Solutions Manual

to accompany

ENGINEERING ELECTROMAGNETICS

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by

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Preface

This Solutions Manual is a supplement to *Engineering Electromagnetics*, which contains 315 problems at the ends of Chapters 2 through 8. We provide here detailed worked-out solutions for *each and every* one of these 315 problems. As educators with a combined >30 years of teaching experience, we firmly believe that practice is the key to learning, and that exams and homeworks are all instruments of teaching, although they are often not regarded as such by students at the time. In our own teaching, we have done our best over the years to provide the students with detailed worked-out solutions of homework and exam problems, rather than cryptic or abbreviated answers. Students in general are greatly appreciative of such a practice and make good use of such solutions in studying the material.

Based on our own experience, we were determined to supply potential users of this book with the ability to readily provide detailed feedback to the students. Accordingly, we took it upon ourselves to prepare thorough and clearly laid-out solutions for every end-of-chapter problem. The entirety of this Solutions Manual has been typeset by ourselves, paying special attention to pedagogical detail.

In the course of preparing this manual, we have uncovered a few errors in problem statements. These and other errata concerning our book is posted at:
http://www2.awl.com/cseng/titles/0-8053-4423-3

We are looking forward to interacting with the users of this book, to collect comments, questions and corrections. We can be most easily reached by EMAIL at
inan@nova.stanford.edu (URL: http//nova.stanford.edu/ vlf) and at ainan@up.edu.

Several of the end-of-chapter problems were initially solved by Teaching Assistants for the Engineering Electromagnetics course taught at Stanford University. We thank these students for their contributions, in particular to Shin-Shiuan Cheng, but also to Anton Lopatinsky, and Arvindh Krishnaswamy. We would also like to express our appreciation of many other students at both Stanford University and University of Portland who have identified errors in earlier versions of the problems.

–Umran S. Inan
–Aziz S. Inan

ERRATA SHEET for Solutions Manual

Engineering Electromagnetics by Inan&Inan

2-7. Observer on line. The last part of this solution should read:

which reaches the center of the line at $t = 1.5t_d$. From the $\mathcal{V}_{\text{ctr}}$ sketch, we have $\mathcal{V}_1^- = 0.3$ V. Substituting above, we find $R_L = 400\Omega$.

(b) For $t_w = 1.5t_d$, the variation of the voltage $\mathcal{V}_{\text{ctr}}$ versus t is as shown.

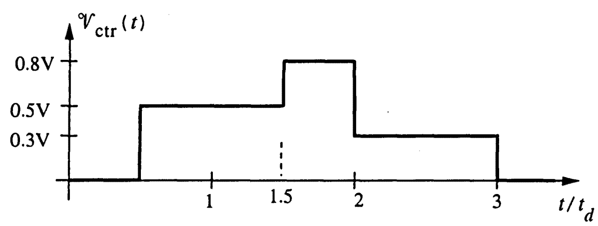


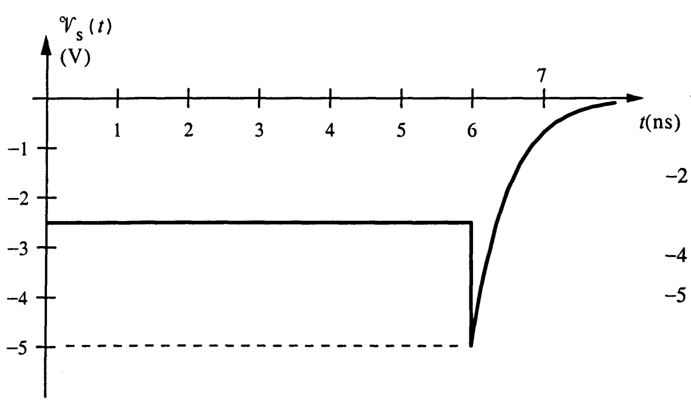
Fig. 1.1. Figure for Problem 2-7. $\mathcal{V}_{\text{ctr}}$ versus t for $t_w = 1.5t_d$.

2-29. Inductive load. The end of part (a) should be:

In a similar fashion, source-end voltage follows as

$$\mathcal{V}_s(t) = -2.5u(t) + \left[2.5 - 5e^{-(t-6 \text{ ns})/(0.5 \text{ ns})}\right] u(t - 6 \text{ ns}) \text{ V}$$

Both $\mathcal{V}_s(t)$ and $\mathcal{V}_L(t)$ are plotted as shown.



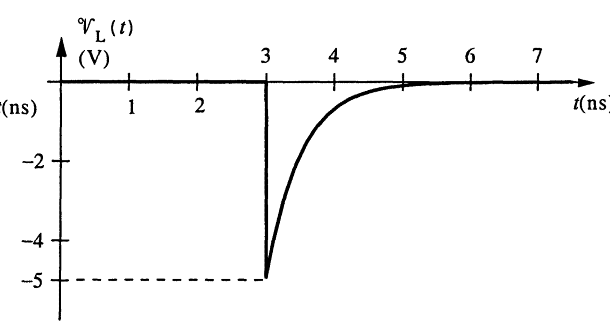


Fig. 1.2. Figure for Problem 2-29a. $\mathcal{V}_s(t)$ and $\mathcal{V}_L(t)$ versus t.

The source-end voltage at the end of part (b) should be:

$$\mathcal{V}_s(t) = -2.5e^{-(t-6 \text{ ns})/(0.25 \text{ ns})}u(t - 6 \text{ ns}) + 2.5u(-t) \text{ V}$$

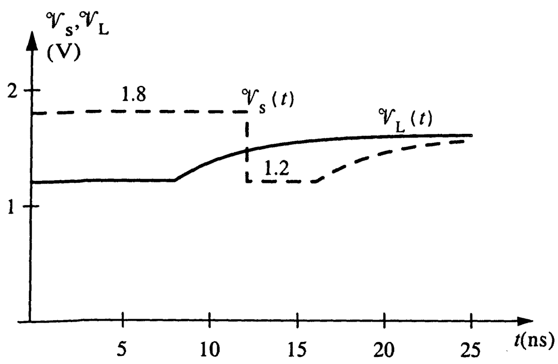


Fig. 1.3. Figure for Problem 2-32. $\mathcal{V}_L$ and $\mathcal{V}_s$ versus t.

2-32. Capacitive load. The plotted curve for $\mathcal{V}_L(t)$ should be shifted up by 1.2 V. The correct plot is shown.

3-19. Power dissipation. The solutions ends without stating the final answer. After finding $P_{in} = 0.5$ W, it should be stated that:

Therefore, $P_L = 0.25$ W.

3-20. Two antennas. In part (b), the numerical value of P_{R_s} is wrong. The correct value is:

$$P_{R_s} = \frac{1}{2}|I_s|^2 R_s \simeq \frac{1}{2}(0.134)^2(100) \simeq 0.898 \text{ W}$$

4-15. Two antennas. The fact that the work required is zero, or $\Phi = 0$ all along the y axis, should be expressed as:

$$\Phi(0, y) = 0$$

4-17. Semi circular line charge. The second half of part (a) should read:

where $\mathbf{r} = 0$, $\rho_l(\mathbf{r}') = \rho_l = \rho_0$, $dl' = a d\phi'$, $(\mathbf{r} - \mathbf{r}') = -(a\cos\phi'\,\hat{\mathbf{x}} + a\sin\phi'\,\hat{\mathbf{y}})$, and $|\mathbf{r} - \mathbf{r}'|^3 = a^3$. Thus,

$$\begin{aligned}\mathbf{E}(0,0,0) &= \frac{\rho_0}{4\pi\epsilon_0}\int_0^{\pi} \frac{-(a\cos\phi'\,\hat{\mathbf{x}} + a\sin\phi'\,\hat{\mathbf{y}})a\,d\phi'}{a^3} \\ &= \frac{-\rho_0\,\hat{\mathbf{y}}}{4\pi\epsilon_0 a}\int_0^{\pi} \sin\phi'\,d\phi' = -\frac{\rho_0}{2\pi\epsilon_0 a}\,\hat{\mathbf{y}}\end{aligned}$$

4-33. Coaxial capacitor with two dielectrics. The capacitor values in parts (b) and (c) should be in pF instead of in nF.

5-17. Resistance of a copper-coated steel wire. The final expression should read:

$$\frac{l}{\sigma_{\text{steel}}\,\pi(0.015)^2} = \frac{l}{\sigma_{\text{copper}}\,\pi[(0.015+a)^2 - (0.015)^2]} \quad \longrightarrow \quad a \simeq 142\ \mu\text{m}$$

5-18. Resistance of an aluminum conductor, steel-reinforced (ACSR) wire. In part (b), the expressions for I_{steel} and I_{aluminum} should both be multiplied with the total current $I_{\text{total}} = I_{\text{steel}} + I_{\text{aluminum}}$.

Also, the numerical values used in parts (b) and (d) are for $I_{\text{total}} = 200$ A, while the given value of the current is 1000 A. To obtain the correct values, the given numerical values should simply be multiplied by 5.

5-23. Leakage resistance. The resulting value for R should be 1.032 kΩ rather than MΩ.

6-2. Forces between two wires. The final expression for I^2 should be

$$\mathbf{F}_{12}^{\text{e}} + \mathbf{F}_{12}^{\text{m}} = 0 \quad \longrightarrow \quad \frac{\mu_0 I^2}{2\pi d} = \frac{\pi\epsilon_0 V_0}{\ln(d/a)} \quad \longrightarrow \quad I^2 = \frac{(2\pi d)\,V_0 \pi\epsilon_0}{\mu_0 \ln(d/a)}$$

Substituting values yields $I = 27.9$ mA.

6-12. Square loop of current. The initial expression for **B** should read:

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\,\frac{\mu_0 I L}{2\pi r\sqrt{r^2 + (L)^2}}$$

B field at a distance r from the center of wire of length $2L$

The rest of the solution is correct.

6-19. Square Helmoltz coils. The procedure followed in this solution is correct. However, the original expression used for the **B** field at a distance r from the center of a current carrying wire of length a should be:

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\frac{\mu_0 I(a/2)}{2\pi r\sqrt{r^2 + (a/2)^2}}$$

This also changes the following expressions for B_{1z} and $B(z)$. However, the result does not change, and the procedure to be followed in order to demonstrate that the optimum distance is $d = 0.5445a$ is correct.

6-36. Inductance of a rectangular toroid. The final expression in part (c) should be:

$$L = \frac{\mu_0 N^2 A}{2\pi r_{\text{m}}} = \frac{4\pi \times 10^{-7}(1000)^2(0.02 - 0.012)(0.015)}{2\pi(0.016)} = 1.5 \times 10^{-3}\ \text{H} = 1.5\ \text{mH}$$

7-16. Induction. recognizing that the problem statement restricts the motion of the loop to be away from both wires, we should define the angle θ is measured downward from the x axis. The second half of part (b) of this problem should then read as follows:

$$\text{Top}: \quad d\mathbf{l} = \hat{\mathbf{x}}dx \quad \longrightarrow \quad (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = v_y B_z dx$$

$$\mathcal{V}_{\text{ind}}^{\text{Top}} = \int_d^{d+a} v_y B_z(x, -d)dx = -v_y \frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{d+a}{d}\right) - \frac{a}{d}\right]$$

$$= +v\sin\theta\frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{d+a}{d}\right) - \frac{a}{d}\right]$$

$$\text{Right}: \quad d\mathbf{l} = -\hat{\mathbf{y}}dy \quad \longrightarrow \quad (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = v_x B_z dy$$

$$\mathcal{V}_{\text{ind}}^{\text{Right}} = \int_{-d}^{-d-b} v_x B_z(d+a, y)dy = v_x\frac{\mu_0 I}{2\pi}\left[\frac{-b}{d+a} + \ln\left(\frac{d+b}{d}\right)\right]$$

$$= v\cos\theta\frac{\mu_0 I}{2\pi}\left[\frac{-b}{d+a} + \ln\left(\frac{d+b}{d}\right)\right]$$

$$\text{Bottom}: \quad d\mathbf{l} = \hat{\mathbf{x}}dx \quad \longrightarrow \quad (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = v_y B_z dx$$

$$\mathcal{V}_{\text{ind}}^{\text{Bottom}} = \int_{d+a}^{d} v_y B_z(x, -d-b)dx = -v_y \frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{d}{d+a}\right) + \frac{a}{d+b}\right]$$

$$= -v \sin\theta \frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{d+a}{d}\right) - \frac{a}{d+b}\right]$$

$$\text{Left}: \quad d\mathbf{l} = -\hat{\mathbf{y}}dy \quad \longrightarrow \quad (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} = v_x B_z dy$$

$$\mathcal{V}_{\text{ind}}^{\text{Left}} = \int_{-d-b}^{-d} v_x B_z(d, y)dy = v_x \frac{\mu_0 I}{2\pi}\left[\frac{b}{d} + \ln\left(\frac{d}{d+b}\right)\right]$$

$$= v\cos\theta \frac{\mu_0 I}{2\pi}\left[\frac{b}{d} - \ln\left(\frac{d+b}{d}\right)\right]$$

The total induced emf is then given by

$$\mathcal{V}_{\text{ind}} = \mathcal{V}_{\text{ind}}^{\text{Top}} + \mathcal{V}_{\text{ind}}^{\text{Right}} + \mathcal{V}_{\text{ind}}^{\text{Bottom}} + \mathcal{V}_{\text{ind}}^{\text{Left}} = \frac{v\mu_0 I}{2\pi}\left[\sin\theta \frac{-ab}{d(b+d)} + \cos\theta \frac{ab}{d(a+d)}\right]$$

To determine the angle θ for which $\mathcal{V}_{\text{ind}}$ is a maximum we can examine the derivative of $\mathcal{V}_{\text{ind}}$ with respect to θ:

$$\frac{\partial \mathcal{V}_{\text{ind}}}{\partial\theta} = \frac{v\mu_0 I}{2\pi}\left[-\cos\theta \frac{ab}{d(b+d)} - \sin\theta \frac{ab}{d(a+d)}\right]$$

For $0 \le \theta \le \pi/2$, the above quantity is always negative, a fact which might lead us to conclude that the motion should be in the $\theta = 0$ direction. However, we should note that our goal is to maximize $|\mathcal{V}_{\text{ind}}|$, without regard to the polarity of $\mathcal{V}_{\text{ind}}$. Thus, it might be the case that as θ varies from 0 to $\pi/2$, $\mathcal{V}_{\text{ind}}$ might start at a positive value but then decrease and eventually go negative with $|\mathcal{V}_{\text{ind}}(\theta = \pi/2)| = |\mathcal{V}_{\text{ind}}(\theta = -\pi/2)| > |\mathcal{V}_{\text{ind}}(\theta = 0)|$. To see this, let $\mathcal{V}_{\text{ind}} = 0$, i.e.,

$$\frac{v\mu_0 I}{2\pi}\left[-\cos\theta \frac{ab}{d(b+d)} - \sin\theta \frac{ab}{d(a+d)}\right] = 0 \quad \longrightarrow \quad \tan\theta_{\text{max}} = \frac{a+d}{b+d}$$

indicating that $\mathcal{V}_{\text{ind}}$ does indeed cross zero. Therefore, we can maximize $|\mathcal{V}_{\text{ind}}|$ in two different ways:

$$\theta = 0 \quad \longrightarrow \quad |\mathcal{V}_{\text{ind}}| = \mathcal{V}_{\text{ind}} = \frac{v\mu_0 I}{2\pi}\frac{ab}{d(a+d)}$$

$$\theta = \frac{\pi}{2} \quad \longrightarrow \quad |\mathcal{V}_{\text{ind}}| = -\mathcal{V}_{\text{ind}} = \frac{v\mu_0 I}{2\pi}\frac{ab}{d(b+d)}$$

Comparing these two cases, it appears that our solution depends on the relative values of a and b. For $a > b$, we should choose $\theta = \pi/2$, with $|\mathcal{V}_{\text{ind}}|_{\text{max}} = v\mu_0 Iab/[2\pi d(b+d)]$, while for $a < b$, we should choose $\theta = 0$, with $|\mathcal{V}_{\text{ind}}|_{\text{max}} = v\mu_0 Iab/[2\pi d(a+d)]$. For $a = b$, either solution produces maximum emf.

Note that for $a \gg b, d$ the result reduces to $\theta_{\text{max}} \to \infty$ or $\theta_{\text{max}} \to \pi/2$, which is equivalent to the sliding bar problem; i.e., the bar has to move in a direction perpendicular to itself in order to induce voltage.

8-28. Uniform plane wave. All of the ϵ_0 terms in this solution should be E_0.

8-33. Air-perfect conductor interface. The spatial variation of the various fields in this problem should be in the form of $e^{-j40\pi x}$ instead of $e^{-j40\pi z}$. The solution is otherwise correct.

8-39. Shielding with copper foil. Part (c) of this problem should read:

(c) For copper with $\sigma = 5.78 \times 10^7$ S-m^{-1}, at 1 GHz we have $\alpha \simeq \sqrt{\omega\mu\sigma/2} \simeq 478513.1$ np-m^{-1}. We then have

$$(\mathbf{S}_{\text{av}})_{\text{emerging}} = \left[(\mathbf{S}_{\text{av}})_{\text{t}}(e^{-2\alpha z})\right]_{z=10\ \mu\text{m}} \simeq 1.07 \quad \text{nW-m}^{-2}$$

8-53. Parallel-plate waveguide modes. The present solution is for an electric field with a spatial variation of $e^{-50\sqrt{2}\pi y}\sin(100\pi z)$ instead of $e^{-60\pi y}\sin(100\pi z)$. The correct solution is as follows: We start with

$$E_x(y, z) = 10e^{-60\pi y}\sin(100\pi z) \quad \text{kV-m}^{-1}$$

(a) The plate separation is given as $a = 2$ cm. We have

$$h = \frac{m\pi}{a} = 100\pi \quad \longrightarrow \quad m = \frac{100\pi a}{\pi} = 100(0.02) = 2 \quad \longrightarrow \quad \text{TE}_2$$

The propagation constant $\bar{\gamma}_m$ appears to be purely real ($\bar{\gamma}_m = 60\pi$) indicating that this is an evanescent, nonpropagating wave.

(b) We have

$$f_{c_2} = \left[\frac{mc}{2a}\right]_{m=2} \simeq \frac{3 \times 10^8}{2 \times 10^{-2}} = 15 \text{ GHz}$$

$$\bar{\gamma}_2 = \bar{\alpha}_2 = 60\pi = \beta\sqrt{\left(\frac{f_{c_2}}{f}\right)^2 - 1}$$

$$60\pi = \frac{2\pi f}{c}\sqrt{\left(\frac{f_{c_2}}{f}\right)^2 - 1} \quad \longrightarrow \quad f \simeq 12 \text{ GHz}$$

(c)

$$f_{c_m} = \frac{mc}{2a} \simeq \frac{m3 \times 10^8}{(2)(2 \times 10^{-2})} = 7.5m \text{ GHz}$$

$$7.5m < 12 \quad \longrightarrow \quad \text{TE}_1 \text{ mode} \quad (\text{since } m = 0 \text{ is not allowed for TE})$$

8-54. Power handling capacity of parallel plate waveguide. The solutions use a value of $a = 1.55$ cm, instead of the given avlue of $a = 1.5$ cm. As a result, the correct value for the cutoff frequency should be 10 GHz (instead of 9.49 GHz), resulting in $\bar{\beta}_1 = 234.16$ rad-m^{-1} and $P_{\text{av}} = 166.82$ kW-(cm)$^{-1}$.

8-56. Power capacity of a parallel-plate waveguide. The correct value of $\bar{\beta}_m$ in parts (a) and (b) should be 0.745 rather than 0.75. The solution of part (c) should read:

(c) For the TM$_1$ mode, we have two electric field components, and breakdown can occur wherever the total electric field magnitude exceeds the breakdown field. From [8.71] we can write

$$\mathbf{E}_{\text{total}} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y = \hat{\mathbf{x}}\frac{\bar{\beta}C_4}{\omega\epsilon}\cos\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z} + \hat{\mathbf{y}}\frac{j\pi C_4}{\omega\epsilon}\sin\left(\frac{\pi}{a}x\right)e^{-j\bar{\beta}z}$$

so that the magnitude of the total field is

$$|\mathbf{E}_{\text{total}}| = \sqrt{\left[\frac{\bar{\beta}C_4}{\omega\epsilon}\cos\left(\frac{\pi}{a}x\right)\right]^2 + \left[\frac{\pi C_4}{\omega\epsilon a}\sin\left(\frac{\pi}{a}x\right)\right]^2}$$

At this point, it is useful to recall the expression [8.74] for $\bar{\beta}$ to rewrite (π/a) in terms of $\bar{\beta}$. We have

$$\bar{\beta}^2 = \omega^2\mu\epsilon - \left(\frac{\pi}{a}\right)^2 \quad \rightarrow \quad \frac{\pi}{a} = \sqrt{\omega^2\mu\epsilon - \bar{\beta}^2} \simeq \sqrt{\omega^2\mu\epsilon - 0.745\sqrt{\mu\epsilon}} \simeq 0.667\omega\sqrt{\mu\epsilon}$$

We now substitute this in the expression for the total electric field magnitude and also set $a = 1$ as before since we are interested in the maximum power transmitted per unit area:

$$\begin{aligned}|\mathbf{E}_{\text{total}}| &\simeq C_4\sqrt{\left[\frac{0.745\omega\sqrt{\mu\epsilon}}{\omega\epsilon}\cos(\pi x)\right]^2 + \left[\frac{0.667\omega\sqrt{\mu\epsilon}}{\omega\epsilon}\sin(\pi x)\right]^2} \\ &= C_4\eta\sqrt{(0.745)^2\cos^2(\pi x) + (0.667)^2\sin^2(\pi x)}\end{aligned}$$

The maximum of the term under the square root occurs at $x = 0$ and $x = 1$ (i.e., at the walls) and is equal to 0.745. Thus, we must have

$$C_4\eta(0.745) \leq 15 \quad \text{kV-cm}^{-1} = 15 \times 10^5 \quad \text{V-m}^{-1} \quad \rightarrow \quad C_4 = 5,338.09 \quad \text{V-m}^{-1}$$

so that the maximum power transmitted per unit area is

$$P_{\text{av}}^{\text{TM}_1} = \frac{\bar{\beta}_1 C_4^2}{4\omega\epsilon} \simeq \frac{(0.745)\omega\sqrt{\mu\epsilon}C_4^2}{4\omega\epsilon} = \frac{(0.745)\eta C_4^2}{4} = 1.99 \times 10^9 = 2 \quad \text{GW-m}^{-2}$$

SOLUTIONS OF PROBLEMS 8-8 to 8-13:

The solutions for these problems were left out from the original camera-ready copy as a result of a printing error:

8-8. Unknown material. Since the intrinsic impedance of the material is real ($\eta = 98\Omega$), we can assume a lossless ($\sigma = 0$) material. The phase velocity and the intrinsic impedance of a lossless material are given by

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \simeq \frac{3 \times 10^8}{\sqrt{\mu_r\epsilon_r}} = 7.8 \times 10^7 \text{ m-s}^{-1}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}} \simeq (377)\sqrt{\frac{\mu_r}{\epsilon_r}} = 98\Omega$$

Solving these two equations simultaneously yields $\mu_r \simeq 1$ and $\epsilon_r \simeq 14.8$ respectively.

8-9. Uniform plane wave. Since $\lambda = 2.5$ cm, $f = c/\lambda \simeq 3 \times 10^8/(0.025) = 1.2 \times 10^{10}$ Hz or 12 GHz. Using $E_0 = 12$ V-m^{-1} and $\eta \simeq 377\Omega$, the magnetic field can be written in phasor form as

$$\mathbf{H} \simeq \hat{\mathbf{x}}\frac{12}{377}e^{-j\frac{\sqrt{3}\beta}{2}y}e^{j\frac{\beta}{2}z}e^{j\theta} \text{ A-m}^{-1}$$

where $\beta = 2\pi/\lambda = 2\pi/(0.025) = 80\pi$ rad-m^{-1}. The corresponding real-time expression follows as

$$\overline{\mathcal{H}}(y, z, t) \simeq \hat{\mathbf{x}}\frac{12}{377}\cos(24\pi \times 10^9 t - 40\sqrt{3}\pi y + 40\pi z + \theta) \text{ A-m}^{-1}$$

When $y = z = 0$ and $t = 0$, the magnetic field is equal to

$$|\overline{\mathcal{H}}(0,0,0)| \simeq \frac{12}{377}\cos\theta = 15.9 \times 10^{-3} \text{ A-m}^{-1}$$

from which we find $\theta \simeq 60° = \pi/3$ rad. The corresponding electric field phasor $\mathbf{E}(y, z)$ can be found from [7.18c] as

$$\mathbf{E}(y, z) = \frac{1}{j\omega\epsilon_0}\nabla \times \mathbf{H}(y, z) = \frac{1}{j\omega\epsilon_0}\left(\hat{\mathbf{y}}\frac{\partial H_x}{\partial z} - \hat{\mathbf{z}}\frac{\partial H_x}{\partial y}\right)$$

Performing the partial derivatives yield

$$\mathbf{E} = \frac{1}{j\omega\epsilon_0}[\hat{\mathbf{y}}j40\pi + \hat{\mathbf{z}}j40\sqrt{3}\pi]\frac{12}{377}e^{-j\frac{\sqrt{3}\beta}{2}y}e^{j\frac{\beta}{2}z}e^{j\pi/3} \text{ V-m}^{-1}$$

Substituting $j\omega\epsilon_0 \simeq j0.668$, we find

$$\mathbf{E}(y, z) = [\hat{\mathbf{y}}6 + \hat{\mathbf{z}}6\sqrt{3}]e^{-j\frac{\sqrt{3}\beta}{2}y}e^{j\frac{\beta}{2}z}e^{j\pi/3} \text{ V-m}^{-1}$$