Solutions Manual to

AN INTRODUCTION TO MATHEMATICAL FINANCE: OPTIONS AND OTHER TOPICS

Sheldon M. Ross

1.1 (a) $1 - p_0 - p_1 - p_2 - p_3 = 0.05$ (b) $p_0 + p_1 + p_2 = 0.80$

1.2 $P\{C \cup R\} = P\{C\} + P\{R\} - P\{C \cap R\} = 0.4 + 0.3 - 0.2 = 0.5$

1.3 (a) $\frac{8}{14}\frac{7}{13} = \frac{56}{182}$ (b) $\frac{6}{14}\frac{5}{13} = \frac{30}{182}$ (c) $\frac{6}{14}\frac{8}{13} + \frac{8}{14}\frac{6}{13} = \frac{96}{182}$

1.4 (a) 27/58 (b) 27/35

1.5

1. The probability that their child will develop cystic fibrosis is the probability that the child receives a CF gene from each of his parents, which is 1/4.

2. Given that his sibling died of the disease, each of the parents much have exactly one CF gene. Let A denote the event that he possesses one CF gene and B that he does not have the disease (since he is 30 years old). Then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{2/4}{3/4} = \frac{2}{3}$$

1.6 Let A be the event that they are both aces and B the event they are of different suits. Then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{4}{52}\frac{3}{51}}{\frac{39}{51}} = \frac{1}{169}$$

1.7

$$\begin{aligned} (a)\ \ P(AB^c) &= P(A) - P(AB) \\ &= P(A) - P(A)P(B) \\ &= P(A)(1 - P(B)) \\ &= P(A)P(B^c) \end{aligned}$$

Part (b) follows from part (a) since from (a) A and B^c are independent, implying from (a) that so are A^c and B^c.

1.8 If the gambler loses both the bets, then $X = -3$. If he wins the first bet, or loses the first bet and wins the second bet, $X = 1$. Therefore,

$$\begin{aligned} P\{X = -3\} &= (\frac{20}{38})^2 = \frac{100}{361} \\ P\{X = 1\} &= \frac{18}{38} + \frac{20}{38}\frac{18}{38} = \frac{261}{361} \end{aligned}$$

1. $P\{X > 0\} = P\{X = 1\} = \frac{261}{361}$

2. $E[X] = 1\frac{261}{361} - 3\frac{100}{361} = \frac{-39}{361}$

1.9

1. $E[X]$ is larger since a bus with more students is more likely to be chosen than a bus with less students.

2.

$$\begin{aligned} E[X] &= \frac{1}{152}(39^2 + 33^2 + 46^2 + 34^2) = \frac{5882}{152} \approx 38.697 \\ E[Y] &= \frac{1}{4}(39 + 33 + 46 + 34) = 38 \end{aligned}$$

1.10 Let N denote the number of sets played. Then it is clear that $P\{N = 2\} = P\{N = 3\} = 1/2$.

1. $E[N] = 2.5$

2. $\mathrm{Var}(N) = \frac{1}{2}(2 - 2.5)^2 + \frac{1}{2}(3 - 2.5)^2 = \frac{1}{4}$

1.11 Let $\mu = E[X]$.

$$\begin{aligned} \mathrm{Var}(X) &= E[(X - \mu)^2] \\ &= E[X^2 - 2\mu X + \mu^2] \\ &= E[X^2] - 2\mu E[X] + \mu^2 \\ &= E[X^2] - \mu^2 \end{aligned}$$

1.12 Let F be her fee if she takes the fixed amount and X when she takes the contingency amount.

$$E[F] = 5,000, \quad SD(F) = 0$$

$$E[X] = 25,000(.3) + 0(.7) = 7,500$$

$$E[X^2] = (25,000)^2(.3) + 0(.7) = 1.875 \times 10^8$$

Therefore,

$$SD(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{1.875 \times 10^8 - (7,500)^2} = \sqrt{1.3125} \times 10^4$$

1.13

$$\begin{aligned} (a)\ E[\bar{X}] &= \frac{1}{n}\sum_{i=1}^{n} E[X_i] \\ &= \frac{1}{n} n\mu = \mu \end{aligned}$$

$$
\begin{aligned}
(b)\ \mathrm{Var}(\bar{X}) &= (\frac{1}{n})^2 \sum_{i=1}^{n} \mathrm{Var}(X_i) \\
&= (\frac{1}{n})^2 n\sigma^2 = \sigma^2/n
\end{aligned}
$$

$$
\begin{aligned}
(c)\ \sum_{i=1}^{n}(X_i - \bar{X})^2 &= \sum_{i=1}^{n}(X_i^2 - 2X_i\bar{X} + \bar{X})^2) \\
&= \sum_{i=1}^{n} X_i^2 - 2\bar{X}\sum_{i=1}^{n} X_i + n\bar{X}^2 \\
&= \sum_{i=1}^{n} X_i^2 - 2\bar{X}n\bar{X} + n\bar{X}^2 \\
&= \sum_{i=1}^{n} X_i^2 - n\bar{X}^2
\end{aligned}
$$

$$
\begin{aligned}
(d)\ E[(n-1)S^2] &= E[\sum_{i=1}^{n} X_i^2] - E[n\bar{X}^2] \\
&= nE[X_1^2] - nE[\bar{X}^2] \\
&= n(\mathrm{Var}(X_1) + E[X_1]^2) - n(\mathrm{Var}(\bar{X}) + E[\bar{X}]^2) \\
&= n\sigma^2 + n\mu^2 - n(\sigma^2/n) - n\mu^2 \\
&= (n-1)\sigma^2
\end{aligned}
$$

1.14

$$
\begin{aligned}
\mathrm{Cov}(X,Y) &= E\left[(X - E[X])(Y - E[Y])\right] \\
&= E\left[XY - XE[Y] - E[X]Y + E[X]E[Y])\right] \\
&= E[XY] - E[Y]E[X] - E[X]E[Y] + E[X]E[Y] \\
&= E[XY] - E[Y]E[X]
\end{aligned}
$$

1.15

$$
\begin{aligned}
(a)\ \mathrm{Cov}(X,Y) &= E\left[(X - E[X])(Y - E[Y])\right] \\
&= E\left[(Y - E[Y])(X - E[X])\right]
\end{aligned}
$$

$$
(b)\ \mathrm{Cov}(X,X) = E[(X - E[X])^2] = \mathrm{Var}(X)
$$

$$
\begin{aligned}
(c)\ \mathrm{Cov}(cX,Y) &= E\left[(cX - E[cX])(Y - E[Y])\right] \\
&= cE\left[(X - E[X])(Y - E[Y])\right] \\
&= c\mathrm{Cov}(X,Y)
\end{aligned}
$$

$$
(d)\ \mathrm{Cov}(c,Y) = E\left[(c - E[c])(Y - E[Y])\right] = 0
$$