Problem 3.1 [Difficulty: 2]

3.1 Because the pressure falls, water boils at a lower temperature with increasing altitude. Consequently, cake mixes and boiled eggs, among other foods, must be cooked different lengths of time. Determine the boiling temperature of water at 1000 and 2000 m elevation on a standard day, and compare with the sea-level value.

Given: Pure water on a standard day

Find: Boiling temperature at (a) 1000 m and (b) 2000 m, and compare with sea level value.

Solution:

We can determine the atmospheric pressure at the given altitudes from table A.3, Appendix A

The data are

Elevation (m)	p/p_o	p (kPa)
0	1.000	101.3
1000	0.887	89.9
2000	0.785	79.5

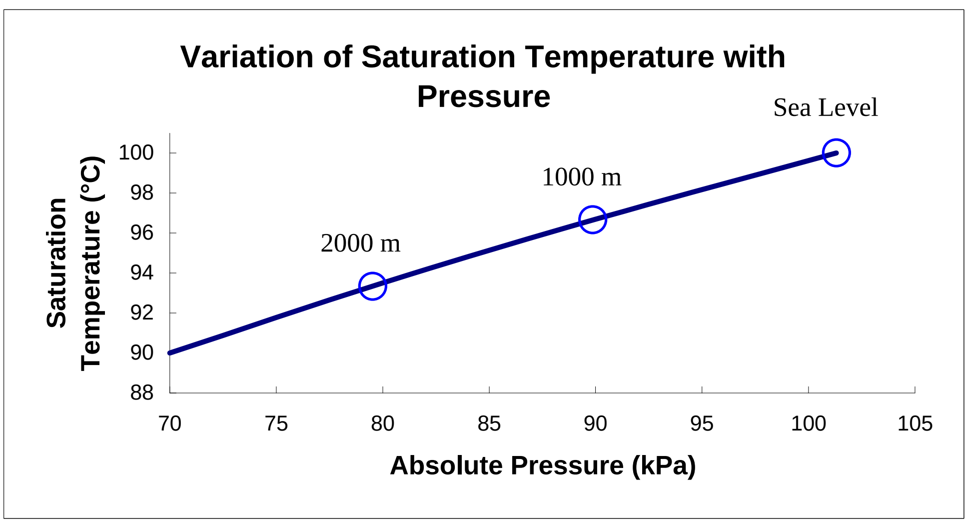
We can also consult steam tables for the variation of saturation temperature with pressure:

p (kPa)	T_{sat} (°C)
70	90.0
80	93.5
90	96.7
101.3	100.0

We can interpolate the data from the steam tables to correlate saturation temperature with altitude:

Elevation (m)	p/p_o	p (kPa)	T_{sat} (°C)
0	1.000	101.3	100.0
1000	0.887	89.9	96.7
2000	0.785	79.5	93.3

The data are plotted here. They show that the saturation temperature drops approximately 3.4°C/1000 m.



Problem 3.2

[Difficulty: 2]

3.2 Ear "popping" is an unpleasant phenomenon sometimes experienced when a change in pressure occurs, for example in a fast-moving elevator or in an airplane. If you are in a two-seater airplane at 3000 m and a descent of 100 m causes your ears to "pop," what is the pressure change that your ears "pop" at, in millimeters of mercury? If the airplane now rises to 8000 m and again begins descending, how far will the airplane descend before your ears "pop" again? Assume a U.S. Standard Atmosphere.

Given: Data on flight of airplane

Find: Pressure change in mm Hg for ears to "pop"; descent distance from 8000 m to cause ears to "pop."

Solution:

Assume the air density is approximately constant constant from 3000 m to 2900 m.
From table A.3

$$\rho_{SL} = 1.225 \cdot \frac{kg}{m^3} \qquad \rho_{air} = 0.7423 \cdot \rho_{SL} \qquad \rho_{air} = 0.909 \frac{kg}{m^3}$$

We also have from the manometer equation,

$$\Delta p = -\rho_{air} \cdot g \cdot \Delta z \quad \text{and also} \quad \Delta p = -\rho_{Hg} \cdot g \cdot \Delta h_{Hg}$$

Combining $\quad \Delta h_{Hg} = \frac{\rho_{air}}{\rho_{Hg}} \cdot \Delta z = \frac{\rho_{air}}{SG_{Hg} \cdot \rho_{H2O}} \cdot \Delta z \qquad SG_{Hg} = 13.55$ from Table A.2

$$\Delta h_{Hg} = \frac{0.909}{13.55 \times 999} \times 100 \cdot m \qquad \Delta h_{Hg} = 6.72 \cdot mm$$

For the ear popping descending from 8000 m, again assume the air density is approximately constant constant, this time at 8000 m.
From table A.3

$$\rho_{air} = 0.4292 \cdot \rho_{SL} \qquad \rho_{air} = 0.526 \frac{kg}{m^3}$$

We also have from the manometer equation

$$\rho_{air8000} \cdot g \cdot \Delta z_{8000} = \rho_{air3000} \cdot g \cdot \Delta z_{3000}$$

where the numerical subscripts refer to conditions at 3000m and 8000m.
Hence

$$\Delta z_{8000} = \frac{\rho_{air3000} \cdot g}{\rho_{air8000} \cdot g} \cdot \Delta z_{3000} = \frac{\rho_{air3000}}{\rho_{air8000}} \cdot \Delta z_{3000} \qquad \Delta z_{8000} = \frac{0.909}{0.526} \times 100 \cdot m \qquad \Delta z_{8000} = 173 m$$

Problem 3.3

[Difficulty: 3]

3.3 When you are on a mountain face and boil water, you notice that the water temperature is 195°F. What is your approximate altitude? The next day, you are at a location where it boils at 185°F. How high did you climb between the two days? Assume a U.S. Standard Atmosphere.

Given: Boiling points of water at different elevations

Find: Change in elevation

Solution:

From the steam tables, we have the following data for the boiling point (saturation temperature) of water

T_{sat} (°F)	p (psia)
195	10.39
185	8.39

The sea level pressure, from Table , is

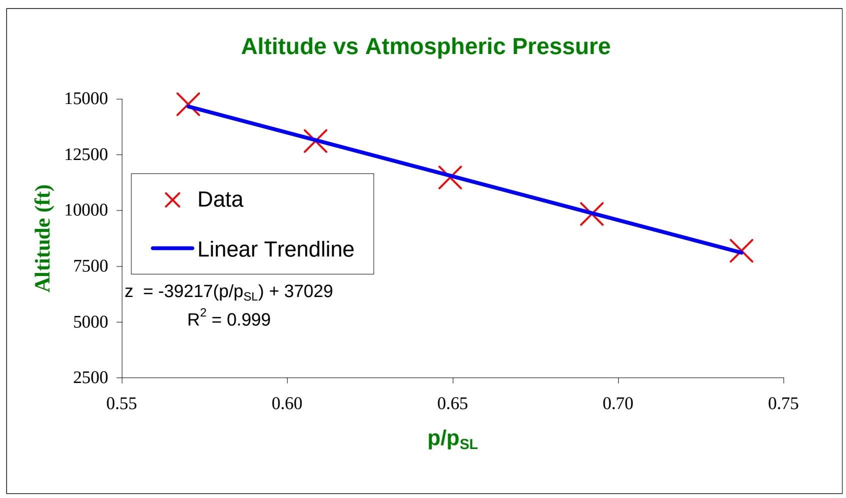
p_{SL} = 14.696 psia

Hence

T_{sat} (°F)	p/p_{SL}
195	0.707
185	0.571

From Table

p/p_{SL}	Altitude (m)	Altitude (ft)
0.7372	2500	8203
0.6920	3000	9843
0.6492	3500	11484
0.6085	4000	13124
0.5700	4500	14765



Then, any one of a number of *Excel* functions can be used to interpolate
(Here we use *Excel*'s *Trendline* analysis)

p/p_{SL}	Altitude (ft)
0.707	9303
0.571	14640

Current altitude is approximately 9303 ft

The change in altitude is then 5337 ft

Alternatively, we can interpolate for each altitude by using a linear regression between adjacent data points

	p/p_{SL}	Altitude (m)	Altitude (ft)	p/p_{SL}	Altitude (m)	Altitude (ft)
For	0.7372	2500	8203	0.6085	4000	13124
	0.6920	3000	9843	0.5700	4500	14765
Then	0.7070	2834	9299	0.5730	4461	14637

The change in altitude is then 5338 ft

Problem 3.4

[Difficulty: 2]

3.4 Your pressure gage indicates that the pressure in your cold tires is 0.25 MPa (gage) on a mountain at an elevation of 3500 m. What is the absolute pressure? After you drive down to sea level, your tires have warmed to 25°C. What pressure does your gage now indicate? Assume a U.S. Standard Atmosphere.

Given: Data on tire at 3500 m and at sea level

Find: Absolute pressure at 3500 m; pressure at sea level

Solution:

At an elevation of 3500 m, from Table

$$p_{SL} = 101 \cdot kPa \qquad p_{atm} = 0.6492 \cdot p_{SL} \qquad p_{atm} = 65.6 \cdot kPa$$

and we have $p_g = 0.25 \cdot MPa$ $\qquad p_g = 250 \cdot kPa \qquad p = p_g + p_{atm} \qquad p = 316 \cdot kPa$

At sea level $p_{atm} = 101 \cdot kPa$

Meanwhile, the tire has warmed up, from the ambient temperature at 3500 m, to 25°C.

At an elevation of 3500 m, $T_{cold} = 265.4 \cdot K$ and $T_{hot} = (25 + 273) \cdot K \qquad T_{hot} = 298\,K$

Hence, assuming ideal gas behavior, $pV = mRT$, and that the tire is approximately a rigid container, the absolute pressure of the hot tire is

$$p_{hot} = \frac{T_{hot}}{T_{cold}} \cdot p \qquad p_{hot} = 354 \cdot kPa$$

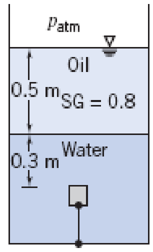
Then the gage pressure is

$$p_g = p_{hot} - p_{atm} \qquad p_g = 253 \cdot kPa$$

Problem 3.5

[Difficulty: 2]

3.5 A 125-mL cube of solid oak is held submerged by a tether as shown. Calculate the actual force of the water on the bottom surface of the cube and the tension in the tether.



Given: Data on system

Find: Force on bottom of cube; tension in tether

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot h$ where h is measured downwards

The absolute pressure at the interface is $p_{interface} = p_{atm} + SG_{oil} \cdot \rho \cdot g \cdot h_{oil}$

Then the pressure on the lower surface is $p_L = p_{interface} + \rho \cdot g \cdot h_L = p_{atm} + \rho \cdot g \cdot \left(SG_{oil} \cdot h_{oil} + h_L\right)$

For the cube $V = 125 \cdot mL$ $\quad V = 1.25 \times 10^{-4} \cdot m^3$

Then the size of the cube is $d = V^{\frac{1}{3}}$ $\quad d = 0.05\,m$ and the depth in water to the upper surface is $h_U = 0.3 \cdot m$

Hence $h_L = h_U + d$ $\quad h_L = 0.35\,m$ where h_L is the depth in water to the lower surface

The force on the lower surface is $F_L = p_L \cdot A$ where $A = d^2$ $\quad A = 0.0025\,m^2$

$$F_L = \left[p_{atm} + \rho \cdot g \cdot \left(SG_{oil} \cdot h_{oil} + h_L\right)\right] \cdot A$$

$$F_L = \left[101 \times 10^3 \cdot \frac{N}{m^2} + 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (0.8 \times 0.5 \cdot m + 0.35 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m}\right] \times 0.0025 \cdot m^2$$

$F_L = 270.894\,N$ Note: Extra decimals needed for computing T later!

For the tension in the tether, an FBD gives $\Sigma F_y = 0$ $\quad F_L - F_U - W - T = 0$ or $T = F_L - F_U - W$

where $F_U = \left[p_{atm} + \rho \cdot g \cdot \left(SG_{oil} \cdot h_{oil} + h_U\right)\right] \cdot A$

Note that we could instead compute $\Delta F = F_L - F_U = \rho \cdot g \cdot SG_{oil} \cdot \left(h_L - h_U\right) \cdot A$ and $T = \Delta F - W$

Using F_U

$$F_U = \left[101 \times 10^3 \cdot \frac{N}{m^2} + 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (0.8 \times 0.5 \cdot m + 0.3 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m}\right] \times 0.0025 \cdot m^2$$

$F_U = 269.668\,N$ Note: Extra decimals needed for computing T later!

For the oak block $SG_{oak} = 0.77$ so $W = SG_{oak} \cdot \rho \cdot g \cdot V$

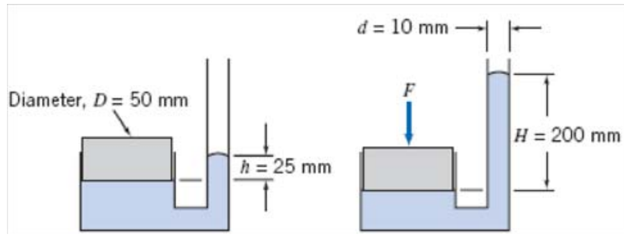
$$W = 0.77 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 1.25 \times 10^{-4} \cdot m^3 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W = 0.944\,N$$

$T = F_L - F_U - W$ $T = 0.282\,N$

Problem 3.6

[Difficulty: 2]

3.6 The tube shown is filled with mercury at 20°C. Calculate the force applied to the piston.



Given: Data on system before and after applied force

Find: Applied force

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $p = p_{atm} - \rho \cdot g \cdot (y - y_0)$ with $p(y_0) = p_{atm}$

For initial state $p_1 = p_{atm} + \rho \cdot g \cdot h$ and $F_1 = p_1 \cdot A = \rho \cdot g \cdot h \cdot A$ (Gage; F_1 is hydrostatic upwards force)

For the initial FBD $\Sigma F_y = 0$ $F_1 - W = 0$ $W = F_1 = \rho \cdot g \cdot h \cdot A$

For final state $p_2 = p_{atm} + \rho \cdot g \cdot H$ and $F_2 = p_2 \cdot A = \rho \cdot g \cdot H \cdot A$ (Gage; F_2 is hydrostatic upwards force)

For the final FBD $\Sigma F_y = 0$ $F_2 - W - F = 0$ $F = F_2 - W = \rho \cdot g \cdot H \cdot A - \rho \cdot g \cdot h \cdot A = \rho \cdot g \cdot A \cdot (H - h)$

$$F = \rho_{H2O} \cdot SG \cdot g \cdot \frac{\pi \cdot D^2}{4} \cdot (H - h)$$

$$SG = 13.54$$

$$F = 1000 \cdot \frac{kg}{m^3} \times 13.54 \times 9.81 \cdot \frac{m}{s^2} \times \frac{\pi}{4} \times (0.05 \cdot m)^2 \times (0.2 - 0.025) \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$

$$F = 45.6 N$$

Problem 3.7

(Difficulty: 1)

3.7 Calculate the absolute pressure and gage pressure in an open tank of crude oil 2.4 m below the liquid surface. If the tank is closed and pressurized to 130 kPa, what are the absolute pressure and gage pressure at this location.

Given: Location: $h = 2.4\ m$ below the liquid surface. Liquid: Crude oil.

Find: The absolute pressure p_a and gage pressure p_g for both open and closed tank .

Assumption: The gage pressure for the liquid surface is zero for open tank and closed tank. The oil is incompressible.

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

The density for the crude oil is:

$$\rho = 856\ \frac{kg}{m^3}$$

The atmosphere pressure is:

$$p_{atmos} = 101000\ Pa$$

The pressure for the closed tank is:

$$p_{tank} = 130\ kPa = 130000\ Pa$$

Using the hydrostatic relation, the gage pressure of open tank 2.4 m below the liquid surface is:

$$p_g = \rho g h = 856\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 2.4\ m = 20100\ Pa$$

The absolute pressure of open tank at this location is:

$$p_a = p_g + p_{atmos} = 20100\ Pa + 101000\ Pa = 121100\ Pa = 121.1\ kPa$$

The gage pressure of closed tank at the same location below the liquid surface is the same as open tank:

$$p_g = \rho g h = 856\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 2.4\ m = 20100\ Pa$$

The absolute pressure of closed tank at this location is:

$$p_a = p_g + p_{tank} = 20100\ Pa + 130000\ Pa = 150100\ Pa = 150.1\ kPa$$

Problem 3.8

(Difficulty: 1)

3.8 An open vessel contains carbon tetrachloride to a depth of 6 ft and water on the carbon tetrachloride to a depth of 5 ft . What is the pressure at the bottom of the vessel?

Given: Depth of carbon tetrachloride: $h_c = 6\ ft$. Depth of water: $h_w = 5\ ft$.

Find: The gage pressure p at the bottom of the vessel.

Assumption: The gage pressure for the liquid surface is zero. The fluid is incompressible.

Solution: Use the hydrostatic pressure relation to detmine pressures in a fluid.

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

The density for the carbon tetrachloride is:

$$\rho_c = 1.59 \times 10^3\ \frac{kg}{m^3} = 3.09\ \frac{slug}{ft^3}$$

The density for the water is:

$$\rho_w = 1.0 \times 10^3\ \frac{kg}{m^3} = 1.940\ \frac{slug}{ft^3}$$

Using the hydrostatic relation, the gage pressure p at the bottom of the vessel is:

$$p = \rho_c g h_c + \rho_w g h_w$$

$$p = 3.09\ \frac{slug}{ft^3} \times 32.2\ \frac{ft}{s^2} \times 6\ ft + 1.940\ \frac{slug}{ft^3} \times 32.2\ \frac{ft}{s^2} \times 5\ ft = 909\ \frac{lbf}{ft^2} = 6.25\ psi$$

Problem 3.9

[Difficulty: 2]

3.9 A hollow metal cube with sides 100 mm floats at the interface between a layer of water and a layer of SAE 10W oil such that 10% of the cube is exposed to the oil. What is the pressure difference between the upper and lower horizontal surfaces? What is the average density of the cube?

Given: Properties of a cube floating at an interface

Find: The pressures difference between the upper and lower surfaces; average cube density

Solution:

The pressure difference is obtained from two applications of these equations:

$$p_U = p_0 + \rho_{SAE10} \cdot g \cdot (H - 0.1 \cdot d) \qquad p_L = p_0 + \rho_{SAE10} \cdot g \cdot H + \rho_{H2O} \cdot g \cdot 0.9 \cdot d$$

where p_U and p_L are the upper and lower pressures, p_0 is the oil free surface pressure, H is the depth of the interface, and d is the cube size

Hence the pressure difference is

$$\Delta p = p_L - p_U = \rho_{H2O} \cdot g \cdot 0.9 \cdot d + \rho_{SAE10} \cdot g \cdot 0.1 \cdot d \qquad \Delta p = \rho_{H2O} \cdot g \cdot d \cdot \left(0.9 + SG_{SAE10} \cdot 0.1\right)$$

From Table $\quad SG_{SAE10} = 0.92$

$$\Delta p = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.1 \cdot m \times (0.9 + 0.92 \times 0.1) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 972\,Pa$$

For the cube density, set up a free body force balance for the cube

$$\Sigma F = 0 = \Delta p \cdot A - W$$

Hence $\quad W = \Delta p \cdot A = \Delta p \cdot d^2$

$$\rho_{cube} = \frac{m}{d^3} = \frac{W}{d^3 \cdot g} = \frac{\Delta p \cdot d^2}{d^3 \cdot g} = \frac{\Delta p}{d \cdot g}$$

$$\rho_{cube} = 972 \cdot \frac{N}{m^2} \times \frac{1}{0.1 \cdot m} \times \frac{s^2}{9.81 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad \rho_{cube} = 991 \frac{kg}{m^3}$$

Problem 3.10

[Difficulty: 2]

3.10 Compressed nitrogen (140 lbm) is stored in a spherical tank of diameter $D = 2.5$ ft at a temperature of 77°F. What is the pressure inside the tank? If the maximum allowable stress in the tank is 30 ksi, find the minimum theoretical wall thickness of the tank.

Given: Data on nitrogen tank

Find: Pressure of nitrogen; minimum required wall thickness

Assumption: Ideal gas behavior

Solution:

Ideal gas equation of state: $$p \cdot V = M \cdot R \cdot T$$

where, from Table A.6, for nitrogen $$R = 55.16 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{R}}$$

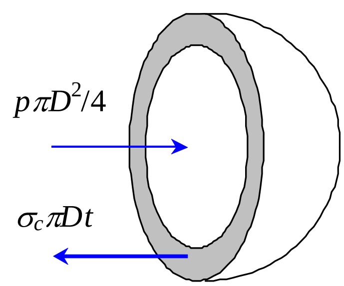
Then the pressure of nitrogen is $$p = \frac{M \cdot R \cdot T}{V} = M \cdot R \cdot T \cdot \left(\frac{6}{\pi \cdot D^3}\right)$$

$$p = 140 \cdot \text{lbm} \times 55.16 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{R}} \times (77 + 460) \cdot \text{R} \times \left[\frac{6}{\pi \times (2.5 \cdot \text{ft})^3}\right] \times \left(\frac{\text{ft}}{12 \cdot \text{in}}\right)^2$$

$$p = 3520 \cdot \frac{\text{lbf}}{\text{in}^2}$$

To determine wall thickness, consider a free body diagram for one hemisphere:

$$\Sigma F = 0 = p \cdot \frac{\pi \cdot D^2}{4} - \sigma_c \cdot \pi \cdot D \cdot t$$



where σ_c is the circumferential stress in the container

Then $$t = \frac{p \cdot \pi \cdot D^2}{4 \cdot \pi \cdot D \cdot \sigma_c} = \frac{p \cdot D}{4 \cdot \sigma_c}$$

$$t = 3520 \cdot \frac{\text{lbf}}{\text{in}^2} \times \frac{2.5 \cdot \text{ft}}{4} \times \frac{\text{in}^2}{30 \times 10^3 \cdot \text{lbf}}$$

$t = 0.0733 \cdot \text{ft}$ $\quad$ $t = 0.880 \cdot \text{in}$

Problem 3.11

(Difficulty: 2)

3.11 If at the surface of a liquid the specific weight is γ_0, with z and p both zero, show that, if $E = constant$, the specific weight and pressure are given $\gamma = \frac{E}{\left(z+\frac{E}{\gamma_0}\right)}$ and $p = -E \ln\left(1 + \frac{\gamma_0 Z}{E}\right)$.
Calculate specific weight and pressure at a depth of 2 km assuming $\gamma_0 = 10.0\ \frac{kN}{m^3}$ and $E = \ 2070\ MPa$.

Given: Depth: $h = 2\ km$. The specific weight at surface of a liquid: $\gamma_0 = 10.0\ \frac{kN}{m^3}$.

Find: The specific weight and pressure at a depth of 2 km.

Assumption:. Bulk modulus is constant

Solution: Use the hydrostatic pressure relation and definition of bulk modulus to detmine pressures in a fluid.

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

Definition of bulk modulus

$$E_v = \frac{dp}{d\rho/\rho} = \frac{dp}{d\gamma/\gamma}$$

Eliminating dp from the hydrostatic pressure relation and the bulk modulus definition:

$$dp = \ -\gamma\, dz = \ E_v \frac{d\gamma}{\gamma}$$

Or

$$dz = -E_v \frac{d\gamma}{\gamma^2}$$

Integrating for both sides we get:

$$z = E_v \frac{1}{\gamma} + c$$

At $z = 0$, $\gamma = \gamma_0$ so:

$$c = -E_v \frac{1}{\gamma_0}$$

$$z = E_v \frac{1}{\gamma} - E_v \frac{1}{\gamma_0}$$

Solving for γ, we have:

$$\gamma = \frac{E_v}{\left(z + \frac{E_v}{\gamma_0}\right)}$$

Solving for the pressure using the hydrostatic relation:

$$dp = -\gamma dz = -\frac{E_v}{\left(z + \frac{E_v}{\gamma_0}\right)} dz$$

Integrating both sides we to get:

$$p = -E_v \ln\left(z + \frac{E_v}{\gamma_0}\right) + c$$

At $z = 0$, $p = 0$ so:

$$c = E_v \ln\left(\frac{E_v}{\gamma_0}\right)$$

$$p = -E_v \ln\left(z + \frac{E_v}{\gamma_0}\right) + E_v \ln\left(\frac{E_v}{\gamma_0}\right) = -E_v \ln\left(1 + \frac{\gamma_0 z}{E_v}\right)$$

For the specific case

$$h = 2\ km$$

$$\gamma_0 = 10.0\ \frac{kN}{m^3}$$

$$E_v = 2070\ MPa$$

The specific weight:

$$\gamma = \frac{E_v}{\left(z + \frac{E_v}{\gamma_0}\right)} = \frac{2070 \times 10^6\ pa}{\left(-2000\ Pa + \frac{2070 \times 10^6\ Pa}{10 \times 10^3\ \frac{N}{m^3}}\right)} = 10100\ \frac{N}{m^3} = 10.1\ \frac{kN}{m^3}$$

Pressure:

$$p = -E_v \ln\left(1 + \frac{\gamma_0 z}{E_v}\right) = -2070 \times 10^6 Pa \times \ln\left(1 + 10000.0\ \frac{kN}{m^3} \times \left(\frac{-2000\ m}{2070 \times 10^6 Pa}\right)\right) = 20100\ kPa$$

Problem 3.12

(Difficulty: 2)

3.12 In the deep ocean the compressibility of seawater is significant in its effect on ρ and p. If $E = 2.07 \times 10^9\ Pa$, find the percentage change in the density and pressure at a depth of 10000 meters as compared to the values obtained at the same depth under the incompressible assumption. Let $\rho_0 = 1020\ \frac{kg}{m^3}$ and the absolute pressure $p_0 = 101.3\ kPa$.

Given: Depth: $h = 10000\ meters$. The density: $\rho_0 = 1020\ \frac{kg}{m^3}$. The absolute pressure: $p_0 = 101.3\ kPa$.

Find: The percent change in density $\rho\%$ and pressure $p\%$.

Assumption: The bulk modulus is constant

Solution: Use the relations developed in problem 3.11 for specific weight and pressure for a compressible liquid:

$$\gamma = \frac{E}{\left(z + \frac{E}{\gamma_0}\right)}$$

$$p = -E \ln\left(1 + \frac{\gamma_0 z}{E}\right)$$

The specific weight at sea level is:

$$\gamma_0 = \rho_0 g = 1020\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} = 10010\ \frac{N}{m^3}$$

The specific weight and density at 10000 m depth are

$$\gamma = \frac{E}{\left(z + \frac{E}{\gamma_0}\right)} = \frac{2.07 \times 10^9}{\left(-10000 + \frac{2.07 \times 10^9}{10010}\right)}\ \frac{N}{m^3} = 10520\ \frac{N}{m^3}$$

$$\rho = \frac{\gamma}{g} = \frac{10520}{9.81}\ \frac{kg}{m^3} = 1072\ \frac{kg}{m^3}$$

The percentage change in density is

$$\rho\% = \frac{\rho - \rho_0}{\rho_0} = \frac{1072 - 1020}{1020} = 5.1\ \%$$

The gage pressure at a depth of 10000m is:

$$p = -E \ln\left(1 + \frac{\gamma_0 z}{E}\right) = 101.3\ kPa - 2.07 \times 10^9\ \times \ln\left(1 + \frac{10010 \times (-10000)}{2.07 \times 10^9}\right)\ Pa = 102600\ kPa$$

The pressure assuming that the water is incompressible is:

$$p_{in} = \rho g h = 1020\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 10000\ m = 100062\ kPa$$

The percent difference in pressure is:

$$p\% = \frac{p - p_0}{p_0} = \frac{102600\ kPa - 100062\ kPa}{100062\ kPa} = 2.54\ \%$$

Problem 3.13

[Difficulty: 4]

3.13 Assuming the bulk modulus is constant for seawater, derive an expression for the density variation with depth, h, below the surface. Show that the result may be written

$$\rho \approx \rho_0 + bh$$

where ρ_0 is the density at the surface. Evaluate the constant b. Then, using the approximation, obtain an equation for the variation of pressure with depth below the surface. Determine the depth in feet at which the error in pressure predicted by the approximate solution is 0.01 percent.

Given: Model behavior of seawater by assuming constant bulk modulus

Find: (a) Expression for density as a function of depth h.
(b) Show that result may be written as

$$\rho = \rho_o + bh$$

(c) Evaluate the constant b
(d) Use results of (b) to obtain equation for p(h)
(e) Determine depth at which error in predicted pressure is 0.01%

Solution: From Table A.2, App. A: $SG_o = 1.025$ $\quad E_v = 2.42 \cdot GPa = 3.51 \times 10^5 \cdot psi$

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g \qquad \text{(Hydrostatic Pressure - h is positive downwards)}$$

$$E_v = \frac{dp}{\frac{d\rho}{\rho}} \qquad \text{(Definition of Bulk Modulus)}$$

Then $\quad dp = \rho \cdot g \cdot dh = E_v \cdot \frac{d\rho}{\rho}$ $\quad$ or $\quad \frac{d\rho}{\rho^2} = \frac{g}{E_v} dh$ $\quad$ Now if we integrate: $\quad \int_{\rho_o}^{\rho} \frac{1}{\rho^2} d\rho = \int_0^h \frac{g}{E_v} dh$

After integrating: $\quad \frac{\rho - \rho_o}{\rho \cdot \rho_o} = \frac{g \cdot h}{E_v}$ $\quad$ Therefore: $\quad \rho = \frac{E_v \cdot \rho_o}{E_v - g \cdot h \cdot \rho_o}$ $\quad$ and $\quad \frac{\rho}{\rho_o} = \frac{1}{1 - \frac{\rho_o \cdot g \cdot h}{E_v}}$

Now for $\quad \frac{\rho_o \cdot g \cdot h}{E_v}$ <<1, the binomial expansion may be used to approximate the density: $\quad \frac{\rho}{\rho_o} = 1 + \frac{\rho_o \cdot g \cdot h}{E_v}$ $\quad$ (Binomial expansion may be found in a host of sources, e.g. *CRC Handbook of Mathematics*)

In other words, $\quad \rho = \rho_o + b \cdot h$ $\quad$ where $\quad b = \frac{\rho_o^2 \cdot g}{E_v}$

Since $\quad dp = \rho \cdot g \cdot dh$ $\quad$ then an approximate expression for the pressure as a function of depth is:

$$p_{approx} - p_{atm} = \int_0^h (\rho_o + b \cdot h) \cdot g \, dh \rightarrow p_{approx} - p_{atm} = \frac{g \cdot h \cdot (2 \cdot \rho_o + b \cdot h)}{2}$$

Solving for p_{approx} we get:

$$p_{approx} = p_{atm} + \frac{g \cdot h \cdot (2 \cdot \rho_o + b \cdot h)}{2} = p_{atm} + \rho_o \cdot g \cdot h + \frac{b \cdot g \cdot h^2}{2} = p_{atm} + \left(\rho_o \cdot h + \frac{b \cdot h^2}{2}\right) \cdot g$$

Now if we subsitiute in the expression for b and simplify, we get:

$$p_{approx} = p_{atm} + \left(\rho_o \cdot h + \frac{\rho_o^2 \cdot g}{E_v} \cdot \frac{h^2}{2}\right) \cdot g = p_{atm} + \rho_o \cdot g \cdot h \cdot \left(1 + \frac{\rho_o \cdot g \cdot h}{2 \cdot E_v}\right) \qquad p_{approx} = p_{atm} + \rho_o \cdot g \cdot h \cdot \left(1 + \frac{\rho_o \cdot g \cdot h}{2E_v}\right)$$

The exact soution for p(h) is obtained by utilizing the exact solution for ρ(h). Thus:

$$p_{exact} - p_{atm} = \int_{\rho_o}^{\rho} \frac{E_v}{\rho} d\rho = E_v \cdot \ln\left(\frac{\rho}{\rho_o}\right)$$ Subsitiuting for $\frac{\rho}{\rho_o}$ we get: $$p_{exact} = p_{atm} + E_v \cdot \ln\left(1 - \frac{\rho_o \cdot g \cdot h}{E_v}\right)^{-1}$$

If we let $x = \frac{\rho_o \cdot g \cdot h}{E_v}$ For the error to be 0.01%: $$\frac{\Delta p_{exact} - \Delta p_{approx}}{\Delta p_{exact}} = 1 - \frac{\rho_o \cdot g \cdot h \cdot \left(1 + \frac{x}{2}\right)}{E_v \cdot \ln\left[(1-x)^{-1}\right]} = 1 - \frac{x \cdot \left(1 + \frac{x}{2}\right)}{\ln\left[(1-x)^{-1}\right]} = 0.0001$$

This equation requires an iterative solution, e.g. Excel's Goal Seek. The result is: $x = 0.01728$ Solving x for h:

$$h = \frac{x \cdot E_v}{\rho_o \cdot g} \qquad h = 0.01728 \times 3.51 \times 10^5 \cdot \frac{lbf}{in^2} \times \frac{ft^3}{1.025 \times 1.94 \cdot slug} \times \frac{s^2}{32.2 \cdot ft} \times \left(\frac{12 \cdot in}{ft}\right)^2 \times \frac{slug \cdot ft}{lbf \cdot s^2} \qquad h = 1.364 \times 10^4 \cdot ft$$

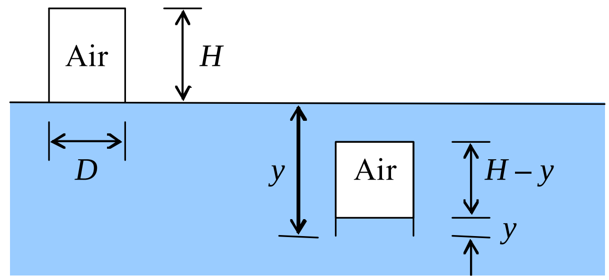
This depth is over 2.5 miles, so the incompressible fluid approximation is a reasonable one at all but the lowest depths of the ocean.

Problem 3.14 [Difficulty: 3]

3.14 An inverted cylindrical container is lowered slowly beneath the surface of a pool of water. Air trapped in the container is compressed isothermally as the hydrostatic pressure increases. Develop an expression for the water height, y, inside the container in terms of the container height, H, and depth of submersion, h. Plot y/H versus h/H.

Given: Cylindrical cup lowered slowly beneath pool surface

Find: Expression for y in terms of h and H.
Plot y/H vs. h/H.



Solution:

Governing Equations: $\frac{dp}{dh} = \rho \cdot g$ (Hydrostatic Pressure - h is positive downwards)

$p \cdot V = M \cdot R \cdot T$ (Ideal Gas Equation)

Assumptions:
(1) Constant temperature compression of air inside cup
(2) Static liquid
(3) Incompressible liquid

First we apply the ideal gas equation (at constant temperature) for the pressure of the air in the cup: $p \cdot V = \text{constant}$

Therefore: $p \cdot V = p_a \cdot \frac{\pi}{4} \cdot D^2 \cdot H = p \cdot \frac{\pi}{4} \cdot D^2 \cdot (H - y)$ and upon simplification: $p_a \cdot H = p \cdot (H - y)$

Now we look at the hydrostatic pressure equation for the pressure exerted by the water. Since ρ is constant, we integrate:

$$p - p_a = \rho \cdot g \cdot (h - y) \quad \text{at the water-air interface in the cup.}$$

Since the cup is submerged to a depth of h, these pressures must be equal:

$$p_a \cdot H = \left[p_a + \rho \cdot g \cdot (h - y)\right] \cdot (H - y) = p_a \cdot H - p_a \cdot y + \rho \cdot g \cdot (h - y) \cdot (H - y)$$

Explanding out the right hand side of this expression:

$$0 = -p_a \cdot y + \rho \cdot g \cdot (h - y) \cdot (H - y) = \rho \cdot g \cdot h \cdot H - \rho \cdot g \cdot h \cdot y - \rho \cdot g \cdot H \cdot y + \rho \cdot g \cdot y^2 - p_a \cdot y$$

$$\rho \cdot g \cdot y^2 - \left[p_a + \rho \cdot g \cdot (h + H)\right] \cdot y + \rho \cdot g \cdot h \cdot H = 0 \qquad y^2 - \left[\frac{p_a}{\rho \cdot g} + (h + H)\right] \cdot y + h \cdot H = 0$$

We now use the quadratic equation: $y = \dfrac{\left[\frac{p_a}{\rho \cdot g} + (h + H)\right] - \sqrt{\left[\frac{p_a}{\rho \cdot g} + (h + H)\right]^2 - 4 \cdot h \cdot H}}{2}$ we only use the minus sign because y can never be larger than H.

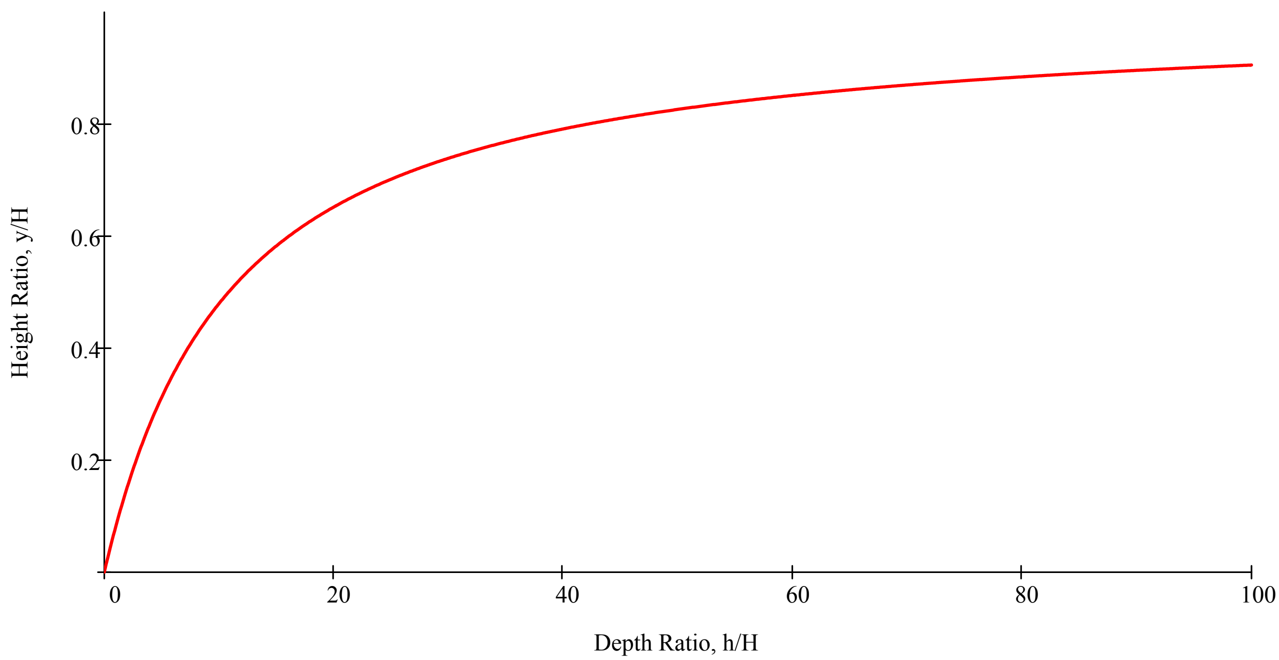
Now if we divide both sides by H, we get an expression for y/H:

$$\frac{y}{H} = \frac{\left(\frac{p_a}{\rho \cdot g \cdot H} + \frac{h}{H} + 1\right) - \sqrt{\left(\frac{p_a}{\rho \cdot g \cdot H} + \frac{h}{H} + 1\right)^2 - 4 \cdot \frac{h}{H}}}{2}$$

The exact shape of this curve will depend upon the height of the cup. The plot below was generated assuming:

$$p_a = 101.3 \cdot kPa$$

$$H = 1 \cdot m$$



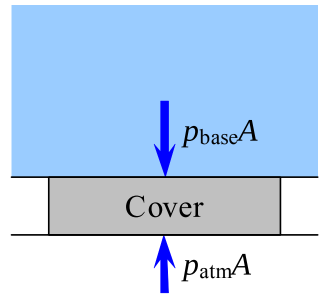
Problem 3.15

[Difficulty: 2]

3.15 A water tank filled with water to a depth of 16 ft has in inspection cover (1 in. × 1 in.) at its base, held in place by a plastic bracket. The bracket can hold a load of 9 lbf. Is the bracket strong enough? If it is, what would the water depth have to be to cause the bracket to break?

Given: Data on water tank and inspection cover

Find: If the support bracket is strong enough; at what water depth would it fail



Assumptions: Water is incompressible and static

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot h$ where h is measured downwards

The absolute pressure at the base is $p_{base} = p_{atm} + \rho \cdot g \cdot h$ where $h = 16 \cdot ft$

The gage pressure at the base is $p_{base} = \rho \cdot g \cdot h$ This is the pressure to use as we have p_{atm} on the outside of the cover.

The force on the inspection cover is $F = p_{base} \cdot A$ where $A = 1 \cdot in \times 1 \cdot in$ $A = 1 \cdot in^2$

$$F = \rho \cdot g \cdot h \cdot A$$

$$F = 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 16 \cdot ft \times 1 \cdot in^2 \times \left(\frac{ft}{12 \cdot in}\right)^2 \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

$F = 6.94 \cdot lbf$ The bracket is strong enough (it can take 9 lbf).

To find the maximum depth we start with $F = 9.00 \cdot lbf$

$$h = \frac{F}{\rho \cdot g \cdot A}$$

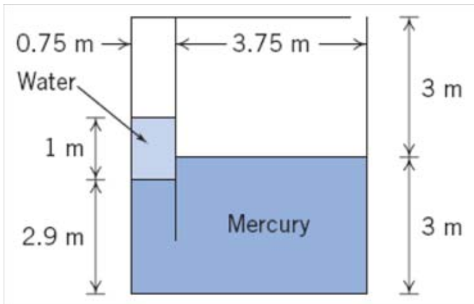
$$h = 9 \cdot lbf \times \frac{1}{1.94} \cdot \frac{ft^3}{slug} \times \frac{1}{32.2} \cdot \frac{s^2}{ft} \times \frac{1}{in^2} \times \left(\frac{12 \cdot in}{ft}\right)^2 \times \frac{slug \cdot ft}{lbf \cdot s^2}$$

$$h = 20.7 \cdot ft$$

Problem 3.16

[Difficulty: 2]

3.16 A partitioned tank as shown contains water and mercury. What is the gage pressure in the air trapped in the left chamber? What pressure would the air on the left need to be pumped to in order to bring the water and mercury free surfaces level?



Given: Data on partitioned tank

Find: Gage pressure of trapped air; pressure to make water and mercury levels equal

Solution:

The pressure difference is obtained from repeated application of Eq. 3.7, or in other words, from Eq. 3.8. Starting from the right air chamber

$$p_{gage} = SG_{Hg} \times \rho_{H2O} \times g \times (3 \cdot m - 2.9 \cdot m) - \rho_{H2O} \times g \times 1 \cdot m$$

$$p_{gage} = \rho_{H2O} \times g \times \left(SG_{Hg} \times 0.1 \cdot m - 1.0 \cdot m\right)$$

$$p_{gage} = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \times 0.1 \cdot m - 1.0 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{gage} = 3.48 \cdot kPa$$

If the left air pressure is now increased until the water and mercury levels are now equal, Eq. 3.8 leads to

$$p_{gage} = SG_{Hg} \times \rho_{H2O} \times g \times 1.0 \cdot m - \rho_{H2O} \times g \times 1.0 \cdot m$$

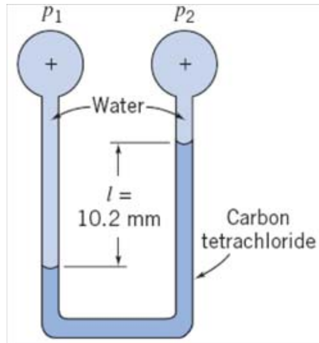
$$p_{gage} = \rho_{H2O} \times g \times \left(SG_{Hg} \times 1 \cdot m - 1.0 \cdot m\right)$$

$$p_{gage} = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times (13.55 \times 1 \cdot m - 1.0 \cdot m) \times \frac{N \cdot s^2}{kg \cdot m} \qquad p_{gage} = 123 \cdot kPa$$

Problem 3.17

[Difficulty: 2]

3.17 Consider the two-fluid manometer shown. Calculate the applied pressure difference.



Given: Two-fluid manometer as shown

$$l = 10.2 \cdot mm \quad SG_{ct} = 1.595$$

Find: Pressure difference

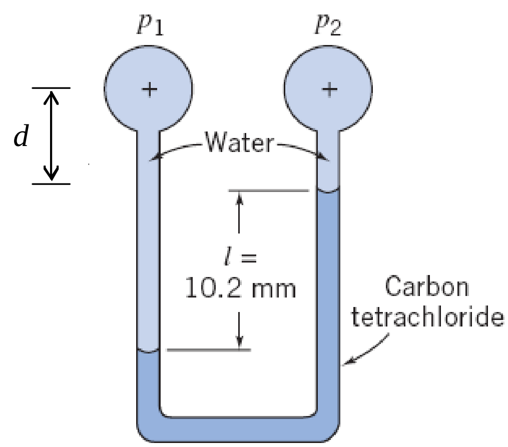
Solution: We will apply the hydrostatics equation.

Governing equations:

$$\frac{dp}{dh} = \rho \cdot g \qquad \text{(Hydrostatic Pressure - h is positive downwards)}$$

$$\rho = SG \cdot \rho_{water} \qquad \text{(Definition of Specific Gravity)}$$

Assumptions: (1) Static liquid
(2) Incompressible liquid



Starting at point 1 and progressing to point 2 we have:

$$p_1 + \rho_{water} \cdot g \cdot (d + l) - \rho_{ct} \cdot g \cdot l - \rho_{water} \cdot g \cdot d = p_2$$

Simplifying and solving for $p_2 - p_1$ we have:

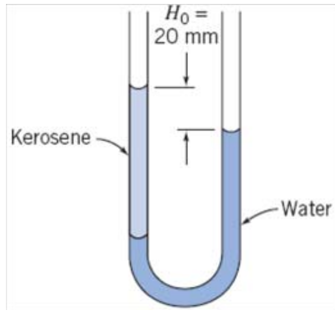
$$\Delta p = p_2 - p_1 = \rho_{ct} \cdot g \cdot l - \rho_{water} \cdot g \cdot l = \left(SG_{ct} - 1\right) \cdot \rho_{water} \cdot g \cdot l$$

Substituting the known data:

$$\Delta p = (1.591 - 1) \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 10.2 \cdot mm \times \frac{m}{10^3 \cdot mm} \qquad \Delta p = 59.1\,Pa$$

Problem 3.18 [Difficulty: 2]

3.18 The manometer shown contains water and kerosene. With both tubes open to the atmosphere, the free-surface elevations differ by $H_0 = 20.0$ mm. Determine the elevation difference when a pressure of 98.0 Pa (gage) is applied to the right tube.



Given: Two fluid manometer contains water and kerosene. With both tubes open to atmosphere, the difference in free surface elevations is known

$$H_o = 20 \cdot mm \quad SG_k = 0.82$$

Find: The elevation difference, H, between the free surfaces of the fluids when a gage pressure of 98.0 Pa is applied to the right tube.

Solution: We will apply the hydrostatics equation.

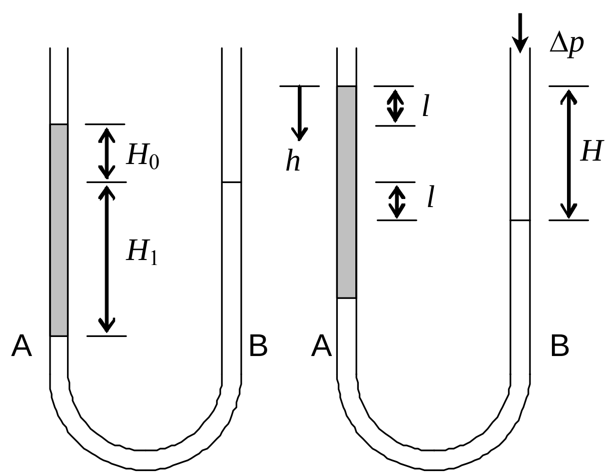
Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$\rho = SG \cdot \rho_{water}$$ (Definition of Specific Gravity)

Assumptions: (1) Static liquid
(2) Incompressible liquid

When the gage pressure Δp is applied to the right tube, the water in the right tube is displaced downward by a distance, l. The kerosene in the left tube is displaced upward by the same distance, l.



Under the applied gage pressure Δp, the elevation difference, H, is:

$$H = H_o + 2 \cdot l$$

Since points A and B are at the same elevation in the same fluid, their pressures are the same. Initially:

$$p_A = \rho_k \cdot g \cdot (H_o + H_1) \qquad p_B = \rho_{water} \cdot g \cdot H_1$$

Setting these pressures equal:

$$\rho_k \cdot g \cdot (H_o + H_1) = \rho_{water} \cdot g \cdot H_1$$

Solving for H_1

$$H_1 = \frac{\rho_k \cdot H_o}{\rho_{water} - \rho_k} = \frac{SG_k \cdot H_o}{1 - SG_k} \qquad H_1 = \frac{0.82 \times 20 \cdot mm}{1 - 0.82} \qquad H_1 = 91.11 \cdot mm$$

Now under the applied gage pressure:

$$p_A = \rho_k \cdot g \cdot (H_o + H_1) + \rho_{water} \cdot g \cdot l \qquad p_B = \Delta p + \rho_{water} \cdot g \cdot (H_1 - l)$$

Setting these pressures equal:

$$SG_k \cdot \left(H_o + H_1\right) + l = \frac{\Delta p}{\rho_{water} \cdot g} + \left(H_1 - l\right) \qquad l = \frac{1}{2}\left[\frac{\Delta p}{\rho_{water} \cdot g} + H_1 - SG_k \cdot \left(H_o + H_1\right)\right]$$

Substituting in known values we get:

$$l = \frac{1}{2} \times \left[98.0 \cdot \frac{N}{m^2} \times \frac{1}{999} \frac{m^3}{kg} \times \frac{1}{9.81} \cdot \frac{s^2}{m} \times \frac{kg \cdot m}{N \cdot s^2} + [91.11 \cdot mm - 0.82 \times (20 \cdot mm + 91.11 \cdot mm)] \times \frac{m}{10^3 \cdot mm}\right] \qquad l = 5.000 \cdot mm$$

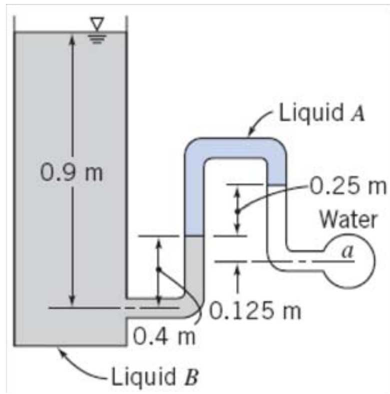
Now we solve for H:

$$H = 20 \cdot mm + 2 \times 5.000 \cdot mm \qquad H = 30.0 \cdot mm$$

Problem 3.19

[Difficulty: 2]

3.19 Determine the gage pressure in kPa at point *a*, if liquid *A* has SG = 1.20 and liquid *B* has SG = 0.75. The liquid surrounding point *a* is water, and the tank on the left is open to the atmosphere.

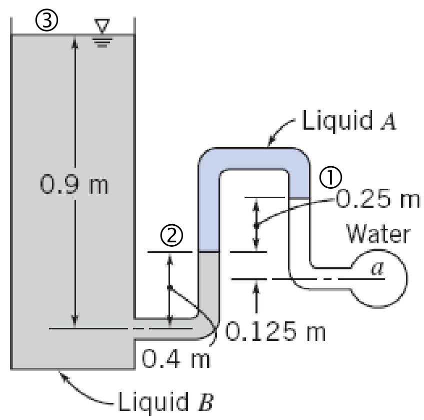


Given: Data on manometer

Find: Gage pressure at point a

Assumption: Water, liquids A and B are static and incompressible

Solution:



Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$

where Δh is height difference

Starting at point a $p_1 = p_a - \rho_{H2O} \cdot g \cdot h_1$ where $h_1 = 0.125 \cdot m + 0.25 \cdot m$ $h_1 = 0.375\,m$

Next, in liquid A $p_2 = p_1 + SG_A \cdot \rho_{H2O} \cdot g \cdot h_2$ where $h_2 = 0.25 \cdot m$

Finally, in liquid B $p_{atm} = p_2 - SG_B \cdot \rho_{H2O} \cdot g \cdot h_3$ where $h_3 = 0.9 \cdot m - 0.4 \cdot m$ $h_3 = 0.5\,m$

Combining the three equations

$$p_{atm} = \left(p_1 + SG_A \cdot \rho_{H2O} \cdot g \cdot h_2\right) - SG_B \cdot \rho_{H2O} \cdot g \cdot h_3 = p_a - \rho_{H2O} \cdot g \cdot h_1 + SG_A \cdot \rho_{H2O} \cdot g \cdot h_2 - SG_B \cdot \rho_{H2O} \cdot g \cdot h_3$$

$$p_a = p_{atm} + \rho_{H2O} \cdot g \cdot \left(h_1 - SG_A \cdot h_2 + SG_B \cdot h_3\right)$$

or in gage pressures $p_a = \rho_{H2O} \cdot g \cdot \left(h_1 - SG_A \cdot h_2 + SG_B \cdot h_3\right)$

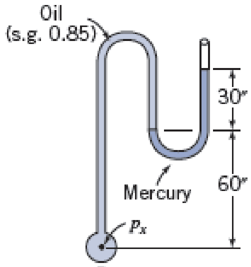
$$p_a = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times [0.375 - (1.20 \times 0.25) + (0.75 \times 0.5)] \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$

$p_a = 4.41 \times 10^3\,Pa$ $p_a = 4.41 \cdot kPa$ (gage)

Problem 3.20

(Difficulty: 1)

3.20 With the manometer reading as shown, calculate p_x.



P3.20

Given: Oil specific gravity: $SG_{oil} = 0.85$ Depth: $h_1 = 60\ inch.$ $h_2 = 30\ inch.$

Find: The pressure p_x.

Assumption: Fluids are incompressible

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

Integrating with respect to z for an incompressible fluid, we have the relation for the pressure difference over a difference in elevation (h):

$$\Delta p = \rho g h$$

Repeated application of this relation yields

$$p_x = SG_{oil}\gamma_{water}h_1 + \gamma_M h_2$$

The specific weight for mercury is:

$$\gamma_M = 845\ \frac{lbf}{ft^3}$$

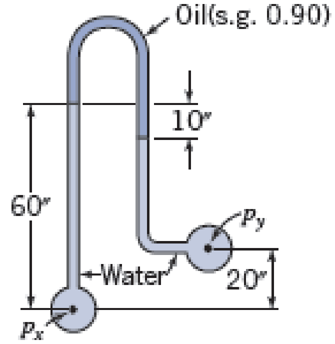
The pressure at the desired location is

$$p_x = 0.85 \times 62.4\ \frac{lbf}{ft^3} \times \left(\frac{60}{12}\right)\ ft + 845\ \frac{lbf}{ft^3} \times \left(\frac{30}{12}\right)\ ft = 2380\ \frac{lbf}{ft^2} = 16.5\ psi$$

Problem 3.21

(Difficulty: 2)

3.21 Calculate $p_x - p_y$ for this inverted U-tube manometer.



Given: Oil specific gravity: $SG_{oil} = 0.90$ Depth: $h_1 = 65\ inch.$ $h_2 = 20\ inch.$ $h_3 = 10\ inch.$

Find: The pressure difference $p_x - p_y$.

Assume: The fluids are incompressible

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

Integrating with respect to z for an incompressible fluid, we have the relation for the pressure difference over a difference in elevation (h):

$$\Delta p = \rho g h$$

Starting at the location of the unknown pressure p_x, we have the following relations for the hydrostatic pressure:

$$p_x - p_1 = \gamma_{water} h_1$$

$$p_1 - p_2 = -SG_{oil}\gamma_{water} h_3$$

$$p_2 - p_y = -\gamma_{water}(h_1 - h_2 - h_3)$$

Adding these three equations together

$$p_x - p_y = \gamma_{water}(h_2 + h_3) - SG_{oil}\gamma_{water} h_3$$

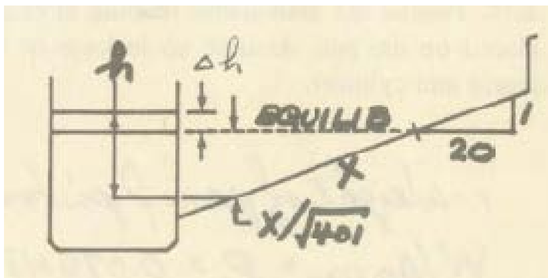
The pressure difference is then

$$p_x - p_y = 62.4\ \frac{lbf}{ft^3} \times \frac{(10+20)}{12} ft - 0.9 \times 62.4\ \frac{lbf}{ft^3} \times \frac{10}{12}\ ft = 109.2\ \frac{lbf}{ft^2} = 0.758\ psi$$

Problem 3.22

(Difficulty: 2)

3.22 An inclined gage having a tube of 3 mm bore, laid on a slope of 1:20, and a reservoir of 25 mm diameter contains silicon oil (SG 0.84). What distance will the oil move along the tube when a pressure of 25 mm of water is connected to the gage?



Given: Silicon oil specific gravity: $SG_{oil} = 0.84$. Diameter: $D_1 = 3\ mm$. $D_2 = 25\ mm$.

Depth: $h_{water} = 25\ mm$. Slope angle: $1:20$.

Find: The distance x of the oil move along the tube.

Assumption: Fluids are incompressible

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

Integrating with respect to z for an incompressible fluid, we have the relation for the pressure difference over a difference in elevation (h):

$$\Delta p = \rho g h$$

We have the volume of the oil as constant, so:

$$A_{reservoir} \Delta h = A_{tube} x$$

or

$$\frac{\Delta h}{x} = \frac{A_{tube}}{A_{reservoir}} = \frac{D_1^2}{D_2^2} = \frac{9}{625}$$

When a pressure of 25 mm of water is connected with the gage we have:

$$\gamma_{water} h_{water} = SG_{oil} \gamma_{water} h$$

$$h = \frac{h_{water}}{SG_{oil}} = 29.8\,mm$$

Using these relations, we obtain, accounting for the slope of the manometer:

$$h = \Delta h + \frac{x}{\sqrt{20^2 + 1^2}} = \left(\frac{9}{625} + \frac{1}{\sqrt{20^2 + 1^2}}\right)x$$

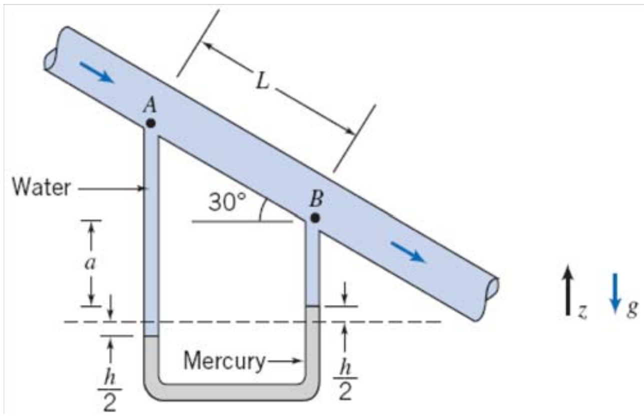
$$h = \Delta h + \frac{x}{\sqrt{401}} = \left(\frac{9}{625} + \frac{1}{\sqrt{401}}\right)x$$

$$x = \frac{h}{\left(\frac{9}{625} + \frac{1}{\sqrt{401}}\right)} = 463\ mm$$

Problem 3.23

[Difficulty: 2]

3.23 Water flows downward along a pipe that is inclined at 30° below the horizontal, as shown. Pressure difference $p_A - p_B$ is due partly to gravity and partly to friction. Derive an algebraic expression for the pressure difference. Evaluate the pressure difference if $L = 5$ ft and $h = 6$ in.



Given: Water flow in an inclined pipe as shown. The pressure difference is measured with a two-fluid manometer

$L = 5 \cdot ft \qquad h = 6 \cdot in \qquad SG_{Hg} = 13.55$

Find: Pressure difference between A and B

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$\rho = SG \cdot \rho_{water}$$ (Definition of Specific Gravity)

Assumptions:

(1) Static liquid
(2) Incompressible liquid
(3) Gravity is constant

Integrating the hydrostatic pressure equation we get:

$$\Delta p = \rho \cdot g \cdot \Delta h$$

Progressing through the manometer from A to B:

$$p_A + \rho_{water} \cdot g \cdot L \cdot \sin(30 \cdot deg) + \rho_{water} \cdot g \cdot a + \rho_{water} \cdot g \cdot h - \rho_{Hg} \cdot g \cdot h - \rho_{water} \cdot g \cdot a = p_B$$

Simplifying terms and solving for the pressure difference:

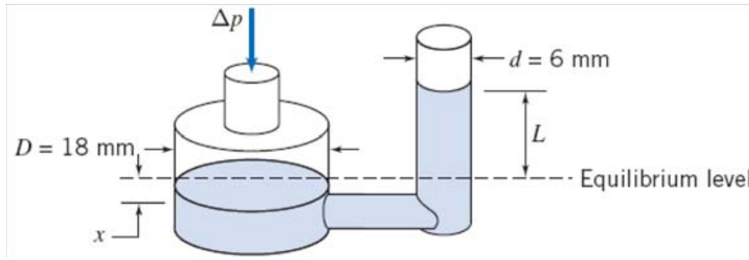
$$\Delta p = p_A - p_B = \rho_{water} \cdot g \cdot \left[h \cdot \left(SG_{Hg} - 1 \right) - L \cdot \sin(30 \cdot deg) \right]$$

Substituting in values:

$$\Delta p = 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \times \left[6 \cdot in \times \frac{ft}{12 \cdot in} \times (13.55 - 1) - 5 \cdot ft \times \sin(30 \cdot deg) \right] \times \frac{lbf \cdot s^2}{slug\,ft} \times \left(\frac{ft}{12 \cdot in} \right)^2 \qquad \Delta p = 1.638 \cdot psi$$

Problem 3.24 [Difficulty: 2]

3.24 A reservoir manometer has vertical tubes of diameter $D = 18$ mm and $d = 6$ mm. The manometer liquid is Meriam red oil. Develop an algebraic expression for liquid deflection L in the small tube when gage pressure Δp is applied to the reservoir. Evaluate the liquid deflection when the applied pressure is equivalent to 25 mm of water (gage).



Given: Reservoir manometer with vertical tubes of knowm diameter. Gage liquid is Meriam red oil

$D = 18 \cdot mm \quad d = 6 \cdot mm \qquad SG_{oil} = 0.827$

Find: The manometer deflection, L when a gage pressure equal to 25 mm of water is applied to the reservoir.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g \qquad \text{(Hydrostatic Pressure - h is positive downwards)}$$

$$\rho = SG \cdot \rho_{water} \qquad \text{(Definition of Specific Gravity)}$$

Assumptions: (1) Static liquid
(2) Incompressible liquid

Integrating the hydrostatic pressure equation we get:

$$\Delta p = \rho \cdot g \cdot \Delta h$$

Beginning at the free surface of the reservoir, and accounting for the changes in pressure with elevation:

$$p_{atm} + \Delta p + \rho_{oil} \cdot g \cdot (x + L) = p_{atm}$$

Upon simplification: $x + L = \dfrac{\Delta p}{\rho_{oil} \cdot g}$ The gage pressure is defined as: $\Delta p = \rho_{water} \cdot g \cdot \Delta h$ where $\Delta h = 25 \cdot mm$

Combining these two expressions: $x + L = \dfrac{\rho_{water} \cdot g \cdot h}{\rho_{oil} \cdot g} = \dfrac{\Delta h}{SG_{oil}}$

x and L are related through the manometer dimensions: $\dfrac{\pi}{4} \cdot D^2 \cdot x = \dfrac{\pi}{4} \cdot d^2 \cdot L \qquad x = \left(\dfrac{d}{D}\right)^2 L$

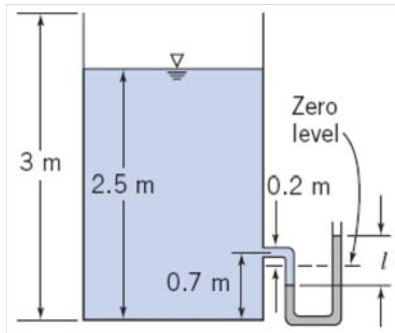
Therefore: $L = \dfrac{\Delta h}{SG_{oil} \cdot \left[1 + \left(\dfrac{d}{D}\right)^2\right]}$ Substituting values into the expression: $L = \dfrac{25 \cdot mm}{0.827 \cdot \left[1 + \left(\dfrac{6 \cdot mm}{18 \cdot mm}\right)^2\right]}$

(Note: $s = \dfrac{L}{\Delta h}$ which yields $s = 1.088$ for this manometer.) $L = 27.2 \cdot mm$

Problem 3.25

[Difficulty: 2]

3.25 A rectangular tank, open to the atmosphere, is filled with water to a depth of 2.5 m as shown. A U-tube manometer is connected to the tank at a location 0.7 m above the tank bottom. If the zero level of the Meriam blue manometer fluid is 0.2 m below the connection, determine the deflection l after the manometer is connected and all air has been removed from the connecting leg.



Given: A U-tube manometer is connected to the open tank filled with water as shown (manometer fluid is Meriam blue)

$D_1 = 2.5 \cdot m \quad D_2 = 0.7 \cdot m \quad d = 0.2 \cdot m \quad SG_{oil} = 1.75$

Find: The manometer deflection, l

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

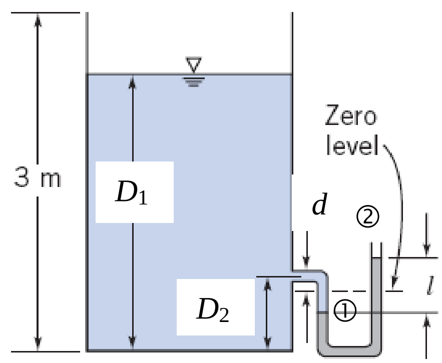
$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$\rho = SG \cdot \rho_{water}$$ (Definition of Specific Gravity)

Assumptions: (1) Static liquid
(2) Incompressible liquid

Integrating the hydrostatic pressure equation we get:

$$\Delta p = \rho \cdot g \cdot \Delta h$$



When the tank is filled with water, the oil in the left leg of the manometer is displaced downward by l/2. The oil in the right leg is displaced upward by the same distance, l/2.

Beginning at the free surface of the tank, and accounting for the changes in pressure with elevation:

$$p_{atm} + \rho_{water} \cdot g \cdot \left(D_1 - D_2 + d + \frac{l}{2} \right) - \rho_{oil} \cdot g \cdot l = p_{atm}$$

Upon simplification:

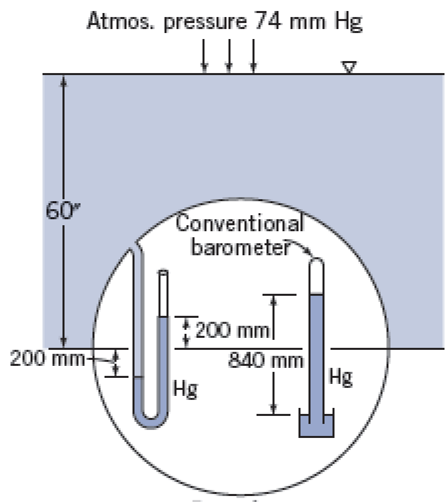
$$\rho_{water} \cdot g \cdot \left(D_1 - D_2 + d + \frac{l}{2} \right) = \rho_{oil} \cdot g \cdot l \qquad D_1 - D_2 + d + \frac{l}{2} = SG_{oil} \cdot l \qquad l = \frac{D_1 - D_2 + d}{SG_{oil} - \frac{1}{2}}$$

$$l = \frac{2.5 \cdot m - 0.7 \cdot m + 0.2 \cdot m}{1.75 - \frac{1}{2}} \qquad l = 1.600\, m$$

Problem 3.26

(Difficulty: 2)

3.26 The sketch shows a sectional view through a submarine. Calculate the depth of submarine, y. Assume the specific weight of the seawater is 10.0 $\frac{kN}{m^3}$.



P3.26

Given: Atmos. Pressure: $p_{atmos} = 740\ mm\ Hg$. Seawater specific weight:$\gamma = 10.0\ \frac{kN}{m^3}$. All the dimensional relationship is shown in the figure.

Find: The depth y.

Assumption: Fluids are incompressible

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

Integrating with respect to z for an incompressible fluid, we have the relation for the pressure difference over a difference in elevation (h):

$$\Delta p = \rho g h$$

Using the barometer reading with 760 mm as atmospheric pressure, the pressure inside the submarine is:

$$p = \frac{840\ mm}{760\ mm} \times 101.3 \times 10^3 Pa = 111.6 \times 10^3\ Pa$$

However, the actual atmosphere pressure is:

$$p_{atmos} = \frac{740\ mm}{760\ mm} \times 101.3 \times 10^3 Pa = 98.3 \times 10^3 Pa$$

For the manometer, using the hydrostatic relation, we have for the pressure, where y is the depth of the submarine:

$$p = p_{atmos} + \gamma y + \gamma \times 200\ mm - \gamma_{Hg} \times 400\ mm$$

$$y = \frac{p + \gamma_{Hg} \times 400\ mm - \gamma \times 200\ mm - p_{atmos}}{\gamma}$$

The specific weight for mercury is:

$$\gamma_{Hg} = 133.1\ \frac{kN}{m^3}$$

So we have for the depth y:

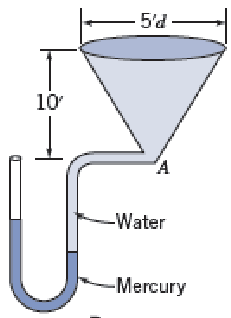
$$y = \frac{111.6 \times 10^3\ Pa + 133.1\ \times 1000\ \frac{N}{m^3} \times 0.4\ m - 1000 \frac{N}{m^3} \times 0.2\ m - 98.3 \times 10^3\ Pa}{1000\ \frac{N}{m^3}}$$

$$y = 6.45\ m$$

Problem 3.27

(Difficulty: 1)

3.27 The manometer reading is 6 in. when the tank is empty (water surface at A). Calculate the manometer reading when the cone is filled with water.



P3.27

Find: The manometer reading when the tank is filled with water.

Assumption: Fluids are static and incompressible

Solution: Use the hydrostatic relations for pressure

When the tank is empty, we have the equation as:

$$h_{MR} \cdot SG_{mercury} \cdot \gamma_{water} = \gamma_{water} h$$

$$SG_{mercury} = 13.57$$

$$h = h_{MR} \cdot SG_{mercury} = 150\ mm \times 13.57 = 2.04\ m$$

When the tank is filled with water, we assume the mercury interface moves by x:

$$\gamma_{water}(h_{tank} + h + x) = \gamma_{water} \cdot SG_{mercury}(h_{MR} + 2x)$$

$$(3\ m + 2.04\ m + x) = 13.57(0.15m + 2x)$$

Thus

$$x = 0.115\ m$$

The new manometer reading is:

$$h'_{MR} = h_{MR} + 2x = 0.15\ m + 2 \times 0.115\ m = 0.38\ m$$

Problem 3.28 [Difficulty: 2]

3.28 A reservoir manometer is calibrated for use with a liquid of specific gravity 0.827. The reservoir diameter is 5/8 in. and the (vertical) tube diameter is 3/16 in. Calculate the required distance between marks on the vertical scale for 1 in. of water pressure difference.

Given: Reservoir manometer with dimensions shown. The manometer fluid specific gravity is given.

$$D = \frac{5}{8}\cdot in \qquad d = \frac{3}{16}\cdot in \quad SG_{oil} = 0.827$$

Find: The required distance between vertical marks on the scale corresponding to Δp of 1 in water.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

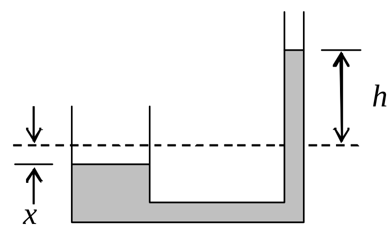
$$\frac{dp}{dz} = -\rho\cdot g$$ (Hydrostatic Pressure - z is positive upwards)

$$\rho = SG\cdot \rho_{water}$$ (Definition of Specific Gravity)

Assumptions: (1) Static liquid
(2) Incompressible liquid

Integrating the hydrostatic pressure equation we get:

$$\Delta p = -\rho\cdot g\cdot \Delta z$$



Beginning at the free surface of the tank, and accounting for the changes in pressure with elevation:

$$p_{atm} + \Delta p - \rho_{oil}\cdot g\cdot(x + h) = p_{atm}$$

Upon simplification: $\Delta p = \rho_{oil}\cdot g\cdot(x + h)$ The applied pressure is defined as: $\Delta p = \rho_{water}\cdot g\cdot l$ where $l = 1\cdot in$

Therefore: $\rho_{water}\cdot g\cdot l = \rho_{oil}\cdot g\cdot(x + h)$ $\qquad x + h = \dfrac{l}{SG_{oil}}$

x and h are related through the manometer dimensions: $\dfrac{\pi}{4}\cdot D^2\cdot x = \dfrac{\pi}{4}\cdot d^2\cdot h \qquad x = \left(\dfrac{d}{D}\right)^2 h$

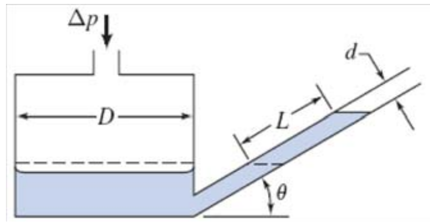
Solving for h: $h = \dfrac{l}{SG_{oil}\cdot\left[1 + \left(\dfrac{d}{D}\right)^2\right]}$ Substituting values into the expression: $h = \dfrac{1\cdot in}{0.827\cdot\left[1 + \left(\dfrac{0.1875\cdot in}{0.625\cdot in}\right)^2\right]}$

$$h = 1.109\cdot in$$

Problem 3.29

[Difficulty: 3]

3.29 The inclined-tube manometer shown has $D = 96$ mm and $d = 8$ mm. Determine the angle, θ, required to provide a 5 : 1 increase in liquid deflection, L, compared with the total deflection in a regular U-tube manometer. Evaluate the sensitivity of this inclined-tube manometer.



Given: Inclined manometer as shown.

$D = 96 \cdot mm \quad d = 8 \cdot mm$

Angle θ is such that the liquid deflection L is five times that of a regular U-tube manometer.

Find: Angle θ and manometer sensitivity.

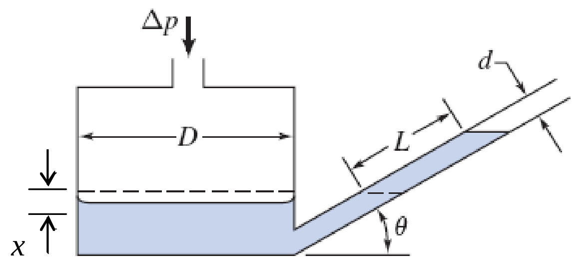
Solution: We will apply the hydrostatics equations to this system.

Governing Equation:

$$\frac{dp}{dz} = -\rho \cdot g$$

(Hydrostatic Pressure - z is positive upwards)

Assumptions: (1) Static liquid
(2) Incompressible liquid



Integrating the hydrostatic pressure equation we get:

$$\Delta p = -\rho \cdot g \cdot \Delta z$$

Applying this equation from point 1 to point 2:

$$p_1 - \rho \cdot g \cdot (x + L \cdot \sin(\theta)) = p_2$$

Upon simplification: $$p_1 - p_2 = \rho \cdot g \cdot (x + L \cdot \sin(\theta))$$

Since the volume of the fluid must remain constant: $$\frac{\pi}{4} \cdot D^2 \cdot x = \frac{\pi}{4} \cdot d^2 \cdot L \qquad x = \left(\frac{d}{D}\right)^2 \cdot L$$

Therefore: $$p_1 - p_2 = \rho \cdot g \cdot L \cdot \left[\left(\frac{d}{D}\right)^2 + \sin(\theta)\right]$$

Now for a U-tube manometer: $$p_1 - p_2 = \rho \cdot g \cdot h$$ Hence: $$\frac{p_{1incl} - p_{2incl}}{p_{1U} - p_{2U}} = \frac{\rho \cdot g \cdot L \cdot \left[\left(\frac{d}{D}\right)^2 + \sin(\theta)\right]}{\rho \cdot g \cdot h}$$

For equal applied pressures: $$L \cdot \left[\left(\frac{d}{D}\right)^2 + \sin(\theta)\right] = h$$ Since L/h = 5: $$\sin(\theta) = \frac{h}{L} - \left(\frac{d}{D}\right)^2 = \frac{1}{5} - \left(\frac{8 \cdot mm}{96 \cdot mm}\right)^2$$

$$\theta = 11.13 \cdot deg$$

The sensitivity of the manometer: $$s = \frac{L}{\Delta h_e} = \frac{L}{SG \cdot h}$$

$$s = \frac{5}{SG}$$

Problem 3.30 [Difficulty: 3]

3.30 The inclined-tube manometer shown has $D = 76$ mm and $d = 8$ mm, and is filled with Meriam red oil. Compute the angle, θ, that will give a 15-cm oil deflection along the inclined tube for an applied pressure of 25 mm of water (gage). Determine the sensitivity of this manometer.

Given: Data on inclined manometer

Find: Angle θ for given data; find sensitivity

Solution:

Basic equation $\frac{dp}{dy} = -\rho \cdot g$ or, for constant ρ $\Delta p = \rho \cdot g \cdot \Delta h$ where Δh is height difference

Under applied pressure $\Delta p = SG_{Mer} \cdot \rho \cdot g \cdot (L \cdot \sin(\theta) + x)$ (1)

From Table A.1 $SG_{Mer} = 0.827$

and Δp = 1 in. of water, or $\Delta p = \rho \cdot g \cdot h$ where $h = 25 \cdot mm$ $h = 0.025\, m$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.025 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 245\, Pa$$

The volume of liquid must remain constant, so $x \cdot A_{res} = L \cdot A_{tube}$ $x = L \cdot \frac{A_{tube}}{A_{res}} = L \cdot \left(\frac{d}{D}\right)^2$ (2)

Combining Eqs 1 and 2 $\Delta p = SG_{Mer} \cdot \rho \cdot g \cdot \left[L \cdot \sin(\theta) + L \cdot \left(\frac{d}{D}\right)^2\right]$

Solving for θ $\sin(\theta) = \frac{\Delta p}{SG_{Mer} \cdot \rho \cdot g \cdot L} - \left(\frac{d}{D}\right)^2$

$$\sin(\theta) = 245 \cdot \frac{N}{m^2} \times \frac{1}{0.827} \times \frac{1}{1000} \cdot \frac{m^3}{kg} \times \frac{1}{9.81} \cdot \frac{s^2}{m} \times \frac{1}{0.15} \cdot \frac{1}{m} \times \frac{kg \cdot m}{s^2 \cdot N} - \left(\frac{8}{76}\right)^2 = 0.186$$

$$\theta = 11 \cdot deg$$

The sensitivity is the ratio of manometer deflection to a vertical water manometer

$$s = \frac{L}{h} = \frac{0.15 \cdot m}{0.025 \cdot m} \qquad s = 6$$

Problem 3.31 [Difficulty: 4]

3.31 A barometer accidentally contains 6.5 inches of water on top of the mercury column (so there is also water vapor instead of a vacuum at the top of the barometer). On a day when the temperature is 70°F, the mercury column height is 28.35 inches (corrected for thermal expansion). Determine the barometric pressure in psia. If the ambient temperature increased to 85°F and the barometric pressure did not change, would the mercury column be longer, be shorter, or remain the same length? Justify your answer.

Given: Barometer with water on top of the mercury column, Temperature is known:

$h_2 = 6.5 \cdot in$ $\quad h_1 = 28.35 \cdot in$ $\quad SG_{Hg} = 13.55$ (From Table A.2, App. A) $\quad T = 70\,°F$

$p_v = 0.363 \cdot psi$ (From Table A.7, App. A)

Find: (a) Barometric pressure in psia
(b) Effect of increase in ambient temperature on length of mercury column for the same barometric pressure: $T_f = 85\,°F$

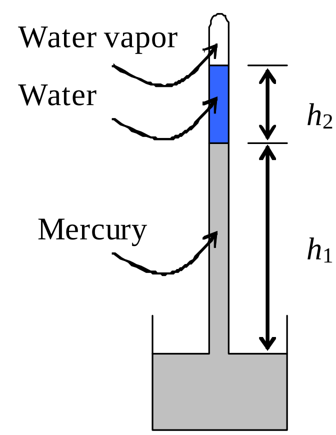
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = -\rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$\rho = SG \cdot \rho_{water}$$ (Definition of Specific Gravity)

Assumptions: (1) Static liquid
(2) Incompressible liquid



Integrating the hydrostatic pressure equation we get:

$$\Delta p = \rho \cdot g \cdot \Delta h$$

Start at the free surface of the mercury and progress through the barometer to the vapor pressure of the water:

$$p_{atm} - \rho_{Hg} \cdot g \cdot h_1 - \rho_{water} \cdot g \cdot h_2 = p_v$$

$$p_{atm} = p_v + \rho_{water} \cdot g \cdot \left(SG_{Hg} \cdot h_1 + h_2\right)$$

$$p_{atm} = 0.363 \cdot \frac{lbf}{in^2} + 1.93 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times (13.55 \times 28.35 \cdot in + 6.5 \cdot in) \times \left(\frac{ft}{12 \cdot in}\right)^3 \qquad p_{atm} = 14.41 \cdot \frac{lbf}{in^2}$$

At the higher temperature, the vapor pressure of water increases to 0.60 psi. Therefore, if the atmospheric pressure were to remain constant, the length of the mercury column would have to decrease - the increased water vapor would push the mercury out of the tube!

Problem 3.32 [Difficulty: 3]

3.32 A water column stands 50 mm high in a 2.5-mm diameter glass tube. What would be the column height if the surface tension were zero? What would be the column height in a 1.0-mm diameter tube?

Given: Water column standin in glass tube

$$\Delta h = 50 \cdot mm \quad D = 2.5 \cdot mm \quad \sigma = 72.8 \times 10^{-3} \frac{N}{m}$$

Find: (a) Column height if surface tension were zero.
(b) Column height in 1 mm diameter tube

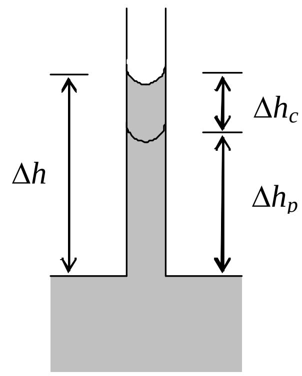
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g \qquad \text{(Hydrostatic Pressure - h is positive downwards)}$$

$$\Sigma F_z = 0 \qquad \text{(Static Equilibrium)}$$

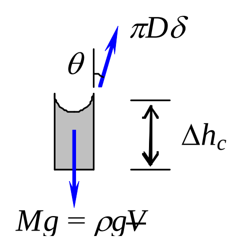
Assumptions:
(1) Static, incompressible liquid
(2) Neglect volume under meniscus
(3) Applied pressure remains constant
(4) Column height is sum of capillary rise and pressure difference



Assumption #4 can be written as: $\Delta h = \Delta h_c + \Delta h_p$

Choose a free-body diagram of the capillary rise portion of the column for analysis:

$$\Sigma F_z = \pi \cdot D \cdot \sigma \cdot \cos(\theta) - \frac{\pi}{4} \cdot D^2 \cdot \rho \cdot g \cdot \Delta h_c = 0 \qquad \text{Therefore:} \quad \Delta h_c = \frac{4 \cdot \sigma}{\rho \cdot g \cdot D} \cdot \cos(\theta)$$



Substituting values:

$$\Delta h_c = 4 \times 72.8 \times 10^{-3} \cdot \frac{N}{m} \times \frac{1}{999} \cdot \frac{m^3}{kg} \times \frac{1}{9.81} \cdot \frac{s^2}{m} \times \frac{1}{2.5} \cdot \frac{1}{mm} \times \frac{kg \cdot m}{N \cdot s^2} \times \left(\frac{10^3 \cdot mm}{m}\right)^2$$

$$\Delta h_c = 11.89 \cdot mm$$

Therefore: $\Delta h_p = \Delta h - \Delta h_c \quad \Delta h_p = 50 \cdot mm - 11.89 \cdot mm \qquad \Delta h_p = 38.1 \cdot mm$ (result for $\sigma = 0$)

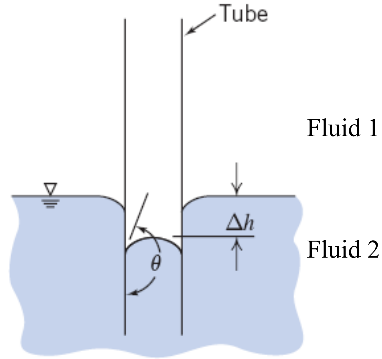
For the 1 mm diameter tube:

$$\Delta h_c = 4 \times 72.8 \times 10^{-3} \cdot \frac{N}{m} \times \frac{1}{999} \cdot \frac{m^3}{kg} \times \frac{1}{9.81} \cdot \frac{s^2}{m} \times \frac{1}{1} \cdot \frac{1}{mm} \times \frac{kg \cdot m}{N \cdot s^2} \times \left(\frac{10^3 \cdot mm}{m}\right)^2 \qquad \Delta h_c = 29.71 \cdot mm$$

$$\Delta h = 29.7 \cdot mm + 38.1 \cdot mm \qquad \Delta h = 67.8 \cdot mm$$

Problem 3.33 [Difficulty :2]

3.33 Consider a small-diameter open-ended tube inserted at the interface between two immiscible fluids of different densities. Derive an expression for the height difference Δh between the interface level inside and outside the tube in terms of tube diameter D, the two fluid densities ρ_1 and ρ_2, and the surface tension σ and angle θ for the two fluids' interface. If the two fluids are water and mercury, find the height difference if the tube diameter is 40 mils (1 mil = 0.001 in.).

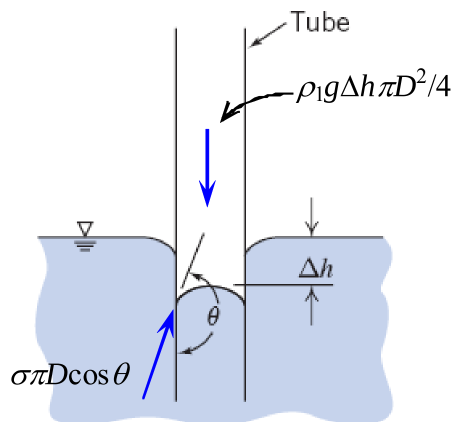


Given: Two fluids inside and outside a tube

Find: (a) An expression for height Δh
(b) Height difference when D =0.040 in for water/mercury

Assumptions: (1) Static, incompressible fluids
(2) Neglect meniscus curvature for column height and volume calculations

Solution:



A free-body vertical force analysis for the section of fluid 1 height Δh in the tube below the "free surface" of fluid 2 leads to

$$\sum F = 0 = \Delta p \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot \sigma \cdot \cos(\theta)$$

where Δp is the pressure difference generated by fluid 2 over height Δh, $\quad \Delta p = \rho_2 \cdot g \cdot \Delta h$

Hence $\quad \Delta p \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} = \rho_2 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} - \rho_1 \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4} = -\pi \cdot D \cdot \sigma \cdot \cos(\theta)$

Solving for Δh $\quad \Delta h = -\frac{4 \cdot \sigma \cdot \cos(\theta)}{g \cdot D \cdot (\rho_2 - \rho_1)}$

For fluids 1 and 2 being water and mercury (for mercury σ = 375 mN/m and $\theta = 140^o$, from Table A.4), solving for Δh when D = 0.040 in

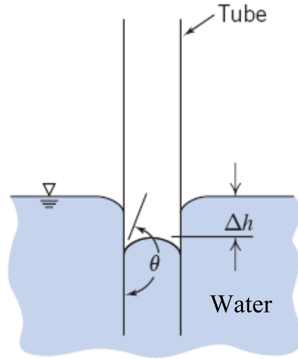
$$\Delta h = -4 \times 0.375 \cdot \frac{N}{m} \times \frac{lbf}{4.448 \cdot N} \times \frac{0.0254m}{in} \times \cos(140 \cdot deg) \times \frac{s^2}{32.2 \cdot ft} \times \frac{1}{0.040 \cdot in} \times \frac{ft^3}{1.94 \cdot slug} \times \left(\frac{12 \cdot in}{ft}\right)^3 \times \frac{1}{(13.6 - 1)} \times \frac{slug\,ft}{lbf \cdot s^2}$$

$$\Delta h = 0.360 \cdot in$$

Problem 3.34

[Difficulty: 2]

3.34 Compare the height due to capillary action of water exposed to air in a circular tube of diameter $D = 0.5$ mm, and between two infinite vertical parallel plates of gap $a = 0.5$ mm.



Given: Water in a tube or between parallel plates

Find: Height Δ*h* for each system

Solution:

a) Tube: A free-body vertical force analysis for the section of water height Δ*h* above the "free surface" in the tube, as shown in the figure, leads to

$$\sum F = 0 = \pi \cdot D \cdot \sigma \cdot \cos(\theta) - \rho \cdot g \cdot \Delta h \cdot \frac{\pi \cdot D^2}{4}$$

Assumption: Neglect meniscus curvature for column height and volume calculations

Solving for Δ*h* $\quad \Delta h = \dfrac{4 \cdot \sigma \cdot \cos(\theta)}{\rho \cdot g \cdot D}$

b) Parallel Plates: A free-body vertical force analysis for the section of water height Δ*h* above the "free surface" between plates arbitrary width *w* (similar to the figure above), leads to

$$\sum F = 0 = 2 \cdot w \cdot \sigma \cdot \cos(\theta) - \rho \cdot g \cdot \Delta h \cdot w \cdot a$$

Solving for Δ*h* $\quad \Delta h = \dfrac{2 \cdot \sigma \cdot \cos(\theta)}{\rho \cdot g \cdot a}$

For water σ = 72.8 mN/m and θ = 0º (Table A.4), so

a) Tube $\quad \Delta h = \dfrac{4 \times 0.0728 \cdot \frac{N}{m}}{999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.005 \cdot m} \times \dfrac{kg \cdot m}{N \cdot s^2} \qquad \Delta h = 5.94 \times 10^{-3} m \qquad \Delta h = 5.94 \cdot mm$

b) Parallel Plates $\quad \Delta h = \dfrac{2 \times 0.0728 \cdot \frac{N}{m}}{999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 0.005 \cdot m} \times \dfrac{kg \cdot m}{N \cdot s^2} \qquad \Delta h = 2.97 \times 10^{-3} m \qquad \Delta h = 2.97 \cdot mm$

Problem 3.35

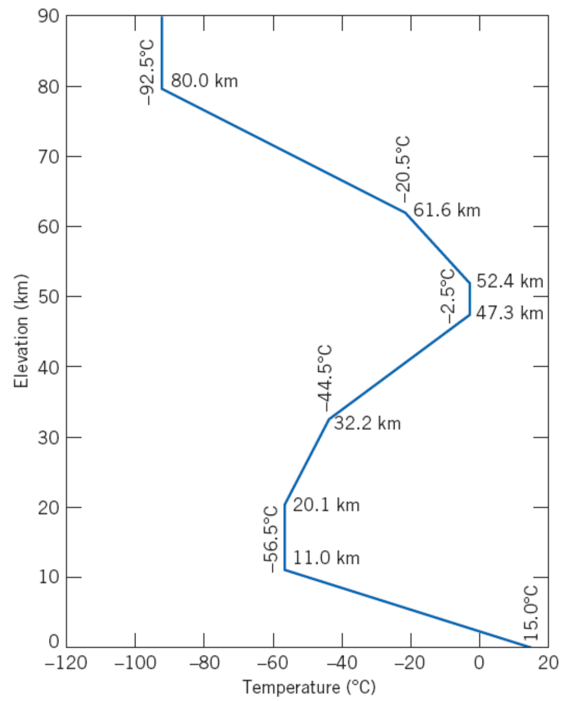
[Difficulty: 3]

3.35 Based on the atmospheric temperature data of the U.S. Standard Atmosphere of Fig. 3.3, compute and plot the pressure variation with altitude, and compare with the pressure data of Table A.3.

Given: Atmospheric temperature data

Find: Pressure variation; compare to Table A.3

Solution:



From Table A.3

From Section 3-3: $\dfrac{dp}{dz} = -\rho \cdot z$

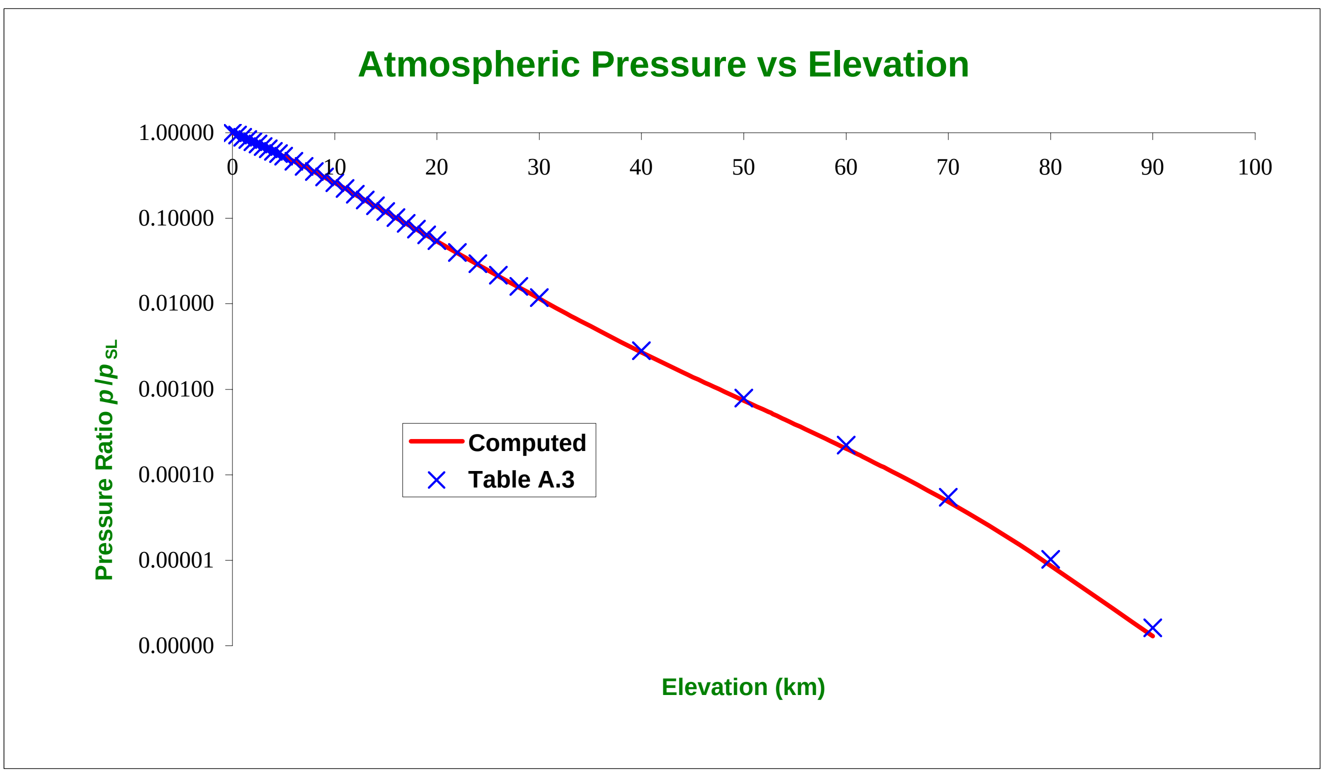
For a linear temperature variation $m = -\dfrac{dT}{dz} = \text{const}$ $\quad p = p_0 \cdot \left(\dfrac{T}{T_0}\right)^{\frac{g}{m \cdot R}}$

For isothermal conditions (Example 3.4) $\quad p = p_0 \cdot e^{-\frac{g \cdot (z - z_0)}{R \cdot T}}$

In these equations p_0, T_0, and z_0 are reference conditions

p_{SL} =	101	kPa
R =	286.9	J/kg.K
ρ =	999	kg/m^3

The temperature can be computed from the data in the figure.
The pressures are then computed from the appropriate equation.



Agreement between calculated and tabulated data is very good (as it should be, considering the table data are also computed!)

z (km)	T (°C)	T (K)		p/p_{SL}
0.0	15.0	288.0	m =	1.000
2.0	2.0	275.00	0.0065	0.784
4.0	-11.0	262.0	(K/m)	0.608
6.0	-24.0	249.0		0.465
8.0	-37.0	236.0		0.351
11.0	-56.5	216.5		0.223
12.0	-56.5	216.5	T = const	0.190
14.0	-56.5	216.5		0.139
16.0	-56.5	216.5		0.101
18.0	-56.5	216.5		0.0738
20.1	-56.5	216.5		0.0530
22.0	-54.6	218.4	m =	0.0393
24.0	-52.6	220.4	-0.000991736	0.0288
26.0	-50.6	222.4	(K/m)	0.0211
28.0	-48.7	224.3		0.0155
30.0	-46.7	226.3		0.0115
32.2	-44.5	228.5		0.00824
34.0	-39.5	233.5	m =	0.00632
36.0	-33.9	239.1	-0.002781457	0.00473
38.0	-28.4	244.6	(K/m)	0.00356
40.0	-22.8	250.2		0.00270
42.0	-17.2	255.8		0.00206
44.0	-11.7	261.3		0.00158
46.0	-6.1	266.9		0.00122
47.3	-2.5	270.5		0.00104
50.0	-2.5	270.5	T = const	0.000736
52.4	-2.5	270.5		0.000544
54.0	-5.6	267.4	m =	0.000444
56.0	-9.5	263.5	0.001956522	0.000343
58.0	-13.5	259.5	(K/m)	0.000264
60.0	-17.4	255.6		0.000202
61.6	-20.5	252.5		0.000163
64.0	-29.9	243.1	m =	0.000117
66.0	-37.7	235.3	0.003913043	0.0000880
68.0	-45.5	227.5	(K/m)	0.0000655
70.0	-53.4	219.6		0.0000482
72.0	-61.2	211.8		0.0000351
74.0	-69.0	204.0		0.0000253
76.0	-76.8	196.2		0.0000180
78.0	-84.7	188.3		0.0000126
80.0	-92.5	180.5	T = const	0.00000861
82.0	-92.5	180.5		0.00000590
84.0	-92.5	180.5		0.00000404
86.0	-92.5	180.5		0.00000276
88.0	-92.5	180.5		0.00000189
90.0	-92.5	180.5		0.00000130

z (km)	p/p_{SL}
0.0	1.000
0.5	0.942
1.0	0.887
1.5	0.835
2.0	0.785
2.5	0.737
3.0	0.692
3.5	0.649
4.0	0.609
4.5	0.570
5.0	0.533
6.0	0.466
7.0	0.406
8.0	0.352
9.0	0.304
10.0	0.262
11.0	0.224
12.0	0.192
13.0	0.164
14.0	0.140
15.0	0.120
16.0	0.102
17.0	0.0873
18.0	0.0747
19.0	0.0638
20.0	0.0546
22.0	0.0400
24.0	0.0293
26.0	0.0216
28.0	0.0160
30.0	0.0118
40.0	0.00283
50.0	0.000787
60.0	0.000222
70.0	0.0000545
80.0	0.0000102
90.0	0.00000162

Problem 3.36 [Difficulty: 3]

3.36 At ground level in Denver, Colorado, the atmospheric pressure and temperature are 83.2 kPa and 25°C. Calculate the pressure on Pike's Peak at an elevation of 2690 m above the city assuming (a) an incompressible and (b) an adiabatic atmosphere. Plot the ratio of pressure to ground level pressure in Denver as a function of elevation for both cases.

Given: Atmospheric conditions at ground level ($z = 0$) in Denver, Colorado are $p_0 = 83.2$ kPa, $T_0 = 25°C$. Pike's peak is at elevation z = 2690 m.

Find: p/p_0 vs z for both cases.

Solution:

Governing Equations: $\frac{dp}{dz} = -\rho\cdot g$ $\qquad p = \rho\cdot R\cdot T$

Assumptions: (1) Static fluid
(2) Ideal gas behavior

(a) For an incompressible atmosphere:

$$\frac{dp}{dz} = -\rho\cdot g \quad \text{becomes} \quad p - p_0 = -\int_0^z \rho\cdot g\,dz \quad \text{or} \quad p = p_0 - \rho_0\cdot g\cdot z = p_0\cdot\left(1 - \frac{g\cdot z}{R\cdot T_0}\right) \qquad (1)$$

At $z = 2690\cdot m$

$$p = 83.2\cdot kPa \times \left(1 - 9.81\cdot\frac{m}{s^2} \times 2690\cdot m \times \frac{kg\cdot K}{287\cdot N\cdot m} \times \frac{1}{298\cdot K} \times \frac{N\cdot s^2}{kg\cdot m}\right) \qquad p = 57.5\cdot kPa$$

(b) For an adiabatic atmosphere: $\frac{p}{\rho^k} = const \qquad \rho = \rho_0\cdot\left(\frac{p}{p_0}\right)^{\frac{1}{k}}$

$$\frac{dp}{dz} = -\rho\cdot g \quad \text{becomes} \quad dp = -\rho_0\cdot\left(\frac{p}{p_0}\right)^{\frac{1}{k}}\cdot g\cdot dz \quad \text{or} \quad \frac{1}{p^{\frac{1}{k}}}dp = -\frac{\rho_0\cdot g}{p_0^{\frac{1}{k}}}\cdot dz$$

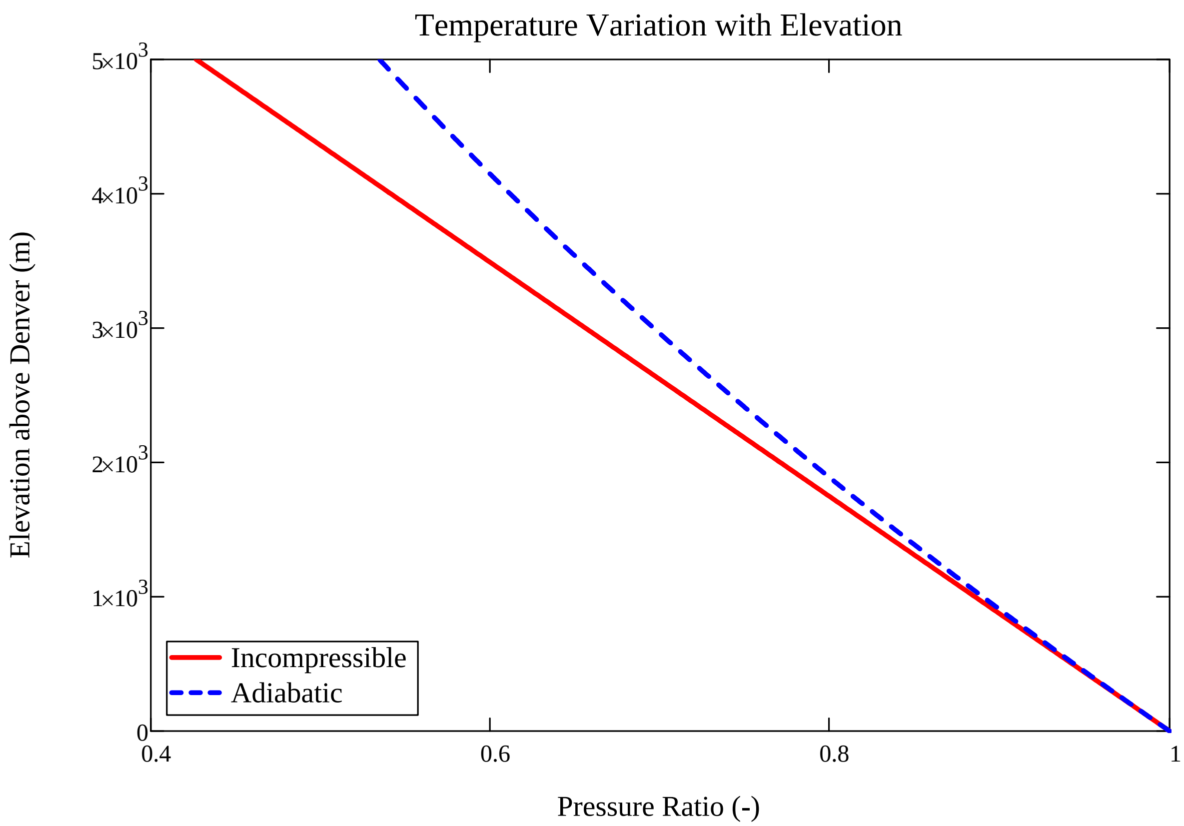
But
$$\int_{p_0}^{p} \frac{1}{p^{\frac{1}{k}}}\,dp = \frac{k}{k-1}\cdot\left(p - p_0\right)^{\frac{k-1}{k}} \quad \text{hence} \quad \frac{k}{k-1}\cdot\left(p^{\frac{k-1}{k}} - p_0^{\frac{k-1}{k}}\right) = -\frac{\rho_0\cdot g}{p_0^{\frac{1}{k}}}\cdot g\cdot z$$

Solving for the pressure ratio
$$\frac{p}{p_0} = \left(1 - \frac{k-1}{k}\cdot\frac{\rho_0}{p_0}\cdot g\cdot z\right)^{\frac{k}{k-1}} \quad \text{or} \quad \frac{p}{p_0} = \left(1 - \frac{k-1}{k}\cdot\frac{g\cdot z}{R\cdot T_0}\right)^{\frac{k}{k-1}} \qquad (2)$$

At $z = 2690\cdot m$

$$p = 83.2\cdot kPa \times \left(1 - \frac{1.4-1}{1.4} \times 9.81\cdot\frac{m}{s^2} \times 2690\cdot m \times \frac{kg\cdot K}{287\cdot N\cdot m} \times \frac{1}{298\cdot K} \times \frac{N\cdot s^2}{kg\cdot m}\right)^{\frac{1.4}{1.4-1}} \qquad p = 60.2\cdot kPa$$

Equations 1 and 2 can be plotted:



Problem 3.37

(Difficulty: 2)

3.37 If atmospheric pressure at the ground is $101.3\ kPa$ and temperature is $15\ °C$, calculate the pressure $7.62\ km$ above the ground, assuming (a) no density variation, (b) isothermal variation of density with pressure, and (c) adiabatic variation of density with pressure.

Assumption: Atmospheric air is stationary and behaves as an ideal gas.

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$\frac{dp}{dz} = -\rho\, g = -\gamma$$

(a) For this case with no density variation, we integrate with respect to z from the ground level pressure p_0 to the pressure at any height h. The pressure is

$$p = p_0 - \gamma h$$

From Table A.10, the density of air at sea level is

$$\rho = 1.23\ \frac{kg}{m^3}$$

Or the specific weight is

$$\gamma = \rho g = 1.23 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} = 12.07\ \frac{N}{m^3}$$

Thus the pressure at 7.62 km is

$$p = 101.3\ kPa - 12.07\ \frac{N}{m^3} \times 7.62 \times 1000\ m = 9.63\ kPa$$

(b) For isothermal condition we have for an ideal gas:

$$\frac{p}{\rho} = \frac{p_0}{\rho_0} = RT = constant$$

Therefore, since $\rho = \gamma$ g and g is a constant

$$\frac{p}{\gamma} = \frac{p_0}{\gamma_0} = \frac{101.3\ kPa}{12.07\ \frac{N}{m^3}} = 8420\ m = constant$$

From the hydrostatic relation we have:

$$dp = -\gamma dz$$

$$\frac{dp}{p} = -\frac{\gamma}{p} dz$$

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{1}{8420m}\int_0^z dz$$

$$\ln\left(\frac{p}{p_0}\right) = -\frac{1}{8420m} z$$

Thus the pressure at 7.62 km is

$$\frac{p}{p_0} = e^{- -\frac{7620\ m}{8420m}} = \ e^{-\ 0.905} = 0.4045$$

$$p = 101.3kPa \times 0.4045 = \ 41.0\ kPa$$

(c) For a reversible and adiabatic variation of density we have:

$$pv^k = \frac{p}{\rho^k} = constant$$

Where k is the specific heat ratio

$$k = 1.4$$

Or, since gravity g is constant, we can write in terms of the specific weight

$$\frac{p}{\gamma^k} = \frac{p_0}{{\gamma_0}^k} = constant$$

Or the specific weight is

$$\gamma = \gamma_0 \left(\frac{p}{p_0}\right)^{1/k}$$

The hydrostatic expression becomes

$$dp = -\gamma_0 \left(\frac{p}{p_0}\right)^{1/k} dz$$

Separating variables

$$\frac{p_0^{1/k}}{\gamma_0}\int_{p_0}^{p} \frac{dp}{(p)^{1/k}} = -\int_0^z dz$$

Integrating between the limits $p=p_0$ at z=0 and p = p at z = z

$$\left(\frac{k}{k-1}\right)\frac{p_0^{1/k}}{\gamma_0}\left[p^{\frac{k-1}{k}} - {p_0}^{\frac{k-1}{k}}\right] = -\ z$$

Or

$$\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = 1 - \left(\frac{k-1}{k}\right)\frac{\gamma_0 z}{p_0}$$

The pressure is then

$$p = p_0\left[1 - \left(\frac{k-1}{k}\right)\frac{\gamma_0 z}{p_0}\right]^{k/_{k-1}} = 101.3kPa\left[1 - \left(\frac{1.4-1}{1.4}\right) \times \frac{12.07\ \frac{N}{m^3} \times 7620m}{101.3 \times 1000\ Pa}\right]^{1.4/_{1.4-1}}$$

$$p = 35.4\ kPa$$

The calculation of pressure depends heavily on the assumption we make about how density changes.

Problem 3.38

(Difficulty: 2)

3.38 If the temperature in the atmosphere is assumed to vary linearly with altitude so $T = T_0 - \alpha z$ where T_0 is the sea level temperature and $\alpha = - dT / dz$ is the temperature lapse rate, find p(z) when air is taken to be a perfect gas. Give the answer in terms of p_0, a, g, R, and z only.

Assumption: Atmospheric air is stationary and behaves as an ideal gas.

Solution: Use the hydrostatic relation to find the pressures in the fluid

Governing equation: Hydrostatic pressure in a liquid, with z measured upward:

$$dp = -\gamma dz$$

The ideal gas relation is

$$\frac{p}{\rho} = RT$$

Or in terms of the specific weight, the pressure is

$$p = \rho RT = \frac{\gamma}{g} RT$$

Relating the temperature to the adiabatic lapse rate

$$p = \frac{\gamma}{g} R(T_0 - \alpha z)$$

Inserting the expression for specific weight into the hydrostatic equation

$$dp = -\frac{gp}{R(T_0 - \alpha z)} dz$$

Separating variables

$$\frac{dp}{p} = -\frac{g}{R} \frac{dz}{(T_0 - \alpha z)}$$

Integrating between the surface and any height z

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R} \int_0^z \frac{dz}{(T_0 - \alpha z)}$$

Or

$$ln\left(\frac{p}{p_0}\right) = -\frac{g}{R} ln\left(\frac{T_0 - \alpha z}{T_0}\right)$$

In terms of p

$$\frac{p}{p_0} = \left(1 - \frac{\alpha z}{T_0}\right)^{g/_{\alpha R}}$$

Problem 3.39

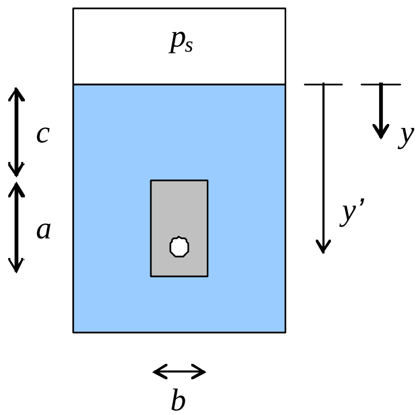
[Difficulty: 3]

3.39 A door 1 m wide and 1.5 m high is located in a plane vertical wall of a water tank. The door is hinged along its upper edge, which is 1 m below the water surface. Atmospheric pressure acts on the outer surface of the door and at the water surface. (a) Determine the magnitude and line of action of the total resultant force from all fluids acting on the door. (b) If the water surface gage pressure is raised to 0.3 atm, what is the resultant force and where is its line of action? (c) Plot the ratios F/F_0 and y'/y_c for different values of the surface pressure ratio p_s/p_{atm}. (F_0 is the resultant force when $p_s = p_{atm}$.)

Given: Door located in plane vertical wall of water tank as shown

$a = 1.5 \cdot m \quad b = 1 \cdot m \qquad c = 1 \cdot m$

Atmospheric pressure acts on outer surface of door.



Find: Resultant force and line of action:

(a) for $\quad p_s = p_{atm}$

(b) for $\quad p_{sg} = 0.3 \cdot atm$

Plot F/Fo and y'/yc over range of ps/patm (Fo is force determined in (a), yc is y-ccordinate of door centroid).

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dy} = \rho \cdot g \qquad \text{(Hydrostatic Pressure - y is positive downwards)}$$

$$F_R = \int p\, dA \qquad \text{(Hydrostatic Force on door)}$$

$$y' \cdot F_R = \int y \cdot p\, dA \qquad \text{(First moment of force)}$$

Assumptions: (1) Static fluid
(2) Incompressible fluid

We will obtain a general expression for the force and line of action, and then simplify for parts (a) and (b).

Since $\quad dp = \rho \cdot g \cdot dh \quad$ it follows that $\quad p = p_s + \rho \cdot g \cdot y$

Now because $\quad p_{atm} \quad$ acts on the outside of the door, $\quad p_{sg} \quad$ is the surface gage pressure: $\quad p = p_{sg} + \rho \cdot g \cdot y$

$$F_R = \int p\, dA = \int_c^{c+a} p \cdot b\, dy = \int_c^{c+a} \left(p_{sg} + \rho \cdot g \cdot y\right) \cdot b\, dy = b \cdot \left[p_{sg} \cdot a + \frac{\rho \cdot g}{2} \cdot \left(a^2 + 2 \cdot a \cdot c\right)\right] \qquad (1)$$

$$y' \cdot F_R = \int y \cdot p\, dA \qquad \text{Therefore:} \qquad y' = \frac{1}{F_R} \int y \cdot p\, dA = \frac{1}{F_R} \cdot \int_c^{c+a} y \cdot \left(p_{sg} + \rho \cdot g \cdot y\right) \cdot b\, dy$$

Evaluating the integral: $\quad y' = \dfrac{b}{F_R} \left[\dfrac{p_{sg}}{2}\left[(c+a)^2 - c^2\right] + \dfrac{\rho \cdot g}{3} \cdot \left[(c+a)^3 - c^3\right]\right]$

Simplifying: $y' = \frac{b}{F_R}\cdot\left[\frac{p_{sg}}{2}\left(a^2 + 2\cdot a\cdot c\right) + \frac{\rho\cdot g}{3}\cdot\left[a^3 + 3\cdot a\cdot c\cdot(a + c)\right]\right]$ (2)

For part (a) we know $p_{sg} = 0$ so substituting into (1) we get: $F_o = \frac{\rho\cdot g\cdot b}{2}\cdot\left(a^2 + 2\cdot a\cdot c\right)$

$$F_o = \frac{1}{2}\times 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\times\left[(1.5\cdot m)^2 + 2\times 1.5\cdot m\times 1\cdot m\right]\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_o = 25.7\cdot kN$$

Substituting into (2) for the line of action we get: $y' = \frac{\rho\cdot g\cdot b}{3\cdot F_o}\cdot\left[a^3 + 3\cdot a\cdot c\cdot(a + c)\right]$

$$y' = \frac{1}{3}\times 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 1\cdot m\cdot\frac{1}{25.7\times 10^3}\cdot\frac{1}{N}\times\left[(1.5\cdot m)^3 + 3\times 1.5\cdot m\times 1\cdot m\times(1.5\cdot m + 1\cdot m)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$y' = 1.9\,m$$

For part (b) we know $p_{sg} = 0.3\cdot atm$. Substituting into (1) we get:

$$F_R = 1\cdot m\times\left[0.3\cdot atm\times\frac{1.013\times 10^5\cdot N}{m^2\cdot atm}\times 1.5\cdot m + \frac{1}{2}\times 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\left[(1.5\cdot m)^2 + 2\times 1.5\cdot m\times 1\cdot m\right]\times\frac{N\cdot s^2}{kg\cdot m}\right]$$

$$F_R = 71.3\cdot kN$$

$$y' = \frac{1\cdot m\times\left[\frac{0.3\cdot atm}{2}\times\frac{1.013\times 10^5\cdot N}{m^2\cdot atm}\times\left[(1.5)^2 + 2\cdot 1.5\cdot 1\right]\cdot m^2 + \frac{999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}}{3}\times\left[(1.5)^3 + 3\cdot 1.5\cdot 1\cdot(1.5 + 1)\right]\cdot m^3\times\frac{N\cdot s^2}{kg\cdot m}\right]}{71.3\times 10^3\cdot N}$$

$$y' = 1.789\,m$$

The value of F/Fo is obtained from Eq. (1) and our result from part (a):

$$\frac{F}{F_o} = \frac{b\cdot\left[p_{sg}\cdot a + \frac{\rho\cdot g}{2}\cdot\left(a^2 + 2\cdot a\cdot c\right)\right]}{\frac{\rho\cdot g\cdot b}{2}\cdot\left(a^2 + 2\cdot a\cdot c\right)} = 1 + \frac{2\cdot p_{sg}}{\rho\cdot g\cdot(a + 2\cdot c)}$$

For the gate $y_c = c + \frac{a}{2}$ Therefore, the value of y'/yc is obtained from Eqs. (1) and (2):

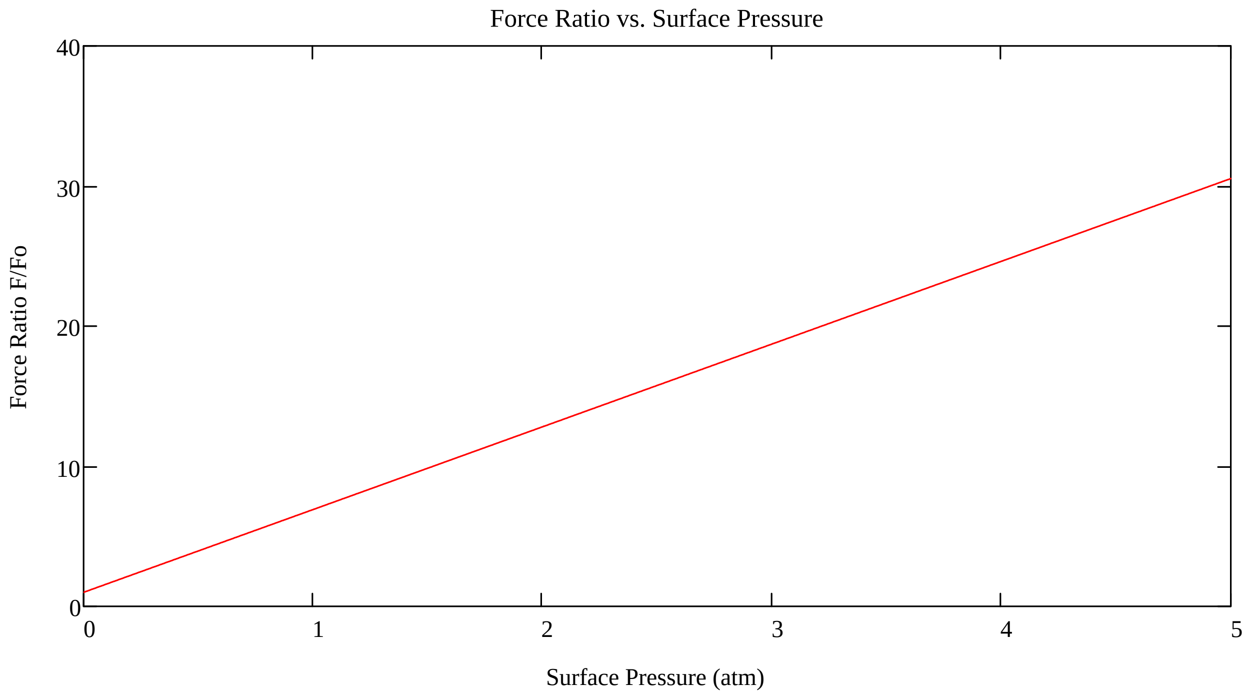
$$\frac{y'}{y_c} = \frac{2\cdot b}{F_R\cdot(2\cdot c + a)}\cdot\left[\frac{p_{sg}}{2}\left(a^2 + 2\cdot a\cdot c\right) + \frac{\rho\cdot g}{3}\cdot\left[a^3 + 3\cdot a\cdot c\cdot(a + c)\right]\right] = \frac{2\cdot b}{(2\cdot c + a)}\cdot\frac{\left[\frac{p_{sg}}{2}\left(a^2 + 2\cdot a\cdot c\right) + \frac{\rho\cdot g}{3}\cdot\left[a^3 + 3\cdot a\cdot c\cdot(a + c)\right]\right]}{\left[b\cdot\left[p_{sg}\cdot a + \frac{\rho\cdot g}{2}\cdot\left(a^2 + 2\cdot a\cdot c\right)\right]\right]}$$

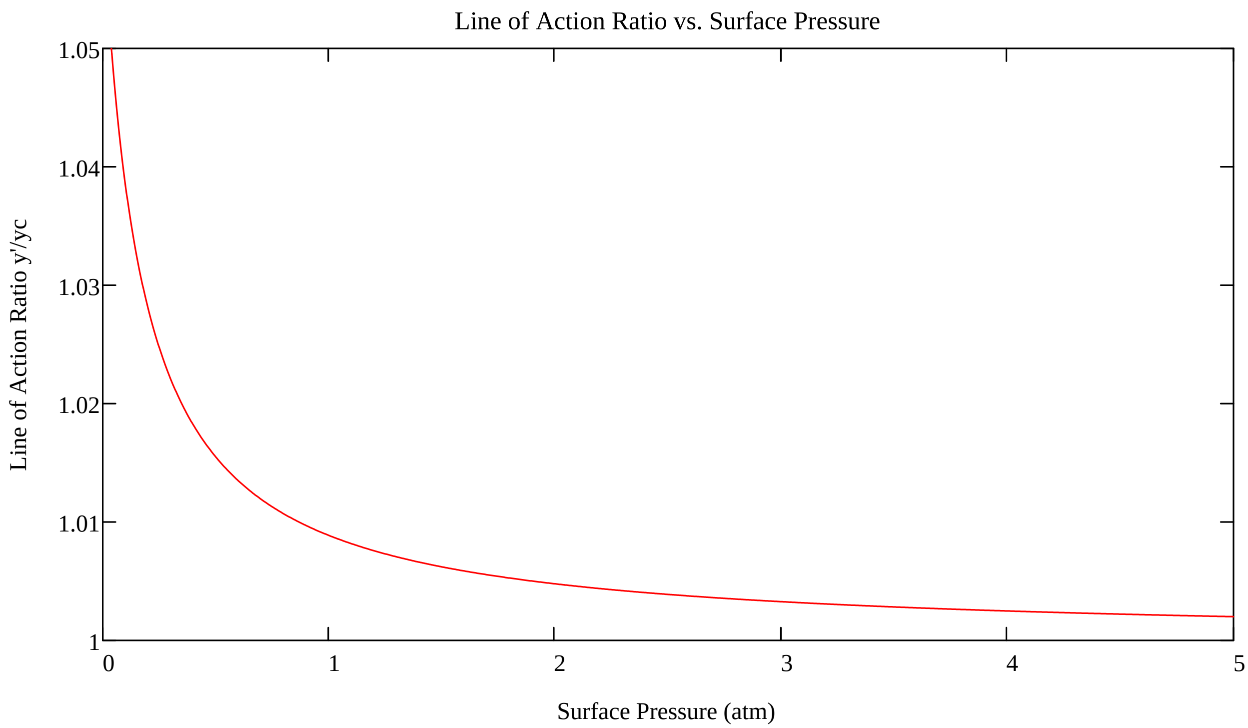
Simplifying this expression we get:

$$\frac{y'}{y_c} = \frac{2}{(2\cdot c + a)}\cdot\frac{\frac{p_{sg}}{2}\left(a^2 + 2\cdot a\cdot c\right) + \frac{\rho\cdot g}{3}\cdot\left[a^3 + 3\cdot a\cdot c\cdot(a+c)\right]}{p_{sg}\cdot a + \frac{\rho\cdot g}{2}\cdot\left(a^2 + 2\cdot a\cdot c\right)}$$

Based on these expressions we see that the force on the gate varies linearly with the increase in surface pressure, and that the line of action of the resultant is always below the centroid of the gate. As the pressure increases, however, the line of action moves closer to the centroid.

Plots of both ratios are shown below:





Problem 3.40 [Difficulty: 5]

3.40 A hydropneumatic elevator consists of a piston-cylinder assembly to lift the elevator cab. Hydraulic oil, stored in an accumulator tank pressurized by air, is valved to the piston as needed to lift the elevator. When the elevator descends, oil is returned to the accumulator. Design the least expensive accumulator that can satisfy the system requirements. Assume the lift is 3 floors, the maximum load is 10 passengers, and the maximum system pressure is 800 kPa (gage). For column bending strength, the piston diameter must be at least 150 mm. The elevator cab and piston have a combined mass of 3000 kg, and are to be purchased. Perform the analysis needed to define, as a function of system operating pressure, the piston diameter, the accumulator volume and diameter, and the wall thickness. Discuss safety features that your company should specify for the complete elevator system. Would it be preferable to use a completely pneumatic design or a completely hydraulic design? Why?

Discussion: The design requirements are specified except that a typical floor height is about 12 ft, making the total required lift about 36 ft. A spreadsheet was used to calculate the system properties for various pressures. Results are presented on the next page, followed by a sample calculation. Total cost dropped quickly as system pressure was increased. A shallow minimum was reached in the 100-110 psig range. The lowest-cost solution was obtained at a system pressure of about 100 psig. At this pressure, the reservoir of 140 gal required a 3.30 ft diameter pressure sphere with a 0.250 in wall thickness. The welding cost was $155 and the material cost $433, for a total cost of $588. Accumulator wall thickness was constrained at 0.250 in for pressures below 100 psi; it increased for higher pressures (this caused the discontinuity in slope of the curve at 100 psig). The mass of steel became constant above 110 psig. No allowance was made for the extra volume needed to pressurize the accumulator. Fail-safe design is essential for an elevator to be used by the public. The control circuitry should be redundant. Failures must be easy to spot. For this reason, hydraulic actuation is good: leaks will be readily apparent. The final design must be reviewed, approved, and stamped by a professional engineer since the design involves public safety. The terminology used in the solution is defined in the following table:

Symbol	Definition	Units
p	System pressure	psig
A_p	Area of lift piston	in^2
$\forall_{oil}$	Volume of oil	gal
D_s	Diameter of spherical accumulator	ft
t	Wall thickness of accumulator	in
A_w	Area of weld	in^2
C_w	Cost of weld	$
M_s	Mass of steel accumulator	lbm
C_s	Cost of steel	$
C_t	Total Cost	$

A sample calculation and the results of the system simulation in Excel are presented below.

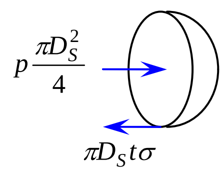
Sample calculation for a pressure of 20 psig:

$$W_t = p \cdot A_p \qquad A_p = \frac{W_t}{p} \qquad A_p = 7500 \cdot lbf \times \frac{1}{20} \cdot \frac{in^2}{lbf} \qquad\qquad A_p = 375 \cdot in^2$$

$$V_{oil} = A_p \cdot L \qquad V_{oil} = 375 \cdot in^2 \times 36 \cdot ft \times \left(\frac{ft}{12 \cdot in}\right)^2 \times \frac{7.48 \cdot gal}{ft^3} \qquad\qquad V_{oil} = 701 \cdot gal$$

$$V_{oil} = V_s = \frac{4}{3} \cdot \pi \cdot R_s^{\,3} = \frac{\pi}{6} \cdot D_s^{\,3} \qquad D_s = \left(\frac{6 \cdot V_{oil}}{\pi}\right)^{\frac{1}{3}} \qquad D_s = \left(\frac{6}{\pi} \times 701 \cdot gal \times \frac{ft^3}{7.48 \cdot gal}\right)^{\frac{1}{3}} \qquad\qquad D_s = 5.64 \cdot ft$$

From a force balance on the sphere:



Thus: $$p \cdot \pi \cdot \frac{D_s^{\,2}}{4} = \pi \cdot D_s \cdot t \cdot \sigma \text{ , so} \qquad t = \frac{p}{\sigma} \cdot \frac{D_s}{4} \qquad t = 20 \cdot \frac{lbf}{in^2} \times \frac{1}{4000} \cdot \frac{in^2}{lbf} \times \frac{5.64 \cdot ft}{4} \times \frac{12 \cdot in}{ft} \qquad\qquad t = 0.085 \cdot in$$

Since the minimum wall thickness is 0.250 in: $$t = 0.250 \cdot in$$

$$A_w = \pi \cdot D_s \cdot t \qquad A_w = \pi \cdot 5.64 \cdot ft \cdot 0.250 \cdot in \cdot \frac{12 \cdot in}{ft} \qquad\qquad A_w = 53.2 \cdot in^2$$

$$C_w = 5.00 \cdot \frac{1}{in^2} \times 53.2 \cdot in^2 \qquad \text{(cost in \$)} \qquad C_w = 266$$

$$M_s = 4 \cdot \pi \cdot R_s^{\,2} \cdot t \cdot \rho_s = \pi \cdot D_s^{\,2} \cdot t \cdot SG_s \cdot \rho_{water}$$

$$M_s = \pi \times (5.64 \cdot ft)^2 \times 0.250 \cdot in \times \frac{ft}{12 in} \times 7.8 \times 62.4 \cdot \frac{lbm}{ft^3} \qquad\qquad M_s = 1013 \cdot lbm$$

$$C_s = 1.25 \cdot \frac{1}{lbm} \times 1013 \cdot lbm \qquad \text{(cost in \$)} \qquad C_s = 1266$$

Therefore the total cost is:

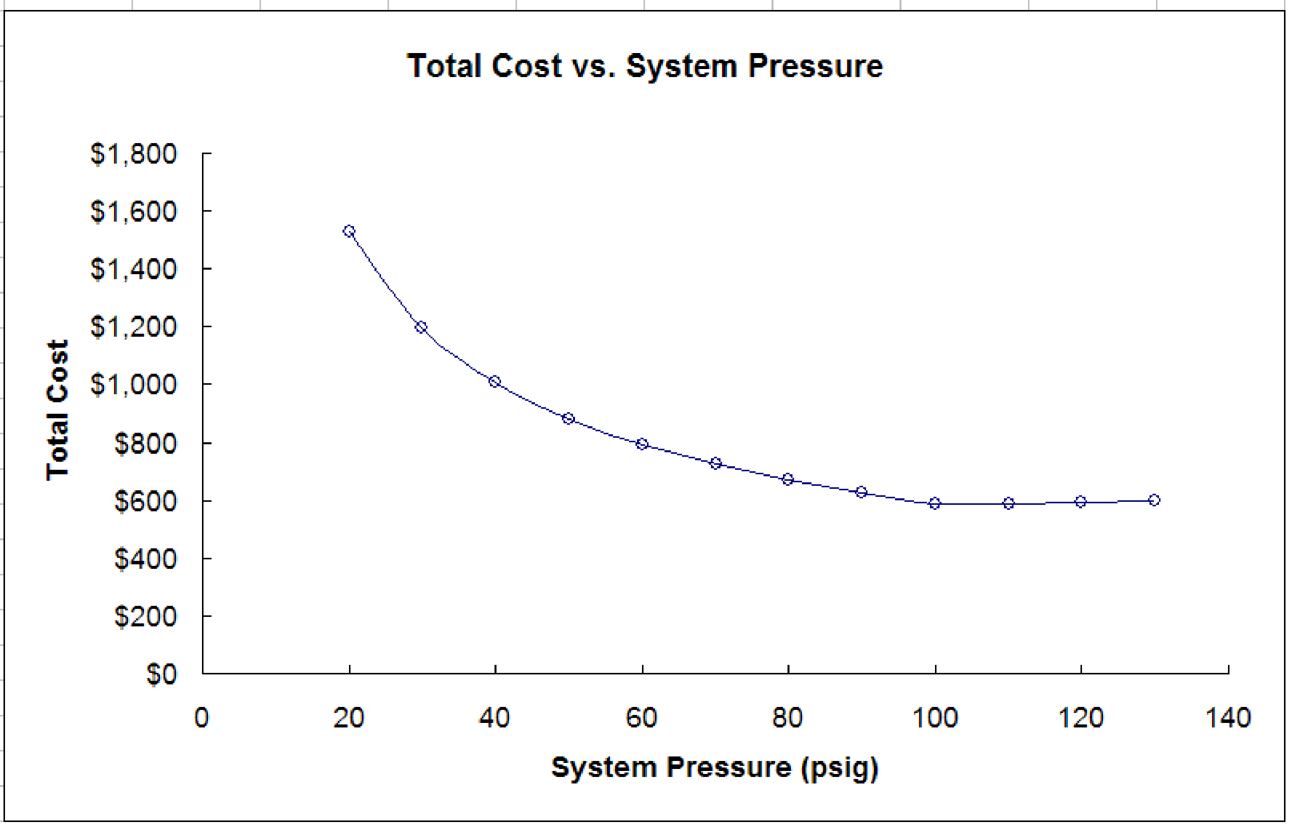
$$C_t = 266 + 1266 \qquad \text{(cost in \$)} \qquad C_t = 1532$$

Results of system simulation:

Input Data:	Cab and piston weight:	W_{cab} =	6000	lbf
	Passenger weight:	W_{pax} =	1500	lbf
	Total weight:	W_{tot} =	7500	lbf
	Allowable stress:	σ =	4000	psi
	Minimum wall thickness:	t =	0.250	in
	Welding cost factor:	cf_w =	5.00	$/in^2
	Steel cost factor:	cf_s =	1.25	$/pound

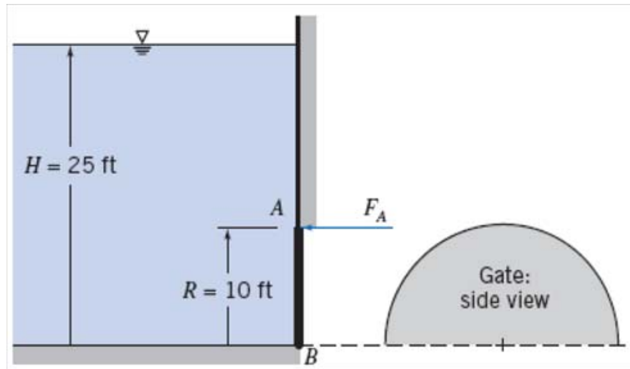
Results:

p (psig)	A_p (in^2)	V_{oil} (gal)	D_s (ft)	t (in)	A_w (in^2)	C_w	M_s (lbm)	C_s	C_t
20	375	701	5.64	0.250	53.1	$266	1012	$1,265	$1,531
30	250	468	4.92	0.250	46.4	$232	772	$965	$1,197
40	187.5	351	4.47	0.250	42.2	$211	638	$797	$1,008
50	150.0	281	4.15	0.250	39.1	$196	549	$687	$882
60	125.0	234	3.91	0.250	36.8	$184	487	$608	$792
70	107.1	200	3.71	0.250	35.0	$175	439	$549	$724
80	93.8	175.3	3.55	0.250	33.5	$167	402	$502	$669
90	83.3	155.8	3.41	0.250	32.2	$161	371	$464	$625
100	75.0	140.3	3.30	0.250	31.1	$155	346	$433	$588
110	68.2	127.5	3.19	0.263	31.7	$159	342	$428	$586
120	62.5	116.9	3.10	0.279	32.6	$163	342	$428	$591
130	57.7	107.9	3.02	0.294	33.5	$168	342	$428	$595



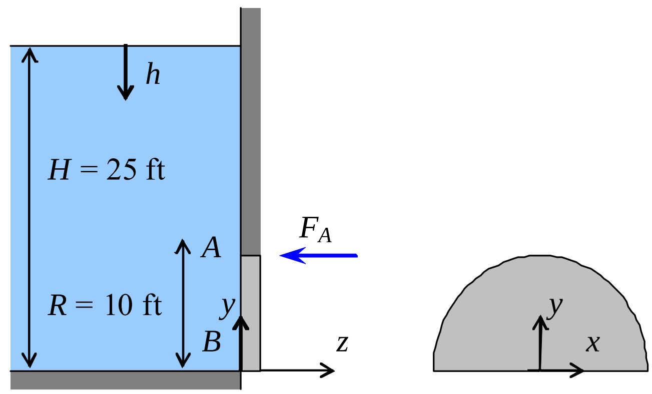
Problem 3.41 [Difficulty: 3]

3.41 Semicircular plane gate AB is hinged along B and held by horizontal force F_A applied at A. The liquid to the left of the gate is water. Calculate the force F_A required for equilibrium.



Given: Geometry of gate

Find: Force F_A for equilibrium



Solution:

Basic equation $\quad F_R = \int p\,dA \qquad \frac{dp}{dh} = \rho\cdot g \qquad \Sigma M_z = 0$

or, use computing equations $\quad F_R = p_c\cdot A \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c}$ where y would be measured from the free surface

Assumptions: static fluid; ρ = constant; p_{atm} on other side; door is in equilibrium

Instead of using either of these approaches, we note the following, using y as in the sketch

$$\Sigma M_z = 0 \qquad F_A\cdot R = \int y\cdot p\,dA \quad \text{with} \quad p = \rho\cdot g\cdot h$$ (Gage pressure, since p = p_{atm} on other side)

$$F_A = \frac{1}{R}\cdot\int y\cdot\rho\cdot g\cdot h\,dA \quad \text{with} \quad dA = r\cdot dr\cdot d\theta \quad \text{and} \quad y = r\cdot\sin(\theta) \quad h = H - y$$

Hence
$$F_A = \frac{1}{R}\cdot\int_0^{\pi}\int_0^{R}\rho\cdot g\cdot r\cdot\sin(\theta)\cdot(H - r\cdot\sin(\theta))\cdot r\,dr\,d\theta = \frac{\rho\cdot g}{R}\cdot\int_0^{\pi}\left(\frac{H\cdot R^3}{3}\cdot\sin(\theta) - \frac{R^4}{4}\cdot\sin(\theta)^2\right)d\theta$$

$$F_R = \frac{\rho\cdot g}{R}\cdot\left(\frac{2\cdot H\cdot R^3}{3} - \frac{\pi\cdot R^4}{8}\right) = \rho\cdot g\cdot\left(\frac{2\cdot H\cdot R^2}{3} - \frac{\pi\cdot R^3}{8}\right)$$

Using given data
$$F_R = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times 32.2\cdot\frac{\text{ft}}{\text{s}^2}\times\left[\frac{2}{3}\times 25\cdot\text{ft}\times(10\cdot\text{ft})^2 - \frac{\pi}{8}\times(10\cdot\text{ft})^3\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$F_R = 7.96\times 10^4\cdot\text{lbf}$$

Problem 3.42

(Difficulty: 2)

3.42 A circular gate 3 m in diameter has its center 2.5 m below a water surface and lies in a plane sloping at 60°. Calculate magnitude, direction and location of total force on the gate.

Find: The direction, magnitude of the total force F.

Assumptions: Fluid is static and incompressible

Solution: Apply the hydrostatic relations for pressure, force, and moments, with y measured from the surface of the liquid:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

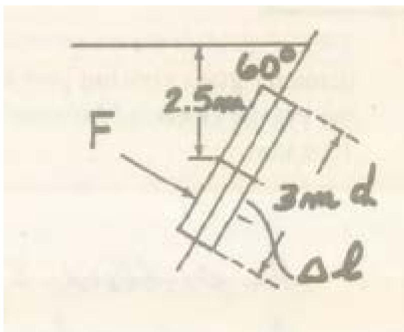
$$F_R = \int p\, dA$$

$$y' F_R = \int y\, p\, dA$$

For the magnitude of the force we have:

$$F = \int_A p dA$$

A free body diagram of the gate is



The pressure on the gate is the pressure at the centroid, which is y_c = 2.5 m. So the force can be calculated as:

$$F = \rho g h_c A = 999\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 2.5\ m \times \frac{\pi}{4} \times (3\ m)^2 = 173200\ N = 173.2\ kN$$

The direction is perpendicular to the gate.

For the location of the force we have:

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{Ay_c}$$

The y axis is along the plate so the distance to the centroid is:

$$y_c = \frac{2.5\ m}{\sin 60°} = 2.89\ m$$

The area moment of inertia is

$$I_{\hat{x}\hat{x}} = \frac{\pi D^4}{64} = \frac{\pi}{64} \times (3\ m)^4 = 3.976\ m^4$$

The area is

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (3\ m)^2 = 7.07\ m^2$$

So

$$y' = 2.89\ m + \frac{3.976\ m^4}{7.07\ m^2 \times 2.89\ m} = 2.89\ m + 0.1946\ m = 3.08\ m$$

The vertical location on the plate is

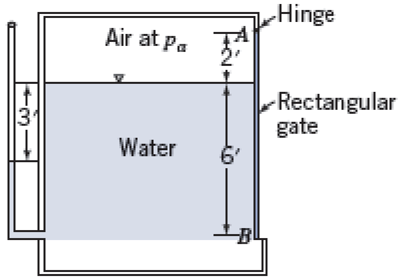
$$h' = y' \sin 60° = 3.08\ m \times \frac{\sqrt{3}}{2} = 2.67\ m$$

The force acts on the point which has the depth of $2.67\ m$.

Problem 3.43

(Difficulty: 2)

3.43 For the situation shown, find the air pressure in the tank in psi. Calculate the force exerted on the gate at the support B if the gate is 10 ft wide. Show a free body diagram of the gate with all the forces drawn in and their points of application located.



P3.43

Assumptions: Fluid is static and incompressible

Solution: Apply the hydrostatic relations for pressure and force, and the static relation for moments:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

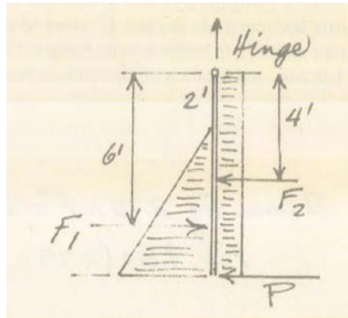
The specfic weight for water is:

$$\gamma = 62.4\ \frac{lbf}{ft^3}$$

The pressure of the air equals that at the surface of the water in the tank. As shown by the manometer, the pressure at the surface is less than atmospheric due to the three foot head of water. The gage pressure of the air is then:

$$p_{air} = -\gamma h = -62.4\ \frac{lbf}{ft^3} \times 3ft = -187.2\ \frac{lbf}{ft^2}$$

A free body diagram for the gate is



For the force in the horizontal direction, we have:

$$F_1 = \gamma h_c A = 62.4\ \frac{lbf}{ft^3} \times 3\ ft \times (6\ ft \times 10\ ft) = 11230\ lbf$$

$$F_2 = p_{air} A = -187.2\ \frac{lbf}{ft^2} \times (8\ ft \times 10\ ft) = 14980\ lbf$$

With the momentume balance about hinge we have:

$$\sum M = F_1 h_c - Ph - F_2 \frac{h}{2} = 11230\ lbf \times 6ft - P \times 8ft - 14980\ lbf \times 4ft = 0$$

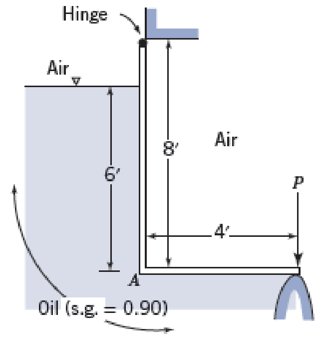
So the force exerted on B is:

$$P = 933\ lbf$$

Problem 3.44

(Difficulty: 3)

3.44 What is the pressure at A? Draw a free body diagram of the 10 ft wide gate showing all forces and locations of their lines of action. Calculate the minimum force P necessary to keep the gate closed.



Given: All the parameters are shown in the figure.

Find: The pressure p_A. The minimum force P necessary to keep the gate closed.

Assumptions: Fluid is static and incompressible

Solution: Apply the hydrostatic relations for pressure, force, and moments, with y measured from the surface of the liquid:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

$$F_R = \int p\, dA$$

$$y' F_R = \int y\, p\, dA$$

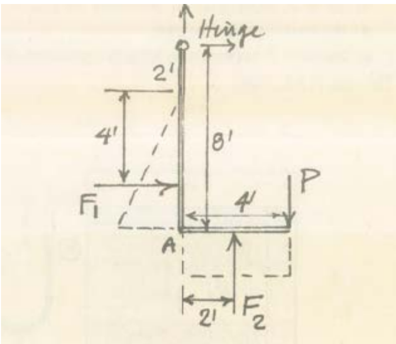
The specfic weight of the water is:

$$\gamma_{water} = 62.4\ \frac{lbf}{ft^3}$$

The gage pressure at A is given by integrating the hydrostatic relation:

$$p_A = \gamma_{oil} h_A = SG \gamma_{oil} h_A = 0.9 \times 62.4\ \frac{lbf}{ft^3} \times 6\ ft = 337\ \frac{lbf}{ft^2}$$

A free body diagram of the gate is



The horizontal force F_1 as shown in the figure is given by the pressure at the centroid of the submerged area (3 ft):

$$F_1 = \gamma_{oil} h_c A = 0.9 \times 62.4\ \frac{lbf}{ft^3} \times 3\ ft \times (6\ ft \times 10\ ft) = 10110\ lbf$$

The vertical force F_2 is given by the pressure at the depth of the surface (4 ft)

$$F_2 = p_A A = 337\ \frac{lbf}{ft^2} \times (4ft \times 10ft) = 13480\ lbf$$

The force F_1 acts two-thirds of the distance down from the water surface and the force F_2 acts at the centroid..

Taking the moments about the hinge:

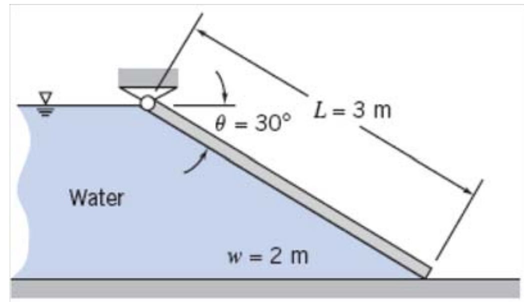
$$-F_1 \times 6\ ft - F_2 \times 2\ ft + P \times 4\ ft = 0$$

So we have for the force at the support:

$$P = \frac{10110\ lbf \times 6ft + 13480\ lbf \times 2ft}{4\ ft} = 21900\ lbf$$

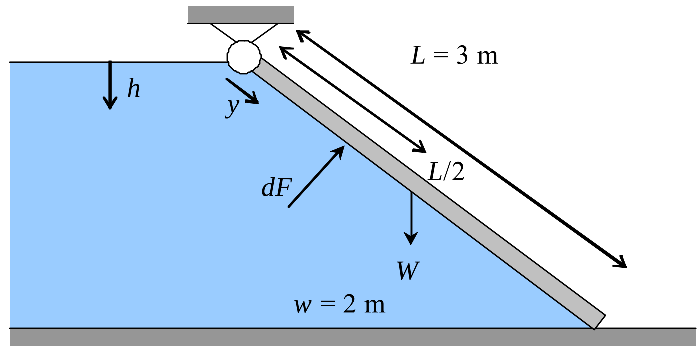
Problem 3.45 [Difficulty: 3]

3.45 A plane gate of uniform thickness holds back a depth of water as shown. Find the minimum weight needed to keep the gate closed.



Given: Geometry of plane gate

Find: Minimum weight to keep it closed



Solution:

Basic equation $F_R = \int p\,dA$ $\qquad \frac{dp}{dh} = \rho\cdot g \qquad \Sigma M_O = 0$

or, use computing equations $\qquad F_R = p_c\cdot A \qquad y' = y_c + \frac{I_{xx}}{A\cdot y_c}$

Assumptions: static fluid; ρ = constant; p_{atm} on other side; door is in equilibrium

Instead of using either of these approaches, we note the following, using y as in the sketch

$$\Sigma M_O = 0 \qquad W\cdot\frac{L}{2}\cdot\cos(\theta) = \int y\,dF$$

We also have $\quad dF = p\cdot dA \quad$ with $\quad p = \rho\cdot g\cdot h = \rho\cdot g\cdot y\cdot\sin(\theta) \quad$ (Gage pressure, since p = p_{atm} on other side)

Hence

$$W = \frac{2}{L\cdot\cos(\theta)}\cdot\int y\cdot p\,dA = \frac{2}{L\cdot\cos(\theta)}\cdot\int y\cdot\rho\cdot g\cdot y\cdot\sin(\theta)\cdot w\,dy$$

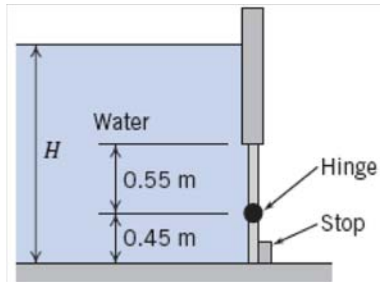
$$W = \frac{2}{L\cdot\cos(\theta)}\cdot\int y\cdot p\,dA = \frac{2\cdot\rho\cdot g\cdot w\cdot\tan(\theta)}{L}\cdot\int_0^L y^2\,dy = \frac{2}{3}\cdot\rho\cdot g\cdot w\cdot L^2\cdot\tan(\theta)$$

Using given data $\quad W = \frac{2}{3}\cdot 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 2\cdot m\times(3\cdot m)^2\times\tan(30\cdot deg)\times\frac{N\cdot s^2}{kg\cdot m} \qquad W = 68\cdot kN$

Problem 3.46

[Difficulty: 3]

3.46 A rectangular gate (width $w = 2$ m) is hinged as shown, with a stop on the lower edge. At what depth H will the gate tip?



Given: Gate geometry

Find: Depth H at which gate tips

Solution:

This is a problem with atmospheric pressure on both sides of the plate, so we can first determine the location of the center of pressure with respect to the free surface,

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} \qquad \text{and} \qquad I_{xx} = \frac{w \cdot L^3}{12} \qquad \text{with} \qquad y_c = H - \frac{L}{2}$$

where $L = 1$ m is the plate height and w is the plate width

Hence

$$y' = \left(H - \frac{L}{2}\right) + \frac{w \cdot L^3}{12 \cdot w \cdot L \cdot \left(H - \frac{L}{2}\right)} = \left(H - \frac{L}{2}\right) + \frac{L^2}{12 \cdot \left(H - \frac{L}{2}\right)}$$

But for equilibrium, the center of force must always be at or below the level of the hinge so that the stop can hold the gate in place. Hence we must have

$$y' > H - 0.45 \cdot m$$

Combining the two equations

$$\left(H - \frac{L}{2}\right) + \frac{L^2}{12 \cdot \left(H - \frac{L}{2}\right)} \geq H - 0.45 \cdot m$$

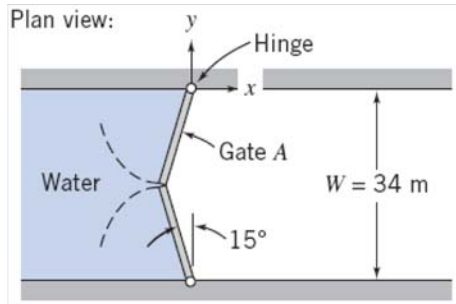
Solving for H

$$H \leq \frac{L}{2} + \frac{L^2}{12 \cdot \left(\frac{L}{2} - 0.45 \cdot m\right)} \qquad H \leq \frac{1 \cdot m}{2} + \frac{(1 \cdot m)^2}{12 \times \left(\frac{1 \cdot m}{2} - 0.45 \cdot m\right)} \qquad H \leq 2.17 \cdot m$$

Problem 3.47

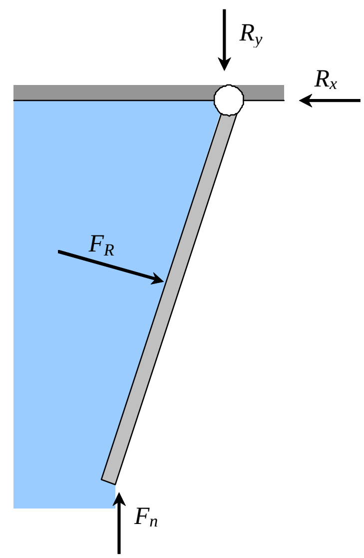
[Difficulty: 3]

3.47 Gates in the Poe Lock at Sault Ste. Marie, Michigan, close a channel $W = 34$ m wide, $L = 360$ m long, and $D = 10$ m deep. The geometry of one pair of gates is shown; each gate is hinged at the channel wall. When closed, the gate edges are forced together at the center of the channel by water pressure. Evaluate the force exerted by the water on gate A. Determine the magnitude and direction of the force components exerted by the gate on the hinge. (Neglect the weight of the gate.)



Given: Geometry of lock system

Find: Force on gate; reactions at hinge



Solution:

Basic equation $\quad F_R = \int p\,dA \qquad \dfrac{dp}{dh} = \rho\cdot g$

or, use computing equation $\quad F_R = p_c\cdot A$

Assumptions: static fluid; ρ = constant; p_{atm} on other side

The force on each gate is the same as that on a rectangle of size

$$h = D = 10\cdot m \quad \text{and} \quad w = \frac{W}{2\cdot\cos(15\cdot deg)}$$

$$F_R = \int p\,dA = \int \rho\cdot g\cdot y\,dA \qquad \text{but} \qquad dA = w\cdot dy$$

Hence
$$F_R = \int_0^h \rho\cdot g\cdot y\cdot w\,dy = \frac{\rho\cdot g\cdot w\cdot h^2}{2}$$

Alternatively
$$F_R = p_c\cdot A \quad \text{and} \quad F_R = p_c\cdot A = \rho\cdot g\cdot y_c\cdot A = \rho\cdot g\cdot\frac{h}{2}\cdot h\cdot w = \frac{\rho\cdot g\cdot w\cdot h^2}{2}$$

Using given data
$$F_R = \frac{1}{2}\cdot 1000\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\frac{34\cdot m}{2\cdot\cos(15\cdot deg)}\times(10\cdot m)^2\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_R = 8.63\cdot MN$$

For the force components R_x and R_y we do the following

$$\Sigma M_{hinge} = 0 = F_R\cdot\frac{w}{2} - F_n\cdot w\cdot\sin(15\cdot deg) \qquad F_n = \frac{F_R}{2\cdot\sin(15\cdot deg)} \qquad F_n = 16.7\cdot MN$$

$$\Sigma F_x = 0 = F_R\cdot\cos(15\cdot deg) - R_x = 0 \qquad R_x = F_R\cdot\cos(15\cdot deg) \qquad R_x = 8.34\cdot MN$$

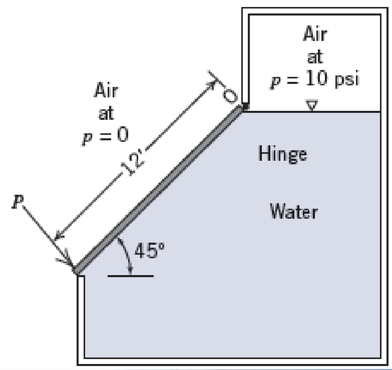
$$\Sigma F_y = 0 = -R_y - F_R\cdot\sin(15\cdot deg) + F_n = 0 \qquad R_y = F_n - F_R\cdot\sin(15\cdot deg) \qquad R_y = 14.4\cdot MN$$

$$R = (8.34\cdot MN, 14.4\cdot MN) \qquad R = 16.7\cdot MN$$

Problem 3.48

(Difficulty: 2)

3.48 Calculate the minimum force P necessary to hold a uniform $12\ ft\ square$ gate weighing $500\ lbf$ closed on a tank of water under a pressure of $10\ psi$. Draw a free body of the gate as part of your solution.



Given: All the parameters are shown in the figure.

Find: The minimum force P to hold the system.

Assumptions: Fluid is static and incompressible

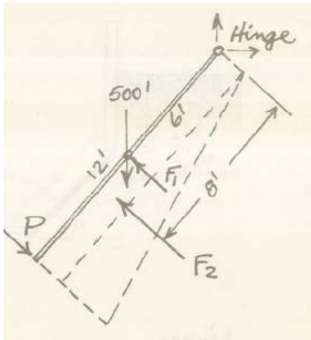
Solution: Apply the hydrostatic relations for pressure, force, and moments, with y measured from the surface of the liquid:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

$$F_R = \int p\, dA$$

$$y' F_R = \int y\, p\, dA$$

A free body diagram of the gate is



The gage pressure of the air in the tank is:

$$p_{air} = 10\ psi = 1440\ \frac{lbf}{ft^2}$$

This produces a uniform force on the gate of

$$F_1 = p_{air}A = 1440\ \frac{lbf}{ft^2} \times (12\ ft \times 12\ ft) = 207360\ lbf$$

This pressure acts at the centroid of the area, which is the center of the gate. In addition, there is a force on the gate applied by water. This force is due to the pressure at the centroid of the area. The depth of the centroid is:

$$y_c = \frac{12\ ft}{2} \times \sin 45°$$

The force is them

$$F_2 = \gamma h_c A = 62.4\ \frac{lbf}{ft^3} \times \frac{12\ ft}{2} \times \sin 45° \times 12\ ft \times 12\ ft = 38123\ lbf$$

The force F_2 acts two-thirds of the way down from the hinge, or $y' = 8\ ft$.

Take the moments about the hinge:

$$-F_B \frac{L}{2} \sin 45° + F_1 \frac{L}{2} + F_2 \times 8\ ft - P \times 12\ ft = 0$$

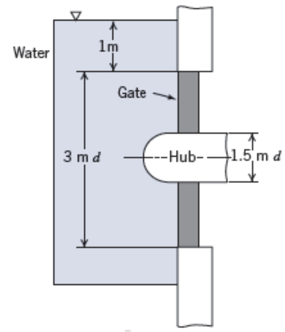
Thus

$$P = \frac{-500\ lbf \times 6\ ft \times \sin 45° + 207360\ lbf \times 6\ ft + 38123\ lbf \times 8\ ft}{12\ ft} = 128900\ lbf$$

Problem 3.49

(Difficulty: 2)

3.49 Calculate magnitude and location of the resultant force of water on this annular gate.



Given: All the parameters are shown in the figure.

Find: Resultant force of water on this annular gate.

Assumptions: Fluid is static and incompressible

Solution: Apply the hydrostatic relations for pressure, force, and moments, with y measured from the surface of the liquid:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

$$F_R = \int p\, dA$$

$$y' F_R = \int y\, p\, dA$$

For the magnitude of the force we have:

$$F = \int_A p dA = \rho g h_c A$$

The pressure is determined at the location of the centroid of the area

$$h_c = 1\ m + 1.5\ m = 2.5\ m$$

$$A = \frac{\pi}{4}(D_2^2 - D_1^2) = \frac{\pi}{4}((3\ m)^2 - (1.5\ m)^2) = 5.3014\ m^2$$

$$F = 999\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 2.5\ m \times 5.3014\ m^2 = 129900\ N = 129.9\ kN$$

The y axis is in the vertical direction. For the location of the force, we have:

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{Ay_c}$$

Where:

$$y_c = 2.5\ m$$

$$I_{\hat{x}\hat{x}} = \frac{\pi(D_2^4 - D_1^4)}{64} = \frac{\pi}{64} \times ((3\ m)^4 - (1.5\ m)^4) = 3.7276\ m^4$$

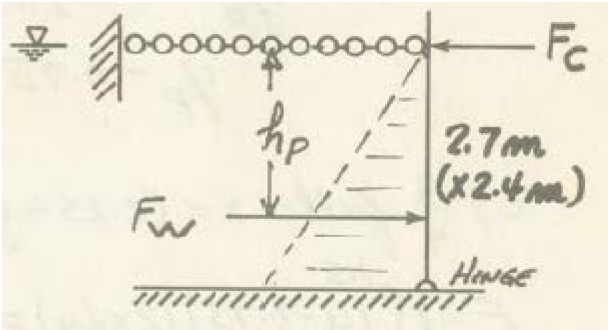
$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{Ay_c} = 2.5\ m + \frac{3.7276\ m^4}{2.5\ m \times 5.3014\ m^2} = 2.78\ m$$

So the force acts on the depth of $y' = 2.78\ m$.

Problem 3.50

(Difficulty: 2)

3.50 A vertical rectangular gate 2.4 m wide and 2.7 m high is subjected to water pressure on one side, the water surface being at the top of the gate. The gate is hinged at the bottom and is held by a horizontal chain at the top. What is the tension in the chain?



Given: The gate wide: $w = 2.4\ m$. Height of the gate: $h = 2.7\ m$.

Find: The tension F_c in the chain.

Assumptions: Fluid is static and incompressible

Solution: Apply the hydrostatic relations for pressure, force, and moments, with y measured from the surface of the liquid:

$$\frac{dp}{dy} = \rho\, g = \gamma$$

$$F_R = \int p\, dA$$

$$y' F_R = \int y\, p\, dA$$

For the magnitude of the force we have:

$$F = \int_A p dA = \rho g h_c A$$

Where h_c is the depth at the centroid

$$h_c = \frac{2.7\ m}{2} = 1.35\ m$$

$$A = wh = 2.4\ m \times 2.7\ m = 6.48\ m^2$$

$$F = 999\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} \times 1.35\ m \times 6.48\ m^2 = 85.7\ kN$$

The y axis is in the vertical direction. For the location of the force, we have:

$$h_p = \frac{2}{3} \times 2.7\ m = 1.8\ m$$

Taking the momentum about the hinge:

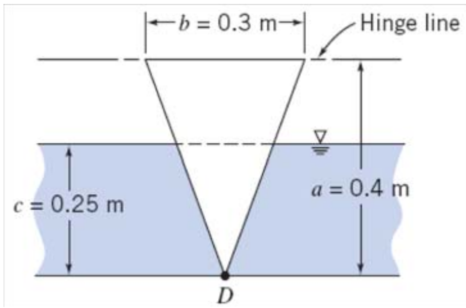
$$F\left(h - h_p\right) - F_c h = 0$$

$$F_c = F\frac{\left(h - h_p\right)}{h} = 85.7\ kN \times \frac{0.9\ m}{2.7\ m} = 28.6\ kN$$

Problem 3.51

[Difficulty: 4]

3.51 A window in the shape of an isosceles triangle and hinged at the top is placed in the vertical wall of a form that contains liquid concrete. Determine the minimum force that must be applied at point D to keep the window closed for the configuration of form and concrete shown. Plot the results over the range of concrete depth $0 \le c \le a$



Given: Window, in shape of isosceles triangle and hinged at the top is located in the vertical wall of a form that contains concrete.

$a = 0.4 \cdot m \quad b = 0.3 \cdot m \quad c = 0.25 \cdot m \quad SG_c = 2.5$

Find: The minimum force applied at D needed to keep the window closed.
Plot the results over the range of concrete depth between 0 and a.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

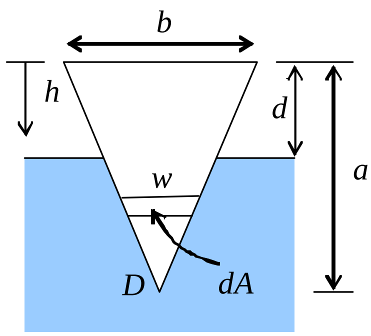
$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$F_R = \int p\, dA$$ (Hydrostatic Force on door)

$$y' \cdot F_R = \int y \cdot p\, dA$$ (First moment of force)

$$\Sigma M = 0$$ (Rotational equilibrium)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts at free surface and on the outside of the window.



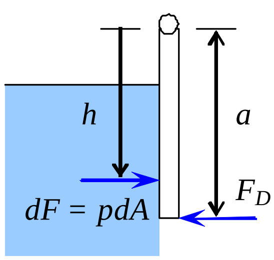
Integrating the pressure equation yields: $p = \rho \cdot g \cdot (h - d)$ for $h > d$

$p = 0$ for $h < d$

where $d = a - c \qquad d = 0.15 \cdot m$

Summing moments around the hinge: $-F_D \cdot a + \int h \cdot p\, dA = 0$

$$F_D = \frac{1}{a} \cdot \int h \cdot p\, dA = \frac{1}{a} \cdot \int_d^a h \cdot \rho \cdot g \cdot (h - d) \cdot w\, dh = \frac{\rho \cdot g}{a} \cdot \int_d^a h \cdot (h - d) \cdot w\, dh$$



From the law of similar triangles: $\frac{w}{b} = \frac{a - h}{a}$ Therefore: $w = \frac{b}{a}(a - h)$

Into the expression for the force at D: $F_D = \frac{\rho \cdot g}{a} \cdot \int_d^a \frac{b}{a} \cdot h \cdot (h - d) \cdot (a - h)\, dh = \frac{\rho \cdot g \cdot b}{a^2} \cdot \int_d^a \left[-h^3 + (a + d) \cdot h^2 - a \cdot d \cdot h \right] dh$

Evaluating this integral we get:

$$F_D = \frac{\rho \cdot g \cdot b}{a^2} \cdot \left[-\frac{\left(a^4 - d^4\right)}{4} + \frac{(a + d) \cdot \left(a^3 - d^3\right)}{3} - \frac{a \cdot d \cdot \left(a^2 - d^2\right)}{2} \right]$$

and after collecting terms:

$$F_D = \rho \cdot g \cdot b \cdot a^2 \cdot \left[-\frac{1}{4} \cdot \left[1 - \left(\frac{d}{a}\right)^4 \right] + \frac{1}{3} \cdot \left(1 + \frac{d}{a}\right) \cdot \left[1 - \left(\frac{d}{a}\right)^3 \right] - \frac{1}{2} \cdot \frac{d}{a} \cdot \left[1 - \left(\frac{d}{a}\right)^2 \right] \right] \qquad (1)$$

The density of the concrete is: $\rho = 2.5 \times 1000 \cdot \frac{kg}{m^3}$ $\qquad \rho = 2.5 \times 10^3 \frac{kg}{m^3}$ $\qquad \frac{d}{a} = \frac{0.15}{0.4} = 0.375$

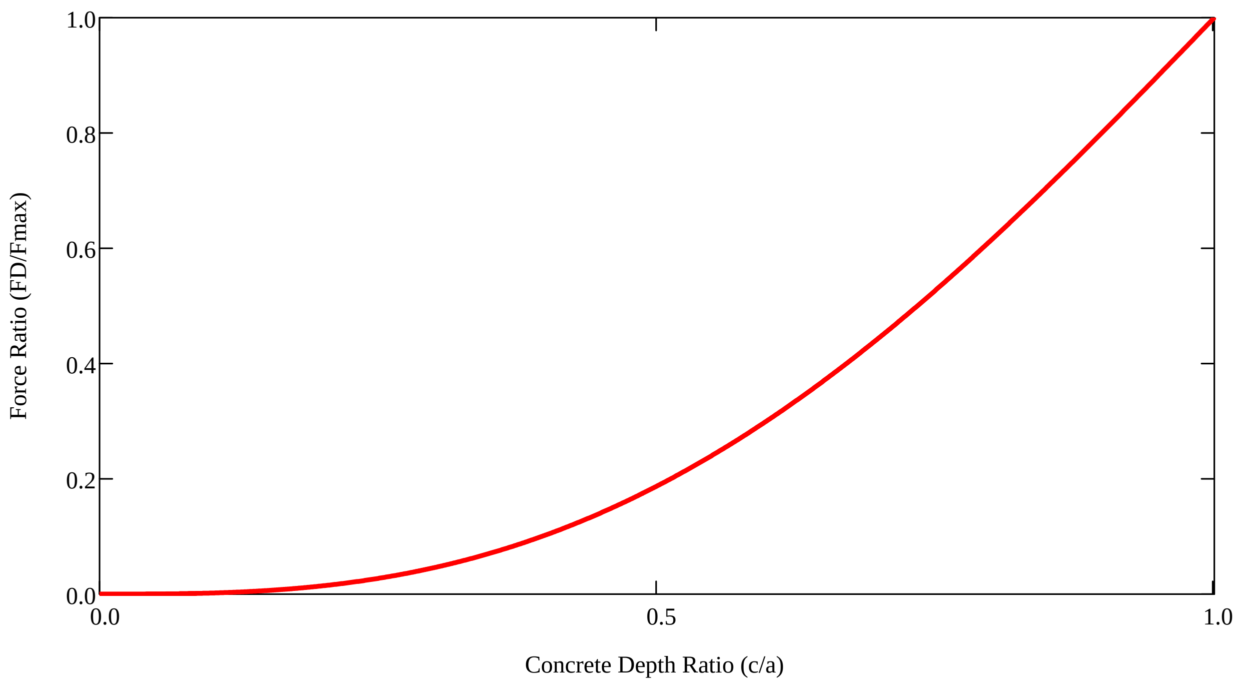
Substituting in values for the force at D:

$$F_D = 2.5 \times 10^3 \cdot \frac{kg}{m^3} \cdot 9.81 \cdot \frac{m}{s^2} \cdot 0.3 \cdot m \cdot (0.4 \cdot m)^2 \cdot \left[-\frac{1}{4} \cdot \left[1 - (0.375)^4 \right] + \frac{1}{3} \cdot (1 + 0.375) \cdot \left[1 - (0.375)^3 \right] - \frac{0.375}{2} \cdot \left[1 - (0.375)^2 \right] \right] \times \frac{N \cdot s^2}{kg \cdot m}$$

To plot the results for different values of c/a, we use Eq. (1) and remember that $d = a - c$ $\qquad F_D = 32.9\,N$

Therefore, it follows that $\frac{d}{a} = 1 - \frac{c}{a}$ In addition, we can maximize the force by the maximum force (when c = a or d = 0):

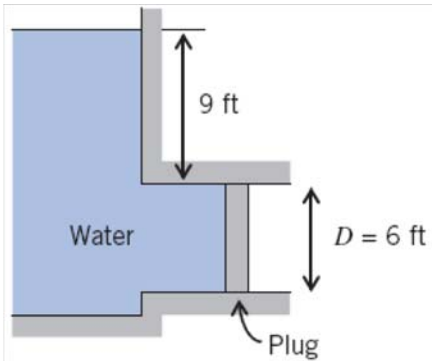
$$F_{max} = \rho \cdot g \cdot b \cdot a^2 \cdot \left(-\frac{1}{4} + \frac{1}{3} \right) = \frac{\rho \cdot g \cdot b \cdot a^2}{12} \qquad \text{and so} \qquad \frac{F_D}{F_{max}} = 12 \cdot \left[-\frac{1}{4} \cdot \left[1 - \left(\frac{d}{a}\right)^4 \right] + \frac{1}{3} \cdot \left(1 + \frac{d}{a}\right) \cdot \left[1 - \left(\frac{d}{a}\right)^3 \right] - \frac{1}{2} \cdot \frac{d}{a} \cdot \left[1 - \left(\frac{d}{a}\right)^2 \right] \right]$$



Problem 3.52

[Difficulty: 2]

3.52 A large open tank contains water and is connected to a 6-ft-diameter conduit as shown. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.



Given: Plug is used to seal a conduit. $\gamma = 62.4 \cdot \frac{lbf}{ft^3}$

Find: Magnitude, direction and location of the force of water on the plug.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \gamma$$ (Hydrostatic Pressure - y is positive downwards)

$$F_R = p_c \cdot A$$ (Hydrostatic Force)

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c}$$ (Location of line of action)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts on the outside of the plug.

Integrating the hydrostatic pressure equation: $p = \gamma \cdot h$ $\quad F_R = p_c \cdot A = \gamma \cdot h_c \cdot \frac{\pi}{4} \cdot D^2$

$$F_R = 62.4 \cdot \frac{lbf}{ft^3} \times 12 \cdot ft \times \frac{\pi}{4} \times (6 \cdot ft)^2 \qquad F_R = 2.12 \times 10^4 \cdot lbf$$

For a circular area: $I_{xx} = \frac{\pi}{64} \cdot D^4$ Therefore: $y' = y_c + \frac{\frac{\pi}{64} \cdot D^4}{\frac{\pi}{4} \cdot D^2 \cdot y_c} = y_c + \frac{D^2}{16 \cdot y_c}$

$$y' = 12 \cdot ft + \frac{(6 \cdot ft)^2}{16 \times 12 \cdot ft}$$

$$y' = 12.19 \cdot ft$$

The force of water is to the right and perpendicular to the plug.

Problem 3.53 [Difficulty: 2]

3.53 The circular access port in the side of a water standpipe has a diameter of 0.6 m and is held in place by eight bolts evenly spaced around the circumference. If the standpipe diameter is 7 m and the center of the port is located 12 m below the free surface of the water, determine (a) the total force on the port and (b) the appropriate bolt diameter.

Given: Circular access port of known diameter in side of water standpipe of known diameter. Port is held in place by eight bolts evenly spaced around the circumference of the port.
Center of the port is located at a know distance below the free surface of the water.

$d = 0.6 \cdot m \quad D = 7 \cdot m \quad L = 12 \cdot m$

Find: (a) Total force on the port
(b) Appropriate bolt diameter

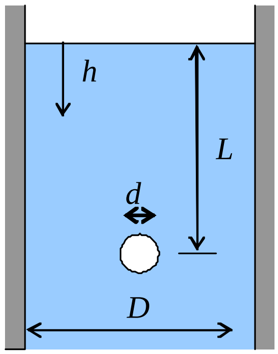
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$\frac{dp}{dh} = \rho \cdot g$ (Hydrostatic Pressure - y is positive downwards)

$F_R = p_c \cdot A$ (Hydrostatic Force)

$\sigma = \frac{F}{A}$ (Normal Stress in bolt)



Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Force is distributed evenly over all bolts
(4) Appropriate working stress in bolts is 100 MPa
(5) Atmospheric pressure acts at free surface of water and on outside of port.

Integrating the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$

The resultant force on the port is: $F_R = p_c \cdot A = \rho \cdot g \cdot L \cdot \frac{\pi}{4} \cdot d^2 \qquad F_R = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 12 \cdot m \times \frac{\pi}{4} \times (0.6 \cdot m)^2 \times \frac{N \cdot s^2}{kg \cdot m}$

$F_R = 33.3 \cdot kN$

To find the bolt diameter we consider: $\sigma = \frac{F_R}{A}$ where A is the area of all of the bolts: $A = 8 \times \frac{\pi}{4} \cdot d_b^2 = 2 \cdot \pi \cdot d_b^2$

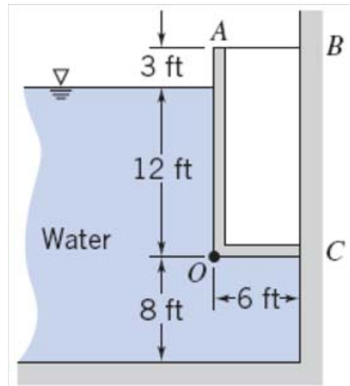
Therefore: $2 \cdot \pi \cdot d_b^2 = \frac{F_R}{\sigma}$ Solving for the bolt diameter we get: $d_b = \left(\frac{F_R}{2 \cdot \pi \cdot \sigma} \right)^{\frac{1}{2}}$

$$d_b = \left(\frac{1}{2 \times \pi} \times 33.3 \times 10^3 \cdot N \times \frac{1}{100 \times 10^6} \cdot \frac{m^2}{N} \right)^{\frac{1}{2}} \times \frac{10^3 \cdot mm}{m} \qquad d_b = 7.28 \cdot mm$$

Problem 3.54

[Difficulty: 3]

3.54 The gate AOC shown is 6 ft wide and is hinged along O. Neglecting the weight of the gate, determine the force in bar AB. The gate is sealed at C.



Given: Gate AOC, hinged along O, has known width;
Weight of gate may be neglected. Gate is sealed at C.

$b = 6 \cdot ft$

Find: Force in bar AB

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

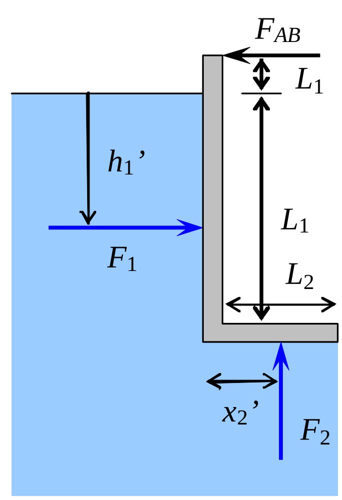
$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$F_R = p_c \cdot A$$ (Hydrostatic Force)

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c}$$ (Location of line of action)

$$\Sigma M_z = 0$$ (Rotational equilibrium)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts at free surface of water and on outside of gate
(4) No resisting moment in hinge at O
(5) No vertical resisting force at C



Integrating the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$

The free body diagram of the gate is shown here:

F_1 is the resultant of the distributed force on AO

F_2 is the resultant of the distributed force on OC

F_{AB} is the force of the bar

C_x is the sealing force at C

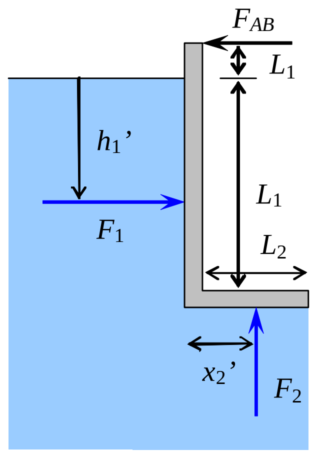
First find the force on AO: $F_1 = p_c \cdot A_1 = \rho \cdot g \cdot h_{c1} \cdot b \cdot L_1$

$$F_1 = 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 6 \cdot ft \times 6 \cdot ft \times 12 \cdot ft \times \frac{lbf \cdot s^2}{slug\,ft} \qquad F_1 = 27.0 \cdot kip$$

$$h'_1 = h_{c1} + \frac{I_{xx}}{A \cdot h_{c1}} = h_{c1} + \frac{b \cdot L_1^{\,3}}{12 \cdot b \cdot L_1 \cdot h_{c1}} = h_{c1} + \frac{L_1^{\,2}}{12 \cdot h_{c1}} \qquad h'_1 = 6 \cdot ft + \frac{(12 \cdot ft)^2}{12 \times 6 \cdot ft} \qquad h'_1 = 8 \cdot ft$$

Next find the force on OC: $F_2 = 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times 12 \cdot ft \times 6 \cdot ft \times 6 \cdot ft \times \frac{lbf \cdot s^2}{slug \cdot ft}$ $\qquad F_2 = 27.0 \cdot kip$

Since the pressure is uniform over OC, the force acts at the centroid of OC, i.e., $x'_2 = 3 \cdot ft$



Summing moments about the hinge gives: $F_{AB} \cdot (L_1 + L_3) - F_1 \cdot (L_1 - h'_1) + F_2 \cdot x'_2 = 0$

Solving for the force in the bar: $F_{AB} = \frac{F_1 \cdot (L_1 - h'_1) - F_2 \cdot x'_2}{L_1 + L_3}$

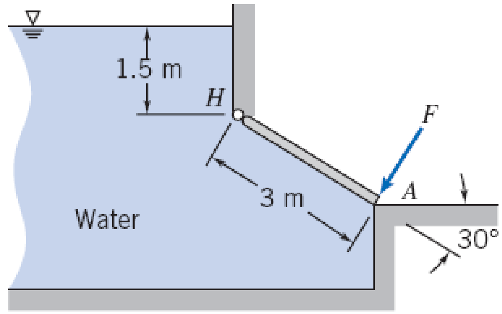
Substituting in values: $F_{AB} = \frac{1}{12 \cdot ft + 3 \cdot ft} \cdot \left[27.0 \times 10^3 \cdot lbf \times (12 \cdot ft - 8 \cdot ft) - 27.0 \times 10^3 \cdot lbf \times 3 \cdot ft\right]$

$F_{AB} = 1800 \cdot lbf$ $\qquad$ Thus bar AB is in compression

Problem 3.55

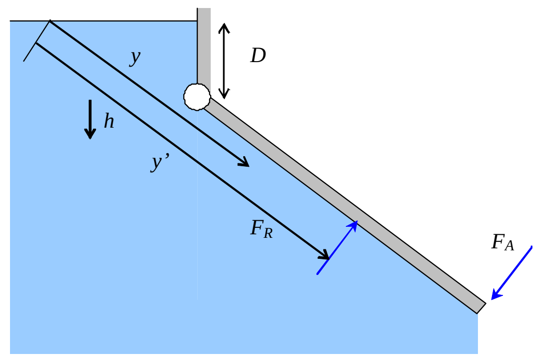
[Difficulty: 3]

3.55 The gate shown is hinged at *H*. The gate is 3 m wide normal to the plane of the diagram. Calculate the force required at *A* to hold the gate closed.



Given: Geometry of gate

Find: Force at A to hold gate closed



Solution:

Basic equation $\frac{dp}{dh} = \rho \cdot g$ $\quad \Sigma M_z = 0$

Computing equations $F_R = p_c \cdot A$ $\quad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$ $\quad I_{xx} = \frac{w \cdot L^3}{12}$

Assumptions: Static fluid; ρ = constant; p_{atm} on other side; no friction in hinge

For incompressible fluid $\quad p = \rho \cdot g \cdot h \quad$ where p is gage pressure and h is measured downwards

The hydrostatic force on the gate is that on a rectangle of size L and width w.

Hence
$$F_R = p_c \cdot A = \rho \cdot g \cdot h_c \cdot A = \rho \cdot g \cdot \left(D + \frac{L}{2} \cdot \sin(30 \cdot deg)\right) \cdot L \cdot w$$

$$F_R = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \left(1.5 + \frac{3}{2} \sin(30 \cdot deg)\right) \cdot m \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_R = 199 \cdot kN$$

The location of this force is given by $y' = y_c + \frac{I_{xx}}{A \cdot y_c}$ where y' and y_c are measured along the plane of the gate to the free surface

$$y_c = \frac{D}{\sin(30 \cdot deg)} + \frac{L}{2} \qquad y_c = \frac{1.5 \cdot m}{\sin(30 \cdot deg)} + \frac{3 \cdot m}{2} \qquad y_c = 4.5\,m$$

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = y_c + \frac{w \cdot L^3}{12} \cdot \frac{1}{w \cdot L} \cdot \frac{1}{y_c} = y_c + \frac{L^2}{12 \cdot y_c} = 4.5 \cdot m + \frac{(3 \cdot m)^2}{12 \cdot 4.5 \cdot m} \qquad y' = 4.67\,m$$

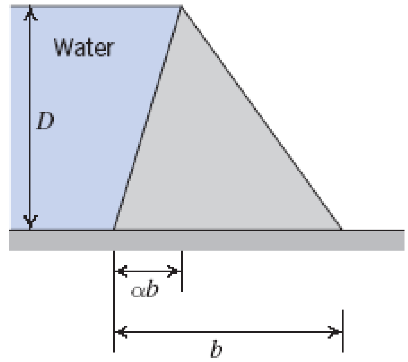
Taking moments about the hinge $\quad \Sigma M_H = 0 = F_R \cdot \left(y' - \frac{D}{\sin(30 \cdot deg)}\right) - F_A \cdot L$

$$F_A = F_R \cdot \frac{\left(y' - \frac{D}{\sin(30 \cdot deg)}\right)}{L} \qquad F_A = 199 \cdot kN \cdot \frac{\left(4.67 - \frac{1.5}{\sin(30 \cdot deg)}\right)}{3} \qquad F_A = 111 \cdot kN$$

Problem 3.56

[Difficulty: 4]

3.56 A solid concrete dam is to be built to hold back a depth D of water. For ease of construction the walls of the dam must be planar. Your supervisor asks you to consider the following dam cross-sections: a rectangle, a right triangle with the hypotenuse in contact with the water, and a right triangle with the vertical in contact with the water. She wishes you to determine which of these would require the least amount of concrete. What will your report say? You decide to look at one more possibility: a nonright triangle, as shown. Develop and plot an expression for the cross-section area A as a function of α, and find the minimum cross-sectional area.



Given: Various dam cross-sections

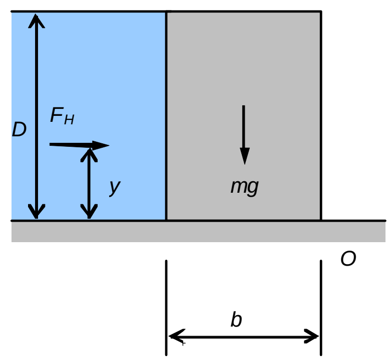
Find: Which requires the least concrete; plot cross-section area A as a function of α

Solution:

For each case, the dam width b has to be large enough so that the weight of the dam exerts enough moment to balance the moment due to fluid hydrostatic force(s). By doing a moment balance this value of b can be found

a) Rectangular dam

Straightforward application of the computing equations of Section 3-5 yields



$$F_H = p_c \cdot A = \rho \cdot g \cdot \frac{D}{2} \cdot w \cdot D = \frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w$$

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{D}{2} + \frac{w \cdot D^3}{12 \cdot w \cdot D \cdot \frac{D}{2}} = \frac{2}{3} \cdot D$$

so $$y = D - y' = \frac{D}{3}$$

Also $$m = \rho_{cement} \cdot g \cdot b \cdot D \cdot w = SG \cdot \rho \cdot g \cdot b \cdot D \cdot w$$

Taking moments about O $$\sum M_{0.} = 0 = -F_H \cdot y + \frac{b}{2} \cdot m \cdot g$$

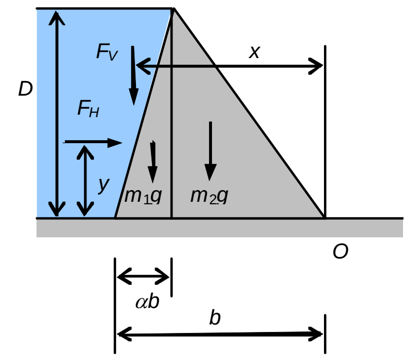
so $$\left(\frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w\right) \cdot \frac{D}{3} = \frac{b}{2} \cdot (SG \cdot \rho \cdot g \cdot b \cdot D \cdot w)$$

Solving for b $$b = \frac{D}{\sqrt{3 \cdot SG}}$$

The minimum rectangular cross-section area is $$A = b \cdot D = \frac{D^2}{\sqrt{3 \cdot SG}}$$

For concrete, from Table A.1, SG = 2.4, so $$A = \frac{D^2}{\sqrt{3 \cdot SG}} = \frac{D^2}{\sqrt{3 \times 2.4}}$$ $$A = 0.373 \cdot D^2$$

b) Triangular dams



Instead of analysing right-triangles, a general analysis is made, at the end of which right triangles are analysed as special cases by setting $\alpha = 0$ or 1.

Straightforward application of the computing equations of Section 3-5 yields

$$F_H = p_c \cdot A = \rho \cdot g \cdot \frac{D}{2} \cdot w \cdot D = \frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w$$

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \frac{D}{2} + \frac{w \cdot D^3}{12 \cdot w \cdot D \cdot \frac{D}{2}} = \frac{2}{3} \cdot D$$

so $$y = D - y' = \frac{D}{3}$$

Also $$F_V = \rho \cdot V \cdot g = \rho \cdot g \cdot \frac{\alpha \cdot b \cdot D}{2} \cdot w = \frac{1}{2} \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w \qquad x = (b - \alpha \cdot b) + \frac{2}{3} \cdot \alpha \cdot b = b \cdot \left(1 - \frac{\alpha}{3}\right)$$

For the two triangular masses

$$m_1 = \frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w \qquad x_1 = (b - \alpha \cdot b) + \frac{1}{3} \cdot \alpha \cdot b = b \cdot \left(1 - \frac{2 \cdot \alpha}{3}\right)$$

$$m_2 = \frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot (1 - \alpha) \cdot b \cdot D \cdot w \qquad x_2 = \frac{2}{3} \cdot b(1 - \alpha)$$

Taking moments about *O*

$$\sum M_{0.} = 0 = -F_H \cdot y + F_V \cdot x + m_1 \cdot g \cdot x_1 + m_2 \cdot g \cdot x_2$$

so $$-\left(\frac{1}{2} \cdot \rho \cdot g \cdot D^2 \cdot w\right) \cdot \frac{D}{3} + \left(\frac{1}{2} \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w\right) \cdot b \cdot \left(1 - \frac{\alpha}{3}\right) \dots \\ + \left(\frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot \alpha \cdot b \cdot D \cdot w\right) \cdot b \cdot \left(1 - \frac{2 \cdot \alpha}{3}\right) + \left[\frac{1}{2} \cdot SG \cdot \rho \cdot g \cdot (1 - \alpha) \cdot b \cdot D \cdot w\right] \cdot \frac{2}{3} \cdot b(1 - \alpha) = 0$$

Solving for *b* $$b = \frac{D}{\sqrt{\left(3 \cdot \alpha - \alpha^2\right) + SG \cdot (2 - \alpha)}}$$

For a right triangle with the hypotenuse in contact with the water, $\alpha = 1$, and

$$b = \frac{D}{\sqrt{3 - 1 + SG}} = \frac{D}{\sqrt{3 - 1 + 2.4}} \qquad b = 0.477 \cdot D$$

The cross-section area is $$A = \frac{b \cdot D}{2} = 0.238 \cdot D^2 \qquad A = 0.238 \cdot D^2$$

For a right triangle with the vertical in contact with the water, $\alpha = 0$, and

$$b = \frac{D}{\sqrt{2 \cdot SG}} = \frac{D}{\sqrt{2 \cdot 2.4}} \qquad b = 0.456 \cdot D$$

The cross-section area is

$$A = \frac{b \cdot D}{2} = 0.228 \cdot D^2 \qquad A = 0.228 \cdot D^2$$

For a general triangle

$$A = \frac{b \cdot D}{2} = \frac{D^2}{2 \cdot \sqrt{\left(3 \cdot \alpha - \alpha^2\right) + SG \cdot (2 - \alpha)}} \qquad A = \frac{D^2}{2 \cdot \sqrt{\left(3 \cdot \alpha - \alpha^2\right) + 2.4 \cdot (2 - \alpha)}}$$

The final result is

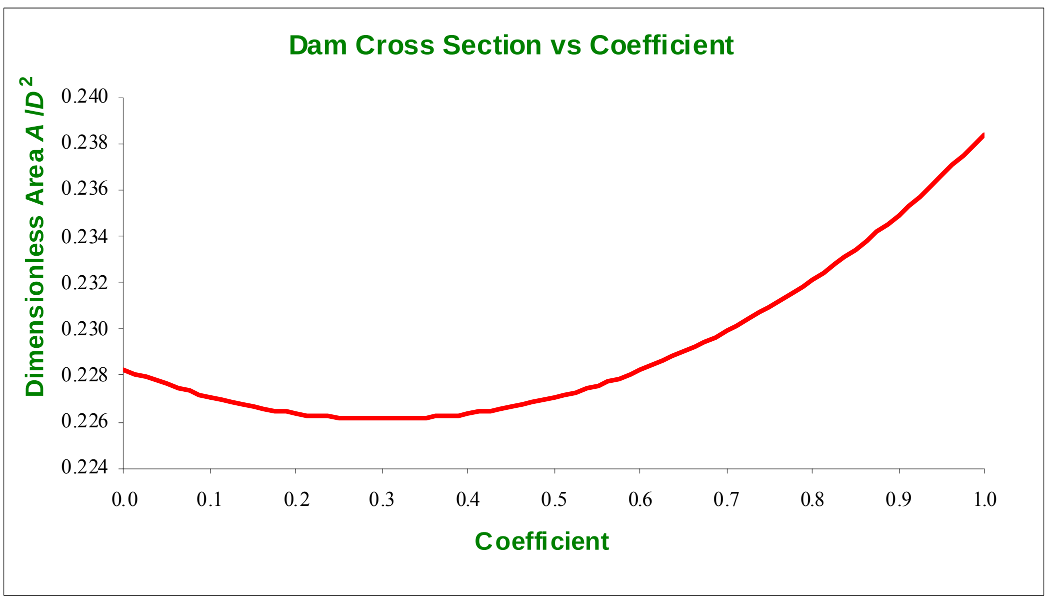
$$A = \frac{D^2}{2 \cdot \sqrt{4.8 + 0.6 \cdot \alpha - \alpha^2}}$$

The dimensionless area, A/D^2, is plotted

Alpha	A/D^2
0.0	0.2282
0.1	0.2270
0.2	0.2263
0.3	0.2261
0.4	0.2263
0.5	0.2270
0.6	0.2282
0.7	0.2299
0.8	0.2321
0.9	0.2349
1.0	0.2384

Solver can be used to find the minimum area

Alpha	A/D^2
0.300	0.2261



From the Excel workbook, the minimum area occurs at $\alpha = 0.3$

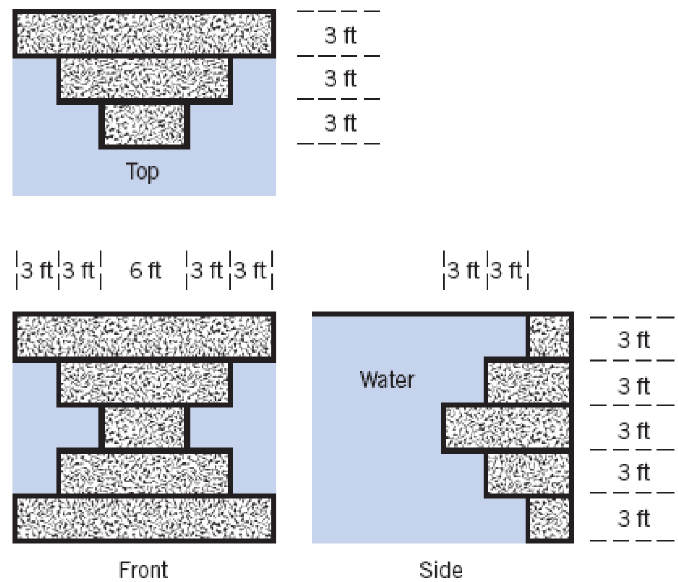
$$A_{min} = \frac{D^2}{2 \cdot \sqrt{4.8 + 0.6 \times 0.3 - 0.3^2}} \qquad A = 0.226 \cdot D^2$$

The final results are that a triangular cross-section with $\alpha = 0.3$ uses the least concrete; the next best is a right triangle with the vertical in contact with the water; next is the right triangle with the hypotenuse in contact with the water; and the cross-section requiring the most concrete is the rectangular cross-section.

Problem 3.57

[Difficulty: 2]

3.57 For the dam shown, what is the vertical force of the water on the dam?



Given: Geometry of dam

Find: Vertical force on dam

Assumptions: (1) water is static and incompressible
(2) since we are asked for the force of the water, all pressures will be written as gage

Solution:

Basic equation: $$\frac{dp}{dh} = \rho \cdot g$$

For incompressible fluid $p = \rho \cdot g \cdot h$ where p is gage pressure and h is measured downwards from the free surface

The force on each horizontal section (depth d and width w) is

$$F = p \cdot A = \rho \cdot g \cdot h \cdot d \cdot w$$ (Note that d and w will change in terms of x and y for each section of the dam!)

Hence the total force is (allowing for the fact that some faces experience an upwards (negative) force)

$$F_T = p \cdot A = \Sigma \rho \cdot g \cdot h \cdot d \cdot w = \rho \cdot g \cdot d \cdot \Sigma h \cdot w$$

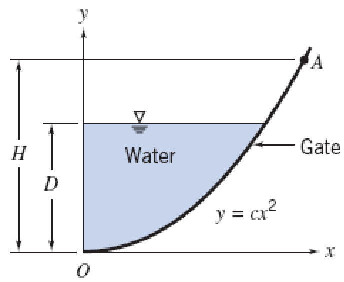
Starting with the top and working downwards

$$F_T = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times 3 \cdot \text{ft} \times [(3 \cdot \text{ft} \times 12 \cdot \text{ft}) + (3 \cdot \text{ft} \times 6 \cdot \text{ft}) - (9 \cdot \text{ft} \times 6 \cdot \text{ft}) - (12 \cdot \text{ft} \times 12 \cdot \text{ft})] \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$F_T = -2.70 \times 10^4 \cdot \text{lbf}$ The negative sign indicates a net upwards force (it's actually a buoyancy effect on the three middle sections)

Problem 3.58 [Difficulty: 3]

3.58 The parabolic gate shown is 2 m wide and pivoted at O; $c = 0.25\ \text{m}^{-1}$, $D = 2$ m, and $H = 3$ m. Determine (a) the magnitude and line of action of the vertical force on the gate due to the water, (b) the horizontal force applied at A required to maintain the gate in equilibrium, and (c) the vertical force applied at A required to maintain the gate in equilibrium.



Given: Parabolic gate, hinged at O has a constant width.

$$b = 2\cdot m \quad c = 0.25\cdot m^{-1} \quad D = 2\cdot m \quad H = 3\cdot m$$

Find:
(a) Magnitude and line of action of the vertical force on the gate due to water
(b) Horizontal force applied at A required to maintain equilibrium
(c) Vertical force applied at A required to maintain equilibrium

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho\cdot g$$ (Hydrostatic Pressure - h is positive downwards)

$$\Sigma M_z = 0$$ (Rotational equilibrium)

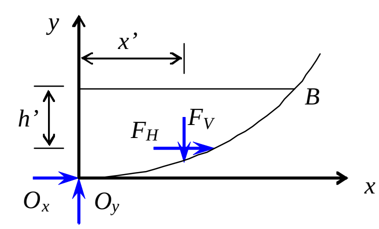
$$F_V = \int p\, dA_y$$ (Vertical Hydrostatic Force)

$$x'\cdot F_V = \int x\, dF_V$$ (Location of line of action)

$$F_H = p_c\cdot A$$ (Horizontal Hydrostatic Force)

$$h' = h_c + \frac{I_{xx}}{A\cdot h_c}$$ (Location of line of action)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts at free surface of water and on outside of gate



Integrating the hydrostatic pressure equation: $p = \rho\cdot g\cdot h$

(a) The magnitude and line of action of the vertical component of hydrostatic force:

$$F_V = \int p\, dA_y = \int_0^{\sqrt{\frac{D}{c}}} \rho\cdot g\cdot h\cdot b\, dx = \int_0^{\sqrt{\frac{D}{c}}} \rho\cdot g\cdot (D - y) b\, dx = \int_0^{\sqrt{\frac{D}{c}}} \rho\cdot g\cdot \left(D - c\cdot x^2\right) b\, dx = \rho\cdot g\cdot b\cdot \int_0^{\sqrt{\frac{D}{c}}} \left(D - c\cdot x^2\right) dx$$

Evaluating the integral: $$F_V = \rho\cdot g\cdot b\cdot \left(\frac{D^{\frac{3}{2}}}{c^{\frac{1}{2}}} - \frac{1}{3}\cdot\frac{D^{\frac{3}{2}}}{c^{\frac{1}{2}}}\right) = \frac{2\cdot\rho\cdot g\cdot b}{3}\cdot\frac{D^{\frac{3}{2}}}{c^{\frac{1}{2}}} \qquad (1)$$

Substituting values: $F_V = \frac{2}{3} \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 2 \cdot m \times (2 \cdot m)^{\frac{3}{2}} \times \left(\frac{1}{0.25} \cdot m\right)^{\frac{1}{2}} \times \frac{N \cdot s^2}{kg \cdot m}$ $\quad F_V = 73.9 \cdot kN$

To find the line of action of this force: $x' \cdot F_V = \int x\, dF_V$ Therefore, $x' = \frac{1}{F_V} \cdot \int x\, dF_V = \frac{1}{F_V} \cdot \int x \cdot p\, dA_y$

Using the derivation for the force: $x' = \frac{1}{F_V} \cdot \int_0^{\sqrt{\frac{D}{c}}} x \cdot \rho \cdot g \cdot \left(D - c \cdot x^2\right) \cdot b\, dx = \frac{\rho \cdot g \cdot b}{F_V} \cdot \int_0^{\sqrt{\frac{D}{c}}} \left(D \cdot x - c \cdot x^3\right) dx$

Evaluating the integral: $x' = \frac{\rho \cdot g \cdot b}{F_V} \cdot \left[\frac{D}{2} \cdot \frac{D}{c} - \frac{c}{4} \cdot \left(\frac{D}{c}\right)^2\right] = \frac{\rho \cdot g \cdot b}{F_V} \cdot \frac{D^2}{4 \cdot c}$ Now substituting values into this equation:

$x' = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 2 \cdot m \times \frac{1}{73.9 \times 10^3} \cdot \frac{1}{N} \times \frac{1}{4} \times (2 \cdot m)^2 \times \frac{1}{0.25} \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$ $\quad x' = 1.061\, m$

To find the required force at A for equilibrium, we need to find the horizontal force of the water on the gate and its line of action as well. Once this force is known we take moments about the hinge (point O).

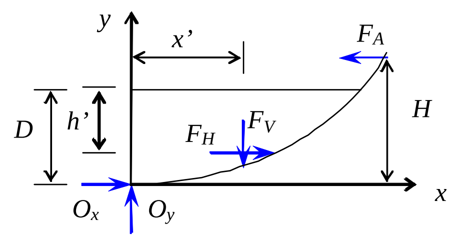
$F_H = p_c \cdot A = \rho \cdot g \cdot h_c \cdot b \cdot D = \rho \cdot g \cdot \frac{D}{2} \cdot b \cdot D = \rho \cdot g \cdot b \cdot \frac{D^2}{2}$ since $h_c = \frac{D}{2}$ Therefore the horizontal force is:

$$F_H = 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 2 \cdot m \times \frac{(2 \cdot m)^2}{2} \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_H = 39.2 \cdot kN$$

To calculate the line of action of this force:

$h' = h_c + \frac{I_{xx}}{A \cdot h_c} = \frac{D}{2} + \frac{b \cdot D^3}{12} \cdot \frac{1}{b \cdot D} \cdot \frac{2}{D} = \frac{D}{2} + \frac{D}{6} = \frac{2}{3} \cdot D$ $\quad h' = \frac{2}{3} \cdot 2 \cdot m$ $\quad h' = 1.333\, m$

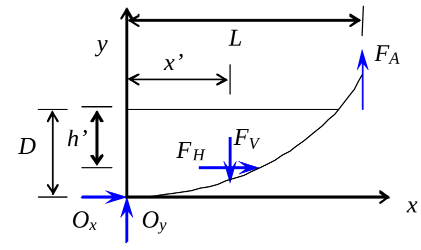
Now we have information to solve parts (b) and (c):
(b) Horizontal force applied at A for equilibrium: take moments about O:



$F_A \cdot H - F_V \cdot x' - F_H \cdot (D - h') = 0$ Solving for F_A $\quad F_A = \frac{F_V \cdot x' + F_H \cdot (D - h')}{H}$

$F_A = \frac{1}{3} \cdot \frac{1}{m} \times [73.9 \cdot kN \times 1.061 \cdot m + 39.2 \cdot kN \times (2 \cdot m - 1.333 \cdot m)]$ $\quad F_A = 34.9 \cdot kN$

(c) Vertical force applied at A for equilibrium: take moments about O:



$F_A \cdot L - F_V \cdot x' - F_H \cdot (D - h') = 0$ Solving for F_A $\quad F_A = \frac{F_V \cdot x' + F_H \cdot (D - h')}{L}$

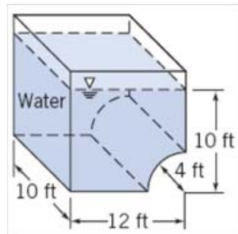
L is the value of x at y = H. Therefore: $L = \sqrt{\frac{H}{c}}$ $\quad L = \sqrt{3 \cdot m \times \frac{1}{0.25} \cdot m}$ $\quad L = 3.464\, m$

$F_A = \frac{1}{3.464} \cdot \frac{1}{m} \times [73.9 \cdot kN \times 1.061 \cdot m + 39.2 \cdot kN \times (2 \cdot m - 1.333 \cdot m)]$ $\quad F_A = 30.2 \cdot kN$

Problem 3.59

[Difficulty: 2]

3.59 An open tank is filled with water to the depth indicated. Atmospheric pressure acts on all outer surfaces of the tank. Determine the magnitude and line of action of the vertical component of the force of the water on the curved part of the tank bottom.



Given: Open tank as shown. Width of curved surface $b = 10 \cdot ft$

Find: (a) Magnitude of the vertical force component on the curved surface
(b) Line of action of the vertical component of the force

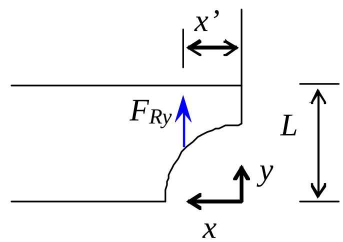
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \gamma \qquad \text{(Hydrostatic Pressure - h is positive downwards)}$$

$$F_V = -\int p\, dA_y \qquad \text{(Vertical Hydrostatic Force)}$$

$$x' \cdot F_V = \int x\, dF_V \qquad \text{(Moment of vertical force)}$$



Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts at free surface of water and on outside of wall

Integrating the hydrostatic pressure equation: $p = \gamma \cdot h$ We can define along the surface $h = L - \left(R^2 - x^2\right)^{\frac{1}{2}}$

We also define the incremental area on the curved surface as: $dA_y = b \cdot dx$ Substituting these into the force equation we get:

$$F_V = -\int p\, dA_y = -\int_0^R \gamma \cdot \left[L - \left(R^2 - x^2\right)^{\frac{1}{2}}\right] \cdot b\, dx = -\gamma \cdot b \cdot \int_0^R \left(L - \sqrt{R^2 - x^2}\right) dx = -\gamma \cdot b \cdot R \cdot \left(L - R \cdot \frac{\pi}{4}\right)$$

$$F_V = -\left[62.4 \cdot \frac{lbf}{ft^3} \times 10 \cdot ft \times 4 \cdot ft \times \left(10 \cdot ft - 4 \cdot ft \times \frac{\pi}{4}\right)\right] \qquad F_V = -17.12 \times 10^3 \cdot lbf \qquad \text{(negative indicates downward)}$$

To find the line of action of the force: $x' \cdot F_V = \int x\, dF_V$ where $dF_V = -\gamma \cdot b \cdot \left(L - \sqrt{R^2 - x^2}\right) \cdot dx$

Therefore:

$$x' = \frac{x' \cdot F_V}{F_V} = \frac{1}{\gamma \cdot b \cdot R \cdot \left(L - R \cdot \frac{\pi}{4}\right)} \cdot \int_0^R x \cdot \gamma \cdot b \cdot \left(L - \sqrt{R^2 - x^2}\right) dx = \frac{1}{R \cdot \left(L - R \cdot \frac{\pi}{4}\right)} \cdot \int_0^R \left(L \cdot x - x \cdot \sqrt{R^2 - x^2}\right) dx$$

Evaluating the integral:

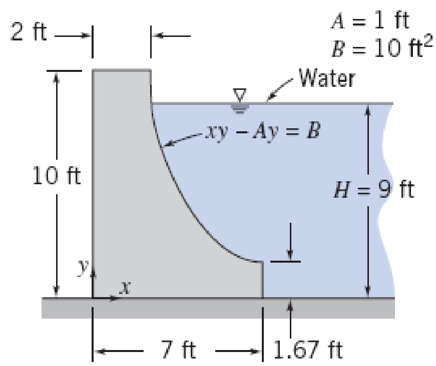
$$x' = \frac{4}{R \cdot (4 \cdot L - \pi \cdot R)} \cdot \left(\frac{1}{2} \cdot L \cdot R^2 - \frac{1}{3} \cdot R^3\right) = \frac{4 \cdot R^2}{R \cdot (4 \cdot L - \pi \cdot R)} \cdot \left(\frac{L}{2} - \frac{R}{3}\right) = \frac{4 \cdot R}{4 \cdot L - \pi \cdot R} \cdot \left(\frac{L}{2} - \frac{R}{3}\right)$$

Substituting known values:

$$x' = \frac{4 \cdot 4 \cdot ft}{4 \cdot 10 \cdot ft - \pi \cdot 4 \cdot ft} \cdot \left(\frac{10 \cdot ft}{2} - \frac{4 \cdot ft}{3}\right) \qquad x' = 2.14 \cdot ft$$

Problem 3.60 [Difficulty: 3]

3.60 A dam is to be constructed using the cross-section shown. Assume the dam width is $w = 160$ ft. For water height $H = 9$ ft, calculate the magnitude and line of action of the vertical force of water on the dam face. Is it possible for water forces to overturn this dam? Under what circumstances will this happen?



Given: Dam with cross-section shown. Width of dam

$b = 160 \cdot ft$

Find: (a) Magnitude and line of action of the vertical force component on the dam
(b) If it is possible for the water to overturn dam

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards from free surface)

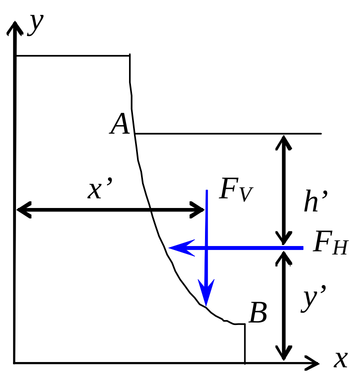
$$F_V = \int p\, dA_y$$ (Vertical Hydrostatic Force)

$$F_H = p_c \cdot A$$ (Horizontal Hydrostatic Force)

$$x' \cdot F_V = \int x\, dF_V$$ (Moment of vertical force)

$$h' = h_c + \frac{I_{xx}}{h_c \cdot A}$$ (Line of action of vertical force)

$$\Sigma M_z = 0$$ (Rotational Equilibrium)



Assumptions: (1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts at free surface of water and on outside of dam

Integrating the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$

Into the vertical force equation: $F_V = \int p\, dA_y = \int_{x_A}^{x_B} \rho \cdot g \cdot h \cdot b\, dx = \rho \cdot g \cdot b \cdot \int_{x_A}^{x_B} (H - y)\, dx$

From the definition of the dam contour: $x \cdot y - A \cdot y = B$ Therefore: $y = \frac{B}{x - A}$ and $x_A = \frac{10 \cdot ft^2}{9 \cdot ft} + 1 \cdot ft$ $\quad x_A = 2.11 \cdot ft$

Into the force equation: $$F_V = \rho\cdot g\cdot b\cdot\int_{x_A}^{x_B}\left(H - \frac{B}{x-A}\right)dx = \rho\cdot g\cdot b\cdot\left[H\cdot\left(x_B - x_A\right) - B\cdot\ln\left(\frac{x_B - A}{x_A - A}\right)\right]$$ Substituting known values:

$$F_V = 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times 160\cdot ft\times\left[9\cdot ft\times(7.0\cdot ft - 2.11\cdot ft) - 10\cdot ft^2\times\ln\left(\frac{7.0-1}{2.11-1}\right)\right]\cdot\frac{lbf\cdot s^2}{slug\cdot ft} \qquad F_V = 2.71\times 10^5\cdot lbf$$

To find the line of action of the force: $x'\cdot F_V = \int x\,dF_V$ where $dF_V = \rho\cdot g\cdot b\cdot\left(H - \frac{B}{x-A}\right)\cdot dx$ Therefore:

$$x' = \frac{x'\cdot F_V}{F_V} = \frac{1}{F_V}\cdot\int_{x_A}^{x_B} x\cdot\rho\cdot g\cdot b\cdot\left(H - \frac{B}{x-A}\right)dx = \frac{1}{H\cdot\left(x_B - x_A\right) - B\cdot\ln\left(\frac{x_B - A}{x_A - A}\right)}\cdot\int_{x_A}^{x_B}\left(H\cdot x - \frac{B\cdot x}{x-A}\right)dx$$

Evaluating the integral: $$x' = \frac{\frac{H}{2}\cdot\left(x_B^2 - x_A^2\right) - B\cdot\left(x_B - x_A\right) - B\cdot A\cdot\ln\left(\frac{x_B - A}{x_A - A}\right)}{H\cdot\left(x_B - x_A\right) - B\cdot\ln\left(\frac{x_B - A}{x_A - A}\right)}$$ Substituting known values we get:

$$x' = \frac{\frac{9\cdot ft}{2}\times\left(7^2 - 2.11^2\right)\cdot ft^2 - 10\cdot ft^2\times(7-2.11)\cdot ft - 10\cdot ft^2\times 1\cdot ft\times\ln\left(\frac{7-1}{2.11-1}\right)}{9\cdot ft\times(7-2.11)\cdot ft - 10\cdot ft^2\times\ln\left(\frac{7-1}{2.11-1}\right)} \qquad x' = 4.96\cdot ft$$

To determine whether or not the water can overturn the dam, we need the horizontal force and its line of action:

$$F_H = p_c\cdot A = \rho\cdot g\cdot\frac{H}{2}\cdot H\cdot b = \frac{\rho\cdot g\cdot b\cdot H^2}{2}$$

Substituting values: $$F_H = \frac{1}{2}\times 1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times 160\cdot ft\times(9\cdot ft)^2\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad F_H = 4.05\times 10^5\cdot lbf$$

For the line of action: $h' = h_c + \frac{I_{xx}}{h_c\cdot A}$ where $h_c = \frac{H}{2}$ $\quad A = H\cdot b \quad I_{xx} = \frac{b\cdot H^3}{12}$

Therefore: $h' = \frac{H}{2} + \frac{b\cdot H^3}{12}\cdot\frac{2}{H}\cdot\frac{1}{b\cdot H} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3}\cdot H \qquad h' = \frac{2}{3}\cdot 9\cdot ft \qquad h' = 6.00\cdot ft$

Taking moments of the hydrostatic forces about the origin:

$$M_w = F_H\cdot(H - h') - F_V\cdot x' \qquad M_w = 4.05\times 10^5\cdot lbf\times(9-6)\cdot ft - 2.71\times 10^5\cdot lbf\times 4.96\cdot ft \qquad M_w = -1.292\times 10^5\cdot lbf\cdot ft$$

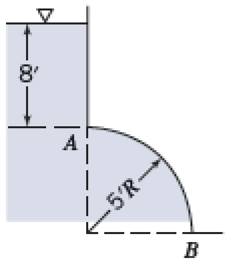
The negative sign indicates that this is a clockwise moment about the origin. Since the weight of the dam will also contribute a clockwise moment about the origin, these two moments should not cause the dam to tip to the left.

Therefore, the water can not overturn the dam.

Problem 3.61

(Difficulty: 2)

3.61 The quarter cylinder AB is $10\ ft$ long. Calculate magnitude, direction, and location of the resultant force of the water on AB.



Given: All the parameters are shown in the figure.

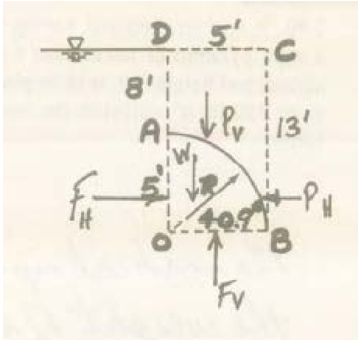
Assumptions: Fluid is incompressible and static

Find: The resultant force.

Solution: Apply the hydrostatic relations for pressure as a function of depth and for the location of forces on submerged objects.

$$\Delta p = \rho g h$$

A freebody diagram for the cylinder is:



The force balance in the horizontal direction yields thathorizontal force is due to the water pressure:

$$F_H = P_H$$

Where the depth is the distance to the centroid of the horizontal area (8 + 5/2 ft):

$$F_H = \gamma h_c A = 62.4\ \frac{lbf}{ft^3} \times \left(8\ ft + \frac{5\ ft}{2}\right) \times (5\ ft \times 10\ ft) = 32800\ lbf$$

$$P_H = 32800\ lbf$$

The force in the vertical direction can be calculated as the weight of a volume of water that is 8 ft + 5 ft = 13 ft deep less the weight of water that would be in the quarter cylinder. This force is then:

$$P_V = F_V - W = 62.4\ \frac{lbf}{ft^3} \times 13\ ft \times (5\ ft \times 10\ ft) - 62.4\ \frac{lbf}{ft^3} \times \frac{\pi}{4} \times (5\ ft)^2 \times (10\ ft) = 28308\ lbf$$

The total resultant force is the vector sum of the two forces:

$$P = \sqrt{{P_H}^2 + {P_V}^2} = \sqrt{(32800\ lbf)^2 + (28308\ lbf)^2} = 43300\ lbf$$

The angle with respect to the horizontal is:

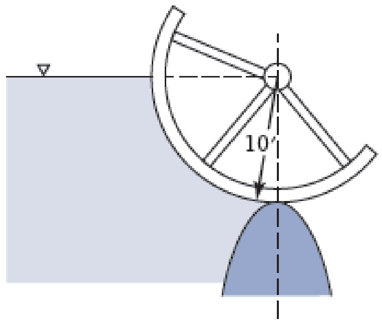
$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{28308\ lbf}{32800\ lbf}\right) = 40.9°$$

So the force acts on the quarter cylinder surface point at an angle of $\theta = 40.9\ °$ with respect to the horizontal.

Problem 3.62

(Difficulty: 2)

3.62 Calculate the magnitude, direction (horizontal and vertical components are acceptable), and line of action of the resultant force exerted by the water on the cylindrical gate 30 ft long.



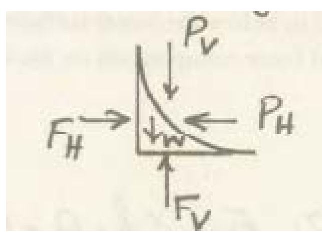
Assumptions: Fluid is incompressible and static

Find: The resultant forces.

Solution: Apply the hydrostatic relations for pressure as a function of depth and for the location of forces on submerged objects.

$$\Delta p = \rho g h$$

A free body diagram of the gate is



The horizontal force is calculated as:

$$P_H = F_H$$

Where the depth is the distance to the centroid of the horizontal area (5 ft):

$$F_H = \gamma h_c A = 62.4 \frac{lbf}{ft^3} \times 5ft \times (10\ ft \times 30\ ft) = 93600\ lbf$$

$$P_H = 93600\ lbf$$

The force in the vertical direction can be calculated as the weight of a volume of water that is 10 ft deep less the weight of water that would be in the quarter cylinder. This force is then:

$$P_V = F_V - W = \gamma h_c A - \gamma \forall$$

$$P_V = 62.4\ \frac{lbf}{ft^3} \times 10\ ft \times (10\ ft \times 30\ ft) - 62.4\ \frac{lbf}{ft^3} \times \left[10\ ft \times (10\ ft \times 30\ ft) - \frac{\pi}{4} \times (10\ ft)^2 \times 30\ ft\right] = 147000\ lbf$$

The total resultant force is the vector sum of the two forces:

$$P = \sqrt{{P_H}^2 + {P_V}^2} = \sqrt{(93600\ lbf)^2 + (147000\ lbf)^2} = 174200\ lbf$$

The direction can be calculated as:

$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) = \tan^{-1}\left(\frac{147000\ \ lbf}{93600\ lbf}\right) = 57.5°$$

Problem 3.63

(Difficulty: 2)

3.63 A hemispherical shell 1.2 m in diameter is connected to the vertical wall of a tank containing water. If the center of the shell is 1.8 m below the water surface, what are the vertical and horizontal force components on the shell? On the top half of the shell?

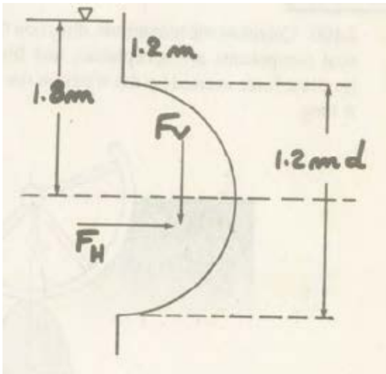
Assumptions: Fluid is incompressible and static

Find: The resultant forces.

Solution: Apply the hydrostatic relations for pressure as a function of depth and for the location of forces on submerged objects.

$$\Delta p = \rho g h$$

A free body diagram of the system is



The force in the horizontal direction can be calculated using the distance to the centroid (1.8 m) as:

$$F_H = \gamma h_c A = 9.81\ \frac{kN}{m^3} \times 1.8\ m \times \left(\frac{1}{4} \times \pi \times (1.2\ m)^2\right) = 19.97\ kN$$

The force in the vertical direction is the buoyancy force due to the volume displaced by the shell:

$$F_V = \gamma V = 9.81\ \frac{kN}{m^3} \times \frac{1}{2} \times \frac{1}{6} \times \pi \times (1.2\ m)^3 = 4.44\ kN$$

For the top shell, the horizontal force acts at:

$$y_c = 1.8\ m - \frac{4 \times 0.6\ m}{3\pi} = 1.545\ m$$

The horizontal force on the top half of the shell is then:

$$F_H = \gamma y_c A = 9.81\ \frac{kN}{m^3} \times 1.545\ m \times \frac{\pi}{8} \times (1.2\ m)^2 = 8.57\ kN$$

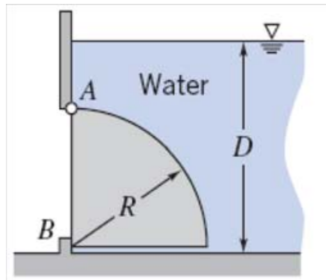
The vertical force on the top half of the shell is the buoyancy force:

$$F_V = pA = 9.81\ \frac{kN}{m^3} \times 1.8\ m \times \frac{\pi}{8} \times (1.2\ m)^2 - 9.81\ \frac{kN}{m^3} \times \frac{1}{4} \times \frac{1}{6} \times \pi \times (1.2\ m)^3 = 7.77\ kN$$

Problem 3.64

[Difficulty: 4]

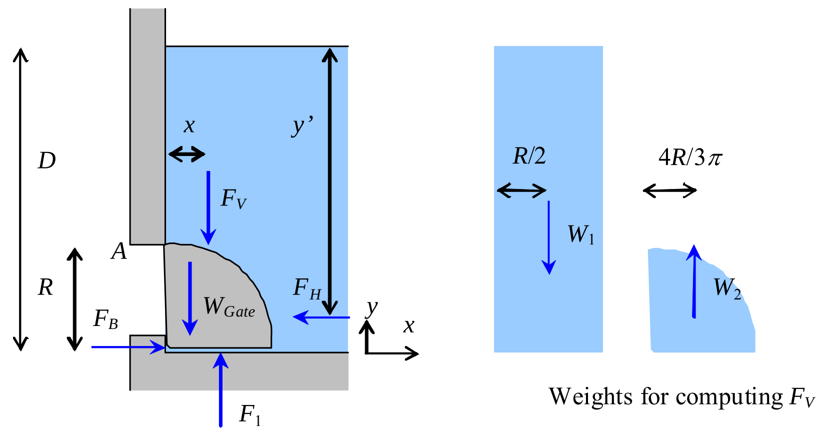
3.64 A gate, in the shape of a quarter-cylinder, hinged at A and sealed at B, is 3 m wide. The bottom of the gate is 4.5 m below the water surface. Determine the force on the stop at B if the gate is made of concrete; $R = 3$ m.



Given: Gate geometry

Find: Force on stop B

Solution:



Weights for computing F_V

Basic equations $\frac{dp}{dh} = \rho \cdot g$ $\quad \Sigma M_A = 0$

Assumptions: static fluid; ρ = constant; p_{atm} on other side

For incompressible fluid $\quad p = \rho \cdot g \cdot h \quad$ where p is gage pressure and h is measured downwards

We need to compute force (including location) due to water on curved surface and underneath. For curved surface we could integrate pressure, but here we use the concepts that F_V (see sketch) is equivalent to the weight of fluid above, and F_H is equivalent to the force on a vertical flat plate. Note that the sketch only shows forces that will be used to compute the moment at A

For F_V $\quad F_V = W_1 - W_2$

with

$$W_1 = \rho \cdot g \cdot w \cdot D \cdot R = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times 4.5 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_1 = 397 \cdot kN$$

$$W_2 = \rho \cdot g \cdot w \cdot \frac{\pi \cdot R^2}{4} = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times \frac{\pi}{4} \times (3 \cdot m)^2 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_2 = 208 \cdot kN$$

$$F_V = W_1 - W_2 \qquad F_V = 189 \cdot kN$$

with x given by $\quad F_V \cdot x = W_1 \cdot \frac{R}{2} - W_2 \cdot \frac{4 \cdot R}{3 \cdot \pi} \quad$ or $\quad x = \frac{W_1}{F_V} \cdot \frac{R}{2} - \frac{W_2}{F_V} \cdot \frac{4 \cdot R}{3 \cdot \pi}$

$$x = \frac{397}{189} \times \frac{3 \cdot m}{2} - \frac{208}{189} \times \frac{4}{3 \cdot \pi} \times 3 \cdot m \qquad x = 1.75\,m$$

For F_H $\quad$ Computing equations $\quad F_H = p_c \cdot A \quad y' = y_c + \frac{I_{xx}}{A \cdot y_c}$

Hence $$F_H = p_c \cdot A = \rho \cdot g \cdot \left(D - \frac{R}{2}\right) \cdot w \cdot R$$

$$F_H = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \left(4.5 \cdot m - \frac{3 \cdot m}{2}\right) \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_H = 265 \cdot kN$$

The location of this force is

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = \left(D - \frac{R}{2}\right) + \frac{w \cdot R^3}{12} \times \frac{1}{w \cdot R \cdot \left(D - \frac{R}{2}\right)} = D - \frac{R}{2} + \frac{R^2}{12 \cdot \left(D - \frac{R}{2}\right)}$$

$$y' = 4.5 \cdot m - \frac{3 \cdot m}{2} + \frac{(3 \cdot m)^2}{12 \times \left(4.5 \cdot m - \frac{3 \cdot m}{2}\right)} \qquad y' = 3.25 \, m$$

The force F_1 on the bottom of the gate is $F_1 = p \cdot A = \rho \cdot g \cdot D \cdot w \cdot R$

$$F_1 = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 4.5 \cdot m \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_1 = 397 \cdot kN$$

For the concrete gate (SG = 2.4 from Table A.2)

$$W_{Gate} = SG \cdot \rho \cdot g \cdot w \cdot \frac{\pi \cdot R^2}{4} = 2.4 \cdot 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3 \cdot m \times \frac{\pi}{4} \times (3 \cdot m)^2 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W_{Gate} = 499 \cdot kN$$

Hence, taking moments about A $$F_B \cdot R + F_1 \cdot \frac{R}{2} - W_{Gate} \cdot \frac{4 \cdot R}{3 \cdot \pi} - F_V \cdot x - F_H \cdot [y' - (D - R)] = 0$$

$$F_B = \frac{4}{3 \cdot \pi} \cdot W_{Gate} + \frac{x}{R} \cdot F_V + \frac{[y' - (D - R)]}{R} \cdot F_H - \frac{1}{2} \cdot F_1$$

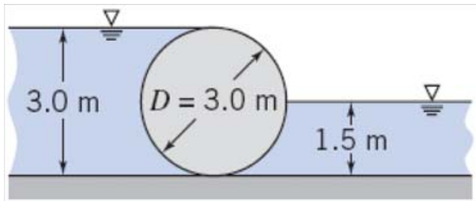
$$F_B = \frac{4}{3 \cdot \pi} \times 499 \cdot kN + \frac{1.75}{3} \times 189 \cdot kN + \frac{[3.25 - (4.5 - 3)]}{3} \times 265 \cdot kN - \frac{1}{2} \times 397 \cdot kN$$

$$F_B = 278 \cdot kN$$

Problem 3.65

[Difficulty: 3]

3.65 A cylindrical weir has a diameter of 3 m and a length of 6 m. Find the magnitude and direction of the resultant force acting on the weir from the water.



Given: Cylindrical weir as shown; liquid is water

Find: Magnitude and direction of the resultant force of the water on the weir

Solution: We will apply the hydrostatics equations to this system.

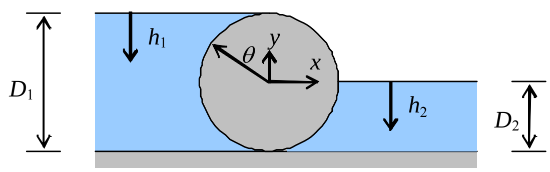
Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards from free surface)

$$\overrightarrow{dF_R} = -p \cdot \overrightarrow{dA}$$ (Hydrostatic Force)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts on free surfaces and on the first quadrant of the cylinder

Using the coordinate system shown in the diagram at the right:



$$F_{Rx} = \vec{F_R} \cdot \hat{i} = -\int p\, \vec{dA} \cdot \hat{i} = -\int p \cdot \cos(\theta + 90 \cdot deg)\, dA = \int p \cdot \sin(\theta)\, dA$$

$$F_{Ry} = \vec{F_R} \cdot \hat{j} = -\int p\, \vec{dA} \cdot \hat{j} = -\int p \cdot \cos(\theta)\, dA$$ Now since $dA = L \cdot R \cdot d\theta$ it follows that

$$F_{Rx} = \int_0^{\frac{3 \cdot \pi}{2}} p \cdot L \cdot R \cdot \sin(\theta)\, d\theta \quad \text{and} \quad F_{Ry} = -\int_0^{\frac{3 \cdot \pi}{2}} p \cdot L \cdot R \cdot \cos(\theta)\, d\theta$$

Next, we integrate the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$ Now over the range $0 \le \theta \le \pi$ $h_1 = R(1 - \cos(\theta))$

Over the range $\pi \le \theta \le \frac{3 \cdot \pi}{2}$ $h_2 = -R \cdot \cos(\theta)$

Therefore we can express the pressure in terms of θ and substitute into the force equations:

$$F_{Rx} = \int_0^{\frac{3 \cdot \pi}{2}} p \cdot L \cdot R \cdot \sin(\theta)\, d\theta = \int_0^{\pi} \rho \cdot g \cdot R \cdot (1 - \cos(\theta)) \cdot L \cdot R \cdot \sin(\theta)\, d\theta - \int_{\pi}^{\frac{3 \cdot \pi}{2}} \rho \cdot g \cdot R \cdot \cos(\theta) \cdot L \cdot R \cdot \sin(\theta)\, d\theta$$

$$F_{Rx} = \rho \cdot g \cdot R^2 \cdot L \cdot \int_0^{\pi} (1 - \cos(\theta)) \cdot \sin(\theta)\, d\theta - \rho \cdot g \cdot R^2 \cdot L \cdot \int_{\pi}^{\frac{3 \cdot \pi}{2}} \cos(\theta) \cdot \sin(\theta)\, d\theta$$

$$F_{Rx} = \rho \cdot g \cdot R^2 \cdot L \cdot \left[\int_0^{\pi} (1 - \cos(\theta)) \cdot \sin(\theta)\, d\theta - \int_{\pi}^{\frac{3\cdot\pi}{2}} \cos(\theta)\cdot\sin(\theta)\, d\theta \right] = \rho\cdot g\cdot R^2\cdot L\cdot\left(2 - \frac{1}{2}\right) = \frac{3}{2}\cdot\rho\cdot g\cdot R^2\cdot L$$

Substituting known values: $F_{Rx} = \frac{3}{2} \times 999\cdot\frac{kg}{m^3} \times 9.81\cdot\frac{m}{s^2} \times (1.5\cdot m)^2 \times 6\cdot m \times \frac{N\cdot s^2}{kg\cdot m}$ $F_{Rx} = 198.5\cdot kN$

Similarly we can calculate the vertical force component:

$$F_{Ry} = -\int_0^{\frac{3\cdot\pi}{2}} p\cdot L\cdot R\cdot\cos(\theta)\, d\theta = -\left[\int_0^{\pi} \rho\cdot g\cdot R\cdot(1-\cos(\theta))\cdot L\cdot R\cdot\cos(\theta)\, d\theta - \int_{\pi}^{\frac{3\cdot\pi}{2}} \rho\cdot g\cdot R\cdot\cos(\theta)\cdot L\cdot R\cdot\cos(\theta)\, d\theta \right]$$

$$F_{Ry} = -\rho\cdot g\cdot R^2\cdot L\cdot\left[\int_0^{\pi} (1-\cos(\theta))\cdot\cos(\theta)\, d\theta - \int_{\pi}^{\frac{3\cdot\pi}{2}} (\cos(\theta))^2\, d\theta \right] = \rho\cdot g\cdot R^2\cdot L\cdot\left(\frac{\pi}{2} + \frac{3\cdot\pi}{4} - \frac{\pi}{2}\right) = \frac{3\cdot\pi}{4}\cdot\rho\cdot g\cdot R^2\cdot L$$

Substituting known values: $F_{Ry} = \frac{3\cdot\pi}{4} \times 999\cdot\frac{kg}{m^3} \times 9.81\cdot\frac{m}{s^2} \times (1.5\cdot m)^2 \times 6\cdot m \times \frac{N\cdot s^2}{kg\cdot m}$ $F_{Ry} = 312\cdot kN$

Now since the weir surface in contact with the water is a circular arc, all elements dF of the force, and hence the line of action of the resultant force, must pass through the pivot. Thus:

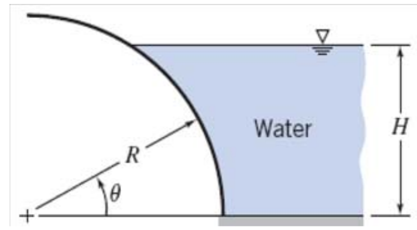
Magnitude of the resultant force: $F_R = \sqrt{(198.5\cdot kN)^2 + (312\cdot kN)^2}$ $F_R = 370\cdot kN$

The line of action of the force: $\alpha = \operatorname{atan}\left(\frac{312\cdot kN}{198.5\cdot kN}\right)$ $\alpha = 57.5\cdot deg$

Problem 3.66

[Difficulty: 3]

3.66 A curved surface is formed as a quarter of a circular cylinder with $R = 0.750$ m as shown. The surface is $w = 3.55$ m wide. Water stands to the right of the curved surface to depth $H = 0.650$ m. Calculate the vertical hydrostatic force on the curved surface. Evaluate the line of action of this force. Find the magnitude and line of action of the horizontal force on the surface.



Given: Curved surface, in shape of quarter cylinder, with given radius R and width w; water stands to depth H.

$R = 0.750 \cdot m \quad w = 3.55 \cdot m \quad H = 0.650 \cdot m$

Find: Magnitude and line of action of (a) vertical force and (b) horizontal force on the curved surface

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards from free surface)

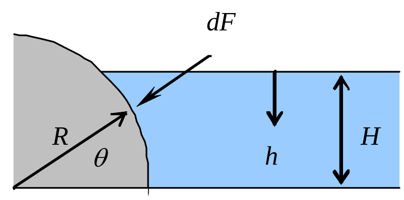
$$F_V = \int p\, dA_y$$ (Vertical Hydrostatic Force)

$$F_H = p_c \cdot A$$ (Horizontal Hydrostatic Force)

$$x' \cdot F_V = \int x\, dF_V$$ (Moment of vertical force)

$$h' = h_c + \frac{I_{xx}}{h_c \cdot A}$$ (Line of action of horizontal force)

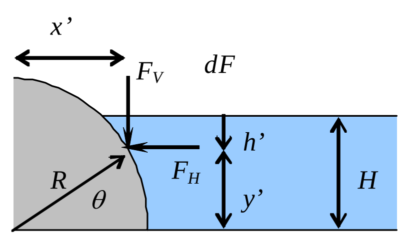
Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts on free surface of the water and on the left side of the curved surface



Integrating the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$

From the geometry: $h = H - R \cdot \sin(\theta) \quad y = R \cdot \sin(\theta) \quad x = R \cdot \cos(\theta) \quad dA = w \cdot R \cdot d\theta$

$$\theta_1 = \operatorname{asin}\left(\frac{H}{R}\right) \quad \theta_1 = \operatorname{asin}\left(\frac{0.650}{0.750}\right) \quad \theta_1 = 1.048 \cdot rad$$



Therefore the vertical component of the hydrostatic force is:

$$F_V = \int p\, dA_y = \int \rho \cdot g \cdot h \cdot \sin(\theta)\, dA = \int_0^{\theta_1} \rho \cdot g \cdot (H - R \cdot \sin(\theta)) \cdot \sin(\theta) \cdot w \cdot R\, d\theta$$

$$F_V = \rho \cdot g \cdot w \cdot R \cdot \int_0^{\theta_1} \left[H \cdot \sin(\theta) - R \cdot (\sin(\theta))^2\right] d\theta = \rho \cdot g \cdot w \cdot R \cdot \left[H \cdot \left(1 - \cos\left(\theta_1\right)\right) - R \cdot \left(\frac{\theta_1}{2} - \frac{\sin\left(2 \cdot \theta_1\right)}{4}\right)\right]$$

$$F_V = 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 3.55\cdot m\times 0.750\cdot m\times\left[0.650\cdot m\times(1-\cos(1.048\cdot rad)) - 0.750\cdot m\times\left(\frac{1.048}{2}-\frac{\sin(2\times 1.048\cdot rad)}{4}\right)\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$F_V = 2.47\cdot kN$$

To calculate the line of action of this force:

$$x'\cdot F_V = \int R\cdot\cos(\theta)\cdot\rho\cdot g\cdot h\cdot\sin(\theta)\,dA = \rho\cdot g\cdot w\cdot R^2\cdot\int_0^{\theta_1}\left[H\cdot\sin(\theta)\cdot\cos(\theta) - R\cdot(\sin(\theta))^2\cdot\cos(\theta)\right]d\theta$$

Evaluating the integral: $x'\cdot F_V = \rho\cdot g\cdot w\cdot R^2\cdot\left[\frac{H}{2}\cdot\left(\sin\left(\theta_1\right)\right)^2-\frac{R}{3}\cdot\left(\sin\left(\theta_1\right)\right)^3\right]$ Therefore we may find the line of action:

$$x' = \frac{x'\cdot F_V}{F_V} = \frac{\rho\cdot g\cdot w\cdot R^2}{F_V}\cdot\left[\frac{H}{2}\cdot\left(\sin\left(\theta_1\right)\right)^2-\frac{R}{3}\cdot\left(\sin\left(\theta_1\right)\right)^3\right]$$

Substituting in known values: $\sin\left(\theta_1\right) = \frac{0.650}{0.750}$

$$x' = 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times 3.55\cdot m\times(0.750\cdot m)^2\times\frac{1}{2.47\times 10^3}\cdot\frac{1}{N}\times\left[\frac{0.650\cdot m}{2}\times\left(\frac{0.650}{0.750}\right)^2-\frac{0.750\cdot m}{3}\times\left(\frac{0.650}{0.750}\right)^3\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$$x' = 0.645\,m$$

For the horizontal force: $F_H = p_c\cdot A = \rho\cdot g\cdot h_c\cdot H\cdot w = \rho\cdot g\cdot\frac{H}{2}\cdot H\cdot w = \frac{\rho\cdot g\cdot H^2\cdot w}{2}$

$$F_H = \frac{1}{2}\times 999\cdot\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times(0.650\cdot m)^2\times 3.55\cdot m\times\frac{N\cdot s^2}{kg\cdot m}$$

$$F_H = 7.35\cdot kN$$

For the line of action of the horizontal force: $h' = h_c + \frac{I_{xx}}{h_c\cdot A}$ where $I_{xx} = \frac{w\cdot H^3}{12}$ $A = w\cdot H$ Therefore:

$$h' = h_c + \frac{I_{xx}}{h_c\cdot A} = \frac{H}{2}+\frac{w\cdot H^3}{12}\cdot\frac{2}{H}\cdot\frac{1}{w\cdot H} = \frac{H}{2}+\frac{H}{6} = \frac{2}{3}\cdot H$$

$$h' = \frac{2}{3}\times 0.650\cdot m$$

$$h' = 0.433\,m$$

3.67 If you throw an anchor out of your canoe but the rope is too short for the anchor to rest on the bottom of the pond, will your canoe float higher, lower, or stay the same? Prove your answer.

Given: Canoe floating in a pond

Find: What happens when an anchor with too short of a line is thrown from canoe

Solution:

Governing equation:

$$F_B = \rho_w g V_{disp} = W$$

Before the anchor is thrown from the canoe the buoyant force on the canoe balances out the weight of the canoe and anchor:

$$F_{B_1} = W_{canoe} + W_{anchor} = \rho_w g V_{canoe_1}$$

The anchor weight can be expressed as

$$W_{anchor} = \rho_a g V_a$$

so the initial volume displaced by the canoe can be written as

$$V_{canoe_1} = \frac{W_{canoe}}{\rho_w g} + \frac{\rho_a}{\rho_w} V_a$$

After throwing the anchor out of the canoe there will be buoyant forces acting on the canoe and the anchor. Combined, these buoyant forces balance the canoe weight and anchor weight:

$$F_{B_2} = W_{canoe} + W_{anchor} = \rho_w g V_{canoe_2} + \rho_w g V_a$$

$$V_{canoe2} = \frac{W_{canoe}}{\rho_w g} + \frac{W_a}{\rho_w g} - V_a$$

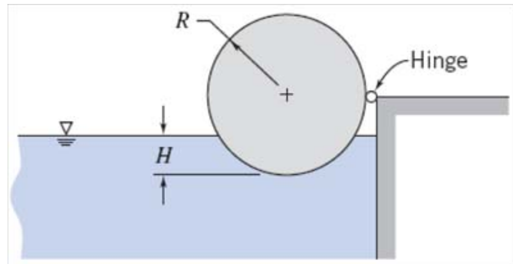
Using the anchor weight,

$$V_{canoe2} = \frac{W_{canoe}}{\rho_w g} + \frac{\rho_a}{\rho_w} V_a - V_a$$

Hence the volume displaced by the canoe after throwing the anchor in is less than when the anchor was in the canoe, meaning that the canoe is floating higher.

Problem 3.68 [Difficulty: 4]

3.68 The cylinder shown is supported by an incompressible liquid of density ρ, and is hinged along its length. The cylinder, of mass M, length L, and radius R, is immersed in liquid to depth H. Obtain a general expression for the cylinder specific gravity versus the ratio of liquid depth to cylinder radius, $\alpha = H/R$, needed to hold the cylinder in equilibrium for $0 \le \alpha < 1$. Plot the results.



Given: Cylinder of mass M, length L, and radius R is hinged along its length and immersed in an incompressilble liquid to deptl

Find: General expression for the cylinder specific gravity as a function of α=H/R needed to hold the cylinder in equilibrium for α ranging from 0 to 1.

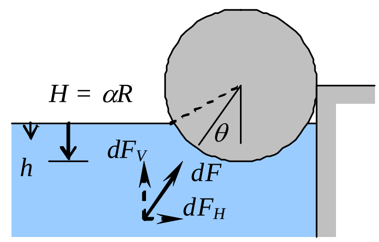
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$\frac{dp}{dh} = \rho \cdot g$$ (Hydrostatic Pressure - h is positive downwards from free surface)

$$F_V = \int p\, dA_y$$ (Vertical Hydrostatic Force)

$$\Sigma M = 0$$ (Rotational Equilibrium)



Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts on free surface of the liquid.

The moments caused by the hydrostatic force and the weight of the cylinder about the hinge need to balance each other.

Integrating the hydrostatic pressure equation: $p = \rho \cdot g \cdot h$

$$dF_V = dF \cdot \cos(\theta) = p \cdot dA \cdot \cos(\theta) = \rho \cdot g \cdot h \cdot w \cdot R \cdot d\theta \cdot \cos(\theta)$$

Now the depth to which the cylinder is submerged is $H = h + R \cdot (1 - \cos(\theta))$

Therefore $h = H - R \cdot (1 - \cos(\theta))$ and into the vertical force equation:

$$dF_V = \rho \cdot g \cdot [H - R \cdot (1 - \cos(\theta))] \cdot w \cdot R \cdot \cos(\theta) \cdot d\theta = \rho \cdot g \cdot w \cdot R^2 \cdot \left[\frac{H}{R} - (1 - \cos(\theta))\right] \cdot \cos(\theta) \cdot d\theta$$

$$dF_V = \rho \cdot g \cdot w \cdot R^2 \cdot \left[(\alpha - 1) \cdot \cos(\theta) + (\cos(\theta))^2\right] \cdot d\theta = \rho \cdot g \cdot w \cdot R^2 \cdot \left[(\alpha - 1) \cdot \cos(\theta) + \frac{1 + \cos(2 \cdot \theta)}{2}\right] \cdot d\theta$$

Now as long as α is not greater than 1, the net horizontal hydrostatic force will be zero due to symmetry, and the vertical force is:

$$F_V = \int_{-\theta_{max}}^{\theta_{max}} 1\, dF_V = \int_0^{\theta_{max}} 2\, dF_V \qquad \text{where} \qquad \cos(\theta_{max}) = \frac{R - H}{R} = 1 - \alpha \qquad \text{or} \qquad \theta_{max} = \mathrm{acos}(1 - \alpha)$$

$$F_V = 2\rho\cdot g\cdot w\cdot R^2\cdot\int_0^{\theta_{max}}\left[(\alpha-1)\cdot\cos(\theta)+\frac{1}{2}+\frac{1}{2}\cdot\cos(2\cdot\theta)\right]d\theta$$

Now upon integration of this expression we have:

$$F_V = \rho\cdot g\cdot w\cdot R^2\cdot\left[\operatorname{acos}(1-\alpha)-(1-\alpha)\cdot\sqrt{\alpha\cdot(2-\alpha)}\right]$$

The line of action of the vertical force due to the liquid is through the centroid of the displaced liquid, i.e., through the center of the cylinde

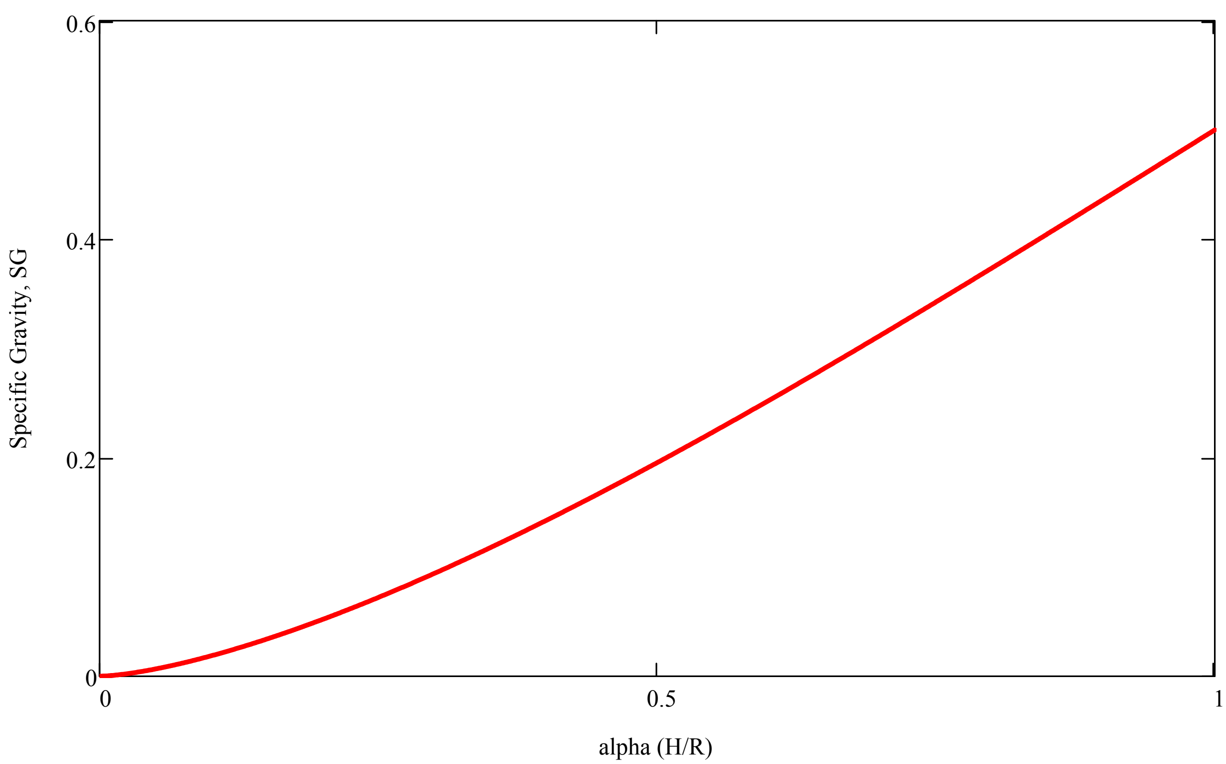
The weight of the cylinder is given by: $W = M\cdot g = \rho_c\cdot V\cdot g = SG\cdot\rho\cdot\pi\cdot R^2\cdot w\cdot g$ where ρ is the density of the fluid and $SG = \dfrac{\rho_c}{\rho}$

The line of action of the weight is also throught the center of the cylinder. Taking moment about the hinge we get:

$\Sigma M_O = W\cdot R - F_V\cdot R = 0$ or in other words $W = F_V$ and therefore:

$$SG\cdot\rho\cdot\pi\cdot R^2\cdot w\cdot g = \rho\cdot g\cdot w\cdot R^2\cdot\left[\operatorname{acos}(1-\alpha)-(1-\alpha)\cdot\sqrt{\alpha\cdot(2-\alpha)}\right]$$

$$SG = \frac{1}{\pi}\cdot\left[\operatorname{acos}(1-\alpha)-(1-\alpha)\cdot\sqrt{\alpha\cdot(2-\alpha)}\right]$$



Problem 3.69

[Difficulty: 2]

3.69 A hydrometer is a specific gravity indicator, the value being indicated by the level at which the free surface intersects the stem when floating in a liquid. The 1.0 mark is the level when in distilled water. For the unit shown, the immersed volume in distilled water is 15 cm^3. The stem is 6 mm in diameter. Find the distance, *h*, from the 1.0 mark to the surface when the hydrometer is placed in a nitric acid solution of specific gravity 1.5.

Given: Hydrometer as shown, submerged in nitric acid. When submerged in water, h = 0 and the immersed volume is 15 cubic cm.

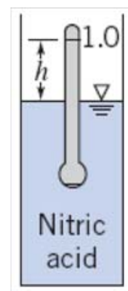
$SG = 1.5 \quad d = 6 \cdot mm$

Find: The distance h when immersed in nitric acid.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations: $F_{buoy} = \rho \cdot g \cdot V_d$ (Buoyant force is equal to weight of displaced fluid)

Assumptions:
(1) Static fluid
(2) Incompressible fluid



Taking a free body diagram of the hydrometer: $\Sigma F_z = 0 \quad -M \cdot g + F_{buoy} = 0$

Solving for the mass of the hydrometer: $M = \dfrac{F_{buoy}}{g} = \rho \cdot V_d$

When immersed in water: $M = \rho_w \cdot V_w$ When immersed in nitric acid: $M = \rho_n \cdot V_n$

Since the mass of the hydrometer is the same in both cases: $\rho_w \cdot V_w = \rho_n \cdot V_n$

When the hydrometer is in the nitric acid: $V_n = V_w - \dfrac{\pi}{4} \cdot d^2 \cdot h \qquad \rho_n = SG \cdot \rho_w$

Therefore: $\rho_w \cdot V_w = SG \cdot \rho_w \cdot \left(V_w - \dfrac{\pi}{4} \cdot d^2 \cdot h \right)$ Solving for the height h:

$V_w = SG \cdot \left(V_w - \dfrac{\pi}{4} \cdot d^2 \cdot h \right) \qquad V_w \cdot (1 - SG) = -SG \cdot \dfrac{\pi}{4} \cdot d^2 \cdot h$

$h = V_w \cdot \left(\dfrac{SG - 1}{SG} \right) \cdot \dfrac{4}{\pi \cdot d^2} \qquad h = 15 \cdot cm^3 \times \left(\dfrac{1.5 - 1}{1.5} \right) \times \dfrac{4}{\pi \times (6 \cdot mm)^2} \times \left(\dfrac{10 \cdot mm}{cm} \right)^3 \qquad h = 177 \cdot mm$

Problem 3.70

(Difficulty: 2)

3.70 A cylindrical can 76 mm in diameter and 152 mm high, weighing 1.11 N, contains water to a depth of 76 mm. When this can is placed in water, how deep will it sink?

Find: The depth it will sink.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

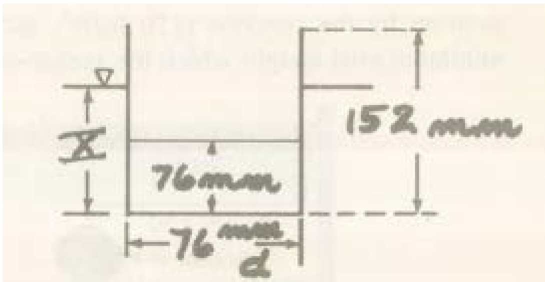
Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\, g\, V$$

A free body diagram on the can is



We have the force balance equation in the vertical direction as:

$$F_b - W_{can} - W_{canwater} = 0$$

The buoyancy force can be calculated as:

$$F_b = \gamma_{water} V_{can} = 9810\ \frac{N}{m^3} \times \frac{\pi}{4} \times (0.076\ m)^2 \times x\ m = 44.50Xx\ N$$

We also have:

$$W_{can} = 1.11\ N$$

$$W_{canwater} = \gamma_{water} V_{canwater} = 9810\ \frac{N}{m^3} \times \frac{\pi}{4} \times (0.076\ m)^3 = 3.38\ N$$

Thus making a force balance for which the net force is zero at equilibrium

$$44.50x = 1.11\ N + 3.38\ N = 4.49\ N$$

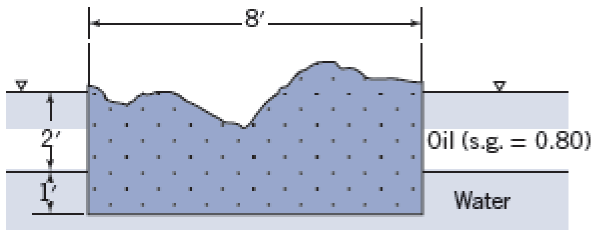
$$x = 0.1009\ m = 100.9\ mm$$

So this can will sink to depth of $100.9\ mm$.

Problem 3.71

(Difficulty: 1)

3.71 If the 10 ft long box is floating on the oil water system, calculate how much the box and its contents must weigh.



Find: The weight of the box and its contents.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\, g\, V$$

The force balance equation in the vertical diretion:

$$F_B - W_B = 0$$

$$F_B = \gamma_{oil} V + \gamma_{water} V$$

Thus

$$F_B = 0.8 \times 62.4\ \frac{lbf}{ft^3} \times 2ft \times 8ft \times 10ft + 62.4\ \frac{lbf}{ft^3} \times 1ft \times 8ft \times 10ft = 12980\ lbf$$

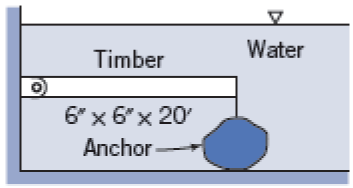
So the box and its contents must weigh:

$$W_B = 12980\ lbf$$

Problem 3.72

(Difficulty: 2)

3.72 The timber weighs $40\ \frac{lbf}{ft^3}$ and is held in a horizontal position by the concrete $\left(150\ \frac{lbf}{ft^3}\right)$ anchor. Calculate the minimum total weight which the anchor may have.



Find: The minimum total weight the anchor may have.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\ g\ V$$

For the buoyancy force we have:

$$F_{bt} = \gamma_{water} V_t$$

$$F_{bt} = 62.4\ \frac{lbf}{ft^3} \times \left(\frac{6}{12}\ ft\right) \times \left(\frac{6}{12}\ ft\right) \times (20\ ft) = 312\ lbf$$

The weight of the timber is:

$$W_t = \gamma_t V_t$$

$$W_t = 40\ \frac{lbf}{ft^3} \times \left(\frac{6}{12}\ ft\right) \times \left(\frac{6}{12}\ ft\right) \times (20\ ft) = 200\ lbf$$

At the horizontal position we take moments about the pivot:

$$F_a L + W_t \frac{L}{2} - F_{bt} \frac{L}{2} = 0$$

$$F_a = \frac{1}{2} F_{bt} - \frac{1}{2} W_t = \frac{1}{2} \times (312\ lbf - 200 lbf) = 56\ lbf$$

$$F_a = F_{ba} - W_a$$

The weight of the anchor is:

$$W_a = \gamma_a V_a$$

The buoyancy force on the anchor is:

$$F_{ba} = \gamma_{water} V_a$$

$$\gamma_a V_a - \gamma_{water} V_a = 56\ lbf$$

$$V_a = \frac{56\ lbf}{\left(150\ \frac{lbf}{ft^3} - 62.4\ \frac{lbf}{ft^3}\right)} = 0.64\ ft^3$$

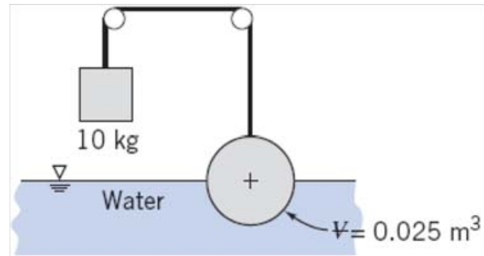
So the weight is:

$$W_a = \gamma_a V_a = 150\ \frac{lbf}{ft^3} \times 0.64\ ft^3 = 96\ lbf$$

Problem 3.73

[Difficulty: 3]

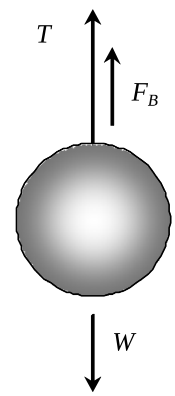
3.73 Find the specific weight of the sphere shown if its volume is 0.025m^3. State all assumptions. What is the equilibrium position of the sphere if the weight is removed?



Given: Data on sphere and weight

Find: SG of sphere; equilibrium position when freely floating

Solution:



Basic equation $\quad F_B = \rho \cdot g \cdot V \quad$ and $\quad \Sigma F_z = 0 \qquad \Sigma F_z = 0 = T + F_B - W$

where $\quad T = M \cdot g \qquad M = 10 \cdot kg \qquad F_B = \rho \cdot g \cdot \frac{V}{2} \qquad W = SG \cdot \rho \cdot g \cdot V$

Hence $\quad M \cdot g + \rho \cdot g \cdot \frac{V}{2} - SG \cdot \rho \cdot g \cdot V = 0 \qquad SG = \frac{M}{\rho \cdot V} + \frac{1}{2}$

$$SG = 10 \cdot kg \times \frac{m^3}{1000 \cdot kg} \times \frac{1}{0.025 \cdot m^3} + \frac{1}{2} \qquad SG = 0.9$$

The specific weight is $\quad \gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{SG \cdot \rho \cdot g \cdot V}{V} = SG \cdot \rho \cdot g \qquad \gamma = 0.9 \times 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \qquad \gamma = 8829 \cdot \frac{N}{m^3}$

For the equilibriul position when floating, we repeat the force balance with T = 0

$$F_B - W = 0 \qquad W = F_B \quad \text{with} \quad F_B = \rho \cdot g \cdot V_{submerged}$$

From references (trying Googling "partial sphere volume") $\quad V_{submerged} = \frac{\pi \cdot h^2}{3} \cdot (3 \cdot R - h)$

where h is submerged depth and R is the sphere radius $\quad R = \left(\frac{3 \cdot V}{4 \cdot \pi}\right)^{\frac{1}{3}} \qquad R = \left(\frac{3}{4 \cdot \pi} \cdot 0.025 \cdot m^3\right)^{\frac{1}{3}} \qquad R = 0.181 \, m$

Hence $\quad W = SG \cdot \rho \cdot g \cdot V = F_B = \rho \cdot g \cdot \frac{\pi \cdot h^2}{3} \cdot (3 \cdot R - h) \qquad h^2 \cdot (3 \cdot R - h) = \frac{3 \cdot SG \cdot V}{\pi}$

$$h^2 \cdot (3 \cdot 0.181 \cdot m - h) = \frac{3 \cdot 0.9 \cdot .025 \cdot m^3}{\pi} \qquad h^2 \cdot (0.544 - h) = 0.0215$$

This is a cubic equation for h. We can keep guessing h values, manually iterate, or use Excel's Goal Seek to find $\quad h = 0.292 \cdot m$

Problem 3.74

[Difficulty: 2]

3.74 The fat-to-muscle ratio of a person may be determined from a specific gravity measurement. The measurement is made by immersing the body in a tank of water and measuring the net weight. Develop an expression for the specific gravity of a person in terms of their weight in air, net weight in water, and SG = $f(T)$ for water.

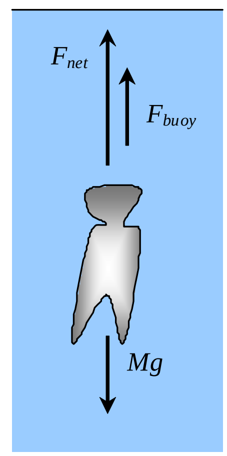
Given: Specific gravity of a person is to be determined from measurements of weight in air and the met weight when totally immersed in water.

Find: Expression for the specific gravity of a person from the measurements.

Solution: We will apply the hydrostatics equations to this system.

Governing Equation: $F_{buoy} = \rho \cdot g \cdot V_d$ (Buoyant force is equal to weight of displaced fluid)

Assumptions: (1) Static fluid
(2) Incompressible fluid



Taking a free body diagram of the body: $\Sigma F_y = 0$ $F_{net} - M \cdot g + F_{buoy} = 0$

F_{net} is the weight measurement for the immersed body.

$F_{net} = M \cdot g - F_{buoy} = M \cdot g - \rho_w \cdot g \cdot V_d$ However in air: $F_{air} = M \cdot g$

Therefore the weight measured in water is: $F_{net} = F_{air} - \rho_w \cdot g \cdot V_d$ and $V_d = \dfrac{F_{air} - F_{net}}{\rho_w \cdot g}$

Now in order to find the specific gravity of the person, we need his/her density:

$F_{air} = M \cdot g = \rho \cdot g \cdot V_d = \rho \cdot g \cdot \dfrac{\left(F_{air} - F_{net}\right)}{\rho_w \cdot g}$ Simplifying this expression we get: $F_{air} = \dfrac{\rho}{\rho_w}\left(F_{air} - F_{net}\right)$

Now if we call the density of water at 4 deg C ρ_{w4C} then: $F_{air} = \dfrac{\left(\dfrac{\rho}{\rho_{w4C}}\right)}{\left(\dfrac{\rho_w}{\rho_{w4C}}\right)}\left(F_{air} - F_{net}\right) = \dfrac{SG}{SG_w} \cdot \left(F_{air} - F_{net}\right)$

Solving this expression for the specific gravity of the person SG, we get: $SG = SG_w \cdot \dfrac{F_{air}}{F_{air} - F_{net}}$

Problem 3.75

[Difficulty: 2]

3.75 An open tank is filled to the top with water. A steel cylindrical container, wall thickness $\delta = 1$ mm, outside diameter $D = 100$ mm, and height $H = 1$ m, with an open top, is gently placed in the water. What is the volume of water that overflows from the tank? How many 1 kg weights must be placed in the container to make it sink? Neglect surface tension effects.

Given: Geometry of steel cylinder

Find: Volume of water displaced; number of 1 kg wts to make it sink

Solution:

The data is For water $\rho = 999 \cdot \frac{kg}{m^3}$

For steel $SG = 7.83$

For the cylinder $D = 100 \cdot mm$ $\quad H = 1 \cdot m$ $\quad \delta = 1 \cdot mm$

The volume of the cylinder is $V_{steel} = \delta \cdot \left(\frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot H \right)$ $\quad V_{steel} = 3.22 \times 10^{-4} \cdot m^3$

The weight of the cylinder is $W = SG \cdot \rho \cdot g \cdot V_{steel}$

$$W = 7.83 \times 999 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times 3.22 \times 10^{-4} \cdot m^3 \times \frac{N \cdot s^2}{kg \cdot m} \qquad W = 24.7\,N$$

At equilibium, the weight of fluid displaced is equal to the weight of the cylinder

$$W_{displaced} = \rho \cdot g \cdot V_{displaced} = W$$

$$V_{displaced} = \frac{W}{\rho \cdot g} = 24.7 \cdot N \times \frac{m^3}{999 \cdot kg} \times \frac{s^2}{9.81 \cdot m} \times \frac{kg \cdot m}{N \cdot s^2} \qquad V_{displaced} = 2.52\,L$$

To determine how many 1 kg wts will make it sink, we first need to find the extra volume that will need to be dsiplaced

Distance cylinder sank $x_1 = \dfrac{V_{displaced}}{\left(\frac{\pi \cdot D^2}{4} \right)}$ $\quad x_1 = 0.321\,m$

Hence, the cylinder must be made to sink an additional distance $x_2 = H - x_1$ $\quad x_2 = 0.679\,m$

We deed to add n weights so that $1 \cdot kg \cdot n \cdot g = \rho \cdot g \cdot \frac{\pi \cdot D^2}{4} \cdot x_2$

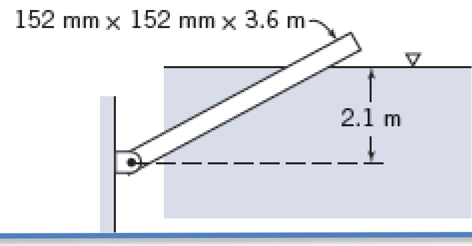
$$n = \frac{\rho \cdot \pi \cdot D^2 \cdot x_2}{4 \times 1 \cdot kg} = 999 \cdot \frac{kg}{m^3} \times \frac{\pi}{4} \times (0.1 \cdot m)^2 \times 0.679 \cdot m \times \frac{1}{1 \cdot kg} \times \frac{N \cdot s^2}{kg \cdot m} \qquad n = 5.33$$

Hence we need $n = 6$ weights to sink the cylinder

Problem 3.76

(Difficulty: 2)

3.76 If the timber weights 670 N, calculate its angle of inclination when the water surface is 2.1 m above the pivot. Above what depth will the timber stand vertically?



Find: Above what depth will the timber stand vertically.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\, g\, V$$

The buoyancy force is:

$$F_b = \gamma_{water} V = 0.152\ m \times 0.152\ m \times x\ m \times 9810\ \frac{N}{m^3} = 226.7x\ (N)$$

Take the moment about pivot we have:

$$M = W \times 0.5 \times 3.6\ m \cos\theta - \frac{x}{2}\ m \times F_b \cos\theta = 0$$

$$670\ N \times 0.5 \times 3.6\ m \times \cos\theta - \frac{x}{2}\ m \times 226.7x \times \cos\theta = 0$$

Soving this equation we have:

$$x = 3.26\ m$$

The angle when water surface $y = 2.1\ m$ is:

$$\theta = \sin^{-1}\left(\frac{2.1\ m}{3.26\ m}\right) = 40.1\ °$$

We have the following relation:

$$x = \frac{y}{\sin\theta}$$

Substitute in to the momentum we have:

$$670\ N \times 0.5 \times 3.6\ m - \frac{y}{2\sin\theta}\ m \times 226.7\frac{y}{\sin\theta} = 0$$

If the timber is vertically, we have:

$$\theta = 90°$$

$$\sin 90° = 1$$

So we have:

$$670\ N \times 0.5 \times 3.6\ m - \frac{y}{2}\ m \times 226.7y = 0$$

Solving this equation we have:

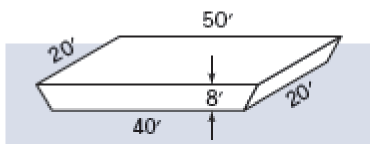
$$y = 3.26\ m$$

When the water surface is $y = 3.26\ m$, the timber will stand vertically.

Problem 3.77

(Difficulty: 2)

3.77 The barge shown weights $40\ tons$ and carries a cargo of $40\ tons$. Calculate its draft in freshwater.



P3.77

Find: The draft, where the draft is the depth to which the barge sinks.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\, g\, V$$

For the barge floating in water we have the buoyancy force as:

$$F_B = \gamma_{water} V = W$$

The weight of the barge is:

$$W = (40 + 40) tons = 80\ tons \times \frac{2000\ lbf}{ton} = 160000\ lbf$$

The volume of water displaced is then:

$$V = \frac{W}{\gamma_{water}} = \frac{160000\ lbf}{62.4\ \frac{lbf}{ft^3}} = 2564\ ft^3$$

The volume in terms of the draft d is:

$$\forall = A_c L = \left(40ft + 40ft + 2 \times \frac{5}{8} d\right) \times \frac{d}{2} \times 20ft = 800d + 12.5d^2$$

Thus we have the relation:

$$800d + 12.5d^2 = 2564$$

Solving this equation we have for the draft:

$$d = 3.06\ ft$$

Problem 3.78

[Difficulty: 2]

3.78 Quantify the experiment performed by Archimedes to identify the material content of King Hiero's crown. Assume you can measure the weight of the king's crown in air, W_a, and the weight in water, W_w. Express the specific gravity of the crown as a function of these measured values.

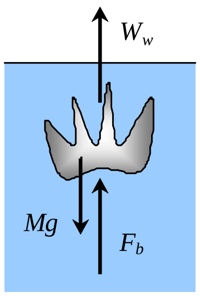
Given: Experiment performed by Archimedes to identify the material conent of King Hiero's crown. The crown was weighed in air and in water.

Find: Expression for the specific gravity of the crown as a function of the weights in water and air.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations: $F_b = \rho \cdot g \cdot V_d$ (Buoyant force is equal to weight of displaced fluid)

Assumptions:
(1) Static fluid
(2) Incompressible fluid



Taking a free body diagram of the body: $\Sigma F_z = 0$ $\quad W_w - M \cdot g + F_b = 0$

W_w is the weight of the crown in water.

$W_w = M \cdot g - F_{buoy} = M \cdot g - \rho_w \cdot g \cdot V_d$ However in air: $W_a = M \cdot g$

Therefore the weight measured in water is: $W_w = W_a - \rho_w \cdot g \cdot V_d$

so the volume is: $V_d = \dfrac{W_a - W_w}{\rho_w \cdot g}$ Now the density of the crown is: $\rho_c = \dfrac{M}{V_d} = \dfrac{M \cdot \rho_w \cdot g}{W_a - W_w} = \dfrac{W_a}{W_a - W_w} \cdot \rho_w$

Therefore, the specific gravity of the crown is: $SG = \dfrac{\rho_c}{\rho_w} = \dfrac{W_a}{W_a - W_w}$ $\qquad SG = \dfrac{W_a}{W_a - W_w}$

Note: by definition specific gravity is the density of an object divided by the density of water at 4 degrees Celsius, so the measured temperature of the water in the experiment and the data from tables A.7 or A.8 may be used to correct for the variation in density of the water with temperature.

Problem 3.79

[Difficulty: 2]

3.79 Hot-air ballooning is a popular sport. According to a recent article, "hot-air volumes must be large because air heated to 150°F over ambient lifts only 0.018 lbf/ft^3 compared to 0.066 and 0.071 for helium and hydrogen, respectively." Check these statements for sea-level conditions. Calculate the effect of increasing the hot-air maximum temperature to 250°F above ambient.

Given: Balloons with hot air, helium and hydrogen. Claim lift per cubic foot of 0.018, 0.066, and 0.071 pounds force per cubic f for respective gases, with the air heated to 150 deg. F over ambient.

Find: (a) evaluate the claims of lift per unit volume
(b) determine change in lift when air is heated to 250 deg. F over ambient.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$$L = \rho_a \cdot g \cdot V - \rho_g \cdot g \cdot V$$ (Net lift force is equal to difference in weights of air and gas)

$$p = \rho \cdot R \cdot T$$ (Ideal gas equation of state)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Ideal gas behavior

The lift per unit volume may be written as: $LV = \frac{L}{V} = g \cdot (\rho_a - \rho_g) = \rho_a \cdot g \cdot \left(1 - \frac{\rho_g}{\rho_a}\right)$ now if we take the ideal gas equation and

we take into account that the pressure inside and outside the balloon are equal: $\frac{L}{V} = \rho_a \cdot g \cdot \left(1 - \frac{R_a \cdot T_a}{R_g \cdot T_g}\right) = \gamma_a \cdot \left(1 - \frac{R_a \cdot T_a}{R_g \cdot T_g}\right)$

At standard conditions the specific weight of air is: $\gamma_a = 0.0765 \cdot \frac{lbf}{ft^3}$ the gas constant is: $R_a = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ and $T_a = 519 \cdot R$

For helium: $R_g = 386.1 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $T_g = T_a$ and therefore: $LV_{He} = 0.0765 \cdot \frac{lbf}{ft^3} \times \left(1 - \frac{53.33}{386.1}\right)$ $LV_{He} = 0.0659 \cdot \frac{lbf}{ft^3}$

For hydrogen: $R_g = 766.5 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $T_g = T_a$ and therefore: $LV_{H2} = 0.0765 \cdot \frac{lbf}{ft^3} \times \left(1 - \frac{53.33}{766.5}\right)$ $LV_{H2} = 0.0712 \cdot \frac{lbf}{ft^3}$

For hot air at 150 degrees above ambient:

$R_g = R_a$ $T_g = T_a + 150 \cdot R$ and therefore: $LV_{air150} = 0.0765 \cdot \frac{lbf}{ft^3} \times \left(1 - \frac{519}{519 + 150}\right)$ $LV_{air150} = 0.0172 \cdot \frac{lbf}{ft^3}$

The agreement with the claims stated above is good.

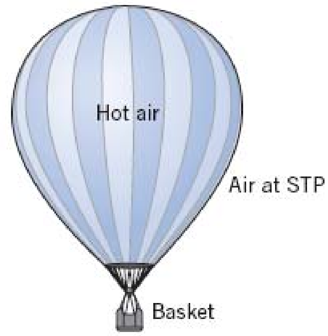
For hot air at 250 degrees above ambient:

$R_g = R_a$ $T_g = T_a + 250 \cdot R$ and therefore: $LV_{air250} = 0.0765 \cdot \frac{lbf}{ft^3} \times \left(1 - \frac{519}{519 + 250}\right)$ $LV_{air250} = 0.0249 \cdot \frac{lbf}{ft^3}$

$\frac{LV_{air250}}{LV_{air150}} = 1.450$ Air at ΔT of 250 deg. F gives 45% more lift than air at ΔT of 150 deg.F!

Problem 3.80 [Difficulty: 3]

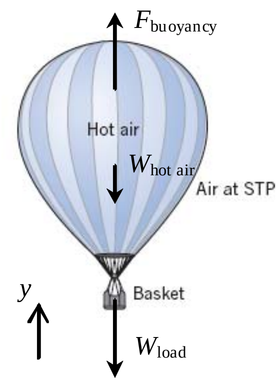
3.80 It is desired to use a hot air balloon with a volume of 320,000 ft^3 for rides planned in summer morning hours when the air temperature is about 48°F. The torch will warm the air inside the balloon to a temperature of 160°F. Both inside and outside pressures will be "standard" (14.7 psia). How much mass can be carried by the balloon (basket, fuel, passengers, personal items, and the component of the balloon itself) if neutral buoyancy is to be assured? What mass can be carried by the balloon to ensure vertical takeoff acceleration of 2.5 ft/s^2? For this, consider that both balloon and inside air have to be accelerated, as well as some of the surrounding air (to make way for the balloon). The rule of thumb is that the total mass subject to acceleration is the mass of the balloon, all its appurtenances, and twice its volume of air. Given that the volume of hot air is fixed during the flight, what can the balloonists do when they want to go down?



Given: Data on hot air balloon

Find: Maximum mass of balloon for neutral buoyancy; mass for initial acceleration of 2.5 ft/s^2.

Assumptions: Air is treated as static and incompressible, and an ideal gas



Solution:

Basic equation $F_B = \rho_{atm}\cdot g\cdot V$ and $\Sigma F_y = M\cdot a_y$

Hence $\Sigma F_y = 0 = F_B - W_{hotair} - W_{load} = \rho_{atm}\cdot g\cdot V - \rho_{hotair}\cdot g\cdot V - M\cdot g$ for neutral buoyancy

$$M = V\cdot\left(\rho_{atm} - \rho_{hotair}\right) = \frac{V\cdot p_{atm}}{R}\cdot\left(\frac{1}{T_{atm}} - \frac{1}{T_{hotair}}\right)$$

$$M = 320000\cdot ft^3 \times 14.7\cdot\frac{lbf}{in^2}\times\left(\frac{12\cdot in}{ft}\right)^2\times\frac{lbm\cdot R}{53.33\cdot ft\cdot lbf}\times\left[\frac{1}{(48+460)\cdot R} - \frac{1}{(160+460)\cdot R}\right] \qquad M = 4517\cdot lbm$$

Initial acceleration $\Sigma F_y = F_B - W_{hotair} - W_{load} = \left(\rho_{atm} - \rho_{hotair}\right)\cdot g\cdot V - M_{new}\cdot g = M_{accel}\cdot a = \left(M_{new} + 2\cdot\rho_{hotair}\cdot V\right)\cdot a$

Solving for M_{new} $\left(\rho_{atm} - \rho_{hotair}\right)\cdot g\cdot V - M_{new}\cdot g = \left(M_{new} + 2\cdot\rho_{hotair}\cdot V\right)\cdot a$

$$M_{new} = V\cdot\frac{\left(\rho_{atm} - \rho_{hotair}\right)\cdot g - 2\cdot\rho_{hotair}\cdot a}{a+g} = \frac{V\cdot p_{atm}}{a+g}\cdot\left[g\cdot\left(\frac{1}{T_{atm}} - \frac{1}{T_{hotair}}\right) - \frac{2\cdot a}{T_{hotair}}\right]$$

$$M_{new} = 320000\cdot ft^3\cdot 14.7\cdot\frac{lbf}{in^2}\cdot\left(\frac{12\cdot in}{ft}\right)^2\cdot\frac{lbm\cdot R}{53.33\cdot ft\cdot lbf}\cdot\frac{s^2}{(2.5+32.2)\cdot ft}\cdot\left[32.2\cdot\left[\frac{1}{(48+460)} - \frac{1}{(160+460)}\right] - 2\cdot 2.5\cdot\frac{1}{(160+460)}\right]\cdot\frac{ft}{s^2\cdot R}$$

$$M_{new} = 1239\cdot lbm$$

To make the balloon move up or down during flight, the air needs to be heated to a higher temperature, or let cool (or let in ambient air).

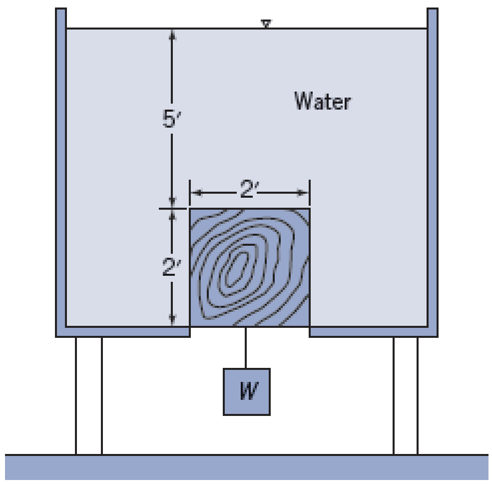
Problem 3.81

(Difficulty: 2)

3.81 The opening in the bottom of the tank is square and slightly less than $2\ ft$ on each side. The opening is to be plugged with a wooden cube $2\ ft$ on a side.

(a) What weight W should be attached to the wooden cube to insure successful plugging of the hole? The wood weighs $40\ \frac{lbf}{ft^3}$,

(b) What upward force must be exerted on the block to lift it and allow water to drain from the tank?



Find: The weight of the block and the force needed to lift it

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\ g\ V$$

(a) Because the wood bottom surface is in the atmosphere so the pressure on the bottom surface is zero in this case and there is no buoyancy force. The force acting on the wood cube in the vertical direction is:

$$F_V = F_p + G$$

$$F_V = \gamma h_1 A + G = 62.4\ \frac{lbf}{ft^3} \times 5\ ft \times 2ft \times 2ft + 40\ \frac{lbf}{ft^3} \times (2\ ft)^3 = 1568\ lbf$$

The direction of F_V is downward. So we do not need any weight W attached to the wood cube.

(b) To lift the block, we need a force larger than F_V, so we have:

$$F_{up} \geq F_V = 1568\ lbf$$

Problem 3.82

(Difficulty: 2)

3.82 A balloon has a weight (including crew but not gas) of 2.2 kN and a gas-bag capacity of 566 m^3. At the ground it is (partially) inflated with 445 N of helium. How high can this balloon rise in the U.S standard atmosphere if the helium always assumes the pressure and temperature of the atmosphere?

Find: How high this balloon will rise.

Assumptions: Fluid is incompressible and static

Solution: Apply the hydrostatic relations:

Pressure as a function of depth

$$\Delta p = \rho g h$$

Buoyancy force:

$$F_b = \rho\, g\, V$$

At the sea level, for helium we have:

$$p = 101.3\ kPa$$

$$T = 288\ K$$

$$R = 2076.8\ \frac{J}{kg \cdot K}$$

According to the ideal gas law:

$$\rho_h = \frac{p}{RT} = \frac{101.3\ kPa}{2076.8\ \frac{J}{kg \cdot K} \times 288\ K} = 0.1694\ \frac{kg}{m^3}$$

$$\gamma_h = \rho g = 0.1694\ \frac{kg}{m^3} \times 9.81\ \frac{m}{s^2} = 1.662\ \frac{N}{m^3}$$

The volume of the helium is:

$$V_h = \frac{W_h}{\gamma_h} = \frac{445\ N}{1.662\ \frac{N}{m^3}} = 268\ m^3$$

The buoyancy force is calculated by:

$$F_B = \gamma_{air} V_b$$

The weight of the whole balloon is:

$$W = 2.2\ kN + W_h$$

We have the following table as (the helium always has the same temperature and pressure as the atmosphere):

Altitude (km)	Pressure (kPa)	Temperature (K)	$\forall\ (m^3)$	$\gamma_{air}\ \left(\frac{N}{m^3}\right)$	W_h (kN)	F_B (kN)	W (kN)
6	47.22	249.2	497	6.46	0.445	3.21	2.65
8	35.70	236.3	566	5.14	0.402	2.91	2.60
10	26.50	223.4	566	4.04	0.317	2.29	2.52

When the maximum volume of the helium is reached, the volume will become a constant for helium.

Equilibrium is reached as:

$$F_B = W$$

At 8 km we have:

$$F_B - W = 0.31\ kN$$

At 10 km we have:

$$F_B - W = -0.23\ kN$$

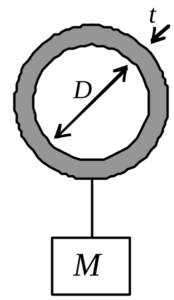
With the interpolation we have the height for equilibrium as:

$$h = 8km + 2km \times \frac{0.31}{0.31 + 0.23} = 9.15\ km$$

3.83 A helium balloon is to lift a payload to an altitude of 40 km, where the atmospheric pressure and temperature are 3.0 mbar and −25°C, respectively. The balloon skin is polyester with specific gravity of 1.28 and thickness of 0.015 mm. To maintain a spherical shape, the balloon is pressurized to a gage pressure of 0.45 mbar. Determine the maximum balloon diameter if the allowable tensile stress in the skin is limited to 62 MN/m². What payload can be carried?

Given: A pressurized balloon is to be designed to lift a payload of mass M to an altitude of 40 km, where p = 3.0 mbar and T = -25 deg C. The balloon skin has a specific gravity of 1.28 and thickness 0.015 mm. The gage pressure of the helium is 0.45 mbar. The allowable tensile stress in the balloon is 62 MN/m²

Find: (a) The maximum balloon diameter
(b) The maximum payload mass



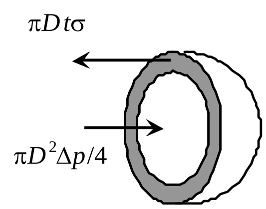
Solution: We will apply the hydrostatics equations to this system.

Governing Equations:

$F_{buoy} = \rho \cdot g \cdot V_d$ (Buoyant force is equal to mass of displaced fluid)

$p = \rho \cdot R \cdot T$ (Ideal gas equation of state)

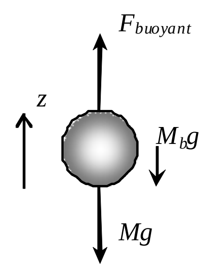
Assumptions: (1) Static, incompressible fluid
(2) Static equilibrium at 40 km altitude
(3) Ideal gas behavior



The diameter of the balloon is limited by the allowable tensile stress in the skin:

$\Sigma F = \frac{\pi}{4} \cdot D^2 \cdot \Delta p - \pi \cdot D \cdot t \cdot \sigma = 0$ Solving this expression for the diameter: $D_{max} = \frac{4 \cdot t \cdot \sigma}{\Delta p}$

$$D_{max} = 4 \times 0.015 \times 10^{-3} \cdot m \times 62 \times 10^{6} \cdot \frac{N}{m^2} \times \frac{1}{0.45 \cdot 10^{-3} \cdot bar} \times \frac{bar \cdot m^2}{10^5 \cdot N} \qquad D_{max} = 82.7 m$$



To find the maximum allowable payload we perform a force balance on the system:

$$\Sigma F_z = F_{buoy} - M_{He} \cdot g - M_b \cdot g - M \cdot g = 0 \qquad \rho_a \cdot g \cdot V_b - \rho_{He} \cdot g \cdot V_b - \rho_s \cdot g \cdot V_s - M \cdot g = 0$$

Solving for M: $M = \left(\rho_a - \rho_{He}\right) \cdot V_b - \rho_s \cdot V_s$ The volume of the balloon is: $V_b = \frac{\pi}{6} \cdot D^3$

The volume of the skin is: $V_s = \pi \cdot D^2 \cdot t$ Therefore, the mass is: $M = \frac{\pi}{6} \cdot \left(\rho_a - \rho_{He}\right) \cdot D^3 - \pi \cdot \rho_s \cdot D^2 \cdot t$

The air density: $\rho_a = \frac{p_a}{R_a \cdot T}$ $\rho_a = 3.0 \times 10^{-3} \cdot bar \times \frac{kg \cdot K}{287 \cdot N \cdot m} \times \frac{1}{(273 - 25) \cdot K} \times \frac{10^5 \cdot N}{bar \cdot m^2}$ $\rho_a = 4.215 \times 10^{-3} \frac{kg}{m^3}$

Repeating for helium: $\rho_{He} = \frac{p}{R \cdot T}$ $\rho_{He} = 6.688 \times 10^{-4} \frac{kg}{m^3}$

The payload mass is:

$$M = \frac{\pi}{6} \times (4.215 - 0.6688) \times 10^{-3} \cdot \frac{kg}{m^3} \times (82.7 \cdot m)^3 - \pi \times 1.28 \times 10^{3} \cdot \frac{kg}{m^3} \times (82.7 \cdot m)^2 \times 0.015 \times 10^{-3} \cdot m$$

$$M = 638 \, kg$$

Problem 3.84 [Difficulty: 3]

3.84 The stem of a glass hydrometer used to measure specific gravity is 5 mm in diameter. The distance between marks on the stem is 2 mm per 0.1 increment of specific gravity. Calculate the magnitude and direction of the error introduced by surface tension if the hydrometer floats in kerosene. (Assume the contact angle between kerosene and glass is 0°.)

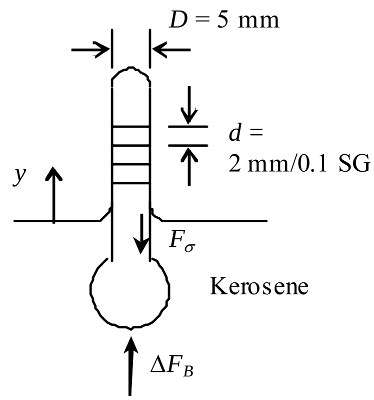
Given: Glass hydrometer used to measure SG of liquids. Stem has diameter D=5 mm, distance between marks on stem is d=2 mm per 0.1 SG. Hydrometer floats in kerosene (Assume zero contact angle between glass and kerosene).

Find: Magnitude of error introduced by surface tension.

Solution: We will apply the hydrostatics equations to this system.

Governing Equations: $F_{buoy} = \rho \cdot g \cdot V_d$ (Buoyant force is equal to weight of displaced fluid)

Assumptions:
(1) Static fluid
(2) Incompressible fluid
(3) Zero contact angle between ethyl alcohol and glass



The surface tension will cause the hydrometer to sink Δh lower into the liquid. Thus for this change:

$$\Sigma F_z = \Delta F_{buoy} - F_\sigma = 0$$

The change in buoyant force is: $\Delta F_{buoy} = \rho \cdot g \cdot \Delta V = \rho \cdot g \cdot \frac{\pi}{4} \cdot D^2 \cdot \Delta h$

The force due to surface tension is: $F_\sigma = \pi \cdot D \cdot \sigma \cdot \cos(\theta) = \pi \cdot D \cdot \sigma$

Thus, $\rho \cdot g \cdot \frac{\pi}{4} \cdot D^2 \cdot \Delta h = \pi \cdot D \cdot \sigma$ Upon simplification: $\frac{\rho \cdot g \cdot D \cdot \Delta h}{4} = \sigma$

Solving for Δh: $\Delta h = \frac{4 \cdot \sigma}{\rho \cdot g \cdot D}$ From Table A.2, SG = 1.43 and from Table A.4, σ = 26.8 mN/m

Therefore, $\Delta h = 4 \times 26.8 \times 10^{-3} \cdot \frac{N}{m} \times \frac{m^3}{1430 \cdot kg} \times \frac{s^2}{9.81 \cdot m} \times \frac{1}{5 \times 10^{-3} \cdot m} \times \frac{kg \cdot m}{s^2 \cdot N}$ $\Delta h = 1.53 \times 10^{-3}\ m$

So the change in specific gravity will be: $\Delta SG = 1.53 \times 10^{-3} \cdot m \times \frac{0.1}{2 \times 10^{-3} \cdot m}$ $\Delta SG = 0.0765$

From the diagram, surface tension acts to cause the hydrometer to float lower in the liquid. Therefore, surface tension results in an indicated specific gravity <u>smaller</u> than the actual specific gravity.

Problem 3.85 [Difficulty:4]

3.85 A sphere, of radius R, is partially immersed, to depth d, in a liquid of specific gravity SG. Obtain an algebraic expression for the buoyancy force acting on the sphere as a function of submersion depth d. Plot the results over the range of water depth $0 \le d \le 2R$.

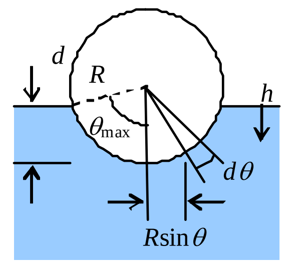
Given: Sphere partially immersed in a liquid of specific gravity SG.

Find: (a) Formula for buoyancy force as a function of the submersion depth d
(b) Plot of results over range of liquid depth

Solution: We will apply the hydrostatics equations to this system.

Governing Equations: $F_{buoy} = \rho \cdot g \cdot V_d$ (Buoyant force is equal to weight of displaced fluid)

Assumptions: (1) Static fluid
(2) Incompressible fluid
(3) Atmospheric pressure acts everywhere



We need an expression for the displaced volume of fluid at an arbitrary depth d. From the diagram we see that:

$d = R\left(1 - \cos\left(\theta_{max}\right)\right)$ at an arbitrary depth h: $h = d - R \cdot (1 - \cos(\theta))$ $r = R \cdot \sin(\theta)$

So if we want to find the volume of the submerged portion of the sphere we calculate:

$$V_d = \int_0^{\theta_{max}} \pi r^2\, dh = \pi \cdot \int_0^{\theta_{max}} R^2 \cdot (\sin(\theta))^2 \cdot R \cdot \sin(\theta)\, d\theta = \pi \cdot R^3 \cdot \int_0^{\theta_{max}} (\sin(\theta))^3\, d\theta$$

Evaluating the integral we get:

$$V_d = \pi \cdot R^3 \cdot \left[\frac{\left(\cos\left(\theta_{max}\right)\right)^3}{3} - \cos\left(\theta_{max}\right) + \frac{2}{3}\right]$$

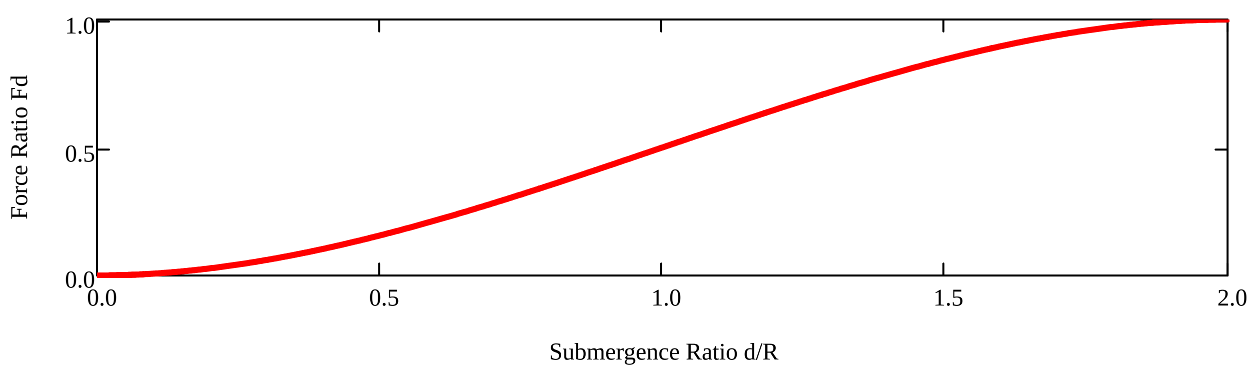
Now since: $\cos\left(\theta_{max}\right) = 1 - \frac{d}{R}$ we get: $V_d = \pi \cdot R^3 \cdot \left[\frac{1}{3} \cdot \left(1 - \frac{d}{R}\right)^3 - \left(1 - \frac{d}{R}\right) + \frac{2}{3}\right]$

Thus the buoyant force is: $F_{buoy} = \rho_w \cdot SG \cdot g \cdot \pi \cdot R^3 \cdot \left[\frac{1}{3} \cdot \left(1 - \frac{d}{R}\right)^3 - \left(1 - \frac{d}{R}\right) + \frac{2}{3}\right]$

If we non-dimensionalize by the force on a fully submerged sphere:

$$F_d = \frac{F_{buoy}}{\rho_w \cdot SG \cdot g \cdot \frac{4}{3} \cdot \pi \cdot R^3} = \frac{3}{4}\left[\frac{1}{3} \cdot \left(1 - \frac{d}{R}\right)^3 - \left(1 - \frac{d}{R}\right) + \frac{2}{3}\right]$$

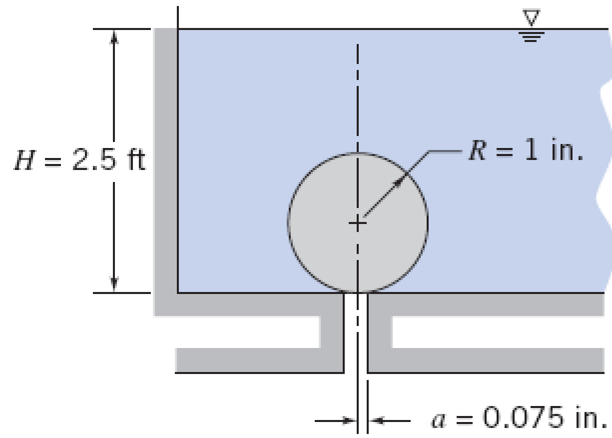
$$F_d = \frac{3}{4}\left[\frac{1}{3} \cdot \left(1 - \frac{d}{R}\right)^3 - \left(1 - \frac{d}{R}\right) + \frac{2}{3}\right]$$



Problem 3.86

[Difficulty: 4]

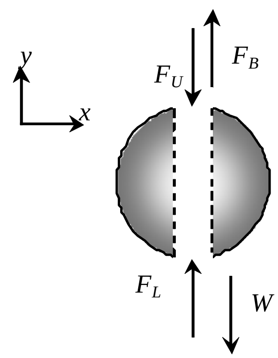
3.86 A sphere of radius 1 in., made from material of specific gravity of SG = 0.95, is submerged in a tank of water. The sphere is placed over a hole of radius 0.075 in., in the tank bottom. When the sphere is released, will it stay on the bottom of the tank or float to the surface?



Given: Data on sphere and tank bottom

Find: Expression for SG of sphere at which it will float to surface; minimum SG to remain in position

Assumptions: (1) Water is static and incompressible
(2) Sphere is much larger than the hole at the bottom of the tank



Solution:

Basic equations $F_B = \rho \cdot g \cdot V$ and $\Sigma F_y = F_L - F_U + F_B - W$

where $F_L = p_{atm} \cdot \pi \cdot a^2$ $F_U = \left[p_{atm} + \rho \cdot g \cdot (H - 2 \cdot R)\right] \cdot \pi \cdot a^2$

$F_B = \rho \cdot g \cdot V_{net}$ $V_{net} = \frac{4}{3} \cdot \pi \cdot R^3 - \pi \cdot a^2 \cdot 2 \cdot R$

$W = SG \cdot \rho \cdot g \cdot V$ with $V = \frac{4}{3} \cdot \pi \cdot R^3$

Now if the sum of the vertical forces is positive, the sphere will float away, while if the sum is zero or negative the sphere will stay at the bottom of the tank (its weight and the hydrostatic force are greater than the buoyant force).

Hence $\Sigma F_y = p_{atm} \cdot \pi \cdot a^2 - \left[p_{atm} + \rho \cdot g \cdot (H - 2 \cdot R)\right] \cdot \pi \cdot a^2 + \rho \cdot g \cdot \left(\frac{4}{3} \cdot \pi \cdot R^3 - 2 \cdot \pi \cdot R \cdot a^2\right) - SG \cdot \rho \cdot g \cdot \frac{4}{3} \cdot \pi \cdot R^3$

This expression simplifies to $\Sigma F_y = \pi \cdot \rho \cdot g \cdot \left[(1 - SG) \cdot \frac{4}{3} \cdot R^3 - H \cdot a^2\right]$

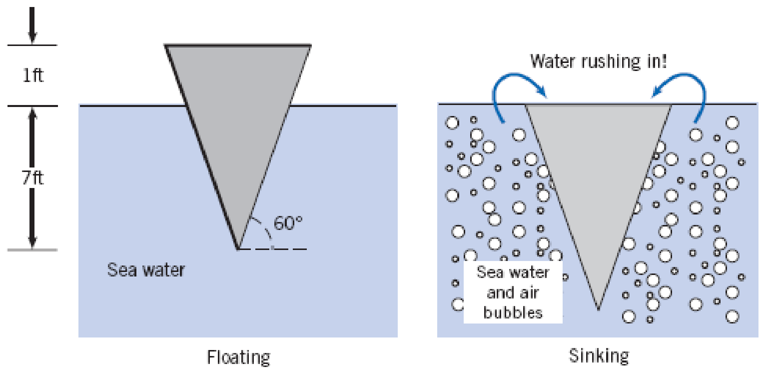
$$\Sigma F_y = \pi \times 1.94 \cdot \frac{slug}{ft^3} \times 32.2 \cdot \frac{ft}{s^2} \times \left[\frac{4}{3} \times (1 - 0.95) \times \left(1 \cdot in \times \frac{ft}{12 \cdot in}\right)^3 - 2.5 \cdot ft \times \left(0.075 \cdot in \times \frac{ft}{12 \cdot in}\right)^2\right] \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

$\Sigma F_y = -0.012 \cdot lbf$ Therefore, the sphere stays at the bottom of the tank.

Problem 3.87

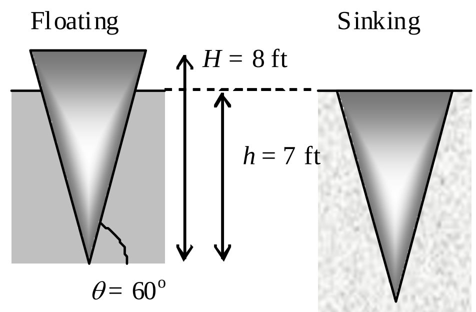
[Difficulty: 3]

3.87 You are in the Bermuda Triangle when you see a bubble plume eruption (a large mass of air bubbles, similar to a foam) off to the side of the boat. Do you want to head toward it and be part of the action? What is the effective density of the water and air bubbles in the drawing on the right that will cause the boat to sink? Your boat is 10 ft long, and weight is the same in both cases.



Given: Data on boat

Find: Effective density of water/air bubble mix if boat sinks



Solution:

Basic equations $\quad F_B = \rho \cdot g \cdot V \quad$ and $\quad \Sigma F_y = 0$

We can apply the sum of forces for the "floating" free body

$$\Sigma F_y = 0 = F_B - W \quad \text{where} \quad F_B = SG_{sea} \cdot \rho \cdot g \cdot V_{subfloat}$$

$$V_{subfloat} = \frac{1}{2} \cdot h \cdot \left(\frac{2 \cdot h}{\tan \cdot \theta}\right) \cdot L = \frac{L \cdot h^2}{\tan(\theta)} \qquad SG_{sea} = 1.024 \qquad \text{(Table A.2)}$$

Hence $\quad W = \dfrac{SG_{sea} \cdot \rho \cdot g \cdot L \cdot h^2}{\tan(\theta)} \quad (1)$

We can apply the sum of forces for the "sinking" free body

$$\Sigma F_y = 0 = F_B - W \quad \text{where} \quad F_B = SG_{mix} \cdot \rho \cdot g \cdot V_{sub} \qquad V_{subsink} = \frac{1}{2} \cdot H \cdot \left(\frac{2 \cdot H}{\tan \cdot \theta}\right) \cdot L = \frac{L \cdot H^2}{\tan(\theta)}$$

Hence $\quad W = \dfrac{SG_{mix} \cdot \rho \cdot g \cdot L \cdot H^2}{\tan(\theta)} \quad (2)$

Comparing Eqs. 1 and 2 $\quad W = \dfrac{SG_{sea} \cdot \rho \cdot g \cdot L \cdot h^2}{\tan(\theta)} = \dfrac{SG_{mix} \cdot \rho \cdot g \cdot L \cdot H^2}{\tan(\theta)}$

$$SG_{mix} = SG_{sea} \cdot \left(\frac{h}{H}\right)^2 \qquad SG_{mix} = 1.024 \times \left(\frac{7}{8}\right)^2 \qquad SG_{mix} = 0.784$$

The density is $\quad \rho_{mix} = SG_{mix} \cdot \rho \qquad \rho_{mix} = 0.784 \times 1.94 \cdot \dfrac{slug}{ft^3} \qquad \rho_{mix} = 1.52 \cdot \dfrac{slug}{ft^3}$

Problem 3.88 [Difficulty: 2]

3.88 Three steel balls (each about half an inch in diameter) lie at the bottom of a plastic shell floating on the water surface in a partially filled bucket. Someone removes the steel balls from the shell and carefully lets them fall to the bottom of the bucket, leaving the plastic shell to float empty. What happens to the water level in the bucket? Does it rise, go down, or remain unchanged? Explain.

Given: Steel balls resting in floating plastic shell in a bucket of water

Find: What happens to water level when balls are dropped in water

Solution: Basic equation $F_B = \rho \cdot V_{disp} \cdot g = W$ for a floating body weight W

When the balls are in the plastic shell, the shell and balls displace a volume of water equal to their own weight - a large volume because the balls are dense. When the balls are removed from the shell and dropped in the water, the shell now displaces only a small volume of water, and the balls sink, displacing only their own volume. Hence the difference in displaced water before and after moving the balls is the difference between the volume of water that is equal to the weight of the balls, and the volume of the balls themselves. The amount of water displaced is significantly reduced, so the water level in the bucket drops.

Volume displaced before moving balls: $V_1 = \dfrac{W_{plastic} + W_{balls}}{\rho \cdot g}$

Volume displaced after moving balls: $V_2 = \dfrac{W_{plastic}}{\rho \cdot g} + V_{balls}$

Change in volume displaced $\Delta V = V_2 - V_1 = V_{balls} - \dfrac{W_{balls}}{\rho \cdot g} = V_{balls} - \dfrac{SG_{balls} \cdot \rho \cdot g \cdot V_{balls}}{\rho \cdot g}$

$$\Delta V = V_{balls} \cdot \left(1 - SG_{balls}\right)$$

Hence initially a large volume is displaced; finally a small volume is displaced ($\Delta V < 0$ because $SG_{balls} > 1$)

Problem 3.89

[Difficulty: 4]

3.89 A proposed ocean salvage scheme involves pumping air into "bags" placed within and around a wrecked vessel on the sea bottom. Comment on the practicality of this plan, supporting your conclusions with analyses.

Open-Ended Problem Statement: A proposed ocean salvage scheme involves pumping air into "bags" placed within and around a wrecked vessel on the sea bottom. Comment on the practicality of this plan, supporting your conclusions with analyses.

Discussion: This plan has several problems that render it impractical. First, pressures at the sea bottom are very high. For example, *Titanic* was found in about 12,000 ft of seawater. The corresponding pressure is nearly 6,000 psi. Compressing air to this pressure is possible, but would require a multi-stage compressor and very high power.

Second, it would be necessary to manage the buoyancy force after the bag and object are broken loose from the sea bed and begin to rise toward the surface. Ambient pressure would decrease as the bag and artifact rise toward the surface. The air would tend to expand as the pressure decreases, thereby tending to increase the volume of the bag. The buoyancy force acting on the bag is directly proportional to the bag volume, so it would increase as the assembly rises. The bag and artifact thus would tend to accelerate as they approach the sea surface. The assembly could broach the water surface with the possibility of damaging the artifact or the assembly.

If the bag were of constant volume, the pressure inside the bag would remain essentially constant at the pressure of the sea floor, e.g., 6,000 psi for *Titanic*. As the ambient pressure decreases, the pressure differential from inside the bag to the surroundings would increase. Eventually the difference would equal sea floor pressure. This probably would cause the bag to rupture.

If the bag permitted some expansion, a control scheme would be needed to vent air from the bag during the trip to the surface to maintain a constant buoyancy force just slightly larger than the weight of the artifact in water. Then the trip to the surface could be completed at low speed without danger of broaching the surface or damaging the artifact.