2.1

a. $\begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{bmatrix}$ 　　　　3 x 3, square, symmetric

b. $\begin{Bmatrix} x \\ x^2 \\ x^3 \\ x^4 \end{Bmatrix}$ 　　　　4 x 1 column

c. $\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$ 　　　　2 x 2, square, diagonal

d. $\begin{bmatrix} 1 & y & y^2 & y^3 \end{bmatrix}$ 　　　　1 x 4, row

e. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 　　　　3 x 3, square, diagonal, identity

f. $\begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 0 & 0 \\ 0 & 4 & 1 & 4 & 0 \\ 0 & 0 & 5 & 4 & 2 \\ 0 & 0 & 0 & 7 & 8 \end{bmatrix}$ 　　　　5 x 5, square, banded

g. $\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 　　　　4 x 4, square, upper triangular

h. $\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$ 　　　　4 x 4, square, diagonal

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a. $[A]+[B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 5 & 0 & -4 \end{bmatrix}$

b. $[A]-[B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix}$

c. $3[A] = 3 \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix}$

d. $[A][B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 18 & 19 & -5 \\ -21 & -21 & 42 \\ -12 & 2 & -37 \end{bmatrix}$

e. $[A]\{C\} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -2 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 4 \\ -21 \\ 23 \end{Bmatrix}$

f. $[A]^2 = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 31 & 3 & -7 \\ 21 & 49 & -14 \\ -28 & -13 & 45 \end{bmatrix}$

g. $[I][A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$

$[A][I] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$

$$[A_{11}] = \begin{bmatrix} 5 & 7 & 2 \\ 3 & 8 & -3 \end{bmatrix} \qquad [A_{12}] = \begin{bmatrix} 0 & 3 & 5 \\ -5 & 0 & 8 \end{bmatrix}$$

$$[A_{21}] = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 10 & 5 \\ 2 & -5 & 9 \end{bmatrix} \qquad [A_{22}] = \begin{bmatrix} 7 & 15 & 9 \\ 12 & 3 & -1 \\ 2 & 18 & -10 \end{bmatrix}$$

$$\{B_{11}\} = \begin{Bmatrix} 2 \\ 8 \\ -5 \end{Bmatrix} \qquad [B_{12}] = \begin{bmatrix} 10 & 0 \\ 7 & 5 \\ 2 & -4 \end{bmatrix}$$

$$\{B_{21}\} = \begin{Bmatrix} 4 \\ 3 \\ 1 \end{Bmatrix} \qquad [B_{22}] = \begin{bmatrix} 8 & 13 \\ 12 & 0 \\ 5 & 7 \end{bmatrix}$$

$$[A_{11}]\{B_{11}\} + [A_{12}]\{B_{21}\} = \begin{Bmatrix} 70 \\ 73 \end{Bmatrix}$$

$$[A_{21}]\{B_{11}\} + [A_{22}]\{B_{21}\} = \begin{Bmatrix} 116 \\ 111 \\ -29 \end{Bmatrix}$$

$$[A_{11}][B_{12}] + [A_{12}][B_{22}] = \begin{bmatrix} 164 & 62 \\ 80 & 43 \end{bmatrix}$$

$$[A_{21}][B_{12}] + [A_{22}][B_{22}] = \begin{bmatrix} 319 & 174 \\ 207 & 179 \\ 185 & -105 \end{bmatrix}$$

$$[A][B] = \begin{bmatrix} 70 & 164 & 62 \\ 73 & 80 & 43 \\ 116 & 319 & 174 \\ 111 & 207 & 179 \\ -29 & 185 & -105 \end{bmatrix}$$

a.

$$[A] = \begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} \qquad [A]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \qquad [B]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix}$$

b.

$$\left[\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \right]^T = \begin{bmatrix} 1 & 9 & 1 \\ 5 & 4 & 13 \\ 9 & 5 & -6 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

$$[A]^T + [B]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} + \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

c.

$$([A][B])^T = \left(\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} -8 & 17 & 19 \\ 3 & 67 & -11 \\ -7 & 28 & 8 \end{bmatrix}^T = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

$$[B]^T[A]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

$\boxed{2\cdot 5}$

a.

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = \begin{aligned}(2)(6)(18)+(10)(14)(12)+(0)(16)(-4) \\ -(10)(16)(18)-(2)(14)(-4)-(6)(6)(12)\end{aligned}$$

$$\underline{\underline{\det(A) = -872}} \qquad \Longleftarrow$$

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = 2\begin{vmatrix} 6 & 14 \\ -4 & 18 \end{vmatrix} -10\begin{vmatrix} 16 & 14 \\ 12 & 18 \end{vmatrix} +0\begin{vmatrix} 16 & 6 \\ 12 & -4 \end{vmatrix}$$

$$= 2\left[(6)(18)-(14)(-4)\right] -10\left[(16)(18)-(14)(12)\right]+0$$

$$\underline{\underline{\det(A) = -872}} \qquad \Longleftarrow$$

matrix [B] is singular because elements of second row and first row are linearly dependent.

$$\underline{\underline{\det(B) = 0}} \qquad \Longleftarrow$$

This result can be shown by direct expansion as well.

b.

$$\underline{\underline{\det([A]^T) = \det(A) = -872}} \qquad \Longleftarrow$$

C.

$$\det(5[A]) = \begin{vmatrix} 10 & 50 & 0 \\ 80 & 30 & 70 \\ 60 & -20 & 90 \end{vmatrix} = \begin{array}{l}(10)(30)(90) + (50)(70)(60)+0 \\ -(50)(80)(90)-(10)(70)(-20)-0\end{array}$$

$$\underline{\underline{\det(5[A]) = -109000}}$$

Since matrix $[A]$ is 3×3, alternatively,

$$\det(5[A]) = 5^3 \det(A) = (125)(-872) = \underline{\underline{-109000}}$$

$$\boxed{2.6}$$

$$\det(A) = \begin{vmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{vmatrix} = \begin{array}{l} (0)(3)(9) + (5)(7)(9) + (0)(8)(-2) \\ -(5)(8)(9) - (0)(7)(-2) - (0)(3)(9) \end{array}$$

$$\underline{\underline{\det(A) = -45}} \qquad \Leftarrow$$

$$\underline{\underline{\det([A]^T) = \det(A) = -45}} \qquad \Leftarrow$$

$$\boxed{2 \cdot 7}$$

$$\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

Following the steps discussed in Section 2.7, we get

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

$$\boxed{2 \cdot 8}$$

$$\begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

Because of the zero elements in Row 1, the lower triangular matrix will not have a triangular form, instead it becomes

$$[L] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Check:

$$[L][U] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[L] = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}$$

$$[u] = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}$$

$$\{z\} = [L]^{-1}\{b\} = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = [u]^{-1}\{z\} = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[A]^{-1} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = [A]^{-1}\{b\} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

$\boxed{2 \cdot 11}$

(a) Using Gaussian method

$$X_1 + X_2 + X_3 = 6$$
$$2X_1 + 5X_2 + X_3 = 15$$
$$-3X_1 + X_2 + 5X_3 = 14$$

$$\begin{cases} 2X_1 + 5X_2 + X_3 = 15 \\ -2X_1 - 2X_2 - 2X_3 = -12 \end{cases}$$
$$\overline{\qquad 3X_2 - X_3 = 3}$$

$$\begin{cases} -3X_1 + X_2 + 5X_3 = 14 \\ 3X_1 + 3X_2 + 3X_3 = 18 \end{cases}$$
$$\overline{\qquad 4X_2 + 8X_3 = 32}$$

$$\begin{cases} 3X_2 - X_3 = 3 \\ 4X_2 + 8X_3 = 32 \end{cases} \rightsquigarrow \begin{array}{l} X_2 - \frac{1}{3}X_3 = 1 \\ 4X_2 + 8X_3 = 32 \end{array}$$

$$\begin{cases} 4X_2 + 8X_3 = 32 \\ -4X_2 + \frac{4}{3}X_3 = -4 \end{cases}$$
$$\overline{\qquad \frac{28}{3}X_3 = 28} \rightsquigarrow \underline{\underline{X_3 = 3}} \Leftarrow$$

$$X_2 = 1 + \frac{1}{3}X_3 = 1 + \frac{1}{3}(3) = 2 \qquad \underline{\underline{X_2 = 2}} \Leftarrow$$

$$X_1 = 6 - X_2 - X_3 = 6 - 2 - 3 = 1 \qquad \underline{\underline{X_1 = 1}} \Leftarrow$$

(b) Using the LU decomposition method

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \qquad \{b\} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$u_{11} = a_{11} = 1 \qquad u_{12} = a_{12} = 1 \qquad u_{13} = a_{13} = 1$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{2}{1} = 2 \qquad l_{31} = \frac{a_{31}}{u_{11}} = \frac{-3}{1} = -3$$

$$u_{22} = a_{22} - l_{21} u_{12} = 5 - (2)(1) = 3$$

$$u_{23} = a_{23} - l_{21} u_{13} = 1 - (2)(1) = -1$$

$$l_{32} = \frac{a_{32} - l_{31} u_{12}}{u_{22}} = \frac{1 - (-3)(1)}{3} = \frac{4}{3}$$

$$u_{33} = a_{33} - (l_{31} u_{13} + l_{32} u_{23}) = 5 - \left[(-3)(1) + \left(\frac{4}{3}\right)(-1)\right] = \frac{28}{3}$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix}}_{[U]}$$

$$[L]\{z\} = \{b\} \quad \longrightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\{z\} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$[U]\{x\} = \{z\} \quad \longrightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$\{x\} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \Longleftarrow$$

(C) by finding the inverse of the Coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}^{-1} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \Longleftarrow$$

$$\boxed{2 \cdot 12}$$

$$[A]^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \quad \Longleftarrow$$

$$[B]^{-1} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \quad \Longleftarrow$$

$$[C] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}}\overbrace{\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}}^{[C]^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \longrightarrow X_{11} = \frac{K_{22}}{K_{11} K_{22} - K_{12} K_{21}}$$

$$X_{21} = \frac{-K_{21}}{K_{11} K_{22} - K_{12} K_{21}}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \longrightarrow X_{12} = \frac{-K_{12}}{K_{11} K_{22} - K_{12} K_{21}} \quad \Longleftarrow$$

$$X_{22} = \frac{K_{11}}{K_{11} K_{22} - K_{12} K_{21}}$$

For a 2×2 matrix:

$$\det\left(\alpha\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) \stackrel{?}{=} \alpha^2\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\alpha\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix}$$

$$\det\begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix} = (\alpha a_{11})(\alpha a_{22}) - (\alpha a_{12})(\alpha a_{21})$$

$$\det\left(\alpha\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}\right) = \alpha^2\left(a_{11}a_{22} - a_{12}a_{21}\right) = \alpha^2\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

For a 3×3 matrix: Q.E.D.

$$\det\left(\alpha\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right) \stackrel{?}{=} \alpha^3\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\det\begin{vmatrix} \alpha a_{11} & \alpha a_{12} & \alpha a_{13} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} \\ \alpha a_{31} & \alpha a_{32} & \alpha a_{33} \end{vmatrix} = (\alpha a_{11})(\alpha a_{22})(\alpha a_{33}) +$$
$$(\alpha a_{12})(\alpha a_{23})(\alpha a_{31}) + \cdots$$
$$\cdots - (\alpha)(a_{13})(\alpha a_{22})(\alpha a_{31})$$

$$\det\left(\alpha\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right) = \alpha^3\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

                                                Q.E.D.

In general for a n×n matrix, we have

$$\det\left(\alpha\begin{bmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & a_{nn} \end{bmatrix}\right) = \alpha^n\begin{vmatrix} a_{11} & a_{12} & \cdot & a_{1n} \\ a_{21} & a_{21} & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdot & a_{nn} \end{vmatrix}$$

$$\boxed{2.14}$$

Using Equation (2.83), we have

$$
\begin{bmatrix}
-\omega^2 + \dfrac{2K}{m_1} & -\dfrac{K}{m_1} \\[2mm]
-\dfrac{K}{m_2} & -\omega^2 + \dfrac{2K}{m_2}
\end{bmatrix}
\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0
$$

$$
\begin{vmatrix}
-\omega^2 + \dfrac{2(100)}{0.1} & -\dfrac{100}{0.1} \\[3mm]
-\dfrac{100}{0.2} & -\omega^2 + \dfrac{2(100)}{0.2}
\end{vmatrix} = 0
$$

$$\omega^4 - 3000\,\omega^2 + 1.5 \times 10^6 = 0$$

$$\omega_1^2 = 2366 \ \left(\text{rad}/s\right)^2 \qquad\qquad \omega_1 = 48.6 \ \text{rad}/s$$

$$\omega_2^2 = 634 \ \left(\text{rad}/s\right)^2 \qquad\qquad \omega_2 = 25.2 \ \text{rad}/s \qquad \Longleftarrow$$

$$(-2366 + 2000)\,X_1 - 1000\,X_2 = 0 \quad \longrightarrow \quad \dfrac{X_2}{X_1} = -0.366$$

$$(-634 + 2000)\,X_1 - 1000\,X_2 = 0 \quad \longrightarrow \quad \dfrac{X_2}{X_1} = 1.366$$

```
>> a=[4 2 1;7 0 -7;1 -5 3]

a =

    4    2    1
    7    0   -7
    1   -5    3

>> b=[1 2 -1;5 3 3;4 5 -7]

b =

    1    2   -1
    5    3    3
    4    5   -7

>> c=[1;-2;4]

c =

    1
   -2
    4

>> a+b

ans =

    5    4    0
   12    3   -4
    5    0   -4

>> a-b

ans =

    3    0    2
    2   -3  -10
   -3  -10   10
```

```
>> 3*a

ans =

    12    6    3
    21    0  -21
     3  -15    9

>> a*b

ans =

    18   19   -5
   -21  -21   42
   -12    2  -37

>> a*c

ans =

     4
   -21
    23

>> a*a

ans =

    31    3   -7
    21   49  -14
   -28  -13   45

>> i=[1 0 0;0 1 0;0 0 1]

i =

     1    0    0
     0    1    0
     0    0    1
```

```
>> i*a

ans =

    4    2    1
    7    0   -7
    1   -5    3

>> a*i

ans =

    4    2    1
    7    0   -7
    1   -5    3

>>
```

```
>> A=[1 4 2;8 3 6;7 1 -2]

A =

    1    4    2
    8    3    6
    7    1   -2

>> B=[0 5 -1;-3 1 7;2 4 -4]

B =

    0    5   -1
   -3    1    7
    2    4   -4

>> A'

ans =

    1    8    7
    4    3    1
    2    6   -2

>> B'

ans =

    0   -3    2
    5    1    4
   -1    7   -4

>> (A+B)'

ans =

    1    5    9
    9    4    5
    1   13   -6
```

>> A'+B'

ans =

```
   1    5    9
   9    4    5
   1   13   -6
```

>> (A*B)'

ans =

```
  -8    3   -7
  17   67   28
  19  -11    8
```

>> B'*A'

ans =

```
  -8    3   -7
  17   67   28
  19  -11    8
```

>>

>> A=[2 10 0;16 6 14;12 -4 18]

A =

```
   2   10    0
  16    6   14
  12   -4   18
```

>> B=[2 10 0;4 20 0;12 -4 18]

B =

```
   2   10    0
   4   20    0
  12   -4   18
```

>> det(A)

ans =

  -872

>> det(B)

ans =

   0

>> det((A)')

ans =

  -872

>> det(5*(A))

ans =

  -109000

>>

```
>> A=[0 5 0;8 3 7;9 -2 9]

A =

    0    5    0
    8    3    7
    9   -2    9

>> det(A)

ans =

  -45

>> det((A)')

ans =

  -45

>>
```

>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

```
   10875000    -1812500          0
   -1812500     6343750    -4531250
          0    -4531250     4531250
```

>> b=[0;0;800]

b =

```
     0
     0
   800
```

>> x=A\b

x =

   1.0e-003 *

```
   0.0883
   0.5297
   0.7062
```

>>

>> A=[0 5 0;8 3 7;9 -2 9]

A =

```
    0    5    0
    8    3    7
    9   -2    9
```

>> [l,u]=lu(A)

l =

```
        0    1.0000         0
   0.8889    0.9556    1.0000
   1.0000         0         0
```

u =

```
    9   -2    9
    0    5    0
    0    0   -1
```

>> l*u

ans =

```
    0    5    0
    8    3    7
    9   -2    9
```

2.21

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

   10875000   -1812500         0
   -1812500    6343750   -4531250
          0   -4531250    4531250

>> b=[0;0;800]

b =

     0
     0
   800

>> [l,u]=lu(A)

l =

   1.0000        0        0
  -0.1667   1.0000        0
        0  -0.7500   1.0000


u =

   1.0e+007 *

   1.0875  -0.1812        0
        0   0.6042  -0.4531
        0        0   0.1133

>> z=inv(l)*b

z =

     0
     0
   800

>> U=inv(u)*z

U =

   1.0e-003 *

   0.0883
   0.5297
   0.7062
```

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]

A =

   10875000   -1812500          0
   -1812500    6343750   -4531250
          0   -4531250    4531250

>> b=[0;0;800]

b =

     0
     0
   800

>> Ainverse=inv(A)

Ainverse =

   1.0e-006 *

   0.1103   0.1103   0.1103
   0.1103   0.6621   0.6621
   0.1103   0.6621   0.8828

>> u=Ainverse*b

u =

   1.0e-003 *

   0.0883
   0.5297
   0.7062

>>
```

2.23

>> A=[1 1 1;2 5 1;-3 1 5]

A =

```
   1    1    1
   2    5    1
  -3    1    5
```

>> b=[6 15 14]

b =

```
   6   15   14
```

>> b=[6;15;14]

b =

```
   6
  15
  14
```

(a) using the Gaussian method

>> x=A\b

x =

```
   1.0000
   2.0000
   3.0000
```

(b) using the LU decomposition method

>> [l,u]=lu(A)

l =

```
  -0.3333    0.2353    1.0000
  -0.6667    1.0000         0
   1.0000         0         0
```

u =

```
  -3.0000    1.0000    5.0000
        0    5.6667    4.3333
        0         0    1.6471
```

```
>> z=inv(l)*b

z =

   14.0000
   24.3333
    4.9412

>> x=inv(u)*z

x =

   1.0000
   2.0000
   3.0000
```

(c) by finding the inverse of the coefficient matrix

```
>> x=inv(A)*b

x =

   1.0000
   2.0000
   3.0000

>>
```

For example, consider the following 4 x 4 matrix, and $\alpha = 2$ and $\alpha = 3$.

>> A=[1 2 1 3;2 1 4 1;5 3 0 1;4 1 5 7]

A =

```
1   2   1   3
2   1   4   1
5   3   0   1
4   1   5   7
```

>> det(A)

ans =

   219

>> det(2*A)

ans =

   3504

Since matrix A is 4 x 4 then let us examine to see if $det(2*A) = 2^4 * det(A)$?

>> 2^4*det(A)

ans =

   3504

>> det(3*A)

ans =

   17739

Or is $det(3*A) = 3^4 * det(A)$?

>> 3^4*det(A)

ans =

   17739

Let us now consider the following 3 x 3 matrix, and $\alpha = 2$ and $\alpha = 3$.

>> B=[1 2 1;2 1 4;5 3 0]

B =

   1   2   1
   2   1   4
   5   3   0

>> det(B)

ans =

   29

>> det(2*B)

ans =

   232

Is det(2*B) = 2^3*det(B)?

>> 2^3*det(B)

ans =

   232

>> det(3*B)

ans =

   783

Or is det(3*B) = 3^3*det(B)?

>> 3^3*det(B)

ans =

   783

>>

>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

7.1100	-1.2300	0	0	0
-1.2300	1.9900	-0.7600	0	0
0	-0.7600	0.8510	-0.0910	0
0	0	-0.0910	2.3110	-2.2200
0	0	0	-2.2200	3.6900

>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

117.6000
      0
      0
      0
102.9000

>> T=A\b

T =

20.5898
23.4091
27.9719
66.0789
67.6410

>>

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

    7.1100  -1.2300     0         0        0
   -1.2300   1.9900  -0.7600      0        0
       0    -0.7600   0.8510   -0.0910     0
       0        0    -0.0910    2.3110  -2.2200
       0        0        0     -2.2200   3.6900

>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

  117.6000
        0
        0
        0
  102.9000

>> Ainverse=inv(A)

Ainverse =

    0.1681    0.1585    0.1430    0.0133    0.0080
    0.1585    0.9160    0.8263    0.0771    0.0464
    0.1430    0.8263    1.9323    0.1803    0.1085
    0.0133    0.0771    0.1803    1.0421    0.6269
    0.0080    0.0464    0.1085    0.6269    0.6482

>> T=Ainverse*b

T =

   20.5898
   23.4091
   27.9719
   66.0789
   67.6410
```

>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]

A =

```
    7.1100   -1.2300        0        0        0
   -1.2300    1.9900   -0.7600        0        0
        0    -0.7600    0.8510   -0.0910        0
        0         0    -0.0910    2.3110   -2.2200
        0         0         0    -2.2200    3.6900
```

>> b=[5.88*20; 0; 0; 0; 1.47*70]

b =

```
  117.6000
        0
        0
        0
  102.9000
```

>> [l,u]=lu(A)

l =

```
    1.0000        0        0        0        0
   -0.1730    1.0000        0        0        0
        0    -0.4276    1.0000        0        0
        0         0    -0.1730    1.0000        0
        0         0         0    -0.9672    1.0000
```

u =

```
    7.1100   -1.2300        0        0        0
        0     1.7772   -0.7600        0        0
        0         0     0.5260   -0.0910        0
        0         0         0     2.2953   -2.2200
        0         0         0         0     1.5428
```

>> z=inv(l)*b

z =

```
  117.6000
   20.3443
    8.6999
    1.5051
  104.3558
```

>> T=inv(u)*z

T =

```
   20.5898
   23.4091
   27.9719
   66.0789
   67.6410
```

>>

>> A=[1 0 0 0 0;-0.0408 0.0888 -0.0408 0 0;0 -0.0408 0.0888 -0.0408 0;0 0 -0.0408 0.0888 -0.0408;
0 0 0 -0.0408 0.04455]

A =

```
   1.0000        0        0        0        0
  -0.0408   0.0888  -0.0408        0        0
        0  -0.0408   0.0888  -0.0408        0
        0        0  -0.0408   0.0888  -0.0408
        0        0        0  -0.0408   0.0445
```

>> b=[100;0.144;0.144;0.144;0.075]

b =

```
 100.0000
   0.1440
   0.1440
   0.1440
   0.0750
```

>> T=A\b

T =

```
 100.0000
  75.0387
  59.7901
  51.5633
  48.9064
```

>>

>> A=10^5*[7.2 0 0 0 -1.49 -1.49;0 7.2 0 -4.22 -1.49 -1.49;0 0 8.44 0 -4.22 0;0 -4.22 0 4.22 0 0;-1.49 -1.49 -4.22 0 5.71 1.49;-1.49 -1.49 0 0 1.49 1.49]

A =

720000	0	0	0	-149000	-149000
0	720000	0	-422000	-149000	-149000
0	0	844000	0	-422000	0
0	-422000	0	422000	0	0
-149000	-149000	-422000	0	571000	149000
-149000	-149000	0	0	149000	149000

>> b=[0;0;0;-500;0;-500]

b =

     0
     0
     0
  -500
     0
  -500

>> U=A\b

U =

  -0.0036
  -0.0103
   0.0012
  -0.0115
   0.0024
  -0.0195

>>

2.30

>> A=[2000 -1000;-500 1000]

A =

    2000   -1000
    -500   1000

The eigenvalues are:

>> eig(A)

ans =

  1.0e+003 *

  2.3660
  0.6340

Note the natural frequencies of the system are equal to the square root of the eigenvalues.

>> sqrt(eig(A))

ans =

  48.6418
  25.1789

The eigenvector and eigenvlaues are given by:

>> [v,e]=eig(A)

v =

  0.9391   0.5907
  -0.3437 0.8069

e =

  1.0e+003 *

  2.3660   0
     0   0.6340

Normalizing the eigenvector with respect to X_1, we get:

>> -0.3437/0.9391

ans =

   -0.3660

Therefore, the first mode is given by $X_2/X_1 = -0.3660$.

>> .8069/0.5907

ans =

   1.3660

The second mode is then given by $X_2/X_1 = 1.3660$.

Problem 2-31

$$[K] = \begin{bmatrix} 7.11 & -1.23 & 0 & 0 & 0 \\ -1.23 & 1.99 & -0.76 & 0 & 0 \\ 0 & -0.76 & 0.851 & -0.091 & 0 \\ 0 & 0 & -0.091 & 2.31 & -2.22 \\ 0 & 0 & 0 & -2.22 & 3.69 \end{bmatrix}$$

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix}$$

$$[F] = \begin{Bmatrix} 117.6 \\ 0 \\ 0 \\ 0 \\ 102.9 \end{Bmatrix}$$

$$[K]^{-1} = \begin{bmatrix} 0.1681 & 0.1585 & 0.143 & 0.0134 & 0.008 \\ 0.1585 & 0.9161 & 0.8264 & 0.0772 & 0.0464 \\ 0.143 & 0.8264 & 1.9324 & 0.1805 & 0.1086 \\ 0.0134 & 0.0772 & 0.1805 & 1.0431 & 0.6276 \\ 0.008 & 0.0464 & 0.1086 & 0.6276 & 0.6486 \end{bmatrix}$$

$$\{T\} = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = [K]^{-1}[F] = \begin{Bmatrix} 20.59 \\ 23.41 \\ 27.98 \\ 66.15 \\ 67.68 \end{Bmatrix}$$

Problem 2-32

$[K] =$

1	0	0	0	0
-0.041	0.0888	-0.0408	0	0
0	-0.0408	0.0888	-0.041	0
0	0	-0.0408	0.0888	-0.0408
0	0	0	-0.041	0.04455

$\{T\} =$

T_1
T_2
T_3
T_4
T_5

$[F] =$

100
0.144
0.144
0.144
0.075

$[K]^{-1} =$

1	0	0	0	0
0.688	16.8623	12.191	9.6701	8.85613
0.4974	12.1906	26.532	21.047	19.2751
0.3945	9.67011	21.047	36.137	33.0956
0.3613	8.85613	19.275	33.096	52.7564

$\{T\} =$

T_1
T_2
T_3
T_4
T_5

$= \quad [K]^{-1} [F] \quad =$

100.00
75.04
59.79
51.56
48.91

Problem 2-33

$[K] =$

7.2	0	0	0	-1.49	-1.49
0	7.2	0	-4.22	-1.49	-1.49
0	0	8.44	0	-4.22	0
0	-4.22	0	4.22	0	0
-1.49	-1.49	-4.22	0	5.71	1.49
-1.49	-1.49	0	0	1.49	1.49

$\{T\} =$

U2x
U2y
U4x
U4y
U5x
U5y

$[F] =$

0
0
0
-0.005
0
-0.005

$[K]^{-1} =$

0.23697	0.23697	0	0.237	0	0.473934
0.23697	0.90811	0	0.9081	0	1.145075
0	-1E-17	0.237	-1E-17	0.23697	-0.23697
0.23697	0.90811	0	1.1451	0	1.145075
0	-2E-17	0.237	-2E-17	0.47393	-0.47393
0.47393	1.14507	-0.237	1.1451	-0.4739	2.764083

$\{U\} =$

U2x
U2y
U4x
U4y
U5x
U5y

$= \quad [K]^{-1} [F] \quad =$

-0.0036
-0.0103
0.0012
-0.0115
0.0024
-0.0195