of equation 1.30) and the molar volume of saturated liquid at 1 bar:

$$\frac{W_s}{M} \approx \hat{V}(P_{out} - P_{in})$$

$$\begin{aligned} \frac{\dot{W}_{s,pump}}{\dot{m}} &\approx \left(0.001043 \frac{\text{m}^3}{\text{kg}}\right) (20 - 1 \text{ bar}) \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}}\right) \left(\frac{1 \frac{\text{N}}{\text{m}^2}}{\text{Pa}}\right) \left(\frac{1 \text{ J}}{\text{Nm}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) \\ &\approx 2 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

The energy balance around the entire heat engine is:

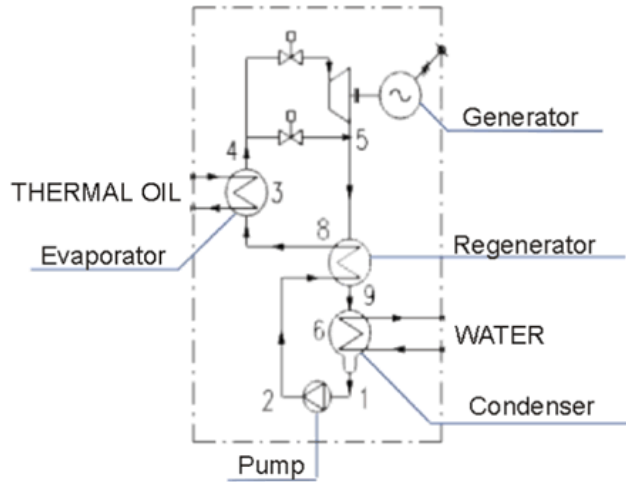
$$\begin{aligned} 0 &= \frac{\dot{Q}_H}{\dot{m}} + \frac{\dot{Q}_C}{\dot{m}} + \frac{\dot{W}_{S,turb}}{\dot{m}} + \frac{\dot{W}_{s,pump}}{\dot{m}} \\ 0 &= \frac{\dot{Q}_H}{\dot{m}} + \left(-2261.1 \frac{\text{kJ}}{\text{kg}}\right) + \left(-459.1 \frac{\text{kJ}}{\text{kg}}\right) + 2 \frac{\text{kJ}}{\text{kg}} \\ \frac{\dot{Q}_H}{\dot{m}} &= 2718.2 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

The efficiency of the heat engine is:

$$\eta = \frac{\left|\frac{\dot{W}_{s,net}}{\dot{m}}\right|}{\left|\frac{\dot{Q}_H}{\dot{m}}\right|} = \frac{\left|\left(-459.1 \frac{\text{kJ}}{\text{kg}}\right) + \left(2 \frac{\text{kJ}}{\text{kg}}\right)\right|}{\left|2718.2 \frac{\text{kJ}}{\text{kg}}\right|} = 0.168$$

C) The steam produced in the ship's boiler room was used for multiple purposes- this steam could be used for anything on the ship that required either heat or work. The captain was trying to maximize the "Q_H" that was being used to power the engine that turned the ship's propellers, so he gave orders that prevented any heat from being diverted to other locations, like the passenger cabins.

5-9) Below is a schematic of a variation on the Rankine Cycle, not for steam but for an organic fluid. This process has been called the “organic Rankine Cycle:”



- A) Do some research and determine the major advantage of using a Rankine cycle with an organic as a working fluid as opposed to water.
- B) There are five unit operations in the process above. Describe what is happening in each of those steps (for the organic working fluid).
- C) There is a valve between line 4→ 5. Why do you think that line exists and why do you think that valve is there?

Solution:

A) The major advantage of the ORC is to obtain useful work from low temperature heat sources. Temperatures too low to boil water will boil many organics, and that is why organic vapor is useful.

B) The turbine/generator is used to convert the internal energy of the organic into useful shaft work. This shaft work is used to create electricity.

The regenerator is used to heat the compressed liquid from the pump. It takes the hot organic from the turbine and reclaims energy so that the boiler does not require as much energy.

The condenser condenses the vapor from the regenerator in order to pressurize it. The purpose of this step is to avoid compressing a vapor because this requires a large amount of work. It is far easier to pressurize a liquid than a vapor.

The pump takes the saturated liquid from the condenser and pressurizes it.

The evaporator uses hot thermal oil to evaporate the compressed liquid past its boiling point. The vapor leaves the boiler as either saturated or superheated.

C) This line and valve gives operators a quick way to re-route some or all of the steam around the turbine rather than sending it through the turbine. This could be used as a safety feature so that the turbine/generator can be turned off quickly, without shutting down the whole system.

5-10) A refrigerator runs on the vapor compression cycle, using R-422A as a refrigerant. The boiler operates at 20°F. The effluent from the condenser is 10°F above ambient temperature. The compressor has an efficiency of 80%.

Find each of the following:

- the temperature and pressure of the gas leaving the compressor
- the fraction of vapor in the stream leaving the expansion valve
- the coefficient of performance
- the mass flow rate of refrigerant needed to attain 10 kJ/sec of cooling.

A) The ambient temperature is 70°F.

B) The ambient temperature is 110°F.

Solution:

A) The boiler operates at 20°F, and the material leaving the boiler in the vapor-compression cycle is conventionally saturated vapor. From the R-422A pressure enthalpy diagram, saturated vapor at this temperature has:

$$\hat{H} = 107 \frac{\text{BTU}}{\text{lb}_m} \quad \hat{S} = 0.223 \frac{\text{BTU}}{\text{lb}_m \cdot ^\circ\text{F}}$$

The condenser temperature is 80°F; ten degrees above ambient. This temperature corresponds to a vapor pressure of **100 psia**, according to the figure. So this is the outlet pressure the compressor must deliver.

If the compressor were reversible, the specific entropy of the exiting vapor would be the same as the specific entropy of the entering vapor. Tracing the line of constant entropy from the inlet condition (saturated vapor at 20°F) to the outlet pressure (100 psia) produces an outlet condition of:

$$\hat{H}_{out,rev} = 117 \frac{\text{BTU}}{\text{lb}_m} \quad \hat{S}_{out,rev} = 0.223 \frac{\text{BTU}}{\text{lb}_m \cdot ^\circ\text{F}}$$

The energy balance for a reversible compressor is:

$$\frac{\dot{W}_{s,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 10 \frac{\text{BTU}}{\text{lb}_m}$$

Applying the 80% efficiency:

$$\eta = \frac{\frac{\dot{W}_{s,rev}}{\dot{M}}}{\frac{\dot{W}_{s,act}}{\dot{M}}}$$

$$\frac{\dot{W}_{s,act}}{\dot{M}} = 12.5 \frac{\text{BTU}}{\text{lb}_m}$$

Closing the energy balance, this means:

$$\hat{H}_{out,act} = 107 + 12.5 \frac{\text{BTU}}{\text{lb}_m} \sim 120 \frac{\text{BTU}}{\text{lb}_m}$$

This specific enthalpy, and $P=100$ psia, corresponds to $T \sim 102^\circ\text{F}$.

Since the condenser operates at 80°F , the stream entering the valve is saturated liquid at 80°F . This has:

$$\hat{H} = 40 \frac{\text{BTU}}{\text{lb}_m}$$

The specific enthalpy does not change in the valve (no heat or work) so the stream exiting the valve has to be a mixture of saturated liquid and saturated vapor at 20°F .

$$\hat{H} = 40 \frac{\text{BTU}}{\text{lb}_m} = q\hat{H}_V + (1 - q)\hat{H}_L$$

$$40 \frac{\text{BTU}}{\text{lb}_m} = q(107 \frac{\text{BTU}}{\text{lb}_m}) + (1 - q)(18 \frac{\text{BTU}}{\text{lb}_m})$$

$$q = 0.247$$

The mixture leaving the valve is **$\sim 25\%$ vapor**.

Calculating the C.O.P. requires knowing the compressor work (12.5 BTU/lb) and the heat added in the boiler. The energy balance for the boiler is:

$$\frac{\dot{Q}_C}{\dot{M}} = \hat{H}_{out} - \hat{H}_{in} = 107 - 40 \frac{\text{BTU}}{\text{lb}_m} = 67 \frac{\text{BTU}}{\text{lb}_m}$$

$$C.O.P. = \frac{\frac{\dot{Q}_C}{\dot{M}}}{\frac{\dot{W}_{s,act}}{\dot{M}}} = \frac{67}{12.5} = 5.4$$

The flow rate needed to provide 10 kJ/sec of cooling can be determined from the known \dot{Q}_C/M :

$$\frac{\dot{Q}_C}{\dot{M}} = 67 \frac{\text{BTU}}{\text{lb}_m}$$

$$\dot{M} = \frac{\dot{Q}_C}{67 \frac{\text{BTU}}{\text{lb}_m}} = \left(\frac{10 \frac{\text{kJ}}{\text{sec}}}{67 \frac{\text{BTU}}{\text{lb}_m}} \right) \left(\frac{0.9486 \text{ BTU}}{1 \text{ kJ}} \right) = \mathbf{0.14 \frac{\text{lb}_m}{\text{sec}}}$$

B) The boiler again operates at 20°F, but now the condenser operates at 120°F. The stream entering the compressor is the same as part A:

$$\hat{H} = 107 \frac{\text{BTU}}{\text{lb}_m} \quad \hat{S} = 0.223 \frac{\text{BTU}}{\text{lb}_m \text{ } ^\circ\text{F}}$$

But now the outlet pressure is about 190 psia, the vapor pressure at 120°F. Assuming a reversible compressor:

$$\hat{H}_{out,rev} = 123 \frac{\text{BTU}}{\text{lb}_m} \quad \hat{S}_{out,rev} = 0.223 \frac{\text{BTU}}{\text{lb}_m \text{ } ^\circ\text{F}}$$

$$\frac{\dot{W}_{s,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 16 \frac{\text{BTU}}{\text{lb}_m}$$

$$\frac{\dot{W}_{s,act}}{\dot{M}} = \frac{16 \frac{\text{BTU}}{\text{lb}_m}}{.8} = 20 \frac{\text{BTU}}{\text{lb}_m}$$

Closing the energy balance, this means:

$$\hat{H}_{out,act} = 107 + 20 \frac{\text{BTU}}{\text{lb}_m} \sim 128 \frac{\text{BTU}}{\text{lb}_m}$$

This specific enthalpy, and P=190 psia, corresponds to **T~160°F**.

The stream entering the valve is saturated liquid at 120°F. This has:

$$\hat{H} = 51 \frac{\text{BTU}}{\text{lb}_m}$$

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The stream exiting the valve is again a mixture of saturated liquid and saturated vapor at 20°F:

$$\hat{H} = 51 \frac{\text{BTU}}{\text{lb}_m} = q\hat{H}_V + (1 - q)\hat{H}_L$$

$$51 \frac{\text{BTU}}{\text{lb}_m} = q(107 \frac{\text{BTU}}{\text{lb}_m}) + (1 - q)(18 \frac{\text{BTU}}{\text{lb}_m})$$

$$q = 0.371$$

The mixture leaving the valve is **~37% vapor**.

The energy balance for the boiler is:

$$\frac{\dot{Q}_C}{\dot{M}} = \hat{H}_{out} - \hat{H}_{in} = 107 - 51 \frac{\text{BTU}}{\text{lb}_m} = 56 \frac{\text{BTU}}{\text{lb}_m}$$

$$C.O.P. = \frac{\frac{\dot{Q}_C}{\dot{M}}}{\frac{\dot{W}_{s,act}}{\dot{M}}} = \frac{56}{20} = \mathbf{2.8}$$

The flow rate needed to provide 10 kJ/sec of cooling can be determined from the known \dot{Q}_C/\dot{M} :

$$\frac{\dot{Q}_C}{\dot{M}} = 56 \frac{\text{BTU}}{\text{lb}_m}$$

$$\dot{M} = \frac{\dot{Q}_C}{56 \frac{\text{BTU}}{\text{lb}_m}} = \left(\frac{10 \frac{\text{kJ}}{\text{sec}}}{56 \frac{\text{BTU}}{\text{lb}_m}} \right) \left(\frac{0.9486 \text{ BTU}}{1 \text{ kJ}} \right) = \mathbf{0.17 \frac{\text{lb}_m}{\text{sec}}}$$

5-11) A heat engine operates on the Rankine cycle, with saturated steam at $T=350^\circ\text{C}$ leaving the boiler, a condenser operating at $T=100^\circ\text{C}$, and a turbine efficiency of 80%.

- A) Find the liquid fraction leaving the turbine.
 B) Find the overall efficiency of the heat engine.
 C) A superheater is inserted into the cycle after the boiler, which increases the steam temperature to 450°C without changing its pressure. All other specifications remain the same. Find the liquid fraction leaving the turbine and the overall efficiency of the cycle, and compare to the answers from A and B.
 D) A heat engine operates with the same boiler, pump and condenser specifications used in parts A and B (no superheater). Instead of a single turbine, there are two turbines, each with 80% efficiency. The steam leaving the first turbine has $P=3$ bar, is sent to a heat exchanger in which its temperature is increased to 200°C , and then continues to the second turbine. Find the liquid fractions in both turbine effluent streams and the overall efficiency of the heat engine, and compare to the answers in A, B and C.

Solution:

A) For saturated steam at 350°C :

$$\hat{H} = 2563.6 \frac{\text{kJ}}{\text{kg}} \quad \hat{S} = 5.211 \frac{\text{kJ}}{\text{kg K}}$$

The condenser operates at 100°C , so the outlet pressure from the turbine must be 1.0142 bar. Following are data for saturated liquid and vapor at these conditions:

$$\hat{H}^V = 2675.6 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^V = 7.3541 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{H}^L = 419.2 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^L = 1.3072 \frac{\text{kJ}}{\text{kg K}}$$

The entropy balance for a reversible turbine (steady state, adiabatic and reversible) is:

$$\hat{S}_{out,rev} = \hat{S}_{in}$$

Thus for the reversible turbine in this case:

$$\hat{S}_{out,rev} = 5.211 \frac{\text{kJ}}{\text{kg K}} = q_{rev} \hat{S}^V + (1 - q_{rev}) \hat{S}^L$$

$$q_{rev} = 0.646$$

This is used to find the enthalpy leaving the reversible turbine:

$$\hat{H}_{out,rev} = q_{rev} \hat{H}^V + (1 - q_{rev}) \hat{H}^L = 1874.8 \frac{\text{kJ}}{\text{kg}}$$

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Reversible work for the turbine is:

$$\frac{\dot{W}_{s,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 1874.8 - 2563.6 \frac{kJ}{kg} = -688.8 \frac{kJ}{kg}$$

For an efficiency of 80%:

$$\frac{\dot{W}_{s,act}}{\dot{M}} = \eta \left(\frac{\dot{W}_{s,rev}}{\dot{M}} \right) = (0.80) \left(-688.8 \frac{kJ}{kg} \right) = -551.0 \frac{kJ}{kg}$$

The actual enthalpy of the stream leaving the turbine can be determined by closing the energy balance:

$$\begin{aligned} \frac{\dot{W}_{s,act}}{\dot{M}} &= \hat{H}_{out,act} - \hat{H}_{in} \\ -551.0 \frac{kJ}{kg} &= \hat{H}_{out,act} - 2563.6 \frac{kJ}{kg} \\ \hat{H}_{out,act} &= 2012.6 \frac{kJ}{kg} \end{aligned}$$

We can use this result to determine the actual quality of the mixture leaving the actual turbine:

$$\begin{aligned} \hat{H}_{out,act} &= q_{act} \hat{H}^V + (1 - q_{act}) \hat{H}^L = 2012.6 \frac{kJ}{kg} \\ q_{act} &= 0.707 \end{aligned}$$

The liquid fraction is $1 - q = \mathbf{0.293}$.

B) The energy balance for the condenser is:

$$\frac{\dot{Q}_C}{\dot{M}} = \hat{H}_{out} - \hat{H}_{in}$$

The stream exiting the ACTUAL turbine is the same as the stream entering the condenser, and the stream exiting the condenser is saturated liquid at 1 bar:

$$\frac{\dot{Q}_C}{\dot{M}} = 419.2 - 2012.6 \frac{kJ}{kg} = -1593.4 \frac{kJ}{kg}$$

In the pump, the pressure is increased from 1.014 bar to the saturation pressure at 350°C, which is 165.29 bar. Using equation 3.77 and the molar volume of saturated liquid at 100°C:

$$\frac{W_s}{M} \approx \hat{V}(P_{out} - P_{in})$$

$$\begin{aligned} \frac{\dot{W}_{s,pump}}{\dot{m}} &\approx \left(0.001043 \frac{\text{m}^3}{\text{kg}}\right) (165.29 - 1.014 \text{ bar}) \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}}\right) \left(\frac{1 \frac{\text{N}}{\text{m}^2}}{\text{Pa}}\right) \left(\frac{1 \text{ J}}{\text{Nm}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) \\ &\approx 17.1 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Note that near the critical point the assumption of constant specific volume for water becomes suspect. At the pump outlet pressure we are comparatively close to the critical pressure (~220 bar) of water, but at ~100°C we are still well away from the critical temperature.

The energy balance around the entire heat engine is:

$$\begin{aligned} 0 &= \frac{\dot{Q}_H}{\dot{m}} + \frac{\dot{Q}_C}{\dot{m}} + \frac{\dot{W}_{s,turb}}{\dot{m}} + \frac{\dot{W}_{s,pump}}{\dot{m}} \\ 0 &= \frac{\dot{Q}_H}{\dot{m}} + \left(-1595.1 \frac{\text{kJ}}{\text{kg}}\right) + \left(-551.0 \frac{\text{kJ}}{\text{kg}}\right) + 17.1 \frac{\text{kJ}}{\text{kg}} \\ \frac{\dot{Q}_H}{\dot{m}} &= 2129.0 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

The efficiency of the heat engine is:

$$\eta = \frac{\left|\frac{\dot{W}_{s,net}}{\dot{m}}\right|}{\left|\frac{\dot{Q}_H}{\dot{m}}\right|} = \frac{\left|\left(-551.0 \frac{\text{kJ}}{\text{kg}}\right) + \left(17.1 \frac{\text{kJ}}{\text{kg}}\right)\right|}{\left|2129.0 \frac{\text{kJ}}{\text{kg}}\right|} = \mathbf{0.251}$$

C) The solution procedure is the same as in parts A and B, except now the temperature of the steam entering the turbine is now 450°C, while the pressure remains 163.29 bar. The steam tables contain data for 450°C at pressure of 150 and 200 bar. Interpolating between these allows us to estimate the specific entropy and enthalpy of the steam entering the turbine:

$$\begin{aligned}\hat{H}_{163.29} &= \hat{H}_{150} + (\hat{H}_{200} - \hat{H}_{150}) \left(\frac{163 - 150 \text{ bar}}{200 - 150 \text{ bar}} \right) \\ \hat{H}_{163.29} &= 3157.9 \frac{\text{kJ}}{\text{kg}} + \left(3061.7 - 3157.9 \frac{\text{kJ}}{\text{kg}} \right) \left(\frac{163 - 150 \text{ bar}}{200 - 150 \text{ bar}} \right) \\ \hat{H}_{163.29} &= 3132.3 \frac{\text{kJ}}{\text{kg}} \\ \hat{S}_{163.29} &= \hat{S}_{150} + (\hat{S}_{200} - \hat{S}_{150}) \left(\frac{163 - 150 \text{ bar}}{200 - 150 \text{ bar}} \right) \\ \hat{S}_{163.29} &= 6.1434 \frac{\text{kJ}}{\text{kgK}} + \left(5.9043 - 6.1434 \frac{\text{kJ}}{\text{kgK}} \right) \left(\frac{163 - 150 \text{ bar}}{200 - 150 \text{ bar}} \right) \\ \hat{S}_{163.29} &= 6.0799 \frac{\text{kJ}}{\text{kgK}}\end{aligned}$$

Using these data for the turbine inlet/boiler outlet stream, while leaving all other specifications and the solution procedure the same, gives the following results:

$$q_{rev} = 0.789$$

$$\frac{W_{s,rev}}{m} = -933.1 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{W_{s,act}}{m} = -746.5 \frac{\text{kJ}}{\text{kg}}$$

$$q_{act} = 0.872$$

$$\frac{Q_C}{m} = -1968.3 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{W_{s,pump}}{\dot{m}} \approx 17.1 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{Q_H}{m} = 2697.7 \frac{\text{kJ}}{\text{kg}}$$

This Q_H accounts for the heat added in *both* the boiler and the superheater. The energy balance for two consecutive heat exchangers isn't substantially different from the energy balance for one.

$$\eta = 0.270$$

The efficiency is fractionally higher and the liquid fraction in the stream leaving the turbine is significantly lower, both of which are good things, but this version of the system requires a heat source that is above 450°C.

D) Here, we have *exactly* the same pump as in part A/B- it takes in saturated liquid at 100°C and emits liquid at 163.29 bar.

$$\frac{\dot{W}_{s,pump}}{\dot{m}} \approx 17.1 \frac{kJ}{kg}$$

Similarly, we have *exactly* the same boiler as in parts A/B—the fluid leaving the pump enters the boiler, and the stream leaving the boiler is saturated steam at 350°C.

$$\frac{\dot{Q}_H}{\dot{m}} = 2129.0 \frac{kJ}{kg}$$

However, we can't assume Q_C , the heat removed in the condenser, is the same—it produces saturated liquid at 100°C as it did in part A, but we as yet have no data for the stream entering the condenser, because the turbine specifications have changed.

Start by modeling the first turbine. As in part A, the steam entering the turbine is saturated steam at 350°C:

$$\hat{H} = 2563.6 \frac{kJ}{kg} \quad \hat{S} = 5.211 \frac{kJ}{kg K}$$

But now the turbine has an outlet pressure of 3 bar, and saturated liquid and vapor data at this pressure are:

$$\hat{H}^V = 2724.9 \frac{kJ}{kg} \quad \hat{S}^V = 6.9916 \frac{kJ}{kg K}$$

$$\hat{H}^L = 561.4 \frac{kJ}{kg} \quad \hat{S}^L = 1.6717 \frac{kJ}{kg K}$$

$$\hat{S}_{out,rev} = 5.211 \frac{kJ}{kg K} = q_{rev} \hat{S}^V + (1 - q_{rev}) \hat{S}^L$$

$$q_{rev} = 0.665$$

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This is used to find the enthalpy leaving the reversible turbine:

$$\hat{H}_{out,rev} = q_{rev}\hat{H}^V + (1 - q_{rev})\hat{H}^L = 2000.8 \frac{kJ}{kg}$$

Reversible work for the turbine is:

$$\frac{\dot{W}_{s,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 2000.8 - 2563.6 \frac{kJ}{kg} = -562.8 \frac{kJ}{kg}$$

For an efficiency of 80%:

$$\frac{\dot{W}_{s,act}}{\dot{M}} = \eta \left(\frac{\dot{W}_{s,rev}}{\dot{M}} \right) = (0.80) \left(-562.8 \frac{kJ}{kg} \right) = -450.3 \frac{kJ}{kg}$$

The actual enthalpy of the stream leaving the turbine can be determined by closing the energy balance:

$$\begin{aligned} \frac{\dot{W}_{s,act}}{\dot{M}} &= \hat{H}_{out,act} - \hat{H}_{in} \\ -450.3 \frac{kJ}{kg} &= \hat{H}_{out,act} - 2563.6 \frac{kJ}{kg} \\ \hat{H}_{out,act} &= 2113.3 \frac{kJ}{kg} \end{aligned}$$

We can use this result to determine the actual quality of the mixture leaving the actual turbine:

$$\begin{aligned} \hat{H}_{out,act} &= q_{act}\hat{H}^V + (1 - q_{act})\hat{H}^L = 2133.3 \frac{kJ}{kg} \\ q_{act} &= 0.717 \end{aligned}$$

The liquid fraction is 0.283.

There is a heat exchanger between the two turbines. The stream exiting it is at 200°C and 3 bar, and thus has $\hat{H} = 2865.9 \frac{kJ}{kg}$ and $\hat{S} = 7.3131 \frac{kJ}{kgK}$. The energy balance, like for any of the other heat exchangers in this process, is:

$$\begin{aligned} \frac{\dot{Q}}{\dot{m}} &= \hat{H}_{out} - \hat{H}_{in} \\ \frac{\dot{Q}_{HE}}{\dot{m}} &= 2865.9 - 2113.3 \frac{kJ}{kg} = 752.6 \frac{kJ}{kg} \end{aligned}$$

For the second turbine, the exiting stream is at 1.014 bar like the turbine in parts A, B and C, and the data for saturated liquid and vapor at these conditions, as used previously, is:

$$\hat{H}^V = 2675.6 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^V = 7.3541 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{H}^L = 419.2 \frac{\text{kJ}}{\text{kg}} \quad \hat{S}^L = 1.3072 \frac{\text{kJ}}{\text{kg K}}$$

Modeling the 2nd turbine reversibly:

$$\hat{S}_{out,2,rev} = 5.211 \frac{\text{kJ}}{\text{kg K}} = q_{3,rev} \hat{S}^V + (1 - q_{2,rev}) \hat{S}^L$$

$$q_{3,rev} = 0.993$$

This is used to find the enthalpy leaving the reversible turbine:

$$\hat{H}_{out,2,rev} = q_{2,rev} \hat{H}^V + (1 - q_{2,rev}) \hat{H}^L = 2660.3 \frac{\text{kJ}}{\text{kg}}$$

Reversible work for the turbine is:

$$\frac{\dot{W}_{s,2,rev}}{\dot{M}} = \hat{H}_{out,rev} - \hat{H}_{in} = 2660.3 - 22865.9 \frac{\text{kJ}}{\text{kg}} = -205.6 \frac{\text{kJ}}{\text{kg}}$$

For an efficiency of 80%:

$$\frac{\dot{W}_{s,2,act}}{\dot{M}} = \eta \left(\frac{\dot{W}_{s,2,rev}}{\dot{M}} \right) = (0.80) \left(-205.6 \frac{\text{kJ}}{\text{kg}} \right) = -164.5 \frac{\text{kJ}}{\text{kg}}$$

The actual enthalpy of the stream leaving the second turbine can be determined by closing the energy balance:

$$\begin{aligned} \frac{\dot{W}_{s,2,act}}{\dot{M}} &= \hat{H}_{out,2,act} - \hat{H}_{in} \\ -164.5 \frac{\text{kJ}}{\text{kg}} &= \hat{H}_{out,act} - 2865.9 \frac{\text{kJ}}{\text{kg}} \\ \hat{H}_{out,act} &= 2701.4 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

This is higher than the saturated vapor specific enthalpy. The stream leaving this turbine isn't a VLE mixture, it's a superheated vapor.

In calculating the efficiency of the heat engine, we have to account for ALL the heat that was added- both Q_H from the boiler and what we called Q_{HE} , for the heat exchanger between the two turbines. Similarly we have to account for all the work added and removed- in this case the pump and both turbines.

$$\eta = \frac{\left| \frac{\dot{W}_{S,net}}{\dot{m}} \right|}{\left| \frac{\dot{Q}_H}{\dot{m}} \right|} = \frac{\left| \left(-164.5 \frac{kJ}{kg} \right) + \left(-450.3 \frac{kJ}{kg} \right) + \left(17.1 \frac{kJ}{kg} \right) \right|}{\left| 2129.0 \frac{kJ}{kg} + 752.6 \frac{kJ}{kg} \right|} = \mathbf{0.207}$$

Section 5.2.3 mentioned that replacing a single turbine with a two-stage turbine and inter-stage heating can be an effective strategy for preventing excessive formation of liquid in the turbine. Here, however, the strategy hasn't really succeeded. The quality of steam leaving the first turbine is only fractionally higher than it was for the turbine in part A, but the efficiency of the cycle is significantly worse.

The first turbine inlet pressure in part D), as in part A), is over 160 bar, while the outlet pressure of that first turbine (3 bar) isn't all that much higher than the outlet pressure in part A (1 bar). If we were going to "split" the turbine from part A into two turbines, it would have worked better to choose an outlet pressure for the first turbine that was more intermediate between 160 and 1 bar.

5-12) Water is the most common working fluid in a Rankine heat engine, but there is no fundamental reason why a Rankine heat engine couldn't be designed with other working fluids. Suppose a heat engine is to operate with the following specifications:

- Quality (q) of fluid exiting turbine must be at least 0.9
- The fluid leaving the boiler is vapor at $T=100^\circ\text{C}$. Its pressure can be specified as needed to meet the constraint that $q>0.9$.
- The fluid leaving the condenser is saturated liquid at $T=40^\circ\text{C}$
- Turbine efficiency is $\eta=0.8$

Find the state of the fluid exiting the turbine, the net work produced by the cycle per kilogram of operating fluid, and the efficiency of the cycle, for each of the following operating fluids:

- A) Water
- B) Ammonia (Use Figure 6-2 for data)

ASSUMPTIONS:

There are no significant pressure changes in the boiler or condenser.

We cannot control the temperature exiting the turbine, but we can control the pressure into the turbine

Solution:

A) We will first start by finding the condition of the stream exiting the turbine as if the boiler effluent was saturated steam.

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{in}\hat{S}_{in} - \dot{m}_{out}\hat{S}_{out} + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Cancelling terms

$$\hat{S}_{in} = \hat{S}_{out}$$

Find \hat{S}_{out}

Saturated Steam at 100°C → 7.354 $\frac{\text{kJ}}{\text{kgK}}$

$$\hat{S}_{in} = \hat{S}_{out} = 7.354 \frac{\text{kJ}}{\text{kgK}}$$

From steam tables

Saturated liquid water at 40°C → 0.5724 $\frac{\text{kJ}}{\text{kgK}}$

Saturated steam at 40°C → 8.256 $\frac{\text{kJ}}{\text{kgK}}$

$$\hat{S}_{out} = \hat{S}_L(1 - q^{\text{rev}}) + \hat{S}_V(q^{\text{rev}})$$

$$7.354 \frac{\text{kJ}}{\text{kgK}} = 0.5724 \frac{\text{kJ}}{\text{kgK}}(1 - q^{\text{rev}}) + 8.256 \frac{\text{kJ}}{\text{kgK}}(q^{\text{rev}})$$

$$q^{\text{rev}} = 0.883$$

Find enthalpy out for the reversible process

$$\hat{H}_{out} = \hat{H}_L(1 - q^{\text{rev}}) + \hat{H}_V(q^{\text{rev}})$$

From steam tables

Saturated liquid water at 40°C → 167.5 $\frac{\text{kJ}}{\text{kg}}$

Saturated steam at 40°C → 2573.5 $\frac{\text{kJ}}{\text{kg}}$

$$\hat{H}_{out} = 167.5 \frac{\text{kJ}}{\text{kg}}(1 - 0.8883) + 2573.5 \frac{\text{kJ}}{\text{kg}}(0.8883) = 2291.2 \frac{\text{kJ}}{\text{kg}}$$

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Set an energy balance on the reversible turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{out} - \hat{H}_{in}$$

Find enthalpies

$$\hat{H}_{in} \rightarrow \text{Saturated Steam at } 100^\circ\text{C} \rightarrow 2675.6 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{out} \text{ was found previously} \rightarrow 2291.2 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 2291.2 \frac{\text{kJ}}{\text{kg}} - 2675.6 \frac{\text{kJ}}{\text{kg}} = -384.3 \frac{\text{kJ}}{\text{kg}}$$

Find actual turbine work

$$\eta_{\text{turbine}} = \frac{W_{s,\text{actual}}}{W_{s,\text{reversible}}}$$

$$W_{s,\text{actual}} = \eta_{\text{turbine}} W_{s,\text{reversible}}$$

$$W_{s,\text{actual}} = 0.8 \left(-384.3 \frac{\text{kJ}}{\text{kg}} \right) = -307.5 \frac{\text{kJ}}{\text{kg}}$$

Find the actual \hat{H}_{out} by setting up another energy balance around the actual turbine.

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{W}_S$$

$$\hat{H}_{out} = \frac{\dot{W}_S}{\dot{m}} + \hat{H}_{in}$$

$$\hat{H}_{out} = -307.5 \frac{\text{kJ}}{\text{kg}} + 2675.6 \frac{\text{kJ}}{\text{kg}} = 2368.1 \frac{\text{kJ}}{\text{kg}}$$

Finding the actual quality

$$\text{Water at } 40^\circ \text{ and } 2368.1 \frac{\text{kJ}}{\text{kg}} \rightarrow q=0.914$$

This quality meets the requirement so there is no reason to lower the turbine inlet pressure, and the turbine inlet pressure can't be raised above the saturation pressure. Therefore the boiler operates at 1.014 bar.

Set an energy balance around the condenser

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{Q}_C$$

$$\frac{\dot{Q}_C}{\dot{m}} = \hat{H}_{out} - \hat{H}_{in}$$

$$\hat{H}_{out} \rightarrow \text{Saturated liquid water at } 40^\circ\text{C} \rightarrow 167.5 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{in} \rightarrow \text{Actual stream leaving turbine} \rightarrow 2368.1 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}_C}{\dot{m}} = 167.5 - 2368.1 \frac{\text{kJ}}{\text{kg}} = -2200.6 \frac{\text{kJ}}{\text{kg}}$$

Next evaluate the work in pump.

$$\frac{\dot{W}_{s,pump}}{\dot{m}} \approx \hat{V}(P_{out} - P_{in})$$

$$\begin{aligned}\frac{\dot{W}_{s,pump}}{\dot{m}} &\approx \left(0.001008 \frac{m^3}{kg}\right) (1.014 - 0.0738 \text{ bar}) \left(\frac{10^5 Pa}{1 \text{ bar}}\right) \left(\frac{1 J}{Nm}\right) \left(\frac{1 \frac{N}{m^2}}{Pa}\right) \left(\frac{1 kJ}{1000 J}\right) \\ &= 0.1 \frac{kJ}{kg}\end{aligned}$$

Energy balance around whole engine:

$$0 = \frac{\dot{Q}_C}{\dot{m}} + \frac{\dot{Q}_H}{\dot{m}} + \frac{\dot{W}_{s,pump}}{\dot{m}} + \frac{\dot{W}_{S,turbine}}{\dot{m}}$$

$$\frac{\dot{Q}_H}{\dot{m}} = 2507.9 \frac{kJ}{kg}$$

$$\eta_{H.E.} = \frac{|W_{net}|}{|Q_{added}|} = \frac{\left|(-307.5 \frac{kJ}{kg}) + (0.1 \frac{kJ}{kg})\right|}{(2507.9 \frac{kJ}{kg})} = \mathbf{0.123}$$

B) First assume the vapor leaving the boiler is saturated ammonia vapor. Notice that, while the vapor pressure of water at 100°C is 1.014 bar, for ammonia it is over 60 bar.

For reversible turbine:

$$\hat{S}_{in} = \hat{S}_{out}$$

Use Figure 6-2 for data

$$\hat{S}_{in} \rightarrow \text{Saturated vapor at } 100^\circ\text{C} \rightarrow 4.52 \frac{kJ}{kgK}$$

$$\hat{S}_{in} = \hat{S}_{out} = 4.52 \frac{kJ}{kgK}$$

$$\text{Saturated liquid water at } 40^\circ\text{C} \rightarrow 1.65 \frac{kJ}{kgK}$$

$$\text{Saturated steam at } 40^\circ\text{C} \rightarrow 5.15 \frac{kJ}{kgK}$$

$$\hat{S}_{out} = \hat{S}_L(1 - q^{rev}) + \hat{S}_V(q^{rev})$$

$$4.52 \frac{kJ}{kgK} = 1.65 \frac{kJ}{kgK} (1 - q^{rev}) + 5.15 \frac{kJ}{kgK} (q^{rev})$$

$$q^{rev} = 0.82$$

Find enthalpy out for the reversible process

$$\hat{H}_{out} = \hat{H}_L(1 - q^{rev}) + \hat{H}_V(q^{rev})$$

Saturated liquid at 40°C → $390 \frac{\text{kJ}}{\text{kg}}$

Saturated vapor at 40°C → $1490 \frac{\text{kJ}}{\text{kg}}$

$$\hat{H}_{\text{out}} = 390 \frac{\text{kJ}}{\text{kg}} (1 - 0.82) + 1490 \frac{\text{kJ}}{\text{kg}} (0.82) = 1292 \frac{\text{kJ}}{\text{kg}}$$

Set an energy balance on the reversible turbine

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$\hat{H}_{\text{in}} \rightarrow$ Saturated vapor at 100°C → $1450 \frac{\text{kJ}}{\text{kg}}$

\hat{H}_{out} was found previously → $1292 \frac{\text{kJ}}{\text{kg}}$

$$\frac{\dot{W}_S}{\dot{m}} = 1292 \frac{\text{kJ}}{\text{kg}} - 1450 \frac{\text{kJ}}{\text{kg}} = -158 \frac{\text{kJ}}{\text{kg}}$$

Find actual turbine work

$$\eta_{\text{turbine}} = \frac{W_{\text{s,actual}}}{W_{\text{s,reversible}}}$$

$$W_{\text{s,actual}} = \eta_{\text{turbine}} W_{\text{s,reversible}}$$

$$W_{\text{s,actual}} = 0.8 \left(-158 \frac{\text{kJ}}{\text{kg}} \right) = -126 \frac{\text{kJ}}{\text{kg}}$$

Find the actual \hat{H}_{out} by setting up another energy balance around the actual turbine.

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

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$$\hat{H}_{\text{out}} = \frac{\dot{W}_S}{\dot{m}} + \hat{H}_{\text{in}}$$

$$\hat{H}_{\text{out}} = -126 \frac{\text{kJ}}{\text{kg}} + 1450 \frac{\text{kJ}}{\text{kg}} = 1324 \frac{\text{kJ}}{\text{kg}}$$

Finding the actual quality

$$\text{Ammonia at } 40^\circ \text{ and } 1324 \frac{\text{kJ}}{\text{kg}} \rightarrow q=0.85$$

This quality does not meet the requirement of $q < 0.9$. One might consider setting the ACTUAL outlet q to 0.9 and back-solving the inlet conditions, but this isn't a realistic strategy because we don't know how much entropy is generated in the turbine. Instead we can use an iterative procedure to find a pressure for the steam that meets the requirement. For example if $P=55$ bar and $T=100^\circ\text{C}$ entering the turbine:

For reversible turbine:

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

$$\hat{S}_{\text{in}} \rightarrow \text{Vapor at } 100^\circ\text{C and } 55 \text{ bar} \rightarrow 4.70 \frac{\text{kJ}}{\text{kgK}}$$

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}} = 4.70 \frac{\text{kJ}}{\text{kgK}}$$

$$\text{Saturated liquid water at } 40^\circ\text{C} \rightarrow 1.65 \frac{\text{kJ}}{\text{kgK}}$$

$$\text{Saturated steam at } 40^\circ\text{C} \rightarrow 5.15 \frac{\text{kJ}}{\text{kgK}}$$

$$\hat{S}_{\text{out}} = \hat{S}_L(1 - q^{\text{rev}}) + \hat{S}_V(q^{\text{rev}})$$

$$4.70 \frac{\text{kJ}}{\text{kgK}} = 1.65 \frac{\text{kJ}}{\text{kgK}}(1 - q^{\text{rev}}) + 5.15 \frac{\text{kJ}}{\text{kgK}}(q^{\text{rev}})$$

$$q^{\text{rev}} = 0.871$$

Find enthalpy out for the reversible process

$$\hat{H}_{\text{out}} = \hat{H}_L(1 - q^{\text{rev}}) + \hat{H}_V(q^{\text{rev}})$$

$$\text{Saturated liquid at } 40^\circ\text{C} \rightarrow 390 \frac{\text{kJ}}{\text{kg}}$$

$$\text{Saturated vapor at } 40^\circ\text{C} \rightarrow 1490 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{out}} = 390 \frac{\text{kJ}}{\text{kg}} (1 - 0.871) + 1490 \frac{\text{kJ}}{\text{kg}} (0.871) = 1349 \frac{\text{kJ}}{\text{kg}}$$

Set an energy balance on the reversible turbine

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$$\hat{H}_{\text{in}} \rightarrow \text{Vapor at } 100^\circ\text{C and } 55 \text{ bar} \rightarrow 1500 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{out}} \text{ was found previously} \rightarrow 1349 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 1349 \frac{\text{kJ}}{\text{kg}} - 1500 \frac{\text{kJ}}{\text{kg}} = -151 \frac{\text{kJ}}{\text{kg}}$$

Find actual turbine work

$$\eta_{\text{turbine}} = \frac{W_{S,\text{actual}}}{W_{S,\text{reversible}}}$$

$$W_{S,\text{actual}} = \eta_{\text{turbine}} W_{S,\text{reversible}}$$

$$W_{S,\text{actual}} = 0.8 \left(-151 \frac{\text{kJ}}{\text{kg}} \right) = -121 \frac{\text{kJ}}{\text{kg}}$$

Find the actual \hat{H}_{out} by setting up another energy balance around the actual turbine.

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\hat{H}_{\text{out}} = \frac{\dot{W}_S}{\dot{m}} + \hat{H}_{\text{in}}$$

$$\hat{H}_{\text{out}} = -121 \frac{\text{kJ}}{\text{kg}} + 1500 \frac{\text{kJ}}{\text{kg}} = 1379 \frac{\text{kJ}}{\text{kg}}$$

Finding the actual quality

$$\text{Ammonia at } 40^\circ \text{ and } 1379 \frac{\text{kJ}}{\text{kg}} \rightarrow q=0.90$$

This meets the criteria.

Set an energy balance around the condenser

$$\frac{\dot{Q}_C}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

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$$\hat{H}_{out} \rightarrow \text{Saturated liquid water at } 40^\circ\text{C} \rightarrow 390 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{in} \rightarrow \text{Actual stream leaving turbine} \rightarrow 1379 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}_C}{\dot{m}} = 390 - 1379 \frac{\text{kJ}}{\text{kg}} = -889 \frac{\text{kJ}}{\text{kg}}$$

Next evaluate the work in pump. Obtain liquid volume from Appendix C.

$$\frac{\dot{W}_{s,pump}}{\dot{m}} \approx \hat{V}(P_{out} - P_{in})$$

$$\begin{aligned} \frac{\dot{W}_{s,pump}}{\dot{m}} \approx & \left(25.0 \frac{\text{cm}^3}{\text{mol}}\right) \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 \left(\frac{1 \text{ mol}}{17 \text{ g}}\right) \left(\frac{1000 \text{ g}}{\text{kg}}\right) (62 \\ & - 15 \text{ bar}) \left(\frac{10^5 \text{ Pa}}{1 \text{ bar}}\right) \left(\frac{1 \text{ J}}{\text{Nm}}\right) \left(\frac{1 \frac{\text{N}}{\text{m}^2}}{\text{Pa}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) = 6.9 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Energy balance around whole engine:

$$0 = \frac{\dot{Q}_C}{\dot{m}} + \frac{\dot{Q}_H}{\dot{m}} + \frac{\dot{W}_{s,pump}}{\dot{m}} + \frac{\dot{W}_{s,turbine}}{\dot{m}}$$

$$\frac{\dot{Q}_H}{\dot{m}} = 1003 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{H.E.}} = \frac{|W_{\text{net}}|}{|Q_{\text{added}}|} = \frac{\left| \left(-121 \frac{\text{kJ}}{\text{kg}}\right) + \left(6.9 \frac{\text{kJ}}{\text{kg}}\right) \right|}{\left(1003 \frac{\text{kJ}}{\text{kg}}\right)} = \mathbf{0.114}$$

5-13) A steady-state liquefaction process generates 100 lb_m/min of saturated liquid methane at $P=10$ psia. Fresh methane enters the process at $T=75^\circ\text{F}$ and $P=10$ psia, and is mixed with the recycled methane to form the “methane feed” stream. This enters a heat exchanger (HX1) that cools the methane feed to $T=-100^\circ\text{F}$ and $P=10$ psia. The process has two compressors (both $\eta=0.75$), the first of which compresses the methane to $P=100$ psia, and the second compresses the methane to $P=1000$ psia. Each compressor is followed by a heat exchanger (HX2 and HX3) that cools the methane to $T=-100^\circ\text{F}$ without changing the pressure. Next, the supercritical methane enters a counter-current heat exchanger (HX4) in which the methane vapor from the flash chamber is used as the coolant. The coolant leaves HX4 at $T=-110^\circ\text{F}$, and is recycled into the “methane feed” stream. The supercritical methane leaves HX4 and enters a flash chamber, where its pressure is reduced to $P=10$ psia.

- Determine the flow rate of the supercritical methane entering the flash chamber.
- Determine W for each of the two compressors.
- Determine Q for each of the four heat exchangers.

Solution:

A) Begin by defining a system that contains both the flash chamber and HX-4, with “in” being the supercritical stream entering HX4, “liq” being the liquid product and “vap” being the recycled vapor.

$$\begin{aligned}\dot{m}_{in} &= \dot{m}_{liq} + \dot{m}_{vap} \\ \dot{m}_{in}\hat{H}_{in} &= \dot{m}_{liq}\hat{H}_{liq} + \dot{m}_{vap}\hat{H}_{vap}\end{aligned}$$

Temperatures and pressures of all three streams are known so all values of specific enthalpy are obtained from P-H diagram for methane (Figure 7-1).

$$\begin{aligned}\dot{m}_{in} &= 100 \frac{\text{lb}_m}{\text{min}} + \dot{m}_{vap} \\ \dot{m}_{in} \left(290 \frac{\text{BTU}}{\text{lb}_m} \right) &= \left(100 \frac{\text{lb}_m}{\text{min}} \right) \left(118 \frac{\text{BTU}}{\text{lb}_m} \right) + \dot{m}_{vap} \left(420 \frac{\text{BTU}}{\text{lb}_m} \right)\end{aligned}$$

These two equations produce:

$$\dot{m}_{in} = 232 \frac{\text{lb}_m}{\text{min}}$$

And also reveals that the vapor flow rate is 132 lb_m/min.

B) The flow rate through each compressor is 232 lb_m/min; the same as the flow rate of supercritical methane entering HX4, since the compressors and HX1, 2 and 3 are simple “one stream in, one stream out” units. For compressor 1, the entering stream has $T=-100^\circ\text{F}$ and $P=10$ psia, so:

$$\hat{H}_{in,c1} = 425 \frac{\text{BTU}}{\text{lb}_m}$$

Assuming the compressor is reversible, $\hat{S}_{in} = \hat{S}_{out}$. Applying this for an outlet pressure of 100 psia produces:

$$\hat{H}_{out,rev,c1} = 560 \frac{\text{BTU}}{\text{lb}_m}$$

$$\frac{\dot{W}_{s,rev,c1}}{\dot{M}} = 560 - 425 \frac{\text{BTU}}{\text{lb}_m} = 135 \frac{\text{BTU}}{\text{lb}_m}$$

For compressor 2, the entering stream has $T = -100^\circ\text{F}$ and $P = 100$ psia, so:

$$\hat{H}_{in,c2} = 415 \frac{\text{BTU}}{\text{lb}_m}$$

Assuming the compressor is reversible, $\hat{S}_{in} = \hat{S}_{out}$. Applying this for an outlet pressure of 1000 psia produces:

$$\hat{H}_{out,rev,c2} = 555 \frac{\text{BTU}}{\text{lb}_m}$$

$$\frac{\dot{W}_{s,rev,c2}}{\dot{M}} = 555 - 415 \frac{\text{BTU}}{\text{lb}_m} = 140 \frac{\text{BTU}}{\text{lb}_m}$$

Applying the efficiency of 75%:

$$\frac{\dot{W}_{s,act,c1}}{\dot{M}} = \frac{135 \frac{\text{BTU}}{\text{lb}_m}}{0.75} = \mathbf{180 \frac{\text{BTU}}{\text{lb}_m}}$$

$$\frac{\dot{W}_{s,act,c2}}{\dot{M}} = \frac{140 \frac{\text{BTU}}{\text{lb}_m}}{0.75} = \mathbf{187 \frac{\text{BTU}}{\text{lb}_m}}$$

Multiplying by the flowrate 232 lb_m/min gives actual work of **41,800 BTU/min** and **43,400 BTU/min** in the two compressors.

Closing the energy balance on the actual compressors then indicates that the ACTUAL outlet streams from the compressors have:

$$\hat{H}_{out,act,c1} = 425 + 180 = 605 \frac{\text{BTU}}{\text{lb}_m}$$

$$\hat{H}_{out,act,c2} = 415 + 187 = 602 \frac{\text{BTU}}{\text{lb}_m}$$

These are the streams entering HX2 and HX3.

C) For HX1, HX2 and HX3, the exact condition of the streams exiting the heat exchanger are given- each have $T = -100^\circ\text{F}$ and pressures of 10, 100 and 1000 psia. Thus the specific enthalpy of each can be obtained from the figure. The specific enthalpy of the inlet streams for HX2 and HX3 were determined in part B. HX1 has two inlet streams- the fresh feed which is at 75°F and 10 psia, and the recycle stream which has specific enthalpy of $420 \text{ BTU}/\text{lb}_m$ as established in part A.

The energy balance for HX2 and HX3 is:

$$\frac{\dot{Q}}{\dot{M}} = \hat{H}_{out} - \hat{H}_{in}$$

And the flow rate through each is again $232 \text{ lb}_m/\text{min}$. For HX1 the energy balance is:

$$\dot{Q} = \dot{M}_{fresh}\hat{H}_{fresh} + \dot{M}_{vap}\hat{H}_{vap} - \dot{M}_{out}\hat{H}_{out}$$

Exchanger	\hat{H}_{in} (BTU/lb _m)	\hat{H}_{out} (BTU/lb _m)	\dot{Q} (BTU/min)
HX1	513 (100 lb _m /min) 420 (132 lb _m /min)	425	-8140
HX2	605	415	-44,100
HX3	602	290	-72,400

In the fourth heat exchanger, if the system includes both sides of the heat exchanger (as it did in part A), $Q=0$; the heat transfer is inside the system boundary. However, we can determine how much heat is exchanged by looking at the recycled vapor only. This stream enters HX4 as saturated vapor at 10 psia and exits with a specific enthalpy of $420 \text{ BTU}/\text{lb}_m$ as established in part A.

$$\frac{\dot{Q}}{\dot{M}} = 420 \frac{\text{BTU}}{\text{lb}_m} - 341 \frac{\text{BTU}}{\text{lb}_m} = 79 \frac{\text{BTU}}{\text{lb}_m}$$

$$\dot{Q} = \left(79 \frac{\text{BTU}}{\text{lb}_m}\right) \dot{M} = \left(79 \frac{\text{BTU}}{\text{lb}_m}\right) \left(132 \frac{\text{lb}_m}{\text{min}}\right) = \mathbf{10,400 \frac{\text{BTU}}{\text{min}}}$$

And this same heat is removed (negative Q) from the supercritical methane in HX4.

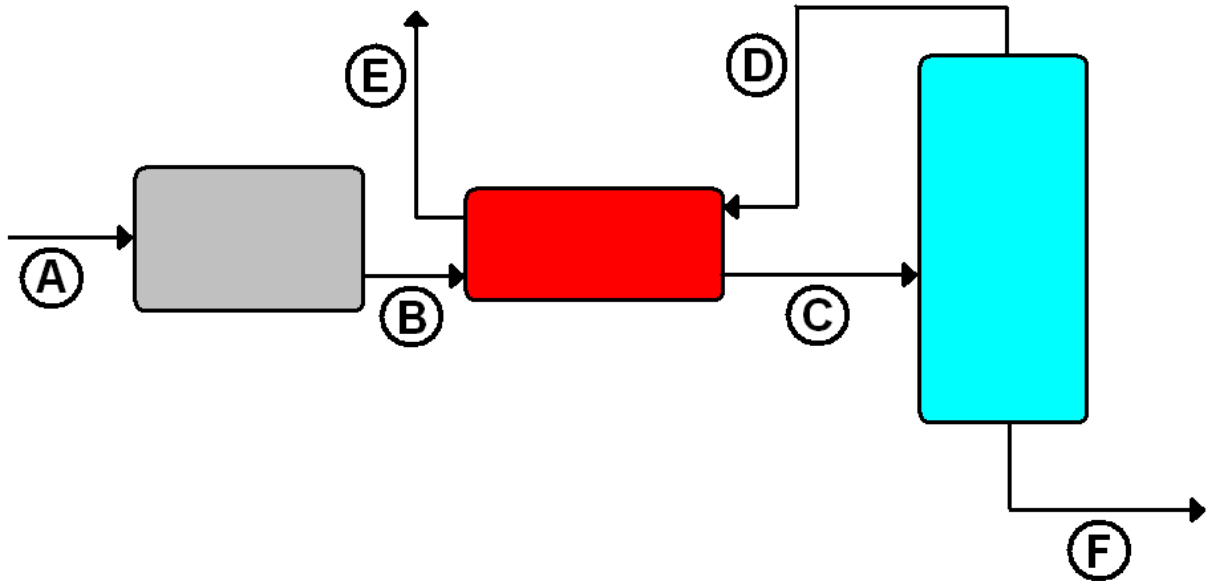
5-14) The boiling point of a compound at $P=0.1$ MPa is 150 K. The Linde liquefaction process will be used to produce saturated liquid at $P=0.1$ MPa, which has a specific enthalpy of 20 kJ/kg. The table below contains some physical properties of the compound. The steady-state process works as follows:

1. 50 kg/minute enters the process at $T=250$ K and $P=0.1$ MPa.
 2. The feed enters a series of compressors and heat exchangers, and it leaves the last heat exchanger at $T=200$ K and $P=10$ MPa. The TOTAL work added by the compressors is 300 kilojoules per kilogram of feed.
 3. The stream leaving step 2 is cooled to $T=175$ K in a heat exchanger.
 4. The stream leaving step 3 enters a flash chamber where it expands to 0.1 MPa and some of it condenses.
 5. The vapor from the flash chamber is used as the coolant for the heat exchanger in step 3.
 6. The vapor stream (at $P=0.1$ MPa) from step 5 is NOT recycled; it exits the process as a by-product.
- A) Find the flow rate of liquid product leaving the flash chamber.
 B) Find the specific enthalpy (kJ/kg) of the vapor by-product described in step 6.
 C) Find the total heat removed by the heat exchangers during step 2, in kJ/min.
 D) What is the heat capacity of the compound, in kJ/kg-K, at ideal gas conditions?

Specific enthalpy in vapor or supercritical phase at various temperatures and pressures, in kJ/kg.

	150 K	175 K	200 K	225 K	250 K	275 K
0.1 MPa	200	215	230	245	260	275
1 MPa	192	202	215	229	241	258
10 MPa	184	190	200	215	228	241

Solution:



A) Set up a mass balance around the entire system

$$M_{\text{final}} - M_{\text{initial}} = \sum_{k=1}^{k=K} \dot{m}_{k,\text{in}} - \sum_{k=1}^{k=K} \dot{m}_{k,\text{out}}$$

Cancelling terms

$$\dot{m}_A = \dot{m}_F + \dot{m}_E$$

Set an energy balance around the flash chamber

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \sum_{j=1}^{j=J} \left\{ \dot{m}_{j,\text{in}} \left(\hat{H}_j + \frac{v_j^2}{2} + gh_j \right) \right\} - \sum_{k=1}^{k=K} \left\{ \dot{m}_{k,\text{out}} \left(\hat{H}_k + \frac{v_k^2}{2} + gh_k \right) \right\} + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_C \hat{H}_C - \dot{m}_F \hat{H}_F - \dot{m}_D \hat{H}_D$$

Find the enthalpies using the table,

$$\hat{H}_C \rightarrow \text{Substance at 175 K and 10 MPa bar} \rightarrow 190 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_F \rightarrow \text{Saturated Liquid substance at 150K} \rightarrow 20 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_D \rightarrow \text{Saturated Vapor at 150 K} \rightarrow 200 \frac{\text{kJ}}{\text{kg}}$$

Plugging in terms to the energy balance around the flash chamber and the mass balance around the entire system,

$$0 = \dot{m}_C 190 \frac{\text{kJ}}{\text{kg}} - \dot{m}_F 20 \frac{\text{kJ}}{\text{kg}} - \dot{m}_D 200 \frac{\text{kJ}}{\text{kg}}$$

$$50 \frac{\text{kg}}{\text{min}} = \dot{m}_F + \dot{m}_E$$

We know that the mass flow rate in and out of the series of heat exchangers and compressors described by Step 2 is the same. We also know that the two streams running in and out of the heat exchanger do not mix. Therefore, $\dot{m}_A = \dot{m}_C = 50 \frac{\text{kg}}{\text{min}}$. Using this same logic, we know $\dot{m}_E = \dot{m}_D$. Applying these facts,

$$50 \frac{\text{kg}}{\text{min}} = \dot{m}_F + \dot{m}_D$$

and

$$0 = \dot{m}_C 190 \frac{\text{kJ}}{\text{kg}} - \dot{m}_F 20 \frac{\text{kJ}}{\text{kg}} + \dot{m}_D 200 \frac{\text{kJ}}{\text{kg}} \rightarrow 0 = \left(50 \frac{\text{kg}}{\text{min}}\right) \left(190 \frac{\text{kJ}}{\text{kg}}\right) - \dot{m}_F 20 \frac{\text{kJ}}{\text{kg}} + \dot{m}_D 200 \frac{\text{kJ}}{\text{kg}}$$

Solving the above system of equations, (two equations, two unknowns), we find

$$\dot{m}_D = 47.22 \frac{\text{kg}}{\text{min}}$$

$$\dot{m}_F = 2.78 \frac{\text{kg}}{\text{min}}$$

B) Set an energy balance around the counter current heat exchanger

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \sum_{j=1}^{j=J} \left\{ \dot{m}_{j,\text{in}} \left(\hat{H}_j + \frac{v_j^2}{2} + gh_j \right) \right\} - \sum_{k=1}^{k=K} \left\{ \dot{m}_{k,\text{out}} \left(\hat{H}_k + \frac{v_k^2}{2} + gh_k \right) \right\} + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = (\dot{m}_D \hat{H}_D + \dot{m}_B \hat{H}_B) - (\dot{m}_C \hat{H}_C + \dot{m}_E \hat{H}_E)$$

We know that the two streams running in and out of the heat exchanger do not mix. Therefore $\dot{m}_E = \dot{m}_D$ and $\dot{m}_B = \dot{m}_C$. Rewriting the simplified energy balance,

$$0 = (\dot{m}_D \hat{H}_D + \dot{m}_C \hat{H}_B) - (\dot{m}_C \hat{H}_C + \dot{m}_D \hat{H}_E)$$

Find the enthalpies using the table,

$$\hat{H}_C \rightarrow \text{Substance at 175 K and 10 MPa} \rightarrow 190 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_B \rightarrow \text{Substance at 10 MPa and 200 K} \rightarrow 200 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_D \rightarrow \text{Saturated Vapor at 150 K} \rightarrow 200 \frac{\text{kJ}}{\text{kg}}$$

We found in Part A) that $\dot{m}_D = 47.22 \frac{\text{kg}}{\text{min}}$. We also established that $\dot{m}_A = \dot{m}_C = 50 \frac{\text{kg}}{\text{min}}$.

Therefore,

$$0 = \left(47.22 \frac{\text{kg}}{\text{min}}\right) \left(200 \frac{\text{kJ}}{\text{kg}}\right) + \left(50 \frac{\text{kg}}{\text{min}}\right) \left(200 \frac{\text{kJ}}{\text{kg}}\right) - \left(50 \frac{\text{kg}}{\text{min}}\right) \left(190 \frac{\text{kJ}}{\text{kg}}\right) + \left(47.22 \frac{\text{kg}}{\text{min}}\right) \hat{H}_E$$

$$\hat{H}_E = \mathbf{210.58 \frac{kJ}{kg}}$$

C) Set up an energy balance around the compressors and heat exchangers described in Step 2.

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_A(\hat{H}_A) - \dot{m}_B(\hat{H}_B) + \dot{W}_S + \dot{Q}$$

Find Enthalpies

$$\hat{H}_B \rightarrow \text{Substance at 10 MPa and 150 K} \rightarrow 200 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_A \rightarrow \text{Substance at 0.1 MPa and 250 K} \rightarrow 260 \frac{\text{kJ}}{\text{kg}}$$

$$0 = \left(50 \frac{\text{kg}}{\text{min}}\right) \left(260 \frac{\text{kJ}}{\text{kg}}\right) - \left(50 \frac{\text{kg}}{\text{min}}\right) \left(200 \frac{\text{kJ}}{\text{kg}}\right) + 300 \frac{\text{kJ}}{\text{min}} + \dot{Q}$$

$$\dot{Q} = \mathbf{-3300 \frac{kJ}{min}}$$

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D) An ideal gas follows the following equation

$$d\hat{H} = C_p^* dT$$

Chose the values from the table that are most likely to act ideally. This is the lowest pressure and the highest temperature.

$$(\hat{H}_2 - \hat{H}_1) = C_p^*(T_2 - T_1)$$

$$\left(275 \frac{\text{kJ}}{\text{kg}} - 260 \frac{\text{kJ}}{\text{kg}}\right) = C_p^*(275\text{K} - 250\text{K})$$

$$C_p^* = 0.6 \frac{\text{kJ}}{\text{kg K}}$$

5-15) A steady-state heat engine operates on the Rankine cycle.

- The steam entering the turbine is 1 kg/s of steam at $P=3.5$ MPa and $T=350^\circ\text{C}$.
- The ACTUAL stream exiting the turbine is a mixture of 95% vapor and 5% liquid at $P=50$ kPa.
- The stream entering the pump is saturated liquid at $P=50$ kPa.

A) Determine the efficiency of the turbine.

B) Estimate the work required by the pump.

C) Estimate the overall efficiency of the heat engine.

D) What is the efficiency of a Carnot heat engine that operates between a high temperature of 350°C and a low temperature that is the same as the temperature of the condenser in this problem?

ASSUMPTIONS:

There are no significant pressure changes in the boiler or condenser.

There is no significant volume change for the liquid in the pump.

The condenser produces saturated liquid, not compressed liquid.

A)

$$\eta_{\text{turbine}} = \frac{W_{s,\text{actual}}}{W_{s,\text{reversible}}}$$

We must find the work done by the turbine if it was reversible. Set up an entropy balance around the reversible turbine

$$(M_2 \hat{S}_2) - (M_1 \hat{S}_1) = m_{\text{in}} \hat{S}_{\text{in}} - m_{\text{out}} \hat{S}_{\text{out}} + \frac{Q}{T} + S_{\text{gen}}$$

Cancelling terms

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

Find \hat{S}_{out}

$$\hat{S}_{\text{in}} \rightarrow \text{Water at 35 bar and } 350^\circ\text{C} \rightarrow 6.660 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}} = 6.660 \frac{\text{kJ}}{\text{kg K}}$$

Set an energy balance around the reversible turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find the Enthalpies

$$\hat{H}_{\text{in}} \rightarrow \text{Water at 35 bar and } 350^\circ\text{C} \rightarrow 3104.8 \frac{\text{kJ}}{\text{kg}}$$

$$\text{At 0.5 bar } \hat{S}^L = 1.0912 \frac{\text{kJ}}{\text{kg K}} \text{ and } \hat{S}^V = 7.5930 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{S} = 6.660 \frac{\text{kJ}}{\text{kg K}} = q_{\text{rev}} \hat{S}^V + (1 - q_{\text{rev}}) \hat{S}^L \rightarrow q_{\text{rev}} = 0.857$$

$$\hat{H}_{\text{out}} \rightarrow \text{VLE mixture at 0.5 bar and } q=0.857 \rightarrow 2314.5 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 2314.5 \frac{\text{kJ}}{\text{kg}} - 3104.8 \frac{\text{kJ}}{\text{kg}} = -790.3 \frac{\text{kJ}}{\text{kg}}$$

Find the work done by the actual turbine

Set an energy balance around the reversible turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

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Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find the Enthalpies

$$\hat{H}_{\text{in}} \rightarrow \text{Water at 35 bar and } 350^\circ\text{C} \rightarrow 3104.8 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{out}} \rightarrow \text{Water at } q=0.95 \text{ and } 0.5 \text{ bar} \rightarrow 2530.0 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 2530.0 \frac{\text{kJ}}{\text{kg}} - 3104.8 \frac{\text{kJ}}{\text{kg}} = -574.8 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{turbine}} = \frac{-574.8 \frac{\text{kJ}}{\text{kg}}}{-790.3 \frac{\text{kJ}}{\text{kg}}} = 72.73\%$$

B) Using the equation obtained from Example 3-8, we will find the shaft work of the pump. We will model the liquid as constant specific volume.

$$\dot{W}_S = \int_{P=P_{\text{in}}}^{P=P_{\text{out}}} \dot{V} dP$$

$$\dot{W}_S = \dot{V} \int_{P=P_{\text{in}}}^{P=P_{\text{out}}} dP = \dot{m} \hat{V} (P_{\text{out}} - P_{\text{in}})$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{V} (P_{\text{out}} - P_{\text{in}})$$

Find the Pressure in and out

The stream leaving the condenser is saturated water. The stream entering the condenser is a mixture. We assume that the condenser does not significantly alter the pressure when the entering stream is a mixture. Therefore $P_{\text{in}} = 0.5 \text{ bar}$

Pressure out can be found by similarly assuming that the boiler does not change the pressure of the steam. Therefore, $P_{\text{out}} = 35 \text{ bar}$

Find specific Volume

$$\hat{V} \rightarrow \text{Saturated liquid water at 0.5 bar} \rightarrow 1.03 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = \left(1.03 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \right) (35\text{bar} - 0.5\text{bar}) = \mathbf{3.55 \frac{kJ}{kg}}$$

C) Set an energy balance around the boiler

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{Q}$$

$$\frac{\dot{Q}}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find the enthalpies

The saturated liquid water leaves the condenser at 0.5 bar (see previous part) and is therefore at $\hat{H} = 340.5 \frac{\text{kJ}}{\text{kg}}$. An energy balance around the pump can be used to demonstrate that the enthalpy leaving the pump is $\hat{H} = 344 \frac{\text{kJ}}{\text{kg}}$. So for the boiler:

$$\hat{H}_{\text{out}} \rightarrow \text{Water at 35 bar and } 350^\circ\text{C} \rightarrow 3104.8 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{in}} \rightarrow \text{Water exiting pump} \rightarrow 344 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}}{\dot{m}} = 3104.8 \frac{\text{kJ}}{\text{kg}} - 344 \frac{\text{kJ}}{\text{kg}} = 2760.8 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{H.E.}} = \frac{-W_{\text{turbine,rev}}}{Q_{\text{added}}}$$

$$\eta_{\text{H.E.}} = \frac{574.8 \frac{\text{kJ}}{\text{kg}}}{2760.8 \frac{\text{kJ}}{\text{kg}}} = \mathbf{0.208}$$

D)

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

$$\eta_{\text{Carnot}} = 1 - \frac{81.32 + 273}{350 + 273} = \mathbf{0.569}$$

5-16) A refrigeration process operates using the vapor-compression cycle, using a proprietary refrigerant that the inventor claims is better than R-422A. At low pressures, the refrigerant can be assumed to act as an ideal gas with constant $C_v=14R$ and reportedly has a molecular weight of 400 g/mol. The steady-state refrigeration cycle reportedly works as follows:

- A boiler produces saturated vapor at $P=0.02$ MPa and $T=5^\circ\text{C}$, which has $\underline{H} = 50$ kJ/mol.
- The vapor is compressed to $P=0.06$ MPa and $T=50^\circ\text{C}$.
- The vapor is then condensed to saturated liquid at $P=0.06$ MPa, which has $\underline{H} = 20$ kJ/mol.
- The liquid undergoes an isenthalpic expansion to $P=0.02$ MPa, and enters the boiler.

- A) What is the efficiency of the compressor?
- B) If we wished to scale the process described above so that it provides 3000 kJ/min of cooling, what is the required flow rate of refrigerant in kg/min?
- C) What is the coefficient of performance of the refrigeration cycle?
- D) Design a refrigeration cycle using R-422A as the refrigerant, in which the rate of cooling is 3000 kJ/min. In designing this process, you should make all specifications and design decisions in such a way as to allow a meaningful comparison between R-422A and the proprietary refrigerant.
- E) Based on the outcomes of questions A-D, comment on the inventor's claim that the proprietary refrigerant is better than R-422A .

A) Set an entropy balance around the reversible compressor

$$\frac{d(N\hat{S})}{dt} = \dot{n}_{\text{in}}\underline{S}_{\text{in}} - \dot{n}_{\text{out}}\underline{S}_{\text{out}} + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Cancelling terms

$$0 = \underline{S}_{\text{in}} - \underline{S}_{\text{out}}$$

Solve for the entropy difference across the compressor modeling the gas as ideal

$$d\underline{S} = C_p^* \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right)$$

$$d\underline{S} = (C_V^* + R) \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_1}{P_2}\right)$$

$$d\underline{S} = (15R) \ln\left(\frac{T_{\text{out}}}{T_{\text{in}}}\right) + R \ln\left(\frac{P_{\text{in}}}{P_{\text{out}}}\right)$$

$$0 = \left(15 \left(8.314 \frac{\text{J}}{\text{mol K}}\right)\right) \ln\left(\frac{T_{\text{out}}}{278\text{K}}\right) + \left(8.314 \frac{\text{J}}{\text{mol K}}\right) \ln\left(\frac{0.02\text{MPa}}{0.06\text{MPa}}\right)$$

$$T_{\text{out}} = 299.1\text{K}$$

To find the enthalpy of the effluent of the reversible compressor (modeling as ideal),

$$d\underline{H} = C_p^* dT$$

$$\underline{H}_{\text{out,rev}} - 50000 \frac{\text{J}}{\text{mol}} = 15 \left(8.314 \frac{\text{J}}{\text{mol K}}\right) (299.1\text{K} - 278\text{K})$$

$$\underline{H}_{\text{out,rev}} = 52.63 \frac{\text{kJ}}{\text{mol}}$$

Set an energy balance around the reversible compressor

$$\begin{aligned} \frac{d}{dt} \left\{ N \left(\underline{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{n}_{\text{in}} \left(\underline{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{n}_{\text{out}} \left(\underline{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{n}_{\text{in}}(\underline{H}_{\text{in}}) - \dot{n}_{\text{out}}(\underline{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_{S,\text{rev}}}{\dot{n}} = \underline{H}_{\text{out,rev}} - \underline{H}_{\text{in}}$$

$$\frac{\dot{W}_{S,\text{rev}}}{\dot{n}} = 52.63 \frac{\text{kJ}}{\text{mol}} - 50 \frac{\text{kJ}}{\text{mol}} = 2.63 \frac{\text{kJ}}{\text{mol}}$$

We know that this gas acts ideal, so we may find the change in enthalpy using the formula

$$d\underline{H} = C_p^* dT$$

This will give us the ACTUAL enthalpy of the exiting stream

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$$dH = (C_V + R)dT$$

$$\hat{H}_{\text{out}} - \hat{H}_{\text{in}} = (14R + R)(T_{\text{out}} - T_{\text{in}})$$

$$\underline{H}_{\text{out}} - 50000 \frac{\text{J}}{\text{mol}} = 15 \left(8.314 \frac{\text{J}}{\text{mol K}} \right) (50^\circ\text{C} - 5^\circ\text{C})$$

$$\underline{H}_{\text{out}} = 55.61 \frac{\text{kJ}}{\text{mol}}$$

Set up an energy balance around the actual compressor

$$\begin{aligned} \frac{d}{dt} \left\{ N \left(\underline{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{n}_{\text{in}} \left(\underline{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{n}_{\text{out}} \left(\underline{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} \\ + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{n}_{\text{in}}(\underline{H}_{\text{in}}) - \dot{n}_{\text{out}}(\underline{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{n}} = \underline{H}_{\text{out}} - \underline{H}_{\text{in}}$$

$$\frac{\dot{W}_S}{\dot{n}} = 55.61 \frac{\text{kJ}}{\text{mol}} - 50 \frac{\text{kJ}}{\text{mol}} = 5.61 \frac{\text{kJ}}{\text{mol}}$$

$$\eta_{\text{compressor}} = \frac{W_{s,\text{reversible}}}{W_{s,\text{actual}}}$$

$$\eta_{\text{compressor}} = \frac{2.63 \frac{\text{kJ}}{\text{mol}}}{5.61 \frac{\text{kJ}}{\text{mol}}} = \mathbf{0.469}$$

B) Set an energy balance around the boiler

$$\begin{aligned} \frac{d}{dt} \left\{ N \left(\underline{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{n}_{\text{in}} \left(\underline{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{n}_{\text{out}} \left(\underline{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{n}_{\text{in}}(\underline{H}_{\text{in}}) - \dot{n}_{\text{out}}(\underline{H}_{\text{out}}) + \dot{Q}$$

$$\frac{\dot{Q}}{\dot{m}} = \underline{H}_{\text{out}} - \underline{H}_{\text{in}}$$

$$\underline{H}_{\text{out}} \rightarrow \text{Given} \rightarrow 50 \frac{\text{kJ}}{\text{mol}}$$

$$\underline{H}_{\text{in}} \rightarrow \text{Given} \rightarrow 20 \frac{\text{kJ}}{\text{mol}}$$

$$\frac{\dot{Q}}{\dot{m}} = 50 \frac{\text{kJ}}{\text{mol}} - 20 \frac{\text{kJ}}{\text{mol}} = 30 \frac{\text{kJ}}{\text{mol}}$$

$$\frac{\left(3000 \frac{\text{kJ}}{\text{min}}\right)}{\left(30 \frac{\text{kJ}}{\text{mol}}\right)} \left(400 \frac{\text{g}}{\text{mol}}\right) \left(\frac{1\text{kg}}{1000\text{g}}\right) = 40 \frac{\text{kg}}{\text{min}}$$

C)

$$\text{C. O. P.} = \frac{\dot{Q}_c}{\dot{W}_s}$$

$$\dot{Q}_c \rightarrow \text{Given as} \rightarrow 3000 \frac{\text{kJ}}{\text{min}}$$

$$\dot{W}_s \rightarrow \text{Found previously} \rightarrow 5.61 \frac{\text{kJ}}{\text{mol}} \left(40 \frac{\text{kg}}{\text{min}}\right) \left(\frac{1000\text{g}}{1\text{kg}}\right) \left(\frac{\text{mol}}{400\text{g}}\right) = 561 \frac{\text{kJ}}{\text{min}}$$

$$\text{C. O. P.} = \frac{3000 \frac{\text{kJ}}{\text{min}}}{561 \frac{\text{kJ}}{\text{min}}} = 5.35$$

D) For valid comparison:

Assume the condenser and boiler temperatures are the same

Assume the compressor efficiency is the same

Set up an entropy balance on the reversible compressor

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

$$\hat{S}_{\text{in}} \rightarrow \text{Saturated vapor R-422A at } 5^\circ\text{C} = 41^\circ\text{F} \rightarrow 0.225 \frac{\text{BTU}}{\text{lb}_m^\circ\text{F}}$$

Condenser operates at $50^\circ\text{C} = 122^\circ\text{F}$ so the compressor outlet pressure must equal vapor pressure at this temperature, which according to the figure is slightly below 200 psia.

Set an energy balance around the reversible compressor

$$\frac{\dot{W}_{s,\text{rev}}}{\dot{m}} = \hat{H}_{\text{out,rev}} - \hat{H}_{\text{in}}$$

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Find enthalpies

$$\hat{H}_{in} \rightarrow \text{Saturated vapor R-422A at } 41^\circ\text{F} \rightarrow 108 \frac{\text{BTU}}{\text{lb}_m}$$

$\hat{H}_{out,rev} \rightarrow$ To find the enthalpy of the outlet, follow the line of constant entropy from inlet condition to $P \sim 195$ psia $\rightarrow 120 \frac{\text{BTU}}{\text{lb}_m}$

$$\frac{\dot{W}_{S,rev}}{\dot{m}} = 120 \frac{\text{BTU}}{\text{lb}_m} - 108 \frac{\text{BTU}}{\text{lb}_m} = 12 \frac{\text{BTU}}{\text{lb}_m}$$

$$\eta_{\text{compressor}} = \frac{W_{s,\text{reversible}}}{W_{s,\text{actual}}}$$

$$W_{s,\text{actual}} = \frac{W_{s,\text{reversible}}}{\eta_{\text{compressor}}} = \frac{12 \frac{\text{BTU}}{\text{lb}_m}}{0.468} = 25.6 \frac{\text{BTU}}{\text{lb}_m}$$

Perform an energy balance on the boiler

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{Q}$$

$$\hat{H}_{in} = \hat{H}_{out} - \frac{\dot{Q}}{\dot{m}}$$

Find \hat{H}_{out} using the P.E diagram

$$\hat{H}_{out} \rightarrow \text{R-422A saturated vapor at } 5^\circ\text{C} \rightarrow 108 \frac{\text{BTU}}{\text{lb}_m}$$

$\hat{H}_{in} \rightarrow$ same as saturated liquid at 50°C , because enthalpy is unchanged in the valve \rightarrow

$$52 \frac{\text{BTU}}{\text{lb}_m}$$

$$\text{C. O. P.} = \frac{\dot{Q}_c}{\dot{W}_s}$$

$$\frac{\dot{W}_s}{\dot{m}} = 25.6 \frac{\text{BTU}}{\text{lb}_m}$$

$$\text{C. O. P.} = \frac{52 \frac{\text{BTU}}{\text{lb}_m}}{25.6 \frac{\text{BTU}}{\text{lb}_m}} = 2.03$$

E) Based solely on the C.O.P. of each cycle, the inventor's refrigerant seems far better, at least at these temperatures and with this compressor efficiency. But neither C.O.P. is especially large, because the inventor's system has a compressor with an efficiency below 50%.

5-17) A steady-state Rankine cycle currently in service operates as follows:

- Steam leaves the boiler at $P=8$ bar and $T=250^\circ\text{C}$.
- Steam leaves the turbine as saturated vapor at $P=0.3$ bar
- Water leaves the condenser as saturated liquid at $P=0.3$ bar
- Water leaving the condenser is pumped up to 8 bar and returned to the boiler
- The power output FROM THE TURBINE is 1 MW

Your company has the opportunity to upgrade the turbine to one that is 85% efficient, at a cost of \$1.2 million. This includes all costs associated with the replacement- the new equipment, installation, instrumentation, etc.

If the upgrade is done it will work this way:

- The flow rate, temperature and pressure of the steam entering the turbine will all be unchanged, but the turbine is expected to produce more work.
- The pressure leaving the turbine will still be $P=0.3$ bar, but won't necessarily be saturated vapor, since the turbine is now removing more energy as work.
- The Q_c in the condenser will be adjusted such that the water leaving the turbine is saturated liquid, allowing the pump and boiler to operate EXACTLY the same in the upgraded cycle as they do in the current cycle.
- The current turbine produces 1 MW of power, 24 hours a day, for 350 days per year. The company values any "extra" work produced beyond this at \$20/GJ.

- A) Determine the flow rate at which water/steam circulates through the process.
- B) Determine the efficiency of the turbine in the CURRENT cycle.
- C) Determine the overall efficiency of the CURRENT cycle.
- D) Determine the power produced by the turbine in the UPGRADED cycle.
- E) Determine the overall efficiency of the UPGRADED cycle.
- F) How long will the new turbine have to operate in order to pay for the \$1.2 million cost of the upgrade?

ASSUMPTIONS:

There are no significant pressure changes in the boiler or condenser.

There are no significant change of water density in the pump.

The condenser produces saturated liquid, not compressed liquid.

Solution:

A) Set up an energy balance around the CURRENT turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{in}(\hat{H}_{in}) - \dot{m}_{out}(\hat{H}_{out}) + \dot{W}_S$$

$$\dot{m} = \frac{\dot{W}_S}{\hat{H}_{out} - \hat{H}_{in}}$$

Find enthalpies

$$\hat{H}_{out} \rightarrow \text{Saturated Steam at 0.3 bar} \rightarrow 2624.5 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{in} \rightarrow \text{Steam at 8 bar and } 250^\circ\text{C} \rightarrow 2950.4 \frac{\text{kJ}}{\text{kg}}$$

We know that \dot{W}_S is 1 MW

$$\dot{m} = \frac{\dot{W}_S}{\hat{H}_{out} - \hat{H}_{in}} = \frac{-1\text{MW} \left(\frac{1000 \frac{\text{kJ}}{\text{sec}}}{1\text{MW}} \right)}{2624.5 \frac{\text{kJ}}{\text{kg}} - 2950.4 \frac{\text{kJ}}{\text{kg}}} = 3.07 \frac{\text{kg}}{\text{sec}}$$

B) Set up an entropy balance on the reversible CURRENT turbine

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{in}\hat{S}_{in} - \dot{m}_{out}\hat{S}_{out} + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Cancelling terms

$$0 = \dot{m}_{in}\hat{S}_{in} - \dot{m}_{out}\hat{S}_{out}$$

$$\hat{S}_{in} = \hat{S}_{out}$$

Find entropies

$$\hat{S}_{\text{in}} \rightarrow \text{Steam at 8 bar and } 250^{\circ}\text{C} \rightarrow 7.040 \frac{\text{kJ}}{\text{kg K}}$$

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}} = 7.040 \frac{\text{kJ}}{\text{kg K}}$$

At $P=0.3$ bar, this specific entropy corresponds to $q=0.893$.

Set up an energy balance on the reversible CURRENT turbine

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$$\hat{H}_{\text{out}} \rightarrow \text{Water at 0.3 bar and } q = 0.893 \rightarrow 2375.6 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{in}} \rightarrow \text{Steam at 8 bar and } 250^{\circ}\text{C} \rightarrow 2950.4 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 2375.6 \frac{\text{kJ}}{\text{kg}} - 2950.4 \frac{\text{kJ}}{\text{kg}} = -574.8 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{turbine}} = \frac{W_{\text{s,actual}}}{W_{\text{s,reversible}}}$$

We know that the reversible turbine produces $574.8 \frac{\text{kJ}}{\text{kg}}$ of work, while the actual turbine produces 1MW of work. We must get these values into the same dimensions. We will bring the reversible turbine value to units of power by multiplying by the mass flow rate found previously.

$$\left(-574.8 \frac{\text{kJ}}{\text{kg}}\right) \left(3.07 \frac{\text{kg}}{\text{sec}}\right) \left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right) = -1.76 \text{ MW}$$

$$\eta_{\text{turbine}} = \frac{1 \text{ MW}}{1.76 \text{ MW}} = \mathbf{56.67\%}$$

C) Find work in pump:

$$\begin{aligned}\frac{\dot{W}_S}{\dot{m}} &= \hat{V}(P_{\text{out}} - P_{\text{in}}) \\ &= \left(0.001022 \frac{\text{m}^3}{\text{kg}}\right) (8 - 0.3 \text{ bar}) \left(\frac{10^5 \text{ Pa}}{\text{bar}}\right) \left(\frac{1 \text{ J}}{1 \text{ Nm}}\right) \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) \left(\frac{1 \frac{\text{N}}{\text{m}^2}}{1 \text{ Pa}}\right) \\ &= 0.8 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies

$$\hat{H}_{\text{in}} \rightarrow \text{Saturated liquid water at } 0.3 \text{ bar} \rightarrow 289.3 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S}{\dot{m}} = 0.8 \frac{\text{kJ}}{\text{kg}}$$

Solve energy balance for:

$$\hat{H}_{\text{out}} = 290.1 \frac{\text{kJ}}{\text{kg}}$$

Set up an energy balance around the boiler

$$\frac{\dot{Q}}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find the enthalpies

$$\hat{H}_{\text{out}} \rightarrow \text{Steam at } 8 \text{ bar and } 250^\circ\text{C} \rightarrow 2950.4 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{\text{in}} \rightarrow \text{same as water leaving pump} \rightarrow 290.1 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}}{\dot{m}} = 2950.4 \frac{\text{kJ}}{\text{kg}} - 290.1 \frac{\text{kJ}}{\text{kg}} = 2660.3 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{H.E.}} = \frac{-W_{\text{net}}}{Q_{\text{added}}}$$

Again, we have the issue of comparing two numbers that are not dimensionally compatible. Therefore, we will multiply the pump work by the mass flow rate again.

$$\eta_{\text{H.E.}} = \frac{-\left(\left(0.8 \frac{\text{kJ}}{\text{kg}}\right)\left(3.07 \frac{\text{kg}}{\text{sec}}\right)\left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right) - 1.0\text{MW}\right)}{\left(2660.3 \frac{\text{kJ}}{\text{kg}}\right)\left(3.07 \frac{\text{kg}}{\text{sec}}\right)\left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right)} = \mathbf{0.122}$$

D) The reversible UPGRADED turbine is the same as the reversible turbine examined in part B.

$$\frac{\dot{W}_s}{\dot{m}} = 2375.6 \frac{\text{kJ}}{\text{kg}} - 2950.4 \frac{\text{kJ}}{\text{kg}} = -574.8 \frac{\text{kJ}}{\text{kg}}$$

Find the energy produced by the actual, UPGRADED turbine

$$\eta_{\text{turbine}} = \frac{W_{s,\text{actual}}}{W_{s,\text{reversible}}}$$

$$W_{s,\text{actual}} = \eta_{\text{turbine}} \times W_{s,\text{reversible}}$$

$$W_{s,\text{actual}} = 0.85 \times -574.8 \frac{\text{kJ}}{\text{kg}} = 488.6 \frac{\text{kJ}}{\text{kg}}$$

Convert this work per mass to power using the mass flow rate

$$\left(488.6 \frac{\text{kJ}}{\text{kg}}\right)\left(3.07 \frac{\text{kg}}{\text{sec}}\right)\left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right) = \mathbf{1.50\text{MW}}$$

E) The pump work and the heat added from the boiler are identical to what they were in the original cycle.

$$\eta_{\text{H.E.}} = \frac{-\left(\left(0.8 \frac{\text{kJ}}{\text{kg}}\right)\left(3.07 \frac{\text{kg}}{\text{sec}}\right)\left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right) - 1.5\text{MW}\right)}{\left(2660.3 \frac{\text{kJ}}{\text{kg}}\right)\left(3.07 \frac{\text{kg}}{\text{sec}}\right)\left(\frac{1\text{MW}}{1000 \frac{\text{kJ}}{\text{sec}}}\right)} = \mathbf{0.183}$$

F) Normally, we would compare the total revenue produced to the start up and operating costs. However, in this problem, the operating costs are exactly the same as the original

cycle. The 1 MW of power being produced originally must also be subtracted from the upgraded power produced. Therefore, the only things being considered in this price analysis are the additional power revenue and the start up costs of the new turbine.

Additional Power Produced

$$= \text{Upgraded Power Produced} - \text{Original Power Produced}$$

$$1.5\text{MW} - 1\text{MW} = 0.5\text{MW}$$

Convert additional power generated to a rate of revenue

$$0.5\text{MW} \left(\frac{1 \frac{\text{GJ}}{\text{sec}}}{1000 \text{ MW}} \right) \left(\frac{20\$}{\text{GJ}} \right) = \frac{\$0.01}{\text{sec}}$$

The upgraded process will generate 1 cent per second. Find how long it will take to generate \$1,200,000.

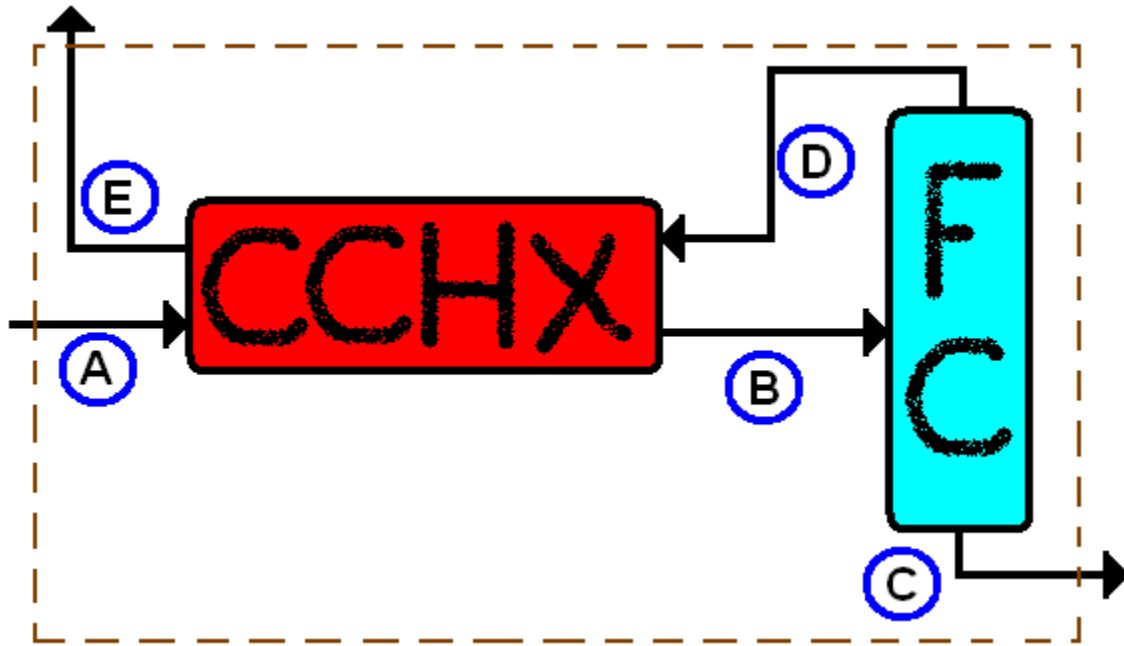
$$\$1200000 \left(\frac{\text{sec}}{\$0.01} \right) \left(\frac{\text{min}}{60\text{sec}} \right) \left(\frac{\text{hour}}{60\text{min}} \right) \left(\frac{\text{day}}{24\text{hour}} \right) \left(\frac{1\text{year}}{350\text{day}} \right) = \mathbf{3.97 \text{ years}}$$

5-18) You are designing a steady state liquefaction process that will manufacture liquid methane. Part A of this problem will focus on two unit operations: the flash separation step itself, and the countercurrent heat exchanger in which the vapor from the flash chamber is used to cool the feed entering the flash chamber. In answering parts B and C, however, consider the entire Linde process, not just the two unit operations for which you performed calculations.

Methane enters the counter-current heat exchanger as supercritical vapor at $P=1000$ psia and $T=-100^\circ\text{F}$. It is cooled and enters the flash chamber, where the pressure is reduced to $P=10$ psia. The flow rate of liquid methane product is $100 \text{ lb}_m/\text{min}$. The methane vapor from the flash is sent to the counter-current heat exchanger. The design parameter that is under your control is the temperature of this methane vapor stream when it leaves the counter-current heat exchanger—it can be -130 , -120 or -110°F .

- A) Find the flow rate of supercritical methane entering the counter current heat exchanger for each of the three possible systems.
- B) Discuss the factors that you would expect to affect the cost OF THE EQUIPMENT ITSELF for the Linde process, and what the results of part A suggest about these costs.
- C) Discuss the factors that you would expect to affect the cost of OPERATION of the process, and what the results of part A suggest about these costs.

Solution:



A) As this is a steady state process, the mass flow rate of A and B are identical, as are the mass flow rates of D and E. They will be used interchangeably during this solution.

FOR STREAM 3 AT -130°F

Set a mass balance around the heat exchanger and flash chamber

$$M_{\text{final}} - M_{\text{initial}} = \sum_{k=1}^{k=K} \dot{m}_{k,\text{in}} - \sum_{k=1}^{k=K} \dot{m}_{k,\text{out}}$$

Cancelling terms

$$\dot{m}_A = \dot{m}_C + \dot{m}_E$$

Set an energy balance around heat exchanger and flash chamber

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \sum_{j=1}^{j=J} \left\{ \dot{m}_{j,\text{in}} \left(\hat{H}_j + \frac{v_j^2}{2} + gh_j \right) \right\} - \sum_{k=1}^{k=K} \left\{ \dot{m}_{k,\text{out}} \left(\hat{H}_k + \frac{v_k^2}{2} + gh_k \right) \right\} + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

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Cancelling terms

$$\dot{m}_A \hat{H}_A = \dot{m}_C \hat{H}_C + \dot{m}_E \hat{H}_E$$

Find enthalpies

$$\hat{H}_A \rightarrow \text{Supercritical methane at 1000psia and } -100^\circ\text{F} \rightarrow 290 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_C \rightarrow \text{Saturated methane at 10psia} \rightarrow 125 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_E \rightarrow \text{Supercritical methane at 10psia and } -130^\circ\text{F} \rightarrow 416 \frac{\text{Btu}}{\text{lb}_m}$$

$$\text{Given } \dot{m}_C = 100 \frac{\text{lb}_m}{\text{min}}$$

Between our two equations, we have two unknowns. Solving for the flow rate of Stream E,

$$\dot{m}_A - \dot{m}_E = 100 \frac{\text{lb}_m}{\text{min}}$$

$$\dot{m}_A \left(290 \frac{\text{Btu}}{\text{lb}_m} \right) - \dot{m}_E \left(416 \frac{\text{Btu}}{\text{lb}_m} \right) = \left(100 \frac{\text{lb}_m}{\text{min}} \right) \left(125 \frac{\text{Btu}}{\text{lb}_m} \right)$$

$$\dot{m}_E = 131 \frac{\text{lb}_m}{\text{min}}$$

FOR STREAM 3 AT -120°F

Set a mass balance around the heat exchanger and flash chamber

$$M_{\text{final}} - M_{\text{initial}} = \sum_{k=1}^{k=K} m_{k,\text{in}} - \sum_{k=1}^{k=K} m_{k,\text{out}}$$

Cancelling terms

$$\dot{m}_A = \dot{m}_C + \dot{m}_E$$

Set an energy balance around heat exchanger and flash chamber

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \sum_{j=1}^{j=J} \left\{ \dot{m}_{j,\text{in}} \left(\hat{H}_j + \frac{v_j^2}{2} + gh_j \right) \right\} - \sum_{k=1}^{k=K} \left\{ \dot{m}_{k,\text{out}} \left(\hat{H}_k + \frac{v_k^2}{2} + gh_k \right) \right\} + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$\dot{m}_A \hat{H}_A = \dot{m}_C \hat{H}_C + \dot{m}_E \hat{H}_E$$

Find enthalpies

$$\hat{H}_A \rightarrow \text{Supercritical methane at 1000psia and } -100^\circ\text{F} \rightarrow 290 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_C \rightarrow \text{Saturated methane at 10psia} \rightarrow 125 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_E \rightarrow \text{Supercritical methane at 10psia and } -120^\circ\text{F} \rightarrow 420 \frac{\text{Btu}}{\text{lb}_m}$$

$$\text{Given } \dot{m}_C = 100 \frac{\text{lb}_m}{\text{min}}$$

Between our two equations, we have two unknowns. Solving for the flow rate of Stream E,

$$\dot{m}_A - \dot{m}_E = 100 \frac{\text{lb}_m}{\text{min}}$$

$$\dot{m}_A \left(290 \frac{\text{Btu}}{\text{lb}_m} \right) - \dot{m}_E \left(420 \frac{\text{Btu}}{\text{lb}_m} \right) = \left(100 \frac{\text{lb}_m}{\text{min}} \right) \left(125 \frac{\text{Btu}}{\text{lb}_m} \right)$$

$$\dot{m}_E = 127 \frac{\text{lb}_m}{\text{min}}$$

FOR STREAM 3 AT -110°F

Set a mass balance around the heat exchanger and flash chamber

$$M_{\text{final}} - M_{\text{initial}} = \sum_{k=1}^{k=K} m_{k,\text{in}} - \sum_{k=1}^{k=K} m_{k,\text{out}}$$

Cancelling terms

$$\dot{m}_A = \dot{m}_C + \dot{m}_E$$

Set an energy balance around heat exchanger and flash chamber

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \sum_{j=1}^{j=J} \left\{ \dot{m}_{j,\text{in}} \left(\hat{H}_j + \frac{v_j^2}{2} + gh_j \right) \right\} - \sum_{k=1}^{k=K} \left\{ \dot{m}_{k,\text{out}} \left(\hat{H}_k + \frac{v_k^2}{2} + gh_k \right) \right\} + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$\dot{m}_A \hat{H}_A = \dot{m}_C \hat{H}_C + \dot{m}_E \hat{H}_E$$

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Find enthalpies

$$\hat{H}_A \rightarrow \text{Supercritical methane at 1000psia and } -100^\circ\text{F} \rightarrow 290 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_C \rightarrow \text{Saturated methane at 10psia} \rightarrow 125 \frac{\text{Btu}}{\text{lb}_m}$$

$$\hat{H}_E \rightarrow \text{Supercritical methane at 10psia and } -110^\circ\text{F} \rightarrow 425 \frac{\text{Btu}}{\text{lb}_m}$$

$$\text{Given } \dot{m}_C = 100 \frac{\text{lb}_m}{\text{min}}$$

Between our two equations, we have two unknowns. Solving for the flow rate of Stream E,

$$\dot{m}_A - \dot{m}_E = 100 \frac{\text{lb}_m}{\text{min}}$$

$$\dot{m}_A \left(290 \frac{\text{Btu}}{\text{lb}_m} \right) - \dot{m}_E \left(425 \frac{\text{Btu}}{\text{lb}_m} \right) = \left(100 \frac{\text{lb}_m}{\text{min}} \right) \left(125 \frac{\text{Btu}}{\text{lb}_m} \right)$$

$$\dot{m}_E = 122 \frac{\text{lb}_m}{\text{min}}$$

B) The most straightforward answer is that the more heat is transferred in the countercurrent heat exchanger, the larger and more expensive the heat exchanger will be. Thus the case in which the vapor stream was -110°F would have the most expensive heat exchanger.

However, an additional point is that the case in which the vapor stream was -110°F had the lowest flow rate of recycle, while all three cases must have the same flow rate of fresh feed. A lower recycle flow rate means a lower total flow rate through all the earlier compressors and heat exchangers in the process, and thus smaller equipment sizes for these pieces of equipment.

C) The main operating costs will be the work added to the compressors and the heat removed in the heat exchangers that precede the counter-current exchanger. While this problem doesn't specify how the heat is being removed in these exchangers, we can assume the coolant isn't "free," since it's well below ambient temperature! Again, a lower recycle flow rate means lower flow rates through these unit operations, which means less work and less cooling is required.

5-19) You are designing a refrigeration cycle, and have the option of using either a compressor with an efficiency of 70%, or a compressor that is more expensive by \$5000, but has an efficiency of 80%. The following specifications are valid regardless of which compressor is used:

- The refrigerant is R-422A.
 - The liquid leaving the condenser is saturated liquid at $T=50^{\circ}\text{C}$.
 - The boiler operates at $T=0^{\circ}\text{C}$.
 - The vapor leaving the boiler is saturated vapor.
 - The cycle must have $Q_C=1000\text{ kJ/min}$ in the condenser.
- A) Determine the flow rate of refrigerant. Is it the same or different in the two cycles?
- B) Determine the compressor work for each of the two possible compressors.
- C) Determine the coefficient of performance for the cycle with each of the two possible compressors.
- D) The compressor runs on electricity, which is available for \$0.10/kWhr. Assuming the refrigeration system runs constantly, how long would the system have to run in order for the higher-efficiency compressor to be cost effective?

Solution:

A) Set an energy balance on the condenser

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{Q}$$

$$\frac{\dot{Q}}{(\hat{H}_{\text{out}} - \hat{H}_{\text{in}})} = \dot{m}$$

Find enthalpies: Both cycles have saturated liquid at 50°C leaving the condenser, which equals 122°F :

$$\text{Saturated liquid R-422A } 50^{\circ}\text{C} \rightarrow 53 \frac{\text{BTU}}{\text{lb}_m}$$

And enthalpy is unchanged by valve so this is both \hat{H}_{in} for the boiler and \hat{H}_{out} for the condenser.

Both cycles have saturated vapor leaving the boiler.

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$$\hat{H}_{\text{out}} \rightarrow \text{Saturated vapor R-422A at } 0^\circ\text{C} = 32^\circ\text{F} \rightarrow 105 \frac{\text{BTU}}{\text{lb}_m}$$

Given that our $\dot{Q} = -1000 \frac{\text{kJ}}{\text{min}}$

$$\dot{m} = \frac{\left(-1000 \frac{\text{kJ}}{\text{min}}\right)}{\left(53 \frac{\text{BTU}}{\text{lb}_m} - 105 \frac{\text{BTU}}{\text{lb}_m}\right)} \left(\frac{1000\text{J}}{1\text{kJ}}\right) \left(\frac{9.486 \times 10^{-4}\text{BTU}}{1\text{J}}\right) = 19.2 \frac{\text{lb}_m}{\text{min}}$$

B) Analysis with 80% Efficiency Compressor

Set an entropy balance on the reversible compressor

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{\text{in}}\hat{S}_{\text{in}} - \dot{m}_{\text{out}}\hat{S}_{\text{out}} + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Cancelling terms

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}}$$

From pressure enthalpy diagram:

$$\hat{S}_{\text{in}} \rightarrow \text{Saturated vapor R-422A at } 32^\circ\text{F} \rightarrow 0.222 \frac{\text{BTU}}{\text{lb}_m^\circ\text{F}}$$

$$\hat{S}_{\text{in}} = \hat{S}_{\text{out}} = 0.222 \frac{\text{BTU}}{\text{lb}_m^\circ\text{F}}$$

Set an energy balance around the reversible compressor

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q} \end{aligned}$$

Cancelling terms

$$0 = \dot{m}_{\text{in}}(\hat{H}_{\text{in}}) - \dot{m}_{\text{out}}(\hat{H}_{\text{out}}) + \dot{W}_S$$

$$\frac{\dot{W}_S}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

Find enthalpies from diagram

$$\hat{H}_{\text{in}} \rightarrow \text{Saturated vapor R-422A at } 32^\circ\text{F} \rightarrow 105 \frac{\text{BTU}}{\text{lb}_m}$$

Following the line of constant entropy from \hat{H}_{in} (recall that $\hat{S}_{in} = \hat{S}_{out}$) until we reach a pressure of about 198 psi, we find

$$\hat{H}_{out} \rightarrow 122 \frac{\text{BTU}}{\text{lb}_m}$$

$$\frac{\dot{W}_s}{\dot{m}} = 122 \frac{\text{BTU}}{\text{lb}_m} - 105 \frac{\text{BTU}}{\text{lb}_m} = 17 \frac{\text{BTU}}{\text{lb}_m}$$

$$\eta_{\text{compressor}} = \frac{W_{s,\text{reversible}}}{W_{s,\text{actual}}}$$

$$W_{s,\text{actual}} = \frac{W_{s,\text{reversible}}}{\eta_{\text{compressor}}}$$

$$\frac{W_{s,\text{actual}}}{\dot{m}} = \frac{17 \frac{\text{BTU}}{\text{lb}_m}}{0.80} = 21.25 \frac{\text{BTU}}{\text{lb}_m}$$

$$W_{s,\text{actual}} = \left(19.2 \frac{\text{lb}_m}{\text{min}}\right) \left(21.25 \frac{\text{BTU}}{\text{lb}_m}\right) = \mathbf{408 \frac{\text{BTU}}{\text{min}}}$$

Analysis with 70% Efficiency Compressor

Reversible work is still 17 BTU/lb_m, but here:

$$W_{s,\text{actual}} = \frac{W_{s,\text{reversible}}}{\eta_{\text{compressor}}}$$

$$\frac{W_{s,\text{actual}}}{\dot{m}} = \frac{17 \frac{\text{BTU}}{\text{lb}_m}}{0.70} = 24.30 \frac{\text{BTU}}{\text{lb}_m}$$

$$W_{s,\text{actual}} = \left(19.2 \frac{\text{lb}_m}{\text{min}}\right) \left(24.3 \frac{\text{BTU}}{\text{lb}_m}\right) = \mathbf{467 \frac{\text{BTU}}{\text{min}}}$$

C)

$$\text{C. O. P.} = \frac{\dot{Q}_c}{\dot{W}_s}$$

For both compressors

$$\dot{Q}_c = \left(1000 \frac{\text{kJ}}{\text{min}}\right) \left(\frac{1000\text{J}}{1\text{kJ}}\right) \left(\frac{9.486 \times 10^{-4}\text{BTU}}{1\text{J}}\right) = 948.6 \frac{\text{BTU}}{\text{min}}$$

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For 80% Efficient Compressor

$$\text{C. O. P.} = \frac{948.6 \frac{\text{BTU}}{\text{min}}}{408 \frac{\text{BTU}}{\text{min}}} = \mathbf{2.33}$$

For 70% Efficient Compressor

$$\text{C. O. P.} = \frac{948.6 \frac{\text{BTU}}{\text{min}}}{467 \frac{\text{BTU}}{\text{min}}} = \mathbf{2.03}$$

D) The difference in prices between the two compressors is \$5000. Since electricity is \$0.1/kWhr, the more expensive compressor would have to save 50,000 kWhr of electricity to be cost effective.

Convert power for both compressors into kW:

For 80%

$$\left(408 \frac{\text{BTU}}{\text{min}}\right) \left(\frac{1 \text{ J}}{9.486 \times 10^{-4} \text{ BTU}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) \left(\frac{1 \text{ W}}{1 \frac{\text{ J}}{\text{ s}}}\right) \left(\frac{1 \text{ kW}}{1000 \text{ W}}\right) = 71.7 \text{ kW}$$

For 70%

$$\left(467 \frac{\text{BTU}}{\text{min}}\right) \left(\frac{1 \text{ J}}{9.486 \times 10^{-4} \text{ BTU}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) \left(\frac{1 \text{ W}}{1 \frac{\text{ J}}{\text{ s}}}\right) \left(\frac{1 \text{ kW}}{1000 \text{ W}}\right) = 82.0 \text{ kW}$$

The two cycles provide the same amount of energy but the 70% efficient compressor uses 10.3 more kW.

$$\left(\frac{50,000 \text{ kWhr}}{10.3 \text{ kW}}\right) \left(\frac{1 \text{ day}}{24 \text{ hr}}\right) = \mathbf{202 \text{ days}}$$

The more efficient compressor “pays for itself” in less than a year.

5-20) You are designing a Rankine heat engine, using water as the working fluid, which must produce 10,000 kJ/min of NET shaft work. Heat will be obtained from a high-temperature reservoir at $T=200^{\circ}\text{C}$ and will be emitted to a low-temperature reservoir at $T=25^{\circ}\text{C}$. The turbine efficiency is 75%, and there are no restrictions on the temperature, pressure or quality of the fluid leaving the turbine. Your job is to design the most cost-effective Rankine engine possible, accounting for the costs of the heat exchangers and the cost of the heat itself.

The boiler can be designed to operate at $T=190$, $T=185$ or $T=180^{\circ}\text{C}$, and the condenser can be designed to operate at $T=35^{\circ}\text{C}$ or $T=40^{\circ}\text{C}$. Costs of these heat exchangers can be determined from the formulas in the table below. Notice that the cost of the heat exchanger goes up as the heat duty goes up, and the cost of the heat exchanger goes down as the ΔT between the heat reservoir and the fluid being boiled or condensed increases.

Exchanger	Operating Temp ($^{\circ}\text{C}$)	Formula
Boiler	180	$C = 10000 + Q$
Boiler	185	$C = 12000 + 1.5Q$
Boiler	190	$C = 15000 + 2Q$
Condenser	35	$C = 10000 + 1.5Q$
Condenser	40	$C = 7000 + Q$

In all formulas C represents the cost in dollars and Q represents the heat addition in kJ/min.

The cost of the heat added to the boiler is \$15/GJ. You can assume the heat removed in the condenser has neither cost nor value. You can also assume that the heat engine will be operating 24 hours per day, 350 days per year.

- For each of the six possible Rankine cycles, determine Q_H , Q_C , the cost of the two heat exchangers, and the yearly cost of the heat.
- Recommend which variation of the Rankine heat engine should be used if it's expected to be in service for 5 years.

For all cases: We want the maximum pressure entering the turbine, so we will assume that the vapor leaving the boiler is saturated vapor in all cases. Similarly we will, as usual in the Rankine cycle, assume the liquid leaving the condenser is saturated.

A) Case 1: Boiler at 180°C – Condenser at 35°C

Set up an entropy balance around the reversible turbine

$$\frac{d(M\hat{S})}{dt} = \dot{m}_{in}\hat{S}_{in} - \dot{m}_{out}\hat{S}_{out} + \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

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Cancelling terms

$$\hat{S}_{in} = \hat{S}_{out}$$

Find entropy (saturated vapor at 180°C)

$$\hat{S}_{in} = \hat{S}_{out} \rightarrow 6.584 \frac{\text{kJ}}{\text{kgK}}$$

At 35°C:

$$\hat{S}^L = 0.5051 \frac{\text{kJ}}{\text{kgK}} \text{ and } \hat{S}^V = 8.3517 \frac{\text{kJ}}{\text{kgK}}$$

$$\hat{S}_{out} = (1 - q_{rev})\hat{S}_L + q_{rev}\hat{S}_V$$

$$q_{rev} = 0.775$$

Set up an energy balance around the reversible turbine

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q} \end{aligned}$$

Cancelling terms

$$\frac{\dot{W}_{S,rev}}{\dot{m}} = \hat{H}_{out} - \hat{H}_{in}$$

Find enthalpies

$$\hat{H}_{out} \rightarrow 35^\circ\text{C and } q=0.775 \rightarrow 2019.8 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{in} \rightarrow \text{Saturated Steam at } 180^\circ\text{C} \rightarrow 2777.2 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_{S,rev}}{\dot{m}} = 2019.8 \frac{\text{kJ}}{\text{kg}} - 2777.2 \frac{\text{kJ}}{\text{kg}} = -757.45 \frac{\text{kJ}}{\text{kg}}$$

$$\eta_{\text{turbine}} = \frac{W_{s,\text{actual}}}{W_{s,\text{reversible}}}$$

$$W_{s,\text{actual}} = W_{s,\text{reversible}} \times \eta_{\text{turbine}}$$

$$W_{s,\text{actual}} = -757.45 \frac{\text{kJ}}{\text{kg}} \times 0.75 = -568.1 \frac{\text{kJ}}{\text{kg}}$$

Set up an energy balance around the actual turbine

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$\hat{H}_{out} = \dot{W}_S + \hat{H}_{in}$$

$$\hat{H}_{out} = -568.1 \frac{\text{kJ}}{\text{kg}} + 2777.2 \frac{\text{kJ}}{\text{kg}} = 2209.1 \frac{\text{kJ}}{\text{kg}}$$

Set up an energy balance around the condenser

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

Cancelling terms

$$\frac{\dot{Q}_c}{\dot{m}} = \hat{H}_{out} - \hat{H}_{in}$$

Find enthalpies

$$\hat{H}_{in} \rightarrow \text{Found previously} \rightarrow 2209.1 \frac{\text{kJ}}{\text{kg}}$$

$$\hat{H}_{out} \rightarrow \text{Saturated liquid water at } 35^\circ\text{C} \rightarrow 146.6 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{Q}_c}{\dot{m}} = 146.6 \frac{\text{kJ}}{\text{kg}} - 2209.1 \frac{\text{kJ}}{\text{kg}} = -2062.5 \frac{\text{kJ}}{\text{kg}}$$

Set an energy balance around the pump

$$\frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{in} \left(\hat{H}_{in} + \frac{v_{in}^2}{2} + gh_{in} \right) - \dot{m}_{out} \left(\hat{H}_{out} + \frac{v_{out}^2}{2} + gh_{out} \right) + \dot{W}_S + \dot{W}_{EC} + \dot{Q}$$

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Cancelling terms

$$\hat{H}_{\text{out}} = \hat{H}_{\text{in}} + \frac{\dot{W}_S}{\dot{m}}$$

Considering liquid water incompressible, we will treat \hat{V} as a constant

$$\dot{W}_{S,\text{pump}} = \int V dP \approx V(P_{\text{out}} - P_{\text{in}})$$

$$\dot{W}_S = 0.001006 \frac{\text{m}^3}{\text{kg}} (10.03 \text{ bar} - 0.0563 \text{ bar}) = 1.00 \frac{\text{kJ}}{\text{kg}}$$

Find \hat{H}_{out} of pump by plugging \dot{W}_S back into the pump energy balance.

$$\hat{H}_{\text{out}} = 146.6 \frac{\text{kJ}}{\text{kg}} + 1 \frac{\text{kJ}}{\text{kg}} = 147.6 \frac{\text{kJ}}{\text{kg}}$$

Set an energy balance around the boiler

$$\begin{aligned} \frac{d}{dt} \left\{ M \left(\hat{U} + \frac{v^2}{2} + gh \right) \right\} \\ = \dot{m}_{\text{in}} \left(\hat{H}_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gh_{\text{in}} \right) - \dot{m}_{\text{out}} \left(\hat{H}_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gh_{\text{out}} \right) + \dot{W}_S + \dot{W}_{\text{EC}} + \dot{Q} \end{aligned}$$

Cancelling terms

$$\frac{\dot{Q}_H}{\dot{m}} = \hat{H}_{\text{out}} - \hat{H}_{\text{in}}$$

$$\frac{\dot{Q}_H}{\dot{m}} = 2777.2 \frac{\text{kJ}}{\text{kg}} - 147.6 \frac{\text{kJ}}{\text{kg}} = 2629.6 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S^{\text{pump}}}{\dot{m}} + \frac{\dot{W}_S^{\text{turbine}}}{\dot{m}} = \frac{\dot{W}_S^{\text{net}}}{\dot{m}}$$

$$\frac{\dot{W}_S^{\text{net}}}{\dot{m}} = 1 \frac{\text{kJ}}{\text{kg}} - 568.1 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_S^{\text{net}}}{\dot{m}} = -567.1 \frac{\text{kJ}}{\text{kg}}$$

Must have a \dot{W}_S^{net} of $-10000 \frac{\text{kJ}}{\text{min}}$

$$\dot{m} = \frac{\left(-10000 \frac{\text{kJ}}{\text{min}} \right)}{\left(-567.1 \frac{\text{kJ}}{\text{kg}} \right)} = 17.63 \frac{\text{kg}}{\text{min}}$$

$$\dot{Q}_H = \frac{\dot{Q}_H}{\dot{m}} \times \dot{m} = \left(2629.6 \frac{\text{kJ}}{\text{kg}}\right) \left(17.63 \frac{\text{kg}}{\text{min}}\right) = \mathbf{46373 \frac{\text{kJ}}{\text{min}}}$$

$$\dot{Q}_C = \frac{\dot{Q}_C}{\dot{m}} \times \dot{m} = \left(-2062.5 \frac{\text{kJ}}{\text{kg}}\right) \left(17.63 \frac{\text{kg}}{\text{min}}\right) = \mathbf{-36373 \frac{\text{kJ}}{\text{min}}}$$

Boiler Cost $\rightarrow T=180^\circ\text{C}$

$$C = 10000 + (46,373) = \mathbf{\$56,373}$$

Condenser Cost $\rightarrow T=35^\circ\text{C}$

$$C = 10000 + 1.5(36,373) = \mathbf{\$64,560}$$

Heat Cost

$$\left(46,370 \frac{\text{kJ}}{\text{min}}\right) \left(\frac{1\text{GJ}}{10^6\text{kJ}}\right) \left(\frac{60\text{min}}{\text{hr}}\right) \left(\frac{24\text{hr}}{\text{day}}\right) \left(\frac{350\text{day}}{\text{year}}\right) = 23,370 \frac{\text{GJ}}{\text{year}}$$

$$\left(23,370 \frac{\text{GJ}}{\text{year}}\right) \left(\frac{\$15}{\text{GJ}}\right) = \mathbf{350,600 \frac{\$}{\text{year}}}$$

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Analogous calculations are completed for each of the six combinations of condenser and boiler temperature. The results are summarized:

	1	2	3	4	5	6
Boiler T (°C)	180	185	190	180	185	190
Condenser T (°C)	35	35	35	40	40	40
Ws,net/M (kJ/kg)	-567.1	-579.3	-591.2	-544.4	-556.8	-569.8
QH/M (kJ/kg)	2629.6	2633.8	2637.7	2608.8	2613.0	2616.9
QC/M (kJ/kg)	-2062.5	-2054.5	-2046.5	-2064.3	-2056.1	-2048.0
M (kg/min)	17.63	17.26	16.92	18.37	17.96	17.58
QH (kJ/min)	46,370	45,466	44,618	47,916	46,926	46,002
QC (kJ/min)	36,362	35,466	34,618	37,916	36,926	36,002
Boiler Cost	\$56,400	\$80,200	\$104,200	\$58,900	\$82,400	\$107,000
Condenser Cost	\$64,600	\$63,200	\$61,900	\$44,900	\$43,900	\$43,000
Energy cost per year	\$350,600	\$343,700	\$337,300	\$362,200	\$354,800	\$347,800
TOTAL COST: Both heat exchangers plus 5 yrs of heat (\$)	1,874,000	1,861,900	1,852,600	1,914,800	1,900,300	1,889,000

Over a 5 year period, it is cost effective to use the most expensive pair of heat exchangers- the boiler operating at 190°C and the condenser operating at 35°C.

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