# CHAPTER 2

## Solutions to Chapter-End Problems

**A. Key Concepts**

Simple Interest:

**2.1** P = 3000
N = 6 months
i = 0.09 per year
= 0.09/12 per month, or 0.09/2 per six months

P + I = P + PiN = P(1 + iN)
= 3000[1 + (0.09/12)(6)] = 3135
*or*
= 3000[1 + (0.09/2)(1)] = 3135

The total amount due is \$3135, which is \$3000 for the principal amount and \$135 in interest.

**2.2** I = 150
N = 3 months
i = 0.01 per month

P = I/(iN) = 150/[(0.01)(3)] = 5000

A principal amount of \$5000 will yield \$150 in interest at the end of 3 months when the interest rate is 1% per month.

**2.3** P = 2000
N = 5 years
i = 0.12 per year

$F = P(1+i)^N = 2000(1+0.12)^5 = 3524.68$

The bank account will have a balance of \$3525 at the end of 5 years.

**2.4** **(a)** P = 21 000
i = 0.10 per year
N = 2 years

$F = P(1+i)^N = 21000(1+0.10)^2 = 25\ 410$

The balance at the end of 2 years will be \$25 410.

**(b)** $P = 2\,900$
$i = 0.12$ per year $= 0.01$ per month
$N = 2$ years $= 24$ months

$F = P(1+i)^N = 2900(1+0.01)^{24} = 3682.23$

The balance at the end of 24 months (2 years) will be $3682.23.

**2.5** From: $F = P(1 + i)^N$

$P = F/(1 + i)^N = 50\,000/(1 + 0.01)^{20} = 40977.22$

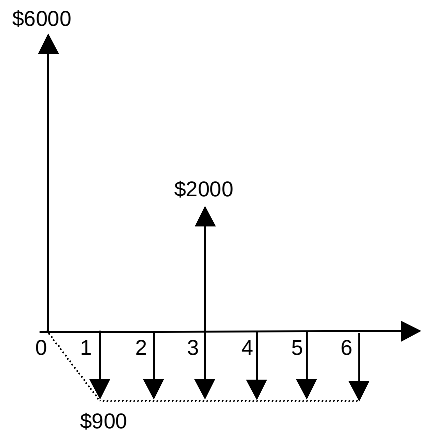
Greg should invest about $40 977.

**2.6** $F = P(1 + i)^N$
$50\,000 = 20\,000(1 + i)^{20}$
$(1+i)^{20} = 5/2$
$i = (5/2)^{1/20} - 1 = 0.04688 = 4.688\%$ per quarter $= 18.75\%$ per year

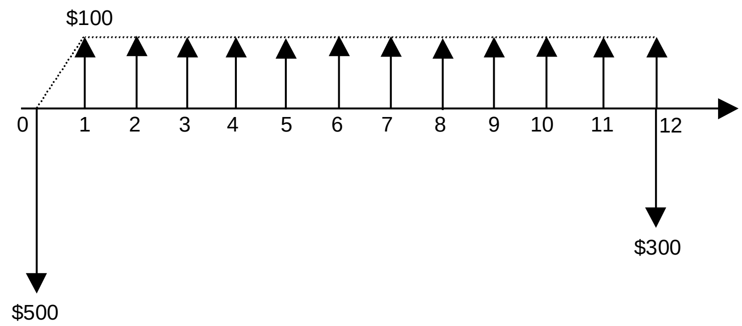
The investment in mutual fund would have to pay at least 18.75% nominal interest, compounded quarterly.

Cash Flow Diagrams:

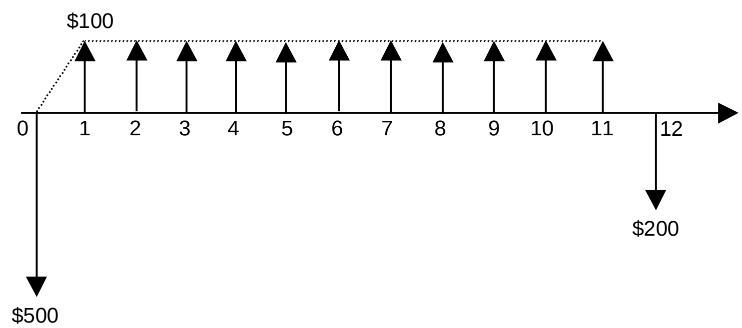
**2.7** Cash flow diagram:



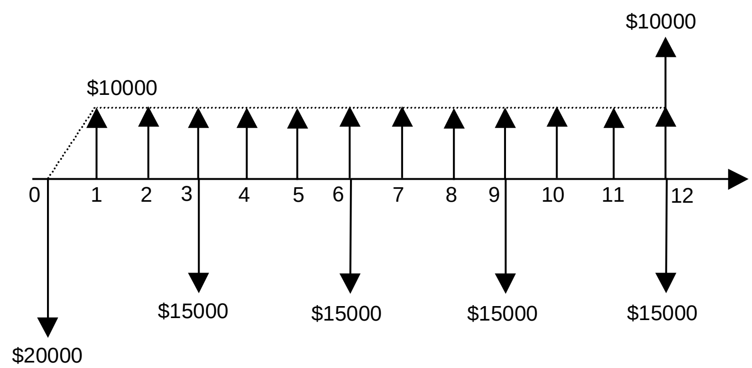

**2.8** Showing cash flow elements separately:



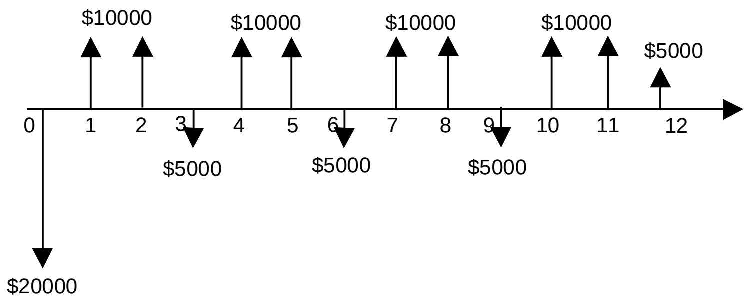

Showing net cash flow:




**2.9** Showing cash flow elements separately:



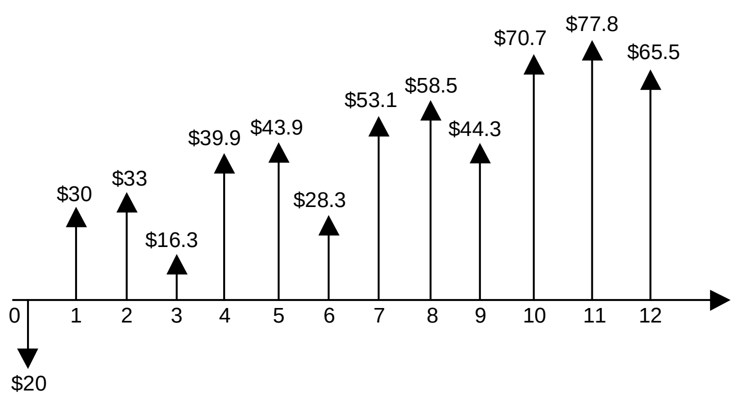

Showing net cash flow:




**2.10** The calculation of the net cash flow is summarized in the table below.

| Time | Payment | Receipt | Net |
|---|---|---|---|
| 0 | 20 | | –20 |
| 1 | | 30 | 30 |
| 2 | | 33 | 33 |
| 3 | 20 | 36.3 | 16.3 |
| 4 | | 39.9 | 39.9 |
| 5 | | 43.9 | 43.9 |
| 6 | 20 | 48.3 | 28.3 |
| 7 | | 53.1 | 53.1 |
| 8 | | 58.5 | 58.5 |
| 9 | 20 | 64.3 | 44.3 |
| 10 | | 70.7 | 70.7 |
| 11 | | 77.8 | 77.8 |
| 12 | 20 | 85.6 | 65.6 |

Cash flow diagram:




**2.11** **(a)** functional loss
**(b)** use-related physical loss
**(c)** functional loss

**(d)** time-related physical loss
**(e)** use-related physical loss
**(f)** use-related physical loss
**(g)** functional loss
**(h)** time-related physical loss

**2.12** **(a)** market value
**(b)** salvage value
**(c)** scrap value
**(d)** market value to Liam, salvage value to Jacque
**(e)** book value

**2.13** The book value of the company is $4.5 based on recent financial statements. The market value is $7 million, assuming that the bid is real and would actually be paid.

**2.14** Since sewing machine technology does not change very quickly nor does the required functionality, functional loss will probably not be a major factor in the depreciation of this type of asset. Left unused, but cared for, the machine will lose some value, and hence time-related loss may be present to some extent. The greatest source of depreciation on a machine will likely be use-related and due to wear and tear on the machine as it is operated.

**2.15** A switch will generally not suffer wear and tear due to use, and thus use-related physical loss is not likely to be a big factor. Nor will there likely be a physical loss due to the passage of time. The primary reason for depreciation will be functional loss - the price of a similar new unit will likely have dropped due to development of new technology and competition in the marketplace.

**2.16** The depreciation is certainly not due to use related physical loss, or other non-physical losses in functionality. The depreciation is a time-related physical loss because it has not being used and maintained over time.

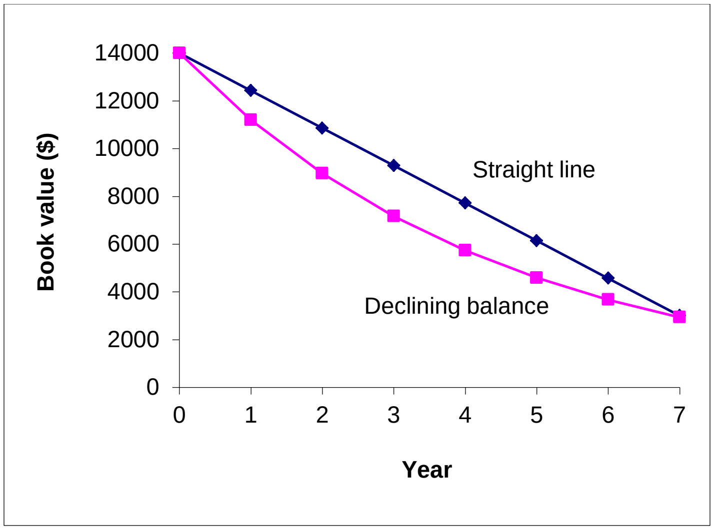
**2.17** **(a)** $BV(1) = 14\ 000 – (14\ 000 – 3000)/7 = \$12\ 429$
**(b)** $BV(4) = 14\ 000 – 4\times(14\ 000 – 3000)/7 = \$7714$
**(c)** $BV(7) = 3000$

**2.18** **(a)** $BV(1) = 14\ 000(1 – 0.2) = \$11\ 200$
**(b)** $BV(4) = 14\ 000(1 – 0.2)^4 = \$5\ 734$
**(c)** $BV(7) = 14\ 000(1 – 0.2)^7 = \$2936$

**2.19** **(a)** $d = 1 – (3000/14\ 000)^{1/7} = 19.75\%$
**(b)** $BV(4) = 14\ 000(1– 0.1975)^4 = \$5806$

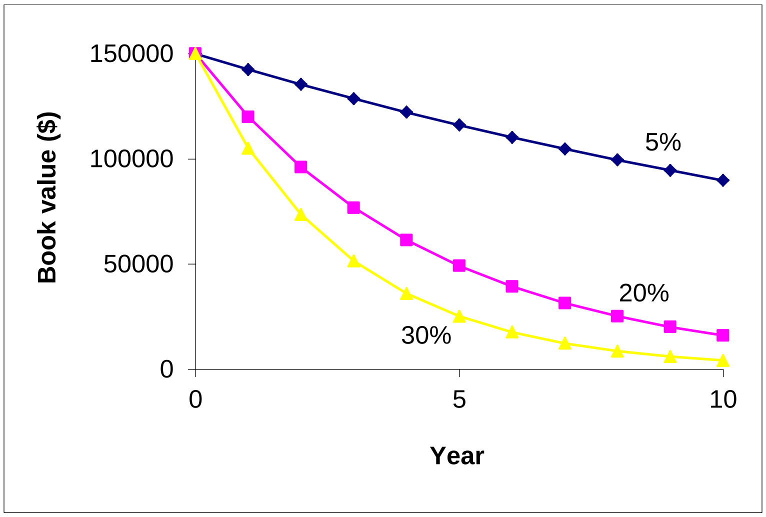
**2.20** Spreadsheet used for chart:

| Year | Straight Line | Declining Balance |
|---|---|---|
| 0 | 14000 | 14000 |
| 1 | 12429 | 11200 |
| 2 | 10857 | 8960 |
| 3 | 9286 | 7168 |
| 4 | 7714 | 5734 |
| 5 | 6143 | 4588 |
| 6 | 4571 | 3670 |
| 7 | 3000 | 2936 |



**2.21** Spreadsheet used for chart:

| Year | d = 5% | d = 20% | d = 30% |
|---|---|---|---|
| 0 | 150000 | 150000 | 150000 |
| 1 | 142500 | 120000 | 105000 |
| 2 | 135375 | 96000 | 73500 |
| 3 | 128606 | 76800 | 51450 |
| 4 | 122176 | 61440 | 36015 |
| 5 | 116067 | 49152 | 25211 |
| 6 | 110264 | 39322 | 17647 |
| 7 | 104751 | 31457 | 12353 |
| 8 | 99513 | 25166 | 8647 |
| 9 | 94537 | 20133 | 6053 |
| 10 | 89811 | 16106 | 4237 |




## B. Applications

**2.22** I = 190.67
P = 550
N = 4 1/3 = 13/3 years

i = I/(PN) = 190.67/[550(13/3)] = 0.08

The simple interest rate is 8% per year.

**2.23** $F = P(1 + i)^N$
$50\ 000 = 20\ 000(1 + 0.01)^N$
$(1.01)^N = 5/2$
N = ln(5/2)/ln(1.01) = 92.09 quarters = 23.02 years

Greg would have to invest his money for about 23.02 years to reach his target.

**2.24** $F = P(1 + i)^N$
$= 20\ 000(1 + 0.01)^{20} = 24\ 403.80$

Greg would have accumulated about \$24 404.

**2.25** **(a)** P = 5000
i = 0.05 per six months
F = 8000

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(8000/5000)/\ln(1 + 0.05) = 9.633$

The answer that we get is 9.633 (six-month) periods. But what does this mean? It means that after 9 compounding periods, the account will not yet have reached \$8000. (You can verify yourself that the account will contain \$7757). Since compounding is done only every six months, we must, in fact, wait 10 compounding periods, or 5 years, for the deposit to be worth *more* than \$8000. At that time, the account will hold \$8144.

**(b)** P = 5000
r = 0.05 (for the full year)
F = 8000
i = r/m = 0.05/2 = 0.025 per six months

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(8000/5000)/\ln(1 + 0.025) = 19.03$

We must wait 20 compounding periods, or 10 years, for the deposit to be worth more than \$8000.

**2.26** P = 500
F = 708.31
i = 0.01 per month

From: $F = P(1 + i)^N$

$N = \ln(F/P)/\ln(1 + i) = \ln(708.31/500)/\ln(1 + 0.01) = 35.001$

The deposit was made 35 months ago.

**2.27** **(a)** P = 1000
i = 0.1
N = 20

$F = P(1 + i)^N = 1000(1+0.1)^{20} = 6727.50$

About \$6728 could be withdrawn 20 years from now.

**(b)** $F = PiN = 1000(0.1)(20) = 2000$

Without compounding, the investment account would only accumulate \$2000 over 20 years.

**2.28** Let $P = X$ and $F = 2X$.

**(a)** By substituting $F = 2X$ and $P = X$ into the formula, $F = P + I = P + PiN$, we get

$2X = X + XiN = X(1 + iN)$
$2 = 1 + iN$
$iN = 1$
$N = 1/i = 1/0.11 = 9.0909$

It will take 9.1 years.

**(b)** From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$. By substituting $F = 2X$ and $P = X$ into this expression of N,

$N = \ln(2X/X)/\ln(1 + 0.11) = \ln(2)/\ln(1.11) = 6.642$

Since compounding is done every year, the amount will not double until the 7th year.

**(c)** Given $r = 0.11$ per year, the effective interest rate is $i = e^r – 1 = 0.1163$.

From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$. By substituting $F = 2X$ and $P = X$ into this expression of N,

$N = \ln(2X/X)/\ln(1 + 0.1163) = \ln(2)/\ln(1.1163) = 6.3013$

Since interest is compounded continuously, the amount will double after 6.3 years.

**2.29** **(a)** $r = 0.25$ and $m = 2$

$i_e = (1 + r/m)^m – 1 = (1 + 0.25/2)^2 – 1 = 0.26563$

The effective rate is approximately 26.6%.

**(b)** $r = 0.25$ and $m = 4$

$i_e = (1 + r/m)^m – 1 = (1 + 0.25/4)^2 – 1 = 0.27443$

The effective rate is approximately 27.4%.

**(c)** $i_e = e^r - 1 = e^{0.25} - 1 = 0.28403$

The effective rate is approximately 28.4%.

**2.30** **(a)** $i_e = 0.15$ and $m = 12$

From: $i_e = (1 + r/m)^m - 1$

$r = m[(1 + i_e)^{1/m} - 1] = 12[(1 + 0.15)^{1/12} - 1] = 0.1406$

The nominal rate is 14.06%.

**(b)** $i_e = 0.15$ and $m = 365$

From: $i_e = (1 + r/m)^m - 1$

$r = m[(1 + i_e)^{1/m} - 1] = 365[(1 + 0.15)^{1/365} - 1] = 0.13979$

The nominal rate is 13.98%.

**(c)** For continuous compounding, we must solve for r in $i_e = e^r - 1$:

$r = \ln(1 + i_e) = \ln(1 + 0.15) = 0.13976$

The nominal rate is 13.98%.

**2.31** $F = P(1 + i)^N$
$14\ 800 = 665(1 + i)^{64}$
$i = 0.04967$

The rate of return on this investment was 5%.

**2.32** The present value of X is calculated as follows:

$F = P(1 + i)^N$
$3500 = X(1 + 0.075)^5$
$X = 2437.96$

The value of X in 10 years is then:

$F = 2437.96(1 + 0.075/365)^{3650} = 4909.12$

The present value of X is \$2438. In 10 years, it will be \$4909.

**2.33** $r = 0.02$ and $m = 365$

$i_e = (1 + r/m)^m – 1 = (1 + 0.02/365)^{365} – 1 = 0.0202$

The effective interest rate is about 2.02%.

**2.34** Effective interest for continuous interest account:
$i_e = e^r – 1 = e^{0.0599} – 1 = 0.08318 = 6.173\%$

Effective interest for daily interest account:
$i_e = (1 + r/m)^m – 1 = (1 + 0.08/365)^{365} – 1 = 0.08328 = 8.328\%$

No, your money will earn less with continuous compounding.

**2.35** $i_e(\text{weekly}) = (1 + r/m)^m – 1 = (1 + 0.055/52)^{52} – 1 = 0.0565 = 5.65\%$
$i_e(\text{monthly}) = (1 + r/m)^m – 1 = (1 + 0.07/12)^{12} – 1 = 0.0723 = 7.23\%$

**2.36** $i_e(\text{Victory Visa}) = (1 + r/m)^m – 1 = (1 + 0.26/365)^{365} – 1 = 0.297 = 29.7\%$
$i_e(\text{Magnificent Master Card}) = (1 + 0.28/52)^{52} – 1 = 0.322 = 32.2\%$
$i_e(\text{Amazing Express}) = (1 + 0.3/12)^{12} – 1 = 0.345 = 34.5\%$

Victory Visa has the lowest effective interest rate, so based on interest rate, Victory Visa seems to offer the best deal.

**2.37** First, determine the effective interest rate that May used to get \$2140.73 from \$2000. Then, determine the nominal interest rate associated with the effective interest:

$F = P(1 + i_e)^N$
$2140.73 = 2000(1 + i_e)^1$
$i_e = 0.070365$

$i_e = e^r – 1$
$0.070365 = e^r – 1$
$r = 0.068$

The *correct* effective interest rate is then:

$i_e = (1 + r/m)^m – 1 = (1 + 0.068/12)^{12} – 1 = 0.07016$

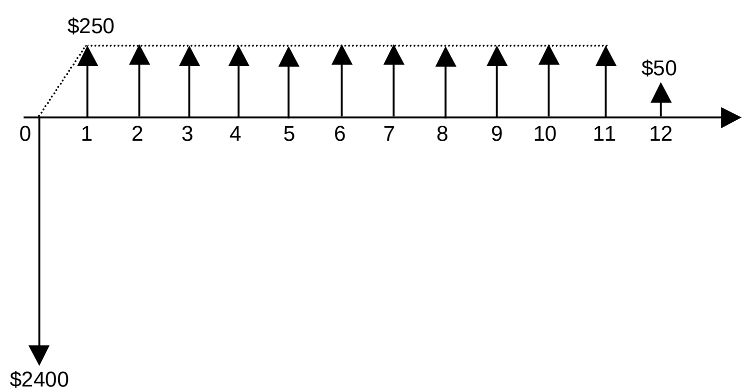
The correct value of \$2000 a year from now is:

$F = P(1 + i_e)^N = 2000(1 + 0.07016)^1 = \$2140.32$

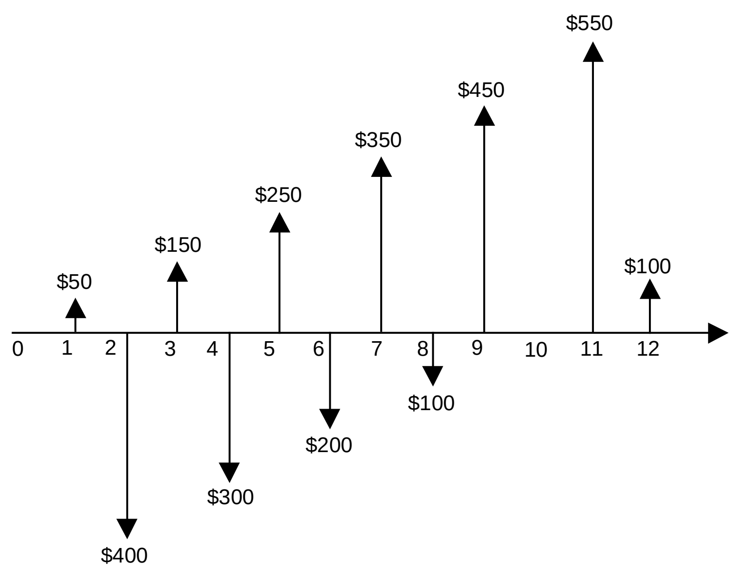
**2.38** The calculation of the net cash flow is summarized in the table below.

| | Investment A | | | Investment B | | |
|---|---|---|---|---|---|---|
| **Time** | **Payment** | **Receipt** | **Net** | **Payment** | **Receipt** | **Net** |
| 0 | 2400 | | –2400 | | | 0 |
| 1 | | 250 | 250 | | 50 | 50 |
| 2 | | 250 | 250 | 500 | 100 | –400 |
| 3 | | 250 | 250 | | 150 | 150 |
| 4 | | 250 | 250 | 500 | 200 | –300 |
| 5 | | 250 | 250 | | 250 | 250 |
| 6 | | 250 | 250 | 500 | 300 | –200 |
| 7 | | 250 | 250 | | 350 | 350 |
| 8 | | 250 | 250 | 500 | 400 | –100 |
| 9 | | 250 | 250 | | 450 | 450 |
| 10 | | 250 | 250 | 500 | 500 | 0 |
| 11 | | 250 | 250 | | 550 | 550 |
| 12 | 200 | 250 | 50 | 500 | 600 | 100 |

Cash flow diagram for investment A:



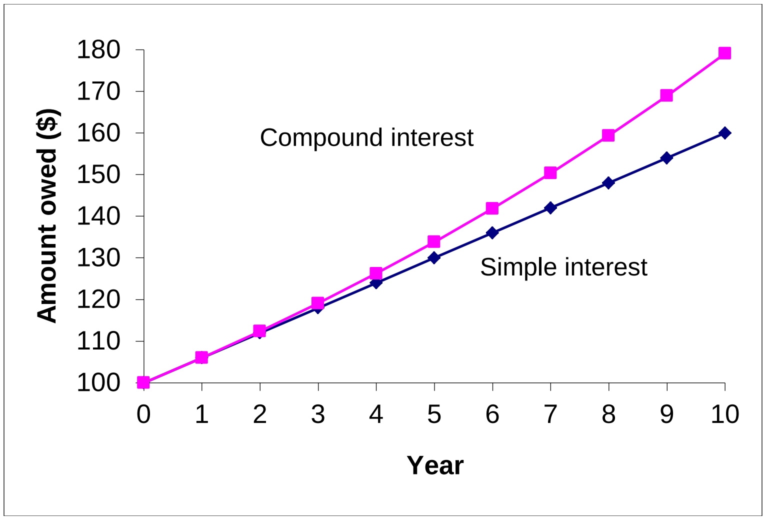

Cash flow diagram for investment B:




Since the cash flow diagrams do not include the time factor (i.e., interest), it is difficult to say which investment may be better by just looking at the diagrams. However, one can observe that investment A offers uniform cash inflows whereas B alternates between positive and negative cash flows for the first 10 months. On the other hand, investment A requires $2400 up front, so it may not be a preferred choice for someone who does not have a lump sum of money now.

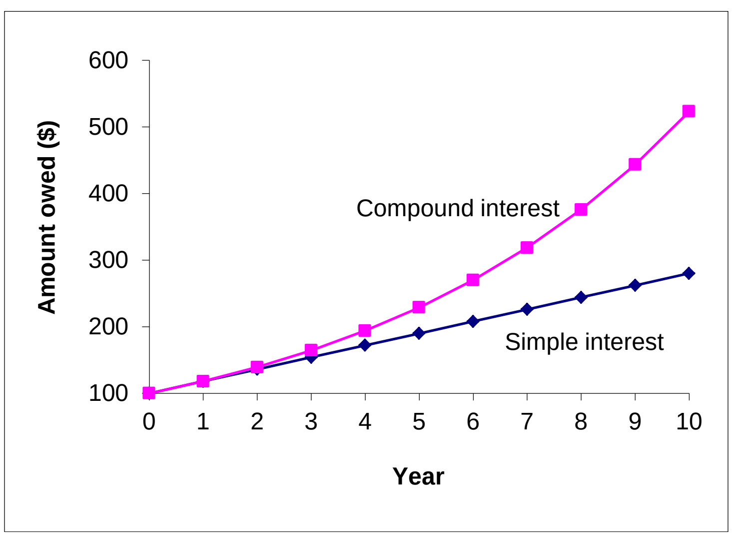
**2.39** **(a)** The amount owed at the end of each year on a loan of $100 using 6% interest rate:

| Year | Simple Interest | Compound Interest |
|---|---|---|
| 0 | 100 | 100.00 |
| 1 | 106 | 106.00 |
| 2 | 112 | 112.36 |
| 3 | 118 | 119.10 |
| 4 | 124 | 126.25 |
| 5 | 130 | 133.82 |
| 6 | 136 | 141.85 |
| 7 | 142 | 150.36 |
| 8 | 148 | 159.38 |
| 9 | 154 | 168.95 |
| 10 | 160 | 179.08 |




**(b)** The amount owed at the end of each year on a loan of $100 using 18% interest rate:

| Year | Simple Interest | Compound Interest |
|---|---|---|
| 0 | 100 | 100.00 |
| 1 | 118 | 118.00 |
| 2 | 136 | 139.24 |
| 3 | 154 | 164.30 |
| 4 | 172 | 193.88 |
| 5 | 190 | 228.78 |
| 6 | 208 | 269.96 |
| 7 | 226 | 318.55 |
| 8 | 244 | 375.89 |
| 9 | 262 | 443.55 |
| 10 | 280 | 523.38 |



**2.40** **(a)** From $F = P(1 + i)^N$, we get $N = \ln(F/P)/\ln(1 + i)$.

At i = 12%:
$N = \ln(1\,000\,000/0.01)/\ln(1 + 0.12) = 162.54$ years

At i = 18%:
$N = \ln(F/P)/\ln(1 + i) = \ln(1\,000\,000/0.01)/\ln(1 + 0.18) = 111.29$ years

**(b)** The growth in values of a penny as it becomes a million dollars:

| Year | At 12% | At 18% |
|---|---|---|
| 0 | 0.01 | 0.01 |
| 10 | 0.03 | 0.05 |
| 20 | 0.10 | 0.27 |
| 30 | 0.30 | 1.43 |
| 40 | 0.93 | 7.50 |
| 50 | 2.89 | 39.27 |
| 60 | 8.98 | 205.55 |
| 70 | 27.88 | 1 075.82 |
| 80 | 86.58 | 5 630.68 |
| 90 | 268.92 | 29 470.04 |
| 100 | 835.22 | 154 241.32 |
| 110 | 2 594.07 | 807 273.70 |
| 120 | 8 056.80 | 4 225 137.79 |
| 130 | 25 023.21 | 22 113 676.39 |
| 140 | 77 718.28 | 115 739 345.70 |
| 150 | 241 381.18 | 605 760 702.48 |
| 160 | 749 693.30 | 3 170 451 901.72 |
| 170 | 2 328 433.58 | 16 593 623 884.84 |
| 180 | 7 231 761.26 | 86 848 298 654.83 |

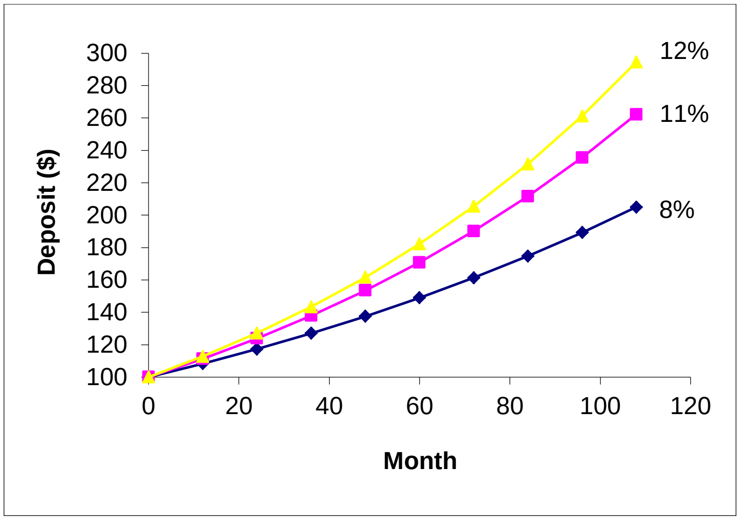
**2.41** From the table and the charts below, we can see that $100 will double in
**(a)** 105 months (or 8.75 years) if interest is 8% compounded monthly

**(b)** 13 six-month periods (6.5 years) if interest is 11% per year, compounded semi-annually

**(c)** 5.8 years if interest is 12% per year compounded continuously

| Month | 8% | 11% | 12% |
|---|---|---|---|
| 0 | 100.00 | 100.00 | 100.00 |
| 12 | 108.30 | 111.30 | 112.75 |
| 24 | 117.29 | 123.88 | 127.12 |
| 36 | 127.02 | 137.88 | 143.33 |
| 48 | 137.57 | 153.47 | 161.61 |
| 60 | 148.98 | 170.81 | 182.21 |
| 72 | 161.35 | 190.12 | 205.44 |

| | | | |
|---|---|---|---|
| 84 | 174.74 | 211.61 | 231.64 |
| 96 | 189.25 | 235.53 | 261.17 |
| 108 | 204.95 | 262.15 | 294.47 |




**2.42** $P(1-d)^n = P - n(P - S)/N$
$245\ 000(1-d)^{20} = 245\ 000 - 20(245\ 000 - 10\ 000)/30$
$(1-d)^{20} = 88\ 333.33/245\ 000 = 0.3605$
$1 - d = 0.9503$
$d = 4.97\%$

The two will be equal in 20 years with a depreciation rate of 4.97%.

**2.43** $780\ 000(1-d)^{20} = 60\ 000$
$(1-d)^{20} = 1/13$
$d = 1 - (1/13)^{1/20} = 1 - 0.8796 = 0.1204$

A depreciation rate of about 12% will produce a book value in 20 years equal to the salvage value of the press.

**2.44** **(a)** $BV(4) = 150\ 000 - 4[(150\ 000 - 25\ 000)/10]$
$= 150\ 000 - 4(12\ 500) = 150\ 000 - 50\ 000 = 100\ 000$
$DC(5) = (150\ 000 - 25\ 000)/10 = 12\ 500$

**(b)** $BV(n) = 150\ 000(1 - 0.2)^4 = 150\ 00(0.8)^4 = 61\ 440$
$DC(5) = BV(4) \times 0.2 = 61\ 440(0.2) = 12\ 288$

**(c)** $d = 1 - (25\ 000/150\ 000)^{1/10} = 0.1640 = 16.4\%$

## C. More Challenging Problems

**2.45** The present worth of each instalment:

| Instalment | F | P |
|---|---|---|
| 1 | 100000 | 100000 |
| 2 | 100000 | 90521 |
| 3 | 100000 | 81941 |
| 4 | 100000 | 74174 |
| 5 | 100000 | 67143 |
| 6 | 100000 | 60779 |
| 7 | 100000 | 55018 |
| 8 | 100000 | 49803 |
| 9 | 100000 | 45082 |
| 10 | 100000 | 40809 |
| | **Total** | 665270 |

Sample calculation for the third instalment, which is received at the end of the second year:

$P = F/(1 + r/m)^N = 100\ 000/(1 + 0.10/12)^{24} = 81\ 941$

The total present worth of the prize is $665 270, not $1 000 000.

**2.46** The present worth of the lottery is $665 270. If you take $300 000 today, that leaves a present worth of $365 270. The future worth of $365 270 in 5 years (60 months) is:

$F = P(1 + r/m)^N = 365\ 270(1 + 0.10/12)^{60} = 600\ 982$

The payment in 5 years will be $600 982.

**2.47** The first investment has an interest rate of 1% per month (compounded monthly), the second 6% per 6 month period (compounding semi-annually).

**(a)** Effective semi-annual interest rate for the first investment:
$i_e = (1 + i_s)^N – 1 = (1 + 0.01)^6 – 1 = 0.06152 = 6.152\%$

Effective semi-annual interest rate for the second investment is 6% as interest is already stated on that time period.

**(b)** Effective annual interest rate for the first investment:
$i_e = (1 + i_s)^N – 1 = (1 + 0.01)^{12} – 1 = 0.1268 = 12.68\%$

Effective annual interest rate for the second investment:
$i_e = (1 + i_s)^N – 1 = (1 + 0.06)^2 – 1 = 0.1236 = 12.36\%$

**(c)** The first investment is the preferred choice because it has the higher effective interest rate, regardless of on what period the effective rate is computed.

**2.48** **(a)** i = 0.15/12 = 0.0125, or 1.25% per month

The effective annual rate is:

$i_e = (1 + i)^m - 1 = (1 + 0.0125)^{12} - 1 = 0.1608$ or 16.08%

**(b)** P = 50 000
N = 12
i = 0.15/12 = 0.0125, or 1.25% per month

$F = P(1 + i)^N = 50\ 000(1 + 0.0125)^{12} = 58\ 037.73$

You will have $58 038 at the end of one year.

**(c)** Adam's Fee = 2% of F = 0.02(58037.73) = 1160.75

Realized F = 58 037.73 – 1160.75 = 56 876.97

The effective annual interest rate is:

$F = P(1 + i)^1$
56 876.97 = 50 000(1 + i)
i = 56 876.97/50 000 – 1 = 0.1375 or 13.75%

The effective interest rate of this investment is 13.75%.

**2.49** Market equivalence does not apply as the cost of borrowing and lending is not the same. Mathematical equivalence does not hold as neither 2% nor 4% is the rate of exchange between the $100 and the $110 one year from now. Decisional equivalence holds as you are indifferent between the $100 today and the $110 one year from now.

**2.50** Decisional equivalence holds since June is indifferent between the two options. Mathematical equivalence does not hold since neither 8% compounded monthly (lending) or 8% compounded daily (borrowing) is the rate of exchange representing the change in the house price ($110 000 now and $120 000 a year later is equivalent to the effective interest rate of 9.09%). Market equivalence also does not hold since the cost of borrowing and lending is not the same.

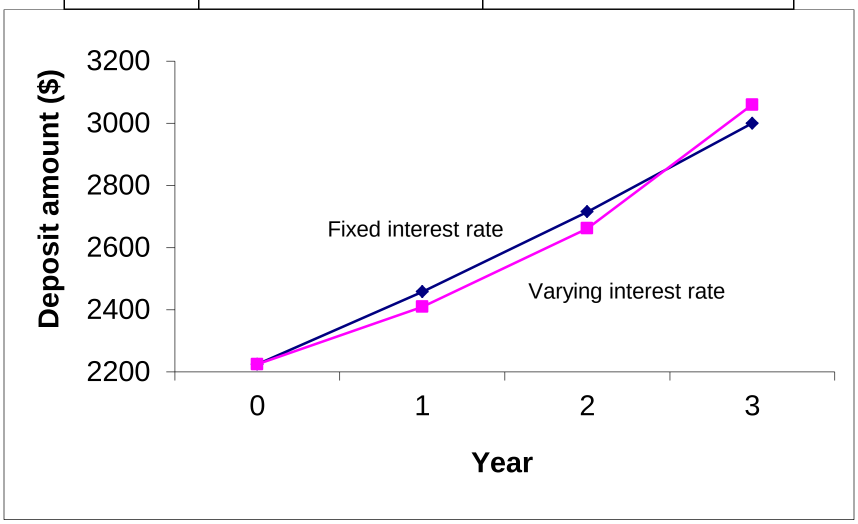
**2.51** **(a)** The amount of the initial deposit, P, can be found from $F = P(1 + i)^N$

with F = \$3000, N = 36, and i = 0.10/12.

**(b)** Having determined P = \$2225, then we can figure out the size of the deposit at the end of years 1, 2 and 3.

If you had not invested in the fixed interest rate investment, you would have obtained interest rates of 8%, 10%, and 14% for each of the three years. The table below shows how much the initial deposit would have been worth at the end of each of the three years if you had been able to reinvest each year at the new rate. Because of the surge in interest rates in the third year, with 20/20 hindsight, you would have been better off (by about \$60) not to have locked in at 10% for three years.

| Year | Fixed Interest Rate | Varying Interest Rate |
|---|---|---|
| 0 | 2225 | 2225 |
| 1 | 2458 | 2410 |
| 2 | 2715 | 2662 |
| 3 | 3000 | 3060 |




**2.52** Interest rate *i* likely has its origins in commonly available interest rates present in Marlee's financial activities such as investing or borrowing money. Interest rate *j* can only be determined by having Marlee choose between X and Y to determine at which interest rate Marlee is indifferent between the choice. Interest rate *k* probably does not exist for Martlee, since it is unlikely that she can borrow and lend money at the same interest rate. If for some reason she could, then *k*=*j*.

Also, *i* could be either greater or less than *j.*

**2.53** BV(0) = 250 000
$BV(6) = 250\ 000 \times (1 - 0.3)^6 = 29\ 412.25$

The book values of the conveyor after 7, 8, 9, and 10 years are:

$BV(7) = 29\ 412.25 - 29\ 412.25/4 \times 1 = 22\ 059.19$
$BV(8) = 29\ 412.25 - 29\ 412.25/4 \times 2 = 14\ 706.13$
$BV(9) = 29\ 412.25 - 29\ 412.25/4 \times 3 = 7353.07$
$BV(10) = 29\ 412.25 - 29\ 412.25/4 \times 4 = 0$

**2.54** $d = 1 - (S/P)^{1/n} = 1 - (8300/12\ 500)^{1/2} = 1 - 0.81486 = 0.18514 = 18.514\%$
$BV_{db}(5) = 12\ 500\ (1 - 0.18514)^5 = 4470.87$

Enrique should expect to get about $4471 for his car three years from now.

# Notes for Case-in-Point 2.1

**1)** Close, if the appropriate depreciation method is being used.

**2)** It makes sense because it is a new technology.

**3)** Because the accounting department is likely using a specific depreciation method that is not particularly accurate in this case. In particular, they may be using a depreciation method required for tax purposes.

**4)** Bill Fisher is probably not doing anything wrong, but it wouldn't hurt to check..

# Notes for Mini-Case 2.1

**3)** Money will always be lost over the year. If money could be gained, everybody would borrow as much money as possible to invest.

# Solutions to All Additional Problems

**Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.**

## 2S.1

You can assume that one month is the shortest interval of time for which the quoted rental rates and salaries apply. Assembling the batteries will require 24 person-months, and the associated rental space. To maximize the interest you receive from your savings, and minimize the interest you pay on your line of credit, you should defer this expenditure till as late in the year as possible. So you leave your money in the bank till December 1, then purchase the necessary materials and rent the industrial space. Assume that salaries will be paid at the end of the month.

As of December 1, you have $100 000(1.005)^{11}$ = $105 640 in the bank.

You need to spend $360 000 on materials and $240 000 to rent space. After spending all you have in the bank, you therefore need to borrow an additional $494 360 against your line of credit.

As of December 31, you owe $494 360(1.01) = $499 304 to the bank, and you owe $240 000 in salaries. So after depositing the government cheque and paying these debts, you have

1 200 000 – 499 304 – 240 000 = $460 696 in the bank.

This example illustrates one of the reasons why Just-in-Time (“JIT”) manufacture has become popular in recent years: You want to minimize the time that capital is tied up. An additional motivation for JIT would become evident if you were to consider the cost of storing the finished batteries before delivery.

Be aware, however, that the JIT approach also carries risks. December is typically a time when labour, space, and credit are in high demand so there is a possibility that the resources you need will be unavailable or more expensive than expected, and there will then be no time to recover. We will look at methods for managing risk in Chapter 12.

## 2S.2

We want to solve the equation

Future worth = Present worth $(1+i)^N$, where the future worth is twice the present worth.

So we have

$2 = (1+i)^N$

Taking logarithms on both sides, we get

$N = \log(2) / \log(1+i)$

For small values of $i$, log $(1+i)$ is approximately $i$ (this can be deduced from the Taylor series).

And log(2) = 0.69315. So, expressing $i$ as a percentage rather than a fraction, we have:

$N = 69.3 / i$

Since this is only an approximation, we will adjust 69.3 to an easily factored integer, 72, thus obtaining

$N = 72 / i$

**2S.3**

Gita is paying 15% on her loan over a two-week period, so the effective annual rate is

$(1.15^{26} - 1) \times 100\% = 3686\%$

The Grameen Bank was awarded the Nobel Peace Prize in 2006 for making loans available to poor investors in Bangladesh at more reasonable rates.

**2S.4**

Five hundred years takes us beyond the scope of the tables in Appendix A, so we employ the formula

$P = F / (1+i)^N$

to find the present value of the potential loss.

In this case, we have P = \$1 000 000 000 / $(1.05)^{500}$ = \$0.025, or two-and-a-half cents. This implies that it is not worth going to any trouble to make the waste repository safe for that length of time.

This is a rather troubling conclusion, because the example is not imaginary; the U.S., for example, is currently trying to design a nuclear waste repository under Yucca Mountain in Nevada that will be secure for ten thousand years—twenty times as long as in our example. It is not clear how the engineers involved in the project can rationally plan how to allocate their funds, since the tool we usually use for that purpose—engineering economics—gives answers that seem irrational.

**2S.5**

There is no “right” answer to this question, which is intended for discussion in class or in a seminar. Some of the arguments that might be advanced are as follows:

One option is to say, “You cannot play the numbers game with human lives. Each life is unique and of inestimable value. Attempting to treat lives on the same basis as dollars is both cold-blooded and ridiculous.”

But this really won’t do. Medical administrators, for example, do have a responsibility to save lives, and they have limited resources to meet this responsibility. If they are to apportion their resources rationally, they must be prepared to compare the results of different strategies.

To support the point of view that future lives saved should be discounted by some percentage in comparison with present lives, the following arguments might be offered:

1. Suppose we make the comparison fifty years in the future. If we spend our resources on traffic police, we will have saved the lives of those who would have died in accidents, and, because we spent the money that way, the world of fifty years hence will also contain the descendants of those who would have died. So the total number of live humans will be increased by more than fifty.

(This argument assumes that creating a new life is of the same value as preserving an existing life. We do not usually accept that assumption; for example, many governments may promote population control by limiting the number of children born, but it would be unacceptable to control the population by killing off the old and infirm. To give another example, if I am accused of murder, it would not be an acceptable defence to argue that I have fathered two children, and have thus made a greater contribution to society than a law-abiding bachelor.)

2. Just as I charge you interest on a loan because of the uncertainty of what might happen between now and the due date—you could go bankrupt, I could die, etc.—so we should discount future lives saved because we cannot anticipate how the world will change before the saving is realized. For example, a cure for cancer could be discovered ten years from now, and then all the money spent on the anti-smoking campaign will have been wasted, when it could have been used to save lives lost in highway accidents.

3. We should not be counting numbers of lives, but years of human life. Thus it is better to spend the money on preventing accidents, because these kill people of all ages, while cancer and heart disease are mostly diseases of later

life; so more years of human life are saved by the first strategy.

4. The world population is growing. Thus, for humanity as a whole, losing a fixed number of lives can more easily be born in the future than it can now. (This is an extension of the argument that it is worse to kill a member of an endangered species than of a species that is plentiful.)


# Engineering Economics

## Chapter 2

## Time Value of Money

# Outline

2.1 Introduction

2.2 Interest and Interest Rates

2.3 Compound and Simple Interest

- 2.3.1 Compound Interest
- 2.3.2 Simple Interest

2.4 Effective and Nominal Interest Rates

2.5 Continuous Compounding

2.6 Cash Flow Diagrams

2.7 Depreciation

- 2.7.1 Reasons for Depreciation
- 2.7.2 Value of an Asset
- 2.7.3 Straight-Line Depreciation
- 2.7.4 Declining Balance Depreciation

2.8 Equivalence

- 2.8.1 Mathematical Equivalence
- 2.8.2 Decisional Equivalence
- 2.8.3 Market Equivalence




# 2.1 Introduction

- Engineering decisions frequently involve tradeoffs among costs and benefits occurring *at different times*
  - Typically, we invest in project today to gain future benefits
- Chapter 2 discusses economic methods used to compare benefits and costs occurring at different times
- The key to making these comparisons is the use of **interest rates** discussed in Sections 2.2 to 2.5
- Section 2.6 introduces Cash Flow Diagrams
- Section 2.7 explains Depreciation models
- Section 2.8 discusses the equivalence of costs and benefits that occur at different times




# 2.2 Interest and Interest Rates

- Interest (*I*) is compensation for giving up use of money
  - difference between the amount loaned and the amount repaid
- An amount of money today, *P*, can be related to a future amount, *F*, by the interest amount *I*, or interest rate *i*:

$$F = P + I = P + Pi = P(1 + i)$$

- Right to *P* at beginning is exchanged for right to *F* at end, where $F = P(1+i)$
- *i* → **interest rate,** *P* → **present worth** of *F*
- *F* → **future worth** of *P*, base period → **interest period**




# 2.2 Interest and Interest Rates (cont'd)

- The dimension of an interest rate is (dollars/dollars)/time.
- i.e. if $1 is lent at a 9% interest rate
  – then $0.09/year would be paid in interest per time period
- period over which interest calculated is **interest period**.

## CLOSE-UP 2.2 Interest Periods

| Interest Period | Interest Is Calculated: |
|---|---|
| **Semiannually** | Twice per year, or once every six months |
| **Quarterly** | Four times a year, or once every three months |
| **Monthly** | 12 times per year |
| **Weekly** | 52 times per year |
| **Daily** | 365 times per year |
| **Continuous** | For infinitesimally small periods |

# 2.3 Compound and Simple Interest

## 2.3.1 Compound Interest

- If amount *P* is lent for one period at interest rate, *i*
  - then amount repaid at the end of the period is $F = P(1 + i)$.
- If more than one period, interest is usually **compounded**
  - (i.e. end of each period, interest is added to principal that existed at the beginning of that period)
- The interest accumulated is:

$$F = P(1 + i)^N$$

$$I_C = F - P = P(1 + i)^N - P$$


# Table 2.1 Compound Interest Computations

**Table 2.1 Compound Interest Computations**

| Beginning of Period | Amount Lent | | Interest Amount | Amount Owed at Period End |
|---|---|---|---|---|
| 1 | $P$ | + | $Pi$ | $= P + Pi = P(1 + i)$ |
| 2 | $P(1 + i)$ | + | $P(1 + i)i$ | $= P(1 + i) + P(1 + i)i = P(1 + i)^2$ |
| 3 | $P(1 + i)^2$ | + | $P(1 + i)^2 i$ | $= P(1 + i)^2 + P(1 + i)^2 i = P(1 + i)^3$ |
| ⋮ | ⋮ | | | |
| $N$ | $P(1 + i)^{N-1}$ | + | $[P(1 + i)^{N-1}]i = P(1 + i)^N$ | |




## Example 2.2

- With $i$ = 10% per year, how much is owed on a loan of $100 at the end of 3 years?
- What is the compound interest amount?

$F = P(1 + i)^N = 100(1 + 0.10)^3 = \$133.10$

$I_C = F - P = \$133.10 - \$100.00 = \$\ \ 33.10$

The amount owed is $133.10 The interest owed is $33.10 (See Table 2.2 for yearly accrual)

What is the amount owed at each year end?




## 2.3 Compound and Simple Interest (cont'd)

### 2.3.2 Simple Interest

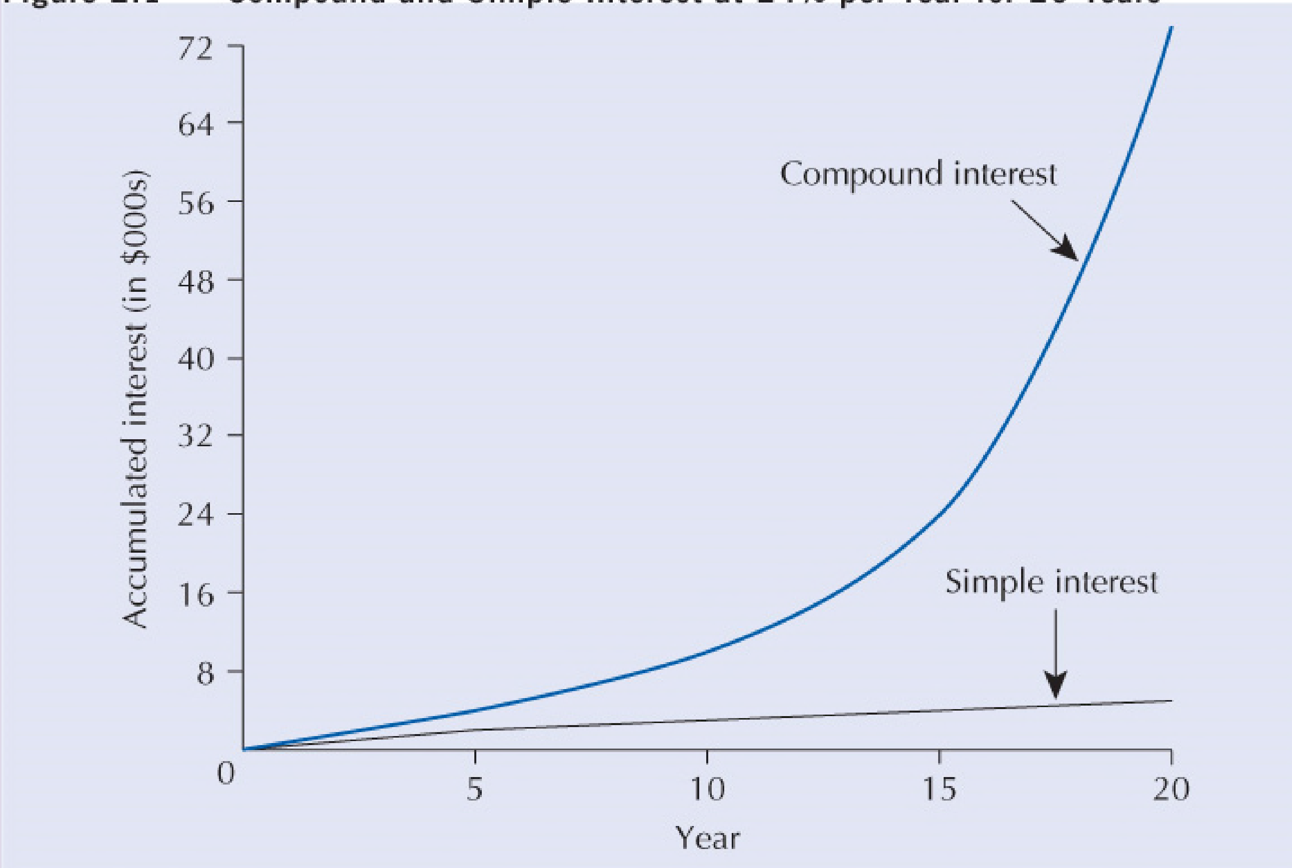
**Simple Interest** – interest without compounding (interest is not added to principal at end of period)

$$I_S = P\,i\,N$$

- Compound and simple interest amounts equal if $N = 1$.
- As $N$ increases, difference between accumulated interest amounts for the two methods increases exponentially
- The conventional approach for computing interest is the **compound interest** method
- Simple interest is rarely used

# Figure 2.1 Compound and Simple Interest at 24% Per Year for 20 Years

Figure 2.1 Compound and Simple Interest at 24% per Year for 20 Years






# 2.4 Effective and Nominal Interest Rates

- Interest rates stated for some period, usually a year
- Computation based on shorter compounding sub-periods
- In this section we consider the relation between:
  - The **nominal interest rate** stated for the full period.
  - The **effective interest rate** that results from the compounding based on the subperiods.
- Unless otherwise noted, rates are **nominal annual rates**
- Suppose: ***r*** is nominal rate stated for a period (1 year) consisting of ***m*** equal compounding periods (sub-periods)
- *If* $i_s = r/m$… then $F = P(1 + i_s)^m = P(1 + i_e)$

## 2.4 Effective and Nominal Interest (cont'd)

- **Effective interest rate, $i_e$**, gives same future amount, $F$, over the full period as when sub-period interest rate, $i_S$, is compounded over $m$ sub-periods $F = P(1 + i_S)^m = P(1 + i_e)$

**EXAMPLE 2.6** Cardex Credit Card Co. charges a nominal 24 percent interest on overdue accounts, compounded daily. **What is the effective interest rate?**

Since $F = P(1 + i_S)^m = P(1 + i_e)$, $i_e = (1 + i_S)^m - 1$

where $i_s = r/m = 0.24/365 = 0.0006575$

*then* $i_e = (1 + i_S)^m - 1 = (1 + 0.0006575)^{365} - 1 = 0.271$

- **The effective interest rate is 27.1%**




## 2.5 Continuous Compounding

- Suppose that the nominal interest rate is 12% and interest is compounded **semi-annually**
- We compute the effective interest rate as follows: where $r = 0.12$, $m = 2$

$$\mathbf{i_s = r/m = 0.12/2 = 0.06}$$

$$i_e = (1 + i_S)^m - 1 = (1 + 0.06)^2 - 1 = .1236\ (12.36\%)$$

What if interest were compounded **monthly?**

$$i_e = (1 + i_S)^m - 1 = (1 + 0.01)^{12} - 1 = 0.1268\ (12.68\%)$$

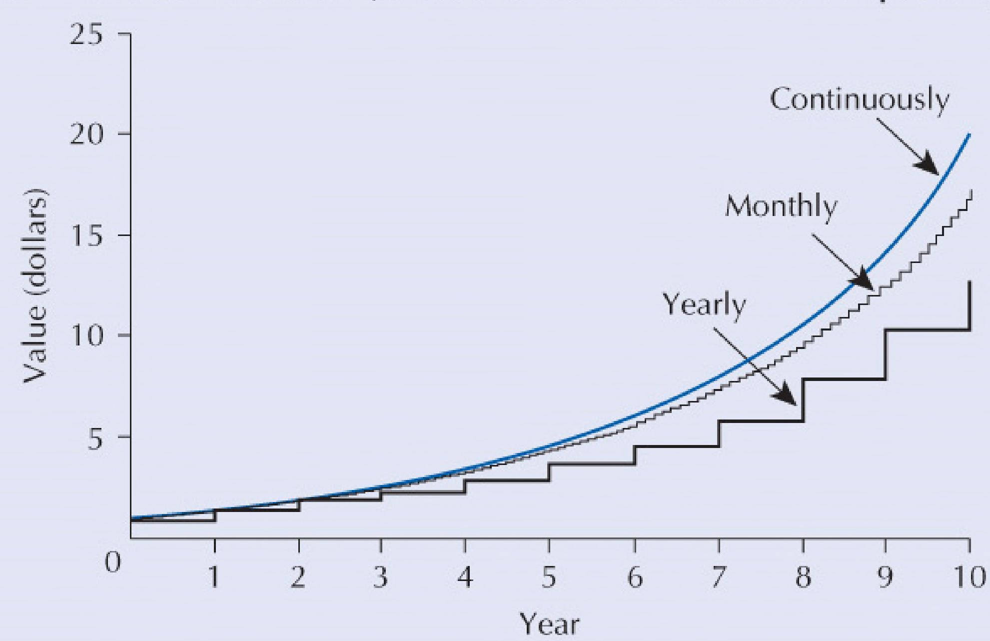
- **Daily**? $i_e = 0.127475$ or about 12.75%
- **More than daily**? …Continuous Compounding




# 2.5 Continuous Compounding (cont'd)

- The effective interest rate under continuous compounding is: $I_e = e^r - 1$

Figure 2.2 Growth in Value of $1 at 30% Interest for Various Compounding Periods



## 2.5 Continuous Compounding (cont'd)

- To compute *effective* interest rate for nominal interest rate of 12% by continuous compounding:

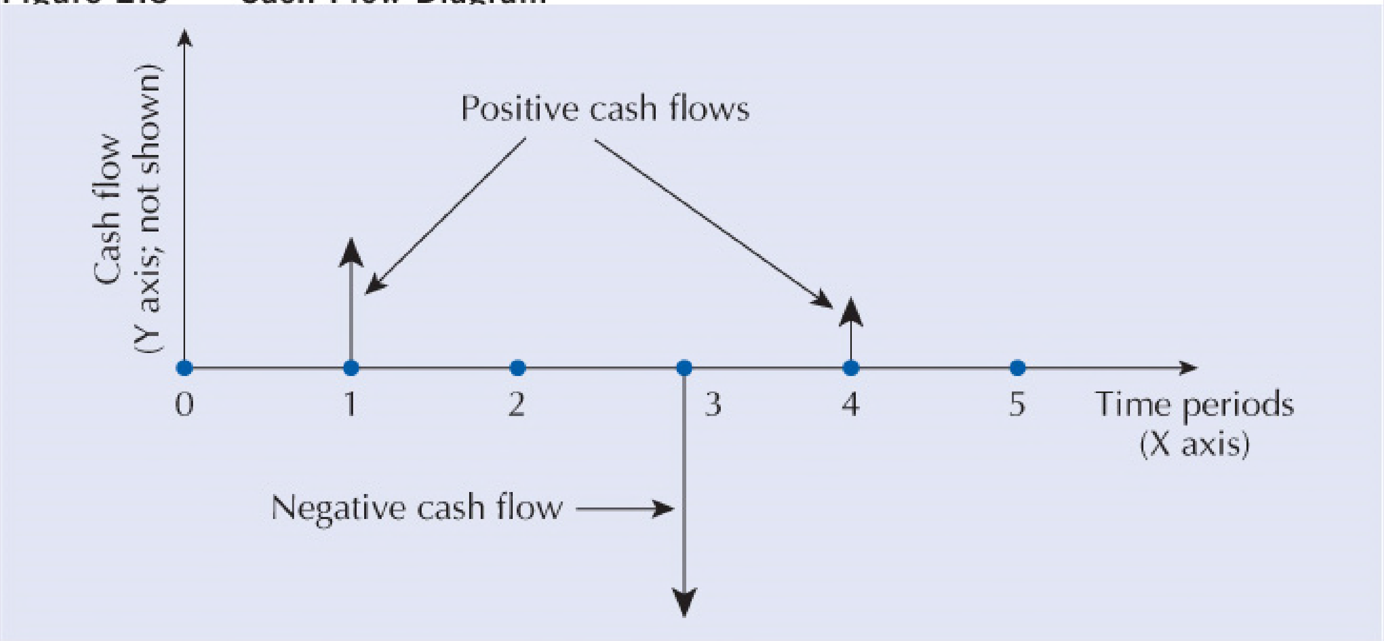
$$i_e = e^r - 1 = e^{0.12} - 1 = 0.12750 = 12.75\%$$

- Continuous compounding makes sense in some situations (i.e large cash flows), but not often used
- Discrete compounding is the norm

# 2.6 Cash Flow Diagrams

- Cash flow diagram is a graphical summary of the timing and magnitude of a set of cash flows

Figure 2.3 Cash Flow Diagram



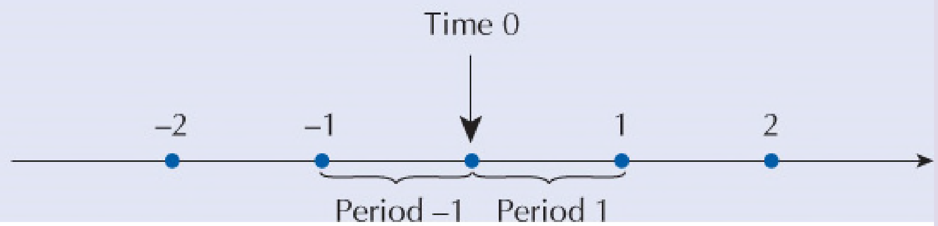



# Close Up 2.3 Beginning and Ending of Periods

**CLOSE-UP 2.3** **Beginning and Ending of Periods**

As illustrated in a cash flow diagram (see Figure 2.3), the end of one period is exactly the same point in time as the beginning of the next period (see Figure 2.4). Now is time 0, which is the end of period –1 and also the beginning of period 1. The end of period 1 is the same as the beginning of period 2, and so on. $N$ years from now is the end of period $N$ and the beginning of period $(N + 1)$.

**Figure 2.4** **Beginning and Ending of Periods**






Assumptions:

- Cash flows occur at the <u>ends of periods.</u>
- End of time period 1 = beginning of time period 2…
- Time 0 = "now"




## 2.7 Depreciation

- Projects involve investment in *assets* (buildings, equipment…) that are put to productive use
- Assets lose value, or depreciate, over time
- Depreciation taken into account when a firm states the value of its assets in a Financial Statement (Chapter 6)
- Also part of decision as to when to replace an aging asset as described in Chapter 7
- It also impacts taxation as shown in Chapter 8




## 2.7.1 Reasons for Depreciation

- Assets depreciate for a variety of reasons:
  1. **Use related physical loss**: usually measured in units of production, kilometres driven, hours of use
  2. **Time related physical loss**: usually measured in units of time as an unused car will rust and lose value over time
  3. **Functional loss:** usually expressed in terms of function lost including fashion, legislative (i.e. pollution control, safety devices) and technical




## 2.7.2 Value of an Asset

- Depreciation models are used to model (estimate) value of an asset at any point in time.
- **Market Value**: value of asset in the open market
- **Book value:** value of an asset calculated from a depreciation model for accounting purposes.
  - This value may be different from the market value.
  - may be several book values given for the same asset (i.e. different for taxation vs. shareholder reports)
- **Salvage Value:** either actual or estimated value at end of its useful life (when sold)
- **Scrap Value:** either actual or estimated value at end of life (when broken up for material value)




## 2.7.2 Value of an Asset (cont)

To state the book value of an asset a good model of depreciation is desirable for the following reasons:

1. To make managerial decisions it's important to know the value of owned assets (i.e. collateral for a loan)
2. One needs an estimate of the value of assets for planning purposes (i.e. keep an asset or replace)
3. Tax legislation requires company tax to be paid on profits. Rules are legislated on how to calculate income and expenses that includes depreciation




## 2.7.2 Value of an Asset (cont)

**CLOSE-UP 2.4** Depreciation Methods

| Method | Description |
|---|---|
| Straight-line | The book value of an asset diminishes by an equal *amount* each year. |
| Declining-balance | The book value of an asset diminishes by an equal *proportion* each year. |
| Sum-of-the-years'-digits | An accelerated method, like declining-balance, in which the depreciation rate is calculated as the ratio of the remaining years of life to the sum of the digits corresponding to the years of life. |
| Double-declining-balance | A declining-balance method in which the depreciation rate is calculated as $2/N$ for an asset with a service life of $N$ years. |
| 150%-declining-balance | A declining-balance method in which the depreciation rate is calculated as $1.5/N$ for an asset with a service life of $N$ years. |
| Units-of-production | The depreciation rate is calculated per unit of production as the ratio of the units produced in a particular year to the total estimated units produced over the asset's lifetime. |




# 2.7.3 Straight-Line Depreciation

- Straight line depreciation (SLD) assumes rate of loss of asset's value is constant over its useful life.

  *P* = purchase price

  *S* = salvage value at the end of *N* periods.

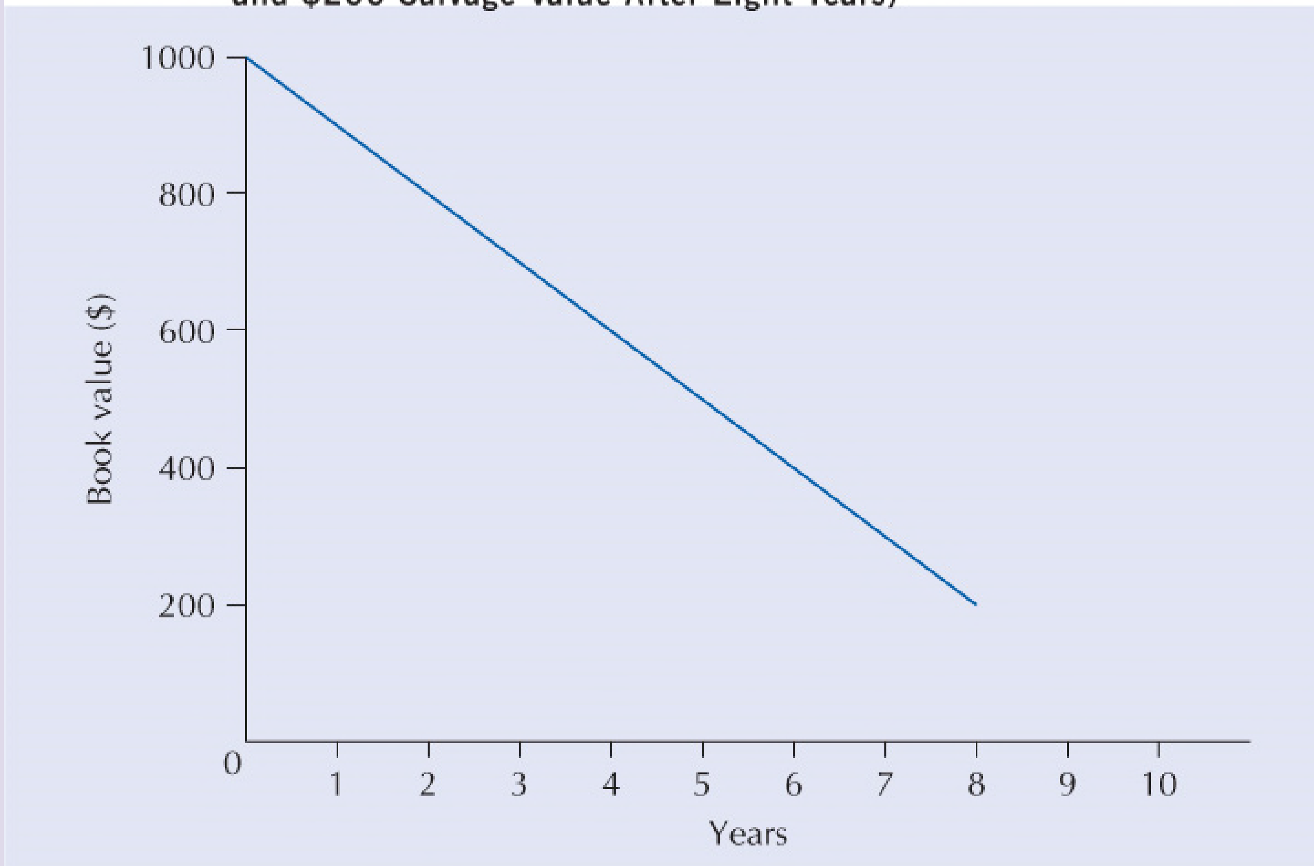
  *N* = useful life of asset

- Advantage: easy to calculate and understand.
- Disadvantage: most assets do not depreciate at a constant rate.
- Hence, market values often differ from book values when SLD is used.




# Figure 2.7 Book Value Under Straight-Line Depreciation

Figure 2.7 Book Value Under Straight-Line Depreciation ($1000 Purchase and $200 Salvage Value After Eight Years)






## 2.7.3 Straight-Line Depreciation (cont'd)

- **Depreciation** in period *n* using SLD:

$$D_{Sl}(n) = \frac{P - S}{N}$$

- **Book Value** of the asset at the end of period *n*:

$$BV_{Sl}(n) = P - n\left(\frac{P - S}{N}\right)$$

- **Accumulated Depreciation** at the end of period *n*:

$$P - BV_{sl}(n) = n\left(\frac{P - S}{N}\right)$$



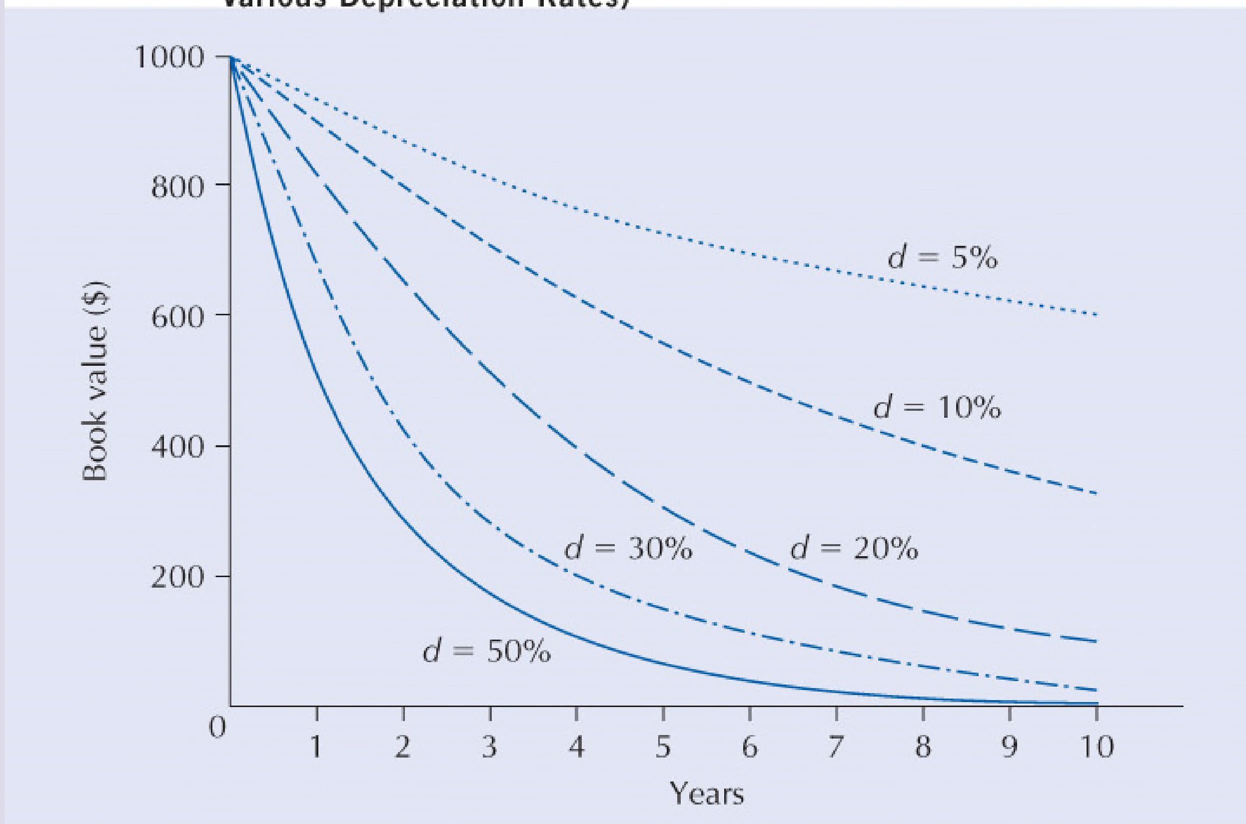

## 2.7.4 Declining-Balance Depreciation

- This DBD method models loss in value of an asset in *a period* as a **constant proportion** of the asset's current value.
  - Initial **Book Value**: $BV_{db}(0) = P$
  - **Book Value** at the end of period *n* using DBD: $BV_{db}(n) = P(1-d)^n$
  - **Depreciation** in period *n* using DBD: $D_{db}(n) = BV_{db}(n-1)\,d$




# Figure 2.8 Book Value Under Declining-Balance Depreciation

Figure 2.8 Book Value Under Declining-Balance Depreciation ($1000 Purchase With Various Depreciation Rates)






# Example 2.11

Sherbrooke Data Services has purchased a new mass storage system for $250 000. It is expected to last six years, with a $10 000 salvage value. Using both the straight-line and declining-balance methods, determine the following:

(a) The depreciation charge in year 1

(b) The depreciation charge in year 6

(c) The book value at the end of year 4

(d) The accumulated depreciation at the end of year 4

• Ideal application for spreadsheet (see Table 2.3)




# Table 2.3 Spreadsheet for Example 2.11

**Table 2.3 Spreadsheet for Example 2.11**

| Straight-Line Depreciation | | | |
|---|---|---|---|
| Year | Depreciation Charge | Accumulated Depreciation | Book Value |
| 0 | | | $250 000 |
| 1 | $40 000 | $ 40 000 | 210 000 |
| 2 | 40 000 | 80 000 | 170 000 |
| 3 | 40 000 | 120 000 | 130 000 |
| 4 | 40 000 | 160 000 | 90 000 |
| 5 | 40 000 | 200 000 | 50 000 |
| 6 | 40 000 | 240 000 | 10 000 |
| **Declining-Balance Depreciation** | | | |
| **Year** | **Depreciation Charge** | **Accumulated Depreciation** | **Book Value** |
| 0 | | | $250 000 |
| 1 | $103 799 | $103 799 | 146 201 |
| 2 | 60 702 | 164 501 | 85 499 |
| 3 | 35 499 | 200 000 | 50 000 |
| 4 | 20 760 | 220 760 | 29 240 |
| 5 | 12 140 | 232 900 | 17 100 |
| 6 | 7 100 | 240 000 | 10 000 |

## 2.8 Equivalence

- Engineering Economics utilises "time value of money" to compare certain values at different points in time.
- Three concepts of equivalence are distinguished underlying comparisons of costs/benefits at different times:
  1. Mathematical Equivalence
  2. Decisional Equivalence
  3. Market Equivalence




## 2.8 Equivalence (cont'd)

- **Mathematical Equivalence**: Decision-makers exchange P dollars now for *F* dollars *N* periods from now using rate *i* and the mathematical relationship: $F = P(1 + i)^N$
- **Decisional Equivalence**: Decision-maker is indifferent as to *P* dollars now or *F* dollars *N* periods from now.
  - We ***infer*** decision-maker's implied interest rate
- **Market Equivalence**: Decision-makers exchange different cash flows in a market at zero cost.
  - In a financial market, individuals/companies are lending and borrowing money.
  - i.e. buying a car and owing $15 000; a lender provides the $15 000 now for $500/month over 36months.




## 2.8 Equivalence (cont'd)

- For the remainder of this text, we assume:
  1. market equivalence holds
  2. decisional equivalence can be expressed in monetary terms
- If these two assumptions are reasonably valid, then mathematical equivalence can be used
- Accurate model of costs/benefits relationship


# Engineering Economics, Sixth Edition

## Summary

- Notion of Interest and Interest Rates
- Compound and Simple Interest
- Effective and Nominal Interest
- Continuous Compounding
- Representing Cash Flows by Diagrams
- Depreciation and Depreciation Accounting
  - Reasons for Depreciation
  - Value of an Asset
  - Straight Line Depreciation
  - Declining Balance Depreciation
- Mathematical, Decisional, Market Equivalence