Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–1. Show that Pascal's law applies within a fluid that is accelerating, provided there are no shearing stresses acting within the fluid.

SOLUTION

Consider the free-body diagram of a triangular element of fluid as shown in Fig. 2–2b. If this element has acceleration components of a_x, a_y, a_z, then since $dm = \rho d\forall$, the equations of motion in the y and z directions give

$$\Sigma F_y = dma_y; \qquad p_y(\Delta x)(\Delta s \sin\theta) - \left[p(\Delta x \Delta s)\right]\sin\theta = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_y$$

$$\Sigma F_z = dma_z; \qquad p_z(\Delta x)(\Delta s\cos\theta) - \left[p(\Delta x\Delta s)\right]\cos\theta - \gamma\left[\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right] = \rho\left(\frac{1}{2}\Delta x(\Delta s\cos\theta)(\Delta s\sin\theta)\right)a_z$$

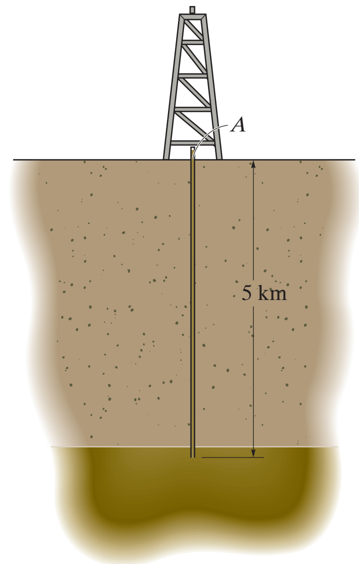
Dividing by $\Delta x \Delta s$ and letting $\Delta s \rightarrow 0$, so the element reduces in size, we obtain

$$p_y = p$$
$$p_z = p$$

By a similar argument, the element can be rotated 90° about the z axis and $\Sigma F_x = dma_x$ can be applied to show $p_x = p$. Since the angle θ of the inclined face is *arbitrary,* this indeed shows that the pressure at a point is the *same in all directions* for any fluid that has no shearing stress acting within it.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–2. The oil derrick has drilled 5 km into the ground before it strikes a crude oil reservoir. When this happens, the pressure at the well head *A* becomes 25 MPa. Drilling "mud" is to be placed into the entire length of pipe to displace the oil and balance this pressure. What should be the density of the mud so that the pressure at *A* becomes zero?



SOLUTION

Consider the case when the crude oil is pushing out at *A* where $p_A = 25(10^6)$ Pa, Fig. *a*. Here, $\rho_o = 880\ \text{kg/m}^3$ (Appendix A), hence $p_o = \rho_o gh = (880\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5000\ \text{m}) = 43.164(10^6)$ Pa

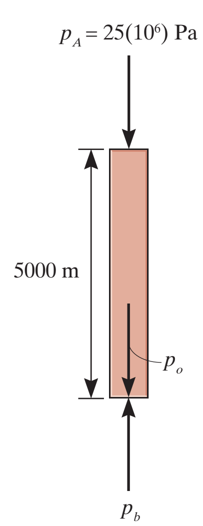
$$p_b = p_A + p_o = 25(10^6)\ \text{Pa} + 43.164(10^6)\ \text{Pa} = 68.164(10^6)\ \text{Pa}$$

It is required that $p_A = 0$, Fig. *b*. Thus,

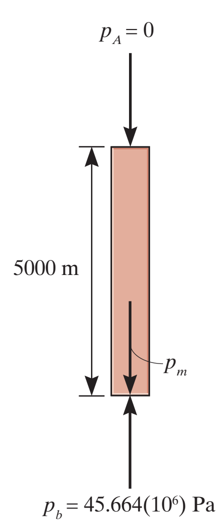
$$p_b = p_m = \rho_m gh$$

$$68.164(10^6)\frac{\text{N}}{\text{m}^2} = \rho_m(9.81\ \text{m/s}^2)(5000\ \text{m})$$

$$\rho_m = 1390\ \text{kg/m}^3 \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\rho_m = 1390\ \text{kg/m}^3$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–3. In 1896, S. Riva-Rocci developed the prototype of the current sphygmomanometer, a device used to measure blood pressure. When it was worn as a cuff around the upper arm and inflated, the air pressure within the cuff was connected to a mercury manometer. If the reading for the high (or systolic) pressure is 120 mm and for the low (or diastolic) pressure is 80 mm, determine these pressures in psi and pascals.

SOLUTION

Mercury is considered to be incompressible. From Appendix A, the density of mercury is $\rho_{Hg} = 13\ 550\ \text{kg/m}^3$. Thus, the systolic pressure is

$$p_s = \rho_{Hg} g h_s = \left(13\ 550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.12\ \text{m}) = 15.95\ \text{kPa}$$

$$= 16.0\left(10^3\right)\ \text{Pa} \qquad \textbf{Ans.}$$

$$p_s = \left[15.95\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\text{ft}}{12\ \text{in.}}\right)^2 = 2.31\ \text{psi} \qquad \textbf{Ans.}$$

The diastolic pressure is

$$p_d = \rho_{Hg} g h_d = \left(13\ 550\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.08\ \text{m}) = 10.63\left(10^3\right)\ \text{Pa}$$

$$= 10.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_d = \left[10.63\left(10^3\right)\frac{\text{N}}{\text{m}^2}\right]\left(\frac{1\ \text{lb}}{4.4482\ \text{N}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.54\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p_s = 16.0\ \text{kPa} = 2.31\ \text{psi}$
$p_d = 10.6\ \text{kPa} = 1.54\ \text{psi}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–4.** Oxygen in a tank has an absolute pressure of 130 kPa. Determine the pressure head in mm of mercury. The atmospheric pressure is 102 kPa.

SOLUTION

The gauge pressure of the oxygen in the tank can be determined from

$$p_{abs} = p_{atm} + p_g$$
$$130\ \text{kPa} = 102\ \text{kPa} + p_g$$
$$p_g = 28\ \text{kPa}$$

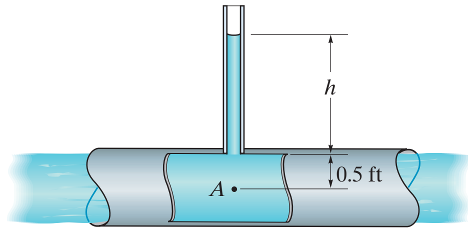
From the table in Appendix A, $\rho_{Hg} = 13\,550\ \text{kg/m}^3$

$$p = \gamma_{Hg} h_{Hg}$$
$$28(10^3)\ \text{N/m}^2 = [(13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)]h_{Hg}$$
$$h_{Hg} = 0.2106\ \text{m} = 211\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$h_{Hg} = 211\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–5. If the piezometer measures a gage pressure of 10 psi at point A, determine the height h of the water in the tube. Compare this height with that using mercury. Take $\rho_w = 1.94\ \text{slug/ft}^3$ and $\rho_{Hg} = 26.3\ \text{slug/ft}^3$.



SOLUTION

Here, the absolute pressure to be measured is

$$p = p_g + p_{atm} = \left(10\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + p_{atm} = (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^2}$$

For the water piezometer, Fig. *a*,

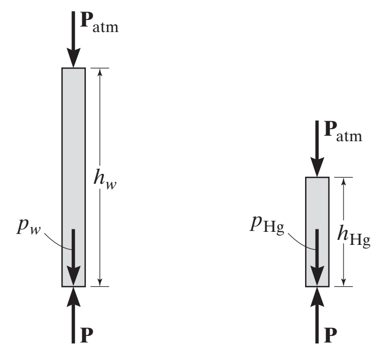
$$p = p_{atm} + p_w;\quad (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^3} = p_{atm} + (1.94\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(h + 0.5\ \text{ft})$$

$$h_w = 22.55\ \text{ft} = 22.6\ \text{ft} \qquad \textbf{Ans.}$$

For the mercury piezometer, Fig. *b*,

$$p = p_{atm} + p_{Hg};\quad (1440 + p_{atm})\ \frac{\text{lb}}{\text{ft}^3} = p_{atm} + (26.3\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(h + 0.5\ \text{ft})$$

$$h_{Hg} = 1.20\ \text{ft} \qquad \textbf{Ans.}$$



(a) (b)

Ans:
$h_w = 22.6\ \text{ft}$
$h_{Hg} = 1.20\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–6. If the absolute pressure in a tank is 140 kPa, determine the pressure head in mm of mercury. The atmospheric pressure is 100 kPa.

SOLUTION

$$p_{\text{abs}} = p_{\text{atm}} + p_g$$

$$140\ \text{kPa} = 100\ \text{kPa} + p_g$$

$$p_g = 40\ \text{kPa}$$

From Appendix A, $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$.

$$p = \gamma_{\text{Hg}} h_{\text{Hg}}$$

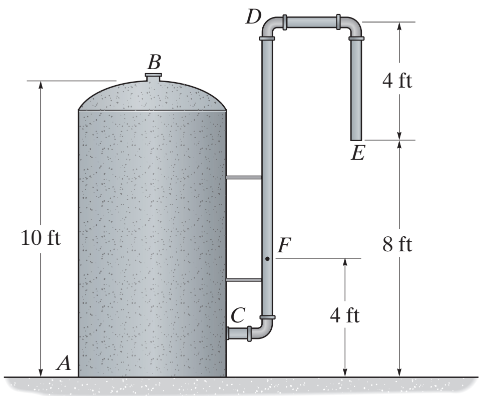
$$40(10^3)\ \text{N/m}^2 = (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)h_{\text{Hg}}$$

$$h_{\text{Hg}} = 0.3009\ \text{m} = 301\ \text{mm}$$ **Ans.**

Ans:
$h_{\text{Hg}} = 301\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–7. The field storage tank is filled with oil. The standpipe is connected to the tank at *C*, and the system is open to the atmosphere at *B* and *E*. Determine the maximum pressure in the tank in psi if the oil reaches a level of *F* in the pipe. Also, at what level should the oil be in the tank, so that the maximum pressure occurs in the tank? What is this value? Take $\rho_o = 1.78\ \text{slug/ft}^3$.



SOLUTION

Since the top of the tank is open to the atmosphere, the free surface of the oil in the tank will be the same height as that of point *F*. Thus, the maximum pressure which occurs at the base of the tank (level *A*) is

$$\begin{aligned}(p_A)_g &= \gamma h \\ &= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4\ \text{ft}) \\ &= 229.26\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.59\ \text{psi} \qquad \textbf{Ans.}\end{aligned}$$

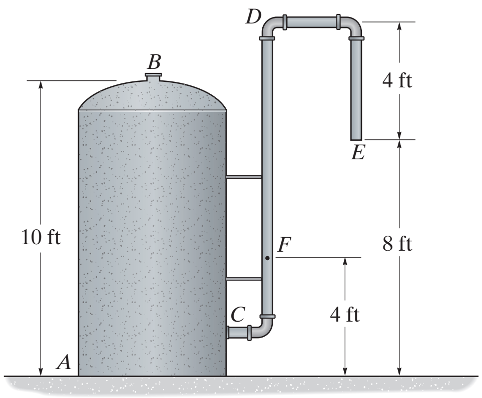
Absolute maximum pressure occurs at the base of the tank (level *A*) when the **oil reaches level B.**

$$\begin{aligned}(p_A)_{\substack{\text{abs}\\ \text{max}}} &= \gamma h \\ &= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(10\ \text{ft}) \\ &= 573.16\ \text{lb/ft}^2\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 3.98\ \text{psi} \qquad \textbf{Ans.}\end{aligned}$$

Ans:
$(p_A)_g = 1.59$ psi
Absolute maximum pressure occurs at base *A* when the oil reaches level *B*.
$(p_A)_{\substack{\text{abs}\\ \text{max}}} = 3.98$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

*__2–8.__ The field storage tank is filled with oil. The standpipe is connected to the tank at C and open to the atmosphere at E. Determine the maximum pressure that can be developed in the tank if the oil has a density of $1.78\ \text{slug/ft}^3$. Where does this maximum pressure occur? Assume that there is no air trapped in the tank and that the top of the tank at B is closed.



SOLUTION

Level D is the highest the oil is allowed to rise in the tube, and the maximum gauge pressure occurs at the base of the tank (level A).

$$(p_{\max})_g = \gamma h$$

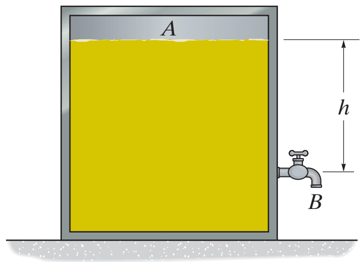
$$= (1.78\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(8\ \text{ft} + 4\ \text{ft})$$

$$= \left(687.79\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 4.78\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$(p_{\max})_g = 4.78$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–9. The closed tank was completely filled with carbon tetrachloride when the valve at B was opened, slowly letting the carbon tetrachloride level drop as shown. If the valve is then closed and the space within A is a vacuum, determine the pressure in the liquid near valve B when $h = 25$ ft. The atmospheric pressure is 14.7 psi.



SOLUTION

From the Appendix, $\rho_{ct} = 3.09\ \text{slug/ft}^3$. Since the empty space A is a vacuum, $p_A = 0$. Thus, the absolute pressure at B when $h = 25$ ft is

$$
\begin{aligned}
(p_B)_{abs} &= p_A + \gamma h \\
&= 0 + (3.09\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(25\ \text{ft}) \\
&= \left(2487.45\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 17.274\ \text{psi}
\end{aligned}
$$

The gauge pressure is given by

$$
\begin{aligned}
(p_B)_{abs} &= p_{atm} + (p_B)_g \\
17.274\ \text{psi} &= 14.7\ \text{psi} + (p_B)_g \\
(p_B)_g &= 2.57\ \text{psi}
\end{aligned}
$$

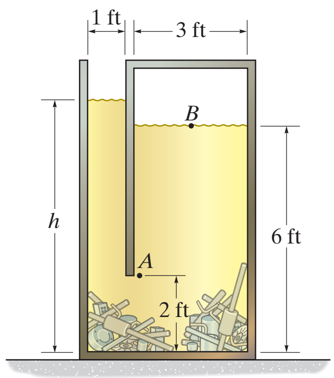
Ans.

Note: When the vacuum is produced, it actually becomes an example of a Rayleigh–Taylor instability. The lower density fluid (air) will migrate up into the valve B and then rise into the space A, increasing the pressure, and pushing some water out the valve. This back-and-forth effect will in time drain the tank.

Ans:
$(p_B)_g = 2.57$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–10. The soaking bin contains ethyl alcohol used for cleaning automobile parts. If $h = 7$ ft, determine the pressure developed at point A and at the air surface B within the enclosure. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressures at points A and B are

$$p_A = \gamma_{ea}\, h_A = \left(49.3\ \frac{\text{lb}}{\text{ft}^3}\right)(7\ \text{ft} - 2\ \text{ft})$$

$$= \left(246.5\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.71\ \text{psi} \qquad \textbf{Ans.}$$

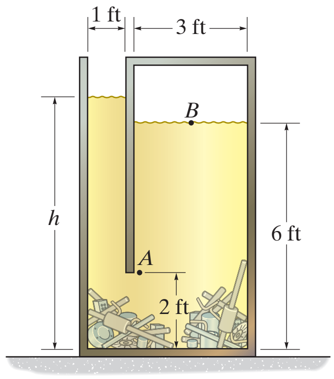
$$p_B = \gamma_{ea}\, h_B = (49.3\ \text{lb/ft}^3)(7\ \text{ft} - 6\ \text{ft})$$

$$= \left(49.3\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 0.342\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p_A = 1.71$ psi, $p_B = 0.342$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–11. The soaking bin contains ethyl alcohol used for cleaning automobile parts. If the gage pressure in the enclosure is $p_B = 0.5$ psi, determine the pressure developed at point A and the height h of the ethyl alcohol level in the bin. Take $\gamma_{ea} = 49.3\ \text{lb/ft}^3$.



SOLUTION

The gauge pressure at point A is

$$(p_A)_g = p_B + \gamma_{ea}\, h_{BA}$$

$$= 0.5\ \text{psi} + \left(49.3\ \frac{\text{lb}}{\text{ft}^3}\right)(6\ \text{ft} - 2\ \text{ft})\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 1.869\ \text{psi} = 1.87\ \text{psi} \qquad \textbf{Ans.}$$

The gauge pressure for the atmospheric pressure is $(p_{\text{atm}})_g = 0$. Thus,

$$(p_B)_g = (p_{\text{atm}})_g + \gamma_{ea}\, h_B$$

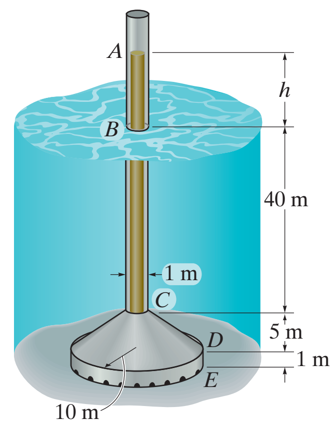
$$\left(0.5\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 0 + \left(49.3\ \frac{\text{lb}}{\text{ft}^3}\right)(h - 6)$$

$$h = 7.46\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$(p_A)_g = 1.87$ psi
$h = 7.46$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–12.** The structure shown is used for the temporary storage of crude oil at sea for later loading into ships. When it is not filled with oil, the water level in the stem is at B (sea level). Why? As the oil is loaded into the stem, the water is displaced through exit ports at E. If the stem is filled with oil, that is, to the depth of C, determine the height h of the oil level above sea level. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

The water level remains at B when empty because the gage pressure at B must be zero. It is required that the pressure at C caused by the water and oil be the same. Then

$$(p_C)_w = (p_C)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

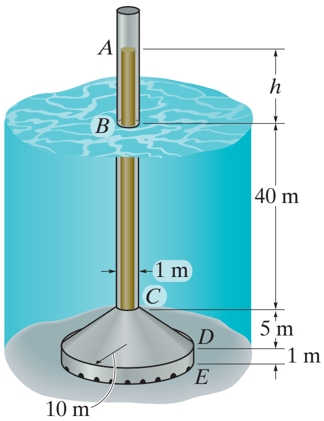
$$(1020\ \text{kg/m}^3)(g)(40\ \text{m}) = (900\ \text{kg/m}^3)g(40\ \text{m} + h)$$

$$h = 5.33\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$h = 5.33\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–13. If the water in the structure in Prob. 2–12 is displaced with crude oil to the level *D* at the bottom of the cone, then how high *h* will the oil extend above sea level? Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1020\ \text{kg/m}^3$.



SOLUTION

It is required that the pressure at *D* caused by the water and oil be the same.

$$(p_D)_w = (p_D)_o$$

$$\rho_w g h_w = \rho_o g h_o$$

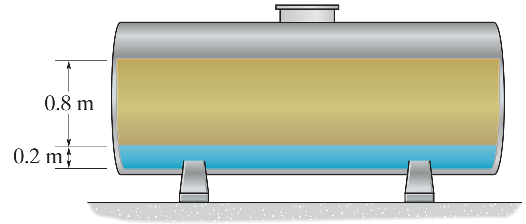
$$(1020\ \text{kg/m}^3)(g)(45\ \text{m}) = (900\ \text{kg/m}^3)(g)(45\ \text{m} + h)$$

$$h = 6.00\ \text{m}$$ **Ans.**

Ans:
$h = 6.00\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–14. The tank is filled with water and gasoline at a temperature of 20°C to the depths shown. If the absolute air pressure at the top of the tank is 200 kPa, determine the gage pressure at the bottom of the tank. Would the results be different if the tank had a flat bottom rather than a curved one? The atmospheric pressure is 101 kPa.



SOLUTION

The absolute pressure at the bottom of tank is

$$(p_b)_{\text{abs}} = p_{\text{air}} + p_g + p_w$$

Here, p_g and p_w can be determined with $\rho_g = 726\ \text{kg/m}^3$ and $\rho_w = 998.3\ \text{kg/m}^3$ at $T = 20°\text{C}$ (table in Appendix A). Then

$$(p_b)_{\text{abs}} = 200(10^3)\ \text{N/m}^2 + (726\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.8\ \text{m}) + (998.3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.2\ \text{m})$$
$$= 207.66(10^3)\ \text{kPa}$$

Thus, the gauge pressure at the bottom of the tank can be determined using

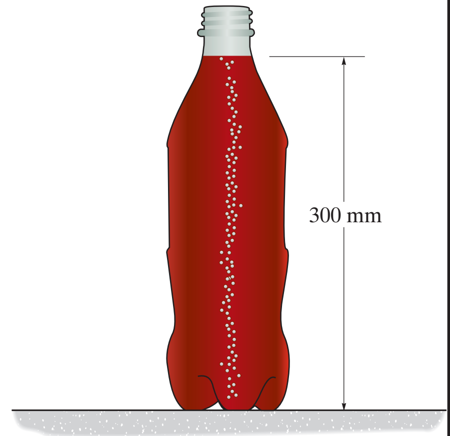
$$(p_b)_{\text{abs}} = (p_b)_g + p_{\text{atm}}$$
$$207.66(10^3)\ \text{kPa} = (p_b)_g + 101\ \text{kPa}$$
$$(p_b)_{\text{gage}} = 106.66\ \text{kPa} = 107\ \text{kPa} \qquad \textbf{Ans.}$$

No, the pressure at the bottom of the tank does not depend on its shape.

Ans:
$(p_b)_{\text{gage}} = 107\ \text{kPa}$
The pressure at the bottom of the tank does not depend on its shape.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–15. At the bottom of a bottle a bubble within carbonated water has a diameter of 0.2 mm. Determine the bubble's diameter when it reaches the surface. The temperature of the water and bubbles is 10°C, and the atmospheric pressure is 101 kPa. Assume that the density of the water is the same as that of pure water.



SOLUTION

Applying the ideal gas law, $p = \rho RT$, of which T is constant in this case. Thus,

$$\frac{p}{\rho} = \text{constant}$$

Since $\rho = \dfrac{m}{\forall}$, where m, the mass of the CO_2 in the bubble, is also constant, then

$$\frac{p}{m/\forall} = \text{constant}$$

$$p\forall = \text{constant} \qquad \textbf{(1)}$$

At $T = 10°\text{C}$, $\rho_w = 999.7\ \text{kg/m}^3$. The pressure due to the static water can be determined using $p = \gamma h$. Then the absolute pressure at the bottom of the bottle can be determined from

$$\begin{aligned} p_b &= p_{\text{atm}} + \gamma_w h_w \\ &= 101(10^3)\ \text{N/m}^2 + (999.7\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m}) \\ &= 103.94(10^3)\ \text{Pa} = 103.94\ \text{kPa} \end{aligned}$$

At the surface of the bottle, the absolute pressure is

$$p_s = p_{\text{atm}} = 101\ \text{kPa}$$

Using Eq. (1), we can write

$$p_b \forall_b = p_s \forall_s$$

$$(103.94\ \text{kPa})\left[\frac{4}{3}\pi(0.1\ \text{mm})^3\right] = (101\ \text{kPa})\left[\frac{4}{3}\pi\left(\frac{d_b}{2}\right)^3\right]$$

$$\begin{aligned} d_b &= 0.201923\ \text{mm} \\ &= 0.202\ \text{mm} \end{aligned} \qquad \textbf{Ans.}$$

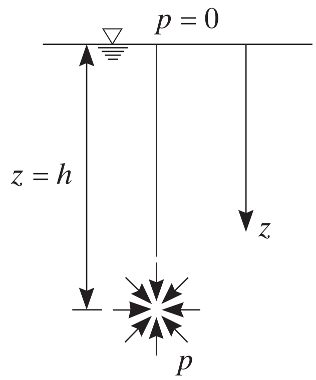
Ans:
$d_b = 0.202\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–16.** The density ρ of a fluid varies with depth h, although its bulk modulus $E_\forall$ can be assumed constant. Determine how the pressure varies with h. The density of the fluid at the surface is ρ_0.

SOLUTION

The fluid is considered compressible.



(a)

$$E_V = -\frac{dp}{d\forall/\forall}$$

However, $\forall = \dfrac{m}{\rho}$. Then,

$$\frac{d\forall}{\forall} = \frac{-(m/\rho^2)\,d\rho}{m/\rho} = -\frac{d\rho}{\rho}$$

Therefore,

$$E_\forall = \frac{dp}{d\rho/\rho}$$

At the surface, where $p = 0$, $\rho = \rho_0$, Fig. a, then

$$E_\forall \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^p dp$$

$$E_\forall \ln\left(\frac{\rho}{\rho_0}\right) = p$$

or

$$\rho = \rho_{0e^{p/E_\forall}}$$

Also,

$$p = p_0 + \rho g z$$

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = g dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = h$, Fig. a.

$$\int_0^p \frac{dp}{\rho_{0e}^{\ p/E_\forall}} = \int_0^h g dz$$

$$\frac{E_\forall}{\rho_0}\left(1 - e^{-p/E_\forall}\right) = gh$$

$$1 - e^{-p/E_\forall} = \frac{\rho_0 g h}{E_\forall}$$

$$p = -E_\forall \ln\left(1 - \frac{\rho_0 g h}{E_\forall}\right) \qquad \textbf{Ans.}$$

Ans:
$p = -E_\forall \ln\left(1 - \dfrac{\rho_o g h}{E_\forall}\right)$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–17. Due to its slight compressibility, the density of water varies with depth, although its bulk modulus $E_V = 2.20$ GPa (absolute) can be considered constant. Accounting for this compressibility, determine the pressure in the water at a depth of 300 m, if the density at the surface of the water is $\rho_0 = 1000\ \text{kg/m}^3$. Compare this result with water that is assumed to be incompressible.

SOLUTION

The water is considered compressible. Using the definition of bulk modulus,

$$E_V = \frac{dp}{dV/V}$$

However, $V = \dfrac{m}{\rho}$. Then

$$\frac{dV}{V} = \frac{-(m/\rho^2)d\rho}{m/\rho} = -\frac{d\rho}{\rho}$$

Therefore,

$$E_V = \frac{dp}{d\rho/\rho}$$

At the surface, $p = 0$ and $\rho = 1000\ \text{kg/m}^3$. Also, $E_V = 2.20$ GPa. Then

$$\left[2.20(10^9)\ \text{N/m}^2\right]\int_{1000\ \text{kg/m}^3}^{\rho}\frac{d\rho}{\rho} = \int_0^{p} dp$$

$$p = 2.20(10^9)\ln\left(\frac{\rho}{1000}\right)$$

$$\rho = 1000\, e^{\frac{p}{2.20(10^9)}} \quad \textbf{(1)}$$

Also,

$$dp = \rho g dz$$

$$\frac{dp}{\rho} = 9.81 dz \quad \textbf{(2)}$$

Substitute Eq. (1) into (2).

$$\frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = 9.81dz$$

Since the pressure $p = 0$ at $z = 0$ and p at $z = 300$ m,

$$\int_0^{p}\frac{dp}{1000e^{\frac{p}{2.20(10^9)}}} = \int_0^{300\ \text{m}} 9.81dz$$

$$-2.2(10^6)e^{-\frac{p}{2.20(10^9)}}\Big|_0^{p} = 9.81z\Big|_0^{300\ \text{m}}$$

2–17. Continued

$$-2.2(10^6)\left[e^{-\frac{p}{2.20(10^9)}} - 1\right] = 2943$$

$$e^{-\frac{p}{2.20(10^9)}} = 0.9987$$

$$\ln e^{-\frac{p}{2.20(10^9)}} = \ln 0.9987$$

$$-\frac{p}{2.20(10^9)} = -1.3386(10^{-3})$$

Compressible:

$$p = 2.945(10^6)\ \text{Pa} = 2.945\ \text{MPa} \qquad \textbf{Ans.}$$

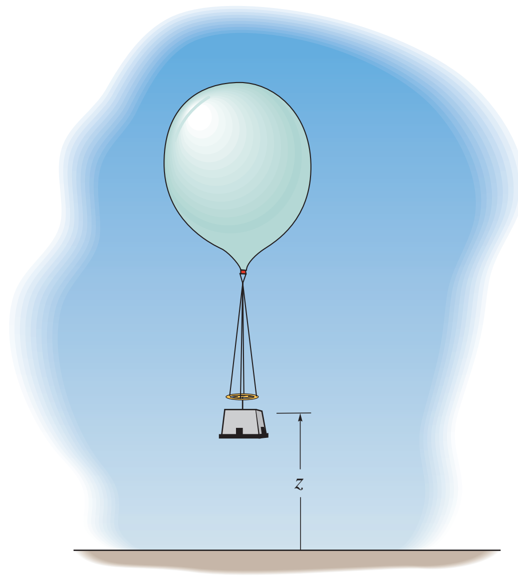
If the water is considered incompressible,

$$p = \rho_o gh = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(300\ \text{m})$$
$$= 2.943(10^6)\ \text{Pa} = 2.943\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
Compressible: $p = 2.945$ MPa
Incompressible: $p = 2.943$ MPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–18. As the weather balloon ascends, measurements indicate that the temperature begins to decrease linearly from T_0 at $z = 0$ to T_f at $z = h$. If the absolute pressure of the air at $z = 0$ is p_0, determine the pressure as a function of z.



SOLUTION

We will first determine the absolute temperature as a function of z:

$$T = \left[T_0 - \left(\frac{T_0 - T_f}{h}\right)z\right] = \frac{T_0 h - (T_0 - T_f)z}{h}$$

Using this result to apply the ideal gas law,

$$p = \rho R T; \qquad \rho = \frac{p}{RT} = \frac{p}{R\left[\dfrac{T_0 h - (T_0 - T_f)z}{h}\right]}$$

$$= \frac{ph}{R[T_0 h - (T_0 - T_f)z]}$$

However,

$$dp = -\gamma dz = -\rho g dz$$

Substitute the result of ρ into this equation:

$$dp = -\frac{phgdz}{R[T_0 h - (T_0 - T_f)]z}$$

$$\frac{dp}{p} = -\frac{gh}{R}\left[\frac{dz}{T_0 h - (T_0 - T_f)z}\right]$$

When $z = 0, p = p_0$. Then

$$\int_{p_0}^{p}\frac{dp}{p} = -\frac{gh}{R}\int_0^z \frac{dz}{T_0 h - (T_0 - T_f)z}$$

$$\ln p\Big|_{p_0}^{p} = -\frac{gh}{R}\left\{-\frac{1}{T_0 - T_f}\ln\left[T_0 h - (T_0 - T_f)z\right]\right\}\Big|_0^z$$

$$\ln\frac{p}{p_0} = \frac{gh}{R(T_0 - T_f)}\ln\left[\frac{T_0 h - (T_0 - T_f)z}{T_0 h}\right]$$

$$\ln\frac{p}{p_0} = \ln\left\{\left[\frac{T_0 h - (T_0 - T_f)z}{T_0 h}\right]^{\frac{gh}{R(T_0 - T_f)}}\right\}$$

$$\frac{p}{p_0} = \left[\frac{T_0 h - (T_0 - T_f)z}{T_0 h}\right]^{\frac{gh}{R(T_0 - T_f)}}$$

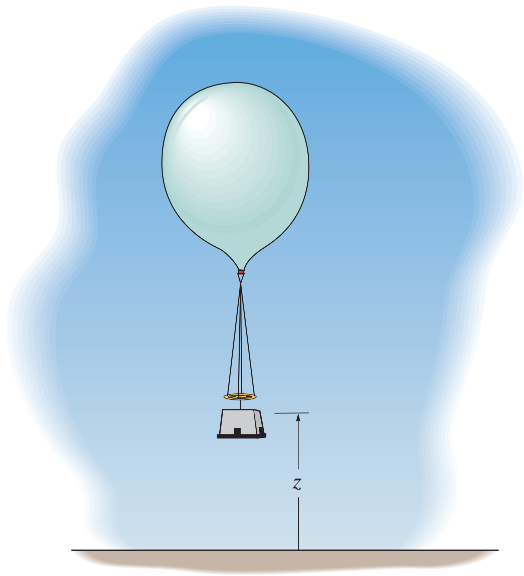
$$p = p_0\left[1 - \left(\frac{T_0 - T_f}{T_0 h}\right)z\right]^{\frac{gh}{R(T_0 - T_f)}} \qquad \textbf{Ans.}$$

Ans:

$$p = p_0\left[1 - \left(\frac{T_0 - T_f}{T_0 h}\right)z\right]^{\frac{gh}{R(T_0 - T_f)}}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–19. As the balloon ascends, measurements indicate that the temperature begins to decrease at a constant rate, from $T = 60°\text{F}$ at $z = 0$ to $T = 50°\text{F}$ at $z = 2500$ ft. If the absolute pressure of the air at $z = 0$ is $p = 14.7$ psi, plot the variation of pressure (vertical axis) versus altitude for $0 \le z \le 2500$ ft. Give values for increments of $\Delta z = 500$ ft.



SOLUTION

We will first determine the absolute temperature as a function of z.

$$T = \left[520 - \left(\frac{520 - 510}{2500}\right)z\right]\ °\text{K} = (520 - 0.004z)\ °\text{K}$$

Using this result to apply the ideal gas law with $R = 1716\ \text{ft}\cdot\text{lb/slug}\cdot°\text{K}$,

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{1716(520 - 0.004z)} = \frac{p}{892.32(10^3) - 6.864z}$$

However,

$$dp = -\gamma dz = -\rho g dz$$

Substitute the result of ρ into this equation:

$$dp = -\frac{p(32.2)}{892.32(10^3) - 6.864z}dz$$

$$\frac{dp}{p} = -\frac{32.2\ dz}{892.32(10^3) - 6.864z}$$

When $z = 0$, $p = \left(14.7\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 2116.8\ \text{lb/ft}^2$. Then

$$\int_{2116.8}^{p} \frac{dp}{p} = -32.2\int_0^z \frac{dz}{892.32(10^3) - 6.864z}$$

$$\ln p\Big|_{2116.8}^{p} = -32.2\left\{-\frac{1}{6.86}\ln\left[892.32(10^3) - 6.864z\right]\right\}\Bigg|_0^z$$

$$\ln\left(\frac{p}{2116.8}\right) = 4.6911\ln\left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)}\right]$$

$$\ln\left(\frac{p}{2116.8}\right) = \ln\left\{\left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)}\right]^{4.6911}\right\}$$

$$\frac{p}{2116.8} = \left[\frac{892.32(10^3) - 6.864z}{892.32(10^3)}\right]^{4.6911}$$

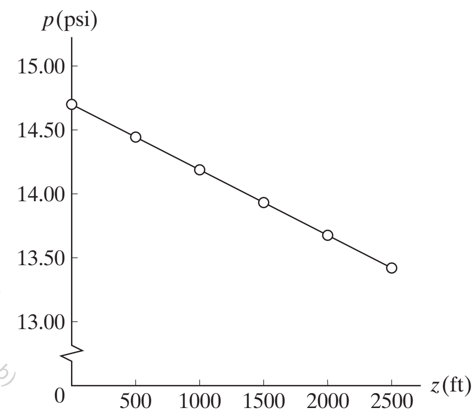
$$p = \left\{2116.8\left[1 - 7.6923(10^{-6})z\right]^{4.6911}\frac{\text{lb}}{\text{ft}^2}\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$p = \{14.7[1 - 7.6923(10^{-6})z]^{4.6911}\}\ \text{psi where } z \text{ is in ft.}$$

For $0 \le z \le 2500$ ft,

z(ft)	0	500	1000	1500	2000	2500
p(psi)	14.70	14.44	14.18	13.92	13.67	13.42

The plot of p vs z is shown in Fig. a.



(a)

Ans:
$p = \{14.7[1 - 7.69(10^{-6})z]^{4.69}\}$ psi where z is in ft.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

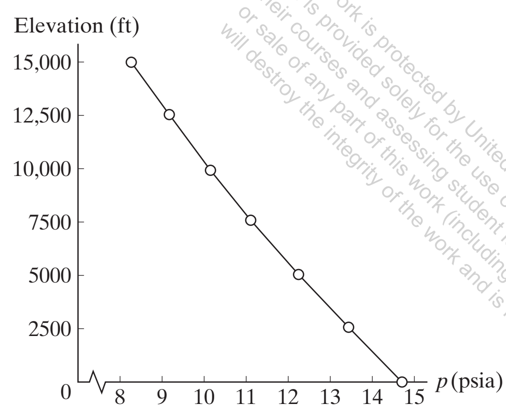
***2–20.** Using the data in Appendix A, make a graph showing how the atmospheric pressure p in psia (horizontal axis) varies with elevation h in feet. Plot values of p every 2500 ft for $0 \le h \le 15000$ ft.

SOLUTION

Elevation (ft)	0	2500	5000	7500	10 000	12 500	15 000
ρ(psf)	2116	1932	1761	1602	1456	1320	1195
p(psia)	14.69	13.42	12.23	11.25	10.11	9.17	8.30

$$\left(\frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = \frac{\text{lb}}{\text{in}^2}$$

The plot of elevation vs. psia is shown in Fig. *a*.



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–21. A liquid has a density that varies with depth h, such that $\rho = (0.001h + 1.60)\ \text{slug/ft}^3$, where h is in feet. Determine the pressure due to the liquid when $h = 80$ ft.

SOLUTION

For the compressible liquid,

$$dp = -\gamma dz = -\rho g dz$$

However, $z = h_0 - h$ where h_0 is a constant, Fig. a. Then $dz = -dh$. Substituting this result into the above equation, it becomes,

$$dp = \rho g dh$$

Integrating this equation using the gage pressure $p = 0$ at $h = 0$ and p at h, then

$$\int_0^p dp = \int_0^h (0.001h + 1.6)(32.2)dh$$

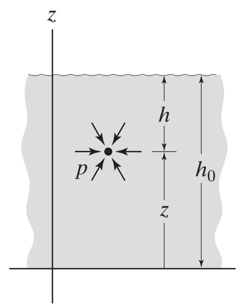
$$p = \left[(0.0161h^2 + 51.52h)\ \frac{\text{lb}}{\text{ft}^2}\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [0.1118(10^{-3})h^2 + 0.3578h]\ \text{psi} \quad \text{where } h \text{ is in feet.}$$

When $h = 80$ ft,

$$p = [0.1118(10^{-3})(80^2) + 0.3578(80)]\ \text{psi}$$

$$= 29.34\ \text{psi} = 29.3\ \text{psi} \qquad \textbf{Ans.}$$



(a)

Ans:
$p = 29.3$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–22. A liquid has a density that varies with depth h, such that $\rho = (0.001h + 1.60)\ \text{slug/ft}^3$, where h is in feet. Plot the variation of the pressure due to the liquid (vertical axis) versus depth for $0 \le h \le 100$ ft. Give values for increments of 20 ft.

SOLUTION

For the compressible liquid,

$$dp = -\gamma dz = -\rho g dz$$

However, $z = h_0 - h$ where h_0 is a constant, Fig. *a*. Then $dz = -dh$. Substituting this result into the above equation, it becomes

$$dp = \rho g dh$$

Integrating this equation using the gauge pressure $p = 0$ at $h = 0$ and p at h, then

$$\int_0^p dp = \int_0^h (0.001h + 1.60)(32.2)dh$$

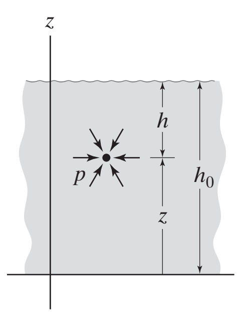
$$p = \left[(0.0161h^2 + 51.52h)\frac{\text{lb}}{\text{ft}^2}\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [0.1118(10^{-3})h^2 + 0.3578h]\ \text{psi} \quad \text{where } h \text{ is in feet.}$$ **Ans.**

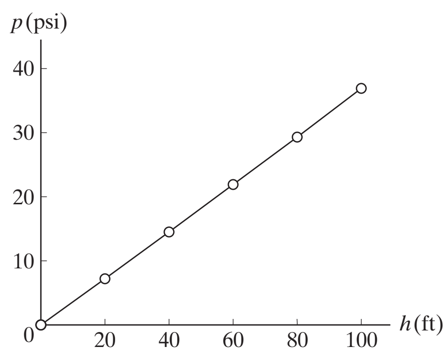
For $0 \le h \le 100$ ft,

h(ft)	0	20	40	60	80	100
p(psi)	0	7.20	14.5	21.9	29.3	36.9

The plot of p vs h is shown in Fig. *b*.



(a)



(b)

Ans:
$p = [0.112(10^{-3})h^2 + 0.358\,h]$ psi where h is in feet.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–23. In the troposphere, which extends from sea level to 11 km, it is found that the temperature decreases with altitude such that $dT/dz = -C$, where C is the constant lapse rate. If the temperature and pressure at $z = 0$ are T_0 and p_0, determine the pressure as a function of altitude.

SOLUTION

First, we must establish the relation between T, and z using $T = T_0$ at $z = 0$.

$$\int_{T_0}^{T} dT = -c\int_0^z dz$$

$$T - T_0 = -Cz$$

$$T = T_0 - Cz$$

Applying the ideal gas law,

$$p = \rho RT; \qquad \rho = \frac{p}{RT} = \frac{p}{R(T_0 - Cz)}$$

$$dp = -\gamma dz = -\rho g dz$$

$$dp = -\frac{gpdz}{R(T_0 - Cz)}$$

$$\frac{dp}{p} = \frac{-g}{R}\left(\frac{dz}{T_0 - Cz}\right)$$

Using $p = p_0$ at $z = 0$,

$$\int_{p_0}^{p}\frac{dp}{p} = \frac{-g}{R}\int_0^z \frac{dz}{T_0 - Cz}$$

$$\ln p\Big|_{p_0}^{p} = -\frac{g}{R}\left[\left(-\frac{1}{C}\right)\ln(T_0 - Cz)\right]\Bigg|_0^z$$

$$\ln\frac{p}{p_0} = \frac{g}{CR}\ln\left(\frac{T_0 - Cz}{T_0}\right)$$

$$\ln\frac{p}{p_0} = \ln\left[\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}\right]$$

$$\frac{p}{p_0} = \left(\frac{T_0 - Cz}{T_0}\right)^{g/CR}$$

$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/CR} \qquad \textbf{Ans.}$$

Ans:
$$p = p_0\left(\frac{T_0 - Cz}{T_0}\right)^{g/RC}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–24.** Determine the temperature at an elevation of $z = 15$ km. Also, what is the pressure at this elevation? Assume the stratosphere begins at $z = 11$ km (see Fig. 2–11).

SOLUTION

Within the troposphere, $0 \le z < 11.0(10^3)$ m, the temperature T_C as a function of z is $T_C = T_0 - Cz$ (the equation of the straight line shown in Fig. 2–11). From this figure, we obtain $T_C = 15°\text{C}$ at $z = 0$. Then

$$15°\text{C} = T_0 - C(0)$$

$$T_0 = 15°\text{C}$$

Also, $T_C = -56.5°\text{C}$ at $z = 11.0(10^3)$ m. Then

$$-56.5°\text{C} = 15°\text{C} - C\left[11.0(10^3)\ \text{m}\right]$$

$$C = 6.50(10^{-3})°\text{C/m}$$

Thus,

$$T_C = \left[15 - 6.50(10^{-3})z\right]°\text{C}$$

The absolute temperature is therefore

$$T = 15 - 6.50(10^{-3})z + 273 = \left[288 - 6.50(10^{-3})z\right]\ \text{K} \qquad \textbf{(1)}$$

Substitute the ideal gas law $p = \rho RT$ or $\rho = \dfrac{p}{RT}$ into $dp = -\gamma dz = -\rho g dz$,

$$dp = -\frac{p}{RT} g dz$$

$$\frac{dp}{p} = -\frac{g}{RT} dz \qquad \textbf{(2)}$$

From the table in Appendix A, the gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$. Also, $p = 101.3$ kPa at $z = 0$. Then substitute Eq. (1) into (2).

$$\int_{101.3(10^3)\ \text{Pa}}^{p} \frac{dp}{p} = -\left(\frac{9.81\ \text{m/s}^2}{286.9\ \text{J/kg}\cdot\text{K}}\right)\int_0^z \frac{dz}{288 - 6.50(10^{-3})z}$$

$$\ln p\Big|_{101.3(10^3)\ \text{Pa}}^{p} = 5.2605 \ln\left[288 - 6.50(10^{-3})z\right]\Big|_0^z$$

$$\ln\left[\frac{p}{101.3(10^3)}\right] = 5.2605 \ln\left[\frac{288 - 6.50(10^{-3})z}{288}\right]$$

$$p = 101.3(10^3)\left[\frac{288 - 6.50(10^{-3})z}{288}\right]^{5.2605}$$

***2–24. Continued**

At $z = 11.0(10^3)$ m,

$$p = 101.3(10^3)\left\{\frac{288 - [6.50(10^{-3})][11.0(10^3)]}{288}\right\}^{5.2605} = 22.58(10^3)\text{ Pa}$$

At $z = 15(10^3)$ m, it is into the stratosphere in the region $11(10^3)\text{ m} < z < 20.1(10^3)$ m, of which the temperature is constant, Fig. 2–11. Thus,

$$T_C = -56.5°\text{C} \qquad \textbf{Ans.}$$

Integrate Eq. (2) using this result and $T = -56.5°\text{C} + 273 = 216.5$ K.

$$\int_{22.58(10^3)\text{ Pa}}^{p} \frac{dp}{p} = -\left[\frac{9.81\text{ m/s}^2}{(286.9\text{ J/kg}\cdot\text{K})(216.5\text{ K})}\right]\int_{11(10^3)\text{ m}}^{z} dz$$

$$\ln p\Big|_{22.58(10^3)\text{ Pa}}^{p} = -0.1579(10^{-3})z\Big|_{11(10^3)\text{ m}}^{z}$$

$$\ln\frac{p}{22.58(10^3)} = 0.1579(10^{-3})[11(10^3) - z]$$

$$p = \left[22.58(10^3)e^{0.1579(10^{-3})[11(10^3)-z]}\right]\text{ Pa}$$

At $z = 15(10^3)$ m,

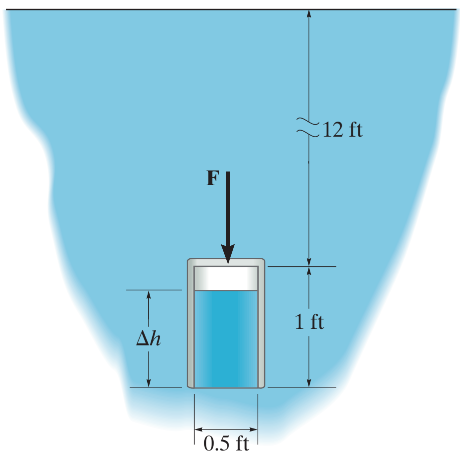
$$p = \left[22.58(10^3)e^{0.1579(10^{-3})[11(10^3)-15(10^3)]}\right]\text{ Pa}$$

$$= 12.00(10^3)\text{ Pa} = 12.0\text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$T_C = -56.5°\text{C}$
$p = 12.0$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–25. A heavy cylindrical glass is inverted and then placed down at the bottom of a 12–ft-deep swimming pool. Determine the height Δh of water within the glass when it is at the bottom. Assume the air in the glass remains at the same temperature as the atmosphere. *Hint:* Account for the change in volume of air in the glass due to the pressure change. The atmospheric pressure is $p_{atm} = 14.7$ psi.



SOLUTION

When submerged, the density and hence the volume of the air change due to pressure changes. Applying the ideal gas law,

$$p = \rho R T$$

However, $\rho = \frac{m}{V}$. Then

$$p = \frac{m}{V} R T$$
$$pV = mRT$$

Since the mass and temperature of the air are constant, mRT is also constant. Thus,

$$p_1 V_1 = p_2 V_2 \qquad \textbf{(1)}$$

When $p_1 = p_{atm} = \left(14.7\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 = 2116.8\ \frac{\text{lb}}{\text{ft}^2}$, $V_1 = \pi(0.25\ \text{ft})^2(1\ \text{ft}) = 0.0625\pi\ \text{ft}^3$. When submerged, the water rises to the height of Δh. Thus,

$$p_2 = p_{atm} + \gamma_w h = 2116.8\ \frac{\text{lb}}{\text{ft}^2} + \left(62.4\ \frac{\text{lb}}{\text{ft}^3}\right)(13\ \text{ft} - \Delta h)$$
$$= (2928 - 62.4\Delta h)\ \frac{\text{lb}}{\text{ft}^2}$$
$$V_2 = \pi(0.25\ \text{ft})^2(1\ \text{ft} - \Delta h) = [0.0625\pi(1 - \Delta h)]\ \text{ft}^3$$

Substitute these values into Eq. (1).

$$\left(2116.8\ \frac{\text{lb}}{\text{ft}^2}\right)(0.0625\pi\ \text{ft}^3) = \left[(2928 - 62.4\Delta h)\ \frac{\text{lb}}{\text{ft}^2}\right][0.0625\pi(1 - \Delta h)]\ \text{ft}^3$$
$$2116.8 = (2928 - 62.4\Delta h)(1 - \Delta h)$$
$$62.4\Delta h^2 - 2990.4\Delta h + 811.2 = 0$$

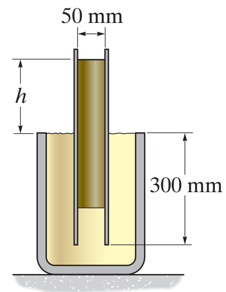
Choose $\Delta h < 1$ ft,

$$\Delta h = 0.2728\ \text{ft}$$
$$= 3.27\ \text{in.} \qquad \textbf{Ans.}$$

Ans:
$\Delta h = 3.27$ in.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–26. The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter pipe is inserted within it to a depth of 300 mm. If $0.00075\ \text{m}^3$ of kerosene is then poured into the pipe, causing the displaced glycerin to overflow, determine the height h to which the kerosene rises from the top of the glycerin.



SOLUTION

The height of the kerosene column in the pipe, Fig. *a*, is

$$h_{ke} = \frac{V_{ke}}{\pi r^2} = \frac{0.00075\ \text{m}^3}{\pi(0.025\ \text{m})^2} = \left(\frac{1.2}{\pi}\right)\text{m}$$

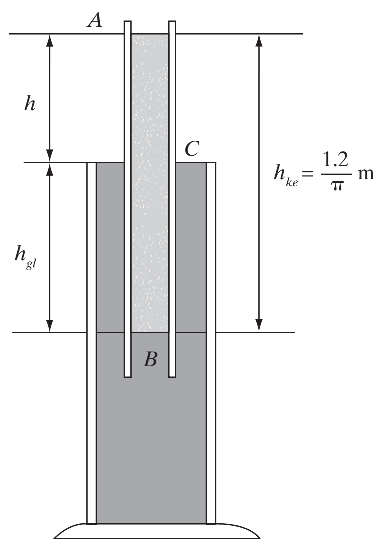
From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$. Writing the manometer equation from $A \rightarrow B \rightarrow C$ by referring to Fig. *a*,

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{gl} = \left(\frac{\rho_{ke}}{\rho_{gl}}\right)h_{ke} = \left(\frac{814\ \text{kg/m}^3}{1260\ \text{kg/m}^3}\right)\left(\frac{1.2}{\pi}\ \text{m}\right) = 0.2468\ \text{m}$$

Thus,

$$h = h_{ke} - h_{gl} = \frac{1.2}{\pi}\ \text{m} - 0.2468\ \text{m} = 0.1352\ \text{m} = 135\ \text{mm} \qquad \textbf{Ans.}$$

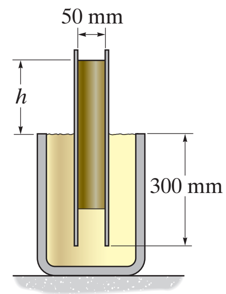


(a)

Ans:
$h = 135\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–27. The 150-mm-diameter container is filled to the top with glycerin, and a 50-mm-diameter pipe is inserted within it to a depth of 300 mm. Determine the maximum volume of kerosene that can be poured into the pipe while causing the displaced glycerin to overflow, so the kerosene does not come out from the bottom end. How high h does the kerosene rise above the glycerin?



SOLUTION

From Appendix A, $\rho_{ke} = 814\ \text{kg/m}^3$ and $\rho_{gl} = 1260\ \text{kg/m}^3$. The kerosene is required to heat the bottom of the tube as shown in Fig. *a*. Write the manometer equation from $A \rightarrow B \rightarrow C$.

$$p_{\text{atm}} + \rho_{ke} g h_{ke} - \rho_{gl} g h_{gl} = p_{\text{atm}}$$

$$h_{ke} = \frac{\rho_{gl}}{\rho_{ke}} h_{gl}$$

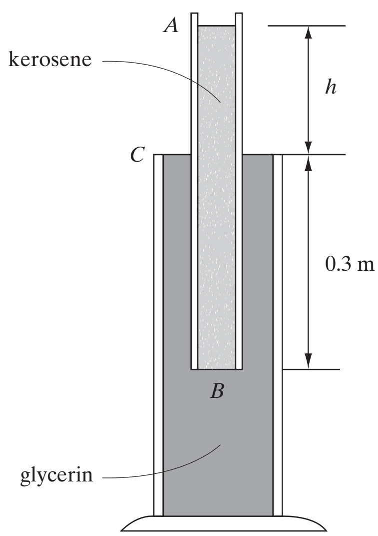
Here, $h_{ke} = (h + 0.3)$ m and $h_{gl} = 0.3$ m. Then

$$(h + 0.3)\ \text{m} = \left(\frac{1260\ \text{kg/m}^3}{814\ \text{kg/m}^3}\right)(0.3\ \text{m})$$

$$h_{ke} = 0.1644\ \text{m} = 164\ \text{mm} \qquad \textbf{Ans.}$$

Thus, the volume of the kerosene in the pipe is

$$V_{ke} = \pi r^2 h_{ke} = \pi(0.025\ \text{m})^2(0.1644\ \text{m} + 0.3\ \text{m}) = 0.9118(10^{-3})\ \text{m}^3$$
$$= 0.912(10^{-3})\ \text{m}^3 \qquad \textbf{Ans.}$$

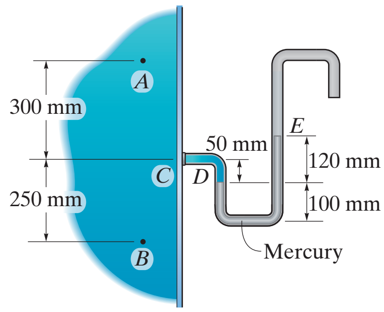


(a)

Ans:
$h_{ke} = 164\ \text{mm}, V_{ke} = 0.912(10^{-3})\ \text{m}^3$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–28.** Butyl carbitol, used in the production of plastics, is stored in a tank having the U-tube manometer. If the U-tube is filled with mercury to level E, determine the pressure in the tank at point B. Take $S_{\text{Hg}} = 13.55$, and $S_{bc} = 0.957$.



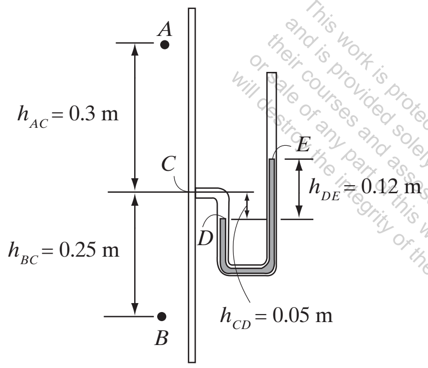
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$p_E + \rho_{\text{Hg}} g h_{DE} + \rho_{bc} g(-h_{CD} + h_{BC}) = p_B$$

$$0 + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.120\ \text{m}) + 0.957(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-0.05\ \text{m} + 0.25\ \text{m}) = p_B$$

$$p_B = 17.83(10^3)\ \text{Pa} = 17.8\ \text{kPa} \qquad \textbf{Ans.}$$

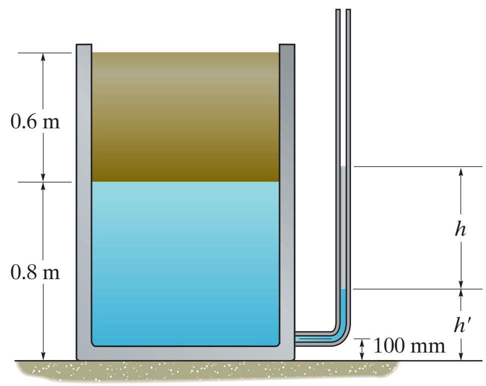


(a)

Ans:
$p_B = 17.8\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–29. Determine the level h' of water in the tube if the depths of oil and water in the tank are 0.6 m and 0.8 m, respectively, and the height of mercury in the tube is $h = 0.08$ m. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, and $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION

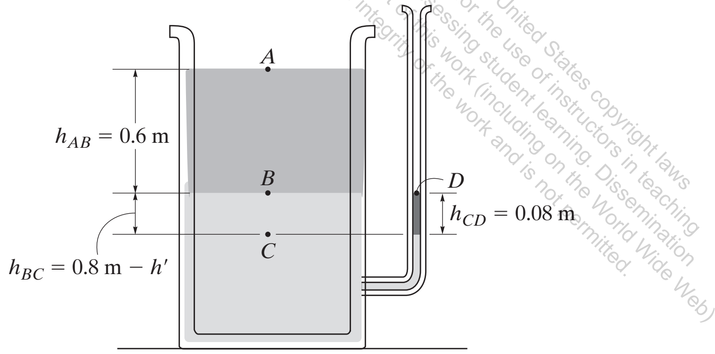
Referring to Fig. *a*, $h_{AB} = 0.6$ m, $h_{BC} = 0.8 - h'$ and $h_{CD} = h = 0.08$ m. Then the manometer rule gives

$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points A and D are exposed to the atmosphere.

$$0 + (900\ \text{kg/m}^3)(g)(0.6\ \text{m}) + (1000\ \text{kg/m}^3)(g)(0.8\ \text{m} - h')$$
$$-(13550\ \text{kg/m}^3)(g)(0.08\ \text{m}) = 0$$
$$h' = 0.256\ \text{m} = 256\ \text{mm} \qquad \textbf{Ans.}$$

Note: Since $0.1\ \text{m} < h' < 0.8\ \text{m}$, the solution is **OK**!

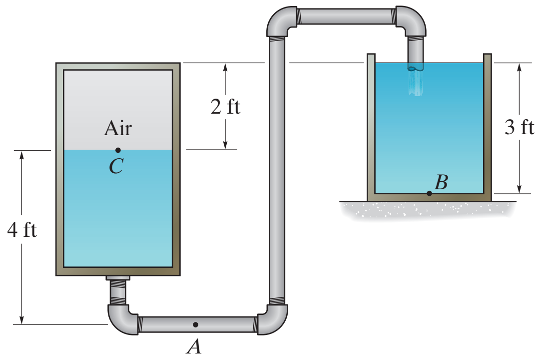


(a)

Ans:
$h' = 256$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–30. Determine the pressures at points A and B. The containers are filled with water.



SOLUTION

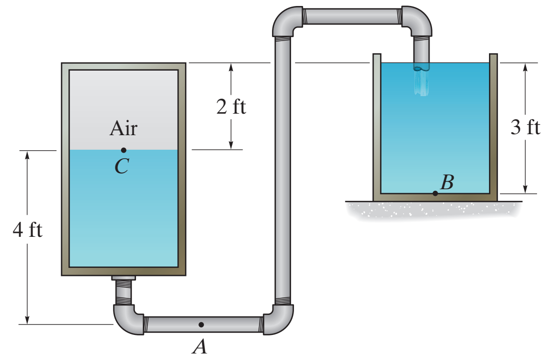
$$p_A = \gamma_A h_A = (62.4\ \text{lb/ft}^3)(2\text{ft} + 4\text{ft}) = \left(374.4\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 2.60\ \text{psi} \quad \textbf{Ans.}$$

$$p_B = \gamma_B h_B = (62.4\ \text{lb/ft}^3)(3\ \text{ft}) = \left(187.2\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.30\ \text{psi} \quad \textbf{Ans.}$$

Ans:
$p_A = 2.60$ psi, $p_B = 1.30$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–31. Determine the pressure at point *C*. The containers are filled with water.



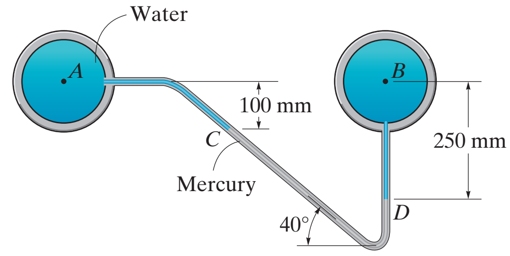
SOLUTION

$$p_C = \gamma h_C = (62.4\ \text{lb/ft}^3)(2\ \text{ft}) = \left(124.8\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right) = 0.870\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p_C = 0.870$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–32.** Determine the difference in pressure $p_B - p_A$ between the centers A and B of the pipes, which are filled with water. The mercury in the inclined-tube manometer has the level shown. Take $S_{Hg} = 13.55$.



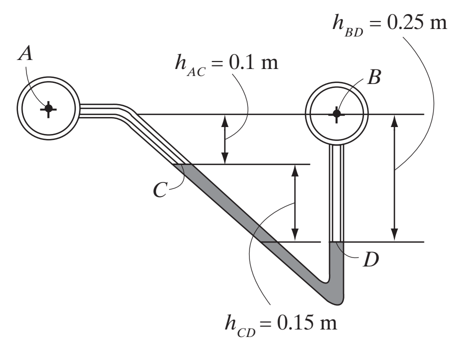
SOLUTION

Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_w g h_{AC} + \rho_{Hg} g h_{CD} - \rho_w g h_{DB} = p_B$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m}) + 13.55(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.15\ \text{m})$$
$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.250\ \text{m}) = p_B$$

$$p_B - p_A = 18.47(10^3)\ \text{Pa} = 18.5\ \text{kPa} \qquad \textbf{Ans.}$$

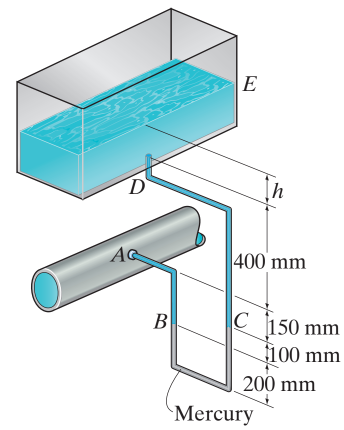


(a)

Ans:
$p_B - p_A = 18.5\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–33. Water in the reservoir is used to control the water pressure in the pipe at A. If $h = 200$ mm, determine this pressure when the mercury is at the elevation shown. Take $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



SOLUTION

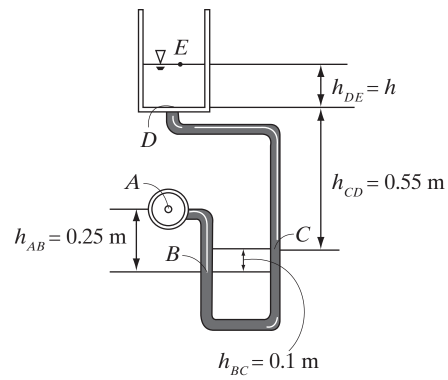
Referring to Fig. *a* with $h = 0.2$ m, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} - \rho_w g (h_{CD} + h_{DE}) = p_E$$

$$p_A + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m}) - (13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m})$$

$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m} + 0.2\text{m}) = 0$$

$$p_A = 18.20(10^3)\ \text{Pa} = 18.2\ \text{kPa}$$ **Ans.**

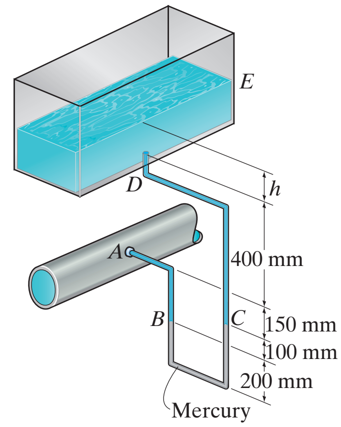


(a)

Ans:
$p_A = 18.2$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–34. If the water pressure in the pipe at A is to be 25 kPa, determine the required height h of water in the reservoir. Mercury in the pipe has the elevation shown. Take $\rho_{\text{Hg}} = 13\ 550\ \text{kg/m}^3$. Neglect the diameter of the pipe.



SOLUTION

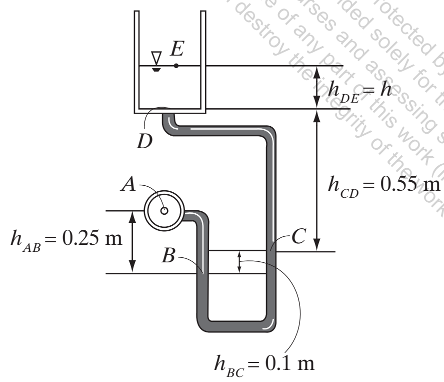
Referring to Fig. a, the manometer rule gives

$$p_A + \rho_w g h_{AB} - \rho_{\text{Hg}} g h_{BC} - \rho_w g (h_{CD} + h_{DE}) = p_E$$

$$25(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m}) - (13\ 550\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.1\ \text{m})$$

$$- (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.550\ \text{m} + h) = 0$$

$$h = 0.8934\ \text{m} = 893\ \text{mm} \qquad \textbf{Ans.}$$

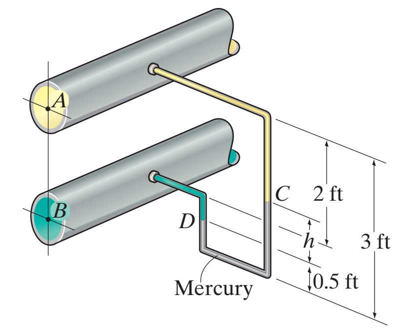


(a)

Ans:
$h = 893$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–35. A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. Determine the pressure in pipe A if the pressure in pipe B is 15 psi. The mercury in the manometer is in the position shown, where $h = 1$ ft. Neglect the diameter of the pipes. Take $S_c = 0.953$, $S_{Hg} = 13.55$, and $S_{el} = 1.03$.



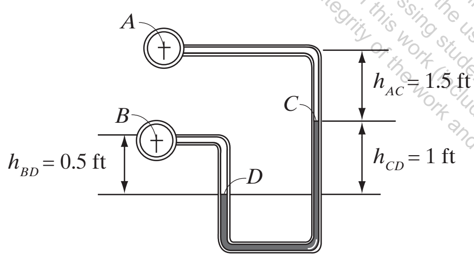
SOLUTION

Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c g h_{AC} + \gamma_{Hg} h_{CD} - \gamma_{el} g h_{BD} = p_B$$

$$p_A + 0.953(62.4\ \text{lb/ft}^3)(1.5\ \text{ft}) + (13.55)(62.4\ \text{lb/ft}^3)(1\ \text{ft}) - (1.03)(62.4\ \text{lb/ft}^3)(0.5\ \text{ft}) = \left(15\ \frac{\text{lb}}{\text{in}^2}\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$p_A = 1257.42\ \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 8.73\ \text{psi} \qquad \textbf{Ans.}$$

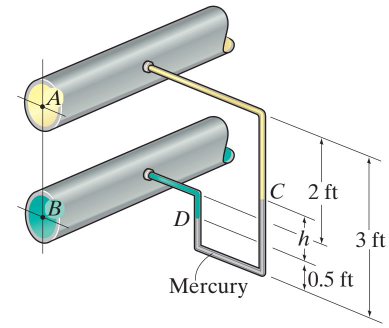


(a)

Ans:
$p_A = 8.73$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–36.** A solvent used for plastics manufacturing consists of cyclohexanol in pipe A and ethyl lactate in pipe B that are being transported to a mixing tank. If the pressure in pipe A is 18 psi, determine the height h of the mercury in the manometer so that a pressure of 25 psi is developed in pipe B. Neglect the diameter of the pipes. Take $S_c = 0.953$, $S_{Hg} = 13.55$, and $S_{el} = 1.03$.



SOLUTION

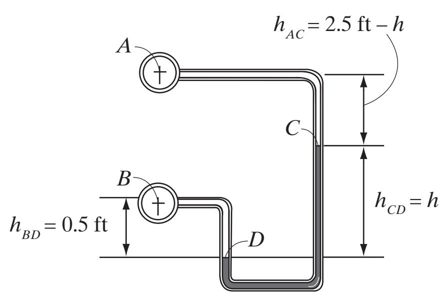
Referring to Fig. a, the manometer rule gives

$$p_A + \gamma_c h_{AC} + \gamma_{Hg} h_{CD} - \gamma_{el} h_{BD} = p_B$$

$$\frac{18\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 + 0.953\left(62.4\ \text{lb/ft}^3\right)(2.5\ \text{ft} - h) + 13.55\left(62.4\ \text{lb/ft}^3\right)(h)$$

$$-(1.03)\left(62.4\ \text{lb/ft}^3\right)(0.5\ \text{ft}) = \frac{25\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2$$

$$h = 1.134\ \text{ft} = 1.13\ \text{ft}$$ **Ans.**

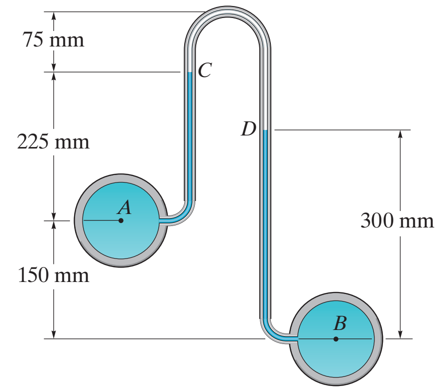


(a)

Ans:
$h = 1.13\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–37. The inverted U-tube manometer is used to measure the difference in pressure between water flowing in the pipes at A and B. If the top segment is filled with air, and the water levels in each segment are as indicated, determine the pressure difference between A and B. $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Notice that the pressure throughout the air in the tube is constant. Referring to Fig. *a*,

$$p_A = (p_w)_1 + p_a = \rho_w g(h_w)_1 + p_a$$

And

$$p_B = (p_w)_2 + p_a = \rho_w g(h_w)_2 + p_a$$

Therefore,

$$p_B - p_A = [\rho_w g(h_w)_2 + p_a] - [\rho_w g(h_w)_1 + p_a]$$

$$= \rho_w g[(h_w)_2 - (h_w)_1]$$

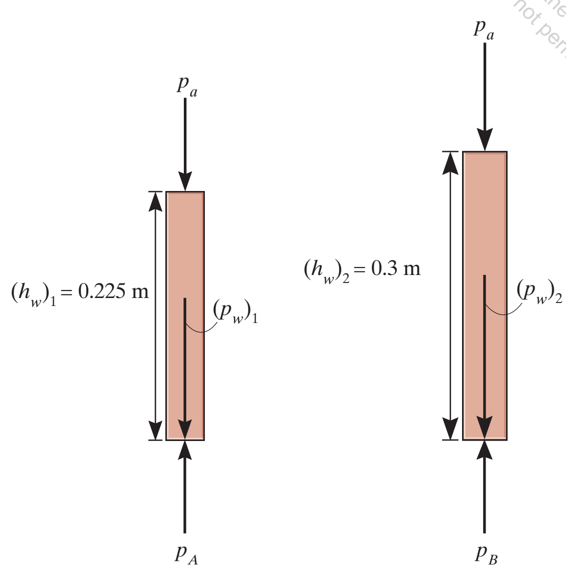
$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m} - 0.225\ \text{m})$$

$$= 735.75\ \text{Pa} = 736\ \text{Pa} \qquad \textbf{Ans.}$$

Also, using the manometer equation,

$$p_A - \rho_w g h_{AC} + \rho_w g h_{DB} = p_B$$

$$p_B - p_A = \rho_w g[h_{DB} - h_{AC}]$$

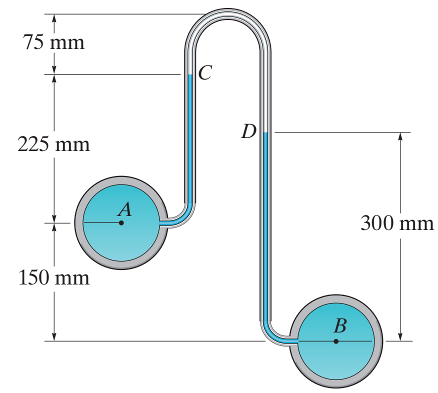


(a)

Ans:
$p_B - p_A = 736\ \text{Pa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–38. Solve Prob. 2–37 if the top segment is filled with an oil for which $\rho_o = 800\ \text{kg/m}^3$.



SOLUTION

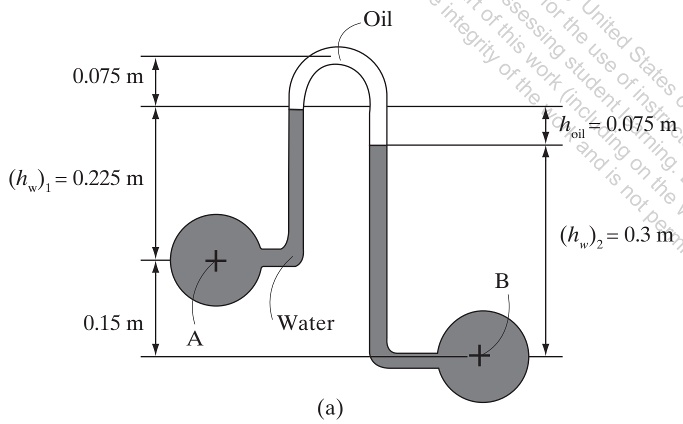
Referring to Fig. *a*, write the manometer equation starting at *A* and ending at *B*.

$$p_A - \rho_w g(h_w)_1 + \rho_{oil} g h_{oil} + \rho_w g(h_w)_2 = p_B$$

$$p_B - p_A = \rho_w g[(h_w)_2 - (h_w)_1] + \rho_{oil} g h_{oil}$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\ \text{m} - 0.225\ \text{m}) + (800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.075\ \text{m})$$

$$= 1.324(10^3)\ \text{Pa} = 1.32\ \text{kPa}$$ **Ans.**

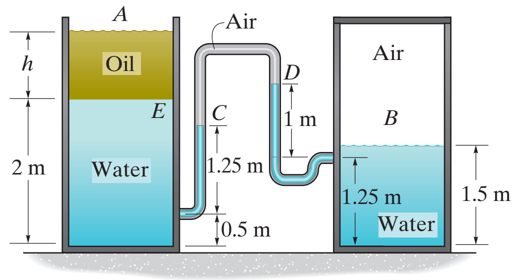


(a)

Ans:
$p_B - p_A = 1.32\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–39. The two tanks A and B are connected using a manometer. If waste oil is poured into tank A to a depth of $h = 0.6$ m, determine the pressure of the entrapped air in tank B. Air is also trapped in line CD as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

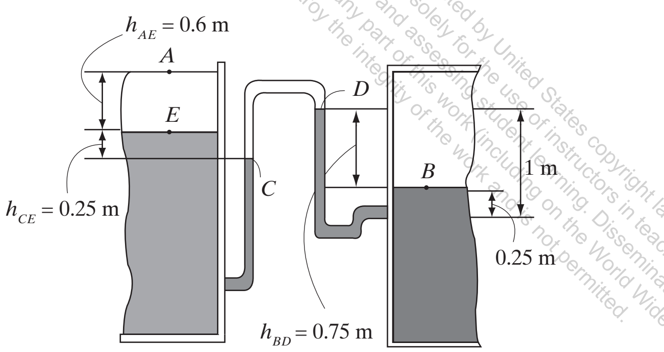
Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.6\ \text{m}) + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.25\ \text{m})$$

$$+ \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m}) = p_B$$

$$p_B = 15.11\left(10^3\right)\ \text{Pa} = 15.1\ \text{kPa} \qquad \textbf{Ans.}$$

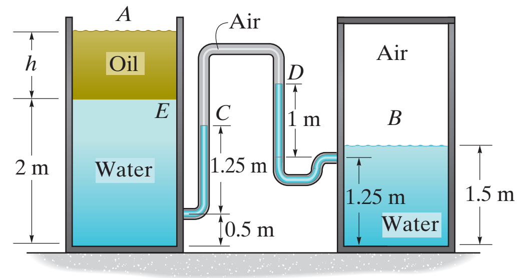


(a)

Ans:
$p_B = 15.1$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–40.** The two tanks *A* and *B* are connected using a manometer. If waste oil is poured into tank *A* to a depth of $h = 1.25$ m, determine the pressure of the trapped air in tank *B*. Air is also trapped in line *CD* as shown. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

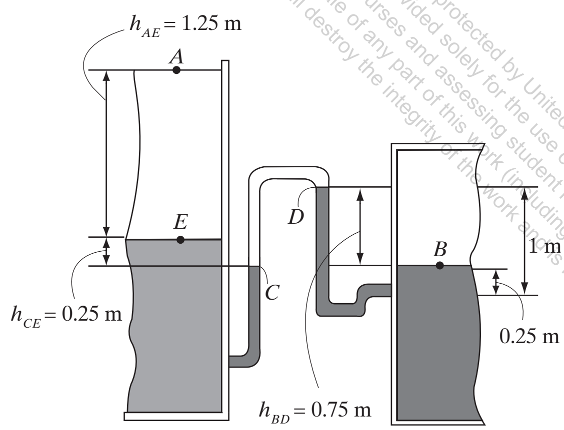
Referring to Fig. *a*, the manometer rule gives

$$p_A + \rho_o g h_{AE} + \rho_w g h_{CE} + \rho_w g h_{BD} = p_B$$

$$0 + (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.25\ \text{m}) + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25\ \text{m})$$

$$+ (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.75\ \text{m}) = p_B$$

$$p_B = 20.846(10^3)\ \text{Pa} = 20.8\ \text{kPa} \qquad \textbf{Ans.}$$

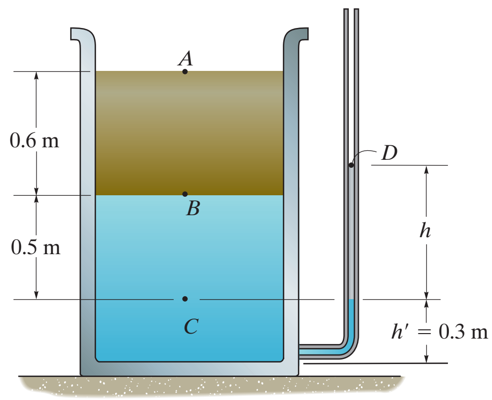


(a)

Ans:
$p_B = 20.8$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–41. Determine the height h of the mercury in the tube if the level of water in the tube is $h' = 0.3$ m and the depths of the oil and water in the tank are 0.6 and 0.5 m, respectively. Take $\rho_o = 900\ \text{kg/m}^3$, $\rho_w = 1000\ \text{kg/m}^3$, and $\rho_{Hg} = 13\,550\ \text{kg/m}^3$.



SOLUTION

Referring to Fig. a, $h_{AB} = 0.6$ m, $h_{BC} = 0.8\text{ m} - 0.3\text{ m} = 0.5$ m and $h_{CD} = h$. Then the manometer rule gives

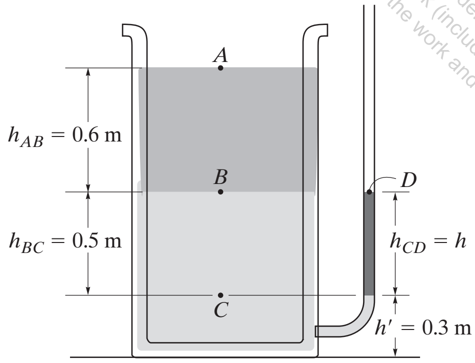
$$p_A + \rho_o g h_{AB} + \rho_w g h_{BC} - \rho_{Hg} g h_{CD} = p_D$$

Here, $p_A = p_D = 0$, since points A and D are exposed to the atmosphere.

$$0 + (900\ \text{kg/m}^3)(g)(0.6\text{ m}) + (1000\ \text{kg/m}^3)(g)(0.5\text{ m})$$

$$-(13550\ \text{kg/m}^3)(g)(h) = 0$$

$$h = 0.07675\text{ m} = 76.8\text{ mm}$$ **Ans.**

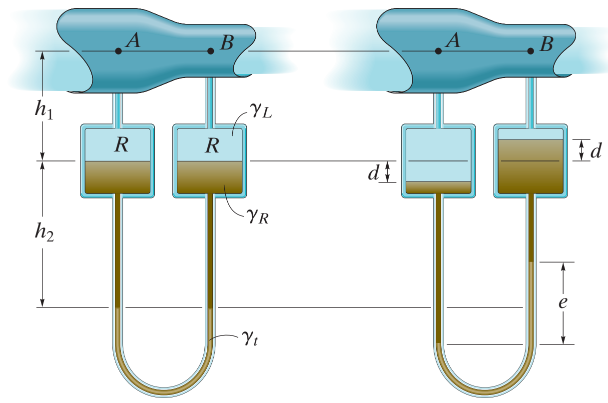


(a)

Ans:
$h = 76.8$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–42. The *micro-manometer* is used to measure small differences in pressure. The reservoirs R and upper portion of the lower tubes are filled with a liquid having a specific weight of γ_R, whereas the lower portion is filled with a liquid having a specific weight of γ_t, Fig. (a). When the liquid flows through the venturi meter, the levels of the liquids with respect to the original levels are shown in Fig. (b). If the cross-sectional area of each reservoir is A_R and the cross-sectional area of the U-tube is A_t, determine the pressure difference $p_A - p_B$. The liquid in the venturi meter has a specific weight of γ_L.



(a) (b)

SOLUTION

Write the manometer equation starting at A and ending at B, Fig. a.

$$p_A + \gamma_L(h_1 + d) + \gamma_R\left(h_2 - d + \frac{e}{2}\right) - \gamma_t e$$

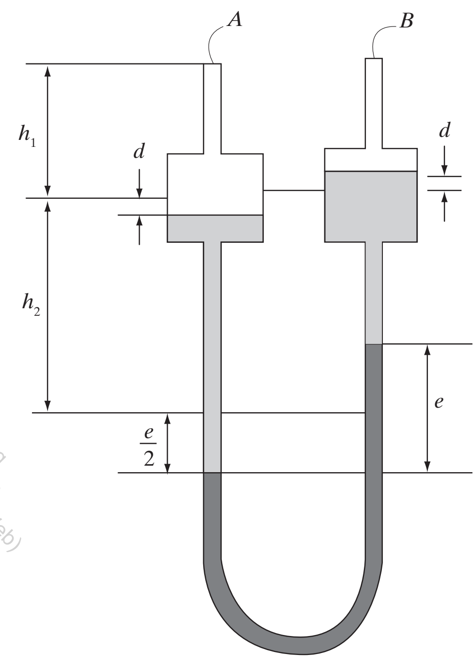
$$-\gamma_R\left(h_2 - \frac{e}{2} + d\right) - \gamma_L(h_1 - d) = p_B$$

$$p_A - p_B = 2\gamma_R d - 2\gamma_L d + \gamma_t e - \gamma_R e \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter into the left tube,

$$A_R d = A_t\left(\frac{e}{2}\right)$$

$$d = \left(\frac{A_t}{2A_R}\right)e$$



(a)

Substitute this result into Eq. (1).

$$p_A - p_B = 2\gamma_R\left(\frac{A_t}{2A_R}\right)e - 2\gamma_L\left(\frac{A_t}{2A_R}\right)e + \gamma_t e - \gamma_R e$$

$$= e\left[\left(\frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L + \gamma_t - \gamma_R\right]$$

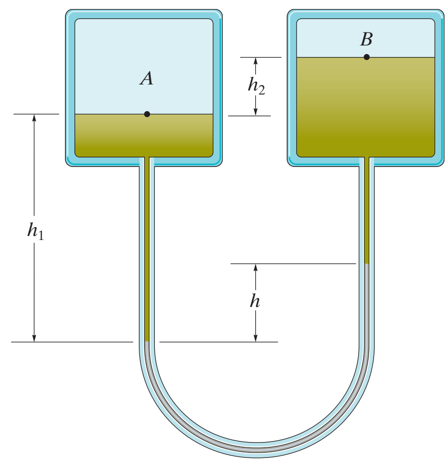
$$= e\left[\gamma_t - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L\right] \qquad \textbf{Ans.}$$

Ans:

$$p_A - p_B = e\left[\gamma_t - \left(1 - \frac{A_t}{A_R}\right)\gamma_R - \left(\frac{A_t}{A_R}\right)\gamma_L\right]$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–43. The Morgan Company manufactures a micro-manometer that works on the principles shown. Here there are two reservoirs filled with kerosene, each having a cross-sectional area of 300 mm². The connecting tube has a cross-sectional area of 15 mm² and contains mercury. Determine h if the pressure difference $p_A = p_B = 40\ \text{Pa}$. Take $\rho_{\text{Hg}} = 13\,550\ \text{kg/m}^3$, $\rho_{ke} = 814\ \text{kg/m}^3$. *Hint*: Both h_1 and h_2 can be eliminated from the analysis.



SOLUTION

Referring to Fig. *a,* write the manometer equation starting at A and ending at B.

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h + h_2) = p_B$$

$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g h_2 \qquad \textbf{(1)}$$

Since the same amount of liquid leaving the left reservoir will enter the left tube,

$$A_R = \left(\frac{h_2}{2}\right) = A_t\left(\frac{h}{2}\right)$$

$$h_2 = \left(\frac{A_t}{A_R}\right)h$$

Substitute this result into Eq. (1).

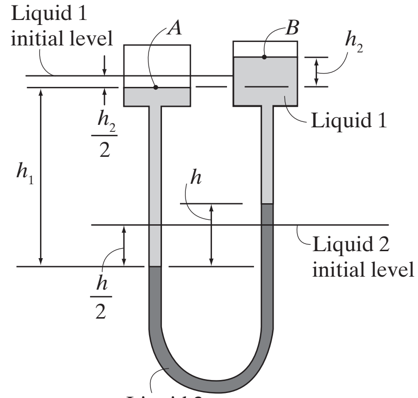
$$p_A - p_B = \rho_2 g h - \rho_1 g h + \rho_1 g\left(\frac{A_t}{A_R}\right)h$$

$$p_A - p_B = h\left[\rho_2 g - \left(1 - \frac{A_t}{A_R}\right)\rho_1 g\right] \qquad \textbf{(2)}$$

When $\rho_1 = \rho_{ke} = 814\ \text{kg/m}^3$, $\rho_2 = \rho_{Hg} = 13\,550\ \text{kg/m}^3$ and $p_A - p_B = 40\ \text{Pa}$,

$$40\ \text{N/m}^2 = h\left[(13\,550\ \text{kg/m}^3)(9.81\ \text{m/s}^2) - \left(1 - \frac{15}{300}\right)(814\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\right]$$

$$h = 0.3191(10^{-3})\ \text{m} = 0.319\ \text{mm} \qquad \textbf{Ans.}$$

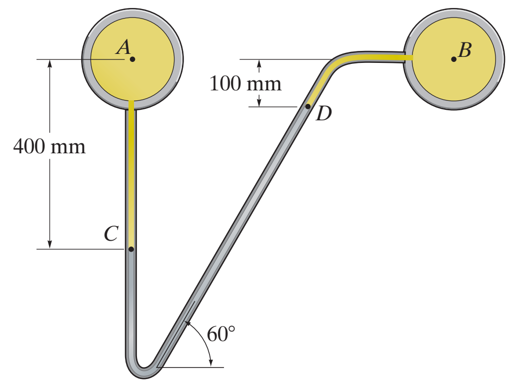


(a)

Ans:
$h = 0.319\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000 \text{ kg/m}^3$ and its specific weight to be $\gamma_w = 62.4 \text{ lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–44.** Determine the difference in pressure $p_A - p_B$ between the centers A and B of the closed pipes, which are filled with kerosene. The mercury in the inclined-tube manometer has the level shown. Take $S_{Hg} = 13.55$ and $S_k = 0.82$.



SOLUTION

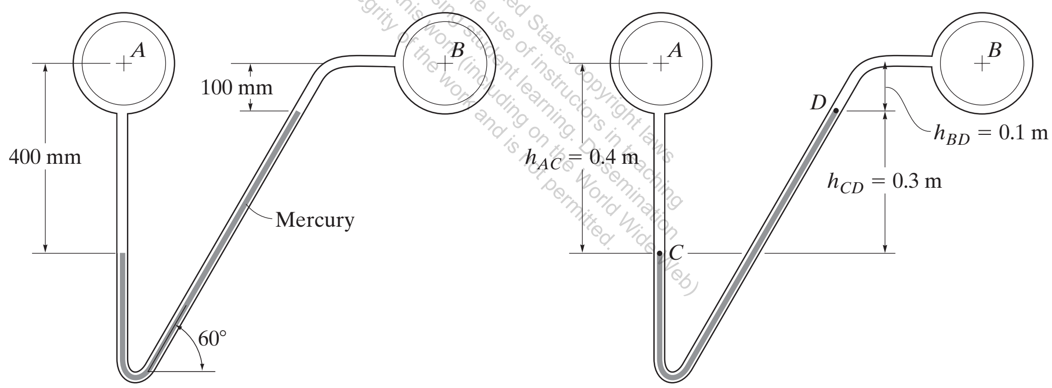
Referring to Fig. a, $h_{AC} = 0.4$ m, $h_{CD} = 0.4 \text{ m} - 0.1 \text{ m} = 0.3$ m and $h_{BD} = 0.1$ m. Then the manometer rule gives

$$p_A + \rho_k g h_{AC} - \rho_{Hg} g h_{CD} - \rho_k g h_{BD} = p_B$$

$$p_A + 0.82(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.4 \text{ m}) - 13.55(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \text{ m})$$

$$-0.82(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m}) = p_B$$

$$p_A - p_B = 37.46(10^3) \text{ Pa} = 37.5 \text{ kPa}$$ **Ans.**

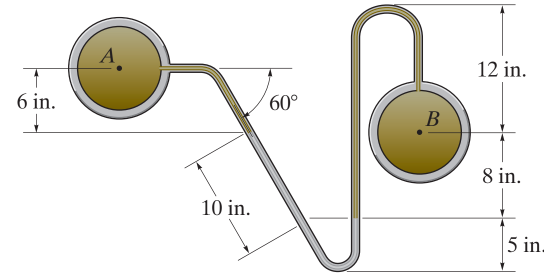


(a)

Ans:
$p_A - p_B = 37.5$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–45. The pipes at A and B contain oil and the inclined-tube manometer is filled with oil and mercury. Determine the pressure difference between A and B. Take $\rho_o = 1.70\ \text{slug/ft}^3$ and $\rho_{Hg} = 26.3\ \text{slug/ft}^3$.



SOLUTION

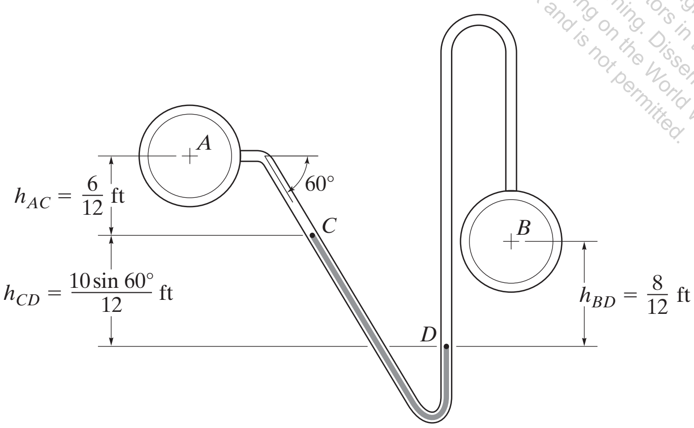
Referring to Fig. a, $h_{AC} = \frac{6}{12}$ ft, $h_{CD} = \frac{10 \sin 60°}{12}$ ft and $h_{BD} = \frac{8}{12}$ ft. Then the manometer rule gives

$$p_A + \rho_o g h_{AC} + \rho_{Hg} g h_{CD} - \rho_o g h_{BD} = p_B$$

$$p_A + \rho_o g (h_{AC} - h_{BD}) + \rho_{Hg} g h_{CD} = p_B$$

$$p_A + (1.70\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)\left(\frac{6}{12}\ \text{ft} - \frac{8}{12}\ \text{ft}\right) + (26.3\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)\left(\frac{10 \sin 60°}{12}\ \text{ft}\right) = p_B$$

$$p_B - p_A = \left(602.05\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 4.18\ \text{psi} \qquad \textbf{Ans.}$$

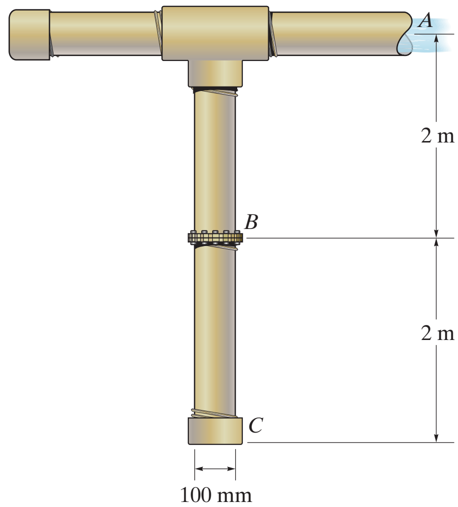


(a)

Ans:
$p_B - p_A = 4.18$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–46. The vertical pipe segment has an inner diameter of 100 mm and is capped at its end and suspended from the horizontal pipe as shown. If it is filled with water and the pressure at A is 80 kPa, determine the resultant force that must be resisted by the bolts at B in order to hold the flanges together. Neglect the weight of the pipe but not the water within it.

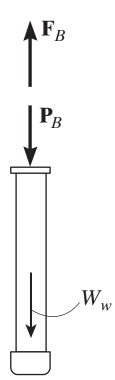


SOLUTION

The forces acting on segment BC of the pipe are indicated on its free-body diagram, Fig. a. Here, $\mathbf{F}_B$ is the force that must be resisted by the bolt, W_w is the weight of the water in segment BC of the pipe, and $\mathbf{P}_B$ is the resultant force of pressure acting on the cross section at B.

$$+\uparrow \Sigma F_y = 0; \qquad F_B - W_w - p_B A_B = 0$$

$$F_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})(\pi)(0.05\ \text{m})^2$$
$$+ \left[80\left(10^3\right)\text{N/m}^2 + 1000\ \text{kg/m}^3\left(9.81\ \text{m/s}^2\right)(2\ \text{m})\right]\pi(0.05\ \text{m})^2$$
$$= 937\ \text{N} \qquad \textbf{Ans.}$$

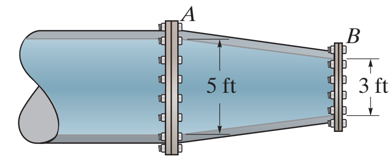


(a)

Ans:
$F_B = 937\ \text{N}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–47. Nitrogen in the chamber is at a pressure of 60 psi. Determine the total force the bolts at joints A and B must resist to maintain the pressure. There is a cover plate at B having a diameter of 3 ft.



SOLUTION

The force that must be resisted by the bolts at A and B can be obtained by considering the free-body diagrams in Figs. a and b, respectively. For the bolts at B, Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad p_B A_B - F_B = 0$$

$$F_B = p_B A_B = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(1.5\ \text{ft})^2\right]$$

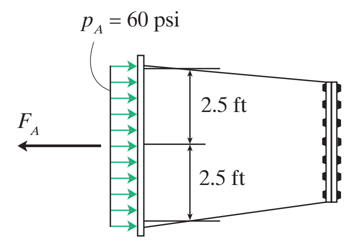
$$= 61\,073\ \text{lb} = 61.1\ \text{kip} \qquad \textbf{Ans.}$$

For the bolts at A, Fig. a,

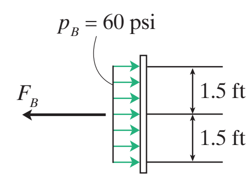
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad p_A A_A - F_A = 0$$

$$F_A = p_A A_A = \left(60\ \text{lb/in}^2\right)\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)^2 \left[(\pi)(2.5\ \text{ft})^2\right]$$

$$= 169\,646\ \text{lb} = 170\ \text{kip} \qquad \textbf{Ans.}$$



(a)

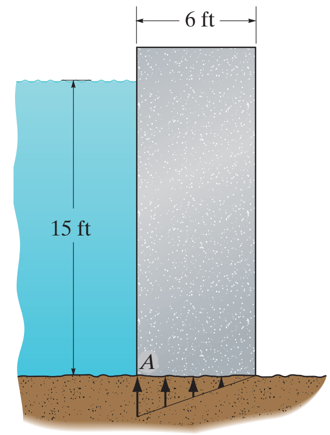


(b)

Ans:
$F_B = 61.1\ \text{kip}, F_A = 170\ \text{kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–48.** Seepage is assumed to occur beneath the concrete wall, producing a linear distribution of hydrostatic pressure as shown. Determine the resultant force on a 1-ft wide portion of the wall and its location, measured to the right and upward from point A.



SOLUTION

Since the width of the dam considered is $b = 1$ ft, the intensity of the distributed load at the base A of the dam is

$$w_A = \gamma_w h_A b = (62.4\ \text{lb/ft}^3)(15\ \text{ft})(1\ \text{ft}) = 936\ \text{lb/ft}$$

Then, the resultant forces of the triangular distributed loads shown in Fig. *a* are

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}(936\ \text{lb/ft})(15\ \text{ft}) = 7020\ \text{lb}$$

$$F_v = \frac{1}{2} w_A L = \frac{1}{2}(936\ \text{lb/ft})(6\ \text{ft}) = 2808\ \text{lb}$$

Then, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2}$$

$$= \sqrt{(7020\ \text{lb})^2 + (2808\ \text{lb})^2} = 7560.77\ \text{lb} = 7.56(10^3)\ \text{lb} \qquad \textbf{Ans.}$$

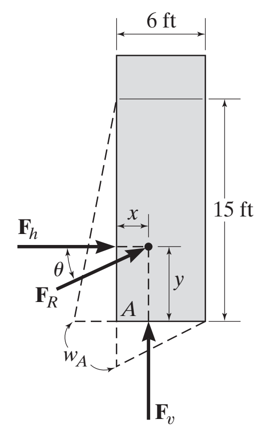
and its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{2808\ \text{lb}}{7020\ \text{lb}}\right) = 21.8^\circ \qquad \textbf{Ans.}$$

The location of the resultant force from A is

$$x = \frac{1}{3}(6\ \text{ft}) = 2\ \text{ft} \qquad \textbf{Ans.}$$

$$y = \frac{1}{3}(15\ \text{ft}) = 5\ \text{ft} \qquad \textbf{Ans.}$$

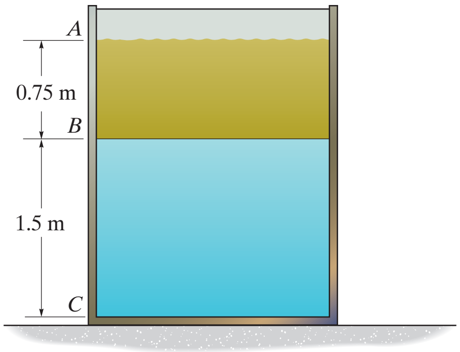


(a)

Ans:
$F_R = 7.56(10^3)$ lb
$\theta = 21.8^\circ$
$x = 2$ ft
$y = 5$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–49. The storage tank contains oil and water acting at the depths shown. Determine the resultant force that both of these liquids exert on the side *ABC* of the tank if the side has a width of 1.25 m. Also, determine the location of this resultant, measured from the top surface of the oil. Take $\rho_o = 900\ \text{kg/m}^3$.



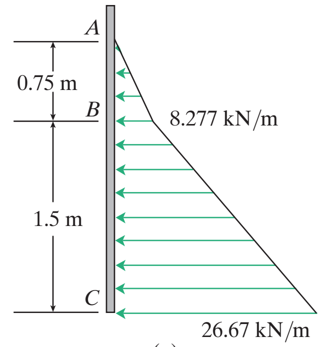
SOLUTION

Loading. Since the side of the tank has a constant width, then the intensities of the distributed loading at *B* and *C*, Fig. 2–28*b*, are

$$w_B = \rho_o g h_{AB} b = \left(900\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})(1.25\ \text{m}) = 8.277\ \text{kN/m}$$

$$w_C = w_B + \rho_w g h_{BC} b = 8.277\ \text{kN/m} + \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.5\ \text{m})(1.25\ \text{m})$$

$$= 26.77\ \text{kN/m}$$



(a)

Resultant Force. The resultant force can be determined by adding the shaded triangular and rectangular areas in Fig. 2–28*c*. The resultant force is therefore

$$F_R = F_1 + F_2 + F_3$$

$$= \frac{1}{2}(0.75\ \text{m})(8.277\ \text{kN/m}) + (1.5\ \text{m})(8.277\ \text{kN/m}) + \frac{1}{2}(1.5\ \text{m})(18.39\ \text{kN/m})$$

$$= 3.104\ \text{kN} + 12.42\ \text{kN} + 13.80\ \text{kN} = 29.32\ \text{kN} = 29.3\ \text{kN} \qquad \textbf{Ans.}$$

As shown, each of these three parallel resultants acts through the centroid of its respective area.

$$y_1 = \frac{2}{3}(0.75\ \text{m}) = 0.5\ \text{m}$$

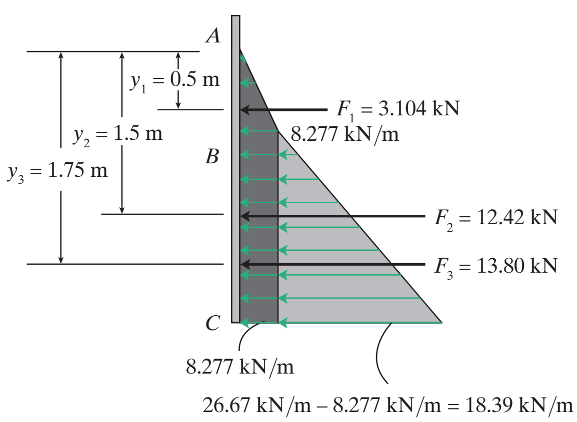
$$y_2 = 0.75\ \text{m} + \frac{1}{2}(1.5\ \text{m}) = 1.5\ \text{m}$$

$$y_3 = 0.75\ \text{m} + \frac{2}{3}(1.5\ \text{m}) = 1.75\ \text{m}$$

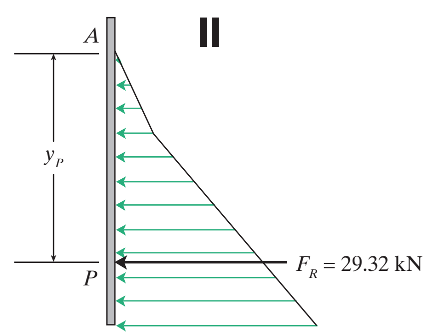
The location of the resultant force is determined by equating the moment of the resultant above *A*, Fig. 2–28*d*, to the moments of the component forces about *A*, Fig. 2–28*c*. We have

$$\bar{y}_P F_R = \Sigma y F; \qquad \bar{y}_P(29.32\ \text{kN}) = (0.5\ \text{m})(3.104\ \text{kN}) + (1.5\ \text{m})(12.42\ \text{kN}) + (1.75\ \text{m})(13.80\ \text{kN})$$

$$\bar{y}_P = 1.51\ \text{m} \qquad \textbf{Ans.}$$



(b)

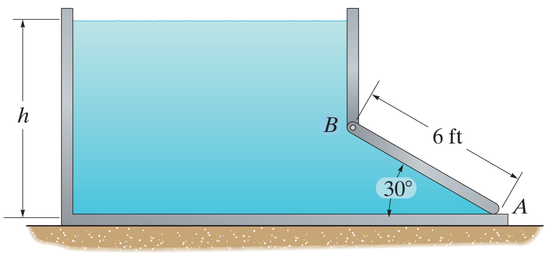


(c)

Ans:
$F_R = 29.3\ \text{kN}, \bar{y}_P = 1.51\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–50. The uniform rectangular relief gate AB has a weight of 8000 lb and a width of 4 ft. Determine the minimum depth h of water within the container needed to open it. The gate is pinned at B and rests on a rubber seal at A.



SOLUTION

Here, $h_B = h - 6 \sin 30° = (h - 3)$ ft and $h_A = h$. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(h - 3\ \text{ft})(4\ \text{ft}) = (249.6h - 748.8)\ \text{lb/ft}$$

$$w_A = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(h)(4\ \text{ft}) = (249.6h)\ \text{lb/ft}$$

Thus,

$$\left(F_p\right)_1 = \left[(249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = (1497.6h - 4492.8)\ \text{lb}$$

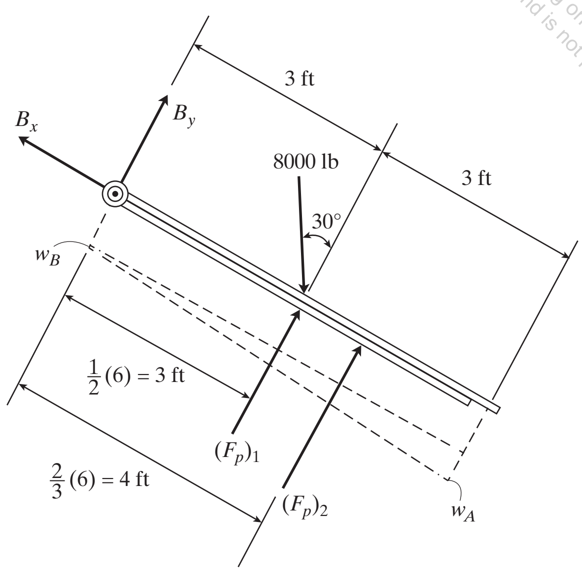
$$\left(F_p\right)_2 = \frac{1}{2}\left[(249.6h\ \text{lb/ft}) - (249.6h - 748.8\ \text{lb/ft})\right](6\ \text{ft}) = 2246.4\ \text{lb}$$

If it is required that the gate is about to open, then the normal reaction at A is equal to zero. Write the moment equation of equilibrium about B, referring to Fig. a.

$$\circlearrowleft + \Sigma M_B = 0;\ \left[(1497.6h - 4492.8\ \text{lb})\right](3\ \text{ft}) + (2246.4\ \text{lb})(4\ \text{ft})$$

$$-(8000\ \text{lb}) \cos 30°(3\ \text{ft}) = 0$$

$$h = 5.626\ \text{ft} = 5.63\ \text{ft}$$ **Ans.**

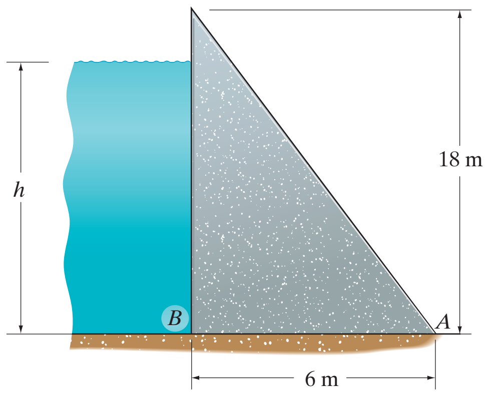


(a)

Ans:
$h = 5.63$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–51. Determine the critical height h of the water level that causes the concrete gravity dam to be on the verge of tipping over due to water pressure. The density of concrete is $\rho_c = 2.40\ \text{Mg/m}^3$. *Hint*: Work the problem using a 1-m width of the dam.



SOLUTION

We will consider the dam having a width of $b = 1$ m. Then the intensity of the distributed load at the base of the dam is

$$w_B = \rho_w g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(1\ \text{m}) = (9810h)\ \text{N/m}$$

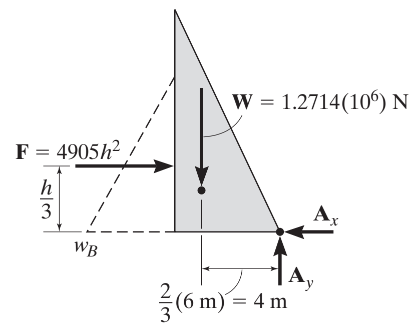
The resulting triangular distributed load is shown on the FBD of the dam, Fig. *a*, and its resultant is

$$F = \frac{1}{2}w_B h = \frac{1}{2}(9810h)h = (4905h^2)\ \text{N}$$

The weight of the concrete dam is

$$W = \rho_C g V = (2400\ \text{kg/m}^3)(9.81\ \text{m/s}^3)\left[\frac{1}{2}(6\ \text{m})(18\ \text{m})(1\ \text{m})\right]$$

$$= 1.2714(10^6)\ \text{N}$$



(a)

The dam will overturn about point A. Write the moment equation of equilibrium about point A by referring to Fig. *a*.

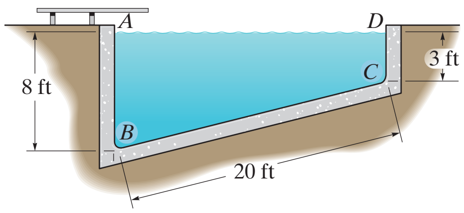
$$\circlearrowleft + \Sigma M_A = 0; \quad [1.2714(10^6)\ \text{N}](4\ \text{m}) - 4905h^2\left(\frac{h}{3}\right) = 0$$

$$h = 14.60\ \text{m} = 14.6\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$h = 14.6$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–52.** A swimming pool has a width of 12 ft and a side profile as shown. Determine the resultant force the water exerts on walls AB and DC, and on the bottom BC.



SOLUTION

Since the swimming pool has a constant width of $b = 12$ ft, the intensities of the distributed load at B and C can be computed from

$$w_B = \gamma h_{AB} b = (62.4\ \text{lb/ft}^3)(8\ \text{ft})(12\ \text{ft}) = 5990.4\ \text{lb/ft}$$

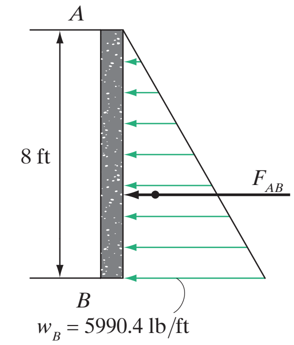
$$w_C = \gamma h_{DC} b = (62.4\ \text{lb/ft}^3)(3\ \text{ft})(12\ \text{ft}) = 2246.4\ \text{lb/ft}$$

Using these results, the distributed loads acting on walls AB and CD and bottom BC are shown in Figs. a, b, and c.

$$F_{AB} = \frac{1}{2} w_B h_{AB} = \frac{1}{2}(5990.4\ \text{lb/ft})(8\ \text{ft}) = 23\,962\ \text{lb} = 24.0\ \text{kip}$$ **Ans.**

$$F_{DC} = \frac{1}{2} w_C h_{CD} = \frac{1}{2}(2246.4\ \text{lb/ft})(3\ \text{ft}) = 3369.6\ \text{lb} = 3.37\ \text{kip}$$ **Ans.**

$$F_{BC} = \frac{1}{2}(w_B + w_C)L_{BC} = \frac{1}{2}\left[5990.4\ \text{lb/ft} + 2246.4\ \text{lb/ft}\right](20\ \text{ft})$$
$$= 82\,368\ \text{lb} = 82.4\ \text{kip}$$ **Ans.**



(a)

SOLUTION II

The same result can also be obtained as follows. For wall AB,

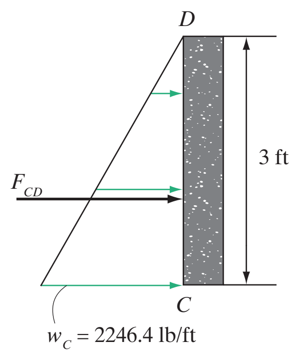
$$F_{AB} = \gamma \bar{h}_{AB} A_{AB} = (62.4\ \text{lb/ft}^3)(4\ \text{ft})\left[8\ \text{ft}(12\ \text{ft})\right] = 23\,962\ \text{lb} = 24.0\ \text{kip}$$ **Ans.**

For wall CD,

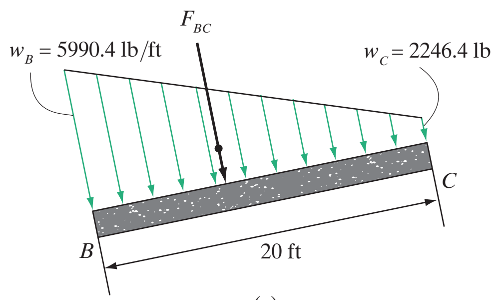
$$F_{CD} = \gamma \bar{h}_{CD} A_{CD} = (62.4\ \text{lb/ft}^3)(1.5\ \text{ft})\left[3\ \text{ft}(12\ \text{ft})\right] = 3369.6\ \text{lb} = 3.37\ \text{kip}$$ **Ans.**

For floor BC,

$$F_{BC} = \gamma \bar{h}_{BC} A_{BC} = (62.4\ \text{lb/ft}^3)(5.5\ \text{ft})\left[20\ \text{ft}(12\ \text{ft})\right] = 82\,368\ \text{lb} = 82.4\ \text{kip}$$ **Ans.**



(b)

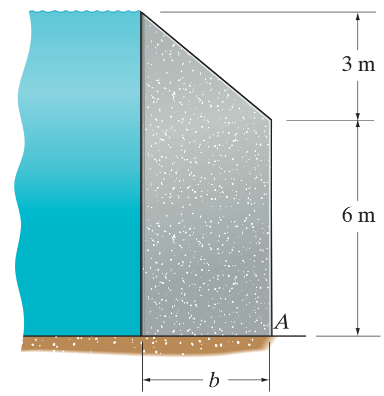


(c)

Ans:
$F_{AB} = 24.0$ kip
$F_{DC} = 3.37$ kip
$F_{BC} = 82.4$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–53. Determine the smallest thickness b of the concrete gravity dam that will prevent the dam from overturning due to water pressure acting on the face of the dam. The density of concrete is $\rho_c = 2.40\ \text{Mg/m}^3$. *Hint:* Work the problem using a 1-m width of the dam.



SOLUTION

If we consider the dam as having a width of $b = 1$ m, the intensity of the distributed load at the base of the dam is

$$w_b = \rho_w g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(9\ \text{m})(1\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

The resultant force of the triangular distributed load shown on the FBD of the dam, Fig. *a*, is

$$F = \frac{1}{2}w_b h = \frac{1}{2}[88.29(10^3)\ \text{N/m}](9\ \text{m}) = 397.305(10^3)\ \text{N}$$

The dam can be subdivided into a triangular and a rectangular part, and the weight of each of these parts is

$$W_t = \rho_c g \forall_t = (2400\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(b)(3\ \text{m})(1\ \text{m})\right] = [35.32(10^3)b]\ \text{N}$$

$$W_r = \rho_c g \forall_r = (2400\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[b(6\ \text{m})(1\ \text{m})] = [141.26(10^3)b]\ \text{N}$$

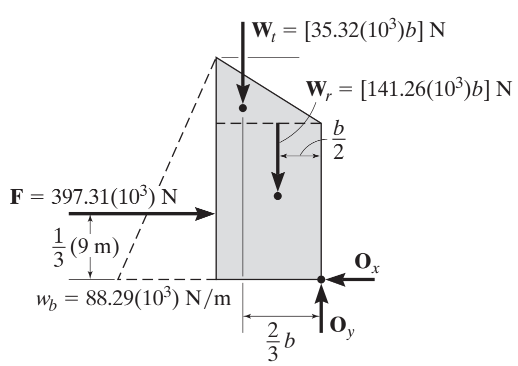
The dam will tip about point O. Writing the moment equation of equilibrium about point O, Fig. *a*,

$$\circlearrowleft + \Sigma M_O = 0; \quad [141.26(10^3)b]\left(\frac{b}{2}\right) + [35.32(10^3)b]\left(\frac{2}{3}b\right)$$

$$-[397.31(10^3)\ \text{N}]\left[\frac{1}{3}(9\ \text{m})\right] = 0$$

$$94{,}176b^2 - 1.191915(10^6) = 0$$

$$b = 3.5576\ \text{m} = 3.56\ \text{m}$$ **Ans.**

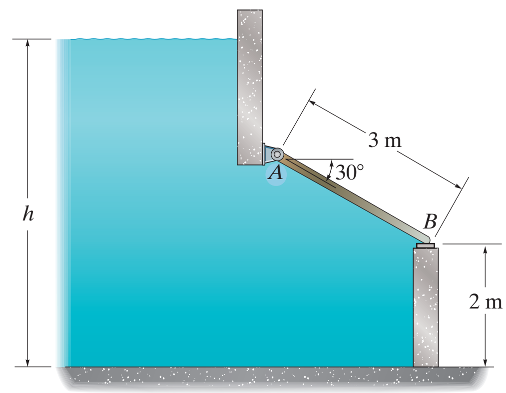


(a)

Ans:
$b = 3.56$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–54. The uniform control gate AB is pinned at A and rests on the smooth surface at B. If the gate has a mass of 8.50 Mg, determine the maximum depth of water h in the reservoir that will cause the gate to be on the verge of opening. The gate has a width of 1 m.



SOLUTION

The depths of points A and B are $h_A = h - 2\text{ m} - 3\text{ m}\sin 30° = (h - 3.5\text{ m})$ and $h_B = (h - 2\text{ m})$. Thus, the intensities of the distributed load at points A and B are

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h - 3.5\ \text{m})(1\ \text{m}) = [9810(h - 3.5)]\ \text{N/m}$$

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h - 2\ \text{m})(1\ \text{m}) = [9810(h - 2)]\ \text{N/m}$$

Thus,

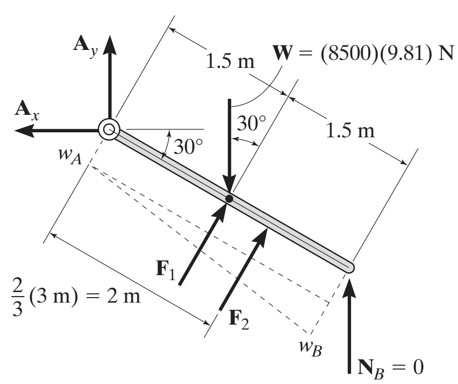
$$F_1 = w_A l_{AB} = [9810(h - 3.5)](3\ \text{m}) = [29.43(10^3)(h - 3.5)]\ \text{N}$$

$$F_2 = \frac{1}{2}(w_B - w_A)l_{AB} = \frac{1}{2}[9810(h - 2) - 9810(h - 3.5)](3\ \text{m}) = 22.0725(10^3)\ \text{N}$$

Since the gate is required to be on the verge to open, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad [29.43(10^3)(h - 3.5)](1.5\ \text{m}) + [22.0725(10^3)\text{N}](2\ \text{m}) - [8500(9.81)\ \text{N}](\cos 30°)(1.5\ \text{m}) = 0$$

$$h = 4.9537\ \text{m} = 4.95\ \text{m} \qquad \textbf{Ans.}$$

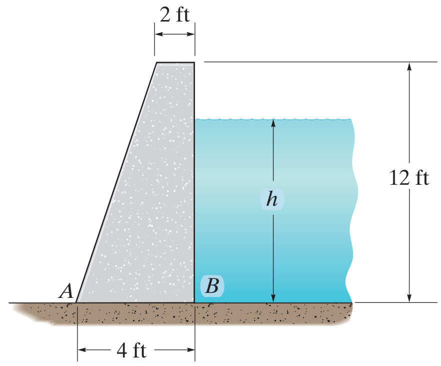


(a)

Ans:
$h = 4.95\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–55. Determine the critical height h of the water level before the concrete gravity dam starts to tip over. The specific weight of concrete is $\gamma_c = 150\ \text{lb/ft}^3$. *Hint:* Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam as having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h b = (62.4\ \text{lb/ft}^3)(h)(1\ \text{ft}) = 62.4h\ \text{lb/ft}$$

The resulting triangular distributed load is shown on the free-body diagram of the dam, Fig. *a*.

$$F = \frac{1}{2} w_B h = \frac{1}{2}(62.4h)h = 31.2h^2$$

It is convenient to subdivide the dam into two parts. The weight of each part is

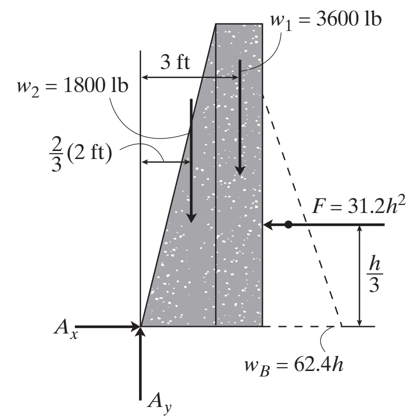
$$W_1 = \gamma_C \mathcal{V}_1 = (150\ \text{lb/ft}^3)\left[2\ \text{ft}(12\ \text{ft})(1\ \text{ft})\right] = 3600\ \text{lb}$$

$$W_2 = \gamma_C \mathcal{V}_2 = (150\ \text{lb/ft}^3)\left[\frac{1}{2}(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 1800\ \text{lb}$$

The dam will overturn about point A. Referring to the free-body diagram of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 31.2h^2\left(\frac{h}{3}\right) - (3600\ \text{lb})(3\ \text{ft}) - (1800\ \text{lb})\left[\frac{2}{3}(2\ \text{ft})\right] = 0$$

$$h = 10.83\ \text{ft} = 10.8\ \text{ft}$$ **Ans.**

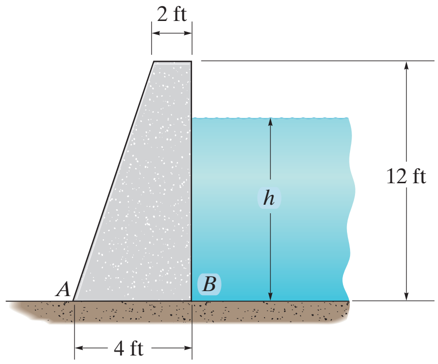


(a)

Ans:
$h = 10.8$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–56.** Determine the critical height h of the water level before the concrete gravity dam starts to tip over. Assume water also seeps under the base of the dam and produces a uniform pressure under the dam. The specific weight of concrete is $\gamma_c = 150\ \text{lb/ft}^3$. *Hint*: Work the problem using a 1-ft width of the dam.



SOLUTION

We will consider the dam having a width of $b = 1$ ft. Then the intensity of the distributed load at the base of the dam is

$$w_B = \gamma_w h_b b = (62.4\ \text{lb/ft}^3)(h)(1) = 62.4h\ \text{lb/ft}$$

The resultant forces of the triangular distributed load and uniform distributed load due the pressure of the seepage water shown on the FBD of the dam, Fig. *a*, are

$$F_1 = \frac{1}{2}w_B h = \frac{1}{2}(62.4h)\ h = 31.2h^2$$

$$F_2 = w_B L_B = 62.4h(4\ \text{ft}) = 249.6h$$

It is convenient to subdivide the dam into two parts. The weight of each part is

$$w_1 = \gamma_C \mathcal{V}_1 = (150\ \text{lb/ft}^3)\left[(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 3600\ \text{lb}$$

$$w_2 = \gamma_C \mathcal{V}_2 = (150\ \text{lb/ft}^3)\left[\frac{1}{2}(2\ \text{ft})(12\ \text{ft})(1\ \text{ft})\right] = 1800\ \text{lb}$$

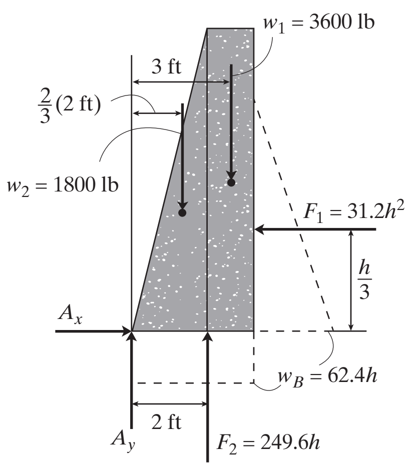
The dam will overturn about point A. Referring to the FBD of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 31.2h^2\left(\frac{h}{3}\right) + 249.6h(2\ \text{ft}) - (3600\ \text{lb})(3\ \text{ft}) - (1800\ \text{lb})\left[\frac{2}{3}(2\ \text{ft})\right] = 0$$

$$10.4h^3 + 499.2h - 13200 = 0$$

Solve numerically,

$$h = 9.3598\ \text{ft} = 9.36\ \text{ft} \qquad \textbf{Ans.}$$

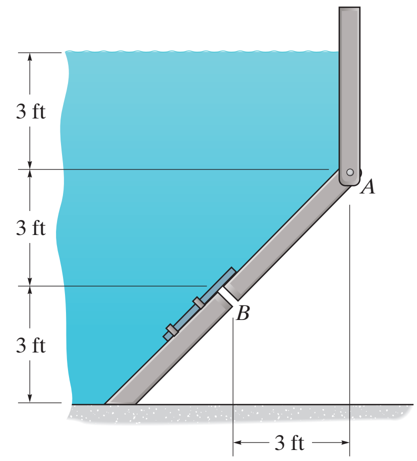


(a)

Ans:
$h = 9.36$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–57. The gate is 2 ft wide and is pinned at A and held in place by a smooth latch bolt at B that exerts a force normal to the gate. Determine this force caused by the water and the resultant force on the pin for equilibrium.



SOLUTION

Since the gate has a width of $b = 2$ ft, the intensities of the distributed loads at A and B can be computed from

$$w_A = \gamma_w h_A b = (62.4\ \text{lb/ft}^3)(3\ \text{ft})(2\ \text{ft}) = 374.4\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(6\ \text{ft})(2\ \text{ft}) = 748.8\ \text{lb/ft}$$

The resulting trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. *a*. This load can be subdivided into two parts. The resultant force of each part is

$$F_1 = w_A L_{AB} = (374.4\ \text{lb/ft})(3\sqrt{2}\ \text{ft}) = 1123.2\sqrt{2}\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{AB} = \frac{1}{2}(748.8\ \text{lb/ft} - 374.4\ \text{lb/ft})(3\sqrt{2}\ \text{ft}) = 561.6\sqrt{2}\ \text{lb}$$

Considering the free-body diagram of the gate, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 1123.2\sqrt{2}\ \text{lb}\left(\frac{1}{2}3\sqrt{2}\ \text{ft}\right) + 561.6\sqrt{2}\ \text{lb}\left(\frac{2}{3}3\sqrt{2}\ \text{ft}\right) - N_B(3\sqrt{2}\ \text{ft}) = 0$$

$$N_B = 1323.7\ \text{lb} = 1.32\ \text{kip} \qquad \textbf{Ans.}$$

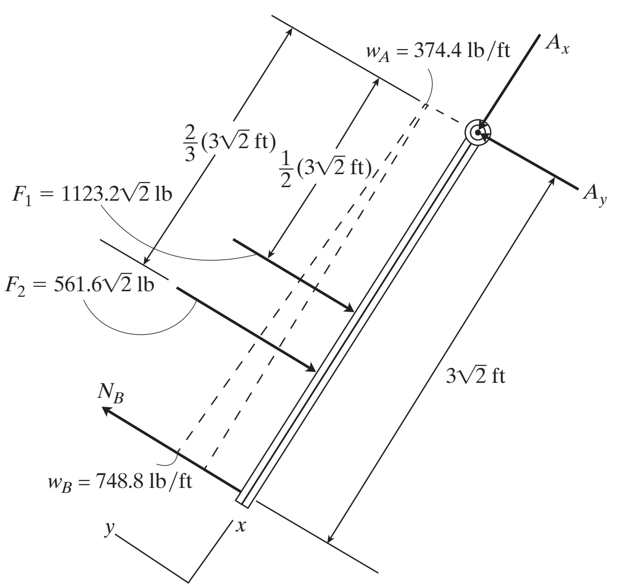
$$\Sigma F_x = 0; \qquad A_x = 0$$

$$\nwarrow + \Sigma F_y = 0; \qquad 1323.7\ \text{lb} - 1123.2\sqrt{2}\ \text{lb} - 561.6\sqrt{2}\ \text{lb} + A_y = 0$$

$$A_y = 1058.96\ \text{lb} = 1.059\ \text{kip}$$

Thus,

$$F_A = \sqrt{(0)^2 + (1.059\ \text{kip})^2} = 1.06\ \text{kip} \qquad \textbf{Ans.}$$

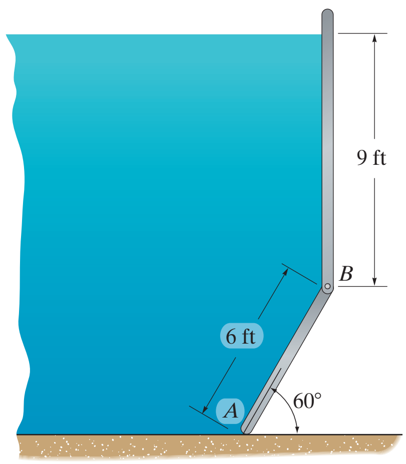


(a)

Ans:
$N_B = 1.32$ kip
$F_A = 1.06$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\,\text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\,\text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–58. The uniform rectangular relief gate AB has a weight of 800 lb and a width of 2 ft. Determine the components of reaction at the pin B and the normal reaction at the smooth support A.



SOLUTION

Here, $h_B = 9$ ft and $h_A = 9\text{ ft} + 6\text{ ft}\sin 60° = 14.20$ ft. Thus, the intensities of the distributed load at B and A are

$$w_B = \gamma_w h_B b = (62.4\,\text{lb/ft}^3)(9\,\text{ft})(2\,\text{ft}) = 1123.2\,\text{lb/ft}$$
$$w_A = \gamma_w h_A b = (62.4\,\text{lb/ft}^3)(14.20\,\text{ft})(2\,\text{ft}) = 1771.68\,\text{lb/ft}$$

Thus,

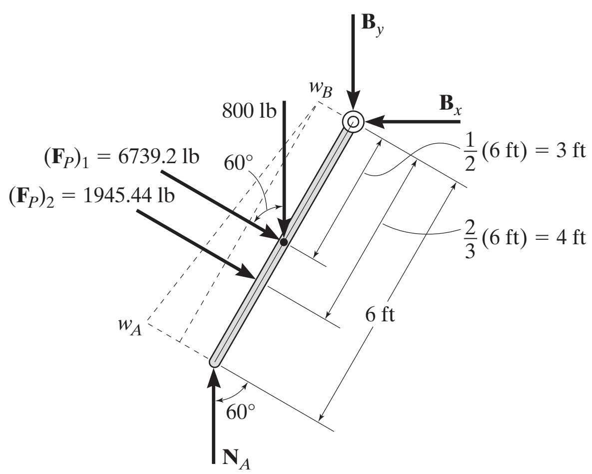
$$(F_p)_1 = (1123.2\,\text{lb/ft})(6\,\text{ft}) = 6739.2\,\text{lb}$$
$$(F_p)_2 = \frac{1}{2}(1771.68\,\text{lb/ft} - 1123.2\,\text{lb/ft})(6\,\text{ft}) = 1945.44\,\text{lb}$$

Write the moment equation of equilibrium about B by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft+\Sigma M_B = 0;\quad (800\,\text{lb})\cos 60°(3\,\text{ft}) + (6739.2\,\text{lb})(3\,\text{ft}) + (1945.44\,\text{lb})(4\,\text{ft}) - N_A\cos 60°(6\,\text{ft}) = 0$$
$$N_A = 9733.12\,\text{lb} = 9.73\,\text{kip} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along x and y axes,

$$\overset{+}{\leftarrow}\Sigma F_x = 0;\quad B_x - (6739.2\,\text{lb})\sin 60° - (1945.44\,\text{lb})\sin 60° = 0$$
$$B_x = 7521.12\,\text{lb} = 7.52\,\text{kip} \qquad \textbf{Ans.}$$
$$+\uparrow\Sigma F_y = 0;\quad 9733.12\,\text{lb} - 800\,\text{lb} - (6739.2\,\text{lb})\cos 60° - (1945.44\,\text{lb})\cos 60° - B_y = 0$$
$$B_y = 4590.80\,\text{lb} = 4.59\,\text{kip} \qquad \textbf{Ans.}$$

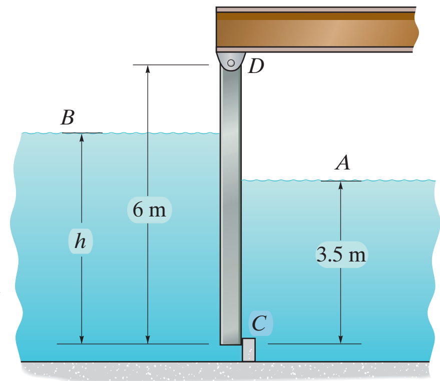


(a)

Ans:
$N_A = 9.73$ kip
$B_x = 7.52$ kip
$B_y = 4.59$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–59. The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. For the water level $h = 4$ m, determine the horizontal reaction at the smooth stop C. The gate has a width of 2 m. At what height h will the gate be on the verge of opening?



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed load on the left and right sides of the gate at C are

$$(w_C)_L = \rho_w g h_{BC}(b) = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4\ \text{m})(2\ \text{m})$$
$$= 78.48\left(10^3\right)\ \text{N/m}$$
$$(w_C)_R = \rho_w g h_{AC}(b) = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3.5\ \text{m})(2\ \text{m})$$
$$= 68.67\left(10^3\right)\ \text{N/m}$$

The resultant triangular distributed load on the left and right sides of the gate is shown on its free-body diagram, Fig. *a*,

$$F_L = \frac{1}{2}(w_C)_L L_{BC} = \frac{1}{2}\left(78.48\left(10^3\right)\ \text{N/m}\right)(4\ \text{m}) = 156.96\left(10^3\right)\ \text{N}$$
$$F_R = \frac{1}{2}(w_C)_R L_{AC} = \frac{1}{2}\left(68.67\left(10^3\right)\ \text{N/m}\right)(3.5\ \text{m}) = 120.17\left(10^3\right)\ \text{N}$$

These results can also be obtained as follows:

$$F_L = \gamma \bar{h}_L A_L = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2\ \text{m})\left[(4\ \text{m})(2\ \text{m})\right] = 156.96\left(10^3\right)\ \text{N}$$
$$F_R = \gamma \bar{h}_R A_R = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.75\ \text{m})\left[3.5\ \text{m}(2\ \text{m})\right] = 120.17\left(10^3\right)\ \text{N}$$

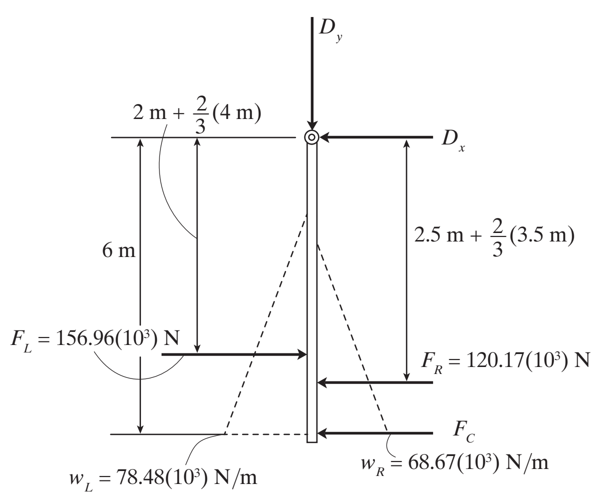
Referring to the free-body diagram of the gate in Fig. *a,*

$$\circlearrowleft + \Sigma M_D = 0; \quad \left[156.96\left(10^3\right)\ \text{N}\right]\left[2\ \text{m} + \frac{2}{3}(4\ \text{m})\right] - \left[120.17\left(10^3\right)\ \text{N}\right]\left[2.5\ \text{m} + \frac{2}{3}(3.5\ \text{m})\right] - F_C(6\ \text{m}) = 0$$

$$F_C = 25.27\left(10^3\right)\ \text{N} = 25.3\ \text{kN} \qquad \textbf{Ans.}$$

When $h = 3.5$ m, the water levels are equal. Since $F_C = 0$, the gate will open.

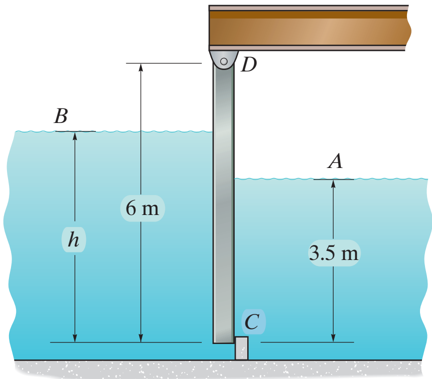
$$h = 3.5\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_C = 25.3$ kN
$h = 3.5$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–60.** The tide gate opens automatically when the tide water at B subsides, allowing the marsh at A to drain. Determine the horizontal reaction at the smooth stop C as a function of the depth h of the water level. Starting at $h = 6$ m, plot values of h for each increment of 0.5 m until the gate begins to open. The gate has a width of 2 m.



SOLUTION

Since the gate has a constant width of $b = 2$ m, the intensities of the distributed loads on the left and right sides of the gate at C are

$$(W_C)_L = \rho_w g h_{BC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(2\ \text{m}) = 19.62(10^3)h$$

$$(W_C)_R = \rho_w g h_{AC} b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})(2\ \text{m}) = 68.67(10^3)\ \text{N/m}$$

The resultant forces of the triangular distributed loads on the left and right sides of the gate shown on its FBD, Fig. a, are

$$F_L = \frac{1}{2}(w_C)_L h_{BC} = \frac{1}{2}\left[19.62(10^3)h\right]h = 9.81(10^3)h^2$$

$$F_R = \frac{1}{2}(w_C)_R h_{AC} = \frac{1}{2}\left[68.67(10^3)\ \text{N/m}\right](3.5\ \text{m}) = 120.17(10^3)\ \text{N}$$

Consider the moment equilibrium about D by referring to the FBD of the gate, Fig. a.

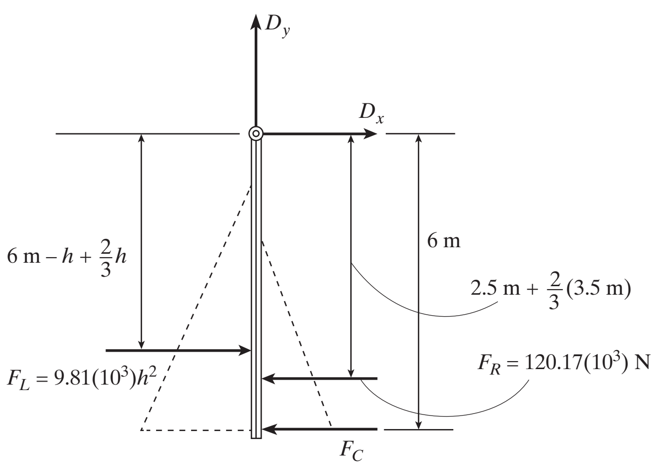
$$\circlearrowleft + \Sigma M_D = 0; \qquad \left[9.81(10^3)h^2\right]\left(6\ \text{m} - h + \frac{2}{3}h\right) - 120.17(10^3)\left[2.5\ \text{m} + \frac{2}{3}(3.5\ \text{m})\right] - F_C(6\ \text{m}) = 0$$

$$58.86(10^3)h^2 - 3.27(10^3)h^3 - 580.83(10^3) - 6F_C = 0$$

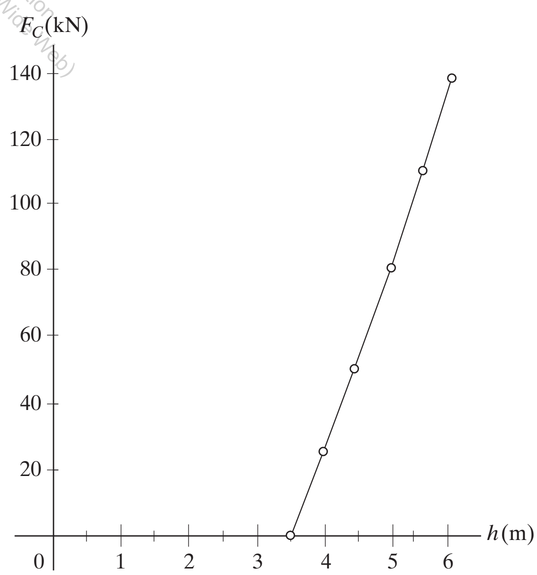
$$F_C = (9.81h^2 - 0.545h^3 - 96.806)(10^3)\ \text{N}$$

$$F_C = (9.81\ h^2 - 0.545\ h^3 - 96.8)\ \text{kN where } h \text{ is in meters.}$$ **Ans.**

The gate will be on the verge of opening when the water level on both sides of the gate are equal, that is, when $h = 3.5$ m. The plot of F_C vs h is shown in Fig. b.



(a)



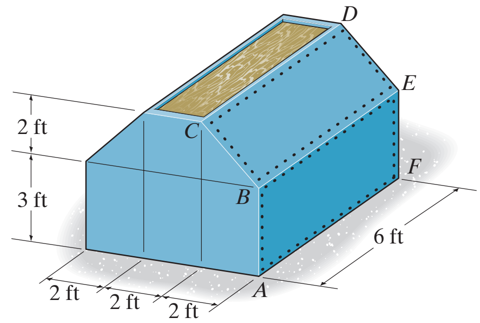
(b)

h(m)	3.5	4	4.5	5.0	5.5	6.0
F_C(kN)	0	25.3	52.2	80.3	109.3	138.6

Ans:
$F_C = (9.81\ h^2 - 0.545\ h^3 - 96.8)$ kN where h is in meters.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–61. The bin is used to store carbon tetrachloride, a cleaning agent for metal parts. If it is filled to the top, determine the magnitude of the resultant force this liquid exerts on each of the two side plates, $AFEB$ and $BEDC$, and the location of the center of pressure on each plate, measured from BE. Take $\rho_{ct} = 3.09\ \text{slug/ft}^3$.



SOLUTION

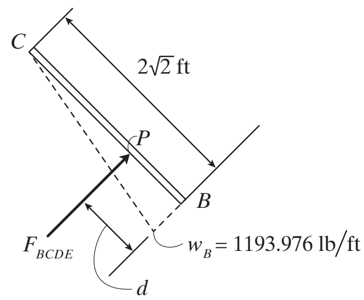
Since the side plate has a width of $b = 6$ ft, the intensities of the distributed load can be computed from

$$w_B = \rho g h_B b = \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(2\ \text{ft})(6\ \text{ft}) = 1193.976\ \text{lb/ft}$$

$$w_A = \rho g h_A b = \left(3.09\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)(5\ \text{ft})(6\ \text{ft}) = 2984.94\ \text{lb/ft}$$

The resulting distributed load on plates $BCDE$ and $ABEF$ are shown in Figs. *a* and *b*, respectively. For plate $BCDE$,

$$F_{BCDE} = \frac{1}{2}(w_B)L_{BC} = \frac{1}{2}(1193.976\ \text{lb/ft})\left(2\sqrt{2}\ \text{ft}\right) = 1688.54\ \text{lb} = 1.69\ \text{kip} \qquad \textbf{Ans.}$$



(a)

And the center of pressure of this plate from BE is

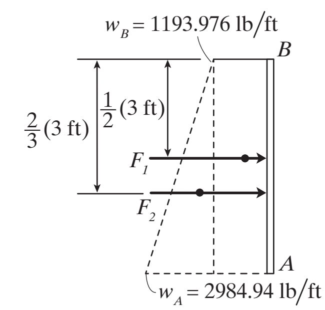
$$d = \frac{1}{3}\left(2\sqrt{2}\ \text{ft}\right) = 0.943\ \text{ft} \qquad \textbf{Ans.}$$

For $ABEF$,

$$F_1 = w_B L_{AB} = (1193.976\ \text{lb/ft})(3\ \text{ft}) = 3581.93\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_A - w_B)L_{AB} = \frac{1}{2}(2984.94\ \text{lb/ft} - 1193.976\ \text{lb/ft})(3\ \text{ft}) = 2686.45\ \text{lb}$$

$$F_{ABEF} = F_1 + F_2 = 3581.93\ \text{lb} + 2686.45\ \text{lb} = 6268.37\ \text{lb} = 6.27\ \text{kip} \qquad \textbf{Ans.}$$

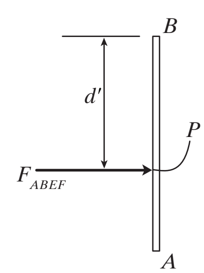


(b)

The location of the center of pressure measured from BE can be obtained by equating the sum of the moments of the forces in Figs. *b* and *c*.

$$\circlearrowleft + M_{R_B} = \Sigma M_B; \quad (6268.37\ \text{lb})d' = (3581.93\ \text{lb})\left[\frac{1}{2}(3\ \text{ft})\right] + (2686.45\ \text{lb})\left[\frac{2}{3}(3\ \text{ft})\right]$$

$$d' = 1.714\ \text{ft} = 1.71\ \text{ft} \qquad \textbf{Ans.}$$

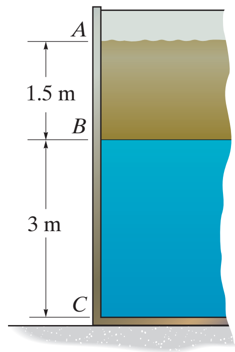


(c)

Ans:
$F_{BCDE} = 1.69$ kip, $d = 0.943$ ft
$F_{ABEF} = 6.27$ kip, $d' = 1.71$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–62. Determine the resultant force that the water and oil together exert on the wall ABC. The wall has a width of 2 m. Also, determine the location of this resultant measured from the top of the tank. Take $\rho_o = 900\ \text{kg/m}^3$.

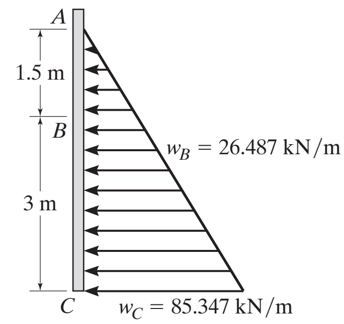


SOLUTION

Since the wall has a constant width, the intensities of the distributed loading at B and C, Fig. a, are

$$w_B = \rho_o g h_{AB} b = (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5\ \text{m})(2\ \text{m}) = 26.487(10^3)\ \text{N/m} = 26.487\ \text{kN/m}$$

$$w_C = w_B + \rho_w g h_{BC} b = 26.487(10^3)\ \text{N/m} + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(2\ \text{m})$$
$$= 85.347(10^3)\ \text{N/m} = 85.347\ \text{kN/m}$$



(a)

The resultant force can be determined by adding the areas of the triangles and rectangle, Fig. b. Here,

$$F_1 = \frac{1}{2}(26.487\ \text{kN/m})(1.5\ \text{m}) = 19.87\ \text{kN}$$

$$F_2 = (26.487\ \text{kN/m})(3\ \text{m}) = 79.46\ \text{kN}$$

$$F_3 = \frac{1}{2}(58.86\ \text{kN/m})(3\ \text{m}) = 88.29\ \text{kN}$$

Thus, the resultant force is

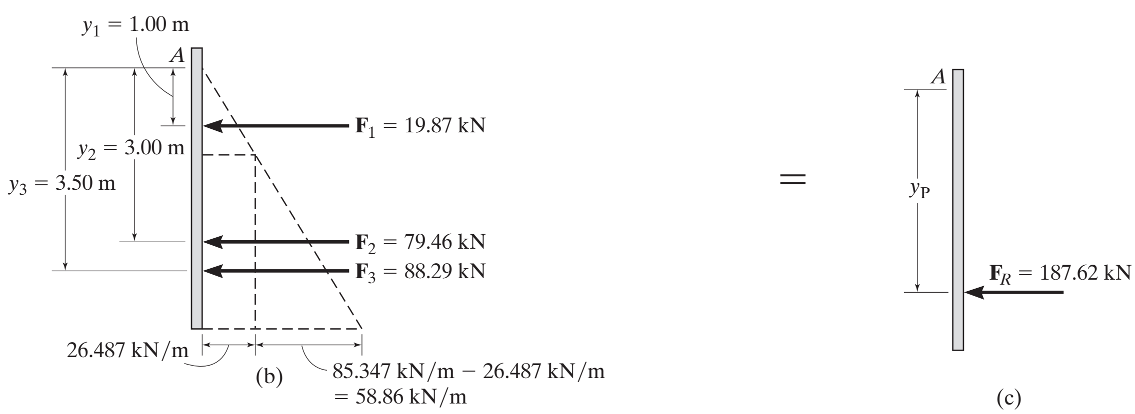
$$F_R = \Sigma F = F_1 + F_2 + F_3$$
$$= 19.87\ \text{kN} + 79.46\ \text{kN} + 88.29\ \text{kN}$$
$$= 187.62\ \text{kN} = 188\ \text{kN} \qquad \textbf{Ans.}$$

Each of the force components measured from the top acts through the centroid of its respective area.

$$y_1 = \frac{2}{3}(1.5\ \text{m}) = 1.00\ \text{m} \quad y_2 = 1.5\ \text{m} + \frac{1}{2}(3\ \text{m}) = 3.00\ \text{m} \quad y_3 = 1.5\ \text{m} + \frac{2}{3}(3\ \text{m}) = 3.50\ \text{m}$$

The location of the resultant force can be determined by equating the moment of the resultant force and the force components about A, by referring to Figs. b and c.

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (187.62\ \text{kN})y_p = (19.87\ \text{kN})(1.00\ \text{m}) + (79.46\ \text{kN})(3.00\ \text{m})$$
$$+ (88.29\ \text{kN})(3.50\ \text{m})$$
$$y_p = 3.024\ \text{m} = 3.02\ \text{m} \qquad \textbf{Ans.}$$

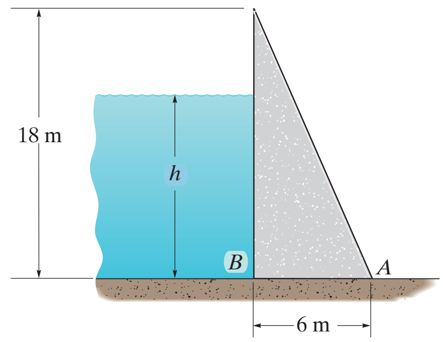


(b) (c)

Ans:
$F_R = 188\ \text{kN}$
$y_p = 3.02\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–63. Determine the critical height h of the water level that causes the concrete gravity dam to be on the verge of tipping over. Assume water also seeps under the base of the dam and produces a uniform pressure under the dam. The density of concrete is $\rho_c = 2400\ \text{kg/m}^3$. *Hint*: Work the problem using a 1-m width of the dam.



SOLUTION

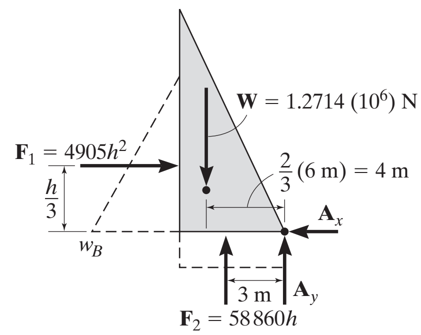
We will consider the dam having a width of $b = 1$ m. Then the intensity of the distributed load at the base of the dam is

$$w_B = \rho_w g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(1\ \text{m}) = (9810h)\ \text{N/m}$$

The resultant forces of the triangular distributed load and uniform distributed load due to the pressure of seepage water shown on the FBD of the dam, Fig. *a*, are

$$F_1 = \frac{1}{2} w_B h = \frac{1}{2}(9810h)(h) = (4905h^2)\ \text{N}$$

$$F_2 = w_B L_B = (9810h)(6\ \text{m}) = (58\,860h)\ \text{N}$$



(a)

The weight of the concrete dam is

$$W = \rho_c g V = (2400\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(6\ \text{m})(18\ \text{m})(1\ \text{m})\right]$$

$$= 1.2714(10^6)\ \text{N}$$

The dam will overturn about point A. Write the moment equation of equilibrium about point A by referring to Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad [1.2714(10^6)\ \text{N}](4\ \text{m}) - 4905h^2\left(\frac{h}{3}\right) - 58\,860h(3\ \text{m}) = 0$$

$$1635h^3 + 17\,658(10^3)h - 5.0855(10^6) = 0$$

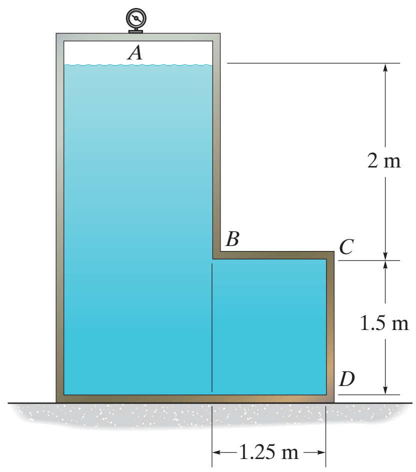
Solving numerically,

$$h = 12.1583\ \text{m} = 12.2\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$h = 12.2$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–64.** The pressure of the air at A within the closed tank is 200 kPa. Determine the resultant force acting on the plates BC and CD caused by the water. The tank has a width of 1.75 m.



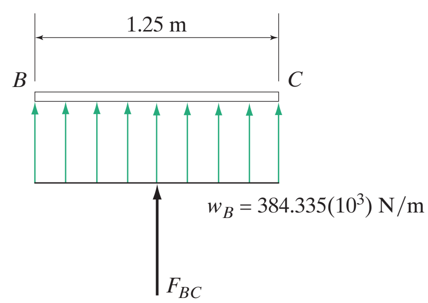
SOLUTION

$$p_C = p_B = p_A + \rho g h_{AB}$$
$$= 200(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})$$
$$= 219.62(10^3)\ \text{Pa}$$

$$p_D = p_A + \rho g h_{AD}$$
$$= 200(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.5\ \text{m})$$
$$= 234.335(10^3)\ \text{Pa}$$

Since plates BC and CD have a constant width of $b = 1.75$ m, the intensities of the distributed load at points B (or C) and D are

$$w_C = w_B = p_B b = (219.62(10^3)\ \text{N/m}^2)(1.75\text{m}) = 384.335(10^3)\ \text{N/m}$$
$$w_D = p_D b = (234.335(10^3)\ \text{N/m}^2)(1.75\ \text{m}) = 410.086(10^3)\ \text{N/m}$$

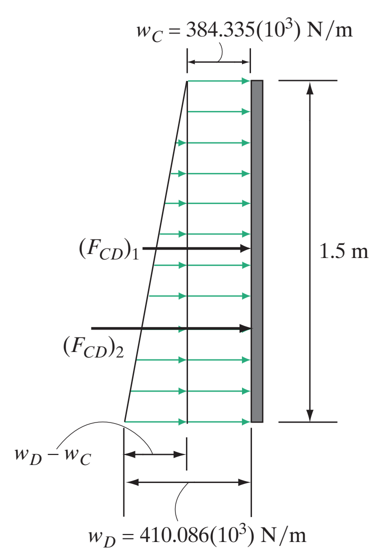


(a)

Using these results, the distributed loads acting on plates BC and CD are shown in Figs. *a* and *b*, respectively.

$$F_{BC} = w_B L_{BC} = \left[384.335(10^3)\ \frac{\text{N}}{\text{m}}\right](1.25\ \text{m}) = 480.42(10^3)\ \text{N} = 480\ \text{kN} \qquad \textbf{Ans.}$$

$$F_{CD} = (F_{CD})_1 + (F_{CD})_2 = w_C L_{CD} + \frac{1}{2}(w_D - w_C)L_{CD}$$
$$= \left[384.335(10^3)\ \frac{\text{N}}{\text{m}}\right](1.5\ \text{m}) + \frac{1}{2}\left[410.086(10^3)\ \frac{\text{N}}{\text{m}} - 384.335(10^3)\ \frac{\text{N}}{\text{m}}\right](1.5\ \text{m})$$
$$= 595.82(10^3)\ \text{N} = 596\ \text{kN} \qquad \textbf{Ans.}$$

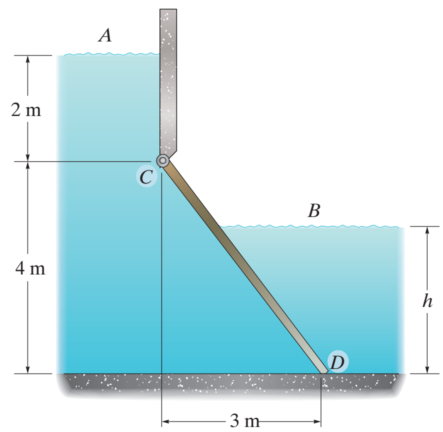


(b)

Ans:
$F_{BC} = 480$ kN, $F_{CD} = 596$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

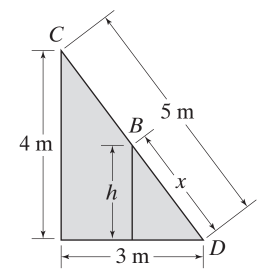
2–65. The uniform plate, which is hinged at C, is used to control the level of the water at A to maintain its constant depth of 6 m. If the plate has a width of 1.5 m and a mass of 30 Mg, determine the required minimum height h of the water at B so that seepage will not occur at D.



SOLUTION

Referring to the geometry shown in Fig. a,

$$\frac{x}{5} = \frac{h}{4}; \qquad x = \frac{5}{4}h$$



(a)

The intensities of the distributed load shown in the FBD of the gate, Fig. b, are

$$w_1 = \rho_w g h_1 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})(1.5\ \text{m}) = 29.43(10^3)\ \text{N/m}$$

$$w_2 = \rho_w g h_2 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(1.5\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

$$w_3 = \rho_w g h_3 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(h)(1.5\ \text{m}) = [14.715(10^3)h]\ \text{N/m}$$

Then, the resultant forces of these distributed loads are

$$F_1 = w_1 l_{CD} = [29.43(10^3)\ \text{N/m}](5\ \text{m}) = 147.15(10^3)\ \text{N}$$

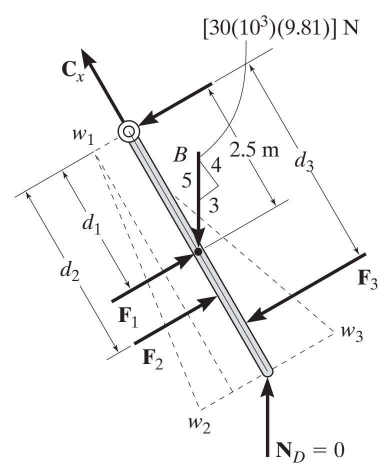
$$F_2 = \frac{1}{2}(w_2 - w_1)l_{CD} = \frac{1}{2}[88.29(10^3)\ \text{N/m} - 2943(10^3)\ \text{N/m}](5\ \text{m}) = 147.15(10^3)\ \text{N}$$

$$F_3 = \frac{1}{2}w_3 l_{BD} = \frac{1}{2}[14.715(10^3)h]\left(\frac{5}{4}h\right) = [9.196875(10^3)h^2]\ \text{N}$$

and act at

$$d_1 = \frac{1}{2}(5\ \text{m}) = 2.5\ \text{m} \quad d_2 = \frac{2}{3}(5\ \text{m}) = 3.3333\ \text{m}$$

$$d_3 = 5\ \text{m} - \frac{1}{3}\left(\frac{5}{4}h\right) = (5 - 0.4167h)\ \text{m}$$



(b)

The seepage is on the verge of occurring when the gate is about to open. Thus, it is required that $N_D = 0$. Write the moment equation of equilibrium about point C by referring to Fig. a.

$$\circlearrowleft + \Sigma M_C = 0; \quad [147.15(10^3)\ \text{N}](2.5\ \text{m}) + [147.15(10^3)\ \text{N}](3.3333\ \text{m})$$

$$- [30(10^3)(9.81)\ \text{N}]\left(\frac{3}{5}\right)(2.5\ \text{m})$$

$$- [9.196875(10^3)h^2](5 - 0.4167h) = 0$$

$$3.8320h^3 - 45.9844h^2 + 416.925 = 0$$

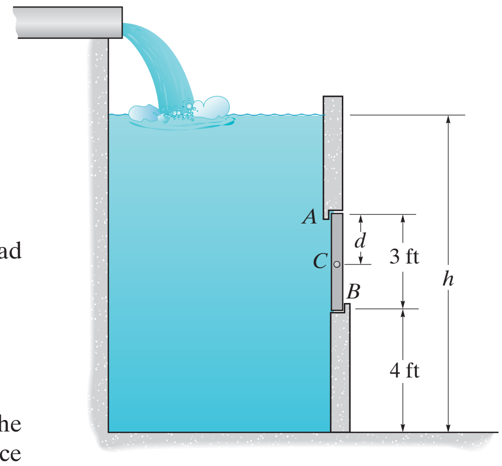
Solving numerically,

$$h = 3.5987\ \text{m} = 3.60\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$h = 3.60\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–66. Determine the placement d of the pin on the 2-ft-wide rectangular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate?



SOLUTION

Since the gate has a constant width of $b = 2$ ft, the intensity of the distributed load at A and B can be computed from

$$w_A = \gamma_w h_A b = (62.4\ \text{lb/ft}^3)(3\ \text{ft})(2\ \text{ft}) = 374.4\ \text{lb/ft}$$

$$w_B = \gamma_w h_B d = (62.4\ \text{lb/ft}^3)(6\ \text{ft})(2\ \text{ft}) = 748.8\ \text{lb/ft}$$

The resultant trapezoidal distributed load is shown on the free-body diagram of the gate, Fig. *a*. This load can be subdivided into two parts for which the resultant force of each part is

$$F_1 = w_A L_{AB} = 374.4\ \text{lb/ft}(3\ \text{ft}) = 1123.2\ \text{lb}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{AB} = \frac{1}{2}(748.8\ \text{lb/ft} - 374.4\ \text{lb/ft})(3\ \text{ft}) = 561.6\ \text{lb}$$

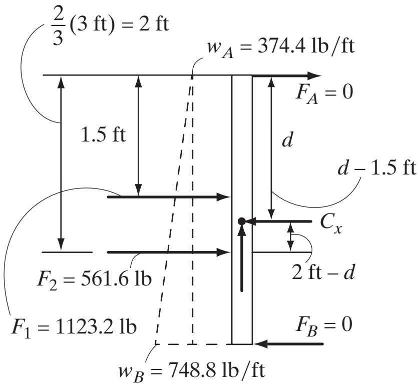
Thus, the resultant force is

$$F_R = F_1 + F_2 = 1123.2\ \text{lb} + 561.6\ \text{lb} = 1684.8\ \text{lb} = 1.68\ \text{kip} \qquad \textbf{Ans.}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_C = 0; \qquad (561.61b)(2\ \text{ft} - d) - (1123.2\ \text{lb})(d - 1.5\ \text{ft}) = 0$$

$$d = 1.67\ \text{ft} \qquad \textbf{Ans.}$$

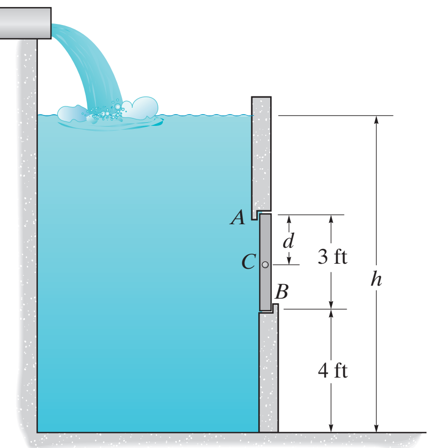


(a)

Ans:
$F_R = 1.68$ kip
$d = 1.67$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–67. Determine the placement d of the pin on the 3-ft-diameter circular gate so that it begins to rotate clockwise (open) when waste water reaches a height $h = 10$ ft. What is the resultant force acting on the gate? Use the formula method.



SOLUTION

Since the gate is circular in shape, it is convenient to compute the resultant force as follows.

$$F_R = \gamma_w \bar{h} A = (62.4\ \text{lb/ft}^3)(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2 = 1984.86\ \text{lb}$$
$$= 1.98\ \text{kip} \qquad \textbf{Ans.}$$

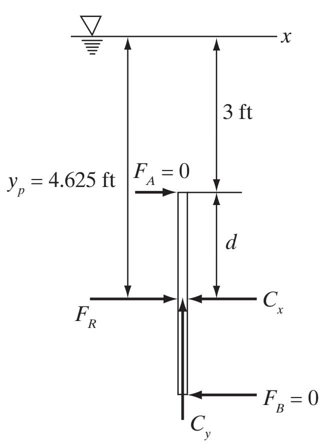
The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\left(\dfrac{\pi(1.5\ \text{ft})^4}{4}\right)}{(10\ \text{ft} - 5.5\ \text{ft})(\pi)(1.5\ \text{ft})^2} + (10\ \text{ft} - 5.5\ \text{ft}) = 4.625\ \text{ft}$$

When the gate is on the verge of opening, the normal force at A and B is zero as shown on the free-body diagram of the gate, Fig. *a*. Summing the moments about point C requires that F_R acts through C. Thus,

$$d = y_p - 3\ \text{ft} = 4.625\ \text{ft} - 3\ \text{ft} = 1.625\ \text{ft} = 1.62\ \text{ft} \qquad \textbf{Ans.}$$

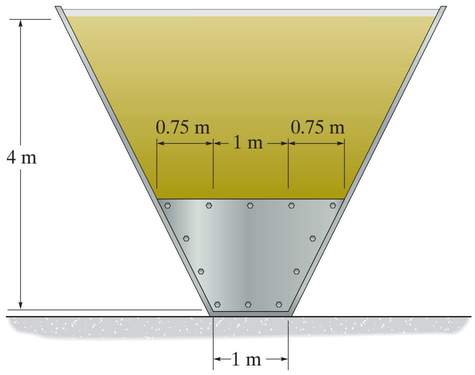


(a)

Ans:
$F_R = 1.98$ kip
$d = 1.62$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–68.** The tapered settling tank is filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its end. How far from the oil surface does this force act on the plate? Use the formula method. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

Referring to the geometry of the plate shown in Fig. *a*

$$A = (1\text{ m})(1.5\text{ m}) + \frac{1}{2}(1.5\text{ m})(1.5\text{ m}) = 2.625\text{ m}^2$$

$$\bar{y} = \frac{(3.25\text{ m})[(1\text{ m})(1.5\text{ m})] + (3\text{ m})\left[\frac{1}{2}(1.5\text{ m})(1.5\text{ m})\right]}{2.625\text{ m}^2} = 3.1429\text{ m}$$

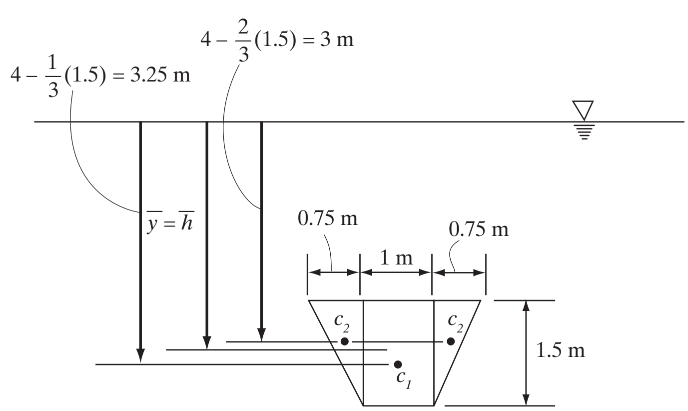
$$\bar{I}_x = \frac{1}{12}(1\text{ m})(1.5\text{ m})^3 + (1\text{ m})(1.5\text{ m})(3.25\text{ m} - 3.1429\text{ m})^2 + \frac{1}{36}(1.5\text{ m})(1.5\text{ m})^3 + \frac{1}{2}(1.5\text{ m})(1.5\text{ m})(3.1429\text{ m} - 3\text{ m})^2 = 0.46205\text{ m}^4$$

The resultant force is

$$F_R = \rho_o g \bar{h} A = (900\text{ kg/m}^3)(9.81\text{ m/s}^2)(3.1429\text{ m})(2.625\text{ m}^2) = 72.84(10^3)\text{ N} = 72.8\text{ kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.46205\text{ m}^4}{(3.1429\text{ m})(2.625\text{ m}^2)} + 3.1429\text{ m} = 3.199\text{ m} = 3.20\text{ m} \qquad \textbf{Ans.}$$

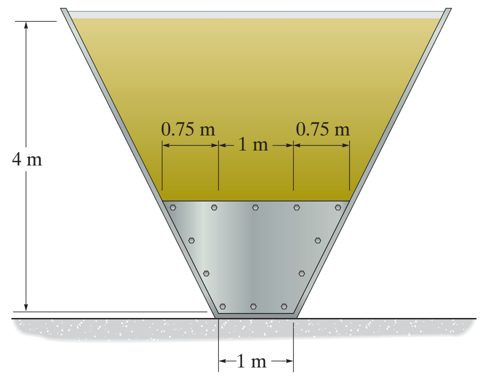


(a)

Ans:
$F_R = 72.8$ kN
$y_P = 3.20$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–69. The tapered settling tank is filled with oil. Determine the resultant force the oil exerts on the trapezoidal clean-out plate located at its end. How far from the oil surface does this force act on the plate? Use the integration method. Take $\rho_o = 900\ \text{kg/m}^3$.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a, is

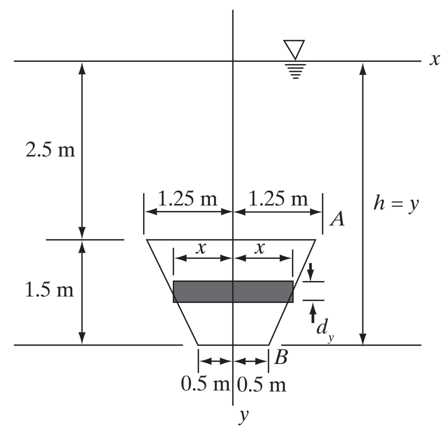
$$\frac{y - 2.5}{x - 1.25} = \frac{4 - 2.5}{0.5 - 1.25}; \qquad 2x = 5 - y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2xdy = 5 - y\,dy$. The pressure acting on this differential element is $p = \rho_0 gh = (900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)y = 8829y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p\,dA = \int_{2.5\text{ m}}^{4\text{ m}} 8829y(5 - y)dy$$

$$= 22\,072.5y^2 - 2943y^3 \Big|_{2.5\text{ m}}^{4\text{ m}}$$

$$= 72.84(10^3)\ \text{N} = 72.8\ \text{kN}$$ **Ans.**



(a)

And it acts at

$$y_P = \frac{\int_A y\,p\,dA}{F_R} = \frac{1}{72.84(10^3)\ \text{N}}\Big|_{2.5\text{ m}}^{4\text{ m}} y(8829y)(5 - y)dy$$

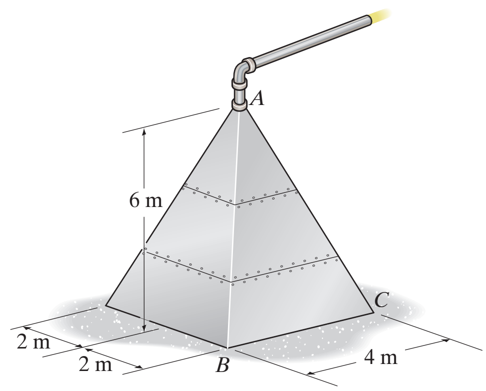
$$= \frac{1}{72.84(10^3)}\left(14\,715y^3 - 2207.25y^4\right)\Big|_{2.5\text{ m}}^{4\text{ m}}$$

$$= 3.199\ \text{m} = 3.20\ \text{m}$$ **Ans.**

Ans:
$F_R = 72.8\ \text{kN}$
$y_P = 3.20\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–70. Ethyl alcohol is pumped into the tank, which has the shape of a four-sided pyramid. When the tank is completely filled, determine the resultant force acting on each side, and its location measured from the top A along the side. Use the formula method. Take $\rho_{ea} = 789\ \text{kg/m}^3$.



SOLUTION

The geometry of the side wall of the tank is shown in Fig. *a*. In this case, it is convenient to calculate the resultant force as follows.

$$F_R = \gamma_{ea}\bar{h}A = (789\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{2}{3}(6\ \text{m})\right]\left(\frac{1}{2}\right)(4\ \text{m})(\sqrt{40})$$

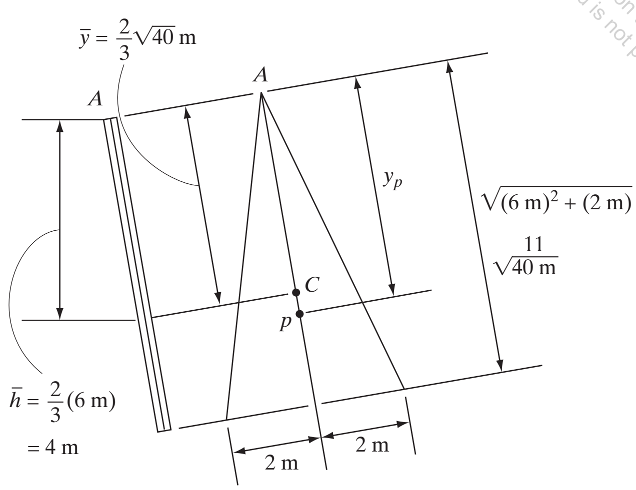
$$= 390.1(10^3)\ \text{N} = 390\ \text{kN} \qquad \textbf{Ans.}$$

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

$$= \frac{\frac{1}{36}(4\ \text{m})(\sqrt{40}\ \text{m})^3}{\frac{2}{3}(\sqrt{40}\ \text{m})\left(\frac{1}{2}(4\ \text{m})(\sqrt{40}\ \text{m})\right)} + \left(\frac{2}{3}\right)(\sqrt{40}\ \text{m})$$

$$= 4.74\ \text{m} \qquad \textbf{Ans.}$$

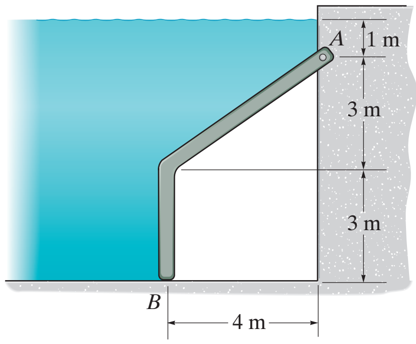


(a)

Ans:
$F_R = 390\ \text{kN}$
$y_P = 4.74\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–71. The bent plate is 2 m wide and is pinned at A and rests on a smooth support at B. Determine the horizontal and vertical components of reaction at A and the vertical reaction at the smooth support B for equilibrium. The fluid is water.



SOLUTION

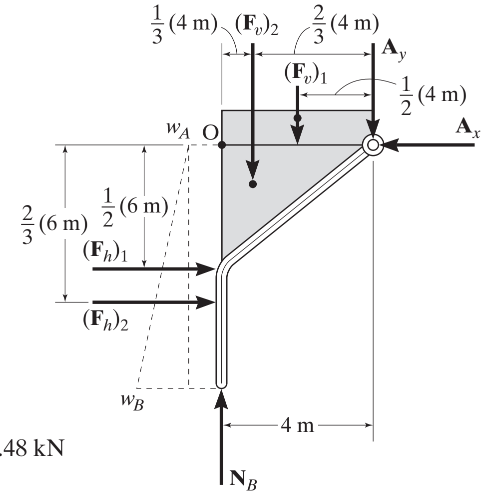
The horizontal loading on the plate is due to the pressure on the vertical projected area of the plate. Since the plate has a constant width of $b = 2$ m, the intensities of the horizontal distributed load at A and B are

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(2\ \text{m}) = 19.62(10^3)\ \text{N/m}$$
$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(7\ \text{m})(2\ \text{m}) = 137.34(10^3)\ \text{N/m}$$

The resultant of the distributed load shown in the FBD of the plate, Fig. *a*, is

$$(F_h)_1 = w_A l_{AB} = [19.62(10^3)\ \text{N/m}](6\ \text{m}) = 117.72(10^3)\ \text{N} = 117.72\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}(w_B - w_A)l_{AB} = \frac{1}{2}[137.34(10^3)\ \text{N/m} - 19.62(10^3)\ \text{N/m}](6\ \text{m})$$
$$= 353.16(10^3)\ \text{N} = 353.16\ \text{kN}$$



$\mathbf{F}_3$ and $\mathbf{F}_4$ are not in the FBD are equal to the weight of water contained in the respective shaded rectangular and triangular blocks, Fig. *a*.

$$(F_v)_1 = \rho_w g V_r = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[4\ \text{m}(1\ \text{m})(2\ \text{m})] = 78.48(10^3)\ \text{N} = 78.48\ \text{kN}$$

$$(F_v)_2 = \rho_w g V_t = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(4\ \text{m})(3\ \text{m})(2\ \text{m})\right] = 117.72(10^3)\ \text{N} = 117.72\ \text{kN}$$

Write the moment equation of equilibrium about points A and O by referring to Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0;\quad (117.72\ \text{kN})\left[\frac{1}{2}(6\ \text{m})\right] + (353.16\ \text{kN})\left[\frac{2}{3}(6\ \text{m})\right] + (78.48\ \text{kN})\left[\frac{1}{2}(4\ \text{m})\right]$$
$$+ (117.72\ \text{kN})\left[\frac{2}{3}(4\ \text{m})\right] - N_B(4\ \text{m}) = 0$$
$$N_B = 559.17\ \text{kN} = 559\ \text{kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_o = 0;\quad (117.72\ \text{kN})\left[\frac{1}{2}(6\ \text{m})\right] + (353.16\ \text{kN})\left[\frac{2}{3}(6\ \text{m})\right] - (78.48\ \text{kN})\left[\frac{1}{2}(4\ \text{m})\right]$$
$$- (117.72\ \text{kN})\left[\frac{1}{3}(4\ \text{m})\right] - A_y(4\ \text{m}) = 0$$
$$A_y = 362.97\ \text{kN} = 363\ \text{kN} \qquad \textbf{Ans.}$$

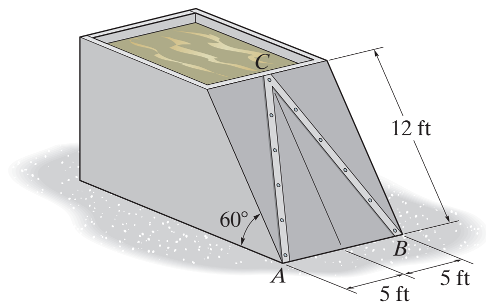
Write the force equation of equilibrium along the x axis.

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 117.72\ \text{kN} + 353.16\ \text{kN} - A_x = 0$$
$$A_x = 470.88\ \text{kN} = 471\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 559$ kN, $A_y = 363$ kN,
$A_x = 471$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–72.** The tank is filled to its top with an industrial solvent, ethyl ether. Determine the resultant force acting on the plate ABC, and its location on the plate measured from the base AB of the tank. Use the formula method. Take $\gamma_{ee} = 44.5\ \text{lb/ft}^3$.



SOLUTION

The resultant force is

$$F_R = \gamma_{ee}\bar{h}A = \left(44.5\ \text{lb/ft}^3\right)(8 \sin 60°\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]$$

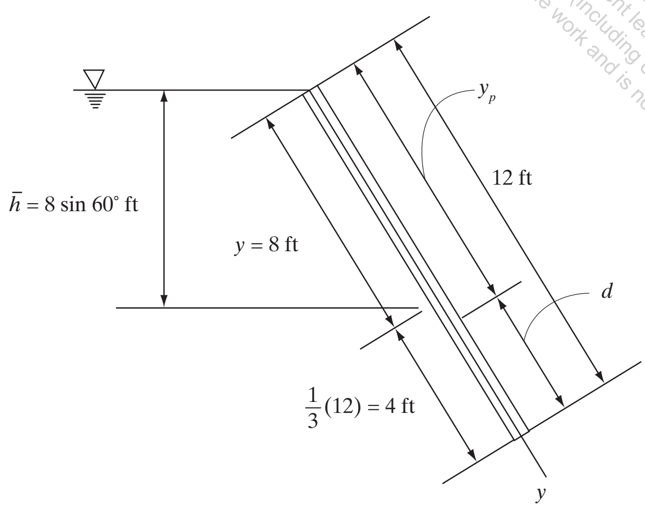
$$= 18.498\left(10^3\right)\ \text{lb} = 18.5\ \text{kip} \qquad \textbf{Ans.}$$

$\bar{I}_x = \dfrac{1}{36}bh^3 = \dfrac{1}{36}(10\ \text{ft})(12\ \text{ft})^3 = 480\ \text{ft}$. Then

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{480\ \text{ft}}{(8\ \text{ft})\left[\frac{1}{2}(10\ \text{ft})(12\ \text{ft})\right]} + 8\ \text{ft} = 9\ \text{ft}$$

Thus,

$$d = 12\ \text{ft} - y_P = 12\ \text{ft} - 9\ \text{ft} = 3\ \text{ft} \qquad \textbf{Ans.}$$

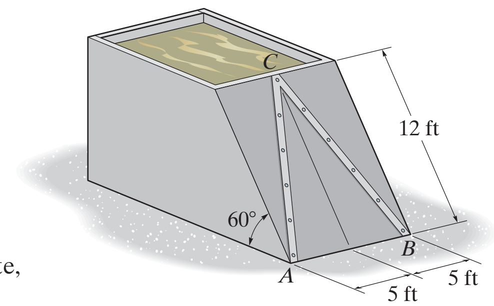


(a)

Ans:
$F_R = 18.5\ \text{kip}$
$d = 3\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–73. Solve Prob. 2–72 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of side AB of the plate, Fig. a, is

$$\frac{y-0}{x-0} = \frac{12-0}{5-0}; \qquad x = \frac{5}{12}y$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\left(\frac{5}{12}y\right)dy = \frac{5}{6}y\,dy$. The pressure acting on this differential element is $p = \gamma h = (44.5\ \text{lb/ft}^3)(y \sin 60°) = 38.54y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p\,dA = \int_0^{12\text{ ft}} (38.54y)\left(\frac{5}{6}y\,dy\right)$$

$$= 10.71\,y^3\Big|_0^{12\text{ ft}}$$

$$= 18.50(10^3)\text{lb} = 18.5\text{ kip} \qquad \textbf{Ans.}$$

And it acts at

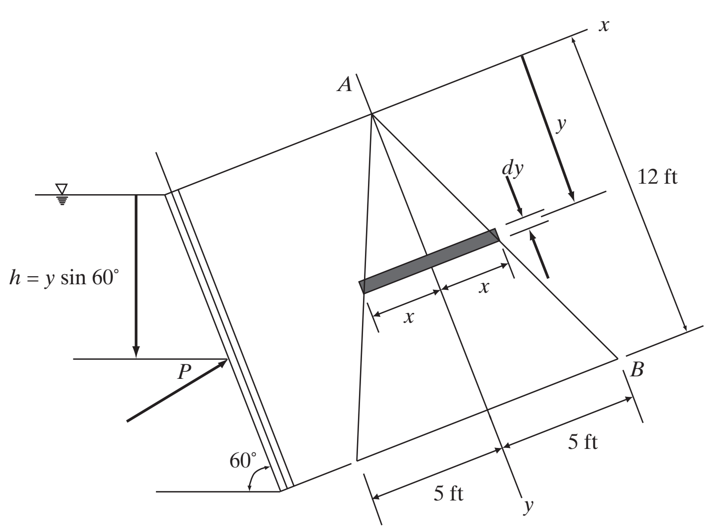
$$y_P = \frac{\int_A y\,p\,dA}{F_R} = \frac{1}{18.50(10^3)}\int_0^{12\text{ ft}} y(38.54y)\left(\frac{5}{6}y\,dy\right)$$

$$= \frac{1}{18.50(10^3)}(8.03y^4)\Big|_0^{12\text{ ft}}$$

$$= 9.00\text{ ft}$$

Thus,

$$d = 12\text{ ft} - y_p = 3.00\text{ ft} \qquad \textbf{Ans.}$$

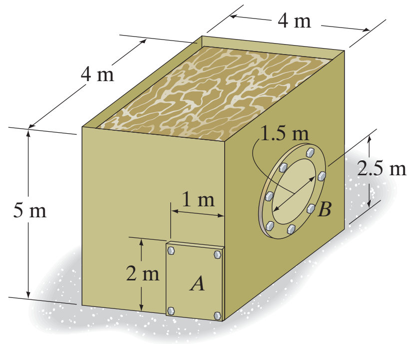


(a)

Ans:
$F_R = 18.5$ kip
$d = 3.00$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–74. If the tank is filled with vegetable oil, determine the resultant force that the oil exerts on plate A, and its location measured from the bottom of the tank. Use the formula method. Take $\rho_{vo} = 932\ \text{kg/m}^3$.



SOLUTION

Since the plate has a width of $b = 1$ m, the intensities of the distributed load at the top and bottom of the plate can be computed from

$$w_t = \rho_{vo}\, g h_t b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(1\ \text{m}) = 27.429(10^3)\ \text{N/m}$$
$$w_b = \rho_{vo}\, g h_b b = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(1\ \text{m}) = 45.715(10^3)\ \text{N/m}$$

The resulting trapezoidal distributed load is shown in Fig. a, and this loading can be subdivided into two parts for which the resultant forces are

$$F_1 = w_t(L) = \left[27.429(10^3)\ \text{N/m}\right](2\ \text{m}) = 54.858(10^3)\ \text{N}$$

$$F_2 = \frac{1}{2}(w_b - w_t)(L) = \frac{1}{2}\left[45.715(10^3)\ \text{N/m} - 27.429(10^3)\ \text{N/m}\right](2\ \text{m}) = 18.286(10^3)\ \text{N}$$

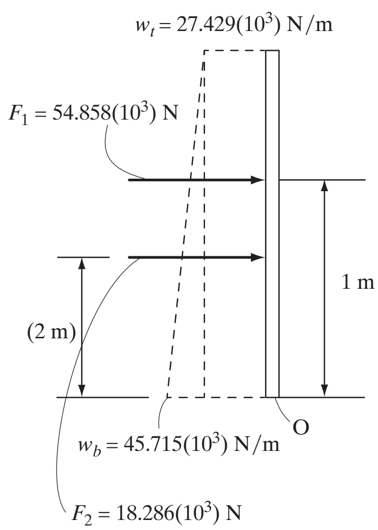
Thus, the resultant force is

$$F_R = F_1 + F_2 = 54.858(10^3)\ \text{N} + 18.286(10^3)\ \text{N} = 73.143(10^3)\ \text{N} = 73.1\ \text{kN} \quad \textbf{Ans.}$$

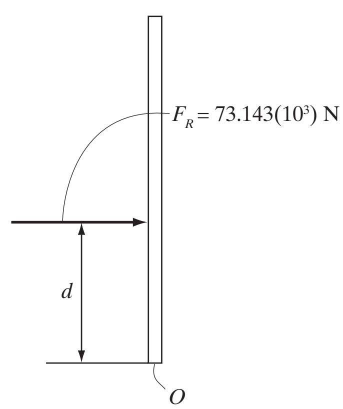
The location of the center of pressure can be determined by equating the sum of the moments of the forces in Figs. a and b about O.

$$\circlearrowleft +(M_R)_O = \Sigma M_O; \qquad \left[73.143(10^3)\ \text{N}\right]d = \left[54.858(10^3)\ \text{N}\right](1\ \text{m}) + \left[18.286(10^3)\ \text{N}\right]\left[\frac{1}{3}(2\ \text{m})\right]$$

$$d = 0.9167\ \text{m} = 917\ \text{mm} \quad \textbf{Ans.}$$



(a)

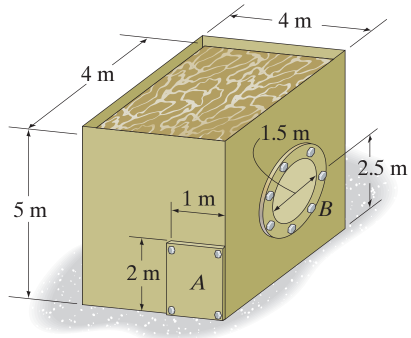


(b)

Ans:
$F_R = 73.1$ kN
$d = 917$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

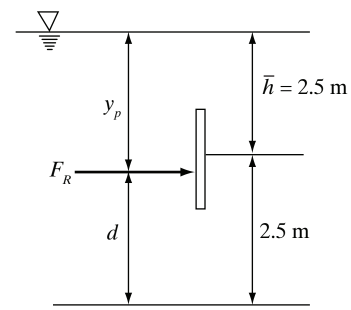
2–75. If the tank is filled with vegetable oil, determine the resultant force that the oil exerts on plate B, and its location measured from the bottom of the tank. Use the formula method. Take $\rho_{vo} = 932\ \text{kg/m}^3$.



SOLUTION

Since the plate is circular in shape, it is convenient to compute the resultant force as follows.

$$F_R = \gamma_{vo}\bar{h}A = \left(932\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.5\ \text{m})\left[\pi(0.75\ \text{m})^2\right]$$
$$= 40.392\left(10^3\right)\ \text{N} = 40.4\ \text{kN} \qquad \textbf{Ans.}$$



(a)

The location of the center of pressure can be determined from

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{\pi\dfrac{(0.75\ \text{m})^4}{4}}{(2.5\ \text{m})(\pi)(0.75\ \text{m})^2} + 2.5\ \text{m}$$
$$= 2.556\ \text{m}$$

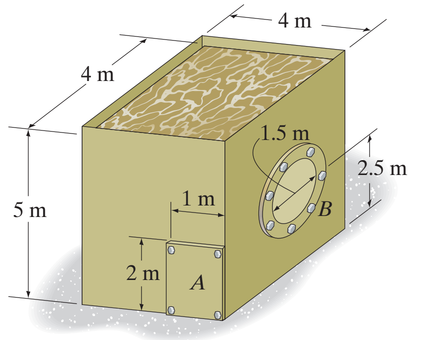
From the bottom of the tank, Fig. *a*,

$$d = 5\ \text{m} - y_P = 5\ \text{m} - 2.556\ \text{m} = 2.44\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 40.4\ \text{kN}$
$d = 2.44\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–76.** Solve Prob. 2–75 using the integration method.



SOLUTION

With respect to x and y axes established, the equation of the circumference of the circular plate is

$$x^2 + y^2 = 0.75^2; \qquad x = \sqrt{0.75^2 - y^2}$$

Thus, the area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\sqrt{0.75^2 - y^2}\,dy$. The pressure acting on this differential element is $p = \rho_{vo}gh = (932\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.5 - y) = 9142.92(2.5 - y)$. Thus, the resultant force acting on the entire plate is

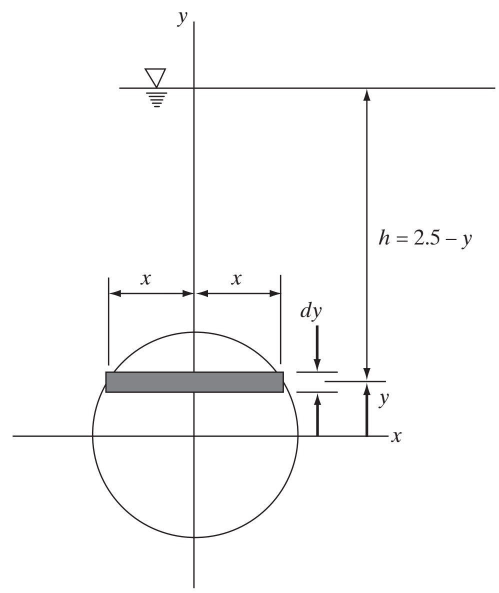
$$F_R = \int_A p\,dA = \int_{-0.75\text{ m}}^{0.75\text{ m}} 9142.92(2.5 - y)\left[2\sqrt{0.75^2 - y^2}\,dy\right]$$

$$= 18\,285.84 \int_{-0.75\text{ m}}^{0.75\text{ m}} (2.5 - y)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 22\,857.3\left[y\sqrt{0.75^2 - y^2} + 0.75^2 \sin^{-1}\frac{y}{0.75}\right]\Bigg|_{-0.75\text{ m}}^{0.75\text{ m}}$$

$$+\ 6095.28\sqrt{(0.75^2 - y^2)^3}\,\Big|_{-0.75\text{ m}}^{0.75\text{ m}}$$

$$= 40.39(10^3)\ \text{N} = 40.4\ \text{kN} \qquad \textbf{Ans.}$$



(a)

And it acts at

$$y_P = \frac{\int_A (2.5 - y)p\,dA}{F_R}$$

$$= \frac{1}{40.39(10^3)}\int_{-0.75\text{ m}}^{0.75\text{ m}} (2.5 - y)\left[9142.92(2.5 - y)\right]\left(2\sqrt{0.75^2 - y^2}\,dy\right)$$

$$= 0.4527\int_{-0.75\text{ m}}^{0.75\text{ m}} (6.25 + y^2 - 5y)\left(\sqrt{0.75^2 - y^2}\right)dy$$

$$= 1.4147\left[y\sqrt{0.75^2 - y^2} + 0.75^2 \sin^{-1}\frac{y}{a}\right]\Bigg|_{-0.75\,m}^{0.75\,m} + 0.4527\left[-\frac{y}{4}\sqrt{(0.75^2 - y^2)^3}\right.$$

$$\left. + \frac{0.75^2}{8}\left(y\sqrt{0.75^2 - y^2} + a^2 \sin^{-1}\frac{y}{0.75}\right)\right]\Bigg|_{-0.75\text{ m}}^{0.75\text{ m}} + 0.75450\Big|_{-0.75\text{ m}}^{0.75\text{ m}}$$

$$= 2.5562\ \text{m}$$

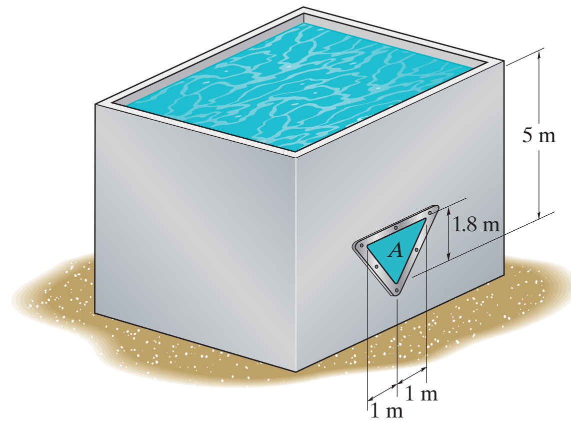
From the bottom of tank is

$$d = 5\ \text{m} - y_p = 5\ \text{m} - 2.5562\ \text{m} = 2.44\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 40.4\ \text{kN},\ d = 2.44\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–77. Determine the resultant force acting on the triangular plate A and the location of the center of pressure, measured from the free water level in the tank. Solve the problem using the formula method.



SOLUTION

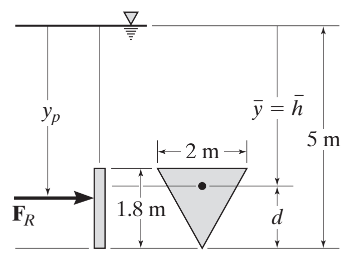
Referring to the dimensions shown in Fig. *a*,

$$d = \frac{2}{3}(1.8\ \text{m}) = 1.20\ \text{m} \quad \bar{y} = \bar{h} = 5\ \text{m} - 1.20\ \text{m} = 3.80\ \text{m}$$

Also,

$$A = \frac{1}{2}bh = \frac{1}{2}(2\ \text{m})(1.8\ \text{m}) = 1.80\ \text{m}^2$$

$$\bar{I}_x = \frac{1}{36}bh^3 = \frac{1}{36}(2\ \text{m})(1.8\ \text{m})^3 = 0.324\ \text{m}^4$$



(a)

The resultant force can now be determined:

$$F_R = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3.80\ \text{m})(1.80\ \text{m}^2)$$
$$= 67.10(10^3)\ \text{N} = 67.1\ \text{kN}$$ **Ans.**

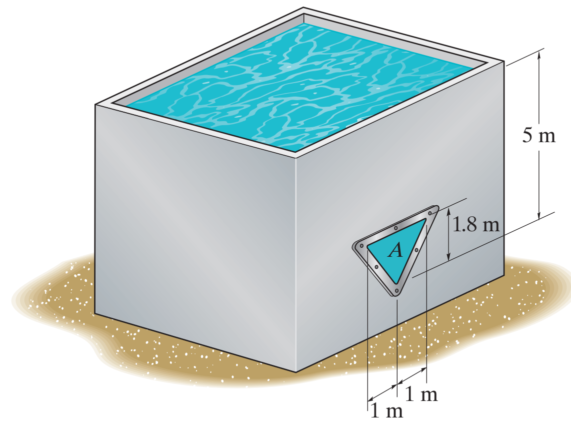
And it acts at

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.324\ \text{m}^4}{(3.80\ \text{m})(1.80\ \text{m}^2)} + 3.80\ \text{m}$$
$$= 3.8474\ \text{m} = 3.85\ \text{m}$$ **Ans.**

Ans:
$F_R = 67.1\ \text{kN},\ y_P = 3.85\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–78. Solve Prob. 2–77 using the integration method.

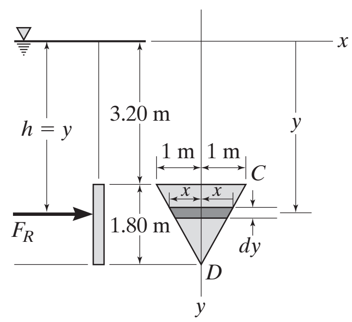


SOLUTION

With respect to x and y axes established, the equation of side CD of the plate, Fig. a, is

$$\frac{y-5}{x-0} = \frac{3.20-5}{1-0}; \quad x = \frac{5}{9}(5-y)$$

The area of the differential element shown shaded in Fig. a is $d_A = 2x\,dy = 2\left[\frac{5}{9}(5-y)\right]dy = \frac{10}{9}(5-y)dy$. The pressure acting on this differential element is $p = \gamma h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)y = 9810y$. Thus, the resultant force acting on the entire plate is



(a)

$$F_R = \int_A p\,dA = \int_{3.20\text{ m}}^{5\text{ m}} (9810y)\left[\frac{10}{9}(5-y)dy\right]$$

$$= 10\,900\int_{3.20}^{5\text{ m}} (5y - y^2)dy$$

$$= 10\,900\left(\frac{5}{2}y^2 - \frac{y^3}{3}\right)\Bigg|_{320\text{ m}}^{5}$$

$$= 67.10(10^3)\ \text{N} = 67.1\ \text{kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_p = \frac{\int_A yp\,dA}{F_R}$$

where

$$\int_A yp\,dA = \int_{3.20\text{ m}}^{5\text{ m}} y(9810y)\left[\frac{10}{9}(5-y)dy\right]$$

$$= 10\,900\int_{3.20\text{ m}}^{5\text{ m}} (5y^2 - y^3)dy$$

$$= 10\,900\left(\frac{5}{3}y^3 - \frac{y^4}{4}\right)\Bigg|_{3.20\text{ m}}^{5\text{ m}}$$

$$= 258.16(10^3)\ \text{N}\cdot\text{m}$$

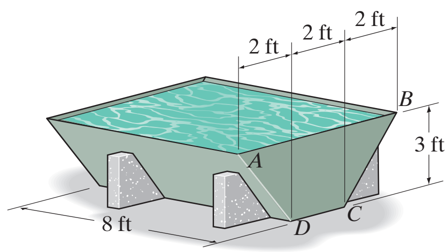
Then

$$y_p = \frac{258.16(10^3)\ \text{N}\cdot\text{m}}{67.10(10^3)\ \text{N}} = 3.8474\ \text{m} = 3.85\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 67.1\ \text{kN}$
$y_P = 3.85\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–79. The open wash tank is filled to its top with butyl alcohol, an industrial solvent. Determine the magnitude of the resultant force on the end plate *ABCD* and the location of the center of pressure, measured from *AB*. Solve the problem using the formula method. Take $\gamma_{ba} = 50.1\ \text{lb/ft}^3$.



SOLUTION

First, the location of the centroid of plate *ABCD*, Fig. *a*, measured from edge *AB* must be determined.

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{(1.5\text{ ft})\left[3\text{ ft}(2\text{ ft})\right] + (1\text{ ft})\left[\frac{1}{2}(4\text{ ft})(3\text{ ft})\right]}{3\text{ ft}(2\text{ ft}) + \frac{1}{2}(4\text{ ft})(3\text{ ft})} = 1.25\text{ ft}$$

Then, the moment of inertia of plate *ABCD* about its centroid $\bar{x}$ axis is

$$\bar{I}_x = \left[\frac{1}{12}(2\text{ ft})(3\text{ ft})^3 + 2\text{ ft}(3\text{ ft})(1.5\text{ ft} - 1.25\text{ ft})^2\right] + \left[\frac{1}{36}(4\text{ ft})(3\text{ ft})^3 + \frac{1}{2}(4\text{ ft})(3\text{ ft})(1.25\text{ ft} - 1\text{ ft})^2\right] = 8.25\text{ ft}^4$$

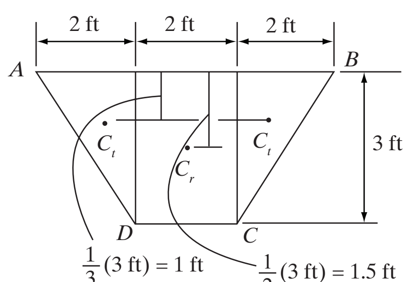
The area of plate *ABCD* is

$$A = 3\text{ ft}(2\text{ ft}) + \frac{1}{2}(4\text{ ft})(3\text{ ft}) = 12\text{ ft}^2$$

Thus,

$$F_R = \gamma\bar{h}A = \left(50.1\text{ lb/ft}^3\right)(1.25\text{ ft})\left(12\text{ ft}^2\right) = 751.5\text{ lb} = 752\text{ lb} \qquad \textbf{Ans.}$$

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{8.25}{1.25(12)} + 1.25 = 1.80\text{ ft} \qquad \textbf{Ans.}$$

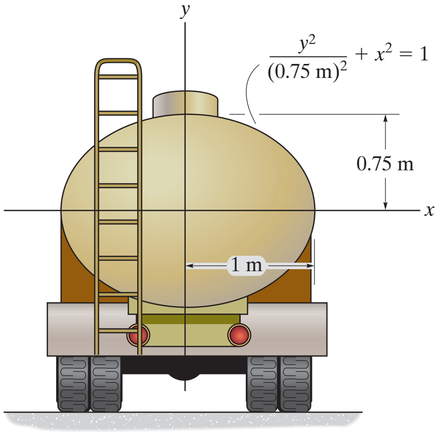


(a)

Ans:
$F_R = 752\text{ lb}$
$y_P = 1.80\text{ ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–80.** The tank truck is filled to its top with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the top of the tank. Solve the problem using the formula method.



SOLUTION

Using Table 2-1 for the area and moment of inertia about the centroidal $\bar{x}$ axis of the elliptical plate, we get

$$F = \rho_w g \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m})(\pi)(0.75\ \text{m})(1\ \text{m})$$

$$= 17.3\ \text{kN} \qquad \textbf{Ans.}$$

The center of pressure is at

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$

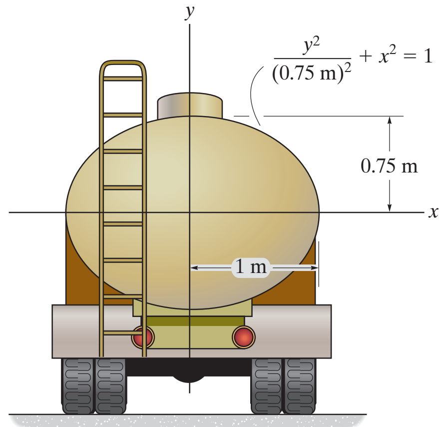
$$= \frac{\left[\frac{1}{4}\pi(1\ \text{m})(0.75\ \text{m})^3\right]}{(0.75\ \text{m})\pi(1\ \text{m})(0.75\ \text{m})} + 0.75\ \text{m}$$

$$= 0.9375\ \text{m} = 0.938\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–81. Solve Prob. 2–80 using the integration method.



SOLUTION

By integration of a horizontal strip of area,

$$dF = p\,dA = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(0.75\ \text{m} - y)(2x\,dy)$$

$$F = 19\,620\int_{-0.75\ \text{m}}^{0.75\ \text{m}}(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy$$

$$= 19\,620\left[\int_{-0.75\ \text{m}}^{0.75\ \text{m}}\sqrt{(0.75)^2 - y^2}\,dy - \frac{1}{0.75}\int_{-0.75\ \text{m}}^{0.75\ \text{m}} y\sqrt{(0.75)^2 - y^2}\,dy\right]$$

$$= \frac{19\,620}{2}\left[y\sqrt{(0.75)^2 - y^2} + (0.75)^2\sin^{-1}\frac{y}{0.75}\right]_{-0.75}^{0.75} - \frac{19\,620}{0.75}\left[-\frac{1}{3}\sqrt{\left((0.75)^2 - y^2\right)^3}\right]_{-0.75}^{0.75}$$

$$= \frac{19\,620\pi(0.75)^2}{2} - 0 = 17\,336\ \text{N} = 17.3\ \text{kN}$$ **Ans.**

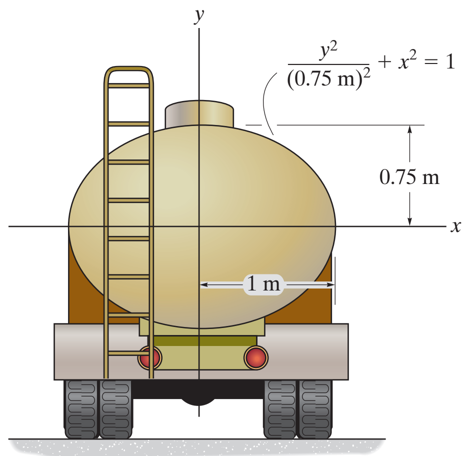
$$y_P = \frac{19\,620\displaystyle\int_{-0.75\ \text{m}}^{0.75\ \text{m}} y(0.75 - y)\left(1 - \frac{y^2}{(0.75)^2}\right)^{\frac{1}{2}}dy}{17\,336\ \text{N}} = -0.1875\ \text{m}$$

$$y_P = 0.75\ \text{m} + 0.1875\ \text{m} = 0.9375\ \text{m} = 0.938\ \text{m}$$ **Ans.**

Ans:
$F = 17.3\ \text{kN}$
$y_P = 0.938\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–82. The tank truck is half filled with water. Determine the magnitude of the resultant force on the elliptical back plate of the tank, and the location of the center of pressure measured from the x axis. Solve the problem using the formula method. *Hint*: The centroid of a semi-ellipse measured from the x axis is $\bar{y} = 4b/3\pi$.



SOLUTION

From Table 2-1, the area and moment of inertia about the x axis of the half-ellipse plate are

$$A = \frac{\pi}{2}ab = \frac{\pi}{2}(1\ \text{m})(0.75\ \text{m}) = 0.375\pi\ \text{m}^2$$

$$I_x = \frac{1}{2}\left(\frac{\pi}{4}ab^3\right) = \frac{1}{2}\left[\frac{\pi}{4}(1\ \text{m})(0.75\ \text{m})^3\right] = 0.05273\pi\ \text{m}^4$$

Thus, the moment of inertia of the half of ellipse about its centroidal $\bar{x}$ axis can be determined by using the parallel-axis theorem.

$$I_x = \bar{I}_x + Ad_y^2$$

$$0.05273\pi\ \text{m}^4 = \bar{I}_x + (0.375\ \pi)\left[\frac{4(0.75\ \text{m})}{3\pi}\right]^2$$

$$\bar{I}_x = 0.046304\ \text{m}^4$$

Since $\bar{h} = \dfrac{4(0.75\ \text{m})}{3\pi} = 0.3183\ \text{m}$, then

$$F_R = \gamma\bar{h}A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3183\ \text{m})(0.375\pi\ \text{m}^2)$$
$$= 3.679(10^3)\ \text{N} = 3.68\ \text{kN} \qquad \textbf{Ans.}$$

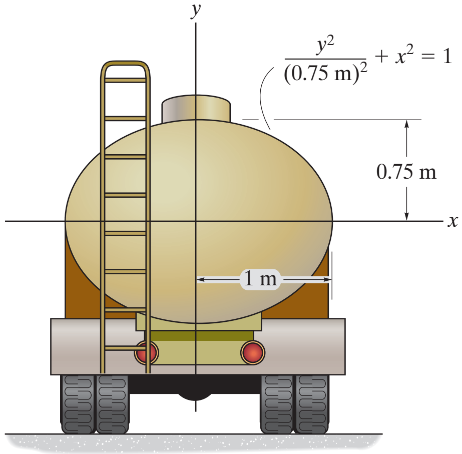
Since $\bar{y} = \bar{h} = 0.3183\ \text{m}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{0.046304\ \text{m}^4}{(0.3183\ \text{m})(0.375\pi\ \text{m}^2)} + 0.3183\ \text{m}$$
$$= 0.4418\ \text{m} = 442\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F_R = 3.68\ \text{kN}$
$y_P = 442\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–83. Solve Prob. 2–82 using the integration method.



SOLUTION

Using a horizontal strip of area dA,

$$dF = p\,dA$$

$$dF = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-y)(2x\,dy)$$

$$F = -19\,620 \int_{-0.75\text{ m}}^{0} (y)\left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}} dy$$

$$= -\frac{19\,620}{0.75} \int_{-0.75\text{ m}}^{0} y\sqrt{0.75^2 - y^2}\,dy$$

$$= \frac{26\,160}{3}\left[\sqrt{(0.75^2 - y^2)^3}\right]_{-0.75\text{ m}}^{0}$$

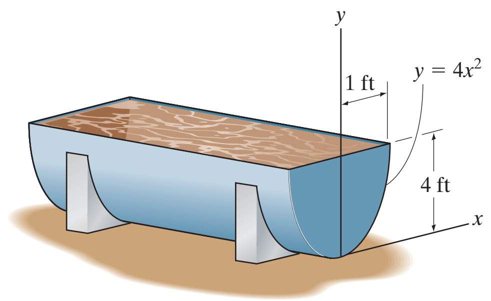
$$= 3.679(10^3)\ \text{N} = 3.68\ \text{kN} \qquad \textbf{Ans.}$$

$$y_P = -\frac{-19\,620 \int_{-0.75\text{ m}}^{0} y(y)\left(1 - \frac{y^2}{0.75^2}\right)^{\frac{1}{2}} dy}{3678.75\ \text{N}} = 0.4418\ \text{m} = 442\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F_R = 3.68\ \text{kN}$
$y_P = 442\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–84.** The trough is filled to its top with carbon disulfide. Determine the magnitude of the resultant force acting on the parabolic end plate, and the location of the center of pressure measured from the top. Solve the problem using the formula method. Take $\rho_{cd} = 2.46\ \text{slug/ft}^3$.



SOLUTION

From Table 2-1, the area and moment of inertia about the centroidal $\bar{x}$ axis of the parabolic plate are

$$A = \frac{2}{3}bh = \frac{2}{3}(2\ \text{ft})(4\ \text{ft}) = 5.3333\ \text{ft}^2$$

$$\bar{I}_x = \frac{8}{175}bh^3 = \frac{8}{175}(2\ \text{ft})(4\ \text{ft})^3 = 5.8514\ \text{ft}^4$$

With $\bar{h} = \frac{2}{5}h = \frac{2}{5}(4\ \text{ft}) = 1.6\ \text{ft}$,

$$F_R = \gamma\bar{h}A = (2.46\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(1.6\ \text{ft})(5.3333\ \text{ft}^2)$$
$$= 675.94\ \text{lb} = 676\ \text{lb} \qquad \textbf{Ans.}$$

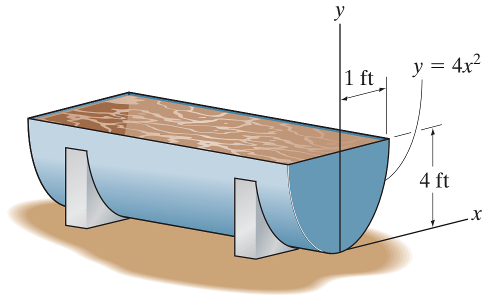
Since $\bar{y} = \bar{h} = 1.6\ \text{ft}$,

$$y_P = \frac{\bar{I}_x}{\bar{y}A} + \bar{y}$$
$$= \frac{5.8514\ \text{ft}^4}{(1.6\ \text{ft})(5.3333\ \text{ft}^2)} + 1.6\ \text{ft}$$
$$= 2.2857\ \text{ft} = 2.29\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 676\ \text{lb}$
$y_P = 2.29\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–85. Solve Prob. 2–84 using the integration method.



SOLUTION

Using a horizontal strip of area,

$$F_R = \int_A p\,dA = \int_0^{4\text{ ft}} (2.46\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(4 - y)2x\,dy$$

$$= 158.424 \int_0^{4\text{ ft}} (4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$= 79.212\left(\int_0^{4\text{ ft}} \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}}\right)dy\right)$$

$$= 79.212\left(\frac{8}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 675.94\ \text{lb} = 676\ \text{lb} \qquad \textbf{Ans.}$$

$$F_R(d) = \int_A y(p\,dA) = 158.424 \int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)\left(y^{\frac{1}{2}}\right)dy$$

$$(675.94\ \text{lb})(d) = 158.424 \int_0^{4\text{ ft}} y(4 - y)\left(\frac{1}{2}\right)y^{\frac{1}{2}}\,dy$$

$$= 79.212 \int_0^{4\text{ ft}} \left(4y^{\frac{3}{2}} - y^{\frac{5}{2}}\right)dy$$

$$= 79.212\left(\frac{8}{5}y^{\frac{5}{2}} - \frac{2}{7}y^{\frac{7}{2}}\right)\Bigg|_0^{4\text{ ft}}$$

$$= 1158.76\ \text{lb}\cdot\text{ft}$$

$$d = \frac{1158.76\ \text{lb}\cdot\text{ft}}{675.94\ \text{lb}} = 1.7143\ \text{ft}$$

$$y_P = 4\ \text{ft} - d$$

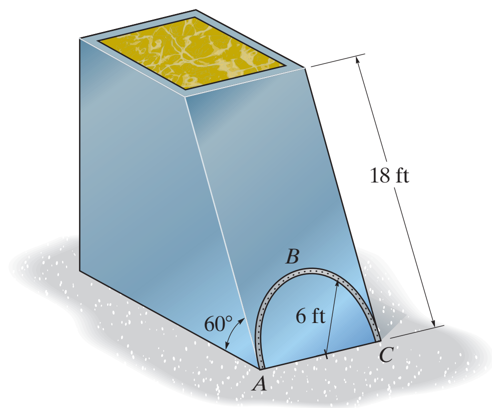
$$= 4\ \text{ft} - 1.7143\ \text{ft}$$

$$= 2.2857\ \text{ft} = 2.29\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 676$ lb
$y_P = 2.29$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–86. The tank is filled to its top with lubricating oil. Determine the resultant force acting on the semicircular plate *ABC*, and its location on the plate measured from the base *AC* of the tank. Use the formula method. Take $\gamma_o = 54.9\ \text{lb/ft}^3$.



SOLUTION

Referring to the dimensions shown in Fig. *a*,

$$\bar{r} = \frac{4r}{3\pi} = \frac{4(6\text{ ft})}{3\pi} = \frac{8}{\pi}\text{ ft}$$

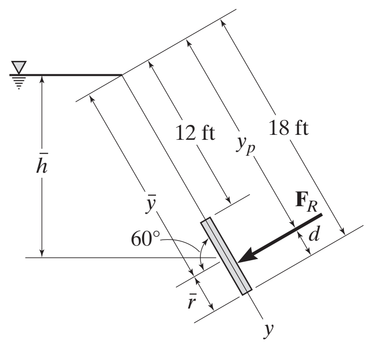
$$\bar{y} = 18\text{ ft} - \frac{8}{\pi}\text{ ft} = 15.4535\text{ ft}$$

$$\bar{h} = (15.4535\text{ ft})\sin 60° = 13.3831\text{ ft}$$

Also,

$$A = \frac{1}{2}(\pi r^2) = \frac{1}{2}[(6\text{ ft})^2] = 18\pi\text{ ft}^2$$

$$\bar{I}_x = \frac{\pi}{8}r^4 - \frac{1}{2}(\pi r^2)\left(\frac{4r}{3\pi}\right) = \left(\frac{9\pi^2 - 64}{72\pi}\right) = \left(\frac{9\pi^2 - 64}{72\pi}\right)(6\text{ ft})^4 = 142.25\text{ ft}^4$$



(a)

The resultant force can now be determined:

$$F_R = \gamma_o\bar{h}A = (54.9\text{ lb/ft}^3)(13.3831\text{ ft})(18\pi\text{ ft}^2)$$
$$= 41.548(10^3)\text{ lb} = 41.5\text{ kip} \qquad \textbf{Ans.}$$

And it acts at

$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{142.25\text{ ft}^4}{(15.4535\text{ ft})(18\pi\text{ ft}^2)} + 15.4535\text{ ft} = 15.6163\text{ ft}$$

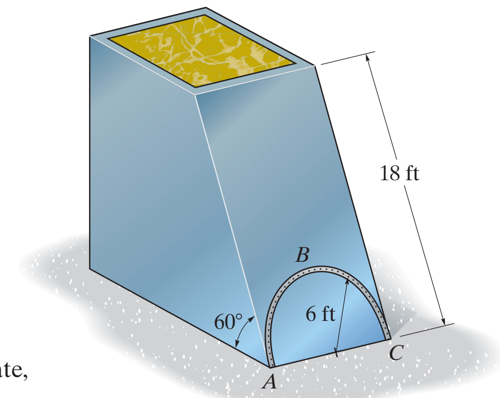
Thus,

$$d = 18\text{ ft} - 15.6163\text{ ft} = 2.3837\text{ ft} = 2.38\text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 41.5$ kip
$d = 2.38$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

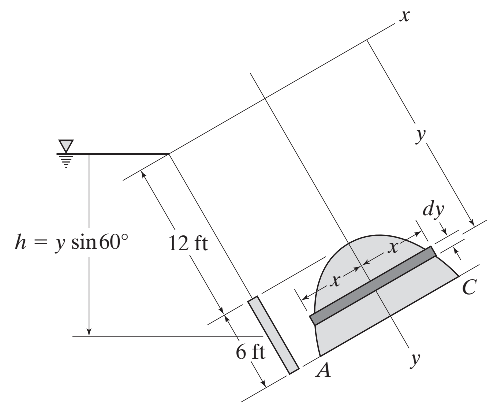
2–87. Solve Prob. 2–86 using the integration method.



SOLUTION

With respect to the x and y axes established, the equation of the semicircular plate, Fig. a, is

$$(x - 0)^2 + (y - 18)^2 = 6^2$$
$$x^2 + (y - 18)^2 = 36$$
$$x = \sqrt{36 - (y - 18)^2}$$



(a)

The area of the differential element shown shaded in Fig. a is $dA = 2x\,dy = 2\sqrt{36 - (y - 18)^2}\,dy$. The pressure acting on this differential element is $p = \gamma h = (54.9\ \text{lb/ft}^3)(y \sin 60°) = 47.5448y$. Thus, the resultant force acting on the entire plate is

$$F_R = \int_A p\,dA = \int_{12\ \text{ft}}^{18\ \text{ft}} (47.5448y)\left[2\sqrt{36 - (y - 18)^2}\,dy\right]$$

$$= 95.0896 \int_{12\ \text{ft}}^{18\ \text{ft}} y\sqrt{36 - (y - 18)^2}\,dy$$

$$= 95.0896\left\{-\frac{1}{3}\sqrt{[36 - (y - 18)^2]^3}\Big|_{12\ \text{ft}}^{18\ \text{ft}} + 9\left[(y - 18)\sqrt{36 - (y - 18)^2} + 36 \sin^{-1}\left(\frac{y - 18}{6}\right)\right]\Big|_{12\ \text{ft}}^{18\ \text{ft}}\right\}$$

$$= 41.548(10^3)\ \text{lb} = 41.5\ \text{kip} \qquad \textbf{Ans.}$$

And it acts at

$$y_p = \frac{\int_A yp\,dA}{F_R}$$

where

$$\int_A yp\,dA = \int_{12\ \text{ft}}^{18\ \text{ft}} y(47.5448y)\left[2\sqrt{36 - (y - 18)^2}\,dy\right]$$

$$= 95.0896 \int_{12\ \text{ft}}^{18\ \text{ft}} y^2\sqrt{36 - (y - 18)^2}\,dy$$

$$= 95.0896\left\{-\left(\frac{y - 18}{4}\right)\sqrt{[36 - (y - 18)^2]^3}\Big|_{12\ \text{ft}}^{18\ \text{ft}} - 12\sqrt{[36 - (y - 18)^2]^3}\Big|_{12\ \text{ft}}^{18\ \text{ft}}\right.$$
$$\left. + \frac{333}{2}\left[(y - 18)\sqrt{36 - (y - 18)^2} + 36 \sin^{-1}\left(\frac{y - 18}{6}\right)\right]\Big|_{12\ \text{ft}}^{18\ \text{ft}}\right\}$$

$$= 648\,829.85\ \text{lb} \cdot \text{ft}$$

Then

$$y_p = \frac{648\,829.85\ \text{lb} \cdot \text{ft}}{41.548(10^3)\ \text{lb}} = 15.6163\ \text{ft}$$

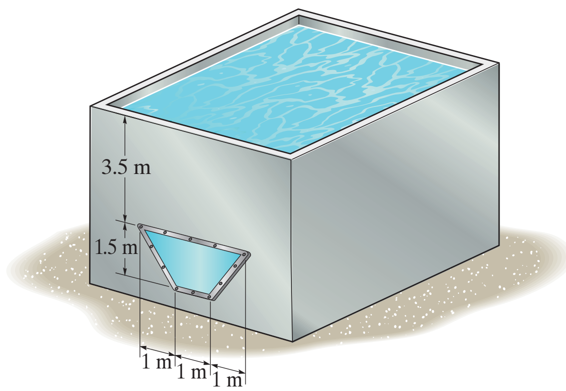
Thus,

$$d = 18 - y_p = 2.3837\ \text{ft} = 2.38\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 41.5$ kip
$d = 2.38$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–88.** The tank is filled with water. Determine the resultant force acting on the trapezoidal plate C and the location of the center of pressure, measured from the top of the tank. Solve the problem using the formula method.

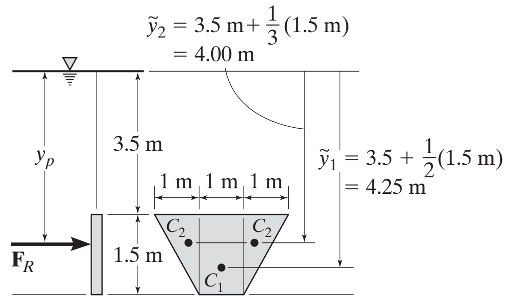


SOLUTION

Referring to the geometry shown in Fig. *a*,

$$A = \frac{1}{2}(2\ \text{m})(1.5\ \text{m}) + (1\ \text{m})(1.5\ \text{m}) = 3.00\ \text{m}^2$$

$$\bar{h} = \bar{y} = \frac{(4.25\ \text{m})[(1\ \text{m})(1.5\ \text{m})] + (4.00\ \text{m})\left[\frac{1}{2}(2\ \text{m})(1.5\ \text{m})\right]}{(1\ \text{m})(1.5\ \text{m}) + \frac{1}{2}(2\ \text{m})(1.5\ \text{m})} = 4.125\ \text{m}$$



(a)

$$\bar{I}_x = \frac{1}{12}(1\ \text{m})(1.5\ \text{m})^3 + (1\ \text{m})(1.5\ \text{m})(4.25\ \text{m} - 4.125\ \text{m})^2$$

$$+\frac{1}{36}(2\ \text{m})(1.5\ \text{m})^3 + \left[\frac{1}{2}(2\ \text{m})(1.5\ \text{m})\right](4.125\ \text{m} - 4.00\ \text{m})^2$$

$$= 0.515625\ \text{m}^4$$

The resultant force can now be determined:

$$F_R = \gamma_w \bar{h} A = (1000\ \text{kg/m}^2)(9.81\ \text{m/s}^2)(4.125\ \text{m})(3.00\ \text{m}^2)$$

$$= 121.40(10^3)\ \text{N} = 121\ \text{kN} \qquad \textbf{Ans.}$$

And it acts at

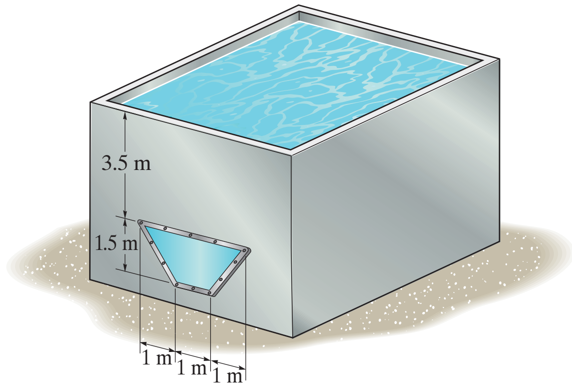
$$y_p = \frac{\bar{I}_x}{\bar{y}A} + \bar{y} = \frac{0.515625\ \text{m}^4}{(4125\ \text{m})(3.00\ \text{m}^2)} + 4.125\ \text{m}$$

$$= 4.1667\ \text{m} = 4.17\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 121\ \text{kN}$
$y_p = 4.17\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–89. Solve Prob. 2–88 using the integration method.

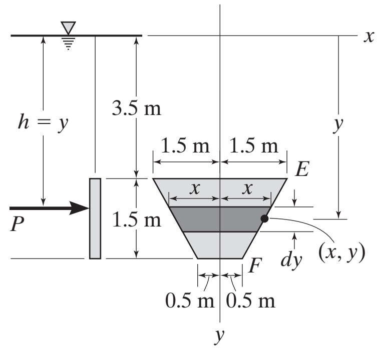


SOLUTION

With respect to the x and y axes established, the equation of side EF of the plate, Fig. a, is

$$\frac{y-3.5}{x-1.5} = \frac{5-3.5}{0.5-1.5}; \quad x = 3.8333 - 0.6667y$$

The area of the differential element shown shaded in Fig. a is $dA = 2x\,dy =$. $2(3.8333 - 0.6667y)dy = (7.6667 - 1.3333y)dy$. The pressure acting on this differential element is $p = \gamma h = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)y = 9810y$. Thus, the resultant force acting on the entire plate is



(a)

$$F_R = \int_A p\,dA = \int_{3.5\text{ m}}^{5\text{ m}} (9810y)(7.6667 - 1.3333y)\,dy$$

$$= 9810\int_{3.5\text{ m}}^{5\text{ m}} (7.6667y - 1.3333y^2)\,dy$$

$$= 9810(3.8333y^2 - 0.4444y^3)\Big|_{3.5\text{ m}}^{5\text{ m}}$$

$$= 121.40(10^3)\ \text{N} = 121\ \text{kN} \qquad \textbf{Ans.}$$

And it acts at

$$y_p = \frac{\int_A yp\,dA}{F_R}$$

where

$$\int_A yp\,dA = \int_{3.5\text{ m}}^{5\text{ m}} y(9810y)(7.6667 - 1.3333y)\,dy$$

$$= 9810\int_{3.5\text{ m}}^{5\text{ m}} (7.6667y^2 - 1.3333y^3)\,dy$$

$$= 9810(2.5556y^3 - 0.3333y^4)\Big|_{3.5\text{ m}}^{5\text{ m}}$$

$$= 505.83(10^3)\ \text{N}\cdot\text{m}$$

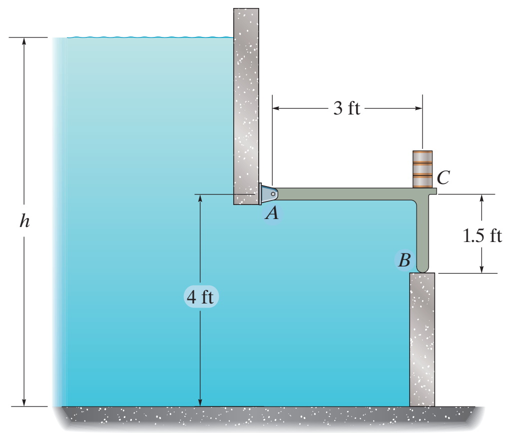
Thus,

$$y_p = \frac{505.83(10^3)\ \text{N}\cdot\text{m}}{121.40(10^3)\ \text{N}} = 4.1667\ \text{m} = 4.17\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 121$ kN
$y_P = 4.17$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–90. The control gate *ACB* is pinned at *A* and rests on the smooth surface at *B*. Determine the amount of weight that should be placed at *C* in order to maintain a reservoir depth of $h = 10$ ft. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed load at *C* and *B* shown in Fig. *a* are

$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(3\ \text{ft}) = 1123.2\ \text{lb/ft}$$

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(7.5\ \text{ft})(3\ \text{ft}) = 1404\ \text{lb/ft}$$

Thus,

$$F_1 = (1123.2\ \text{lb/ft})(3\ \text{ft}) = 3369.6\ \text{lb}$$

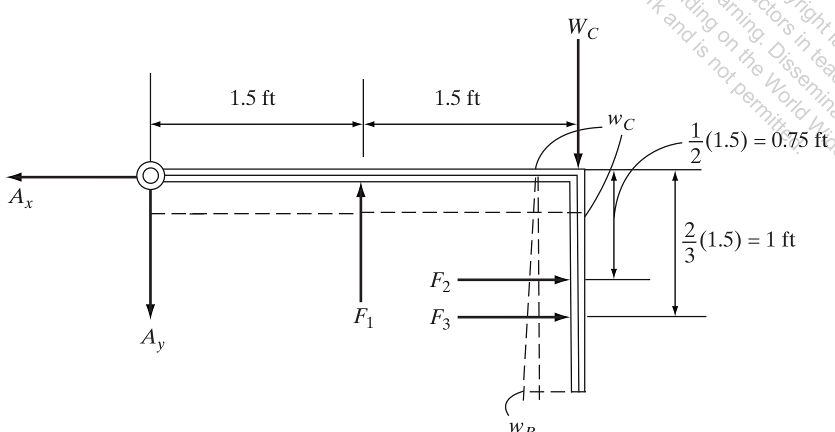
$$F_2 = (1123.2\ \text{lb/ft})(1.5\ \text{ft}) = 1684.8\ \text{lb}$$

$$F_3 = \frac{1}{2}\left[(1404 - 1123.2\ \text{lb/ft})\right](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is about to be opened, $N_B = 0$. Write the moment equation of equilibrium about point *A* by referring to Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad (3369.6\ \text{lb})(1.5\ \text{ft}) + (1684.8\ \text{lb})(0.75\ \text{ft}) + (210.6\ \text{lb})(1\ \text{ft}) - w_C(3\ \text{ft}) = 0$$

$$W_C = 2176.2\ \text{lb} = 2.18\ \text{kip} \qquad \textbf{Ans.}$$

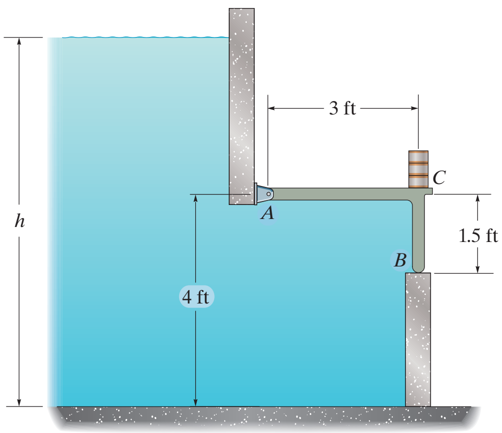


(a)

Ans:
$W_C = 2.18$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–91. The control gate *ACB* is pinned at *A* and rests on the smooth surface at *B*. If the counterweight *C* is 2000 lb, determine the maximum depth of water *h* in the reservoir before the gate begins to open. The gate has a width of 3 ft. Neglect its weight.



SOLUTION

The intensities of the distributed loads at *C* and *B* are shown in Fig. *a*.

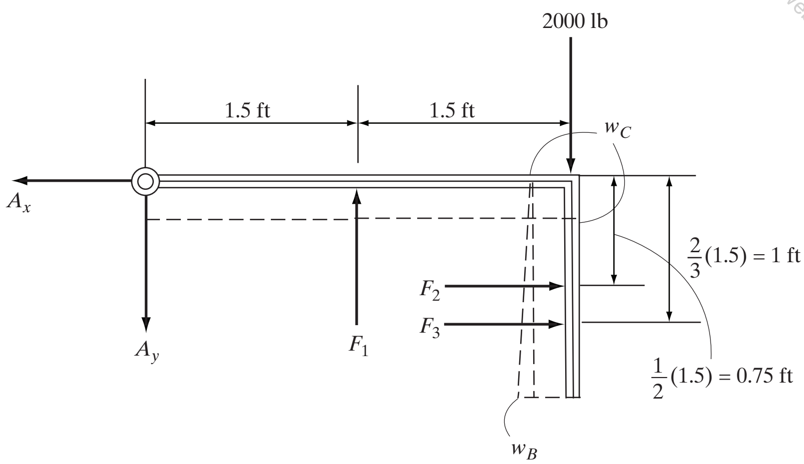
$$w_C = \gamma_w h_C b = \left(62.4\ \text{lb/ft}^3\right)(h - 4\ \text{ft})(3\ \text{ft}) = \left[187.2(h-4)\right]\ \text{lb/ft}$$
$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(h - 2.5\ \text{ft})(3\ \text{ft}) = \left[187.2(h-2.5)\right]\ \text{lb/ft}$$

Thus,

$$F_1 = (187.2(h-4)\ \text{lb/ft})(3\ \text{ft}) = 561.6(h-4)\ \text{lb}$$
$$F_2 = (187.2(h-4)\ \text{lb/ft})(1.5\ \text{ft}) = 280.8(h-4)\ \text{lb}$$
$$F_3 = \frac{1}{2}\left[187.2(h-2.5)\ \text{lb/ft} - (187.2(h-4)\ \text{lb/ft}\right](1.5\ \text{ft}) = 210.6\ \text{lb}$$

Since the gate is required to be opened, $N_B = 0$. Write the moment equation of equilibrium about point *A* by referring to Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad \left[561.6(h-4)\ \text{lb}\right](1.5\ \text{ft}) + \left[280.8(h-4)\ \text{lb}\right](0.75\ \text{ft})$$
$$+ (210.6\ \text{lb})(1\ \text{ft}) - (2000\ \text{lb})(3\ \text{ft}) = 0$$
$$1053(h-4) = 5789.4$$
$$h = 9.498\ \text{ft} = 9.50\ \text{ft} \qquad \textbf{Ans.}$$

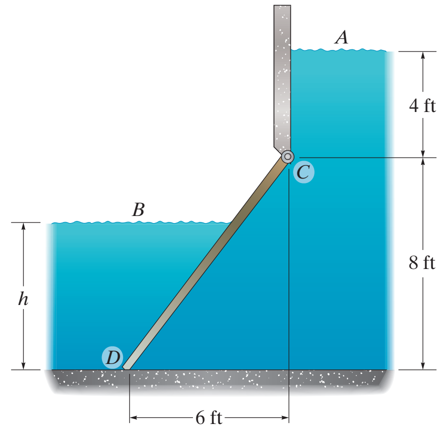


(a)

Ans:
$h = 9.50$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

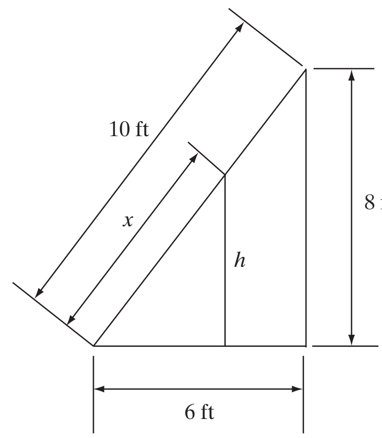
***2–92.** The uniform plate, which is hinged at C, is used to control the level of the water at A to maintain its constant depth of 12 ft. If the plate has a width of 8 ft and a weight of $50(10^3)$ lb, determine the minimum height h of the water at B so that seepage will not occur at D.



SOLUTION

Referring to the geometry in Fig. *a*,

$$\frac{x}{10} = \frac{h}{8}; \qquad x = \frac{5}{4}h$$



(a)

The intensities of the distributed load shown in Fig. *b* are

$$w_1 = \gamma_w h_1 b = \left(62.4\ \text{lb/ft}^3\right)(4\ \text{ft})(8\ \text{ft}) = 1996.8\ \text{lb/ft}$$
$$w_2 = \gamma_w h_2 b = \left(62.4\ \text{lb/ft}^3\right)(12\ \text{ft})(8\ \text{ft}) = 5990.4\ \text{lb/ft}$$
$$w_3 = \gamma_w h_3 b = \left(62.4\ \text{lb/ft}^3\right)(h)(8\ \text{ft}) = (499.2h)\ \text{lb/ft}$$

Thus, the resultant forces of these distributed loads are

$$F_1 = (1996.8\ \text{lb/ft})(10\ \text{ft}) = 19\,968\ \text{lb}$$
$$F_2 = \frac{1}{2}(5990.4\ \text{lb/ft} - 1996.8\ \text{lb/ft})(10\ \text{ft}) = 19\,968\ \text{lb}$$
$$F_3 = \frac{1}{2}(499.2h\ \text{lb/ft})\left(\frac{5}{4}h\right) = \left(312h^2\right)\ \text{lb}$$

and act at

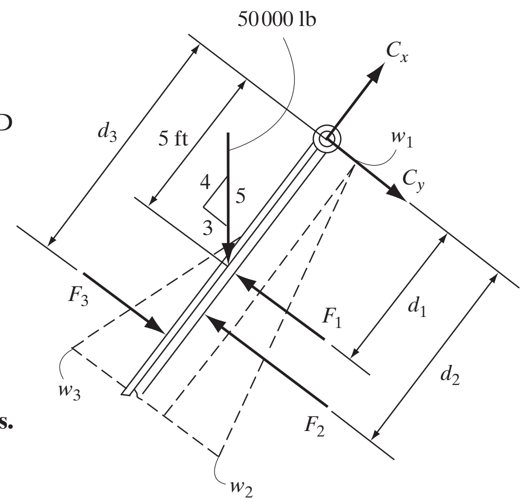
$$d_1 = \frac{10\ \text{ft}}{2} = 5\ \text{ft}$$
$$d_2 = \frac{2}{3}(10\ \text{ft}) = 6.667\ \text{ft}$$
$$d_3 = 10\ \text{ft} - \frac{1}{3}\left(\frac{5}{4}h\right) = (10 - 0.4167h)\ \text{ft}$$

For seepage to occur, the reaction at D must be equal to zero. Referring to the FBD of the gate, Fig. *b*,

$$\zeta + \Sigma M_C = 0; \qquad (50\,000\ \text{lb})\left(\frac{3}{5}\right)(5\ \text{ft}) + \left(312h^2\ \text{lb}\right)(10 - 0.4167h)\ \text{ft}$$
$$- (19968\ \text{lb})(5\ \text{ft}) - (19968\ \text{lb})(6.667\ \text{ft}) = 0$$
$$-130h^3 + 3120h^2 - 82\,960 = 0$$

Solving numerically,

$$h = 5.945\ \text{ft} = 5.95\ \text{ft} < 8\ \text{ft} \qquad \textbf{Ans.}$$

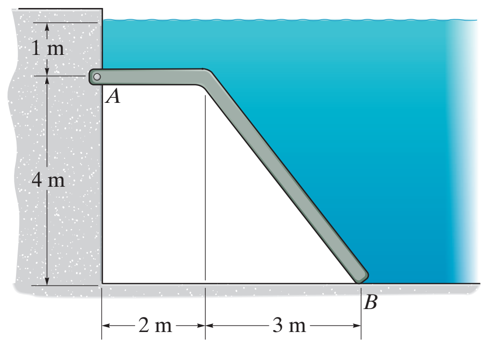


(b)

Ans:
$h = 5.95\ \text{ft} < 8\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–93. The bent plate is 1.5 m wide and is pinned at A and rests on a smooth support at B. Determine the horizontal and vertical components of reaction at A and the vertical reaction at the smooth support B. The fluid is water.



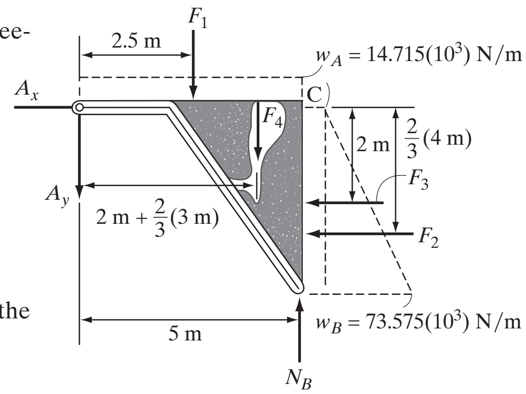
SOLUTION

Since the gate has a width of $b = 1.5$ m, the intensities of the distributed loads at A and B can be computed from

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(1.5\ \text{m}) = 14.715(10^3)\ \text{N/m}$$

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1\ \text{m})(1.5\ \text{m}) = 14.715(10^3)\ \text{N/m}$$

Using these results, the distributed load acting on the plate is shown on the free-body diagram of the gate, Fig. *a*.



(a)

$$F_1 = w_A L_{AB} = (14.715(10^3)\ \text{N/m})(5\ \text{m}) = 73.575(10^3)\ \text{N}$$

$$F_2 = \frac{1}{2}(w_B - w_A)L_{BC} = \frac{1}{2}(73.575(10^3)\ \text{N/m} - 14.715(10^3)\ \text{N/m})(4\ \text{m})$$

$$= 117.72(10^3)\ \text{N}$$

$$F_3 = w_A L_{BC} = (14.715(10^3)\ \text{N/m})(4\ \text{m}) = 58.86(10^3)\ \text{N}$$

$\mathbf{F}_4$ on the free-body diagram is equal to the weight of the water contained in the shaded triangular block, Fig. *a*.

$$F_4 = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(3\ \text{m})(4\ \text{m})(1.5\ \text{m})\right] = 88.29(10^3)\ \text{N}$$

Considering the free-body diagram of the gate, Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B(5\ \text{m}) - 73.575(10^3)\ \text{N}(2.5\ \text{m}) - 58.86(10^3)\ \text{N}(2\ \text{m}) - 117.72(10^3)\ \text{N}\left(\frac{2}{3}(4\ \text{m})\right)$

$$- 88.29(10^3)\ \text{N}\left(2\ \text{m} + \frac{2}{3}(3\ \text{m})\right) = 0$$

$N_B = 193.748(10^3)\ \text{N} = 194\ \text{kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 58.86(10^3)\ \text{N} - 117.72(10^3)\ \text{N} = 0$

$A_x = 176.58(10^3)\ \text{N} = 177\ \text{kN}$ **Ans.**

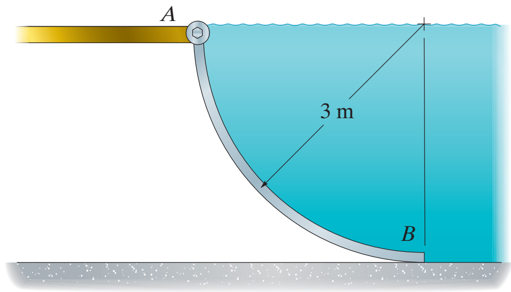
$+\uparrow \Sigma F_y = 0;$ $\quad -A_y - 73.575(10^3)\ \text{N} - 88.29(10^3)\ \text{N} + 193.748(10^3)\ \text{N} = 0$

$A_y = 31.88(10^3)\ \text{N} = 31.9\ \text{kN}$ **Ans.**

Ans:
$N_B = 194$ kN
$A_x = 177$ kN
$A_y = 31.9$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–94. The gate is 1.5 m wide, is pinned at A, and rests on the smooth support at B. Determine the reactions at these supports due to the water pressure.



SOLUTION

The horizontal loading on the plate is due to the pressure on the vertical projected area of the plate, Fig. *a*. Since the plate has a constant width of $b = 1.5$ m, the intensity of the horizontal distributed load at B is given by

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2(3\ \text{m})(1.5\ \text{m}) = 44.145(10^3)\ \text{N/m}$$

And its resultant force is

$$F_h = \frac{1}{2} w_B l_{DB} = \frac{1}{2}[44.145(10^3)\ \text{N/m}](3\ \text{m}) = 66.2175(10^3)\ \text{N} = 66.2175\ \text{kN}$$

The vertical force acting on the plate is equal to the weight of the water contained in the block shown shaded in Fig. *a*.

$$F_v = \rho_w g \mathcal{V}_{AOB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(3\ \text{m})^2(1.5\ \text{m})\right]$$

$$= 104.01(10^3)\ \text{N} = 104.01\ \text{kN}$$

Write the moment equations of equilibrium about points A and O by referring to the FBD of the plate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(3\ \text{m}) - (66.2175\ \text{kN})\left[\frac{2}{3}(3\ \text{m})\right] - (104.01\ \text{kN})\left(3 - \frac{4}{\pi}\right)\text{m} = 0$$

$$N_B = 104.01\ \text{kN} = 104\ \text{kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_O = 0; \quad A_y(3\ \text{m}) + (104.01\ \text{kN})\left(\frac{4}{\pi}\ \text{m}\right) - (66.2175\ \text{kN})\left[\frac{2}{3}(3\ \text{m})\right] = 0$$

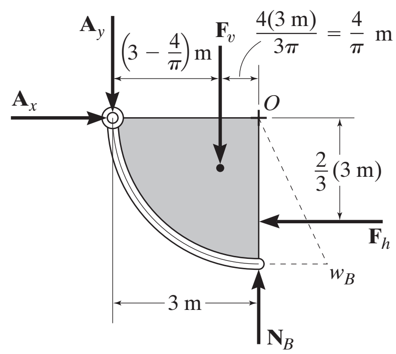
$$A_y = 0 \qquad \textbf{Ans.}$$

Write the force equation of equilibrium along the x axis.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 66.2175\ \text{kN} = 0$$

$$A_x = 66.2175\ \text{kN}$$

$$= 66.2\ \text{kN} \qquad \textbf{Ans.}$$

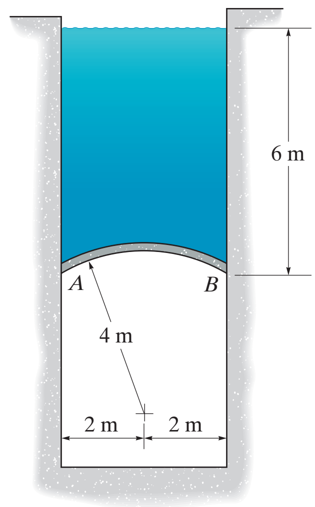


(a)

Ans:
$N_B = 104$ kN
$A_y = 0$, $A_x = 66.2$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–95. Water is confined in the vertical chamber, which is 2 m wide. Determine the resultant force it exerts on the arched roof AB.



SOLUTION

Due to symmetry, the resultant force that the water exerts on arch AB will be vertically downward, and its magnitude is equal to the weight of water of the shaded block in Fig. *a*. This shaded block can be subdivided into two parts as shown in Figs. *b* and *c*. The block in Fig. *c* should be considered a negative part since it is a hole. From the geometry in Fig. *a*,

$$\theta = \sin^{-1}\left(\frac{2\ \text{m}}{4\ \text{m}}\right) = 30°$$

$$h = 4\cos 30°\ \text{m}$$

Then, the area of the parts in Figs. *b* and *c* are

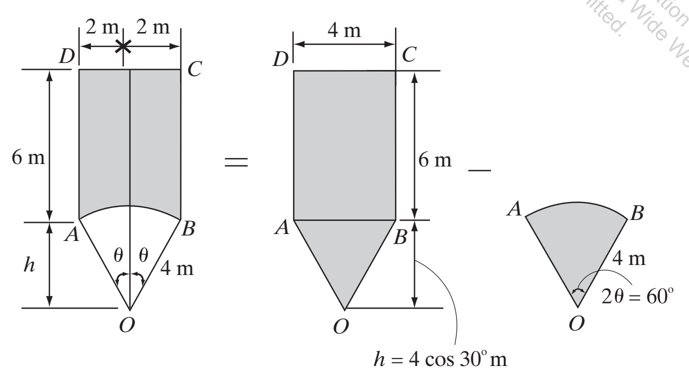
$$A_{OBCDAO} = 6\ \text{m}(4\ \text{m}) + \frac{1}{2}(4\ \text{m})(4\cos 30°\ \text{m}) = 30.928\ \text{m}^2$$

$$A_{OBAO} = \frac{60°}{360°}(\pi r^2) = \frac{60°}{360°}\left[\pi(4\ \text{m})^2\right] = 2.6667\pi\ \text{m}^2$$

Therefore,

$$F_R = W = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(30.928\ \text{m}^2 - 2.6667\pi\ \text{m}^2)(2\ \text{m})\right]$$

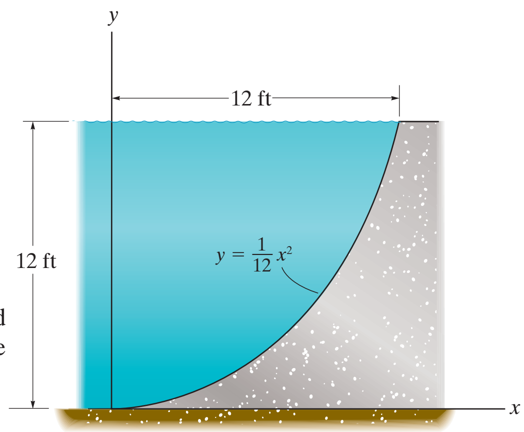
$$= 442.44(10^3)\ \text{N} = 442\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$F_R = 442\ \text{kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–96.** The wall is in the form of a parabola. Determine the magnitude and direction of the resultant force on the wall if it is 8 ft wide.



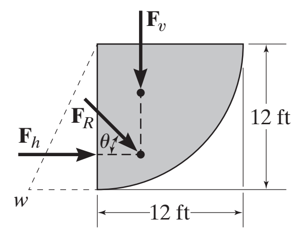
SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. *a*. Since the wall has a constant width of $b = 8$ ft, the intensity of the horizontal distributed load at the base of the wall is

$$w = \gamma_w hb = (62.4\ \text{lb/ft}^3)(12\ \text{ft})(8\ \text{ft}) = 5.9904(10^3)\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}wh = \frac{1}{2}[5.9904(10^3)\ \text{lb/ft}](12\ \text{ft}) = 35.9424(10^3)\text{lb}$$



(a)

The vertical force acting on the wall is equal to the weight of the water contained in the block above the wall (shown shaded in Fig. *a*). From the inside back cover of the text, the volume of this block (parabolic cross-section) is

$$\forall = \frac{2}{3}ahb = \frac{2}{3}(12\ \text{ft})(12\ \text{ft})(8\ \text{ft}) = 768\ \text{ft}^3$$

Thus,

$$F_v = \gamma_w \forall = (62.4\ \text{lb/ft}^3)(768\ \text{ft}^3) = 47.9232(10^3)\ \text{lb}$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{[35.9424(10^3)\ \text{lb}]^2 + [47.9232(10^3)\ \text{lb}]^2}$$
$$= 59.904(10^3)\ \text{lb} = 59.9\ \text{kip} \qquad \textbf{Ans.}$$

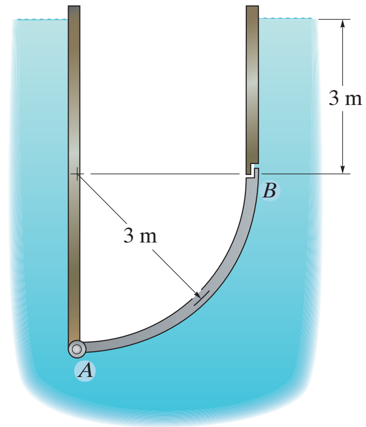
And its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left[\frac{47.9232(10^3)\ \text{lb}}{35.9424(10^3)\ \text{lb}}\right] = 53.13° = 53.1° \searrow \qquad \textbf{Ans.}$$

Ans:
$F_R = 59.9$ kip
$\theta = 53.1° \searrow$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–97. Determine the horizontal and vertical components of reaction at the hinge A and the horizontal normal reaction at B caused by the water pressure. The gate has a width of 3 m.



SOLUTION

The horizontal component of the resultant force acting on the gate is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(3\ \text{m}) = 176.58(10^3)\ \text{N/m}$$

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(3\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

Thus,

$$(F_h)_1 = \left[88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[176.58(10^3)\ \text{N/m} - 88.29(10^3)\ \text{N/m}\right](3\ \text{m}) = 132.435(10^3)\ \text{N} = 132.435\ \text{kN}$$

They act at

$$\tilde{y}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{y}_2 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

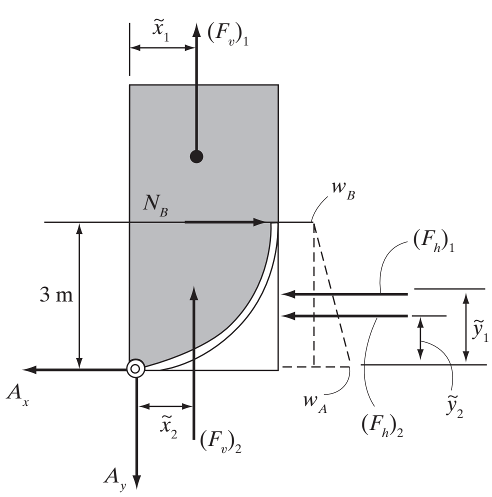
The vertical component of the resultant force acting on the gate is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*), but acts upward.

$$(F_v)_1 = \rho_w g \mathcal{V}_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(3\ \text{m})(3\ \text{m})(3\ \text{m})\right] = 264.87(10^3)\ \text{N} = 264.87\ \text{kN}$$

$$(F_v)_2 = \rho_w g \mathcal{V}_2 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(3\ \text{m})^2(3\ \text{m})\right] = 66.2175\pi(10^3)\ \text{N} = 66.2175\pi\ \text{kN}$$

They act at

$$\tilde{x}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{x}_2 = \frac{4(3\ \text{m})}{3\pi} = \left(\frac{4}{\pi}\right)\text{m}$$



(a)

2–97. Continued

Considering the equilibrium of the FBD of the gate in Fig. *a*

$$\circlearrowleft +\Sigma M_A = 0; \quad (264.87 \text{ kN})(1.5 \text{ m}) + (132.435 \text{ kN})(1 \text{ m}) + (264.87 \text{ kN})(1.5 \text{ m}) + (66.2175\pi \text{ kN})\left(\frac{4}{\pi}\text{ m}\right) - N_B(3 \text{ m}) = 0$$

$$N_B = 397.305 \text{ kN} = 397 \text{ kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 397.305 \text{ kN} - 264.87 \text{ kN} - 132.435 \text{ kN} - A_x = 0$$

$$A_x = 0 \qquad \textbf{Ans.}$$

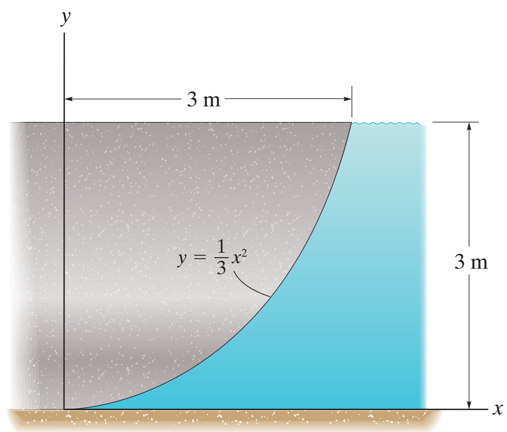
$$+\uparrow \Sigma F_y = 0; \quad 264.87 \text{ kN} + 66.2175\pi \text{ kN} - A_y = 0$$

$$A_y = 472.90 \text{ kN} = 473 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 397$ kN, $A_x = 0$, $A_y = 473$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–98. The 5-m-wide overhang is in the form of a parabola. Determine the magnitude and direction of the resultant force on the overhang.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})(5\ \text{m}) = 147.15\left(10^3\right)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}w_A h_A = \frac{1}{2}\left[147.15\left(10^3\right)\ \text{N/m}\right](3\ \text{m}) = 220.725\left(10^3\right)\ \text{N} = 220.725\ \text{kN}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above surface *AB* of the wall (shown shaded in Fig. *a*), but acts upward. The volume of this column of water is

$$\forall = \frac{2}{3}ahb = \frac{2}{3}(3\ \text{m})(3\ \text{m})(5\ \text{m}) = 30\ \text{m}^3$$

Thus,

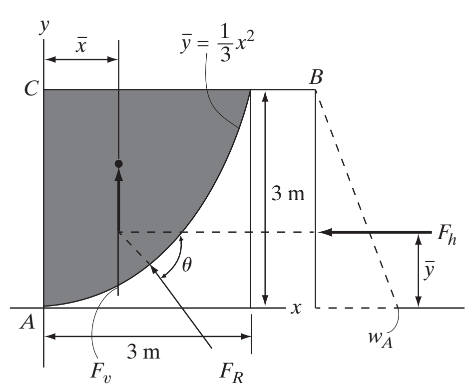
$$F_v = \rho_w g \forall = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left(30\ \text{m}^3\right) = 294.3\left(10^3\right)\ \text{N} = 294.3\ \text{kN}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(220.725\ \text{kN})^2 + (294.3\ \text{kN})^2} = 367.875\ \text{kN} = 368\ \text{kN} \quad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{294.3\ \text{kN}}{220.725\ \text{kN}}\right) = 53.13^\circ = 53.1^\circ \quad \textbf{Ans.}$$

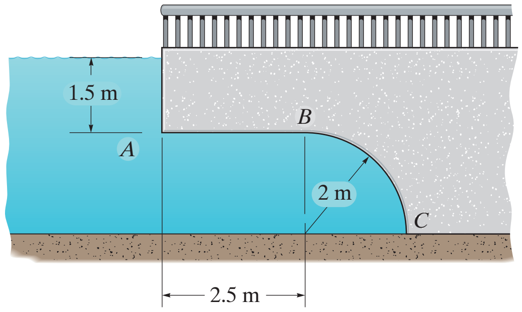


(a)

Ans:
$F_R = 368\ \text{kN}$
$\theta = 53.1^\circ$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

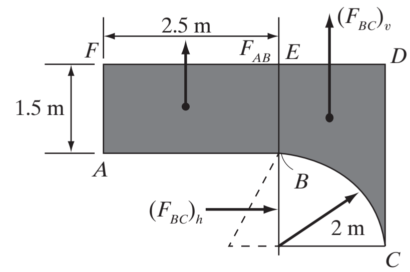
2–99. Determine the resultant force that water exerts on the overhanging sea wall along *ABC*. The wall is 2 m wide.



SOLUTION

Horizontal Component. Since *AB* is along the horizontal, no horizontal component exists. The horizontal component of the force on *BC* is

$$(F_{BC})_h = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(1.5\ \text{m} + \frac{1}{2}(2\ \text{m})\right)(2\ \text{m}(2\ \text{m})) = 98.1(10^3)\ \text{N}$$



(a)

Vertical Component. The force on *AB* and the vertical component of the force on *BC* is equal to the weight of the water contained in blocks *ABEFA* and *BCDEB* (shown shaded in Fig. *a*), but it acts upwards. Here, $A_{ABEFA} = 1.5\ \text{m}(2.5\ \text{m}) = 3.75\ \text{m}^2$ and $A_{BCDEB} = (3.5\ \text{m})(2\ \text{m}) - \frac{\pi}{4}(2\ \text{m})^2 = (7 - \pi)\ \text{m}^2$. Then,

$$F_{AB} = \gamma_w V_{ABEFA} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(3.75\ \text{m}^2)(2\ \text{m})\right]$$
$$= 73.575(10^3)\ \text{N} = 73.6\ \text{kN}$$

$$(F_{BC})_v = \gamma_w V_{BCDEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(7 - \pi)\ \text{m}^2(2\ \text{m})\right]$$
$$= 75.702(10^3)\ \text{N}$$

Therefore,

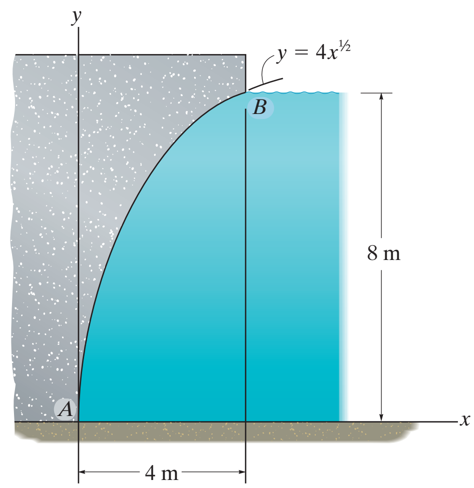
$$F_{BC} = \sqrt{(F_{BC})_h^2 + (F_{BC})_v^2} = \sqrt{\left[98.1(10^3)\ \text{N}\right]^2 + \left[75.702(10^3)\ \text{N}\right]^2}$$
$$= 123.91(10^3)\ \text{N} = 124\ \text{kN}$$

$$F_R = \sqrt{(F_{BC})_h^2 + \left[F_{AB} + (F_{BC})_v\right]^2}$$
$$= \sqrt{\left[98.1(10^3)\ \text{N}\right]^2 + \left[73.6(10^3)\ \text{N} + 75.702(10^3)\ \text{N}\right]^2}$$
$$= 178.6(10^3)\ \text{N} = 179\ \text{kN}$$ **Ans.**

Ans:
$F_R = 179\ \text{kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–100.** Determine the magnitude and direction of the resultant hydrostatic force the water exerts on the parabolic face AB of the wall if it is 3 m wide.



SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. *a*. Since the wall has a constant width of $b = 3$ m, the intensity of the horizontal distributed load at the base of the wall is

$$w = \rho_w g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(8\ \text{m})(3\ \text{m}) = 235.44(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2}wh = \frac{1}{2}[235.44(10^3)\ \text{N/m}](8\ \text{m}) = 941.76(10^3)\ \text{N} = 941.76\ \text{kN}$$

The vertical force acting on the wall is equal to the weight of water contained in the imaginary block above the wall (shown shaded in Fig. *a*), but acts vertically upward. From the inside back cover of the text, the volume of this block (parabolic cross-section) is

$$\mathcal{V} = \frac{1}{3}ahb = \frac{1}{3}(4\ \text{m})(8\ \text{m})(3\ \text{m}) = 32.0\ \text{m}^3$$

Thus,

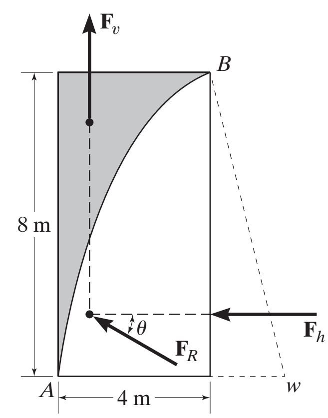
$$F_v = \rho_w g \mathcal{V} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(32.0\ \text{m}^3)$$
$$= 313.92(10^3)\ \text{N} = 313.92\ \text{kN}$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_n^2 + F_v^2} = \sqrt{(941.76\ \text{kN})^2 + (313.92\ \text{kN})^2} = 992.70\ \text{kN} = 993\ \text{kN}\ \textbf{Ans.}$$

And its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F}\right) = \tan^{-1}\left(\frac{313.92\ \text{kN}}{941.76\ \text{kN}}\right) = 18.43° = 18.4°\ \textbf{Ans.}$$

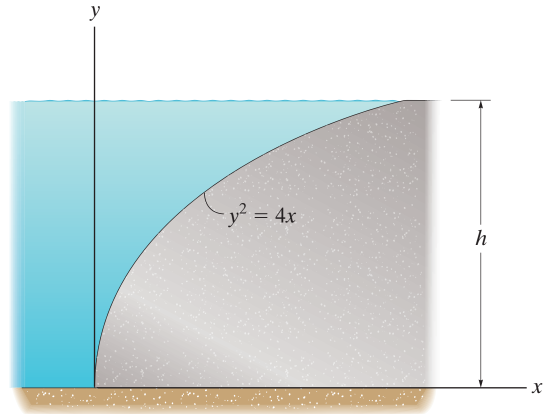


(a)

Ans:
$F_R = 993$ kN
$\theta = 18.4°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–101. The 5-m-wide wall is in the form of a parabola. If the depth of the water is $h = 4$ m, determine the magnitude and direction of the resultant force on the wall.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. *a*,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4\ \text{m})(5\ \text{m}) = 196.2(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[196.2(10^3)\ \text{N/m}\right](4\ \text{m}) = 392.4(10^3)\ \text{N} = 392.4\ \text{kN}$$

It acts at

$$\tilde{y} = \frac{1}{3} h_A = \frac{1}{3}(4\ \text{m}) = \frac{4}{3}\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. *a*). The volume of this column of water is

$$V = \frac{1}{3} a h b = \frac{1}{3}(4\ \text{m})(4\ \text{m})(5\ \text{m}) = 26.67\ \text{m}^3$$

Thus,

$$F_r = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(26.67\ \text{m/s}) = 261.6(10^3)\ \text{N} = 261.6\ \text{kN}$$

It acts at

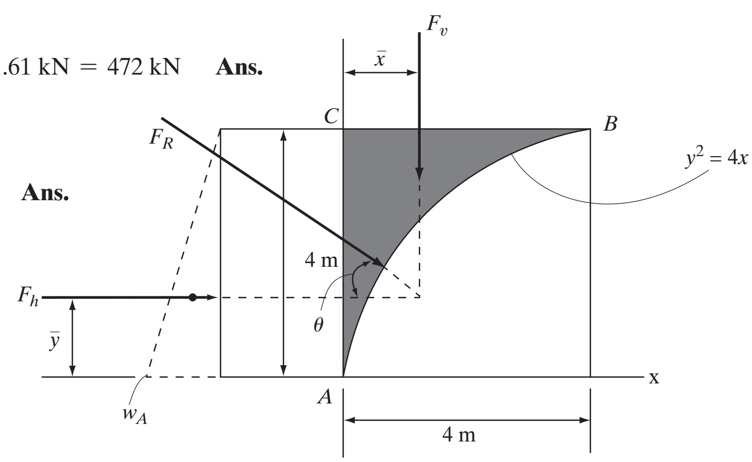
$$\tilde{x} = \frac{3}{10} a = \frac{3}{10}(4\ \text{m}) = \frac{6}{5}\ \text{m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(392.4\ \text{kN})^2 + (261.6\ \text{kN})^2} = 471.61\ \text{kN} = 472\ \text{kN} \qquad \textbf{Ans.}$$

And its direction is ⦪

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{261.6\ \text{kN}}{392.4\ \text{kN}}\right) = 33.69° \qquad \textbf{Ans.}$$



(a)

Ans:
$F_R = 472\ \text{kN},\ \theta = 33.7°$

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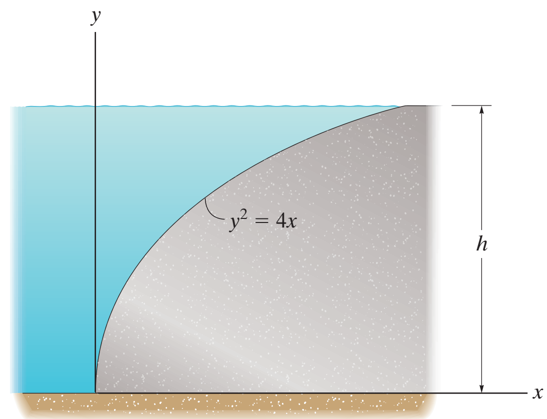
Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–102. The 5-m-wide wall is in the form of a parabola. Determine the magnitude of the resultant force on the wall as a function of depth h of the water. Plot the results of force (vertical axis) versus depth h for $0 \le h \le 4$ m. Give values for increments of $\Delta h = 0.5$ m.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the wall. Referring to Fig. a,

$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\text{m/s}^2)(h)(5\ \text{m}) = 49.05(10^3)h$$

Thus,

$$F_h = \frac{1}{2} w_A h_A = \frac{1}{2}\left[49.05(10^3)h\right]h = 24.525(10^3)h^2$$

The vertical component of the resultant force is equal to the weight of the column of water above surface AB of the wall (shown shaded in Fig. a). The volume of this column of water is

$$\forall = \frac{1}{3} a h b = \frac{1}{3}\left(\frac{h^2}{4}\right)(h)(5\ \text{m}) = \frac{5}{12}h^3$$

Thus,

$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{5}{12}h^3\right) = 4087.5h^3$$

Then the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}}$$

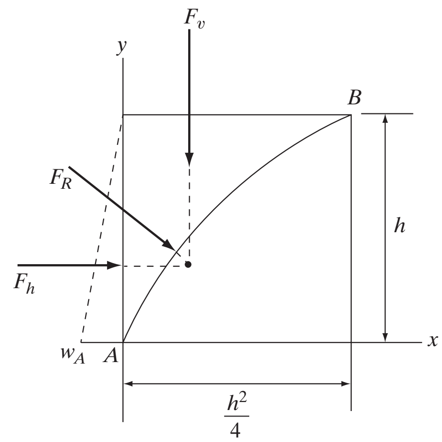
$$F_R = \sqrt{\left[24.525(10^3)h^2\right]^2 + \left[4087.5h^3\right]^2}$$

$$F_R = \sqrt{601.48(10^6)h^4 + 16.71(10^6)h^6}$$

The plot of F_R vs h is shown in Fig. b

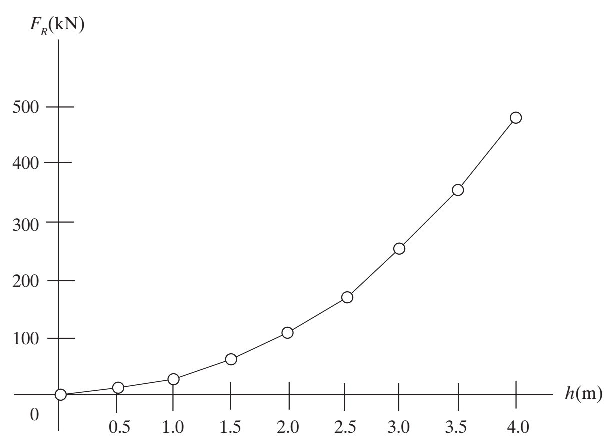
$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\ \text{N}$$

where h is in m.



(a)

$h(m)$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
F_R(kN)	0	6.15	24.9	56.9	103.4	166.1	246.8	347.8	471.6



(b)

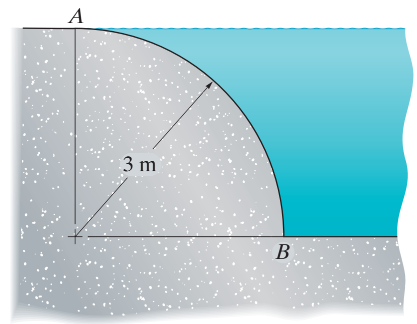
Ans:

$$F_R = \left[\sqrt{601(10^6)h^4 + 16.7(10^6)h^6}\right]\ \text{N}$$

where h is in m.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–103. Determine the resultant force the water exerts on the quarter-circular wall AB if it is 3 m wide.



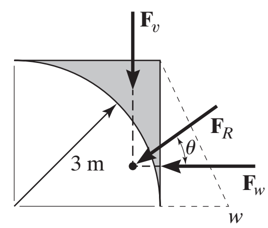
SOLUTION

The horizontal loading on the wall is due to the pressure on the vertical projected area of the wall, Fig. *a*. Since the wall has a constant width of $b = 3$ m, the intensity of the horizontal distributed load at the base of the wall is

$$w = \rho_w g h b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(3\ \text{m}) = 88.29(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w h = \frac{1}{2}[88.29(10^3)\text{N/m}](3\ \text{m}) = 132.435(10^3)\ \text{N} = 132.435\ \text{kN} \qquad \textbf{Ans.}$$



(a)

The vertical force acting on the wall is equal to the weight of the water contained in the shaded block above the wall, Fig. *a*. The cross-sectional area of the block is

$$A = (3\ \text{m})(3\ \text{m}) - \frac{1}{4}[\pi(3\ \text{m})^2] = (9 - 2.25\pi)\ \text{m}^2$$

Thus,

$$F_v = \rho_w g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(9 - 2.25\pi)\ \text{m}^2](3\ \text{m}) = 56.84(10^3)\ \text{N} = 56.84\ \text{kN}$$

Then, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(132.435\ \text{kN})^2 + (56.84\ \text{kN})^2} = 144.12\ \text{kN} = 144\ \text{kN} \qquad \textbf{Ans.}$$

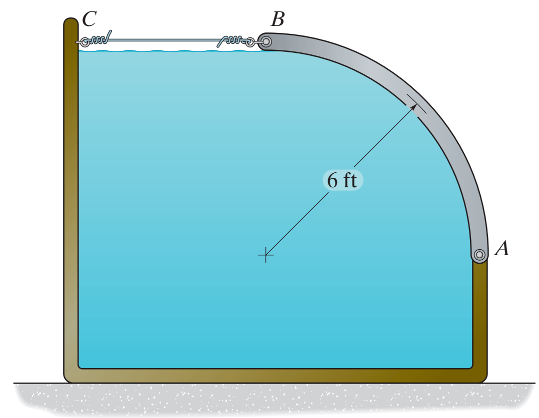
And its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{56.84\ \text{kN}}{132.435\ \text{kN}}\right) = 23.23° = 23.2° \qquad \textbf{Ans.}$$

Ans:
$F_R = 144\ \text{kN}, \theta = 23.2°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–104.** A quarter-circular plate is pinned at A and tied to the tank's wall using the cable BC. If the tank and plate are 4 ft wide, determine the horizontal and vertical components of reaction at A, and the tension in the cable due to the water pressure.



SOLUTION

Referring to the geometry shown in Fig. a,

$$A_{ADB} = (6\ \text{ft})(6\ \text{ft}) - \frac{\pi}{4}(6\ \text{ft})^2 = (36 - 9\pi)\ \text{ft}^2$$

$$\bar{x} = \frac{(3\ \text{ft})\left[(6\ \text{ft})(6\ \text{ft})\right] - \left[\left(6 - \frac{8}{\pi}\right)\text{ft}\right]\left[\frac{\pi}{4}(6\ \text{ft})^2\right]}{(36 - 9\pi)\ \text{ft}^2} = 1.3402\ \text{ft}$$

The horizontal component of the resultant force acting on the shell is equal to the pressure force on the vertically projected area of the shell. Referring to Fig. b,

$$w_{\bar{A}} = \gamma_w h_A b = \left(62.4\ \text{lb/ft}^3\right)(6\ \text{ft})(4\ \text{ft}) = 1497.6\ \text{lb/ft}$$

Thus,

$$F_h = \frac{1}{2}(1497.6\ \text{lb/ft})(6\ \text{ft}) = 4492.8\ \text{lb}$$

The vertical component of the resultant force acting on the shell is equal to the weight of the imaginary column of water above the shell (shown shaded in Fig. b), but acts upwards.

$$F_v = \gamma_w \forall = \gamma_w A_{ADB} b = \left(62.4\ \text{lb/ft}^3\right)\left[(36 - 9\pi)\ \text{ft}^2\right](4\ \text{ft}) = 1928.33\ \text{lb}$$

Write the moment equation of equilibrium about A by referring to Fig. b.

$$\circlearrowleft + \Sigma M_A = 0;\quad T_{BC}(6\ \text{ft}) - (1928.33\ \text{lb})(1.3402\ \text{ft}) - (4492.8\ \text{lb})(2\ \text{ft}) = 0$$

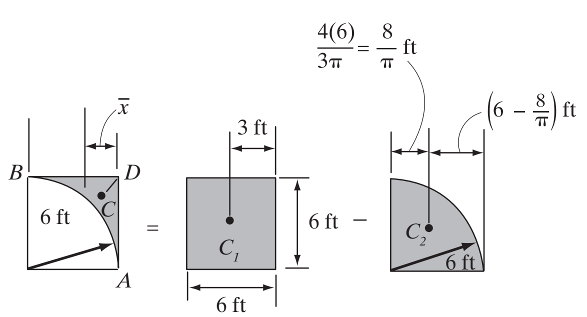
$$T_{BC} = 1928.33\ \text{lb} = 1.93\ \text{kip} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + 4492.8\ \text{lb} - 1928.32\ \text{lb} = 0$$

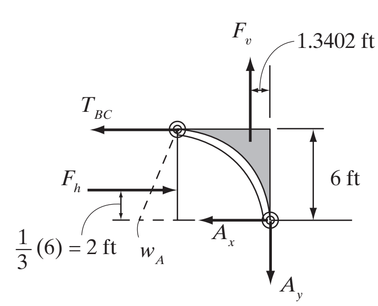
$$A_x = 2564.5\ \text{lb} = 2.56\ \text{kip} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0;\quad 1928.33\ \text{lb} - A_y = 0$$

$$A_y = 1928.33\ \text{lb} = 1.93\ \text{kip} \qquad \textbf{Ans.}$$



(a)

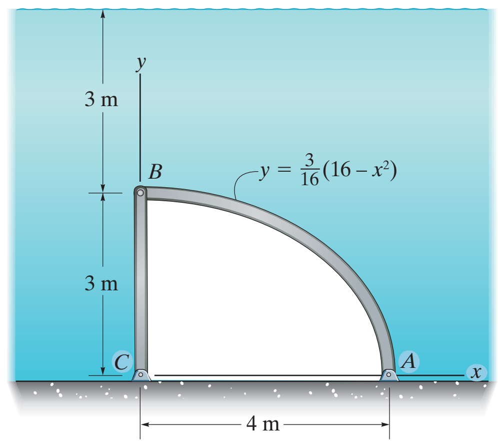


(b)

Ans:
$T_{BC} = 1.93$ kip
$A_x = 2.56$ kip
$A_y = 1.93$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures.

2–105. The parabolic and flat plates are pin connected at A, B, and C. They are submerged in water at the depth shown. Determine the horizontal and vertical components of reaction at pin B. The plates have a width of 4 m.

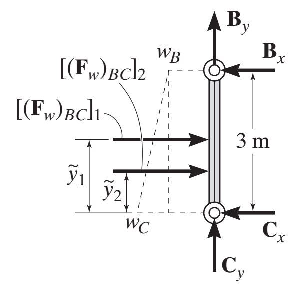


SOLUTION

The horizontal loadings on the plates are due to the pressure on the vertical projected areas of the plates, Fig. *a* and *b*. Since the plates have a constant width of $b = 4$ m, the intensities of the horizontal distributed load at points B, C and A are

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(4\ \text{m}) = 117.72(10^3)\ \text{N/m}$$

$$w_A = w_C = \rho_w g h_C b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(4\ \text{m}) = 235.44(10^3)\ \text{N/m}$$



(a)

Thus,

$$[(F_h)_{AB}]_1 = [(F_h)_{BC}]_1 = w_B h_{BC} = [117.72(10^3)\ \text{N/m}](3\ \text{m}) = 353.16(10^3)\ \text{N} = 353.16\ \text{kN}$$

$$[(F_h)_{AB}]_2 = [(F_h)_{BC}]_2 = \frac{1}{2}(w_C - w_B)h_{BC} = \frac{1}{2}[235.44(10^3)\ \text{N/m} - 117.72(10^3)\ \text{N/m}](3\ \text{m})$$

$$= 176.58(10^3)\ \text{N} = 176.58\ \text{kN}$$

And they act at

$$\tilde{y}_3 = \tilde{y}_1 = \frac{1}{2}(3\ \text{m}) = 1.5\ \text{m} \qquad \tilde{y}_2 = \tilde{y}_4 = \frac{1}{3}(3\ \text{m}) = 1\ \text{m}$$

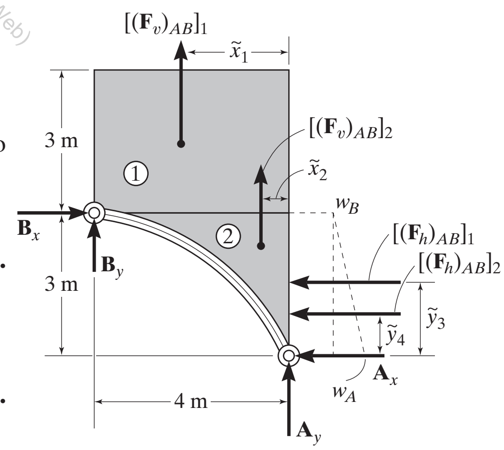
The vertical force acting on plate AB is equal to the weight of the water contained in the imaginary block above the plate (shown shaded in Fig. *b*).

$$[(F_v)_{AB}]_1 = \rho_w g V_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(4\ \text{m})(3\ \text{m})(4\ \text{m})] = 470.88(10^3)\ \text{N} = 470.88\ \text{kN}$$

$$[(F_v)_{AB}]_2 = \rho_w g V_2 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{3}(3\ \text{m})(4\ \text{m})(4\ \text{m})\right] = 156.96(10^3)\ \text{N} = 156.96\ \text{kN}$$

And they act at

$$\tilde{x}_1 = \frac{1}{2}(4\ \text{m}) = 2\ \text{m} \qquad \tilde{x}_2 = \frac{1}{4}(4\ \text{m}) = 1\ \text{m}$$



(b)

Write the moment equations of equilibrium about points C and A by referring to the FBDs of plates BC, Fig. *a*, and AB, Fig. *b*, respectively.

$\zeta + \Sigma M_C = 0;\quad B_x(3\ \text{m}) - (353.16\ \text{kN})(1.5\ \text{m}) - (176.58\ \text{kN})(1\ \text{m}) = 0$

$B_x = 235.44\ \text{kN} = 235\ \text{kN}$ **Ans.**

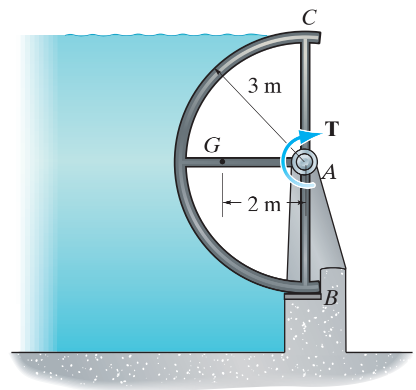
$\zeta + \Sigma M_A = 0;\quad -B_y(4) + (353.16\ \text{kN})(1.5\ \text{m}) + (176.58\ \text{kN})(1\ \text{m})$
$- (235.44\ \text{kN})(3\ \text{m}) + (470.88\ \text{kN})(2\ \text{m}) + (156.96\ \text{kN})(1\ \text{m}) = 0$

$B_y = 274.68\ \text{kN} = 275\ \text{kN}$ **Ans.**

Ans:
$B_x = 235\ \text{kN},\ B_y = 275\ \text{kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–106. The semicircular gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque **T** that must be applied at the pin A in order to open the gate. The gate has a mass of 8 Mg with center of mass at G. It is 4 m wide.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 4$ m, the intensity of the distributed load at point B is

$$w_B = \rho_w g h_C b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(4\ \text{m}) = 235.44(10^3)\ \text{N/m}$$

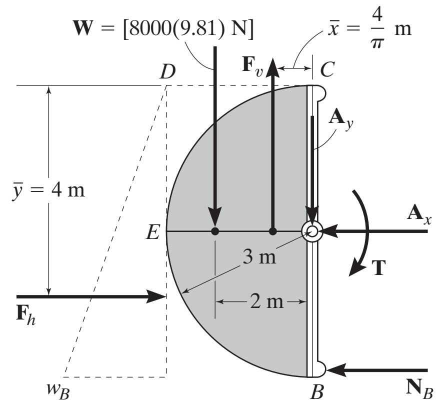
Thus,

$$F_h = \frac{1}{2} w_C h_C = \frac{1}{2}[235.44(10^3)\ \text{N/m}](6\ \text{m}) = 706.32(10^3)\ \text{N}$$

and it acts at

$$\bar{y} = \frac{2}{3}(6\ \text{m}) = 4\ \text{m}$$

The upward force on BE and downward force on CE is equal to the weight of water contained in blocks $BACDEB$ (imaginary) and $CDEC$, respectively. Thus, the net upward force on BEC is equal to the weight of water contained in block $BACEB$ shown shaded in Fig. *a*. Thus,



(a)

$$F_v = \rho_w g \forall_{BACEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{2}(3\ \text{m})^2(4\ \text{m})\right]$$

$$= 176.58(10^3)\pi\ \text{N}$$

And it acts at

$$\bar{x} = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m}$$

When the gate is on the verge of opening, $N_B = 0$. Write the moment equation of equilibrium about point A by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad [8000(9.81)\ \text{N}](2\ \text{m}) + [706.32(10^3)\ \text{N}](4\ \text{m} - 3\ \text{m}) - [176.58(10^3)\pi\ \text{N}]\left(\frac{4}{\pi}\ \text{m}\right) - T = 0$$

$$T = 156.96(10^3)\ \text{N}\cdot\text{m} = 157\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

This solution can be simplified if one realizes that the resultant force due to the water pressure on the gate will act perpendicular to the circular surface, thus acting through center A of the semicircular gate and so producing no moment about this point.

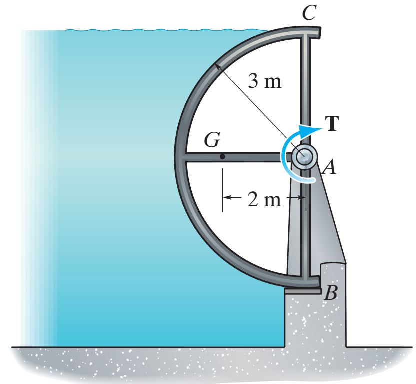
$$\circlearrowleft + \Sigma M_A = 0; \quad [8000(9.81)\text{N}](2\ \text{m}) - T = 0$$

$$T = 156.96(10^3)\ \text{N}\cdot\text{m} = 157\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T = 157\ \text{kN}\cdot\text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–107. The semicircular gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the normal reaction at B. The gate has a weight of 8 Mg with center of mass at G. It is 4 m wide. Take $T = 0$.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 4$ m, the intensity of the distributed load at point B is

$$w_B = \rho_w g h_C b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6\ \text{m})(4\ \text{m}) = 235.44(10^3)\ \text{N/m}$$

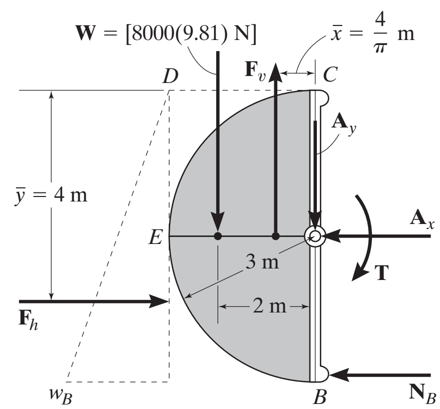
Thus,

$$F_h = \frac{1}{2} w_C h_C = \frac{1}{2}[235.44(10^3)\ \text{N/m}](6\ \text{m}) = 706.32(10^3)\ \text{N}$$

and it acts at

$$\bar{y} = \frac{2}{3}(6\ \text{m}) = 4\ \text{m}$$

The upward force on BE and downward force on CE is equal to the weight of water contained in blocks $BACDEB$ (imaginary) and $CDEC$, respectively. Thus, the net upward force on BEC is equal to the weight of water contained in block $BACEB$ shown shaded in Fig. *a*. Thus,



(a)

$$F_v = \rho_w g \forall_{BACEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{2}(3\ \text{m})^2(4\ \text{m})\right]$$

$$= 176.58(10^3)\pi\ \text{N}$$

And it acts at

$$\bar{x} = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m}$$

Write the moment equation of equilibrium about point A and B with $T = 0$ by referring to the FBD of the gate, Fig. *a*.

$$\zeta + \Sigma M_A = 0; \quad [8000(9.81)\ \text{N}](2\ \text{m}) + [70632(10^3)\ \text{N}](4\ \text{m} - 3\ \text{m})$$

$$- [176.58(10^3)\pi\ \text{N}]\left(\frac{4}{\pi}\ \text{m}\right) - N_B(3\ \text{m}) = 0$$

$$N_B = 52.32(10^3)\ \text{N} = 52.3\ \text{kN} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_B = 0; \quad A_x(3\ \text{m}) + [8000(9.81)\ \text{N}](2\ \text{m}) - [176.58(10^3)\pi\ \text{N}]\left(\frac{4}{\pi}\ \text{m}\right)$$

$$- [706.32(10^3)\ \text{N}](6\ \text{m} - 4\ \text{m}) = 0$$

$$A_x = 654(10^3)\ \text{N} = 654\ \text{kN} \qquad \textbf{Ans.}$$

Write the force equation of equilibrium along y axis.

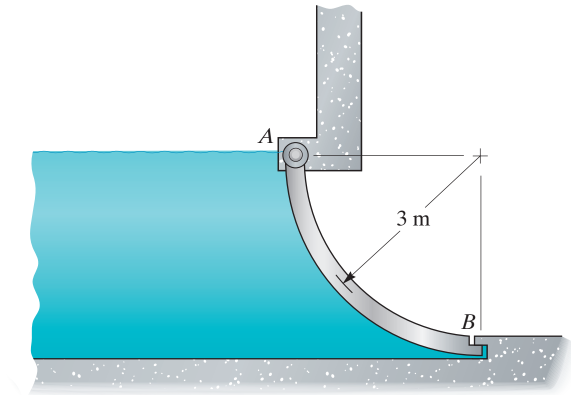
$$+\uparrow \Sigma F_y = 0; \quad 176.58(10^3)\pi\ \text{N} - 8000(9.81)\ \text{N} - A_y = 0$$

$$A_y = 476.26(10^3)\ \text{N} = 476\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$N_B = 52.3$ kN
$A_x = 654$ kN
$A_y = 476$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–108.** Plate AB has a width of 1.5 m and a radius of 3 m. Determine the horizontal and vertical components of reaction at the pin A and the vertical reaction at the *smooth* stop B due to the water pressure.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 1.5$ m, the intensity of the horizontal distributed load at point B is

$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(1.5\ \text{m}) = 44.145(10^3)\ \text{N/m}$$

Thus,

$$F_h = \frac{1}{2} w_B h_B = \frac{1}{2}[44.145(10^3)\ \text{N/m}](3\ \text{m}) = 66.2175(10^3)\ \text{N} = 66.2175\ \text{kN}$$

And it acts at

$$\bar{y} = \frac{2}{3}(3\ \text{m}) = 2\ \text{m}$$

The vertical force acting on the gate is equal to the weight of water contained in the imaginary block (shown shaded in Fig. *a*) above the gate, but acts upward.

$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{4}(\pi)(3\ \text{m})^2\right](1.5\ \text{m})$$
$$= 104.01(10^3)\ \text{N} = 104.01\ \text{kN}$$

And it acts at

$$x_1 = \frac{4(3\ \text{m})}{3\pi} = \frac{4}{\pi}\ \text{m} \quad x_2 = \left(3 - \frac{4}{\pi}\right)\text{m} = 1.7268\ \text{m}$$

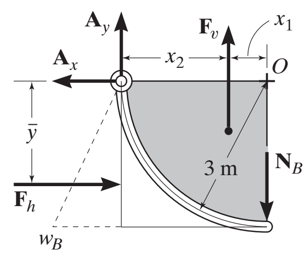
Write the moment equations of equilibrium about points A and O by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \quad (66.2175\ \text{kN})(2\ \text{m}) + (104.01\ \text{kN})(1.7268\ \text{m}) - N_B(3\ \text{m}) = 0$$
$$N_B = 104.01\ \text{kN} = 104\ \text{kN}$$ **Ans.**

$$\circlearrowleft + \Sigma M_O = 0; \quad (66.2175\ \text{kN})(2\ \text{m}) - (104.01\ \text{kN})\left(\frac{4}{\pi}\ \text{m}\right) - A_y(3\ \text{m}) = 0$$
$$A_y = 0$$ **Ans.**

Write the force equation of equilibrium along the x axis.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 66.2175\ \text{kN} - A_x = 0$$
$$A_x = 66.2175\ \text{kN} = 66.2\ \text{kN}$$ **Ans.**

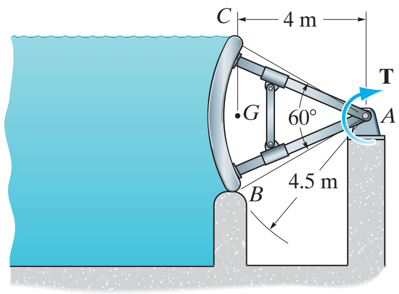


(a)

Ans:
$N_B = 104$ kN
$A_y = 0$
$A_x = 66.2$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–109. The Tainter gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the torque **T** that must be applied at the pin A in order to open the gate. The gate has a mass of 5 Mg and a center of mass at G. It is 3 m wide.



SOLUTION

Horizontal Component. This component can be determined by applying

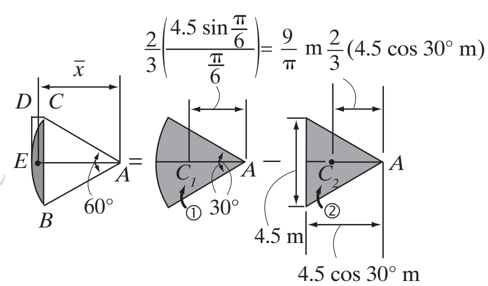
$$(F_{BC})_h = \gamma_w \bar{h} A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(4.5 \sin 30°\ \text{m})\left[2(4.5 \sin 30°\ \text{m})(3\ \text{m})\right]$$
$$= 297.98\left(10^3\right)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$, shown shaded in Fig. a. This block can be subdivided into parts (1) and (2), Figs. a and b, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5 \cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5 \cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$
$$= 4.1397\ \text{m}$$

The magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w V_{BCEB} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\left[1.8344\ \text{m}^2(3\ \text{m})\right]$$
$$= 53.985\left(10^3\right)\ \text{N}$$



(a) (b) (c)

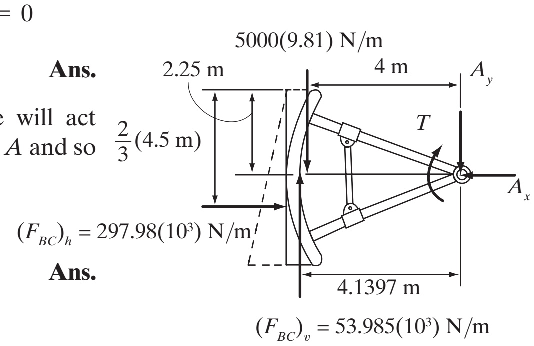
When the gate is on the verge of opening, $N_B = 0$. Referring to the free-body diagram of the gate in Fig. d,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) + \left[297.98\left(10^3\right)\ \text{N}\right]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right]$$
$$- \left[53.985\left(10^3\right)\ \text{N}\right](4.1397\ \text{m}) - T = 0$$

$$T = 196.2\left(10^3\right)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

This solution can be simplified if one realizes that the resultant force will act perpendicular to the circular surface. Therefore, F_{BC} will act through point A and so produces no moment about this point. Hence,

$$\circlearrowleft + \Sigma M_A = 0; \quad \left[5000(9.81)\ \text{N}\right](4\ \text{m}) - T = 0$$
$$T = 196.2\left(10^3\right)\ \text{N}\cdot\text{m} = 196\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

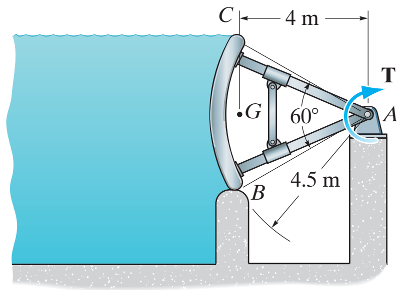


(d)

Ans:
$T = 196\ \text{kN}\cdot\text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–110. The Tainter gate is used to control the flow of water over a spillway. If the water is at its highest level as shown, determine the horizontal and vertical components of reaction at pin A and the vertical reaction at the smooth spillway crest B. The gate has a mass of 5 Mg and a center of mass at G. It is 3 m wide. Take $T = 0$.



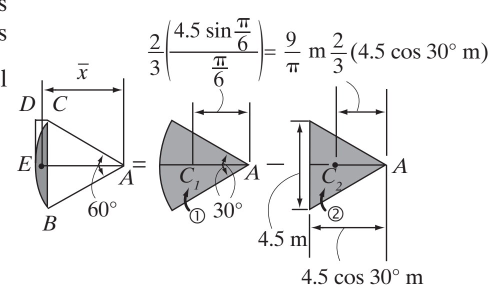
SOLUTION

Horizontal Component. This component can be determined from

$$(F_{BC})_h = \gamma_w \bar{h} A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.5\ \sin 30°\ \text{m})[2(4.5\ \sin 30°\ \text{m})(3\ \text{m})]$$

$$= 297.98(10^3)\ \text{N}$$

Vertical Component. The upward force on BE and downward force on CE is equal to the weight of the water contained in blocks $BCDEB$ and $CEDC$, respectively. Thus, the net upward force on BEC is equal to the weight of the water contained in block $BCEB$, shown shaded in Fig. *a*. This block can be subdivided into parts (1) and (2), Figs. *a* and *b*, respectively. However, part (2) is a hole and should be considered as a negative part. The area of block $BCEB$ is $\Sigma A = \left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m}) = 1.8344\ \text{m}^2$ and the horizontal distance measured from its centroid to point A is



(a) (b) (c)

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\left(\frac{9}{\pi}\ \text{m}\right)\left[\frac{\pi}{6}(4.5\ \text{m})^2\right] - \frac{2}{3}(4.5\cos 30°\ \text{m})\left[\frac{1}{2}(4.5\ \text{m})(4.5\cos 30°\ \text{m})\right]}{1.8344\ \text{m}^2}$$

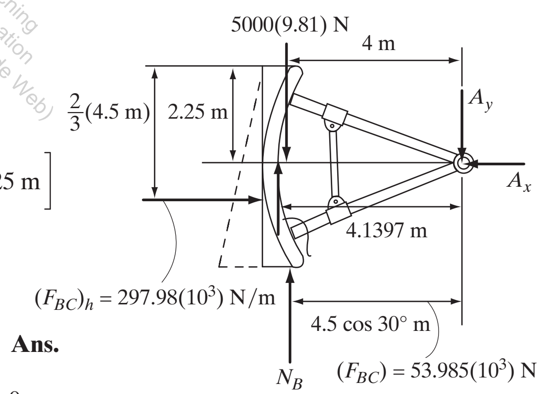
$$= 4.1397\ \text{m}$$

Thus, the magnitude of the vertical component is

$$(F_{BC})_v = \gamma_w \forall_{BCEB} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[1.8344\ \text{m}^2(3\ \text{m})]$$

$$= 53.985(10^3)\ \text{N}$$

Considering the free-body diagram of the gate in Fig. *d*,



(d)

$\circlearrowleft + \Sigma M_A = 0;$ $$[5000(9.81)\ \text{N}](4\ \text{m}) + [297.98(10^3)\ \text{N}]\left[\frac{2}{3}(4.5\ \text{m}) - 2.25\ \text{m}\right] - [53.985(10^3)\ \text{N}](4.1397\ \text{m}) - N_B(4.5\ \cos 30°\ \text{m}) = 0$$

$$N_B = 50.345(10^3)\ \text{N} = 50.3\ \text{kN}$$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $$50.345(10^3)\ \text{N} + 53.985(10^3)\ \text{N} - 5000(9.81)\ \text{N} - A_y = 0$$

$$A_y = 55.28(10^3)\ \text{N} = 55.3\ \text{kN}$$ **Ans.**

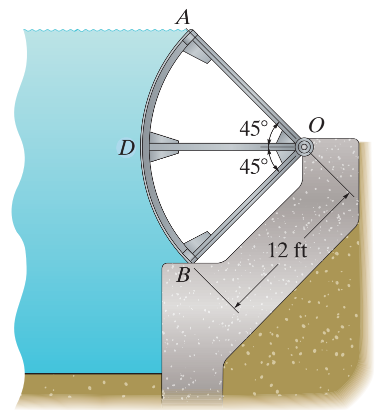
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $$297.98(10^3)\ \text{N} - A_x$$

$$A_x = 297.98(10^3)\ \text{N} = 298\ \text{kN}$$ **Ans.**

Ans:
$N_B = 50.3\ \text{kN}$
$A_y = 55.3\ \text{kN}$
$A_x = 298\ \text{kN}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–111. The 6-ft-wide Tainter gate in the form of a quarter-circular arc is used as a sluice gate. Determine the magnitude and direction of the resultant force of the water on the bearing O of the Tainter gate. What is the moment of this force about the bearing?



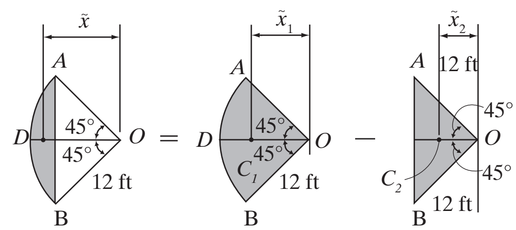
SOLUTION

Referring to the geometry in Fig. *a*,

$$A_{ADB} = \frac{\pi}{4}(12\ \text{ft}^2) - \frac{1}{2}\left[(2)(12 \sin 45^\circ\ \text{ft})(12 \cos 45^\circ\ \text{ft})\right] = 41.097\ \text{ft}^2$$

$$\tilde{x}_1 = \frac{2}{3}\left(\frac{12\ \sin 45^\circ\ \text{ft}}{\pi/4}\right) = 7.2025\ \text{ft}$$

$$\tilde{x}_2 = \frac{2}{3}(12 \cos 45^\circ\ \text{ft}) = 5.6569\ \text{ft}$$



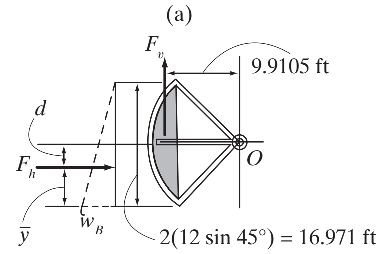
(a)

$$\bar{x} = \frac{(7.2025\ \text{ft})\left[\frac{\pi}{4}(12\ \text{ft})^2\right] - (5.6569\ \text{ft})\left[\frac{1}{2}(2)(12 \sin 45^\circ\ \text{ft})(12 \cos 45^\circ\ \text{ft})\right]}{41.097\ \text{ft}^2}$$

$$= 9.9105\ \text{ft}$$

The horizontal component of the resultant force is equal to the pressure force on the vertically projected area of the gate. Referring to Fig. *b*,

$$w_B = \gamma_w h_B b = \left(62.4\ \text{lb/ft}^3\right)(16.971\ \text{ft})(6\ \text{ft}) = 6353.78\ \text{lb/ft}$$



(b)

Thus,

$$F_h = \frac{1}{2}(6353.78\ \text{lb/ft})(16.971\ \text{ft}) = 53.9136\left(10^3\right)\ \text{lb} = 53.9136\ \text{kip}$$

It acts at

$$\bar{y} = \frac{1}{3}(16.971\ \text{ft}) = 5.657\ \text{ft}$$

$$d = 12\ \sin 45^\circ\ \text{ft} - 5.657\ \text{ft} = 2.8284\ \text{ft}$$

The vertical component of the resultant force is equal to the weight of the block of water contained in sector ADB, shown in Fig. *a*, but acts upward.

$$F_v = \gamma_w V_{ADB} = \gamma_w A_{ADB} b = \left(62.4\ \text{lb/ft}^3\right)\left(41.097\ \text{ft}^2\right)(6\ \text{ft}) = 15.3868\left(10^3\right)\ \text{lb} = 15.3868\ \text{kip}$$

Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{(53.9136\ \text{kip})^2 + (15.3868\ \text{kip})^2} = 56.07\ \text{kip} = 56.1\ \text{kip} \qquad \textbf{Ans.}$$

Its direction is

$$\theta = \tan^{-1}\left(\frac{F_v}{F_h}\right) = \tan^{-1}\left(\frac{15.3868\ \text{kip}}{53.9136\ \text{kip}}\right) = 15.93^\circ = 15.9^\circ \qquad \textbf{Ans.}$$

By referring to Fig. *b*, the moment of F_R about O is

$$\zeta + (M_R)_O = \Sigma M_O;\ (M_R)_O = (53.9136\ \text{kip})(2.8284\ \text{ft}) - (15.3868\ \text{kip})(9.9105\ \text{ft})$$

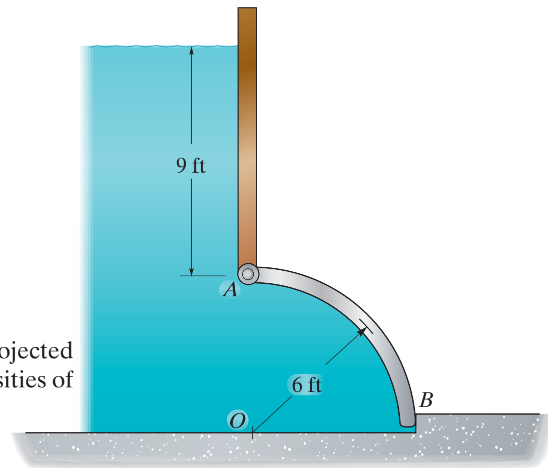
$$= 0 \qquad \textbf{Ans.}$$

This result is expected since the gate is circular in shape. Thus, F_R is always directed toward center O of the circular gate.

Ans:
$F_R = 56.1$ kip
$\theta = 15.9^\circ$
$(M_R)_O = 0$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–112.** Determine the horizontal and vertical components of reaction at the hinge A and the horizontal reaction at the smooth surface B *caused* by the water pressure. The plate has a width of 4 ft.



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width $b = 4$ ft, the intensities of the horizontal distributed load at A and B are

$$w_A = \gamma_w h_A b = (62.4\ \text{lb/ft}^3)(9\ \text{ft})(4\ \text{ft}) = 2246.4\ \text{lb/ft}$$
$$w_B = \gamma_w h_B b = (62.4\ \text{lb/ft}^3)(15\ \text{ft})(4\ \text{ft}) = 3744\ \text{lb/ft}$$

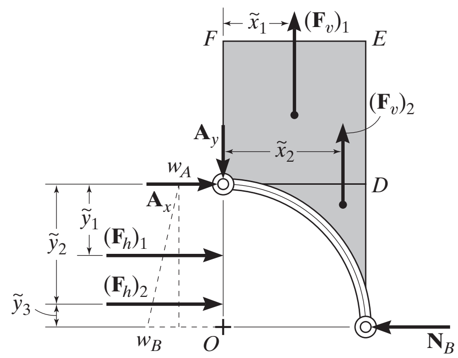
Thus,

$$(F_h)_1 = w_A l_{AD} = (2246.4\ \text{lb/ft})(6\ \text{ft}) = 13.4784(10^3)\ \text{lb}$$

$$(F_h)_2 = \frac{1}{2}(w_B - w_A)l_{AD} = \frac{1}{2}(3744\ \text{lb/ft} - 2246.4\ \text{lb/ft})(6\ \text{ft}) = 4.4928(10^3)\ \text{lb}$$

and they act at

$$\tilde{y}_1 = \frac{1}{2}(6\ \text{ft}) = 3\ \text{ft} \quad \tilde{y}_2 = \frac{2}{3}(6\ \text{ft}) = 4\ \text{ft} \quad \tilde{y}_3 = \frac{1}{3}(6\ \text{ft}) = 2\ \text{ft}$$



(a)

The vertical force acting on the gate is equal to the weight of the water contained in the imaginary block above the gate (shown shaded in Fig. *a*), but acts upward. For $(F_v)_1$,

$$(F_v)_1 = \gamma_w \forall_{ADEF} = (62.4\ \text{lb/ft}^3)[(6\ \text{ft})(9\ \text{ft})(4\ \text{ft})] = 13.4784(10^3)\ \text{lb}$$

And it acts at

$$\bar{x}_1 = \frac{1}{2}(6\ \text{ft}) = 3\ \text{ft}$$

For $(F_v)_2$, we need to refer to the geometry shown in Fig. *b*.
Here,

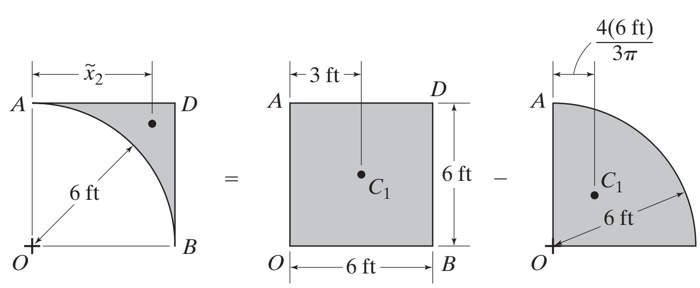
$$A_{ADB} = A_{ADBO} - A_{ABO} = (6\ \text{ft})(6\ \text{ft}) - \frac{1}{4}[\pi(6\ \text{ft})^2] = (36 - 9\pi)\ \text{ft}^2$$

Then,

$$(F_v)_2 = \gamma_w \forall_{ADB} = (62.4\ \text{lb/ft}^3)[(36 - 9\pi)\ \text{ft}^2](4\ \text{ft}) = 1.9283(10^3)\ \text{lb}$$

And it acts at

$$\tilde{x}_2 = \frac{(3\ \text{ft})[(6\ \text{ft})(6\ \text{ft})] - \dfrac{4(6\ \text{ft})}{3\pi}\left[\dfrac{\pi}{4}(6\ \text{ft})^2\right]}{(36 - 9\pi)\ \text{ft}^2} = 4.6598\ \text{ft}$$



(b)

2–112. Continued

Write the moment equation of equilibrium about points A and B by referring to the FBD of the gate, Fig. a.

$$\zeta+ \Sigma M_A = 0; \quad [13.4784(10^3)\text{ lb}](3\text{ ft}) + [4.4928](10^3)\text{ lb}](4\text{ ft})$$
$$+ [13.4784(10^3)\text{ lb}](3\text{ ft}) + [1.9283(10^3)\text{ lb}](4.6598\text{ ft}) - N_B(6\text{ ft}) = 0$$
$$N_B = 17.9712(10^3)\text{ lb} = 18.0\text{ kip} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_o = 0; \quad [13.4784(10^3\text{ lb})(3\text{ ft}) + [1.9283(10^3)\text{ lb}](4.6598\text{ ft})$$
$$- [13.4784(10^3)\text{ lb}](3\text{ ft}) - [4.4928(10^3)\text{ lb}](2\text{ ft}) - A_x(6\text{ ft}) = 0$$
$$A_x = 0 \qquad \textbf{Ans.}$$

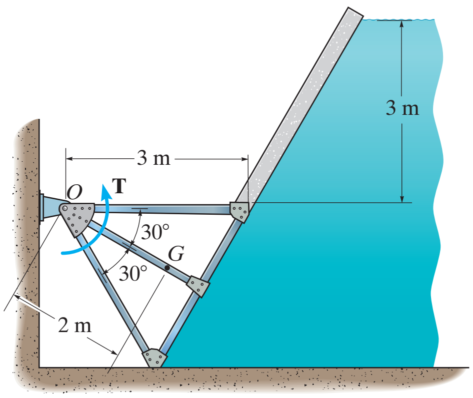
Write the force equation of equilibrium along the y axis.

$$+\uparrow\Sigma F_y = 0; \quad 13.4784(10^3)\text{ lb} + 1.9283(10^3)\text{ lb} - A_y = 0$$
$$A_y = 15.4067(10^3)\text{ lb} = 15.4\text{ kip} \qquad \textbf{Ans.}$$

Ans:
$N_B = 18.0$ kip
$A_x = 0$
$A_y = 15.4$ kip

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–113. The sluice gate for a water channel is 2 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Also, what is the smallest torque **T** that must be applied to open the gate if its mass is 6 Mg with its center of mass at G?



SOLUTION

The horizontal loading on the gate is due to the pressure on the vertical projected area of the gate, Fig. *a*. Since the gate has a constant width of $b = 2$ m, the intensities of the horizontal distributed load at points A and B are

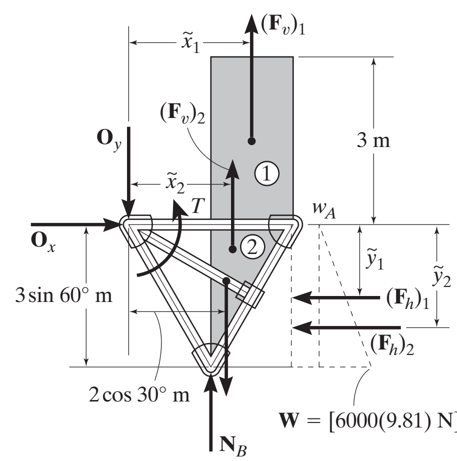
$$w_A = \rho_w g h_A b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})(2\ \text{m}) = 58.86(10^3)\ \text{N/m}$$
$$w_B = \rho_w g h_B b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\text{m} + 3\sin 60°\ \text{m})(2\ \text{m}) = 109.83(10^3)\ \text{N/m}$$

Thus,

$$(F_h)_1 = w_4 h_{AB} = [58.86(10^3)\ \text{N/m}](3\sin 60°\ \text{m}) = 152.92(10^3)\ \text{N}$$
$$(F_h)_2 = \frac{1}{2}(w_B - w_A)h_{AB} = \frac{1}{2}[109.83(10^3)\ \text{N/m} - 58.86(10^3)\ \text{N/m}](3\sin 60°\ \text{m})$$
$$= 66.22(10^3)\ \text{N}$$

Then

$$F_h = (F_h)_1 + (F_h)_2 = 152.92(10^3)\ \text{N} + 66.22(10^3)\ \text{N}$$
$$= 219.14(10^3)\ \text{N} = 219.14\ \text{kN}$$



(a)

Here, $(F_h)_1$ and $(F_h)_2$ act at

$$\tilde{y}_1 = \frac{1}{2}(3\sin 60°\ \text{m}) = 1.2990\ \text{m} \qquad \tilde{y}_2 = \frac{2}{3}(3\sin 60°\ \text{m}) = 1.7320\ \text{m}$$

The vertical force is equal to the weight of the water contained in the imaginary block above the gate (shown shaded in Fig. *a*), but acts upward.

$$(F_v)_1 = \rho_w g V_1 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})(1.5\ \text{m})(2\ \text{m})] = 88.29(10^3)\ \text{N}$$
$$(F_v)_2 = \rho_w g V_2 = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{1}{2}(3\sin 60°\ \text{m})(1.5\ \text{m})(2\ \text{m})\right] = 38.23(10^3)\ \text{N}$$

Then

$$(F_v) = (F_v)_1 + (F_v)_2 = 88.29(10^3) + 38.23(10^3) = 126.52(10^3)\ \text{N} = 126.52\ \text{kN}$$

Here, $(F_v)_1$ and $(F_v)_2$ act at

$$\tilde{x}_1 = 1.5\ \text{m} + \frac{1}{2}(1.5\ \text{m}) = 2.25\ \text{m} \qquad \tilde{x}_2 = 1.5\ \text{m} + \frac{1}{3}(1.5\ \text{m}) = 2\ \text{m}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(219.14\ \text{kN})^2 + (126.52\ \text{kN})^2} = 253.04\ \text{kN} = 253\ \text{kN} \quad \textbf{Ans.}$$

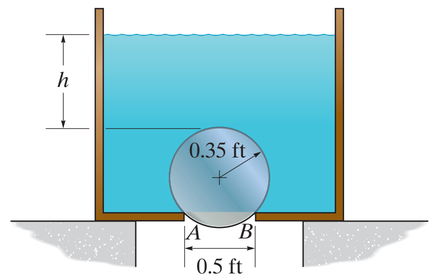
When the gate is on the verge of opening, $N_B = 0$. Write the moment equation of equilibrium about point O by referring to the FBD of the gate, Fig. *a*.

$$\circlearrowleft + \Sigma M_O = 0; \quad T + [88.29(10^3)\ \text{N}](2.25\ \text{m}) + [38.23(10^3)\ \text{N}](2\ \text{m})$$
$$- [152.92(10^3)\ \text{N}](1.2990\ \text{m}) - [66.22(10^3)\ \text{N}](1.7320\ \text{m})$$
$$- [6000(9.81)\ \text{N}](2\cos 30°\ \text{m}) = 0$$
$$T = 140.18(10^3)\ \text{N}\cdot\text{m} = 140\ \text{kN}\cdot\text{m} \quad \textbf{Ans.}$$

Ans:
$F_R = 253$ kN
$T = 140$ kN·m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–114. The steel cylinder has a specific weight of 490 lb/ft^3 and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant vertical force the bottom of the tank exerts on the cylinder when the water in the tank is at a depth of $h = 2$ ft.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$V_1 = \left[2.35\ \text{ft}(0.7\ \text{ft}) - \frac{\pi}{2}(0.35\ \text{ft})^2\right](1\ \text{ft}) = 1.4526\ \text{ft}^3$$

The volume of the shaded block in Fig. *b* is

$$V_2 = 2\left\{0.1\ \text{ft}(2.35\ \text{ft}) + \left[\frac{44.42^\circ}{360^\circ}(\pi)(0.35\ \text{ft})^2 - \frac{1}{2}(0.25\ \text{ft})(0.2449\ \text{ft})\right]\right\}(1\ \text{ft})$$
$$= 0.5037\ \text{ft}^3$$

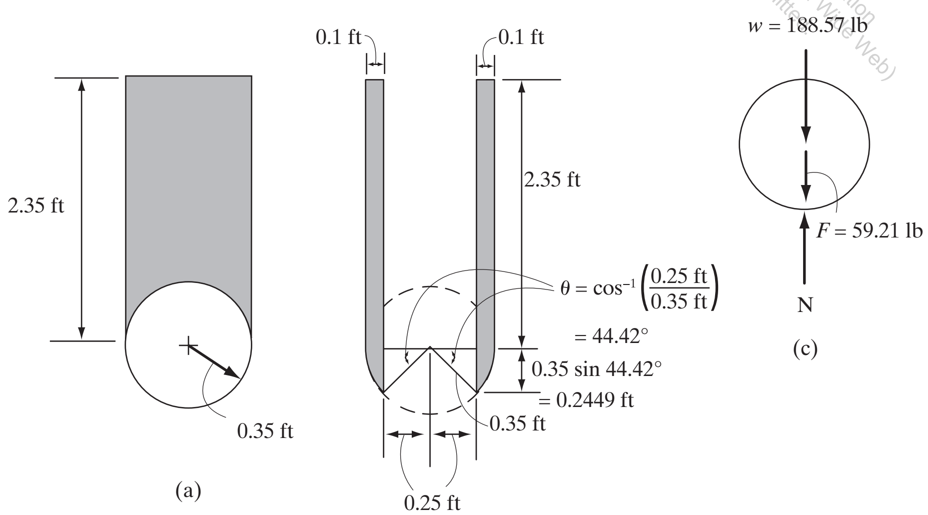
Then,

$$F = \gamma_w(V_1 - V_2)$$
$$= (62.4\ \text{lb/ft}^3)(1.4526\ \text{ft}^3 - 0.5037\ \text{ft}^3)$$
$$= 59.21\ \text{lb} \downarrow$$

The weight of the cylinder is $W = \gamma_{st}V_C = (490\ \text{lb/ft}^2)\left[\pi(0.35\ \text{ft})^2(1\ \text{ft})\right] = 188.57$ lb. Considering the free-body diagram of the cylinder, Fig. *c*, we have

$+\uparrow\Sigma F_y = 0;\qquad N - 59.21\ \text{lb} - 188.57\ \text{lb} = 0$

$N = 247.78\ \text{lb} = 248\ \text{lb}$ **Ans.**

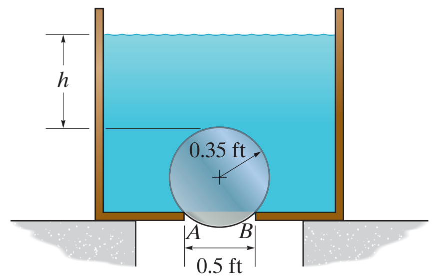


(a) (b) (c)

Ans:
$N = 248$ lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–115. The steel cylinder has a specific weight of $490\ \text{lb/ft}^3$ and acts as a plug for the 1-ft-long slot in the tank. Determine the resultant vertical force the bottom of the tank exerts on the cylinder when the water in the tank just covers the top of the cylinder, $h = 0$.



SOLUTION

The vertical downward force and the vertical upward force are equal to the weight of the water contained in the blocks shown shaded in Figs. *a* and *b*, respectively. The volume of the shaded block in Fig. *a* is

$$\forall_1 = \left[0.35\ \text{ft}(0.7\ \text{ft}) - \frac{\pi}{2}(0.35\ \text{ft})^2\right](1\ \text{ft}) = 0.05258\ \text{ft}^3$$

The volume of the shaded block in Fig. *b* is

$$\forall_2 = 2\left\{0.35\ \text{ft}(0.1\ \text{ft}) + \left[\frac{44.42°}{360°}(\pi)(0.35\ \text{ft})^2 - \frac{1}{2}(0.25\ \text{ft})(0.2449\ \text{ft})\right]\right\}(1\ \text{ft})$$
$$= 0.10372\ \text{ft}^3$$

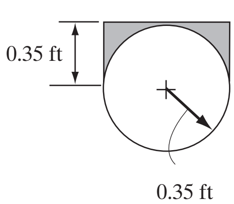
Then,

$$F = \gamma_w(\forall_1 - \forall_2)$$
$$= \left(62.4\ \text{lb/ft}^3\right)\left(0.10372\ \text{ft}^3 - 0.05258\ \text{ft}^3\right)$$
$$= 3.192\ \text{lb} \uparrow$$

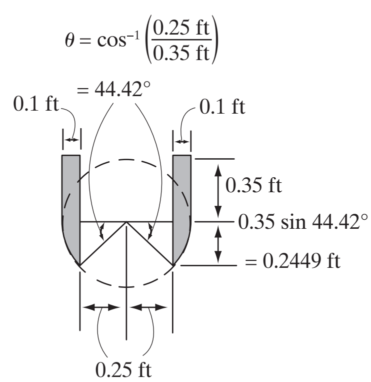
The weight of the cylinder is $W = \gamma_{st}\forall_C = \left(490\ \text{lb/ft}^2\right)\left[\pi(0.35\ \text{ft})^2(1\ \text{ft})\right] = 188.57\ \text{lb}$. Considering the force equilibrium vertically by free-body diagram of the cylinder, Fig. *c*, we have

$+\uparrow\Sigma F_y = 0;\qquad N + 3.192\ \text{lb} - 188.57\ \text{lb} = 0$

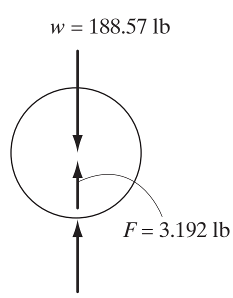
$N = 185.38\ \text{lb} = 185\ \text{lb}$ **Ans.**



(a)



(b)

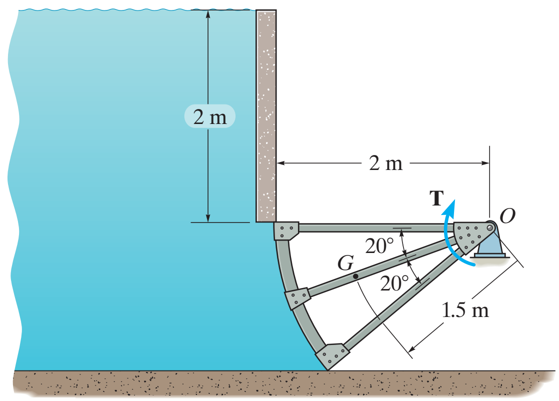


(c)

Ans:
$N = 185\ \text{lb}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–116.** The Tainter gate for a water channel is 1.5 m wide and in the closed position, as shown. Determine the magnitude of the resultant force of the water acting on the gate. Also, what is the smallest torque **T** that must be applied to open the gate if its weight is 30 kN and its center of gravity is at G.



SOLUTION

The horizontal component of the resultant force is equal to the pressure force acting on the vertically projected area of the gate. Referring to Fig. *a*,

$$w_1 = \rho_w g h_1 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m})(1.5\ \text{m}) = 29.43(10^3)\ \text{N/m}$$

$$w_2 = \rho_w g h_2 b = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2\ \text{m} + 2\ \text{m}\sin 40°)(1.5\ \text{m}) = 48.347(10^3)\ \text{N/m}$$

Then

$$(F_h)_1 = \left[29.43(10^3)\ \text{N/m}\right](2\sin 40°\text{m}) = 37.834(10^3)\ \text{N} = 37.834\ \text{kN}$$

$$(F_h)_2 = \frac{1}{2}\left[(48.347 - 29.43)(10^3)\ \text{N/m}\right](2\ \sin\ 40°\text{m}) = 12.160(10^3)\ \text{N} = 12.160\ \text{kN}$$

$$F_h = (F_h)_1 + (F_h)_2 = 37.834(10^3)\ \text{N} + 12.160(10^3)\ \text{N} = 49.994(10^3)\ \text{N} = 49.994\ \text{kN}$$

Also,

$$\tilde{y}_1 = \frac{1}{2}(2\ \text{m}\sin 40°) = 0.6428\ \text{m} \quad \text{and} \quad \tilde{y}_2 = \frac{2}{3}(2\ \text{m}\sin 40°) = 0.8571\ \text{m}$$

The vertical component of the resultant force is equal to the weight of the imaginary column of water above the gate (shown shaded in Fig. *a*), but acts upward. The volume of this column of water is

$$\forall = \left[(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40°}{180°}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right](1.5\ \text{m})$$
$$= 2.0209\ \text{m}^3$$

$$F_v = \rho_w g \forall = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.0209\ \text{m}^3) = 19.825(10^3)\ \text{N} = 19.825\ \text{kN}$$

Referring to Fig *b* and *c*,

$$\bar{r} = \frac{2}{3}\left(\frac{2\ \text{m}\sin 20°}{\frac{20}{180}\pi}\right) = 1.3064\ \text{m} \qquad \tilde{x}_2 = 1.3064\ \text{m}\cos 20° = 1.2276\ \text{m}$$

$$\tilde{x}_1 = 2\ \text{m}\cos 40° + \left(\frac{2\ \text{m} - 2\ \text{m}\cos 40°}{2}\right) = 1.7660\ \text{m}$$

$$\tilde{x}_3 = \frac{2}{3}(2\ \text{m}\cos 40°) = 1.0214\ \text{m}$$

Thus, F_v acts at

$$\bar{x} = \frac{(1.7660\ \text{m})(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + (1.2276\ \text{m})\left[\frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right)\right] - (1.0214\ \text{m})\left[\frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)\right]}{(2\ \text{m} - 2\ \text{m}\cos 40°)(2\ \text{m}) + \frac{1}{2}(2\ \text{m})^2\left(\frac{40}{180}\pi\ \text{rad}\right) - \frac{1}{2}(2\ \text{m}\cos 40°)(2\ \text{m}\sin 40°)}$$

$$= 1.7523\ \text{m}$$

***2–116. Continued**

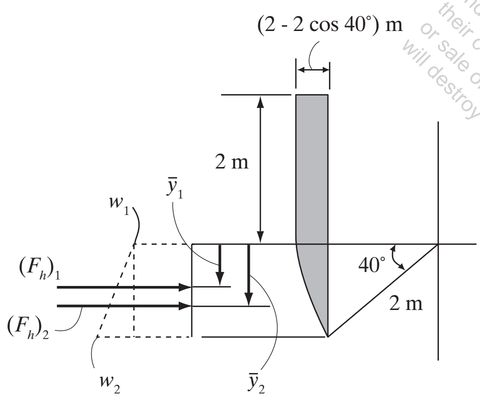
The magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994 \text{ kN})^2 + (19.825 \text{ kN})^2} = 53.78 \text{ kN} = 53.8 \text{ kN} \quad \textbf{Ans.}$$

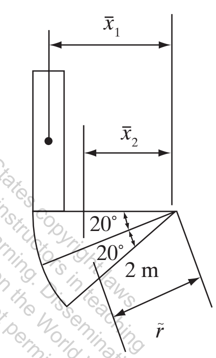
Referring to the FBD of the gate shown in Fig *d*,

$$\circlearrowleft + \Sigma M_0 = 0; \quad (30 \text{ kN})(1.5 \cos 20°\text{m}) + (37.834 \text{ kN})(0.6428 \text{ m}) + (12.160 \text{ kN})(0.8571 \text{ m})$$
$$-(19.825 \text{ kN})(1.7524 \text{ m}) - T = 0$$
$$T = 42.29 \text{ kN}\cdot\text{m} = 42.3 \text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$

Note that the resultant force of the water acting on the gate must act normal to its surface, and therefore it will pass through the pin at *O*. Therefore, it produces moment about the pin.



(a)



(b)

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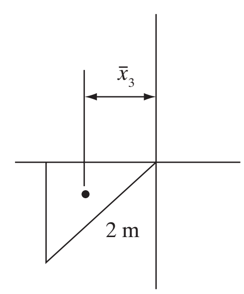
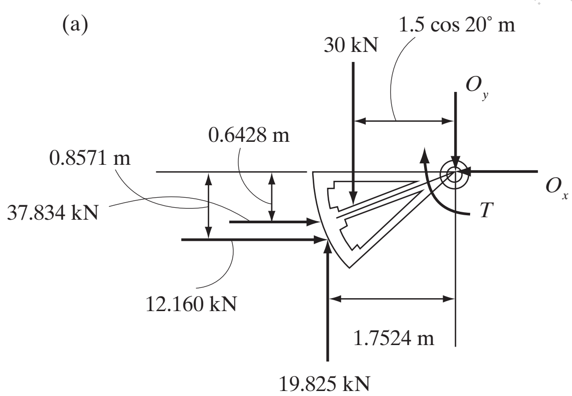


(c)

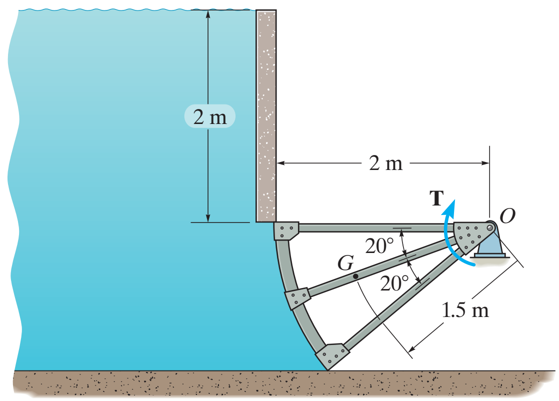


(d)

Ans:
$F_R = 53.8$ kN
$T = 42.3$ kN·m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–117. Solve the first part of Prob. 2–116 by the integration method using polar coordinates.

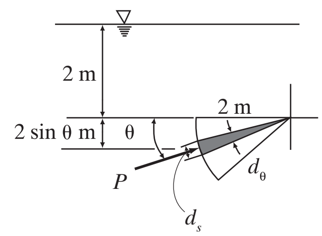


SOLUTION

Referring to Fig a, $h = (2 + 2\sin\theta)$ m. Thus, the pressure acting on the gate as a function of θ is

$$p = \rho_w g h = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2 + 2\sin\theta)\ \text{m} = [19620(1 + \sin\theta)]\ \text{N/m}^2$$

This pressure is acting on the element of area $dA = b\,ds = 1.5\,ds = 1.5(2\,d\theta) = 3\,d\theta$.



(a)

Thus,

$$dF = p\,d_A = 19620(1 + \sin\theta)(3\,d\theta) = 58.86\left(10^3\right)(1 + \sin\theta)\,d\theta$$

The horizontal and vertical components of $d\mathbf{F}$ are

$$(dF)_h = 58.86\left(10^3\right)(1 + \sin\theta)\cos\theta\,d\theta$$

$$= 58.86\left(10^3\right)(\cos\theta + \sin\theta\cos\theta)\,d\theta$$

$$(dF)_v = 58.86\left(10^3\right)(1 + \sin\theta)\sin\theta\,d\theta$$

$$= 58.86\left(10^3\right)\left(\sin\theta + \sin^2\theta\right)d\theta$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$, then

$$(dF)_h = 58.86\left(10^3\right)\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$(dF)_v = 58.86\left(10^3\right)\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

The horizontal and vertical components of the resultant force are

$$F_h = \int (dF)_h = 58.86\left(10^3\right)\int_0^{\frac{2\pi}{9}}\left(\cos\theta + \frac{1}{2}\sin 2\theta\right)d\theta$$

$$= 58.86\left(10^3\right)\left[\sin\theta - \frac{1}{4}\cos 2\theta\right]\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 49.994\left(10^3\right)\ \text{N} = 49.994\ \text{kN}$$

$$F_v = \int (dF)_v = 58.86\left(10^3\right)\int_0^{\frac{2\pi}{9}}\left(\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$$

$$= 58.86\left(10^3\right)\left(-\cos\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Bigg|_0^{\frac{2\pi}{9}}$$

$$= 19.825\left(10^3\right)\ \text{N} = 19.825\ \text{kN}$$

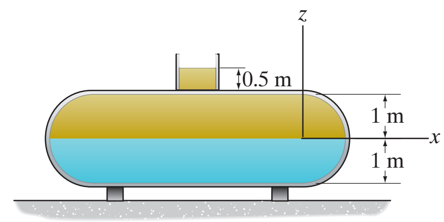
Thus, the magnitude of the resultant force is

$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{(49.994\ \text{kN})^2 + (19.825\ \text{kN})^2} = 53.78\ \text{kN} = 53.8\ \text{kN} \quad \textbf{Ans.}$$

Ans:
$F_R = 53.8$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–118. The cylindrical tank is filled with gasoline and water to the levels shown. Determine the horizontal and vertical components of the resultant force on its hemispherical end. Take $\rho_g = 726\ \text{kg/m}^3$.



SOLUTION

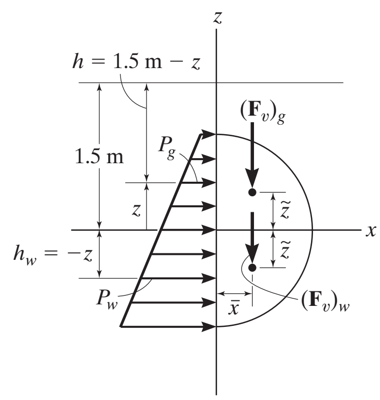
The vertical component of the resultant force is equal to the total weight of the gasoline and water contained in the hemisphere. Here,

$$(F_v)_g = \rho_g g V_g = [(726\ \text{kg/m}^3)(9.81\ \text{m/s}^2)]\left[\frac{1}{4}\left(\frac{4}{3}\right)(\pi)(1\ \text{m})^3\right] = 2.374(10^3)\pi\ \text{kN}$$

$$(F_v)_w = \rho_w g V_w = [(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)]\left[\frac{1}{4}\left(\frac{4}{3}\right)(\pi)(1\ \text{m})^3\right] = 3.270(10^3)\pi\ \text{kN}$$

Then

$$F_v = (F_v)_g + (F_v)_w = 2.374(10^3)\pi\ \text{N} + 3.270(10^3)\pi\ \text{N}$$
$$= 5.644(10^3)\pi\ \text{N} = 17.7\ \text{kN} \qquad \textbf{Ans.}$$



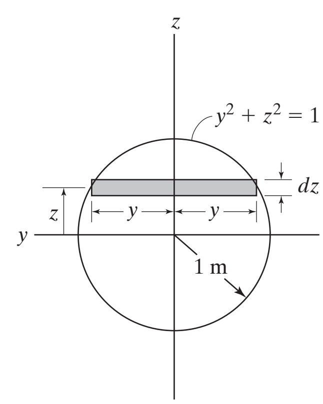
(a)

The equation of the pressure as a function of z for gasoline, Fig. a, is

$$P_g = \rho_g g h_g = (726\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.5\ \text{m} - z) = 7.122(10^3)(1.5 - z)$$

When $h_g = 1.5\ \text{m}(z = 0)$, $p_g|_{z=0} = 7.122(10^3)(1.5 - 0) = 10.683(10^3)\ \text{N/m}^2$. Then for water realizing that $h_w = -z$,

$$p_w = p_g|_{z=0} + \rho_w g h_w$$
$$= 10.683(10^3)\ \text{N/m}^2 + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(-z)$$
$$= [10.683(10^3) - 9.81(10^3)z]\ \text{N/m}^2$$



(b)

Here, the differential force dF acting on the differential element of area $dA = 2y\,dz = 2\sqrt{1 - z^2}dz$ is $df = p\,dA = p[2\sqrt{1 - z^2}dz]$. For gasoline,

$$dF_g = 7.123(10^3)(1.5 - z)[2\sqrt{1 - z^2}dz]$$
$$= 14.244(10^3)[1.5\sqrt{1 - z^2} - z\sqrt{1 - z^2}]dz$$

Integrating,

$$(F_g)_h = 14.244(10^3)\int_0^{1\ \text{m}}[1.5\sqrt{1 - z^2} - z\sqrt{1 - z^2}]dz$$

$$= 14.244(10^3)\left\{\frac{1.5}{2}[z\sqrt{1 - z^2} + \sin^{-1}z] - \left[-\frac{1}{3}\sqrt{(1 - z^2)^3}\right]\right\}\Bigg|_0^{1\ \text{m}}$$

$$= 12.033(10^3)\ \text{N}$$

For water,

$$dF_w = (10.683(10^3) - 9.81(10^3)z)[2\sqrt{1 - z^2}dz]$$
$$= 19.62(10^3)[1.089\sqrt{1 - z^2} - z\sqrt{1 - z^2}]dz$$

2–118. Continued

Integrating,

$$(F_w)_h = 19.62(10^3)\int_{-1\text{ m}}^{0}[1.089\sqrt{1-z^2} - z\sqrt{1-z^2}]dz$$

$$= 19.62(10^3)\left\{\frac{1.089}{2}[z\sqrt{1-z^2} + \sin^{-1}z] - \left[-\frac{1}{3}\sqrt{(1-x)^3}\right]\right\}\Bigg|_{-1\text{ m}}^{0}$$

$$= 23.321(10^3)\text{ N}$$

Then

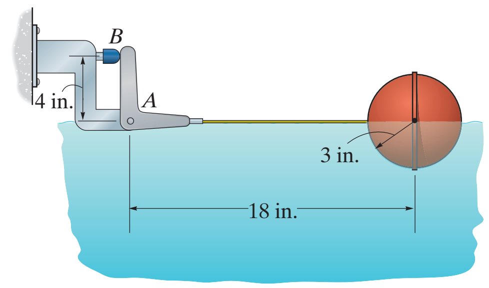
$$F_h = (F_g)_h + (F_w)_h = 12.033(10^3)\text{ N} + 23.321(10^3)\text{ N}$$

$$= 35.354(10^3)\text{ N} = 35.4\text{ kN}$$ **Ans.**

Ans:
$F_v = 17.7$ kN
$F_h = 35.4$ kN

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–119. The hollow spherical float controls the level of water within the tank. If the water is at the level shown, determine the horizontal and vertical components of the force acting on the supporting arm at the pin A, and the normal force on the smooth support B. Neglect the weight of the float.



SOLUTION

The vertical buoyant force acting on the spherical float is

$$F_v = \gamma_w \forall_{\text{sub}} = (62.4\ \text{lb/ft}^3)\left\{\frac{1}{2}\left[\frac{4}{3}\pi\left(\frac{3}{12}\ \text{ft}\right)^3\right]\right\} = 0.65\pi\ \text{lb}$$

Consider the equilibrium of the FBD of the floating system shown in Fig. *a*.

$$\curvearrowleft + \Sigma M_A = 0; \qquad (0.65\pi\ \text{lb})(18\ \text{in.}) - N_B(4\ \text{in.}) = 0$$

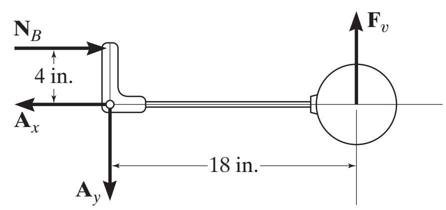
$$N_B = 2.925\pi\ \text{lb} = 9.19\ \text{lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2.925\pi\ \text{lb} - A_x = 0$$

$$A_x = 2.925\pi\ \text{lb} = 9.19\ \text{lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 0.65\pi\ \text{lb} - A_y = 0$$

$$A_y = 0.65\pi\ \text{lb} = 2.04\ \text{lb} \qquad \textbf{Ans.}$$

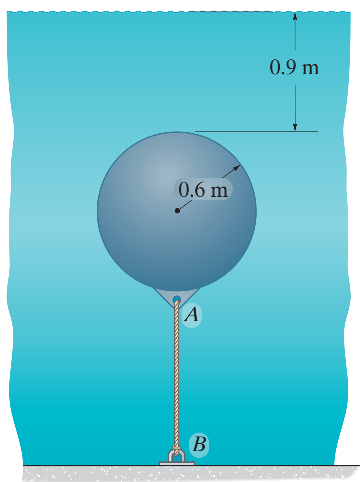


(a)

Ans:
$N_B = 9.19$ lb
$A_x = 9.19$ lb
$A_y = 2.04$ lb

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–120.** The solid ball is made of plastic having a density of $\rho_p = 48\ \text{kg/m}^3$. Determine the tension in the cable AB if the ball is submerged in the water at the depth shown. Will this force increase, decrease, or remain the same if the cord is shortened? Why? *Hint:* The volume of a ball is $V = \frac{4}{3}\pi r^3$.



SOLUTION

The weight of the ball is

$$W_b = \rho_p g V_b = (48\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{4}{3}\pi(0.6\ \text{m})^3\right] = 426.04\ \text{N}$$

The submerged volume is equal to the volume of the ball since it is fully submerged. Thus, the buoyant force is

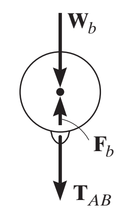
$$F_b = \rho_w g V_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{4}{3}\pi(0.6\ \text{m})^3\right] = 8875.88\ \text{N}$$

Referring to the FBD of the ball, Fig. *a*, the equilibrium requires

$$+\uparrow \Sigma F_y = 0; \qquad 8875.88\ \text{N} - 426.04\ \text{N} - T_{AB} = 0$$

$$T_{AB} = 8449.84\ \text{N} = 8.45\ \text{kN} \qquad \textbf{Ans.}$$

The tension in cable AB **remains the same** since the buoyant force does not change once a body is fully submerged, which means that it is independent of the submerged depth.



(a)

Ans:
$T_{AB} = 8.45\ \text{kN}$

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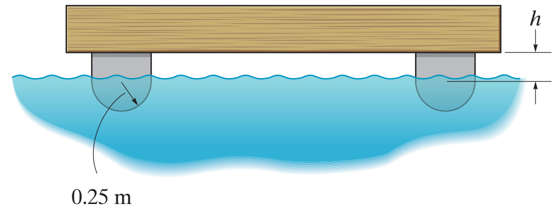
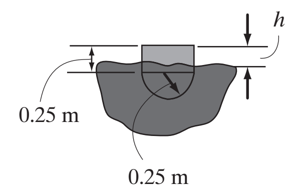
Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–121. The raft consists of a uniform platform having a mass of 2 Mg and four floats, each having a mass of 120 kg and a length of 4 m. Determine the height h at which the platform floats from the water surface. Take $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION



(a)

Each float must support a weight of

$$W = \left[\frac{1}{4}(2000\ \text{kg}) + 120\ \text{kg}\right]9.81\ \text{m/s}^2 = 6082.2\ \text{N}$$

For equilibrium, the buoyant force on each float is required to be

$$+\uparrow\Sigma F_y = 0; \qquad F_b - 6082.2\ \text{N} = 0 \qquad F_b = 6082.2\ \text{N}$$

Therefore, the volume of water that must be displaced to generate this force is

$$F_b = \gamma \mathcal{V}; \qquad 6082.2\ \text{N} = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\mathcal{V}$$

$$\mathcal{V} = 0.620\ \text{m}^3$$

Since the semicircular segment of a float has a volume of $\frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) = 0.3927\ \text{m}^3 < 0.620\ \text{m}^3$, then it must be fully submerged to develop F_b. As shown in Fig. *a*, we require

$$0.620\ \text{m}^3 = \frac{1}{2}(\pi)(0.25\ \text{m})^2(4\ \text{m}) + (0.25\ \text{m} - h)(0.5\ \text{m})(4\ \text{m})$$

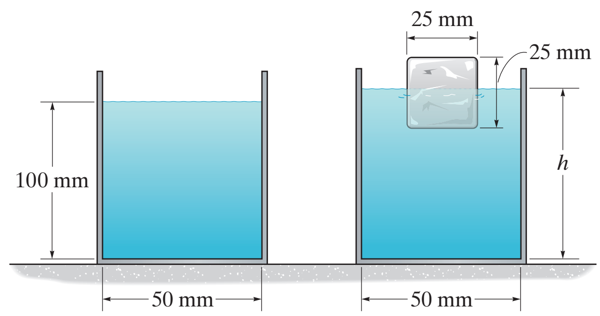
Thus,

$$h = 0.136\ \text{m} = 136\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$h = 136\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–122. A glass having a diameter of 50 mm is filled with water to the level shown. If an ice cube with 25-mm sides is placed into the glass, determine the new height h of the water surface. Take $\rho_w = 1000\ \text{kg/m}^3$ and $\rho_{ice} = 920\ \text{kg/m}^3$. What will the water level h be when the ice cube completely melts?



SOLUTION

Since the ice floats, the buoyant force is equal to the weight of the ice cube, which is

$$F_b = W_i = \rho_i \forall_i g = (920\ \text{kg/m}^3)(0.025\ \text{m})^3(9.81\ \text{m/s}^2) = 0.1410\ \text{N}$$

This buoyant force is also equal to the weight of the water displaced by the submerged ice cube at a depth h_s.

$$F_b = \rho_w g \forall_s; \qquad 0.1410\ \text{N} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[(0.025\ \text{m})^2 h_s\right]$$

$$h_s = 0.023\ \text{m}$$

Referring to Fig. a,

$$\forall_1 = \forall_2 - \forall_3$$

$$\left[\pi(0.025\ \text{m})^2\right](0.1\ \text{m}) = \left[\pi(0.025\ \text{m})^2\right]h - (0.02\ \text{m})^2(0.023\ \text{m})$$

$$h = 0.1073\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

The mass of the ice cube is

$$M_i = \rho_i \forall_i = (920\ \text{kg/m}^3)(0.025\ \text{m})^3 = 0.014375\ \text{kg}$$

Thus, the rise in water level due to the additional water of the melting ice cubes can be determined from

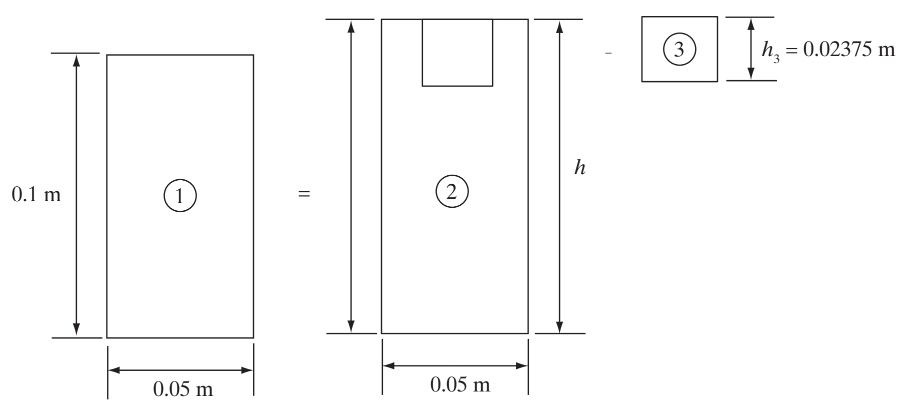
$$M_i = \rho_w \forall_w; \qquad 0.014375\ \text{kg} = (1000\ \text{kg/m}^3)\left[\pi(0.025\ \text{m})^2 \Delta h\right]$$

$$\Delta h = 0.007321$$

Thus,

$$h' = 0.1\ \text{m} + 0.007321\ \text{m} = 107\ \text{mm} \qquad \textbf{Ans.}$$

Note: The water level h remains unchanged as the cube melts.

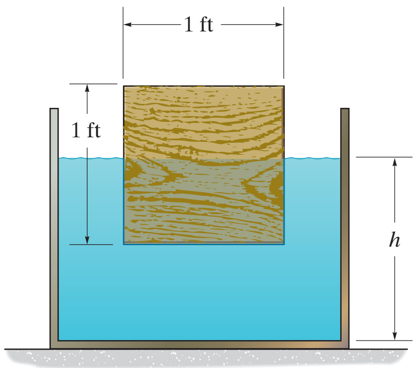


(a)

Ans:
$h = 107\ \text{mm}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–123. Water in the container is originally at a height of $h = 3$ ft. If a block having a specific weight of $50\ \text{lb/ft}^3$ is placed in the water, determine the new level h of the water. The base of the block is 1 ft square, and the base of the container is 2 ft square.



SOLUTION

The weight of the block is

$$W_b = \gamma_b \forall_b = \left(50\ \text{lb/ft}^3\right)\left[(1\ \text{ft})^3\right] = 50\ \text{lb}$$

Equilibrium requires that the buoyancy force equal the weight of the block so that $F_b = 50$ lb. Thus, the displaced volume is

$$F_b = \gamma_w \forall_{\text{Disp}} \qquad 50\ \text{lb} = \left(62.4\ \text{lb/ft}^3\right)\forall_{\text{Disp}}$$

$$\forall_{\text{Disp}} = 0.8013\ \text{ft}^3$$

The volume of the water is

$$\forall_w = 2\ \text{ft}(2\ \text{ft})(3\ \text{ft}) = 12\ \text{ft}^3$$

When the level of the water in the container has a height of h,

$$\forall_w = \forall' - \forall_{\text{Disp}}$$

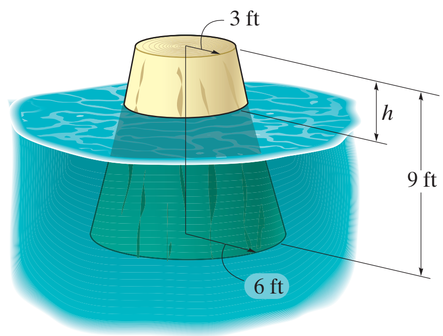
$$12\ \text{ft}^3 = 4h\ \text{ft}^3 - 0.8013\ \text{ft}^3$$

$$h = 3.20\ \text{ft}$$ **Ans.**

Ans:
$h = 3.20$ ft

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–124.** Determine the height at which the oak block will float above the water surface. The specific weight of the wood is $\gamma_w = 48\ \text{lb/ft}^3$.



SOLUTION

Referring to the geometry shown in Fig. *a*,

$$\frac{y}{9+y} = \frac{3}{6}; \qquad y = 9\ \text{ft}$$

And

$$\frac{v}{3} = \frac{h+9}{9}; \qquad r = \frac{1}{3}(h+9)$$

Thus, the volume of the oak block is

$$V_O \frac{1}{3}\pi(6\ \text{ft})^2(18\ \text{ft}) - \frac{1}{3}\pi(3\ \text{ft})^2(9\ \text{ft}) = 189\pi\ \text{ft}^3$$

And the submerged volume of the wooden block is

$$V_{\text{sub}} = \frac{1}{3}\pi(6\ \text{ft})^2(18\ \text{ft}) - \frac{1}{3}\pi\left[\frac{1}{3}(h+9)\right]^2(h+9) = \frac{\pi}{27}\left[5832 - (h+9)^3\right]$$

Then the weights of the wooden block and the buoyant force are

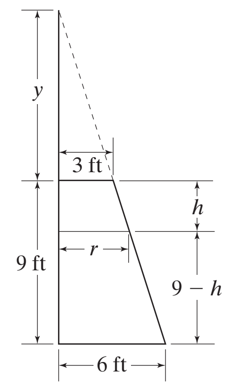
$$W_w = \gamma_O V_O = (48\ \text{lb/ft}^3)(189\pi\ \text{ft}^3) = 9072\pi\ \text{lb}$$

$$F_b = \gamma_w V_{\text{sub}} = (62.4\ \text{lb/ft}^3)\left\{\frac{\pi}{27}\left[5832 - (h+9)^3\right]\right\} = 2.3111\pi\left[5832 - (h+9)^3\right]$$

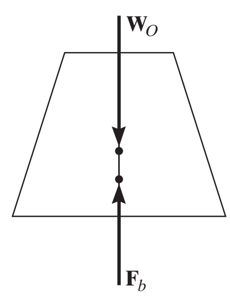
Referring to the FBD of oak block, Fig. *b*, equilibrium requires

$$+\uparrow\Sigma F_y = 0; \qquad 2.3111\pi\left[5832 - (h+9)^3\right] - 9072\pi\ \text{lb} = O$$

$$h = 3.4000\ \text{ft} = 3.40\ \text{ft} \qquad \textbf{Ans.}$$



(a)

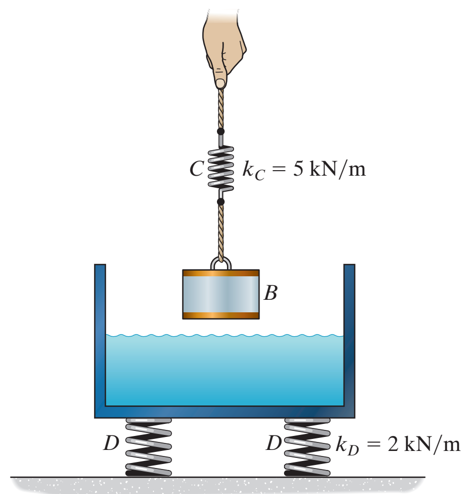


(b)

Ans:
$h = 3.40\ \text{ft}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–125. The container of water has a mass of 20 kg. Block B has a density of $7840\ \text{kg/m}^3$ and a mass of 30 kg. Determine the total compression or elongation of each spring when the block is fully submerged in the water.



SOLUTION

The volume of block B is determined from

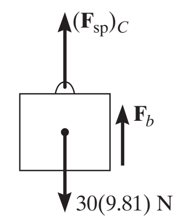
$$\rho_B = \frac{M_B}{\mathcal{V}_B}; \qquad 7840\ \text{kg/m}^3 = \frac{30\ \text{kg}}{\mathcal{V}_B} \qquad \mathcal{V}_B = 3.8265(10^{-3})\ \text{m}^3$$

Here, block B is fully submerged. Then $\mathcal{V}_{sub} = \mathcal{V}_B$. Thus, the buoyant force

$$F_b = \rho_w g \mathcal{V}_{sub} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[3.8265(10^{-3})\ \text{m}^3] = 37.54\ \text{N}$$

Referring to the FBD of block B, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad (F_{sp})_C + 37.54\ \text{N} - 30(9.81)\ \text{N} = 0 \qquad (F_{sp})_C = 256.76\ \text{N}$$



(a)

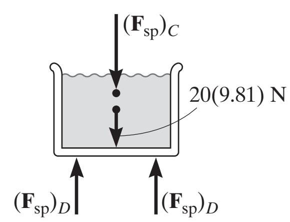
Referring to the FBD of the container, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad 2(F_{sp})_D - 37.54\ \text{N} - 20(9.81)\ \text{N} = 0 \qquad (F_{sp})_D = 116.87\ \text{N}$$

Thus, the change in length of springs C and D are

$$\delta_C = \frac{(F_{sp})_C}{K_C} = \frac{256.76\ \text{N}}{5(10^3)\ \text{N/m}} = 0.05135\ \text{m} = 51.4\ \text{mm} \qquad \textbf{Ans.}$$

$$\delta_D = \frac{(F_{sp})_D}{K_C} = \frac{116.87\ \text{N}}{2(10^3)\ \text{N/m}} = 0.05843\ \text{m} = 58.4\ \text{mm} \qquad \textbf{Ans.}$$

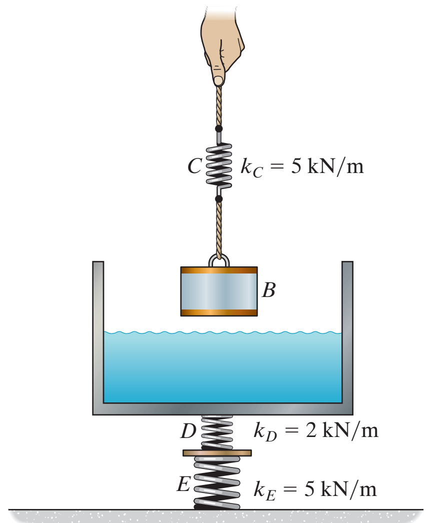


(b)

Ans:
$\delta_C = 51.4$ mm
$\delta_D = 58.4$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–126. The container of water has a mass of 20 kg. Block B has a density of $7840\ \text{kg/m}^3$ and a mass of 30 kg. Determine the total compression or elongation of each spring when the block is fully submerged in the water.



SOLUTION

The volume of block B is determined from

$$\rho_B = \frac{M_B}{V_B};\qquad 7840\ \text{kg/m}^3 = \frac{30\ \text{kg}}{V_B}\qquad V_B = 3.8265(10^{-3})\ \text{m}^3$$

Here, block B is fully submerged. Then $V_{\text{sub}} = V_B$. Thus, the buoyant force is

$$F_b = \rho_w g V_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[3.8265(10^{-3})\ \text{m}^3] = 37.54\ \text{N}$$

Referring to the FBD of block B, Fig. *a*,

$$+\uparrow\Sigma F_y = 0;\qquad (F_{\text{sp}})_C + 37.54\ \text{N} - 30(9.81)\ \text{N} = 0\qquad (F_{\text{sp}})_C = 256.76\ \text{N}$$

Thus, the change in length of spring C is

$$\delta_C = \frac{(F_{\text{sp}})_C}{K_C} = \frac{256.76\ \text{N}}{5(10^3)\ \text{N/m}} = 0.05135\ \text{m} = 51.4\ \text{mm}\qquad \textbf{Ans.}$$

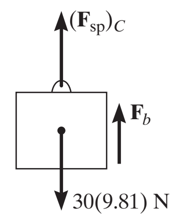
Springs D and E are subjected to the same force, $(F_{\text{sp}})_D = (F_{\text{sp}})_E$, since both are arranged in series. Referring to the FBD of the container, Fig. *b*,

$$+\uparrow\Sigma F_y = 0;\qquad (F_{\text{sp}})_D - 37.54\ \text{N} - 20(9.81)\ \text{N} = 0\qquad (F_{\text{sp}})_D = 233.74\ \text{N}$$

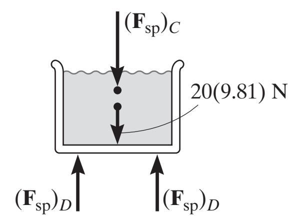
Thus, the change in length of spring D and E are

$$\delta_D = \frac{(F_{\text{sp}})_D}{K_D} = \frac{233.74\ \text{N}}{2(10^3)\ \text{N/m}} = 0.1169\ \text{m} = 117\ \text{mm}\qquad \textbf{Ans.}$$

$$\delta_E = \frac{(F_{\text{sp}})_D}{K_E} = \frac{233.74\ \text{N}}{5(10^3)\ \text{N/m}} = 0.04675\ \text{m} = 46.7\ \text{mm}\qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\delta_C = 51.4$ mm
$\delta_D = 117$ mm
$\delta_E = 46.7$ mm

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–127. The hot-air balloon contains air having a temperature of 180°F, while the surrounding air has a temperature of 60°F. Determine the maximum weight of the load the balloon can lift if the volume of air it contains is $120(10^3)\ \text{ft}^3$. The empty weight of the balloon is 200 lb.

SOLUTION

From the Appendix, the densities of the air inside the balloon, where $T = 180°\text{F}$ and outside the balloon where $T = 60°\text{F}$, are

$$\rho_a\big|_{T=60°\text{F}} = 0.00237\ \text{slug/ft}^3$$

$$\rho_a\big|_{T=180°\text{F}} = 0.00193\ \text{slug/ft}^3$$

Thus, the weight of the air inside the balloon is

$$W_a\big|_{T=180°\text{F}} = \rho_a\big|_{T=180°\text{F}}\, g\Psi = \left(0.00193\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 7457.52\ \text{lb}$$

The buoyancy force is equal to the weight of the displaced air outside of the balloon. This gives

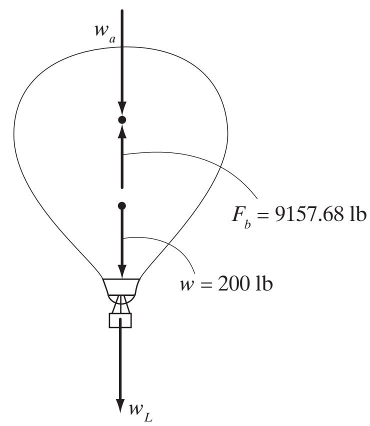
$$F_b = \rho_a\big|_{T=60°\text{F}}\, g\Psi = \left(0.00237\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left[120\left(10^3\right)\text{ft}^3\right]$$

$$= 9157.68\ \text{lb}$$

Considering the free-body diagram of the balloon in Fig. *a*,

$+\uparrow\Sigma F_y = 0;\qquad 9157.68\ \text{lb} - 7457.52\ \text{lb} - 200\ \text{lb} - W_L = 0$

$W_L = 1500.16\ \text{lb} = 1.50\ \text{kip}$ **Ans.**

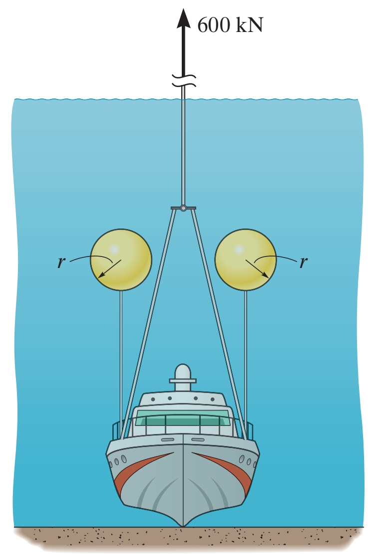


(a)

Ans:
$W_L = 1.50\ \text{kip}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–128.** A boat having a mass of 80 Mg rests on the bottom of the lake and displaces 10.25 m^3 of water. Since the lifting capacity of the crane is only 600 kN, two balloons are attached to the sides of the boat and filled with air. Determine the smallest radius r of each spherical balloon that is needed to lift the boat. What is the mass of air in each balloon if the air and water temperature is 12°C? The balloons are at an average depth of 20 m. Neglect the mass of the air and the balloon. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.



SOLUTION

The bouyant force acting on the boat and a balloon are

$$(F_b)_B = \rho_w g(V_B)_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(10.25\ \text{m}^3) = 100.55(10^3)\ \text{N}$$
$$= 100.55\ \text{kN}$$

$$(F_b)_b = \rho_w g(V_b)_{\text{sub}} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{4}{3}\pi r^3\right] = 13.08\pi r^3(10^3)\ \text{N}$$
$$= 13.08\pi r^3\ \text{kN}$$

Referring to the FBD of the boat, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2T + 100.55\ \text{kN} + 600\ \text{kN} - [80(9.81)\ \text{kN}] = 0$$
$$T = 42.124\ \text{kN}$$

Referring to the FBD of the balloon, Fig. *b*

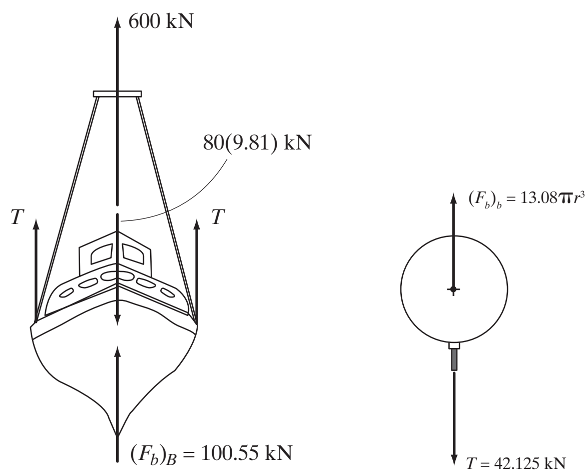
$$+\uparrow \Sigma F_y = 0; \qquad 13.08\pi r^3 - 42.125\ \text{kN} = 0$$
$$r = 1.008\ \text{m} = 1.01\ \text{m} \qquad \textbf{Ans.}$$

Here, $p = p_{\text{atm}} + \rho_w g h = 101(10^3)\ \text{Pa} + (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(20\ \text{m}) = 297.2(10^3)\ \text{Pa}$ and $T = 12°\text{C} + 273 = 285\ \text{K}$. From Appendix A, $R = 286.9\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

$$p = \rho R T; \qquad \rho = \frac{p}{RT} = \frac{297.2(10^3)\ \text{N/m}^2}{(286.9\ \text{J/kg}\cdot\text{K})(285\ \text{K})} = 3.6347\ \text{kg/m}^3$$

Thus,

$$m = \rho V = (3.6347\ \text{kg/m}^3)\left[\frac{4}{3}\pi(1.008\ \text{m})^3\right] = 15.61\ \text{kg} = 15.6\ \text{kg} \qquad \textbf{Ans.}$$

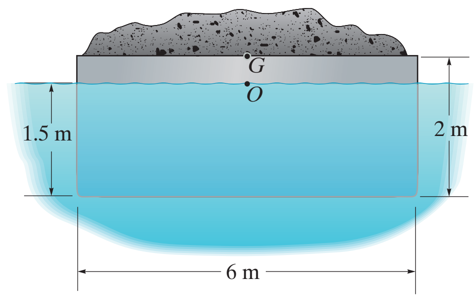


(a) (b)

Ans:
$r = 1.01$ m
$m = 15.6$ kg

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–129. When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is at G, determine whether the barge will restore itself when a wave causes it to roll slightly at 9°.



SOLUTION

When the barge tips 9°, the submerged portion is trapezoidal in shape, as shown in Fig. *a*. The new center of buoyancy, $C_b{}'$, is located at the centroid of this area. Then

$$x = \frac{(0)(6)(1.0248) + (1)\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.3168 \text{ m}$$

$$y = \frac{\frac{1}{2}(1.0248)(6)(1.0248) + \left[1.0248 + \frac{1}{3}(0.9503)\right]\left[\frac{1}{2}(6)(0.9503)\right]}{(6)(1.0248) + \frac{1}{2}(6)(0.9503)} = 0.7751 \text{ m}$$

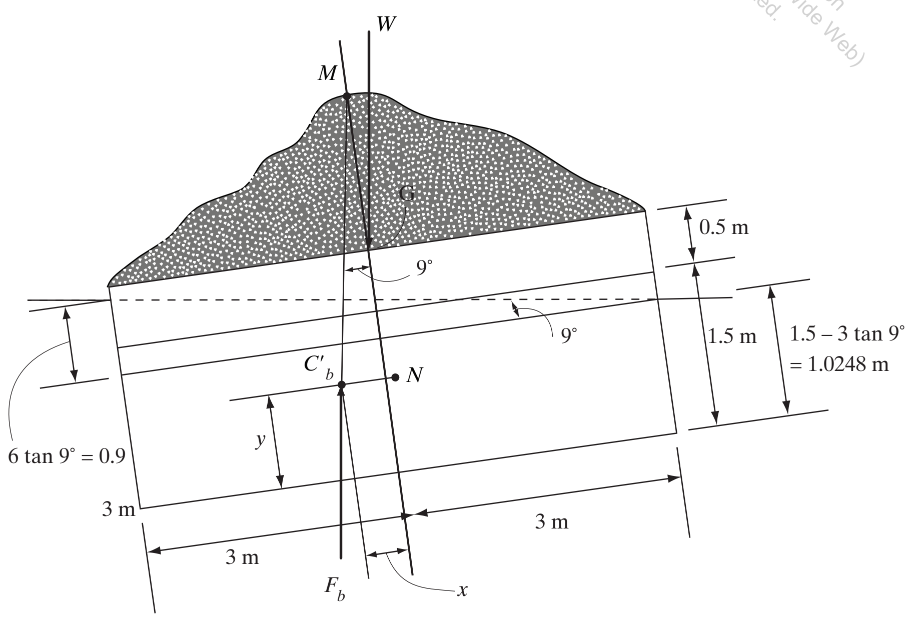
The intersection point M of the line of action of $\mathbf{F}_b$ and the centerline of the barge is the metacenter, Fig. *a*. From the geometry of triangle $MNC_b{}'$, we have

$$MN = \frac{x}{\tan 9^\circ} = \frac{0.3168}{\tan 9^\circ} = 2 \text{ m}$$

Also,

$$GN = 2 - y = 2 - 0.7751 = 1.2249 \text{ m}$$

Since $MN > GN$, point M is above G. Therefore, **the barge will restore itself.**

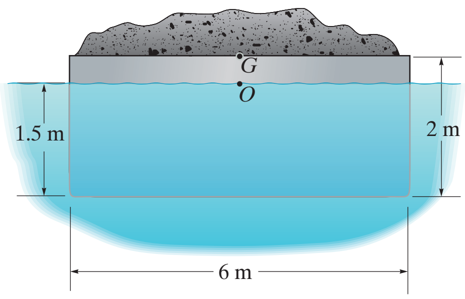


(a)

Ans:
It will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–130. When loaded with gravel, the barge floats in water at the depth shown. If its center of gravity is at G, determine whether the barge will restore itself when a wave causes it to tip slightly.



SOLUTION

The barge is tilted counterclockwise slightly and the new center of buoyancy $C_b{}'$ is located to the left of the old one. The metacenter M is at the intersection point of the center line of the barge and the line of action of $\mathbf{F}_b$, Fig. *a*. The location of $C_b{}'$ can be obtained by referring to Fig. *b*.

$$\bar{x} = \frac{(1\ \text{m})\left[\frac{1}{2}(6\ \text{m})(6\tan\phi\ \text{m})\right]}{(1.5\ \text{m})(6\ \text{m})} = 2\tan\phi\ \text{m}$$

Then

$$\delta = \bar{x}\cos\phi = 2\ \text{m}\tan\phi\ \cos\phi = (2\ \text{m})\left(\frac{\sin\phi}{\cos\phi}\right)(\cos\phi) = (2\sin\phi)\ \text{m}$$

Since ϕ is very small $\sin\phi = \phi$, hence

$$\delta = 2\phi\ \text{m} \qquad \textbf{(1)}$$

From the geometry shown in Fig. *a*,

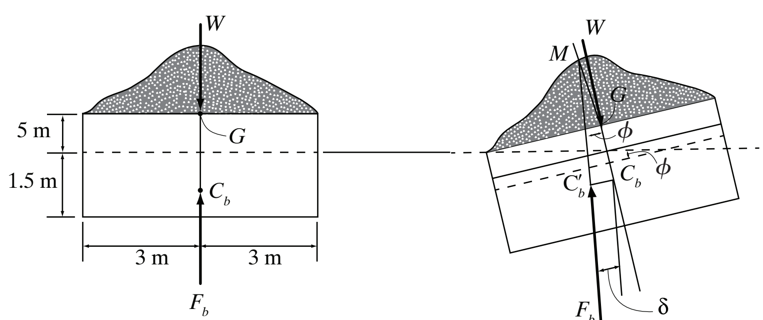
$$\delta = MC_b\ \sin\phi = MC_b\phi \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

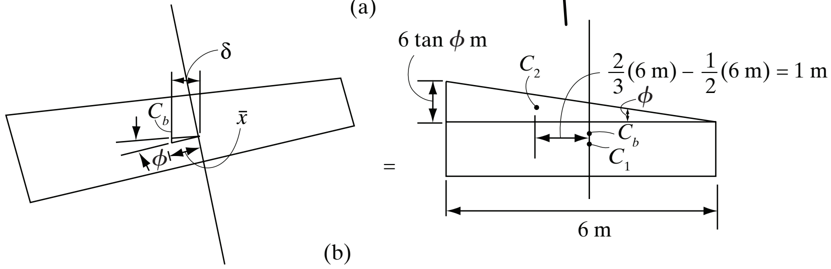
$$2\phi = MC_b\phi$$

$$MC_b = 2\ \text{m}$$

Here, $GC_b = 2\ \text{m} - 0.75\ \text{m} = 1.25\ \text{m}$. Since $MC_b > GC_b$, the barge is in stable equilibrium. Thus, it will restore itself if tilted slightly. **Ans.**



(a)

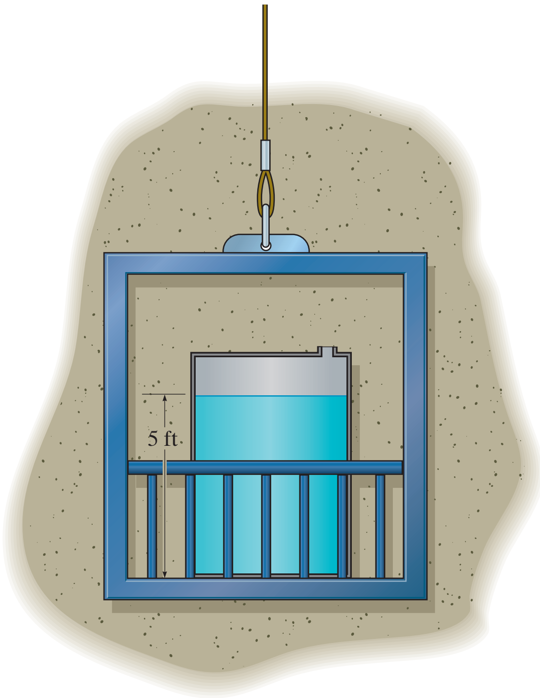


(b)

Ans:
It will restore itself.

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–131. The can of alcohol rests on the floor of a hoist. Determine the maximum pressure developed at the base of the can if the hoist is moving upward with (a) a constant velocity of 10 ft/s, and (b) a constant upward acceleration of 5 ft/s². Take $\gamma_{al} = 49.3\ \text{lb/ft}^3$.



SOLUTION

a) Since the hoist is travelling with a constant velocity, it is in equilibrium. Thus,

$$p = \gamma_a h = (49.3\ \text{lb/ft}^3)(5\ \text{ft})$$

$$= (246.5\ \text{lb/ft}^2)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.71\ \text{psi} \qquad \textbf{Ans.}$$

b) Since the hoist is accelerating,

$$p = \gamma_a h\left(1 + \frac{a_c}{g}\right)$$

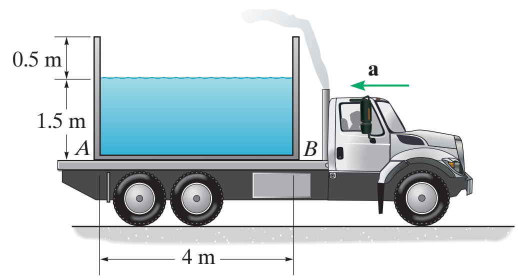
$$= (49.3\ \text{lb/ft}^3)(5\ \text{ft})\left(1 + \frac{5\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}\right)$$

$$= (284.78\ \text{lb/ft}^2)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 1.98\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
a. $p = 1.71$ psi
b. $p = 1.98$ psi

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–132.** The truck carries an open container of water. If it has a constant deceleration $1.5\ \text{m/s}^2$, determine the angle of inclination of the surface of the water and the pressure at the bottom corners *A* and *B*.

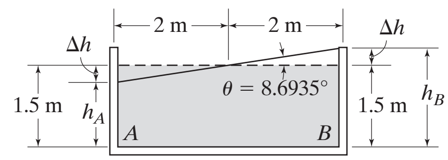


SOLUTION

The free surface of the water in the decelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{1.5\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 8.6935° = 8.69° \qquad \textbf{Ans.}$$



(a)

From the geometry in Fig. *a*,

$$\Delta h = (2\ \text{m})\tan 8.6935° = 0.3058\ \text{m}$$

Since $\Delta h < 0.5$ m, the water will not spill. Thus,
$h_A = 1.5\ \text{m} - 0.3058\ \text{m} = 1.1942\ \text{m}$ and $h_B = 1.5\ \text{m} + 0.3058\ \text{m} = 1.8058\ \text{m}$. Then

$$p_A = \rho_w g h_A = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.1942\ \text{m})$$

$$= 11.715(10^3)\ \text{N/m}^2 = 11.7\ \text{kPa} \qquad \textbf{Ans.}$$

And

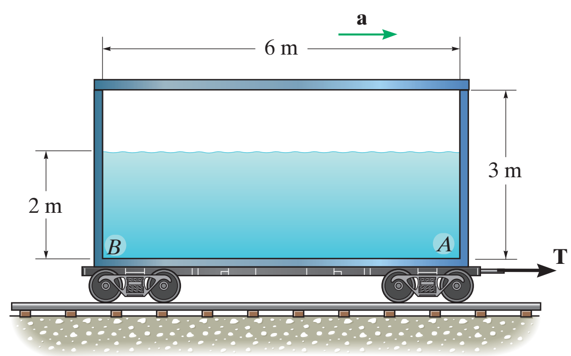
$$p_B = \rho_w g h_B = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.8058\ \text{m})$$

$$= 17.715(10^3)\ \text{N/m}^2 = 17.7\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$\theta = 8.69°$, $p_A = 11.7$ kPa,
$p_B = 17.7$ kPa

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

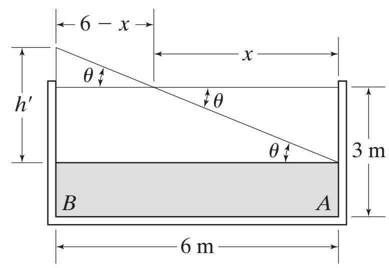
2–133. The closed rail car is 2 m wide and filled with water to the level shown. Determine the pressure that acts at A and B when the car has a constant acceleration of $4\ \text{m/s}^2$.



SOLUTION

$$\tan\theta = \frac{4}{9.81} = \frac{h'}{6}$$

$$h' = 2.446\ \text{m}$$



Also,

$$\theta = \tan^{-1}\frac{4}{9.81} = 22.183°$$

Empty volume in tank is $2(6)(1) = 12\ \text{m}^3$. During accelerating we require

$$12 = 2\left(\frac{1}{2}x\right)(x \tan 22.183°)$$

$$x = 5.425\ \text{m}$$

$$p_A = (1000)9.81(3 - 5.425 \tan 22.183°)$$

$$= 7.73\ \text{kPa} \qquad \textbf{Ans.}$$

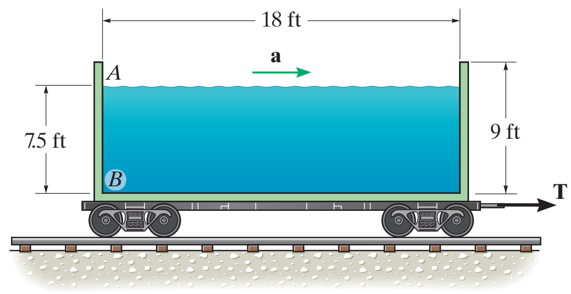
$$p_B = 1000(9.81)(3 + 16 - 5.425)(\tan 22.183°)$$

$$= 31.7\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
$p_A = 7.73\ \text{kPa}$, $p_B = 31.7\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–134. The open rail car is 6 ft wide and filled with water to the level shown. Determine the pressure that acts at point B both when the car is at rest, and when the car has a constant acceleration of $10\ \text{ft/s}^2$. How much water spills out of the car?



SOLUTION

When the car is at rest, the water is at the level shown by the dashed line shown in Fig. *a*.

At rest: $p_B = \gamma_w h_B = (62.4\ \text{lb/ft}^3)(7.5\ \text{ft}) = 468\ \text{lb/ft}^2$ **Ans.**

When the car accelerates, the angle θ the water level makes with the horizontal can be determined.

$$\tan\theta = \frac{a_c}{g} = \frac{10\ \text{ft/s}^2}{32.2\ \text{ft/s}^2}; \quad \theta = 17.25^\circ$$

Assuming that the water will spill out, then the water level when the car accelerates is indicated by the solid line shown in Fig. *a*. Thus,

$$h = 9\ \text{ft} - 18\ \text{ft}\tan 17.25^\circ = 3.4099\ \text{ft}$$

The original volume of water is

$$V = (7.5\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 810\ \text{ft}^3$$

The volume of water after the car accelerates is

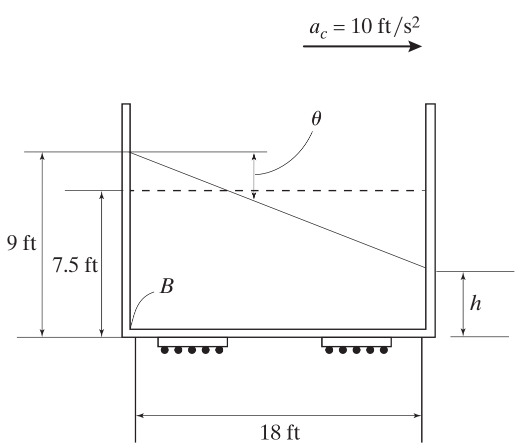
$$V' = \frac{1}{2}(9\ \text{ft} + 3.4099\ \text{ft})(18\ \text{ft})(6\ \text{ft}) = 670.14\ \text{ft}^3 < 810\ \text{ft}^3 \quad \textbf{(OK!)}$$

Thus, the amount of water spilled is

$$\Delta V = V - V' = 810\ \text{ft}^3 - 670.14\ \text{ft}^3 = 139.86\ \text{ft}^3 = 140\ \text{ft}^3 \quad \textbf{Ans.}$$

The pressure at B when the car accelerates is

With acceleration: $p_B = \gamma_w h_B = (62.4\ \text{lb/ft}^3)(9\ \text{ft}) = 561.6\ \text{lb/ft}^2 = 562\ \text{lb/ft}^2$ **Ans.**

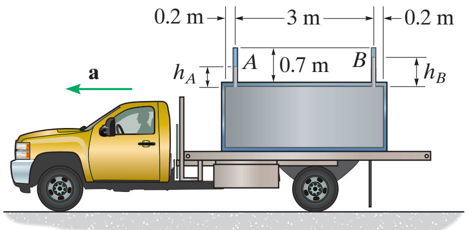


(a)

Ans:
At rest: $p_B = 468\ \text{lb/ft}^2$
With acceleration: $\Delta V = 140\ \text{ft}^3$
$p_B = 562\ \text{lb/ft}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–135. A large container of benzene is transported on the truck. Determine the level in each of the vent tubes A and B if the truck accelerates at $a = 1.5\ \text{m/s}^2$. When the truck is at rest, $h_A = h_B = 0.4$ m.



SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_c}{g} = \frac{1.5\ \text{m/s}^2}{9.81\ \text{m/s}^2}$$

$$\theta = 8.6935°$$

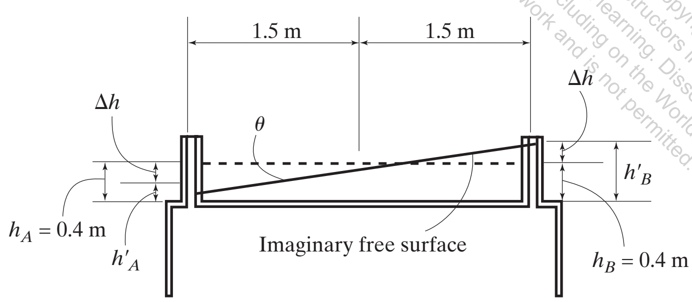
Then,

$$\Delta h = (1.5\ \text{m})\ \tan\ 8.6935° = 0.2294\ \text{m}$$

Thus,

$$h'_A = h_A - \Delta h = 0.4\ \text{m} - 0.2294\ \text{m} = 0.171\ \text{m}$$ **Ans.**

$$h'_B = h_B + \Delta h = 0.4\ \text{m} + 0.2294\ \text{m} = 0.629\ \text{m}$$ **Ans.**

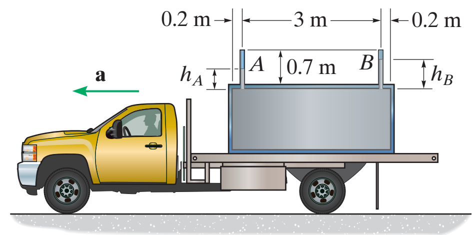


(a)

Ans:
$h'_A = 0.171$ m, $h'_B = 0.629$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–136.** A large container of benzene is being transported by the truck. Determine its maximum constant acceleration so that no benzene will spill from the vent tubes A or B. When the truck is rest, $h_A = h_B = 0.4$ m.

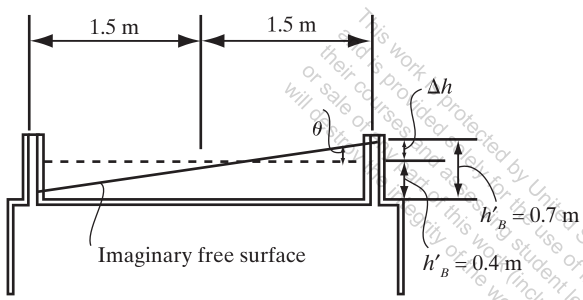


SOLUTION

The imaginary surface of the benzene in the accelerated tank is shown in Fig. *a*. Under this condition, the water will spill from vent B. Thus, $\Delta h = h'_B - h_B = 0.7\ \text{m} - 0.4\ \text{m} = 0.3\ \text{m}$.

$$\tan\theta = \frac{0.3\ \text{m}}{1.5\ \text{m}} = 0.2 = \frac{a_c}{g}$$

$$a_c = 0.2\left(9.81\ \text{m/s}^2\right) = 1.96\ \text{m/s}^2 \qquad \textbf{Ans.}$$

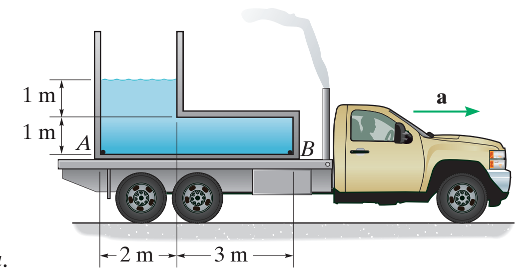


(a)

Ans:
$a_c = 1.96\ \text{m/s}^2$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–137. If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners A and B of the water tank.



SOLUTION

The imaginary free surface of the water in the accelerated tank is shown in Fig. *a*.

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. *a*,

$$\Delta h_A = (1\ \text{m})\tan\theta = (1\ \text{m})(0.2039) = 0.2039\ \text{m}$$

$$\Delta h_B = (1\ \text{m} + 3\ \text{m})\tan\theta = (4\ \text{m})(0.2039) = 0.8155\ \text{m}$$

Then

$$h_A = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.2039\ \text{m} = 2.2039\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.8155\ \text{m} = 1.1845\ \text{m}$$

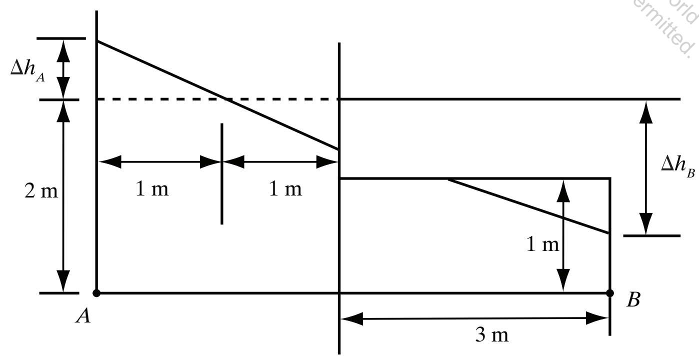
Finally,

$$p_A = \rho_w g h_A = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.2039\ \text{m})$$

$$= 21.62\left(10^3\right)\ \text{Pa} = 21.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.1845\ \text{m})$$

$$= 11.62\left(10^3\right)\ \text{Pa} = 11.6\ \text{kPa} \qquad \textbf{Ans.}$$

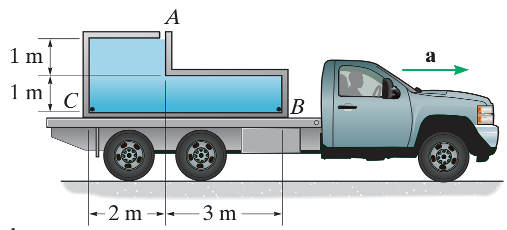


(a)

Ans:
$p_A = 21.6\ \text{kPa}$, $p_B = 11.6\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–138. If the truck has a constant acceleration of $2\ \text{m/s}^2$, determine the water pressure at the bottom corners B and C of the water tank. There is a small opening at A.



SOLUTION

Since the water in the tank is confined, the imaginary free surface must pass through A as shown in Fig. a. We have

$$\tan\theta = \frac{a_C}{g} = \frac{2\ \text{m/s}^2}{9.81\ \text{m/s}^2} = 0.2039$$

From the geometry in Fig. a,

$$\Delta h_C = (2\ \text{m})\tan\theta = (2\ \text{m})(0.2039) = 0.4077\ \text{m}$$

$$\Delta h_B = (3\ \text{m})\tan\theta = (3\ \text{m})(0.2039) = 0.6116\ \text{m}$$

Then

$$h_C = 2\ \text{m} + \Delta h_A = 2\ \text{m} + 0.4077\ \text{m} = 2.4077\ \text{m}$$

$$h_B = 2\ \text{m} - \Delta h_B = 2\ \text{m} - 0.6116\ \text{m} = 1.3884\ \text{m}$$

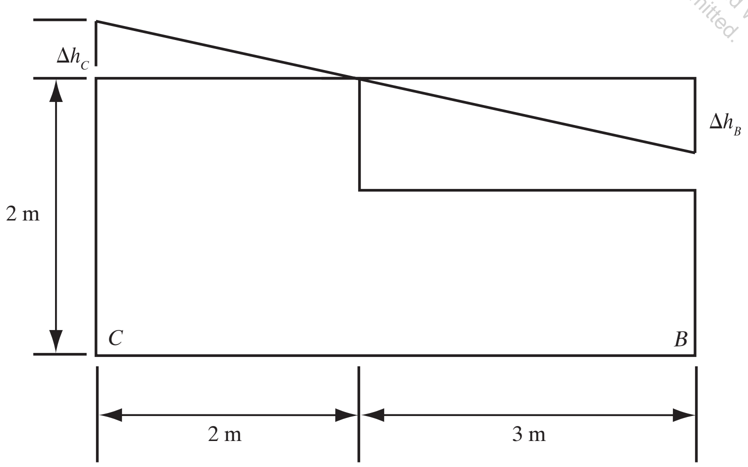
Finally,

$$p_C = \rho_w g h_C = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(2.4077\ \text{m})$$

$$= 23.62\left(10^3\right)\ \text{Pa} = 23.6\ \text{kPa} \qquad \textbf{Ans.}$$

$$p_B = \rho_w g h_B = \left(1000\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(1.3884\ \text{m})$$

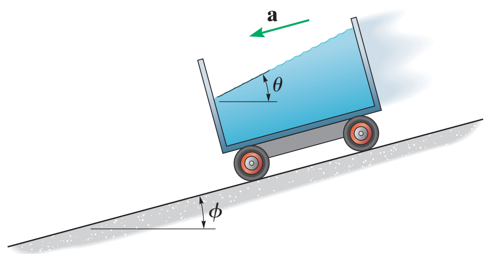
$$= 13.62\left(10^3\right)\ \text{Pa} = 13.6\ \text{kPa} \qquad \textbf{Ans.}$$



Ans:
$p_C = 23.6\ \text{kPa}$, $p_B = 13.6\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–139. The cart is allowed to roll freely down the inclined plane due to its weight. Show that the slope of the surface of the liquid, θ, during the motion is $\theta = \phi$.



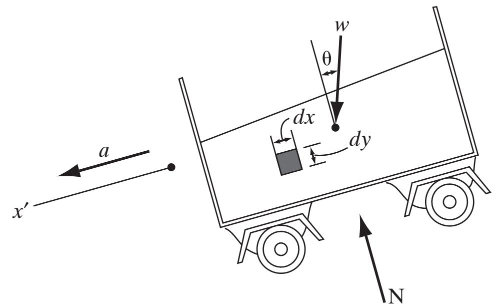
SOLUTION

Referring to the free-body diagram of the container in Fig. *a*,

$\swarrow + \Sigma F_{x'} = ma_{x'}$

$$w \sin\phi = \frac{w}{g} a$$

$$a = g \sin\phi$$

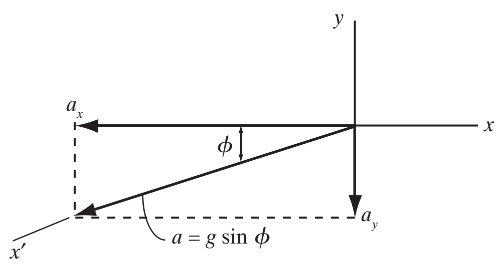


(a)

Referring to Fig. *b*,

$$a_x = -(g \sin\phi) \cos\phi$$

$$a_y = -(g \sin\phi) \sin\phi$$



(b)

We will now apply Newton's equations of motion, Fig. *c*.

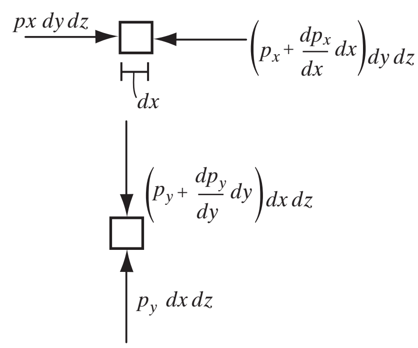
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad -\left(p_x + \frac{\partial p_x}{\partial x} dx\right) dy\,dz + p_x\,dy\,dz = \frac{\gamma(dx\,dy\,dz)}{g} a_x$

$$dp_x = -\frac{\gamma\,dx}{g} a_x$$

In y direction,

$+\uparrow \Sigma F_y = ma_y;\qquad p_y\,dx\,dz - \left(p_y + \frac{\partial p_y}{\partial y} dy\right) dx\,dz - \gamma\,dx\,dy\,dz = \frac{\gamma\,dx\,dy\,dz}{g} a_y$

$$dp_y = -\gamma\,dy\left(1 + \frac{a_y}{g}\right)$$



(c)

At the surface, p is constant, so that $dp_x + dp_y = 0$, or $dp_x = -dp_y$.

$$\frac{\gamma\,dx}{g} a_x = -\gamma\,dy\left(1 + \frac{a_y}{g}\right)$$

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} = \frac{g \sin\phi \cos\phi}{g - g \sin\phi \sin\phi} = \frac{\sin\phi \cos\phi}{\cos^2\phi} = \frac{\sin\phi}{\cos\phi} = \tan\phi$$

Since at the surface,

$$\frac{dy}{dx} = -\tan\theta$$

then

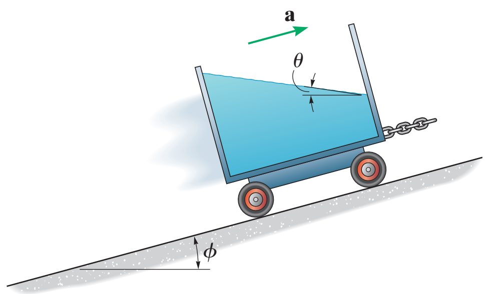
$$\tan\theta = \tan\phi$$

or

$$\theta = \phi \qquad \textbf{Q.E.D.}$$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–140.** The cart is given a constant acceleration **a** up the inclined plane. Show that the lines of constant pressure *within* the liquid have a slope of $\tan\theta = (a\cos\phi)/(a\sin\phi + g)$.



SOLUTION

As in the preceding solution, we determine that

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y} \qquad \textbf{(1)}$$

Here, the slope of the surface of the liquid, Fig. *a*, is

$$\frac{dy}{dx} = -\tan\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2), we obtain

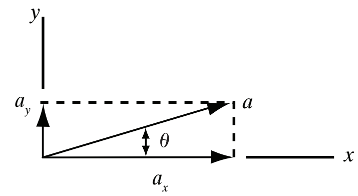
$$\tan\theta = \frac{a_x}{g + a_y} \qquad \textbf{(3)}$$

By establishing the *x* and *y* axes shown in Fig. *a*,

$$a_x = a\cos\phi \qquad a_y = a\sin\phi$$

Substituting these values into Eq. (3),

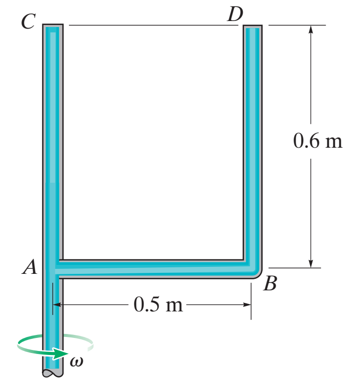
$$\tan\theta = \frac{a\cos\phi}{a\sin\phi + g} \qquad \textbf{Q.E.D}$$



(a)

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–141. The sealed tube assembly is completely filled with water, such that the pressures at *C* and *D* are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between *C* and *D*.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2(0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

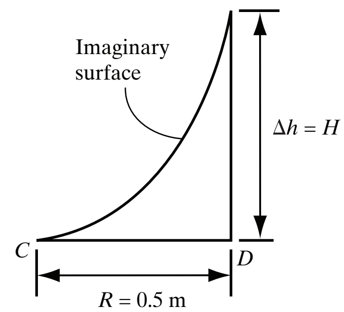
$$= 2.867\ \text{m}$$

From Fig. *a*, $\Delta h = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_D - p_C = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa}$$ **Ans.**

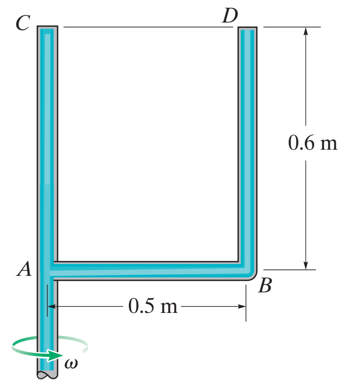


(a)

Ans:
$p_D - p_C = 28.1\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–142. The sealed tube assembly is completely filled with water, such that the pressures at C and D are zero. If the assembly is given an angular velocity of $\omega = 15\ \text{rad/s}$, determine the difference in pressure between A and B.



SOLUTION

$$H = \frac{\omega^2 R^2}{2g}$$

$$= \frac{(15\ \text{rad/s})^2 (0.5\ \text{m})^2}{2(9.81\ \text{m/s}^2)}$$

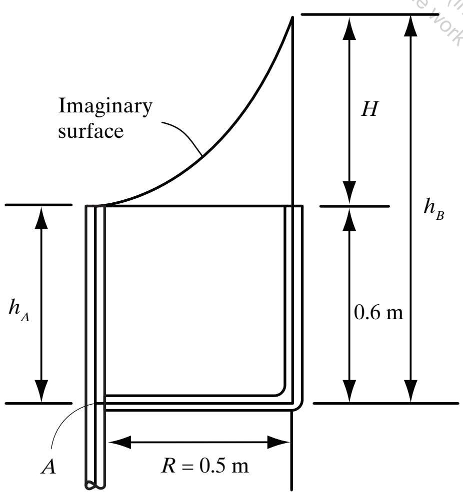
$$= 2.867\ \text{m}$$

From Fig. a, $\Delta h = h_B - h_A = H = 2.867\ \text{m}$. Then,

$$\Delta p = p_B - p_A = \rho_w g \Delta h$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(2.867\ \text{m})$$

$$= 28.13(10^3)\ \text{Pa} = 28.1\ \text{kPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$p_B - p_A = 28.1\ \text{kPa}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–143. A woman stands on a horizontal platform that is rotating at 0.75 rad/s. If she is holding a cup of tea, and the center of the cup is 2 ft from the axis of rotation, determine the slope angle of the liquid's surface. Neglect the size of the cup.

SOLUTION

Since the cup is rotating at a constant velocity about the vertical axis of rotation, then its acceleration is always directed horizontally toward the axis of rotation and its magnitude is given by

$$a_r = \omega^2 r = (0.75\ \text{rad/s})^2(2\ \text{ft}) = 1.125\ \text{ft/s}^2$$

Thus, the slope of the tea surface is

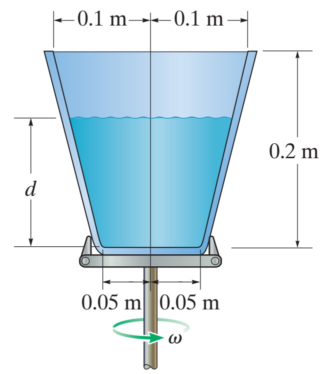
$$m = \tan\theta = \frac{a_r}{g} = \frac{1.125\ \text{ft/s}^2}{32.2\ \text{ft/s}^2} = 0.3494$$

$$\theta = 2.00° \qquad \textbf{Ans.}$$

Ans:
$\theta = 2.00°$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

***2–144.** Determine the maximum height d the glass can be filled with water so that no water spills out when the glass is rotating at 15 rad/s.



SOLUTION

From the geometry shown in Fig. a,

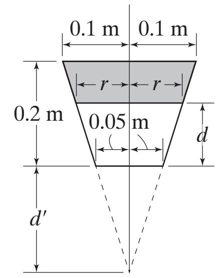
$$\frac{d'}{d' + 0.2\ \text{m}} = \frac{0.05\ \text{m}}{0.1\ \text{m}}; \quad d' = 0.2\ \text{m}$$

Then

$$\frac{r}{0.05\ \text{m}} = \frac{d + 0.2\ \text{m}}{0.2\ \text{m}}; \qquad r = 0.25(d + 0.2)$$

Thus, the volume of the empty space in the container shown shaded in Fig. a is

$$V_{es} = \frac{1}{3}\pi(0.1\ \text{m})^2(0.4\ \text{m}) - \frac{1}{3}\pi[0.25(d + 0.2)]^2(d + 0.2)$$
$$= \frac{1}{3}\pi[0.004 - 0.0625(d + 0.2)^3]\text{m}^3$$

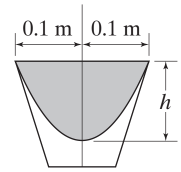


(a)

For the condition that the water is about to spill, the parabolic profile of the free water surface is shown in Fig. b.

$$h = \left(\frac{\omega^2}{2g}\right)r^2$$

$$h = \left[\frac{(15\ \text{rad/s})^2}{2(9.81\ \text{m/s}^2)}\right](0.1\ \text{m})^2$$
$$= 0.1147\ \text{m} < 0.2\ \text{m} \qquad \textbf{(O.K!)}$$



(b)

Since the empty space in the glass must remain the same, the volume of the paraboloid shown shaded in Fig. b must be equal to this volume. Here, the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height.

$$V_{\text{parab}} = V_{\text{es}}$$

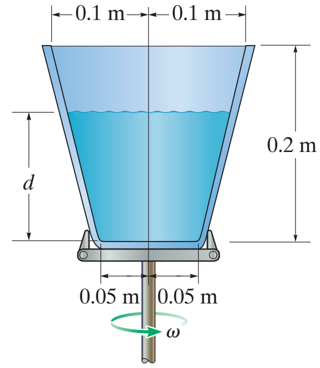
$$\frac{1}{2}[\pi(0.1\ \text{m})^2](0.1147\ \text{m}) = \frac{1}{3}\pi[0.004 - 0.0625(d + 0.2)^3]$$

$$d = 0.1316\ \text{m} = 0.132\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$d = 0.132\ \text{m}$

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–145. The glass is filled with water to a height of $d = 0.1$ m. To what height $d = d'$ does the water rise against the wall of the glass when the platform has an angular velocity of $\omega = 15$ rad/s?



SOLUTION

From the geometry shown in Fig. *a*,

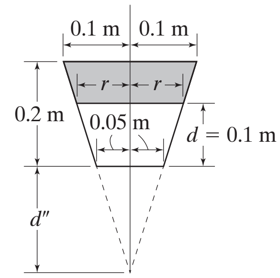
$$\frac{d''}{d'' + 0.2\text{ m}} = \frac{0.05\text{ m}}{0.1\text{m}}; \qquad d'' = 0.2\text{ m}$$

Then,

$$\frac{r}{0.05\text{ m}} = \frac{0.3\text{ m}}{0.2\text{ m}}; \qquad r = 0.075\text{ m}$$

Thus, the volume of the empty space in the container shown shaded in Fig. *a* is

$$\forall_{es} = \frac{1}{3}\pi(0.1\text{ m})^2(0.4\text{ m}) - \frac{1}{3}\pi(0.075\text{ m})^2(0.3\text{ m}) = 0.77083\pi(10^{-3})\text{ m}$$



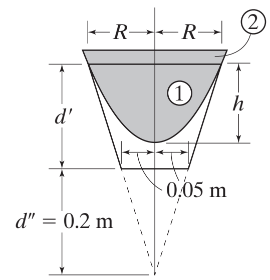
(a)

The parabolic profile of the free water surface is shown in Fig. *a*. Then

$$h = \left(\frac{\omega^2}{2g}\right)r^2$$

$$h = \left[\frac{(15\text{ rad/s})^2}{2(9.81\text{ m/s}^2)}\right]R^2$$

$$= 11.4679R^2$$



(b)

Since the empty space in the glass must remain the same, the shaded volume shown in Fig. *b* [paraboloid segment (1) and segment (2)] must be equal to this volume. Here, the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height.

$$\forall_1 + \forall_2 = \forall_{es}$$

$$\frac{1}{2}[\pi R^2(11.4679R^2)] + \left[\frac{1}{2}\pi(0.1\text{ m})^2(0.4\text{ m}) - \frac{1}{3}\pi R^1(d' + 0.2)\right] = 0.77083\pi(10^{-3})\text{ m}$$

$$5.7339R^4 - \frac{1}{3}R^2(d' + 0.2) + 0.5625(10^{-3}) = 0 \qquad \textbf{(1)}$$

From the geometry shown in Fig. *b*,

$$\frac{d' + 0.2\text{ m}}{0.2} = \frac{R}{0.05}; \qquad d' + 0.2 = 4R$$

Substitute this result into Eq. (1).

$$5.7339R^4 - \frac{4}{3}R^3 + 0.5625(10^{-3}) = 0$$

Solving numerically,

$$R = 0.087852\text{ m}$$

Thus,

$$d' + 0.2 = 4(0.087852)$$

$$d' + 0.1514\text{ m} = 0.151\text{ m} \qquad \textbf{Ans.}$$

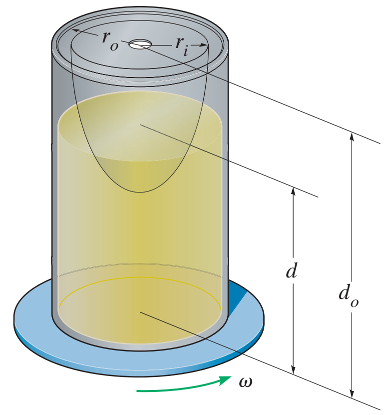
Also,

$$h = 11.4679(0.087852^2) = 0.0885\text{ m} < d' \qquad \textbf{(O.K!)}$$

Ans:
$d' = 0.151$ m

Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–146. The drum has a hole in the center of its lid and is filled to a height d with a liquid having a density ρ. If the drum is then placed on the rotating platform and it attains an angular velocity of ω, determine the inner radius r_i of the liquid where it contacts the lid.



SOLUTION

The volume of the paraboloid empty space must be the same as the volume of the empty space when the liquid is not spinning. Since the volume of the paraboloid is equal to one half the volume of the cylinder of the same radius and height, then

$$\mathcal{V}_{\text{parab}} = \mathcal{V}_{\text{es}}$$

$$\frac{1}{2}(\pi r_i^2 h) = \pi r_o^2 (d_o - d)$$

$$r_o^2 h = 2 r_o^2 (d_o - d) \qquad \textbf{(1)}$$

So then,

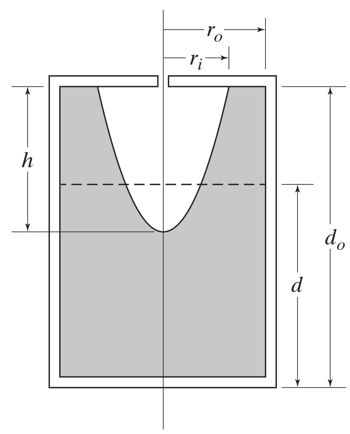
$$h = \frac{\omega^2}{2g} r_i^2 \qquad \textbf{(2)}$$

Substitute Eq. (2) into (1).

$$r_i^2 \left(\frac{\omega^2}{2g}\right) r_i^2 = 2 r_o^2 (d_o - d)$$

$$r_i^4 = \frac{4 g r_o^2 (d_o - d)}{\omega^2}$$

$$r_i = \left[\frac{4 g r_o^2 (d_o - d)}{\omega^2}\right]^{1/4} \qquad \textbf{Ans.}$$



(a)

Ans:

$$r_i = \left[\frac{4 g r_o^2 (d_o - d)}{\omega^2}\right]^{1/4}$$

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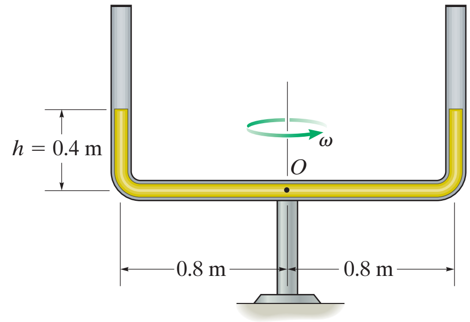
Unless otherwise stated, take the density of water to be $\rho_w = 1000\ \text{kg/m}^3$ and its specific weight to be $\gamma_w = 62.4\ \text{lb/ft}^3$. Also, assume all pressures are gage pressures unless stated otherwise.

2–147. The tube is filled with oil to the level $h = 0.4$ m. Determine the angular velocity of the tube so that the pressure at O becomes -15 kPa. Take $S_o = 0.92$.



SOLUTION

The level of oil in the tube will not change. Therefore, the imaginary surface of the oil will be that shown in Fig. *a*. The pressure of point O is negative, since it is located above the imaginary surface.

$$p_o = -\gamma h_o; \qquad -15(10^3)\ \text{N/m}^2 = -0.92(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)h_o$$
$$h_o = 1.6620\ \text{m}$$

Thus

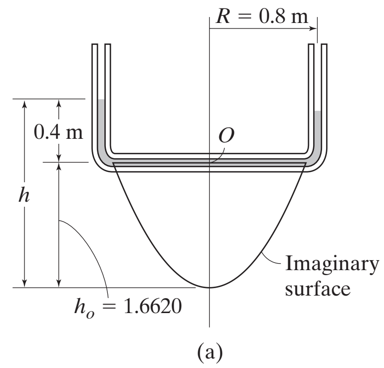
$$h = h_o + 0.4\ \text{m} = 1.6620\ \text{m} + 0.4\ \text{m} = 2.0620\ \text{m}$$

So then,

$$h = \left(\frac{\omega^2}{2g}\right)r^2$$

$$2.0620\ \text{m} = \left[\frac{\omega^2}{2(9.81\ \text{m/s}^2)}\right](0.8\ \text{m})^2$$

$$\omega = 7.9506\ \text{rad/s} = 7.95\ \text{rad/s} \qquad \textbf{Ans.}$$



(a)

Ans:
$\omega = 7.95$ rad/s