3.1: PROBLEM DEFINITION

Apply the grid method to cases a, b, c and d.

a.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (kPa) corresponding to 15 in. H_2O (vacuum).

Solution:

$$p = \left(\frac{15\,\text{in H}_2\text{O}}{}\right)\left(\frac{\text{Pa}}{0.00402\,\text{in-H}_2\text{O}}\right)\left(\frac{\text{kPa}}{1000\,\text{Pa}}\right)$$

$$\boxed{p = 3.73\,\text{kPa-vacuum} = -3.73\,\text{kPa-gage}}$$

b.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (psig) corresponding to 140 kPa-abs.

Properties:

$p_{\text{atm}} = 14.70$psi.

Solution:

$$p_{\text{abs}} = \left(\frac{140\text{ kPa}}{1}\right)\left(\frac{14.70\text{ psi}}{101.3\text{ kPa}}\right) = 20.32\text{ psia}$$

$$p_{\text{gage}} = p_{\text{abs}} - p_{\text{atm}} = (20.32\text{ psia}) - (14.70\text{ psia}) = 5.62\text{ psi}$$

$$\boxed{p_{\text{gage}} = 5.62\text{ psig}}$$

c.)

Situation:

Pressure values need to be converted.

Find:

Calculate the absolute pressure (psia) corresponding to a pressure of 0.55 bar (gage).

Properties:

$p_{atm} = 14.70$psi.

Solution:

$$p_{\text{gage}} = \left(\frac{0.55\text{ bar}}{1}\right)\left(\frac{14.70\text{ psi}}{1.013\text{ bar}}\right) = 7.98\text{ psig}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (7.98\text{ psig}) + (14.70\text{ psia}) = 22.7\text{ psia}$$

$$\boxed{p_{\text{abs}} = 22.7 \text{ psia}}$$

d.)

Situation:

Pressure values need to be converted.

Find:

Calculate the pressure (kPa abs) corresponding to a blood pressure of 119 mm-Hg gage.

Properties:

Solution:

$$p_{\text{gage}} = \left(\frac{119 \text{ mm-Hg}}{1}\right)\left(\frac{101.3 \text{ kPa}}{760 \text{ mm-Hg}}\right) = 15.86 \text{ kPa-gage}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (101.3 \text{ kPa}) + (15.86 \text{ kPa-gage}) = 117.2 \text{ kPa abs}$$

$$\boxed{p_{\text{abs}} = 117 \text{ kPa abs}}$$

3.2: PROBLEM DEFINITION

Apply the grid method to:

a.)

Situation:

A sphere of 93 mm diameter contains an ideal gas.

$T = 20\,^\circ\mathrm{C} = 293.2\,\mathrm{K}$

Find:

Calculate the density of helium at a gage pressure of 36 in. H_2O.

Properties:

From Table A.2 (EFM11e): $R_{\mathrm{helium}} = 2077\,\mathrm{J/\,kg \cdot K}$.

Solution:

$$p_{\mathrm{abs}} = p_{\mathrm{atm}} + p_{\mathrm{gage}} = 101.3\,\mathrm{kPa} + \left(\frac{20\ \mathrm{in.\ H2O}}{1}\right)\left(\frac{248.8\,\mathrm{Pa}}{1.0\ \mathrm{in.\ H2O}}\right) = 110.26\,\mathrm{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{110.26\,\mathrm{kPa}}{1}\right)\left(\frac{\mathrm{kg\,K}}{2077\,\mathrm{J}}\right)\left(\frac{1}{293.2\,\mathrm{K}}\right)\left(\frac{1000\,\mathrm{Pa}}{1\,\mathrm{kPa}}\right)\left(\frac{\mathrm{J}}{\mathrm{N\,m}}\right)\left(\frac{\mathrm{N}}{\mathrm{Pa\,m^2}}\right)$$

$$\boxed{\rho = 0.181\ \mathrm{kg/m^3}}$$

b.)

Situation:

A sphere of 93 mm diameter contains an ideal gas.

$T = 20\,^\circ\mathrm{C} = 293.2\,\mathrm{K}$

Find:

Calculate the density of argon at a vacuum pressure of 8.8 psi.

Properties:

From Table A.2 (EFM11e): $R_{\mathrm{methane}} = 518\,\mathrm{J/\,kg \cdot K}$.

Solution:

$$p_{\mathrm{abs}} = p_{\mathrm{atm}} - p_{\mathrm{vacuum}} = 101.3\,\mathrm{kPa} - \left(\frac{8.8\ \mathrm{psi}}{1}\right)\left(\frac{101.3\,\mathrm{kPa}}{14.696\ \mathrm{psi}}\right) = 40.64\,\mathrm{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{40.64\,\mathrm{kPa}}{1}\right)\left(\frac{\mathrm{kg\,K}}{518\,\mathrm{J}}\right)\left(\frac{1}{293.2\,\mathrm{K}}\right)\left(\frac{1000\,\mathrm{Pa}}{1\,\mathrm{kPa}}\right)\left(\frac{\mathrm{J}}{\mathrm{N\,m}}\right)\left(\frac{\mathrm{N}}{\mathrm{Pa\,m^2}}\right)$$

$$\boxed{\rho = 0.268\ \mathrm{kg/m^3}}$$

3.3: PROBLEM DEFINITION

Situation:
For the questions below, assume standard atmospheric pressure.
a. For a vacuum pressure of 43 kPa, what is the absolute pressure? Gage pressure?
b. For a pressure of 15.6 psig, what is the pressure in psia?
c. For a pressure of 190 kPa gage, what is the absolute pressure in kPa?
d. Give the pressure 100 psfg in psfa.

SOLUTION

a.)

Consulting Fig. 3.4 in EFM11e,

$$\begin{aligned} p_{abs} &= 101.3 - 43 = 58.3\,\text{kPa} \\ & \boxed{P_{gage} = -43\,\text{kPa or } 43\,\text{kPa vacuum}} \end{aligned}$$

b.)

Consulting Fig. 3.4 in EFM11e,

$$\begin{aligned} p_{abs} &= 15.6 \text{ psig} + 14.7 \text{ psi } = 30.3 \text{ psia} \\ & \boxed{P_{abs} = 30.3 \text{ psia}} \end{aligned}$$

c.)

Consulting Fig. 3.4 in EFM11e,

$$\begin{aligned} p_{abs} &= 190\,\text{kPa gage } + 101.3\,\text{kPa} = 291.3\,\text{kPa abs} \\ & \boxed{P_{abs} = 291\,\text{kPa abs}} \end{aligned}$$

d.)

Consulting Fig. 3.4 in EFM11e,

$$\begin{aligned} p_{abs} &= \frac{100\,\text{lbf gage}}{\text{in}^2} + \left(\frac{14.7\,\text{lbf}}{\text{in}^2}\right)\left(\frac{144\,\text{in}^2}{\text{ft}^2}\right) = 2216.8 \text{ psfa} \\ & \boxed{p_{abs} = 2220 \text{ psfa}} \end{aligned}$$

3.4: PROBLEM DEFINITION

Situation:
The local atmospheric pressure is 91.0 kPa. A gage on an oxygen tank reads a pressure of 250 kPa gage.

Find:
What is the pressure in the tank in kPa abs?

PLAN

Consult Fig. 3.4 in EFM11e

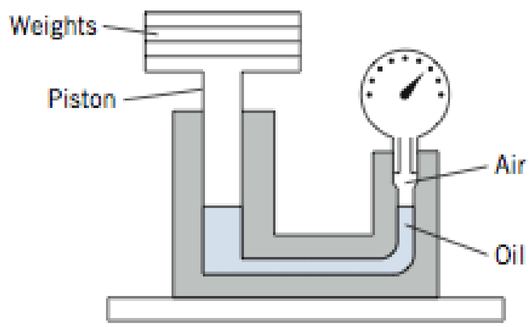
SOLUTION

$$
\begin{aligned}
P_{abs} &= P_{gage} + P_{atm} \\
P_{abs} &= 250\,\text{kPa} + 91\,\text{kPa} \\
& \boxed{P_{abs} = 341\,\text{kPa abs}}
\end{aligned}
$$

3.5: PROBLEM DEFINITION

Situation:

A Crosby gage tester is applied to calibrate a pressure gage.
Indicated pressure on the gage is $p = 197\,\text{kPa}$ gage.
$W = 132\,\text{N}$, $D = 0.03\,\text{m}$.



Find:

Percent error in gage reading.

PLAN The oil exerts an upward force on the piston to support the weights. Thus, we can calculate the true pressure and then compare this with indicated reading to obtain the error in the gage reading. The steps are

1. Calculate the true pressure by applying force equilibrium to the piston and weights.
2. Calculate the error in the gage reading.

SOLUTION

1. Force equilibrium (apply to piston + weights)

$$\begin{aligned} F_{\text{pressure}} &= W \\ p_{\text{true}} A &= W \\ p_{\text{true}} &= \frac{W}{A} \\ &= \frac{132\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}^2} \\ p_{\text{true}} &= 186,742\,\text{Pa} \end{aligned}$$

2. Percent error

$$\begin{aligned} \%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\ &= \frac{(197\,\text{kPa} - 186.7\,\text{kPa})\,100}{186.7\,\text{kPa}} \\ &= 5.49\% \end{aligned}$$

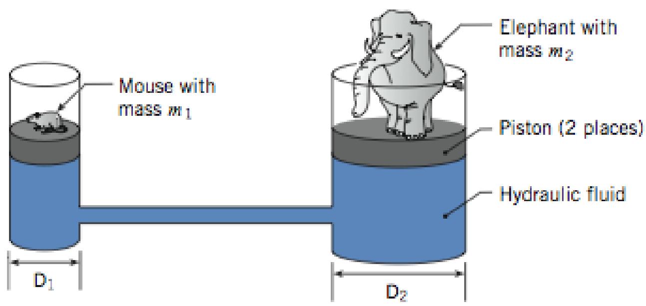
$$\boxed{\%\ \text{Error} = 1.01\%}$$

3.6: PROBLEM DEFINITION

Situation:

A hydraulic machine is used to provide a mechanical advantage.

$m_1 = 0.025\,\text{kg}$, $m_2 = 7500\,\text{kg}$.



Find:

(a) Derive an algebraic equation for the mechanical advantage.

(b) Calculate D_1 and D_2 so the mouse can support the elephant.

Assumptions:

- Neglect the mass of the pistons.
- Neglect the friction between the piston and the cylinder wall.
- The pistons are at the same elevation; thus, the pressure acting on the bottom of each piston is the same.
- A mouse can fit onto a piston of diameter $D_1 = 70$ mm.

PLAN

1. Define "mechanical advantage."
2. Derive an equation for the pressure acting on piston 1.
3. Derive an equation for the pressure acting on piston 2.
4. Derive an equation for mechanical advantage by combining steps 2 and 3.
5. Calculate D_2 by using the result of step 4.

SOLUTION

1. Mechanical advantage.

$$\left\{ \begin{array}{c} \text{Mechanical} \\ \text{advantage} \end{array} \right\} = \frac{\text{Weight "lifted" by the mouse}}{\text{Weight of the mouse}} = \frac{W_2}{W_1} \tag{1}$$

where W_2 is the weight of the elephant, and W_1 is the weight of the mouse.

2. Equilibrium (piston 1):

$$\begin{aligned} W_1 &= p\left(\frac{\pi D_1^2}{4}\right) \\ p &= W_1\left(\frac{4}{\pi D_1^2}\right) \end{aligned} \tag{2}$$

3. Equilibrium (piston 2):

$$\begin{aligned} W_2 &= p\left(\frac{\pi D_2^2}{4}\right) \\ p &= W_2\left(\frac{4}{\pi D_2^2}\right) \end{aligned} \tag{3}$$

4. Combine Eqs. (2) and (3):

$$p = W_1\left(\frac{4}{\pi D_1^2}\right) = W_2\left(\frac{4}{\pi D_2^2}\right) \tag{5}$$

Solve Eq. (5) for mechanical advantage:

$$\boxed{\frac{W_2}{W_1} = \left(\frac{D_2}{D_1}\right)^2}$$

5. Calculate D_2.

$$\begin{aligned} \frac{W_2}{W_1} &= \left(\frac{D_2}{D_1}\right)^2 \\ \frac{(7500\,\text{kg})\,(9.80\,\text{m/s}^2)}{(0.025\,\text{kg})\,(9.80\,\text{m/s}^2)} &= 300000 = \left(\frac{D_2}{0.07\,\text{m}}\right)^2 \\ D_2 &= 38.3\,\text{m} \end{aligned}$$

$$\boxed{\text{The ratio of } (D_2/D_1) \text{ needs to be } \sqrt{300,000}. \text{ If } D_1 = 70\,\text{mm, then } D_2 = 38.3\,\text{m.}}$$

REVIEW

1. Notice. The mechanical advantage varies as the diameter ratio squared.
2. The mouse needs a mechanical advantage of 300,000:1. This results in a piston that is impractical (diameter = 38.3 m = 126 ft !).

3.7: PROBLEM DEFINITION

Situation:

To work the problem, data was recorded from a parked vehicle. Relevant information:

- Left front tire of a parked VW Passat 2003 GLX Wagon (with 4-motion).
- Bridgestone snow tires on the vehicle.
- Inflation pressure = 36 psig. This value was found by using a conventional "stick-type" tire pressure gage.
- Contact Patch: $5.88\,\text{in} \times 7.5\,\text{in}$. The 7.5 inch dimension is across the tread. These data were found by measuring with a ruler.
- Weight on the front axle = 2514 lbf. This data was recorded from a sticker on the driver side door jamb. The owners manual states that this is maximum weight (car + occupants + cargo).

Assumptions:

- The weight on the car axle without a load is 2000 lbf. Thus, the load acting on the left front tire is 1000 lbf.
- The thickness of the tire tread is 1 inch. The thickness of the tire sidewall is 1/2 inch.
- The contact path is flat and rectangular.
- Neglect any tensile force carried by the material of the tire.

Find:

Measure the size of the contact patch.
Calculate the size of the contact patch.
Compare the measurement with the calculation and discuss.

PLAN

To estimate the area of contact, apply equilibrium to the contact patch.

SOLUTION

Equilibrium in the vertical direction applied to a section of the car tire

$$p_i A_i = F_{\text{pavement}}$$

where p_i is the inflation pressure, A_i is the area of the contact patch on the inside of the tire and F_{pavement} is the normal force due to the pavement. Thus,

$$
\begin{aligned}
A_i &= \frac{F_{\text{pavement}}}{p_i} \\
&= \frac{1000\,\text{lbf}}{36\,\text{lbf}/\,\text{in}^2} \\
&= 27.8\,\text{in}^2
\end{aligned}
$$

Comparison. The actual contact patch has an area $A_o = 5.88\,\text{in} \times 7.5\,\text{in} = 44.1\,\text{in}^2$. Using the assumed thickness of rubber, this would correspond to an inside contact area of $A_o = 4.88\,\text{in} \times 5.5\,\text{in} = 26.8\,\text{in}^2$.Thus, the predicted contact area $\left(27.8\,\text{in}^2\right)$ and the measured contact area $\left(26.8\,\text{in}^2\right)$ agree to within about 1 part in 25 or about 4%.

REVIEW

The comparison between predicted and measured contact area is highly dependent on the assumptions made.

3.8: PROBLEM DEFINITION

Situation:
To derive the hydrostatic equation, which of the following must be assumed? (Select all that are correct.)
a. the specific weight is constant
b. the fluid has no charged particles
c. the fluid is at equilibrium

SOLUTION

The answers are (a) and (c); see §3.2

3.9: PROBLEM DEFINITION

Situation:

Two tanks.
Tank A is filled to depth h with water.
Tank B is filled to depth h with oil.

Find:

Which tank has the largest pressure?
Why?
Where in the tank does the largest pressure occur?

SOLUTION

In both tanks, pressure increases with depth, according to $p = -\gamma z$.
At the bottom of each tank, pressure is given by $p = \gamma h$.
At the bottom of Tank A, $p = \gamma_{water} h$.
At the bottom of Tank B, $p = \gamma_{oil} h$.
Because $\gamma_{oil} < \gamma_{water}$, the pressure in Tank A has the largest pressure.
The reason is because water has a larger specific weight than oil.
The largest pressure occurs at the bottom of the tank.

3.10: PROBLEM DEFINITION

Situation:

Consider Figure 3.11 in §3.2 of EFM11e.

a. Which fluid has the larger density?

b. If you graphed pressure as a function of z in these two layered liquids, in which fluid does the pressure change more with each incremental change in z?

SOLUTION

a. Water has the larger density, and thus the larger specific weight.

b. To pressure as a function of fluid, you would use $p = -\gamma z$. The pressure changes more with each incremental change in z in the water than in the oil because $\gamma_{oil} < \gamma_{water}$.

Problem 3.11

Apply the grid method to calculations involving the hydrostatic equation:

$$\Delta p = \gamma \Delta z = \rho g \Delta z$$

Note: Unit cancellations are not shown in this solution.

a.)

Situation:

Pressure varies with elevation.
$\Delta z = 6.8\,\text{ft}$.

Find:

Pressure change (kPa).

Properties:

$\rho = 90\ \text{lbm/ft}^3$.

Solution:

Convert density to units of kg/m^3:

$$\rho = \left(\frac{90\ \text{lbm}}{\text{ft}^3}\right)\left(\frac{35.315\,\text{ft}^3}{\text{m}^3}\right)\left(\frac{1.0\,\text{kg}}{2.2046\ \text{lbm}}\right) = 1442\ \frac{\text{kg}}{\text{m}^3}$$

Calculate the pressure change:

$$\Delta p = \rho g \Delta z = \left(\frac{1442\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{6.8\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.2808\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\ \text{m}\cdot\ \text{s}^2}{\text{kg}}\right)$$

$$\boxed{\Delta p = 29300\,\text{Pa} = 29.3\,\text{kPa}}$$

b.)

Situation:

Pressure varies with elevation.
$\Delta z = 22\,\text{m}$, $SG = 1.3$.

Find:

Pressure change (psf).

Properties:

$\gamma = 62.4\,\text{lbf/ft}^3$.

Solution:

$$\Delta p = \gamma \Delta z = S\ \gamma_{H2O}\ \Delta z = \left(\frac{(1.3 \times 62.4)\ \text{lbf}}{\text{ft}^3}\right)\left(\frac{22\,\text{m}}{1.0}\right)\left(\frac{3.2808\,\text{ft}}{\text{m}}\right)$$

$$\boxed{\Delta p = 5860\ \text{psf}}$$

c.)

Situation:

Pressure varies with elevation.
$\Delta z = 2500\,\text{ft}$.

Find:

Pressure change (in H_2O).

Properties:

air, $\rho = 1.2\,\text{kg/m}^3$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{2500\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)\left(\frac{\text{in.-H2O}}{249.1\,\text{Pa}}\right)$$

$$\boxed{\Delta p = 36.1 \text{ in } H_2O}$$

d.)

Situation:

Pressure varies with elevation.
$\Delta p = 1/6\,\text{atm}$, $SG = 1.4$.

Find:

Elevation change (mm).

Properties:

$\gamma = 9810\,\text{N/m}^3$, $p_{atm} = 101.3\,\text{kPa}$.

Solution:

d. Calculate Δz (mm) corresponding to $S = 1.4$ and $\Delta p = 1/6\,\text{atm}$.

$$\Delta z = \frac{\Delta p}{\gamma} = \frac{\Delta p}{S\gamma_{H2O}} = \left(\frac{1/6\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{(1.4\times 9810)\ \text{N}}\right)\left(\frac{101.3\times 10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{1000\,\text{mm}}{1.0\,\text{m}}\right)$$

$$\boxed{\Delta z = 1230\,\text{mm}}$$

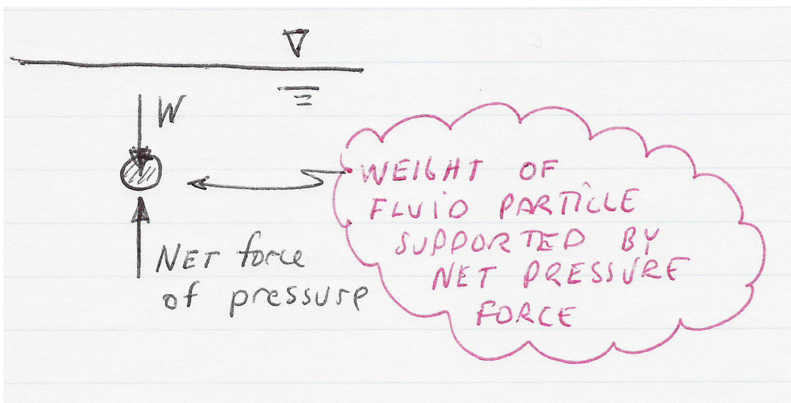
Problem 3.12

Using Section 3.2 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

---

a. What does hydrostatic mean? How do engineers identify if a fluid is hydrostatic?

- Each fluid particle within the body is in force equilibrium(z-direction) with the net force due to pressure balancing the weight of the particle. Here, the z-direction is aligned with the gravity vector.



- Engineers establish hydrostatic conditions by analyzing the forces acting in the z-direction.

---

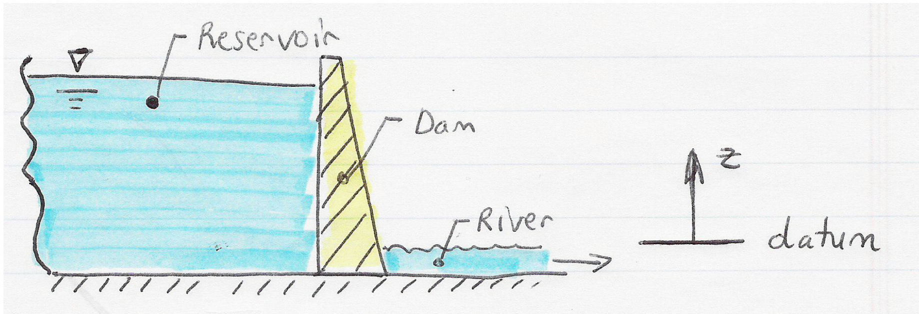
b. What are common forms of the hydrostatic equation? Are the forms equivalent or are they different?

- There are three common forms; these are given in Table F.2 (front of book).
- These equations are equivalent because you can start with any of the equations and derive the other two.

---

c. What is a datum? How do engineers establish a datum?

- A datum is a fixed reference point from which elevations are measured.
- Engineers select a datum that makes calculations easy. For example, select a datum on the free surface of a river below a dam so that all elevations are positive.



d. What are the main ideas of Eq. (3.10) EFM11e? That is, what is the meaning of this equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation means that the sum of $(p + \gamma z)$ has the same numerical value at every location within a body of fluid.

e. What assumptions need to be satisfied to apply the hydrostatic equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation is valid when

- the density of the fluid is constant at all locations.
- equilibrium is satisfied in the z-direction (net force of pressure balances weight of the fluid particle.

Problem 3.13

Apply the grid method to each situation below. Unit cancellations are not shown in these solutions.

a.)

Situation:

Pressure varies with elevation.
$\Delta z = 8\,\text{ft}$.

Find:

Pressure change (Pa).

Properties:

air, $\rho = 1.2\,\text{kg}/\,\text{m}^3$.

Solution:

$$\Delta p = \rho g \Delta z$$

$$\begin{aligned} \Delta p &= \rho g \Delta z \\ &= \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{8\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right) \end{aligned}$$

$$\boxed{\Delta p = 28.7\,\text{Pa}}$$

b.)

Situation:

Pressure increases with depth in the ocean.
Pressure reading is 1.5 atm gage.

Find:

Water depth (m).

Properties:

Seawater, Table A.4 (EFM11e), $SG = 1.03$, $\gamma = 10070\,\text{N}/\,\text{m}^3$.

Solution:

$$\Delta z = \frac{\Delta p}{\gamma} = \left(\frac{1.5\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{10070\,\text{N}}\right)\left(\frac{101.3\times10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{\text{N}}{\text{Pa}\,\text{m}^2}\right)$$

$$\boxed{\Delta z = 15.1\,\text{m}}$$

c.)

Situation:

Pressure decreases with elevation in the atmosphere.
$\Delta z = 1240\,\text{ft}$.
Starting air pressure = 960 mbar

Find:

Pressure (mbar).

Assumptions:
Density of air is constant.

Properties:
Air, $\rho = 1.1\,\mathrm{kg/\,m^3}$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.1\,\mathrm{kg}}{\mathrm{m}^3}\right)\left(\frac{9.81\,\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{-1240\,\mathrm{ft}}{1.0}\right)\left(\frac{\mathrm{m}}{3.2808\,\mathrm{ft}}\right)\left(\frac{\mathrm{Pa\cdot\ m\cdot\ s^2}}{\mathrm{kg}}\right) = -4078\,\mathrm{Pa}$$

Pressure at summit:

$$p_{\text{summit}} = p_{\text{base}} + \Delta p = 960\,\mathrm{mbar} + \left(\frac{-4078\,\mathrm{Pa}}{1.0}\right)\left(\frac{10^{-2}\,\mathrm{mbar}}{\mathrm{Pa}}\right)$$

$$\boxed{p_{\text{summit}} = 919\,\mathrm{mbar\ (absolute)}}$$

d.)

Situation:
Pressure increases with depth in a lake.
$\Delta z = 370\,\mathrm{m}$.

Find:
Pressure (MPa).

Properties:
Water, $\gamma = 9810\,\mathrm{N/\,m^3}$.

Solution:

$$\begin{aligned}\Delta p &= \gamma \Delta z \\ &= \left(\frac{9810\,\mathrm{N}}{\mathrm{m}^3}\right)\left(\frac{370\,\mathrm{m}}{1.0}\right)\left(\frac{\mathrm{Pa\cdot\ m^2}}{\mathrm{N}}\right)\left(\frac{\mathrm{MPa}}{10^6\,\mathrm{Pa}}\right)\end{aligned}$$

$$\boxed{p_{\max} = 3.63\,\mathrm{MPa\ (gage)}}$$

e.)

Situation:
Pressure increase with water depth in a standpipe.
$\Delta z = 55\,\mathrm{m}$.

Find:
Pressure (kPa).

Properties:
Water, $\gamma = 9810\,\mathrm{N/\,m^3}$.

Solution:

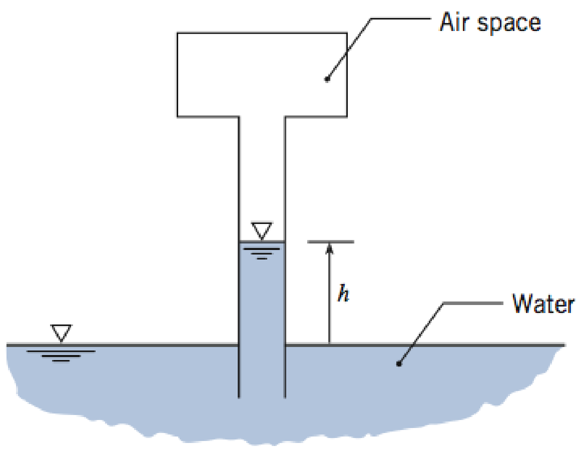
$$\begin{aligned}\Delta p &= \gamma \Delta z \\ &= \left(\frac{9810\,\text{N}}{\text{m}^3}\right)\left(\frac{55\,\text{m}}{1.0}\right)\left(\frac{\text{Pa}\cdot\,\text{m}^2}{\text{N}}\right)\left(\frac{\text{kPa}}{10^3\,\text{Pa}}\right)\end{aligned}$$

$$\boxed{p_{\text{max}} = 540\,\text{kPa (gage)}}$$

3.14: PROBLEM DEFINITION

Situation:

Air above a long tube is pressurized.
Initial state: $p_{\text{air1}} = 50\,\text{kPa-vacuum}$
Final state: $p_{\text{air2}} = 25\,\text{kPa-vacuum}$.



Find:

Will h increase or decrease?
The change in water column height (Δh) in meters.

Assumptions:

Atmospheric pressure is 100 kPa.
Surface tension can be neglected.

Properties:

Water (20 °C), Table A.5 (EFM11e), $\gamma = 9790\,\text{N/m}^3$.

PLAN

Since pressure increases, the water column height will decrease. Use absolute pressure in the hydrostatic equation.

1. Find h (initial state) by applying the hydrostatic equation.
2. Find h (final state) by applying the hydrostatic equation.
3. Find the change in height by $\Delta h = h(\text{final state}) - h\,(\text{initial state})$.

SOLUTION

1. Initial State. Locate point 1 on the reservoir surface; point 2 on the water surface inside the tube:

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\
\frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{50\,\text{kPa}}{9790\,\text{N/m}^3} + h \\
h\,(\text{initial state}) &= 5.107\,\text{m}
\end{aligned}$$

2. Final State:

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{100\,\mathrm{kPa}}{9790\,\mathrm{N/\,m^3}} + 0 &= \frac{75\,\mathrm{kPa}}{9790\,\mathrm{N/\,m^3}} + h \\ h\,(\text{final state}) &= 2.554\,\mathrm{m} \end{aligned}$$

3. Change in height:

$$\begin{aligned} \Delta h &= h(\text{final state}) - h\,(\text{initial state}) \\ &= 2.554\,\mathrm{m} - 5.107\,\mathrm{m} = -2.55\,\mathrm{m} \end{aligned}$$

The height has decreased by 2.55 m.

REVIEW

Tip! In the hydrostatic equation, use gage pressure or absolute pressure. Using vacuum pressure will give a wrong answer.

3.15: PROBLEM DEFINITION

Situation:

Open tank, $d_{oil} = 1\,\text{m}$
$d_{brine} = 0.55\,\text{m}$
$p_{bottom} = 14\,\text{kPa} = 14,000\,\text{Pa}$

Properties:

$\rho_{brine} = 1,050\,\text{kg/m}^3$

Find:

ρ_{oil}

PLAN

Use the Hydrostatic Equation, and the knowledge that the oil is floating on the brine. According to the Fluid Interface Rule (§ 3.2 in EFM 11e), the pressure at the brine/water interface is constant.
Therefore, pressure over the total depth of the tank is given by

$$\begin{aligned} \Delta p &= \Delta p_{oil} + \Delta p_{brine} \\ \text{where } p &= \rho g h \quad \text{generally} \\ \text{and } p_{bottom} &= \rho_{brine} g d_{brine} + \rho_{oil} g d_{oil} \quad \text{for this case} \end{aligned}$$

SOLUTION

$$\begin{aligned} \rho_{oil} &= \frac{p_{bottom} - \rho_{brine} g d_{brine}}{g d_{oil}} \\ \rho_{oil} &= \frac{14,000\,\text{Pa} - 1,050\,\text{kg/m}^3 \times 9.81\,\text{m/s}^2 \times 0.55\,\text{m}}{9.81\,\text{m/s}^2 \times 1\,\text{m}} \end{aligned}$$

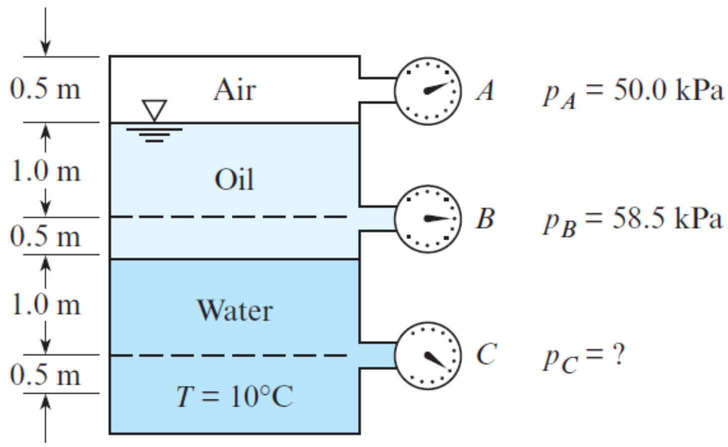
$$\boxed{\rho_{oil} = 850\,\text{kg/m}^3}$$

3.16: PROBLEM DEFINITION

Situation:
A closed tank contains air, oil, and water.

Find:
Specific gravity of oil.
Pressure at C (kPa-gage).

Sketch:



Properties:
Water (10 °C), Table A.5, $\gamma = 9810\,\mathrm{N/\,m^3}$.

PLAN

1. Find the oil specific gravity by applying the hydrostatic equation from A to B.
2. Apply the hydrostatic equation to the water.
3. Apply the hydrostatic equation to the oil.
4. Find the pressure at C by combining results for steps 2 and 3.

SOLUTION

1. Hydrostatic equation (from oil surface to elevation B):

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50,000\ \mathrm{N/m^2} + \gamma_{\mathrm{oil}}\,(1\ \mathrm{m}\ ) &= 58,500\ \mathrm{N/m^2} + \gamma_{\mathrm{oil}}\,(0\ \mathrm{m}) \\ \gamma_{\mathrm{oil}} &= 8500\ \mathrm{N/m^3} \end{aligned}$$

Specific gravity:

$$S = \frac{\gamma_{\mathrm{oil}}}{\gamma_{\mathrm{water}}} = \frac{8500\ \mathrm{N/m^3}}{9810\ \mathrm{N/m^3}}$$

$$\boxed{S_{\mathrm{oil}} = 0.87}$$

2. Hydrostatic equation (in water):

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}}\,(1\,\text{m})$$

3. Hydrostatic equation (in oil):

$$p_{\text{btm of oil}} = (58,500\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

4. Combine equations:

$$\begin{aligned} p_c &= (58,500\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}}\,(1\,\text{m}) \\ &= \left(58,500\,\text{Pa} + 8500\,\text{N/m}^3 \times 0.5\,\text{m}\right) + 9810\,\text{N/m}^3\,(1\,\text{m}) \\ &= 43,310\ \text{N/m}^2 \end{aligned}$$

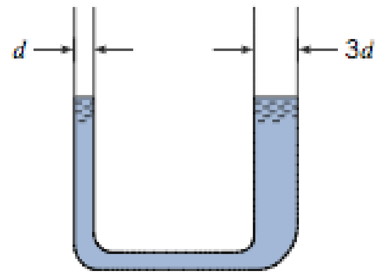
$$\boxed{p_c = 43.3\ \text{kPa-gage}}$$

3.17: PROBLEM DEFINITION

Situation:

A manometer is described in the problem statement.

$d_{\text{left}} = 1\,\text{mm}$, $d_{\text{right}} = 3\,\text{mm}$.



Find:

Water surface level in the left tube as compared to the right tube.

SOLUTION

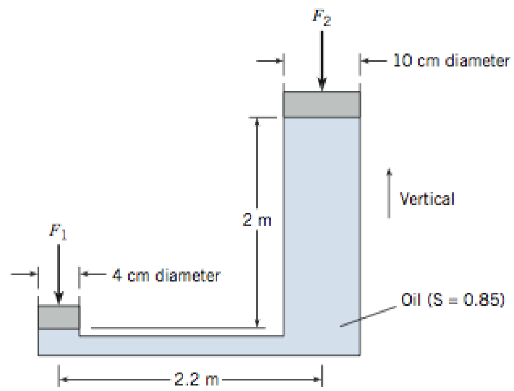
(a) The water surface level in the left tube will be higher because of greater surface tension effects for that tube.

3.18: PROBLEM DEFINITION

Situation:

A force is applied to a piston.

$F_1 = 390\,\text{N}$, $d_1 = 4\,\text{cm}$, $d_2 = 10\,\text{cm}$, $SG = 0.85$



Find:

Force resisted by piston.

Assumptions:

Neglect piston weight.

PLAN

Apply the hydrostatic equation and equilibrium.

SOLUTION

1. Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 390\,\text{N}}{\pi \cdot (0.04\,\text{m})^2\ \text{m}^2} \\ &= 310,352\,\text{Pa} \end{aligned}$$

2. Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\ &= 310,352\,\text{Pa} + \left(0.85 \times 9810\,\text{N/m}^3\right)(-2\,\text{m}) \\ &= 293,680\,\text{Pa} \end{aligned}$$

3. Equilibrium (piston 2)

$$\begin{aligned} F_2 &= p_2 A_2 \\ &= \left(293,680\,\mathrm{N/\,m^2}\right)\left(\frac{\pi\left(0.1\,\mathrm{m}\right)^2}{4}\right) \\ &= 2310\,\mathrm{N} \end{aligned}$$

$$\boxed{F_2 = 2310\ \mathrm{N}}$$

3.19: PROBLEM DEFINITION

Situation:

Regarding the hydraulic jack in Problem 3.18 (EFM11e), which ideas were used to analyze the jack? (select all that apply)

a. pressure = (force)(area)

b. pressure increases linearly with depth in a fluid with a constant density

c. the pressure at the bottom of the 4-cm chamber is larger than the pressure at the bottom of the 10-cm chamber

d. when a body is stationary, the sum of forces on the body is zero

e. when a body is stationary, the sum of moments on the body is zero

f. differential pressure = (weight/volume)(change in elevation)

SOLUTION

Correct answers are a, b, d, e and f.
Statement c is incorrect because the two chambers are connected, therefore the pressure at the flat bottom is everywhere the same. Pressure is a scalar, and is transferred continuously in all directions. It increases with depth; however at the same depth (of a fluid with a constant density that is not being accelerated to the left or right) it is the same.

3.20: PROBLEM DEFINITION

Situation:

A diver goes underwater.

$\Delta z = 50\,\text{m}$.

Find:

Gage pressure (kPa).

Ratio of pressure to normal atmospheric pressure.

Properties:

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Apply the hydrostatic equation.
2. Calculate the pressure ratio (use absolute pressure values).

SOLUTION

1. Hydrostatic equation

$$\begin{aligned} p &= \gamma \Delta z = 9790\,\text{N/m}^3 \times 50\,\text{m} \\ &= 489,500\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p = 490\ \text{kPa gage}}$$

2. Calculate pressure ratio

$$\frac{p_{50}}{p_{\text{atm}}} = \frac{489.5\,\text{kPa} + 101.3\,\text{kPa}}{101.3\,\text{kPa}}$$

$$\boxed{\frac{p_{50}}{p_{\text{atm}}} = 5.83}$$

3.21: PROBLEM DEFINITION

Situation:

Water and kerosene are in a tank.
$z_{\text{water}} = 1.2\,\text{m}$, $z_{\text{kerosene}} = 0.8\,\text{m}$.

Find:

Gage pressure at bottom of tank (kPa-gage).

Properties:

Water (20 °C), Table A.5, $\gamma_w = 9790$ N/m^3.
Kerosene (20 °C), Table A.4, $\gamma_k = 8010$ N/m^3.

SOLUTION

Manometer equation (add up pressure from the top of the tank to the bottom of the tank).

$$p_{\text{atm}} + \gamma_{\text{k}}\, z_{\text{kerosene}} + \gamma_{\text{w}}\, z_{\text{water}} = p_{\text{bottom}}$$

Solve for pressure

$$\begin{aligned} p_{\text{bottom}} &= 0 + \gamma_{\text{k}}\ (0.8\,\text{m}) + \gamma_{\text{w}}\ (1.2\,\text{m}) \\ &= \left(8010\,\text{N/ m}^3\right)(0.8\,\text{m}) + \left(9790\,\text{N/ m}^3\right)(1.2\,\text{m}) \\ &= 18.2\,\text{kPa} \end{aligned}$$

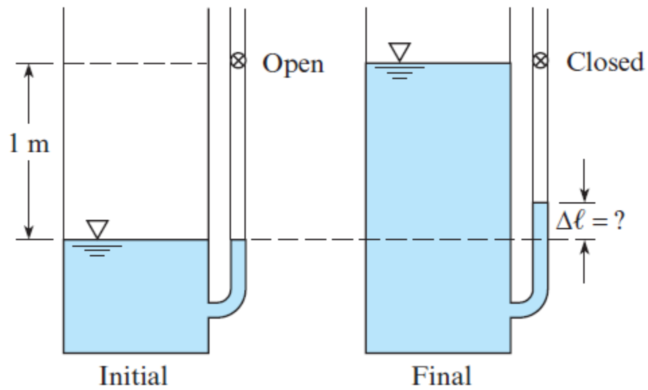
$$\boxed{p_{\text{bottom}} = 18.2\,\text{kPa gage}}$$

3.22: PROBLEM DEFINITION

Situation:

Initial State: Water levels as shown. Valve in open.

Final State: Water is added to the tank with the valve closed.



Find:

Increase of water level $\Delta\ell$ in manometer (in meters).

Properties:

Water (20 °C), Table A.5, γ_w=9790 N/m^3.

$p_{atm} = 100\,\text{kPa}$.

Assumptions: Ideal gas.

PLAN

Apply the hydrostatic equation and the ideal gas law.

SOLUTION

Ideal gas law (mole form; apply to air in the manometer tube)

$$p\forall = n\Re T$$

Because the number of moles (n) and temperature (T) are constants, the ideal gas reduces to Boyle's equation.

$$p_1\forall_1 = p_2\forall_2 \tag{1}$$

State 1 (before air is compressed)

$$\begin{aligned} p_1 &= 100,000 \text{ N/m}^2 \text{ abs} \\ \forall_1 &= 1 \text{ m} \times A_{\text{tube}} \end{aligned} \tag{a}$$

State 2 (after air is compressed)

$$\begin{aligned} p_2 &= 100,000 \text{ N/m}^2 + \gamma_w(1 \text{ m} - \Delta\ell) \\ \forall_2 &= (1 \text{ m} - \Delta\ell)A_{\text{tube}} \end{aligned} \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned} p_1 \mathcal{V}_1 &= p_2 \mathcal{V}_2 \\ \left(100,000\,\mathrm{N/\,m^2}\right)(1\ \mathrm{m} \times A_{\mathrm{tube}}) &= \left(100,000\ \mathrm{N/m^2} + \gamma_{\mathrm{w}}(1\ \mathrm{m} - \Delta\ell)\right)(1\ \mathrm{m} - \Delta\ell)A_{\mathrm{tube}} \\ 100,000\,\mathrm{N/\,m^2} &= \left(100,000\,\mathrm{N/\,m^2} + 9790\,\mathrm{N/\,m^3}\,(1 - \Delta\ell)\right)(1 - \Delta\ell) \end{aligned}$$

Solving for $\Delta\ell$

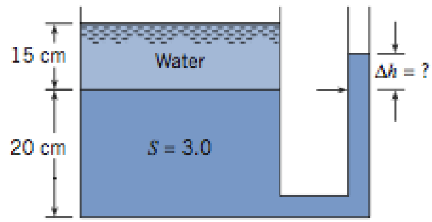
$$\boxed{\Delta\ell = 0.0824\ \mathrm{m}}$$

3.23: PROBLEM DEFINITION

Situation:

A tank is fitted with a manometer.

$SG = 3$, $z_1 = 0.15\,\text{m}$.



Find:

Deflection of the manometer (cm).

Properties:

γ_{water}=9810 N/m^3.

PLAN

Apply the hydrostatic principle to the water and then to the manometer fluid.

SOLUTION

1. Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned}
\frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\
\frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\
p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\
&= 1472\,\text{Pa}
\end{aligned}$$

2. Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

$$\begin{aligned}
\frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\
\frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h
\end{aligned}$$

3. Solve for Δh

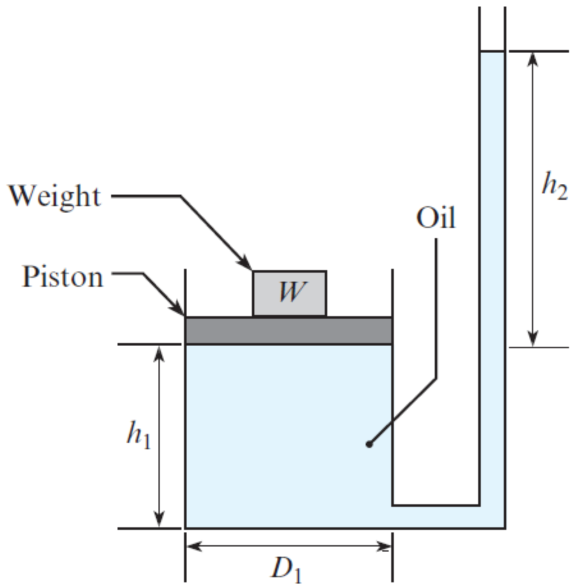
$$\begin{aligned}
\Delta h &= \frac{1472\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} \\
&= 0.0500\,\text{m}
\end{aligned}$$

$$\boxed{\Delta h = 5.00\text{ cm}}$$

3.24: PROBLEM DEFINITION

Situation:

A weight sits on top of a piston situated above a reservoir of oil.



Find:

Derive an equation for h_2 in terms of the specified parameters.

Assumptions:

Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate w to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$w = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ \ h_2 \left(\frac{\pi D_1^2}{4} \right)$$

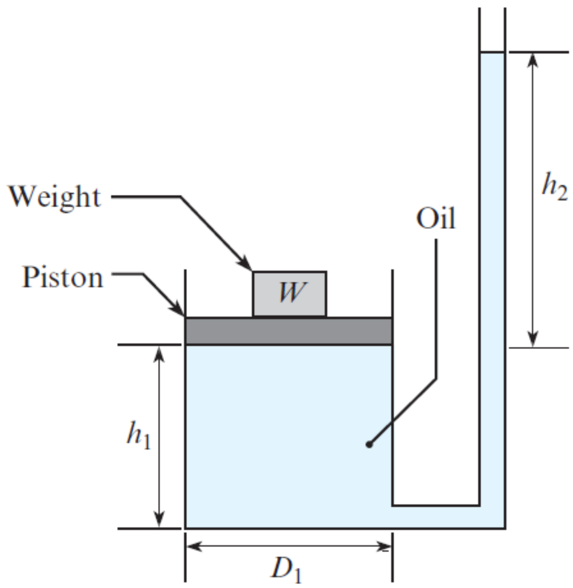
$$\boxed{\text{Answer:}} \qquad h_2 = \frac{4w}{(S)\,(\gamma_{\text{water}})\,(\pi D_1^2)}$$

REVIEW

1. Notice. Column height h_2 increases linearly with increasing weight w.Similarly, h_2 decreases linearly with S and decreases quadratically with D_1.
2. Notice. The apparatus involved in the problem could be used to create an instrument for weighing an object.

3.25: PROBLEM DEFINITION

Situation:
A weight sits on top of a piston situated above a reservoir of oil.
$m = 5\,\text{kg}$, $SG = 0.8$, $h_1 = 42\,\text{mm}$.
$D_1 = 120\,\text{mm}$, $D_2 = 5\,\text{mm}$.



Find:
Calculate h_2 (m).

Assumptions:
Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate mass m to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$mg = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\
\frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\
p_1 &= S\ \gamma_{\text{water}}\ h_2
\end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4}\right)$$

$$h_2 = \frac{4mg}{(S)\,(\gamma_{\text{water}})\,(\pi D_1^2)} = \frac{4\,(5\,\text{kg})\,(9.81\,\text{m/s}^2)}{(0.8)\,(9810\,\text{N/m}^3)\,(\pi)\,(0.12^2\,\text{m}^2)}$$

$$\boxed{h_2 = 0.553\,\text{m}}$$

3.26: PROBLEM DEFINITION

Situation:

An odd tank contains water, air and a liquid.

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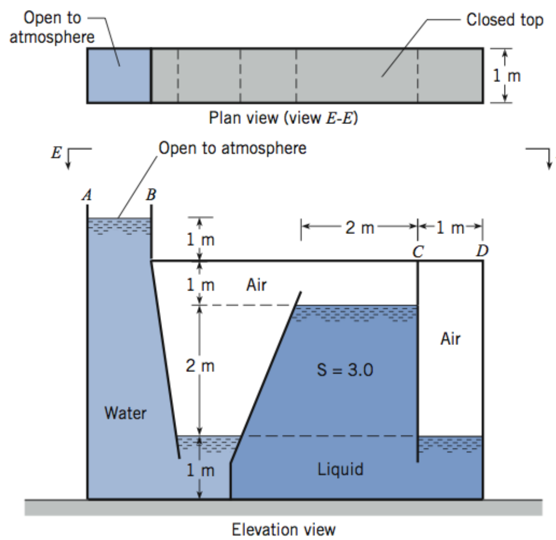


Find:

Maximum gage pressure (kPa).
Where will maximum pressure occur.
Pressure force (in kN) on top of the last chamber, surface CD.

Properties:

$\gamma_{\text{water}} = 9810 \text{ N/m}^3$.

PLAN

1. To find the maximum pressure, apply the manometer equation.
2. To find the hydrostatic force, multiply pressure times area.

SOLUTION

1. Manometer eqn. (start at surface AB; neglect pressure changes in the air; end at the bottom of the liquid reservoir)

$$\begin{aligned} 0 + 4 \times \gamma_{\text{H}_2\text{O}} + 3 \times 3\gamma_{\text{H}_2\text{O}} &= p_{\max} \\ p_{\max} &= 13\,\text{m} \times 9,810\,\text{N/m}^3 \\ &= 127,530 \text{ N/m}^2 \end{aligned}$$

$$\boxed{p_{\max} = 128 \text{ kPa}}$$

Answer ⇒ Maximum pressure will be at the bottom of the liquid that has a specific gravity of $S = 3$.

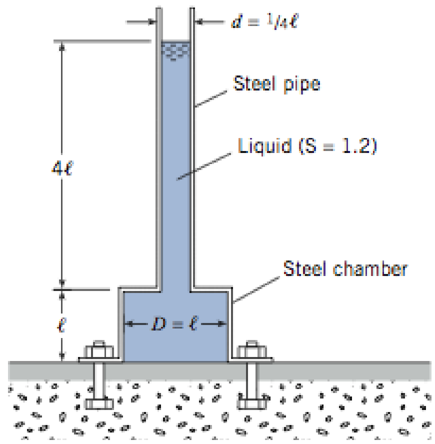
2. Hydrostatic force

$$
\begin{aligned}
F_{CD} &= pA \\
&= (127,530\,\mathrm{N/\,m^2} - 1\,\mathrm{m} \times 3 \times 9810\,\mathrm{N/\,m^3}) \times 1\ \mathrm{m}^2 \\
&\boxed{F_{CD} = 98.1\ \mathrm{kN}}
\end{aligned}
$$

3.27: PROBLEM DEFINITION

Situation:

A steel pipe is connected to a steel chamber.
$\ell = 4\,\text{ft}$, $W = 700\,\text{lbf}$.
$D_1 = 0.25\ell$, $z_1 = 4\ell$.
$D_2 = \ell$, $SG = 1.2$.



Find:

Force exerted on chamber by bolts (lbf).

Properties:

$\gamma_{\text{water}} = 62.4\,\text{lbf/ ft}^3$.

PLAN

Apply equilibrium and the hydrostatic equation.

SOLUTION

1. Equilibrium. (system is the steel structure plus the liquid within)

$$(\text{Weight of the steel}) = \begin{array}{l}(\text{Force exerted by bolts}) + (\text{Weight of the liquid}) + \\ (\text{Pressure force acting on the bottom of the free body })\end{array}$$

$$F_B + W_{\text{liquid}} + W_s = p_2 A_2 \tag{1}$$

2. Hydrostatic equation (location 1 is on surface; location 2 at the bottom)

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\ 0 + 4\ell &= \frac{p_2}{1.2\gamma_{\text{water}}} + 0 \\ p_2 &= 1.2\gamma_{\text{water}} 4\ell \\ &= 1.2 \times 62.4 \times 4 \times 2.5 \\ &= 1497.6 \text{ psfg} \end{aligned}$$

3. Area

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi \ell^2}{4} = \frac{\pi \times (4\,\mathrm{ft})^2}{4} = 12.57\,\mathrm{ft}^2$$

4. Weight of liquid

$$\begin{aligned} W_{\text{liquid}} &= \left(A_2\ell + \frac{\pi d^2}{4}4\ell\right)\gamma_{\text{liquid}} = \left(A_2\ell + \frac{\pi \ell^3}{16}\right)(1.2)\,\gamma_{\text{water}} \\ &= \left(\left(4.909\,\mathrm{ft}^2\right)(4\,\mathrm{ft}) + \frac{\pi\,(4\,\mathrm{ft})^3}{16}\right)(1.2)\left(62.4\frac{\mathrm{lbf}}{\mathrm{ft}^3}\right) \\ &= 4704.9\,\mathrm{lbf} \end{aligned}$$

5. Substitute numbers into Eq. (1)

$$\begin{aligned} F_B + (4704.9\,\mathrm{lbf}) + (700\,\mathrm{lbf}) &= \left(1497.6\,\mathrm{lbf}/\,\mathrm{ft}^2\right)\left(12.57\,\mathrm{ft}^2\right) \\ F_B &= 13400\,\mathrm{lbf} \end{aligned}$$

$$\boxed{F_B = 13400\ \mathrm{lbf}}$$

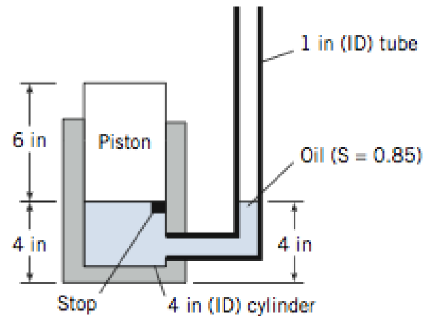
3.28: PROBLEM DEFINITION

Situation:

Oil is added to the tube so the piston rises 1 inch.

$W_{\text{piston}} = 8\,\text{lbf}$, $SG = 0.85$.

$D_p = 4\,\text{in}$, $D_{\text{tube}} = 1\,\text{in}$.

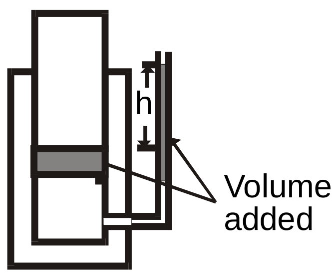


Find:

Volume of oil (in^3) that is added.

SOLUTION

Notice that the oil fills the apparatus as shown below.



Pressure acting on the bottom of the piston

$$\begin{aligned} p_p A_p &= 8 \text{ lbf} \\ p_p &= \frac{8\,\text{lbf}}{A_p} = \frac{8\,\text{lbf}}{\pi\,(4\,\text{in})^2/4} \\ &= 0.6366 \text{ psig } = 91.67 \text{ psfg} \end{aligned}$$

Hydrostatic equation (apply to liquid in the tube)

$$\begin{aligned} \gamma_{\text{oil}} h &= 91.67 \text{ psfg} \\ h &= 91.67/(62.4 \times 0.85) = 1.728 \text{ ft} = 20.74 \text{ in} \end{aligned}$$

Calculate volume

$$\begin{aligned} V\!\!\!/_{\text{added}} &= V\!\!\!/_{\text{left}} + V\!\!\!/_{\text{right}} \\ &= \frac{\pi\,(4\,\text{in})^2\,(1\,\text{in})}{4} + \frac{\pi\,(1\,\text{in})^2\,(1\,\text{in} + 20.74\,\text{in})}{4} \end{aligned}$$

$$\boxed{V\!\!\!/_{\text{added}} = 29.6 \text{ in.}^3}$$

3.29: PROBLEM DEFINITION

Situation:

An air bubble rises from the bottom of a lake.

$z_{34} = 34\,\text{ft}$, $z_8 = 8\,\text{ft}$.

Find:

Ratio of the density of air within the bubble at different depths.

Assumptions:

Air is ideal gas.

Temperature is constant.

Neglect surface tension effects.

Properties:

$\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

PLAN

Apply the hydrostatic equation and the ideal gas law.

SOLUTION

Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
\rho_{34} &= \frac{p_{34}}{RT};\ \rho_8 = \frac{p_8}{RT} \\
\frac{\rho_{34}}{\rho_8} &= \frac{p_{34}}{p_8}
\end{aligned}$$

where p is absolute pressure (required in ideal gas law).

Hydrostatic equation

$$\begin{gathered}
p_8 = p_{\text{atm}} + \gamma\,(8\,\text{ft}) \\
= 2120\,\text{lbf}/\,\text{ft}^2 + \ \left(62.4 \text{ lbf/ft}^3\right)(8 \text{ ft}) \\
= 2619 \text{ lbf/ft}^2
\end{gathered}$$

$$\begin{gathered}
p_{34} = p_{\text{atm}} + \gamma\,(34\,\text{ft}) \\
= 2120\,\text{lbf}/\,\text{ft}^2 + \ \left(62.4 \text{ lbf/ft}^3\right)(34 \text{ ft}) \\
= 4241.6 \text{ lbf/ft}^2
\end{gathered}$$

Density ratio

$$\begin{aligned}
\frac{\rho_{34}}{\rho_8} &= \frac{4241.6 \text{ lbf/ft}^2}{2619 \text{ lbf/ft}^2} \\
&= 1.620
\end{aligned}$$

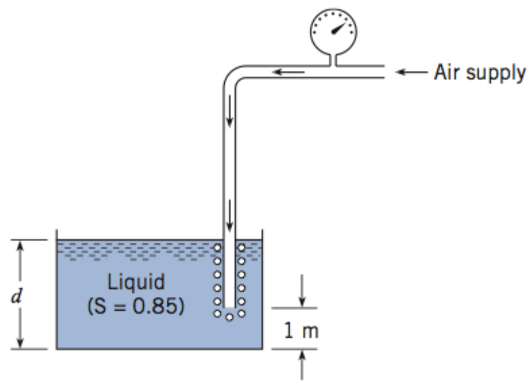
$$\boxed{\rho_{34}/\rho_8 = 1.62}$$

3.30: PROBLEM DEFINITION

Situation:

Air is injected into a tank of liquid.

Pressure reading the Bourdon tube gage is $p_{\text{gage}} = 15\,\text{kPa}$



Find:

Depth d of liquid in tank (m).

Assumptions:

Neglect the change of pressure due to the column of air in the tube.

Properties: $\gamma\,(\text{water}) = 9810\,\text{N/}\,\text{m}^3$, $S = 0.85$.

PLAN

1. Find the depth corresponding to $p = 15\,\text{kPa}$ using the hydrostatic equation.
2. Find d by adding 1.0 m to value from step 1.

SOLUTION

1. Hydrostatic equation

$$\begin{aligned}\Delta p &= \gamma_{\text{liquid}}\Delta z \\ \Delta z &= \frac{\Delta p}{\gamma_{\text{liquid}}} = \frac{15000\,\text{Pa}}{0.85\,(9810\,\text{N/}\,\text{m}^3)} = 1.80\,\text{m}\end{aligned}$$

2. Depth of tank

$$\begin{aligned}d &= \Delta z + 1\,\text{m} \\ &= 1.80\,\text{m} + 1\,\text{m} \\ &= 2.8\,\text{m}\end{aligned}$$

$$\boxed{d = 2.80\ \text{m}}$$

3.31: PROBLEM DEFINITION

Situation:

Match the following pressure-measuring devices with the correct name. The device names are: barometer, Bourdon gage, piezometer, manometer, and pressure transducer.

a. A U-shaped tube where changes in pressure cause changes in relative elevation of a liquid that is usually denser than the fluid in the system measured; can be used to measure vacuum.

b. Typically contains a diaphragm, a strain gage, and conversion to an electric signal.

c. A round face with a scale to measure needle deflection, where the needle is deflected by changes in extension of a coiled hollow tube.

d. A vertical tube where a liquid rises in response to a positive gage pressure.

e. An instrument used to measure atmospheric pressure; of various designs.

SOLUTION

a. manometer

b. pressure transducer

c. Bourdon gage

d. piezometer

e. barometer

3.32: PROBLEM DEFINITION

Situation:
Which is the more correct way to describe the two summation ($\sum$) terms of the manometer equation, Eqn. 3.21, in §3.3 of EFM11e?

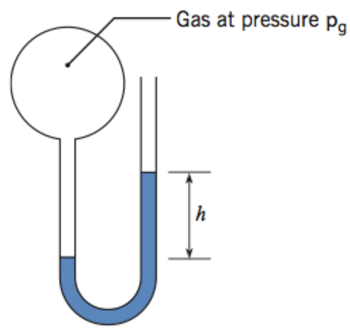
a. Add the downs and subtract the ups.
b. Subtract the downs and add the ups.

SOLUTION

The correct answer is a.
As you go down in a given fluid, the pressure gets larger; as you move your point of inquiry up higher in elevation, the pressure gets smaller.

Problem 3.33

Apply the grid method to a U-tube manometer.



The working equation (i.e. the hydrostatic equation) is:

$$p_{\text{gas}} = \gamma_{\text{liquid}} h$$

a.)

Situation:

Water in a manometer.

$h = 2.3\,\text{ft}$.

Find:

Absolute pressure (psig).

Properties:

$SG = 1.4$, $\gamma_{\text{H2O}} = 62.4\,\text{lbf}/\,\text{ft}^3$.

Solution:

First, find the gage pressure in the gas:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = SG \times \gamma_{\text{H2O}} \times h \\ &= (1.4)\left(\frac{62.4\,\text{lbf}}{\text{ft}^3}\right)\left(\frac{2.3\,\text{ft}}{1.0}\right)\left(\frac{1\,\text{ft}}{12\,\text{in.}}\right)^2 = 1.395 \text{ psig} \end{aligned}$$

Now, find the absolute pressure:

$$\begin{aligned} p_{\text{abs}} &= p_{\text{atm}} + p_{\text{gage}} \\ &= 14.7 \text{ psi } + 1.395 \text{ psi } ; \quad \boxed{p_{\text{abs}} = 16.1 \text{ psia}} \end{aligned}$$

b.)

Situation:

Mercury in a manometer.

Find:

Column rise (mm).

Properties:
Table A.4, $\gamma = 133000\,\mathrm{N/\,m^3}$.
$p_{gas} = 0.5\,\mathrm{atm}$, $p_{atm} = 101.3\,\mathrm{kPa}$.

Solution:
b. Find column rise in mm. The manometer uses mercury. The gas pressure is .25 atm.

$$h = \frac{p_{\text{gas}}}{\gamma_{\text{liquid}}} = \left(\frac{0.5\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{133000\,\text{N}}\right)\left(\frac{101.3\times10^3\,\text{N}}{1\,\text{atm}\cdot\,\text{m}^2}\right) = 0.381\,\text{m}$$

$$\boxed{h = 381\,\text{mm}}$$

c.)

Situation:
Liquid in manometer.
$h = 6\,\mathrm{in}$.

Find:
Pressure (psfg).

Properties:
$\rho = 22\ \mathrm{lbm/\,ft^3}$.

Solution:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = \rho g h \\ &= \left(\frac{22\ \text{lbm}}{\text{ft}^3}\right)\left(\frac{32.2\,\text{ft}}{\text{s}^2}\right)\left(\frac{6\,\text{in}}{1.0}\right)\left(\frac{\text{lbf}\cdot\,\text{s}^2}{32.2\ \text{lbm}\cdot\ \text{ft}}\right)\left(\frac{1.0\,\text{ft}}{12\,\text{in}}\right) \\ &\boxed{p_{\text{gas}} = 11\ \text{psfg}} \end{aligned}$$

d.)

Situation:
Liquid in manometer.
$h = 2.3\,\mathrm{m}$.

Find:
Gage pressure (bar).

Properties:
$\rho = 800\,\mathrm{kg/\,m^3}$.

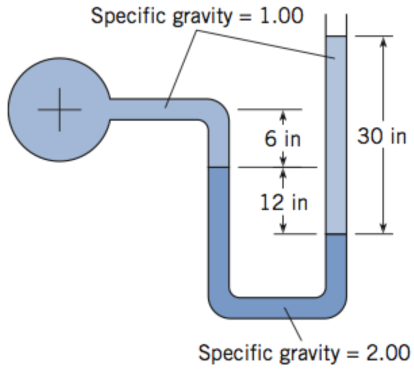
Solution:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = \rho g h \\ &= \left(\frac{800\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{2.3\,\text{m}}{1.0}\right)\left(\frac{\text{Pa}\cdot\ \text{m}\cdot\ \text{s}^2}{\text{kg}}\right)\left(\frac{1\,\text{bar}}{10^5\,\text{Pa}}\right) \\ &= \boxed{0.181\,\text{bar}} \end{aligned}$$

3.34: PROBLEM DEFINITION

Situation:

A manometer is connected to a pipe which is going in and out of the page; pipe center is at the "+" symbol.



Find:

Determine if the gage pressure at the center of the pipe is:
(a) negative
(b) positive
(c) zero

PLAN

Apply the manometer equation and justify the solution using calculations.

SOLUTION

Manometer equation. (add up pressures from the pipe center to the open end of the manometer)

$$p_{\text{pipe}} + (0.5 \text{ ft})(62.4 \text{ lbf/ft}^3) + (1 \text{ ft})(2 \times 62.4 \text{ lbf/ft}^3) - (2.5 \text{ ft})(62.4 \text{ lbf/ft}^3) = 0 \quad (1)$$

Solve Eq. (1) for the pressure in the pipe

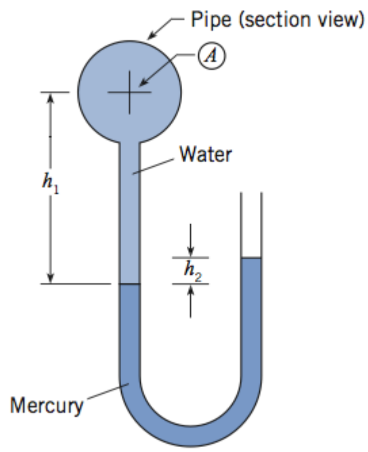
$$p_{\text{pipe}} = (-0.5 - 2 + 2.5) \text{ ft } (62.4 \text{ lbf/ft}^3) = 0$$

$$\boxed{p\,(\text{center of pipe}) = 0.0\,\text{lbf/ft}^2}$$

3.35: PROBLEM DEFINITION

Situation:

A manometer is connected to a pipe.
$h_1 = 16\,\text{in}$, $h_2 = 2\,\text{in}$.



Find:

Gage pressure at the center of the pipe in units of psig.

Properties:

Mercury (68 °F), Table A.4, $\gamma_{\text{Hg}} = 847\,\text{lbf/ft}^3$.
Water (70 °F), Table A.5, $\gamma_{\text{H2O}} = 62.3\,\text{lbf/ft}^3$.

PLAN

Find pressure (p_A) by applying the manometer equation from point A to the top of the mercury column.

SOLUTION

Manometer equation:

$$p_A + \left(\frac{16}{12}\,\text{ft}\right)(62.3\ \text{lbf/ft}^3) - \left(\frac{2}{12}\,\text{ft}\right)(847\ \text{lbf/ft}^3) = 0$$

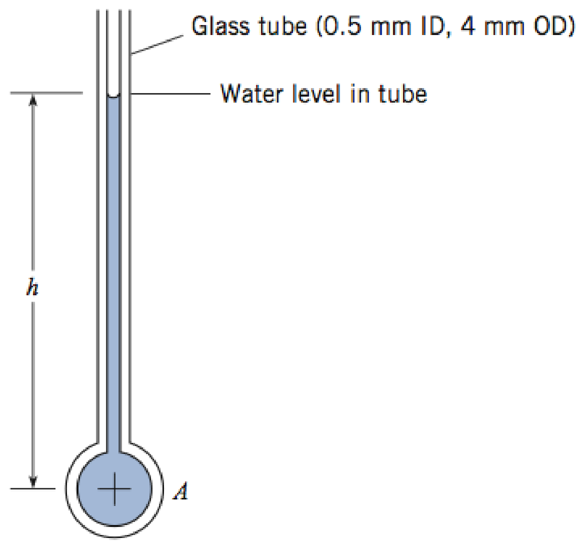
$$p_A = \left(\frac{58.1\,\text{lbf}}{\text{ft}^2}\right)\left(\frac{1.0\,\text{ft}}{12\,\text{in}}\right)^2$$

$$\boxed{p_A = 0.403\ \text{psig}}$$

3.36: PROBLEM DEFINITION

Situation:

A glass tube ($d = 0.5\,\text{mm}$) is connected to a pipe containing water.
Column rise ($h = 120\,\text{mm}$) is due to pressure and surface tension.



Find:

Gage pressure at the center of the pipe (Pa-gage).

Assumptions:

The contact angle is small so $\cos\theta \approx 1$ in the capillary rise equation.

Properties:

Water (20 °C), Table A-5: $\gamma = 9790\,\text{N/m}^3, \sigma = 0.073\,\text{N/m}$.

PLAN

1. Find the column rise due to surface tension by applying the capillary rise equation.
2. Know that the total column rise observed is due to the sum of the hydrostatic effect plus the capillary rise effect.
3. Subtract capillary head from total observed head to get the head associated with the hydrostatic pressure.
4. Calculate pressure from the component of head that was due only to hydrostatics.

SOLUTION

1. Capillary rise equation (from chapter 2):

$$\Delta h_1 = \frac{4\sigma}{\gamma d} \tag{1}$$

2a. Hydrostatic equation.

$$\Delta h_2 = \frac{p_A}{\gamma} \tag{2}$$

2b. Total column rise:

$$\Delta h = \Delta h_1 + \Delta h_2 = \frac{4\sigma}{\gamma d} + \frac{p_A}{\gamma} \tag{3}$$

3. Subtraction:

$$\Delta h_1 = \frac{4\sigma}{\gamma d} = 4\left(\frac{0.073\,\mathrm{N}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}^3}{9790\,\mathrm{N}}\right)\left(\frac{1.0}{0.5 \times 10^{-3}\,\mathrm{m}}\right) = 0.05965\,\mathrm{m}$$

4. Calculate pressure.

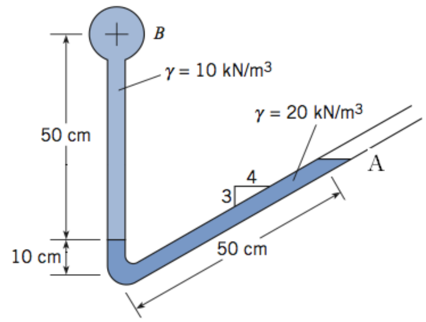
$$p_A = (\Delta h - \Delta h_1)\,\gamma = (0.120\,\mathrm{m} - 0.05965\,\mathrm{m})\left(9790\,\mathrm{N/\,m^3}\right) = 590.8\,\mathrm{Pa}$$

$$\boxed{p_A = 591\,\mathrm{Pa\ gage}}$$

3.37: PROBLEM DEFINITION

Situation:

A tube (manometer) is connected to a pipe.



Find:

Pressure at the center of pipe B in units of kPa gage.

Properties:

$\gamma_1 = 10\,\text{kN/m}^3$, $\gamma_2 = 20\,\text{kN/m}^3$.

PLAN

Apply the manometer equation from point A (open leg of manometer, at the right side) to point B (center of pipe)

SOLUTION

Manometer equation

$$p_B = p_A + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$$

were h_i denotes the vertical deflection in the i$^{\text{th}}$ section of the manometer

$$\begin{aligned} p_B &= (0\,\text{Pa}) \\ &\quad + \left(0.30\,\text{m} \times 20,000\,\text{N/m}^3\right) \\ &\quad - \left(0.1\,\text{m} \times 20,000\,\text{N/m}^3\right) \\ &\quad - \left(0.5\,\text{m} \times 10,000\,\text{N/m}^3\right) \\ &= -1000\,\text{Pa} \end{aligned}$$

$$\boxed{p_B = -1.00 \text{ kPa gage}}$$

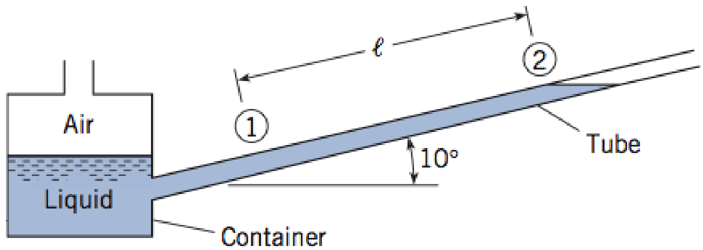
REVIEW

Tip! Note that a manometer that is open to atmosphere will read gage pressure.

3.38: PROBLEM DEFINITION

Situation:

State 1: air at p_{atm}, liquid in tube at elevation 1.
State 2: air is pressurized; liquid at elevation 2.
$\ell = 0.4\,\text{m}$, $D_{\text{container}} = 8D_{\text{tube}}$.



Find:

Pressure in the air within the container (Pa).

Properties:

Liquid, $\rho = 1200\,\text{kg/m}^3$.

PLAN

1. Find the decrease in liquid level in the container by applying conservation of mass.
2. Find the air pressure by applying the hydrostatic equation.

SOLUTION

1. Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
V\!\!\!/_{\text{tube}} &= V\!\!\!/_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= (1/8)^2 \times 40 \\
&= 0.625 \text{ cm}
\end{aligned}$$

2. Hydrostatic equation

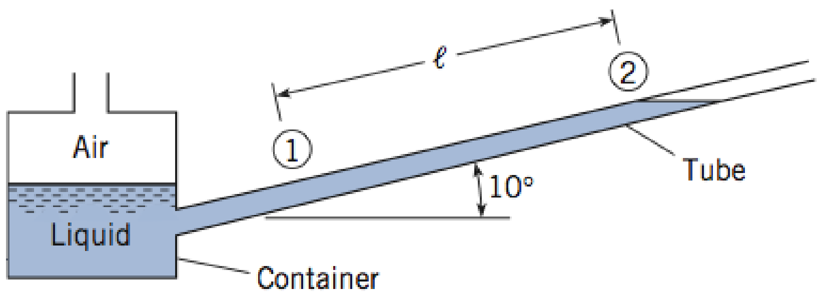
$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\rho g \\
&= [(0.4\,\text{m})\sin 10^o + 0.00625\,\text{m}]\left(1200\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)
\end{aligned}$$

$$\boxed{p_{\text{container}} = 891\,\text{Pa gage}}$$

3.39: PROBLEM DEFINITION

Situation:

State 1: air at p_{atm}, liquid in tube at elevation 1.
State 2: air is pressurized; liquid at elevation 2.
$D_{\text{container}} = 10D_{\text{tube}}$, $\ell = 3\,\text{ft}$.



Find:

Pressure in the air within the container (psfg).

Properties:

liquid, $\gamma = 50\,\text{lbf/ft}^3$.

PLAN

1. Find the decrease in liquid level in the container by using conservation of mass.
2. Find the pressure in the container by apply the manometer equation.

SOLUTION

1. Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
\mathcal{V}_{\text{tube}} &= \mathcal{V}_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= \left(\frac{1}{10}\right)^2 \times 3\,\text{ft} \\
&= 0.03\,\text{ft}
\end{aligned}$$

2. Manometer equation (point 1 = free surface of liquid in the tube; point 2 = free surface of liquid in the container)

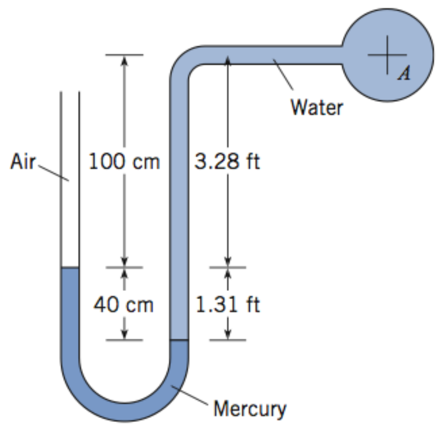
$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\gamma \\
&= (3 \sin 10^\circ + .03)\,\text{ft} \times 50\,\text{lbf/ft}^3 \\
&= 27.548\,\text{lbf/ft}^2
\end{aligned}$$

$$\boxed{p_{\text{container}} = 27.5 \text{ psfg}}$$

3.40: PROBLEM DEFINITION

Situation:

A pipe system has a manometer attached to it.



Find:

Gage pressure at center of pipe A (psi, kPa).

Properties:

Mercury, Table A.4: $\gamma = 1.33 \times 10^5\,\mathrm{N/m^3}$.
Water, Table A.5: $\gamma = 9810\,\mathrm{N/m^3}$.

PLAN

Apply the manometer equation.

SOLUTION

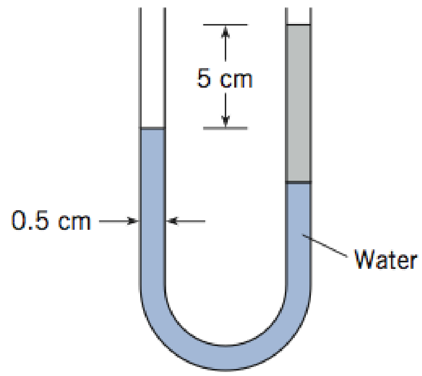
Manometer equation

$$
\begin{aligned}
p_A &= 1.31\,\mathrm{ft} \times 847\,\mathrm{lbf/ft^3} - 4.59\,\mathrm{ft} \times 62.4\,\mathrm{lbf/ft^3} \\
&= 823.2 \text{ psf} \\
& \boxed{p_A = 5.72 \text{ psig}} \\
p_A &= 0.4\,\mathrm{m} \times 1.33 \times 10^5\,\mathrm{N/m^3} - 1.4\,\mathrm{m} \times 9810\,\mathrm{N/m^3} \\
& \boxed{p_A = 39.5 \text{ kPa gage}}
\end{aligned}
$$

3.41: PROBLEM DEFINITION

Situation:

A U-tube manometer can be used to measure γ.
Initial state: A U-tube manometer contains water.
Final state: An unknown liquid ($\not{V} = 2\,\text{cm}^3$) is added to the right leg
$d = 0.5\,\text{cm}$, $\Delta h = 5\,\text{cm}$.



Find:

Specific weight of unknown fluid (N/m^3).

SOLUTION

1. Find the length of the column of the unknown liquid.

$$\not{V} = (\pi/4)(0.5\,\text{cm})^2\ell = 2\,\text{cm}^3$$

Solve for ℓ

$$\ell = 10.186 \text{ cm}$$

2. Manometer equation (from water surface in left leg to liquid surface in right leg)

$$0 + (10.186 \text{ cm - } 5 \text{ cm})(10^{-2} \text{ m/cm})(9810 \text{ N/m}^3)$$
$$-(10.186 \text{ cm})(10^{-2} \text{ m/cm})\gamma_{\text{liq.}} = 0$$

Solve for $\gamma_{\text{liq.}}$

$$508.7 \text{ Pa } - 0.10186\gamma_{\text{liq.}} = 0$$

$$\boxed{\gamma_{\text{liq.}} = 4995 \text{ N/m}^3}$$

3.42: PROBLEM DEFINITION

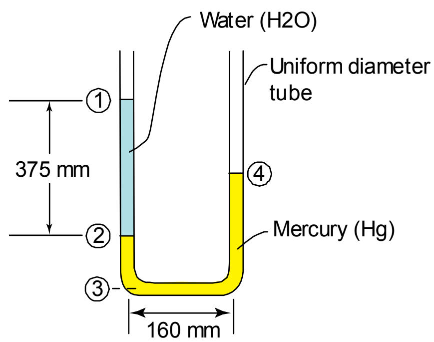
Situation:

Mercury and water are poured into a tube.
$\ell_{\text{mercury}} = \ell_{\text{water}} = 375\,\text{mm}$.

Find:

Locate the water surface (mm).
Locate the mercury surface (mm).
Find the maximum pressure in the U-tube (kPa gage).

Sketch:



Assumptions:

Uniform diameter tube.

Properties:

Mercury (20 °C), Table A.4, $\gamma_{Hg} = 133000\,\text{N/m}^3$.
Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Find p_2 by applying the hydrostatic equation.
2. Find $(z_4 - z_2)$ by applying the hydrostatic equation.
3. Solve $(z_2 - z_3)$ by using the fact that the mercury column has a fixed length.
4. Locate the liquid surfaces by using lengths from steps 2 and 3.
5. Solve for the maximum pressure by applying the hydrostatic equation to the mercury.

SOLUTION

1. Hydrostatic equation (apply to water column):

$$\begin{aligned}
\frac{p_1}{\gamma_{H2O}} + z_1 &= \frac{p_2}{\gamma_{H2O}} + z_2 \\
0 + z_1 &= \frac{p_2}{9710\,\text{N/m}^3} + z_2 \\
p_2 &= \left(9710\,\text{N/m}^3\right)(z_1 - z_2) = \left(9710\,\text{N/m}^3\right)(0.375\,\text{m}) = 3641\,\text{N/m}^2
\end{aligned}$$

Since the pressure across the water/mercury interface is constant, $p_{2,\,\text{H2O}} = p_{2,\,\text{Hg}}$.

2. Hydrostatic equation (apply to Hg column):

$$\begin{aligned}\frac{p_4}{\gamma_{\mathrm{Hg}}}+z_4 &= \frac{p_2}{\gamma_{\mathrm{Hg}}}+z_2 \\ 0+z_4 &= \frac{3641\,\mathrm{N/\,m^2}}{133000\,\mathrm{N/\,m^3}}+z_2 \\ (z_4-z_2) &= 27.38\,\mathrm{m}\end{aligned}$$

3. Length constraint (length of Hg column is 375 mm):

$$(z_2-z_3)+160\,\mathrm{mm}+(z_2-z_3)+27.38\,\mathrm{mm}=375\,\mathrm{mm}$$
$$(z_2-z_3)=93.18\,\mathrm{mm}$$

4. Locate surfaces:

Water: $(z_1-z_2)+(z_2-z_3)=375\,\mathrm{mm}+93.18\,\mathrm{mm}=468\,\mathrm{mm}$

The surface of the water is located 468 mm above the centerline of the horizontal leg

Mercury: $(z_4-z_2)+(z_2-z_3)=27.38\,\mathrm{mm}+93.18\,\mathrm{mm}=121\,\mathrm{mm}$

The surface of the mercury is located 121 mm above the centerline of the horizontal leg

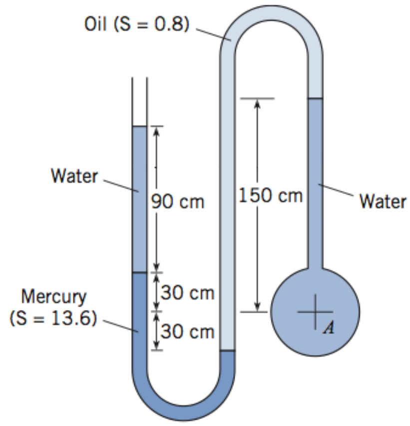
5. Hydrostatic Equation:

$$\begin{aligned}\frac{p_3}{\gamma_{\mathrm{Hg}}}+z_3 &= \frac{p_4}{\gamma_{\mathrm{Hg}}}+z_4 \\ p_3 &= \frac{p_4}{\gamma_{\mathrm{Hg}}}+\gamma_{\mathrm{Hg}}(z_4-z_3)=0+\left(133000\,\mathrm{N/\,m^3}\right)(0.121\,\mathrm{m}) \\ & \boxed{p_3=p_{\max}=16.1\,\mathrm{kPa\ gage}}\end{aligned}$$

3.43: PROBLEM DEFINITION

Situation:

A manometer is used to measure pressure at the center of a pipe.



Find:

Pressure at center of pipe A (kPa).

Properties:

Water (10 °C, 1 atm), Table A.5, $\gamma_{\text{water}} = 9810\,\text{N/m}^3$.
Mercury (20 °C, 1 atm), Table A.4, $\gamma_{\text{Hg}} = 133000\,\text{N/m}^3$
(assume this value applies at 10 °C).
Oil. Since $S = 0.8$, $\gamma_{\text{oil}} = (0.8)(9810\,\text{N/m}^3) = 7848\,\text{N/m}^3$.

PLAN

Since the manometer is applied to measure pressure at point A, select Eq. (3.21) of EFM 10e in Table 3.2 (EFM 10e).

$$p_1 = p_2 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$$

Let point B be situated at the top of the water column. Then, let A = 1 and B =2

$$p_A = p_B + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$$

Substitute terms

$$p_A = p_B + \gamma_{\text{water}} h_{\text{water1}} + \gamma_{\text{Hg}} h_{\text{Hg}} - \gamma_{\text{oil}} h_{\text{oil}} + \gamma_{\text{water}} h_{\text{water2}}$$

Since all terms on the right side of this equation are know, calculations may be done.

SOLUTION

$$
\begin{aligned}
p_A &= 0 + \left(9810\,\mathrm{N/\,m^3}\right)(0.9\,\mathrm{m}) + \left(133000\,\mathrm{N/\,m^3}\right)(0.6\,\mathrm{m}) \\
&\quad - \left(7848\,\mathrm{N/\,m^3}\right)(1.8\,\mathrm{m}) + \left(9810\,\mathrm{N/\,m^3}\right)(1.5\,\mathrm{m}) \\
&= 89.2\,\mathrm{kPa}
\end{aligned}
$$

$$
\boxed{p_A = 89.2\,\mathrm{kPa\ gage}}
$$

REVIEW

1. To validate, look at left most leg of manometer. The pressure in the pipe is holding up 600 mm of Hg and 0.9 m of water. Since 1 atm = 760 mm Hg = 10.33 ft of water, we can estimate the pressure as $600/760 + 1/10 \approx 0.9\,\mathrm{atm}$.If we assume that effects of the oil column (center leg) and water (right most leg) cancel, we expect the pressure at A to be about 0.9 atm (which it is).

2. An alternative approach is to use specific gravity as shown below. This approach makes calculations easier.

$$
\begin{aligned}
p_A &= p_B + \gamma_{\text{water}} h_{\text{water1}} + \gamma_{\text{Hg}} h_{\text{Hg}} - \gamma_{\text{oil}} h_{\text{oil}} + \gamma_{\text{water}} h_{\text{water2}} \\
&= p_B + \gamma_{\text{water}} h_{\text{water1}} + S_{\text{Hg}} \gamma_{\text{water}} h_{\text{Hg}} - S_{\text{oil}} \gamma_{\text{water}} h_{\text{oil}} + \gamma_{\text{water}} h_{\text{water2}} \\
&= p_B + \gamma_{\text{water}} \left(h_{\text{water1}} + S_{\text{Hg}} h_{\text{Hg}} - S_{\text{oil}} h_{\text{oil}} + h_{\text{water2}}\right) \\
&= 0 + \left(9810\,\mathrm{N/\,m^3}\right)(0.9 + 13.6 \times 0.6 - 0.8 \times 1.8 + 1.5)\ \mathrm{m} = 89.5\,\mathrm{kPa}
\end{aligned}
$$

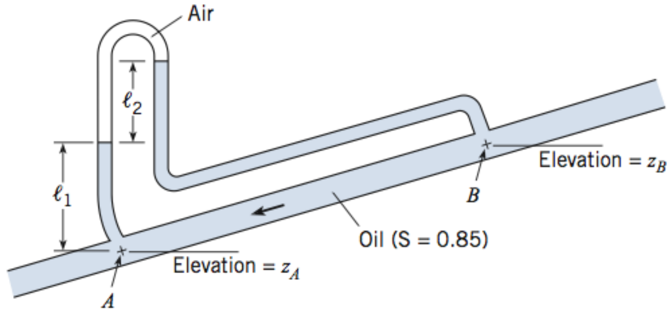
3.44: PROBLEM DEFINITION

Situation:

A system is described in the problem statement.

$\ell_1 = 1\,\text{m}$, $\ell_2 = 0.5\,\text{m}$.

$z_A = 10\,\text{m}$, $z_B = 11\,\text{m}$.



Find:

(a) Difference in pressure between points A and B (kPa).

(b) Difference in piezometric head between points A and B (m).

Properties:

$\gamma = 9810\,\text{N/m}^3$, $S = 0.85$.

PLAN

Apply the manometer equation.

SOLUTION

Manometer equation (apply from A to B)

$$\begin{aligned} p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) &= p_B \\ p_A - p_B &= 4169\,\text{Pa} \end{aligned}$$

$$\boxed{p_A - p_B = 4.17 \text{ kPa}}$$

Piezometric head

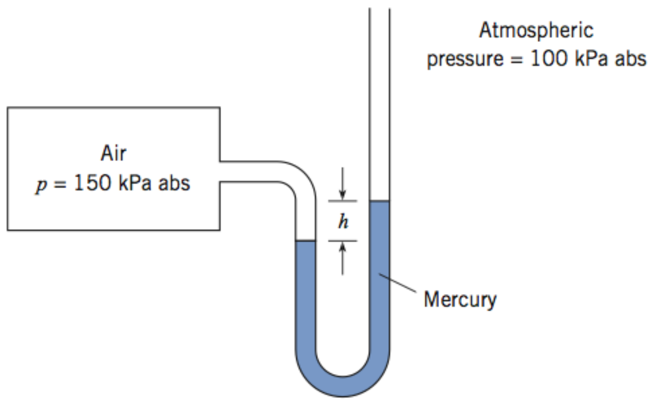
$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5 \text{ m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50 \text{ m}}$$

3.45: PROBLEM DEFINITION

Situation:

A manometer attached to a tank.



Find:

Manometer deflection when pressure in tank is doubled.

Properties:

$p_{atm} = 100\,\text{kPa}$, $p = 150\,\text{kPa}$.

SOLUTION

$$p - p_{atm} = \gamma h$$

For 150 kPa absolute pressure and an atmospheric pressure of 100 kPa,

$$\gamma h = 150 - 100 = 50 \text{ kPa}$$

For an absolute pressure of 300 kPa

$$\gamma h_{new} = 300 - 100 = 200 \text{ kPa}$$

Divide equations to eliminate the specific weight

$$\frac{h_{new}}{h} = \frac{200}{50} = 4.0$$

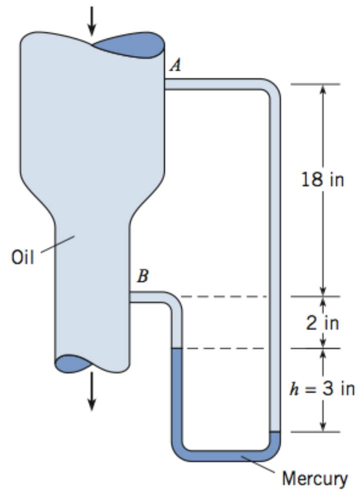
so

$$\boxed{h_{new} = 4.0h}$$

3.46: PROBLEM DEFINITION

Situation:

A manometer is tapped into a vertical conduit.



Find:

Difference in pressure between points A and B (psf).

Difference in piezometric head between points A and B (ft).

Properties:

From Table A.4, $\gamma_{Hg} = 847$ lbf/ft^3.

$$\begin{aligned}\gamma_{\text{oil}} &= (0.95)(62.4 \text{ lbf/ft}^3)\\ &= 59.28 \text{ lbf/ft}^3\end{aligned}$$

SOLUTION

Manometer equation

$$p_A + \left(\tfrac{18}{12}\right) \text{ ft } (\gamma_{\text{oil}}) + \left(\tfrac{2}{12}\right) \text{ ft. } \gamma_{\text{oil}} + \left(\tfrac{3}{12}\right) \text{ ft } \gamma_{\text{oil}} - \left(\tfrac{3}{12}\right) \text{ ft } \gamma_{\text{Hg}} - \left(\tfrac{2}{12}\right) \text{ ft } \gamma_{\text{oil}} = p_B$$

thus

$$p_A - p_B = (-1.75 \text{ ft.})(59.28 \text{ lbf/ft}^3) + (0.25 \text{ ft.})(847 \text{ lbf/ft}^3)$$

$$\boxed{p_A - p_B = 108 \text{ psf}}$$

Piezometric head

$$h_A - h_B = \frac{p_A - p_B}{\gamma_{\text{oil}}} + z_A - z_B$$

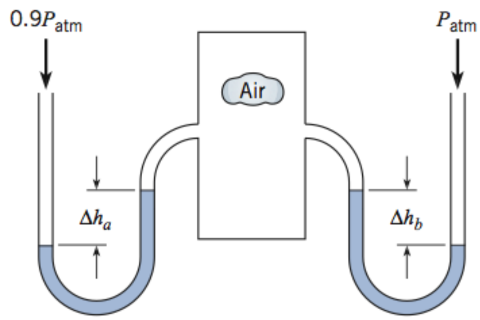
$$h_A - h_B = \frac{108.01 \text{ lbf/ft}^2}{59.28 \text{ lbf/ft}^3} + (1.5 - 0) \text{ ft}$$

$$\boxed{h_A - h_B = 3.32 \text{ ft}}$$

3.47: PROBLEM DEFINITION

Situation:

Two manometers attached to an air tank.



Find:

Difference in deflection between manometers (m).

Properties:

$p_{\text{left}} = 0.9p_{\text{atm}}$, $p_{\text{right}} = p_{\text{atm}} = 100\,\text{kPa}$.
$\gamma_w = 9810\,\text{N/m}^3$.

SOLUTION

The pressure in the tank using manometer b is

$$p_t = p_{atm} - \gamma_w \Delta h_b$$

and using manometer a is

$$p_t = 0.9p_{atm} - \gamma_w \Delta h_a$$

Combine equations

$$p_{atm} - \gamma_w \Delta h_b = 0.9p_{atm} - \gamma_w \Delta h_a$$

or

$$0.1p_{atm} = \gamma_w(\Delta h_b - \Delta h_a)$$

Solve for the difference in deflection

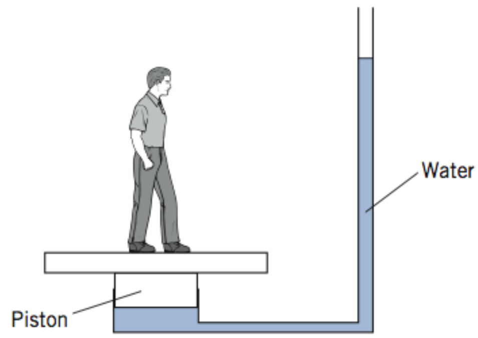
$$\begin{aligned} \Delta h_b - \Delta h_a &= \frac{0.1p_{atm}}{\gamma_w} \\ &= \frac{0.1 \times 10^5\,\text{Pa}}{9.81 \times 10^3\,\text{N/m}^3} \end{aligned}$$

$$\boxed{\Delta h_b - \Delta h_a = 1.02\text{ m}}$$

3.48: PROBLEM DEFINITION

Situation:

A piston scale is used to measure weight.
Weight range: $60 = 250\,\text{lbf}$.
Height range: $4 - 6\,\text{ft}$ tall.



Find:

Select a piston size and standpipe diameter.

SOLUTION

First of all neglect the weight of the piston and find the piston area which will give reasonable manometer deflections. Equating the force on the piston, the piston area and the deflection of the manometer gives

$$W = \Delta h \gamma A$$

where γ is the specific weight of the water. Thus, solving for the area one has

$$A = \frac{W}{\gamma \Delta h}$$

For a four foot person weighing 60 lbf, the area for a 4 foot deflection (manometer near eye level of person) would be

$$A = \frac{60\,\text{lbf}}{62.4\,\text{lbf}/\,\text{ft}^3 \times 4\,\text{ft}^2} = 0.24\ \text{ft}^2$$

while for a 250 lbf person 6 feet tall would be

$$A = \frac{250\,\text{lbf}}{62.4\,\text{lbf}/\,\text{ft}^3 \times 6\,\text{ft}} = 0.66\ \text{ft}^2$$

It will not be possible to maintain the manometer at the eye level for each person so take a piston area of 0.5 ft^2. This would give a deflection of 1.92 ft for the 4-foot, 60 lbf person and 8 ft for the 6-foot, 250 lbf person. This is a good compromise.

The size of the standpipe does not affect the pressure. The pipe should be big enough so the person can easily see the water level and be able to read the calibration on the scale. A 1/2 inch diameter tube would probably suffice. Thus the ratio of the standpipe area to the piston area would be

$$\frac{A_{\text{pipe}}}{A_{\text{piston}}} = \frac{\frac{\pi}{4} \times (0.5\,\text{in})^2}{0.5\,\text{ft}^2 \times 144\,\text{in}^2/\,\text{ft}^2} = 0.0027$$

This means that when the water level rises to 8 ft, the piston will only have moved by $0.0027 \times 8 = 0.0216$ ft or 0.26 inches.
The weight of the piston will cause an initial deflection of the manometer. If the piston weight is 5 lbf or less, the initial deflection of the manometer would be

$$\Delta h_o = \frac{W_{\text{piston}}}{\gamma A_{\text{piston}}} = 0.16 \text{ ft or } 1.92 \text{ inches}$$

This will not significantly affect the range of the manometer (between 2 and 8 feet). The system would be calibrated by putting knows weights on the scale and marking the position on the standpipe. The scale would be linear.

Problem 3.49

Using Section 3.4 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication. There are many possible good answers to these questions. Here, we give some examples.

---

a. For hydrostatic conditions, what do typical pressure distributions on a panel look like? Sketch three examples that correspond to different situations.

- Arrows (which represent normal stress) are compressive.
- Arrows are normal to the panel.
- Pressure varies linearly with elevation.
- Slope of pressure with respect to elevation (dp/dz) equal the negative of specific weight ($dp/dz = -\gamma$).

---

b. What is a center of pressure? What is a centroid of area?

- The center of pressure is an imaginary point. If pressure distribution is replaced with a statically equivalent "point force," then this resultant force acts at the "center of pressure."
- The centroid of area is the "geometric center." For a flat plate, the centroid of area is at the same location as the center of gravity for a thin uniform-density plate of that shape,

---

c. In Eq. (3.23), what does $\overline{p}$ mean? What factors influence the value of $\overline{p}$?

- P-bar ($\overline{p}$)is the pressure evaluated at the elevation of the centroid of area.
- Typically $\overline{p} = \gamma\overline{z}$. Since this equation has two variable, there are two factors that influence the value of $\overline{p}$:
  - The specific weight of the liquid.
  - The vertical distance $\overline{z}$ from liquid surface to the centroid of the panel.

---

d. What is the relationship between the pressure distribution on a panel and the resultant force?

$$\left|\overrightarrow{F}\right| = \int_{\text{panel area}} p dA$$

---

e. How far is the center of pressure from the centroid of area? What factors influence this distance?

- Distance is given by $\overline{I}/(\overline{y}A)$. Thus
  - The shape of the panel determines $\overline{I}$.
  - The depth of liquid and the angle of the panel determine $\overline{y}$.
  - The size of the panel determines A.

3.50: PROBLEM DEFINITION

Situation:

Part 1. Consider the equation for the distance between the CP and the centroid of a submerged panel (Eq. 3.33 in §3.4, EFM11e). In that equation, y_{cp} is

a. the vertical distance from the water surface to the CP.

b. the slant distance from the water surface to the CP.

Part 2. Next, consider the figure shown. For case 1 as shown, the viewing window on the front of a submersible exploration vehicle is at a depth of y_1. For case 2, the submersible has moved deeper in the ocean, to y_2. As a result of this increased overall depth of the submersible and its window, does the spacing between the CP and centroid

(a) get larger,

(b) stay the same, or

(c) get smaller?

SOLUTION

Part 1. The correct answer is (b), the slant difference from the water surface to the CP.

Part 2. The answer is (c), get smaller. Consider equation 3.33 in §3.4 (10e) for the spacing of the CP and centroid, given by $y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$. On the right-hand side of the equation, the moment of inertia $\bar{I}$ and the area A do not change. Only $\bar{y}$ changes. As both the CP and centroid get deeper, $\bar{y}$ increases, so the space between them decreases.

3.51: PROBLEM DEFINITION

Situation:

Which of these assumptions and/or limitations must be known when using Eq. 3.33 of §3.4 (EFM11e) for a submerged surface or panel to calculate the distance between the centroid of the panel and the center of pressure of the hydrostatic force (select all that apply):

a. The equation only applies to a single fluid of constant density

b. The pressure at the surface must be $p = 0$ gage

c. The panel must be vertical

d. The equation gives only the vertical location (as a slant distance) to the CP, not the lateral distance from the edge of the body

SOLUTION

The correct answers are a, b, and d

3.52: PROBLEM DEFINITION

Situation:

Two cylindrical tanks have bottom areas A and $4A$ respectively, and are filled with water to the depths shown in the problem statement.

Find:

a. Which tank has the higher pressure at the bottom of the tank?

b. Which tank has the greater force acting downward on the bottom circular surface?

PLAN

Use the hydrostatic equation,

$$P = -\gamma \Delta z$$

SOLUTION

Part a) Which tank has the higher pressure at the bottom?

Tank 1

$$P_1 = \gamma h \text{ at the bottom}$$

Tank 2

$$P_2 = \gamma(\frac{1}{2}h) = \frac{1}{2}\gamma h \text{ at the bottom}$$

Solution:

$$\boxed{\text{Tank 1 has the higher pressure at the bottom}}$$

Part b) Which tank has the greater force acting downward at the bottom?

PLAN

$$\text{Use } F = pA$$

SOLUTION

Tank 1

$$F_1 = pA = \gamma hA$$

Tank 2

$$P_2 = pA = \left(\frac{1}{2}\gamma h\right)(4A) = 2\gamma hA$$

Solution:

$$\boxed{\text{Tank 2 has the greater force acting downard on the tank bottom}}$$

3.53: PROBLEM DEFINITION

Situation:

Irrigation ditch and gate
2 ft wide, 2 ft deep, and the ditch is completely full of water
There is no water on the other side of the gate
Hot weather, so the water is 70°F

Find:

F_H acting on the gate

Properties:

From Table A.5 (EFM 10e): $\gamma_{water} = 62.3\,\mathrm{lbf/ft^3}$ at 70°F

PLAN

Use the concept of calculating the magnitude of a force acting on a panel.

SOLUTION

Use Eq. 3.28 in §3.4

$$
\begin{aligned}
F_p &= pA \\
F_p &= \gamma h A \text{ where } h = \text{ depth to centroid} = 1\,\mathrm{ft} \\
F_p &= \left(\frac{62.3\,\mathrm{lbf}}{\mathrm{ft}^3}\right)\left(\frac{1\,\mathrm{ft}}{}\right)\left(\frac{2\,\mathrm{ft}\times 2\,\mathrm{ft}}{}\right) \\
&\boxed{F_p = 249\,\mathrm{lbf}}
\end{aligned}
$$

3.54: PROBLEM DEFINITION

Situation:
Irrigation ditch and gate
2 m wide, 1.5 m deep, and the ditch is competely full of water
There is no water on the other side of the gate
Winter; water is 5 °C

Find:
(a) F_p acting on the gate (lbf)
(b) Distance from y_{cp} to bottom of channel(ft)

Properties:
From Table A.5 (EFM 10e): $\gamma_{water} = 9810\,\mathrm{N/m^3}$ at 5 °C

PLAN

(a) Use $F_p = \overline{p}A$;where $\overline{p} = \gamma\overline{z}$.
All variables are known except F; so these equations can be solved for F.
The steps in detail are as follows:

- Locate depth to the centroid ($\overline{z}$)by inspection (center of the panel).
- Know the pressure at the depth of the centroid comes from the hydrostatic equation.
- Find the resultant force using $F_p = \overline{p}A$.

(b) Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$
All variables are known.
Then, the distance from y_{cp} to bottom of channel is given by $\frac{h}{2} - (y_{cp} - \bar{y})$

SOLUTION

(a) Depth of the centroid of area:

$$\begin{aligned} \overline{z} &= \frac{h}{2} = (1.5\,\mathrm{m})/2 \\ &\overline{z} = 0.75\,\mathrm{m} \end{aligned}$$

Force at the depth of the centroid

$$\begin{aligned} F_p &= \bar{p}A \\ &= (\gamma\Delta z)\,A \\ &= (9810\,\mathrm{N/m^3})(0.75\,\mathrm{m})(2\,\mathrm{m}\times 1.5\,\mathrm{m}) \\ &\boxed{F = 22.1\ \mathrm{kN}} \end{aligned}$$

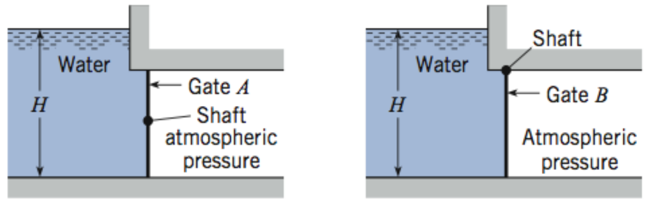
(b) Distance from y_{cp} to bottom of channel; first calculate $y_{cp} - \bar{y}$

$$\begin{aligned}
y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\
&= \frac{\left(\left(2 \times 1.5^3\right)/12\right)\ \mathrm{m}^4}{(0.75\,\mathrm{m})\,(2\,\mathrm{m} \times 1.5\,\mathrm{m})} \\
y_{cp} - \bar{y} &= 0.25\,\mathrm{m} \\
\text{Distance from } y_{cp} \text{ to bottom of channel} &= 0.75\,\mathrm{m} - 0.25\,\mathrm{m} \\
&\boxed{\text{Distance} = 0.50\,\mathrm{m}}
\end{aligned}$$

3.55: PROBLEM DEFINITION

Situation:

Two submerged gates are described in the problem statement.



Find:

Determine which statements are true.

(a) T_A increases with H.

(b) T_B increases with H.

(c) T_A does not change with H.

(d) T_B does not change with H.

PLAN

Apply equilibrium equations. Apply hydrostatic force equations.

SOLUTION

Let the horizontal gate dimension be given as b and the vertical dimension, h.

Torque (Gate A). Equilibrium. Sum moments about the hinge:

$$T_A = F(y_{cp} - \bar{y}) \tag{1}$$

Hydrostatic force equation (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= \gamma\left(H - \frac{h}{2}\right)bh \end{aligned} \tag{2}$$

Hydrostatic force equation (center of pressure)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh} \end{aligned} \tag{3}$$

Combine eqns. 1 to 3:

$$\begin{aligned} T_A &= F(y_{cp} - \bar{y}) \\ &= \left[\gamma\left(H - \frac{h}{2}\right)bh\right]\left[\frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh}\right] \\ &= \gamma\frac{bh^3}{12} \end{aligned} \tag{4}$$

Therefore, T_A does not change with H.

Torque (gate B). Equilibrium. Sum moments about the hinge:

$$T_B = F\left(\frac{h}{2} + y_{cp} - \bar{y}\right) \tag{5}$$

Combine eqns. 2, 3, and 5:

$$\begin{aligned} T_B &= F\left(\frac{h}{2} + y_{cp} - \bar{y}\right) \\ &= \left[\gamma\left(H - \frac{h}{2}\right)bh\right]\left[\frac{h}{2} + \frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh}\right] \\ &= \frac{\gamma h^2 b\,(3H - h)}{6} \end{aligned} \tag{6}$$

Thus, T_A is constant but T_B increases with H.

Case (b) is a correct choice.

Case (c) is a correct choice.

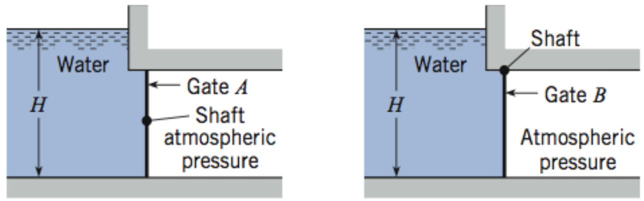
REVIEW

Case A provides an example of how to design a gate so that the torque to hold the gate closed is independent of water depth.

3.56: PROBLEM DEFINITION

Situation:

This problem involves Gate A (see sketch).



Find:

Choose the statements that are valid for Gate A.

(a) The hydrostatic force acting on the gate increases as H increases.

(b) The distance between the CP on the gate and the centroid of the gate decreases as H increases.

(c) The distance between the CP on the gate and the centroid of the gate remains constant as H increases.

(d) The torque applied to the shaft to prevent the gate from turning must be increased as H increases.

(e) The torque applied to the shaft to prevent the gate from turning remains constant as H increases.

SOLUTION

Let the horizontal gate dimension be given as b and the vertical dimension, h.

Torque (Gate A). Sum moments about the hinge:

$$T_A = F(y_{cp} - \bar{y}) \tag{1}$$

Hydrostatic force equation (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= \gamma\left(H - \frac{h}{2}\right)bh \end{aligned} \tag{2}$$

Hydrostatic force equation (center of pressure)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh} \end{aligned} \tag{3}$$

Combine eqns. 1 to 3:

$$\begin{aligned} T_A &= F(y_{cp} - \bar{y}) \\ &= \left[\gamma\left(H - \frac{h}{2}\right)bh\right]\left[\frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh}\right] \\ &= \gamma\frac{bh^3}{12} \end{aligned} \tag{4}$$

Therefore, T_A does not change with H. The correct answers are obtained by reviewing the above solution.

$$\boxed{\text{a, b, and e are valid statements.}}$$

3.57: PROBLEM DEFINITION

Situation:

Water exerts a load on square panel.
$d = 2.3\,\mathrm{m}$, $h = 2\,\mathrm{m}$

Find:

(a) Depth of the centroid (m).
(b) Resultant force on the panel (kN).
(c) Distance from the centroid to the center of pressure (m).

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\mathrm{N/\,m^3}$.

PLAN

1. Locate the centroid by inspection (center of the panel).
2. Find the pressure at the depth of the centroid using the hydrostatic equation.
2. Find the resultant force using $F = \overline{p}A$.
3. Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$

SOLUTION

1. Depth of the centroid of area:

$$\begin{aligned} \overline{z} &= d + h/2 = 2.3\,\mathrm{m} + (2\,\mathrm{m})/2 \\ &\boxed{\overline{z} = 3.3\,\mathrm{m}} \end{aligned}$$

2. Hydrostatic equation:

$$\overline{p} = \gamma\overline{z} = \left(9800\,\mathrm{N/\,m^3}\right)(3.3\,\mathrm{m}) = 32.3\,\mathrm{kPa}$$

3. Resultant force:

$$\begin{aligned} F &= \overline{p}A = (32.3\,\mathrm{kPa})(2\,\mathrm{m})(2\,\mathrm{m}) \\ &\boxed{F = 129\ \mathrm{kN}} \end{aligned}$$

4. Distance between centroid and CP:

- Find $\overline{I}$ using formula from Fig. A.1 (EFM11e).

$$\overline{I} = \frac{bh^3}{12} = \frac{(2\,\mathrm{m})(2\,\mathrm{m})^3}{12} = 1.333\,\mathrm{m}^4$$

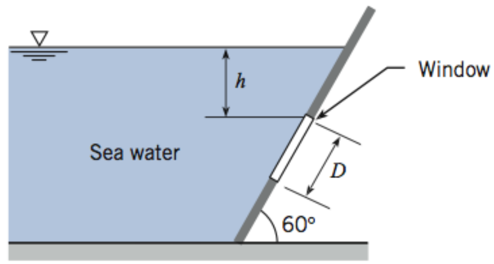
- Recognize that $\bar{y} = \overline{z} = 2\,\mathrm{m}$.
- Final calculation:

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\overline{I}}{\bar{y}A} = \frac{(1.333\,\mathrm{m}^4)}{(3.3\,\mathrm{m})(2\,\mathrm{m})^2} \\ &\boxed{y_{cp} - \bar{y} = 0.101\ \mathrm{m}} \end{aligned}$$

3.58: PROBLEM DEFINITION

Situation:

Seawater exerts a load on a round viewing window.
$h = 2.0\,\mathrm{m}$, $\theta = 60^0$, $D = 0.8\,\mathrm{m}$



Find:

Hydrostatic force on the window (kN).
Locate the CP (center of pressure).

Properties:

Seawater: $SG = 1.03$, $\gamma = 1.03 \times 9810\,\mathrm{N/\,m^3} = 10100\,\mathrm{N/\,m^3}$.

PLAN

1. Find distances using trig.
2. Find the pressure at the depth of the centroid using the hydrostatic equation.
2. Find the resultant force using $F = \overline{p}A$.
3. Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$

SOLUTION

1. Distances:

- Slant height

$$\bar{y} = \frac{D}{2} + \frac{h}{\sin\theta} = \frac{0.8\,\mathrm{m}}{2} + \frac{2\,\mathrm{m}}{\sin 60^o} = 2.709\,\mathrm{m}$$

- Depth of centroid

$$\Delta z = h + \frac{D}{2}\sin 60^o = 2\,\mathrm{m} + \frac{0.8\,\mathrm{m}}{2}\sin 60^o = 2.346\,\mathrm{m}$$

2. Hydrostatic equation:

$$\overline{p} = \gamma\Delta z = \left(10100\,\mathrm{N/\,m^3}\right)(2.346\,\mathrm{m}) = 23.7\,\mathrm{kPa}$$

3. Resultant force:

$$F = \overline{p}A = (23.7\,\mathrm{kPa})\,\frac{\pi\,(0.8\,\mathrm{m})^2}{4} = 11.9\ \mathrm{kN}$$

$$\boxed{F = 11.9\ \mathrm{kN}}$$

4. Distance to CP:

- Find $\bar{I}$ using formula from Fig. A.1 (EFM11e).

$$\bar{I} = \frac{\pi r^4}{4} = \frac{\pi\,(0.4\,\mathrm{m})^4}{4} = 0.0201\,\mathrm{m}^4$$

- Final calculation:

$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{(0.0201\,\mathrm{m}^4)}{(2.709\,\mathrm{m})\left(\frac{\pi(0.8\,\mathrm{m})^2}{4}\right)} = 0.01476\,\mathrm{m}$$

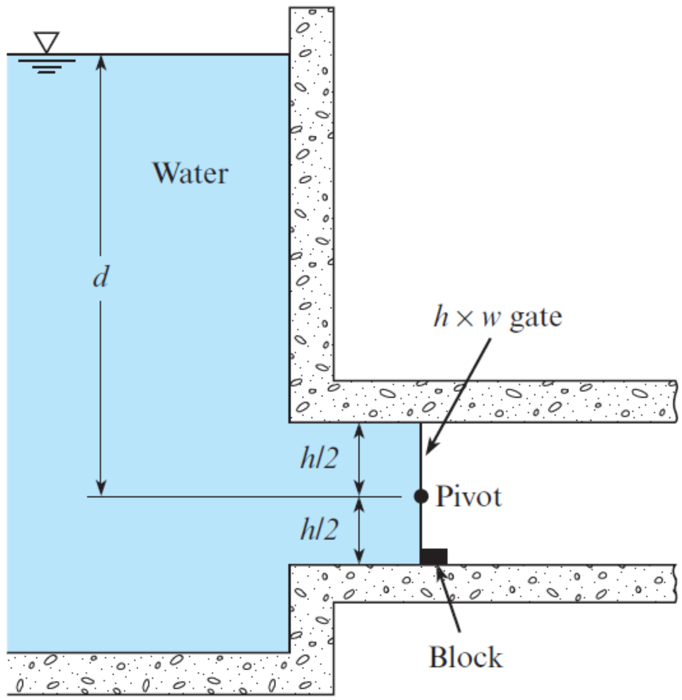
$$\boxed{y_{cp} - \bar{y} = 14.8\ \ \mathrm{mm}}$$

3.59: PROBLEM DEFINITION

Situation:

Water exerts a load on a submerged gate.

$d = 12\ \text{m}$, $h = 6\ \text{m}$, $w = 6\ \text{m}$



Find:

Force of gate on block (kN).

SOLUTION

Hydrostatic force

$$\begin{aligned} F_{\text{p}} &= \bar{p}A \\ &= \bar{y}\gamma A \\ &= (12\,\text{m}) \times \left(9810\,\text{N}/\,\text{m}^3\right) \times (6 \times 6)\ \text{m}^2 \\ &= 4.238 \times 10^6\,\text{N} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{bh^3/12}{\bar{y}A} \\ &= \frac{(6 \times 6^3/12)\ \text{m}^4}{(12\,\text{m})\,(6 \times 6)\ \text{m}^2} \\ &= 0.250\,\text{m} \end{aligned}$$

Equilibrium (sum moments about the pivot)

$$\begin{aligned} F_{\mathrm{p}}\left(y_{cp}-\bar{y}\right)-F_{\mathrm{block}}\left(\frac{h}{2}\,\mathrm{m}\right) &= 0 \\ \left(4.238\times 10^{6}\,\mathrm{N}\right)(0.250\,\mathrm{m})-F_{\mathrm{block}}\,(3\,\mathrm{m}) &= 0 \\ F_{\mathrm{block}} &= 353160\,\mathrm{N}\ \text{(acts to the left)} \end{aligned}$$

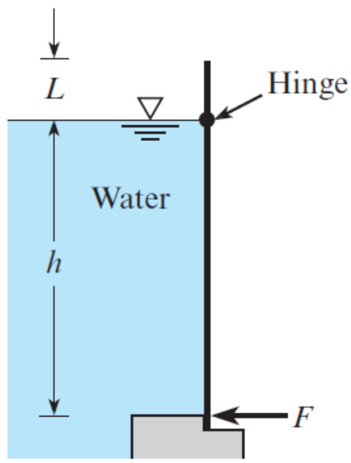
$$\boxed{F_{\text{on gate}} = 353\,\mathrm{kN}}$$

3.60: PROBLEM DEFINITION

Situation:

A rectangular gate is hinged at the water line.

$L = 1\,\text{ft}$, $h = 4\,\text{ft}$, $b = 5.8\,\text{ft}$.



Find:

Force to keep gate closed.

Properties:

From Table A.4, $\gamma_{\text{Water}} = 62.4\ \text{lbf/ft}^3$.

SOLUTION

Hydrostatic Force (magnitude):

$$\begin{aligned} F_p &= \bar{p}A \\ &= \left(\gamma_{\text{H}_2\text{O}} \times \bar{y}\right)\left(h \times b\ \text{ft}^2\right) \\ &= \left(62.4\,\text{lbf}/\,\text{ft}^3 \times 2\,\text{ft}\right)\left(4 \times 5.8\,\text{ft}^2\right) \\ &= 2895\,\text{lbf} \end{aligned}$$

Center of pressure. Since the gate extends from the free surface of the water, F_G acts at 2/3 depth or 8/3 ft below the water surface.

Moment Equilibrium - sum moments about the hinge.

$$\begin{aligned} \sum M &= 0 \\ (F_p \times y_{cp}) - (h \times F) &= 0 \end{aligned}$$

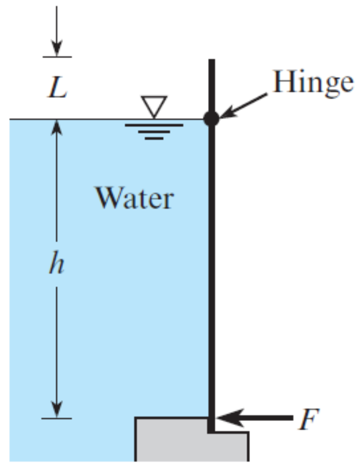
$$F = \frac{2895\,\text{lbf} \times 8/3\,\text{ft}}{4\,\text{ft}}$$

$$\boxed{F = 1930\ \text{lbf}}$$

3.61: PROBLEM DEFINITION

Situation:

A rectangular gate is hinged at the water line.
$L = 0.3\,\text{m}$, $h = 2\,\text{m}$, $b = 2\,\text{m}$



Find:

Force to keep gate closed.

Properties:

From Table A.4, $\gamma_{\text{Water}} = 9810\,\text{N/m}^3$

SOLUTION

Hydrostatic Force (magnitude):

$$\begin{aligned} F_p &= \bar{p}A \\ &= \left(\gamma_{\text{H}_2\text{O}} \times \bar{y}\right)\left(h \times b \ \text{m}^2\right) \\ &= \left(9810\,\text{N/m}^3 \times 1\,\text{m}\right)\left(2 \times 2\,\text{m}^2\right) \\ &= 39,240\,\text{N} \end{aligned}$$

Center of pressure. Since the gate extends from the free surface of the water, F_G acts at 2/3 depth or 4/3 m below the water surface.

Moment Equilibrium - sum moments about the hinge.

$$\begin{aligned} \sum M &= 0 \\ (F_p \times 4/3\,\text{m}) - (h \times F) &= 0 \end{aligned}$$

$$F = \frac{39,240\,\text{N} \times 4/3\,\text{m}}{2\,\text{m}}$$

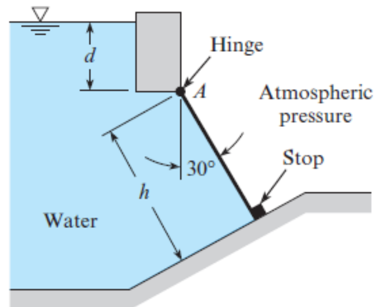
$$\boxed{F = 26.2\,\text{kN}}$$

.

3.62: PROBLEM DEFINITION

Situation:

A submerged gate sits at an angle.
$h = 6\,\text{m}$, $b = 4\,\text{m}$, $d = 3\,\text{m}$, $\theta = 30\,°$.



Find:

Force at point A - acting normal to the gate

Assumptions:

Gate is weightless.

Properties:

Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

PLAN

The reaction at A can be found by summing moments about the stop. The steps are
1. Find the hydrostatic force.
2. Locate the center of pressure.
3. Sum moments about the stop.

SOLUTION

1. Hydrostatic force (magnitude)

$$\begin{aligned} F_p &= \bar{p}A \\ &= [d + (\frac{h}{2})\cos 30°] \times \gamma \times bh \\ &= (1\text{ m } + 2.5\text{ m} \times \cos 30°)(9810\text{ N/m}^3) \times 20\text{ m}^2 \\ F_p &= 620,986\text{ N} \end{aligned}$$

2. Center of pressure:

$$
\begin{aligned}
\bar{y} &= d + \frac{h/2}{\cos 30^{\circ}} \\
\bar{y} &= 1 + \frac{2.5}{\cos 30^{\circ}} \\
&= 3.887 \text{ m} \\
y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\
&= \frac{(4 \times 5^3/12)\,\text{m}^4}{3.887 \text{ m} \times 20 \text{ m}^2} \\
&= 0.5360 \text{ m}
\end{aligned}
$$

3. Moment equilibrium about the stop:

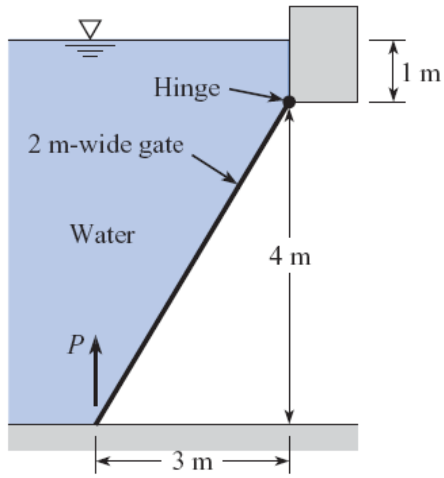
$$
\begin{aligned}
\sum M_{\text{stop}} &= 0 \\
(h)\,R_A - (h/2 - (y_{cp} - \bar{y})) \times F_p &= 0 \\
(5\,\text{m})\,R_A - (2.5\,\text{m} - 0.5360\,\text{m}) \times 620,986\,\text{N} &= 0
\end{aligned}
$$

Thus

$$
\boxed{R_A = 244\,\text{kN}}
$$

3.63: PROBLEM DEFINITION

Situation:
A submerged gate holds back water.
$b = 2\,\text{m}$



Find:
Force P required to begin to open gate (kN).

Assumptions:
Gate is weightless.

Properties:
Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

The length of gate is $\sqrt{4^2+3^2} = 5$ m
Hydrostatic force

$$\begin{aligned} F &= \bar{p}A \\ &= (\gamma \Delta z)\,A \\ &= (9810\,\text{N/m}^3)(3\,\text{m})(2\,\text{m}\times 5\,\text{m}) \\ &= 294.3\text{ kN} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{\left(\left(2\times 5^3\right)/12\right)\text{ m}^4}{(2.5\,\text{m} + 1.25\,\text{m})\,(2\,\text{m}\times 5\,\text{m})} \\ &= 0.555\,6\,\text{m} \end{aligned}$$

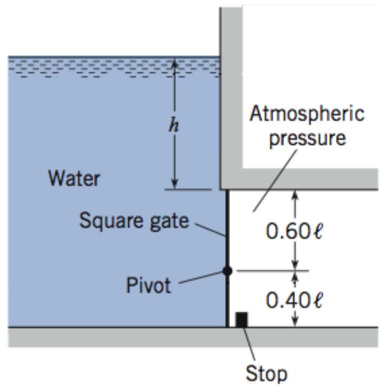
Equilibrium

$$\sum M_{\text{hinge}} = 0$$
$$294.3\,\text{kN} \times (2.5\,\text{m} + 0.555\,6\,\text{m}) - (3\,\text{m})\,P = 0$$
$$P = 299.\,75\,\text{kN}$$
$$\boxed{P = 300\,\text{kN}}$$

3.64: PROBLEM DEFINITION

Situation:

A submerged gate opens when the water level reaches a certain value.



Find:

h in terms of ℓ to open gate.

PLAN

As depth of water increase, the center of pressure will move upward. The gate will open when the center of pressure reaches the pivot.

SOLUTION

Center of pressure (when the gate opens)

$$\begin{aligned} y_{cp} - \bar{y} &= 0.60\ell - 0.5\ell \\ &= 0.10\ell \end{aligned} \tag{1}$$

Center of pressure (formula)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

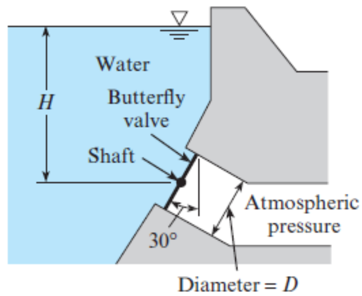
$$\begin{aligned} 0.10\ell &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \\ 0.10 &= \frac{\ell}{12(h + \ell/2)} \\ h &= \frac{5}{6}\ell - \frac{1}{2}\ell \\ &= \frac{1}{3}\ell \end{aligned}$$

$$\boxed{h = \ell/3}$$

3.65: PROBLEM DEFINITION

Situation:

A butterfly valve is described in the problem statement.
$D = 12\,\text{ft},\ H = 60\,\text{ft},\ \theta = 30\,^\circ$



Find:

Torque required to hold valve in position (ft-lbf).

SOLUTION Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \overline{y}\gamma A \\ &= (H\text{ ft} \times 62.4\text{ lb/ft}^3)\left(\pi \times \frac{D^2}{4}\right)\text{ ft}^2 \\ &= \left(60\,\text{ft} \times 62.4\,\text{lbf/ ft}^3 \times \pi \times \frac{(12\,\text{ft})^2}{4}\right) \\ &= 423,436\text{ lbf} \end{aligned}$$

Center of pressure, where I for a circle is given by $\pi r^4/4$

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\ &= \frac{\pi r^4/4}{\bar{y}\pi r^2} \\ &= \frac{(6\,\text{ft})^2/4}{60\,\text{ft}/0.866} \\ &= 0.1299\text{ ft} \end{aligned}$$

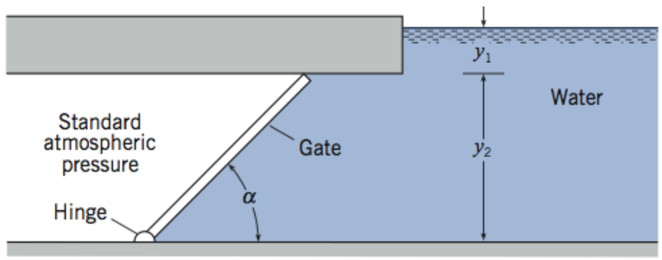
Torque

$$\begin{aligned} \text{Torque} &= 0.1299\,\text{ft} \times 423,436\,\text{lbf} \\ & \boxed{T = 55,000\text{ ft-lbf}} \end{aligned}$$

3.66: PROBLEM DEFINITION

Situation:

A submerged gate may fall due to its weight (or be held in place by pressure).
$y_1 = 1\,\text{m}$, $y_2 = 4\,\text{m}$, $w = 1\,\text{m}$.
$W = 150\,\text{kN}$, $\alpha = 45^o$.



Find:

Will the gate fall or stay in position?

Properties:

Water (10 °C), Table A.5, $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

1. Geometry

- Slant height:

$$\bar{y} = \frac{y_1 + y_2/2}{\sin\alpha} = \frac{(1 + 4/2)\,\text{m}}{\sin 45^o} = 4.243\,\text{m}$$

- Depth of centroid:

$$\Delta z = y_1 + \frac{y_2}{2} = \left(1 + \frac{4}{2}\right)\,\text{m} = 3\,\text{m}$$

- Panel surface area

$$A = \left(\frac{y_2}{\sin\alpha}\right) w = \left(\frac{4\,\text{m}}{\sin 45^o}\right)(1\,\text{m}) = 5.657\,\text{m}^2$$

2. Pressure at Centroid:

$$\bar{p} = \gamma\Delta z = \left(9810\,\text{N/m}^3\right)(3\,\text{m}) = 29.43\,\text{kPa}$$

3. Hydrostatic force:

$$F = \bar{p}A = (29.43\,\text{kPa})\left(5.657\,\text{m}^2\right) = 166.5\,\text{kN}$$

4. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1 (EFM11e):

$$\begin{aligned}
\bar{I} &= \frac{wh^3}{12} \\
h &= \frac{y_2}{\sin\alpha} = \frac{4\,\mathrm{m}}{\sin 45^o} = 5.657\,\mathrm{m} \\
\bar{I} &= \frac{wh^3}{12} = \frac{(1\,\mathrm{m})\,(5.657\,\mathrm{m})^3}{12} = 15.09\,\mathrm{m}^4
\end{aligned}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{(15.09\,\mathrm{m}^4)}{(4.243\,\mathrm{m})\,(5.657\,\mathrm{m}^2)} = 0.6287\,\mathrm{m}$$

5. Torques:

- Torque caused by hydrostatic force:

$$x_h = \frac{h}{2} - (y_{cp} - \bar{y}) = \frac{5.657\,\mathrm{m}}{2} - 0.6287\,\mathrm{m} = 2.200\,\mathrm{m}$$

$$T_{\mathrm{HS}} = Fx_h = (166.5\,\mathrm{kN})\,(2.2\,\mathrm{m}) = 366\,\mathrm{kN}\cdot\,\mathrm{m}$$

- Torque caused by the weight:

$$x_w = \frac{y_2/2}{\tan\alpha} = \frac{4\,\mathrm{m}/2}{\tan 45^o} = 2\,\mathrm{m}$$

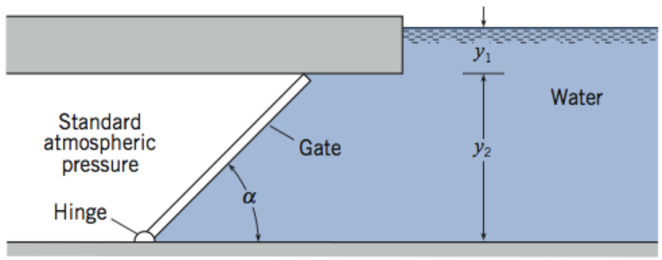
$$T_{\mathrm{W}} = Wx_w = (150\,\mathrm{kN})\,(2\,\mathrm{m}) = 300\,\mathrm{kN}\cdot\,\mathrm{m}$$

The torque caused by the hydrostatic force exceeds the torque caused by the weight: So the $\boxed{\text{gate will stay in position.}}$

3.67: PROBLEM DEFINITION

Situation:

A submerged gate may fall due to its weight.
$y_1 = 3\,\text{ft}$, $y_2 = 6\,\text{ft}$, $w = 3\,\text{ft}$.
$W = 18000\,\text{lbf}$, $\alpha = 45^o$.



Find:

Will gate fall or stay in position?

Properties:

Water (50 °F), Table A.5, $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

SOLUTION

1. Hydrostatic Force:

- Area:

$$A = \frac{y_2}{\sin\alpha} \times w = \frac{6\,\text{ft}}{\sin 45^o} \times 3\,\text{ft} = 25.46\,\text{ft}^2$$

- Depth of the centroid of the plate:

$$\Delta z = y_1 + \frac{y_2}{2} = 3\,\text{ft} + \frac{6\,\text{ft}}{2} = 6\,\text{ft}$$

- Final Calculation:

$$F = \bar{p}A = \gamma \Delta z A = \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(6\,\text{ft})\left(25.46\,\text{ft}^2\right) = 9532\,\text{lbf}$$

2. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1:

$$\begin{aligned} \bar{I} &= \frac{wh^3}{12} \\ h &= \frac{y_2}{\sin\alpha} = \frac{6\,\text{ft}}{\sin 45^o} = 8.485\,\text{ft} \\ \bar{I} &= \frac{wh^3}{12} = \frac{(3\,\text{ft})(8.485\,\text{ft})^3}{12} = 152.7\,\text{ft}^4 \end{aligned}$$

- Slant height:

$$\bar{y} = \frac{h}{2} + \frac{y_1}{\sin\alpha} = \frac{8.485\,\text{ft}}{2} + \frac{3\,\text{ft}}{\sin 45^o} = 8.485\,\text{ft}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{\left(152.7\,\text{ft}^4\right)}{\left(8.485\,\text{ft}\right)\left(25.46\,\text{ft}^2\right)} = 0.7069\,\text{ft}$$

3. Torque due to weight:

- Moment arm:

$$x_1 = \frac{y_2 \tan\alpha}{2} = \frac{\left(6\,\text{ft}\right)\left(\tan 45^o\right)}{2} = 3\,\text{ft}$$

- Final calculation:

$$M_1 = W x_1 = \left(18000\,\text{lbf}\right)\left(3\,\text{ft}\right) = 54000\,\text{ft}\,\text{lbf}$$

4. Torque due hydrostatic pressure:

- Moment arm:

$$x_2 = h/2 - \left(y_{cp} - \bar{y}\right) = \frac{8.485\,\text{ft}}{2} - \left(0.7069\,\text{ft}\right) = 3.536\,\text{ft}$$

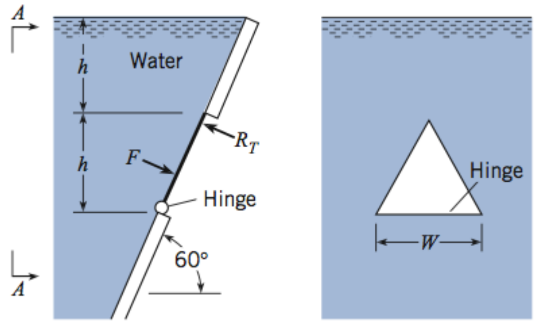
- Final calculation:

$$M_1 = F x_2 = \left(9532\,\text{lbf}\right)\left(3.536\,\text{ft}\right) = 33705\,\text{ft}\,\text{lbf}$$

Since the torque due to weight exceeds the torque due to hydrostatic pressure: **the gate will fall**.

3.68: PROBLEM DEFINITION

Situation:
A submerged gate is described in the problem statement.



View A-A

Find:
Hydrostatic force (F) on gate.
Ratio (R_T/F) of the reaction force to the hydrostatic force.

SOLUTION

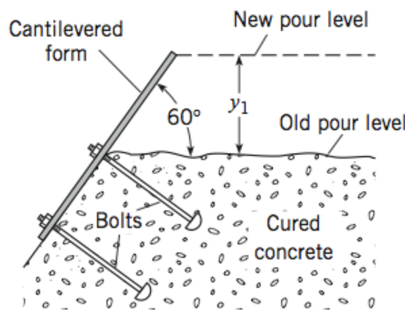
$$
\begin{aligned}
F &= \bar{p}A \\
&= \left(h+\frac{2h}{3}\right)\gamma\left(\frac{Wh/\sin 60^\circ}{2}\right) \\
& \boxed{F=\frac{5\gamma W h^2}{3\sqrt{3}}} \\
y_{cp}-\bar{y} &= \frac{I}{\bar{y}A}=\frac{W(h/\sin 60^\circ)^3}{(36\times(5h/(3\sin 60^\circ))}\times\frac{Wh}{2\sin 60^\circ} \\
&= \frac{h}{15\sqrt{3}} \\
\Sigma M &= 0 \\
R_T h/\sin 60^\circ &= F\left[\left(\frac{h}{3\sin 60^\circ}\right)-\left(\frac{h}{15\sqrt{3}}\right)\right] \\
& \boxed{\frac{R_T}{F}=\frac{3}{10}}
\end{aligned}
$$

3.69: PROBLEM DEFINITION

Situation:

A concrete form is described in the problem statement.

$y_1 = 1.8\,\mathrm{m}$, $b = 1\,\mathrm{m}$, $\theta = 60\,^\circ$



Find:

Moment at base of form per meter of length (kN·m/m).

Properties:

Concrete, $\gamma = 24\,\mathrm{kN/\,m^3}$.

Assumptions:

Assume that the form has a width of $w = b$ =1 meter into the paper.

PLAN

Find the moment by muliplying the hydrostatic force by its moment arm. The plan for reaching the goal is:

1. Calculate the hydrostatic force.
2. Calculate the centroid of area using $I = bh^3/12$.
3. Calculate the center of pressure.
4. Use results from steps 1 to 4 to calculate the moment.

SOLUTION

1. Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \gamma z_c h w \\ h &= \text{height of panel} = \left(\frac{1.8\,\mathrm{m}}{\sin 60^\circ}\right) = 2.078\,\mathrm{m} \\ F &= \left(24000\,\mathrm{N/\,m^3}\right)\left(\frac{1.8}{2}\,\mathrm{m}\right)(2.078\,\mathrm{m})\,(1\,\mathrm{m}) = 44895\,\mathrm{N} \end{aligned}$$

2. Moment of Inertia of a rectangle

$$I = \frac{bh^3}{12} = \frac{(1\,\mathrm{m})\,(2.078\,\mathrm{m})^3}{12} = 0.7483\,\mathrm{m}^4$$

3. Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\ \bar{y} &= (2.078\,\mathrm{m})\,/2 = 1.0392\,\mathrm{m} \\ A &= hw = (2.078\,\mathrm{m})\,(1\,\mathrm{m}) = 2.078\,\mathrm{m}^2 \\ y_{cp} - \bar{y} &= \frac{(0.7483\,\mathrm{m}^4)}{(1.0392\,\mathrm{m})\,(2.078\,\mathrm{m}^2)} = 0.3464\,\mathrm{m} \end{aligned}$$

4. Moment at base

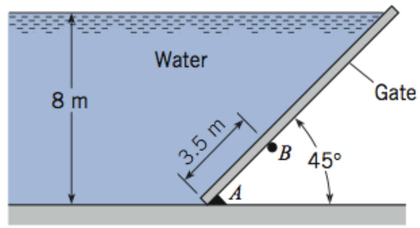
$$\begin{aligned} M &= (\text{force})\,(\text{moment arm}) \\ &= (44895\,\mathrm{N})\,(2.078\,\mathrm{m}/2 - 0.3464\,\mathrm{m}) \\ & \boxed{M = 31.1 \text{ kN}\cdot\text{m per meter of form}} \end{aligned}$$

3.70: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.

$\theta = 45\,^{\circ}$.



Find:

Is the gate stable or unstable.

SOLUTION

$$y_{cp} = \frac{2}{3} \times \frac{8}{\cos 45^{\circ}} = 7.54 \text{ m}$$

Point B is (8/cos 45°) m - 3.5 m = 7.81 m along the gate from the water surface; therefore, the gate is **unstable.**

3.71: PROBLEM DEFINITION

Situation:

Two hemispherical shells are sealed together.
$r_o = 10.5\,\mathrm{cm}$, $r_i = 10.75\,\mathrm{cm}$.

Find:

Force required to separate the two shells.

Assumptions:

The pressure seal is at the average radius ($r = 10.6\,\mathrm{cm}$)

Properties:

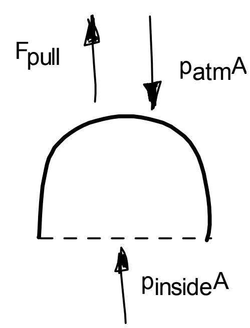
$p_{\mathrm{atm}} = 101.3\,\mathrm{kPa}$, $p_i = 0.25 p_{\mathrm{atm}}$.

PLAN

1. Apply equilibrium to a freebody comprised of the top half of the shell plus the air inside.
2. Calculate the force.

SOLUTION

1. Equilibrium.



$$
\begin{aligned}
\sum F_y &= 0 \\
F_{\mathrm{pull}} + p_i A - p_{\mathrm{atm}} A &= 0
\end{aligned}
$$

2. Force to separate shells.

$$
\begin{aligned}
F_{\mathrm{pull}} &= (p_{\mathrm{atm}} - p_i)\,A = p_{\mathrm{atm}}\,(1 - 0.25)\,A \\
&= (1 - 0.25)\left(101000\,\mathrm{N/\,m^2}\right)\left(\pi\,(0.106\,\mathrm{m})^2\right) \\
&= 2670\,\mathrm{N}
\end{aligned}
$$

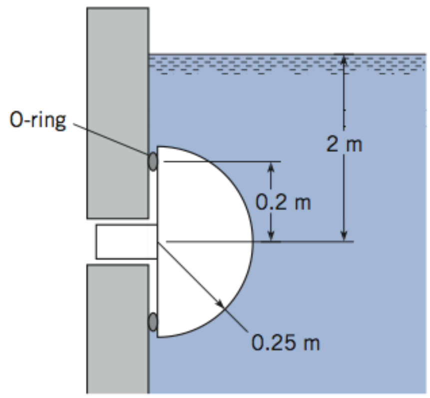
$$
\boxed{F_{\mathrm{pull}} = 2670\,\mathrm{N}}
$$

3.72: PROBLEM DEFINITION

Situation:

A plug sits in a hole in the side of a tank.

$z = 2\,\text{m}$, $r_{o-ring} = 0.2\,\text{m}$, $r_{plug} = 0.25\,\text{m}$.



Find:

Horizontal and vertical forces on plug.

Properties:

Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

Hydrostatic force

$$\begin{aligned} F_h &= \bar{p}A \\ &= \gamma z A \\ &= 9810\,\text{N/m}^3 \times 2\,\text{m} \times \pi \times (0.2\,\text{m})^2 \\ &\boxed{F_h = 2465\ \text{N}} \end{aligned}$$

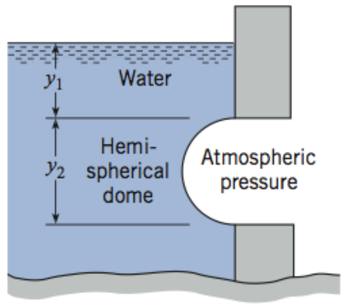
The vertical force is simply the buoyant force.

$$\begin{aligned} F_v &= \gamma V\!\!\!/ \\ &= 9810\,\text{N/m}^3 \times \frac{4}{6} \times \pi \times (0.25\,\text{m})^3 \\ &\boxed{F_v = 321\ \text{N}} \end{aligned}$$

3.73: PROBLEM DEFINITION

Situation:

A dome below the water surface is described in the problem statement.
$y_1 = 1\,\text{m}$, $y_2 = 2\,\text{m}$.



Find:

Magnitude and direction of force to hold dome in place.

Properties:

Water (10 °C), Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

1. Horizontal component of force.

$$\begin{aligned} F_H &= (1\,\text{m} + 1\,\text{m})9810\,\text{N/m}^3 \times \pi \times (1\,\text{m})^2 \\ &= 61,640\ \text{N}\ = 61.64\ \text{kN} \end{aligned}$$

2. Center of pressure.

$$\begin{aligned} (y_{cp} - \bar{y}) &= \frac{I}{\bar{y}A} \\ &= \frac{\pi \times (1\,\text{m})^4/4}{2\,\text{m} \times \pi \times (1\,\text{m})^2} \\ &= 0.125\,\text{m} \end{aligned}$$

3. Vertical component of force

$$\begin{aligned} F_V &= \left(\frac{1}{2}\right)\left(\frac{4\pi \times (1\,\text{m})^3}{3}\right) 9,810\,\text{N/m}^3 \\ &= 20,550\ \text{N} \\ F_V &= 20.6\,\text{kN} \end{aligned}$$

4. Answer

$$\boxed{F_{\text{horizontal}} = 61.6\,\text{kN (applied to the left to hold dome in place)}}$$

Line of action is 0.125 m below a horizontal line passing through the dome center

$F_{\text{vertical}} = 20.6\,\text{kN}$ (applied downward to hold dome in place)

3.74: PROBLEM DEFINITION

Situation:

Three spheres of the same diameter are submerged in the same body of water. One sphere is steel, one is a spherical balloon filled with water, and one is a spherical balloon filled with air.

a. Which sphere has the largest buoyant force?

b. If you move the steel sphere from a depth of 1 ft to 10 ft, what happens to the magnitude of the buoyant force acting on that sphere?

c. If all 3 spheres are released from a cage at a depth of 1 m, what happens to the 3 spheres, and why?

SOLUTION

Answers:

a. All 3 spheres have the same bouyant force, because they all have the same diameter, and all displace the same volume of water (assuming the air-fillled balloon doesn't compress under the compressive inward force that would be incurred, and would increase as you moved the balloon deeper). Remember, F_b is a function of the specific weight of the displaced fluid, not the specific weight of the displacing body.

b. When you move the steel sphere deeper, F_b does not change.

c. To answer this question about releasing the 3 spheres from a cage, do a force balance on each of the 3 bodies. For all 3 cases, the force balance results in W acting down, and F_b acting up. For the case of the steel sphere, W is greater than F_b, so the steel sphere sinks. For the case of the water balloon, W and F_b are essentially the same, so the water balloon hovers at the same depth, assuming the weight of the ballon's rubber is negligible. Since rubber is lighter than water, the water-filled ballon might appear to hover, or move just perceptibly upward. For the case of the air-filled balloon, W is significantly less than F_b, so the air-filled ballon would move upward. If the air-filled balloon were compressed due to the inward-acting water pressure, this would make F_b even smaller because the volume would decrease.

3.75: PROBLEM DEFINITION

Situation:
In air, a rock weighs $W_{\text{air}} = 980\,\text{N}$.
In water, a rock weighs $W_{\text{water}} = 609\,\text{kg}$.

Find:
The volume of the rock (liters).

Properties:
Water (15^oC) ,Table A.5, $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Apply equilibrium to the rock when it is submerged in water.
2. Solve the equation from step 1 for volume, convert volume to L.

SOLUTION

1. Equilibrium:

$$\left\{\begin{array}{c}\text{Force to hold}\\ \text{rock stationary in water}\\ \text{(apparent weight)}\end{array}\right\}+\left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on rock}\end{array}\right\}=\left\{\begin{array}{c}\text{Weight of rock}\\ \text{in air}\end{array}\right\}$$

$$\begin{aligned} W_{\text{water}} + F_B &= W_{\text{air}} \\ W_{\text{water}} + \gamma V\!\!\!- &= W_{\text{air}} \\ 609\,\text{N} + 9800\,\text{N/m}^3 V\!\!\!- &= 980\,\text{N} \end{aligned}$$

2. Solve for volume, report volume in L.

$$V\!\!\!- = \frac{980\,\text{N} - 609\,\text{N}}{9800\,\text{N/m}^3} = 0.0378\,\text{m}^3$$

$$\boxed{V\!\!\!- = 37.8\text{ L}}$$

3.76: PROBLEM DEFINITION

Situation:

A gold pendant is submerged in water
$m_{\text{in air}} = 100\,\text{g} = 0.1\,\text{kg}; W_{\text{in air}} = 0.981\,\text{N}.$
$m_{\text{in water}} = 94.8 = 0.0948\,\text{kg}; W_{\text{in water}} = 0.930\,\text{N}.$
SG of gold is 19.3

Find:

The SG of the pendant, and decide to bid if $SG > 19.0$

Properties:

Water, $\gamma = 9810\,\text{N/m}^3$.

PLAN

To find the SG, find ρ_{pendant}, using the definition $\rho = \frac{mass}{vol}$.
For density, unknown is volume of the pendant.
To get $V\!\!\!/_{pend}$ use Archimedes Principle: $F_B = \gamma_{\text{H2O}} V\!\!\!/_{disp}$.
Find F_B from a free-body diagram on the body in water
Relate ρ_{pendant} to ρ_{water} to check whether $SG > 19.0$.

SOLUTION

Free-body diagram on pendant in equilbrium (pendant submerged in water):

$$\left\{\begin{array}{c}\text{Weight of pendant}\\ \text{in water}\\ \text{(apparent weight)}\end{array}\right\} + \left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on pendant}\end{array}\right\} = \left\{\begin{array}{c}\text{Weight of pendant}\\ \text{in air}\end{array}\right\}$$

$$W_{\text{in water}} + \gamma_{\text{H2O}} V\!\!\!/ = W_{\text{ in air}}$$

Solve for volume:

$$\begin{aligned} V\!\!\!/ &= \frac{W_{\text{in air}} - W_{\text{in water}}}{\gamma_{\text{H2O}}} \\ &= \frac{0.981\,\text{N} - 0.930\,\text{N}}{9810\,\text{N/m}^3} = 5.2 \times 10^{-6}\,\text{m}^3 \end{aligned}$$

Solve for density (definition):

$$\begin{aligned} \rho_{\text{pendant}} &= \frac{0.1\,\text{kg}}{5.2 \times 10^{-6}\,\text{m}^3} \\ \rho_{\text{pendant}} &= 19,261\,\text{kg/m}^3 \end{aligned}$$

Specific gravity (definition):

$$SG = \frac{\rho_{\text{pendant}}}{\rho_{\text{water}}} = \frac{19,261\,\text{kg/m}^3}{1000\,\text{kg/m}^3} = 19.26$$

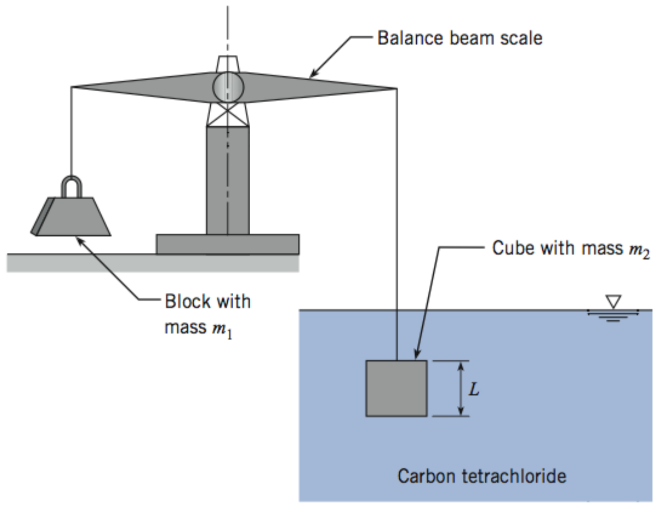
$$\boxed{SG > 19.0\text{, so decide to bid}}$$

3.77: PROBLEM DEFINITION

Situation:

A cube is suspended in carbon tetrachloride.

$m_1 = 610\,\text{g},\ L = 0.094\,\text{m}$



Find:

The mass of the cube (kg).

Properties:

Carbon Tetrachloride (20^oC) ,Table A.4, $\gamma = 15600\,\text{N/m}^3$.

PLAN

1. Find the force on the balance arm scale by finding the weight of the block.
2. Find m_2 by applying equilibrium to the cube.

SOLUTION

1. Force on balance arm:

$$\left\{\begin{array}{c}\text{Force on}\\ \text{balance arm}\end{array}\right\} = \left\{\begin{array}{c}\text{Weight}\\ \text{of block}\end{array}\right\} = mg = (0.61\,\text{kg})\left(9.81\,\text{m/s}^2\right) = 5.984\,\text{N}$$

2. Equilibrium (applied to cube):

$$\begin{aligned}\left\{\begin{array}{c}\text{Force on}\\ \text{balance arm}\end{array}\right\} + \left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on cube}\end{array}\right\} &= \{\text{Weight of cube}\}\\ F + \gamma (L_2)^3 &= m_2 g\end{aligned}$$

Solve for m_2 :

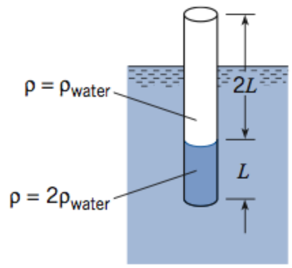
$$m_2 = \frac{F + \gamma (L_2)^3}{g} = \frac{(5.984\,\text{N}) + (15600\,\text{N/m}^3)(0.094\,\text{m})^3}{9.81\,\text{m/s}^2}$$

$$\boxed{m_2 = 1.93\,\text{kg}}$$

3.78: PROBLEM DEFINITION

Situation:

A rod is floating in a liquid.



Find:

Determine if the liquid is

a. lighter than water

b. must be water

c. heavier than water.

SOLUTION

Rod weight

$$\begin{aligned} W &= (2LA\gamma_W + LA2\gamma_W) \\ W &= 4LA\gamma_W \end{aligned} \tag{1}$$

Since part of the rod extends above the liquid,

$$F_B < \forall\gamma_{\text{Liq}} = 3LA\gamma_{\text{Liq}} \tag{2}$$

Equilibrium applied to the rod

$$W = F_B \tag{3}$$

Combine Eqs. (1), (2) and (3).

$$\begin{aligned} 4LA\gamma_W &< 3LA\gamma_{\text{Liq}} \\ \gamma_{\text{Liq}} &> \frac{4}{3}\gamma_W. \end{aligned}$$

The liquid is more dense than water so is answer $\boxed{\text{c).}}$

3.79: PROBLEM DEFINITION

Situation:

A ship is sailing from salt to fresh water.

$W = 40000$ tons $= 80 \times 10^6\,\text{lbf}$.

$A = 38000\,\text{ft}^2$, $L = 800\,\text{ft}$.

Find:

Will the ship rise or settle?

Amount (ft) the ship will rise or settle.

Properties:

Seawater, $\gamma_s = (1.03)62.4\,\text{lbf/ft}^3$

PLAN

1. To establish whether the ship will rise or settle, apply the equilibrium equation.
2. Determine the volume displaced in both salt and fresh water.
3. Calculate the distance the ship moves up or down.

SOLUTION

1. Equilibrium. The weight of the ship is balanced by the buoyant force

$$W = F_B = \gamma \forall$$

As the ship moves into freshwater, the specific weight of the water decreases Thus, the volume of the displaced water will increase as shown below.

$$W = F_B = (\gamma \downarrow)(\forall \uparrow)$$

Thus $\boxed{\text{the ship will settle}}$.

2. Volume displaced (salt water):

$$W = F_B = \gamma_s \forall_s$$

$$\forall_s = \frac{W}{\gamma_s} = \frac{80 \times 10^6\,\text{lbf}}{1.03\left(62.4\,\text{lbf/ft}^3\right)}$$

Volume displaced (fresh water):

$$W = F_B = \gamma_f \forall_f$$

$$\forall_f = \frac{W}{\gamma_f} = \frac{80 \times 10^6\,\text{lbf}}{62.4\,\text{lbf/ft}^3}$$

3. Distance Moved. The distance moved Δh is given by

$$\Delta \forall = A \Delta h$$

where $\Delta \mathcal{V}$ is the change in displaced volume and A is the section area of the ship at the water line. Thus:

$$\left(\frac{80 \times 10^6\,\mathrm{lbf}}{62.4\,\mathrm{lbf/\,ft^3}}\right) - \left(\frac{80 \times 10^6\,\mathrm{lbf}}{1.03\left(62.4\,\mathrm{lbf/\,ft^3}\right)}\right) = \left(38000\,\mathrm{ft^2}\right)\Delta h$$

Thus:

$$\boxed{\Delta h = 0.983\,\mathrm{ft}}$$

3.80: PROBLEM DEFINITION

A ship is sailing from fresh to salt water.
$W = 300 \text{ MN} = 300 \times 10^6 \text{ N}$.
$A = 2600 \text{ m}^2$, $L = 150 \text{ m}^2$.

Find:
Will the ship rise or settle?
Amount (m) the ship will rise or settle.

Properties:
Seawater, $\gamma_s = (1.03) 9810 \text{ N/m}^3$

PLAN

1. To establish whether the ship will rise or settle, apply the equilibrium equation.
2. Determine the volume displaced in both salt and fresh water.
3. Calculate the distance the ship moves up or down.

SOLUTION

1. Equilibrium. The weight of the ship is balanced by the buoyant force

$$W = F_B = \gamma \forall$$

As the ship moves into saltwater, the specific weight of the water increases. Thus, the volume of the displaced water will decrease as shown below.

$$W = F_B = (\gamma \uparrow)(\forall \downarrow)$$

Thus $\boxed{\text{the ship will rise}}$.

2. Volume displaced (salt water):

$$W = F_B = \gamma_s \forall_s$$

$$\forall_s = \frac{W}{\gamma_s} = \frac{300 \times 10^6 \text{ N}}{1.03 \,(9810 \text{ N/m}^3)}$$

Volume displaced (fresh water):

$$W = F_B = \gamma_f \forall_f$$

$$\forall_f = \frac{W}{\gamma_f} = \frac{300 \times 10^6 \text{ N}}{9810 \text{ N/m}^3}$$

3. Distance Moved. The distance moved Δh is given by

$$\Delta \forall = A \Delta h$$

where $\Delta \forall$ is the change in displaced volume and A is the section area of the ship at the water line. Thus:

$$\left(\frac{300\times10^{6}\,\mathrm{N}}{9810\,\mathrm{N/\,m^{3}}}\right)-\left(\frac{300\times10^{6}\,\mathrm{N}}{1.03\,(9810\,\mathrm{N/\,m^{3}})}\right)=\left(2600\,\mathrm{m^{2}}\right)\Delta h$$

Thus:

$$\boxed{\Delta h=0.343\,\mathrm{ft}}$$

3.81: PROBLEM DEFINITION

Situation:

A spherical buoy is anchored in salt water.
$W_b = 1800\,\text{N}$, $D = 1.2\,\text{m}$.
$T = 5000\,\text{N}$, $y = 50\,\text{m}$.

Find:

Weight of scrap iron (N) to be sealed in the buoy.

Properties:

Seawater, Table A.4 $\gamma_s = 10070\,\text{N/m}^3$.

PLAN

1. Find the buoyant force using the buoyant force equation.
2. Find the weight of scrap iron by applying equilibrium.

SOLUTION

1. Buoyant force equation:

$$F_B = \gamma_s \not{V} = \left(10070\,\text{N/m}^3\right) \frac{\pi\,(1.2\,\text{m})^3}{6} = 9111\,\text{N}$$

2. Equilibrium

$$\begin{aligned} \Sigma F_y &= 0 \\ F_B &= W_{\text{buoy}} + W_{\text{scrap}} + T \\ 9111\,\text{N} &= 1800\,\text{N} + W_{\text{scrap}} + 5000\,\text{N} \end{aligned}$$

$$\boxed{W_{\text{scrap}} = 2310\,\text{N}}$$

3.82: PROBLEM DEFINITION

Situation:

A block is submerged in water.

$W_{\text{water}} = 390\,\text{N}$, $W_{\text{air}} = 700\,\text{N}$.

Find:

The volume of the block (liters).

The specific weight of the material that was used to make the block (N/m^3).

Properties:

Water (15^oC) ,Table A.5, $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Find the block's volume by applying equilibrium to the block.
2. Find the specific weight by using the definition.

SOLUTION

1. Equilibrium (block submerged in water):

$$\left\{ \begin{array}{c} \text{Force to hold} \\ \text{block in water} \\ \text{(apparent weight)} \end{array} \right\} + \left\{ \begin{array}{c} \text{Buoyant Force} \\ \text{on block} \end{array} \right\} = \left\{ \begin{array}{c} \text{Weight of block} \\ \text{in air} \end{array} \right\}$$

$$\begin{aligned} W_{\text{water}} + F_B &= W_{\text{air}} \\ W_{\text{water}} + \gamma_{\text{H2O}} \forall &= W_{\text{air}} \end{aligned}$$

Solve for volume:

$$\begin{aligned} \forall &= \frac{W_{\text{air}} - W_{\text{water}}}{\gamma_{\text{H2O}}} \\ &= \frac{700\,\text{N} - 390\,\text{N}}{9800\,\text{N/m}^3} = 0.0316\,\text{m}^3 \end{aligned}$$

$$\boxed{\forall = 31.6\text{ L}}$$

2. Specific weight (definition):

$$\gamma_{\text{block}} = \frac{\text{weight of block}}{\text{volume of block}} = \frac{700\,\text{N}}{0.0316\,\text{m}^3} = 22129\,\text{N/m}^3$$

$$\boxed{\gamma_{\text{block}} = 22.1\,\text{kN/m}^3}$$

3.83: PROBLEM DEFINITION

Situation:

A cylindrical tank is filled with water up to depth D_{tank}.

A cylinder of wood is set afloat in the water.

$D_{\text{tank}} = 1\,\text{ft}$, $D_{\text{wood}} = 5\,\text{in}$.

$W_{\text{wood}} = 3.5\,\text{lbf}$, $L_{\text{wood}} = 6\,\text{in}$.

Find:

Change of water level in tank.

Properties:

Water, Table A.5: $\gamma = 62.4\,\text{lbf/ ft}^3$.

PLAN

When the wood enters the tank, it will displace volume. This volume can be visualized as adding extra water to the tank. Thus, find this volume and use it to determine the increase in water level.

1. Find the buoyant force by applying equilibrium.
2. Find the displaced volume by applying the buoyancy equation.
3. Find the increase in water level by equating volumes

SOLUTION

1. Equilibrium

$$\text{weight of block} = \text{buoyant force on block}$$

$$F_B = W_{\text{block}} = 3.5\,\text{lbf}$$

2. Buoyancy equation

$$F_B = \gamma_{\text{H}_2\text{O}} \forall_D = 3.5\,\text{lbf}$$

$$\forall_D = \frac{3.5\,\text{lbf}}{62.4\,\text{lbf/ ft}^3} = 0.05609\,\text{ft}^3$$

3. Find Δh of the tank resulting from volume change

$$\forall_D = (\text{volume change}) = (\text{tank section area})\,(\text{height change})$$

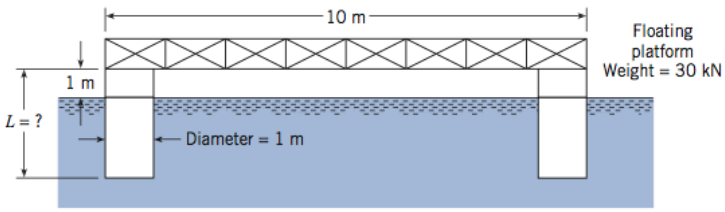
$$0.05609\,\text{ft}^3 = \frac{\pi\,(1\,\text{ft})^2}{4}\Delta h$$

$$\boxed{\Delta h = 0.0714\,\text{ft} = 0.86\,\text{in}}$$

3.84: PROBLEM DEFINITION

Situation:

A platform floats in water.
$W_{\text{platform}} = 30\,\text{kN}$, $W_{\text{cylinder}} = 1\,\text{kN/m}$.
$y = 1\,\text{m}$, $D_{\text{cylinder}} = 1\,\text{m}$.



Find:

Length of cylinder so that the platform floats 1 m above water surface.

Properties:

$\gamma_{\text{water}} = 10,000\,\text{N/m}^3$.

SOLUTION

1.Equilibrium (vertical direction)

$$\begin{pmatrix}\text{Weight of}\\ \text{platform}\end{pmatrix} + 4\begin{pmatrix}\text{Weight of}\\ \text{a cylinder}\end{pmatrix} = 4\begin{pmatrix}\text{Buoyant force}\\ \text{on a cylinder}\end{pmatrix}$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\left(\gamma \forall_D\right)$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\left(\frac{10000\,\text{N}}{\text{m}^3}\right)\left(\frac{\pi\,(1\,\text{m})^2}{4}\,(L - 1\,\text{m})\right)$$

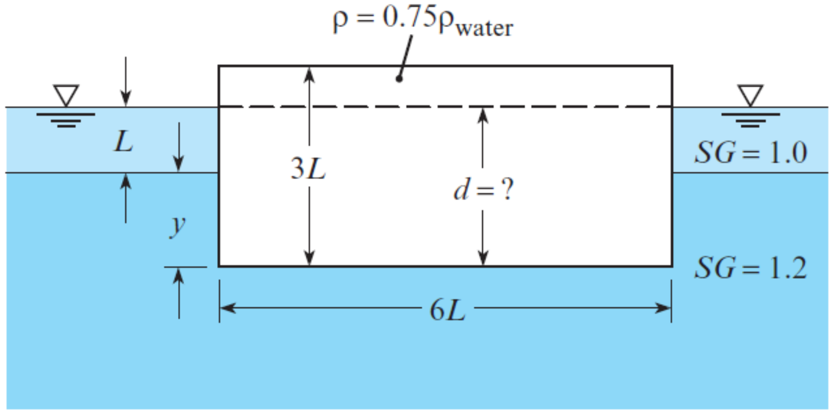
1. Solve for L

$$\boxed{L = 2.24\,\text{m}}$$

3.85: PROBLEM DEFINITION

Situation:

A block floats in two layered liquids.
$b = 6L$, $h = 3L$.



Find:

Depth block will float.

Assumptions:

The block will sink a distance y into the fluid with $S = 1.2$.

Properties:

$\rho_{block} = 0.75\rho_{water}$.

SOLUTION

1. Equilibrium.

$$\begin{aligned} \sum F_y &= 0 \\ -\begin{pmatrix} \text{Weight} \\ \text{of block} \end{pmatrix} + \begin{pmatrix} \text{Pressure force} \\ \text{on btm of block} \end{pmatrix} &= 0 \end{aligned}$$

$$-\left(\forall_{\text{block}}\right)\left(\gamma_{\text{block}}\right) + p_{\text{btm}}A = 0$$

$$-(6L)^2 \times 3L \times 0.75\gamma_{\text{water}} + (L \times \gamma_{\text{water}} + y \times 1.2\gamma_W)36L^2 = 0$$

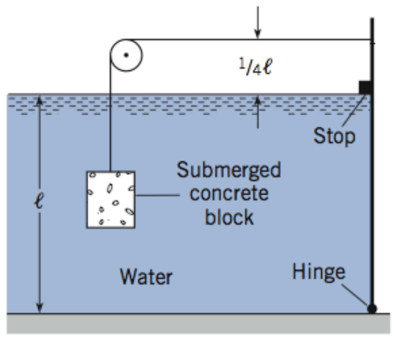
$$\begin{gathered} y = 1.105L \\ d = y + L \\ \boxed{d = 2.105L} \end{gathered}$$

3.86: PROBLEM DEFINITION

Situation:

A submerged gate has a concrete block attached to it.

$b = 1\,\text{m}$, $\ell = 3\,\text{m}$.



Find:

Minimum volume of concrete to keep gate in closed position (m^3).

Properties:

Concrete $\gamma = 23.6\,\text{kN/m}^3$.

SOLUTION

Hydrostatic force on gate and CP

$$\begin{aligned} F &= \bar{p}A = 0.5\ell\,\text{m} \times 9,810\,\text{N/m}^3 \times 3\,\text{m} \times 1\,\text{m} = 44,145 \text{ N} \\ y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} = \frac{1\,\text{m} \times (3\,\text{m})^3}{12 \times 0.5\ell\,\text{m} \times 3\,\text{m} \times 1\,\text{m}} = 0.5 \text{ m} \end{aligned}$$

Sum moments about the hinge to find the tension in the cable

$$T = 44,145 \times \frac{1 - 0.5}{1.25\ell} = 11772 \text{ N}$$

Equilibrium applied to concrete block

$$\begin{pmatrix} \text{Tension} \\ \text{in cable} \end{pmatrix} + \begin{pmatrix} \text{Buoyant} \\ \text{force} \end{pmatrix} = (\text{Weight})$$

$$T + V\gamma_{H_2O} = V\gamma_c$$

Solve for volume of block

$$\begin{aligned} V &= \frac{T}{\gamma_c - \gamma_{H_2O}} \\ &= \frac{11772\,\text{N}}{23,600\,\text{N/m}^3 - 9,810\,\text{N/m}^3} \end{aligned}$$

$$\boxed{V = 0.854 \text{ m}^3}$$

3.87: PROBLEM DEFINITION

Situation:

Ice is added to a cylindrical tank holding water.

$d = 2\,\text{ft}$, $h = 4\,\text{ft}$.

$W_{ice} = 5\,\text{lb}$.

Find:

Change of water level in tank after ice is added.

Change in water level after the ice melts.

Explain all processes.

Properties:

$\gamma_{water} = 62.4\,\text{lbf}/\,\text{ft}^3$.

SOLUTION

Change in water level (due to addition of ice)

$$\begin{aligned} W_{\text{ice}} &= F_{\text{buoyancy}} \\ &= \Delta V_W \gamma_W \end{aligned}$$

So

$$\begin{aligned} \Delta V_W &= \frac{W_{\text{ice}}}{\gamma_W} = \frac{5\,\text{lbf}}{62.4\,\text{lbf}/\,\text{ft}^3} \\ &= 0.0801\ \text{ft}^3 \end{aligned}$$

Rise of water in tank (due to addition of ice)

$$\begin{aligned} \Delta h &= \frac{\Delta V_W}{A_{\text{cyl}}} \\ &= \frac{0.0801\ \text{ft}^3}{(\pi/4)(2\ \text{ft})^2} = 0.02550\,\text{ft} = 0.3060\,\text{in} \end{aligned}$$

$$\boxed{\Delta h = 0.306\,\text{in} \quad <== \text{(due to addition of ice)}}$$

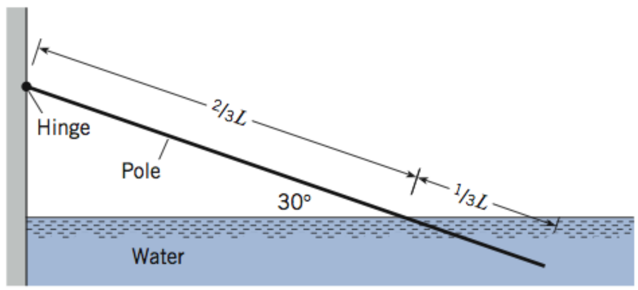
Answer ⇒ When the ice melts, the melted water will occupy the same volume of water that the ice originally displaced; therefore, there will be no change in water surface level in the tank after the ice melts.

3.88: PROBLEM DEFINITION

Situation:

A partially submerged wood pole is attached to a wall.

$\theta = 30\,°$.



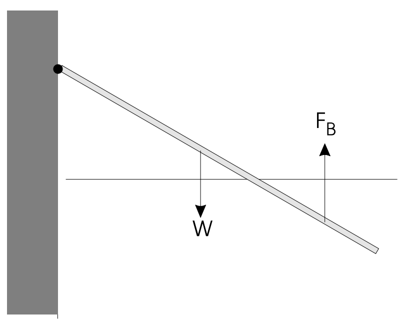
Find:

Density of wood.

Properties:

$\gamma = 9810\,\mathrm{N/\,m^3}$.

SOLUTION



$$
\begin{aligned}
M_{\text{hinge}} &= 0 \\
-W_{\text{wood}} \times (0.5L\cos 30°) + F_{B.} \times (5/6)L\cos 30° &= 0 \\
-\gamma_{\text{wood}} \times AL \times (0.5L\cos 30°) + \left(\frac{1}{3}AL\gamma_{\mathrm{H_2O}}\right) \times \left(\frac{5}{6}L\cos 30°\right) &= 0
\end{aligned}
$$

$$\gamma_{\text{wood}} = \left(\tfrac{10}{18}\right)\gamma_{\mathrm{H_2O}}$$

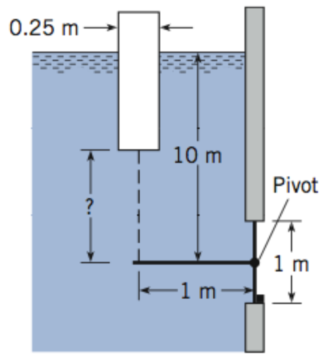
$$\boxed{\gamma_{\text{wood}} = 5,450\ \mathrm{N/m^3}}$$

$$\boxed{\rho_{\text{wood}} = 556\ \mathrm{kg/m^3}}$$

3.89: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.
$d = 25\,\text{cm}$, $W = 200\,\text{N}$.
$y = 10\,\text{m}$, $L = 1\,\text{m}$.



Find:

Length of chain so that gate just on verge of opening.

PLAN

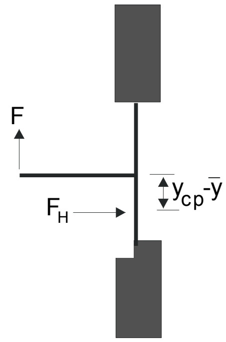
Apply hydrostatic force equations and then sum moments about the hinge.

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A = 10\,\text{m} \times 9{,}810\,\text{N/m}^3 \times \frac{\pi D^2}{4} \\
&= 98{,}100\,\text{N/m}^2 \times \pi\frac{\pi}{4}\left(1\,\text{m}\right)^2 \\
&= 77{,}048\ \text{N} \\
y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\
&= \frac{\pi r^4/4}{10\,\text{m} \times \pi D^2/4} \\
y_{cp} - \bar{y} &= \frac{r^2}{40} = 0.00625\ \text{m}
\end{aligned}
$$

Equilibrium



$$\begin{aligned}\sum M_{\text{Hinge}} &= 0 \\ F_H \times (0.00625 \text{ m}) - 1\,\text{m} \times F &= 0\end{aligned}$$

$$\begin{aligned}\text{But } F &= F_{\text{buoy}} - W \\ &= A(10 \text{ m} - \ell)\gamma_{\text{H}_2\text{O}} - 200 \\ &= \frac{\pi}{4}(0.25\,\text{m})^2(10-\ell)(9,810\,\text{N/m}^3) - 200\,\text{N} \\ &= 4815.5 \text{ N} - 481.5\ell \text{ N} - 200 \text{ N} \\ &= (4615.5 - 481.5\ell) \text{ N} \\ \text{where } \ell &= \text{length of chain}\end{aligned}$$

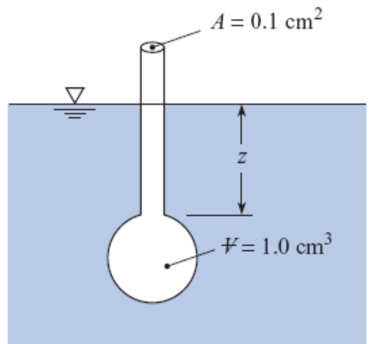
$$\begin{aligned}77,048\,\text{N} \times 0.00625\,\text{m} - 1\,\text{m} \times (4615.5 - 481.5\ell)\,\text{N} &= 0 \\ (481.55 - 4615.5 + 481.5\ell)\ \text{N}\,\text{m} &= 0\end{aligned}$$

$$\boxed{\ell = 8.59 \text{ m}}$$

3.90: PROBLEM DEFINITION

Situation:

A hydrometer is floating in oil.
$\forall_{\text{bulb}} = 1\,\text{cm}^3$, $A_{\text{stem}} = 0.1\,\text{cm}^2$.
$z = 7.2\,\text{cm}$, $W = 0.015\,\text{N}$.



Find:

Specific gravity of oil.

Properties:

$\gamma_W = 9810\,\text{N/m}^3$.

SOLUTION

1. Equilibrium

$$\begin{aligned} F_{\text{buoy.}} &= W \\ \forall_D \gamma_{\text{oil}} &= W \end{aligned}$$

2. Calculations

$$\begin{aligned} (1\text{ cm}^3 + (7.2\text{ cm})(0.1\text{ cm}^2))(0.01^3)\text{ m}^3/\text{cm}^3 \gamma_{\text{oil}} &= 0.015\text{ N} \\ (1 + 0.6) \times 10^{-6}\text{ m}^3 \gamma_{\text{oil}} &= 0.015\text{ N} \\ \gamma_{\text{oil}} &= 8781\text{ N/m}^3 \end{aligned}$$

3. Definition of SG

$$\begin{aligned} SG &= \frac{\gamma_{\text{oil}}}{\gamma_{H_2O}} \\ &= \frac{8781\,\text{N/m}^3}{9810\,\text{N/m}^3} \end{aligned}$$

$$\boxed{SG = 0.89}$$

3.91: PROBLEM DEFINITION

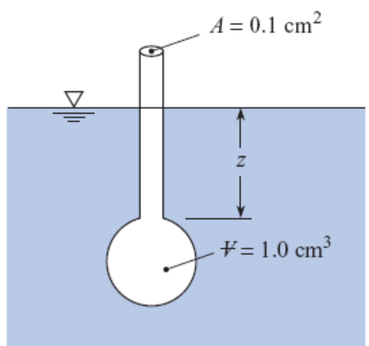
Situation:

A hydrometer in water.

$\forall_{\text{bulb}} = 1\,\text{cm}^3$, $A_{\text{stem}} = 0.1\,\text{cm}^2$.

$z = 4.7\,\text{cm}$.

$T = 15\,°\text{C}$



Find:

Weight of hydrometer.

Properties:

Water, Table A.5 at $T = 15\,°\text{C}$, $\gamma_W = 9800\,\text{N/m}^3$.

SOLUTION

Equilibrium

$$\begin{aligned} F_{\text{buoy.}} &= W \\ \forall_D \gamma_W &= W \end{aligned}$$

Calculations

$$\begin{aligned} (1\text{ cm}^3 + (4.7\text{ cm})(0.1\text{ cm}^2))(0.01\text{m})^3/\text{cm}^3(\gamma_W) &= W \\ (1.47\text{ cm}^3)(10^{-6}\text{ m}^3/\text{cm}^3)(9800\text{ N/m}^3) &= W \end{aligned}$$

$$\boxed{W = 0.0148\text{ N}}$$

3.92: PROBLEM DEFINITION

Situation:

A hydrometer is described in the problem statement.

Find:

Weight of each ball.

Properties:

$SG_{10\%} = 1.012$, $SG_{50\%} = 1.065$.

$\gamma_{water} = 9810\,\mathrm{N/m^3}$.

SOLUTION

Equilibrium (for a ball to just float, the buoyant force equals the weight)

$$F_B = W \tag{1}$$

Buoyancy force

$$F_B = \left(\frac{\pi D^3}{6}\right)\gamma_{\text{fluid}} \tag{2}$$

Combine Eq. (1) and (2) and let $D = 0.01\,\mathrm{m}$.

$$\begin{aligned} W &= \left(\frac{\pi D^3}{6}\right) \times SG \times \gamma_{\text{water}} \\ &= \left(\frac{\pi\,(0.01)^3}{6}\right) \times SG \times (9810) \\ &= 5.136 \times 10^{-3} \times SG \end{aligned} \tag{3}$$

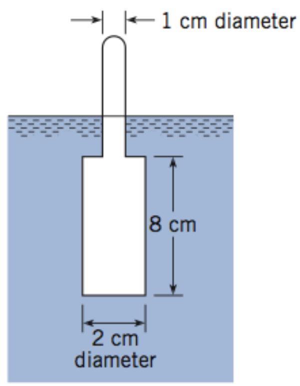
The following table (from Eq. 3) shows the weights of the balls needed for the required specific gravity intervals.

ball number	1	2	3	4	5	6
SG	1.01	1.02	1.03	1.04	1.05	1.06
weight (mN)	5.19	5.24	5.29	5.34	5.38	5.44

3.93: PROBLEM DEFINITION

Situation:

A hydrometer floats in a liquid.
Liquid levels range from btm to top of stem.
$d_1 = 1\,\text{cm}$, $d_2 = 2\,\text{cm}$.
$L_1 = 8\,\text{cm}$, $L_2 = 8\,\text{cm}$.
$W = 40\,\text{g}$.



Find:

Range of specific gravities.

Properties:

$\gamma_{H_2O} = 9810\,\text{N/m}^3$.

SOLUTION

When only the bulb is submerged

$$\begin{aligned} F_B &= W \\ \forall_D \gamma_{H_2O} &= W \\ \frac{\pi}{4}\left[(0.02\,\text{m})^2 \times 0.08\,\text{m}\right] \times 9810\,\text{N/m}^3 \times SG &= 0.040\,\text{kg} \times 9.81\,\text{m/s}^2 \end{aligned}$$

$$SG = 1.59$$

When the full stem is submerged

$$\frac{\pi}{4}\left[(0.02\,\text{m})^2 \times (0.08\,\text{m}) + (0.01\,\text{m})^2 \times (0.08\,\text{m})\right] 9,810\,\text{N/m}^3 \times SG = 0.040\,\text{kg} \times 9.81\,\text{m/s}^2$$

$$SG = 1.27$$

Thus, the range is

$$\boxed{1.27 \leq SG \leq 1.59}$$

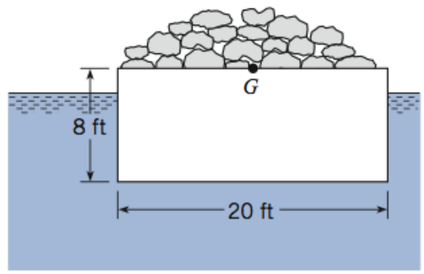
3.94: PROBLEM DEFINITION

Situation:

A barge is floating in water.

$l = 40\,\text{ft}$, $b = 20\,\text{ft}$.

$W = 400,000\,\text{lbf}$.



Find:

Stability of barge.

Properties:

$\gamma_{water} = 62.4\,\text{lbf/ft}^3$.

SOLUTION

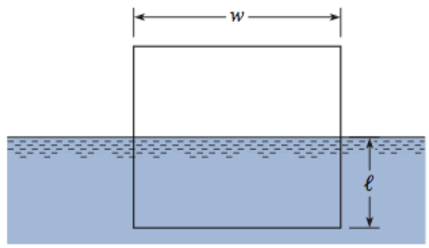
$$\begin{aligned} \text{Draft} &= \frac{400000\,\text{lbf}}{40\,\text{ft} \times 20\,\text{ft} \times 62.4\,\text{lbf/ft}^3} \\ &= 8.01\ \text{ft} > 8\ \text{ft} \end{aligned}$$

Will Sink

3.95: PROBLEM DEFINITION

Situation:

A floating body is in water.



Find:

Location of water line for stability.

Specific gravity (SG) of material.

SOLUTION

For neutral stability, the distance to the metacenter is zero. In other words

$$GM = \frac{I_{oo}}{V\!\!\!\!-} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy.

Moment of inertia at the waterline

$$I_{oo} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is $\ell w L$ so

$$GC = \frac{w^3 L}{12 \ell w L} = \frac{w^2}{12\ell}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{\ell}{2}$$

So

$$\frac{w}{2} - \frac{\ell}{2} = \frac{w^2}{12\ell}$$

or

$$\left(\frac{\ell}{w}\right)^2 - \frac{\ell}{w} + \frac{1}{6} = 0$$

Solving for ℓ/w gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

$$\boxed{\frac{\ell}{w} = 0.211}$$

The weight of the body is equal to the weight of water displaced.

$$\gamma_b \forall_b = \gamma_f \forall_f$$

Therefore

$$SG = \frac{\gamma_b}{\gamma_f} = \frac{w\ell L}{w^2 L} = \frac{\ell}{w} = 0.211$$

$$\boxed{SG = 0.211}$$

The specific gravity is smaller than this value, thus the body will be unstable (floats too high).

3.96: PROBLEM DEFINITION

Situation:

A block of wood.

$d = 1\,\mathrm{m}$, $L = 1\,\mathrm{m}$.

Find:

Stability.

Properties:

$\gamma_{wood} = 7500\,\mathrm{N/\,m^3}$.

SOLUTION

$$\begin{aligned}
\text{draft} &= \frac{1 \times 7500\,\mathrm{N/\,m^3}}{9,810\,\mathrm{N/\,m^3}} = 0.7645\ \mathrm{m} \\
c_{\text{from bottom}} &= \frac{0.7645\,\mathrm{m}}{2} = 0.3823\,\mathrm{m}
\end{aligned}$$

Metacentric height

$$\begin{aligned}
G &= 0.500\ \mathrm{m};\ \mathrm{CG}\ = 0.500 - 0.3823 = 0.1177\ \mathrm{m} \\
GM &= \frac{I}{V\!\!\!/} - \mathrm{CG} \\
&= \frac{\pi R^4/4}{0.7645 \times \pi R^2} - 0.1177 \\
&= 0.0818\ \mathrm{m} - 0.1177\ \mathrm{m}\ \text{(negative)}
\end{aligned}$$

Thus, block is **unstable with axis vertical.**

3.97: PROBLEM DEFINITION

Situation: A block of wood.
$d = 1\,\mathrm{m}$, $L = 1\,\mathrm{m}$.

Properties:
$\gamma_{wood} = 5000\,\mathrm{N/\,m^3}$.

Find:
Stability.

SOLUTION

$$\begin{aligned}\text{draft} &= 1\,\mathrm{m} \times \frac{5000\,\mathrm{N/\,m^3}}{9810\,\mathrm{N/\,m^3}} \\ &= 0.5097\ \mathrm{m}\end{aligned}$$

Metacentric height

$$\begin{aligned}\mathrm{GM} &= \frac{I_{00}}{\forall} - \mathrm{CG} \\ &= \left[\frac{\pi \times (0.5\,\mathrm{m})^4/4}{0.5097\,\mathrm{m} \times \pi \times (0.5\,\mathrm{m})^2}\right] - \left(0.5 - \frac{0.5097}{2}\right)\ \mathrm{m} \\ &= -0.122\ \mathrm{m},\ \text{negative}\end{aligned}$$

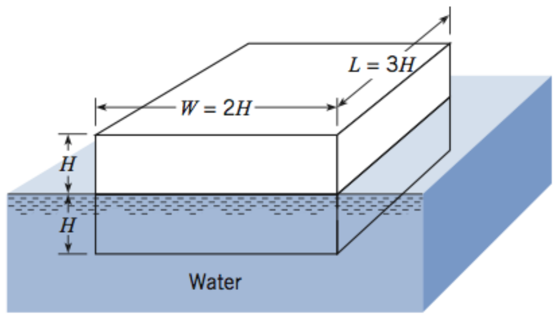
So **will not float stable with its ends horizontal.**

3.98: PROBLEM DEFINITION

Situation:

A floating block is described in the problem statement.

$W = 2H$, $L = 3H$.



Find:

Stability.

SOLUTION

Analyze longitudinal axis

$$\begin{aligned} \text{GM} &= \frac{I_{00}}{\mathcal{V}} - \text{CG} \\ &= \frac{3H(2H)^3}{12 \times H \times 2H \times 3H} - \frac{H}{2} \\ &= -\frac{H}{6} \end{aligned}$$

Not stable about longitudinal axis.

Analyze transverse axis.

$$\begin{aligned} \text{GM} &= \frac{2H \times (3H)^3}{12 \times H \times 2H \times 3H} - \frac{3H}{4} \\ &= 0 \end{aligned}$$

Neutrally stable about transverse axis.

Not stable