Chapter 3
What Do Interest Rates Mean and What Is Their Role in Valuation?

■ Overview and Teaching Tips

In my years of teaching financial markets and institutions, I have found that students have trouble with what I consider to be easy material because they do not understand what an interest rate is—that it is negatively associated with the price of a bond, that it differs from the return on a bond, and that there is an important distinction between real and nominal interest rates.

This chapter spends more time on these issues than does any other competing textbook. My experience has been that giving this material so much attention is well rewarded. After putting more emphasis on this material in my financial markets and institutions course, I witnessed a dramatic improvement in students' understanding of portfolio choice and asset and liability management in financial institutions.

An innovative feature of the textbook is the set of over twenty special applications called, "The Practicing Manager." These applications introduce students to real-world problems that managers of financial institutions have to solve and make the course both more relevant and exciting to students. They are not meant to fully prepare students for jobs in financial institutions—it is up to more specialized courses such as bank or financial institutions management to do this—but these applications teach them some of the special analytical tools that they will need when they enter the business world.

This chapter contains the Practicing Manager application on "Calculating Duration to Measure Interest-Rate Risk." The application shows how to quantify interest-rate risk using the duration concept and is a basic tool for managers of financial institutions. For those instructors who do not want a managerial emphasis in their financial markets and institutions course, this and other Practicing Manager applications can be skipped without loss of continuity.

■ Answers to End-of-Chapter Questions

1. Yield to maturity (YTM) is the rate of return on a bond if held until the end of its lifetime. YTM is also the rate which, when used for discounting, leads to a zero Present Value (PV). YTM is considered a long-term bond yield expressed as an annual rate. The YTM calculation takes into account the bond's current market price, par value, coupon interest rate and time to maturity.

2. Due to the fixed income nature of bonds and debentures, they are often referred to as "fixed-income securities". When an investor purchases a corporate bond, they are actually purchasing a portion of a company's debt. This debt is issued with specific details regarding periodic coupon payments, the principal amount of the debt, and the time period until the bond's maturity. Another concept that is important for understanding interest rate risk in bonds is that bond prices are inversely related to interest rates. When interest rates go up, bond prices go down, and vice versa.
   There are two primary reasons why long-term bonds are subject to greater interest rate risk than short-term bonds:

   - There is a greater probability that interest rates will rise (and thus negatively affect a bond's market price) within a longer time period than a shorter one. As a result, investors who buy long-term bonds and then attempt to sell them before maturity may be faced with a deeply discounted market price. With short-term bonds, this risk is not as significant because interest rates are less likely to substantially change in the shorter time period. Short-term bonds are also easier to hold until maturity, thereby alleviating an investor's concern about the effect of interest rate driven changes in the price of bonds.

   - Since long-term bonds have greater duration than short-term bonds, a change in the interest rate will have greater impact on them. This concept of duration can be difficult to conceptualize, but just think of it as the length of time that your bond will be affected by an interest rate change. For example, suppose interest rates rise today by 0.25%. A bond with one coupon payment left until maturity will be underpaying the investor by 0.25% for only that one. On the other hand, a bond with 20 coupon payments left will be underpaying the investor for a much longer period. This difference in remaining payments will cause a greater drop in a long-term bond's price than it will in a short-term bond's price when interest rates rise.

3. While bonds and savings accounts are fundamentally distinct, they do share some key similarities. When an investor deposits money in a savings account the investor is essentially loaning money to the bank, and the bank pays interest to the investor on her deposit. The same thing happens with bonds. An investor who buys bonds issued by a company or government entity is loaning money to the issuer in return for interest payments. When a bond reaches its maturity date, the investor receives his initial investment back just as he would if he had withdrawn it from a savings account.

Valuation

Money invested in savings accounts maintains a steady value, which does not go down unless the owner makes withdrawals from the account. Bonds rarely stay at their par value in the secondary market. At certain times during the life of a bond, it may trade for more or less than its par value. Bond prices fluctuate up and down based on factors such as interest rate changes, the creditworthiness of the issuer and the length of time to maturity.

Default Risk

Savings accounts up to a certain amount are guaranteed by deposit insurance in the event of a bank going out of business. But bond investors can run the risk of losing money if a bond issuer defaults. Bonds issued by the federal government are considered risk-free because governments will always make its interest and principal payments to bondholders. Bonds issued by state and local governments, however, depend on the financial health of the issuer. Bonds issued by corporations run the risk of default if the company falls on hard times.

Call Risk

Savings account investors can leave their money in the bank for as long as they like and withdraw it at any time that is convenient. Bond investors are faced with call risk. This means that even though an investor may have purchased a bond paying 6 percent for 10 years, the corporation or municipality may repurchase the bonds from its investors before the maturity date. This is likely to happen if market interest rates drop below the rate the bond is paying. When an issuer calls a bond, it pays off the old bonds it sold to investors and reissues new bonds at a lower rate. This is bad for bond investors because they lose the higher rate they were receiving, but good for bond issuers because they end up paying less interest to investors.

4. During fluctuating interest rates, people who have taken a variable interest rate loan would have fluctuating monthly installments which mean instability in the repayment schedule for the duration of the variable loan. This is defined as interest rate risk.

■ Quantitative Problems

1. Calculate the present value of a $1,000 zero-coupon bond with six years to maturity if the yield to maturity is 7%.

Solution:

$$\text{Bond price present value} = C\left[1 - \frac{1}{(1+\text{YTM})^{M}}\right] + \frac{\text{FV}}{(1+\text{YTM})^{M}}$$

Since there are no coupon payments as this is a zero-coupon bond, the first term in the equation above is zero.

$$\text{Bond price present value} = \frac{\text{FV}}{(1+\text{YTM})^{M}}$$

$$\text{Bond price present value} = \frac{1{,}000}{(1+0.07)^{6}}$$

$$\text{Bond price present value} = \$666.34.$$

2. A lottery claims its grand prize is $20 million, payable over 40 years at $1,000,000 per year. If the first payment is made immediately, what is the grand prize worth? Use an interest rate of 12%.

Solution:

Using a Financial Calculator

Set: Begin

N = 40

PMT = 1,000,000

FV = 0

I = 12%

Compute PV

PV = $9,233,029.88

3. Consider a bond with an 8% annual coupon and a face value of $1,000. Complete the following table:

Years to maturity	Yield to maturity	Current Price
4	6	
5	8	
7	9	
10	11	
11	12	

Solution:

Current price = PV

Using a Financial Calculator

CMPD

Set: End

Years to maturity	Yield to maturity (%)	Current Price ($)
4	6	1069.30
5	8	1000.00
7	9	949.67
10	11	823.32
11	12	762.49

4. Consider a coupon bond which has a $1,000 par value and a coupon rate of 20%. The bond is currently selling for $2,300 and has 16 years to maturity. Calculate the bond's yield to maturity.

Solution:

Using a Financial Calculator

N = 16

I = ?

PV = 2300

PMT = 200

FV = 1000

Compute I

I = 6.6%

5. You are willing to pay \$31,250 now to purchase a perpetuity that will pay you and your heirs \$2,500 each year, forever, starting at the end of this year. If your required rate of return does not change, how much would you be willing to pay if this were a 40-year, annual payment, ordinary annuity instead of perpetuity?

Solution: To find your yield to maturity, Perpetuity Value = PMT/I

So, 31,250 = 2500 / I

I = 0.08

Using a financial calculator:

N = 40

I = 8

PMT = 2,500

FV = 0

PV = press 'solve'

PV = 29,811.53

6. The price would be \$50/.025 = \$2000. If the yield to maturity doubles to 5%, the price would fall to half its previous value, to \$1000 = \$50/.05.

7. Property taxes in DeKalb County are roughly 2.66% of the purchase price every year. If you just bought a \$100,000 home, what is the *PV* of *all* the future property tax payments? Assume that the house remains worth \$100,000 forever, property tax rates never change, and that a 9% discount rate is used for discounting.

   Solution: The taxes on a \$100,000 home are roughly 100,000 × 0.0266 = 2,660.
   The *PV* of all future payments = 2,660/0.09 = \$29,555.55 (a perpetuity).

8. Assume you just deposited \$1,000 into a bank account. The current real interest rate is 2% and inflation is expected to be 6% over the next year. What nominal interest rate would you require from the bank over the next year? How much money will you have at the end of one year? If you are saving to buy a stereo that currently sells for \$1,050, will you have enough to buy it?

Solution: The required nominal rate would be:

$$\begin{aligned} i &= i_r + \pi^e \\ &= 2\% + 6\% = 8\%. \end{aligned}$$

At this rate, you would expect to have \$1,000 × 1.08, or \$1,080 at the end of the year. Can you afford the stereo? In theory, the price of the stereo will increase with the rate of inflation. So, one year later, the stereo will cost \$1,050 × 1.06, or \$1,113. You will be short by \$33.

9. A 10-year, 7% coupon bond with a face value of \$1,000 is currently selling for \$871.65. Compute your rate of return if you sell the bond next year for \$880.10.

Solution:

$$R = \frac{C + P_{t+1} - P_t}{P_t} = \frac{70 + 880.10 - 871.65}{871.65} = 0.09, \quad \text{or} \quad 9\%.$$

10. You have paid \$980.30 for an 8% coupon bond with a face value of \$1,000 that mature in five years. You plan on holding the bond for one year. If you want to earn a 9% rate of return on this investment, what price must you sell the bond for? Is this realistic?

Solution: To find the price, solve:

$$\frac{80 + P_{t+1} - 980.30}{980.30} = 0.09 \quad \text{for} \quad P_{t+1}.\ P_{t+1} = 988.53.$$

Although this appears possible, the yield to maturity when you purchased the bond was 8.5%. At that yield, you only expect the price to be \$983.62 next year. In fact, the yield would have to drop to 8.35% for the price to be \$988.53.

11. Calculate the duration of a \$1,000 6% coupon bond with three years to maturity. Assume that all market interest rates are 7%.

Solution:

Year	**1**	**2**	**3**	**Sum**
Payments	60.00	60.00	1060.00	
PV of Payments	56.07	52.41	865.28	973.76
Time Weighted *PV* of Payments	56.07	104.81	2595.83	
Time Weighted *PV* of Payments Divided by Price	0.06	0.11	2.67	2.83

This bond has a duration of 2.83 years. Note that the current price of the bond is \$973.76, which is the sum of the individual "*PV* of payments."

12. Consider the bond in the previous question. Calculate the expected price change if interest rates drop to 6.75% using the duration approximation. Calculate the actual price change using discounted cash flow.

Solution: Using the duration approximation, the price change would be:

$$\Delta P = -\text{DUR} \times \frac{\Delta i}{1+i} \times P = -2.83 \times \frac{-0.0025}{1.07} \times 973.76 = 6.44.$$

The new price would be \$980.20. Using a discounted cash flow approach, the price is 980.23—only \$.03 different.

Year	**1**	**2**	**3**	**Sum**
Payments	60.00	60.00	1060.00	
PV of payments	56.21	52.65	871.3	980.23

13. The duration of a \$100 million portfolio is 10 years. \$40 million dollars in new securities are added to the portfolio, increasing the duration of the portfolio to 12.5 years. What is the duration of the \$40 million in new securities?

Solution: First, note that the portfolio now has \$140 million in it. The duration of a portfolio is the weighted average duration of its individual securities. Let D equal the duration of the \$40 million in new securities. Then, this implies:

$$12.5 = (100/140 \times 10) + (40/140 \times \text{D})$$

$$12.5 = 7.1425 + 0.2857\,D$$

$$18.75 = D$$

The new securities have a duration of 18.75 years.

14. A bank has two, 3-year commercial loans with a present value of \$70 million. The first is a \$30 million loan that requires a single payment of \$37.8 million in 3 years, with no other payments until then. The second is for \$40 million. It requires an annual interest payment of \$3.6 million. The principal of \$40 million is due in 3 years.
    a. What is the duration of the bank's commercial loan portfolio?
    b. What will happen to the value of its portfolio if the general level of interest rates increased from 8% to 8.5%?

Solution: The duration of the first loan is 3 years since it is a zero-coupon loan. The duration of the second loan is as follows:

Year	**1**	**2**	**3**	**Sum**
Payment	3.60	3.60	43.60	
PV of Payments	3.33	3.09	34.61	41.03
Time Weighted *PV* of Payments	3.33	6.18	103.83	
Time Weighted *PV* of Payments Divided by Price	0.08	0.15	2.53	2.76

The duration of a portfolio is the weighted average duration of its individual securities. So, the portfolio's duration = 3/7 × (3) + 4/7 × (2.76) = 2.86

If rates increased, $\Delta P = -\text{DUR} \times \dfrac{\Delta i}{1+i} \times P = -2.86 \times \dfrac{0.005}{1.08} \times 70{,}000{,}000 = -926{,}852.$

15. Consider a bond that promises the following cash flows. The required discount rate is 12%.

Year	0	1	2	3	4
Promised Payments		160	170	180	230

You plan to buy this bond, hold it for 2½ years, and then sell the bond.

a. What total cash will you receive from the bond after the 2½ years? Assume that periodic cash flows are reinvested at 12%.

b. If immediately after buying this bond, all market interest rates drop to 11% (including your reinvestment rate), what will be the impact on your total cash flow after 2½ years? How does this compare to part (a)?

c. Assuming all market interest rates are 12%, what is the duration of this bond?

Solution:

a. You will receive 160 reinvested for 1.5 years, and 170 reinvested for 0.5 years. Then you will sell the remaining cash flows, discounted at 12%. This gives you:

$$160\times(1.12)^{1.5}+170\times(1.12)^{0.5}+\frac{180}{1.12^{0.5}}+\frac{230}{1.12^{1.5}}=\$733.69.$$

b. This is the same as part (a), but the rate is now 11%.

$$160\times(1.11)^{1.5}+170\times(1.11)^{0.5}+\frac{180}{1.11^{0.5}}+\frac{230}{1.11^{1.5}}=\$733.74.$$

Notice that this is only $0.05 different from part (a).

c. The duration is calculated as follows:

Year	1	2	3	4	Sum
Payments	160.00	170.00	180.00	230.00	
PV of Payments	142.86	135.52	128.12	146.17	552.67
Time Weighted *PV* of Payments	142.86	271.05	384.36	584.68	
Time Weighted *PV* of Payments Divided by Price	0.26	0.49	0.70	1.06	2.50

Since the duration and the holding period are the same, you are insulated from immediate changes in interest rates! It doesn't always work out this perfectly, but the idea is important.